

MODELING AND OPTIMIZATION OF A
TOWER-TYPE ACTIVATED SLUDGE SYSTEM

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Chapter I

INTRODUCTION

As the population grows and stricter water quality standards are legislated, the amount of resources allocated to wastewater treatment will grow. The design of treatment plants thus becomes more important as an improved design could lead to substantial savings. By developing mathematical models of the treatment processes, one can use the computer to simulate the performance of different designs under a variety of operating conditions. The role of the computer in designing treatment plants has been reviewed by Mishra et. al.¹

To meet stringent water quality standards, almost all the soluble organic nutrients must be removed. Because of the multistage design of the tower fermentor, high substrate consumption can be achieved in a smaller volume than that required by a single stage aerator. Another advantage of the tower fermentor over the single stage aerator is the ability to have step aeration and step feeding of return sludge. Therefore, the tower fermentor should be considered as an alternative in designing a treatment plant.

In Chapter II, the mathematical modeling of the secondary clarifier is considered. Clarification and sludge thickening are the two functions of the clarifier. Previous models treated only one of these functions. When both functions are considered, there is interaction between the clarifier and aeration vessel designs. Design and operation of an activated sludge system (completely mixed aeration vessel and secondary clarifier) are considered.

Chapter III presents the tower-type activated sludge system model. This model includes the structural parameter method for optimizing the

distribution of feed and sludge recycle to the tower stages. Optimal designs of systems with one, two and three tower stages are presented.

In Chapter IV, the tower fermentor model is modified to include the effect of organism sedimentation within the tower stages. This sedimentation has been observed in experimental studies and is shown to have a desirable effect on the system design.

Chapter V presents some ideas for future research. An improved model for organism sedimentation in the tower fermentor is presented.

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1. Mishra, P. N., Fan, L. T. and Erickson, L. E., paper 4a presented at Fourth Joint Chemical Engineering Conference, Vancouver (1973).

Chapter II

SIMULTANEOUS USE OF CLARIFICATION AND SLUDGE THICKENING MODELS

INTRODUCTION

In the optimal design of an activated sludge system, interaction between the design of the aeration vessel and that of the secondary clarifier should be taken into account. The performance of the aeration vessel varies significantly with the return sludge concentration and flow rate while the clarification and thickening functions of the clarifier depend on the effluent of the aeration vessel.

The recent attention given to the optimal design of waste treatment systems has underscored the need for accurate process models. To fully describe the secondary clarifier, both the overflow and underflow should be modeled. Thus equations describing the clarification and thickening are required.

A review of research concerned with optimal waste treatment system design was presented recently by Mishra et al.¹ In the work by Fan et al.² and Chen et al.,³ sludge thickening was not included in the clarifier model. Jennings and Grady,⁴ Berthouex⁵ and Berthouex and Polkowski⁶ modeled sludge thickening but not clarification. These references stress the need to design an integrated system rather than separate units. The paper by Jennings and Grady also presents a useful review of the literature about design of secondary clarifiers.

In the present work, an activated sludge system consisting of a completely mixed aeration vessel and a secondary clarifier is designed to minimize the total volume of the system. The secondary clarifier model

includes a description of clarification and sludge thickening so that both the overflow and underflow streams can be characterized.

SECONDARY CLARIFIER MODELS

The development of mathematical models to describe the clarification and thickening processes in secondary clarifiers has received considerable attention. Sludge thickening has been considered recently by Dick,⁷ Dick and Javaheri,⁸ Alkema,⁹ Tracy and Keinath,¹⁰ and Rex Chainbelt, Inc.¹¹ The limiting downward flux of solids can be determined graphically following a procedure first presented by Yoshioka et al.¹² or analytically by finding the organism concentration at which the total flux is a minimum.

Models for the clarification process have been presented by Naito et al.¹³ and by Rex Chainbelt, Inc.¹¹ Takamatsu et al.^{14,15} have also considered mixing and scouring in clarifiers.

Figure 1 is a schematic of the activated sludge system. The flow balance around the secondary clarifier is:

$$q(1 + r) = q_e + b \quad (1)$$

Assuming that there is no growth of organisms in the clarifier, the organism balance is given by:

$$q(1 + r) X = q_e X_e + b X_b \quad (2)$$

Since there is no growth, the substrate concentration is unchanged. The clarification function is described by the following empirical equation developed by Naito et al.:¹³

$$X_e = p X_c^\alpha \exp(-\beta_c t) \quad (3)$$

This equation is applicable only to clarifiers with a depth of 9 to 10 feet and to normal clarifier operation.

These three equations form the basic clarifier model now used for computer-aided design of waste treatment systems (Mishra et al.¹). The underflow organism concentration is either fixed at a reasonable value or found from the organism balance (Equation 2) subject to the constraint of a practical upper bound (e.g., 10,000 mg/l).

The addition of equations describing thickening of the sludge will complete the model used in this work. In the secondary clarifier, the organisms move downward because of the bulk fluid flow and also because of the density difference between the organisms and the fluid. At any point in the clarifier, the total downward flux of organisms is the sum of the bulk and settling fluxes.

$$\begin{array}{ll} \text{Bulk flux} & F_B = X v \end{array} \quad (4)$$

$$\begin{array}{ll} \text{Settling flux} & F_S = X v_S \end{array} \quad (5)$$

$$\begin{array}{ll} \text{Total flux} & F = X (v + v_S) \end{array} \quad (6)$$

A reasonable choice for the bulk fluid velocity in the downward direction is the average velocity in that direction. The average velocity is defined as the volumetric flow rate divided by the cross-sectional area, i.e.,

$$v = \frac{b}{A_c} \quad (7)$$

The settling velocity of activated sludge has been obtained from batch data by many investigators. The data presented by Dick⁷ was modeled by Alkema⁹ as

$$X \leq X_L \quad v_S = k_1 - k_2 X \quad (8)$$

$$X > X_L \quad v_S = k_3 X^{k_4} \quad (9)$$

The original data was said to be representative of sludge with poor settling characteristics. Settling of more typical sludge was modeled by shifting the

curve to the right. The constants for sludge with both poor and typical settling characteristics are given in Table 1. For concentrations between 1500 mg/l and X_L , the power law model (Equation 9) predicts a slower settling velocity than does the linear model (Equation 8). Using the power law model for all concentrations greater than 1500 mg/l gives a conservative estimate of the settling velocity.

The flux equations (Equations 4, 5, and 6) are shown graphically in Fig. 2. At concentration X^* , the total flux is a minimum and thus

$$\left. \frac{dF}{dX} \right|_{X^*} = 0 \quad (10)$$

Since

$$F = X(v + k_3 X^{k_4}), \quad (11)$$

$$\frac{dF}{dX} = k_3(k_4 + 1) X^{k_4} + v \quad (12)$$

Setting this derivative equal to zero and solving for X^* , one obtains

$$X^* = \left[\frac{-v}{k_3(k_4 + 1)} \right]^{1/k_4} \quad (13)$$

For a given value of v , the limiting flux of organisms in the clarifier is given by

$$F(X^*) = X^* \left[v + k_3 (X^*)^{k_4} \right] \quad (14)$$

The flow of organisms in the underflow stream must be less than or equal to the product of the limiting flux and the clarifier area.

$$b X_b \leq F(X^*) A_c \quad (15)$$

When designing for the minimum required clarifier area, one assumes that the clarifier is operating at its maximum capacity. Equation (15) is thus

treated as an equality

$$b X_b = F (X^*) A_c \quad (16)$$

Solving for X_b and substituting for b from equation (7) yields

$$X_b = \frac{F (X^*)}{v} \quad (17)$$

Figure 3 shows the organism concentration in the underflow predicted by this model as a function of the average downward fluid velocity for three values of the parameter k_3 .

OPTIMAL DESIGN OF A WASTE TREATMENT SYSTEM

The basic activated sludge waste treatment system is shown schematically in Fig. 4. For simplicity, the aeration vessel is treated as a completely-mixed, stirred-tank reactor. As shown by Chiu et al.,¹⁶ the kinetics can be modeled by the following two equations:

$$\frac{dX}{dt} = \frac{\mu_{\max} SX}{(K_S + S)} - k_d X \quad (18)$$

$$\frac{dS}{dt} = \frac{\mu_{\max} SX}{Y (K_S + S)} \quad (19)$$

Values of parameters in the kinetic expressions are given in Table 1. The cell and substrate material balances for the completely mixed aeration tank can be generally expressed as

$$q X_f + qr X_b - q(1 + r) X + V \left[\frac{\mu_{\max} SX}{K_S + S} - k_d X \right] = 0 \quad (20)$$

$$q S_f + qrS - q(1 + r) S - V \left[\frac{\mu_{\max} SX}{Y (K_S + S)} \right] = 0 \quad (21)$$

The inlet substrate concentration was assumed to be 300 mg/l. The effluent

quality was fixed by the following equation:

$$S + 0.5 X_e = 20 \text{ mg/l} \quad (22)$$

The optimization of this system requires a set of algebraic equations having two degrees of freedom when all four clarifier equations, equations (1), (2), (3), and (17), are used, and the inlet conditions, effluent quality, and clarifier depth are specified. The substrate concentration in the aeration tank and the recycle ratio were chosen to be the independent variables. The computational procedure was to assume a value of X_b (e.g., 10,000 mg/l) for each choice of the independent variables. The area of and the underflow from the secondary clarifier could then be computed and used to predict a new concentration of organisms in the underflow. The calculation was then repeated with this new value of X_b . This iterative procedure was continued until the new prediction differed from the previous value by less than one percent.

Minimization of the total volume of the system (aeration vessel and secondary clarifier volumes) was repeated for values of k_3 . Table 2 shows the effect of k_3 on the optimal design.

ACTUAL CLARIFIER OPERATING CONDITIONS

In practice, a secondary clarifier may be underloaded, operated at its maximum capacity or overloaded. If the clarifier is operated at its maximum capacity, equations (1), (2), (3), and (17) can be used as in the design problem.

If the clarifier is underloaded, the thickening will not be as great as that predicted by equation (17). In this case, equation (3) is used to describe the clarification but the underflow concentration must be determined

from the organism balance, equation (2), subject to an upper bound set by equation (17).

If the clarifier is overloaded, equation (3) will not correctly describe the clarification. Equation (17) is used to determine the underflow concentration while the overflow concentration is calculated from the organism balance, subject to a lower bound set by equation (3).

Optimal steady state operation of an activated sludge system can be achieved by properly adjusting the recycle flow rate and the sludge wasting rate. Overloading of the clarifier can usually be avoided by wasting sufficient sludge. However, if the entire system is overloaded this action may lead to insufficient oxidation of the waste in the aeration tank. The operating conditions which minimize the effluent BOD may be found by using the model presented here with the recycle flow rate as the decision variable. The optimal result will include the sludge wasting rate which corresponds to the maximum thickening capacity of the secondary clarifier. In practice a wasting rate slightly larger than this value should be used to prevent the clarifier from overloading.

SAFETY FACTORS IN DESIGN

In designing a waste treatment system, parameters which represent slow growing organisms and poor sludge settling characteristics may be used in the kinetic models for cell growth, substrate consumption, and sludge thickening. With these parameters, the designed system will be larger than needed for organisms with average growth rates and sludge with average settling characteristics. Once the system is constructed and the parameters which characterize the growth rate, substrate consumption rate, and settling rate are

known, operating conditions may be chosen to minimize the effluent BOD or to meet some other objective, such as minimum operating cost.

In Table 3, results are presented for a conservative design in which the parameter values $\mu_{\max} = 0.05 \text{ hr}^{-1}$ and $k_3 = 0.88 \times 10^7 \text{ m/hr}$ were used. The operation of this system based on minimization of the effluent BOD was then determined for the parameter values $\mu_{\max} = 0.1 \text{ hr}^{-1}$ and $k_3 = 2.53 \times 10^7 \text{ m/hr}$. The effluent BOD is reduced to 7.33 mg/l at the optimal operating conditions. The return sludge flow rate is reduced by more than 40 percent.

NOMENCLATURE

A_c	area of secondary clarifier, m^2
b	flow rate of underflow stream, m^3/hr
F	total flux of organisms in secondary clarifier, $(mg/l)(m/hr)$
F_B	bulk flux of organisms in secondary clarifier, $(mg/l)(m/hr)$
F_S	settling flux of organisms in secondary clarifier, $(mg/l)(m/hr)$
K_S	saturation constant, mg/l
k_d	specific endogenous organism attrition rate, hr^{-1}
k_1, k_2, k_3, k_4	constants in settling velocity equation
p	constant in clarification equation
q	flow rate of feed stream, m^3/hr
q_e	flow rate of overflow stream, m^3/hr
r	ratio of recycle stream to feed stream
S	substrate concentration in aeration vessel, mg/l
S_f	substrate concentration in feed stream, mg/l
t	time, hrs
v	average downward fluid velocity in secondary clarifier, m/hr
v_s	settling velocity of organisms in secondary clarifier, m/hr
w	ratio of wasting stream to feed stream
X	organism concentration in aeration vessel, mg/l
X_b	organism concentration in underflow stream, mg/l
X_e	organism concentration in overflow stream, mg/l
X_f	organism concentration in feed stream, mg/l
X_L	organism concentration at which settling velocity model changes, mg/l
X^*	organism concentration at which total flux is a minimum, mg/l

Y	yield factor
α	constant in clarification equation
β_c	time constant in clarification equation, hr^{-1}
μ_{\max}	maximum specific growth of organisms, hr^{-1}

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Table 1. Parameter values employed in
optimal design studies.

1) Constants for settling velocity equation.

	Settling Character	
	<u>Poor</u>	<u>Average</u>
k_3	$0.88 \times 10^7 \text{ m/hr}$	$2.53 \times 10^7 \text{ m/hr}$
k_4	-2.16	-2.16
x_L	2200 mg/l	3540 mg/l

2) Constants for kinetic expressions.

$$\begin{aligned}\mu_{\max} &= 0.1 \text{ hr}^{-1} \\ K_s &= 100.0 \text{ mg/l} \\ k_d &= 0.002 \text{ hr}^{-1} \\ Y &= 0.5\end{aligned}$$

3) Constants for clarification equation.

$$\begin{aligned}p &= 2.1 \\ \alpha &= 0.5 \\ \beta_c &= 0.74 \text{ hr}^{-1}\end{aligned}$$

Table 2. Effect of k_3 on the optimal design.

	Values of k_3		
	<u>0.88×10^7</u>	<u>1.71×10^7</u>	<u>2.53×10^7</u>
Aeration tank			
cell conc.	1935	2450	2820
substrate conc.	16.1	15.3	15.0
dimensionless volume	0.529	0.438	0.389
Secondary clarifier			
underflow conc.	5630	7940	10000
overflow conc.	7.8	9.4	10.1
dimensionless volume	0.457	0.463	0.447
Dimensionless total volume	1.03	0.901	0.836
<u>Aerator vol.</u>			
Total vol.	0.515	0.486	0.465
Recycle ratio	0.489	0.424	0.375

(All concentrations in mg/l)

Table 3. Comparison of design and actual operating conditions for oversized system.

	<u>Design</u>	<u>Actual</u>
μ_{\max} , hr ⁻¹	0.05	0.10
k_3 , m/hr	0.88×10^7	2.53×10^7
Aeration tank		
cell conc.	2280	3220
substrate conc.	17.7	5.82
Secondary clarifier		
underflow conc.	5820	12400
overflow conc.	4.8	3.1
Effluent BOD	20.0	7.37
Recycle ratio (r)	0.613	0.340
Wasting ratio (w)	0.0170	0.00730

(All concentrations in mg/l)

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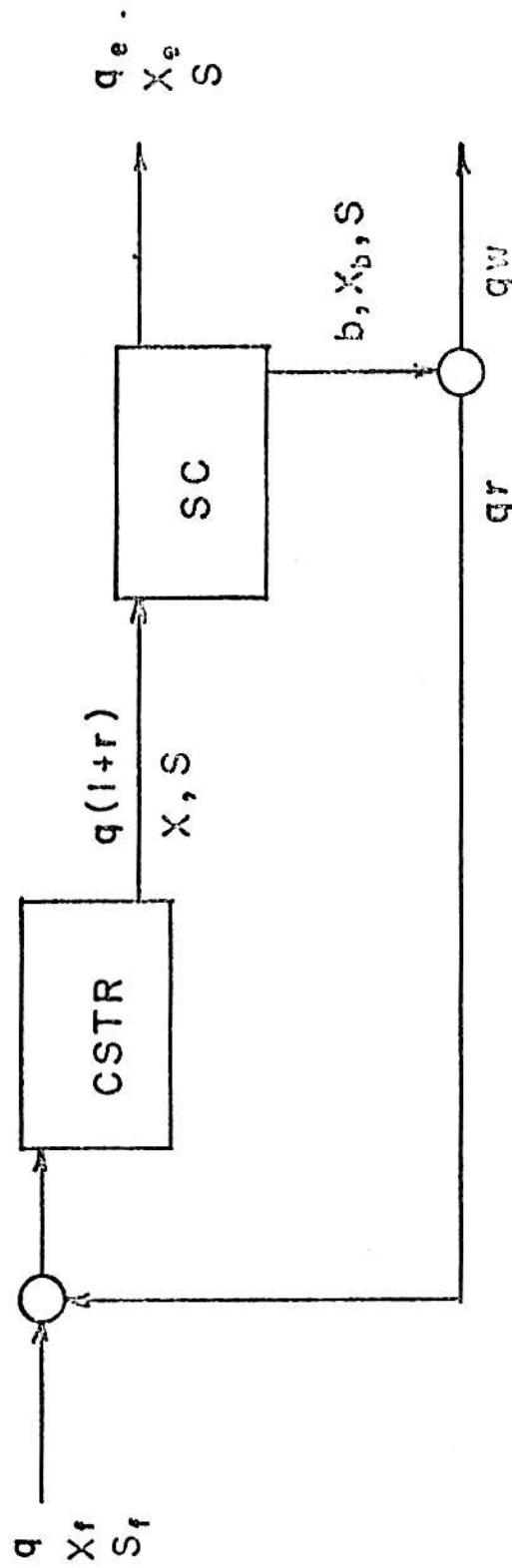


Fig.1. Activated sludge waste treatment system schematic.

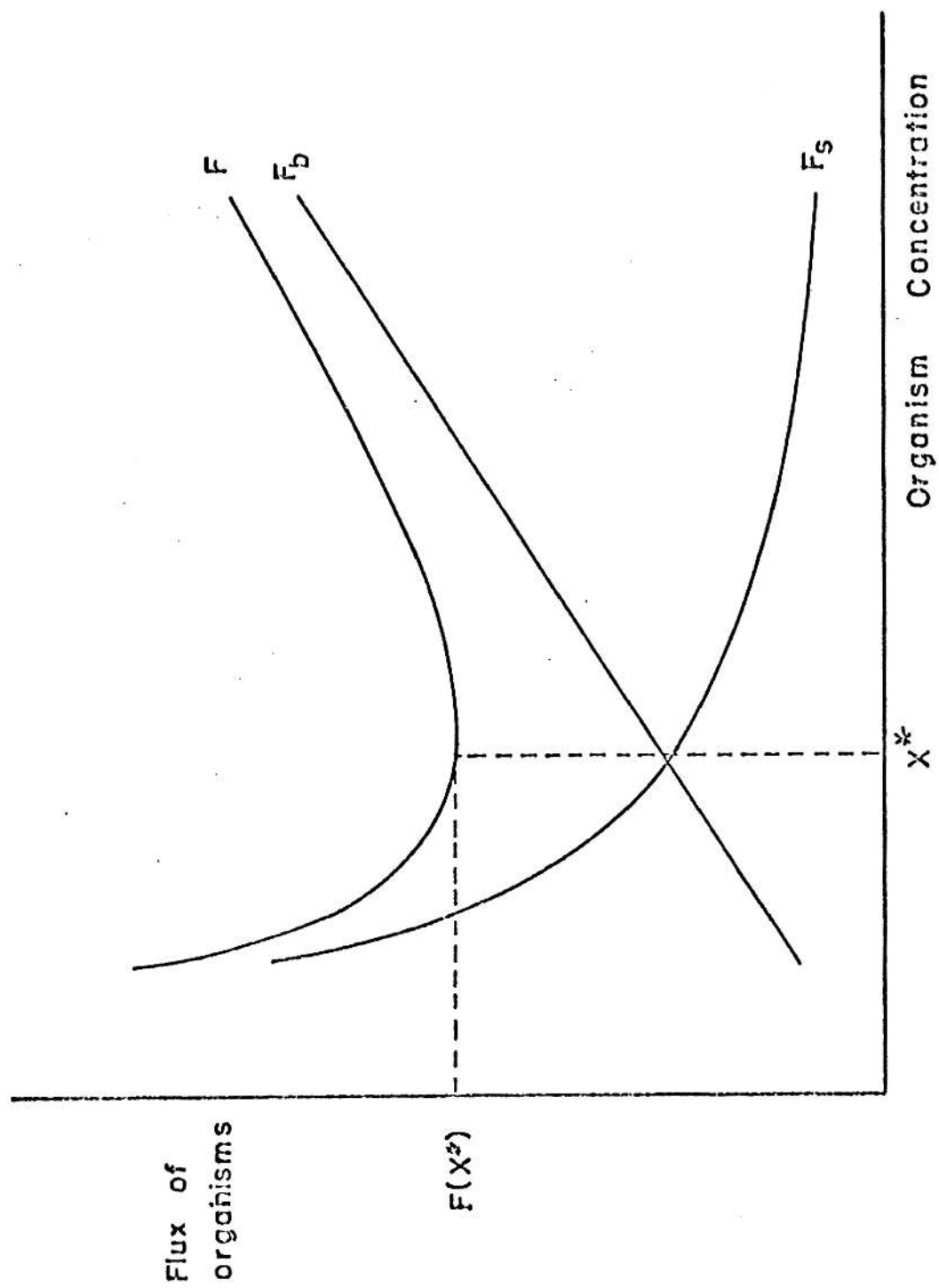


Fig. 2. Flux of organisms versus organism concentration.

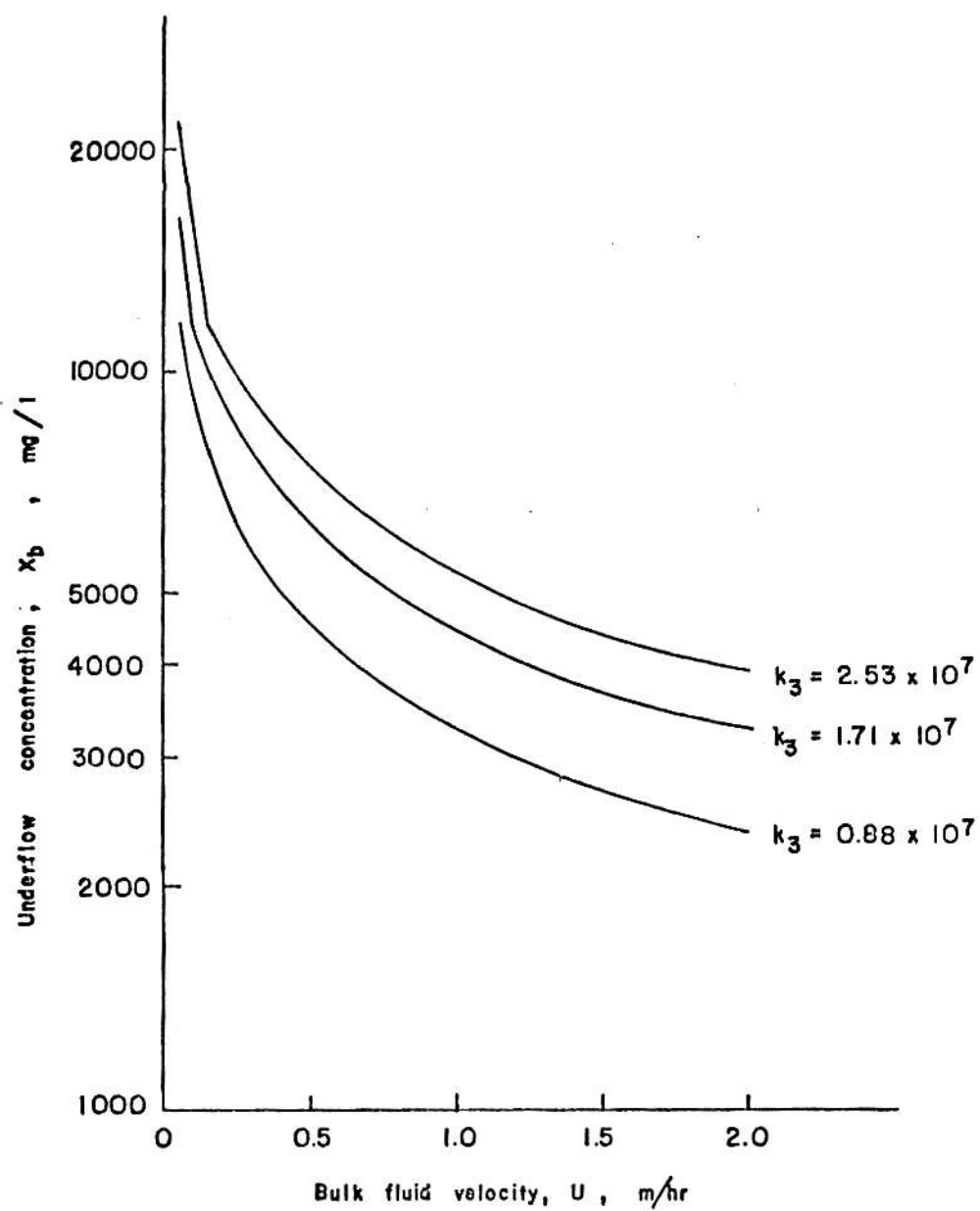


Fig. 3. Organism concentration in underflow as a function of downward fluid velocity in secondary clarifier.

Chapter III

SYNTHESIS OF OPTIMAL DESIGNS

INTRODUCTION

The tower-type activated sludge system to be optimized is composed of a tower fermentor for aeration and a secondary clarifier. The tower fermentor is a column divided into stages by sieve trays or other devices to promote oxygen transfer to the liquid phase. In the tower fermentor considered here, both the gas and liquid phases flow upward in a cocurrent fashion. Cocurrent tower fermentors with sieve trays have been investigated by Prokop et al.¹ and Kitai et al.^{2,3} Both groups found the backflow of liquid and organism sedimentation to be important factors in modeling their data while the latter group also correlated the backflow rate to the gas phase flow rate and sieve tray design. Additional modeling has been performed by Erickson et al.^{4,5}

By operating a pilot scale activated sludge system with a counter-current tower fermentor, Besik⁶ has demonstrated the efficiency of a tower system. Lee et al.⁷ have modeled and optimized an activated sludge system with a cocurrent tower fermentor. In this work, the system model will be expanded to include feed and sludge recycle to each tower stage. In addition, an improved model of the secondary clarifier describing both clarification and sludge thickening is used, and the interaction between the clarifier and fermentor designs is considered.

Previous studies by Gould⁸ and Erickson et al.⁹ have indicated that under certain conditions, it is desirable to use step aeration in an activated sludge system. By including a feed and sludge recycle stream to each tower stage in this model, it is possible to have step aeration and also

step feeding of sludge recycle. The splitting of the feed and sludge recycle streams will be optimized by using structural parameters, as discussed by Ichikawa and Fan.¹⁰ The application of this technique to the design of waste treatment systems has been demonstrated by Mishra et al.¹¹ System synthesis techniques, as reviewed by Hendry et al.,¹² have not previously been applied to the design of a tower-type activated sludge system.

SYSTEM MODELING

Figure 1a shows a schematic diagram of a tower-type activated sludge system with one tower stage while Fig. 1b shows the model representation of this system. Following the convention of the structural parameter method,¹⁰ material entering (leaving) the system model goes directly to (from) a stage. Stages 1, 4 and 5 are just part of the modeling format and do not represent actual processing units. The model representations of systems with two and three tower stages are shown in Figs. 2 and 3.

Before each tower stage, fractions of the feed and sludge recycle streams are mixed and added to the exit stream from the previous stage if there is one. A typical mixing point is shown in Fig. 4. The flow, organism and substrate balances around the mixing point are:

$$q_{n-1} + \alpha_{n1}q_1 + \alpha_{nj} f_j - q_n^0 = 0 \quad (1)$$

$$q_{n-1} X_{n-1} + \alpha_{n1}q_1 X_1 + \alpha_{nj} f_j X_j - q_n^0 X_n^0 = 0 \quad (2)$$

$$q_{n-1} S_{n-1} + \alpha_{n1}q_1 S_1 + \alpha_{nj} f_j S_j - q_n^0 S_n^0 = 0 \quad (3)$$

The feed and sludge recycle streams are split and distributed to several locations. Figure 5 shows a typical distribution point. As the organism and

substrate concentrations are unchanged, only the following flow balance is necessary.

$$q_i - \alpha_{ki} q_i - \alpha_{li} q_i - \alpha_{mi} q_i = 0 \quad (4)$$

This equation may be simplified and generalized to the following:

$$\sum_j \alpha_{ji} = 1 \quad (5)$$

In the tower fermentor, the growth of organisms and depletion of substrate are described by Monod kinetics.^{13,14} Figure 6 shows a typical tower stage. For complete mixing within the tower stage, the steady-state balances are:

$$q_n^o + f_{n+1} - q_n - f_n = 0 \quad (6)$$

$$q_n^o X_n^o + f_{n+1} X_{n+1} - q_n X_n - f_n X_n + V_n X_n \left(\frac{\mu_{\max} S_n}{K_s + S_n} - k_d \right) = 0 \quad (7)$$

$$q_n^o S_n^o + f_{n+1} S_{n+1} - q_n S_n - f_n S_n - V_n X_n \frac{\mu_{\max} S_n}{Y(K_s + S_n)} = 0 \quad (8)$$

The flow balance for the secondary clarifier, shown in Fig. 7, is given by:

$$q_{j-1} - q_j - f_j = 0 \quad (9)$$

If the assumption of no organism growth in the clarifier is made, the substrate concentration is unchanged and the organism balance is given by

$$q_{j-1} X_{j-1} - q_j \bar{X}_j - f_j X_j = 0 \quad (10)$$

For a clarifier depth of nine to ten feet, Naito et al.¹⁵ developed the following empirical equation to describe clarification.

$$\bar{X}_j = p X_{j-1}^\alpha \exp(-\beta_c T_s) \quad (11)$$

Utilizing the concept of a limiting flux, Erickson et al.¹⁶ (see Chapter II) have described the organism concentration in the underflow by the following equation:

$$\underline{X}_j = X^* \left[1 + \frac{A_c}{f_j} k_3 (X^*)^{k_4} \right] \quad (12)$$

where

$$X^* = \left[\frac{-f_j}{A_c k_3 (k_4 + 1)} \right]^{1/k_4}$$

The effluent BOD is fixed by the following water quality standard:

$$S_e + m X_e = C_d \quad (13)$$

where subscript e denotes the effluent stream.

The equations are made more general by introducing the following dimensionless variables:

$$\begin{aligned} d_n &= \frac{q_n}{\mu_{\max} V} & ; & & b_n &= \frac{f_n}{\mu_{\max} V} \\ y_n &= \frac{X_n}{Y S^0} & ; & & z_n &= \frac{S_n}{S^0} \\ K &= \frac{K_s}{S^0} & ; & & k &= \frac{k_d}{\mu_{\max}} \\ \beta &= \frac{\beta_c}{\mu_{\max}} & ; & & t &= T_s \mu_{\max} \\ a &= \frac{A_c}{H^2} & ; & & v_i &= \frac{V_i}{V} \end{aligned}$$

After making these substitutions in all the balances, one obtains a set of dimensionless, non-linear algebraic equations. The feed conditions and clarifier depth are specified for all three systems. For a system with more

than one tower stage, equal volumes of the tower stages are specified. There are two degrees of freedom for the system with one tower stage, five for the system with two tower stages and eight for the system with three tower stages. For a system with any number of tower stages, there are two degrees of freedom for the first tower stage and three more for each additional tower stage.

COMPUTATIONAL ASPECTS

For all three systems, an iterative procedure is employed to determine the organism concentration in the clarifier underflow. Initially, this concentration is fixed at an arbitrary value. Equation (12) is removed from the set of equations so that the degrees of freedom are unchanged. After solving the set of remaining equations, the values of A_c and f_j are used in equation (12) to predict a new underflow organism concentration. The set of equations is solved again and then another underflow organism concentration is predicted. This procedure is continued until the predicted concentration does not differ significantly from the previous value.

For all three systems, the set of equations (minus equation (12)) could be solved directly with the proper choice of independent variables. As a computational scheme, the direct solution is practical only for the system with one tower stage. For this system, the independent variables selected are the substrate concentrations in the effluent and the sludge recycle flow rate. When the backflow(s), the downward flow(s) between tower stages, approach zero, the direct solution procedures become unstable for the systems with two and three tower stages. With these direct procedures, it is also difficult to find a set of independent variables which do not violate a constraint on the dependent variables.

With the proper choice (discussed later) of independent variables for the systems with two and three tower stages, the only non-linear terms are the kinetic expressions in the organism and substrate balances for the tower stages. This choice also allows the balances on the tower stages and mixing points between the tower stages to be solved simultaneously. Since some of the equations are non-linear due to the kinetic terms, it is desirable to linearize these equations. A truncated Taylor's series expansion about the point (v', y', z') gives the following linearized kinetic terms.

$$\frac{vyz}{K+z} \doteq \frac{y'z'}{K+z'} v + \frac{v'z'}{K+z'} y + \frac{K v'y'}{(K+z)^2} z - \frac{v'y'z'}{K+z'} - \frac{K v'y'z'}{(K+z')^2} \quad (14)$$

$$-kvy \doteq -k y'v - k v' y + kv'y' \quad (15)$$

An iterative procedure is used so that the solution to the resulting set of linear equations converges to the solution of the original set of non-linear equations. The previously mentioned difficulties with the direct solution procedure were not encountered using this linearized procedure.

For the system with two tower stages, the independent variables were the substrate concentration in the effluent, the backflow rate, the fraction of the feed sent to the first tower stage and the sludge recycle flow rates to both tower stages. The effluent substrate concentration, the two backflow rates, the fractions of the feed sent to the first and second tower stages and the sludge recycle flow rates to all three tower stages were the independent variables for the system with three tower stages. By comparing the structures of these two systems, one can see the similarity of these two sets of independent variables. Also, considering the system with one tower stage leads to the conclusion that for each additional tower stage, one must specify the fraction of the feed sent to the preceding stage, the backflow

rate from the additional to the preceding stage and the sludge recycle flow rate to the additional stage.

Following the procedures outlined, it is possible to simulate a system at any point (set of values of the independent variables). At many points, a constraint on a dependent variable is violated. The simulation routine must check that all concentrations, flow rates and volumes are positive and that all structural parameters are between zero and one. If no constraints are violated, the total volume of the system is computed and used as the objective function in the search for the optimal design.

RESULTS AND DISCUSSION

Tables 2, 3, and 4 give the values of the dimensionless variables for the best designs of the systems with one, two and three tower stages. Table 5 shows the effect of the number of tower stages on the clarifier, fermentor and total volumes, the fraction of the system volume used for aeration, and the sludge recycle and wasting rates. The effect of the number of tower stages on the volumes is shown graphically in Fig. 8.

In performing the design optimization, it was found that the tower fermentor volume was a very weak function of the backflow rate(s). For this reason, equal backflow rates were specified for the system with three tower stages. This condition is a reasonable assumption for the case of single feed and sludge recycle streams. This assumption reduces the degrees of freedom by one.

As indicated in Table 5, the system with three tower stages meets the effluent quality standard with the smallest tower, clarifier and total volumes. Changing from one tower stage to two reduces the total volume by 30.4% while a 10.7% reduction is caused by changing from two to three tower

stages. A single tower stage is mathematically the same as the model generally used to describe an agitated aeration vessel. Therefore, a multistage tower system offers a large savings in volume over a completely mixed activated sludge system. Tables 2, 3, and 4 show that the fraction of organic waste which is metabolized increases as the number of tower stages is increased. For the systems with one, two and three tower stages, the fractions of entering substrate which leaves in the effluent are .050, .037 and .033. Since the total BOD of the effluent is fixed by equation (13), the overflow of suspended solids increases as the number of tower stages is increased.

For the given objective function and parameter values, the optimal designs of the systems with two and three tower stages both specify single feed and sludge recycle streams to the first tower stage and no backflow between stages. Thus step aeration and step feeding of sludge recycle are not desirable and the general tower fermentor flow model has been reduced to a tanks in series model with no backflow at the optimal design. These results are in agreement with the results of Erickson et al.⁹ which show that step aeration is not desirable when the dimensionless saturation constant is greater than 0.2 and more than 95% of the substrate is to be metabolized. Namkoong et al.¹⁷ modeled the aeration vessel by a tanks in series model which allowed step feeding of the return sludge. Their results show the optimal policy to be the feeding of all return sludge to the first tank and also show the desirability of multi-tank systems. When oxygen transfer and organism sedimentation are not considered, the tanks in series model with no backflow also represents a counter-current tower fermentor.

As the number of tower stages is increased, the behavior of the tower fermentor is less like a completely mixed reactor and more like a plug flow

reactor. Fan et al.¹⁸ have shown that increasing the Peclet number of the aeration vessel causes a decrease in the area required for clarification for a fixed organism concentration in the clarifier underflow as well as a decrease in the total volume of the system. In this work, the area required for clarification also fixes the clarifier size but the organism concentration in the underflow is determined by the sludge thickening capacity for that fixed size. For this clarifier model, increasing the Peclet number of the aeration vessel (i.e., increasing the number of tower stages) also caused a decrease in the clarifier size and total volume of the system.

The locations of the optimal designs are influenced by the choice of the objective function, the values of the parameters and the assumptions made in developing the models of the clarifier and tower fermentor. Objective functions based on capital and operating costs could be used in place of the total volume used here.

NOMENCLATURE

A_c	area of clarifier, m^2
a	dimensionless area of clarifier
b_n	dimensionless backflow rate from stage n
C_d	effluent quality standard, mg/ℓ
d_n	dimensionless flow rate from stage n
d_n^o	dimensionless flow rate to stage n
f_n	volumetric backflow rate from stage n , m^3/hr
F	downward flux of organisms in clarifier, $(mg/\ell)(m/hr)$
H	height of clarifier, m
K	dimensionless saturation constant
K_s	saturation constant, mg/ℓ
k	dimensionless endogenous organism attrition rate
k_d	specific endogenous organism attrition rate, hr^{-1}
k_3, k_4	constants in empirical settling velocity equation
m	conversion factor, $(mg/\ell \text{ BOD})/(mg/\ell \text{ sludge dry weight})$
p	constant in empirical clarification equation
q_n	volumetric flow rate from stage n , m^3/hr
q_n^o	volumetric flow rate to stage n , m^3/hr
S_e	substrate concentration in effluent stream, mg/ℓ
S_n	substrate concentration in stage n , mg/ℓ
S_n^o	substrate concentration entering stage n , mg/ℓ
t	dimensionless holding time of clarifier
T_s	holding time of clarifier, hr
U	downward velocity of liquid in clarifier, m/hr
U_s	settling velocity of organisms, m/hr

V	volume used to define dimensionless variables; equal to H^3
V_n	volume of tower stage n , m^3
V_T	dimensionless total volume of system
v_n	dimensionless volume of tower stage n
X^*	organism concentration at which clarifier flux is a minimum, mg/ℓ
X_e	organism concentration in effluent, mg/ℓ
\bar{X}_j	organism concentration in clarifier overflow, mg/ℓ
\underline{X}_j	organism concentration in clarifier underflow, mg/ℓ
X_n	organism concentration in stage n , mg/ℓ
Y	yield factor in Monod kinetics
y_n	dimensionless organism concentration stage n
z_n	dimensionless substrate concentration in stage n
α	constant in empirical clarification equation
α_{ij}	structural parameter representing fraction of flow from stage j to stage i
β	dimensionless constant in clarification equation
β_c	constant in clarification equation, hr^{-1}
μ_{max}	maximum specific growth rate of organisms, hr^{-1}

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Table 1. Parameter values used for optimization.

I. Kinetic

$$\begin{aligned}\mu_{\max} &= 0.1 \text{ hr}^{-1} \\ K_s &= 100.0 \text{ mg/l} \\ k_d &= 0.002 \text{ hr}^{-1} \\ Y &= 0.5\end{aligned}$$

II. Clarifier performance

$$\begin{aligned}k_3 &= 2.53 \times 10^7 \text{ m/hr} \\ k_4 &= -2.16 \\ p &= 2.1 \\ \alpha &= 0.5 \\ \beta_c &= 0.74 \text{ hr}^{-1}\end{aligned}$$

III. Effluent quality

$$\begin{aligned}m &= 0.5 \frac{\text{mg/l BOD}}{\text{mg/l sludge dry weight}} \\ C_d &= 20 \text{ mg/l}\end{aligned}$$

IV. Dimensionless variables

$$\begin{aligned}S^0 &= 300 \text{ mg/l} \\ H &= 10 \text{ ft} = 3.05 \text{ m} \\ V &= H^3 = 1000 \text{ ft}^3 = 28.3 \text{ m}^3\end{aligned}$$

V. Feed conditions

$$\begin{aligned}d_1^0 &= 1.0 \\ y_1^0 &= 0.0 \\ z_1^0 &= 1.0\end{aligned}$$

Table 2. Optimal design for system with
one tower stage (Figure 1b).

I. Flow rates

$$\begin{array}{ll} d_1 = 1.000 & d_4 = 0.989 \\ d_2 = 1.375 & d_5 = 0.011 \end{array}$$

II. Cell concentrations

$$\begin{array}{ll} y_1 = 0.000 & y_4 = 0.067 \\ y_2 = 18.750 & y_5 = 66.667 \end{array}$$

III. Substrate concentrations

$$\begin{array}{ll} z_1 = 1.000 & z_4 = 0.050 \\ z_2 = 0.050 & z_5 = 0.050 \end{array}$$

IV. Structural parameters

$$\alpha_{23} = 0.971$$

V. Volumes

$$\begin{array}{ll} v = 0.389 \\ a = 0.447 \\ V_T = 0.836 \end{array}$$

Table 3. Optimal design for system with
two tower stages (Figure 2).

I. Flow rates

$$\begin{array}{ll} d_1 = 1.000 & d_5 = 0.988 \\ d_2 = 1.285 & d_6 = 0.012 \\ d_3 = 1.285 & b_3 = 0.000 \end{array}$$

II. Cell concentrations

$$\begin{array}{ll} y_1 = 0.000 & y_5 = 0.117 \\ y_2 = 14.390 & y_6 = 62.368 \\ y_3 = 14.518 & \end{array}$$

III. Substrate concentrations

$$\begin{array}{ll} z_1 = 1.000 & z_5 = 0.037 \\ z_2 = 0.197 & z_6 = 0.037 \\ z_3 = 0.037 & \end{array}$$

IV. Structural parameters

$$\begin{array}{ll} \alpha_{21} = 1.000 \\ \alpha_{24} = 0.9559 \\ \alpha_{34} = 0.000 \end{array}$$

V. Volumes

$$\begin{array}{ll} v = 0.141 \\ a = 0.298 \\ V_T = 0.580 \end{array}$$

Table 4. Optimal design for system with
three tower stages (Figure 3).

I. Flow rates

$d_1 = 1.000$	$d_6 = 0.986$
$d_2 = 1.323$	$d_7 = 0.014$
$d_3 = 1.323$	$b_3 = 0.000$
$d_4 = 1.323$	$b_4 = 0.000$

II. Cell concentrations

$y_1 = 0.000$	$y_4 = 14.043$
$y_2 = 13.787$	$y_6 = 0.148$
$y_3 = 14.043$	$y_7 = 54.800$

III. Substrate concentrations

$z_1 = 1.000$	$z_4 = 0.030$
$z_2 = 0.322$	$z_6 = 0.030$
$z_3 = 0.104$	$z_7 = 0.030$

IV. Structural parameters

$\alpha_{21} = 1.000$	$\alpha_{25} = 0.959$
$\alpha_{31} = 0.000$	$\alpha_{35} = 0.000$
	$\alpha_{45} = 0.000$

V. Volumes

$v = 0.0861$
$a = 0.262$
$V_T = 0.521$

Table 5. Effect of number of tower stages
on the optimal design.

	Number of Tower Stages		
	<u>1</u>	<u>2</u>	<u>3</u>
Dimensionless volumes			
Fermentor	0.389	0.282	0.258
Clarifier	0.447	0.298	0.262
Total	0.836	0.580	0.520
<u>Fermentor volume</u> Total volume	0.465	0.486	0.496
Dimensionless biomass			
Recycled	25.0	17.8	17.7
Wasted	0.733	0.748	0.767

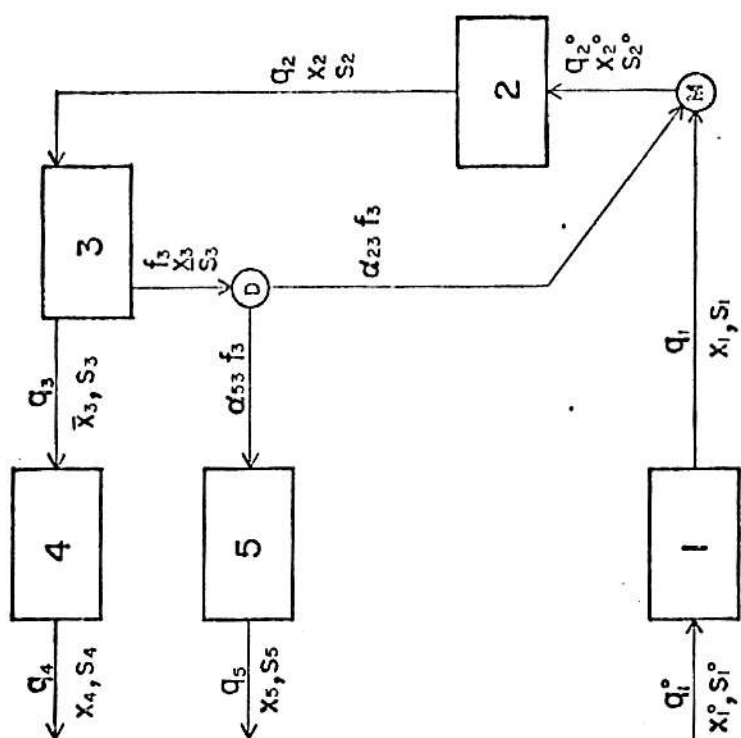


Figure 1b. System Model
With One Tower Stage.

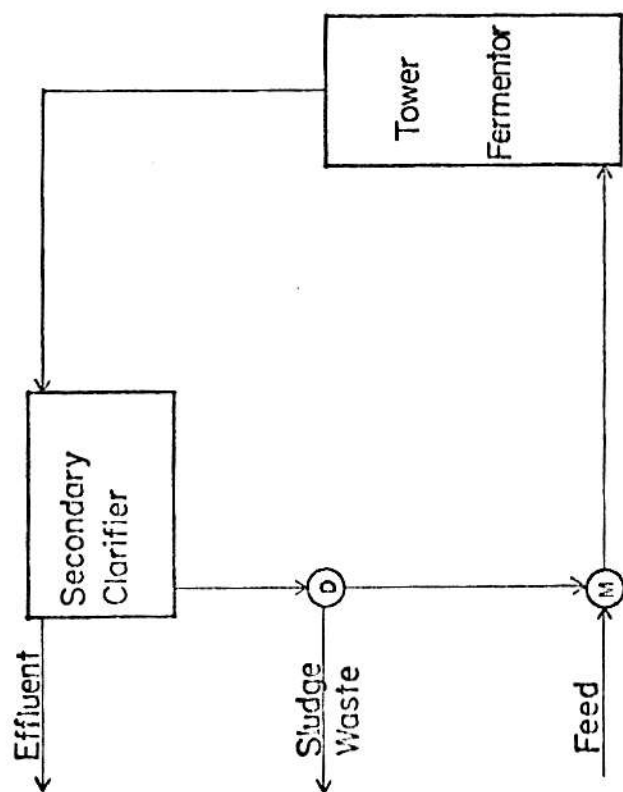


Figure 1a. System With
One Tower Stage.

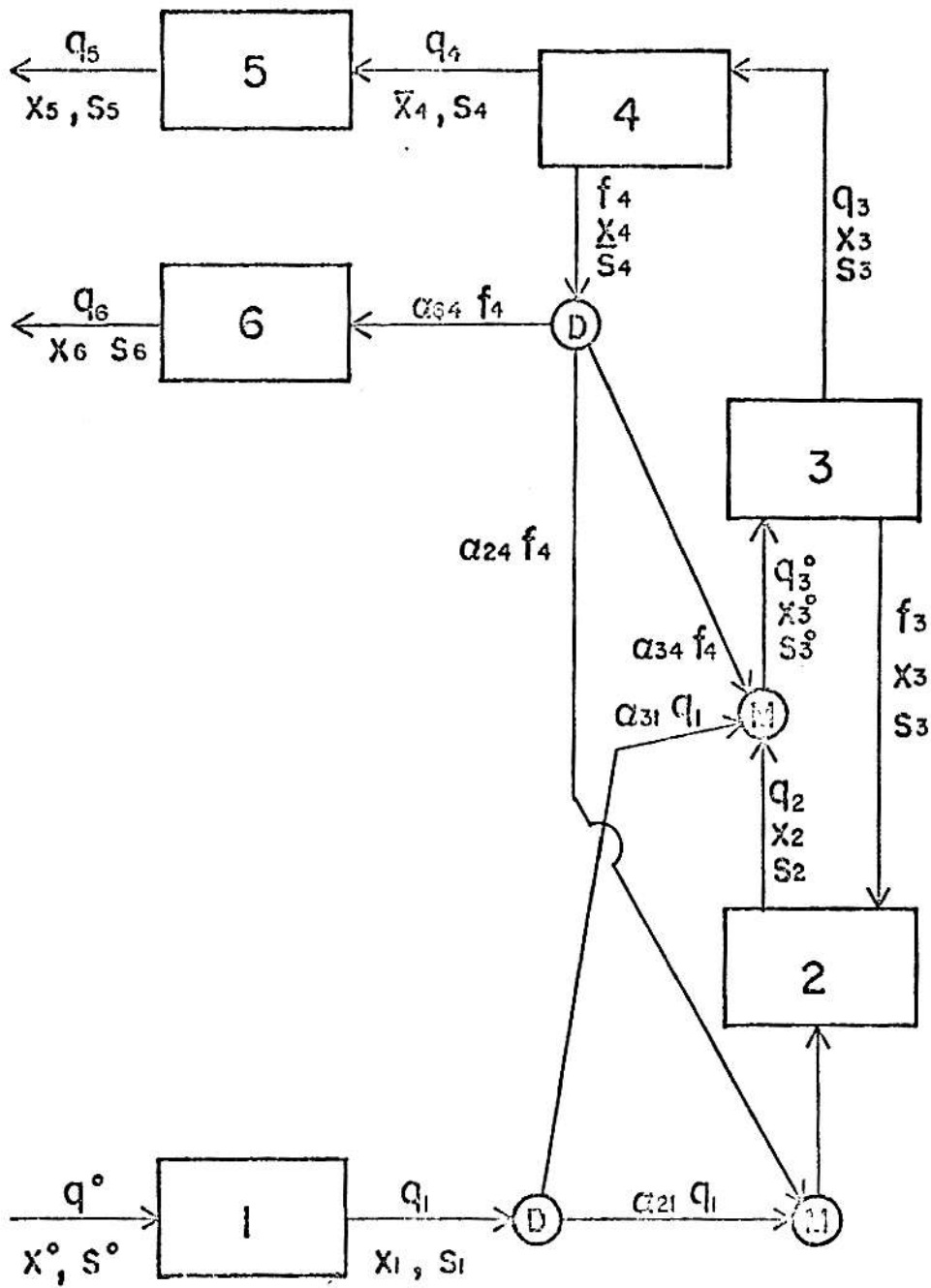


Figure 2. System Model With Two Tower Stages.

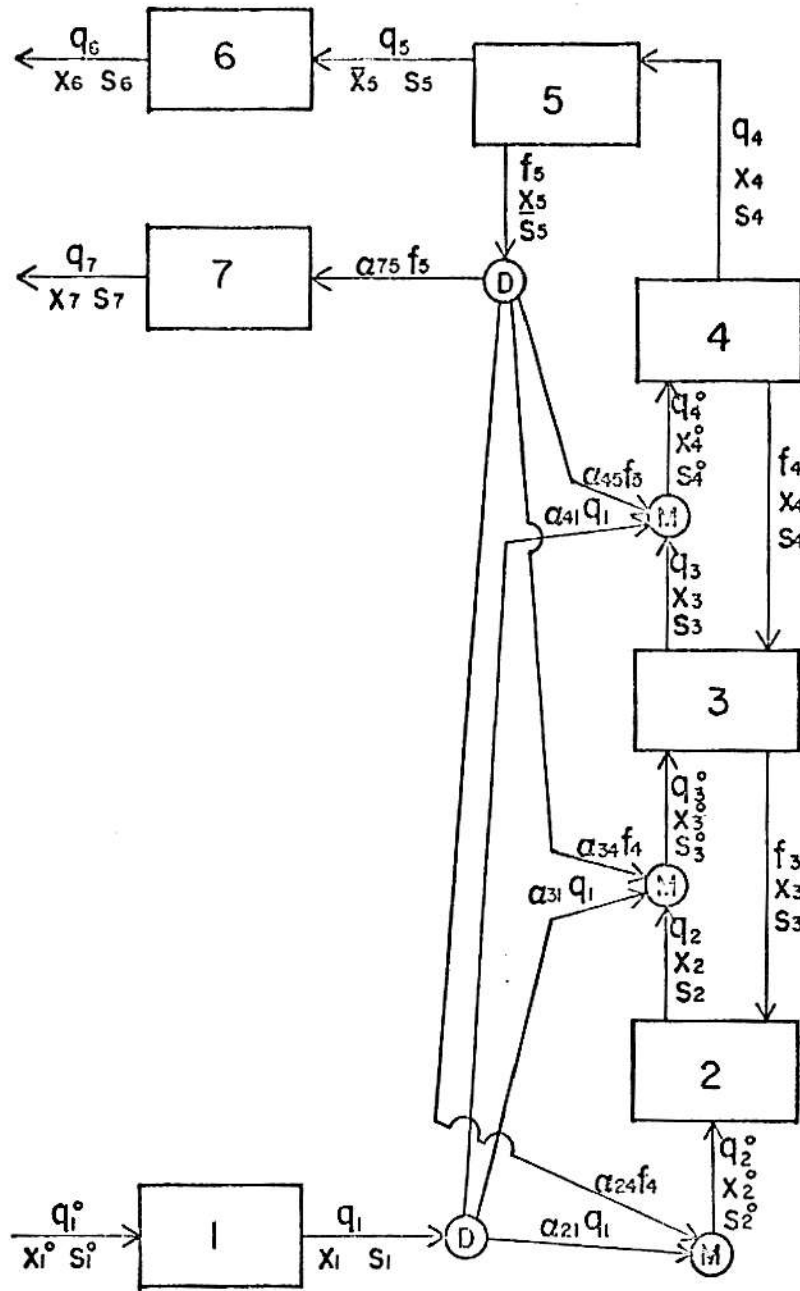


Figure 3. System Model With Three Tower Stages

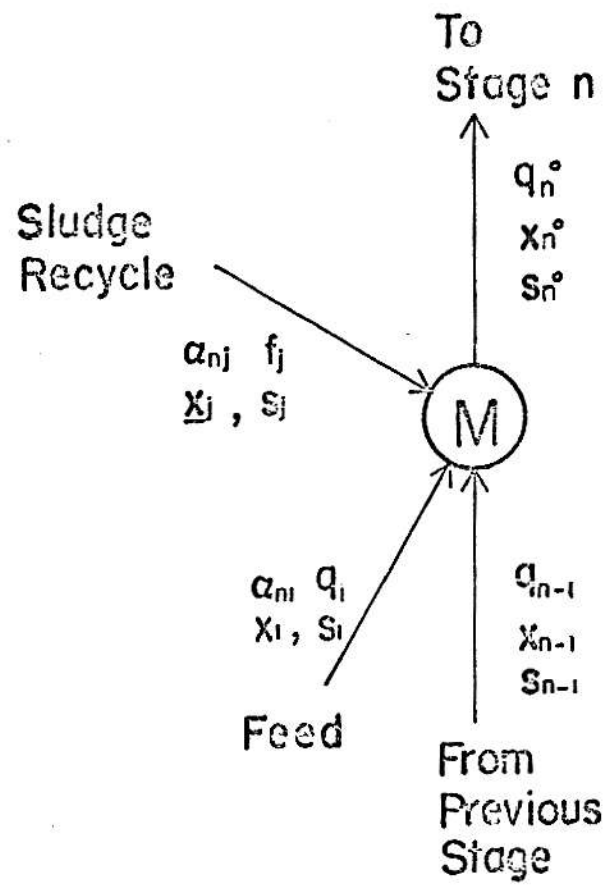


Figure 4. Typical Mixing Point

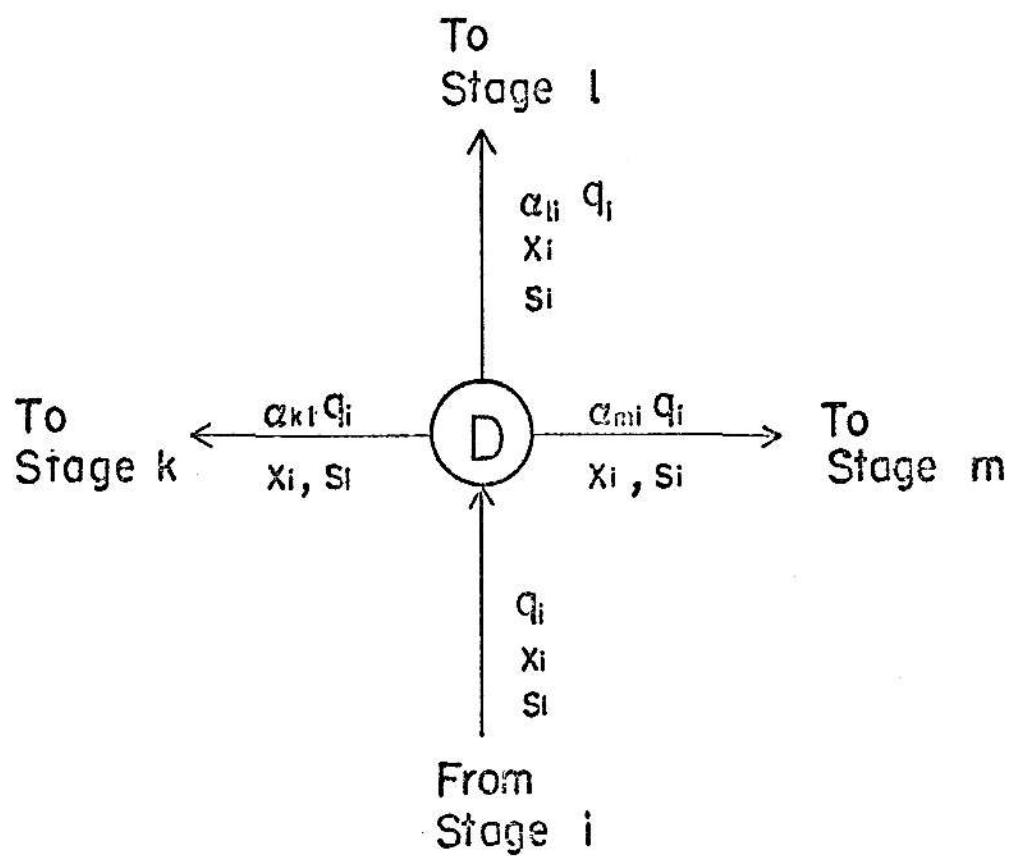


Figure 5. Typical Distribution Point.

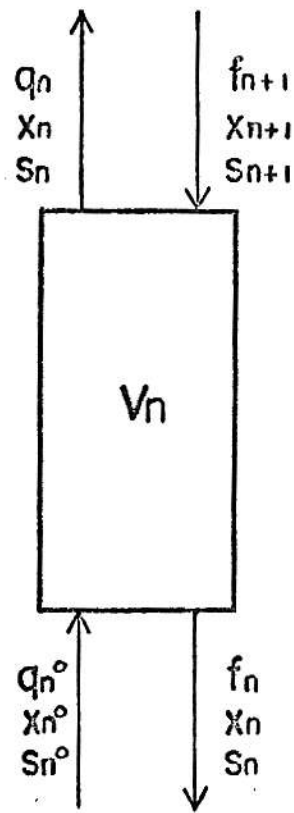


Figure 6. Typical Tower Fermentor Stage.

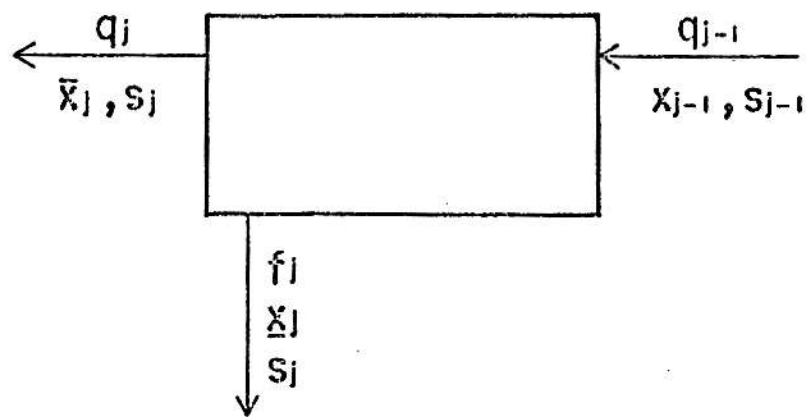


Figure 7. Secondary Clarifier

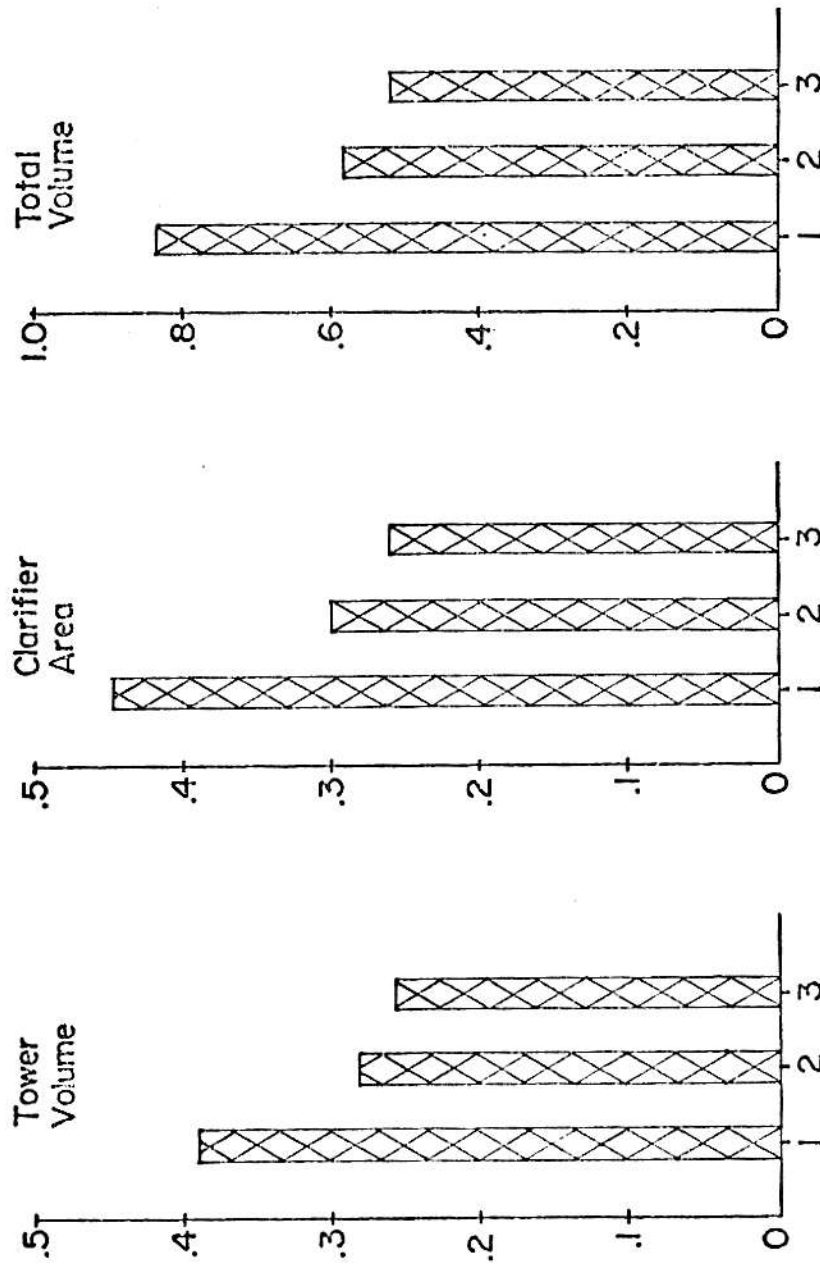


Figure 8. Effect of Number of Tower Stages on the Optimal Design

Chapter IV

EFFECT OF ORGANISM SEDIMENTATION

INTRODUCTION

Sedimentation of organisms in cocurrent tower fermentors has been reported by Prokop et al.¹ and Kitai et al.^{2,3} Although no experiments on sedimentation in activated sludge tower systems have been reported, Lee et al.⁴ included sedimentation in their model by employing a parameter relating the organism concentration at the top of a stage to the average organism concentration in that stage. This parameter was treated as a constant for all tower stages.

In this work, two sedimentation parameters will be calculated for each stage by using the dispersion model to characterize the flow. The two parameters will relate the organism concentrations at the top and bottom of a stage to the average concentration in that stage. The degree of sedimentation in each stage will depend on the organism concentration as well as the size and flow characteristics of the stage. In the previously mentioned work by Lee et al.,⁴ the tower-type activated sludge system was optimized for a fixed value of the sedimentation parameter. In this work, the sedimentation parameters are dependent on the system variables and thus are an integral part of the system optimization. For systems with two and three tower stages, the designs which minimize the total aeration and clarification volume are determined for three values of the Peclet number.

DISPERSION MODEL PREDICTION OF SEDIMENTATION IN A TOWER STAGE

The model of a tower-type activated sludge system with three tower stages is shown in Fig. 1. A one dimensional dispersion model is used to estimate the organism sedimentation within each tower stage. A typical stage with flows indicated is shown in Fig. 2.

The one dimensional dispersion model for a reactant is given by:

$$E \frac{d^2 C_A}{d\zeta^2} - U \frac{dC_A}{d\zeta} + r_A = 0 \quad (1)$$

For the tower-type activated sludge system, the reaction is the growth of organisms. The kinetics of this growth follow the Monod equation (with endogenous term).^{5,6}

$$\frac{dX_n}{dt} = \frac{\mu_{\max} S_n X_n}{K_s + S_n} - k_d X_n \quad (2)$$

The characteristic organism velocity is the upward velocity of the organisms by fluid flow minus the settling velocity of the organisms. The latter has been reported by Dick and Javaheri⁷ to be a function of organism concentration. Analysis of their data leads to an equation of the form

$$U_s(X) = ae^{-bX} \quad (3)$$

The dispersion model for organisms in stage n of the tower is given by:

$$E \frac{d^2 X_n}{d\zeta^2} - [U_n - U_s(X_n)] \frac{dX_n}{d\zeta} + \left(\frac{\mu_{\max} S_n}{K_s + S_n} - k_d \right) X_n = 0 \quad (4)$$

This equation can be made dimensionless by introducing the dimensionless variables of Chapter III and the following variables.

$$d_s(y_n) = \frac{\pi D^2}{4\mu_{\max} V} U_s(y_n) \quad ; \quad \ell = \frac{\zeta}{L}$$

The resulting equation is:

$$\frac{1}{Pe} \frac{d^2 y_n}{d\ell^2} - \left[1 - \frac{d_s (y_n)}{d_n} \right] \frac{dy_n}{d\ell} + \frac{v_n}{d_n} \left(\frac{z_n}{K + z_n} - k \right) y_n = 0 \quad (5)$$

where

$$Pe = \frac{U_n L}{E}$$

The Peclet number, Pe , is a parameter which will be treated as a constant for all tower stages. Ghai⁸ reported experimental data on salt tracer concentration as a function of time in a tower fermentor stage. Analysis of this data gives a Peclet number of 1.5 which is between 0 for one completely mixed tank and 2.6 for two completely mixed tanks in series.

Equation (5) is a non-linear, second-order differential equation which cannot be solved analytically. It would be time consuming to numerically solve this equation for each set of independent variables generated in a direct search routine for the optimal system design. An estimate of the sedimentation can be obtained by solving a related form of the equation linearized about the average organism concentration. Note that fixing the average organism concentration in a stage also fixes the average substrate concentration.

$$\frac{1}{Pe} \frac{d^2 y_n}{d\ell^2} - \left[1 - \frac{d_s (\langle y_n \rangle)}{d_n} \right] \frac{dy_n}{d\ell} + \frac{v_n}{d_n} \left(\frac{z_n}{K + z_n} - k \right) y_n = 0 \quad (6)$$

This linearized form of the equation is a linear, second-order differential equation which can be solved analytically with the proper choice of two boundary conditions.

Figure 3 shows the entrance region to a stage. An organism balance for this region is given by:

$$q_n^0 x_n^0 + f_n(0^+) x_n(0^+) - q_n(0^+) x_n(0^+) - f_n(0) x_n(0) + \frac{\pi D^2}{4} [U_s x_n]_{0^+} + \frac{\pi D^2}{4} E \frac{dx_n}{d\zeta} \bigg|_{\epsilon} = 0 \quad (7)$$

where $0 \leq \epsilon \leq 0^+$. After rearranging and letting 0^+ approach 0, the equation becomes

$$E \frac{dx_n}{d\zeta} \bigg|_0 = \frac{4q_n^0 [x_n(0) - x_n^0]}{\pi D^2} - [U_s x_n]_0 \quad (8)$$

Replacing U_s by $U_s (\langle x_n \rangle)$ and introducing the dimensionless variables leads to

$$\frac{1}{Pe} \frac{dy_n}{d\ell} \bigg|_0 = \frac{d_n^0}{d_n} [y_n(0) - y_n^0] - \frac{d_s (\langle y_n \rangle)}{d_n} y_n(0) \quad (9)$$

The second boundary condition to be used requires that the average organism concentration calculated from the solution to equation (6) must correspond to the specified average organism concentration (i.e., the value about which the equation is linearized).

$$\frac{\int_0^L x_n(\zeta) d\zeta}{\int_0^L d\zeta} = \langle x_n \rangle \quad (10)$$

In dimensionless form, this becomes

$$\int_0^1 y_n(\ell) d\ell = \langle y_n \rangle \quad (11)$$

There are three possible solutions to equation (6) depending on the values of the constants. Let

$$\alpha = \frac{1}{Pe} \quad (12)$$

$$\beta = 1 - \frac{d_s (\langle y_n \rangle)}{d_n} \quad (13)$$

$$\gamma = \frac{v_n}{d_n} \left(\frac{z_n}{k + z_n} - k \right) \quad (14)$$

For $\beta^2 < 4\alpha\gamma$,

$$y_n(\ell) = e^{m_1 \ell} (C_1 \cos m_2 \ell + C_2 \sin m_2 \ell) \quad (15)$$

For $\beta^2 = 4\alpha\gamma$,

$$y_n(\ell) = C_1 e^{m \ell} + C_2 \ell e^{m \ell} \quad (16)$$

For $\beta^2 > 4\alpha\gamma$,

$$y_n(\ell) = C_1 e^{m_1 \ell} + C_2 e^{m_2 \ell} \quad (17)$$

A computational problem arises in equation (17) when γ approaches zero. For this case,

$$y_n(\ell) = \frac{C_1}{m} e^{m \ell} + C_2 \quad (18)$$

Expressions for the constants C_1 , C_2 , m , m_1 , and m_2 , are given in Table 2.

The sedimentation parameters for each stage δ_n and ϵ_n are calculated from the following definitions.

$$\delta_n \equiv \frac{y_n(1)}{\langle y_n \rangle} \quad (19)$$

$$\epsilon_n \equiv \frac{\langle y_n \rangle}{y_n(0)} \quad (20)$$

These definitions lead to values of δ_n and ϵ_n between zero and one.

COMPUTATIONAL SCHEME

The sedimentation parameters are estimated using an iterative procedure which is independent of the solution procedure described in Chapter III. To use the procedure of Chapter III, equal tower volumes must be specified. The only modifications to the procedure of Chapter III are replacing y_n by \bar{y}_n or z_n in the balances which include exit streams from a tower stage. The average organism concentration, $\langle y_n \rangle$, is used in the kinetic terms. For the tower stage shown in Fig. 2, the balances in dimensionless form are:

$$d_n^o + b_{n+1} - d_n - b_n = 0 \quad (21)$$

$$d_n^o y_n^o + b_{n+1} z_{n+1} - d_n \bar{y}_n - b_n z_n + v \langle y_n \rangle \left(\frac{z_n}{K + z_n} - k \right) = 0 \quad (22)$$

$$d_n^o z_n^o + b_{n+1} z_{n+1} - d_n z_n - b_n z_n - v \langle y_n \rangle \left(\frac{z_n}{K + z_n} \right) = 0 \quad (23)$$

where

$$\bar{y}_n = \delta_n \langle y_n \rangle$$

$$z_n = \frac{\langle y_n \rangle}{\epsilon_n}$$

To start the iteration for a given set of independent variables, δ_n and ϵ_n are set at arbitrary values. Using the procedure of Chapter III, the average organism and substrate concentrations and liquid volume of each tower stage and the flow rates are calculated. Based on the gas hold-up data of Prokop et al.,¹ the liquid volume is assumed to be seventy percent of the actual volume of each stage. To be consistent with the geometry used in estimating the Peclet number, the length is chosen to be twice the diameter. Equation (6) is then solved to predict new values of δ_n and ϵ_n . With these

new values, the design calculations are repeated. New values of δ_n and ϵ_n are computed from those results. This iterative procedure is continued until there is no significant change in the sedimentation parameters.

RESULTS

For each value of the Peclet number and each number of tower stages, the system is designed to minimize the total volume. Tables 3 and 4 give the optimal designs for systems with two and three tower stages for Peclet numbers of 0, 0.5, and 1.0. A Peclet number of zero corresponds to the assumption of complete mixing made in Chapter III. The effect of Peclet number on the clarifier, fermentor and total volumes for the two optimal systems is shown in Figs. 4 and 5.

To show the effects of effluent substrate concentration, return sludge flow rate and backflow rate on the total volume, results of simulation near the optimum design for two tower stages and a Peclet number of 0.5 are presented in Fig. 6. The organism concentration profiles for the optimal designs with two tower stages are given in Fig. 7.

DISCUSSION

As discussed in Chapter III, the optimal design is influenced by the choice of the objective function, the parameter values used and the assumptions inherent in the system model. Introduction of the sedimentation parameters, estimated from the dispersion model, made the objective function surface less smooth than it had been with the assumption of complete mixing made in Chapter III. Locating the optimal designs was more difficult and time consuming.

Although a Peclet number of 1.5 was estimated from the residence time distribution data presented by Ghai,⁸ smaller values (0.5 and 1.0) are used in

this work because the dispersion model equation is linearized about the completely mixed state. Use of Peclet numbers of 0.5 and 1.0 lead to predicted values of δ_n that were generally in the range of 0.4 to 0.9. Using larger values of the Peclet number made it difficult to find a set of values of the independent variables which did not violate some constraint on the system.

Tables 3 and 4 and Figs. 4 and 5 show that the optimal volume requirements decrease as the Peclet number increases. A larger Peclet number implies more organism sedimentation which has the effect of reducing the loss of organisms from a tower stage by flow. If fewer organisms leave a tower stage, there is less need for sludge recycle and thus a smaller upward flow rate through the tower. Fewer organisms leaving the upper tower stage reduces the area required for clarification. Lee et al.⁴ showed that under certain conditions, increasing the backflow rate reduced the volume of the tower when organism sedimentation was considered. Their results were for a fixed sedimentation parameter and for dimensionless saturation constants less than 0.05. With the larger dimensionless saturation constant (0.333) used here, backflow is undesirable because it makes the tower behave less like a plug flow reactor. When the dimensionless saturation constant is large, the kinetics are nearly first order with respect to substrate concentration and thus plug flow is desirable. Because of the higher organism concentration, sludge recycle is more efficient than backflow in providing organisms to the bottom tower stage.

For all three values of the Peclet number, both systems contain single feed and sludge recycle streams at the optimal design. The optimal amount of backflow is zero in all cases so that the optimal system designs are tanks in

series models. The Peclet number influences the volumes of the tower fermentor, clarifier and total system as shown in Figs. 4 and 5. As the Peclet number is an indication of the extent of organism sedimentation, these results actually show the effect of this sedimentation. This effect is significant and thus shows the need for experimental data on which to base a more accurate model of organism sedimentation.

For systems with single feed and recycle streams, there are only three degrees of freedom. The independent variables remaining are the effluent substrate concentration, the return sludge flow rate and the backflow rate (equal rates are assumed for three tower stages). The effects of these variables on the total volume of a system with two tower stages are shown in Fig. 6. Near the optimum point, the dependence of the objective function on the effluent substrate concentration is strong while the dependence on the return sludge flow rate and backflow rate is weak. As both the return sludge flow and backflow bring organisms to the lower tower stage and both have a small effect on the objective function, there are many combinations of these two variables which produce roughly the same value of the objective function for a given value of the effluent substrate concentration.

In developing the approximate solution to the dispersion model equation, an organism balance around the exit region of a tower stage could have been used as the second boundary condition. This balance leads to the following boundary condition:

$$\left. E \frac{dX_n}{d\zeta} \right|_L = \frac{4 f_{n+1} [X_{n+1} - X_n(L)]}{\pi D^2} - [U_s X_n]_L \quad (24)$$

This boundary condition was not used as it would have changed the average organism concentration and it would have resulted in two non-linear equations

to solve for the integration constants. When there is no backflow, this boundary condition predicts a negative value of the derivative at the top of the tower stage. The organism concentration profiles shown in Fig. 7 have this negative slope.

The sedimentation parameters predicted for the system with two tower stages were much larger than those predicted for the system with three tower stages for the same value of the Peclet number. A large value (close to one) for the sedimentation parameter implies a small amount of sedimentation. Because of the fixed geometry of the tower stages, the upward fluid velocity increases as the number of stages increases. Since the fluid velocity increases and the mean residence time decreases within each tower stage, the flow pattern changes away from complete mixing towards plug flow as the number of stages increases. Thus, the smaller sedimentation parameters for the system with three tower stages appear reasonable for this model and have the effect of making this system more sensitive to a change in the Peclet number.

CONCLUSIONS

For the activated sludge process with a large dimensionless saturation constant, the tower fermentor with its multi-stage design can metabolize more substrate than a single stage aeration vessel of the same volume. The most efficient design of the tower-type activated sludge system has single feed and sludge recycle streams to the bottom tower stage and minimizes the backflow between tower stages. With a cocurrent tower fermentor, the sedimentation of organisms is advantageous because it reduces the loss of organisms from the tower. Ignoring the effect of organism sedimentation gives a conservative estimate of the required volume of the tower system.

NOMENCLATURE

a	constant in settling velocity equation
b	constant in settling velocity equation
b_n	dimensionless backflow rate from stage n
C_A	concentration of species A
C_1, C_2	constants in organism concentration profiles
D	diameter of stage, m
d_n	dimensionless flow rate from stage n
d_n^o	dimensionless flow rate to stage n
d_s	dimensionless flow rate due to settling organisms
E	dispersion coefficient
f_n	volumetric backflow rate from stage n , m^3/hr
K	dimensionless saturation constant
K_s	saturation constant, mg/l
k	dimensionless endogenous organism attrition rate
k_d	specific endogenous organism attrition rate, hr^{-1}
L	height of stage, m
l	normalized vertical coordinate
m, m_1, m_2	roots of characteristic equation
Pe	Peclet number
q_n	volumetric flow rate from stage n , m^3/hr
q_n^o	volumetric flow rate to stage n , m^3/hr
r_A	rate of appearance of species A by reaction
S_n	substrate concentration in stage n , mg/l
t	time, hr
U	average velocity of fluid

U_n	average velocity of fluid in stage n, m/hr
U_s	settling velocity of organisms, m/hr
v	dimensionless volume of tower stage n
x_n	organism concentration in stage n, mg/l
x_n^o	organism concentration entering stage n, mg/l
$\langle x_n \rangle$	average value of organism concentration in stage n, mg/l
y_n	dimensionless organism concentration in stage n
\bar{y}_n	dimensionless organism concentration at top of stage n
\underline{y}_n	dimensionless organism concentration at bottom of stage n
y_n^o	dimensionless organism concentration entering stage n
$\langle y_n \rangle$	average value of dimensionless organism concentration in stage n
z_n	dimensionless substrate concentration in stage n
α	constant in linearized dispersion equation
β	constant in linearized dispersion equation
γ	constant in linearized dispersion equation
δ_n	parameter relating \bar{y}_n to y_n
ϵ_n	parameter relating \underline{y}_n to y_n
ζ	vertical coordinate
μ_{\max}	maximum specific growth rate of organisms, hr ⁻¹

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Table 1. Settling velocity parameters.

$$a = 6.42 \text{ m/hr}$$

$$b = 0.000379 \frac{\ell}{\text{mg}}$$

Table 2. Constants for equations (15) to (18).

$$\text{Let } \phi = \frac{d_s \langle y_n \rangle}{16 v_n}$$

$$\text{I. } y_n(\ell) = e^{m_1 \ell} (c_1 \cos m_2 \ell + c_2 \sin m_2 \ell) \quad (15)$$

$$m_1 = \frac{\beta}{2\alpha} \quad m_2 = \frac{\sqrt{4\alpha\gamma - \beta^2}}{2\alpha}$$

$$\text{Let } \theta = (m_1 \sin m_2 - m_2 \cos m_2) e^{m_1} + m_2$$

$$c_1 = \frac{(m_1^2 + m_2^2) \langle y_n \rangle + \frac{Pe}{m_2} \frac{d_n^0}{d_n} y_n^0 \theta}{(m_1 \cos m_2 + m_2 \sin m_2) e^{m_1} - m_1 + \frac{\theta}{m_2} \left[\frac{Pe}{d_n} (d_n^0 - \phi) - m_1 \right]}$$

$$c_2 = \frac{\frac{Pe}{d_n} [d_n^0 (c_1 - y_n^0) - c_1 \phi] - c_1 m_1}{m_2}$$

$$\text{II. } y_n(\ell) = c_1 e^{m\ell} + c_2 \ell e^{m\ell} \quad (16)$$

$$m = \frac{\beta}{2\alpha}$$

$$c_1 = \frac{m \langle y_n \rangle + Pe y_n^0 \frac{d_n^0}{d_n} \left(e^m - \frac{e^m}{m} + \frac{1}{m} \right)}{e^m - 1 + \left(e^m - \frac{e^m}{m} + \frac{1}{m} \right) \left[\frac{Pe}{d_n} (d_n^0 - \phi) - m \right]}$$

$$c_2 = \frac{Pe}{d_n} [d_n^0 (c_1 - y_n^0) - c_1 \phi] - c_1 m$$

Table 2. Continued

$$\text{III. } y_n(\ell) = c_1 e^{m_1 \ell} + c_2 e^{m_2 \ell} \quad (17)$$

$$m_1 = \frac{\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad m_2 = \frac{\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$

$$c_1 = \frac{(b_n - \frac{d_n m_2}{Pe}) \langle y_n \rangle - (\frac{e^{m_2} - 1}{m_2}) d_n^o y_n^o}{(b_n - \frac{d_n m_2}{Pe}) (\frac{e^{m_1} - 1}{m_1}) - (b_n - \frac{d_n m_2}{Pe}) (\frac{e^{m_2} - 1}{m_2})}$$

$$c_2 = \frac{d_n^o y_n^o - c_1 (b_n - \frac{d_n m_1}{Pe})}{b_n - \frac{d_n m_1}{Pe}}$$

$$\text{IV. } y_n(\ell) = \frac{c_1}{m} e^{m\ell} + c_2 \quad (18)$$

$$m = \frac{\beta}{\alpha}$$

$$c_1 = \frac{(d_n^o - \phi) \langle y_n \rangle d_n^o y_n^o}{(\frac{d_n^o - \phi}{m^2}) (e^m - 1 - m) + \frac{d_n}{Pe}}$$

$$c_2 = \frac{c_1 [\frac{1}{m} (d_n^o - \phi) - \frac{d_n}{Pe}] - d_n^o y_n^o}{\phi - d_n^o}$$

Table 3. Optimal designs for two tower stages.

		Peclet Number		
		<u>0</u>	<u>0.5</u>	<u>1.0</u>
I. Flow rates				
	d_1	1.000	1.000	1.000
	d_2	1.285	1.165	1.244
	d_3	1.285	1.165	1.244
	d_5	0.988	0.989	0.988
	d_6	0.012	0.011	0.012
	b_3	0.000	0.000	0.000
II. Cell concentrations				
	y_1	0.000	0.00	0.00
	y_2	14.4	16.0	20.6
	y_3	14.5	16.1	20.8
	y_5	0.117	0.120	0.149
	y_6	62.4	72.1	59.91
III. Substrate concentrations				
	z_1	1.000	1.000	1.000
	z_2	0.197	0.208	0.178
	z_3	0.037	0.037	0.029
	z_5	0.037	0.037	0.029
	z_6	0.037	0.037	0.029

Table 3. Continued

	Peclet Number		
	<u>0</u>	<u>0.5</u>	<u>1.0</u>
IV. Structural parameters			
α_{21}	1.00	1.00	1.00
α_{24}	0.959	0.940	0.952
α_{34}	0.00	0.00	0.00
V. Sedimentation parameters			
δ_2	1.00	0.678	0.600
δ_3	1.00	0.682	0.600
ϵ_3	1.00	0.732	0.714
VI. Volumes			
v	0.141	0.125	0.109
a	0.298	0.245	0.236
V_T	0.580	0.494	0.455

Table 4. Optimal designs for three tower stages.

		Peclet Number		
		<u>0</u>	<u>0.5</u>	<u>1.0</u>
I. Flow rates				
	d_1	1.000	1.000	1.000
	d_2	1.323	1.100	1.121
	d_3	1.323	1.100	1.121
	d_4	1.323	1.100	1.121
	d_6	0.986	0.990	0.988
	d_7	0.014	0.010	0.012
	b_3	0.000	0.000	0.000
	b_4	0.000	0.000	0.000
II. Cell concentrations				
	y_1	0.00	0.00	0.00
	y_2	13.8	14.9	20.7
	y_3	14.0	15.4	20.8
	y_4	14.0	15.5	20.8
	y_6	0.148	0.141	0.200
	y_7	54.8	77.8	61.0
III. Substrate concentrations				
	z_1	1.000	1.000	1.000
	z_2	0.322	0.386	0.305
	z_3	0.104	0.119	0.076
	z_4	0.030	0.031	0.017

Table 4. Continued

	Peclet Number		
	<u>0</u>	<u>0.5</u>	<u>1.0</u>
III. Substrate concentrations (continued)			
z_6	0.030	0.030	0.017
z_7	0.030	0.031	0.017
IV. Structural parameters			
α_{21}	1.00	1.00	1.00
α_{31}	0.00	0.00	0.00
α_{25}	0.959	0.911	0.913
α_{35}	0.00	0.00	0.00
α_{45}	0.00	0.00	0.00
V. Sedimentation parameters			
δ_2	1.000	0.508	0.345
δ_3	1.00	0.507	0.353
δ_4	1.00	0.510	0.355
ε_3	1.00	0.667	0.498
ε_4	1.00	0.668	0.498
VI. Volumes			
v	0.0861	0.0722	0.0669
a	0.262	0.183	0.128
V_T	0.521	0.400	0.329

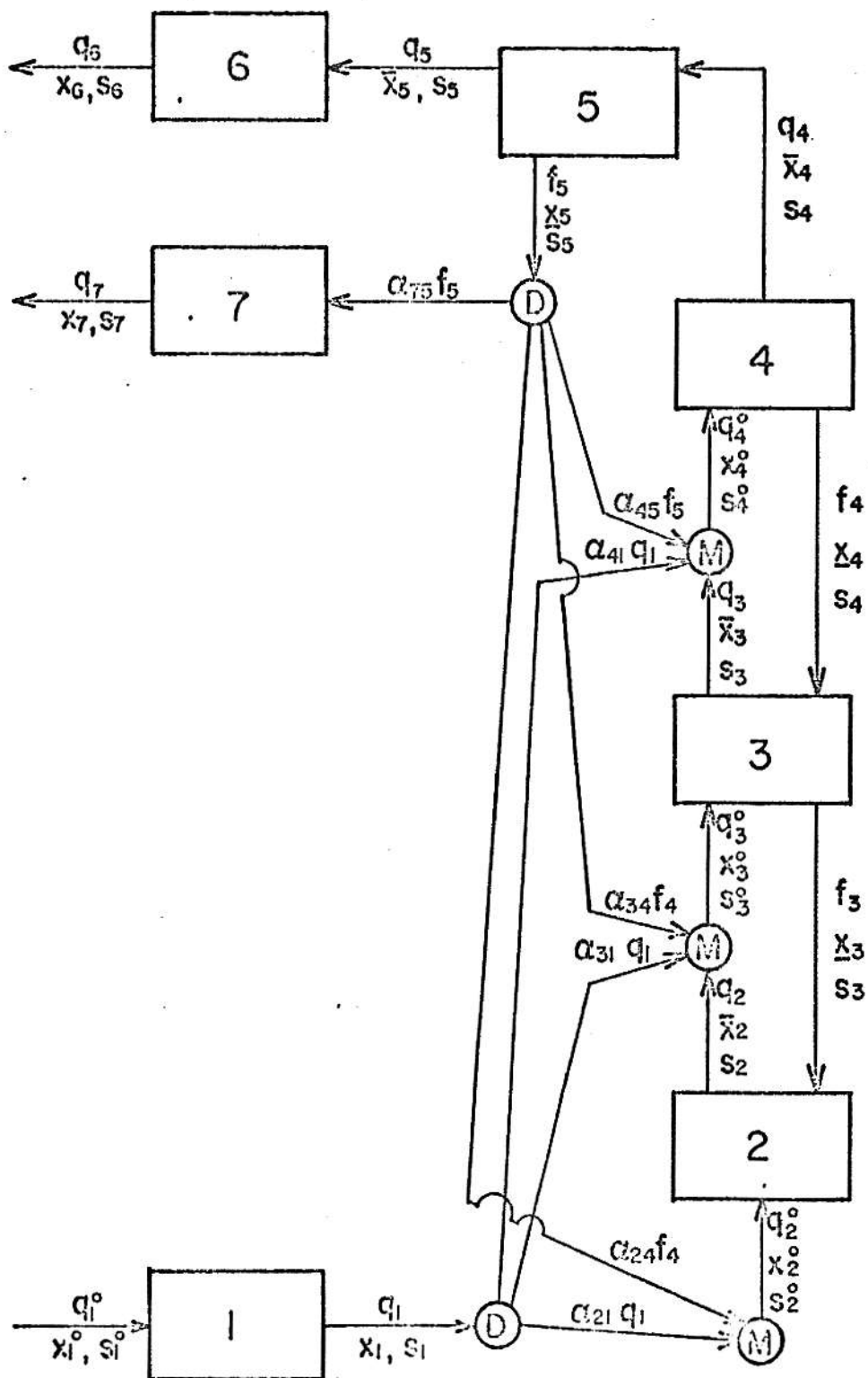


Figure 1. System Model With Three Tower Stages.

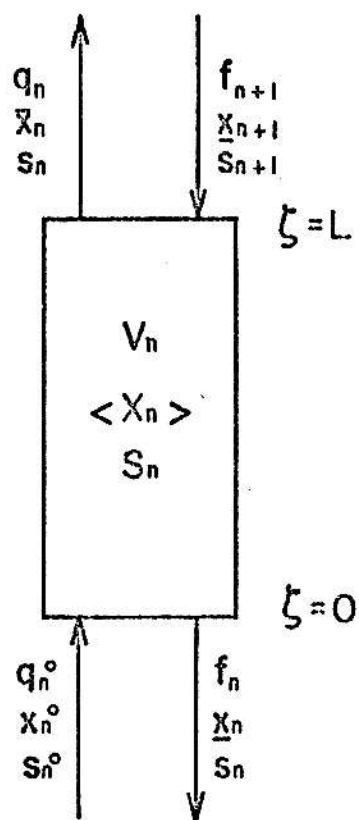


Figure 2. Typical Tower Fermentor Stage.

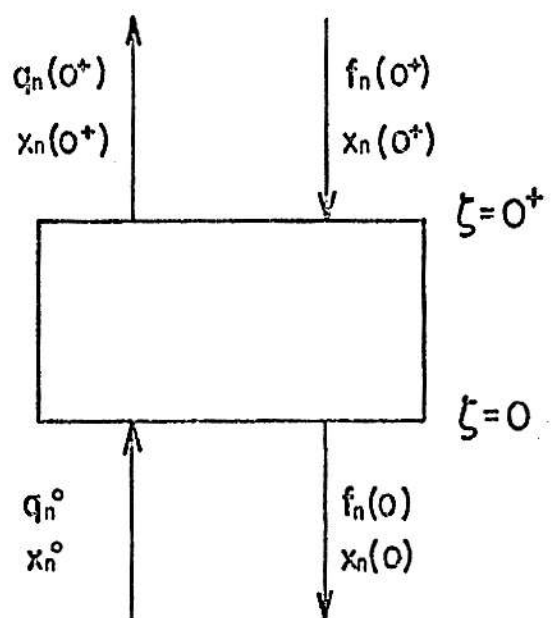


Figure 3. Entrance Region to Tower Fermentor Stage.

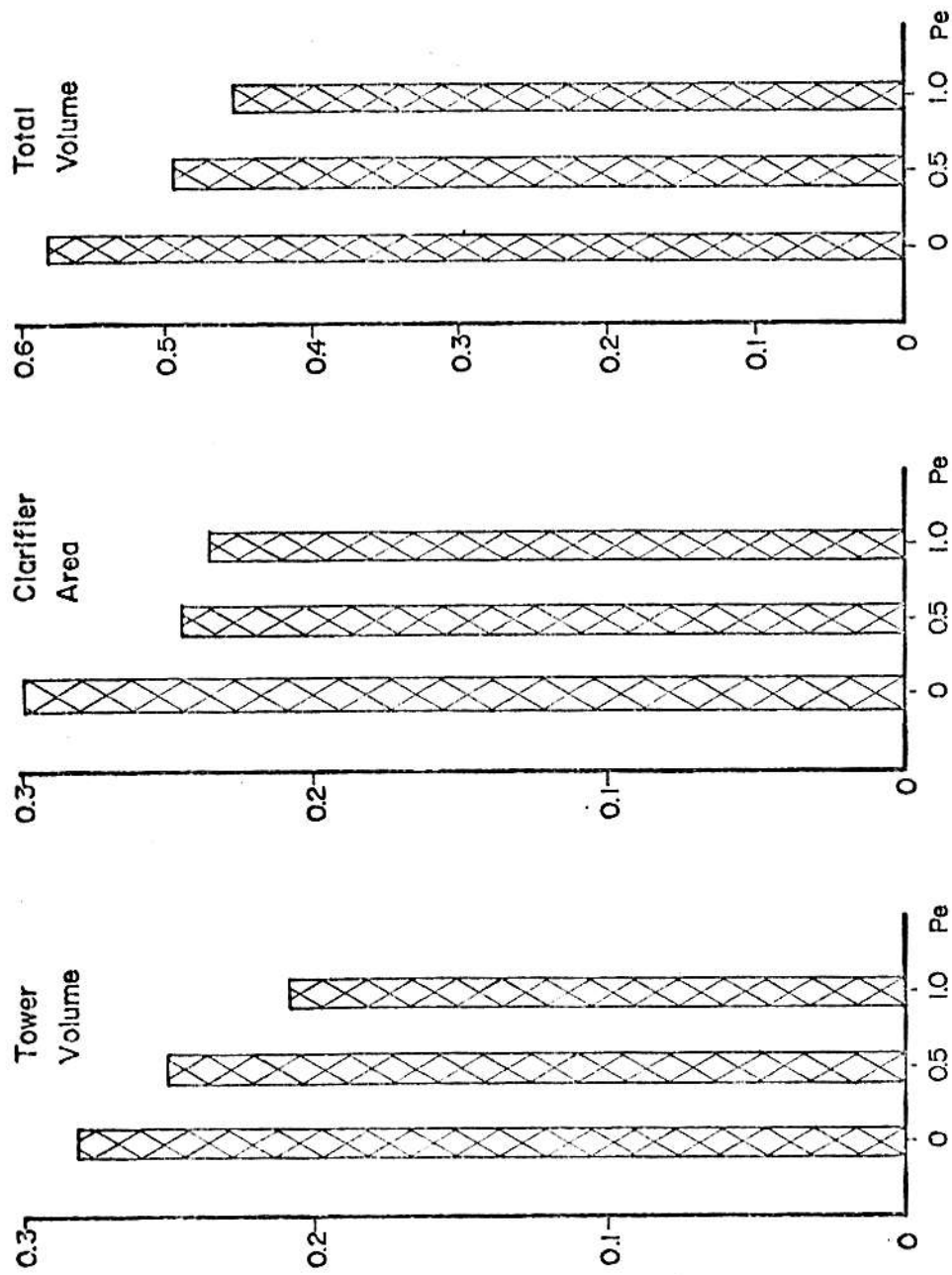


Figure 4. Effect of the Peclet Number on the Optimal Designs of Systems with Two Tower Stages.

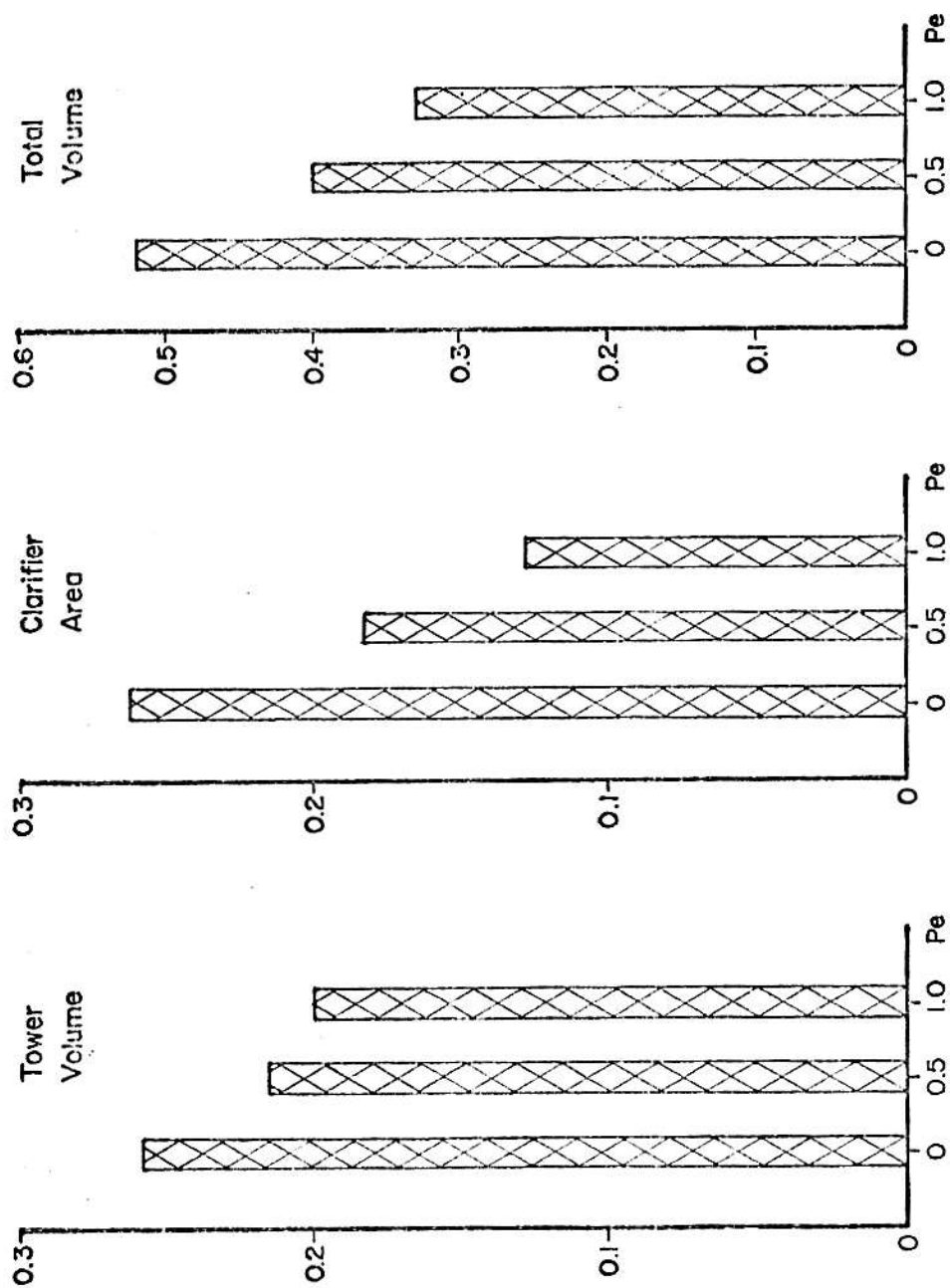


Figure 5. Effect of the Peclet Number on the Optimal Designs of Systems with Three Tower Stages.

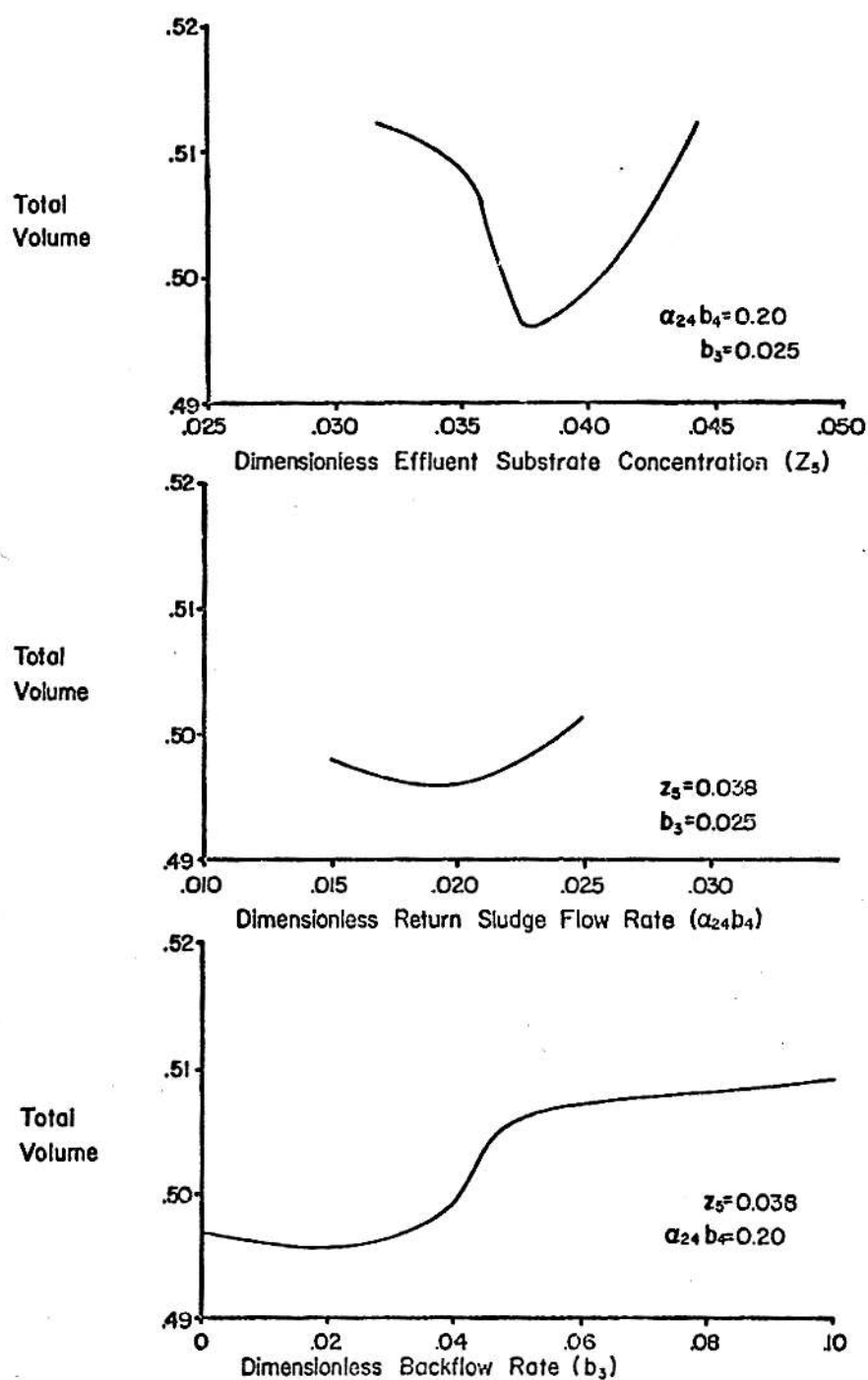


Figure 6. Simulation Near Optimum Point for Two Tower Stages and $Pe = 0.5$.

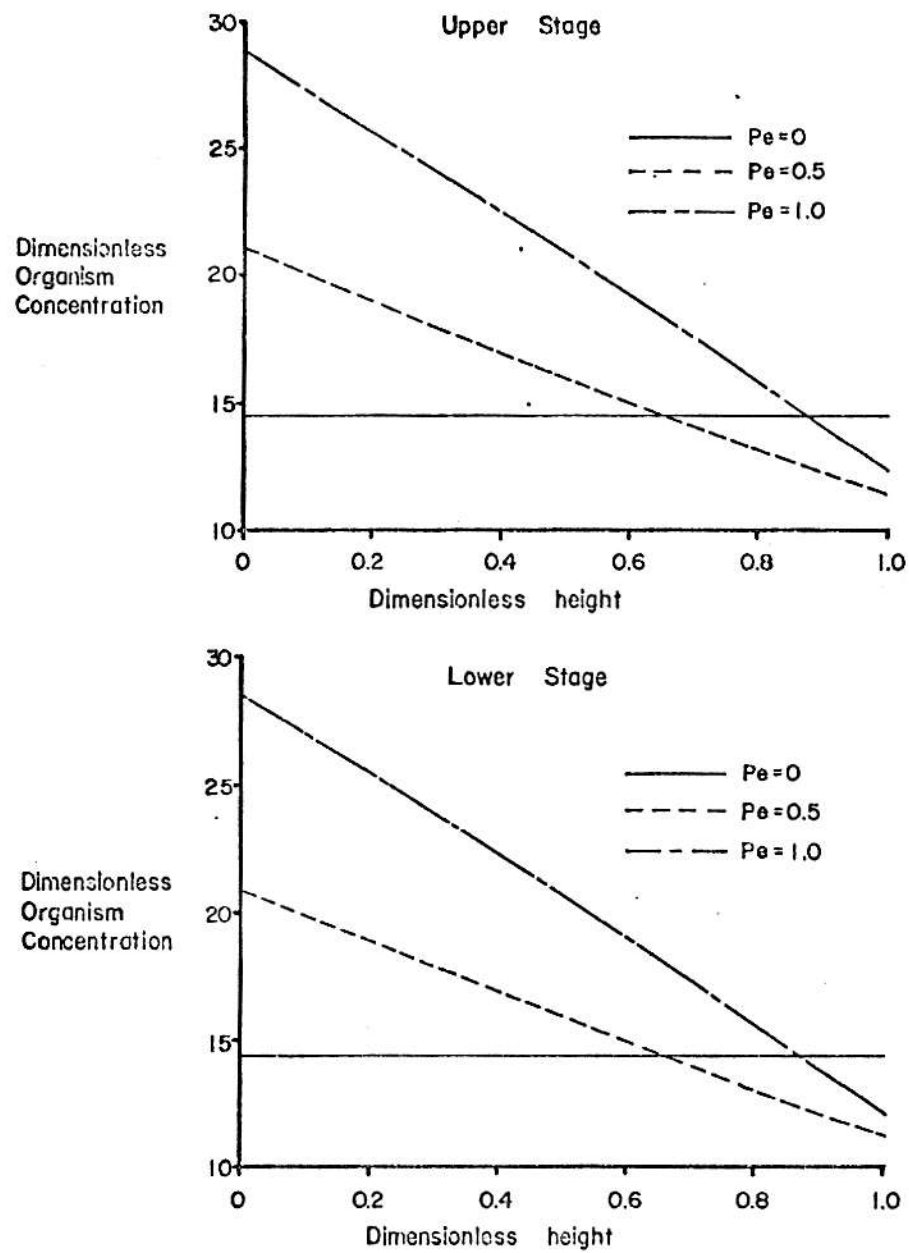


Figure 7. Predicted Organism Concentration Profiles at Optimum Points For Two Tower Stages.

Chapter V

FUTURE WORK

The results presented in Chapters III and IV were for fixed feed conditions and fixed values of the parameters in the kinetic expressions, sedimentation equations, clarifier performance equations, and the effluent quality constraint. The sensitivity of the system design to these parameters and the stability of the system under varying feed conditions have not been considered. Sensitivity studies are very helpful in selecting design safety factors to incorporate into the design equations. The development of optimal operating conditions for the oversized system should also be investigated. The stability of operation, which usually improves with oversize, should also be examined.

The model of the tower fermentor needs to be improved. The present model does not consider the gas phase. Oxygen transfer from the gas to the liquid phase should be included so that the minimum air flow rate can be calculated. Correlations between the gas flow rate and degree of mixing, gas hold-up and backflow rate are needed. If experimental data existed, the nonlinear dispersion model equation could be solved numerically to fit the parameters to the data. These parameters could then be used in the approximate solution used for design purposes.

An alternate approximate solution to the dispersion model for organism sedimentation developed in Chapter IV is presented here. The dispersion model equation is given by:

$$E \frac{d^2 X_n}{dz^2} - [U_n - U_s(X_n)] \frac{dX_n}{dz} + \left(\frac{\mu_{\max} S_n}{K_s + S_n} - k_d \right) X_n = 0 \quad (1)$$

In most cases, the reaction term is the smallest contribution. Therefore, fixing the reaction term at a constant value will lead to less error than fixing $U_s(X_n)$ at a constant value. The reaction term is approximated by:

$$R_n = \left(\frac{\mu_{\max} S_n}{K_s + S_n} - k_d \right) \langle X_n \rangle \quad (2)$$

where

$$\langle X_n \rangle \equiv \frac{1}{L} \int_0^L X_n d\zeta$$

As in Chapter IV, the settling velocity is given by:

$$U_s(X_n) = ae^{-bX_n} \quad (3)$$

Substituting these expressions into equation (1) gives:

$$E \frac{d^2 X_n}{d\zeta^2} - (U_n - ae^{-bX_n}) \frac{dX_n}{d\zeta} + R_n = 0 \quad (4)$$

This equation may be integrated to yield:

$$E \frac{dX_n}{d\zeta} - U_n X_n - \frac{a}{b} e^{-bX_n} + R_n \zeta = C_1 \quad (5)$$

In order to integrate this equation, the exponential is linearized using a truncated Taylor series expansion about the average value, $\langle X_n \rangle$.

$$e^{-bX_n} \doteq p_1 X_n + p_2 \quad (6)$$

where

$$p_1 = -be^{-b\langle X_n \rangle}$$

$$p_2 = e^{-b\langle X_n \rangle} (1 + b\langle X_n \rangle)$$

Substituting this expression into equation (4) and integrating yields the following equation:

$$X_n = -\frac{1}{\theta E} \left(C_1 + \frac{ap_2}{b} - \frac{R_n}{\theta} - R_n \zeta \right) + C_2 e^{\theta \zeta} \quad (7)$$

where

$$\theta = \left(U_n + \frac{ap_1}{b} \right) / E$$

For the special case when θ approaches zero, the predicted organism profile is given by:

$$X_n = \frac{1}{E} \left(C_1 + \frac{ap_2}{b} \right) \zeta - \frac{R_n}{2} \zeta^2 + C_2 \quad (8)$$

The two integration constants, C_1 and C_2 , are found from the boundary conditions. From organism balances around the entrance and exit regions of the tower stage, the following two boundary conditions can be developed:

$$E \frac{dX_n}{d\zeta} \bigg|_0 = \frac{4q_n^0 [X_n(0) - X_n^0]}{D^2} - [U_s X_n]_0 \quad (9)$$

$$E \frac{dX_n}{d\zeta} \bigg|_L = \frac{4f_{n+1} [X_{n+1} - X_n(L)]}{D^2} - [U_s X_n]_L \quad (10)$$

where $\zeta = 0$ is the bottom of the stage and $\zeta = L$ is the top. Application of these two boundary conditions leads to two non-linear equations to solve for C_1 and C_2 . Both equations are of the following form:

$$g_{i1} C_1^2 + g_{i2} C_1 + g_{i3} C_1 C_2 + g_{i4} C_2 + g_{i5} C_2^2 = g_{i6} \quad i = 1, 2 \quad (11)$$

where g_{ij} is a constant. After determining the constants, the appropriate organism concentration profile, given by equation (7) or (8), can be used to estimate the organism sedimentation parameters discussed in Chapter IV.

Using this approach, the average organism concentration, $\langle X_n \rangle$, changes along with \bar{X}_n and X_n .

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MODELING AND OPTIMIZATION OF A
TOWER-TYPE ACTIVATED SLUDGE SYSTEM

by

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ABSTRACT

The optimal design of a wastewater treatment system consisting of a tower fermentor for aeration and a secondary clarifier is studied. A new secondary clarifier model which describes both clarification and sludge thickening is developed. The optimal design and optimal operation of an activated sludge system composed of a completely mixed aeration vessel and a secondary clarifier is investigated using the new clarifier model. With the new model, there is interaction between the aerator and clarifier designs. The importance of this interaction is illustrated by comparing the optimal designs obtained for different values of the parameters in the equation describing the sludge thickening.

The modeling of a tower-type activated sludge system with one, two and three tower stages is considered using the new clarifier model. Application of the structural parameter method of system synthesis allows the distribution of the feed and sludge recycle streams to the tower stages to be optimized. For complete mixing within the tower stages, increasing the number of tower stages decreases the clarifier, fermentor and total volumes of the system.

A one-dimensional dispersion model is developed for predicting the sedimentation of organisms within a tower stage and is used to calculate two parameters. These two parameters are used to estimate the organism concentrations in the exit streams from a tower stage. Using the Peclet number as a parameter, it is shown that increasing the amount of sedimentation decreases the clarifier, fermentor and total volumes of the systems at the optimal designs.