A STUDY OF THE MAXIMUM POWER DESIGN TECHNIQUE FOR DESIGNING HYDRAULIC ACTUATING SYSTEMS

by

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TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

C_d = coefficient of discharge

P_s = supply pressure, psig

P_d = discharge pressure, psig

P_l = pressure (left side), psig

P_r = pressure (right side), psig

F_L = load force, lb_f

M_p = mass of piston, lb_m

M_L = mass of load, lb_m

B_p = damping coefficient of piston, $\frac{lb_f\text{-sec}}{in}$

B_L = damping coefficient of load, $\frac{lb_f\text{-sec}}{in}$

F_c = coulomb type friction, lb_f

K = spring constant, lb_f/in

F_u = external force, lb_f

A = area of piston, in^2

X = valve opening, in

d = diameter of spool, in

Q_l = flow rate (left side), in^3/sec

Q_r = flow rate (right side), in^3/sec

ρ = density of working fluid, $\frac{lb_f\text{-sec}}{in^4}$

β = bulk modulus of working fluid, lb_f/in^2

NOMENCLATURE (Contd.)

X_{max} = maximum valve opening, in

C = piston travel, in

L = length of cylinder, in

b = piston width, in

$\dot{C}$ = velocity of piston, in/sec

$\ddot{C}$ = rate of change of velocity of piston, in/sec^2

HP = horse power

C_o = initial position of piston from reference surface, in

C_{max} = amplitude of load motion, in

ω_d = design frequency of load motion, rad/sec

V_v = volume of valve, in^3

V_l = volume of transmission lines, in^3

$\bar{P}_l$ = dimensionless pressure (left side)

$\bar{P}_r$ = dimensionless pressure (right side)

$\bar{F}$ = dimensionless force on piston

$\bar{\dot{C}}$ = dimensionless piston velocity

$\overline{HP}$ = dimensionless horse power

$\bar{X}$ = dimensionless valve opening

$\bar{Q}_l$ = dimensionless flow rate (left side)

$\bar{Q}_r$ = dimensionless flow rate (right side)

h = time interval in computer solution

CHAPTER I

INTRODUCTION

Before electric power was invented, hydraulic power (water as the working fluid) was widely used in industries which used steam driven engines and mills. The ease of transmitting electric power from generating station to farthest point left the hydraulic power far behind in the race. Only for last few decades the hydraulic power (oil as the working fluid) is again put to numerous uses. Developments and research in the field of hydraulic power started again at rapid rate when demand power and types of performances which are impossible to obtain with the use of electric power grew more and more. The present demand for large power and high speed of response drew the attention of scientists and engineers towards the development of hydraulic power. The present sophisticated aircraft and numerous defense applications require large power to weight ratio and high frequency response. In case of earth moving equipment and numerous other applications it may be necessary to hold the load at a particular position for some length of time. Among all the presently available techniques hydraulic power is the best suited for these applications. The hydraulic power system has some disadvantages but the advantages in most applications outweigh the disadvantages.

Generally, the hydraulic power system can be divided into four major segments. The pressure generation or power input segment, the power transmission segment, the power control segment and the power output segment. The power input segment consists of pump and accumulators, power transmission segment consists of tubing etc., power control segment consists of directional, pressure, or flow control valves and power output segment consists of linear or rotary actuator, or hydraulic motor.

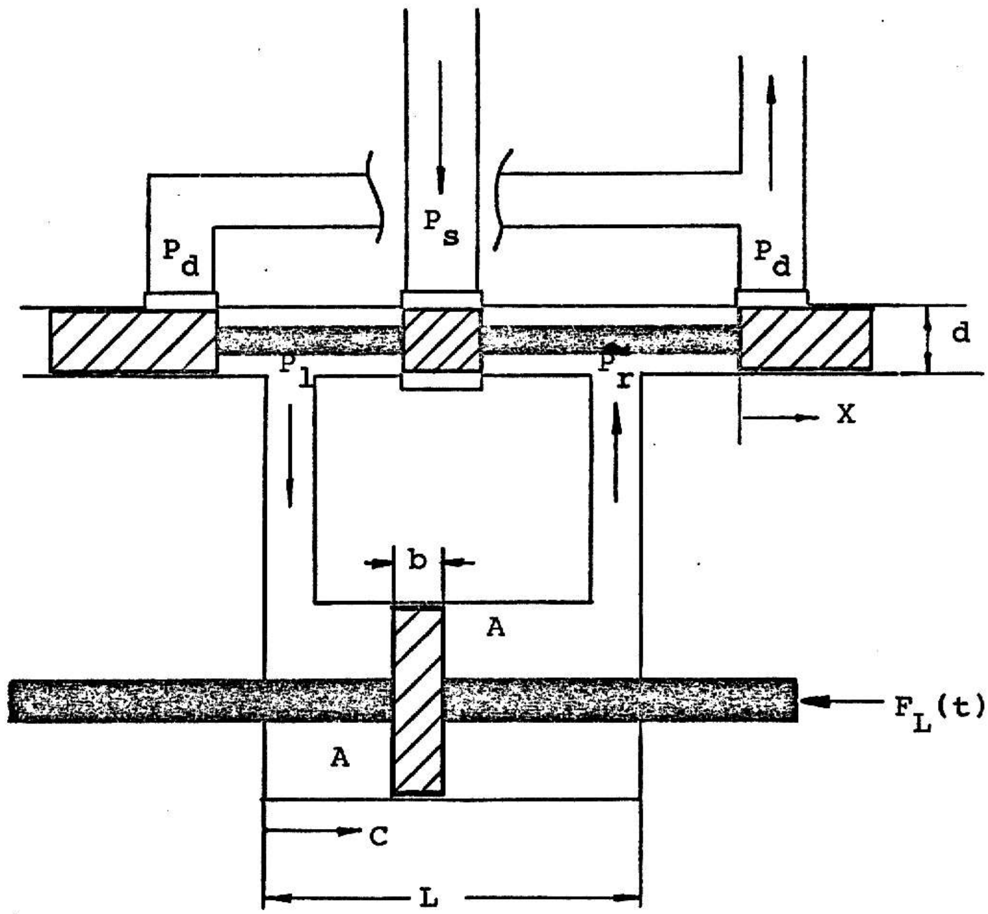
The purpose of this report is to study the use of the <u>maximum power design technique</u> to design the directional - type valve power control segment and the linear actuator type power output segment and to make some inference that if a system is designed using this technique then how closely will the actual system perform to desired design performance specifications? In this report a zero lapped, four-way, directional-type control valve and double-acting double-ended piston-type actuator has been used for the purpose of studying the maximum power design technique.

CHAPTER II

DERIVATION OF DESIGN EQUATIONS

The four-way zero lapped directional type control valve and linear actuator chosen for study is shown in Figure 1. The valve shown in the figure is spool-type valve, the actuator is double-acting double-ended piston-type actuator and in general the load could consist of mass, viscous damping, coulomb friction, spring and external force. The system shown here is an open loop system. The maximum power design technique considered in this report is used to determine the spool diameter d, the maximum valve opening X_{max} and piston area A. The technique derives its name due to the fact that design is based on the maximum power the load will require for a sinusoidal motion of amplitude and frequency, determined by the dynamic performance requirement desired for the designed system. For example, the frequency selected will be related to desired bandwidth if the valve and actuator are to become part of a closed loop servo system.

Operation of the actuating system is as follows. The load initially is at rest when valve opening X is zero. Now assume a sinusoidal motion is applied to X. As X increases in one direction, say to the right, the flow rates through the valve openings change and the pressure at the left side, and also at the right side, of the piston changes. This



In General

$$F_L(t) = M_L\ddot{C} + B_L\dot{C} + F_c\frac{\dot{C}}{|\dot{C}|} + KC + F_u$$

Figure 1. Four-way Zero-lapped and Double Acting Actuator

pressure difference exerts a force on piston and load starts to move towards right. (Since the system is not linear, the load motion C(t), will not be exactly sinusoidal for sinusoidal X(t). But in using the maximum power design technique this motion is assumed to be sinusoidal). Let the velocity at some instant be $\dot{C}$. As the valve opening goes on increasing the pressure at the left side and the right side of the piston changes. This change in pressure changes the force F acting on ram which in turn changes the velocity $\dot{C}$ at which ram moves. A graph can be plotted between force on the piston at any instant and corresponding velocity $\dot{C}$ at that instant. This graph is called load locus. Figure 2. shows a typical load locus. The dotted curve in Figure 2. is a constant power curve, $HP = F\dot{C}$ = constant, which is a hyperbola. A family of constant power curves can be plotted for different values of horsepower. One such constant power curve will touch the load locus at a single point. This point is the maximum power point.

Summary of Maximum Power Design Technique

Steps required in applying the maximum power technique can be summarized as follows:

1. Using equations for valve flow-pressure and for force acting on piston, develop velocity-force equation for valve-actuator system. Determine equations for spool diameter d, maximum valve opening X_{max} and piston area A, based on maximum

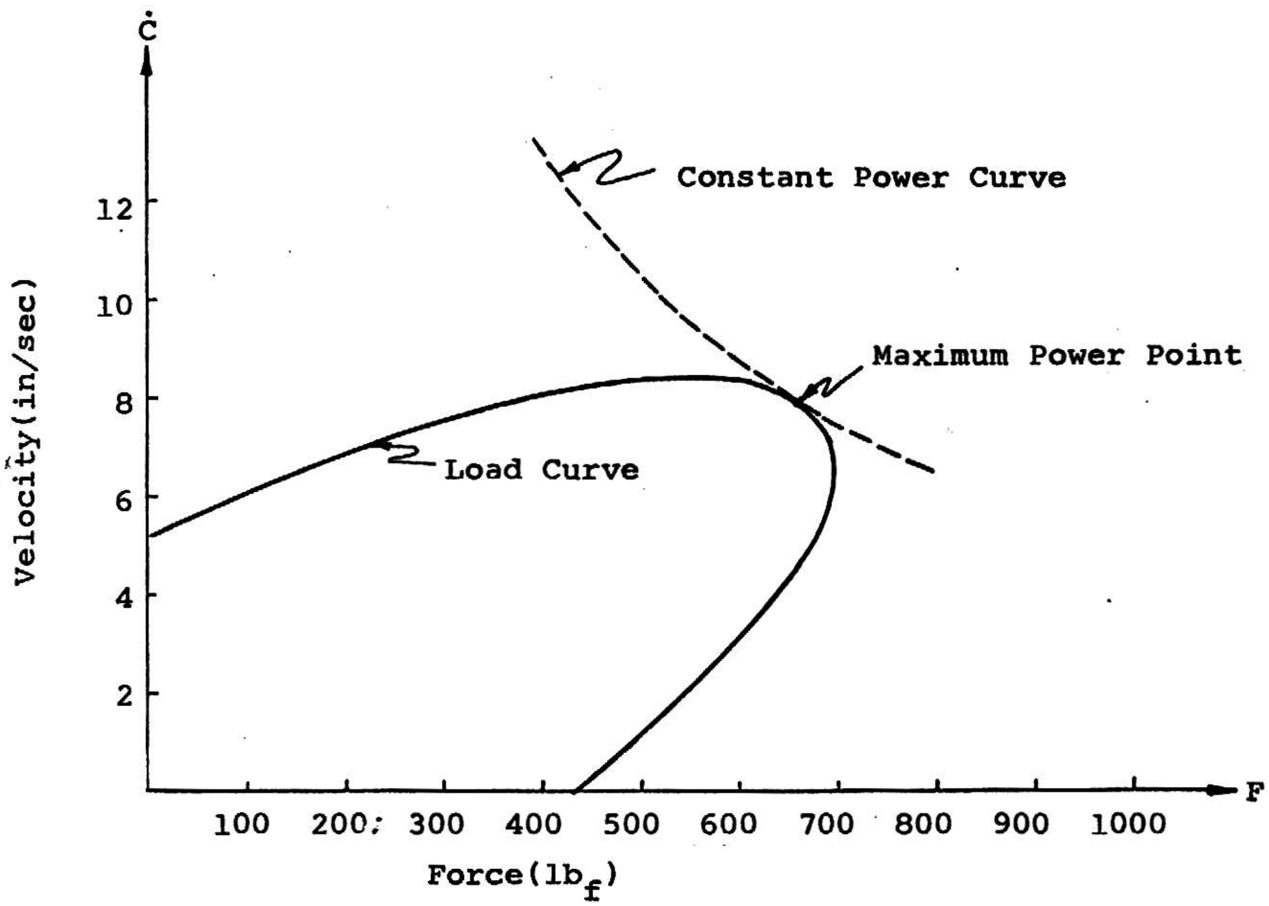


Figure 2. Velocity-Force Curve

power point, from these equations.

2. Assume sinusoidal load motion, $C = C_o + \frac{C_{max}}{2} \sin\omega_d t.$, with specified design values for C_{max} and ω_d. Then substitute the derivatives of C in the force equation,

$$F_L = (M_p + M_L)\ddot{C} + (B_p + B_L)\dot{C} + F_c \frac{\dot{C}}{|\dot{C}|} + KC + F_u$$

This will give F for any value of time. Now construct the velocity-force characteristics for load and use this to determine point of maximum power required to drive the load with the assumed sinusoidal motion.

3. Substitute the values of $\dot{C}$ and F obtained for the maximum power point in step 2 into the equations for d, X_{max} and A derived in step 1 and calculate values of d, X_{max} and A.

Assumptions for Derived Equations

In order to derive the valve flow-pressure equations and actuator force-velocity equations the following assumptions have been made.

P_s = const.

C_d = 0.625 = const. (same for all flow areas)

P_d = 0 psig = const.

Leakage is negligible

Valve body and cylinder walls are rigid

Transmission lines are rigid

Pressure drop in lines is negligible.

Valve Flow-Pressure Equations

1. Case I. $X>0$

   a. Left hand side $P_s>P_1$

$$Q_1 = -C_d \pi d X \sqrt{\frac{2}{\rho}(P_s - P_1)} \tag{1}$$

   b. Left hand side $P_s<P_1$

$$Q_1 = -C_d \pi d X \sqrt{2(P_1 - P_s)/\rho} \tag{2}$$

   c. Right hand side $P_r>0$

$$Q_r = -C_d \pi d X \sqrt{2 P_r/\rho} \tag{3}$$

   d. Right hand side $P_r<0$

$$Q_r = C_d \pi X \sqrt{2 |P_r| /\rho} \tag{4}$$

2. Case II. $X<0$

   a. Right hand side $P_s>P_r$

$$Q_r = -C_d \pi d X \sqrt{2(P_s - P_r) /\rho} \tag{5}$$

   b. Right hand side $P_s<P_r$

$$Q_r = C_d \pi d X \sqrt{2(P_r - P_s)/\rho} \tag{6}$$

c. Left hand side; $P_l > 0$

$$Q_l = C_d \pi d X \sqrt{2 P_l / \rho} \tag{7}$$

d. Left hand side; $P_l < 0$

$$Q_l = -C_d \pi d X \sqrt{2 |P_l| / \rho} \tag{8}$$

Now, to make the equations(1) through (8) dimensionless define:

$$Q' = C_d \pi d X_{max} \sqrt{2 P_s / \rho} \tag{9}$$

also let

$$Q_l / Q' = \bar{Q}_l$$

$$Q_r / Q' = \bar{Q}_r$$

$$P_l / P_s = \bar{P}_l$$

$$P_r / P_s = \bar{P}_r$$

and

$$X / X_{max} = \bar{X}$$

Dimensionless Valve Flow-Pressure Equations

Dividing equations(1) through (8) by Q'

1. Case I. $X>0$

a. if $P_s>P_l$

$$\bar{Q}_l = \bar{X} \sqrt{1 - \bar{P}_l} \tag{10}$$

b. if $P_s<P_l$

$$\bar{Q}_l = -\bar{X} \sqrt{\bar{P} - 1} \tag{11}$$

c. if $P_r>0$

$$\bar{Q}_r = -\bar{X} \sqrt{\bar{P}_r} \tag{12}$$

d. if $P_r<0$

$$\bar{Q}_r = \bar{X} \sqrt{\bar{P}_r} \tag{13}$$

2. Case II. $X<0$

a. if $P_s>P_r$

$$\bar{Q}_r = -\bar{X} \sqrt{1 - \bar{P}_r} \tag{14}$$

b. if $P_s<P_r$

$$\bar{Q}_r = \bar{X} \sqrt{\bar{P}_r - 1} \tag{15}$$

c. if $P_l>0$

$$\bar{Q}_l = \bar{X} \sqrt{\bar{P}_l} \tag{16}$$

d. if $P_1 < 0$

$$\overline{Q}_1 = -\overline{X}\sqrt{|\overline{P}_1|} \tag{17}$$

Velocity-Force Equation

Taking summation of the forces acting on piston

$$A\,P_1(t) - A\,P_r(t) - M_p\ddot{C} - B_p\dot{C} - F_L(t) = 0 \tag{18}$$

Sum of the forces on load

$$F_L(t) - M_L\ddot{C} - B_L\dot{C} - F_c\frac{\dot{C}}{|\dot{C}|} - F_u(t) - KC = 0 \tag{19}$$

Solving equation (19) for F_L and substituting this value of F_L in equation (18) gives

$$\begin{aligned} &A\,P_1(t) - A\,P_r(t) - (M_p + M_L)\ddot{C} - (B_p + B_L)\dot{C} \\ &- F_c\frac{\dot{C}}{|\dot{C}|} - F_u(t) - KC = 0 \end{aligned} \tag{20}$$

Let $F(t) = (M_p + M_L)\ddot{C} + (B_p + B_L)\dot{C} + F_c\dfrac{\dot{C}}{|\dot{C}|} + F_u(t) + KC$ (21)

Therefore equation (20) becomes

$$A\,P_1(t) - A\,P_r(t) = F(t) \tag{22}$$

In this report it will be assumed that $F_c\dfrac{\dot{C}}{|\dot{C}|}$ is small compared to inertia and viscous damping and can be neglected.

Further it will be assumed that there is no external force ($F_u = 0$) and no spring ($K = 0$). Then $F(t)$ becomes

$$F(t) = (M_p + M_L)\ddot{C} + (B_p + B_L)\dot{C} \tag{23}$$

Therefore equation (22) can now be written as

$$A\ P_l(t) - A\ P_r(t) = (M_p + M_L)\ddot{C} + (B_p + B_L)\dot{C} \tag{24}$$

Dimensionless Velocity-Force Equation

To make equation (22) dimensionless define

$$A\ P_s = F'$$

$$F/F' = \overline{F}$$

Divide equation (22) by $F' = A\ P_s$

then

$$\overline{P}_l - \overline{P}_r = \overline{F} \tag{25}$$

Control Volume Flow (Left side)

Consider a control volume on left side

$$- Q_l\rho + \frac{\partial}{\partial t}\ \rho\ (V_v + V_l + AC) = 0$$

or

$$- Q_l\rho + \rho A\dot{C} + (V_v + V_l + AC)\ \frac{\dot{P}_l\,\rho}{\beta} = 0$$

where

$$\dot{\rho} = \frac{\rho \dot{P}_1}{\beta}$$

Assuming $\left[\frac{(V_v + V_1 + AC)}{\beta}\right]^{-1} >> \dot{P}_1$, i.e. neglecting compressibility effect since β is 200,000 or greater

$$-Q_1 + A\dot{C} = 0$$

$$Q_1 = A\dot{C} \tag{26}$$

To make equation (26) dimensionless, let

$$Q' = A\dot{C}'$$

Dividing equation (26) by $Q' = A\dot{C}'$ gives

$$\overline{Q}_1 = \overline{\dot{C}} \tag{27}$$

Control Volume Flow (Right Side)

Consider control volume on right hand side

$$\rho Q_r + \frac{\partial}{\partial t}\left[V_v + V_1 + A\ (L - C - b)\right]\rho = 0$$

or

$$\rho Q_r + \rho A(-\dot{C}) + \left[V_v + V_1 + A(L - C - b)\right] \frac{\dot{P}_r \rho}{\beta} = 0$$

where

$$\dot{\varrho} = \frac{\varrho \dot{P}_r}{\beta}$$

Assuming $\left[\frac{V_v + V_l + A(L-C - b)}{\beta}\right]^{-1} >> \dot{P}_r$

then

$$Q_r = A\dot{C} \tag{28}$$

To make equation (28) dimensionless divide $Q' = A\dot{C}'$

Therefore

$$\overline{Q}_r = \overline{\dot{C}} \tag{29}$$

Condition for Maximum Power

Substituting the values of $\overline{Q}_l$ and $\overline{Q}_r$ from equations (10) and (12) in equations (28) and (29) gives

$$\overline{Q}_l = \overline{X} \sqrt{1- \overline{P}_l} = \overline{\dot{C}} \qquad (\text{for } \overline{X} > 0) \tag{30}$$

$$\overline{Q}_r = -\overline{X}\sqrt{\overline{P}_r} = \overline{\dot{C}} \qquad (\text{for } \overline{X} > 0) \tag{31}$$

solving equation (30) for $\overline{P}_l$, gives

$$\overline{P}_l = 1 - \frac{\overline{\dot{C}}^2}{\overline{X}^2} \tag{32}$$

Solving equation (31) for $\overline{P}_r$

$$\overline{P}_r = \frac{\overline{C}^2}{\overline{X}^2} \tag{33}$$

Substituting values of $\overline{P}_l$ and $\overline{P}_r$ from equations (32) and (33) into equation (25) gives

$$1 - \frac{\overline{C}^2}{\overline{X}^2} - \frac{\overline{C}^2}{\overline{X}^2} = \overline{F} \tag{34}$$

Solving equation (34) for $\overline{C}$ gives

$$\overline{C} = \overline{X}\sqrt{\frac{1 - \overline{F}}{2}} \qquad (\overline{X} > 0) \tag{35}$$

Dimensionless power can be defined as

$$\overline{HP} = \overline{F}\,\overline{C}$$

or

$$\overline{HP} = \overline{F}\,\overline{X}\sqrt{\frac{1 - \overline{F}}{2}}$$

Assume maximum power is developed when $X = X_{max}$, then for zero lapped valve $\overline{X}_{max} = 1$. Therefore

$$\overline{HP} = \sqrt{\frac{\overline{F}^2(1 - \overline{F})}{2}}$$

Now maximize power with respect to force

$$\frac{d\,\overline{HP}}{d\,\overline{F}} = \frac{1}{2}\left[\frac{2}{\overline{F}^2(1-\overline{F})}\right]^{\frac{1}{2}}(2\overline{F} - 3\overline{F}^2) = 0$$

This gives

$$\overline{F} = 2/3 \;@\; \overline{HP} = \overline{HP}_{max} \tag{36}$$

Deriving Design Equation for d, X_{max} and A

$$\overline{F}_{(HP\,=\,max)\ actuator} = \frac{F_{(HP\,=\,max)\ actuator}}{P_s\ A}$$

Substituting the value of $\overline{F}$ for $\overline{HP}$ = max from equation (36) gives

$$\frac{2}{3} = \frac{F_{(HP\,=\,max)\ actuator}}{P_s\ A}$$

or

$$A = \frac{3\ F_{(HP\,=\,max)\ actuator}}{2\ P_s}$$

If load is directly connected to piston, then

$$F_{(HP\,=\,max)\ actuator} = F_{(HP\,=max)\ load}$$

therefore

$$A = \frac{3F_{(HP\,=\,max)\ load}}{2\ P_s} \tag{37}$$

Equation (37) is the design equation for calculating the size of piston required. Substituting value of $\overline{F}$ from equation (36) in equation (35) ($\overline{X}$ = 1) gives

$$\bar{\dot{C}}_{(\overline{HP} = max)\ actuator} = \sqrt{1/6}$$

but

$$\bar{\dot{C}} = A\dot{C}/Q'$$

or

$$\frac{A\dot{C}_{(HP = max)\ actuator}}{Q'} = \sqrt{1/6}$$

or, substituting equation (9) for Q' and solving for X_{max} gives

$$X_{max} = \frac{\sqrt{6}\ A\dot{C}_{(HP = max)\ actuator}}{C_d\ \pi\ d\ \sqrt{\frac{2}{\rho}\ P_s}}$$

If load is directly connected to piston, then

$$\dot{C}_{(HP = max)\ actuator} = \dot{C}_{(HP = max)\ load}$$

therefore

$$X_{max} = \frac{\sqrt{6}\ A\dot{C}_{(HP = max)\ load}}{C_d\ \pi\ d\ \sqrt{\frac{2}{\rho}\ P_s}} \qquad (38)$$

This is the design equation for calculating X_{max}. Note that a value for d must be assumed before X_{max} can be calculated.

Equations for Plotting Load Locus

From equation (23)

$$F = (M_p + M_L)\ddot{C} + (B_p + B_L)\dot{C} \tag{23R}$$

Assuming that load moves sinusoidally with amplitude equal to C_{max} and frequency equal to ω_d.

Hence

$$C = C_o + \frac{C_{max}}{2} \sin \omega_d t \tag{39}$$

Differentiating equation (39) gives

$$\dot{C} = \frac{C_{max}}{2} \omega_d \cos \omega_d t \tag{40}$$

and

$$\ddot{C} = - \frac{C_{max}}{2} \omega_d^{\ 2} \sin \omega_d t \tag{41}$$

Substituting these values of $\ddot{C}$ and $\dot{C}$ in equation (23) gives

$$F = -(M_p + M_L)\left(\frac{C_{max}}{2}\omega_d^2 \sin \omega_d t\right) + (B_p + B_L)\frac{C_{max}}{2}\omega_d \cos \omega_d t \qquad (42)$$

If the values of M_p, M_L, B_L, ω_d and C_{max} are known the force F can be calculated from equation (42) and $\dot{C}$ from equation (40) for different values of time t: these values of F and $\dot{C}$ can now be plotted to obtain a graph known at the load locus.

Now the design equations have been derived and a method to construct load locus has been discussed. These equations will now be applied (hence maximum power design technique) to an example problem consisting of mass and viscous damper. Also since the compressibility effects were neglected in getting the design equations an attempt will be made to determine limitations, if any, when this assumption is made.

CHAPTER III

APPLYING MAXIMUM POWER TECHNIQUE TO EXAMPLE PROBLEM

In Chapter II the design equations were derived and procedure to plot load locus was discussed. Here in this chapter an example problem will be assumed and the values of maximum valve opening X_{max}, spool diameter d and piston area A will be calculated for the example problem.

Constructing Load Locus for Example Problem

Assume that for the example problem

$$C_{max} = 0.5 \text{ inches}$$

$$\omega_d = 10\,\pi \text{rad/sec}$$

$$M_L = 575 \text{ lb}_m$$

$$M_p = 25 \text{ lb}_m$$

$$B_L = 90 \frac{\text{lb}_f - \text{sec}}{\text{in}}$$

$$B_p = 10 \frac{\text{lb}_f - \text{sec}}{\text{in}}$$

Also assume

$$P_s = 1000 \text{ psig} = \text{const.}$$

$$C_d = 0.625$$

Note: The values of M_p and B_p are not known when design is started. Therefore some reasonable values of M_p and B_p have to be assumed. If assumed values prove to be unsatisfactory for an actual physical design they can be changed and the problem can be reworked.

Now substituting the values of C_{max}, ω_d, M_p, M_L, B_p and B_L in equations (40) and (41) gives

$$\dot{C} = (0.5/2)\ (10\ \pi)\cos\ (10\ \pi\ t) \tag{42}$$

and

$$F = -(\frac{600}{386}\ \frac{0.5}{2})(10\ \pi)^2 \sin\ (10\ \pi\ t) + (100)\ (\frac{0.5}{2})$$
$$(10\ \pi)\cos\ (10\ \pi\ t) \tag{43}$$

These two equations can now be used to calculate the values of F and $\dot{C}$ for different values of t. A graph (load locus) can now be plotted for these values of F and $\dot{C}$. Figure 3 shows the load locus for the example problem. From the load locus the maximum power point can now be obtained by plotting a constant power curve, HP = $F\dot{C}$ = const., which is tangent to load locus plot at one point. This point gives HP_{max} (load) which establishes values for $F(HP = max)_{load}$ and $\dot{C}$ (HP = max) $_{load}$ to be used in design equations (37) and (38). In equation (38) there are two unknown, X_{max} and d. Any combination of

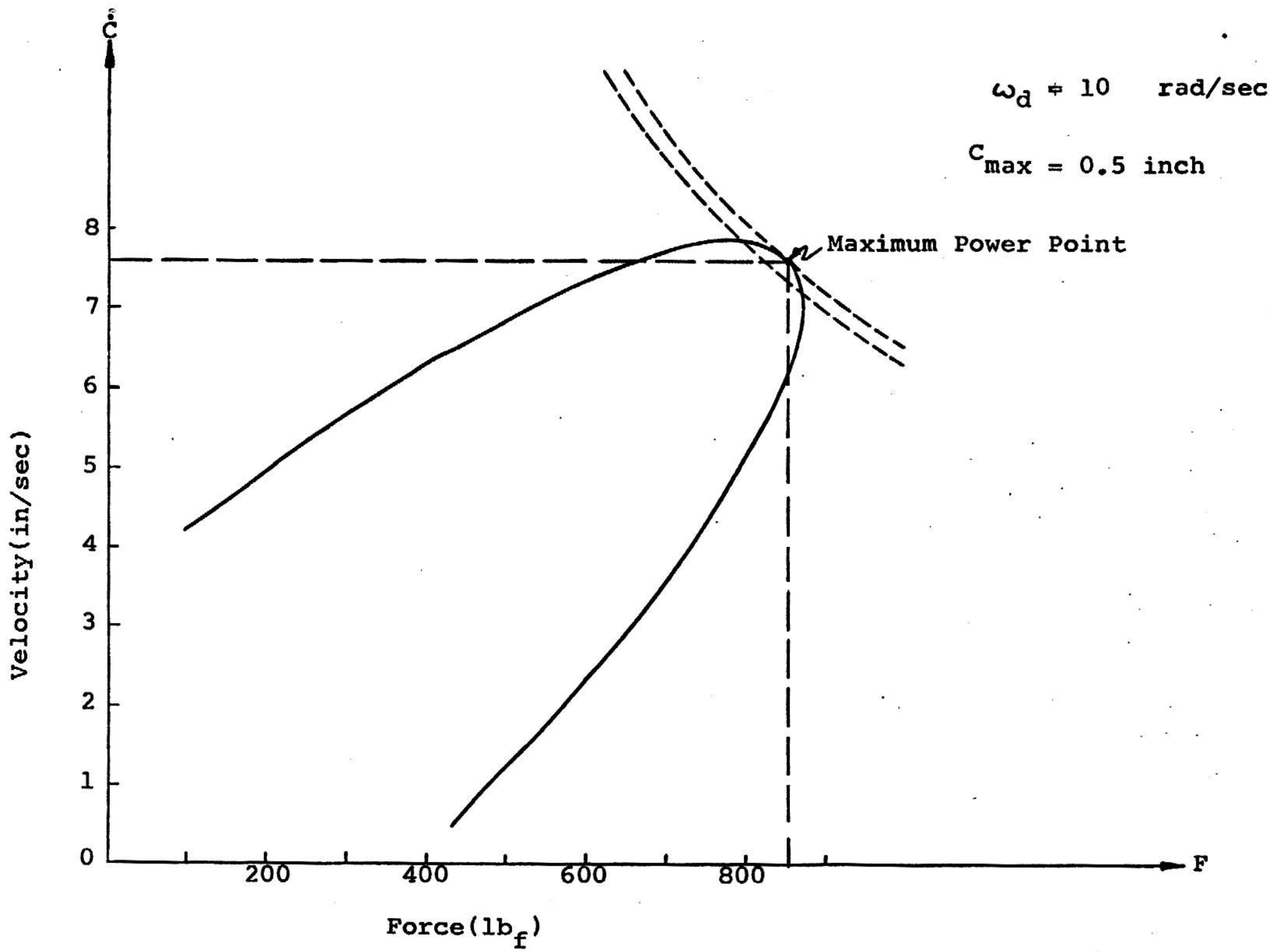


Figure 3. Velocity-Force Curve For Example Problem

X_{max} and d which satisfy equation (38) will give a valve of correct size.

In choosing the valve spool dimensions the following condition should be kept in mind.

Spool diameter d is inversely proportional to X_{max}. Therefore if X_{max} is large d will be small. But due to fine surface finish required for spool and the mating surfaces of valve body, d should probably not be chosen less than 0.25 inch in order to facilitate its fabrication.

Design Calculation for Example Problem

From Fig. 3 at maximum power point

$$F(HP = max)_{load} = 856.0 \text{ lb}_f$$

$$\dot{C}(HP = max)_{load} = 7.6 \text{ in/sec}$$

1. Piston Area

Substituting the value for $F(HP = max)_{load}$ in equation (37) and specifying P_s = 1000 psig as the system supply pressure to be used, the piston area A can be calculated as

$$A = \frac{3x856}{2x1000} = 1.284 \text{ in}^2$$

2. Valve Size

Substituting the calculated value of A and the value of $\acute{C}(HP = max)_{load}$ in equation (38) gives

$$X_{max} = \frac{\sqrt{6} \times 1.284 \times 7.6}{0.625 \times 3.14 \times \sqrt{2000/\rho} \times d}$$

For the example problem the value of ρ was assumed to be

$$\rho = 8.13 \times 10^{-5} \frac{lb_f - sec^2}{in^4}$$

and if diameter of spool is taken as d = 0.25 in, then

$$X_{max} = 0.00982 \text{ in}$$

This is an appropriate value for X_{max} if the valve and piston are to be used in a closed loop servo system. The reason is that in such a system the displacement X is generated by a force motor which is limited to displacements on the order of plus and minus 0.010 inches.

CHAPTER IV

EVALUATION OF MAXIMUM POWER DESIGN TECHNIQUE WHEN APPLIED TO EXAMPLE PROBLEM

In Chapter II, the maximum power technique used for establishing the design point was discussed and the design equations to calculate the piston area, spool diameter and the maximum valve opening were derived. In implementing this technique it was necessary to neglect the compressibility of oil. Therefore a prime question is, will a valve-actuator designed (using this technique) to drive a given load with a specified sinusoidal load motion, be able to reproduce the specified load motion? Here in this chapter an attempt is made to answer this question by solving the dynamic equations for a "designed" valve-actuator system using a digital computer.

Derivation of Dynamic Equations Using Control Volumes

1. Consider a control volume for the left hand side of the valve-actuator system of Figure 1. For this control volume, using continuity equation and assuming $X > 0$

$$- Q_1\rho + \frac{\partial}{\partial t} \rho\,(V_v + V_1 + AC) = 0$$

or it can be shown

$$Q_l = A\dot{C} + \frac{1}{\beta}(V_v + V_l + AC)\,\dot{P}_l \qquad (43)$$

2. Similarly consider a control volume for the right hand side. Using continuity equation and assuming $X>0$.

$$\rho Q_r - \frac{\partial}{\partial t}\left[\rho V_v + V_l + A(L - b - C)\right] = 0$$

or it can be shown

$$Q_r = -A\dot{C} + \left[V_v + V_l + A(L - b - C)\right]\frac{\dot{P}_r}{\beta} \qquad (44)$$

Equations (43) and (44) are the general equations. For $X>0$, Q_l will be positive and Q_r will be negative, for $X = 0$, Q_l and Q_r will both be zero and for $X<0$ Q_l will be negative and Q_r will be positive. The values of Q_l and Q_r, from equations (1) through (8) can now be substituted in equations (43) and (44) to obtain all the dynamic equations for different cases.

Combining Control Volume and Valve Flow Equations

1. Case I, ($X>0$, $P_s>P_l$)

   a. Left hand side

$$C_d \, \pi \, d \, X \sqrt{\frac{2}{\rho} (P_s - P_1)}$$

$$= A\dot{C} + \frac{1}{\beta} (V_v + V_1 + AC) \, \dot{P}_1 \tag{45}$$

b. Right hand side ($P_r > 0$)

$$-C_d \, \pi \, d \, X \sqrt{\frac{2}{\rho} P_r}$$

$$= -A\dot{C} + \frac{1}{\beta} \left[V_v + V_1 + A(L - b - C) \right] \dot{P}_r \tag{46}$$

c. Right hand side ($P_r < 0$)

$$C_d \, \pi \, d \, X \sqrt{\frac{2}{\rho} |P_r|}$$

$$= -A\dot{C} + \frac{1}{\beta} \left[V_v + V_1 + A(L - b - C) \right] \dot{P}_r \tag{47}$$

2. Case II, ($X > 0$, $P_s < P_1$)

a. Left hand side

$$-C_d \, \pi \, d \, X \sqrt{\frac{2}{\rho} (P_1 - P_s)}$$

$$= A\dot{C} + \frac{1}{\beta} (V_v + V_1 + AC) \, \dot{P}_1 \tag{48}$$

b. Right hand side ($P_r>0$)

$$-C_d \pi d X \sqrt{\frac{2}{\rho} P_r} =$$

$$-A\dot{C} + \frac{1}{\beta}\left[V_v + V_1 + A(L - b - C)\right] \dot{P}_r \tag{49}$$

c. Right hand side ($P_r<0$)

$$C_d \pi d X \sqrt{\frac{2}{\rho} |P_r|} =$$

$$-A\dot{C} + \frac{1}{\beta}\left[V_v + V_1 + A(L - b - C)\right] \dot{P}_r \tag{50}$$

3. Case III, ($X<0$, $P_s>P_r$)

a. Left hand side ($P_1>0$)

$$C_d \pi d X \sqrt{\frac{2}{\rho} P_1} =$$

$$A\dot{C} + \frac{1}{\beta} (V_v + V_1 + AC) \dot{P}_1 \tag{51}$$

b. Left hand side ($P_1<0$)

$$-C_d \pi d X \sqrt{\frac{2}{\rho} |P_1|} =$$

$$A\dot{C} + \frac{1}{\beta} (V_v + V_1 + AC) \dot{P}_1 \tag{52}$$

c. Right hand side

$$-C_d \pi d X \sqrt{\frac{2}{\rho}(P_s - P_r)} =$$

$$-A\dot{C} + \frac{1}{\beta}\left[V_v + V_l + A(L - b - C)\right] \dot{P}_r \tag{53}$$

4. Case IV, $(X<0,\ P_s<P_r)$

a. Left hand side $(P_l>0)$

$$C_d \pi d X \sqrt{\frac{2}{\rho} P_l} =$$

$$A\dot{C} + \frac{1}{\beta}(V_v + V_l + AC) \dot{P}_l \tag{54}$$

b. Left hand side $(P_l<0)$

$$-C_d \pi d X \sqrt{\frac{2}{\rho}|P_l|} =$$

$$A\dot{C} + \frac{1}{\beta}(V_v + V_l + AC) \dot{P}_l \tag{55}$$

c. Right hand side

$$C_d \pi d X \sqrt{\frac{2}{\rho}(P_r - P_s)} =$$

$$-A\dot{C} + \frac{1}{\beta}\left[V_v + V_l + A(L - b - C)\right] \dot{P}_r \tag{56}$$

5. Case V, X = 0

a. Left hand side

$$0 = A\dot{C} + \frac{1}{\beta}(V_v + V_1 + AC)\,\dot{P}_1 \tag{57}$$

b. Right hand side

$$0 = -A\dot{C} + \frac{1}{\beta}\left[V_v + V_1 + A(L - b - C)\right]\dot{P}_r \tag{58}$$

Summation of Forces Acting on Piston

Forces acting on Piston, including load consisting of mass and viscous damping, are:

$$A\,P_1(t) - A\,P_r(t) = (M_p + M_1)\ddot{C} + (B_p + B_1)\dot{C} \tag{59}$$

Dynamic Response of Actuating System Designed For Example Problem

For the purpose of proceeding with evaluation of the maximum power design technique, the example problem considered in Chapter III will be used here.

1. Specified design values for example problem

$$M_p + M_L = 600 \ lb_m$$

$$B_p + B_L = 100 \ \frac{lb_f - sec}{in}$$

$$C_o = 0.5 \ in$$

$$C_{max} = 0.5 \ in$$

$$\omega_d = 10\pi rad/sec$$

2. Assumed values for example problem

$$V_v + V_l = 100 \ cu. \ in$$

$$L = 4 \ in$$

$$b = 0.5 \ in$$

$$\rho = 8.13 \times 10^{-5} \ \frac{lb_f - sec^2}{in^2}$$

$$\beta = 365{,}000 \ lb_f/in^2$$

3. Calculated values for example problem

$A = 1.284 \text{ in}^2$

$d = 0.25 \text{ in}$

$X_{max} = 0.00982 \text{ in}$

Maximum Power = 0.981 H.P.

A complete computer program to calculate C, $\dot{C}$, P_l, P_r and force, $F = A P_l - A P_r$, is given in Appendix 1. In this program a standard scientific subroutine RKG has been used. This program works in the following way:

First initial values (time $t = 0$) of C, $\dot{C}$, P_l and P_r must be specified.

Here these values were assumed to be

$C(0) = 0.5 \text{ in}$

$\dot{C}(0) = 0.0 \text{ in/sec}$

$P_l(0) = 0.0 \text{ psig}$

$P_r(0) = 0.0 \text{ psig}$

With these initial values of C, $\dot{C}$, P_l and P_r, the values of $\ddot{C}$, $\dot{P}_l$ and $\dot{P}_r$, for time $t = 0$ can be calculated using equations (57), (58) and (59). Then these values are multiplied by h to get new values of $\dot{C}$, P_l, P_r and C. At this point the following checks have to be made.

Is $X > 0$?	Is $X = 0$?	Is $X < 0$?
Is $P_s > P_l$? or $P_s < P_l$?		Is $P_s > P_r$? or $P_s < P_r$?
Is $P_r > 0$? or $P_r < 0$?		Is $P_l > 0$? or $P_l < 0$?

According to conditions existing a proper choice of equations has to be made and then new values of $\dot{C}$, $\ddot{C}$, $\dot{P}_l$ and $\dot{P}_r$ calculated. Then multiplying these values by h, new values of C, $\dot{C}$, P_l and P_r can be calculated. Proceeding this way all the values of C, $\dot{C}$, P_l and P_r can be calculated for any duration of time desired. Computer program given in Appendix 1 does all this. In the computer program the following definitions have been used:

$C = Y(1)$ $\qquad$ $\ddot{C} = DY(2)$

$\dot{C} = Y(2)$ $\qquad$ $\dot{P}_l = DY(3)$

$P_l = Y(3)$ $\qquad$ $\dot{P}_r = DY(4)$

$P_r = Y(4)$

Verification of the Calculated Response
of the "Designed" System

To initially verify that computed results were correct, the frequency response of the designed system (for example problem) was calculated for input frequencies to the spool ranging from 2πrad/sec to 150πrad/sec and was compared to the theoretical frequency response which is expected for such a system. The theoretical open loop transfer function for the actuating system of Figure 1, can be obtained by linearizing the non-linear dynamic equations. This transfer function is of the form

$$G(s) = \frac{C(s)}{X(s)} = \frac{K_1}{s(K_2 s^2 + K_3 s + 1)} \tag{60}$$

The Bode diagram for a system whose transfer function is given above will be similar to that shown in Fig. 4. Bode diagram is method of analysing frequency response of the system. This is a plot between log-magnitude and angle vs log frequency. The advantages of Bode plot for frequency response analysis are as follows:

1. The multiplication and division of transfer function which are particularly tedious are simplified. Multiplication is replaced by addition while division is replaced by subtraction; phase angle is also added for multiplication and subtracted for division.

2. The phase angle is related to the slope of the log

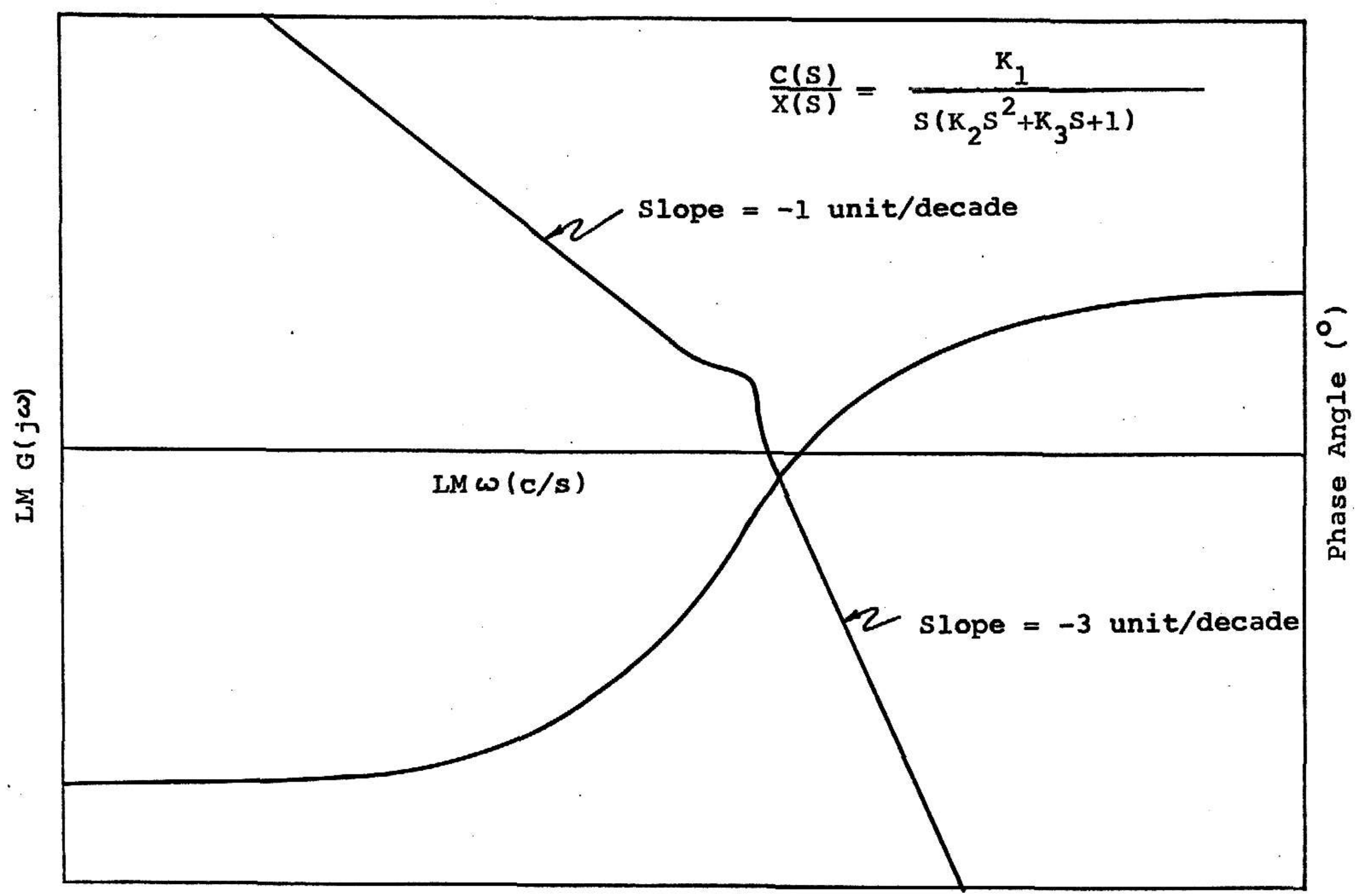


Figure 4. Theoretical Bode Diagram

modulus characteristics.

3. Gain and phase margin may be established from information on Bode diagram.

4. Bode attentuation plots and phase attentuation plot may be obtained from experimental frequency analysis of components for which actual transfer functions are not known. The plots may then be used to establish actual transfer function.

The Figure (5) shows the Bode diagram as obtained by calculation. The solid line shows the log-magnitude vs log frequency plot and the dotted line shows the phase angle vs log frequency plot. If the theoretical Bode plot of Fig. (4) and Bode plot of Figure (5) obtained by calculation are compared, it can be seen that the two are nearly same. Fig. (5) shows that the slope of log-magnitude is -1 unit/decade for frequency of 6 c/s or less. As the frequency is further increased the slope of the log-magnitude plot changes and at frequency 25 c/s and more the slope becomes -3 unit/decade. This is what is theoretically expected from the transfer function of type given by equation (60). The log-magnitude plot Fig. (4) obtained from the theoretical transfer function has a slope of -1 unit/decade for low frequencies and the slope changes to -3 unit/decade for higher frequencies. Therefore the two

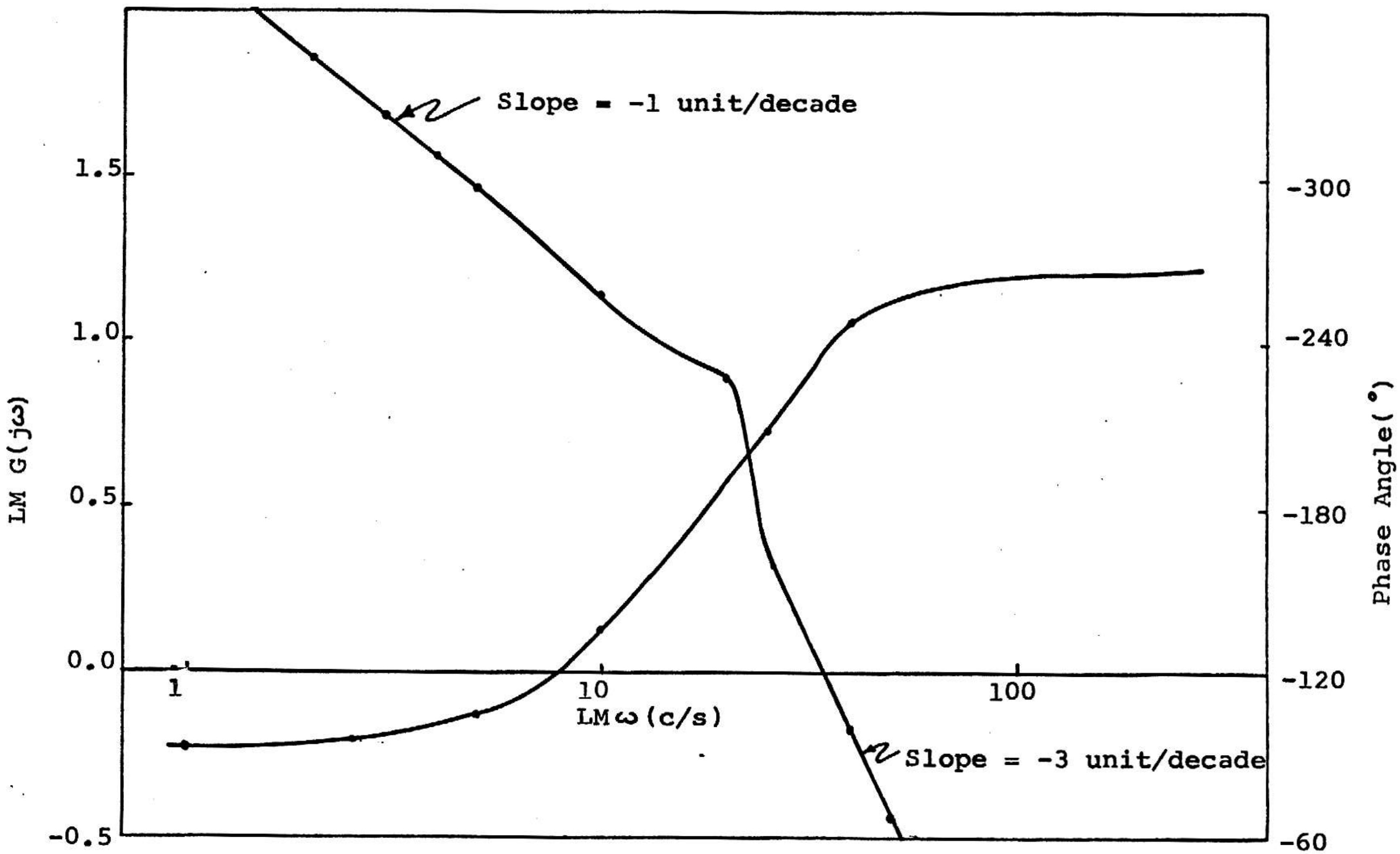


Figure 5. Bode Diagram for Example Problem

plots are similar. The phase angle is -90° at zero frequency and changes rapidly in intermediate range of frequencies and goes to -270° as the frequency approaches infinity. Phase angle plot of Fig.4 (obtained from transfer function of equation(60))also starts at -90° for zero frequency and goes to -270° as frequency approaches infinity. This verifies that the calculated output response of the designed system is right.

Evaluate Results Obtained Using Design Technique

In the last section it was established that the equations and calculations are correct. In this section results will be evaluated in the following way:

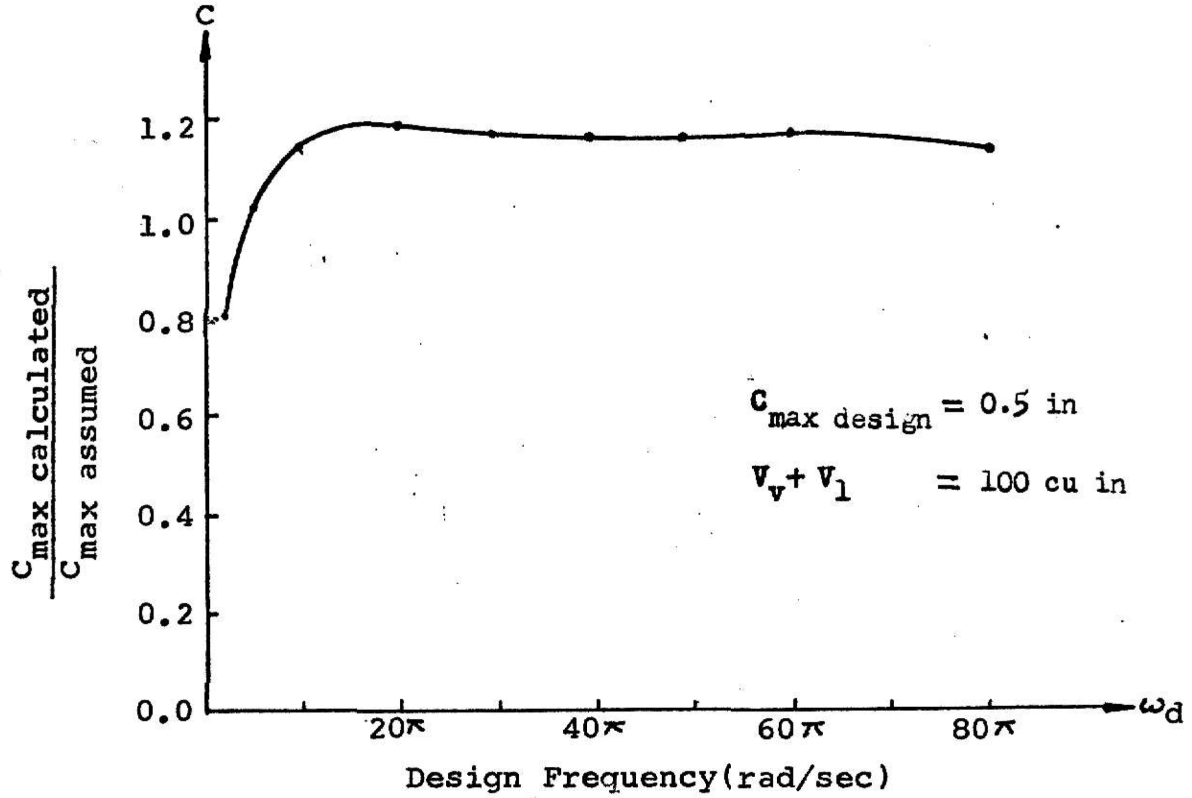
(1). By comparing the amplitude obtained by calculation with specified design amplitude. This will be done by taking different values of specified design frequency, keeping design amplitude constant, and by taking different values of design amplitude, keeping design frequency constant.

(2). By comparing the maximum power obtained by calculation with maximum power for assumed sinusoidal load motion for the cases mentioned in step 1.

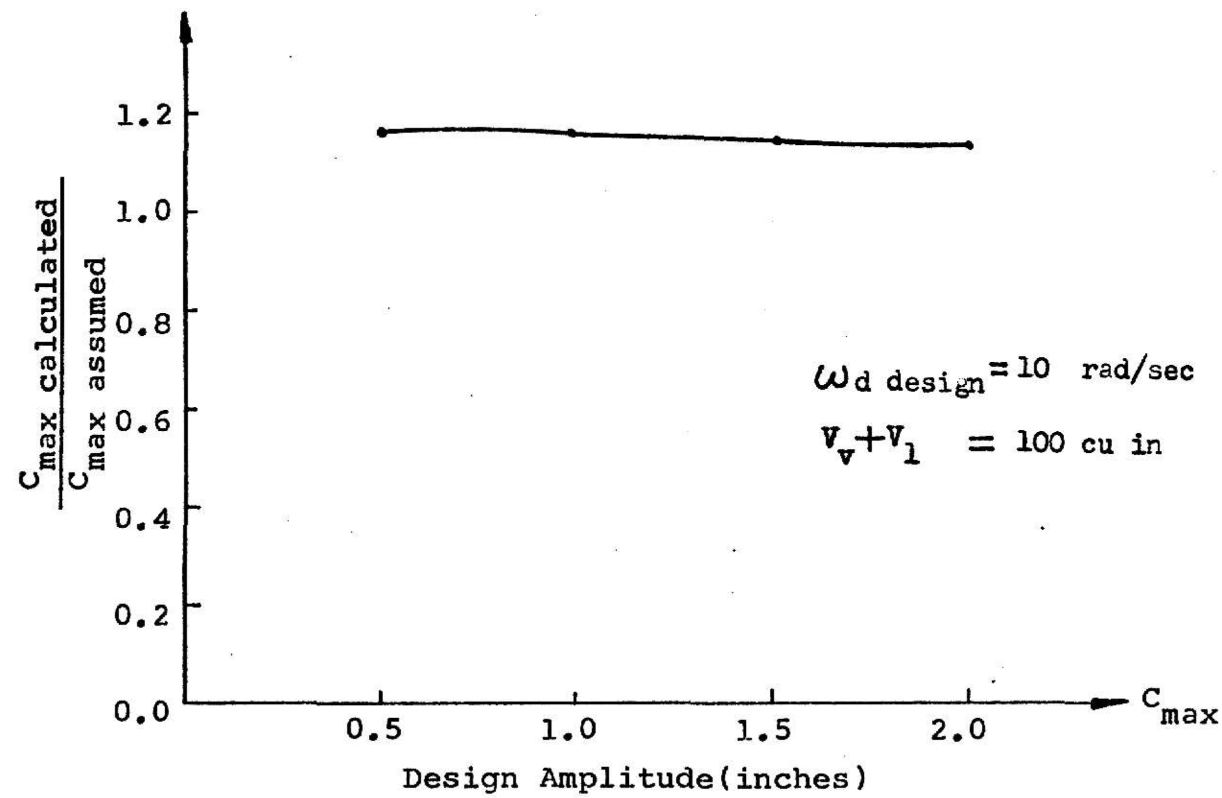
This evaluation is done in order to determine the feasibility of using the maximum power design technique to design an actuating system which can satisfy the desired design specifications. Results obtained here are based on the behaviour observed from the graphs. It is beyond the scope of this report to give any mathematical or experimental explanation for these behaviours because the equations involved are non-linear.

1. Comparison between calculated amplitude and specified design amplitude for different design frequencies.

Figure 6(a) shows a plot between different design frequencies and the ratio of amplitude obtained by calculation to specified design amplitude (same for all design frequencies). The figure shows that when design frequency is low, 2πrad/sec or less the ratio is less than 1. But, for the design frequencies of 4πrad/sec and more the ratio is always greater than 1. This means that for the design frequencies of 2πrad/sec and less the amplitude obtained by calculation is less than specified design amplitude and for design frequencies of 4πrad/sec and more the amplitude obtained by calculations is more than the specified design amplitude. For design frequencies of 4πrad/sec or more this gives an added "factor of safety".



(a) Amplitude Ratio Vs Design Frequency



(b) Amplitude Ratio Vs Design Amplitude

Figure 6.

This is desirable since the calculated results are based on several assumptions which are not 100% realizable in a real phsical actuating systems i.e., constant-supply pressure, constant discharge coefficient, etc.

2. Comparison between calculated amplitude and different specified design amplitudes for same design frequency.

Figure 6(b) shows the plot between different design amplitudes and the ratio of amplitude obtained by calculation to specified design amplitude (design frequency of 10πrad/sec for all cases). The figure shows that when the specified design amplitude is small (of the order of 0.5 in or less) the amplitude ratio is higher than when specified design amplitude is large. It shows that this ratio decreases slightly as the specified design amplitude is increased. But the change obtained is small and it could be concluded that a system designed using maximum power method will (based on calculated results) reproduce the specified load motion with an amplitude at least 10% greater than the specified design amplitude and that this is true for design amplitudes ranging from 0.5 to 2.0 inches.

3. Comparison of calculated and specified load velocity-force characteristic for different design frequencies.

Figures (7) and (8) show the velocity-force curves for the

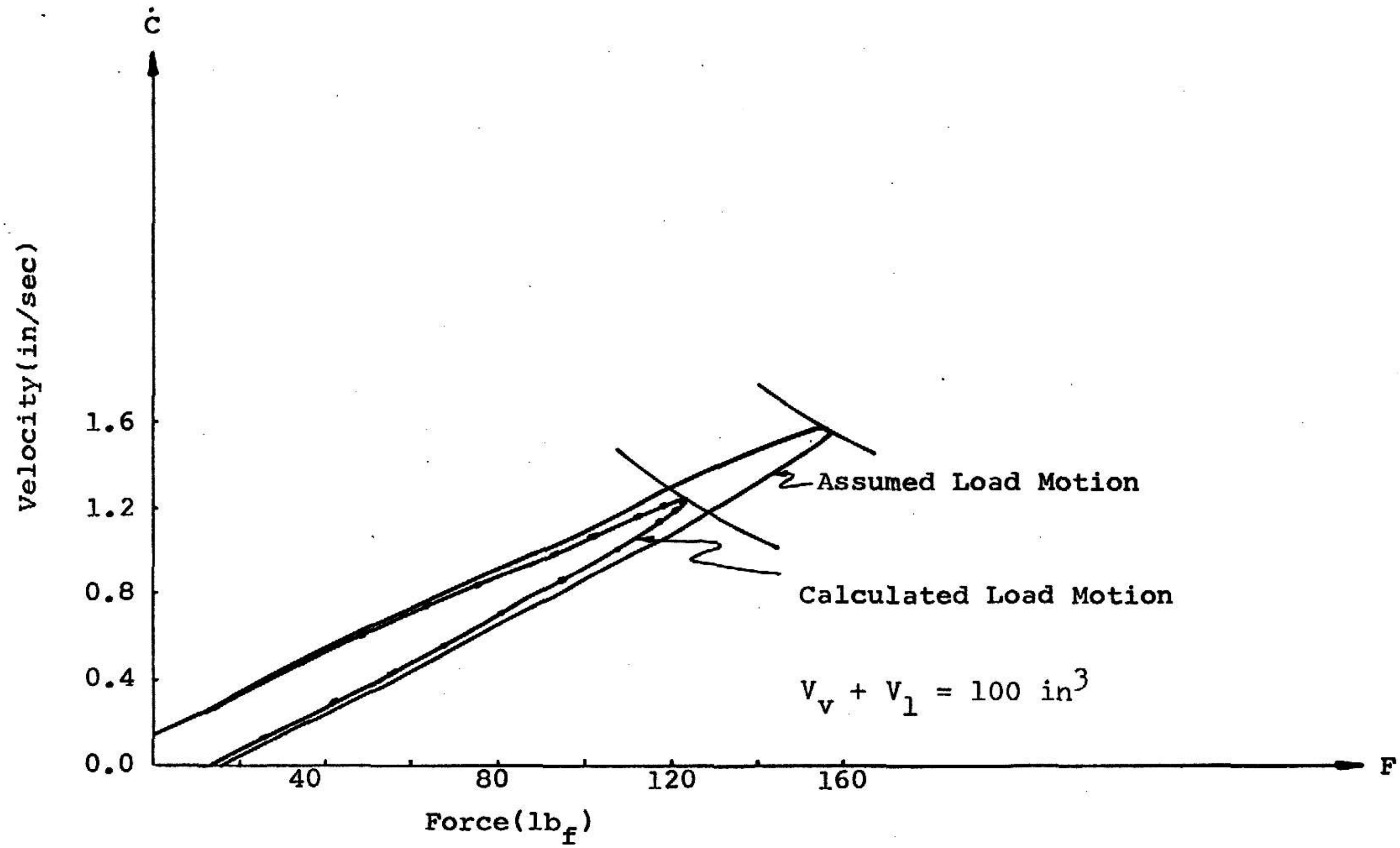


Figure 7. Velocity-Force Curve ($\omega_d = 2\pi rad/sec, C_{max} = 0.5$ in.)

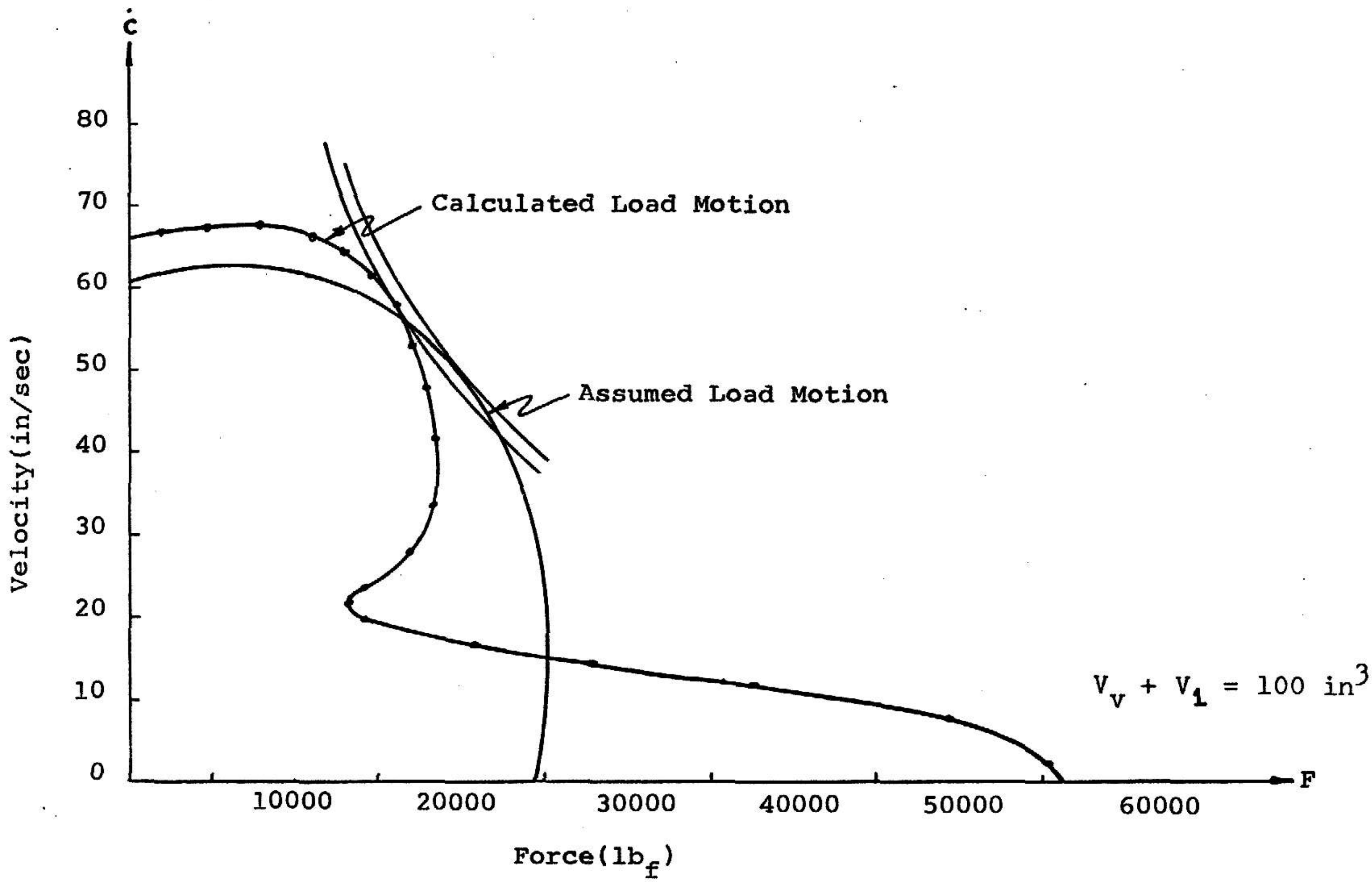
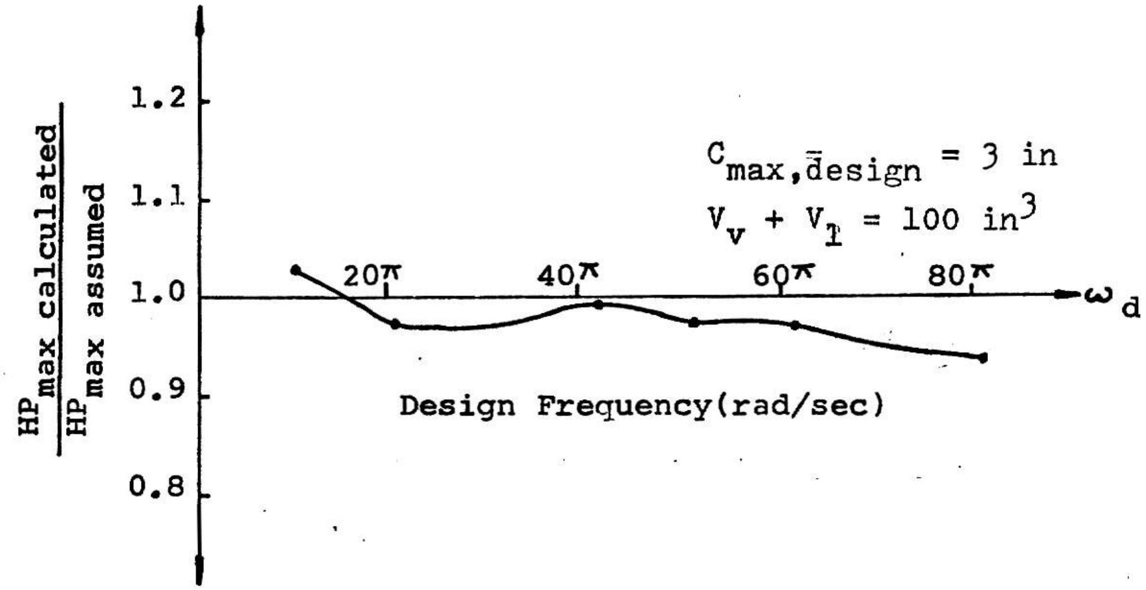
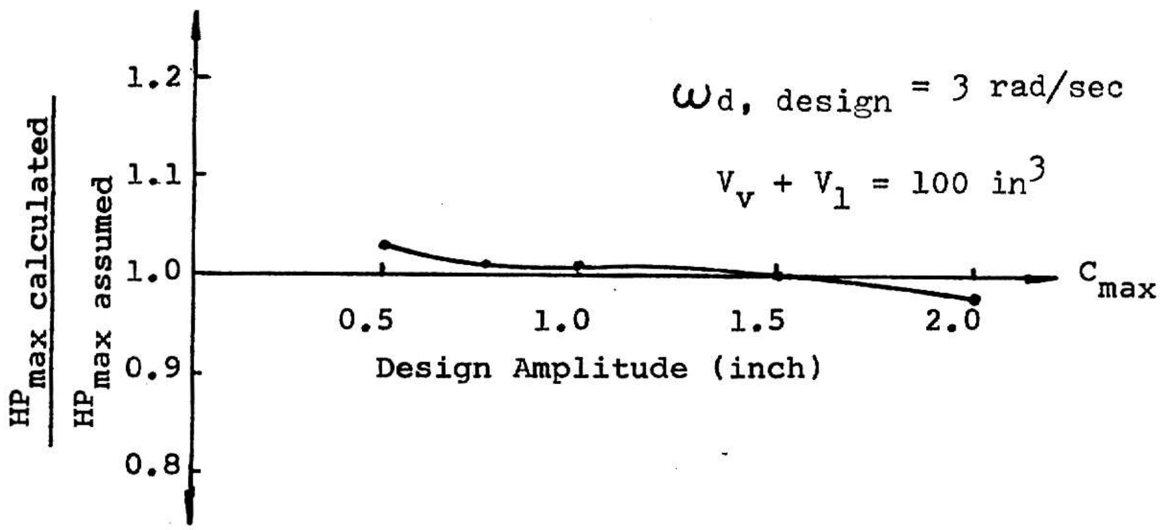


Figure 8. Velocity-Force Curve ($\omega_d = 80\pi$ rad/sec , $C_{max} = 0.5$ in.)

same specified design amplitude, but different design frequencies. Figure 7 shows that when the design frequency is low (2πrad/sec) the shape of the load velocity-force curve, obtained by calculation, is almost the same as that for the assumed sinusoidal load motion, but for higher design frequencies, as shown by Figure 8, it varies considerably. This could mean that the higher the design frequency the more prominent are the non linear and compressibility effects. No definite explanation is given here for this behaviour of load curve because analysis of non-linear equations is very complex and beyond the scope of this report. But it could be said that this behaviour of load curve is certainly undesirable. Fig. 9(a) shows the plot between ratio of maximum power obtained by calculation and the maximum power for assumed sinusoidal load motion and design frequency. It shows that maximum power obtained by calculation is higher than the maximum power for assumed motion for design frequencies less than 10πrad/sec. For design frequencies higher than 10πrad/sec this ratio is less than 1. Though the variation obtained is small (within 10%), it desirable to have the maximum power obtained by calculation higher than maximum power for assumed load motion, because such a system will not change the required performance specification due to small fluctuations is load.



(a) HP_{max} Ratio Vs Design Frequency



(b) HP_{max} Ratio Vs Design Amplitude

Figure 9.

4. Comparison of calculated and specified velocity-force characteristics for different design amplitudes.

Figure 10 shows the velocity-force curves obtained by caculation and based on assumed sinusoidal load motion for two different design amplitudes, 0.5 inches and 2.0 inches. Note that the design frequency remains the same, 10πrad/sec for both cases. It is evident from the figure that the assumed load motion velocity-force curve follows closely the velocity-force curve obtained by calculation for relatively small design amplitudes but varies considerably for fairly large design amplitudes. This variation in velocity-force curves for the larger design amplitude could be because of the nonlinearity of the dynamic equations and fluid compressibility effects. Figure 9(b) shows the plot between ratio of maximum power obtained by calculation and maximum power the system is designed for and design amplitude. This figure indicates that as the design amplitude is increased, the maximum power obtained by calculation decreases. Therefore a system designed using the maximum power technique can be expected to deliver more power than it was designed for, at small design amplitudes. The consequent additional "factor of safety" is certainly desirable. Therefore it is advantageous to base the design on small design amplitudes.

In evaluating the maximum power design technique in this chapter, the results obtained indicate that with this technique,

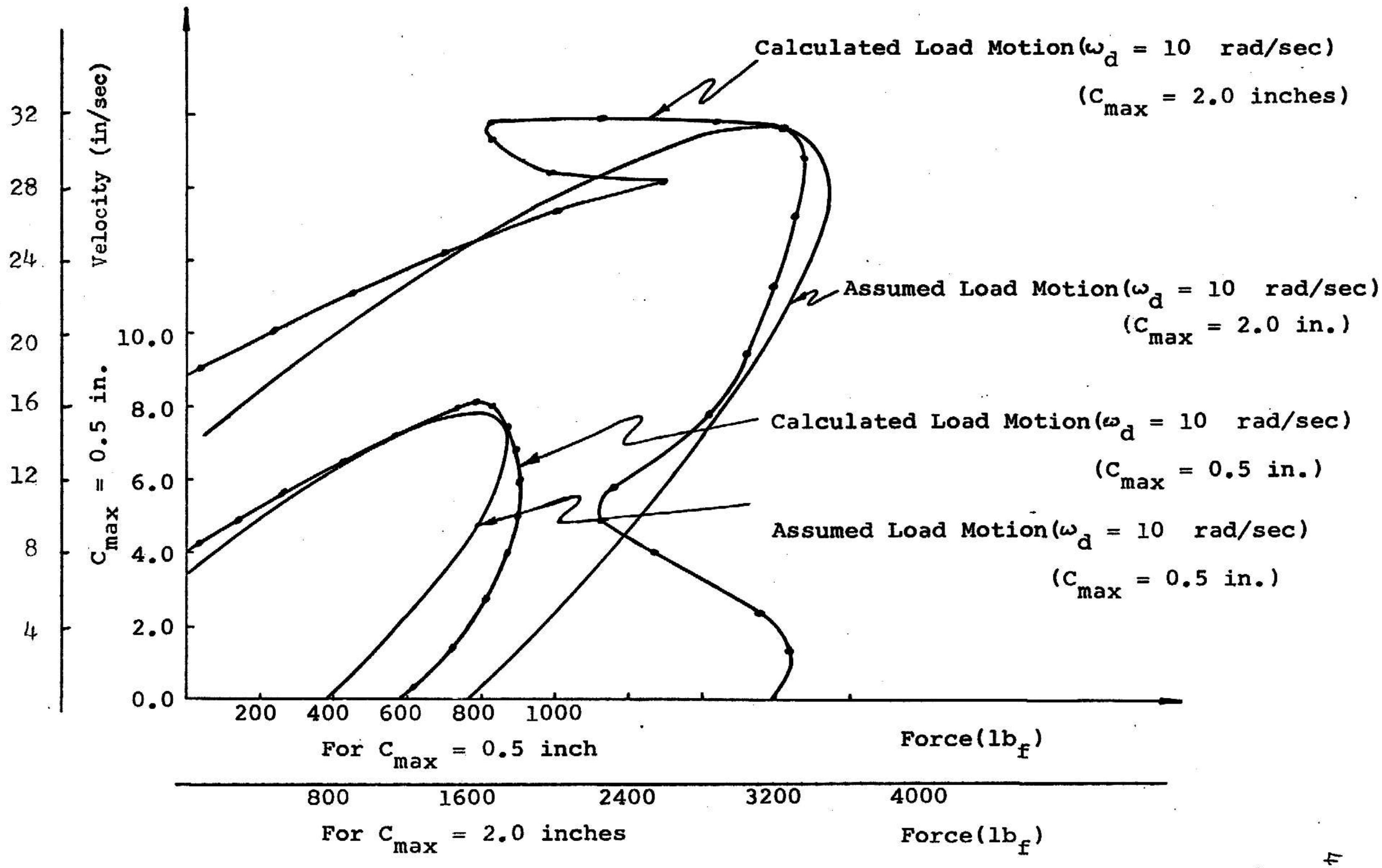


Figure 10. Velocity Vs Force For Different Design Amplitudes

the designed system very nearly reproduces the assumed sinusoidal load motion for low design frequency and small design amplitude. No attempt has been made here to specify exactly how low the design frequency and how small the design amplitude should be.

However, Figures 7, 8, 9 and 10 can be used as guides for insight into the differences between assumed and calculated motion for different design frequencies and different design amplitudes.

CHAPTER V

COMPRESSIBILITY EFFECTS

In the previous chapters a value of 100 cu in was assumed for the volume of oil in the transmission line and valve body. All the calculations were then made using this value for compressed volume. But if equations (45) and (46) and the equations which could be obtained by solving these two equations for $\dot{P}_l$ and $\dot{P}_r$ are examined, then it can be seen that compressed volume does play an important part in compressibility factor[1] of these equations. From equation (45) it can be seen that the compressed volume and $\dot{P}_l$ or $\dot{P}_r$ are in the numerator in the compressibility factor. But in equations for $\dot{P}_l$ and $\dot{P}_r$ the compressed volume is in denominator, which indicates that the values of $\dot{P}_l$ and $\dot{P}_r$ will be reduced if compressed volume is increased. This presents two opposing situations and it is difficult to predict, just by looking at these equations, what compressed volume will be most satisfactory in order that the compressibility effects are negligible.

1"Compressibility factor" is the factor which was originally omitted in applying maximum power design technique. The term compressibility factor as is used here stands for
$\frac{1}{\beta}(V_v + V_l + AC)\ \dot{P}_l$ or $\frac{1}{\beta}\left[V_v + V_l + A(L - b - C)\right]\ \dot{P}_r$
in equations (45) and (46) respectively.

An attempt was made here to obtain a compromise between these two opposing situations and make some recommendation about limitations on value of compressed volume for systems designed by use of the maximum power design technique. This was done by selecting different values of compressed volume and then studying effect of such changes on calculated amplitude of load motion and calculated maximum power at the load.

Effect of Changes in Compressed Volume on Amplitude of Load Motion

Figure 11 shows calculated load motion C for a design frequency of 10π rad/sec and a design amplitude of 0.5 inch, but for different compressed volumes. As can be seen from Figure 11, the load motion tends to become unstable as the compressed volume is increased. Further it can also be seen the amplitude of motion goes on decreasing as volume is increased. Figure 12(a) shows the ratio of calculated amplitudes to design amplitude plotted against compressed volume. This figure shows that for smaller compressed volumes the ratio is greater than 1, but for larger compressed volumes, this ratio becomes less than 1.

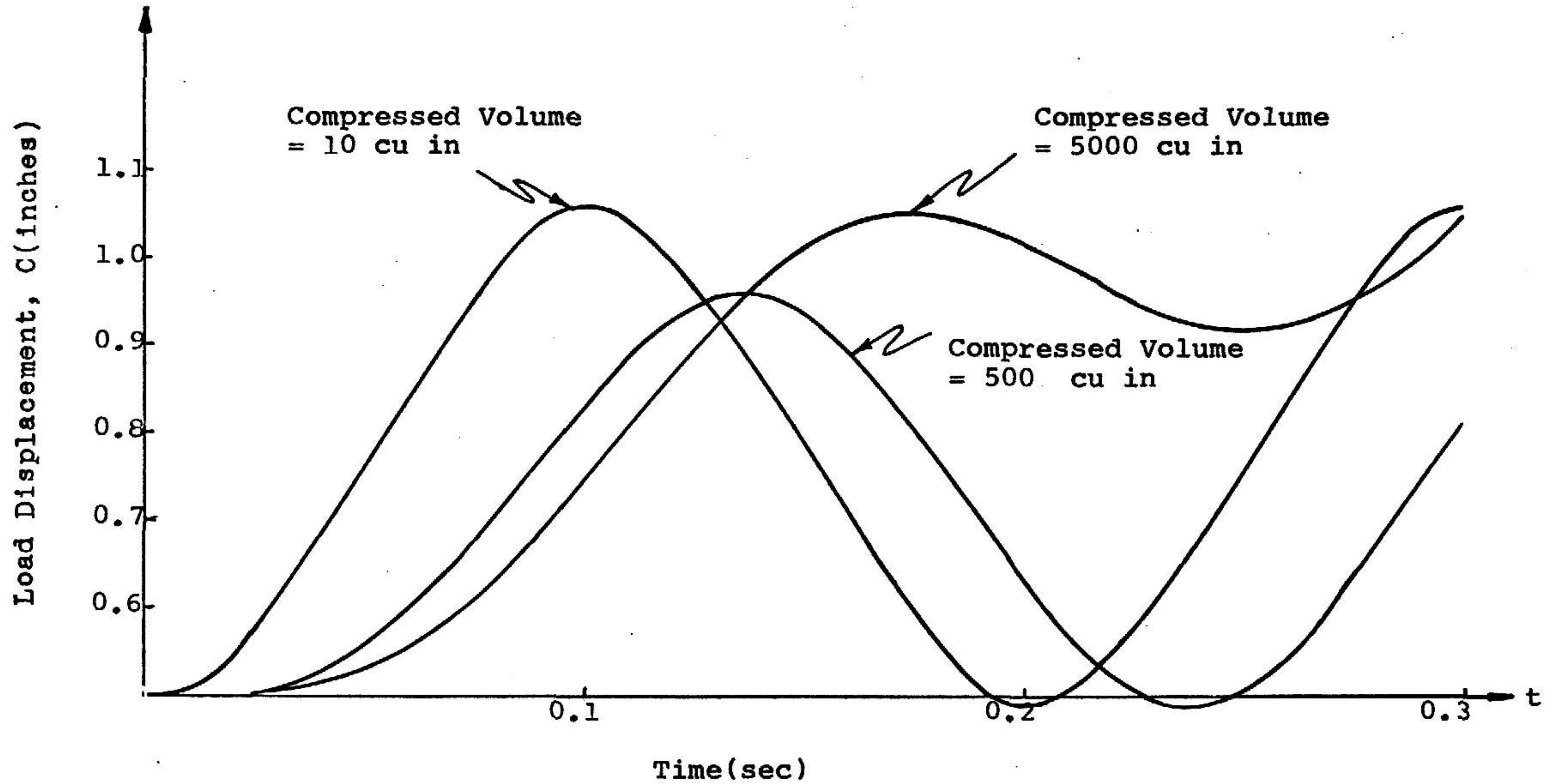
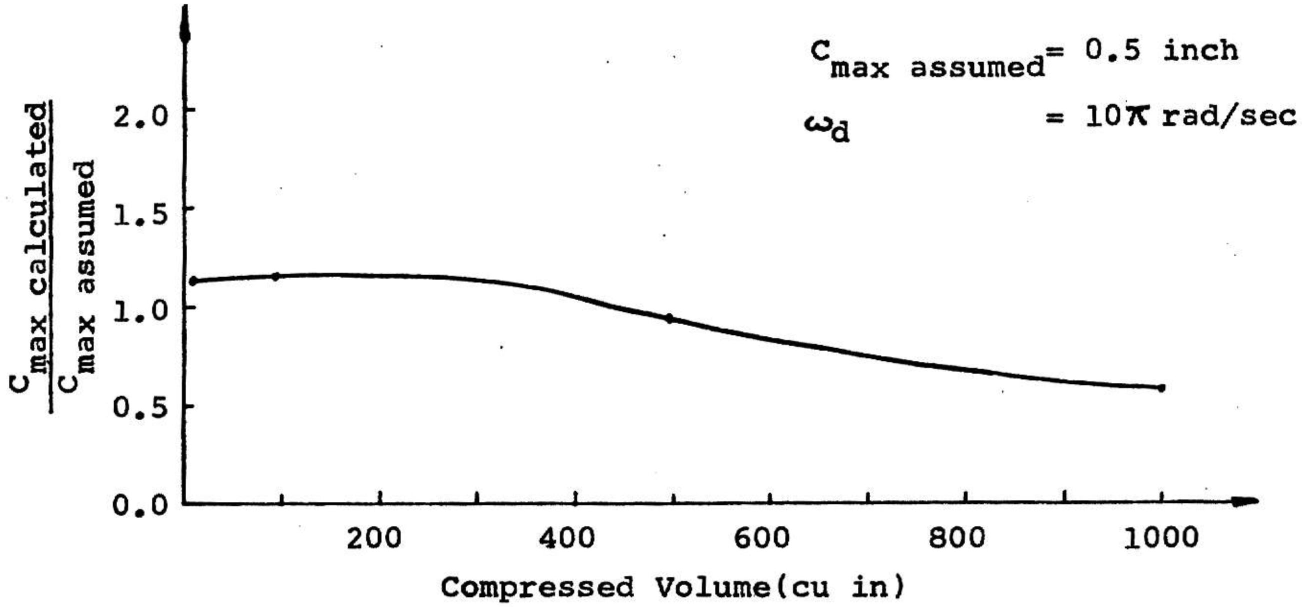
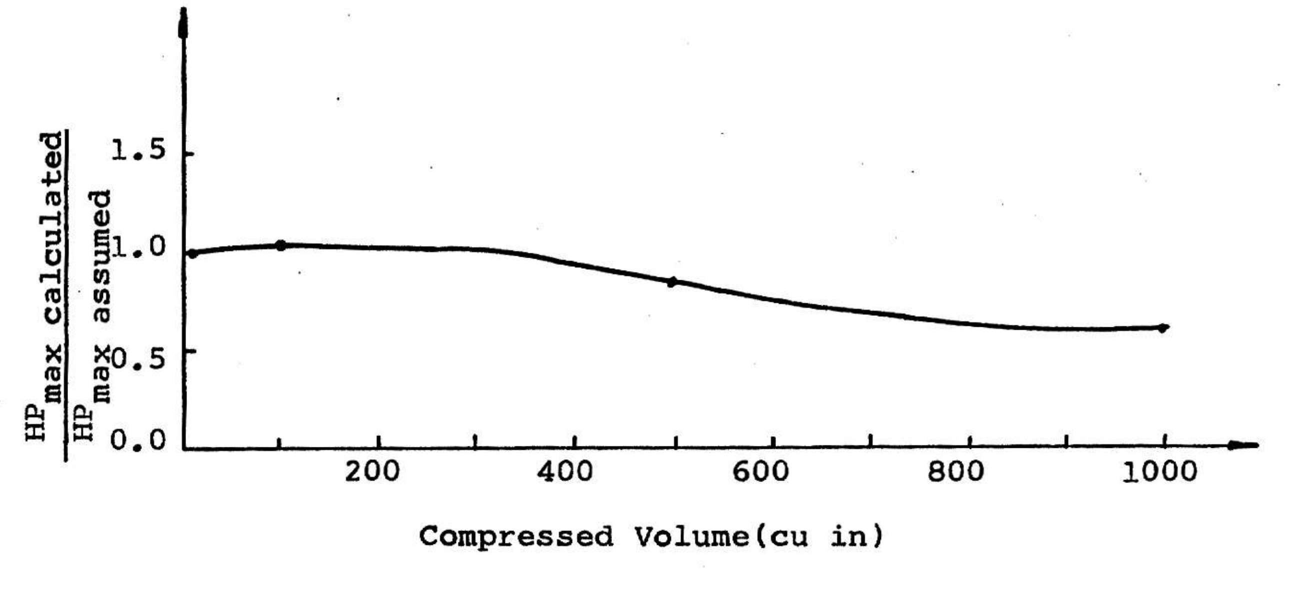


Figure 11. Load Displacement Vs Time for Different Values of Compressed Volume($\omega_d = 10\pi$ rad/sec, $C_{max} = 0.5$ inch)



(a) Amplitude Ratio Vs Compressed Volume



(b) HP_{max} Ratio Vs Compressed Volume

Figure 12.

Effect of Changes in Compressed Volume on Ratio of Calculated Power to Assumed Design Power

Figure 13 shows the velocity-force curves for different compressed volumes. This figure shows that the shape of the load locus obtained by calculation differ considerably from the shape of the load locus (velocity-force curve) as shown by Figure 10 for assumed sinusoidal load motion at small volume, but at higher compressed volume the shape of both the curves is nearly same. Figure 12(b) shows the graph between ratio of maximum power, based on calculated load motion, to power based on assumed sinusoidal load motion, and the compressed volume. The figure shows that this ratio is greater than 1 for small compressed volumes and decreases steadily as the volume is increased.

From the above discussion it is clear that as the compressed volume is increased the load motion tends to become unsteady and the amplitude of load motion decreases. Also the maximum power obtained at load decreases as the compressed volume is increased. Both these effects are undesirable for satisfactory performance of the designed system. This indicates that compressibility effects are more pronounced when compressed volume is large. Therefore it could be concluded that for the designed system to behave closely to the required design specifications, compressed volume should be kept as small as possible (to realize this locate valve close to cylinder, i.e., keep lines short).

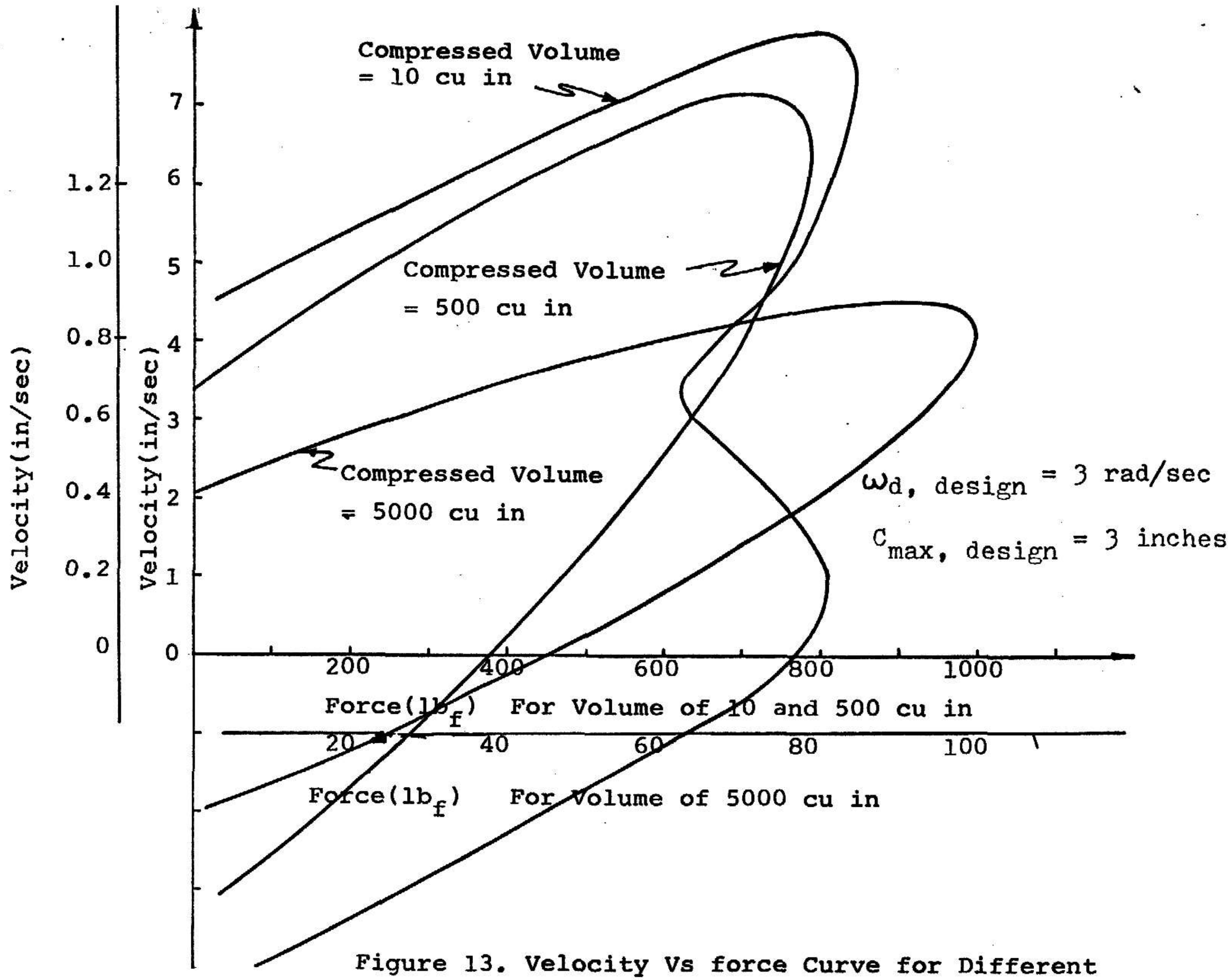


Figure 13. Velocity Vs force Curve for Different Compressed Volume

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

It is concluded that the maximum power design technique can be successfully applied in designing a hydraulic actuating system, (1) if the specificed design frequency of the assumed sinusoidal load motion is low (5π to 10πrad/sec) and (2) specified design amplitude is small (0.5 in). It is further concluded that for satisfactory performance of such a designed system the compressed volume should be as low as possible (keep transmission lines short).

In the process of evaluating maximum power design technique, in this report, it was assumed that the system is free of coulomb friction and stiction force. Also, it was assumed that transmission lines are rigid. In an actual physical system all these assumptions are seldom true. Further work could be done by taking all these factors in consideration. No attempt was made in this report to study the effect of change in supply pressure. Results obtained in this report could not be varified experimentally because of the author's inability to obtain a suitable system for experimentation.

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APPENDIX I

(Computer Program)

```
      DIMENSION Y(4),DY(4),Q(4)
      NEQ=4
      DO 10 I=1,NEQ
   10 Q(I)=0.0
      XMAX=0.02211
      A=1.926
      WD=10.0
      AN=0.2
      H=AN/100
      Y(1)=0.50
      Y(2)=0.0
      Y(3)=C.00
      Y(4)=0.0
      T=0.00
      DO 20 L=1,400
      CALL RKG(NEQ,H,T,Y,DY,Q)
      F=(Y(3)-Y(4))*A
      POWER=Y(2)*F
      PRINT 11, T,Y(1),Y(2),Y(3),Y(4),F,POWER
   11 FORMAT(//,7F18.7)
   20 CONTINUE
      STOP
      END
      SUBROUTINE RKG(NEQ,H,X,Y,DY,Q)
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     THE INDEPENDENT VARIABLE X IS INCREMENTED IN THIS PROGRAM
C     Y(I) AND DY(I) ARE THE DEPENDENT VARIABLE AND ITS DERIVATIVE
C     ALL THE Q(I) MUST BE INITIALLY SET TO ZERO IN THE MAIN PROGRAM
C     NEQ = NUMBER OF FIRST ORDER EQUATIONS
C     H = INTERVAL SIZE
C     A SUBROUTINE DERIV(NEQ,X,Y,DY) MUST BE PROVIDED
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION  A(2)
      DIMENSION Y(NEQ),DY(NEQ),Q(NEQ)
      A(1)=0.292893218813452475
      A(2)=1.70710678118654752
      H2=.5*H
      CALL DERIV(NEQ,X,Y,DY)
      DO 13 I=1,NEQ
      B=H2*DY(I)-Q(I)
      Y(I)=Y(I)+B
   13 Q(I)=Q(I)+3.*B-H2*DY(I)
      X=X+H2
      DO 20 J=1,2
      CALL DERIV(NEQ,X,Y,DY)
      DO 20 I=1,NEQ
      B=A(J)*(H*DY(I)-Q(I))
      Y(I)=Y(I)+B
```

```
20 Q(I)=Q(I)+3.*B-A(J)*H*DY(I)
   X=X+H2
   CALL DERIV(NEQ,X,Y,DY)
   DO 26 I=1,NEQ
   B=0.16666666666666666*(H*DY(I)-2.*Q(I))
   Y(I)=Y(I)+B
26 Q(I)=Q(I)+3.*B-H2*DY(I)
   RETURN
   END
   SUBROUTINE DERIV(NEQ,T,Y,DY)
   DIMENSION Y(NEQ),DY(NEQ)
   XMAX=0.02211
   AREA=1.926
   WD=10.0
   AMTOT=386.0/600.0
   BM=365000.0
   PHI=WD*3.1415926*T
   X=XMAX*SIN(PHI)
   DIA=0.25
   AX=X
   IF(X.LT.0.0) AX=-X
   IF(AX.LT.0.000001) X=0.0000000
   IF(X.LT.0.00) GO TO 75
   GO TO 80
75 PSD=Y(3)-Y(4)
   PDIFS=Y(3)
   PDIF=1000.0-Y(4)
   GO TO 85
80 PSD=Y(3)-Y(4)
   PDIFS=1000.0-Y(3)
   PDIF=Y(4)
85 IF(PDIF.LT.0.0) PDIF=-PDIF
   IF(PDIFS.LT.0.0) PDIFS =-PDIFS
   DY(1)=Y(2)
   DY(2)=(AREA*PSD-100.0*Y(2))*AMTOT
   SFLOW=0.625*3.14159*DIA*X*SQRT((2.0*100000./8.13)*PDIFS)
   DFLOW=0.625*3.14159*DIA*X*SQRT((2.0*100000./8.13)*PDIF)
   IF(X.GT.0.0.AND.1000.0.LT.Y(3))  SFLOW=-SFLOW
   IF(X.GT.0.0.AND.Y(4).LT.0.0)  DFLOW=-DFLOW
   IF(X.LT.0.0.AND.Y(3).LT.0.0)  SFLOW=-SFLOW
   IF(X.LT.0.0.AND.1000.0.LT.Y(4)) DFLOW=-DFLOW
   DY(3)=(SFLOW-AREA*Y(2))*BM/(100.0+AREA*Y(1))
   DY(4)=(AREA*Y(2)-DFLOW)*BM/(100.0+AREA*(3.5-Y(1)))
   RETURN
   END
```

A STUDY OF THE MAXIMUM POWER DESIGN TECHNIQUE
FOR DESIGNING HYDRAULIC ACTUATING SYSTEMS

by

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ABSTRACT

In this report the maximum power design technique to design a hydraulic actuating system is discussed. First the design equations for maximum power point were derived from flow-pressure equations for valve and velocity-force equation for actuator. Then assuming a sinusoidal load motion a load locus was plotted to obtain the values of force and velocity for maximum power point. The design values were obtained by substituting these values of force and velocity in design equations. The dynamic equations for the designed valve-actuator systems were derived. These equations were solved using digital computer. Calculations were checked by comparing the Bode diagram, for theoretical transfer function for the actuating system and Bode diagram obtained from the calculations for the designed system. Results were then evaluated by comparing amplitude obtained by calculation with specified design amplitude and by comparing the maximum power obtained from calculation with maximum power for assumed sinusoidal load motion. Finally the effect of compressed volume on performance of the design system was evaluated by taking different values for compressed volume. Results indicate that the maximum power design technique could be successfully applied for designing hydraulic actuating system if the design frequency is low $(5\pi$ to 10π rad/sec$)$ and design amplitude is small (0.5 in)