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CHARACTERISTICS OF A VELOCITY SELECTOR USED WITH A TANDEM VAN DE GRAAFF ACCELERATOR

by 1264

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INTRODUCTION

This paper discusses the uses and limitations of velocity selector systems for tandem Van de Graaff accelerators used with heavy ions. In the usual "tandem" accelerator, which is described below, it is often not possible to select a given final mass and energy from several that may exist, or to separate a desired beam from contamination which may be present. As will be shown, these problems can be termed energy resolution, mass resolution, and contamination removal. It will also be shown that a velocity selector can be useful when placed either before or after the accelerator. The trajectory equations for the beam line elements are presented in the Penner formulism. The trajectory equations for the velocity selector show that the element does not effect the primary beam desired by the experimentalist. There are no known commercial vendors for a velocity selector. However, the design need not be elaborate, and a suitable device could be constructed by the personnel at Kansas State University.

First it is necessary to describe the operation of a tandem Van de Graaff accelerator. Plate I shows a schematic diagram of the usual tandem Van de Graaff system. Negative ions are produced in the ion source. These ions leave the ion source with energies usually of fifty to one hundred fifty keV. Due to the techniques involved, electrons are also emitted. The beam is passed through an inflection magnet so that the beam is deflected by 30°. The electrons are bent much more sharply than the desired ions and are eliminated from the beam as they collide with the walls of the beam tube. The negative ions are accelerated to the positively charged terminal. They acquire an energy of q^-V where q^- is the value of the charge on the ion and V is the potential of the terminal above ground. Once inside the stripping canal, a

EXPLANATION OF PLATE I

Plate I A schematic drawing showing the major elements of a tandem Van de Graaff accelerator system.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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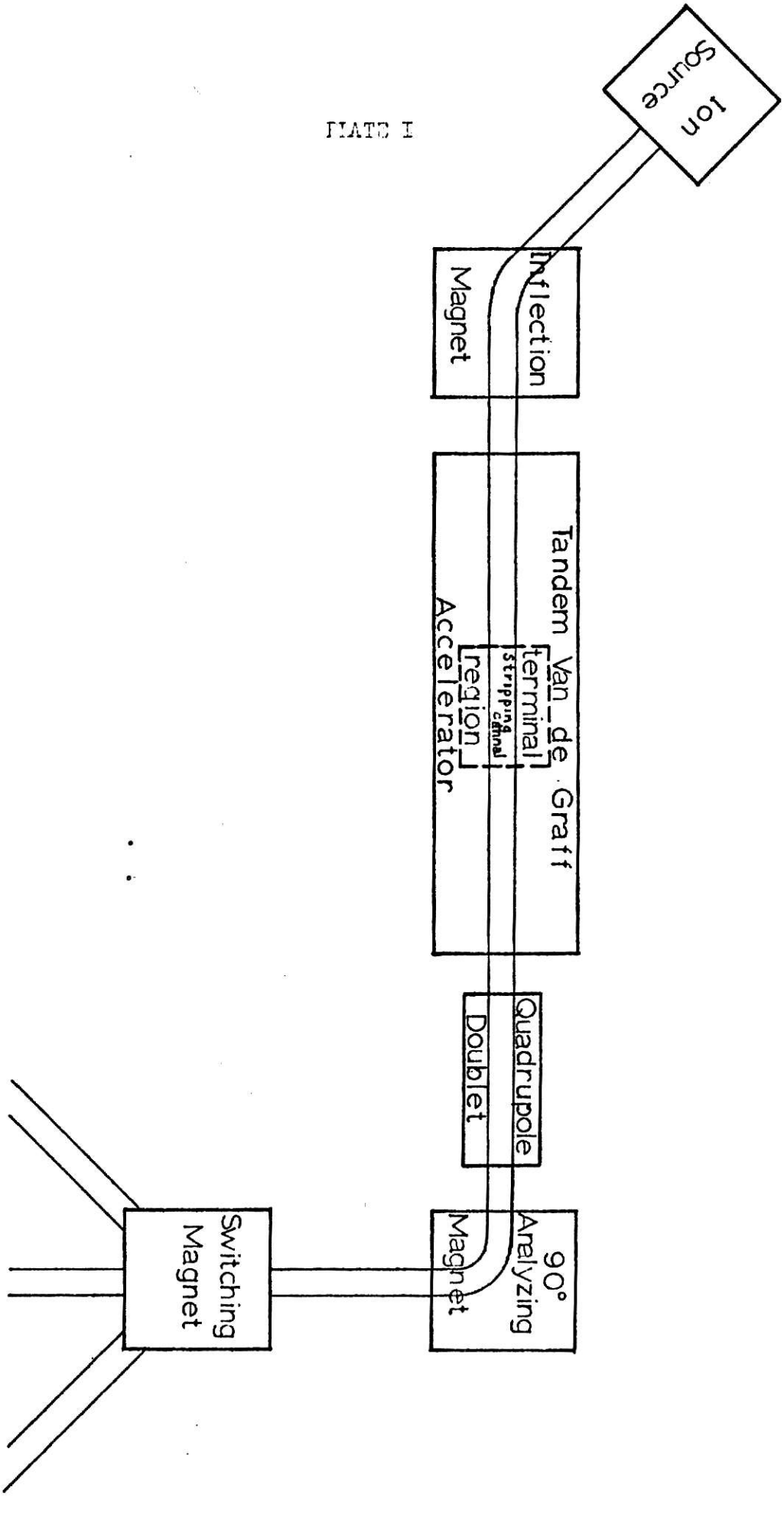
PLATE I


Ion Source
Inflection Magnet
Tandem Van de Graff
terminal
Stripping element
region
Accelerator
Quadrupole Doublet
90° Analyzing Magnet
Switching Magnet

gas[1] (oxygen or some other appropriate gas) is introduced here to serve as a medium of electron exchange. Some of the negative ions lose electrons as they pass through the stripping canal and become positively charged ions. These positive ions are then accelerated away from the positive terminal. They acquire an additional energy of q^+V. The total energy of the particles then is $(q^- + q^+)V$, where q^+ is the charge state during the high energy acceleration.[2] Plate II shows the relation of the final charge state to the total energy during acceleration for the usual case of $q^- = -1$.

The beam is next focused by a magnetic lens which consists of a magnetic quadrupole doublet. Following the lens the beam is deflected by the analyzing magnet. The magnetic field and the radius of curvature of the ion in the field (as set by entrance and exit slits) determine the ratio of the momentum to the charge of the ion. Thus, for one particular charge state the analyzing magnet determines the energy of the beam. Finally, the beam passes through a switching magnet which selects the experimental beam tube.

The "tandem" system has been used principally to accelerate protons to energies of ten to fifteen MeV. Interest is developing in acceleration of heavier ions using tandem techniques. Initial attention has been directed to the development of ion sources for producing negative ions of heavy atoms.

Once the negative ion is produced, one must consider various aspects of energy and mass selection. When the negative ion reaches the stripping canal and is changed to a positive ion, many different charge states are produced, depending upon how many electrons are removed. (The distribution

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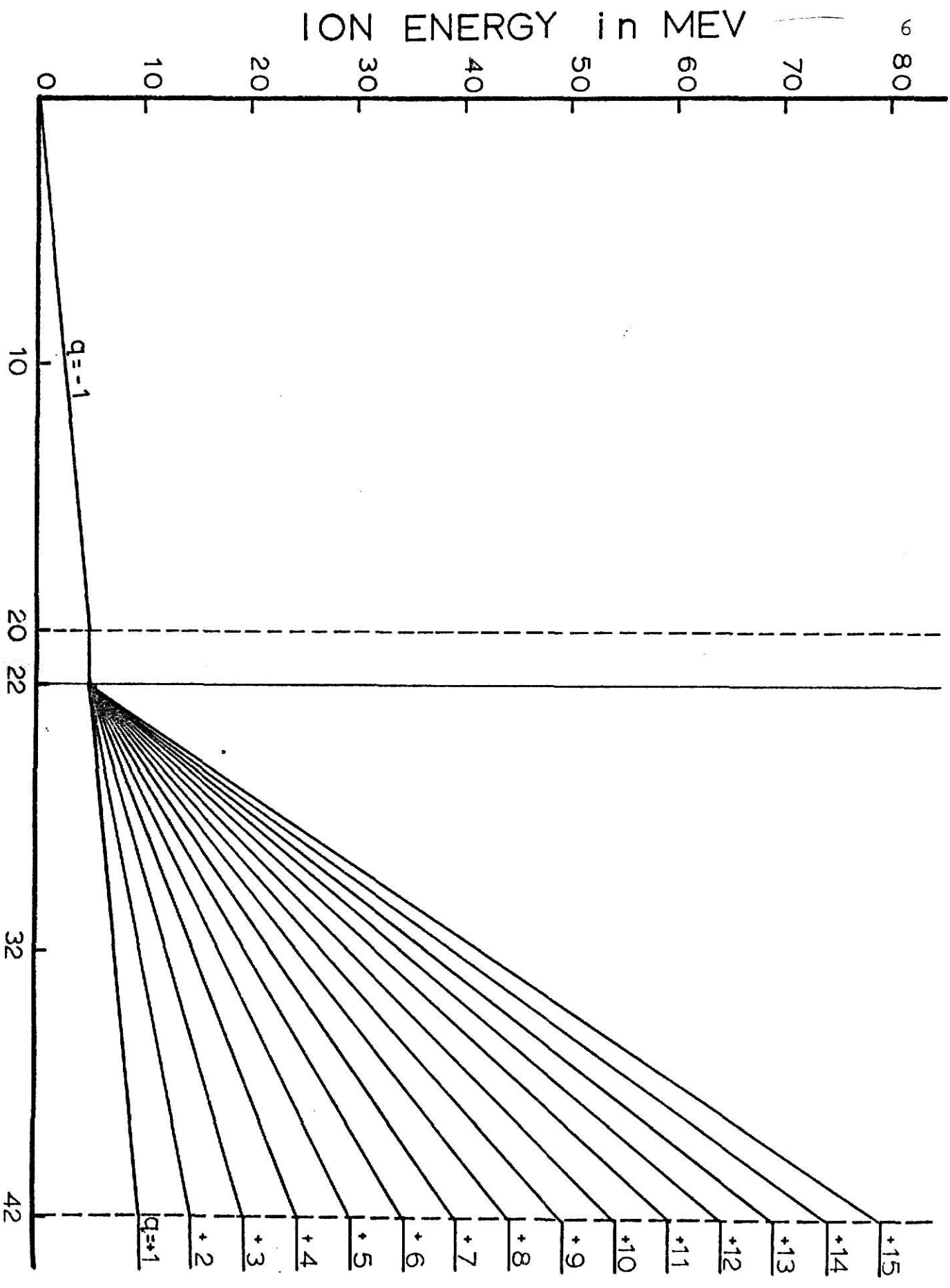
1. In some systems a thin foil is used instead of a gas.

2. The energy may be known under normal conditions to about 10 keV.

EXPLANATION OF PLATE II

Plate II Energy vs. Distance in the tandem Van de Graaff for several exiting charge states. The plate shows a terminal voltage of 5 MV and an incident charge state of -1.


ION ENERGY in MEV
0
10
20
30
40
50
60
70
80
10
20
22
32
42
q=-1
q=+1
+2
+3
+4
+5
+6
+7
+8
+9
+10
+11
+12
+13
+14
+15

of charge states that results is discussed in the next chapter.) Thus different final energies are obtained, as can be seen in Plate II. However, only one energy passes through the analyzing magnet. The magnet selects $(Br)^2 = ME/q^2$ (the momentum-to-charge ratio squared) where B is the magnetic field, r the radius, M the mass of the ion, E the energy, and q the charge. The energy E is given by $E = (q + 1)V$ so that the magnet selects $MV(q + 1)/q^2$ which can be satisfied by only one charge state.

However, the phenomenon of "continuous stripping," in which the charge of the ion is changed in collisions with residual gas atoms during acceleration in the high energy accelerating tube, leads to a range of energies for each final charge state. Thus several energies are passed by the analyzing magnet.

Further, if one is accelerating a beam of some element, in which there are several naturally occurring isotopes, the difference in $(Br)^2$ may be very small and not differentiated by the analyzing magnet. For example, isotopes of nickel -58 and nickel -60 at charge states of 19 and 20 yield $(Br)^2 = 3.21$ and 3.15.

Another consideration is that the output of an ion source is often a function of its history, and different species of ions may be added to the energy spectrum to give another dimension to the problem of mass resolution.

All of these problems can be eliminated or reduced by the use of a velocity selector. Energy selection problems from continuous stripping can be solved by placing a velocity selector at some point after the accelerator, either before or after the analyzing magnet. Mass resolution and removal of contaminants can be improved by placing a velocity selector either before or after the accelerator. This thesis discusses these cases and the design of a

velocity selector or Wien filter,[3] consisting of crossed electric and magnetic fields.

Since one is concerned with calculations of ion orbits in the tandem accelerator system in order to optimize focusing conditions, it is relevant to consider the ion trajectory in a velocity selector. A chapter has been included which discusses the ion trajectories in a velocity selector.

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3. L. Wahlin, Nucl. Instr. and Meth., 27(1964)55

ENERGY RESOLUTION AND CHARGE EXCHANGE

When fast ions are passed through matter the process of electron loss and capture causes the charge on the ion to fluctuate. If the material is suitably thick, a dynamically stable charge distribution will be established.[4] This is the mechanism used to change the negative ions to positive in the stripping canal, and it is the desired effect. However, the presence of gases in the high energy acceleration tubes causes continuous stripping, an effect which is usually undesirable. It is termed continuous stripping because the charge state can change from stripping at any point during acceleration after the stripping canal and give a continuous energy spectrum.

The average charge on stripped heavy ions can be calculated with good results. One theory due to Betz[5] presents the empirical formula

$$\bar{\xi}/z = 1 - C \exp(-\delta \beta/\alpha)$$

where z is the atomic number, C and δ are parameters derived from measurement. β/α is called the specific velocity. ($\beta = v/c$, α = Fine structure constant.) The average charge $\bar{\xi}$ is dependent upon which ion is used as a projectile and its velocity, but not the initial or incident charge state.

This empirical formula describes the average charge state for a beam which has traversed enough material so that the beam is in a dynamic

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4. I. S. Dmitriev, and V. S. Nikolaev, Soviet Phys. JETP 20,(1965)409

5. H. -D. Betz, G. Hortig, E. Leischner, Ch. Schmelzer, B. Stadler, and J. Weihrauch, Physics Letters 22,(1966)643

equilibrium. The formula is also valid for projectiles in a field free region. The charge exchange processes in continuous stripping occur during the acceleration process where the specific velocity β/α is not constant.

Plate III shows an experimental nonequilibrium charge distribution for the sulfur ion.[6] The intensity of a given charge state is highly dependent upon the amount of material the beam has passed through. When the beam has passed 10^{18} atoms/cm^2, the intensity of each of the charge states has stabilized, and the beam has passed through the equilibrium thickness.

The distribution of residual particles in the acceleration tubes is difficult to measure. However, some data are available. There is a pressure gradient in the acceleration tubes with the highest pressure at the terminal. This occurs since in normal operation the vacuum is achieved in the acceleration tubes by operating diffusion pumps at the high and low energy ends of the accelerator.[7] There is a natural outgassing of the accelerator from the electrodes, the insulation, and the sealing material. This outgassing has been measured, and it is approximately 2×10^{-4} torr l/sec for a typical accelerator tube.[8]

This volume of material is in addition to that which may be introduced at the stripping canal. Under maximum gas flow the pressure in the terminal can be as high as 10^{-4} torr but more typically it is 5×10^{-6} torr.

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6. H.-D. Betz et al., op. cit.

7. F. Chmara, G. Ryding and J. Shaw, 'A Study of Vacuum Requirements for the Acceleration of Heavy Ions in an MP Tandem Van de Graaff,' High Voltage Engineering Corporation, Burlington, Massachusetts

8. H.-D. Betz et al., op. cit.

EXPLANATION OF PLATE III

Plate III "Experimental nonequilibrium charge distribution for S-ions, stripped in air at $\beta/\alpha = 6.2$."[9] The intensities of the different charge states vary considerably until an equilibrium distribution is produced.

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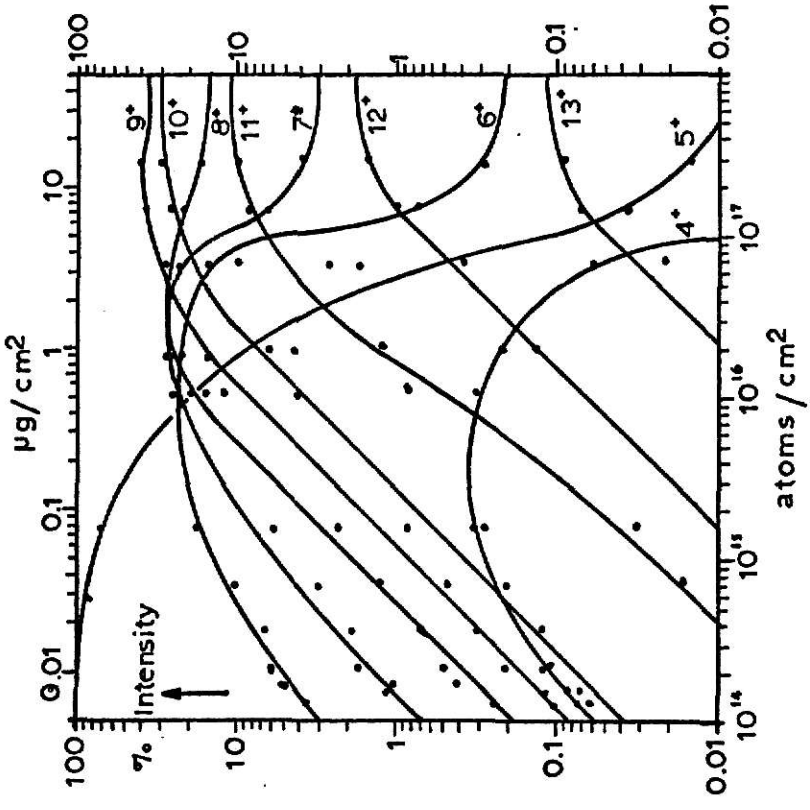
9. Ibid.


μg/cm²
atoms/cm²
Intensity
η₀
9⁺
10⁺
8⁺
11⁺
7⁺
12⁺
6⁺
13⁺
5⁺
4⁺

The operating pressure at the base of the high energy tubes is usually near 5×10^{-7} torr.[10]

Plates IV - VII show the calculated cross sections vs incident velocity for sulfur, arsenic, iodine, and uranium ions.[11] These cross sections are calculated assuming single electron loss or capture and that the equilibrium charge distribution is gaussian. These graphs show that the electron loss cross sections decrease rapidly with increased energy. The mean charge is also shown on the abscissa showing the relationship of mean charge to velocity. The mean charge of the equilibrium distribution increases with increasing velocities. Also, the uranium ions have higher mean charge values than sulfur at comparable velocities. The widths of the charge distributions will also be different, with more charge states available to the heavier ion. In the region $.2 \leq \bar{\xi} \leq .8$ the width of the charge distribution Γ has been experimentally found to be $\Gamma = k_i Z$ where $k_i = 0.57 \pm 0.02$.[12]

The charge distributions depend on the ion used as the projectile, the velocity of the projectile, and the thickness of the target. The equilibrium charge distribution does not depend on the initial charge state. In continuous stripping, the velocity of the projectile is not constant, and the amount of residual gas in the acceleration tubes is not constant along the length of the tubes. Consequently, charge distributions for continuously stripped heavy ions are not yet predicted.

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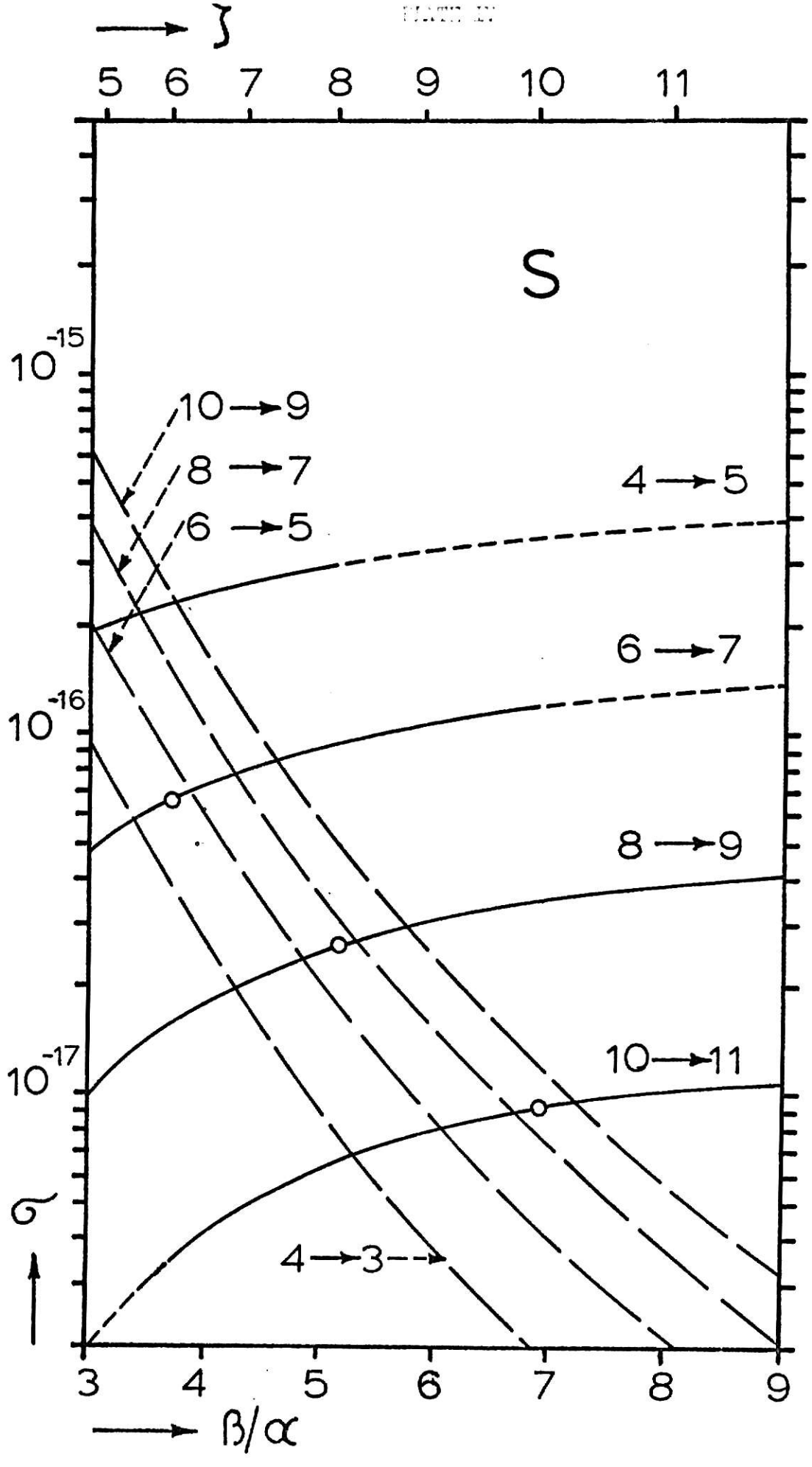
10. Ibid

11. H. -D. Betz, Ch. Schmelzer, UNILAC-Bericht N. 6-66

12. Ibid

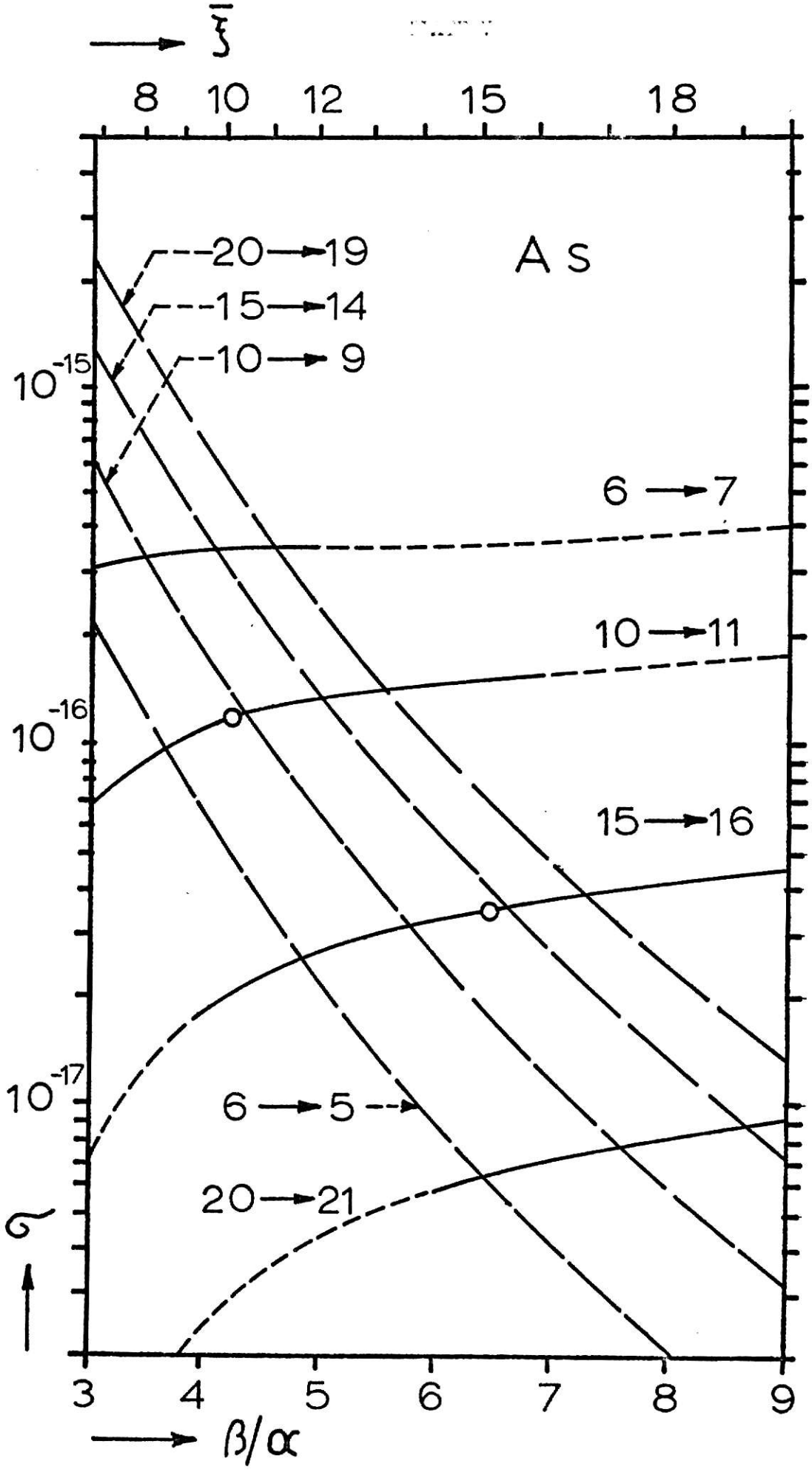
EXPLANATION OF PLATE IV

Plate IV Calculated electron loss and capture cross sections for sulfur.


J
5 6 7 8 9 10 11
S
10^{-15}
10^{-16}
10^{-17}
10→9
8→7
6→5
4→5
6→7
8→9
10→11
4→3
σ
3 4 5 6 7 8 9
β/α

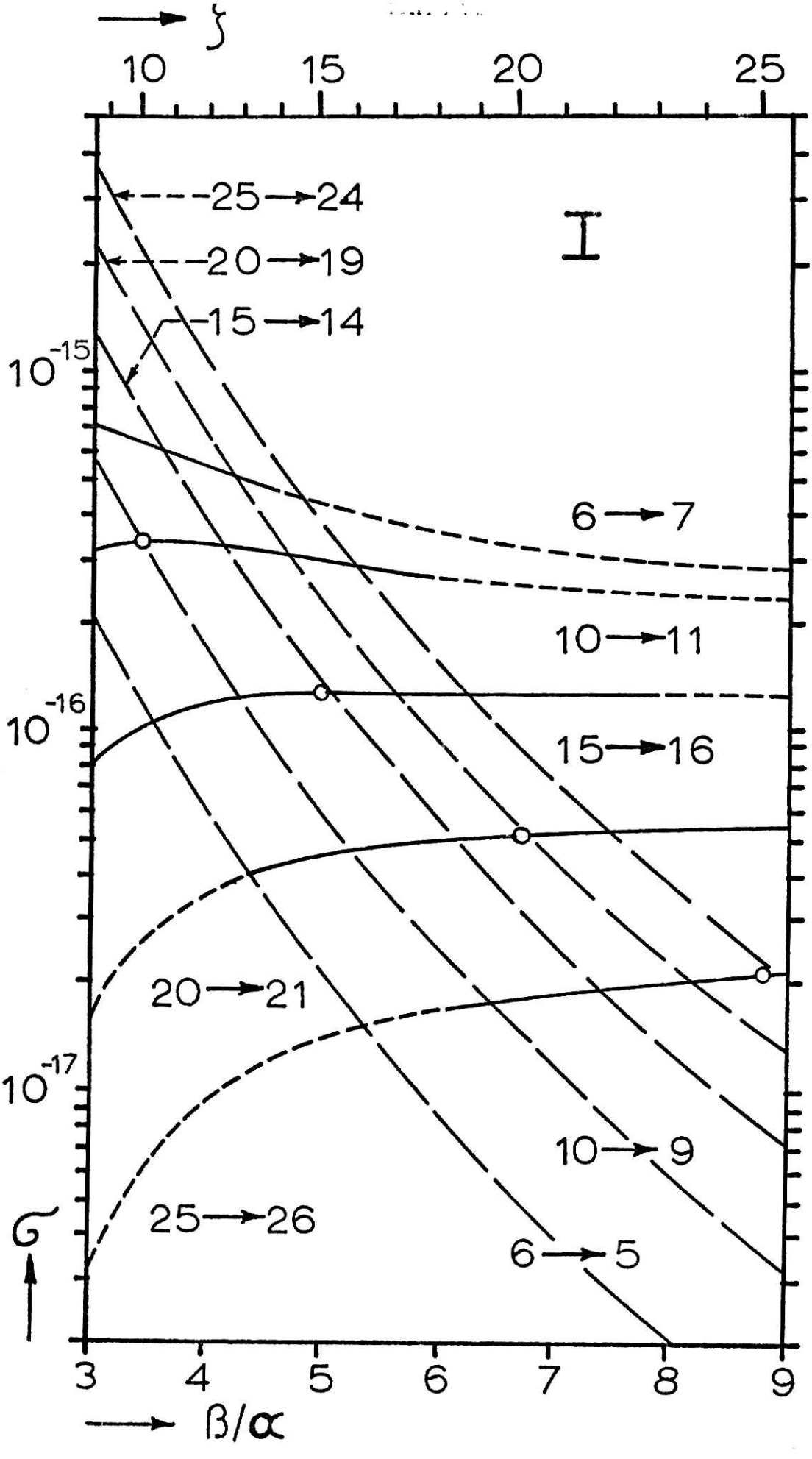
EXPLANATION OF PLATE V

Plate V Calculated electron loss and capture cross sections for arsenic.


ξ̄
8
10
12
15
18
As
20→19
15→14
10→9
6→7
10→11
15→16
6→5
20→21
10^-15
10^-16
10^-17
ζ
3
4
5
6
7
8
9
β/α

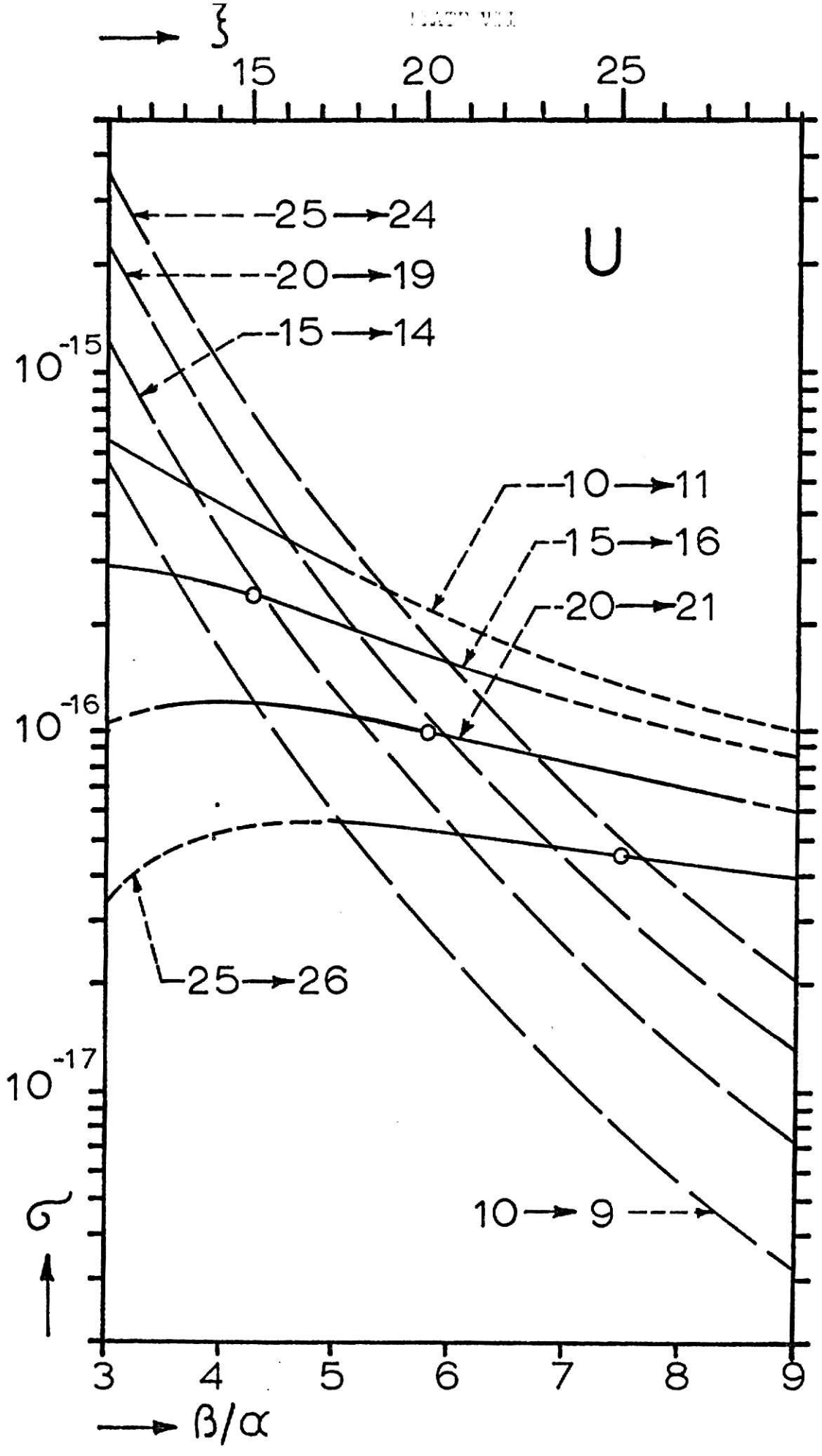
EXPLANATION OF PLATE VI

Plate VI Calculated electron loss and capture cross sections for iodine.


ζ
10
15
20
25
25→24
20→19
15→14
I
10^-15
6→7
10→11
10^-16
15→16
20→21
10^-17
10→9
25→26
6→5
σ
3
4
5
6
7
8
9
β/α

EXPLANATION OF PLATE VII

Plate VII Calculated electron loss and capture cross section for uranium.


ξ
15
20
25
U
25→24
20→19
15→14
10→11
15→16
20→21
25→26
10→9
10^-15
10^-16
10^-17
σ
3
4
5
6
7
8
9
β/α

When ions suffer charge exchange reactions during acceleration, the final charge distribution is affected. More importantly the energy spectrum is affected. Instead of a unique energy corresponding to each exiting charge state, there is an entire range of energies emerging from the Van de Graaff for each charge state. The analyzing magnet selects $(Br)^2 = ME/q^2$. It is possible for ions with many different energies, and corresponding charge states, to satisfy this constraint. Plate VIII shows the relation of ME/q^2 to the final charge state when the energy is given by $E = (q+1)V$. The arrows show the range of the mass energy products that are available when only one charge exchange occurs during acceleration.

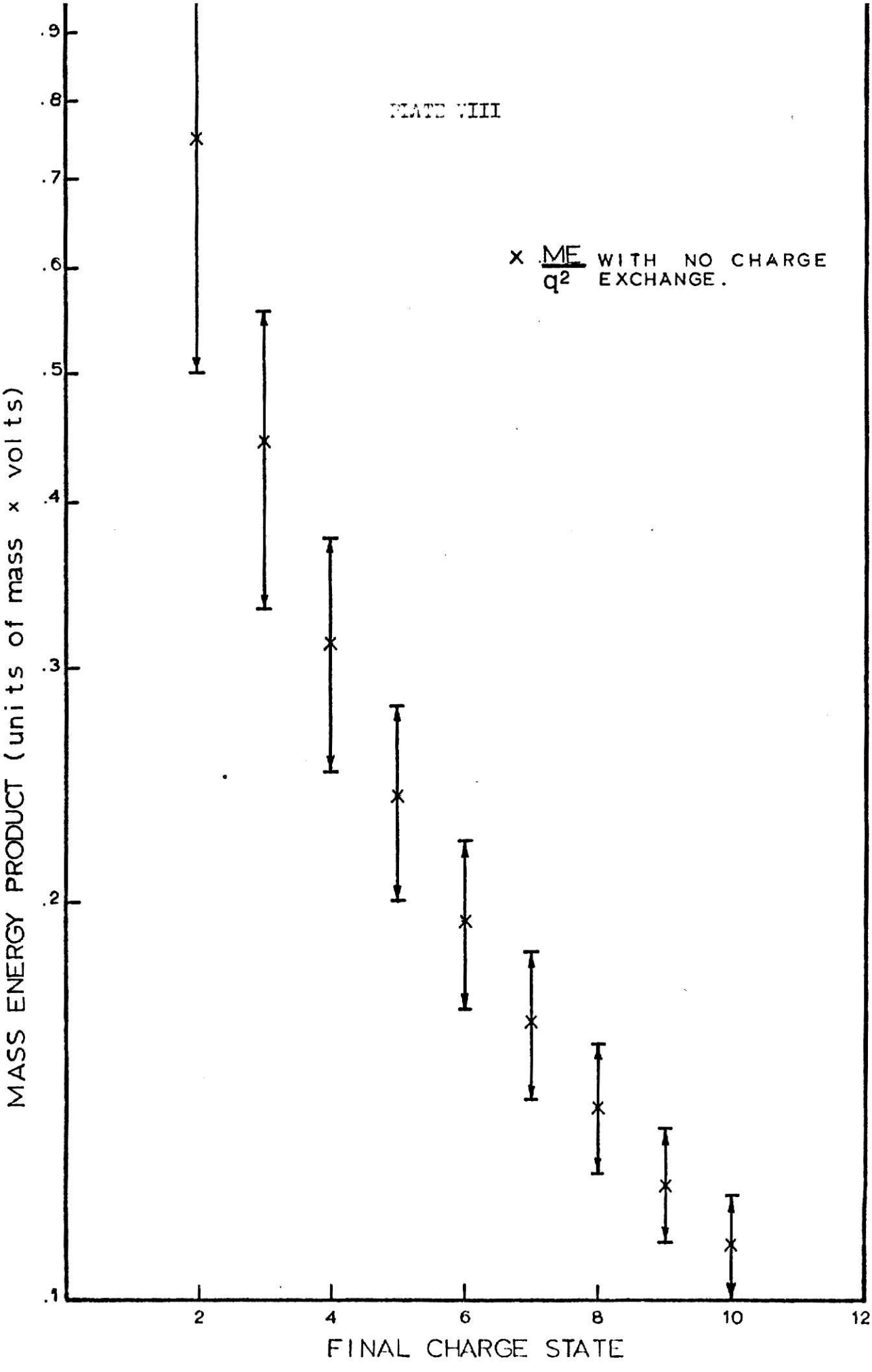
It may be seen by examining these curves that the loss or capture of one electron effects a spread in the mass energy product; however, the mass energy products for adjacent charge states of normally accelerated ions do not overlap. The analyzing magnet could be expected to resolve the various charge components. Plate IX shows a similar display of the maximum and minimum values of loss and capture of up to five electrons. It is seen that at least two charge exchange reactions are required to produce equal mass energy products for different exiting charge states. The analyzing magnet passes all components of the beam that have the same mass energy product. Consequently, more than one charge state and energy is allowed to continue on toward the experimental site.

This poor definition of energy can be eliminated by placing a velocity selector in the beam line after acceleration. The velocity selector, consisting of perpendicular electric and magnetic fields, passes or rejects particles independently of their charge. Consequently, the undesired components of the beam may be rejected prior to reaching the experimental site.

EXPLANATION OF PLATE VIII

Plate VIII The mass energy product or momentum to charge ratio vs the exiting charge state is plotted. The arrows indicate the range of the momentum to charge ratio when charge exchange of ± 1 is considered during acceleration.

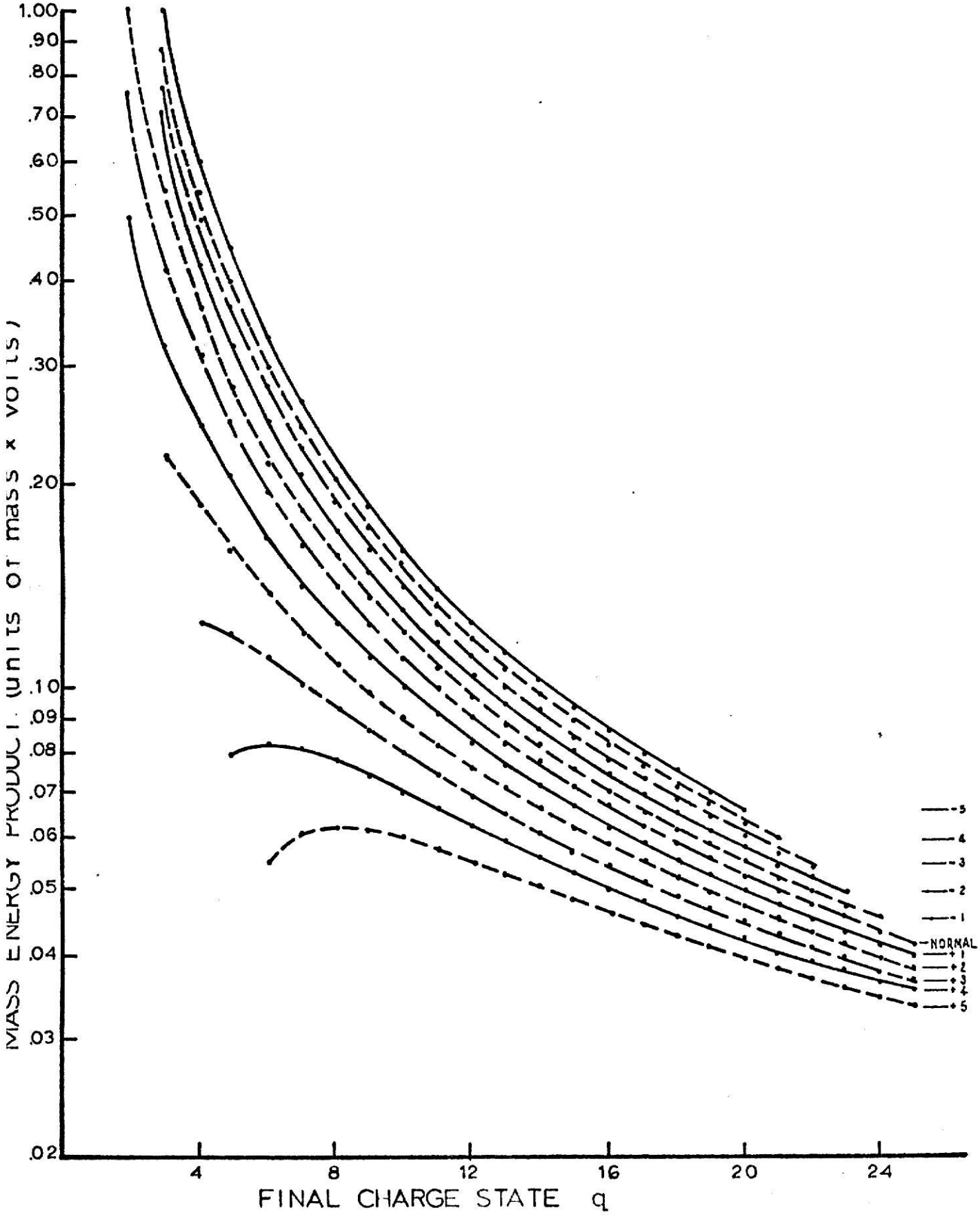
PLATE VIII


× ME/q² WITH NO CHARGE EXCHANGE.
MASS ENERGY PRODUCT (units of mass × volts)
.9
.8
.7
.6
.5
.4
.3
.2
.1
2
4
6
8
10
12
FINAL CHARGE STATE

EXPLANATION OF PLATE IX

Plate IX Mass Energy Product vs Final Charge State is plotted for several charge exchanges. The values of q are discrete. The lines are drawn for identification.

PLATE IX


1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.09
.08
.07
.06
.05
.04
.03
.02
MASS ENERGY PRODUCT (UNITS OF mass x VOLTS)
4
8
12
16
20
24
FINAL CHARGE STATE q
-5
-4
-3
-2
-1
NORMAL
+1
+2
+3
+4
+5

The position of the velocity selector in the beam line is still open for discussion. Calculations have been made which show that to first order the desired beam component is unaffected by the passage through the velocity selector. That is, no focusing or defocusing occurs. This was done for nonrelativistic uncoupled fields. In addition, the fields were assumed to be uniform and fringing fields were neglected.

One prime position for the velocity selector is prior to the analyzing magnet but after the quadrupole lenses. In this position the slits of the bending magnet can be used to discard the unwanted beam; no additional slits are needed in the system. In this position any unpredicted defocusing of the beam could be corrected by adjusting the quadrupole lenses on down the beam line.

The velocity selector could be positioned directly after the bending magnet prior to the switching magnet, or just prior to the experimental site. In the latter position, defocusing effects would be small and only the experimentalist desiring its use would need to be familiar with its operation.

However, if the purpose of a velocity selector is only to select a single charge state out of several, then this can be accomplished by an electrostatic deflection in front of a slit at the experimental apparatus.

MASS RESOLUTION

It is not always economical to buy commercially prepared material consisting of only a single isotope. Often it is impossible. In many cases it is not necessary to use isotopically pure source material since the analyzing magnet can usually resolve the components of an accelerated beam. However, a complication may arise from the method presently employed in regulating the beam. Regulation to maintain constant beam energy is carried out by measuring the amount of charge intercepted by the slits following the bending magnet. Since the energy distribution is approximately symmetric around a given central energy E, equal amounts of charge discarded onto each side of the slit indicates that the central trajectory contains the mean energy. This may not be the case when the source material contains two or more isotopes. The undesired isotope will be rejected onto one or the other slit making a nonbalance. The regulation system will attempt to correct the situation by altering the terminal energy slightly until equal amounts of charge are discarded on each side of the slit. Consequently the desired beam will not pass through the center of the slits.

A velocity selector in the beam line ahead of the regulating system would be useful in minimizing the difficulty just described. This is done by rejecting the undesired component, or components onto a slit at the velocity selector. Then only the desired component is passed through the analyzing magnet.

Wahlin[13] has described a mass spectrometer which uses crossed electric and magnetic fields which has been used to separate low energy ion

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13. L. Wahlin, op. cit.

beams (up to a hundred keV). He calculates the dispersion for the device and finds that

$$D = \tfrac{1}{2}\, l\, a\, \mathcal{E}\, [\sqrt{M_4/M} - 1]/V$$

$$D \approx \tfrac{1}{4}\, l\, a\, \frac{\mathcal{E}}{V}\, \frac{\Delta M}{M}$$

where l is the distance from the center of the velocity selector to the target; a is the length of the device; and $\mathcal{E}$ is the magnitude of the electric field between the electrostatic plates. This formulation is valid for the case of an acceleration through a potential difference V when no charge exchange occurs. In the case of the tandem Van de Graaff, there is the stripping process at the terminal. In the normal case where the energy after acceleration is given by $(q + 1)V$, the Wahlin expression must be multiplied by the factor $\frac{q}{q+1}$ to account for the extra energy provided by the effect. For the tandem Van de Graaff the dispersion is given by

$$D_T \approx \frac{1}{4}\, l\, a\, \frac{q\,\mathcal{E}}{(q+1)V}\, \frac{\Delta M}{M}.$$

The geometry of the system can be adjusted to accomodate any particular V whether it is on the order of 10^2 kV in the case described by Wahlin, or 10 mV, the order of magnitude of tandem terminal voltages. Once the geometry is defined, the dispersion D or D_T is approximately proportional to $\Delta M/M$, the mass resolution of the device.

Rearranging the expression, the mass resolution becomes

$$\frac{\Delta M}{M} \approx \frac{4\,D\,V}{l\,a\,\mathcal{E}}$$

for the case described by Wahlin, or

$$\frac{\Delta M}{M} \approx \frac{8+1}{8} \frac{4DV}{laE}$$

for the extension to the tandem system.

It is instructive to obtain the order of magnitudes of the mass resolution for the two cases. For the low energy case Wahlin's geometry is appropriate.

Let $a \approx 40$ cm

$l \approx 350$ cm

$D \approx .1$ cm

$E \approx 10^3$ V/cm

$V \approx 10^5$ V.

Then

$$\frac{\Delta M}{M} \approx \frac{4 \times .1 \times 10^5}{350 \times 40 \times 10^3}$$

$$\frac{\Delta M}{M} \approx 3 \times 10^{-3}$$

for the low energy design. For the high energy design take

$a \approx 10^2$ cm

$l \approx 400$ cm

$D_T \approx .1$ cm

$E \approx 10^4$ V/cm

$V \approx 5 \times 10^6$ V.

Then

$$\frac{\Delta M}{M} \approx \frac{8+1}{8} \frac{4 \times .1 \times 5 \times 10^6}{4 \times 10^2 \times 10^2 \times 10^4}$$

$$\frac{\Delta M}{M} \approx 5 \frac{8+1}{8} \times 10^{-3}.$$

For heavy ions, chlorine for example, charge states of -7 are obtainable, then

$$\frac{8+1}{8} \approx 1.1$$

and

$$\frac{\Delta M}{M} \approx 6 \times 10^{-3}.$$

ION SOURCE CONTAMINATION

There is very little specific information available concerning ion source contamination. However, if there is residual material remaining in the source from a previous run, it is likely that the resulting beam will contain some of that material. For example, the diode source at the Kansas State University tandem is not a widely used instrument at this time, so there is little experience concerning contamination. It is known that hydride and flouride ions are easily formed in this device. If these are accelerated they would certainly contaminate the beam.

Once contamination is in the beam, from any contributing condition, the problem reduces to the one described under the section concerned with mass resolution. The analyzing magnet measures Br, the magnetic rigidity, and the particle is rejected or accepted by the system only on the basis of whether Br is within given bounds.

Since doubly charged negative ions are unlikely to occur, the mass energy product is given by,

$$\frac{ME}{q^2} = 2MV.$$

The magnitude of q is one so the magnetic rigidity is only a function of mass and ion source extraction voltage. Since the latter is fixed, the magnetic field in the inflection magnet is just proportional to the square root of the mass passing through

$$B \propto \sqrt{M}.$$

The mass resolution of these magnets is typically $\frac{1}{20}$, but it is hoped that this resolution can be doubled upon the introduction of slits preceding and following the magnet. Even so, elements having isotopes with closely spaced atomic weights will be a problem. A velocity selector placed after the

inflection magnet, before the accelerator, can be used to improve the mass resolution.

The formation of negative hydride ions is an even more formidable problem. For example, consider two isotopes HX^n and X^{n+1}. If $(HX^n)^-$ is formed in the ion source it will be extracted with the same energy, momentum, charge state and velocity as $(X^{n+1})^-$. The two ions will be indistinguishable for analysis. A velocity selector prior to injection would not be at all useful in the last case, since the two species have the same velocity.

TRAJECTORY CALCULATIONS

S. Penner[14] has outlined a method of calculating the trajectory of an ion beam through a magnetic beam handling system. This method relies on the well known matrix techniques used in optics and electron optics. The method is applicable to first order calculations assuming small angle approximations. Under this condition the equations of the trajectories are linear.

The validity of the solutions are of course dependent upon the validity of the small angle approximations.[15] Also, the operator of an actual machine has control over additional parameters for which provisions are not made when a calculation is made. However, the first order calculations are useful for making initial judgments concerning the placement of the magnetic elements.

The linear equations may be placed in the following form,

$$X = a_{11} X_0 + a_{12} \theta_0 + a_{13} \Delta P/P_0$$

$$\theta = a_{21} X_0 + a_{22} \theta_0 + a_{23} \Delta P/P_0$$

$$\Delta P/P_1 = a_{31} X_0 + a_{32} \theta_0 + a_{33} \Delta P/P_0$$

$$Y = b_{11} Y_0 + b_{12} \phi_0 + b_{13} \Delta P/P_0$$

$$\phi = b_{21} Y_0 + b_{22} \phi_0 + b_{23} \Delta P/P_0$$

$$\Delta P/P_2 = b_{31} Y_0 + b_{32} \phi_0 + b_{33} \Delta P/P_0$$

where

x is the horizontal displacement

θ is the angular divergence in the horizontal plane

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14. S. Penner, Rev. Sci., Instr. 32,(1961)150

15. As an indication of the validity of the small angle approximation the usual divergence at the exit of a tandem Van de Graaff is about .003 radians. The small angle approximation should be valid to at least .1 radian.

y is the vertical displacement

φ is the angular divergence in the vertical plane

$\Delta p/p$ is the momentum spread.

In matrix notation

$$M_H = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$M_V = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}.$$

When there is more than one element to be considered, which is the usual case, the order of the matrix multiplication is important since

$$A \cdot B \neq B \cdot A$$

for many matrix products. If the first three elements of a system could be represented by the matricies M_{H1}, M_{H2} and M_{H3} respectively for the horizontal component, then the final horizontal co-ordinates are found by

$$M_H = M_{H3} \cdot M_{H2} \cdot M_{H1}$$

and

$$\begin{pmatrix} x \\ \alpha \\ \Delta p/p \end{pmatrix} = M_H \begin{pmatrix} x_0 \\ \alpha_0 \\ \Delta p/p_0 \end{pmatrix}$$

The simplest beam line element to consider is the drift length. The linear equations for this case may be found by inspection.

$$x = x_0 + L\alpha_0$$

$$\theta = \theta_0$$

$$y = y_0 + L\varphi_0$$

$$\varphi = \varphi_0$$

$$\Delta p/p = \Delta p/p_0 .$$

The corresponding matricies are

$$M_{HD} = \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{VD} = \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

Where L is the length of the drift space.

The development of the linear equations for the magnetic quadrupole and the bending magnet are derived by Penner, as well as in many ion optics references. Penner is able to derive the equations of the trajectory of a charged particle from the geometry of the fields, and this leads to satisfactory solutions. The derivations of the linear equations are not presented here since they are readily accessible in the literature.

The bending magnet is a dispersive element and consequently the position co-ordinates of a particle is dependent upon its momentum. In particular the dependence is on the deviation away from the momentum which the instrument is supposed to pass in the central trajectory. For this reason the parameter $\Delta p/p$ was chosen as a co-ordinate. Fortunately there are few elements which are dispersive. The bending magnet with normal pole

faces effects only the horizontal component of the beam. The matrix is

$$M_{HB} = \begin{pmatrix} \cos\alpha & r\sin\alpha & r(1-\cos\alpha) \\ \sin\alpha & \cos\alpha & \sin\alpha \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{VB} = \begin{pmatrix} 1 & r\alpha & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where α is the angle that the central trajectory is bent and r is the radius of curvature in the magnet. Then αr is the effective length of the element. The vertical component is given by a drift length since there is no focusing of the beam. Plate X shows how the angle of a pole face rotation is defined. The appropriate matrix form to provide for a pole face rotation is

$$M_{HR} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\tan\beta}{r} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

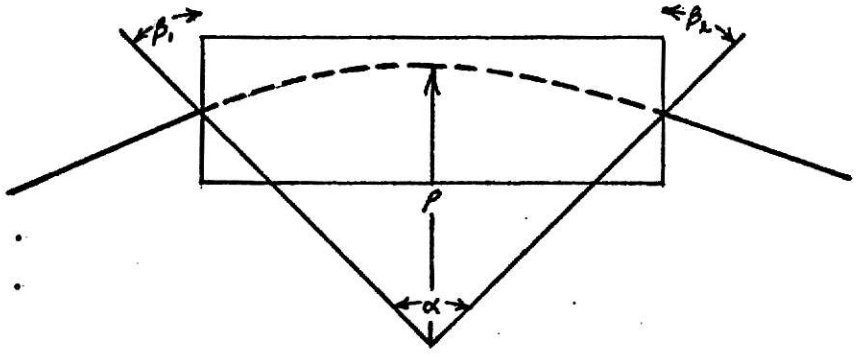
$$M_{VR} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{\tan\beta}{r} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Plate XI shows the geometry of the quadrupole magnetic lens. The element is not momentum dispersive so there is no dependence upon the momentum spread. The nature of the fields in the quadrupole are such that they focus one component. Consequently, it is necessary to use two or more quadrupoles (a quadrupole doublet, or triplet), in order to achieve a focusing action on both components of the beam. The quadrupole doublet is an astigmatic device

EXPLANATION OF PLATE X

Plate X The geometry of a bending magnet with pole face rotations of β_1 and β_2.

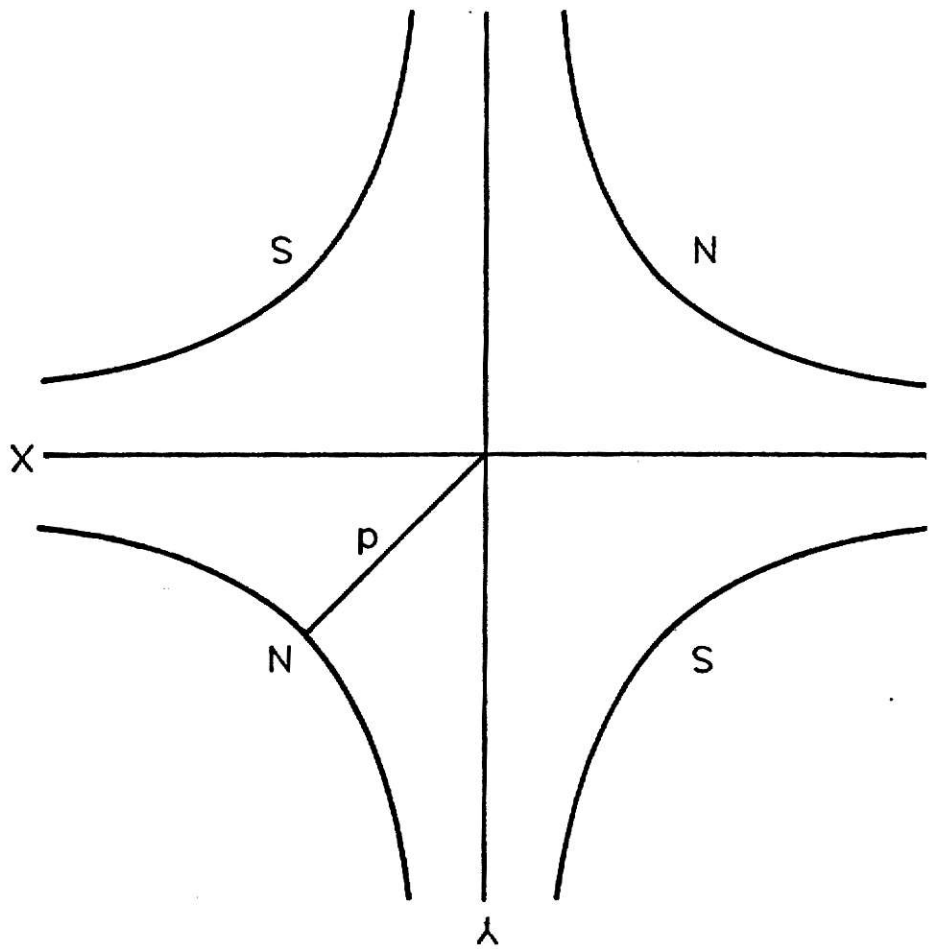
PLATE X



EXPLANATION OF PLATE XI

Plate XI A cross section of a quadrupole magnet showing hyperbolic pole faces.

PLATE XI



since a parallel incident beam is focused at two different locations corresponding to a horizontal focus and vertical focus. This follows since the centers of the lenses are physically displaced. The matrices describing the trajectories in a quadrupole magnetic lens are

$$M_c = \begin{pmatrix} \cos kL & \frac{1}{k}\sin kL & 0 \\ -k\sin kL & \cos kL & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_D = \begin{pmatrix} \cosh kL & \frac{1}{k}\sinh kL & 0 \\ k\sinh kL & \cosh kL & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The matrix M_D is the matrix which describes a diverging beam, and M_c describes the converging beam. These matrices are used as the vertical or horizontal matrix when the corresponding element is horizontally or vertically converging or diverging. The parameter k is given by

$$k = \left[\frac{1}{Br} \frac{\partial B_y}{\partial x} \right]^{1/2}.$$

The $\partial B_y/\partial x$ is the field gradient of the lens and L is the effective length of the magnet.[16]

When an electric field is set perpendicular to a magnetic field the polarities may be set so that the fields exert forces in opposite directions on a moving charge. A device utilizing this principle may be used to select particles of a given velocity since the magnetic force is velocity dependent. Wien[17] first described and used such a system and the velocity selector or

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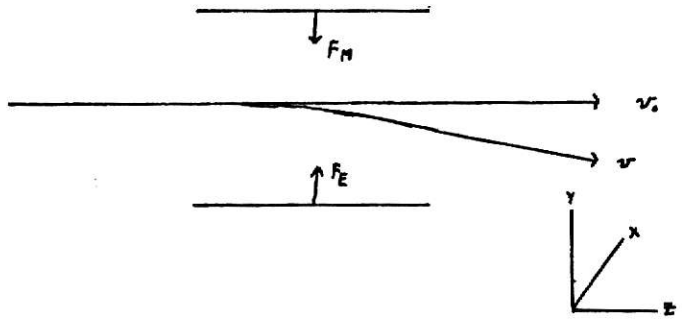
16. is the length of an ideal magnet with no fringe field.

17. W. Wien, Ann. Physik 65,(1898)440;8,(1902)260

'ion filter has been used (by Oliphant et al.)[18] as a low intensity isotope separator.

The first order equations describing the trajectories in the velocity selector are easily developed. Consider an electric field in the y (vertical) direction.



The forces on the particle due to the electric field and magnetic fields are

$$\bar{F}_E = q\mathcal{E} \qquad F_M = q\,\bar{v} \times \bar{B} .$$

There are no forces in the x direction so the horizontal co-ordinates are simply

$$x = x_0 + L\,\varphi_0$$

$$\varphi = \varphi_0 .$$

In the vertical direction

$$y = y_0 + L\,\varphi_0 + \tfrac{1}{2} a t^2$$

$$y = y_0 + L\,\varphi_0 + \frac{F t^2}{2M} .$$

The time that the forces are acting is $t = L/v_0$ where v_0 is the component of the velocity down the beam line. Then substituting

$$y = y_0 + L\,\varphi_0 + \frac{F L^2}{2 m v_0^2}$$

or

$$y = y_0 + L\,\varphi_0 + \frac{F L^2}{4E}$$

---

18. M. L. Oliphant, E. S. Shire and B. M. Crowther, Proc. Roy. Soc. (London) A146(1934)922.

where E is the kinetic energy of the particles. The force F is the sum of the electric and magnetic fields.

It is convenient to have the desired particles pass through the velocity selector without any deflection. This will occur when the sum of the forces on the particles in the beam is zero. Then

$$\bar{F}_e + \bar{F}_m = 0$$

$$qE - qv_0 B = 0$$

$$B = E/v_0$$

where v_0 is the velocity of the undeflected particles. The vertical displacement may be written

$$y = y_0 + L\varphi_0 + \frac{L^2}{4E}\left[F_e + F_m\right]$$

$$y = y_0 + L\varphi_0 + \frac{L^2}{4E}\left[qE - qv\,E/v_0\right]$$

$$y = y_0 + L\varphi_0 + \frac{qL^2E}{4E}\left[\frac{v_0 - v}{v_0}\right]$$

or in Penner's notation

$$y = y_0 + L\varphi_0 - \frac{qL^2E}{4E}\,\Delta p/p_0 .$$

The φ co-ordinate is found to first order by

$$\varphi \approx \tan\varphi = \frac{dy}{dz} .$$

Identifying z with L

$$\varphi = \varphi_0 - \frac{qLE}{2E}\,\Delta p/p_0 .$$

Then the matrix representing a velocity selector is

$$M_{VS} = \begin{pmatrix} 1 & L & -\frac{qL^2E}{4E} \\ 0 & 1 & -\frac{qLE}{2E} \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{VH} = \begin{pmatrix} 1 & L & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

DESCRIPTION OF A VELOCITY SELECTOR

The Wien filter consists of an electric field and a magnetic field set perpendicular to each other and perpendicular to the direction of the ion transmission. The magnetic fields already exist in the usual beam handling system. The inflection magnet, analyzing magnet or the switching magnet could be used, with the addition of an electric field, to make a Wien filter. However, this is impractical. The addition of the electric field produces an extra bend in the system which requires the relocation of most of the equipment that comprises the system. This impracticality lies primarily with a post acceleration velocity selector system.

Wahlin[19] has designed and built a practical velocity selector which would be appropriate for the energies used for injection into the tandem Van de Graaff. Plate XII shows an assembly drawing of the velocity selector. Briefly, Wahlin's design consists of an iron core magnet with a maximum magnetic field of 12,000 gauss. The magnet pole faces and electrostatic deflection plates are of the same area. The pole faces are insulated from the electrostatic plates by a set of guard rings. The potential of the shims can be adjusted to provide the optimum beam. Magnetic shorts are placed before and after the element to insure symmetric fields and also to terminate the fields at the entrance and exit of the selector.

An ion traveling at a velocity v_o will not be deflected from the beam if the magnetic and electrostatic forces are equal in magnitude. When this is the case,

$$v_o = E/B.$$

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19. L. Wahlin, op. cit.

EXPLANATION OF PLATE XII

Plate XII Assembly drawing of the Wahlin isotope separator.[20]

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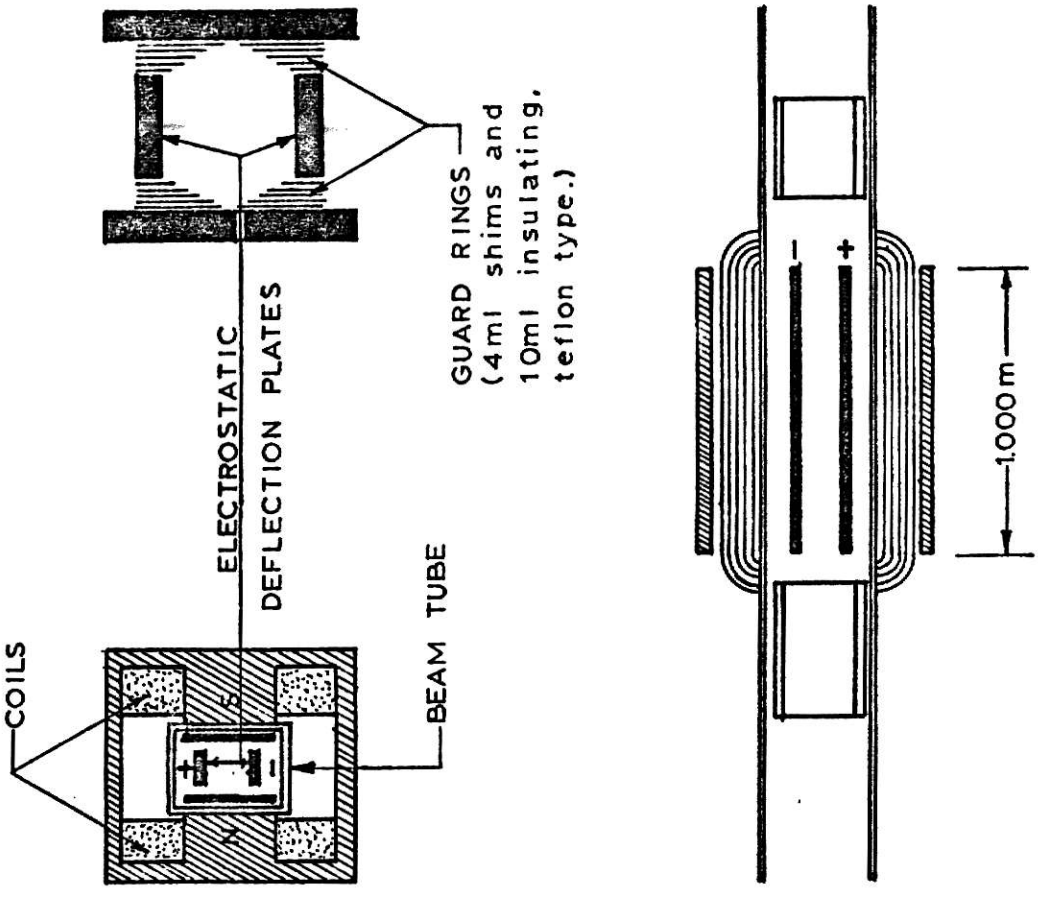
20. Ibid.

PLATE XII


COILS
ELECTROSTATIC
DEFLECTION PLATES
BEAM TUBE
GUARD RINGS
(4ml shims and
10ml insulating,
teflon type.)
1.000 m

The velocity may be used to find the magnitude of the fields used in the Wien filter. Since the deflection of the undesired component is directly proportional to the electric field and consequently proportional to $v B$, these fields should be as large as practical. When the electric field is set, the range of the required magnetic field is set by the velocity of the desired projectiles. To find the extremes for the magnetic field, the velocities are shown for representative energies for protons and uranium.

energy	100 keV	5 meV	12 meV	100 meV
proton velocity	4.5×10^6 m/sec	3.2×10^7 m/sec	4.9×10^7 m/sec	
uranium velocity	2.8×10^5 m/sec	2.0×10^6 m/sec		9.1×10^6 m/sec

The corresponding magnetic fields are shown for the electric field set at a reasonable value, 10 kV/cm.

energy	100 keV	5 meV	12 meV	100 meV
B for protons	2.2×10^3 gauss	3.1×10^2 gauss	2.0×10^3 gauss	
B for uranium	3.6×10^4 gauss	5.0×10^3 gauss		1.1×10^3 gauss

The mass separator designed by Wahlin is certainly adequate for injection energies, although for very heavy ions either the magnetic field or the extraction voltage will have to be increased. For post acceleration velocities, the Wahlin design is adequate for the very heavy ions; however, the weak magnetic fields required by fast light ions may be difficult to control with an iron core magnet. For lighter ions, it might be convenient to have the iron core magnet designed so that it can be removed and replaced by an air core magnet.

CONCLUSION

The tandem Van de Graaff accelerator has been used for many years to accelerate protons and helium nuclei. However, more recently there has been extensive interest in much heavier ion beams. The techniques of acceleration and energy selection used for proton and helium beams are not adequate to provide heavy ion beams of a single energy at the experimental site. This is due to the mechanism of continuous stripping in the high energy acceleration tubes.

It has been shown in this paper that the multiple beam resulting from continuous stripping can be eliminated by the use of a velocity selector. The velocity selector, or Wien filter, consists of crossed electric and magnetic fields. It has maximum effectiveness when placed between the accelerator and the object slits for the 90° analyzing magnet.

Further, the use of a velocity selector between the ion source and the accelerator can serve to minimize difficulties in separating small mass differences which might not be resolved by the inflection magnet.

BIBLIOGRAPHY

Banford, A. P., The Transport of Charged Particle Beams, SPON, 1966

Betz, H. -D., Hortig, G., Leischner, E., Schmelzer, Ch., Stadler, E., and Weihrauch, J., Physics Letter, 22,(1966)643

Betz, H. -D., Schmelzer, Ch., UNILAC - Bericht N. 6-66

Chmara, F., Ryding, G., and Chaw, J., "A Study of Vacuum Requirements for the Acceleration of Heavy Ions" High Voltage Engineering Corporation, Burlington, Massachusetts.

Dmitrier, I. S., and Nikolaev, V. S., Soviet Phys. JETP 20,409(1965)

Grivet, P., Electron Optics, Pergamon Press, 1965

Hemenway, C. L., Henry, R. W., Caultron, M., Physical Electronics, John Wiley and Sons Inc., 1962

Kirstein, Kino, and Walters, Space-Charge Flow, McGraw-Hill, 1967

Livingston, M. S., and Blewett, J. P., Particle Accelerators, McGraw-Hill, 1962

Pierce, J. R., Theory and Design of Electron Beams, Van Nostrand, 1949

Steffen, K. G., High Energy Beam Optics, Interscience Publishers, 1965

CHARACTERISTICS OF A VELOCITY SELECTOR USED WITH A TANDEM VAN DE GRAAFF ACCELERATOR

by

DENNIS KEITH MACDONALD

A. B., Grinnell College, 1966

AN ABSTRACT OF A MASTER'S THESIS

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Manhattan, Kansas

1970

After many years of using tandem Van de Graaff accelerators for producing beams primarily of protons and alpha particles, interest is shifting toward accelerating heavy ions. Several problems arise in obtaining beams of a single energy and of a single ionic species. This thesis discusses the use of a velocity selector, consisting of crossed electric and magnetic fields, to eliminate some of these problems.