Relativistic electron scattering from a freely movable proton in a strong laser field

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We study the electron scattering from the freely movable spin-$\frac{1}{2}$ proton in the presence of a linearly polarized laser field in the first Born approximation. The dressed state of the electron is described by a time-dependent wave function derived from a perturbation treatment (in a laser field). With the aid of numerical results we explore the dependencies of the differential cross section (DCS) on the electron-impact energy. Due to the mobility of the target, the DCS of this process is modified compared to the Mott scattering, especially in large scattering angles.

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Physics related to the radiative processes experienced by free electrons inside a strong electromagnetic field were studied since the advent of laser sources in the early 1960s [1,2]. An overview on this field can be found in the textbooks by Faisal [3], Mittleman [4], Fedorov [5], and some other recent papers [6–8]. Most of these studies are carried out in the regime of nonrelativistic collisions and for low- or moderate-field intensities. There are also some studies that have been carried out to investigate theoretically the relativistic potential. In the presence of ultrastrong lasers, a relativistic treatment becomes imperative (even for slow electrons). In the treatments of Refs. [9–11], effects related to the electron spin have been neglected, and the electron has been considered as a Klein-Gordon particle. Based on the theory of Refs. [12,13], Szymanowski *et al.* [14] and Szymanowski and Maquet [15] investigated the spin effect in the relativistic potential scattering in the presence of a circularly polarized field. However, as they stated, the resulting expression for the circularly polarized field turned out to be more tractable than for the general case of elliptical or linear polarizations, and then Li *et al.* studied the case of a linearly polarized field [16], Attaourti *et al.* studied the cases of circularly and elliptically polarized fields [17], and Manaut *et al.* investigated the case of polarized electrons [18].

The present Brief Report addresses the problem of an electron scattering off the freely movable target in the presence of a monochromatic linearly polarized homogeneous laser field. The aim of this Brief Report is to add some physical insight and to show the modification of a differential cross section (DCS) due to the movable target and compare it to the case of Mott scattering. We investigate, to be specific, the recoil effect in relativistic scattering of an electron from a freely movable proton. A DCS is derived by the trace procedure with the aid of FEYNCALC [19] and *Mathematica*, and a simplified form is given for specific cases.

Unless specifically stated, atomic units $\hbar = m = e = 1$ are used throughout, and the metric tensor is $g^{\mu\nu} = \text{diag}(1,-1,-1,-1)$.

In the regime of laser field intensity as considered in this Brief Report, the field can be treated as classically, which does not allow pair creation [20], its four-potential that satisfies the Lorenz condition $\partial A(x) = 0$ is described by (linear polarization): $A(x) = a\cos(kx)$, where $a = (0,\mathbf{a})$ and $\mathbf{a}$ is the amplitude of vector potential of the field. The four-wave-vector of the field is $k = (\omega/c,\mathbf{k})$, where ω and k are the frequency and wave number, respectively.

This scattering process involves two fermions, including an electron (e^-) and a proton (p). This e-p scattering can be expressed as

$$p + e^- \rightarrow p + e^-. \tag{1}$$

Different from Mott scattering, the target (p) in e-p scattering now is no longer fixed, therefore we should expect a recoil effect on the scattering DCS.

The relativistic asymptotic electron state in the laser field can be described by the Dirac-Volkov function [21] when normalized in volume V, considering the linearly polarized field, it reads

$$\psi_q(x) = \psi_p(x) = \left(1 + \frac{\not{k}\not{A}}{2c(kp)}\right)\frac{u(p,s)}{\sqrt{2QV}}e^{iS(x)}, \tag{2}$$

$$S(x) = -qx - \frac{a\cdot p}{c(k\cdot p)}\sin(kx), \tag{3}$$

where u represents a bispinor for the free electron which is normalized as $\overline{u}u = 2c^2$ and $q^\mu = (Q/c,\mathbf{q})$ is the averaged four-momentum of the electron in the presence of the laser field $q^\mu = p^\mu - \frac{\overline{a^2}}{2c^2(k\cdot p)}k^\mu$, where $\overline{a^2}$ is the time-averaged square of the four-potential of the laser field and Q is the energy of the electron in the laser field. The square of this momentum q^μ is $q^\mu q_\mu = m_*^2c^2$. The parameter $m_* = \sqrt{1 - \frac{\overline{a^2}}{c^4}}$ is an effective mass of the electron in a radiative field.

In a first approximation the proton will be treated as a structureless spin-$\frac{1}{2}$ Dirac particle. Comparing to the electron, it is much heavier (1836 times), therefore its modification in the presence of a laser is not so notable, and we can use plane wave $\psi_P(x)$ to describe it,

$$\psi_P(x) = \frac{u(P,s)}{\sqrt{2EV}}e^{-iPx}. \tag{4}$$

Then the S-matrix element for the scattering process takes the form

$$S_{fi} = -i \int d^4x\, \overline{\psi}_{q_f}(x) \not{\Lambda}(x) \psi_{q_i}(x), \quad (5)$$

with the quantum potential,

$$\Lambda^\mu(x) = -\int d^4y\, D_F(x-y)\overline{\psi}_{P_f}(y)\gamma_\mu \psi_{P_i}(y), \quad (6)$$

where D_F is the Feynman propagator for electromagnetic radiation, given by [22]

$$D_F(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \frac{-1}{q^2+i\epsilon}. \quad (7)$$

For a useful result we calculated $d\sigma$ in the laboratory frame of reference in which the initial proton is at rest and set $q_f = (Q', \mathbf{q}')$, $q_i = (Q, \mathbf{q}_i)$, and $P_i = (Mc, \mathbf{0})$. To get the unpolarized cross section we must average over initial states and sum over final ones. Then using the relation $d^3q_f = \frac{1}{c^2} q' Q' dQ' d\Omega'$ and integrating over the final-state energy Q_f, we get for the scattering DCS [23],

$$\frac{d\overline{\sigma}}{d\Omega'} = \sum_l \frac{d\overline{\sigma}^l}{d\Omega'} = \frac{1}{(2\pi)^2}\frac{1}{16Mc^6}\sum_l \frac{\mathbf{q}'}{\mathbf{q}_i}\frac{1}{q^4} \times \frac{\overline{|M_{fi}^{(l)}|^2}}{Mc^2 + Q + l\omega - \frac{Q'}{c^2 q'}(q_i \cos\theta + lk \sin\theta \cos\phi)}, \quad (8)$$

where

$$\overline{|M_{fi}^{(l)}|} = \frac{1}{4}\sum_{l=-\infty}^{\infty} \mathrm{Tr}\left[(c\not{p}_f + c^2)\Gamma^\mu_{(l)}(c\not{p}_i + c^2)\overline{\Gamma}^\nu_{(l)}\right] \times \mathrm{Tr}[(c\not{P}_f + c^2)\gamma^{(l)}_\mu (c\not{P}_i + c^2)\gamma^{(l)}_\nu], \quad (9)$$

with $\Gamma^\mu = \Delta_0\gamma^\mu + \Delta_1\gamma^\mu \not{k}\not{a} + \Delta_2 \gamma^\mu \not{a}\not{k} + \Delta_3 \gamma^\mu \not{a}\not{k}\gamma^\mu \not{k}\not{a}$. Here Δ_0, Δ_1, Δ_2, and Δ_3, have the same meanings as those of Ref. [16].

The energy conservation relation derived from the δ function is

$$Q'(Mc^2 + Q + l\omega) - c^2(\mathbf{q}'\cdot\mathbf{q}_i + l\mathbf{q}'\cdot\mathbf{k}) = m_*^2 c^4 + Mc^2 Q + lc^2(M\omega - \mathbf{q}_i\cdot\mathbf{k}). \quad (10)$$

In the limit case of $Q \ll Mc^2$, the scattering potential is approached by the fixed Coulomb potential, and there is a laser field assisted. The energy conservation relation here becomes

$$Q' = Q + l\omega. \quad (11)$$

This is exactly the Mott scattering that has been discussed in Ref. [16].

In Table I we display the laser-assisted e-p scattering DCS at the field strength $\mathcal{E}_0 = 1.0 \times 10^8$ V/cm and photon energy $\hbar\omega = 1.17$ eV for both geometries $\mathcal{E}_0 \perp \mathbf{p}$ and $\mathcal{E}_0 \parallel \mathbf{p}$, respectively (at the scattering angle $\theta = 90^\circ$ the azimuthal angle is $\phi = 0^\circ$). It is shown that the DCS for scattering is greatly enhanced with the application of the laser field. Due to the fact that the target is movable, there is a small modification on the DCS, and the cross section is a little smaller that that of the laser-assisted Mott scattering [16]. With the increase in impact energy, the modifications become larger. At a 90° scattering angle, the mobility of the target results in a smaller DCS compared to the Mott scattering. For instance, when an electron has 320 impact energies, the DCS of the Mott scattering is $10^{-11.7881}$, and it is $10^{-11.8469}$ for freely movable proton scattering. On the other hand, in the backwards direction scattering with scattering angle 180°, the modification of the DCS is most significant. The recoil effect of the free proton acts as a "softening" to the collision, which can increase the interaction time, and therefore we can observe enhancement. The stronger the recoil effect, the more enhancement in interaction time and DCS. In low impact energy, the enhancement is small. But with increases in electron impact energy, the enhancement can be up to more than 100 times.

TABLE I. The DCS for laser-assisted e-p scattering at the field strength $\mathcal{E}_0 = 1.0 \times 10^8$ V/cm and frequency $\hbar\omega = 1.17$ eV. $\log_{10}\frac{d\sigma_0}{d\Omega}$: result for the laser free; $\log_{10}\frac{d\sigma_{\mathrm{Mott}}}{d\Omega}$: result for the Mott scattering; $\log_{10}\frac{d\sigma_\parallel}{d\Omega}$: result for $\mathcal{E}_0 \parallel \mathbf{p}$.

θ (deg)	E_{T_i} (m_0c^2)	$\log_{10}\frac{d\sigma_0}{d\Omega}$	$\log_{10}\frac{d\sigma_{Mott}}{d\Omega}$	$\log_{10}\frac{d\sigma_\parallel}{d\Omega}$
90	1	−9.1035	−7.0904	−7.0907
90	10	−10.9225	−8.8490	−8.8515
90	20	−11.4943	−9.4169	−9.4218
90	40	−12.0821	−10.0000	−10.0094
90	80	−12.6826	−10.6378	−10.6558
90	160	−13.4947	−11.2346	−11.4219
90	320	−13.9197	−11.7881	−11.8469
180	1	−10.1032	−8.7878	−8.7879
180	10	−13.3037	−11.9318	−11.9282
180	20	−14.3907	−13.0591	−13.0139
180	40	−15.1959	−14.2225	−13.8190
180	80	−15.4298	−15.4539	−14.1010
180	160	−15.5149	−16.6473	−14.1861
180	320	−15.6363	−17.7977	−14.2591

In this Brief Report we study the electron scattering from the free proton in the presence of a radiation field. The theoretical results for the linear polarization show that the scattering DCS is greatly enhanced by the presence of a strong laser field. Comparing to the Mott scattering of the same situation, the recoil effect reduces the DCS at small scattering angles (for instance, $\theta = 90^\circ$) but enhances the DCS at large scattering angles (for instance, $\theta = 180^\circ$). The treatment can be readily extended to the case of a general polarization of the field and even to the cases of Müller scattering and Bhabha scattering, which involve two fermions.

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