CRITERIA FOR COMPARING COMMERCIAL TEXTBOOKS WITH THE SCHOOL MATHEMATICS STUDY GROUP MATERIALS AND AN APPLICATION OF THESE CRITERIA TO CHARLES E. MERRILL ALGEBRA TEXTS

by

NELSON C. STAFFORD

B. S., East Central State College, 1960

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1966

Approved by:

J. Harvey Littell

Major Professor

ACKNOWLEDGMENT

The writer wishes to express sincere thanks to Dr. J. Harvey Littrell, major advisor in this study, for his valuable assistance, constructive criticisms, and more specifically, for his personal interest during the writing of this report.

TABLE OF CONTENTS

INTRODUCTION

The Problem

Many schools in the past decade have been using one of the several mathematic series which have been developed by study groups of mathematics curricula. It was found by Holland Payne that some form of a modern program in mathematics had been tried in various schools throughout the United States for at least the past twelve years.[1] In many schools the materials have been those developed by the School Mathematics Study Group and published in 1961 by the Yale University Press.

With so much interest being shown in modern mathematics materials, many textbook publishing companies began to publish textbooks featuring these materials for market. Payne also found that nearly all major publishing companies now have on the market textbooks with modern orientation.[2]

Many of the schools which had been using SMSG materials began to examine textbooks as possible replacements for these materials. With such a variety of texts on the market, this examination was no easy task. Curriculum committees charged with the task of examination needed some guidance in making such choices.

The Purpose of the Study

The purpose of the study was to develop a set of criteria to be used for comparing commercial textbooks with SMSG materials and by using

---

[1] Holland Payne, "What About Modern Programs in Mathematics," The Mathematics Teacher, 58:422, May, 1965.

[2] Ibid.

this criteria to compare the algebra textbooks published by the Charles E. Merrill Books, Inc., with the materials developed by the School Mathematics Study Group to determine whether or not the Merrill texts would be suitable replacements for the SMSG materials.

It was decided that the methods used in this report for comparison would be of aid to schools across the nation as a guide for comparing SMSG materials with other texts they might wish to adopt.

Definition of Terms

In this report the initials SMSG were used to refer to the algebra program developed by the School Mathematics Study Group and published in 1961 by the Yale University Press. The term Merrill Algebra Series was used to refer to the books, Algebra One: A Modern Course and Algebra Two: A Modern Course, written by Glen D. Vannatta, A. Wilson Goodwin and Harold P. Fawcett and published by Charles E. Merrill Books, Inc., in 1962.

Methods of Research and Comparison

A direct approach was used in most of the research for this paper. Copies of the textbooks, teacher's commentaries, and supplementary materials were obtained from the representative presses and were directly compared.

The items which were compared in this report were items the author thought were basic since one text was being examined as a possible replacement for another. These criteria were developed following a careful review of the literature relating to the important items to be considered in the evaluation of textbooks. They were also based on experience

gained by the author in teaching both modern and traditional mathematics during the past six years, and from completing forty-five semester hours of college mathematics. These items were developed with the aid of very competent advisors and instructors in the field of education.

After the criteria to be used for comparison were developed, the SMSG and the Merrill textbooks and materials were studied and carefully compared with regard to the set of criteria presented in another section of this report.

REVIEW OF LITERATURE

The selection of textbooks has been one of the complicated problems that has faced our public schools probably as long as there has been more than one publishing company in competition for the public funds used to purchase textbooks. With the introduction of the modern theory of teaching mathematics of the past decade this problem has become even more acute. According to Herbert F. Spitzer:

> The extensive use of textbooks in arithmetic instruction has made a good publisher's market and as is usually true in such situations, a large number of different books are available. All books on the market have some good features though, of course, some have more than others. What is good in books is not, however, a matter of common agreement among teachers, administrators, and others concerned with the instruction in arithmetic.[1]

One of the problems which faces committees for selecting school textbooks is the selection of the criteria by which the books are to be judged. Writers differ in their opinions as to the basis to be used in

---

[1]Herbert F. Spitzer, The Teaching of Arithmetics. (Cambridge: Houghton Mifflin Company, 1954), p. 322.

judging texts. Spitzer found that, unfortunately, no satisfactory non-objective way of rating textbooks has yet been devised.[1]

During recent years the public schools have made selections from several modern mathematics series developed by various mathematics study groups that have sought to upgrade the public schools' mathematics curriculum. Dwain L. Thatcher pointed out that The School Mathematics Study Group probably has gained more popularity than any of the other experimental programs.[2] Because schools adopted the materials formulated by one of these pilot experimental groups, some textbook publishing companies began to place textbooks on the market that were designed to contain ideas expressed by these study groups. In a recent survey by Blenard C. Wilson of the modern mathematics programs in Kansas Schools he found that SMSG's textbook, A First Course In Algebra was used in eleven, or 14.8 per cent of the schools.[3] He also found that other modern algebra texts were also popular. The textbook most commonly used was, Modern Algebra, Book I by Dolciani.[4] This survey made no mention of any criteria that was used as a basis for deciding what textbook to adopt.

---

[1] Ibid., p. 323.

[2] Dwain L. Thatcher, "A Comparison of UICSM and SMSG Algebra Units with Accompanying Suggestions" (unpublished Master's Report, Kansas State University, Manhattan, Kansas, 1964), p. 4.

[3] Blenard C. Wilson, "Modern Mathematics in the Comprehensive High Schools and Selected Junior High Schools in Kansas" (unpublished Master's Report, Kansas State University, Manhattan, Kansas, 1965), p. 25.

[4] Ibid.

Textbook selections are limited in some states or counties by regulations which specify that selection must be made from a given list of books. Many schools though are not limited by these regulations; therefore the schools have to devise one means or another whereby textbook selections can be made. The textbook selection committees need to have some basis or criteria by which they can be guided in making their selections. One writer related:

> The basic consideration in evaluating an arithmetic series should be as follows: To what extent does the series of textbooks make possible, or fit into, a complete program of arithmetic? The textbook should be geared to the use of exploratory and visual materials. It should contain the necessary visualization of new steps and adequate explanations of each new process to be learned. The textbook should contain the practice material to develop skill with operations. It should also contain an abundance of material dealing with the applications of number operations.[1]

Any committee charged with the selection of textbooks which include topics of modern mathematics would need also to consider the proposals of the College Entrance Examinations Board's Commission on mathematics as stated by Hlavaty:

> The Commission formulated and proposed a nine point program for college-capable students:
>
> 1. Strong preparation, both in concepts and in skills for college mathematics at the level of calculus and analytic geometry.
>
> 2. Understanding of the nature and role of deductive reasoning--in algebra; as well as in geometry.
>
> 3. Appreciation of mathematical structure--for example properties of natural, rational, real and complex numbers.
>
> 4. Judicious use of unifying ideas--sets, variables, functions and relations.

---

[1]Leo J. Brueckner, Foster E. Grossnickle, and John Reckzeh, Developing Mathematical Understanding in the Upper Grades (New York: Holt, Rinehart & Winston, Inc., 1957), p. 121.

5. Treatment of inequalities along with equations.

6. Incorporation with plane geometry of some co-ordinate and space perception.

7. Introduction in grade eleven of fundamental trigonometry--centered on co-ordinates, vectors, and complex numbers.

8. Emphasis in grade twelve on elementary functions.

9. Recommendation of additional alternative units for grade twelve: either introductory probability with statistical application or an introduction to modern algebra.[1]

The selection of textbooks for any school program has always brought with it another important question that is answered in several different ways by different school systems. Spitzer puts it this way:

> The question "Who shall select the pupil's textbook?" must be answered first. In the past, the superintendent, board of education, supervisor, principal, or other individuals with administrative power chose most of the books. That method of selection is still in use in many small school systems, in some county schools, and in a few larger school systems. The common alternative is the committee system. Each year such a system seems to be becoming more popular. This popularity is deserved, for the committee plan has many advantages over the plan of having some administrative officer select the books. The chief advantages lie in the fact that the selection is placed in the hands of those who are to use the books, the committee is appointed for this one task so that selection is not influenced by previously incurred obligations, and the final decision requiring agreement of members is usually the result of careful study.[2]

It must be recognized that no set of criteria could be devised which would answer the problems faced by all textbook selection committees. Such a set of criteria most likely would not even be desirable. This

---

[1]Julius H. Hlavaty, "Mathematics in Transition," The Mathematics Teacher, 52:265-67, April, 1959.

[2]Spitzer, op. cit., p. 324.

undesirability was pointed out by the members of one committee in writing aids for textbook evaluators:

> Throughout its deliberation, this committee felt that it must guard against a set of criteria which would tend to establish a national curriculum. At all times the local community's particular characteristics must be considered.[1]

The selection of textbooks in any school system must be done with regard to the specific needs and in accordance with the particular philosophies of that specific school system.

Much has been written about the place of modern mathematics in the schools of today and about the merits of one system as compared to other systems. The advantages and disadvantages of modern mathematics over traditional mathematics and vice-versa have been discussed frequently in the literature, but so far as could be found not much has been done toward establishing criteria for comparing one mathematics system with another objectively.

THE CRITERIA USED FOR COMPARISON

In a report to The National Council of Teachers of Mathematics, a committee headed by Philip Peak recommended to evaluators of textbooks, two major areas for examination. In the area concerning Presentation and Content, the committee recommended considerations be given to (1) Structure, (2) Rigor, (3) Vocabulary, (4) Definitions and undefined terms, (5) Correctness, (6) Theorems and proofs, (7) Generalizations, (8) Ordering, (9) Tests, exercises, and reviews, (10) Illustrative examples, and (11) Optional topics. The other major area related to

---

[1]Philip Peak and others, "Aids for Evaluators of Mathematics Textbooks," The Mathematics Teacher, 38:467-73, May, 1965.

Physical Characteristics and Services. In this area the committee recommended examinations be made of (1) The general format, (2) Index and references, (3) Usability, (4) Services, and (5) Teachers' Manuals.[1]

Under each of these headings the committee listed questions that should be answered regarding the book's treatment of these materials and cited examples of items presented properly and others presented improperly.

Other authors have expressed ideas for items which should be considered in textbook selection. Two such authors, Brueckner and Grossnickle, suggested the following categories as points of importance in textbook selection.

1. The point of view of the author.
2. The presentation of the review work of previous years at the beginning of the new year.
3. Visualization and other aids used in developing processes new to the grade.
4. Considerations of social applications of arithmetic.
5. Means provided by the textbook for evaluating the pupil progress and for diagnosing their difficulties.
6. Provision for individual differences.
7. Special features
   a. Character and quality of illustrations
   b. Size of type, appearance of page, readability, margins, and paper used.
   c. Factors facilitating teaching.
8. Mechanical features.
   a. Attractiveness of text.
   b. Durability of mechanical structure of text.
   c. Extent to which pictures contribute to text.
   d. Index and table of contents.
   e. Heavy type, italics, and boxing.
   f. Organization as page units.[2]

---

[1] Ibid.

[2] Leo J. Brueckner and Foster E. Grossnickle, Making Arithmetic Meaningful (New York: Holt, Rinehart, & Winston, Inc., 1953), pp. 547-50.

In this report the Merrill materials were compared to the SMSG materials to determine whether or not they were suitable substitutes for the SMSG materials. Direct comparisons were made according to the following set of criteria.

I. Comparisons of physical characteristics of materials.

A. General format

1. Does the cover of the text identify it as one on algebra?
2. Does the page arrangement give the reader a feeling of continuity?
3. Do the headings clearly portray the sequence of subject matter?
4. Is the type style and size of such a nature so as not to cause the reader difficulty?
5. Does the construction of the materials facilitate reasonable usability?

B. Indexes and References

1. Are the indexes accurate and easy to use?
2. Does the table of contents provide descriptive information about the materials to which they refer?
3. Does the text contain a glossary of essential terms?

C. Usability of materials

1. Are important terms and concepts presented in such a manner that they are easy to remember?
2. Are adequate tables included and conveniently located so as to support concept presentation?
3. Are the materials free of typographical errors?

D. Teachers' Manuals

1. Does the format provide for effective use?
2. Does the manual contain background material that is helpful to teachers?
3. Does the manual contain an answer key to problems that are difficult or which have several methods by which they may be solved?
4. Does the manual contain answers to all exercises found in the text?
5. Are the pages of the manual keyed to the pages of the text?

II. Comparisons of content and presentations.

A. Important concepts contained in the materials.

1. Do the texts cover a satisfactory number of the same concepts for one to replace the other?
2. Do the texts provide approximately the same sequence of topics and concepts?
3. Are their differences of such a nature that one text could not be substituted for the other?

B. Notation Used by the Systems

1. In what ways are the notations used by the texts similar?
2. What are the major differences in notation and symbolism used?
3. Are these differences of such a nature that one text should not replace the other?

C. Methods of presenting topics

1. In what ways do the methods of presentation used by each text favorably compare.
2. In what ways do the presentations differ?
3. Are the methods of presentation such that one text might substitute for the other?

D. Use made of illustrations.

1. How does each text use illustrations to supplement their topics?
2. In what manner are their illustrations alike?
3. In what manner are they different?
4. Are their differences important enough to be of consequence in their replacement of each other?

E. Use made of exercises

1. Are the exercises used of such a nature that they enhance the reader's understanding of ideas presented by the text?
2. Are the selection of problem items adequate?
3. Are the headings of exercise sections presented in a way that makes them easily identified?

F. Treatment of modern mathematics concepts

1. Do the texts present modern ideas and concepts in a manner that makes them easy to understand?
2. In what ways do the texts similarly present these concepts?
3. In what ways do the presentations of these ideas differ?
4. Could the Merrill texts successfully substitute for the SMSG materials in regard to the presentations of these concepts?

COMPARISONS OF PHYSICAL CHARACTERISTICS OF MATERIALS

General Format of SMSG

The textbooks provided by SMSG for the students consisted of two paper-back textbooks which were designed to cover a normal high school course in algebra one and by two paper-back textbooks designed for a course in algebra two.

The covers of these textbooks easily identified them as mathematics books. The titles used for the algebra one textbooks were, First Course in Algebra, Part I and First Course in Algebra, Part II. The titles used for the second year algebra course were somewhat confusing in that they did not directly show that they were algebra textbooks. The titles used were, Intermediate Mathematics, Part I and Intermediate Mathematics, Part II.

The materials of SMSG contained no pictures but the use of drawings and illustrations to clarify and explain concepts were used extensively.

The use of exercises and page arrangement for the most part tended to give the reader a clear feeling of continuity as he traveled from one concept to the other.

The headings for the topics portrayed the sequence of the subject matter in such a way that the reader was not left in doubt as to the concepts that were to be discussed in that topic.

The printing for the entire set of materials was black print on white paper with the print resembling the print of a typewriter. The pages were printed in paragraph form in such a manner that reading was made easy. The size of the type was such that no eye strain should have been experienced by the readers. No use was made anywhere, in any of the SMSG materials, of colored ink either in making definitions of terms stand out or in making illustrations clearer.

The paper back construction of the texts of SMSG did not seem to be desirable from the standpoint of remaining functional for a reasonal period of use. The author taught in one school system which had used these materials on a rental basis and the materials had to be replaced after two years use.

General Format of the Merrill Algebra Series

The Merrill Algebra Series consisted of one hardback bound copy to cover the course in algebra one and one hardback issue for the course in algebra two. The titles used readily identified them as being books for the intended subject matter. The book for algebra one was entitled, Algebra One: A Modern Course and for algebra two, Algebra Two: A Modern Course. The cover of the algebra one book was a combination of green with white lettering in the title and gray with green lettering for the author's names. The cover also contained an illustration of the graphs of the rectangular co-ordinate system on which the functions $x + y > 4$, $x + y = 4$ and $x + y < 4$, were graphed. The algebra two text was a

combination of brown with white lettering over gray with brown lettering and featured a graph of the co-ordinate system on which the functions $y = \left(\frac{1}{2}\right)^x$ and $y = 2^x$ were graphed.

Photographs and colored pictures were not contained in the Merrill textbooks but drawings, graphs and illustrations were abundant throughout the texts in places appropriate to clarifying the concepts under consideration at the time.

Chapter, topic, and page arrangements were of such a nature in these texts that the reader had no trouble sensing the feeling of concept continuity as his reading progressed from concept to concept and topic to topic.

Chapter and topic headings left little doubt as to what concepts were to be discussed and explained in that particular section. The reader was led very smoothly from one concept to the other.

The printing in these texts was somewhat smaller than was used in the SMSG materials, but not so small that it would tend to produce eye strain. For the most part the lettering was black ink on a very good quality white paper with words and concepts that were being defined printed in red ink in the algebra one text and in brown ink in the algebra two text to make them stand out. Red and brown ink was also used in the illustrations to make their meaning clear. The pages were printed in paragraph form with illustrations immediately accompanying concepts being discussed.

The size of the textbooks were about the same as most high school textbooks; therefore, there would be no trouble storing them in the

places most high schools provide for storage.

Comments

The general format of the Merrill materials were not of such a nature as to prevent their being used as replacements for the SMSG materials. In some cases the Merrill texts would probably prove to be advantageous. By providing one single textbook to cover a complete algebra course to replace two copies of SMSG would be an improvement since books have to be checked out to students and students tend to lose copies of them. The hardback construction probably would tend to make the Merrill texts last longer than the SMSG texts.

Indexes and References of SMSG

Both parts of the SMSG algebra one texts featured indexes covering the two complete parts. The same was true for the two sections of texts that were used for the algebra two texts, This made reference to any concepts of the individual courses easy. The indexes proved to be both complete and accurate regarding the materials to which they referred.

Each book contained a complete table of contents which covered only the topics presented in that particular section. For example, the book, First Course In Algebra, Part I featured a table of contents covering only materials found in Part I. For the books to which these tables pertained, they were found to be adequately detailed and descriptive of the materials to which they referred. These tables of content employed terminology commonly found in modern mathematics materials.

SMSG materials did not contain a glossary section for definitions of essential terms and symbols nor were cross-references provided.

Indexes and References for the Merrill Algebra Series

Each textbook of the Merrill Algebra Series contained an index for the material in that text alone. The indexes were found to be complete, concise, and accurate for the materials for which they were designed to cover.

Both books contained very accurate tables of contents which were found to be very descriptive of the materials to which they referred.

Like SMSG, these books did not feature a glossary section for the definitions of terms or symbols and did not offer any cross-references.

Comments

The materials provided by both texts in the way of indexes and references were very similar and no differences were noted which would hinder in any way the replacement of the SMSG materials with the Merrill Algebra Series.

Usability of SMSG Materials

Definitions, theorems, axioms, and other important items in the SMSG texts were presented first by leading the reader along a course to the point where he saw the need for these items. The items were then presented in such a manner as to aid the student in remembering them. Definitions and important concepts were underlined to make them stand out in the reader's mind.

The books used as textbooks in the algebra one course of the SMSG series did not provide any tables of any kind, probably because the

contents of these books were not of a nature that such tables would enhance their clarity. The first book of the algebra two series contained no tables and the second book contained only tables of common logarithms and of trigonometric functions. These tables were located at the points in the table where the concepts they supplemented were presented. The entire series contained no tables of squares and approximate square roots.

The texts were found to be free from typographical errors, although in some places the printing was run together especially when use was made of the radical sign, ($\sqrt{}$).

Usability of the Merrill Algebra Series

The use of colored ink in defining terms and the use of italics for important concepts were featured in the Merrill texts to make these points stand out. These definitions were presented by the Merrill texts in a more formal manner than employed by the SMSG materials in that they were introduced before the reader was made to feel a particular need for them. The presentations of the Merrill books resembled those used in traditional texts more than did the presentations made by SMSG.

The algebra one textbook of the Merrill Algebra Series contained a table of squares and approximate square roots for the positive integers from zero to one-hundred, inclusively. It also contained a table of the sine, cosine, and tangent trigonometric functions for angle sizes from zero degrees to ninety degrees, inclusively. The algebra two text also contained the table of squares and approximate square roots exactly like the one in the algebra one text. This book also featured a table of values correct to four decimal places for radians, and the sine, cosine,

tangent, and cotangent trigonometric functions for every ten minute intervals from zero degrees to ninety degrees, inclusively. This text also contained a table of logarithms of three place numbers correct to four decimal places. In both texts these tables were located in a section near the end of the book where they were quickly accessible.

These texts were also found to be free from typographical errors and in no way was the printing such that reading was made difficult.

Comments

As far as the usability was concerned, there was no evidence to indicate that the Merrill texts would not serve well as replacements for the SMSG materials. Their inclusion of tables of squares and approximate square roots should make them more usable, in fact, than the SMSG texts. The fact that the type used was in no way confusing was also an improvement.

Teachers' Manuals of SMSG

The teachers' manuals for the SMSG materials were contained in separate books from the students textbooks. There was a separate teachers' manual for each part making four manuals in all for the two courses in algebra. These manuals were of approximately the same size as the students' texts.

The commentaries serve the purpose of being both an answer book for the exercises found in the text and as a teaching guide. They featured explanations regarding methods of approach for most materials, an overview of the extent of the course, the organization of the materials, and a correlation of future topics with present topics that were being considered.

The manuals made only a rough estimate as to the number of days that should be spent on the individual chapters. They noted that such a wide variety of abilities existed in classes that a definite set number of days for particular topics would be inadvisable.[1]

The manuals were arranged according to the chapters that appeared in the students' text and the pages being discussed from the students' texts were printed at the bottom of the pages on which the discussions appeared.

Solutions to the more difficult problems were provided in the commentaries. Answers were provided for all exercises. They were appropriately numbered according to the problem set of the text to which they referred.

Teachers' Manuals for the Merrill Algebra Series

The teacher's guide and answer key for the Merrill textbooks were provided in the first part of an annotated text which featured many aids to assist the teacher of the courses.

These guides contained answers for all exercises that appeared in the student's textbook labeled according to the pages on which they appeared. They contained solutions for the more difficult problems and for some of them gave more than one solution. The teacher's guide itself was not as complete as those of the SMSG but the annotated edition provided longhand aids and suggestions printed directly on the pages of the text for aiding the teacher on specific items. They also had references to pages of the teacher's guide printed directly on the pages of the texts.

---

[1] School Mathematics Study Group, Teachers' Commentary. (New Haven: Yale University Press, 1961), p. xvi.

The guides gave specific information regarding a timetable suggested for both minimum courses as well as for a more advanced study of the material depending on the abilities of the students taking the course. This information was presented in a table showing the number of weeks to be spent on the individual topics for an average class and the number to be spent by classes with higher abilities. These instructions also aid the instructor in choosing topics to be discussed and those to be omitted when a minimum course is to be presented.

Comments

The teacher's manuals for the SMSG materials were found to be more complete than the guides for the Merrill Algebra Series in their explanations as to why the materials were presented in the manner in which they appeared. The SMSG manuals covered almost every section of the textbooks in complete detail and were very descriptive in their explanations. While the Merrill guides were not as thorough in their explanations, their supplementation by materials printed directly on the annotated textbook pages provided an end result as complete as that of SMSG. When teaching from texts with such information printed directly on the pages, the author of this report found use of such materials confusing. The teacher was provided with fingertip information that the students did not have. He tended to forget the students did not have this material and thus he covered the materials too lightly.

The teacher's guide was bound as a part of the textbook along with the information that was available to the students which should be an improvement compared to the separate manuals provided by SMSG in that they should be easier to use.

Nothing found in the physical characteristics of the Merrill Algebra Series materials were of the nature that they could not be used successfully as substitutes for the SMSG textbooks and materials.

COMPARISONS OF CONTENT AND PRESENTATIONS

Important Concepts Contained in the Materials

A comparison of the pertinent concepts common to both the SMSG materials and the Merrill units have been presented in Table I. This table also indicates concepts found in one system, but not in the other. If a concept was presented by both series, an x was placed under both the SMSG heading and the Merrill heading, whereas, concepts or items appearing in only one of the series were indicated by placing an x under the appropriate heading.

It was discovered that the sequence of concepts presented in the Merrill texts roughly paralleled those of SMSG, except for some that have been noted later in this report.

While Table I indicates that SMSG did not present a section regarding the nature and use of axioms, SMSG did present this material as properties of the number system without calling them axioms. SMSG did not devote a section to the use of formulas, as did the Merrill texts, but the same result was accomplished by use made of exercises.

As shown by Table I, the Merrill texts presented a chapter relating to the discussion of statistics in its algebra one text which was not presented in the entire SMSG algebra series. A section devoted to numerical trigonometry and another about complex numbers was included in Merrill algebra one, while SMSG did not introduce these items until algebra two.

TABLE I

CONCEPTS PRESENTED BY THE SMSG MATERIALS,
THE MERRILL ALGEBRA SERIES OR BOTH

Concept	SMSG	Merrill
Sets and subsets	x	x
Use of the number line	x	x
Symbols	x	x
The natural numbers	x	x
The integers	x	x
The rational numbers	x	x
The irrational numbers	x	x
The real numbers	x	x
The complex numbers	x	x
The nature and use of axioms		x
Graphing on the number line	x	x
Use of formulas		x
Closure	x	x
Commutativity	x	x
Associativity	x	x
Identity elements	x	x
Inverse elements	x	x
Distributivity	x	x
Ordering of the number system	x	x

TABLE I (continued)

Concept	SMSG	Merrill
Algebraic sentences compared to English sentences	x	x
Absolute value	x	x
Subtraction and division for real numbers	x	x
Use of variables	x	x
Truth or solution sets of algebraic equations	x	x
Solutions and graphs of inequalities	x	x
Opposites or signed numbers	x	x
Reciprocals or inverses	x	x
Factors and exponents	x	x
Prime numbers and factorization	x	x
Addition, subtraction, multiplication, and division of algebraic fractions	x	x
Radicals and their simplification	x	x
Square roots	x	x
Polynomials and factoring	x	x
Factoring by the distributive property	x	x
Differences of squares	x	x
Perfect square polynomials	x	x
Quadratic polynomials	x	x
Addition, subtraction, multiplication, and division of polynomials	x	x
Equivalent equations and inequalities	x	x
Fractional equations	x	x

TABLE I (continued)

Concept	SMSG	Merrill
Polynomial inequalities	x	x
Graphing equations and inequalities containing two variables	x	x
Systems of equations and inequalities	x	x
Algebraic solutions of systems of equations and inequalities	x	x
Graphs of quadratic polynomials	x	x
Definition of a relation		x
Definition of a function	x	x
Domain and range of a function	x	x
Linear functions and equations	x	x
Quadratic functions and equations	x	x
The rectangular coordinate system	x	x
Ratio and proportion	x	x
Direct and inverse variation	x	x
Methods in Statistics		x
Scientific notation		x
Square root tables		x
Solutions of quadratic equations by graphing	x	x
Solutions of quadratic equations by factoring	x	x
Solutions of quadratic equations by completing the square	x	x
Solutions of quadratic equations by the quadratic formula	x	x
Properties or axioms of equality	x	x

TABLE I (continued)

Concept	SMSG	Merrill
Decimal representation of rational and irrational numbers	x	x
The distance between two points in the coordinate plane	x	x
The slope of a line	x	x
Sketching graphs of equations and inequalities	x	x
Analytic proofs of geometric theorems	x	
Sets satisfying geometric conditions	x	x
Notation for a function	x	x
The general form of the quadratic equation	x	x
The nature of the roots of quadratic equations	x	x
Equations and graphs of the parabola	x	x
Equations and graphs of the circle	x	x
Equations and graphs of the ellipse	x	x
Equations and graphs of the hyperbola	x	x
The reflexive property of equality	x	x
The symmetrical property of equality	x	x
The transitive property of equality	x	x
Equations transformable to quadratic equations	x	
Addition, multiplication, subtraction, and division of complex numbers	x	x
Standard form of complex numbers	x	x

TABLE I (continued)

Concept	SMSG	Merrill
Graphic representation of complex numbers	x	x
Quadratic equations with complex roots	x	x
Equations of first and second degree in two variables	x	x
The straight line	x	x
The general definition of the conic	x	x
Equivalent equations and inequalities	x	x
Solutions and graphs of first degree equations with three variables	x	
Algebraic solutions of first degree equations with three variables	x	x
Intersection of two intersecting planes	x	
The graphic approach to the solution of two first degree equations in three variables (Appendix)	x	
Equivalent systems of equations in three variables (Appendix)	x	
The exponential function	x	x
The logarithmic function	x	x
Operations with logarithmic functions	x	x
Tables of common logarithms	x	x
Interpolation	x	x
Computation with common logarithms	x	x
Logarithms with an arbitrary base	x	x
Graphs of logarithmic functions	x	x
Graphs of exponential functions	x	x

TABLE I (continued)

Concept	SMSG	Merrill
Trigonometric functions	x	x
Signed angles	x	x
Radian measure	x	x
Units of angle measure	x	x
Sine, cosine, and tangent functions	x	x
Cotangent, cosecant, and secant	x	x
Tables of the trigonometric functions	x	x
Functions of special angles	x	x
Graphs of the trigonometric functions	x	x
The law of sines	x	x
The law of cosines	x	x
The area of a triangle		x
The addition formulas	x	
Identities	x	x
Inverse trigonometric functions	x	x
Trigonometric form of complex numbers	x	x
The system of vectors	x	x
Directed line segments	x	x
Applications to geometry	x	
Vectors and scalars	x	x
Inner product	x	
Applications of vectors in physics	x	x
Vectors as a formal mathematical system	x	
Vectors in three dimension	x	

TABLE I (continued)

Concept	SMSG	Merrill
Polar forms of complex numbers	x	x
Products in polar form	x	x
Integral powers; deMoivre's Theorem	x	x
Quadratic equations with complex coefficients	x	
Roots of order n	x	x
Fractional powers of complex numbers (Appendix; SMSG)	x	
The meaning of a matrix		x
The meaning of a determinant		x
Solving systems of linear equations by determinants		x
Determinants of the third order		x
Solving systems of three linear equations in three variables by determinants		x
The meanings of sequences and series	x	x
Arithmetic sequences and series	x	x
Geometric sequences and series	x	x
Limits of sequences	x	x
The sum of infinite series	x	x
Ordered m-tuples	x	
Permutations	x	x
Combinations	x	x
The Binomial Theorem	x	x
Probability		x
Arrangements and partitions	x	

TABLE I (concluded)

Concept	SMSG	Merrill
Partitions of a set	x	
Probability of more than one event		x
Empirical probability		x
Algebraic structures	x	
Internal operations	x	
Group	x	
Some general properties of groups	x	
An example of a non-abelian group	x	
Field	x	
Subfield	x	

SMSG did not give the definition of a relation in its discussion of functions, whereas, the authors of the Merrill texts defined a relation and then defined functions as being relations with special significant properties.

Table I indicated that both SMSG and the Merrill texts presented concepts involving polar coordinates. On this topic their sequence of presentation varied to a significant extent. SMSG presented an entire chapter devoted to the concepts involving polar coordinates. The Merrill texts presented most of the same material in a section called Trigonometric form of complex numbers contained in the chapter of algebra two devoted to complex numbers.

As indicated by the table, SMSG did not take up discussion of matrices and determinants while the Merrill book for algebra two devoted a chapter to them.

SMSG followed their presentation of permutations and combinations with a section thoroughly explaining the binomial theorem. Merrill's materials followed these items with a section on probability and only mentioned verification of a binomial expansion in an exercise.

Series and sequences followed about the same pattern in both of the texts. SMSG ended the algebra two course with a section (contained in the Appendix) presenting a discussion about the structures of algebra and some aspects of abstract algebra which the Merrill texts did not offer.

Notation Used by the Systems

The notation used by each algebra system was, for the most part, standard to that used by most mathematics textbook publishers.

Table II was prepared to give a comparison of the basic items of notation each system used. The meaning of the symbol was given in the first column followed by a column with the symbol used by SMSG and then a column of Merrill's symbol for the same item.

The Merrill texts presented a far more complete section relating to sets, as can be seen in the table. SMSG used only enough set notation to relate sets to the number system.

The symbol used by SMSG to indicate "approximately equal to" was a symbol that is not often, if ever, found in other texts.

It was interesting to note that SMSG also used an uncommon method to indicate whether a number was positive or negative. For this, SMSG used a small plus or minus sign, ($^{+}$) or ($^{-}$), preceding the number or letter. This method was used only for a few pages after such numbers were defined and was then dropped in favor of the more conventional notation. SMSG used this notation for only twelve pages.

For grouping, SMSG made use of large parentheses containing small parentheses, whereas, the Merrill texts used combinations of braces and brackets with parentheses.

Methods of Presenting Topics (SMSG)

The SMSG materials introduced a new topic by making the reader aware of the need for knowing the concepts which were to be treated in that topic. The materials recalled previous concepts that were related to the new concepts and then gave formal definitions of the new terms. Problem sets were presented at points where practice was needed to supplement the new concepts. Often completion of these exercises was needed in order for the reader to gain a full understanding of other

TABLE II

COMPARISON OF THE NOTATION USED IN THE SMSG AND THE MERRILL ALGEBRA SERIES

Meaning of Symbol	SMSG Notation	Merrill Notation
Addition	+	+
Subtraction	-	-
Multiplication	$\cdot$, $\times$, ( )( )	$\cdot$, $\times$, ( )( )
Division a divided by b	$\div$ or $\frac{a}{b}$	$\div$ or $\frac{a}{b}$
All included items to be treated as one numeral		
Parentheses	( )	( )
Brackets	[ ]	[ ]
Braces	{ }	{ }
Line or vinculum	———	———
The number a is positive	a, $^{+}$a, or +a	a or +a
The number a is negative	-a or $^{-}$a	-a
Is equal to	=	=
Is not equal to	$\neq$	$\neq$
Is greater than	>	>
Is less than	<	<
Is greater than or equal to	$\geq$	$\geq$
Is less than or equal to	$\leq$	$\leq$
Is not greater than	$\ngtr$	
Is not less than	$\nless$	
Set	{ }	{ }
Member		$\in$

TABLE II (concluded)

Meaning of Symbol	SMSG Notation	Merrill Notation
Not a member		$\notin$
Is contained in		$\subset$
Not contained in		$\not\subset$
Contains		$\supset$
Solution set		S
Universal set		U
Empty set		E
Null set		$\emptyset$
Set Union		$\cup$
Intersection		$\cap$
Complement		( )'
The principal square root of	$\sqrt{}$	$\sqrt{}$
The value of the function f at x	f(x)	f(x)
Matrix		$\begin{pmatrix} \end{pmatrix}$
Determinant		$\begin{vmatrix} \end{vmatrix}$
The absolute value of a	|a|	|a|
Is approximately equal to	$\approx$	
The sum of a series	Σ	
Infinity	∞	∞
Addition of angles	$\oplus$	
Operation	o	
Ray AB	$\overrightarrow{AB}$	
Vector AB	$\overrightarrow{AB}$	v_{AB}

concepts presented at a later time. When needed, drawings and illustrations were used in appropriate places to make explanations more meaningful and to aid in their understanding. Miscellaneous exercises were presented at convenient locations and review problems were to be found at points where review of concepts were needed before continuing to new concepts. Lists of concepts were presented at intervals after these items, such as the properties of the number system, had been discussed individually.

Methods of Presenting Topics (Merrill)

The Merrill texts introduced a chapter by a section in which they gave the reader a preview of what the chapter was to contain and a background setting needed for the sections included in that chapter.

Topics and sections were numbered in such a manner that the first number gave the chapter number. This number was followed by a number which indicated the section of the chapter being presented. These numbers were printed in red ink in the algebra one text and in brown ink in the algebra two text. The topic headings were in heavy black type. The topics contained exercises designed to supplement the concepts discussed in that topic.

When an important principle was presented, it was first presented as part of the discussion and then was given as a principle by setting it off between two red lines of arrowheads, labeling it principle, and printing it in dark black type.

At the end of each chapter, the Merrill materials presented a list of words contained in the chapter which they called a "Vocabulary List." Here all the new and important words were listed for easy reference. This section was followed with a section entitled "Chapter Review"

featuring problems designed to aid the student in reviewing the entire chapter. Last, in each chapter, came a section called "Test on Chapter (Number)" containing test items covering the concepts of that chapter.

Comments

Both the SMSG system and the Merrill materials provided adequate formats for the topics they presented. The use of color ink, the vocabulary list, and the chapter test items should make the Merrill texts easier to use than the SMSG materials. The necessity of completing some items contained in the exercises of SMSG to understand concepts could be frustrating.

Use Made of Illustrations

Both SMSG and the Merrill materials used illustrations liberally in their presentation of ideas. The use of such drawings in each system was found to be very similar.

Most of the illustrations found in both SMSG and the Merrill texts were items relating to either the number line or the coordinate plane. The number line was used as a graph for the number system and each text pointed out that there existed a "one to one correspondence" between the real number system and the points on the number line. Each text also pointed out the "one to one correspondence" existing between the points of the coordinate plane and the set of ordered pairs of real numbers. Only SMSG used illustrations using the idea of matching the points of space to the sets of ordered triples of real numbers. Both systems featured methods, using illustrations, to graph complex numbers on a plane and showed geometric concepts of explaining vectors by use of illustrations of the coordinate plane.

SMSG made use of illustrations regarding the number line much earlier than did the Merrill texts. Figure 1 is an example of one of the early illustrations in SMSG in relating numbers to points. By using other illustrations whereby smaller fractional intervals were shown, SMSG illustrated that each rational number corresponded to a unique point on the number line.

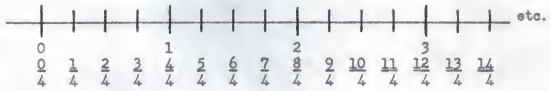

Figure 1

The authors of the Merrill texts introduced number line illustrations after they had presented signed numbers in their second chapter of their algebra one text. Figure 2 is an example of one of Merrill's illustrations of the number line relating rational numbers to the points on the line. The numbers to the left of "0" were printed in red ink to make understanding of negative rational numbers more noticeable.

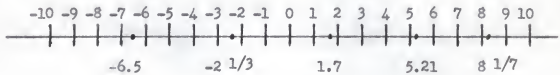

Figure 2

Figure 3 is an example of how the Merrill texts graphed the inequality $x + 2 > 5$ followed by Figure 4 showing their method of graphing $x + 2 \geqq 5$. Since the number 3 did not satisfy the first inequality, an open circle was drawn around it and since all other numbers greater than 3 did satisfy the inequality, a heavy dark arrow was drawn to the right from the point 3. In Figure 4, the point whose coordinate is 3 is included by a heavy black dot and all the points to the right are included in the heavy black arrow to indicate that for this inequality the point 3 is included in the solution set. SMSG used practically the same diagrams except their numbers were placed below the line instead of above it and they did not use colored ink for negative numbers.

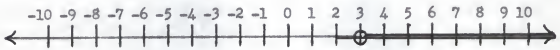

Figure 3

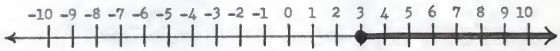

Figure 4

Illustrations involving the rectangular coordinate axis were employed by both algebra series as indicated in Figure 5. The intersection of the two axis was a point that both systems referred to by the ordered pair "(0,0)". Points to the right of the origin, (0,0) corresponded to ordered pairs with positive abscissas and points to the left matched ordered pairs with negative abscissas. Points above the horizontal axis had positive ordinates while points below this axis

were assigned negative ordinates. This proceedure was adhered to by both the SMSG illustrations, as well as the Merrill illustrations. Again the Merrill texts utilized the use of colored ink printing by making the light line of Figure 5 in red or brown ink. In the SMSG texts only black ink was used. Each system used the coordinate plane in illustrating graphs of functions such as linear functions, quadratic functions, inequalities, and conic sections. The coordinate plane was by far the most used illustrative example in both algebra series.

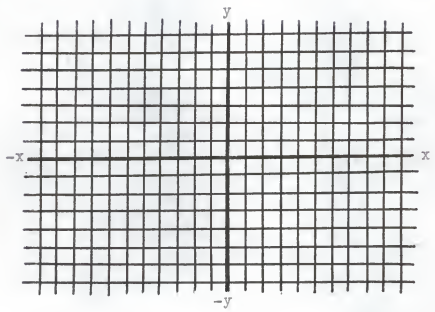

Figure 5

Other types of illustrations, such as drawings and pictures were used more extensively in the Merrill texts than they were in the SMSG materials. Merrill texts featured a number of circle graphs, line graphs, bar graphs, etc., to illustrate points of discussion.

The author of this report thought SMSG utilized illustrations to the point of explaining items in a better manner than Merrill, but that the color contrasts used in the Merrill texts made a greater impression on the reader.

Use Made of Exercises

Both SMSG and the Merrill system used exercises to aid the reader in learning the materials and concepts that each presented.

Exercises contained in SMSG were headed by the title "Problem Set" followed by the number of the chapter, a dash, and then by the number relating to the set of problems in that chapter. The Merrill texts used only the word "Exercises" with a red or brown arrowhead pointing at the word from either side.

Both systems selected problems for their exercises that were designed to help the reader understand concepts that had just been presented and which would put the reader in the right frame of mind for concepts to be presented next.

In both texts, exercises were arranged generally in an ascending order of difficulty. Merrill ended many of their exercises with some "Honors Exercises" to challenge the more able students. SMSG did about the same thing by presenting, at ideal intervals, problem sets they called "Challenge Problems."

Miscellaneous or review problems were used liberally by both texts to keep the reader from forgetting earlier concepts and to produce continuity of understanding.

The Merrill texts used "discovery" exercises early in the text to motivate the student to investigate and draw his own conclusions concerning the topic under consideration. SMSG utilized this learning technique in the wording of its explanations and in using such problems in its exercises. This technique tended to make the reader more interested in the concepts to be presented later.

It was the opinion of the writer that both systems chose problems for their exercises that did an excellent job of supplementing the concepts being discussed.

Treatment of Modern Mathematics Concepts

Both of the algebra systems presented the modern mathematics concepts, although SMSG succeeded in weaving the concepts into their discussions more smoothly. The properties of the number system such as closure, commutativity, associativity, distributivity, identies, inverses, and order were presented adequately by both texts, although SMSG continued to cause the reader to refer to these properties throughout the text to a greater extent than did the authors of the Merrill materials.

Merrill's authors did a better job of explaining to the reader the reasons for the use of undefined terms and for the careful and accurate definitions used than did SMSG. SMSG defined their terms adequately, but did not explain the importance of using undefined

terms and for accurately defining other terms. SMSG used the term, "property" to refer to ideas that the Merrill texts referred to as both axioms and properties of the number system.

The Merrill texts presented their materials in a fashion that greater resembled the presentation of traditional texts than did SMSG, but the modern concepts and terminology were not left out. They just presented them in the manner of listing them, while SMSG worked them in to their discussion and then presented them formally.

It was the opinion of the author of this report that the modern concepts of mathematics were presented better by SMSG, but that the presentation by the Merrill texts was adequate enough that they could serve to replace the SMSG materials.

SUMMARY AND CONCLUSIONS

The purpose of this report was to develop a set of criteria whereby textbooks could be compared with the books designed by the School Mathematics Study Group to determine whether or not these commercial texts would serve as suitable replacements for the materials of SMSG. It was also the purpose of this paper to use this set of criteria to determine if the algebra texts published by the Charles E. Merrill Books, Inc., in 1962 would serve satisfactorily as such a replacement.

The criteria developed consisted of examination of the physical features of the materials including, the general format, indexes and references, usability, and the teacher's manuals. It also consisted of items relating to content and presentation consisting of, examination

of concepts presented, the use of notation and symbols, methods of presenting topics, illustrations, uses made of exercises, and treatment of modern concepts and terminology.

Copies of the Merrill materials were directly compared with those of SMSG according to the above criteria and it was concluded that, for the most part, they were adequate as replacements for the SMSG materials.

It should be pointed out that, in the author's opinion, other texts probably can be found that would serve as well, or better, than the Merrill texts and that any committee considering replacing SMSG materials with commercial texts should give them consideration.

The Merrill texts definitely offer the advantage of presenting the algebra one course and the algebra two course each in a single textbook as compared to two books for each of the courses by SMSG. They also provide the teacher with an annotated copy of the text including a teacher's guide and answer key. SMSG provided these in separate books, thus requiring the teacher to look after eight books to cover the two courses.

The SMSG materials did a better job of presenting and using the ideas and concepts of modern mathematics, although the author thought the Merrill presentation of these materials were adequate.

BIBLIOGRAPHY

BIBLIOGRAPHY

Brueckner, Leo J., Foster E. Grossnickle, and John Reckzeh, Developing Mathematical Understanding in the Upper Grades. New York: Holt, Rinehart, & Winston, Inc., 1957.

Brueckner, Leo J., and Foster E. Grossnickle, Making Mathematics Meaningful. New York: Holt, Rinehart, & Winston, 1953.

Hlavaty, Julius H., "Mathematics in Transition." The Mathematics Teacher, 52:265-67, April, 1959.

Payne, Holland, "What About Modern Programs in Mathematics?" The Mathematics Teacher, 58:422-24, May, 1965.

Peak, Philip, and Others. "Aids For Evaluators of Mathematics Textbooks." The Mathematics Teacher, 58:467-73, May, 1965.

School Mathematics Study Group. First Course In Algebra. New Haven: Yale University Press, 1961.

School Mathematics Study Group. First Course In Algebra, Teacher's Commentary. New Haven: Yale University Press, 1961.

School Mathematics Study Group. Intermediate Mathematics. New Haven: Yale University Press, 1961.

School Mathematics Study Group. Intermediate Mathematics, Teacher's Commentary. New Haven: Yale University Press, 1961.

Spitzer, Herbert F. The Teaching of Arithmetic. Cambridge: Houghton Mifflin Company, 1954.

Thatcher, Dwain L. "A Comparison of UICSM and SMSG Algebra Units with Accompanying Suggestions." Unpublished Master's Report, Kansas State University, Manhattan, Kansas, 1964.

Vannatta, Glen D., A. Wilson Goodwin, and Harold P. Fawcett. Algebra One: A Modern Course. Columbus: The Charles E. Merrill Books, Inc., 1962.

Vannatta, Glen D., A. Wilson Goodwin, and Harold P. Fawcett. Algebra One: A Modern Course. (Annotated Edition). Columbus: The Charles E. Merrill Books, Inc., 1962.

Vannatta, Glen D., A. Wilson Goodwin, and Harold P. Fawcett. Algebra Two: A Modern Course. Columbus: The Charles E. Merrill Books, Inc., 1962.

Vannatta, Glen D., A. Wilson Goodwin, and Harold P. Fawcett, Algebra Two: A Modern Course. (Annotated Edition). Columbus: The Charles E. Merrill Books, Inc., 1962.

Wilson, Elenard C. "Modern Mathematics in the Comprehensive High Schools and Selected Junior High Schools in Kansas." Unpublished Master's Report, Kansas State University, Manhattan, Kansas, 1965.

CRITERIA FOR COMPARING COMMERCIAL TEXTBOOKS WITH THE SCHOOL MATHEMATICS STUDY GROUP MATERIALS AND AN APPLICATION OF THESE CRITERIA TO CHARLES E. MERRILL ALGEBRA TEXTS

by

NELSON C. STAFFORD

B. S., East Central State College, 1960

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

College of Education

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1966

The purposes of this report were to develop a set of criteria by which commercial textbooks could be compared to the materials developed by the School Mathematics Study Group and by use of these criteria to compare the algebra textbooks offered by the Charles E. Merrill publishing company to the SMSG materials.

A review of literature pertaining to textbook evaluation and selection was made. The set of criteria to be used for comparison was developed from the views of other writers, the experiences of the author of this paper, and from suggestions of competent advisors in the field of education.

The SMSG textbooks and the Merrill textbooks were obtained from the representative presses and were carefully studied and compared according to the set of criteria that had been developed.

Comparisons were made of the physical characteristics of the materials including the (1) general formats, (2) indexes and references, (3) usability, and (4) teacher's manuals.

Comparisons were also made of the contents and methods of presentations in the areas of (1) important concepts contained by the materials, (2) notation used by the texts, (3) methods of presenting topics, (4) use made of illustrations, (5) use made of exercises, and (6) the treatment of modern mathematics concepts.

It was found that the Merrill algebra texts would serve satisfactorily as substitutes for the SMSG materials in most respects and that in some instances they would serve to improve SMSG's offerings.

The two system's similarities and differences were noted and their strengths and weaknesses were compared, and comments were made concerning substituting one for the other.

It was found that the Merrill texts would serve to yield approximately the same end result as would the SMSG materials, but that other texts should be examined for possible substitutes which would present the modern mathematics concepts more nearly in the manner that SMSG presented them.