

LINEAR FREE VIBRATIONS OF ORTHOTROPIC
ANNULAR PLATES OF VARIABLE THICKNESS

by 613-8301

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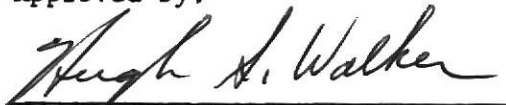
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Nomenclature

a, b	outer and inner radii of the plate, respectively
A_1, A_2, A_3, A_4	known coefficients of governing equations
a_{ij}	elastic constants
c	$= a_{11}/a_{22}$
D	flexural rigidity of the plate
D_0	$h_0^3/12a_{22}$
f_1	dimensionless frequency parameter
g	dimensionless transverse displacement
h	plate thickness
h_0	thickness of plate at $r = 0$
M, N	coefficient matrices
M_r, M_θ	radial and circumferential bending moments per unit length
$p(r, t)$	transverse load
r, θ, z	cylindrical coordinates (see fig. 1)
t	time variable
w	transverse displacement of the middle plane
$\bar{Y}, \bar{Z}, \bar{H}$	(4×1) vector functions
β	exponent of thickness expressions
η	function of radial distance
ν	$= -a_{12}/a_{22}$
λ	$= (c - \nu^2)\omega^2$
ρ	mass density per unit volume
ξ, τ	dimensionless space and time variables, respectively
$\epsilon_r, \epsilon_\theta$	components of strain
σ_r, σ_θ	components of stress

ζ admissible variational function

$\bar{\eta}$ adjustable data in the related initial value problem

1. INTRODUCTION

The axisymmetric flexural vibrations of a cylindrically aeolotropic plate has been analysed by several authors [1-6]. The basic differential equations of motion and associated boundary conditions of the nonlinear axisymmetric flexural vibrations of a cylindrically anisotropic circular plate of varying thickness have been developed by Huang [7], using the method of variational calculus. The advantage of this method is that both the differential equations and appropriate boundary conditions can be obtained simultaneously. The present analysis deals with the linear free vibrations of orthotropic annulus of variable thickness.

In this report, the change in frequency parameter is analysed by varying $c = a_{11}/a_{22}$, the ratio of elastic constants. A circular plate with fixed immovable inside and free outside conditions is considered. The governing equation for the free vibration of such a plate is solved by the method of numerical integration by shooting. The thickness of the plate is varied in the radial direction only both linearly and nonlinearly.

Finally, a brief discussion of the results is presented.

* [] Numbers in brackets designate references at the end of report.

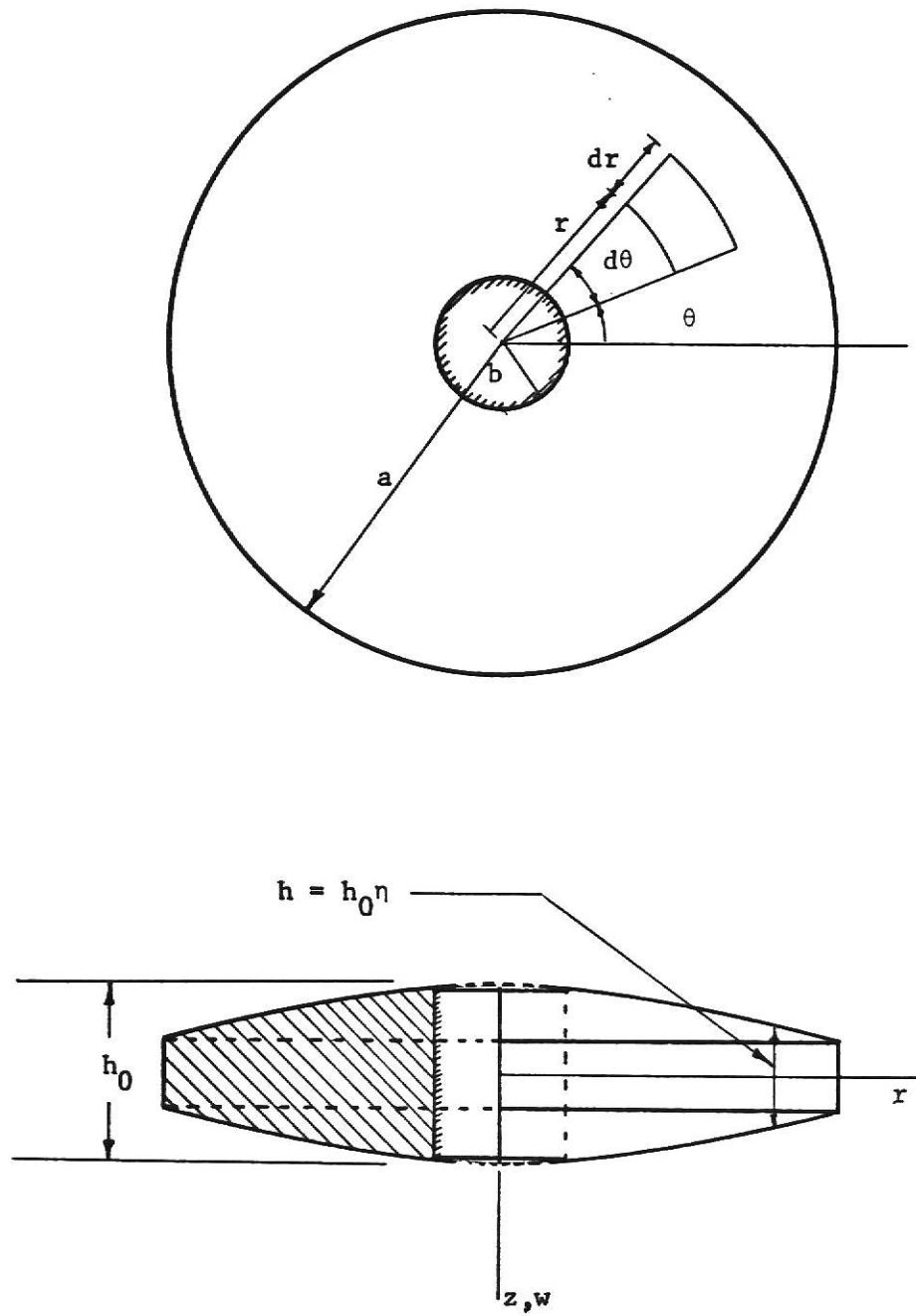
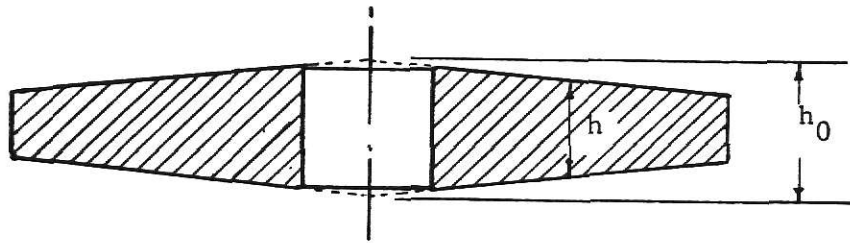
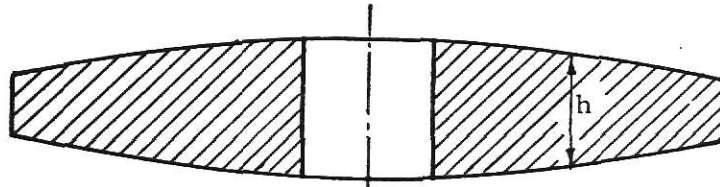


Fig. 1. Variable thickness plate with fixed inside boundary.



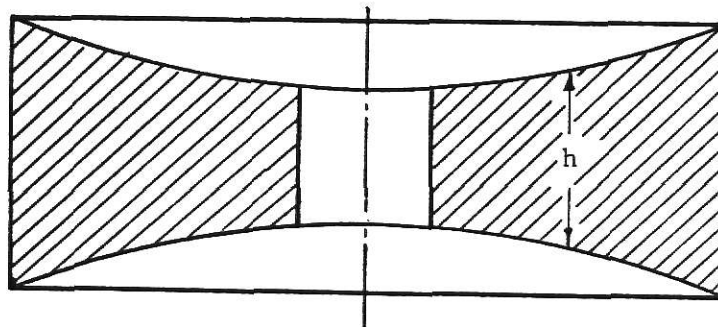
Case I, Straight tapered plate.

$$h = h_0 \eta; \eta = (1 - k\xi), k = 0.5$$



Case II, Circular plate (curved form)

$$h = h_0 \eta; \eta = e^{-\beta \xi^2 / 6}, \beta = 4$$



Case III, Circular plate, (curved form)

$$h = h_0 \eta; \eta = e^{-\beta \xi^2 / 6}, \beta = -4$$

Fig 2. Different cases of plate shapes.

2. DERIVATION OF GOVERNING EQUATION

In what follows, the circular plate is analysed by small deflection theory of plate and the governing equation of motion is derived along with appropriate boundary conditions. The following simplifying assumptions are made:

- (i) Normals to the middle plane before bending remain straight and normal to the middle plane after bending.
- (ii) Normal stresses, σ_z , are small compared with other stress components and may be neglected in stress-strain relations.
- (iii) The stretching of the median surface of the plate is considered negligible, since the deflections are small as compared with the local thickness of the plate.

In accordance with these assumptions, the components of strain for axisymmetric case are found to be

$$\begin{aligned}\epsilon_r &= -zw_{rr} \\ \epsilon_\theta &= -zr^{-1}w_r \\ \gamma_{r\theta} &= 0\end{aligned}\tag{1}$$

In the above equations, $w(r,t)$ is the transverse component of the mid-plane displacement and w_r and w_{rr} are first and second derivatives of w with respect to r , respectively.

The stress strain relationships in polar coordinates for orthotropic axisymmetric plates are [10]

$$\begin{aligned}
\epsilon_{\theta} &= a_{11}\sigma_{\theta} + a_{12}\sigma_r \\
\epsilon_r &= a_{12}\sigma_{\theta} + a_{22}\sigma_r \\
\gamma_{r\theta} &= 0
\end{aligned} \tag{2}$$

where a_{ij} are elastic constants, $(\sigma_r, \sigma_{\theta})$ and $(\epsilon_r, \epsilon_{\theta})$ denote stress and strain components, respectively. The intensities of bending moments are obtained by integrating the inplane stresses over the plate thickness.

$$\begin{aligned}
M_r &= \int_{-h/2}^{h/2} \sigma_r z dz = -D[zw_{rr} + \nu r^{-1}w_r] \\
M_{\theta} &= \int_{-h/2}^{h/2} \sigma_{\theta} z dz = -D[r^{-1}w_r + \nu w_{rr}]
\end{aligned} \tag{3}$$

where M_r and M_{θ} are the radial and circumferential bending moments per unit length, respectively and $h = h(r)$ is the local thickness of the plate.

The flexural rigidity of the plate is expressed as

$$D = \frac{a_{22}h^3}{12(a_{11}a_{22} - a_{12}^2)} = \frac{h^3}{12a_{22}(c - \nu^2)}$$

where $c = a_{11}/a_{22}$ and $\nu = -a_{12}/a_{22}$

The total kinetic energy of the plate is given by

$$T = \frac{1}{2} \iint_S (\rho h w_t^2) r dr d\theta \tag{4}$$

where ρ is the mass density per unit volume.

The potential energy of the axisymmetric time varying transverse load $p(r,t)$, is obtained as

$$\Omega = - \iint_S p(r,t) w r dr d\theta \quad (5)$$

The strain energy due to bending of the plate is expressed [8] as

$$U_b = - \frac{1}{2} \iint_S [M_r w_{rr} + M_\theta r^{-1} w_r] r dr d\theta$$

By substitution of M_r and M_θ from Eqs. (3) we obtain

$$U_b = \frac{1}{2} \iint_S D [c w_{rr}^2 + 2(\nu/r) w_r w_{rr} + (w_r/r)^2] r dr d\theta \quad (6)$$

According to Hamilton's principle it is required that $\delta I = 0$, where

$$I = \int_{t_1}^{t_2} [T - U_b - \Omega] dt \quad (7)$$

Substitution of Eqs (4), (5) and (6) into Eq. (7) yields

$$I = \int_{t_1}^{t_2} \iint_S f(t, r, w, w_r, w_{rr}, w_t) r dr d\theta dt \quad (8)$$

where

$$f = \frac{1}{2} [\rho h r w_t^2 - D(c r w_{rr}^2 + 2\nu w_r w_{rr} + \frac{w_r^2}{r}) + 2p(r,t) r w] \quad (9)$$

The extremization of the functional I can be done by giving the function w the admissible variation $\epsilon \zeta(r,t)$, where ϵ is any arbitrary constant and ζ is an arbitrary function. Then the first variation δI can be written [8] as

$$\begin{aligned}
\delta I = & \varepsilon \int_{t_1}^{t_2} \iint_s \left[\frac{\partial f}{\partial w} - \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial w_r} \right) - \frac{\partial}{\partial t} \left(\frac{\partial f}{\partial w_t} \right) + \frac{\partial^2}{\partial r^2} \left(\frac{\partial f}{\partial w_{rr}} \right) \right] \zeta r dr d\theta dt \\
& + \varepsilon \oint_{\bar{c}} \left[\frac{\partial f}{\partial w_r} - \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial w_{rr}} \right) \right] \zeta r dt + \varepsilon \oint_{\bar{c}} \left(\frac{\partial f}{\partial w_{rr}} \right) \zeta_r r dt \\
& + \varepsilon \oint_{\bar{c}} \left(\frac{\partial f}{\partial w_t} \right) \zeta r dr
\end{aligned}$$

where $\bar{c}$ denotes the bounding curve of the region s . The integral must vanish independently of the line integrals, since δI vanishes for all admissible functions $\zeta(r,t)$. This condition yields the governing equation of motion as

$$\frac{\partial f}{\partial w} - \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial w_r} \right) - \frac{\partial}{\partial t} \left(\frac{\partial f}{\partial w_t} \right) + \frac{\partial^2}{\partial r^2} \left(\frac{\partial f}{\partial w_{rr}} \right) = 0 \quad (11)$$

The natural boundary conditions are given by following relations.

$$\zeta \left[\frac{\partial f}{\partial w_r} - \frac{\partial}{\partial r} \left(\frac{\partial f}{\partial w_{rr}} \right) \right] = 0 \quad \text{on edge } r = \text{const}$$

$$\zeta_r \left[\frac{\partial f}{\partial w_{rr}} \right] = 0 \quad \text{on edge } r = \text{const.} \quad (12)$$

Substituting Eq. (9) into Eqs. (11) and (12), the governing differential equations of motion and associated boundary conditions are obtained as follows

$$\begin{aligned}
& D[cw_{rrrr} + 2cr^{-1}w_{rrr} - r^{-2}w_{rr} + r^{-3}w_r] \\
& + D_r[2cw_{rrr} + (2c+v)r^{-1}w_{rr} - r^{-2}w_r] \\
& + D_{rr}[cw_{rr} + vr^{-1}w_r] + \rho hw_{tt} = p(r,t)
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
& \{\zeta r D[cw_{rrrr} + cr^{-1}w_{rrr} - r^{-2}w_{rr}] + \zeta r D_r[cw_{rr} + vr^{-1}w_r]\}_b^a = 0 \\
& \{D_r \zeta_r[cw_{rr} + vr^{-1}w_r]\}_b^a = 0
\end{aligned} \tag{14}$$

where a and b are outer and inner radii, respectively.

For the present case in consideration, that is, the linear free vibrations of an orthotropic annular plate, the axisymmetric transverse load, $p(r,t) = 0$. Hence, the governing differential equation becomes

$$\begin{aligned}
& D[cw_{rrrr} + 2cr^{-1}w_{rrr} - r^{-2}w_{rr} + r^{-3}w_r] \\
& + D_r[2cw_{rrr} + (2c+v)r^{-1}w_{rr} - r^{-2}w_r] \\
& + D_{rr}[cw_{rr} + vr^{-1}w_r] + \rho hw_{tt} = 0
\end{aligned} \tag{15}$$

and the boundary conditions for a plate with fixed-immovable inside are

$$w = 0 \quad \text{and} \quad w_r = 0 \quad \text{at } r = b \tag{16}$$

$$cw_{rr} + vr^{-1}w_r = 0 \quad \text{at } r = a \tag{17}$$

and

$$D[cw_{rrr} + cr^{-1}w_{rr} - r^{-2}w_r] + D_r[cw_{rr} + vr^{-1}w_r] = 0 \quad \text{at } r = a_r \quad (18)$$

For the present annular plate, the thickness is varied only in the radial direction and it is expressed by

$$h = h_0 \eta \quad (19)$$

where η is the function of radial distance from the center of the plate (see Fig. 1). Three different cases of thickness variation are considered (see Fig. 2).

First, in attempting to solve the governing equation (15), non-dimensional variables have been introduced as

$$g = w/a, \quad \xi = r/a$$

and

$$\tau = t \left(\frac{D_0}{\rho h_0 a^4} \right)^{1/2} \quad (20)$$

where

$$D_0 = h_0^3 / 12 a_{22}$$

Using equations (15) and (20), the governing equation of motion in dimensionless form is found to be

$$A_1 g'''' + A_2 g''' + A_3 g'' + A_4 g' - \eta \lambda g = 0 \quad (21)$$

where

$$A_1 = c\eta^3$$

$$A_2 = (2c/\xi)\eta^3 + 6c\eta^2\eta'$$

$$A_3 = -\xi^{-2}\eta^3 + 3(2c+v)\xi^{-1}\eta^2\eta' + 3c\eta''\eta^2 + 6c\eta(\eta')^2$$

$$A_4 = \xi^{-3}\eta^3 - 3\xi^{-2}\eta^2\eta' + (v/\xi)[3\eta^2\eta'' + 6\eta(\eta')^2]$$

and

$$\lambda = (c-v^2)\omega^2$$

The dimensionless frequency parameter is expressed as

$$f_1 = \lambda^{1/2} = (c-v^2)^{1/2} \omega \quad (22)$$

The primes over the symbols denote differentiation with respect to ξ

The boundary conditions (16), (17) and (18) are conveniently expressed as

$$g = 0 \quad \text{and} \quad g' = 0 \quad \text{at} \quad \xi = R \quad (23)$$

$$cg'' + vg' = 0 \quad \text{at} \quad \xi = 1 \quad (24)$$

and

$$cg''' + c[1 + 3(\eta'/\eta)]g'' - [1 - 3v(\eta'/\eta)]g' = 0 \quad \text{at} \quad \xi = 1 \quad (25)$$

where $R = b/a$

The normalization of outer radius ($a=1$) is done to simplify the numerical computations.

2. METHOD OF SOLUTION AND RESULTS

The governing equation (21) along with a set of boundary conditions (23-25) forms a two-point boundary value problem which can be solved by the application of numerical integration techniques.

The governing equation (21) can be written as the system of four first order equations,

$$\frac{d\bar{Y}}{d\xi} = \bar{H}(\xi, \bar{Y}; c, \lambda), \quad R < \xi < 1 \quad (26a)$$

where

$$\bar{Y}(\xi) = \begin{pmatrix} g \\ g' \\ g'' \\ g''' \\ g \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

and $\bar{H}$ is the (4×1) vector function defined by,

$$\frac{dy_1}{d\xi} = y_2$$

$$\frac{dy_2}{d\xi} = y_3$$

$$\frac{dy_3}{d\xi} = y_4 \quad (26b)$$

$$\frac{dy_4}{d\xi} = -\frac{A_2}{A_1} y_4 - \frac{A_3}{A_1} y_3 - \frac{A_4}{A_1} y_2 + \frac{\lambda \eta}{A_1} y_1$$

The equations (26) contain an additional unknown parameter λ , hence a component of $\bar{Y}(1)$ is normalized, i.e. $g(1) = y_1(1) = 1$

Thus, the boundary conditions in the generalized form are written as

$$N\bar{Y}(R) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (26c)$$

and

$$M\bar{Y}(1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (26d)$$

where

$$N = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & v & c & 0 \\ 0 & -k_2 & k_1 & c \end{pmatrix}$$

where

$$k_1 = c[1+3(\eta'/\eta)_{\xi=1}] \quad \text{and} \quad k_2 = [1 - 3v(\eta'/\eta)_{\xi=1}]$$

The first row of M normalizes $y_1(1)$ and remaining rows of M and N are constructed from the boundary conditions (23), (24) and (25).

The numerical solution of equations (26) is initiated by introducing the related initial value problem,

$$\frac{d\bar{Z}}{d\xi} = \bar{H}(\xi, \bar{Z}; c, \lambda), \quad R < \xi < 1 \quad (27a)$$

$$\bar{Z}(1) = \bar{\gamma}^*(\eta_1) = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}_{\xi=1} = \begin{pmatrix} 1.0 \\ \eta_1 \\ -(v/c)\eta_1 \\ (\frac{1+v}{c})\eta_1 \end{pmatrix} \quad (27b)$$

Eqs (27b) represents the initial values that satisfy the boundary conditions (26d). A solution of the initial value problem (27) is obtained on the closed interval $[1, R]$ and it is denoted by

$$\bar{Z} = \bar{Z}(\xi; \bar{\eta}, c); \quad \bar{\eta} = \begin{Bmatrix} \eta_1 \\ \lambda \end{Bmatrix}$$

Thus, solving the boundary value problem (26) is equivalent to obtaining a continuous solution to the related initial value problem (27) which satisfies,

$$N\bar{Z}(R, \bar{\eta}, z) = \bar{0} \quad (28)$$

If the unknown boundary values at $\xi=1$ were assigned arbitrary values $\bar{\eta}_1$, then step-by-step integration of equations (27) from $\xi=1$ to $\xi=R$ would be possible. Generally, the values of $\bar{Z}(R, \bar{\eta}, z)$, thus obtained would not satisfy the given conditions at $\xi=R$. Hence, corrections to initial values $\bar{\eta}_1$ is made by application of Newton's Method and following iterative sequence is employed,

$$\bar{\eta}_{k+1} = \bar{\eta}_k + \Delta\bar{\eta}_k; \quad k = 1, 2, 3, \dots \quad (29a)$$

The correction vector $\Delta\bar{\eta}_k$ is obtained by,

$$\Delta\bar{\eta}_k = -[N(J_R)_k]^{-1} N\bar{Z}(R, \bar{\eta}_k, c) \quad (29b)$$

The (4×2) matrix (J_R) is defined as the change of final values with respect to change of data, $\bar{\eta}$.

$$J_R = \begin{pmatrix} \frac{\partial z_1}{\partial \eta_1} & \frac{\partial z_1}{\partial \lambda} \\ \frac{\partial z_2}{\partial \eta_1} & \frac{\partial z_2}{\partial \lambda} \\ \frac{\partial z_3}{\partial \eta_1} & \frac{\partial z_3}{\partial \lambda} \\ \frac{\partial z_4}{\partial \eta_1} & \frac{\partial z_4}{\partial \lambda} \end{pmatrix}_{\xi=R} \quad (30)$$

The (J_R) matrix at inner-fixed boundary can be constructed by variational problem which contains eight first order equations and corresponding set of initial values at outer-free boundary. It is stated as follows

$$\frac{d}{d\xi} \left(\frac{\partial z_1}{\partial \eta_1} \right) = \frac{\partial z_2}{\partial \eta_1}$$

$$\frac{d}{d\xi} \left(\frac{\partial z_2}{\partial \eta_1} \right) = \frac{\partial z_3}{\partial \eta_1}$$

(31a)

$$\frac{d}{d\xi} \left(\frac{\partial z_3}{\partial \eta_1} \right) = \frac{\partial z_4}{\partial \eta_1}$$

$$\frac{d}{d\xi} \left(\frac{\partial z_4}{\partial \eta_1} \right) = -\frac{A_2}{A_1} \left(\frac{\partial z_4}{\partial \eta_1} \right) - \frac{A_3}{A_1} \left(\frac{\partial z_3}{\partial \eta_1} \right) - \frac{A_4}{A_1} \left(\frac{\partial z_2}{\partial \eta_1} \right) + \frac{\lambda \eta}{A_1} \left(\frac{\partial z_1}{\partial \eta_1} \right)$$

and

$$\frac{d}{d\xi} \left(\frac{\partial z_1}{\partial \lambda} \right) = \frac{\partial z_2}{\partial \lambda}$$

$$\frac{d}{d\xi} \left(\frac{\partial z_2}{\partial \lambda} \right) = \frac{\partial z_3}{\partial \lambda}$$

(31b)

$$\frac{d}{d\xi} \left(\frac{\partial z_3}{\partial \lambda} \right) = \frac{\partial z_4}{\partial \lambda}$$

$$\frac{d}{d\xi} \left(\frac{\partial z_4}{\partial \lambda} \right) = -\frac{A_2}{A_1} \left(\frac{\partial z_4}{\partial \lambda} \right) - \frac{A_3}{A_1} \left(\frac{\partial z_3}{\partial \lambda} \right) - \frac{A_4}{A_1} \left(\frac{\partial z_2}{\partial \lambda} \right) + \frac{\lambda \eta}{A_1} \left(\frac{\partial z_1}{\partial \lambda} \right)$$

The initial conditions for the variational problem (31) are

$$\frac{\partial}{\partial \eta_1} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}_{\xi=1} = \begin{pmatrix} 0 \\ 1 \\ -v/c \\ \frac{1+v}{c} \end{pmatrix} \quad (32a)$$

and

$$\frac{\partial}{\partial \lambda} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}_{\xi=1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (32b)$$

This variational problem and initial value problem (27) may be numerically integrated, simultaneously, on the interval $[1, R]$. The

solution of variational problem at $\xi = R$ gives the elements of (J_R) matrix corresponding to the given values of $\bar{\eta}$. Thus, by setting $\bar{\eta} = \bar{\eta}_1$ and integrating equations (27) and (31) from $\xi=1$ to $\xi=R$, the first corrected vector, $\bar{\eta}_2$, can be calculated according to equations (29). Successive repetition of this procedure yields the desired sequence, $\bar{\eta}_k$, provided the value of c is held fixed in this process.

After obtaining a root, $\bar{\eta}^0$, corresponding to $c = c^0$, the value of c can be perturbed,

$$c_1 = c^0 + \Delta c$$

For this value of $c = c_1$ iteration is initiated from $\bar{\eta} = \bar{\eta}^0$. The value of Δc is kept small (0.2) and the value of c is varied from 0.4 to 5.2. In this investigation the value of $c^0 = 1$ (Isotropic case) is picked up to start the iterative procedure.

This method of solving the governing equations and boundary conditions was applied to several clamped plates of various shapes. The change in frequency parameter, f_1 , is analysed by varying the ratio of elastic constants, $c = a_{11}/a_{22}$. In case of straight tapered plates, the influence of the ratio of inside radius of the plate to the outer radius is studied, (see Table (1-4) and Fig. 3). Fig. 4. illustrates the variation of frequency parameter for the annular plate with fixed-immovable inside, having non-linear thickness variation.

The complete analysis is presented in the following tables and figures.

Table 1.

Data for straight tapered plates, $R = 0.1$

Case Ia, $h = h_0(1-k\xi)$; $k = 0.5$, $\nu = 1/3$

c	f_1	c	f_1
0.4	3.5481	3.0	5.3057
0.6	3.7191	3.2	5.4163
0.8	3.8791	3.4	5.5247
1.0	4.0314	3.6	5.6309
1.2	4.1773	3.8	5.7352
1.4	4.3179	4.0	5.8376
1.6	4.4538	4.2	5.9383
1.8	4.5854	4.4	6.0372
2.0	4.7133	4.6	6.1345
2.2	4.8377	4.8	6.2304
2.4	4.9584	5.0	6.3247
2.6	5.0772	5.2	6.4177
2.8	5.1927		

Table 2.

Data for straight tapered plate, $R = 0.2$

Case Ib, $h = h_0(1-k\xi)$; $k = 0.5$, $\nu = 1/3$

c	f_1	c	f_1
0.4	3.9309	3.0	6.7683
0.6	4.2255	3.2	6.9374
0.8	4.4966	3.4	7.1024
1.0	4.7505	3.6	7.2637
1.2	4.9904	3.8	7.4215
1.4	5.2188	4.0	7.5759
1.6	5.4372	4.2	7.7273
1.8	5.6470	4.4	7.8758
2.0	5.8490	4.6	8.0214
2.2	6.0442	4.8	8.1645
2.4	6.2332	5.0	8.3051
2.6	6.4166	5.2	8.4434
2.8	6.5948		

Table 3.

Data for straight tapered plates, $R = 0.3$

Case Ic, $h = h_0(1-k\xi)$; $k = 0.5$, $\nu=1/3$

c	f_1	c	f_1
0.4	4.5775	3.0	8.7691
0.6	5.0379	3.2	9.0094
0.8	5.4532	3.4	9.2434
1.0	5.8363	3.6	9.4716
1.2	6.1943	3.8	9.6944
1.4	6.5319	4.0	9.9122
1.6	6.8523	4.2	10.1253
1.8	7.1582	4.4	10.3340
2.0	7.4512	4.6	10.5385
2.2	7.7330	4.8	10.7391
2.4	8.0047	5.0	10.9361
2.6	8.2674	5.2	11.1295
2.8	8.5220		

Table 4.

Data for straight tapered plate, $R = 0.4$

Case Id, $h = h_0(1-k\xi)$; $k = 0.5$, $\nu = 1/3$

c	f_1	c	f_1
0.4	5.6107	3.0	11.6961
0.6	5.3070	3.2	12.0354
0.8	6.9256	3.4	12.3654
1.0	7.4899	3.6	12.6868
1.2	8.0128	3.8	13.0002
1.4	8.5026	4.0	13.3061
1.6	8.9651	4.2	13.6052
1.8	9.4045	4.4	13.8979
2.0	9.8240	4.6	14.1845
2.2	10.2261	4.8	14.4654
2.4	10.6129	5.0	14.7409
2.6	10.9859	5.2	15.0114
2.8	11.3466		

Table 5.

Data for circular Plate (curved form), $R = 0.2$

Case II

$$h = h_0 \eta; \eta = e^{-\beta \xi^2 / 6}, \beta = 4, \nu = 1/3$$

c	f_1	c	f_1
0.4	4.4543	3.0	7.6389
0.6	4.7826	3.2	7.8293
0.8	5.0859	3.4	8.0151
1.0	5.3704	3.6	8.1967
1.2	5.6396	3.8	8.3743
1.4	5.8961	4.0	8.5483
1.6	6.1416	4.2	8.7187
1.8	6.3773	4.4	8.8859
2.0	6.6046	4.6	9.0500
2.2	6.8241	4.8	9.2111
2.4	7.0368	5.0	9.3695
2.6	7.2431	5.2	9.5252
2.8	7.4437		

Table 6.

Data for circular plate (curved form), $R = 0.2$

Case III,

$$h = h_0 \eta; \eta = e^{-\beta \xi^2 / 6}, \beta = -4, \nu = 1/3$$

c	f_1	c	f_1
0.4	4.4467	3.0	6.8913
0.6	4.7899	3.2	7.0217
0.8	5.0598	3.4	7.1491
1.0	5.2896	3.6	7.2738
1.2	5.4945	3.8	7.3959
1.4	5.6822	4.0	7.5157
1.6	5.8575	4.2	7.6333
1.8	6.0231	4.4	7.7489
2.0	6.1810	4.6	7.8625
2.2	6.3326	4.8	7.9743
2.4	6.4787	5.0	8.0844
2.6	6.6202	5.2	8.1928
2.8	6.7576		

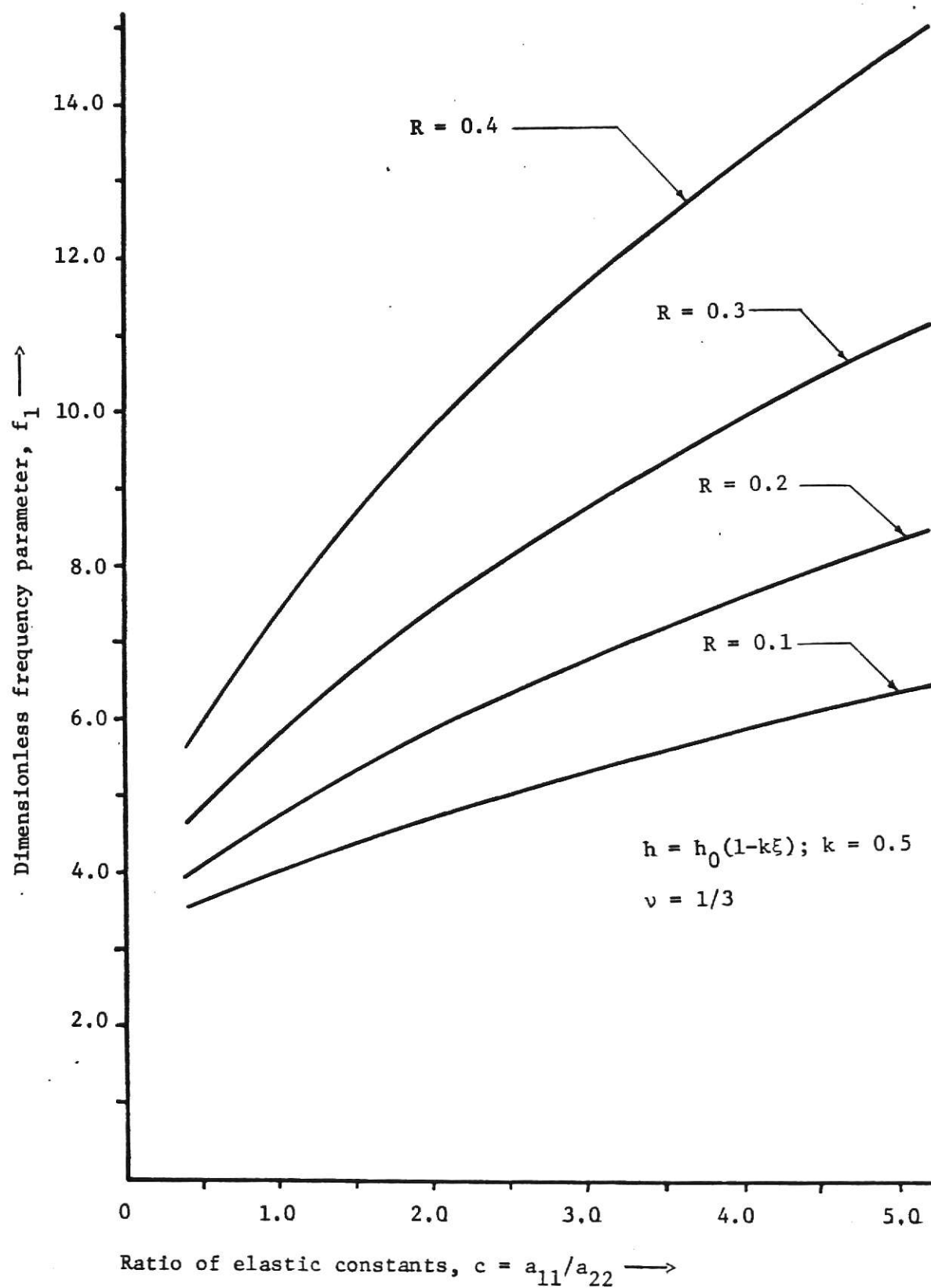


Fig 3. Variation of frequency parameter for straight tapered plates.

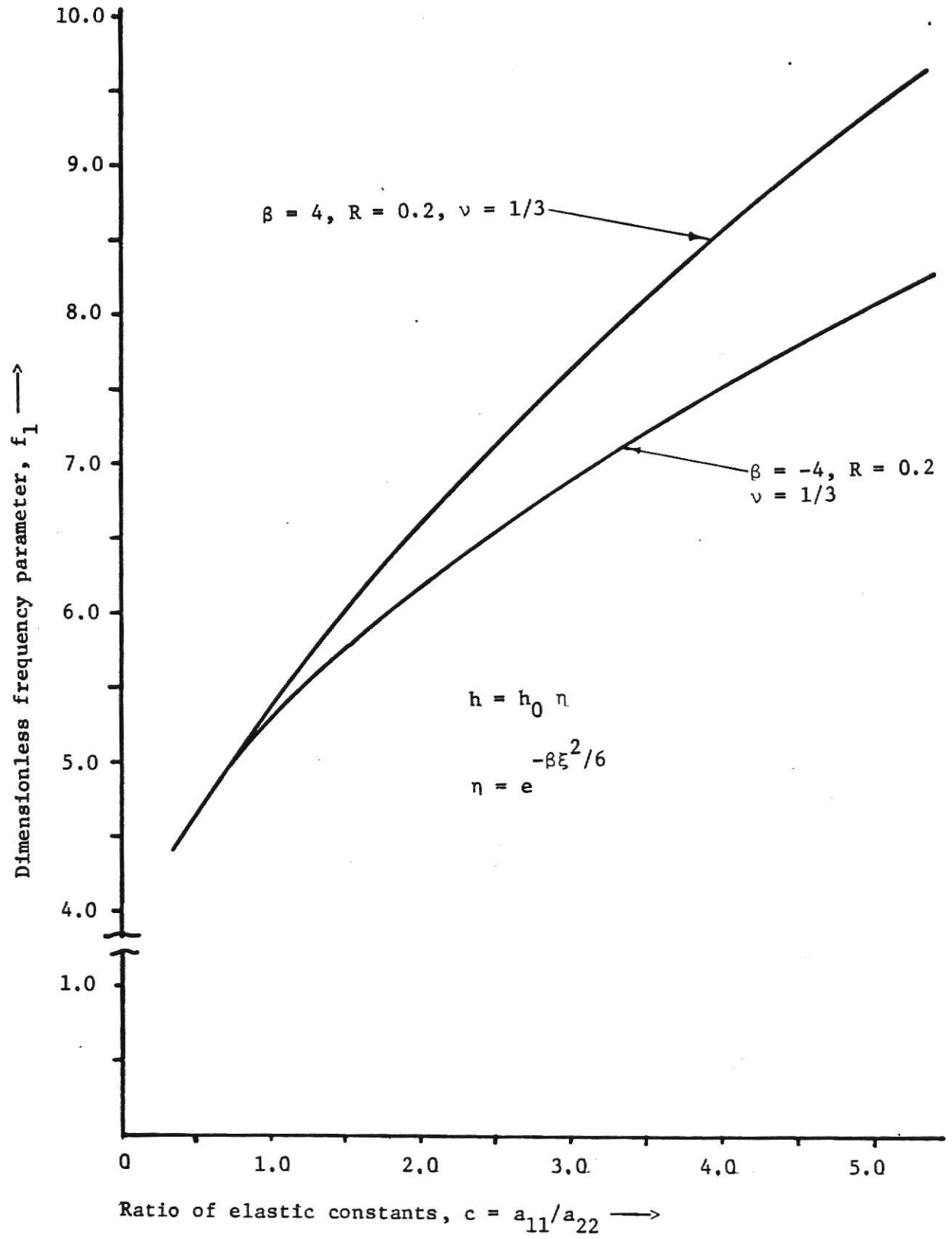


Fig 4. Variation of frequency parameter (Non-linear thickness variation).

4. DISCUSSION OF RESULTS

This report deals primarily with the linear free vibrations of orthotropic annular plates of varying thickness. In particular, the variations in frequency parameter, f_1 , by varying the ratio of elastic constants, $c = a_{11}/a_{22}$, have been presented. The variation of c , is achieved by varying the elastic constant a_{11} and keeping a_{22} fixed.

It may be concluded from Fig. 3. that the dimensionless frequency parameter f_1 , tends to increase as the ratio of elastic constants, $c = a_{11}/a_{22}$, increases, and tends to decrease as the ratio of inner to outer radii, R , decreases. Fig. 4. illustrates that for the plates with curved profiles, the frequency parameter for a concave plate is less than that of a convex plate.

It would be of further interest to carry out similar investigations for non-linear flexural vibrations of circular plates of variable thickness.

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LINEAR FREE VIBRATIONS OF ORTHOTROPIC
ANNULAR PLATES OF VARIABLE THICKNESS

by

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ABSTRACT

The present investigation is concerned with the linear free vibrations of orthotropic annular plates of variable thickness. In particular, the variations in frequency parameter, f_1 , by varying the ratio of elastic constants, $c = a_{11}/a_{22}$, have been presented. The derivation of the governing differential equation is based on the method of variational calculus. The solution of this equation is obtained by the application of numerical integration by shooting. The numerical results are presented for several plates of various shapes.