# BEARING CAPACITY OF SHALLOW FOUNDATIONS ON SAND AND ON CLAY

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# I. INTRODUCTION:

## 1. Statement of the Problem:

Shallow foundations are defined by most suthors as foundations whose depth is not greater than the width. These are used to support structures on deposits of cohesionless or highly cohesive soils. It is recognized that every foundation problem necessitates the study of the ultimate bearing capacity of soils. This study, by both mathematical and practical procedures, is required to determine the load which a footing, with given shape, dimensions and depth, can safely impart to the soil.

The bearing capacity of sand is a function of its inherent resistance to frictional shear which is expressed in terms of angle of internal friction $\phi$, while for cohesive soils the cohesion c is the controlling factor of strength. Water table rising to the ground surface decreases the bearing capacity of sand by one half, but it has little influence on clay soil. The effect of the width of foundation on the bearing capacity is completely different in these two cases. A result of small area involved in a loading test may lead to a serious error for the bearing capacity of sand while it gives a reasonable value for clay. Application of surcharge increases the bearing capacity of sand more than for clay. Both sand and clay will exhibit a progressive failure in some cases but will also fail suddenly in other cases.

Since a bearing capacity failure usually results in a complete failure of the structure, a scientific treatment of the subject of bearing capacity of sand and clay, with the aim of developing a true understanding of the factors upon which it depends, is significantly required for the practicing soil engineer.

## 2. Purpose of the Study:

Since no exact mathematical analysis of the shear failure of sand or clay beneath the foundation has been derived, a number of approximate methods, both analytical and experimental, based on some simplifying assumptions of the complex failure surface and of the soil properties, will first be presented. From the result obtained above, the variables that the bearing capacity depends on are thus clearly shown and may be used as a guide for design. The purpose of this study is to derive some known formulas and to explain some experimental methods for the calculation of bearing capacity of sand or clay, as well as to determine the factors that affect the bearing capacity.

## 3. Scope of the Study:

The scope of this study includes: a careful and intensive review of the important literature on ultimate bearing capacity; a discussion of bearing capacity under assumed cases with numercial examples; and, the writer's conclusions concerning ultimate bearing capacity based on the literature review and on the results obtained.

## II. REVIEW OF THE LITERATURE:

### 1. Coulomb's (1) Shear Strength Equation:

The shearing strength of soil was first mathematically defined by Coulomb, based on the force required to slide wooden blocks over each other and an intuitive grasp of the mechanics of granular media. In this work, Coulomb developed the formula

$$s = c + \sigma \tan\phi$$

where

$s$ = shearing strength

$c$ = unit cohesion

$\sigma$ = normal stress on the rupture plane

$\tan\phi$ = the coefficient of solid friction

This clearly indicates that two seperate components of soil strength exist in soil; cohesion and friction. In purely cohesive soils friction is absent and in granular soils, of course, cohesion is absent.

### 2. Rankine's (2,3) Formula:

The Rankine theory on lateral earth pressure can be used for developing the relationship between the size and depth below surface of a footing. Rankine assumed that two elements of soil are considered, one immediately beneath the footing and the other just beyond the edge of the footing as shown in Fig. (1).

Based on theory of stress limit equilibrium condition in an ideal elastic soil, the ratio of major and minor principal stresses, and a plane rupture surface, Rankine calculated the bearing capacity for the cohesionless soil as follows:

For the weight of a structure there is a pressure $\sigma_I$ on the area AB of its footing. To sustain the direct pressure on the earth below the footing, a lateral pressure $\sigma_{III}$ is necessary, according to Rankine's theory.

$$\sigma_{III} = \sigma_I \tan^2(45^o - \phi/2)$$

where $\phi$ is the angle of internal friction.

The lateral pressure $\sigma_{III}$ or $\sigma_1$, must be sustained by a third pressure $\sigma_3$ at right angle to $\sigma_1$ therefore

$$\sigma_3 = \sigma_1 \tan^2(45^o - \phi/2)$$

where $\sigma_3$ = rD, r is the unit weight of soil, D is the depth of foundation, for $\sigma_1 = \sigma_{III}$ it becomes

$$\sigma_3 = \sigma_I \tan^4(45^o - \phi/2)$$

hence $$\sigma_I = rD\tan^4(45^o + \phi/2)$$

When the element II under the footing attains a state of shear failure, $\sigma_I$ = the ultimate bearing capacity $q_f$, thus

$$q_f = rD\tan^4(45^o + \phi/2)$$

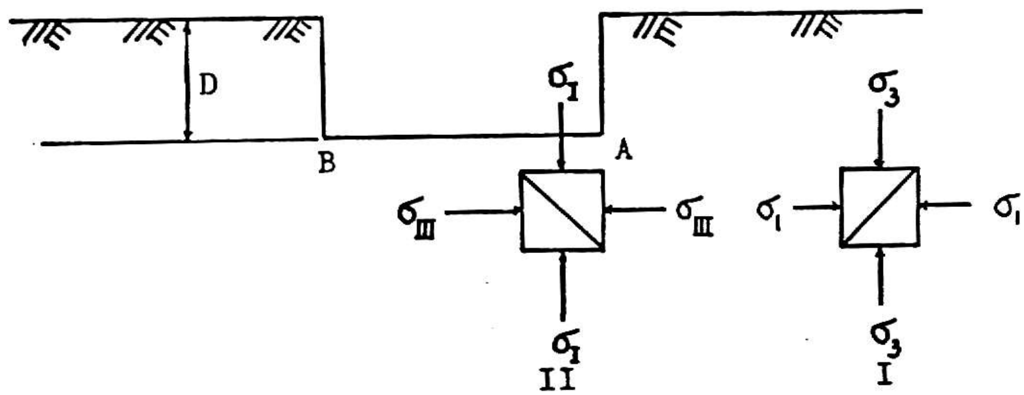



Fig. 1. Rankine & Bell's system.

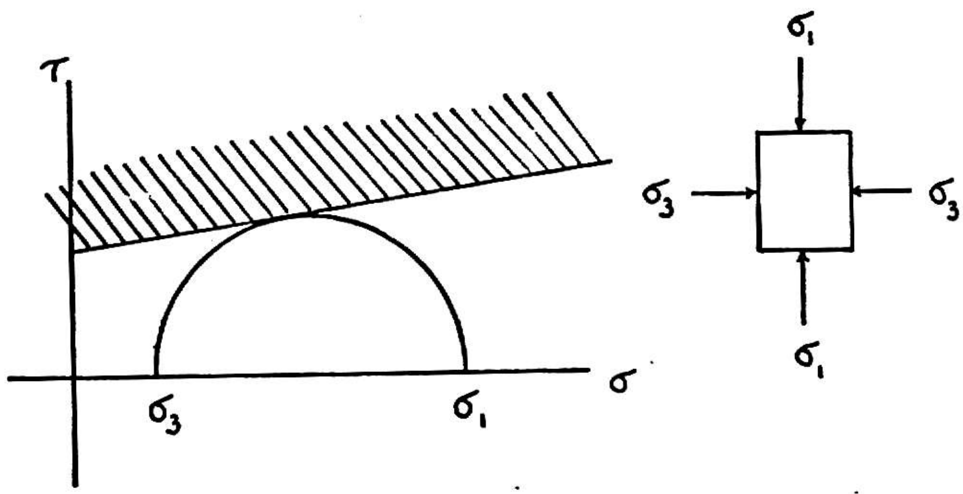



Fig. 2. Mohr envelope of rupture.

3. <u>Mohr's (4) Theory of Rupture:</u>

Mohr contributed in 1871 the so-called "rupture theory" to the subject of materials and gave a graphical representation of stress at a point, popularly known as "Mohr's stress circle". In soil mechanics, Mohr's stress circles are extensively used in the analysis of the shear strength of soils.

Mohr reasoned that yield or failure within a material was caused by critical combination of both shear and normal stresses. These stresses plotted on the $\sigma, \tau$ coordinates form a line known as the Mohr's envelope of rupture as shown in Figure (2). Failure will occur if for a given value of $\sigma$ the shear stress exceeds that shown by the envelope.

Shear in Cohesionless Soil:

A cohesionless soil is composed largely of quartz and similar rigid, strong particles. The grain strength is sufficient that the grains themselves do not fail until extremely high stresses are reached. Failure of such a soil therefore requires that the grains roll or slide over one another.

The results of tests on cohesionless soil show that the shear stress of failure, termed the shear strength s, follows the equation

$$s = \sigma \tan\phi$$

where

$\sigma$ = normal stress on the failure surface

$\phi$ = angle of internal friction

The Mohr envelope for the test shown in Figure (3). is approximately a straight line through the origin. The angle of failure plane $\alpha$, can be found graphically from the circle, that

$$\alpha = 45^{\circ}+\phi/2$$

Direct Shear Test:

In a direct shear test the plane of shear failure is predetermined, a number of identical specimens are tested under increasing normal loads and the required maximum shear force is recorded. A graph is plotted between the normal stress as abscissa and the shear strength as ordinate. The inclination to the horizontal of the strength envelope so obtained is the angle of shearing resistance and the intercept on the Y-axis is taken as the cohesion. Figure (4) shows the result of the test.

Triaxial Test:

The most reliable shear test is the triaxial shear test. Its important advantages are the relative uniform stress distribution on the failure plane and the freedom of the soil

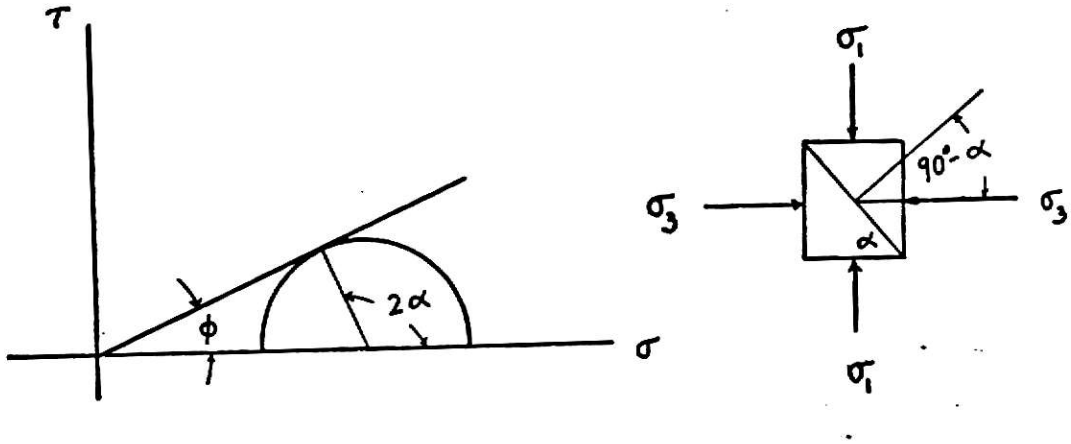



Fig. 3. Mohr envelope for cohesionless soil.

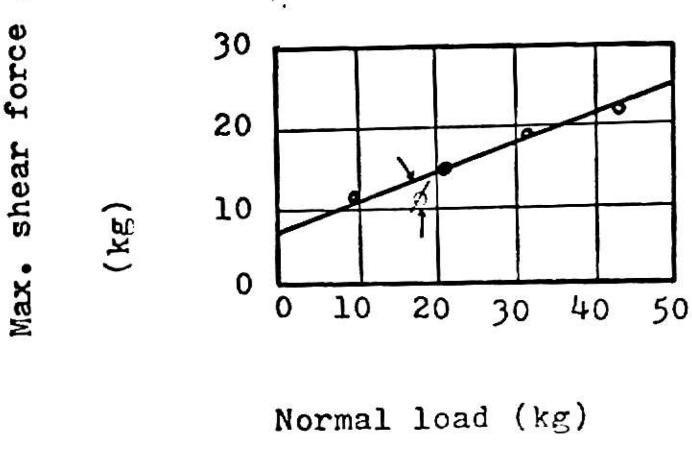



Fig. 4. Result of direct shear test.

to fail along the weakest plane in the specimen. Water can be drained from the soil during the test to simulate actual conditions in the ground. The result can be shown again by Mohr circle as in Figure (5). From these, cohesion c and friction angle $\phi$ are obtained.

Unconfined Compression Test:

The unconfined compression test is one of the simplest and quickest tests used for the determination of the shear strength of cohesive soils. The failures occur under an axial compressive stress with zero lateral stress. It is assumed that no moisture is lost from the specimen during the test. Since

$$\sigma_1 = 2c\tan(45^\circ + \phi/2)$$

if $\phi = 0$

$$\sigma_1 = 2c$$

$\sigma_1$ is also called the unconfined compression strength $q_u$, thus

$$\sigma_1 = q_u = 2c$$

or

$$c = q_u/2$$

Figure (6) gives the resulting cohesion c from the unconfined compression test by drawing the Mohr circle.

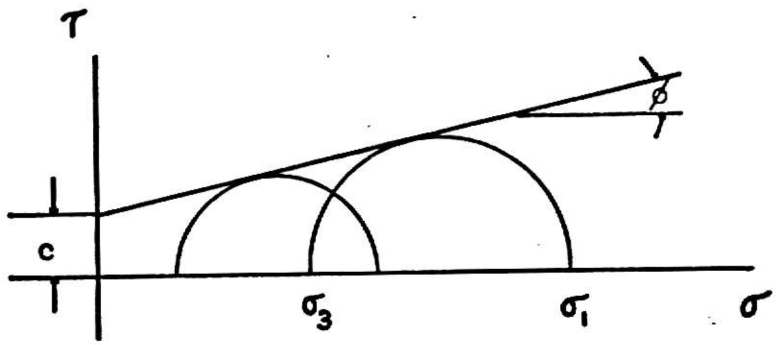



Fig. 5. Result of triaxial test explained by Mohr circle.

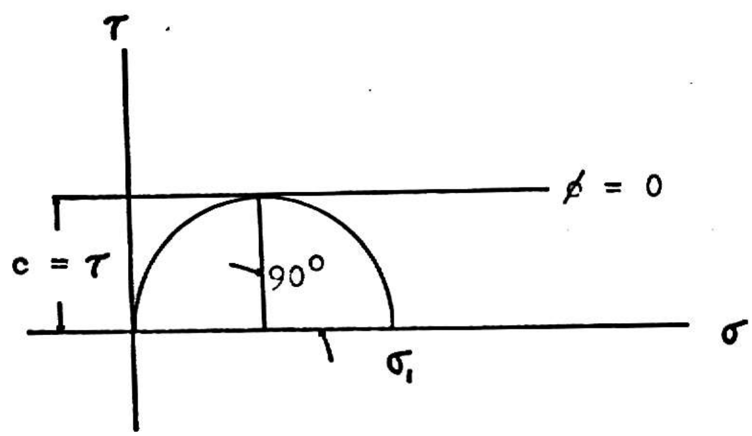



Fig. 6. Unconfined compression test for clay.

4. Bell's (5) Equation:

Rankine's formula was modified by Bell to be applicable for cohesive soils. In Bell's equation both friction and cohesion are considered. According to theory of earth pressure for cohesive soil, the stresses in Figure (1) become

$$\sigma_{III} = \sigma_I \tan^2(45^\circ - \phi/2) - 2c\tan(45^\circ - \phi/2)$$

$$\sigma_1 = \sigma_3 \tan^2(45^\circ + \phi/2) + 2c\tan(45^\circ + \phi/2)$$

Set

$\sigma_3 = rD, \qquad \sigma_I = q_f \qquad$ then

$$q_f = rD\tan^4(45^\circ + \phi/2) + 2c\tan(45^\circ + \phi/2)\left[\tan^2(45^\circ + \phi/2) + 1\right]$$

If $c = 0 \qquad q_f = rD\tan^4(45^\circ + \phi/2)$

Which is the Rankine formula for the ultimate bearing capacity of noncohesive soil.

If $\phi = 0$, for purely cohesive soil

$$q_f = rD + 4c$$

Without the effect of surcharge, $D = 0$

$$q_f = 4c$$

5. Housel's (6) Method:

Housel suggested a method of determining bearing capacity by field plate loading tests, which is particularly applicable in case the soil is reasonably homogeneous in depth. Housel assumed the foundation load is transmitted to the soil as the sum of two parts, (1) compression of the soil directly beneath the foundation, (2) shear along the perimeter of the footing.

This concept is expressed by an equation,

$$W = An + Pm = Ap$$

Where

$W$ = total load

$P$ = length of perimeter

$A$ = Area

$m$ = perimeter shear

$n$ = compressive stress on soil column

$p$ = bearing capacity

Let

$P/A = x$ = perimeter-area ratio, then

$$p = mx + n$$

m,n are constants which vary for different soils. At the same horizon in the soil, two test plates with different areas and perimeters, say, $A_1$, $P_1$ and $A_2$, $P_2$ are loaded to failure and the total loads $W_1$, $W_2$ when the maximum allowable settlement is developed are measured. This results in two simultaneous equations,

$$W_1 = A_1n + P_1m$$

$$W_2 = A_2n + P_2m$$

From which m,n are determined. Using the perimeter-area ratio of actual footing x, the bearing capacity is obtained by equation $p = mx + n$

6. <u>Krey(7) Method</u>:

The Krey method is a graphical procedure which can be applied to determine the safe load which a foundation can carry by giving a specified factor of safety.

1st Consideration: Excluding Cohesion

(1) The resultant passive resistance pressure Hp can be determined by the triangle of soil CDE, as shown in Figure (8), where

$P_p$ = the weight of soil in the triangle, acting through the centroid of the triangle

$R_p$ = the resultant of the shearing and normal forces acting on plane CD, acting at angle $\phi$ with the normal to CD

The magnitude and direction of both $P_p$ and $R_p$ are known, the passive pressure $H_p$ is solved by drawing a triangle of forces

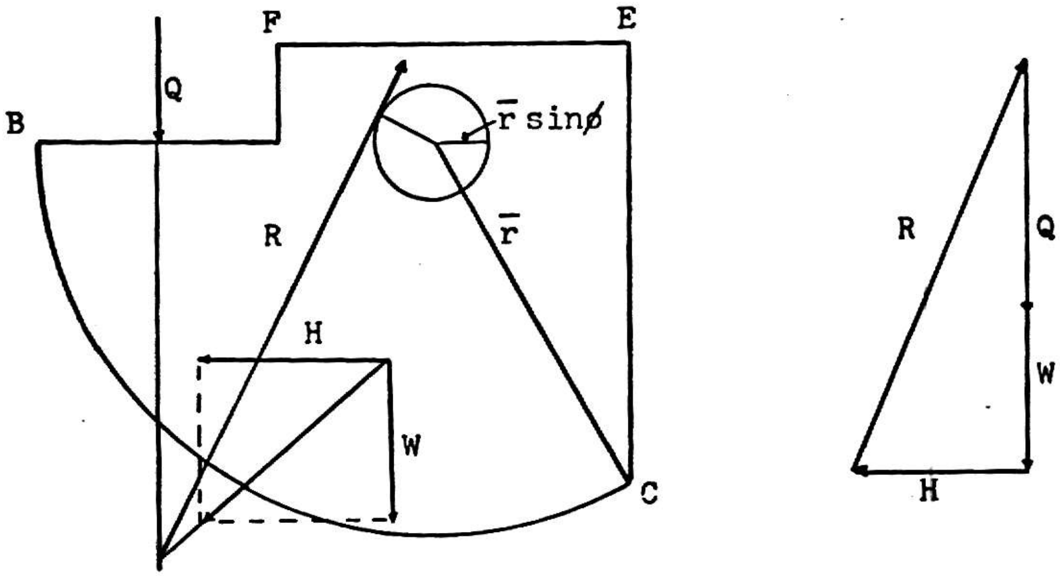



Fig. 7. Forces acting on active zone, excluding cohesion.

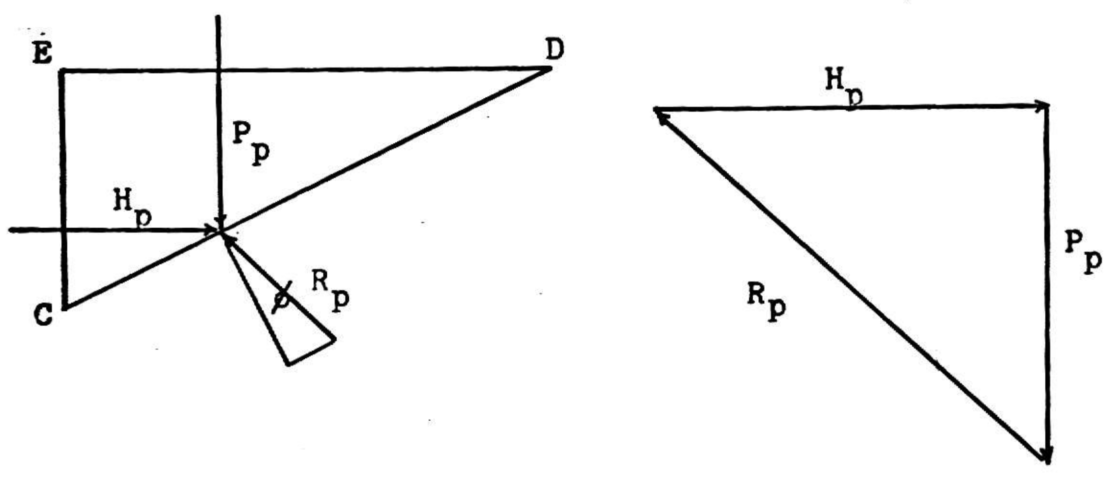



Fig. 8. Forces acting on passive zone, excluding cohesion.

(2) If the active zone is considered as a free body, as shown in Figure (7), the following forces exist

W = the weight of the soil in the active zone

R = the resultant of the normal and shearing forces on the arc BC

Q = the foundation load, which is to be solved

H = the active horizontal thrust, equal to $H_p$ divided by factor of safety

The magnitude and line of action of H and W are known, and the line of action of Q is known, the line of action of R can be determined by extending the resultant of H and W until it intersects the line of action of Q. Then R will pass through this intersection and is tangent to the $\phi$-circle. With the line of action of R known, the value of Q can be determined by drawing a force polygon.

Several trial failure surfaces should be investigated, and the minimum value of Q is the allowable foundation load.

2nd Consideration: Including Cohesion

(1) The total cohesion on the passive zone is equal to the unit cohesion c times the length L of the plane CD, as shown in Figure (9). It acts along the plane CD and is directed toward C. A polygon of forces acting on the passive zone is drawn.

(2) The total cohesion on the active zone as shown in

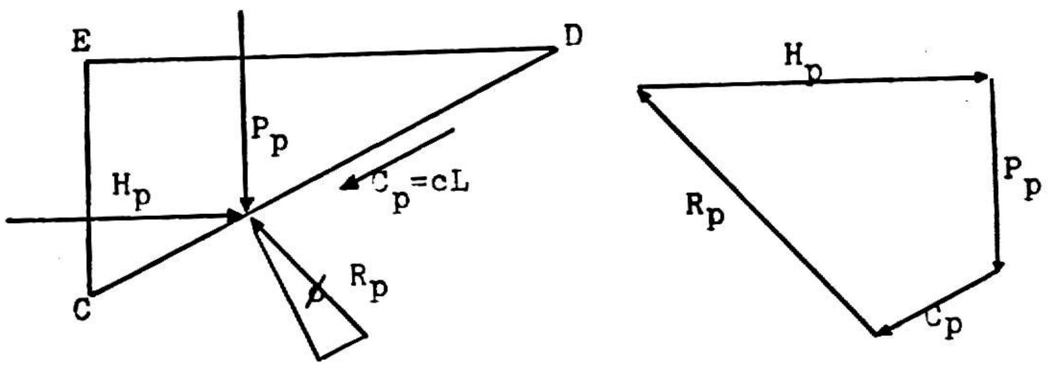



Fig. 9. Forces acting on passive zone, including cohesion.

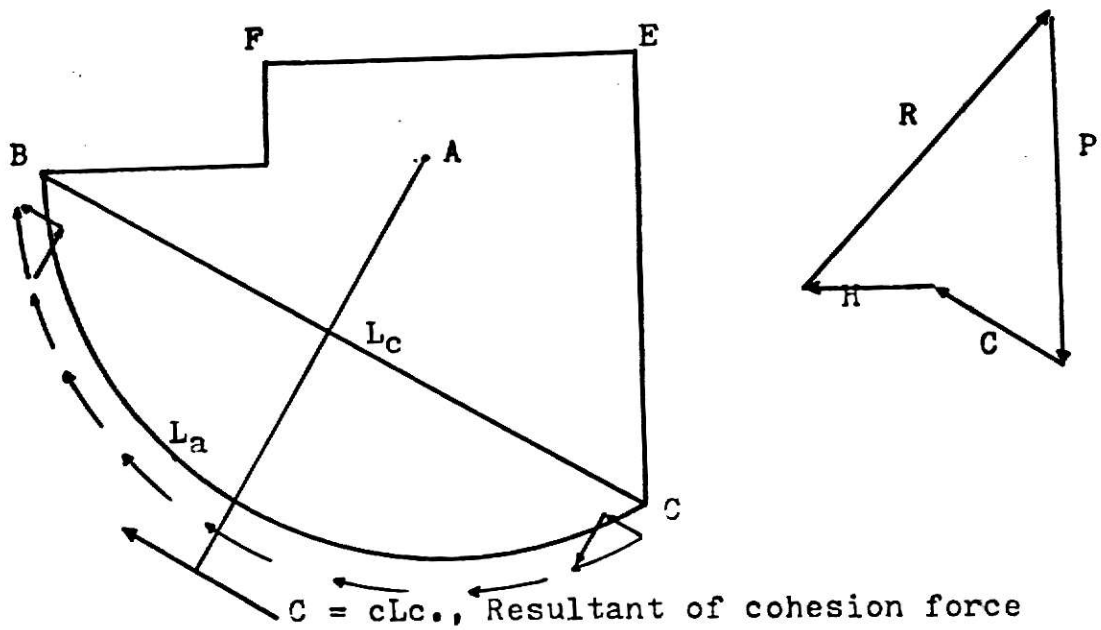



Fig. 10. Forces acting on active zone, including cohesion.

Figure (10), is equal to the unit cohesion c times the length La of arc BC. It is assumed uniformly distributed over this length and is directed toward B. The cohesion force on each increment of length of arc BC is resolved into parallel and normal components to chord BC, the parallel components are additive and the normal compenents cancel out. Therefore the magnitude of the resultant of all the cohesive forces is cLc, where Lc is the length of the chord BC.

The line of action of the resultant of the cohesive forces is determined by taking a moment about A,

$$\bar{r}cLa = dcLc$$

$$d = \bar{r}\,\frac{La}{Lc}$$

in which

d = distance from center of arc to line of action of the resultant of the cohesion forces on arc BC

$\bar{r}$ = radius of arc BC

La = length of arc BC

Lc = length of chord BC

A polygon of forces applicable to the 1st consideration (R, P, H) and including cohesion is also shown in Figure (10).

(3) The allowable foundation load Q can be determined by the following procedures and Figure (11).

(a) Draw the line of action of the resultant of Ha and W, this is line N

(b) Draw the line M, the resultant of N and C

(c) Draw R through the intersection of M and Q and tangent to the $\phi$-circle

(d) Draw force polygon to obtain Q

7. <u>Terzaghi's (8) Analysis</u>:

Terzaghi (1943) has developed a method of bearing capacity analysis for a long footing, which is based extensively on the theory of plastic failure and modern principles of soil mechanics. It is a very useful method and has gained wide acceptance in engineering circles. The basic concept is that, as a footing settles, the prism of soil beneath the structure exerts a lateral pressure against an adjacent soil mass. This adjacent mass provides resistance to the lateral pressure and thereby contributes to the support of the footing.

The calculation of the ultimate bearing capacity of soil under a uniformly loaded strip footing as shown in Figure (12) is based on the following assumptions:

(1) The soil is homogeneous, isotropic and its shear strength can be described by Coulomb's equation

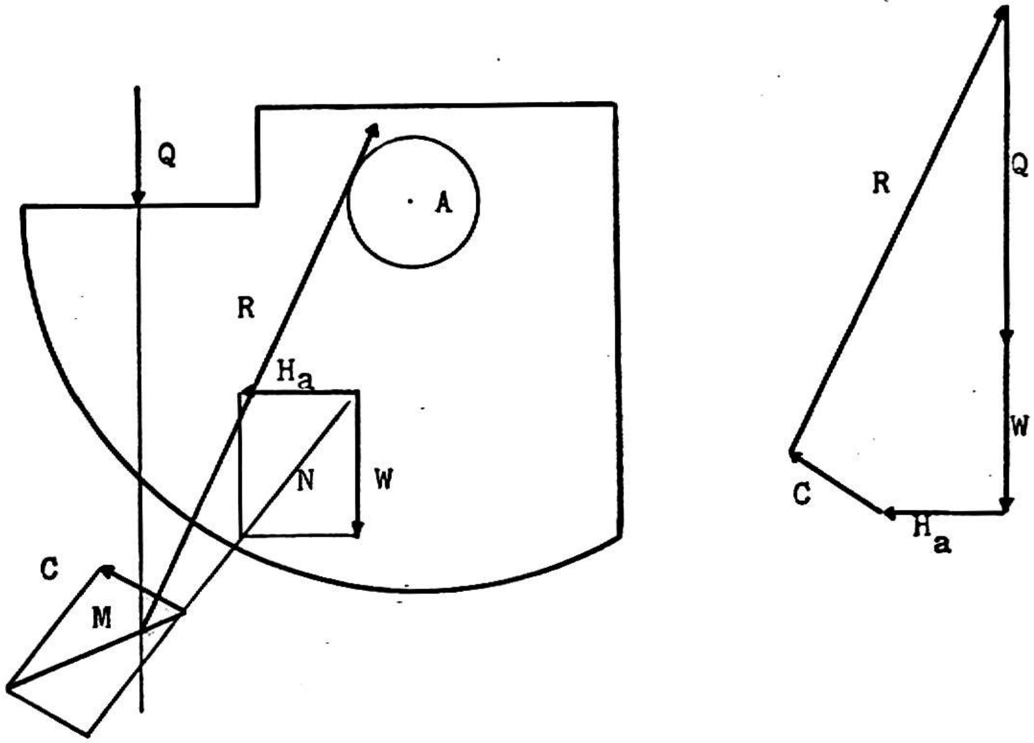



Fig. 11. Forces acting on active zone, including cohesion.

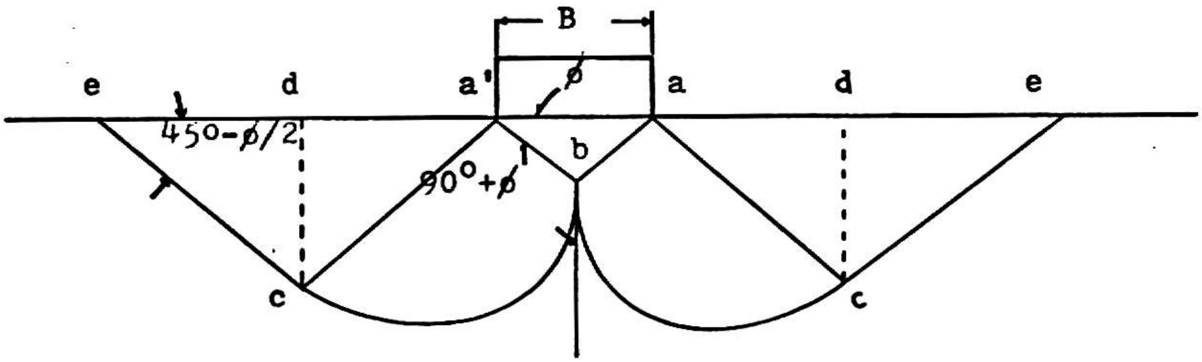



Fig. 12. Terzaghi bearing capacity theory, showing the slip surface beneath a strip footing.

$$s = c + \sigma \tan\phi$$

(2) The base of the footing is rough. Friction between the footing and the soil is greater than the shearing resistance of the soil.

(3) The failure surface is composed of a straight line ce and the logarithmic spiral bc.

(4) The soil wedge aba' remains in elastic equilibrium and behaves as if it were a part of the footing.

(5) The line ab makes an angle $\phi$ with horizontal.

(6) The spiral portion of the failure surface bc must be vertical at point b, because ab is also a failure surface, and failure surfaces intersect each other at angle of $90^{\circ}-\phi$.

(7) The linear shear planes of passive zones are inclined at an angle $45^{\circ}-\phi/2$ with the horizontal.

(8) The shear resistance of soil located above the level of the base of the footing is neglected, and the effect of the soil is equivalent to a surcharge D, where D is the depth of the footing.

(9) The depth D is not greater than the width of the footing.

(10) The shape of the spiral as shown in Figure (13), is given by

$$r_x = r_o e^{\theta \tan\phi}$$

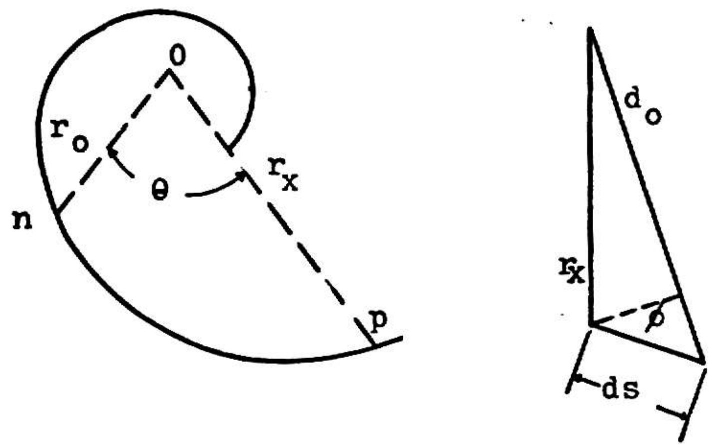



Fig. 13. Log spiral.

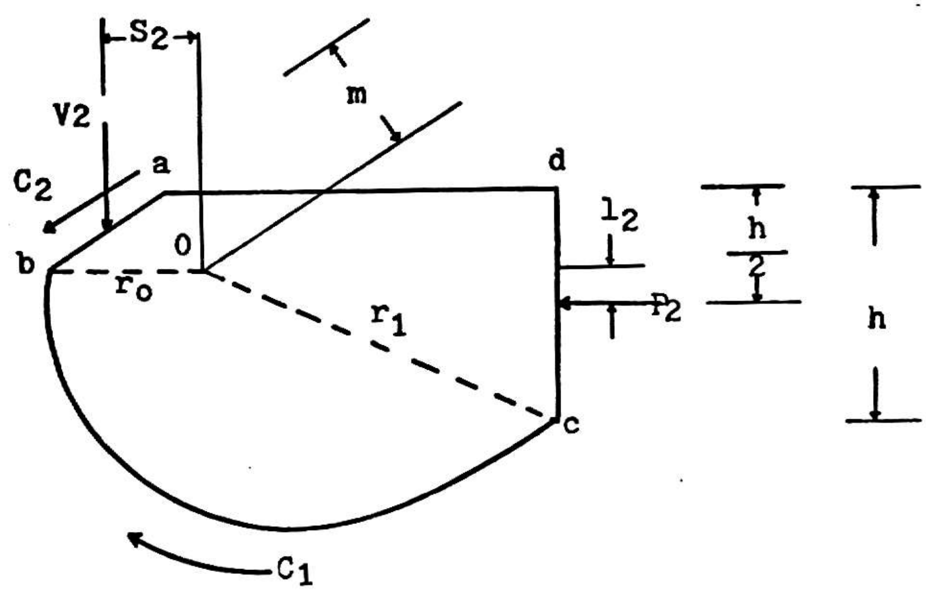



Fig. 15. Forces on mass abcd, with cohesion.

where

$r_x$ = the distance from the origin to the point p

$r_o$ = the distance along any chosen axis

$\theta$ = the angle between On and Op

At any point p, the radius Op makes an angle $\phi$ with the normal at that point.

Case 1: $c = 0$, $s = \sigma \tan\phi$

Since failure planes occur on dce, as shown in Figure (14), the passive earth pressure according to Rankine's theory,

$$P_1 = 1/2rh^2\tan^2(45^o+\phi/2)$$

The weight of the mass abcd = W, acts through the centroid of abcd, and F is the resultant of $\Sigma\sigma$ and $\Sigma\sigma\tan\phi$, in which

$\Sigma\sigma$ = resultant of all the forces normal to spiral bc

$\Sigma\sigma \tan\phi$ = resultant of shearing forces along bc

$V_1$ is the pressure acting on the failure plane ab, it acts through the lower third point.

Taking moments about point O, we have

$$M_o = P_1 l_1 + Wu - V_1 s_1 = 0,$$

$$V_1 = \frac{P_1 l_1 + Wu}{s_1}$$

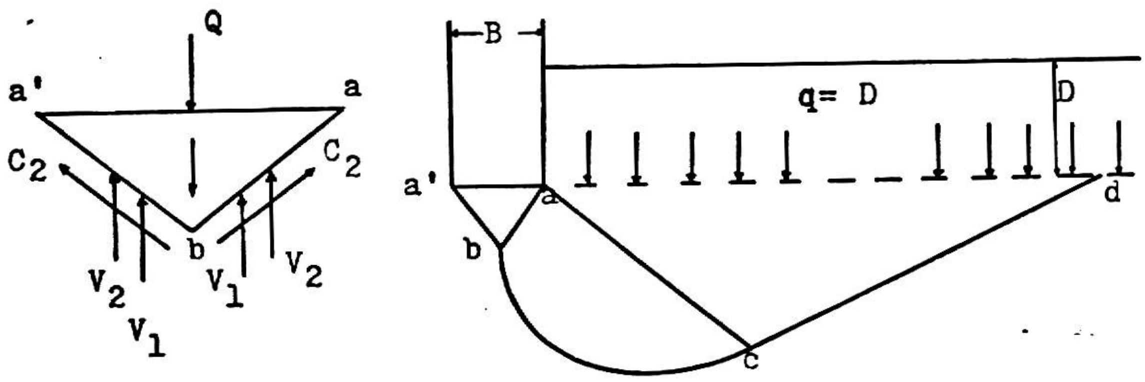



Fig. 16. Terzaghi's system for bearing capacity.

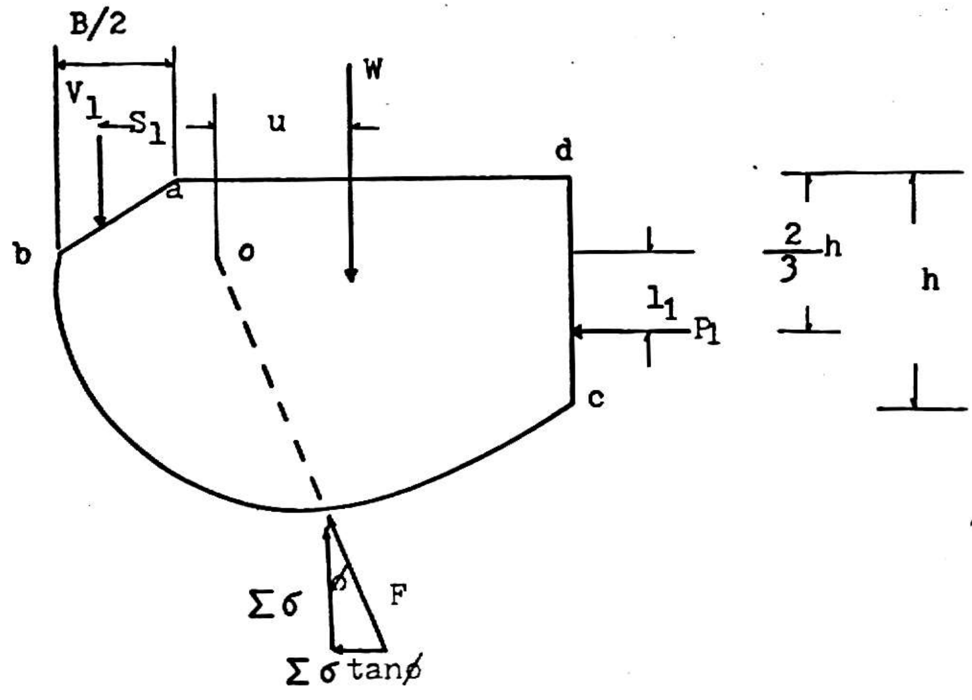



Fig. 14. Forces on mass abcd.

Case 2: Find resistance to failure, $V_2$, developed by cohesion c only

The passive pressure due to cohesion c is

$$P_2 = 2ch\tan(45^o + \phi/2)$$

and acts at the midpoint of dc as shown in Figure (15). The moment about O caused by cohesion along may be found by integrating the unit stress c, referring to Figure (13), thus

$$M_c = \int_o^{\alpha} dM_c = \int_o^{\alpha} (cds\cos\phi) r_x$$

$$= \int_o^{\alpha} r_x c \frac{r_x d\theta}{\cos\phi} \cos\phi$$

$$= \int_o^{\alpha} c r_x^2 d\theta = \frac{c}{2\tan\phi} (r_1^2 - r_o^2)$$

Since $C_2$ is equal to the unit stress c times the distance $\overline{ab}$, $V_2$ acts at the midpoint of ab, that the summation of moments about O becomes

$$M_o = P_2 l_2 + M_c - C_2 m - V_2 s_2 = 0$$

$$V_2 = \frac{P_2 l_2 + M_c - C_2 m}{s_2}$$

Case 3: Find load Q acting on footing

Having the values of $V_1$ and $V_2$, the load Q is determined by summing the vertical forces acting on mass aba' equal to zero as indicated in Figure (16).

$$\sum F_v = Q + r1/2B(1/2B\tan\phi) - 2(V_1 + V_2) - 2C_2\sin\phi = 0$$

or

$$Q = 2(V_1 + V_2) + 2C_2\sin\phi - r(B/2)^2\tan\phi$$

Case 4: Find bearing capacity

The part of the soil mass above ad in Figure (16) is treated as a surcharge exerting a pressure q=rD on the surface ad. From the analysis in case 2 and case 3, both W and $P_1$ are proportional to $(B/2)^2$. Thus they contribute bearing capacity $Q_f/B = q_f$ which is proportional to (B/2). W and $P_1$ are also proportional to unit weight r, and if this portion of bearing capacity is denoted as $q_r$, then

$$q_r = 1/2\ rBN_r$$

By similar examination, $V_2$ and $C_2$ are also found to be proportional to B/2 and c. This contribution to bearing capacity is

$$q_c = cN_c$$

The effect of surcharge is independent of B,

$$q_q = rDN_q$$

N, $N_c$ and $N_q$ are dimensionless coefficients that are governed by the value of $\phi$. For various values of $\phi$, $N_r$, $N_c$ and $N_q$ are presented graphically as shown in Figure (17). The total bearing capacity can be obtained by summing each part of contribution, and

$$q_f = 1/2rBN_r + cN_c + rDN_q$$

8. Plate Loading Test:

A plate loading test is an in situ test which is performed with the object of determining the ultimate bearing capacity of soil. This test as Sower (9) recommended in 1950, was made to the expected foundation level by loading a rigid bearing plate, usually from 1 ft to 2 1/2 ft square, with gradual load increments. During loading the settlement of the plate is measured and a load settlement curve similar to that show in Figure (18) is obtained.

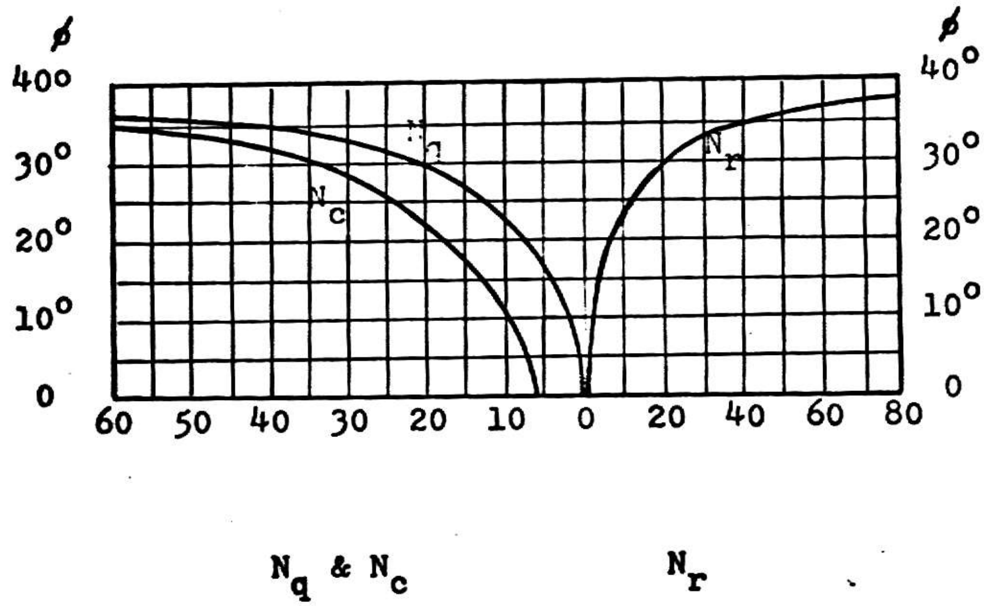



Fig. 17. Bearing capacity factors.

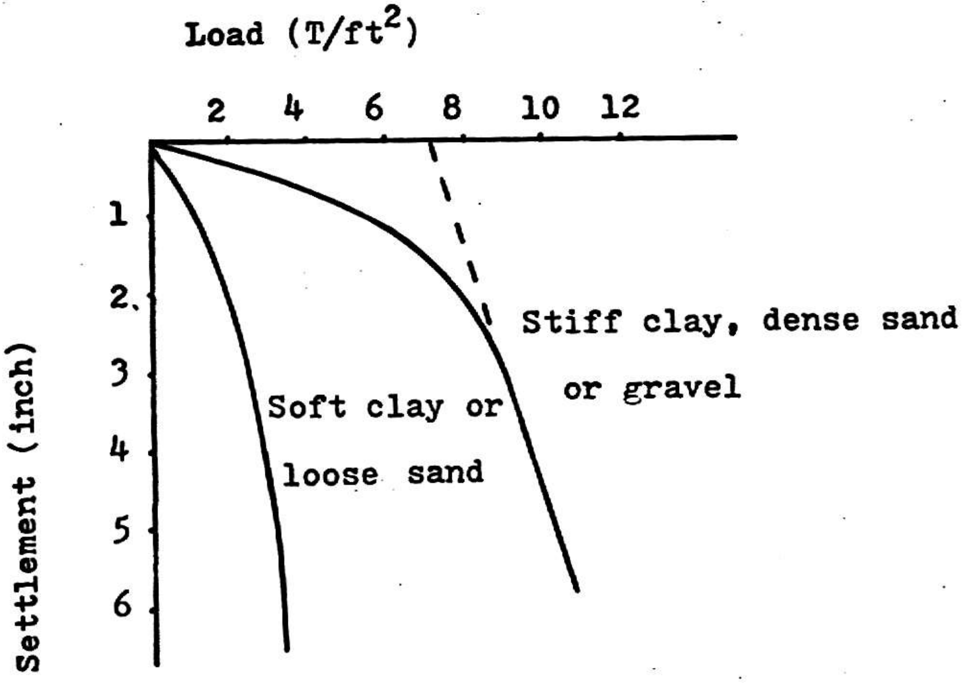



Fig. 18. Typical plate loading test result.

As would be expected, the settlement of a square footing kept at a constant pressure, increases as the size of the footing increases. Terzaghi & Peck (11) investigated this effect and produced the relationship,

$$S = S_1\left(\frac{2B}{B+1}\right)^2$$

where

$S_1$ = settlement of a loading area 1 $ft^2$ under a given loading intensity p

$S$ = settlement of a square or rectangular footing of width B under the same pressure p

If we assume S = 1.0 inch and B = 10 ft, $S_1$ can be obtained. From plate loading test results, we have a curve which shows the relation between P and $S_1$, hence the value of P corresponding to the calculated value of $S_1$, is the allowable bearing capacity.

9. Standard Penetration Test:

The bearing capacity of a cohesionless soil, which is difficult and expensive if sampling in the undisturbed state, can be determined most ecnomically from in situ standard penetration tests. The results of the test, as Meyerhof (10)

| N | Relative Density | |
|---|---|---|
| | Terzaghi & Peck | Gibbs & Holtz |
| 0-4 | Very loose | 0 - 15% |
| 4-10 | Loose | 15 - 35 |
| 10-30 | Medium | 35 - 65 |
| 30-50 | Dense | 65 - 85 |
| Over 50 | Very dense | 85 -100 |

Table. 1. Relationship between relative density (R.D.) & penetration value N.

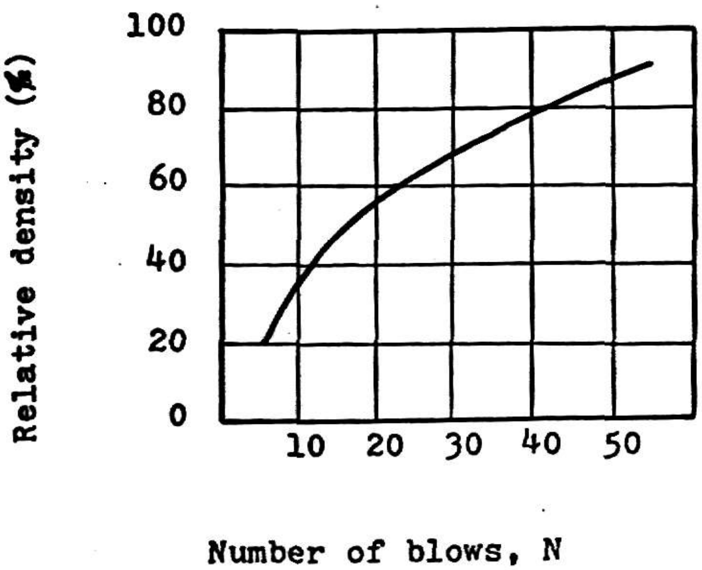



Fig. 19. Relationship between R.D. and N.

stated, have been correlated to the density index, angle of shearing resistance and the corresponding bearing capacity of footings. This test is conducted with a split spoon sampler sometimes known as the Raymond spoon sampler. The sampler is driven into the cohesionless soil for its full length of 18 inches. The number of blows required to drive the last 12 inches is recorded as the N value of the soil. Terzaghi & Peck (11) evolved a qualitative relationship between relative density and N as shown in Table (1). Gibbs & Holtz (12) gave quantative values for it. Gibbs & Holtz's values are also plotted in curve as shown in Figure (19). Coffman (13) interpreted Gibb & Holtz's results in a simpler form as illustrated by Figure (20).

Terzaghi & Peck (11) also pointed out that in saturated sands ( i.e. below the water table), the N value can be altered by the low permeability of the soil. They suggested an empirical rule,

$$N' = 15 + 1/2(N-15)$$

where

$N$ = actual number of blows obtained from the test

$N'$ = number of blows to be assumed for design purposes

Having the value of N', the allowable bearing capacity can be determined from the curves evolved by Terzaghi & Peck as shown

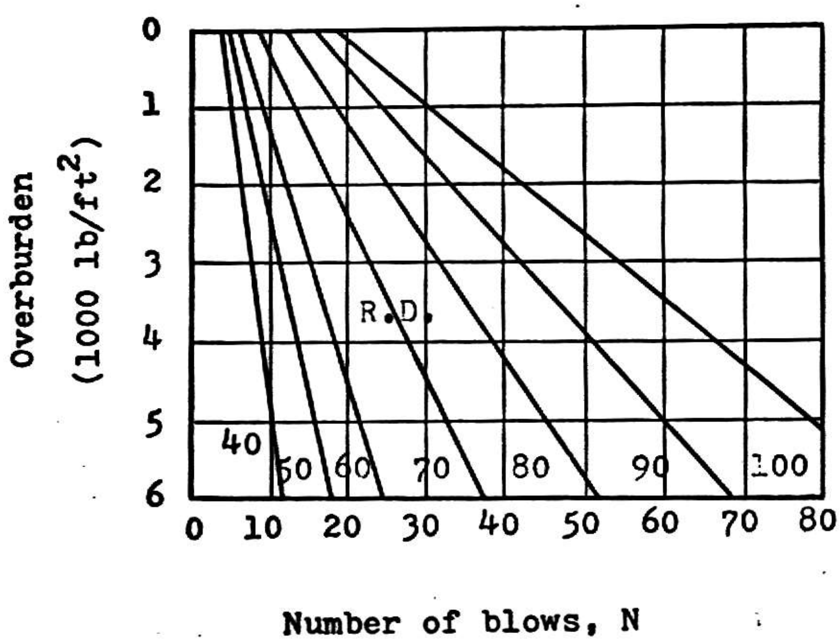



Fig. 20. Relation between effective overburden pressure, N and R.D.

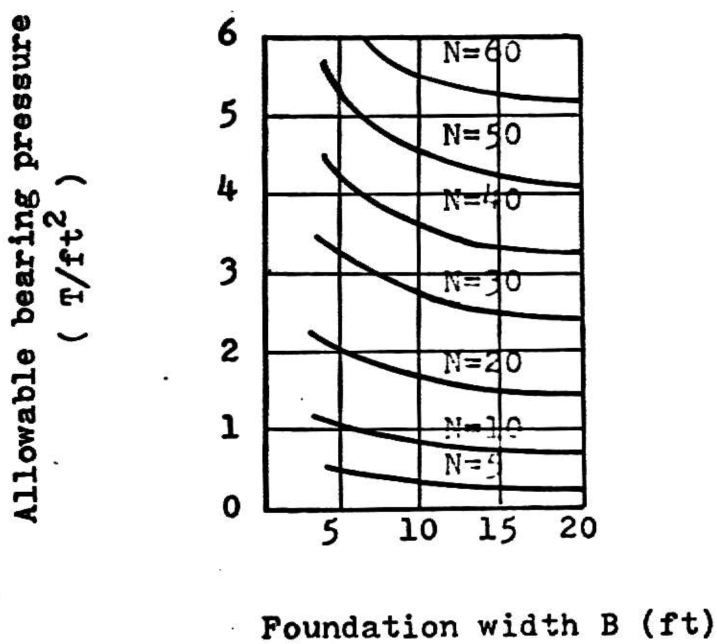



Fig. 21. Relation between N, B and allowable bearing capacity.

In Figure (21). They also found that the pressure will not cause a settlement greater than 1.0 inch.

For example, if a granular soil at a depth of 12 ft was subjected to a standard penetration test, and if the ground water table occurred at 5 ft below the surface of the soil with an average density 120 lb/ft$^3$, and the N value from test was 15, determine the allowable bearing capacity, assuming foundation width B = 10 ft.

Solution:

Effective overburden pressure = 12 x 120 - 5 x 62.4 = 1128 lb/ft$^2$

From Coffman's diagram, Eff. O.P. = 1128, N = 15, R.D. = 72%

From Gibbs & Holtz's chart, R.D. = 72%, the corresponding N = 36

Using Terzaghi & Peck's correction for saturated soil,

N' = 15 + 1/2 (36 - 15) = 25 blows per foot.

Having N' = 25, B = 10 ft, the allowable bearing capacity as obtained from Terzaghi & Peck's curves, is 2.2 T/ft$^2$.

# III. DISCUSSION AND EXAMPLES

## 1. Discussion:

### A. Cohesionless Soil (sand), c=0

(1) Rankine's formula

$$q_f = r\,D\tan^4(45^\circ+\phi/2)$$

It shows the effect of depth D and angle of internal friction $\phi$.

(2) Terzaghi's formula

$$q_f = 1/2\,rBN_r + cN_c + r\,DN_q$$

at the surface, c=0, D=0

$$q_f = 1/2\,rBN_r$$

below the surface, c=0

$$q_f = 1/2\,rBNr + rDN_q$$

Assume surcharge ratio C'=D/B, then

$$D=C'B$$

$$q_f = 1/2\, r\, BN_r + r\, C' BN_q$$

$$= 1/2\, r\, BN_r (1 + 2C' \frac{N_q}{N_r})$$

$(1+2C'\frac{N_q}{N_r})$ is called the depth factor, it becomes

$1 + 2.2C'$ when $\phi = 28°$

$1 + 1.7C'$ when $\phi = 38°$

It appears reasonable to accept $1 + 2C'$ as a conservative expression for the depth factor. For the contribution to $q_f$ of depth D is $2C'$, we can say the ultimate bearing capacity increases greatly as the depth increases in cohesionless soil.

In general we find from the above result, the ultimate bearing capacity in a cohesionless soil is proportional to the breadth of the footing at the ground surface. The increase in bearing capacity due to an increase in width of footing is shown in Figure (22).

(3) Effect of water table on cohesionless soil

A rise of water table below the foundation influences the bearing capacity by reducing cohesion (the reduction in cohesion is small, it is not considered.) and the effective unit weight of soil. When the water table is between the ground and footing level, a reduction factor W should be applied to the surcharge, if the water table is between the base of the footing and the depth of the failure zone, this reduction factor W' should be applied to the $1/2rBN_r$ term. Therefore we write the bearing capacity equation of cohesionless soil as

$$q_f = rDN_qW + 1/2BrN_rW$$

The reduction factors W, W' are graphically represented in Figure (23). A rise of water table to the ground surface, W = 0.5, W' = 0.5

$$q_f = 0.5(rDN_q + 1/2N_rBr)$$

This reduces $q_f$ by approximate 50% and is in accordance since the submerged unit weight of soil is roughly half of its unit weight when not submerged.

(4) Krey Method

According to Figure (24), the forces can be expressed in the following equation.

$$P_p = 1/2\gamma\left[R\sin(\pi/4 + \phi/2) + Z\right]^2\tan^2(\pi/4 + \phi/2)$$

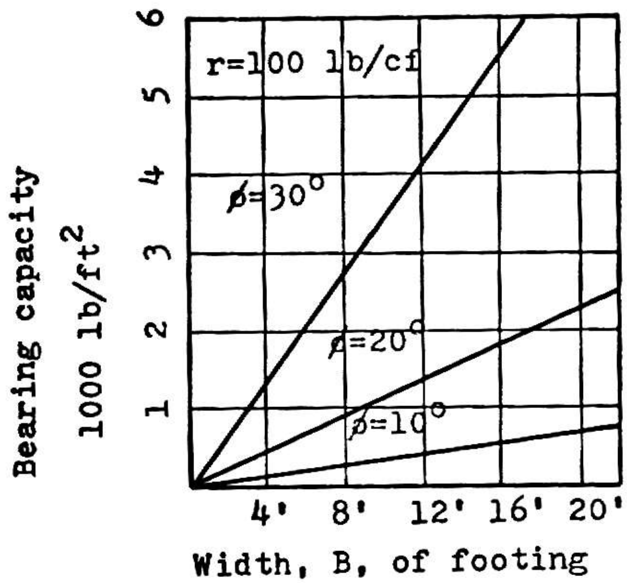



Fig. 22. Cohesionless soil, no surcharge.

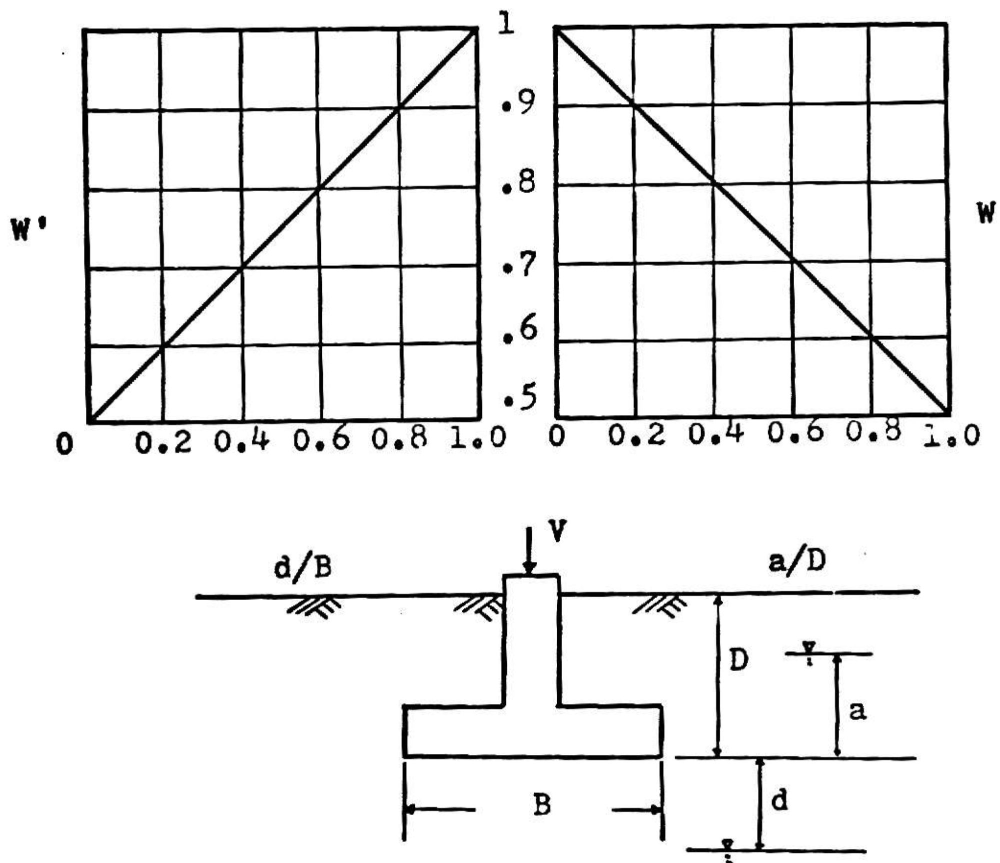



Fig. 23. Diagram for calculating bearing capacity, under effect of water table.

$W_1 = 1/4rR^2\cos\phi$

$W_2 = r\left[a + R\cos(\pi/4 + \phi/2)\right] Z$

$W_3 = \pi R^2 r(3/8 - \phi/4\pi)$

$F_v = W_1 + W_2 + W_3$

For $\phi = 30^\circ$, $Z = 4$ft, $r = 110$ lb/cf,

let $B = 2$ft, $a = 0.2$ft, then $q_f = 5{,}500$ lb/ft$^2$

$B = 4$ft, $a = 0.5$ft, then $q_f = 8{,}100$ lb/ft$^2$

*This indicates that* $q_f$ *for cohesionless soil increases* greatly as the width of the foundation increases.

(5) Plate Loading Test

In sand and gravel the bearing capacity as Sower (9) stated, increases directly proportional to the width of the loaded area, that $q_f$ of foundation becomes

$$q_f \text{ (foundation)} = q_f \text{ (load test)} \frac{\text{width of foundation}}{\text{width of test plate}}$$

(6) Standard Penetration Test

The factors $N_r$ and $N_q$ are functions only of $\phi$ which, in turn, is primarily a function of relative density of the sand. The most expedient procedure for investigating the relative density is the standard penetration test. The N value as determined by means of this test can be fairly reliably correlated with the values of $\phi$, $N_r$ and $N_q$. The results of such correlations are shown in Figure (25). If the N values are known, $q_f$ can be evaluated with the aid of a diagram. It should be understood that the relationship between N and $\phi$ is only approximate, and the relationship between $\phi$ and $N_r$ or $N_q$ is based primarily on theory and is much more reliable.

B. Purely Cohesive Soil (clay), $\phi = 0$

(1) Bell's equation

$$q_f = rD\tan^4(45^\circ + \frac{\phi}{2}) + 2c\tan(45^\circ + \frac{\phi}{2}) \left[ \tan^2(45^\circ + \frac{\phi}{2}) + 1 \right]$$

If $\phi = 0$

$$q_f = rD + 4c$$

Without the effect of surcharge

$$q_f = 4c$$

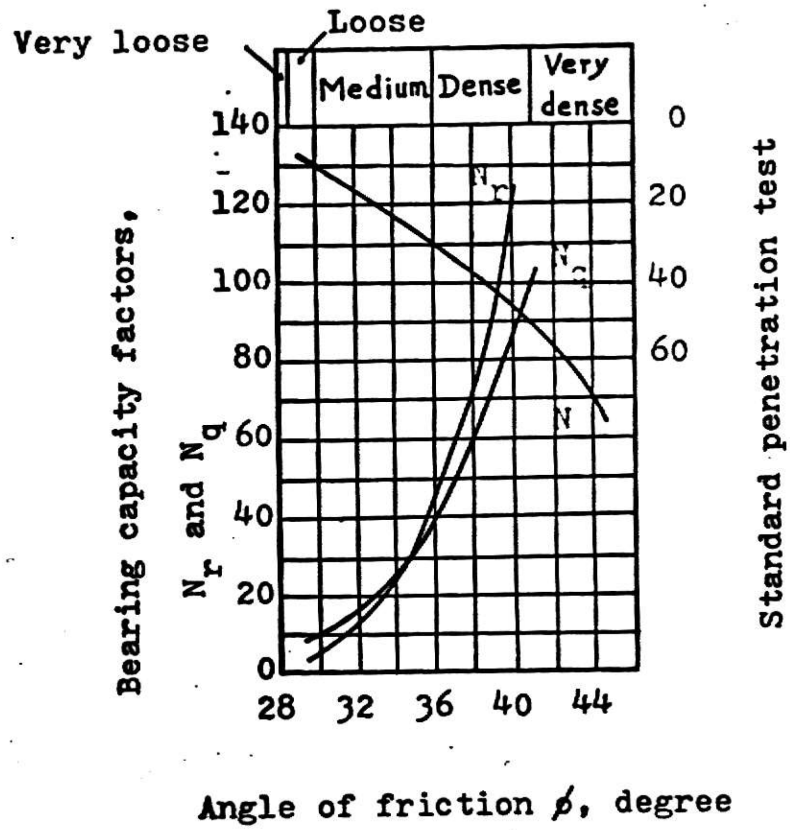
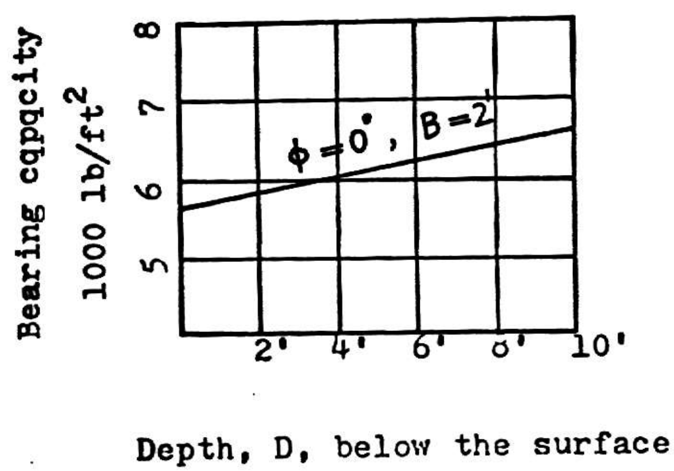



Fig. 25. Relationship between $\phi$, bearing capacity factors and N.




Fig, 26. Infuence of surcharge of clay,
$c = 1400$ lb/ft$^2$, $r = 100$ lb/ft3

(2) Terzaghi's formula

$$q_f = \frac{1}{2} rBN_r + cN_c + rDN_q$$

When $\phi = 0$, then $N_r = 0$, $N_q = 1$, $N_c = 5.7$

$$q_f = 5.7c + rD$$

again if $\phi = 0$, $D = 0$

$$q_f = 5.7c$$

From the result above, we state that for footings below the surface, the bearing capacity $q_f$ is affected both by cohesion c and depth D. Suppose $c = 1400$ lb/ft$^2$, $r = 100$ lb/cf, the relationship between D and $q_f$ can be shown in Figure (26). It appears that $q_f$ is slightly greater below the surface than it is at the surface, but the increase is small and is of minor importance.

If the footing is at the surface, $q_f$ is proportional to the cohesion c.

Both cases as stated above are independent of the footing width B. For clay soil, cohesion c can be determined from unconfined compression strength $q_u$ for $c = q_u/2$.

(3) Krey Method

The bearing capacity of clay soil can be determined by using a similar diagram as shown in Figure (24) for the cohesionless soil except that $\phi=0$, $c\neq0$.

Simplifying equations for calculating passive earth pressure, cohesive force and soil weight are obtained as follows.

$$P_p = 1/2(0.7R+Z)^2 + 2c(0.7R+Z)$$

$$W_1 = 1/4\pi r R^2$$

$$W_2 = r\,(a+0.7R)Z$$

$$W_3 = 3/8 r R^2 \pi$$

$$C_v = 0.7Rc$$

$$C_H = 1.7Rc$$

$$d = 1.275R$$

$$Fv = W_1 + W_2 + W_3 - Cv$$

$$F_H = P_p + C_H$$

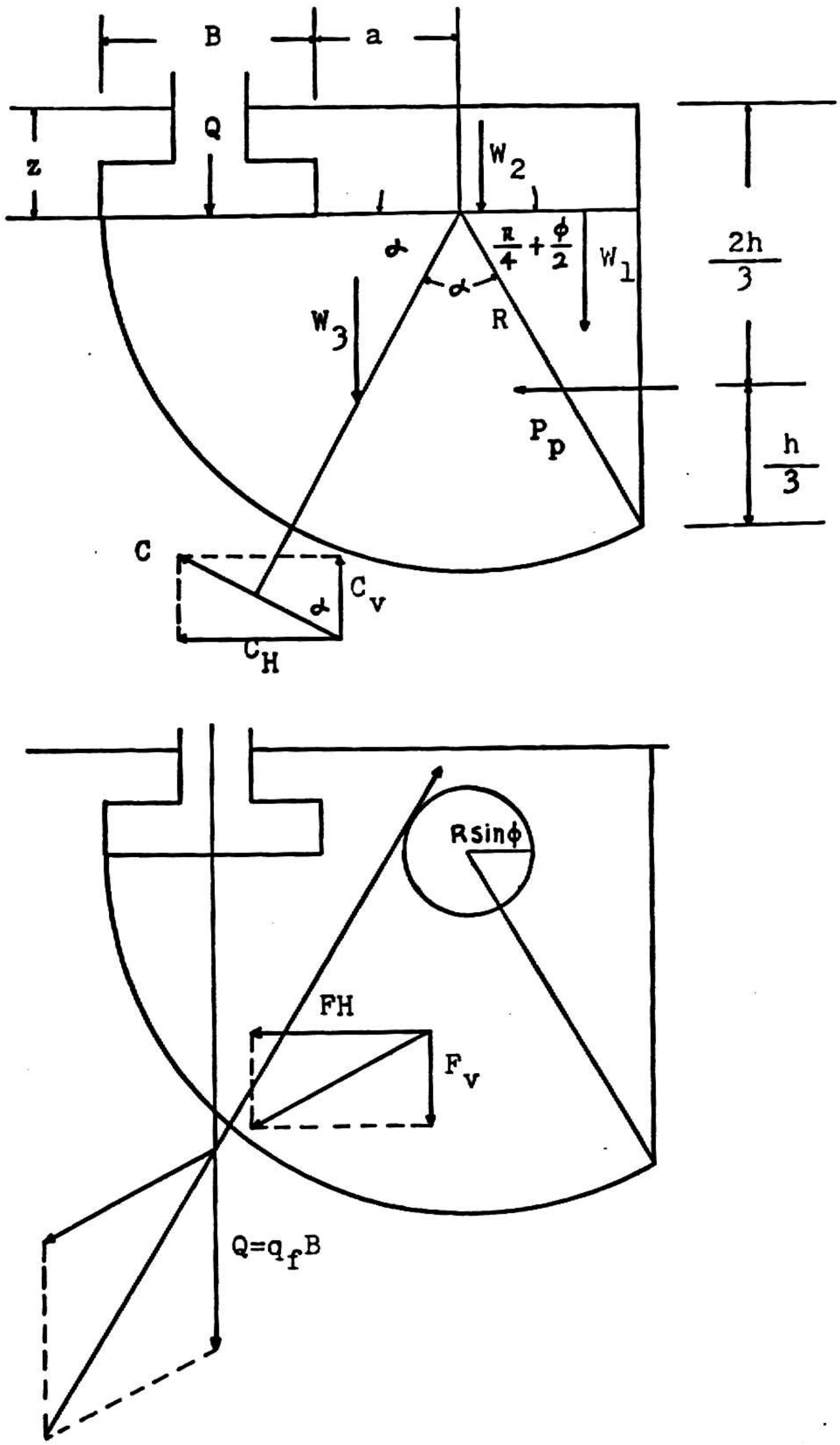



Fig. 24. Forces on soil mass.

(4) Plate Loading Test

Since the bearing capacity of footings on highly cohesive soil is independent of the width of the loaded area, the critical pressure $q_f$ determined by the load test is the same for all footing sizes, i.e.

$$q_f \text{ (foundation)} = q_f \text{ (load test)}$$

2. Numerical Examples

Example 1

Given: A strip footing is laid on a saturated clay, $r = 110 \text{ lb/ft}^3$, $c = 1130 \text{ lb/ft}^2$, if

(1) $B = 5\text{ft}$, $D = 0$

(2) $B = 10\text{ft}$, $D = 0$

(3) $B = 5\text{ft}$, $D = 3\text{ft}$

(4) $B = 10\text{ft}$, $D = 6\text{ft}$

Find: the ultimate bearing capacity

Solution:

From Terzaghi's formula:

$\phi=0, \quad N_q=1, \quad N_c=5.7, \quad N_r=0$

$$q_f=cN_c + rD$$

(1) $q_f = 1,130 \times 5.7 = 6,440 \text{ lb/ft}^2$

(2) $q_f = 1,130 \times 5.7 = 6,440 \text{ lb/ft}^2$

The bearing capacity is independent of the width of the footing.

(3) $q_f = 1,130 \times 5.7 + 110 \times 3 = 6,770 \text{ lb/ft}^2$

(4) $q_f = 1,130 \times 5.7 + 110 \times 6 = 7,100 \text{ lb/ft}^2$

The bearing capacity increases as the depth increases.

Rankine & Bell's equation:

$$q_f = rD + 4c$$

(1) $q_f = 1,130 \times 4 = 4,520 \text{ lb/ft}^2$

(2) $q_f = 1,130 \times 4 = 4,520 \text{ lb/ft}^2$

The bearing capacity is also unaffected by the width of footing.

(3) $q_f = 1{,}130 \times 4 + 110 \times 3 = 4{,}850 \text{ lb/ft}^2$

(4) $q_f = 1{,}130 \times 4 + 110 \times 6 = 5{,}180 \text{ lb/ft}^2$

This shows a little increase due to surcharge.

- Krey Method:

Using equations obtained for clay soil in discussion part, and solving it graphically, we find the bearing capacity as presented in the following.

(1) $P_p = 10{,}450$ lb, $F_v = 920$ lb, $F_H = 21{,}950$ lb

$q_f = 6{,}300 \text{ lb/ft}^2$

(2) $P_p = 20{,}660$ lb, $F_v = 10{,}330$ lb, $F_H = 41{,}860$ lb

$q_f = 6{,}220 \text{ lb/ft}^2$

(3) $P_p = 12{,}360$ lb, $F_v = 2{,}640$ lb, $F_H = 23{,}860$ lb

$q_f = 6{,}550 \text{ lb/ft}^2$

(4) $P_p = 27{,}600$ lb, $F_v = 16{,}070$ lb, $F_H = 48{,}800$ lb

$q_f = 6{,}790 \text{ lb/ft}^2$

Example 2

Given: A long footing resting on a sand weighing 110 $lb/ft^3$, having an angle of internal friction, $\phi = 28^{\circ}$, if

(1) $D = 0$, $B = 4ft$

(2) $D = 0$, $B = 8ft$

(3) $D = 3ft$, $B = 5ft$

(4) $D = 5ft$, $B = 5ft$

(5) $D = 3ft$, $B = 5ft$ and water table is at the ground surface

Find: the ultimate bearing capacity

Solution:

Terzaghi's formula:

When $\phi = 28^{\circ}$, $N_q = 17$, $N_r = 15$

$$q_f = 1/2\ rBN_r + rDN_q$$

(1) $q_f = 1/2 \times 100 \times 4 \times 15 = 3300\ lb/ft^2$

(2) $q_f = 1/2 \times 110 \times 8 \times 15 = 6600\ lb/ft^2$

This shows that $q_f$ is directly proportional to the width of the foundation at the surface.

(3) $q_f = 1/2 \times 110 \times 5 \times 15 + 110 \times 3 \times 17 = 9740 \ lb/ft^2$

(4) $q_f = 1/2 \times 110 \times 5 \times 15 + 110 \times 5 \times 17 = 13500 \ lb/ft^2$

This increase in depth of the footing influences the bearing capacity of sand greatly.

(5) $q_f = 1/2rBN_rW' + rDN_qW$

$= 1/2 \times 5 \times 110 \times 15 \times 0.5 + 110 \times 3 \times 17 \times 0.5$

$= 4870 \ lb/ft^2$

This is half the amount obtained in (3).

Rankine's formula:

$q_f = rD\tan^4(45^\circ + \phi/2)$

(1) $q_f = 0$

(2) $q_f = 0$

This indicates that Rankine's formula is not true for bearing capacity at D = 0, because according to experience, it's not zero at the surface.

(3) $q_f = 110 \times 3 \times \tan^4(45^\circ + 14^\circ) = 2500 \ lb/ft^2$

(4) $q_f = 110 \times 5 \times \tan^4 59^\circ = 4160 \ lb/ft^2$

(5) $q_f = (110-62.4) \times 3 \times \tan^4 59^\circ = 1080 \ lb/ft^2$

A rise of water table to the ground surface decreases about 57% of that at or below the base of the footing.

Krey Method:

(1) $P_p = 2{,}260$ lb, $F_v = 3{,}030$ lb

$$q_f = 3{,}120 \text{ lb/ft}^2$$

(2) $P_p = 10{,}900$ lb, $F_v = 14{,}930$ lb

$$q_f = 5{,}630 \text{ lb/ft}^2$$

(3) $P_p = 10{,}000$ lb, $F_v = 6{,}740$ lb

$$q_f = 10{,}200 \text{ lb/ft}^2$$

(4) $P_p = 15{,}600$ lb, $F_v = 7{,}650$ lb

$$q_f = 14{,}600 \text{ lb/ft}^2$$

(5) $P_p = 4{,}500$ lb, $F_v = 3{,}030$ lb

$$q_f = 4{,}590 \text{ lb/ft}^2$$

Example 3

Given: A footing 6 ft square for a maximum allowable settlement of 1/4 inch. Field plate load tests give the data when settlement of 1/4 inch is developed.

| | W/A | A | P |
|---|---|---|---|
| 12x12 inch plate | 6.5 | 1 | 4 |
| 18x18 inch plate | 4.6 | 2.25 | 6 |
| 24x24 inch plate | 3.7 | 4 | 8 |

Where W, A, P are in kips, $ft^2$, ft respectively.

Find: the bearing capacity

Solution:

$$W = An + Pm$$

$$6.5 \times 1 = n \times 1 + m \times 4$$

$$4.6 \times 2.25 = n \times 2.25 + m \times 6$$

$$3.7 \times 4 = n \times 4 + m \times 8$$

Simultaneous solution of these equations gives the following sets of values,

$$n = 0.77, \quad 0.90, \quad 0.63$$

$$m = 1.43, \quad 1.40, \quad 1.52$$

Take the average value for m, n, the general equation becomes

$$W = 0.77A + 1.45P$$

$$\frac{W}{A} = 0.77 + 1.45\frac{P}{A}$$

or

$$p = 0.77 + 1.45x$$

For the footing to be designed, the perimeter-area ratio $x = 24/36 = 0.67$, thus the bearing capacity

$$p = 0.77 + 1.45 \times 0.67$$
$$= 1.74 \text{ kip/ft}^2$$

## IV. CONCLUSIONS:

For cohesionless soil (sand):

1. The value of $N_r$ and $N_q$ increase rapidly as $\phi$ increases. Since $\phi$ depends to a considerable extent upon the relative density of the sand, but is practically independent of the grain size, the bearing capacity is greatly influenced by the relative density but only slightly by the grain size.

2. A rise of the water table from a depth greater than about B (the width of footing) below the base of the footing up to the top of the surcharge would have the effect of reducing the bearing capacity to about half of its value of moist, dry or saturated sand.

3. In the sand with high angle of internal friction $\phi$, a small amount of surcharge produces a large increase in bearing capacity.

4. The bearing capacity is proportional to the width of the footing at the surface, so an estimation of the bearing capacity of sand by small scale tests can be misleading because the bearing capacity of a full size foundation will be much larger.

For purely cohesive soil (clay):

1. The bearing capacity is practically independent of the width of the footing. Therefore at a given soil pressure large footings on clay are not safer than small ones.

2. For $\phi = 0$, $N_q = 1$, the contribution of D to the bearing capacity is small.

3. The bearing capacity is chiefly a function of cohesion c, at the surface, $q_f = 5.7c$

4. The decrease of cohesion in a rise of water table is of little consequence since the value of bearing capacity is virtually unaffected by the presence of ground water.

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# BEARING CAPACITY OF SHALLOW FOUNDATIONS ON SAND AND ON CLAY

by

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AN ABSTRACT OF A MASTER'S REPORT

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requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

ABSTRACT

The purpose of this report is to present both mathematical and practical methods for the calculation of bearing capacity of footings on sand and clay. It also explains various other factors on the bearing capacity such as a changing water table, surcharge pressure and etc.

The first process is the derivation of formulas based on theories of elastic and plastic equilibrium. A number of simplifying assumptions are used in these theories. The bearing capacity of sand is a function of unit weight , friction angle $\phi$, depth D, and width of footing B, while it is only a function of cohesion c and depth D for clay.

The discussion indicates that the bearing capacity of sand depends to a considerable extent upon the relative density of the sand, since the angle of internal friction $\phi$ is dependent upon the relative density. A rising of water table decreases the bearing capacity by approximate one half that where the water table is at least a depth of B (width of footing) below the base of footing. The bearing capacity in sand is increased greatly as the depth of surcharge increases, and is also directly proportional to the width of the footing.

For clay cohesion dominates the bearing capacity though a small amount of increasing of bearing capacity will

be contributed by surcharge. Since the width B disappears from the formula, the bearing capacity is independent of the width of the footing. The bearing capacity may be easily obtined by setting $c = q_u/2$, where $q_u$ is the unconfined compression strength.