

THE APPLICATION OF SYMBOLIC LOGIC TO  
PLANT LOCATION

by

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## INTRODUCTION

Many industrial firms in the United States are relocating because of expansion, shifting markets, steadily mounting freight costs, the need for new labor reservoirs, the desire to operate in small low-cost communities, or special requirements for new production facilities. Geographic location of firms, as a result, has become increasingly important to industry when compared with the accepted importance of sound management, modern plant structure, and shrewd merchandising policies. Some firms realize a differential of as much as ten per cent of total manufacturing and distribution costs simply by virtue of geography.<sup>1</sup>

Unfortunately, few real advances can be claimed in the method of determining where new plants should be built. Far too often, decisions are made with little regard for the minute organizational details considered to be so essential in actual production. In spite of the seriousness of the problem, there have been developed no definite guide lines to follow in the field of plant location. Over 13 thousand agencies, such as industrial development commissions, utility and railroad development groups, local chambers of commerce, et cetera, operate in the United States today seeking to attract industry to the areas represented by these agencies.<sup>2</sup> Such agencies, together with the manufacturing firms themselves, are still searching for a concise method of manipulating the great number of factors required to reach a decision in plant location.

At this point it should be mentioned that many of the industrial giants

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<sup>1</sup> Leonard C. Yassen, Plant Location, p. 5.

<sup>2</sup> F. E. Le Van, "What Du Pont Looks for in Site Selection," Industrial Development, December 1959, 128:6.

appear to have developed a workable technique of plant location by virtue of the number of plants which they have located and which are in operation. The techniques used by such firms are neither available for critical analysis nor for common usage, and therefore contribute little to our knowledge of the subject.

In considering the problem of plant location, the actual analysis may be broken down into the following areas:

- 1) Determination of factors to be considered.
- 2) Determination of the relative importance of these factors.
- 3) Manipulation and analysis of the factors necessary to reach a decision.

This thesis is an attempt to examine only the method of analyzing and manipulating location factors. In this study the use of symbolic logic in the field of plant location will be introduced. The system of symbolic logic appears to lend itself to this type of problem, although no evidence has been found of previous applications in this field. It is felt that the characteristics of symbolic logic, which enable one to form concise and unambiguous statements of complex propositions, qualify this technique as one method of solving the plant location problem. It is hoped that this preliminary work may lead to the later development of an effecient technique of analysis, usable by firms interested in relocating and by communities seeking new industry.

## PLANT LOCATION IN INDUSTRY TODAY

### Importance

The problem of plant location is of special interest to two distinct groups. The first group includes firms seeking a location at which their business can be successfully carried on. The second group includes those persons seeking to attract industry to a given place. This group is comprised of railroad and utility area development groups, promoters of industrial parks, local chambers of commerce, real estate interests, and others.

Although these groups approach the problem from opposite sides, they share a common goal. This goal is the successful location of an industrial firm. With this sharing of goals it is not uncommon to find the efforts of the groups pooled to reach their objective.

Increased activity concerned with plant location problems is shown by the number of firms "on the move" and by the number of groups engaged in attempting to influence the location decisions of these firms. This heightened activity indicates increased importance being assigned to plant location problems, and there seems to be a sound basis for this increase.

From the standpoint of the first group, changes in the relative importance of plant location considerations are largely a result of technological changes.

- 1) Improvements in transportation may drastically reduce the time required to get a product to the market from distant points of production, thus decreasing the advantage of local suppliers. At the same time, however, transportation costs have increased in the past decade.

- 2) Technical changes have also changed the skills required in the workforce producing an item. Division of labor has made operations possible in an area with a work force having little or no industrial back-

ground because the limited number of skilled personnel needed can be imported.

3) The greater mobility of the population as a whole has caused massive shifts in population centers and radically altered market potentialities, thus causing a constant change in the competitive value of a given location.

A recent study cited specific differences which were found to exist among communities studied.<sup>1</sup> Measured from one extreme condition to the opposite extreme, the differences were as follows:

- 1) Four and one half year differential in median school years completed.
- 2) 700 per cent difference in the number of engineers and scientists per 1000 population.
- 3) 45 to 1 ratio of mental rejects by Selective Service.
- 4) Difference in average hourly wages in manufacturing of as much as 77 cents per hour.
- 5) Crime ratio 40 times higher in some locations.
- 6) Per Capita debt of \$350 in some places, one dollar in others.
- 7) Per cent non-farm workers organized by unions six times as great in one location as in another.
- 8) Work stoppages per 1,000,000 non-farm employees were 12 times as great in one place as another.
- 9) Accidental death rate four times as high in some locations.
- 10) Voting rate varied from 22.1 to 77.3 per cent.
- 11) Community chest contributions 12 times as high in some places.

These figures serve as a representative picture of the variations which management might encounter in their plant location studies. Realization of these variations must be added to the factors influencing manufacturers to place new emphasis on obtaining the most advantageous relocation, initial location or branch plant location.

From the standpoint of the second group, the increased interest in plant location is a result of a new awareness of the worth of industry as a part

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<sup>1</sup> "Area Development Guide," by publishers of Industrial Development, vol. 1, p. 3-1.

of the community. As a result of improved transportation and production facilities, companies found that they were able to relocate in an area where cheaper labor was available or where the type of labor needed was available, where there was ample space for expansion, or where a greater share of the market for some particular item might be captured. As such moves took place, communities that formerly had little interest in industrial firms became aware of the economic value of attracting new industry. Community, area, and utility development groups were formed in an effort to lure companies to their particular locations. As proof of the worth of such efforts, the following figures were compiled by the United States Chamber of Commerce.<sup>1</sup>

For every 100 new factory employees, the following will be created:

- 112 more households in the community
- 296 additional people
- \$360,000 annual rental gains, exclusive of plant purchases
- 51 more school children
- Four more retail establishments
- 174 more workers employed
- 107 more passenger cars registered
- \$270,000 more bank deposits
- \$590,000 more personal income per year

Thus, considering these expected values, it can be seen that a significant addition to the economic welfare of the site chosen can be anticipated.

If site selection is so important to both the industrial and the community groups, the study of decision-making techniques used in determining locations appears to provide a worthwhile research area.

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<sup>1</sup> "What New Industrial Jobs Mean to a Community," United States Chamber of Commerce, 1954.

### Present Methods

Perhaps the most significant discovery to be made in a study of plant location problems is the infinite combination of factors requested by industry, which are to be filled by communities possessing equally widely varying characteristics. The needs of industrial firms differ greatly, as do the offerings of communities. What is good for one company may be wholly unacceptable for another. Probably for this reason there has been no standard approach to the problem throughout industry.

Plant location factors, in general, have been divided into two separate groups. The first group is composed of those factors which can be directly evaluated in terms of dollars, such as transportation costs, materials costs, and labor costs. This group is sometimes called tangible factors. The second group of factors is termed the intangibles and includes factors such as the attitude of a city toward new industry, the quality of schools, climate, recreation facilities, and other factors not capable of direct monetary evaluation.

Because of difficulty in evaluating the intangibles, most present systems of plant location stress heavily the former group of factors.

A great many of the discussions of plant location techniques by present authors assert that transportation cost is the primary production cost which is a function of plant location, and base the location decision on consideration of this cost. It is said that all other things being equal, a plant will want to locate at the point having the lowest aggregate transportation costs. New facilities should be established at that point where a freight advantage can be reached competitively. A plant location should attempt to neutralize the freight advantage of competitors.

Working from these premises, plans have been formulated for calculation



of the cost of purchase of raw materials at various points under consideration, and calculation of the cost of shipping goods to market from these various points. By simple arithmetic operations the cost of transportation for these locations can be compared, given the tonnage shipped, the means of shipping, and the shipping rates.<sup>1</sup>

Where shipping goods to market is a major cost and raw materials transportation costs are less critical, the approximate location desired can be obtained by computation of the weighted center of the market area. This is a graphical procedure and is carried out as follows:<sup>2</sup>

- 1) An overlay of graph paper is made to cover a large map of the area to be served.
- 2) Customer locations are plotted on the graph paper according to their locations on the map.
- 3) Horizontal and vertical axis lines are arbitrarily drawn with their origin in the lower left hand corner of the graph paper so that all points plotted fall within quadrant 1, the upper right hand sector of the coordinate system.
- 4) A uniform scale is laid out along the horizontal (X) axis and along the vertical (Y) axis.
- 5) The number of distance units along the X axis and Y axis of each destination are found and recorded in tabular form.
- 6) Multiply the number of distance units along each axis for each entry by the volume of tonnage moved to that destination during a representative period.

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<sup>1</sup> Leonard C. Yassen, Plant Location, p. 36.

<sup>2</sup> Ibid, p. 39.

- 7) Determine the arithmetic mean as follows: a) Sum up the values obtained for step six so that a value is obtained for  $\sum_0^n (X_i T_i)$  and for  $\sum_0^n (Y_i T_i)$  where  $n$  cities are considered and  $X_i$  is the distance from the origin to the  $i^{\text{th}}$  city and  $T_i$  is the tonnage moving to the  $i^{\text{th}}$  city.
- b) Divide the summation values ( $\sum_0^n [X_i T_i]$  and  $\sum_0^n [Y_i T_i]$ ) by  $\sum_0^n T_i$ , the tonnage moving to all points considered.

The weighted center of the market is found by constructing a perpendicular line from the  $X$  axis at the point  $X$  is equal to the mean value of distance along the  $X$  axis, and similarly constructing a perpendicular on the  $Y$  axis at the mean value of  $Y$ . The intersection of the two perpendiculars is the weighted center of the market.

Actual transportation costs for shipping goods to market are found by applying specific freight rates for those locations under consideration that lie close to the weighted center previously calculated.

Recently, further sophistication has been added to the approach mentioned above by the use of linear programming techniques.<sup>1</sup> Linear programming methods are useful primarily in finding the most economical location of an addition to present facilities in terms of variable costs incurred in marketing an item. They can also be used, however, in locating a new plant by examining the plant's competitive position. The data required for this analysis is the same as in the previous method. The transportation costs incurred in supplying customers in a given city or area must be known. If there are any further costs which are subject to change due to distance or method of shipping, these could also be included in the cost figure for an item moved from the factory to

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<sup>1</sup> Robert E. Johnson, "Science and Site Selection," Industrial Development, July 1959, 128:6.

its destination. Also, the capacity of each plant or proposed plant must be known along with the requirements of customers in each area. Using these data a transportation problem may be constructed for each proposed location, and the optimal solution obtained for each proposed location. The minimum cost figure of the optimal solutions would indicate the most economical location in terms of transportation costs.

It can be appreciated that many firms would not consider transportation costs to be the most important location factor. It would be readily admitted by proponents of the former plans that many other factors must be considered.

Other plans go a step further, and consider a larger number of factors in reaching a decision. Here again, however, the factors considered are only those which can be evaluated in terms of dollar costs. A typical schedule of items which might be considered in a plant location study is shown in the comparison chart on the following page.<sup>1</sup>

Although such a comparison would provide a fairly clear picture of the relative cost of operation in communities under consideration, there are many other intangible factors to be considered. Such an analysis might be considered to provide information concerning the most desirable area (speaking in general terms) in which to locate, since many of the cost factors will not vary greatly from one community to another in a given area. A study of the intangible factors is required in pinpointing any location decision.

Many firms and location groups have compiled extensive lists of the intangible or "non-cost" factors to consider. Mere lists, however, tend to result in large volumes of unmanagable data. In partial answer to this problem one

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<sup>1</sup> H. B. Maynard, Industrial Engineering Handbook, ch. 7, p. 23.

Comparison Chart

Basic Factors	Present Location	City A	City B	City C	City D	City E	City F
Total Transportation Costs							
Inbound Materials							
Outbound Materials							
Total	\$						
Labor							
Direct Production							
Non-productive							
Total	\$						
Plant Overhead							
Rent or Carrying Costs excluding taxes							
Additional costs due to inefficient layout, lack of siding, etc.							
Real Estate Taxes							
Personal Property Taxes, etc.							
Fuel for heating purposes only							
Total	\$						
Utilities							
Power							
Gas							
Water							
Sewage							
Total	\$						
State Factors							
State Taxes							
Workman's Compensation							
Insurance							
Total	\$						
Miscellaneous							
Other cost Factors Inher- ent or Peculiar to Pre- sent Location(s)							
Total	\$						
Grand Total	\$						

firm proposed a system of "rating" factors.<sup>1</sup> In preliminary studies aimed at evaluating existing operations of the firm, a list of factors or criteria in the selection of a new plant site was drawn, and a weight or numerical value was assigned to each criterion according to the importance of that criterion to successful operations. Next, the communities which were being considered as possible plant sites were studied and an attempt was made to determine to what extent each community possessed each of the previously determined criteria. Each community was rated on each of the criteria on the basis of poor (0), fair (1), good (2), or excellent (3). The rating given each criteria for each community was multiplied by the previously determined weight assigned to that criteria, and the community totals for the weighted evaluations were obtained as a sum (for each community) of these products.

It is acknowledged by industry men that these previously developed aids provide only guides to follow. The difficulty in the development of a more refined methodological approach to plant location problems is due to the complexity of the problem. Even though it is possible for industry to predict its needs with some accuracy, the situation rarely arises where these needs can be ideally fulfilled. In actual practice, the plant location decision usually is found to hinge on a series of compromises. This, also, marks the weakness of present numerical methods of analysis.. It has appeared to be impossible to design into a system the concept of the interrelationship among factors of location. There has been no adequate means of representing the logical decisions through which management actually reached a solution. Therefore, the expression of the relationship of values among factors was left

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<sup>1</sup> Walter E. Froehlich, "Associated Spring," Industrial Development, March 1960, 129:12.

unstated except in the minds of the men performing the work.

This seeming void in plant location procedures suggests the application of a relatively new tool in the industrial world: symbolic logic. The following section of this thesis is a general explanation of the history and methods of symbolic logic, which in turn is followed by the presentation of a proposed technique for applying those methods to plant location solutions.

## SYMBOLIC LOGIC

### History

Symbolic logic can be thought of as a language that manipulates ideas as algebra manipulates numbers. It has been said that without algebra, the ancient Egyptian arithmeticians struggled in vain up to 1600 B.C. to find an answer for the following simple problem: What number, plus one-fifth of itself, equals twenty one? By modern algebraic methods we write the problem as

$$X + X/5 = 21,$$

and solve for

$$X = 17\frac{1}{2}.$$

The difficulty involved in finding the answer without algebraic procedures can be traced to the fact that the early Egyptians lacked a convenient symbolic method of stating the problem, such as letting  $X$  represent an unknown, and digits (0 to 9) represent numbers.<sup>1</sup>

Following a similar train of thought, logicians have sought to develop an algebra of ideas, or as it is more commonly referred to in modern literature, a calculus of propositions.

Formal logic, as such, began with the syllogisms of Aristotle. A syllogism consists of a major premise and a minor premise, the first making a statement about a "predicate term" and a "middle term", the second about the same middle term and a "subject term." By elimination of the middle term, one arrives at the correct conclusion as to the relation of the subject to the predicate. These syllogisms were generally stated as follows: "All men are

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<sup>1</sup> John E. Pfeiffer, "Symbolic Logic," Scientific American, December 1950, 183:22.

mortal; Socrates is a man; Therefore Socrates is mortal." Greek philosophers set forth 14 such syllogisms and believed that they had summed up most of the operations of reasoning. Latter medieval theologians added five more syllogisms, and for hundreds of years these nineteen syllogisms were the foundations of the teaching of logic. All during this period there was very little use of symbols. Occasionally some symbols were used as a kind of shorthand for terms and propositions, but there was no use of symbols for logical relations.

Not until the Nineteenth Century did anyone successfully apply symbols and algebra to logic, in the place of verbalisms of Aristotle and his followers. In 1847 George Boole first presented his algebra in the paper, The Mathematical Analysis of Logic--Being an Essay Toward a Calculus of Deductive Reasoning. This paper has largely been the basis of the development of symbolic logic since then. For the first time operations of a mathematical type were systematically and successfully applied to logic. In his paper Boole stated a set of axioms from which more complex statements should be deduced. Statements were in algebraic terms, with symbols such as X and Y representing classes of object. Through this and other related papers Boole became known as the inventor of symbolic logic.

Following the presentation of Boole's system of algebra, numerous logicians and mathematicians, led by Pierce, Schroeder, and Peano, attacked the study of logic with renewed interest.

By 1913 Alfred North Whitehead and Bertrand Russell had developed a formal "mathematical logic" using a system of symbols developed by Peano. The Mathematica Principia, in addition to providing an elaborate derivation of mathematical concepts, provides the first significant bridge in the gap between Boole's class concept of logic to the more usable concept of propositional calculus.



In spite of the new interest in logic which was generated by Boole, Whitehead, Russell, and others, the study of symbolic logic remained one of only academic interest for many years. One of the first applications of symbolic logic to a business problem was credited to a mathematician employed by a life insurance company. It was felt that conflicting sets of policy clauses had been established over a period of years, and symbolic logic was used to trace such conflicts and to simplify statements where possible.<sup>1</sup>

Other more recent and widespread applications have been found in the design of the logical circuitry of computing machines.<sup>2</sup> Occasional applications have been made in product engineering problems involving the design of complex arrangements of switches or valves needed to control an operation,<sup>3</sup> and in simplifying the statement of production requirements.<sup>4</sup>

In recent years modern logicians have punched the classical Aristotelian system of logic full of holes and have greatly refined the system of logical development. Logicians have now rejected some of the original syllogisms, and reduced the others to only a few theorems. In doing this, logicians have abandoned one of Aristotle's basic principles, that a statement must be either true or false. Some of the men who have led the way with the development of refined logical systems are Post, Rosser, Turquette, Lukasiewicz, and Tarski,

<sup>1</sup> Sanford S. Ackerman, "Symbolic Logic: A Summary of the Subject and its Application to Industrial Engineering," Journal of Industrial Engineering, September-October 1957, 8:263.

<sup>2</sup> J. P. Laird, "Design by Logic - Automatic Chemical Batching," Mechanical Engineering, January 1958, 80:38.

<sup>3</sup> Martin Gardner, "Logic Machines," Scientific American, March 1952, 186:68

<sup>4</sup> Joseph F. McCloskey, and Florence N. Trefethen, editors, Operations Research for Management, vol. 1, p. 191.

to mention a few.

Having followed the general historical development of the systems of mathematical logic to present times, one can but speculate as to future developments. It is fairly certain that new applications will be forthcoming from this virtually untapped body of theory, even though the avenues of extension are at this time unknown.

### Principles of Symbolic Logic

Logic provides techniques for examination of the logical make-up of any formal structure. Symbolic logic might well be known as the "algebra of logic" and is often referred to as mathematical logic. In place of word-propositions, or functional statements, it substitutes symbols. Relations between propositions and functions are, furthermore, concisely and unambiguously expressed in symbols. The rules of a formal system, given in symbolic form, then define permissible transformations from one statement to another. By using such transformations one is enabled to operate upon algebraic expressions.

One of the primary areas of interest in symbolic logic, and one with which this development is primarily concerned, is the calculus of propositions.

The expression of this discipline is accomplished through the use of two types of primitive symbols, or symbols which are accepted by definition.

The first of these two is the propositional symbol. A proposition, briefly defined, is a sentence which says something of a subject. As can readily be appreciated, there are an endless number of possible propositions which could be asserted. For the sake of clarity any capital letter will be chosen to represent a proposition. For example, the letter A can stand for "The grass is green", or for a more complex statement, such as "Symbolic logic is a

means of concisely and unambiguously expressing complex verbal statements".

The second type of symbol is the operator symbol. For these, certain special symbols are used to show relations between propositions. These special symbols and their meaning can be defined within the framework of the traditional two-valued calculus of propositions. A calculus of propositions, by definition, examines the implications of several statements made concurrently. The calculus of propositions is known as two-valued if any proposition considered can exist in only one of two states -- true or false. In other words, the possibility of only two states is admitted for any given proposition.

One of the simplest methods of presenting a description of any operation is by means of truth tables. Truth tables merely present in matrix form all possible values of the relations between two asserted propositions, based on the values of the original propositions asserted.<sup>1</sup> The truth values of all operationally constructed elements (propositions joined by operator symbols) are determined by the truth-value of their constituents. This definition of truth tables can be appreciated in light of the uses which follow.

#### Primitive Operator Symbols

Disjunction ( v ). The first special symbol to be considered is that of disjunction. A disjunction is formed by the linking of two or more statements with the word "or", and is shown by "v". In case of a two-valued system the symbol "v" is taken to mean that either one or the other, or both of the statements asserted are true. This is shown by truth tables as follows:

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<sup>1</sup> Susanne K. Langer, Symbolic Logic, 2nd revised edition, p. 352.

Table 1. Disjunction truth table.

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

In Table 1, the letters T and F represent the truth values of true and false respectively as assigned to the general propositions A and B for a two-valued system.

The truth values of the asserted disjunction of the propositions ( $A \vee B$ ) are given in the third column and are dependent on the truth values of the original elements. For example, it can be seen that the disjunction of the two statements "The sun is shining" and "The sky is overcast" is true when either or both of the original statements are true. In this case, one might suggest that a disjunction of the above mentioned statements could have meaning only under certain conditions, as for example, a pilot flying above clouds and observing both the sun and clouds, or either one of the two.

It is important to point out here that operator symbols have nothing to do with the truth or falsity of the propositions themselves, just as algebra is not concerned with the particular physical quantity designated by its symbols. The operations of symbolic logic can only show that, given certain premises, certain conclusions are valid. The establishment of factually accurate premises is outside the province of logic; its concern is with the validity of the conclusions drawn from a given set of facts or assumptions.

Conjunction ( $\cdot$ ). The second operational symbol indicates a conjunction of statements, or joining two statements by inserting the word "and" between the statements. To assert the conjunction ( $A \cdot B$ ) is equivalent to

saying that both A and B are asserted. That is, the conjunctive statement can be true only if both parts of the conjunction (both A and B) are true. In the example used above, the conjunction of the propositions "The sun is shining" and "The sky is overcast" is true only when both the propositions are true. This operation is designated by a dot ( $\cdot$ ). Its truth value is as follows:

Table 2. Conjunction truth table.

A	B	A $\cdot$ B
T	T	T
T	F	F
F	T	F
F	F	F

Thus it is seen above that the conjunction of two statements is true only when the truth values of each of the constituent statements are true.

Negation ( $\sim$ ). In a two-valued system of logic the operation of negation reverses the value or contradicts a statement. Therefore the negation of a true statement is false (has the value of false) and vice versa. To assert the negation of B is equivalent to saying "not B" is asserted. If a proposition B can have only two possible values, true or false, and if it is taken to have the value of true, then  $\sim B$  is the contradiction of B, and has the value of false. Thus, if B = the sun is shining,  $\sim B$  = it is not true that the sun is shining. The truth table representation of this operation is as follows:

Table 3. Negation truth table.

A	$\sim A$
T	F
F	T

### Additional Operator Symbols

Using the above symbols as primitive or defined operations, several other operations have been developed from these, but are represented by separate operator symbols because of their frequency of use in some applications of the propositional calculus.

Implication ( $\supset$ ). This operation is generally defined as expressing the relationship that if the first statement asserted is true, then the second statement is true. The truth tables indicate that the operation  $(A \supset B)$  is equivalent to  $(\sim A \vee B)$ , and they are in fact equal by definition. This can be seen by observing the truth values given in the columns below the appropriate operation heading.

Table 4. Implication truth table.

A	B	$A \supset B$
T	T	T
T	F	F
F	T	T
F	F	T

A	$\sim A$	B	$\sim A \vee B$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

Equivalence ( $\equiv$ ). Two statements are said to be equivalent when they have the same truth value, and the symbolization that two statements are equivalent is shown by inserting ( $\equiv$ ) between the statements. The truth values are as shown in Table 5.

Table 5. Equivalence truth table.

A	B	$A \equiv B$
T	T	T
T	F	F
F	T	F
F	F	T

This operation could be used to express the relationship between the two propositions  $(A \supset B)$  and  $(\sim A \vee B)$ , which is  $(A \supset B) \equiv (\sim A \vee B)$ , as shown in the section on implication. This operation is especially useful in expressing some of the laws governing transformations which may be carried out.

From these concepts, an extensive framework of theorems, rules, and argument forms has been developed in providing a rigid logical system for academic considerations. The above operations are sufficient for describing the operations to be performed in this thesis. The presentation of any laws necessary to follow simplification steps will be made later if needed to understand the steps taken. The derivation of such transformation theorems involves a great deal of study in logical developments that is not pertinent to this thesis.

An example of the application of symbolic logic in a production engineering problem using a modified form of truth tables is shown in the appendix. This example is intended to show the usefulness of symbolic logic in dealing with complex problems.<sup>1</sup>

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<sup>1</sup> McCloskey and Trefethen, loc cit.

## APPLICATION OF SYMBOLIC LOGIC TO PLANT LOCATION

## General Development

The technique to be presented is based on the laws of symbolic logic. The conclusions reached through the application of symbolic logic to the problem of plant location are not unique in the sense that they can be derived through no other procedures. On the contrary, symbolic logic itself is merely an ordered method of handling propositions, and its methods of solution are characterized by deductive reasoning and symbolic notation of propositions. Therefore, anyone possessing the powers of deductive reasoning, and the ability to arrive at symbolic notations for propositions could possibly reach solutions in a given problem as well as could a person schooled in symbolic logic. The same thing is true of the technique presented here, and in fact, of most techniques of problem solution. This, however, does not detract from the usefulness of analytical techniques in general, or the use of symbolic logic in plant location in particular.

The behavior of logical relations, such as  $\vee$  and  $\cdot$ , (disjunction and conjunction) is the basis of this development. These relations hold among propositions, the terms of such logical relations are propositions themselves, and the propositions to which these relations give rise are in fact propositions about propositions.

These relations have formal properties as all relations do. It can be set forth initially that we shall denote propositions by the symbols of the alphabet

A, B, C, . . . . .

A<sub>1</sub>, B<sub>1</sub>, C<sub>1</sub>, . . . . .



$$A_2, B_2, C_2, \dots$$

$$A_3, B_3, C_3, \dots$$

as needed in the expressions which they symbolize.

The operations which will be used to relate the propositions concerning plant location factors are as follows:

(  $\vee$  ) Disjunction

(  $\cdot$  ) Conjunction

(  $\sim$  ) Negation

These operations on logical statements can be carried out for the case involving more than two possible alternatives of a given proposition as well as when there are only two such possible alternatives, as was the case in the truth tables seen earlier.<sup>1</sup>

The need for a many-valued system of logic can be recognized when one realizes that the factors of plant location problems are always present in varying degrees rather than either being present or not present. The very core of the location problem lies in the proper matching of the various degrees of all factors available. For this reason the many-valued system of logic is used to provide the more complex framework needed for the problem.

In the development of a many-valued system, the concept of probability is introduced.<sup>2</sup> The various levels or possible levels of truth values which are admitted of a particular system are based on the probability of occurrence of each level. Thus, while T (truth value sometimes given as 1) was thought of as being the symbol for truth in a two-valued system, it is thought of in a many-valued system as the segment of a class of items which is most likely to

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<sup>1</sup> J. B. Rosser, and A. R. Turquette, Many-Valued Logics, p. 15.

<sup>2</sup> C. I. Lewis and C. H. Langford, Symbolic Logic, p. 230.

be realized or the most likely occurrence out of the possible occurrences. The situation to be studied requires a definition along the lines of the level of a given location factor which will most probably give success. For example, the particular city size which management feels will best suit its operations is rated as (1), and less desirable sizes given correspondingly higher numerical ratings.

The definition of operations involving many-valued systems will be shown through the use of truth tables, as was done for two-valued systems.<sup>1</sup> It is also desirable to develop a verbal explanation of these operations lest it be felt that the truth tables represent an arbitrary manipulation in themselves.

Disjunction. The operation of disjunction allows a choice of one proposition, or the other, out of two propositions, or the choice of both propositions. Any time such a choice is offered, the better of the two choices is taken. That is, if there is a factor A ranked as 3, and a factor B ranked as 1, then the value of  $(A \vee B)$  would be taken as 1, where 1 is the evaluation of proposition B and is the better of the two choices. Following this line of reasoning, the truth table for disjunction involving a universe having  $n$  possible values for each factor is given as follows:

Table 6. Disjunction truth table.

(v)	1	2	3	...	n
1	1	1	1	...	1
2	1	2	2	...	2
3	1	2	3	...	3
4	1	2	3	...	4
⋮	⋮	⋮	⋮	⋮	⋮
n	1	2	3	...	n

<sup>1</sup> Paul C. Rosenbloom, The Elements of Mathematical Logic, P. 52.

Table 6 is read by locating the desired truth value for one proposition in the first row, the desired truth value of another proposition in the first column, and the truth value of the disjunction of propositions is read at the intersection of the selected column and row.

The construction of the table assumes that the assertion of a proposition concerning a specific level of a factor also includes the assertion of all less desirable levels of the factor.<sup>1</sup> To say that a community possesses the most desirable level of a factor also implies that the less desirable levels of a factor are possessed. The value of the disjunction of two propositions is given by the highest individual ranking.

Conjunction. The operation of conjunction was previously said to be true, in the case of only two alternative values of a proposition, when both propositions asserted were true and false in all other cases. For the case allowing  $n$  possible alternatives, the combination of two propositions can be no better than either of the asserted propositions. If a factor A is ranked as 3, and a factor B is ranked as 1, as in the previous example, the conjunction ( $A \cdot B$ ) would have the ranking of 3, where 3 is the evaluation given to proposition A. The combination of A and B, limited by the low ranking of A, follows the rule that the conjunction of two propositions has the ranking of their highest common ranking. If there are five possible levels of a proposition, a ranking of 2 for one factor and a ranking of 3 for another factor are considered to have the ranking of 5, 4, and 3 in common. A greater rank can be thought of as containing all lesser ranks.

The conjunctive truth table is given on the following page.

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<sup>1</sup> Louis Couturat, The Algebra of Logic, p. 10.

Table 7. Conjunction truth table.

(:)	1	2	3	. . .	n
1	1	2	3	. . .	n
2	2	2	3	. . .	n
3	3	3	3	. . .	n
4	4	4	4	. . .	n
⋮	⋮	⋮	⋮	⋮ ⋮ ⋮	⋮
n	n	n	n	. . .	n

Negation. The operation of negation asserts the contradiction of a given statement. When a two-valued system of logic was considered, the negation of one value, of necessity, was the assertion of the other remaining value. That is, when only two values are possible in a system, such as 1 and 0, or true and false, the negation of the value 1, or true, automatically means that the one remaining value of 0, or false, is being asserted. This is the same as saying an object is either black or white, and that it is not black. Therefore, we are saying it is white. One could also say a person has two coins, a nickel and a dime, in his pocket. If one is withdrawn, and the statement is made that the value of the coin withdrawn was not 10 cents, this coin must be the nickel, and the coin still in the pocket is the dime.

The situation becomes more complex in the case of a many-valued system, and cannot be represented by one simple expression.<sup>1</sup> For example, one might visualize a person having different coins in his pocket, ranging in value from a penny to a half dollar. To say that a coin withdrawn from the group does not have a value of 10 cents (meaning equal to or greater than 10 cents)

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<sup>1</sup> Rosser and Turquette, loc cit.

means only that the coin in mind is not a dime, quarter, or half dollar. Using the convention adopted by the authors of the many-valued system of logic, it would be said that we could have a nickel, since the lesser value of the penny is common to the value of the nickel. Therefore, as shown in the table, the negation of level 3 of a given factor means that it is not possible to have levels 1, 2, or 3, since level 3 is a common part of each of these higher levels. It is possible to have level 4, because 3 is not a common part of 4. The negation of level or ranking  $n$  is a special case where  $n$  acts as 0 in a two-valued system. By the logicians' definition the negation of  $n$  is 1, as shown in the table.<sup>1</sup>

Generally speaking, the negation of any level  $(n-1)$  is equivalent to assertion of level  $n$ , and the negation of  $n$  is taken as the assertion of level one (1).

Table 8. Negation truth table.

A	$\sim A$
1	2
2	3
3	4
4	5
$\vdots$	$\vdots$
$n-1$	$n$
$n$	1

Because of the added complexity of the negation operation in the many-valued system of logic, propositions should be stated in positive terms when possible. An example of a situation involving such a statement would be the

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<sup>1</sup> Rosenbloom, op cit. p. 53.

verbalization of the factor of "percent of foreign element in work force". This factor can be handled more easily if it is positively stated (i.e., it is most desirable to have a certain small percentage of the work force made up of foreign born workers) rather than negatively stated (i.e., it is most desirable not to have a certain higher percentage of the work force made up of foreign born workers).

Using the above notations, in various combinations, most propositions can be expressed in symbolic language.

It is proposed that the basic laws applying to the calculus of propositions be utilized to present the logical statements written concerning the various plant location factors, and that operations as defined by truth tables of a many-valued system be used to relate these factors.

#### Approach to Problem

The object of the study is to provide a means of simplifying the decision making process involved in site selection. The desired end in mind is to develop a technique whereby the actual handling and processing of plant location data can be reduced to mere clerical work rather than work requiring the time of management personnel. It is further believed that such a technique would in fact reduce the decision making duties of management for the following reasons:

- 1) Concise statement and presentation of decisions concerning the relative importance of location factors would eliminate much redundancy in management's task of site selection. A final decision of the relative importance of each factor must be made if management is to be able to select a site, but at present these decisions are often not clearly stated. This is especially true with factors which cannot be compared in terms of

dollars.

2) Once a series of decisions has been made by management concerning plant location, the use of a technique as proposed allows comparisons to be made of many cities either by mechanical processing equipment or by clerical employees.

The initial step in attacking a location problem is to determine the factors which will be making some contribution, either positive or negative, to the success of the firm to be located. For the purpose of this thesis, location "factors" are defined as a rather broad designation, such as transportation, utilities, taxes, et cetera. "Sub-factors", which are combined to form the factors, are more narrowly defined, with examples of these being air transportation, rail transportation, et cetera, making up the transportation factor. Further subdivision might be made into the elements making up each sub-factor. Examples of this would be cost of rail service, warehousing facilities, frequency of service, reliability of service, special inducements offered, et cetera.

As indicated earlier, the variability of the importance of specific factors, even within a given industry, is such that selection of factors for location must be made on an individual basis geared to the objectives of the individual firm.

Immediately following the determination by the firm of the factors, sub-factors, and elements of importance in location, the use of symbolic logic enters into the plant location problem.

In an effort to present the technique as clearly as possible, a brief example is given below, using five rankings as desirability levels within each location element. Assuming that the management of a firm knows what is best for its firm, numerical values were assigned to each level of a given location

element to indicate the relative importance within an element.

Example 1.

Population of Metropolitan Area (Element)

- 1) 90,000 to 110,000
- 2) 110,000 to 150,000 or 50,000 to 90,000
- 3) Over 150,000 or 30,000 to 50,000
- 4) 20,000 to 30,000
- 5) Below 20,000

In the example above the level 1) is the most desirable level, and corresponds to the 90,000 to 110,000 population bracket. Similarly, the second most desirable level is number 2), corresponding to a larger population, 110,000 to 130,000 and also including a lower population bracket, 50,000 to 90,000. The other levels are interpreted in the same manner.

Following this pattern for each location element to be considered within each sub-factor, a tabular listing, as in the example shown, of the relative importance or desirability of each level in a given location element is set up.

Having developed the relative importance levels for the various elements of the sub-factors to be considered, it is then necessary to make a statement relating these propositions (importance levels) within each sub-factor of the location problem. Statements must be made relating sub-factors within factors of plant location and finally factors must be related to reach an overall rating factor.

Example 2. Consider the sub-factor of "population of the area in which to locate", which might be within the larger factor of "community characteristics", including such other sub-factors as housing, schools, hospitals, et cetera.



The sub-factor can be broken down into the following four areas for the purpose of consideration: Population of metropolitan area; population within 30 mile radius in addition to metropolitan area; relation of 1950 to 1960 population of total area mentioned above; and distance to a major city.

The levels of importance for each element can be assumed to have been selected by management as follows:

A) Population of Metropolitan Area

- 1) 90,000 to 110,000
- 2) 110,000 to 150,000 or 50,000 to 90,000
- 3) Over 150,000 or 30,000 to 50,000
- 4) 20,000 to 30,000
- 5) Below 20,000

B) Population within 30 mile radius (excluding metropolitan area)

- 1) 30,000 and above
- 2) 20,000 to 30,000
- 3) 10,000 to 20,000
- 4) 5,000 to 10,000
- 5) Below 5,000

C) Relation of 1950 to 1960 population

- 1) Increase of from 10 to 20 per cent
- 2) Increase of from 20 per cent on up
- 3) Increase of from 0 to 10 per cent
- 4) No change
- 5) Any decrease

D) Distance to Major City

- 1) Up to 40 miles
- 2) 40 to 75 miles
- 3) 75 to 100 miles
- 4) 100 to 150 miles
- 5) 150 miles or above

As a means of relating these decisions which it has been assumed that management has made, a statement must be made concerning all such propositions as are given above. Such a statement might read as follows: If a city is to be acceptable, then it must have the most desirable population bracket possible, or be in the most desirable distance bracket possible from a major city, and

have the best possible area population and show specified growth tendencies.

Written in symbolic language, this statement would be given as

$$(A \vee B) \cdot C \cdot D.$$

This expression can then be evaluated by use of the truth tables for conjunction and disjunction. The conjunctive and disjunctive truth tables for a five-valued system are given below.

Table 9. Disjunctive truth table.

(v)	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Table 10. Conjunctive truth table.

(.)	1	2	3	4	5
1	1	2	3	4	5
2	2	2	3	4	5
3	3	3	3	4	5
4	4	4	4	4	5
5	5	5	5	5	5

Carrying the case further, three hypothetical cities are considered and the various levels of the elements being studied are shown below for each city.

Table 11. Table of elements A, B, C, and D possessed by cities X, Y, and Z.

City	Elements			
	A	B	C	D
X	1	2	3	1
Y	2	1	4	2
Z	3	3	2	1

The overall rating of each city for the elements of "population of area" can be evaluated using the tables presented above.

It should first be pointed out that, by the principle of association, the placement of parenthesis in separating similar operations has no effect on the result of the operations. For this reason the answer obtained is in no way dependent on the sequence in which the operations are carried out. The principle of association states that an expression consisting entirely of a number of like operations is not affected by the placement of the parenthesis. It is symbolically written

$$P \cdot (Q \cdot R) \equiv (P \cdot Q) \cdot R \equiv P \cdot Q \cdot R$$

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R \equiv P \vee Q \vee R.$$

Due to this freedom the answer could be obtained in many ways, but the order of occurrence of the symbols in the statement will be used as the order of solution in the example.

The first step is to find the value of the disjunction of statements A and B. Using city X as an example, it can be seen that the city possesses level 1 of element A and level 2 of element B. The disjunction of A and B, using these values, is found in Table 9 by letting the first column (or row) of numbers represent the five levels of element A and letting the first row (or column) of numbers represent the levels of element B. The value of the disjunction of these two elements is found by reading across in the appropriate row (or column) for element A (level 1), and reading down the appropriate column (or row) for element B (level 2) until finding the square lying at the intersection of the row and column chosen. The value or ranking found in this square is 1. This is another way of saying that since either A or B was acceptable, it is concluded that the most desirable of the two elements, or the element contributing the more positive effect, shall determine the ranking for

that part of the evaluation.

$$\begin{aligned} A &= 1 \\ B &= 2 \\ (A \vee B) &= (1 \vee 2) = 1 \end{aligned}$$

The next part of the statement to be evaluated is the conjunction of  $(A \vee B) \cdot C$ . Here the statement is made that both A or B, and C are desired. Using the conjunctive truth table as the disjunctive truth table was used, the first step is to select the first row (or column) figures to represent the possible levels of  $(A \vee B)$  and select the first column (or row) of values to represent the possible values or levels of C. Reading the value or ranking in the square lying at the intersection of the row indicating level 1 and the column representing level 3, the value 3 is found.

$$\begin{aligned} (A \vee B) &= 1 \\ C &= 3 \\ (A \vee B) \cdot C &= 1 \cdot 3 = 3 \end{aligned}$$

Finishing the statement by evaluating the conjunction of  $(A \vee B) \cdot C = 3$  and  $D = 1$  in exactly the same manner as was done in the preceding step, the overall rating is found to be 3. Thus city X has a rating of 3 on the sub-factor of "population of the area considered for location". The rating for cities Y and Z are found in the same manner, and are found to have the overall ratings as shown below.

$$\begin{aligned} \text{City X's rating} &= 3 \\ \text{City Y's rating} &= 4 \\ \text{City Z's rating} &= 3 \end{aligned}$$

When ratings are obtained for each sub-factor of a factor, a statement must be made relating the sub-factors within a factor and an overall rating given to the factor. Finally, the same process is repeated in relating the factors and an overall rating obtained.

The example presented is admittedly quite simple. Going a step further, consider the case involving eight sub-factors making up a factor. These sub-

factors and the ranking of each for a given city is given below.

Table 12. Ranking of eight sub-factors for a hypothetical city.

Factor	A	B	C	D	E	F	G	H
Ranking	1	2	1	3	4	2	5	3

Assume the following relationship was found to exist: We must have [A and (B or C) and (D or E) and F and (H or G)] or, we must have [(A and B) or (C and D and E)] and (F and G and H) or [(A and H) or (B and G)] and [(C and D) or (E and F)].

Using logical symbols, this is written as follows:

$[A \cdot (B \vee C) \cdot (D \vee E) \cdot F \cdot (H \vee G)] \vee [\{(A \cdot B) \vee (C \cdot D \cdot E)\} \cdot (F \cdot G \cdot H)]$   
 $\vee [\{(A \cdot H) \vee (B \cdot G)\} \cdot \{(C \cdot D) \vee (E \cdot F)\}]$ . Evaluating the relationship by using truth tables, the following result is obtained.

$[1 \cdot (2 \vee 1) \cdot (3 \vee 4) \cdot 2 \cdot (3 \vee 5)] \vee [\{(1 \cdot 2) \vee (1 \cdot 3 \cdot 4)\} \cdot (2 \cdot 5 \cdot 3)]$   
 $\vee [\{(1 \cdot 3) \vee (2 \cdot 5)\} \cdot \{(1 \cdot 3) \vee (4 \cdot 2)\}]$

Further simplification yields the following:

$[1 \cdot (1) \cdot (3) \cdot 2 \cdot (3)] \vee [\{(2) \vee (4)\} \cdot (5)] \vee [\{(3) \vee (5)\} \cdot \{(3) \vee (4)\}]$   
 $3 \qquad \qquad \vee \quad (2 \qquad \cdot \quad 5) \vee (3 \qquad \cdot \qquad 3)$   
 $3 \vee 5 \vee 3$

Overall rating = 3.

The complexity of the problem can be appreciated as the number of sub-factors being considered increases. However, because the different components can be reduced to relatively simple tables, offering a concise means of handling the variables involved, the difficulty of handling need not become prohibitive.

## SUMMARY AND CONCLUSIONS

The application of symbolic logic to industrial plant location problems is believed to offer a useful step in the development of more effective decision-making methods. A great volume of literature is available discussing the influence of each of the many factors of plant location of the selection of a site. In sharp contrast, it was found that very little was offered in the form of a technique yielding a final solution. This leaves management's decision-maker with little more to guide him than his own judgement or "feel" for the problem.

This difficulty is compounded in the case of the small industry executive, since plant location requires skills and knowledge which are needed only once in a lifetime in the operations of many firms.

A study of plant location problems indicated that the greatest difficulty was found in attempting to systematically analyze the large quantities of data required to reach a decision in site selection. This difficulty led to consideration of symbolic logic as a possible means of handling the data. It is the feeling of the writer that the system presented offers two valuable additions to techniques applicable to plant location problems.

First, the method presented provided for setting down in rigid form the procedure of relating factors as is done in deductive reasoning. By developing a logical structure of propositional statements, the mental operations necessary to evaluate the overall ranking of a city or to evaluate the ranking of a factor and sub-factor can be symbolically stated, objectively examined, and sometimes simplified by logical transformations.

It is important to remember that each factor, sub-factor, and element considered under this technique is also considered in some way at the present time, under present methods. In many cases, however, the difficulties arising

in the analysis of factors which cannot be evaluated in terms of dollars forces management to accept a superficial analysis of these factors.

The second addition to present techniques offered in this thesis is a means of evaluating the complex propositions which are formulated stating a given firm's location needs. The worth of such a method can be appreciated when considering the complexity of the situation being dealt with. Some plant location handbooks suggest approximately 700 different points to consider in locating a plant.<sup>1</sup> Compound this number by the statements of relationships among elements within a sub-factor, those among sub-factors within factors, and finally the relationships existing among factors. The difficulties arising in seeking to analyze even a small portion of these relationships by verbal or mental images is easily appreciated.

It is the feeling of the writer that the application of symbolic logic to the analysis of plant location factors shows more promise in the area of qualitative factors than in the area of quantitative factors. The reasoning behind this is quite simple. If a firm possesses adequate quantitative information to determine the actual cost to the company as a result of utilizing a given factor, sub-factor, or element, there is little reason to translate this information into qualitative terms. This statement suggests further that the specific factors, sub-factors, and elements, to which symbolic logic could be most usefully applied depends on the information available to the company. The factors to which a qualitative evaluation is applicable are believed to be analyzed effectively by the method presented.

It is not the purpose of this thesis to study either the forces which lead management to identify given factors as being important to the success of

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<sup>1</sup> "Outline for Expansion," Industrial Development, October 1959, 128:84.

their firm, nor to study the actual effect which these factors have on the firm's success. It is felt that a great deal could be contributed to the "science" of plant location by applying the principles of decision theory to the general problem of determining the relative ranking of location factors. Also, a great deal of study is needed in the determination of the actual contribution of a given factor to the success of the firm. This could perhaps be studied by breaking down industries into their Standard Industrial Classification categories, and would involve study of location factors by industries.<sup>1</sup>

As a final comment on future work it is suggested that work could be done to adapt this procedure to solution by electronic computing devices. It can be seen that only a limited number of basic operations and rules for performing transformations on logical propositions are needed. Such a repetitive system would seem to lend itself readily to computer solution.

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<sup>1</sup> Refers to industrial code numbers used to classify products, as presented in the "Standard Industrial Classification Manual," published by the United States Bureau of the Budget, 1957.



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## APPENDIX

## SYMBOLIC LOGIC IN PRODUCTION ENGINEERING

To illustrate the manner in which symbolic logic might be used in an actual problem, an example from the field of production engineering might be described.

One manufacturer knew that he could produce a certain three-piece mechanism of higher quality and at a lower cost by redesigning. The problem was to decide which metal to use for each of the three parts. On the basis of engineering decisions, it is determined that there are five substances which might be used in component #1 of the product, six substances which might be used in component #2, and three which might be used component #3. Mathematically, 90 possible new combinations should have been developed for testing and evaluation. Research and development of all these combinations would have been too costly, so symbolic logic was used to simplify the selection.

Certain combinations of material created frictional or electrical characteristics that could not be tolerated. A list was prepared of the restrictions to be placed on combinations. For example, if metal A were the first component, then metals D or E could not be used for the second and metal J was unsuitable for the third. Similarly, if metal B were the first component, then metals D or E could not be used as the second and J was unsuitable for the third. The full list of suitable and unsuitable combinations is given below. The first five propositions correspond to requirements of the five materials to be used for the #1 component, the next six propositions refer to the materials used in #2 component, and the last three propositions apply to the materials used for #3 component. The materials which can be used in a given component and the designation of the proposition applying to each material are shown in Table 13 prior to the statements of materials restrictions.

Table 13. Propositions pertaining to materials available for each component.

Proposition	Substance in Component		
	#1	#2	#3
a b c d e	A B C D E		
f g h i j k		E F G H I J	
l m n			A F G

Statements of materials restrictions are given below.

- (1)  $a \supset (\sim d \cdot \sim j \cdot \sim k \cdot \sim l)$
- (2)  $b \supset (\sim j \cdot \sim k \cdot \sim n)$
- (3)  $c \supset (\sim f \cdot \sim j \cdot \sim m)$
- (4)  $d \supset (\sim f \cdot \sim h \cdot \sim m)$
- (5)  $e \supset (\sim f \cdot \sim g \cdot \sim i \cdot \sim j \cdot \sim k \cdot \sim l \cdot \sim m)$
- (6)  $f \supset (\sim c \cdot \sim d \cdot \sim e \cdot \sim n)$
- (7)  $g \supset (\sim e \cdot \sim m \cdot \sim n)$
- (8)  $h \supset (\sim d)$
- (9)  $i \supset (\sim a \cdot \sim e \cdot \sim l \cdot \sim m)$
- (10)  $j \supset (\sim a \cdot \sim b \cdot \sim c \cdot \sim e)$
- (11)  $k \supset (\sim a \cdot \sim b \cdot \sim e \cdot \sim m)$
- (12)  $l \supset (\sim a \cdot \sim e \cdot \sim h)$
- (13)  $m \supset (\sim c \cdot \sim d \cdot \sim e \cdot \sim g \cdot \sim i \cdot \sim k)$
- (14)  $n \supset (\sim b \cdot \sim f \cdot \sim g)$

For example, statement (2) says that if substance B (refer to table above) is used, substance I and substance J cannot be used for #2 component, and substance G cannot be used for the #3 component.

Setting these propositions given above into the form of Table 14, it becomes a matter of routine examination to eliminate the combinations of elements that are not possible from the total list of combinations available.

Examination of the table shows that only 19 out of the possible 90 combination remain as usable alternatives after logical analysis in terms of the materials restrictions.

Table 14. Tabular form of materials propositions.

Proposition	Substance in component			Substances for Component													
	#1	#2	#3	A a	B b	C c	D d	E e	F f	G g	H h	I i	J j	K k	L l	M m	N n
a	A											N					
b	B											N	N				N
c	C								N				N	N		N	
d	D								N		N			N			
e	E																
f		E				N	N	N		N							N
g		F															
h		G														N	N
i		H		N													
j		I		N	N	N		N									
k		J		N	N			N								N	
l			A									N					
m			F			N	N			N							
n			G		N		N	N	N			N	N				

Table 14 above was constructed by placing an N at each point where the materials restrictions statements indicated that a material could not be used. For example, propositions b indicates that materials I and J cannot be used in #2 component, and that material G cannot be used in #3 component, when material B is used in #1 component. Therefore, on the row designated as proposition b, and N is placed under propositions (substances) j, k, and n.

The analysis of the propositions is performed by checking each component of each possible combination that could be used to manufacture a given item. These combinations would be the groupings of propositions a.f.l, a.f.m, a.f.n, a.g.l, et cetera, until all combinations were formed. Then the three components of a given group (i.e. a.f.m) are examined by means of the chart to see

if any of the initial statements of the materials restrictions prevent the use of this particular grouping of materials for production. Examination of the chart shows that proposition (a) allows the use of materials E and F for #2 and #3 components respectively. Similar examination of proposition (f) for materials A and G for #1 and #3 components and proposition (n) for materials A and E in #1 and #2 components reveals that the group of materials specified by propositions a-f-m is suitable for production purposes. If an N had been encountered at any point in the previous examination of the grouping that particular grouping of materials would have been eliminated from the possible production combinations.

In addition to the restrictions first stated, the cost accounting department then calculated production and material costs for the remaining 19 combinations. The cost of materials will be considered to be acceptable for those metals costing no more than 10% above present material costs. In other words, the cost of materials will be considered a true proposition for figures of 110% or less. The conversion cost proposition will be considered true, (acceptable) for figures of \$80,000 or less. The operating cost proposition will be considered true for figures of 105% or less. The Y's in the performance characteristics will be considered true. With the lower case letters again used for the propositional components, the additional propositions are given below.

- (1)  $a \supset (\sim p \cdot q \cdot \sim r \cdot s \cdot t \cdot u)$
- (2)  $b \supset (\sim p \cdot q \cdot \sim r \cdot s \cdot t \cdot \sim u)$
- (3)  $c \supset (p \cdot \sim q \cdot r \cdot s \cdot t \cdot \sim u)$
- (4)  $d \supset (\sim p \cdot q \cdot \sim r \cdot s \cdot t \cdot u)$
- (5)  $e \supset (\sim p \cdot q \cdot r \cdot s \cdot \sim t \cdot u)$
- (6)  $f \supset (\sim p \cdot q \cdot r \cdot s \cdot t \cdot u)$
- (7)  $g \supset (p \cdot q \cdot r \cdot s \cdot \sim t \cdot u)$
- (8)  $h \supset (p \cdot q \cdot r \cdot s \cdot \sim t \cdot \sim u)$
- (9)  $i \supset (p \cdot q \cdot r \cdot s \cdot t \cdot \sim u)$
- (10)  $j \supset (p \cdot q \cdot r \cdot s \cdot t \cdot \sim u)$
- (11)  $k \supset (p \cdot q \cdot r \cdot s \cdot \sim t \cdot u)$
- (12)  $l \supset (p \cdot q \cdot r \cdot s \cdot \sim t \cdot u)$



- (13)  $m \supset (p \cdot q \cdot r \cdot s \cdot t \cdot u)$   
 (14)  $n \supset (p \cdot \sim q \cdot r \cdot \sim s \cdot t \cdot \sim u)$   
 (15)  $s$   
 (16)  $q$   
 (17)  $p \vee r$   
 (18)  $t \vee u$

These propositions can be placed in table form for easier reference as the propositions were which were given first.

Table 15. Table of cost accounting restrictions.

Proposition	Substance in component			Cost of each Material, %	Conversion Cost, \$1000	Operating Cost, %	Performance Characteristic		
	#1	#2	#3	$p \vee \sim p$	$q \vee \sim q$	$r \vee \sim r$	$s$	$t$	$u$
a	A			110 -- Y	75 -- Y	130 -- N	Y	Y	Y
b	B			95 -- Y	80 -- Y	150 -- N	Y	Y	N
c	C			80 -- Y	105 -- N	95 -- Y	Y	Y	N
d	D			120 -- N	60 -- Y	110 -- N	Y	Y	Y
e	E			170 -- N	150 -- N	60 -- Y	Y	N	Y
f		E		170 -- N	20 -- Y	60 -- Y	Y	Y	Y
g		F		95 -- Y	30 -- Y	85 -- Y	Y	N	Y
h		G		80 -- Y	15 -- Y	100 -- Y	Y	N	N
i		H		95 -- Y	35 -- Y	90 -- Y	Y	Y	N
j		I		110 -- Y	10 -- Y	90 -- Y	Y	Y	N
k		J		105 -- Y	40 -- Y	75 -- Y	Y	Y	N
l			A	110 -- Y	90 -- N	130 -- N	Y	Y	N
m			F	95 -- Y	80 -- Y	85 -- Y	Y	Y	Y
n			G	80 -- Y	90 -- N	100 -- Y	N	Y	N

Using the above restrictions as stated in the additional propositions, the list of the groups or combinations of materials that are acceptable from the production standpoint can be reduced from 19 to 2. Using the combination examined in the previous table, a.f.m, and checking Table 15 against the requirements set down in propositions (15) to (18), it can be seen that each of the materials in the group meets the test of having (s) true, (q) true, having either or both (p v r) true, and having either or both (t v u) true. Therefore this combination of a.f.m is one of the acceptable combinations.

The final analysis leaves only two combinations to study, as compared to

the original 90 combinations presented for consideration. The problem of development and testing these two combinations before making a final choice was simple compared with the perplexing problem of 90 combinations the company first faced.<sup>1</sup>

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<sup>1</sup> McCloskey and Trefethen, loc cit.

THE APPLICATION OF SYMBOLIC LOGIC TO  
PLANT LOCATION

by

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The problem of plant location continues to be a vexing one to industrialists and to communities. Traditional methods of determining location have not proven entirely satisfactory, especially in attempting to include the qualitative elements and their interrelationships.

In this thesis, a technique of handling these qualitative factors is proposed, analyzed, and explained. The basic concept is the application of symbolic logic methods to the location problem. While older methods are at least as satisfactory for handling quantitative factors such as transportation costs, taxes, and utilities, they fail to provide for inclusion of such qualitative factors as the effect of city size, community attitudes, schools, and professional services.

Thus, some new approach was required in order to advance the decision-making techniques in this area. Symbolic logic seems to hold promise for this purpose.

The method developed involves the logical relations of conjunction, disjunction, and negation. A brief explanation of these relations is as follows:

conjunction: The assertion of factors common to two propositions.  
( $\cdot$ )

The assertion of the conjunction of factors A and B is interpreted as asserting whatever possesses both A and B. To say that we want  $(A \cdot B)$  is to say that we want whatever possesses both A and B.

disjunction: The assertion of factors stated in either of two propositions stated, or in both propositions. The  
( $\vee$ )

assertion of the disjunction of A and B is interpreted as asserting whatever possess either A, or B, or both A and B. To say that we want  $(A \vee B)$  is to say that

we want whatever possesses A, or B, or both A and B.

negation : The contradiction or denial of a factor asserted in  
 ( $\sim$ ) a proposition. To say that we want  $\sim A$  is to say that  
 we do not want A.

Using the above relations, propositions can be written expressing the relations which management believes to exist. These propositions are first written to express the desired relationships among elements within a sub-factor, such as elements of the area surrounding the city, and growth trends within the sub-factor of total area population. Propositions are then written to express the relationships among sub-factors within factors, and finally the relationships among factors are evaluated to provide an over-all rating.

A ranking procedure, applied to the levels of a given factor which might be encountered is utilized in a special form of truth tables. Through the use of these truth tables the propositions which related elements, sub-factor, and factors can be evaluated and analyzed for the purpose of decision-making.

The combined use of traditional methods and symbolic logic techniques is suggested for the actual decision process as required for industry. The procedures set forth are viewed as practical tools rather than merely as theoretical developments.