SPECTRAL ANALYSIS OF APERIODIC DIGITAL SIGNALS

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CHAPTER I

INTRODUCTION

This report can essentially be divided into two parts. In the first part, the notion of the time-varying Fourier transform which was introduced recently [1] for noise-free discrete signals is extended to the case when noise is present. This is achieved by introducing some damping in the filtering process associated with the time-varying Fourier transform. The second part presents a study of Arnold's on-line computational algorithm [2].

It is demonstrated that Arnold's algorithm can be used to good advantage prior to classification for signal representation of transient waveforms which are embedded in noise. It is especially useful for those cases in which the signal to noise ratio is low. Examples of such applications include the following [2]:

(1) The classification of moving traffic in the presence of a noisy background.

(2) The discrimination between classes of seismic events in a noisy background.

(3) The classification of both man-made and biologically generated transients in an ocean background.

If the transient waveforms are well above the noise to the extent that their epoch and duration can be easily determined, then the fast Fourier transform (FFT) technique [3] may be used as a means of signal representation prior to classification. However, in applications of the type cited above, the transients are usually well below the level of the noise. Consequently their epochs and durations are not known. To overcome this, one could choose an upper bound T_{max} as the duration of any transient of interest and then use the FFT technique. However, this is not satisfactory since the arbitrary segmentation could result in spectral distortion.

Next, it is demonstrated that the time-varying Fourier transform, on being modified as cited above, can be used as an effective tool to estimate the Fourier power and phase spectra of transients in the presence of noise. Again, it is shown that such estimation can be achieved with any desired frequency resolution. In conclusion, it is demonstrated that the spectral estimates so obtained are superior to those obtained by standard smoothing techniques.

The above applications are considered in Chapter IV of this report. In Chapters II and III some fundamental aspects of Arnold's algorithm and the time-varying Fourier transform are presented. Conclusions pertaining to the study undertaken and corresponding recommendations for future work are presented in Chapter V.

CHAPTER II

TIME-VARYING FOURIER TRANSFORM AND RELATED SPECTRA

2.1 Fourier Transform of a Discrete Signal

Let X(t) be a continuous real-valued signal. It can be approximated to a series of rectangular segments of width Δt as shown in Figure 2.1 (a). Now the signal can be represented by the sequence of impulses X*(t) shown in Figure 2.1(b). The impulse at the point $m\Delta t$ has a strength Δt X(m), which is area of a rectangle formed by the base Δt and the height X(m) at $m\Delta t$ in Figure 2.1(a).

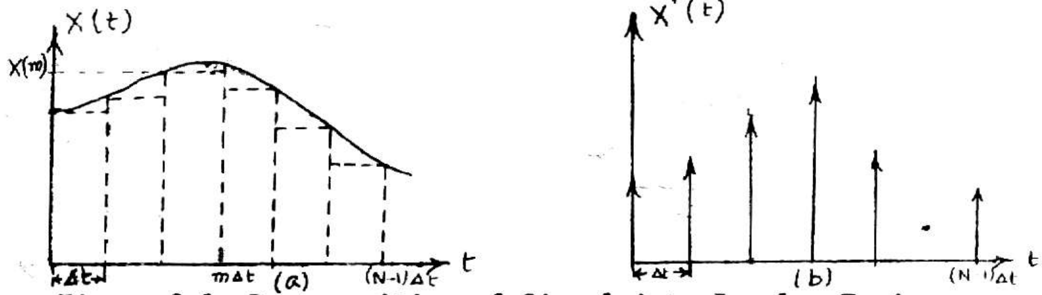

Figure 2.1 Decomposition of Signal into Impulse Train

The discrete signal X*(t) can be written as*

$$X^*(t) \simeq \sum_{m=0}^{N-1} \left[\Delta t\, X(m)\right] \delta(t-m\Delta t) \qquad (2\text{-}1a)$$

where $\delta(t)$ is the Dirac or impulse function. It X(t) has a bandwidth of B Hz, then from Shannon's Sampling theorem Δt must be less than or equal to 1/2B sec.

Fourier transform $F_x{*}(\omega)$ of X*(t) is given by

$$F_x{*}(\omega) = \Delta t \int_{-\infty}^{\infty} \sum_{m=0}^{N-1} X(m)\, \delta(t-m\Delta t)\, e^{-i\omega t}\, dt \qquad (2\text{-}1b)$$

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*See for example, "Lin. Network Analysis", by Seshu and Balabanian, John Wiley, 1959, p.205.

Interchanging the order of integration and summation in Eq. (2-1b) we obtain

$$F_{x*}(\omega) = \Delta t \sum_{m=0}^{N-1} X(m) \int_{-\infty}^{\infty} \delta(t-M\Delta t)e^{-i\omega t}dt$$

$$= \Delta t \sum_{m=0}^{N-1} X(m)e^{-im\omega\Delta t} \tag{2-2}$$

$$=\Delta t \sum_{m=0}^{N-1} X(m)\text{Cos}(\omega m\Delta t)-i \sum_{m=0}^{N-1} X(m)\text{Sin}(\omega m\Delta t) \tag{2-3}$$

that is,

$$F_{x*}(\omega)=\Delta t\{R(\omega)-iI(\omega)\} \tag{2-4}$$

where

$$R(\omega)= \sum_{m=0}^{N-1} X(m)\text{Cos}(\omega m\Delta t)$$

and

$$I(\omega)= \sum_{m=0}^{N-1} X(m)\text{Sin}(\omega m\Delta t)$$

$F_{x*}(\omega)$ in Eq. (2-4) is the desired Fourier transform of a given discrete signal $X*(t)$.

2.2 Time Varying Fourier Transform [1]

$F_{x*}(\omega)$ in Eq. (2-4) can be expressed in terms of a (2x1) vector to obtain,

$$\underline{F}_{x*}(\omega)=\Delta t\begin{bmatrix} R(\omega) \\ I(\omega) \end{bmatrix} \tag{2-5}$$

Now, consider the (2x2) matrix

$$\left[L(\omega)\right]=\begin{bmatrix} \text{Cos}(\omega\Delta t) & -\text{Sin}(\omega\Delta t) \\ \text{Sin}(\omega\Delta t) & \text{Cos}(\omega\Delta t) \end{bmatrix} \tag{2-6}$$

Since $\left[L(\omega)\right]$ is orthogonal, it has the property

$$\left[L(\omega)\right]^m = \begin{bmatrix} \mathrm{Cos}(m\omega\Delta t) & -\mathrm{Sin}(m\omega\Delta t) \\ \mathrm{Sin}(m\omega\Delta t) & \mathrm{Cos}(m\omega\Delta t) \end{bmatrix} \tag{2-7}$$

Thus, from Eqs. (2-5) and (2-7) it follows that,

$$\underline{F}_{X*}(\omega) = \Delta t \sum_{m=0}^{N-1} \left[L(\omega)\right]^m \underline{b} X(m) \tag{2-8}$$

where

$$\underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We introduce the recurrence relation

$$\begin{aligned} &\underline{Z}(\omega,k) = \left[L(\omega)\right] \underline{Z}(\omega,k-1) + \underline{b}X(N-1-k) \\ &k=0,1,2,\ldots,(N-1) \end{aligned} \tag{2-9}$$

where k denotes the instant of time $t=k\Delta t$ and

$$\underline{Z}(\omega,-1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With k=0, and k=1, Eq. (2-9) yields

$$\underline{Z}(\omega,0) = \left[L(\omega)\right] \underline{Z}(\omega,-1) + \underline{b}X(N-1) \tag{2-10}$$

and

$$\begin{aligned} \underline{Z}(\omega,1) &= \left[L(\omega)\right] \underline{Z}(\omega,0) + \underline{b}X(N-2) \\ &= \left[L(\omega)\right]\{ \left[L(\omega)\right] \underline{Z}(\omega,-1) + \underline{b}X(N-1) \} + \underline{b}X(N-2) \\ &= \left[L(\omega)\right]^2 \underline{Z}(\omega,-1) + \left[L(\omega)\right] \underline{b}X(N-1) + \underline{b}X(N-2) \end{aligned} \tag{2-11}$$

Proceeding on similar lines we arrive at the fundamental relationship

$$\underline{Z}(\omega,N-1) = \sum_{m=0}^{N-1} \left[L(\omega)\right]^m \underline{b}X(m) = \frac{1}{\Delta t} \underline{F}_{X*}(\omega) \tag{2-12}$$

where $\underline{F}_{X*}(\omega)$ is the (2x1) vector defined in Eq. (2-5). Eq. (2-12) states that, at the instant $k=(N-1)$, $\underline{Z}(\omega,k)$ in Eq. (2-9) yields the conventional Fourier transform defined in Eq. (2-4). $\underline{Z}(\omega,k)$ can therefore be defined as the time-varying Fourier transform. Again if we denote

$$\underline{Z}(\omega,k)=\begin{bmatrix}Z_1(\omega,k)\\Z_2(\omega,k)\end{bmatrix} \tag{2-13}$$

then the time-varying power and phase spectra can be defined as

$$P(\omega,k)=(\Delta t)^2\,||\underline{Z}(\omega,k)||^2 \tag{2-14a}$$

and $$\Psi(\omega,k)=\tan^{-1}\left\{\frac{Z_2(\omega,k)}{Z_1(\omega,k)}\right\} \tag{2-14b}$$

respectively. Eq. (2-9) yields an efficient recursive algorithm to compute the time-varying power and phase spectra. This is illustrated in Figure 2.2 for the case of computing the time-varying power spectrum at a set of points $\omega=\omega_n$, n=1,2,...M. From Figure 2.2 it follows that the computational process can be associated with a bank of filters characterized by the (2x2) matrices $\left[L(\omega_n)\right]$, n=1,2,...M.

2.3 Noise Considerations [2]

In the above analysis it is assumed that the signal $X^*(t)$ is noise-free. In many real-world problems, however, one does not have isolated processes, but usually a combination, such as periodic and transient signals in a noisy background. In such cases, the spectra discussed in the previous section need smoothing if they are to yield consistent estimates. The recurrence relation in Eq. (2-9) can be extended to handle the noisy signals with some modifications. This is discussed in what follows.

The desired frequency range is covered by a bank of constant Q filters whose frequencies are equally spaced on a logarithmic scale. The matrix considered in the Eq. (2-6) is redefined for the n-th filter in the filter bank as,

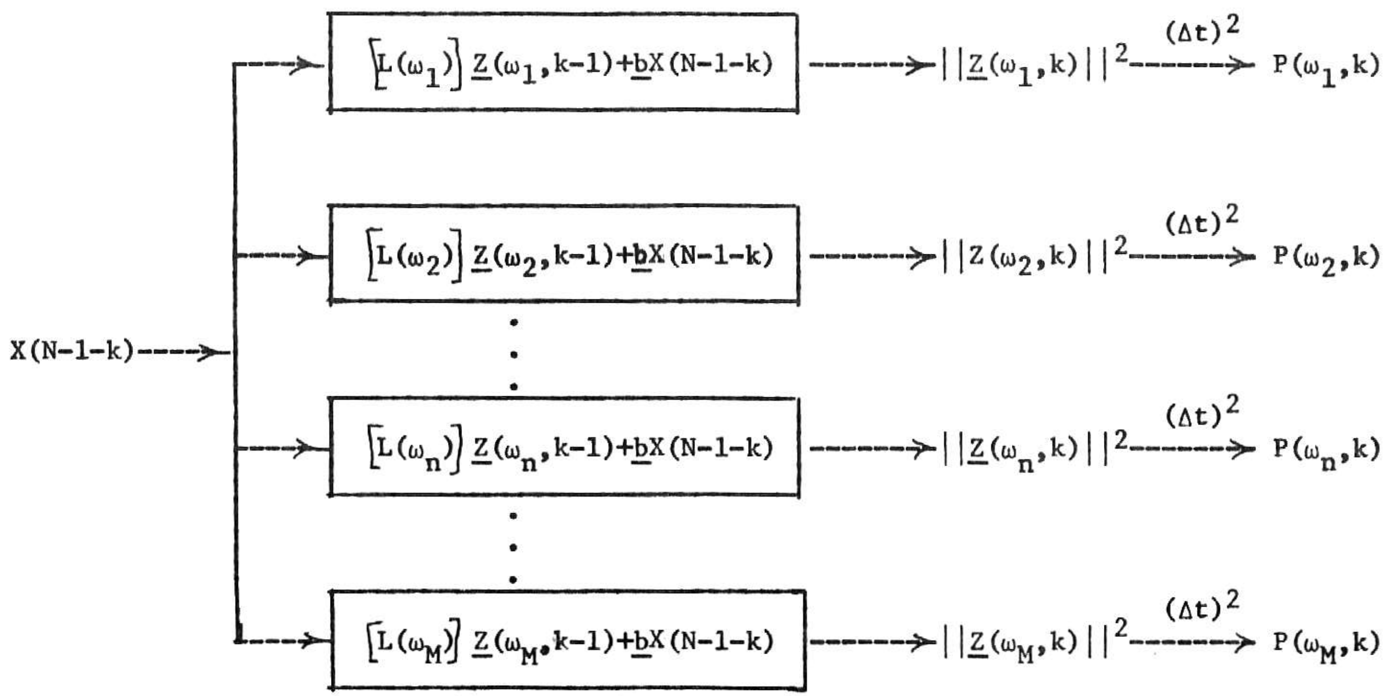

Figure 2.2 Computation of the Time-Varying Power Spectrum

$$\left[L(\omega_n)\right]=\begin{bmatrix} e^{-\alpha\Delta t}\mathrm{Cos}(\omega_o\Delta t) & -e^{-\alpha\Delta t}\mathrm{Sin}(\omega_o\Delta t) \\ e^{-\alpha\Delta t}\mathrm{Sin}(\omega_o\Delta t) & e^{-\alpha\Delta t}\mathrm{Cos}(\omega_o\Delta t) \end{bmatrix} \tag{2-15}$$

where $\alpha=\zeta\omega_n$, ζ is called the damping factor

and $\omega_o=\omega_n\sqrt{1-\zeta^2}$ (2-16)

Then, corresponding to Eq. (2-9) we obtain

$$\underline{Z}(\omega_n,k)=\left[L(\omega_n)\right]\underline{Z}(\omega_n,k-1)+\underline{b}X(N-1-k)$$
$$k=0,1,2,\ldots N-1 \tag{2-17}$$

Again, the time varying power and phase spectra are respectively given by

$$P(\omega_n,k)=(\Delta t)^2\left|\left|\underline{Z}(\omega_n,k)\right|\right|^2 \tag{2-18a}$$

and

$$\Psi(\omega_n,k)=\tan^{-1}\left\{\frac{Z_2(\omega_n,k)}{Z_1(\omega_n,k)}\right\} \tag{2-18b}$$

The computational process is identical to that shown in Figure 2.2 except that each $\left[L(\omega_n)\right]$ defined in Eq. (2-9) is replaced by that defined in Eq. (2-15). The introduction of the damping has the effect of smoothing the spectrum as each filter has finite bandwidth Δf given by the relation

$$Q=\frac{\omega}{\Delta\omega}=\frac{f}{\Delta f}=\frac{1}{2\zeta} \tag{2-19}$$

The various contiguous filters overlap at their -3dB points as shown in Figure 2.3. For a given Q or equivalently for a given damping factor ζ, the set of center frequencies for the filter bank are readily derived using the algorithm [2],

$$f_1=(1+\zeta)\ f_{base}$$
$$f_{n+1}=\frac{1+\zeta}{1-\zeta}f_n,\ n=1,2,\ldots M \tag{2-20}$$

The frequency points generated by Eq. (2-20) are equally spaced on a logarithmic scale as shown in Figure 2.3.

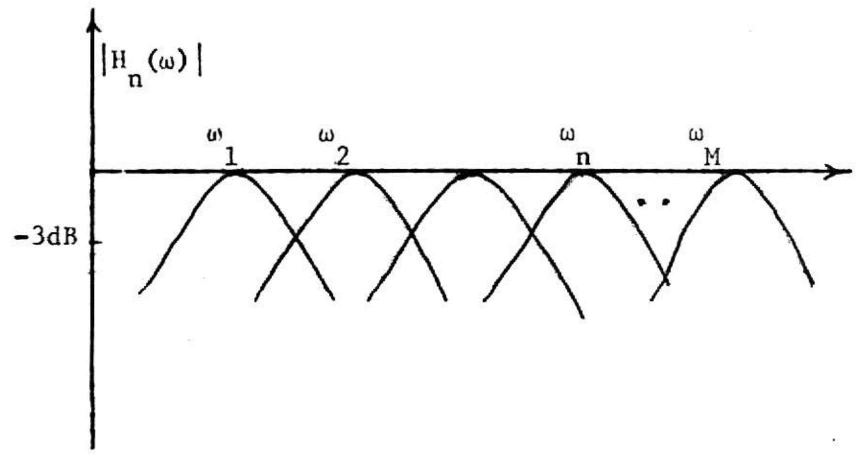

Figure 2.3 Transfer Grain Characteristics of Filter Bank

The base frequency f base is obtained as

$$f_{base} = f_{NY} / 10^{DEC} \tag{2-21}$$

where f_{NY} is the Nyquist or folding frequency given by

$$f_{NY} = 1/2\Delta t \tag{2-22}$$

and DEC is the desired frequency range in decades.

If damping is not considered, then $\zeta=0$, and hence one considers the bank of filters as having **zero bandwidth at a set of chosen frequencies. The corresponding matrix associated with the n-th filter is given by**

$$\left[L(\omega_n)\right] = \begin{bmatrix} \text{Cos}(\omega_n\Delta t) & -\text{Sin}(\omega_n\Delta t) \\ \text{Sin}(\omega_n\Delta t) & \text{Cos}(\omega_n\Delta t) \end{bmatrix} \tag{2-23}$$

Clearly, $\left[L(\omega_n)\right]$ in Eq. (2-23) is the same as $\left[L(\omega)\right]$ in Eq. (2-6).

2.4 Arnold's Recursive Algorithm [2]

Arnold's recursive algorithm is very similar to Eqs. (2-9) and (2-17). Specifically it is defined as

$$\underline{\hat{Z}}(\omega_n,k) = \left[L(\omega_n)\right] \underline{\hat{Z}}(\omega_n,k-1)+\underline{b}X(k)$$

$$k=0,1,\ldots,N-1 \qquad (2\text{-}24)$$

where $\left[L(\omega_n)\right]$ is given by Eqs. (2-15) and (2-23) for the damped (i.e. noisy) and undamped (i.e. noise free) cases respectively. Comparing Eq. (2-24) with Eq. (2-9) or Eq. (2-17) it is clear that while the recurrence relation in Eq. (2-24) enables an on-line computation of $||\underline{\hat{Z}}(\omega_n,k)||^2$ (see Figure 2.4), $||\underline{\hat{Z}}(\omega_n,k)||^2 \neq ||\underline{Z}(\omega_n,k)||^2$ where $\underline{Z}(\omega_n,k)$ is defined in (2-13). At best, it can be shown that $||\underline{\hat{Z}}(\omega_n,N-1)||^2= ||\underline{Z}(\omega_n,N-1)||^2$, for the undamped case. Thus the quantity $||\underline{\hat{Z}}(\omega_n,k)||^2$ can be thought of a sort of instantaneous power of the n-th filter of the filter bank. However Arnold's algorithm has been successfully used in applications such as earthquake detection, where some sort of on-line spectral estimation is preferred. This is because the epoch of a transient caused by the occurence of a certain event is unknown in such problems.

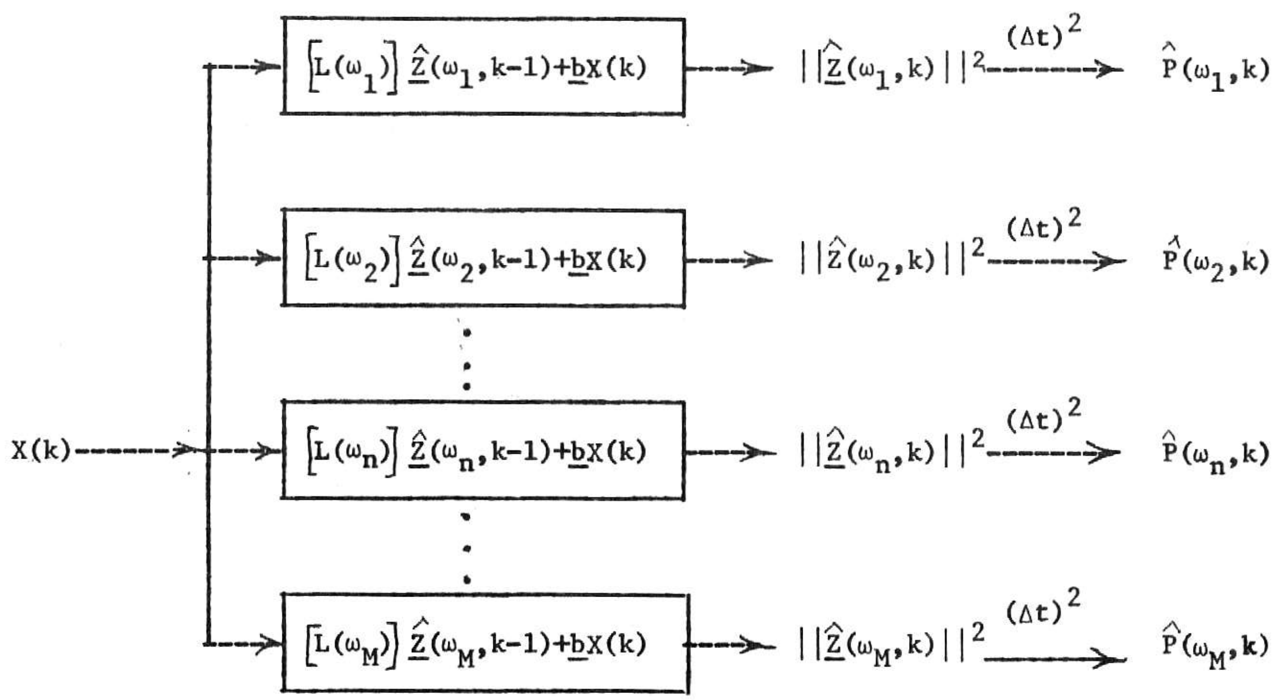

Figure 2.4 Illustration of Arnold's Algorithm

CHAPTER III

RELATIONSHIP BETWEEN A CLASS OF SPECTRA

3.1 Relation Between the Time-Varying and Discrete Fourier Spectra

If $\{X(m)\}$ denotes a sequence $X(m), m=0,1,...,N-1$ of N finite, real-valued numbers, then its discrete [3] Fourier transform is defined as

$$C_x(k) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) W^{km}, k = 0,1,...N-1 \tag{3-1}$$

where $W=e^{-i2\pi/N}$

Eq. (3-1) can be rewritten as

$$C_x(k) = \frac{1}{N}\left\{\sum_{m=0}^{N-1} X(m) \operatorname{Cos}\left(\frac{2\pi km}{N}\right) - i \sum_{m=0}^{N-1} X(m) \operatorname{Sin}\left(\frac{2\pi km}{N}\right)\right\}$$

$$k=0,1,...(N-1) \tag{3-2}$$

For the real-valued data sequence $\{X(m)\}$ the DFT power spectrum is defined as

$$|C_x(k)|^2, \ k = 0,1,...N/2 \tag{3-3}$$

From Eq. (3-2) it follows that,

$$|C_x(k)|^2 = \frac{1}{N^2}\left[\left|\sum_{m=0}^{N-1} X(m) \operatorname{Cos}\left(\frac{2\pi km}{N}\right)\right|^2 + \left|\sum_{m=0}^{N-1} X(m) \operatorname{Sin}\left(\frac{2\pi km}{N}\right)\right|^2\right] \tag{3-4}$$

$$k=0,1,...,N/2$$

represents the DFT power spectrum

Now from Eqs. (2-8) and (2-12) we have

$$\underline{F}_{x^*}(\omega) = (\Delta t) \ \underline{Z}(\omega, N-1) = (\Delta t) \sum_{m=0}^{N-1} \left[L(\omega)\right]^m \underline{b} X(m) \tag{3-5}$$

where,

$$\left[L(\omega)\right]^m = \begin{bmatrix} \mathrm{Cos}(m\omega\Delta t) & -\mathrm{Sin}(m\omega\Delta t) \\ \mathrm{Sin}(m\omega\Delta t) & \mathrm{Cos}(m\omega\Delta t) \end{bmatrix} \tag{3-6}$$

and $\underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

In particular, we consider the following set of values taken by ω in Eq. (3-5)

$$\omega_n = \frac{2\pi n}{N\Delta t}, \quad n=0,1,\ldots,N/2 \tag{3-7}$$

Then the total power at the frequency ω_n is given by Eq. (2-14) as

$$P(\omega_n,N-1) = (\Delta t)^2 \left|\left|\underline{Z}(\omega_n,N-1)\right|\right|^2 \tag{3-8}$$

where

$$\underline{Z}(\omega_n,N-1) = \begin{bmatrix} \sum_{m=0}^{N-1} X(m)\ \mathrm{Cos}(m\omega_n\Delta t) \\ \\ \sum_{m=0}^{N-1} X(m)\ \mathrm{Sin}(m\omega_n\Delta t) \end{bmatrix} \tag{3-9}$$

Substituting Eq. (3-7) in Eqs. (3-8) and (3-9), the power spectrum can be written as

$$P(\omega_n,N-1) = (\Delta t)^2 \left\{ \left| \sum_{m=0}^{N-1} X(m)\mathrm{Cos}\left(\frac{2\pi mn}{N}\right)\right|^2 + \left| \sum_{m=0}^{N-1} X(m)\mathrm{Sin}\left(\frac{2\pi mn}{N}\right)\right|^2 \right\}$$

$$n=0,1,\ldots N/2 \tag{3-10}$$

which is the same as the DFT power spectrum defined in Eq. (3-4), except for the scale factor $\left(\frac{1}{N\Delta t}\right)^2$. Thus we conclude that the DFT power spectrum is obtained as a special case of the time-varying power spectrum. Similarly, it can be shown that the DFT phase spectrum can also be obtained as a special case of the time-varying phase spectrum.

3.2 Time-Varying BIFORE Transform Power Spectrum [4]

The BIFORE (Binary Fourier Representation) transform* power spectrum is related to DFT power spectrum as follows [3]

$$P(0) = |C_x(0)|^2$$

$$P(S) = \sum_{k=2^{S-1}}^{2^S-1} |C_x(\langle k\rangle)|^2$$

$$S=1,2,..,n; \; n=\log_2 N \tag{3-11}$$

where <k> denotes the decimal number corresponding to the bit reversal of k_{binary}. For example if k=6, k_{binary}=110 and hence <k>=3.

From the preceding section it follows that

$$|C_x(k)|^2 = \frac{1}{N^2} ||Z(\omega_n, N-1)||^2 \tag{3-12}$$

Comparing (3-11) and (3-12) it follows that the time-varying BIFORE transform power spectrum can be defined as

$$P_B(0,m) = \frac{1}{(m+1)^2} ||\underline{Z}(\omega_o, m)||^2$$

$$P_B(S,m) = \frac{1}{(m+1)^2} \sum_{k=2^{S-1}}^{2^S-1} ||\underline{Z}(\omega_{\langle k\rangle}, m)||^2$$

$$\text{for } S=1,2,\ldots n, \; n=\log_2 N, \; m=0,1,2,..(N-1) \tag{3-13}$$

It follows that by combining groups of the $||\underline{Z}(\omega_{\langle k\rangle}, m)||^2$ terms given by Eq. (3-13), the BIFORE transform time-varying power spectrum can be computed as shown in Figure 3.1 for the Case N=8.

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*Also known as the Walsh-Hadamard transform.

$$\underline{Z}(\omega_0,m) = [L(\omega_0)]\ \underline{Z}(\omega_0,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_0,m)\|^2 \xrightarrow{\frac{1}{(m+1)^2}} P(0,m)$$

$$\underline{Z}(\omega_4,m) = [L(\omega_4)]\ \underline{Z}(\omega_4,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_4,m)\|^2 \xrightarrow{\frac{1}{(m+1)^2}} P(1,m)$$

$$\underline{Z}(\omega_2,m) = [L(\omega_2)]\ \underline{Z}(\omega_2,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_2,m)\|^2$$

$$\Sigma \xrightarrow{\frac{1}{(m+1)^2}} P(2,m)$$

$$\underline{Z}(\omega_6,m) = [L(\omega_6)]\ \underline{Z}(\omega_6,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_6,m)\|^2$$

$$\underline{Z}(\omega_1,m) = [L(\omega_1)]\ \underline{Z}(\omega_1,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_1,m)\|^2$$

$$\underline{Z}(\omega_5,m) = [L(\omega_5)]\ \underline{Z}(\omega_5,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_5,m)\|^2$$

$$\Sigma \xrightarrow{\frac{1}{(m+1)^2}} P(3,m)$$

$$\underline{Z}(\omega_3,m) = [L(\omega_3)]\ \underline{Z}(\omega_3,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_3,m)\|^2$$

$$\underline{Z}(\omega_7,m) = [L(\omega_7)]\ \underline{Z}(\omega_7,m-1) + \underline{b}\ X(7-m) \longrightarrow \|\underline{Z}(\omega_7,m)\|^2$$

Figure 3.1 Computation of Time-Varying BIFORE Transform Power Spectrum

CHAPTER IV

APPLICATION CONSIDERATIONS

4.1 Introductory Remarks

Two applications pertaining to the recurrence relations defined in Eqs. (2-17) and (2-24) are considered in this chapter. First, it is demonstrated that Arnold's algorithm defined in Eq. (2-24) can be used to secure on-line signal representation (i.e. feature extraction) in pattern recognition problems. On-line signal representation is desirable in cases where the epochs of transients embedded in noise are not known. Such situations are encountered in the processing of seismic and sonar signals [2]. Second, it is demonstrated that damping associated with the recurrence relation in Eq. (2-17) can be used for the spectral estimation of transients embedded in noise whose statistics may be known or unknown.

4.2 On-Line Signal Representation

An experiment is simulated to classify two classes of signals, A and B. The discrete signals belonging to class A and class B are represented by the waveforms in Figure 4.1 and Figure 4.2 respectively. In Figures 4.1 and 4.2, A_j and B_j denote the j-th sample of A and B respectively. The discrete values, 50 for each sample of A, are normally distributed random variables with zero mean and unit standard deviation, which are generated using two scientific subroutines, SUBROUTINE GAUSS and SUBROUTINE RANDU.

The discrete values for each sample of B are obtained by adding the discrete data given by

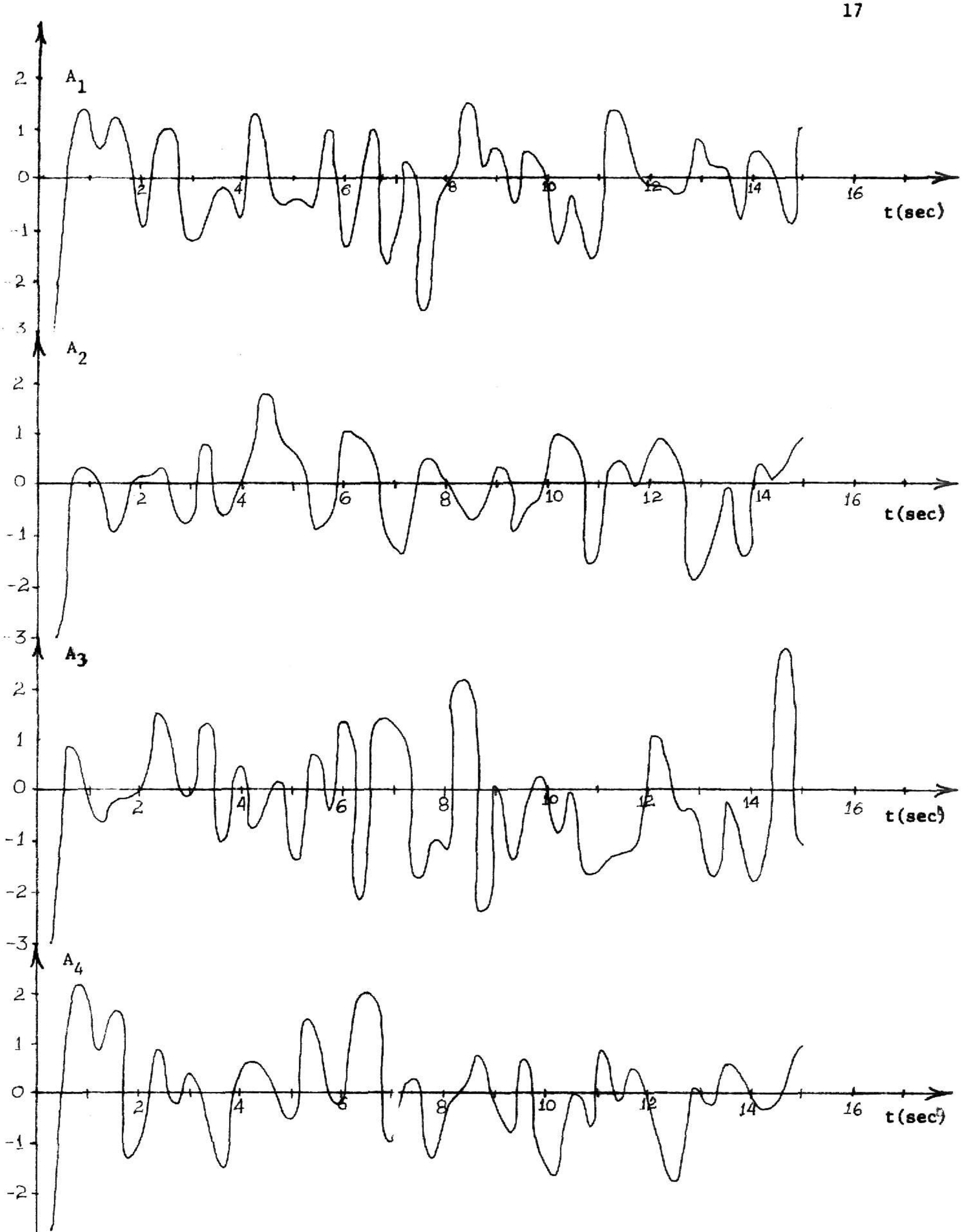

Figure 4-1(a) Waveforms of Signals Belonging to Class A

Time scale factor=$\left(\frac{50}{26\times 15}\right)$

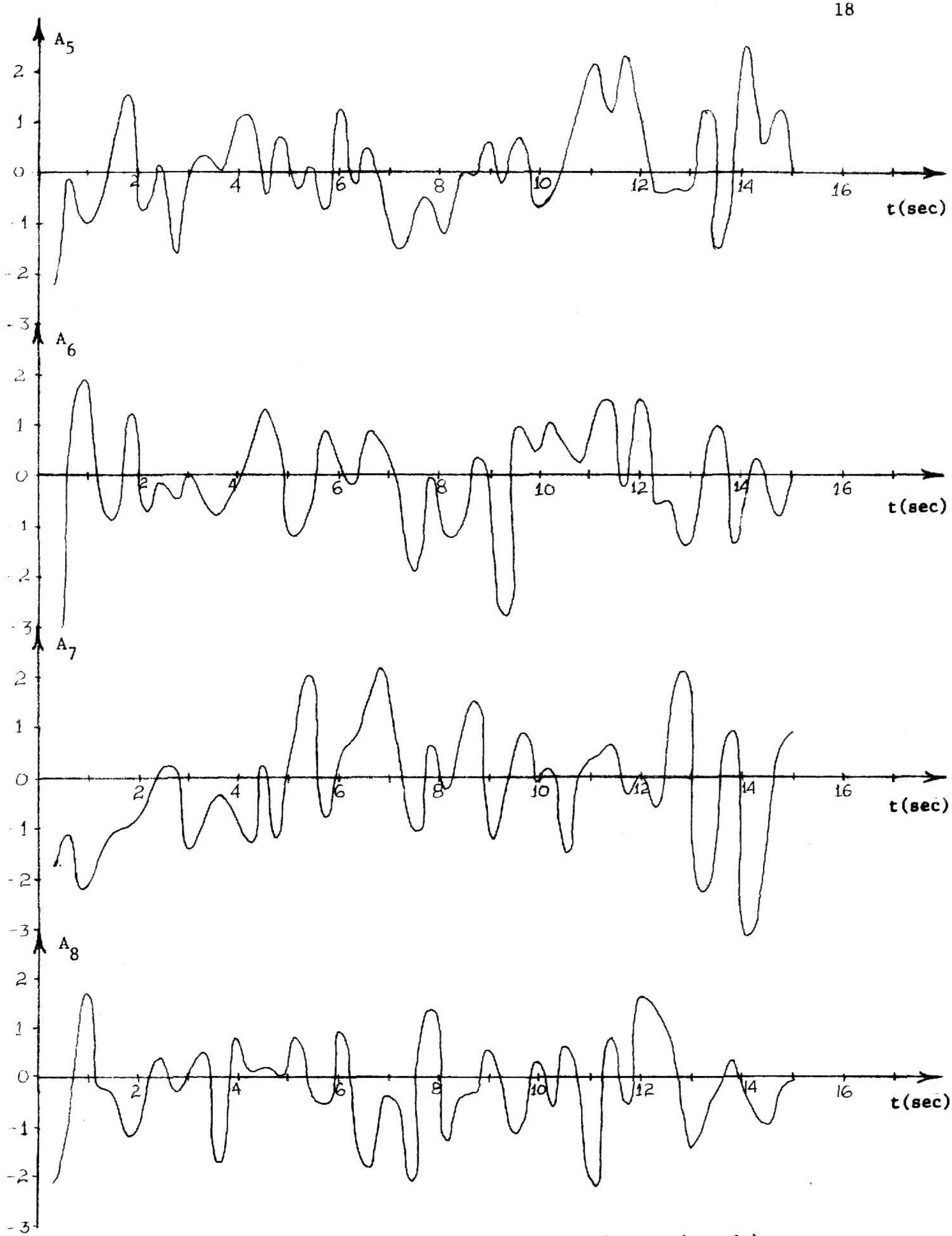

Figure 4.1(b) Waveforms of Signals Belonging to Class A (contd.)

Time scale factor $=(\frac{50}{26\text{x}15})$

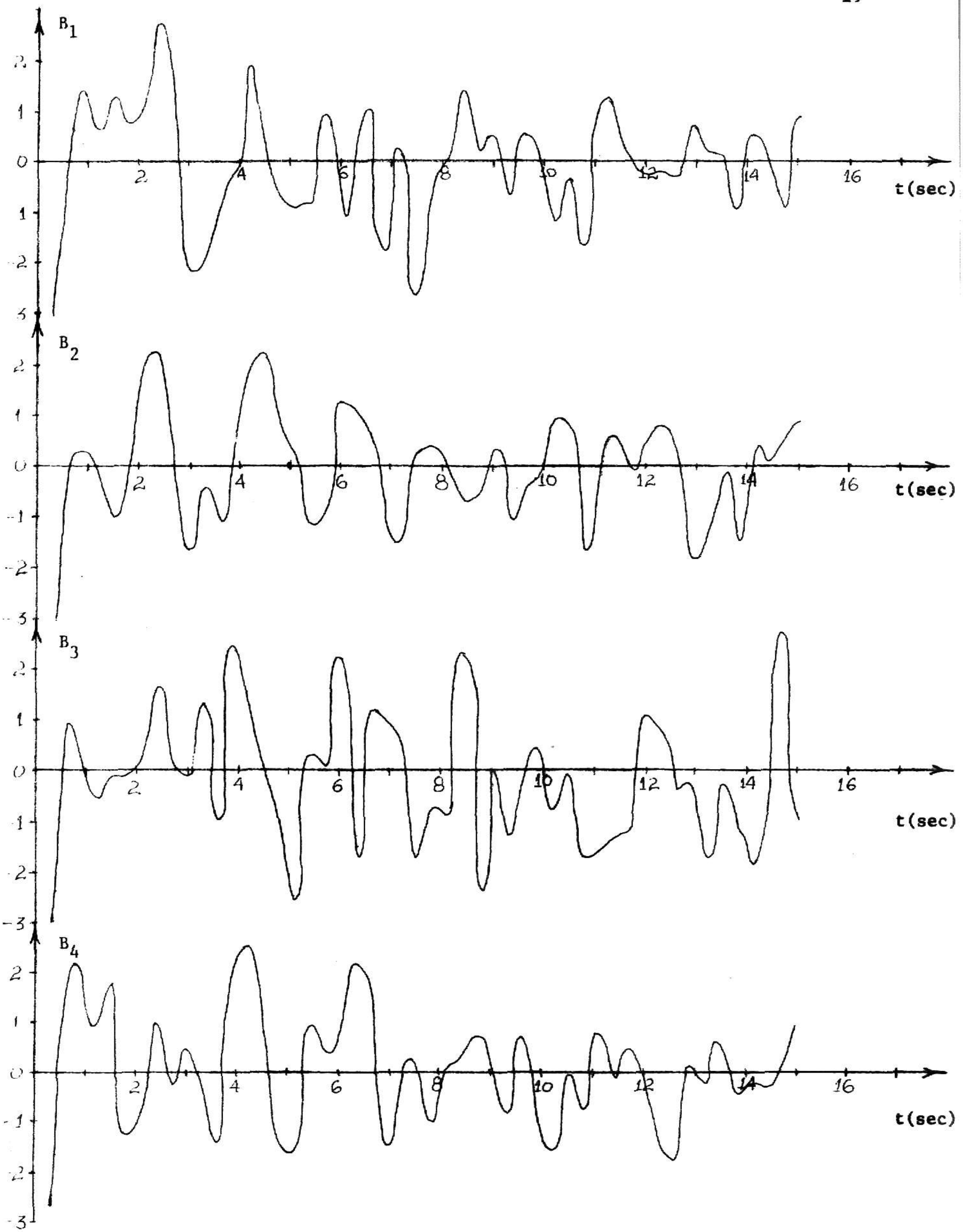

Figure 4•2(a) Waveforms of Signals Belonging to Class B

Time scale factor=(50 / [illegible])

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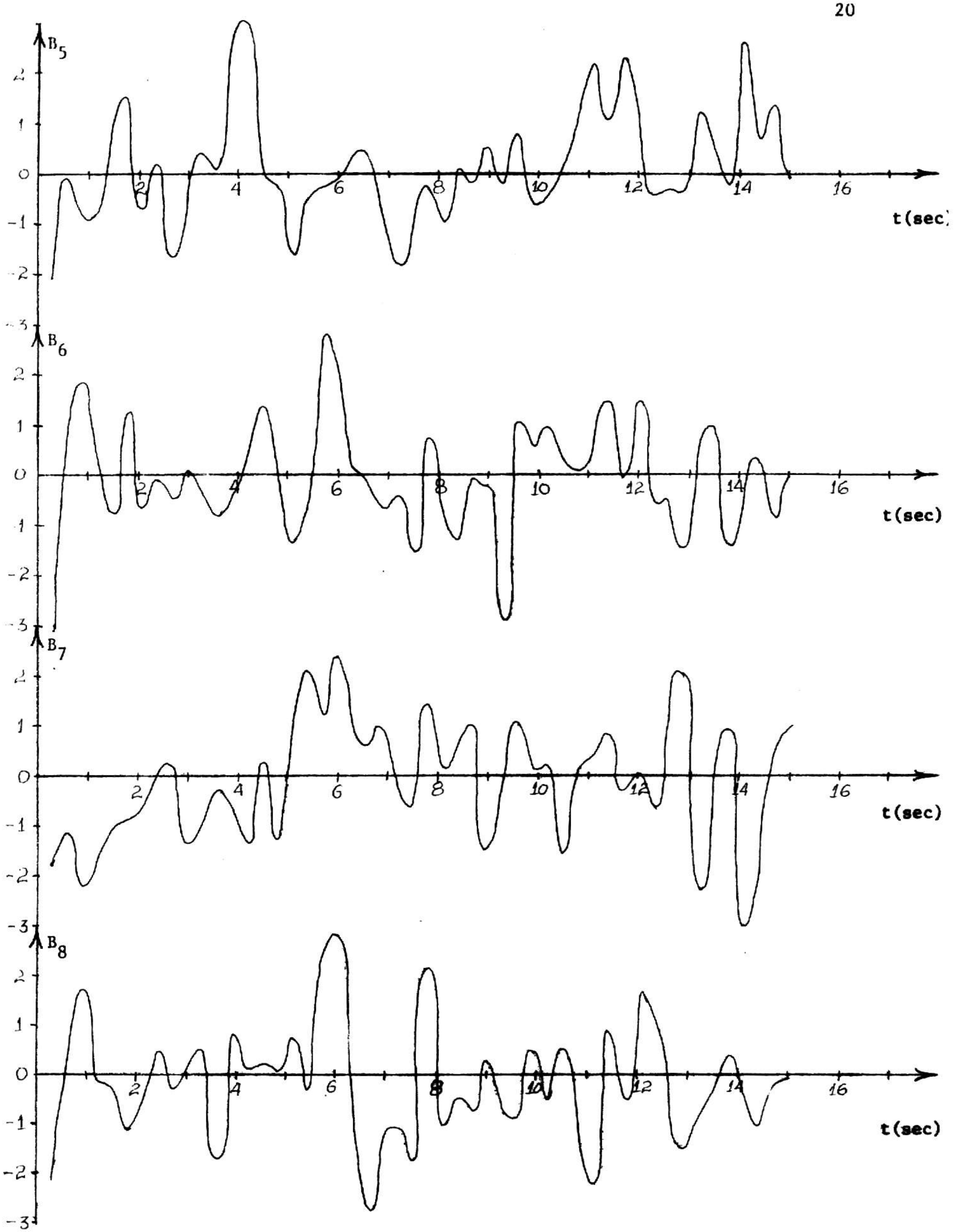

Figure 4·2(b) Waveforms of Signals Belonging to Class B (contd.)

Time scale factor=(50)

$$f(m) = 2.8 \times e^{-m/6} \text{ Sin } (2\pi \times 4 \times \frac{m}{26})$$

$$m=1,2,\ldots,26 \qquad (4\text{-}1)$$

to the discrete values of each sample of A in various positions. The signal-to-noise ratio for the class B was not evaluated. However, the scale factor 2.8 associated with f(m) in Eq. (4-1) was chosen such that the mean-square value of the signal f(m) is approximately <u>one-half</u> times that of the samples of A. That is,

$$\frac{1}{26\Delta t} \sum_{m=1}^{26} f^2(m) \approx \frac{0.5}{8} \left\{ \frac{1}{50\Delta t} \sum_{j=1}^{8} \sum_{m=1}^{50} A^2{}_j(m) \right\} \qquad (4\text{-}2)$$

where $\Delta t = \frac{1}{26}$ sec.

The corresponding Nyquist frequency is given by $0.5/\Delta t$=13Hz from Eq. (2-22)

A computer program* which implements Arnold's algorithm given in Eq. (2-24) was used to compute the instantaneous spectra of the discrete signals of A and B over a frequency range of 1 decade. Since the Nyquist frequency is 13 Hz, the base frequency is given by 13/10=1.3 Hz from Eq. (2-21). The damping factor ζ was chosen as 0.0588. Substituting this value of ζ in Eq. (2-20) there results 20 filters equally spaced on logarithmic scale between 1.3 Hz and 13 Hz. The center frequencies associated with these 20 filters are listed in Table(4-1).

The corresponding instantaneous power points at the filters having center frequencies 3.97, 4.47 and 5.03 Hz were used as the training sets of the two classes A and B for a training algorithm which uses a least-square mapping technique [3]. The basic idea used is to map (in the least-squares sense) the training samples of a class k, k=1,2,..., K into a unit vector

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*For a listing of the program, see Appendix I.

TABLE 4-1

Filter #	Center Frequency Hz
1	1.38
2	1.55
3	1.74
4	1.96
5	2.20
6	2.48
7	2.79
8	3.14
9	3.53
10	3.97
11	4.47
12	5.03
13	5.65
14	6.36
15	7.15
16	8.05
17	9.05
18	10.19
19	11.46
20	12.89

V_k in a K-dimensional decision space. All the components of V_k are zero except for the K-th one, which is unity. The corresponding mapping matrix is obtained during the training process. Then the trained classifier assigns an incoming pattern to class i_0 if the pattern is mapped closest to the unit vector V_{i_0} in the decision space. **The training vectors used are listed in the Table (4-2).**

The resulting "confusion matrix" obtained from the classification of the training sets using the above training algorithm (see Appendix II) is as follows:

$$F = \begin{bmatrix} 8 & 0 \\ 1 & 7 \end{bmatrix}$$

From the matrix F it can be observed that one member of class B is classified as belonging to class A. That is, only one error was incurred in 16 decisions. Thus the above simulated experiment demonstrates the manner in which Arnold's algorithm (see Eq. (2-24)) can be used as an effective tool to secure on-line signal representation in pattern recognition problems.

4.3 Spectral Estimation of Transients

This section demonstrates the smoothing effect of using damped filters characterized by the $[L(\omega_n)]$ matrices defined in Eq. (2-15). A discrete signal was generated as follows:

$$S(m) = 3.97 \times e^{-m/6} \sin\left(2\pi \times 4 \times \frac{m}{26}\right) \qquad (4\text{-}3)$$

$$m = 1, 2, \ldots, 26$$

Figure 4.3 shows the waveform corresponding to this discrete signal. This signal was then perturbed by additive noise which consists of 26 normally distributed random variables with zero mean and unit variance. An ensemble of 8 samples of perturbed signal was generated, the waveforms corresponding to which are shown in Figure 4.4. We denote the j-th member j=1,2,...,8, of

TABLE 4-2

Class	Sample #	Instantaneous Power P_i at the i^{th} Instant							
		Fitler #10,3.97 Hz			Filter #11,4.47 Hz			Filter #12,5.03 Hz	
		P_{20}	P_{30}	P_{40}	P_{20}	P_{30}	P_{40}	P_{20}	P_{30}
	1	0.6	12.9	0.9	3.6	6.7	12.6	14.8	6.0
	2	7.3	3.8	11.1	17.0	18.9	11.0	7.5	10.3
	3	0.8	4.3	15.2	5.6	4.4	20.2	3.5	15.0
Class A	4	5.7	13.0	4.8	7.5	4.9	3.0	15.3	2.6
	5	6.3	0.2	7.2	0.0	0.6	1.9	2.6	0.9
	6	2.6	7.7	3.5	17.6	0.6	8.0	14.8	3.6
	7	3.0	9.1	0.6	4.2	18.9	1.7	4.2	22.0
	8	4.3	1.5	8.3	9.9	3.4	8.8	3.8	5.4
	1	16.4	31.0	2.8	28.4	4.5	16.7	37.6	9.9
	2	39.4	25.0	26.0	35.2	35.4	11.0	8.5	13.7
	3	35.3	7.5	30.4	46.3	1.8	9.1	32.5	10.5
Class B	4	8.8	17.0	0.3	42.7	13.6	10.8	59.4	7.1
	5	51.5	18.8	12.7	20.6	5.5	7.9	13.9	3.6
	6	9.1	5.1	14.4	39.3	19.8	11.1	47.3	14.4
	7	18.6	11.2	7.9	12.2	2.0	7.3	7.6	4.1
	8	2.6	20.3	30.0	23.1	35.8	4.4	25.0	21.8

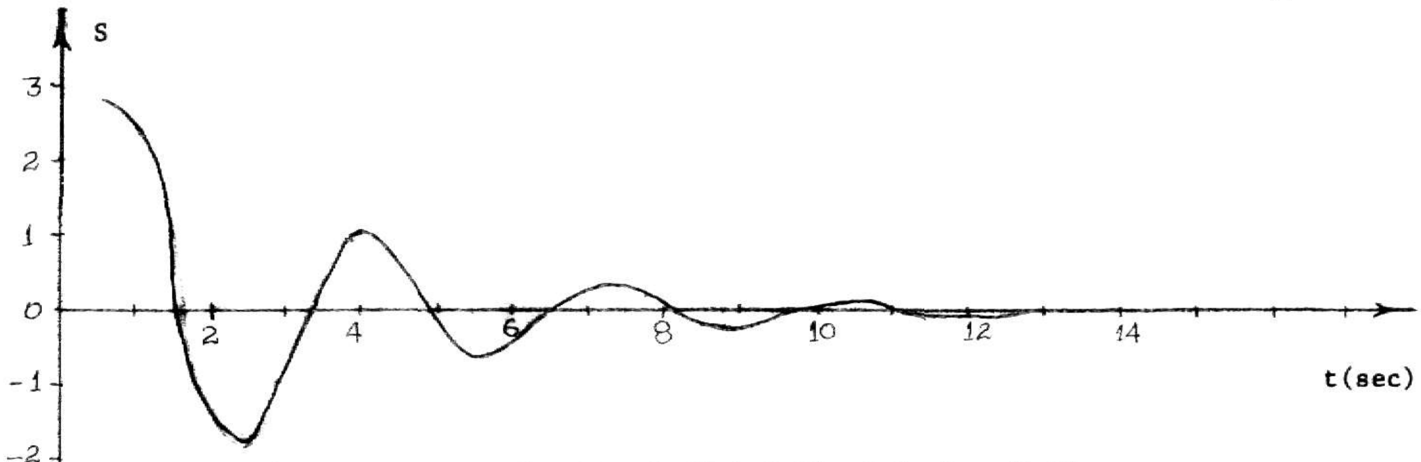

Figure 4-3 Waveform of the Signal S(m) Defined in Eq. (4-3)

Time scale factor=($\frac{1}{13}$)

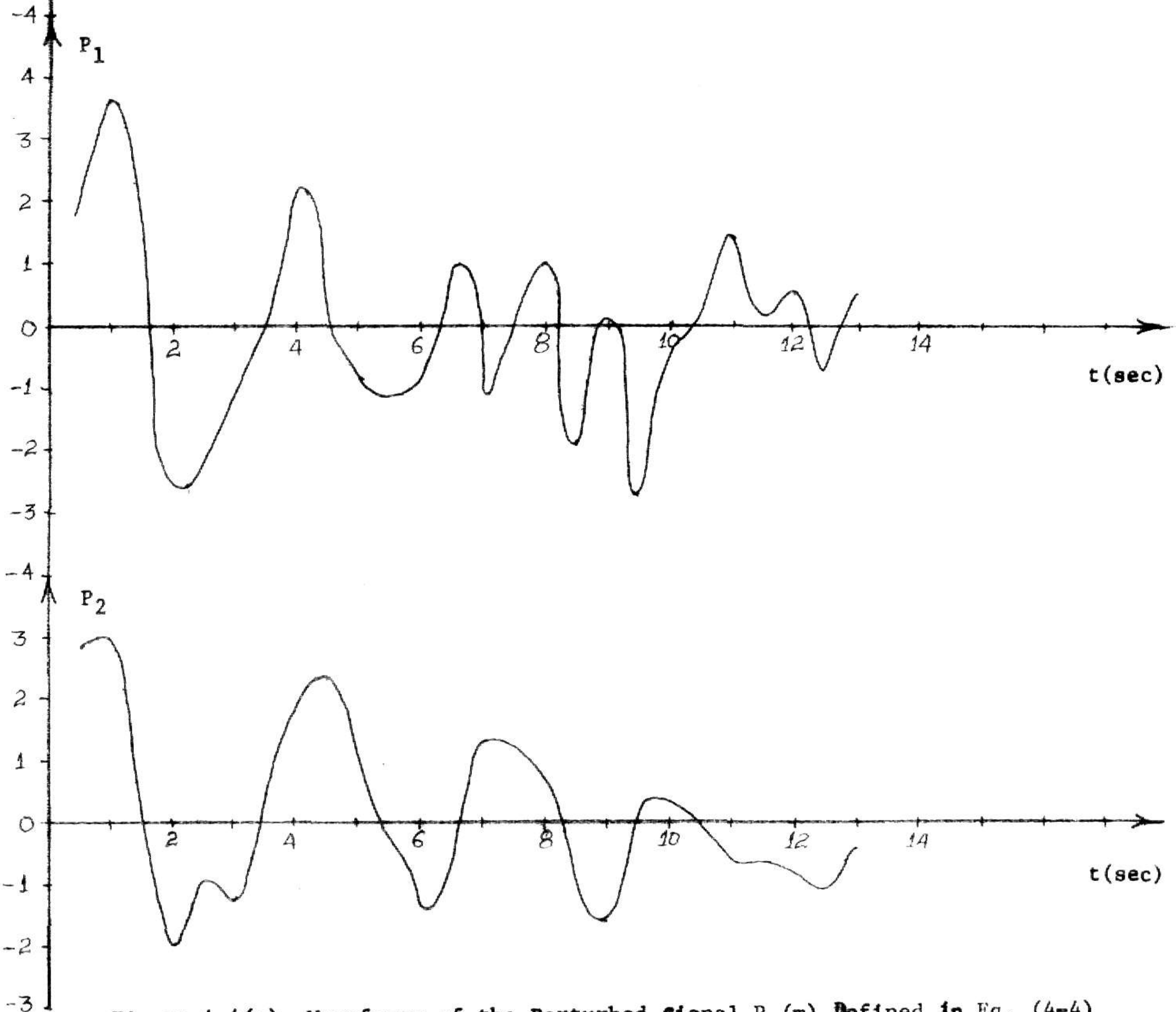

Figure 4-4(a) Waveforms of the Perturbed Signal P_j(m) Defined in Eq. (4-4)

Time scale factor=($\frac{1}{13}$)

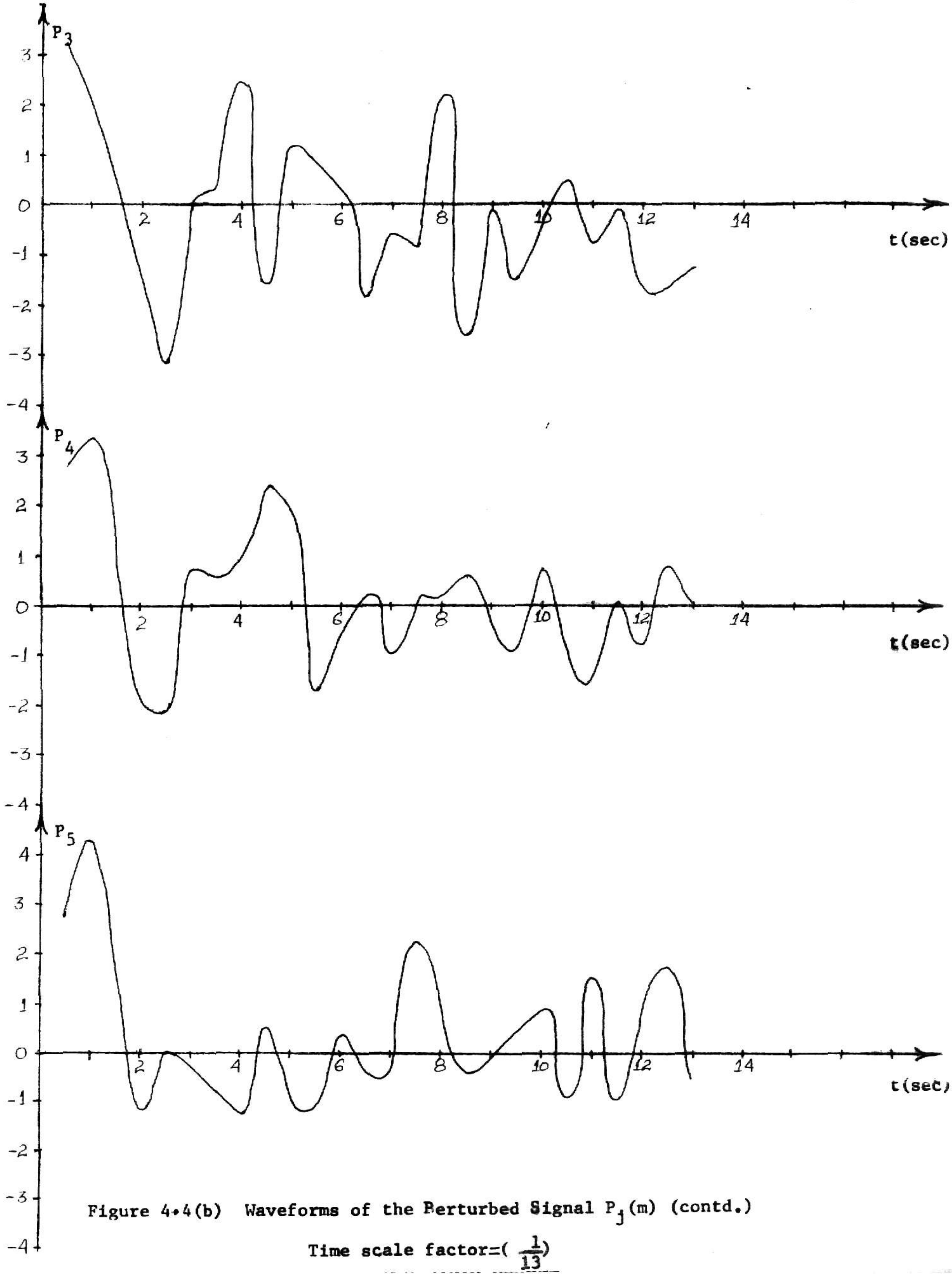

Figure 4.4(b) Waveforms of the Berturbed Signal $P_j(m)$ (contd.)

Time scale factor=$(\frac{1}{13})$

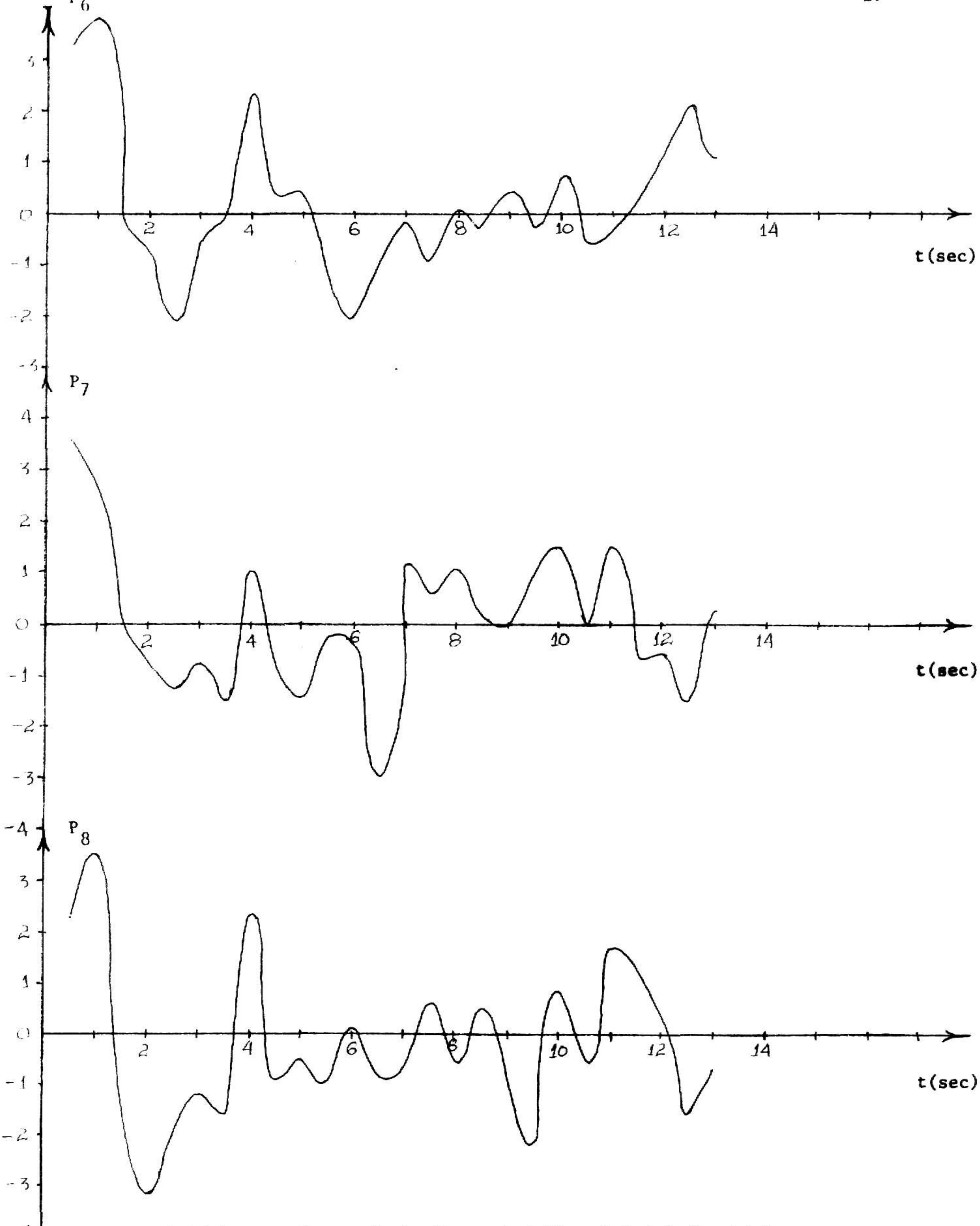

Figure 4·4(c) Waveforms of the Perturbed Signal $P_j(m)$ (contd.)

Time scale factor=$(\frac{1}{13})$

the ensemble of perturbed signal as

$$P_j(m) = S(m)+N_j(m) \qquad m=1,2,\ldots,26 \tag{4-4}$$

where $N_j(m)$ denotes the j-th noise sample.

The scale factor 3.97 associated with S(m) in Eq. (4-3) was chosen such that the mean-square value of S(m) is approximately <u>equal</u> to that of the noise ensemble.

That is,

$$\sum_{m=1}^{26} S^2(m) \approx \frac{1}{8} \sum_{j=1}^{8} \sum_{m=1}^{26} N_j^{\,2}(m) \tag{4-5}$$

The Fourier power spectrum of the signal S(m) at 20 filters having center frequences listed in Table 4-1 is given by

$$P(\omega_n,N-1) = (\Delta t)^2 \, ||\underline{Z}(\omega_n,N-1)||^2 \qquad n=1,2,\ldots,20 \qquad N=26, \Delta t=\frac{1}{26} \text{ sec} \tag{4-6}$$

where

$$\underline{Z}(\omega_n,N-1) = \sum_{m=0}^{N-1} \left[L(\omega_n)\right]^m \underline{b}\; S(m) \tag{4-7}$$

$\underline{Z}(\omega_n,N-1)$ is obtained using the recurrence relation in Eq. (2-9) as

$$\underline{Z}(\omega_n,k) = \left[L(\omega_n)\right] \underline{Z}(\omega_n,k-1) + \underline{b}\; X(N-1-k) \qquad k=0,1,\ldots,26 \tag{4-8}$$

where

X(N-1-k) is replaced by S(27-m), m=1,2,...,26

and

$$\left[L(\omega_n)\right] = \begin{bmatrix} \cos(\omega_n\Delta t) & -\sin(\omega_n\Delta t) \\ \sin(\omega_n\Delta t) & \cos(\omega_n\Delta t) \end{bmatrix} \tag{4-9}$$

Next, the following computations were performed*:

(1) The Fourier power spectrum of $S(m)$ defined in Eq. (4-6) was obtained using the recurrence relation in Eq. (4-8).

(2) The Fourier power spectra of the samples of the perturbed signal $P_j(m), j=1,2,...,8$ were obtained using the recurrence relation defined in Eq. (4-8) with $X(N-1-k)$ replaced by $P_j(27-m), m=1,2,...,26, j=1,2,...,8$ at the frequencies listed in Table (4-1).

(3) The smoothed power spectra of the samples of the perturbed signal $P_j(m), j=1,2,...,8$ were obtained using 20 damped filters having the <u>same</u> center frequencies using the recurrence relation in Eq. (4-8) with $X(N-1-k)$ replaced by $P_j(27-m)$, $m=1,2,...,26$, $j=1,2,...,8$ and

$$\left[L(\omega_n)\right] = \begin{bmatrix} e^{-\alpha\Delta t}\cos(\omega_c\Delta t) & -e^{-\alpha\Delta t}\sin(\omega_c\Delta t) \\ e^{-\alpha\Delta t}\sin(\omega_c\Delta t) & e^{-\alpha\Delta t}\cos(\omega_c\Delta t) \end{bmatrix}$$

where

$$\alpha=\zeta\omega_n$$

$$\omega_c=\omega_n\sqrt{1-\zeta^2}$$

$$\zeta = 0.0588 \tag{4-10}$$

The above value of ζ was obtained using Eq. (2-20). Thus Eq. (2-19) implies that each filter has a $Q=1/2\zeta=8.5$.

The Fourier power spectrum points of the signal $S(m)$ and the power spectra of a typical perturbed signal $P_1(m)$ without smoothing and with smoothing are tabulated in Table (4-3). Also, the mean-square errors of the above spectra with respect to the Fourier power spectrum of the signal $S(m)$ are tabulated in Table (4-3).

*For a listing of the computer program, see Appendix III.

TABLE 4-3

Frequency Hz	1.38	1.5	1.7	1.96	2.2	2.48	2.8	3.14	3.5
Fourier Power Spectrum of the Signal-S(m) $\left[\cdot\left(\frac{1}{26}\right)^2\right]$	17.5	18.9	20.0	22.0	26.0	34.0	43.0	63.0	103.0
Unsmoothed Power Spectrum of the First Sample of the Perturbed Signal-P_1(m) $\left[\cdot\left(\frac{1}{26}\right)^2\right]$	10.0	0.9	14.0	54.0	88.0	71.0	190	69.0	249.0
Mean Square Error	.0151	.0132	.0080	.0038	.0068	.0075	.0058	.0219	.0266
Smoothed Power Spectrum (ζ=0.0588) of P_1(m) $\left[\cdot\left(\frac{1}{26}\right)^2\right]$	10.7	4.7	14.0	38.0	58.0	52.0	34.0	61.0	140.0
Mean Square Error	0.0082	0.0069	.0043	.0025	.0028	.0020	.0013	.0037	.0033
Power Spectrum of P_1(m) Smoothed Using Hanning's Algorithm $\left[\cdot\left(\frac{1}{26}\right)^2\right]$	16.22	6.45	20.72	52.5	75.25	62.25	44.5	101.5	218.5
Mean Square Error	.0094	.0118	.0073	.0042	.0052	.0046	.0055	.0142	.0163

TABLE 4-3 (Cont.)

3.97	4.47	5.	5.65	6.36	7.2	8.	9.	10.2	11.5	12.89
137.0	91.0	37.0	17.0	8.7	4.7	2.9	1.9	1.4	1.2	1.0
307.0	97.0	122.0	72.0	7.0	16.0	22.0	2.4	57.0	28.0	44.0
.0242	.0097	.0088	.0053	.0005	.0058	.0049	.0004	.0035	.0012	.0061
157.0	90.0	72.0	36.0	10.8	12.0	8.7	2.1	1.8	0.5	0.02
.0042	.0029	.0011	.0005	.0001	.0005	.0003	.0000	.0000	.0000	.0000
240.	155.75	103.25	68.25	25.5	15.25	15.6	20.95	36.1	39.25	31.5
.0092	.0036	.0033	.0032	.0016	.0027	.0027	.0014	.0015	.0020	0.0050

The usual practice to smooth the power spectrum against a random process is to average the spectrum over neighbouring frequencies. A particular case is the discrete-time Hanning smoothing algorithm given by,

$$P(\omega_n,N-1)=\tfrac{1}{4}P(\omega_{n-1},N-1)+\tfrac{1}{2}P(\omega_n,N-1)+\tfrac{1}{4}P(\omega_{n+1},N-1) \qquad (4-11)$$

The Hanning smoothed power spectrum points for a typical perturbed signal $P_1(m)$ and the corresponding mean-square errors are also listed in the Table (4-3). It is remarked that Hanning smoothing algorithm is as effective as any of the other smoothing algorithms for run-of-the-mill problems [2].

The mean-square errors corresponding to the unsmoothed power spectra, the smooth power spectra and Hanning smoothed power spectra of the set of 8 perturbed signals (see Figure 4.4) are plotted in Figure 4.5.

From the Figure 4.5 it can be observed that the smoothed power spectrum of the perturbed signal $P_j(m)$ obtained using the damped filters gives the best spectral estimate of $P_j(m)$ with the least error.

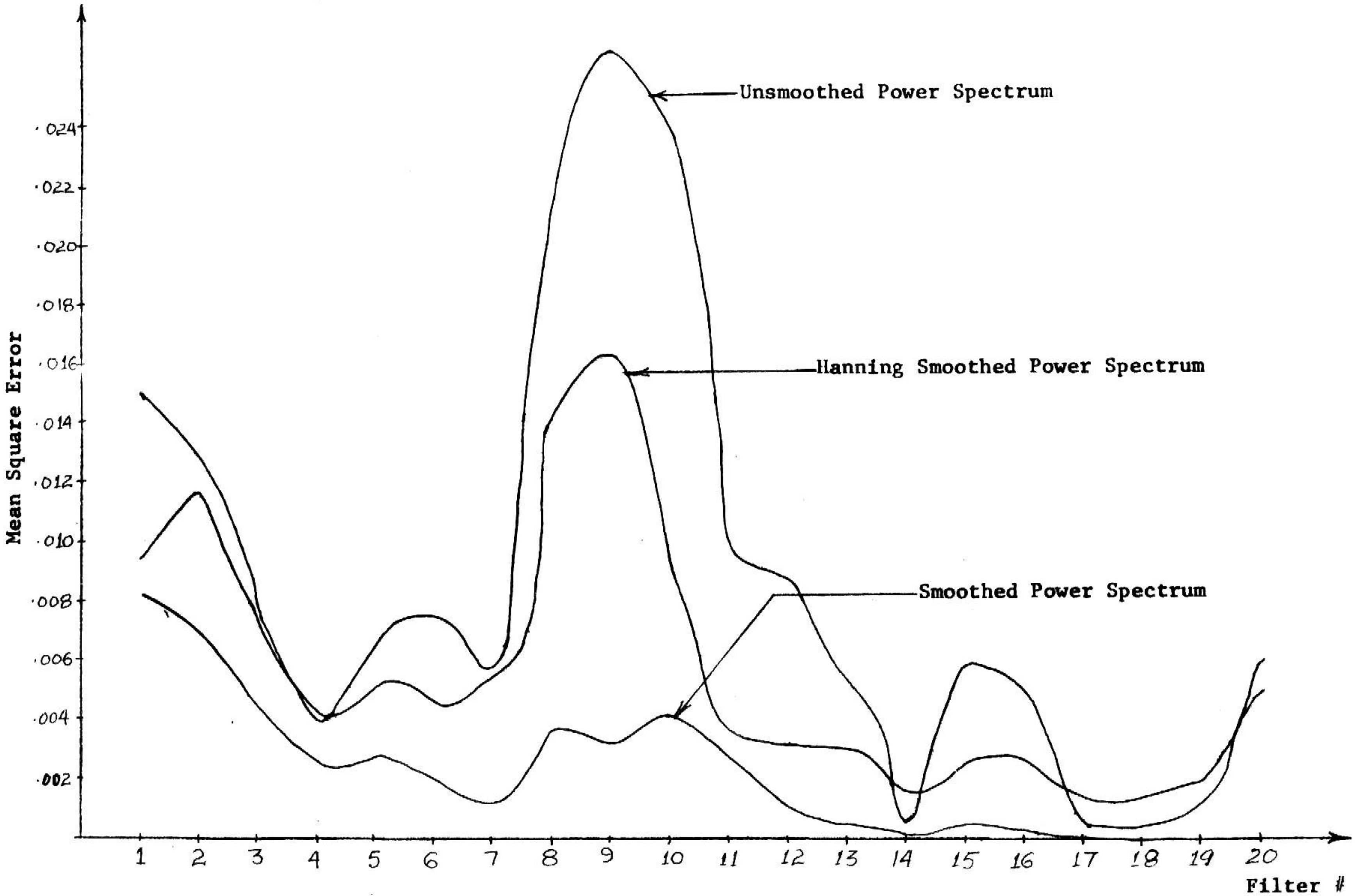

Figure 4.5 Summary of Errors in Spectral Estimation

CHAPTER V

CONCLUSION

This report has presented a study of the possible applications of Arnold's algorithm and the time-varying Fourier transform. It has been demonstrated that Arnold's algorithm can be used in situations where transient waveforms, embedded in noise, have to be classified via pattern classification techniques. Then it is shown that the time-varying Fourier transform can be used for two purposes: (1) to compute the Fourier transform of a discrete signal, and (2) to estimate the Fourier power and phase spectra of transient waveforms embedded in noise. Both of these spectra can be obtained recursively with any desired frequency resolution.

Several areas are open for further work. Arnold's on-line computational algorithm might be extended to two-dimensions and hence applied to image processing. Specifically it could prove to be useful in classifying two-dimensional patterns, embedded in noise.

It is apparent that the time-varying Fourier transform could be used to estimate the correlation and convolution functions associated with discrete signals in the presence of noise. As such it could prove useful in the area of digital filtering and hence deserves to be investigated and compared with the techniques which are presently available.

By means of a simulated experiment, it has been demonstrated that the time-varying Fourier transform yields superior spectral estimates when compared with a standard smoothing technique such as that attributed to Hanning. The logical question which arises is whether this is the case in general at <u>various</u> signal-to-noise ratios. It would be very worthwhile to investigate this problem using methods which are used in the area of statistical communication theory.

APPENDIX I

In this appendix, a listing of FORTRAN IV Computer program which implements Arnold's on-line computational algorithm (Eq. (2-24)) to compute the instantaneous power points of the first sample of the signals belonging to Class A (see Figure 4-1) is provided.

```
              C
              C
              C      COMPUTATION OF TIME-VARYING POWER SPECTRUM
              C
              C
              C      THE FIRST DATA CARD CONTAINS...
              C            N,THE NUMBER OF INPUT DATA POINTS IN COLS.1-3,IN I3 FORMAT
              C
              C      THE SECOND DATA CARD CONTAINS...
              C            DT,THE SAMPLING INTERVAL IN COLS.1-8,IN F8.4 FORMAT
              C            ZETA,THE DAMPING FACTOR IN COLS.9-16,IN F8.4 FORMAT.ZETA=0.
              C                  IMPLIES NO SMOOTHING
              C            IDFT,FLAG PARAMETER,IN COL.17,IN I1 FORMAT.IDFT=1 GIVES THE
              C                  POWER SPECTRUM AT THE HARMONIC FREQUENCIES WITH
              C                  ZETA=0.,IDFT=0 GIVES THE POWER SPECTRUM AT THE
              C                  COMPUTED FREQUENCIES WITH THE GIVEN ZETA OR AT THE
              C                  CHOSEN FREQUENCIES WITH ZETA=0.
              C            INT,THE INTERVAL OF PRINTING THE TIME-VARYING POWER IN
              C                  COLS.18-20,IN I3 FORMAT.INT=M IMPLIES THAT THE TIME-
              C                  VARYING POWER WILL BE PRINTED EVERY M-TH INSTANT
              C      THE THIRD DATA CARD CONTAINS...
              C            DEC,THE DESIRED FREQUENCY RANGE IN DECADES,IN COLS.1-3,IN
              C                  F3.0 FORMAT.THIRD DATA CARD IS REQUIRED WHEN ZETA IS
              C                  NOT EQUAL TO ZERO
 0001                DIMENSION X(100),Z1(100),Z2(100),FREQ(100),POWER(100)
              C ***  FIRST DATA CARD
 0002                READ 10,N
 0003             10 FORMAT(I3)
              C ***  SECOND DATA CARD
 0004                READ 15,DT,ZETA,IDFT,INT
 0005             15 FORMAT(2F8.4,I1,I3)
 0006                IF (ZETA.EQ.0.) GO TO 60
              C ***  THIRD DATA CARD
 0007                READ 20,DEC
 0008             20 FORMAT(F3.0)
 0009                PRINT 25,DT,ZETA,DEC
 0010             25 FORMAT('1',30X,'SMOOTHED POWER SPECTRUM'//' ',10X,'DELTA T =',F6.4
                    1,5X,'ZETA =',F6.4,5X,'FREQUENCY RANGE =',F3.0,' DECADE(S)')
              C      CALCULATE FILTER CENTER FREQUENCIES
 0011                FRNQ=0.5/DT
              C      COMMENT-FRNQ IS THE COMPUTED NYQUIST FREQUENCY
 0012                FRBS=FRNQ/10.**DEC
              C      COMMENT-FRBS IS THE BASE FREQUENCY
 0013                ONPZ=1.+ZETA
 0014                ONMZ=1.-ZETA
 0015                FOX=ONPZ*FRBS
 0016                KK=1
 0017                FACT=ONPZ/ONMZ
 0018                FREQ(1)=FOX
 0019             30 FOX=FACT*FOX
 0020                IF(FRNQ.LT.FOX) GO TO 35
 0021                KK=KK+1
 0022                FREQ(KK)=FOX
 0023                GO TO 30
              C      PRINT THE COMPUTED FREQUENCIES
 0024             35 PRINT 40,FRBS,FRNQ
 0025             40 FORMAT('0',9X,'THE COMPUTED FREQUENCIES ARE.... '//13X,'BASE=',F6.
                    13,//10X,'NYQUIST=',F10.3,///10X'TABLE OF FILTER CENTER FREQUENCIES
```

```
                 2',//10X,'COUNT',10X,'CENTER FRECUENCY',18X,'BANDWIDTH',18X,'3 DB C
                 3ROSS OVER PT.'//)
0026                DO 100 K=1,KK
0027                BAND=2.C*ZETA*FREQ(K)
0028                KP1=K+1
0029                IF(K.NE.KK) GO TO 45
0030                FRQP1=FACT*FREQ(KK)
0031                CROSS=SQRT(FREQ(KK)*FROP1)
0032                GO TO 50
0033             45 CROSS=SQRT(FREQ(K)*FREQ(KP1))
0034             50 PRINT 55,K,FREQ(K),BAND,CROSS
0035             55 FORMAT(' ',10X,I3,10X,F12.6,20X,F10.4,20X,F10.4)
0036            100 CONTINUE
0037                GO TO 75
0038             60 IF(IDFT.NE.1) GO TO 65
0039                PRINT 160
0040            160 FORMAT('1',10X,'FOURIER POWER SPECTRUM AT HARMONIC FREQUENCIES')
              C     CALCULATE THE HARMONIC FREQUENCIES
0041                FREQ(1)=0.
0042                KK=N/2+1
0043                AN=N
0044                DO 150 I=2,KK
0045                AK=K
0046                FREC(K)=(AK-1.)/(AN*DT)
0047            150 CONTINUE
0048                GO TO 75
              C     READ THE CHOSEN FREQUENCIES
0049             65 READ 70,KK,(FREQ(K),K=1,KK)
0050             70 FORMAT(I3/(8F10.2))
0051                PRINT 165,(FREC(K),K=1,KK)
0052            165 FORMAT('1',10X,'FOURIER POWER SPECTRUM AT THE CHOSEN FREQUENCIES'/
                 1/' ','THE CHOSEN FREQUENCIES ARE...'//(' ',10F10.2))
0053             75 TPDT=6.28318*DT
              C     COMMENT-TPDT IS TWO PI DELTA T
0054                ROMZ=SQRT(1.0-ZETA**2)
              C     READ THE INPUT DATA
0055             80 READ (5,200,END=1000) (X(I),I=1,N)
0056            200 FORMAT(16F5.2)
0057                PRINT 250,(X(I),I=1,N)
0058            250 FORMAT('0','INPUT SAMPLED DATA:'/(' ',10F10.2))
0059                DO 500 K=1,KK
0060                ARG=TPDT*FREQ(K)
0061                EXX=EXP(-ZETA*ARG)
0062                CC=EXX*COS(ROMZ*ARG)
0063                SS=EXX*SIN(ROMZ*ARG)
0064                L=N+1
              C     INITIALISE THE FILTERS
0065                DO 300 M=1,L
0066                Z1(M)=0.
0067                Z2(M)=0.
0068            300 CONTINUE
0069                PRINT 85,K,FREQ(K)
0070             85 FORMAT('0','FILTER #=',I3,10X,'FREQUENCY =',F10.3)
0071                IF(INT.EQ.0) GO TO 95
0072                PRINT 90
0073             90 FORMAT('0',10X,'INSTANT OF TIME',12X,'INSTANTANEOUS PWOER')
0074             95 CONTINUE
0075                DO 400 M=1,N
```

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(Arnold's Algorithm)

```
C      COMPUTE THE TIME-VARYING POWER
       Z1(M+1)=CC*Z1(M)-SS*Z2(M)
       Z2(M+1)=SS*Z1(M)+CC*Z2(M)+X(M)
       POWER(K)=Z1(M+1)**2+Z2(M+1)**2
       IF(INT.EQ.0) GO TO 400
       IF(INT.EQ.1) GO TO 110
       AM=M
       AINT=INT
       RINT=AM/AINT
       KR=RINT
       RKR=KR
       IF(RINT.NE.RKR) GO TO 400
   110 PRINT 115,M,POWER(K)
   115 FORMAT(' ',15X,I3,20X,F12.6)
   400 CONTINUE
       PRINT 120,POWER(K)
   120 FORMAT(' ',10X ,'TOTAL POWER=',F12.4)
   500 CONTINUE
C      RETURN TO READ NEXT SET OF INPUT DATA
       GO TO 60
  1000 CONTINUE
       STOP
       END
```

SMOOTHED POWER SPECTRUM

DELTA T =0.0385   ZETA =0.0538   FREQUENCY RANGE = 1. DECADE(S)

THE COMPUTED FREQUENCIES ARE....

BASE= 1.300

NYQUIST= 13.000

TABLE OF FILTER CENTER FREQUENCIES

COUNT	CENTER FREQUENCY	BANDWIDTH	3 DB CROSS OVER PT.
1	1.376456	0.1619	1.4599
2	1.548440	0.1821	1.6423
3	1.741910	0.2048	1.8475
4	1.959554	0.2304	2.0784
5	2.204391	0.2592	2.3381
6	2.479819	0.2916	2.6302
7	2.789661	0.3281	2.9586
8	3.138217	0.3691	3.3285
9	3.530323	0.4152	3.7444
10	3.971421	0.4670	4.2122
11	4.467632	0.5254	4.7385
12	5.025843	0.5910	5.3306
13	5.653799	0.6649	5.9966
14	6.360216	0.7480	6.7459
15	7.154897	0.8414	7.5887
16	8.048869	0.9465	8.5359
17	9.054540	1.0648	9.6036
18	10.185864	1.1979	10.8035
19	11.458544	1.3475	12.1533
20	12.890239	1.5159	13.6718

INPUT SAMPLED DATA:

-3.23	-0.03	1.38	-0.62	1.24	0.24	-0.94	0.97	0.91	-1.19
-0.70	-0.17	-0.66	1.23	-0.13	-0.52	-0.46	-0.52	0.98	-1.38
-0.07	0.98	-1.69	0.39	-2.60	-0.41	-0.06	1.36	0.27	0.62
-0.59	0.56	0.27	-1.21	-0.31	-1.56	0.60	1.29	0.17	-0.11
-0.11	-0.29	0.72	0.24	0.20	-0.88	0.60	0.26	-0.97	0.98

FILTER #= 1   FREQUENCY = 1.376

INSTANT OF TIME	INSTANTANEOUS PWOER
10	8.123537
20	9.300437
30	8.603025
40	3.545267
50	3.309862

TOTAL POWER= 3.3099

FILTER #= 2   FREQUENCY = 1.546

INSTANT OF TIME	INSTANTANEOUS PWOER
10	10.041385
20	7.455791
30	1.888959

```
            40                          1.880012
            50                          1.262900
        TOTAL POWER=       1.2629

FILTER #=  3          FRECUENCY =      1.742

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         12.083050
            20                          5.088265
            30                          7.986492
            40                          0.718280
            50                          0.474867
        TOTAL POWER=       0.4749

FILTER #=  4          FRECUENCY =      1.960

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         13.962616
            20                          4.703764
            30                         26.822968
            40                         13.409464
            50                          8.144091
        TOTAL POWER=       8.1441

FILTER #=  5          FRECUENCY =      2.204

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         15.203162
            20                          8.282079
            30                         33.921699
            40                         34.612427
            50                         21.490356
        TOTAL POWER=      21.4904

FILTER #=  6          FRECUENCY =      2.460

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         15.191533
            20                         13.645615
            30                         13.069179
            40                         23.580948
            50                         11.518242
        TOTAL POWER=      11.5182

FILTER #=  7          FRECUENCY =      2.790

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         13.439216
            20                         14.174773
            30                          4.908806
            40                         15.703625
            50                          4.990037
        TOTAL POWER=       4.9900

FILTER #=  8          FRECUENCY =      3.138

        INSTANT OF TIME                INSTANTANEOUS PWOER
            10                         10.131757
            20                          7.319141
            30                         26.963715
```

```
          40                            29.386429
          50                             7.059221
     TOTAL POWER=        7.0592

FILTER #=  9          FREQUENCY =     3.530

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                             6.780526
          20                             1.114478
          30                            18.599991
          40                             5.026380
          50                             4.346114
     TOTAL POWER=        4.3461

FILTER #= 10          FREQUENCY =     3.971

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                             6.168399
          20                             0.556690
          30                            12.951870
          40                             0.906883
          50                             1.882741
     TOTAL POWER=        1.8827

FILTER #= 11          FREQUENCY =     4.468

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                            10.279009
          20                             3.619137
          30                             6.695562
          40                            12.623339
          50                             6.998896
     TOTAL POWER=        6.9989

FILTER #= 12          FREQUENCY =     5.026

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                            16.341553
          20                            14.792209
          30                             6.084167
          40                            12.380816
          50                             3.897155
     TOTAL POWER=        3.8972

FILTER #= 13          FREQUENCY =     5.654

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                            16.204559
          20                            10.572622
          30                             6.975285
          40                             8.724563
          50                             3.246077
     TOTAL POWER=        3.2461

FILTER #= 14          FREQUENCY =     6.360

     INSTANT OF TIME                 INSTANTANEOUS PWOER
          10                             6.629095
          20                             2.027371
          30                             1.646444
```

```
          40                               29.386429
          50                                7.059221
      TOTAL POWER=       7.0592

FILTER #=  9          FREQUENCY =     3.530

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                                6.780526
          20                                1.114478
          30                               18.599991
          40                                5.026360
          50                                4.346124
      TOTAL POWER=       4.3461

FILTER #= 10          FREQUENCY =     3.971

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                                6.168399
          20                                0.556690
          30                               12.951870
          40                                0.906883
          50                                1.882741
      TOTAL POWER=       1.8827

FILTER #= 11          FREQUENCY =     4.468

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                               10.279009
          20                                3.619137
          30                                6.695562
          40                               12.623339
          50                                6.998896
      TOTAL POWER=       6.9989

FILTER #= 12          FREQUENCY =     5.026

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                               16.341553
          20                               14.792209
          30                                6.084167
          40                               12.380816
          50                                3.897155
      TOTAL POWER=       3.8972

FILTER #= 13          FREQUENCY =     5.654

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                               16.204559
          20                               10.572622
          30                                6.975285
          40                                8.724563
          50                                3.246077
      TOTAL POWER=       3.2461

FILTER #= 14          FREQUENCY =     6.360

      INSTANT OF TIME                  INSTANTANEOUS PWUER
          10                                6.829095
          20                                2.027371
          30                                1.046444
```

```
           40                                1.851954
           50                                2.103821
        TOTAL POWER=        2.1030

FILTER #= 15           FREQUENCY =      7.155

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                                2.197970
           20                                3.302446
           30                                0.027491
           40                                1.209677
           50                                3.062873
        TOTAL POWER=        3.0629

FILTER #= 16           FREQUENCY =      8.049

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                               12.663522
           20                                6.609526
           30                                1.992380
           40                                2.375566
           50                                6.960953
        TOTAL POWER=        6.9610

FILTER #= 17           FREQUENCY =      9.055

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                               14.562389
           20                                1.293921
           30                                1.710649
           40                                2.491132
           50                                4.971066
        TOTAL POWER=        4.9711

FILTER #= 18           FREQUENCY =     10.186

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                                1.978825
           20                                7.870586
           30                                1.462383
           40                                2.011731
           50                                6.918830
        TOTAL POWER=        6.9189

FILTER #= 19           FREQUENCY =     11.459

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                                2.122783
           20                                7.284651
           30                                5.017647
           40                                1.728312
           50                                3.050838
        TOTAL POWER=        3.0508

FILTER #= 20           FREQUENCY =     12.890

        INSTANT OF TIME                INSTANTANEOUS PWUER
           10                                0.500082
           20                                3.724787
           30                                7.516002
```

40		0.035832
50		0.884828
TOTAL POWER=	0.8846	

APPENDIX II

This appendix provides a listing of a computer program for a training algorithm which uses the least-square mapping principle to classify two classes of signals mentioned in Section 4.2.

```
                C
                C     # OF CLASSES: MAX. 10
                C     #OF SAMPLES IN EACH CLASS: MAX. 100
                C     DIMENSION OF SAMPLES & UNIT VECTORS: MAX. 20
                C     FIRST CARD IN THE MAIN PROGRAM SHOULD BE CHANGED DEPENDING ON THE
                C     COMPILER USED
                C     FIRST DATA CARD: # OF CLASSES,DIMENSIONS OF SAMPLES & UNIT VECTORS IN 3I5
                C     SECOND CARD : # OF SAMPLES IN EACH CLASS IN FORMATS OF I5
                C     THIRD CARD : PROBABILITY OF EACH CLASS IN FORMATS OF F8.4
                C     READ IN UNIT VECTORS , ONE PER CARD, EACH COMPONENT IN F8.4
                C     SAMPLES , ONE PER CARD,WITH COMPONENTS IN F8.4
                C
0001                  DATA NR,NW/5,6/
0002                  DIMENSION NSAMP(10),PROB(10),V(10,10),X(21,100),P(2100,10)
0003                  DIMENSION Y(21,21),XX(21,21),A(10,21),E(10,21),VX(10,21),C(10,100)
0004                  DIMENSION ICF(10,10)
0005                  READ(NR,1)NCLAS,NCOMP,LV
0006                1 FORMAT(10I5)
0007                  WRITE(NW,2)NCLAS,NCOMP
0008                2 FORMAT(10X,'THIS PROBLEM HAS',I5,'  CLASSES & EACH CLASS IS',I5,
                     1' DIMENSIONAL'///)
0009                  N=NCLAS
0010                  READ(NR,1)(NSAMP(I),I=1,N)
0011                  READ(NR,20)(PROB(I),I=1,N)
0012                3 FORMAT(///)
0013                  DO 4 I=1,N
0014                4 WRITE(NW,5)I,NSAMP(I),PROB(I)
0015                5 FORMAT(10X,'CLASS',I5,'  HAS',I5,'  SAMPLES &  ITS PROBABILITY
                     1IS',F10.4)
0016                  L=NCOMP
0017                  DO 10 J=1,N
0018               10 READ (NR,20)(V(I,J),I=1,LV)
0019               20 FORMAT(10F8.4)
0020                  WRITE(NW,3)
0021                  WRITE(NW,25)
0022               25 FORMAT(10X,'  V  MATRIX'///)
0023                  DO 30 I=1,LV
0024               30 WRITE(NW,35)(V(I,J),J=1,N)
0025               35 FORMAT(10X,11F10.3)
0026                  L1=L+1
0027                  DO 80 K=1,N
0028                  NI=NSAMP(K)
0029                  FI=FLOAT(NI)
0030                  FI=PROB(K)
0031                  DO 40 J=1,NI
0032                  READ(NR,20)(X(I,J),I=1,L)
0033               40 X(L1,J)=-1.0
0034                  M=1
0035                  DO 41 J=1,NI
0036                  DO 41 I=1,L1
0037                  P(M,K)=X(I,J)
0038               41 M=M+1
0039                    WRITE(NW,3)
0040                  WRITE(NW,45)K
0041               45 FORMAT(10X,'AUGMENTED PATTERNS OF CLASS',I5///)
0042                  DO 50 I=1,L1
0043               50 WRITE(NW,35)(X(I,J),J=1,NI)
0044                  DO 60 I1=1,L1
```

```
      DO 60 I2=1,L1
      Y(I1,I2)=0.0
      DO 55 J=1,NI
   55 Y(I1,I2)=Y(I1,I2)+X(I1,J)*X(I2,J)
      Y(I1,I2)=PI*Y(I1,I2)/FI
      IF(K.EQ.1)XX(I1,I2)=0.0
   60 XX(I1,I2)=XX(I1,I2)+Y(I1,I2)
      DO 65 I=1,L1
      DO 65 J=2,NI
   65 X(I,1)=X(I,1)+X(I,J)
      DO 70 I=1,LV
      DO 70 J=1,L1
      A(I,J)=PI*V(I,K)*X(J,1)/FI
      IF(K.EQ.1)VX(I,J)=0.0
   70 VX(I,J)=VX(I,J)+A(I,J)
   80 CONTINUE
      CALL INVERS(XX,L1)
      DO 120 I=1,LV
      DO 120 J=1,L1
      A(I,J)=0.0
      DO 120 K=1,L1
  120 A(I,J)=A(I,J)+VX(I,K)*XX(K,J)
      WRITE(NW,3)
      WRITE(NW,125)
  125 FORMAT(10X,'TRANSFORMATION MATRIX A'//)
      DO 130 I=1,LV
  130 WRITE(NW, 35)(A(I,J),J=1,L1)
      DO  135 I=1,N
      DO  135 J=1,L1
      E(I,J)=0.0
      DO 135 K=1,LV
  135 E(I,J)=E(I,J)+V(K,I)*A(K,J)
        WRITE(NW,3)
      WRITE(NW,140)
  140 FORMAT(10X,'DECISION MATRIX'///)
      DO 142 I=1,N
  142 WRITE(NW,35)(E(I,J),J=1,L1)
      DO 145 I=1,N
      DO 145 J=1,N
  145 ICF(I,J)=0
      DO 185 K=1,N
      NI=NSAMP(K)
      M=1
      DO 150 J=1,NI
      DO 150 I=1,L1
      X(I,J)=P(M,K)
  150 M=M+1
      DO 160 I=1,N
      DO 160 J=1,NI
      D(I,J)=0.0
      DO 160 M=1,L1
  160 D(I,J)=D(I,J)+E(I,M)*X(M,J)
      WRITE(NW,3)
      DO 161 ID=1,N
  161 WRITE(NW,35)(D(ID,JD),JD=1,NI)
      DO 180 J=1,NI
      LS=1
      C=D(1,J)
```

```
      CC 17C I=1,N
      IF(C.LT.D(I,J))GO TO 165
      CC TC 170
165   LS=I
      C=D(I,J)
17C   CONTINUE
18C   ICF(K,LS)=ICF(K,LS)+1
185   CCNTINUE
        WRITE(NW,3)
      WRITE(NW,190)
19C   FORMAT(10X,'CCNFUSICN MATRIX'///)
      CO 195 I=1,N
195   WRITE(NW,2CC)(ICF(I,J),J=1,N)
200   FCRMAT(10X,1CI5)
      STCP
      END
```

FORTRAN IV G LEVEL 18 INVERS DATE = 72094 16/35/34 PAGE 0001

```
      SUBROUTINE INVERS(Z,N)
      DIMENSION U(21,21),V(21,21),Y(21,21),Z(21,21)
      DO 5 I=1,N
      DO 5 J=1,N
      U(I,J)=0.0
    5 IF (I.EQ.J)U(I,J)=1.0
    1 L=1
   10 K=L
   11 Q=ABS(Z(K,L))
      IF(Q-0.000001)20,20,30
   20 Z(K,L)=0.0
      K=K+1
      IF(K-N)11,11,90
   30 DO 31 M=1,N
      Y(K,M)=Z(K,M)
      Z(K,M)=Z(L,M)
      Z(L,M)=Y(K,M)
      V(K,M)=U(K,M)
      U(K,M)=U(L,M)
   31 U(L,M)=V(K,M)
   40 Y(L,L)=Z(L,L)
      DO 41 M=1,N
      Z(L,M)=Z(L,M)/Y(L,L)
   41 U(L,M)=U(L,M)/Y(L,L)
      K=1
   50 IF(K-L)52,51,52
   51 K=K+1
      IF(K-N)50,50,70
   52 Q=ABS(Z(K,L))
      IF(Q-0.000001)53,53,54
   53 Z(K,L)=0.0
      K=K+1
      IF(K-N)50,50,60
   54 Y(K,L)=Z(K,L)
      DO 55 M=1,N
      Z(K,M)=Z(K,M)-Z(L,M)*Y(K,L)
   55 U(K,M)=U(K,M)-U(L,M)*Y(K,L)
      K=K+1
      IF(K-N)50,50,60
   60 L=L+1
      IF(L-N)10,10,70
   70 DO 80 I=1,N
      DO 80 J=1,N
   80 Z(I,J)=U(I,J)
   90 RETURN
      END
```

THIS PROBLEM HAS 2 CLASSES & EACH CLASS IS 8 DIMENSIONAL

CLASS 1 HAS 8 SAMPLES & ITS PROBABILITY IS 0.5000
CLASS 2 HAS 8 SAMPLES & ITS PROBABILITY IS 0.5000

V MATRIX

1.000	0.0
0.0	1.000

AUGMENTED PATTERNS OF CLASS 1

0.600	7.300	0.800	5.700	6.300	2.600	3.000	4.300
12.900	3.800	4.300	13.000	0.200	7.700	9.100	1.500
0.900	11.100	15.200	4.800	7.200	3.500	0.600	8.300
3.600	17.000	5.600	7.500	0.0	17.600	4.200	9.900
6.700	18.900	4.400	4.900	0.600	0.600	18.900	3.400
12.600	11.000	20.200	3.000	1.900	8.000	1.700	8.800
14.800	7.500	3.500	15.300	2.600	14.800	4.200	3.800
6.000	10.300	15.000	2.600	0.900	3.600	22.000	5.400
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000

AUGMENTED PATTERNS OF CLASS 2

16.400	39.400	35.300	8.800	51.500	9.100	18.600	2.600
31.000	25.000	7.500	17.000	18.800	5.100	11.200	20.300
2.800	26.000	30.400	0.300	12.700	14.400	7.900	30.000
28.400	35.200	46.300	42.700	20.600	39.300	12.200	23.100
4.500	35.400	1.800	13.600	5.500	19.800	2.000	35.800
16.700	11.000	9.100	10.800	7.900	11.100	7.300	4.400
37.600	8.500	37.500	59.400	13.900	47.300	7.600	25.000
9.900	13.700	10.500	7.100	3.600	14.400	4.100	21.800
-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000

TRANSFORMATION MATRIX A

-0.015	-0.013	-0.014	0.004	-0.001	0.004	-0.018	-0.005	-1.274
0.015	0.013	0.014	-0.004	0.001	-0.004	0.018	0.005	0.274

DECISION MATRIX

-0.015	-0.013	-0.014	0.CC4	-C.CC1	C.CC4	-C.C18	-C.C05	-1.274
C.015	0.C13	0.014	-0.004	0.001	-0.004	0.018	C.CC5	C.274

DECISION NUMBERS

CLASS 1	0.848	0.887	0.972	0.701	1.034	0.908	0.937	1.C6C
	0.152	0.113	C.028	0.299	-0.034	C.C92	0.063	-0.060
CLASS 2	0.035	-0.C43	-0.178	0.012	-0.C77	C.143	0.662	0.099
	0.965	1.043	1.178	0.988	1.077	0.857	0.338	0.901

CONFUSION MATRIX

8	C
1	7

APPENDIX III

In this appendix, a listing of the Computer program which implements the algorithm defined in Eq. (2-9) to compute the Fourier transform power spectrum. The outputs consist of the following (see Table 4-3):

1. Fourier power spectrum of signal S(m) defined in Eq. (4-3).

2. Unsmoothed power spectrum of the first sample of the perturbed signals -- $P_1(m)$ defined in Eq. (4-4).

3. Smoothed power spectrum of $P_1(m)$.

```
$JOB
C
C
C     COMPUTATION OF TIME-VARYING POWER SPECTRUM
C
C
C     THE FIRST DATA CARD CONTAINS...
C           N,THE NUMBER OF INPUT DATA POINTS IN COLS.1-3,IN I3 FORMAT
C
C     THE SECOND DATA CARD CONTAINS...
C           DT,THE SAMPLING INTERVAL IN COLS.1-8,IN F8.4 FORMAT
C           ZETA,THE DAMPING FACTOR IN COLS.9-16,IN F8.4 FORMAT.ZETA=0.
C                 IMPLIES NO SMOOTHING
C           IDFT,FLAG PARAMETER,IN COL.17,IN I1 FORMAT.IDFT=1 GIVES THE
C                 POWER SPECTRUM AT THE HARMONIC FREQUENCIES WITH
C                 ZETA=0.,IDFT=0 GIVES THE POWER SPECTRUM AT THE
C                 COMPUTED FREQUENCIES WITH THE GIVEN ZETA OR AT THE
C                 CHOSEN FREQUENCIES WITH ZETA=0.
C           INT,THE INTERVAL OF PRINTING THE TIME-VARYING POWER IN
C                 COLS.18-20,IN I3 FORMAT.INT=M IMPLIES THAT THE TIME-
C                 VARYING POWER WILL BE PRINTED EVERY M-TH INSTANT
C     THE THIRD DATA CARD CONTAINS...
C           DEC,THE DESIRED FREQUENCY RANGE IN DECADES,IN COLS.1-3,IN
C                 F3.0 FORMAT.THIRD DATA CARD IS REQUIRED WHEN ZETA IS
C                 NOT EQUAL TO ZERO
      DIMENSION X(100),Z1(100),Z2(100),FREQ(100),POWER(100)
C *** FIRST DATA CARD
      READ 10,N
   10 FORMAT(I3)
C *** SECOND DATA CARD
      READ 15,DT,ZETA,IDFT,INT
   15 FORMAT(2F8.4,I1,I3)
      IF (ZETA.EQ.0.) GO TO 60
C *** THIRD DATA CARD
      READ 20,DEC
   20 FORMAT(F3.0)
      PRINT 25,DT,ZETA,DEC
   25 FORMAT('1',30X,'SMOOTHED POWER SPECTRUM'//' ',10X,'DELTA T =',F6.4
     1,5X,'ZETA =',F6.4,5X,'FREQUENCY RANGE =',F3.0,' DECADE(S)')
C     CALCULATE FILTER CENTER FREQUENCIES
      FRNQ=0.5/DT
C     COMMENT-FRNQ IS THE COMPUTED NYQUIST FREQUENCY
      FRBS=FRNQ/10.**DEC
C     COMMENT-FRBS IS THE BASE FREQUENCY
      CNPZ=1.+ZETA
      CNMZ=1.-ZETA
      FCX=CNPZ*FRBS
      KK=1
      FACT=CNPZ/CNMZ
      FREQ(1)=FCX
   30 FCX=FACT*FCX
      IF(FRNQ.LT.FCX) GO TO 35
      KK=KK+1
      FREQ(KK)=FCX
      GO TO 30
C     PRINT THE COMPUTED FREQUENCIES
   35 PRINT 40,FRBS,FRNQ
   40 FORMAT('0',5X,'THE COMPUTED FREQUENCIES ARE.... '//13X,'BASE=',F6.
     13,//10X,'NYQUIST=',F10.3,///10X'TABLE OF FILTER CENTER FREQUENCIES
     2',//10X,'COUNT',10X,'CENTER FREQUENCY',18X,'BANDWIDTH',18X,'3 DB C
```

```
         3RCSS CVER PT.'//)
          CC 1CC K=1,KK
          BANC=2.0*ZETA*FREQ(K)
          KP1=K+1
          IF(K.NE.KK) GO TC 45
          FRCP1=FACT*FREQ(KK)
          CROSS=SCRT(FREC(KK)*FRCP1)
          GO TC 5C
       45 CROSS=SQRT(FREQ(K)*FREQ(KP1))
       5C PRINT 55,K,FREQ(K),BANC,CRCSS
       55 FORMAT(' ',1CX,I3,1CX,F12.6,20X,F1C.4,20X,F10.4)
      1CC CCNTINUE
          GC TC 75
       6C IF(IDFT.NE.1) GC TC 65
          PRINT 16C
      160 FCRMAT('1',1CX,'FOURIER PCWER SPECTRUM AT HARMONIC FREQUENCIES')
C         CALCULATE THE HARMCNIC FREQUENCIES
          FREQ(1)=C.
          KK=N/2+1
          AN=N
          DC 15C I=2,KK
          AK=K
          FREC(K)=(AK-1.)/(AN*CT)
      15C CCNTINUE
          GO TC 75
C         REAC THE CHOSEN FREQUENCIES
       65 REAC 7C,KK,(FREC(K),K=1,KK)
       7C FORMAT(I3/(8F10.2))
          PRINT 165,(FREQ(K),K=1,KK)
      165 FCRMAT('1',10X,'FOURIER POWER SPECTRUM AT THE CHCSEN FRECUENCIES'/
         1/' ','THE CHCSEN FRECUENCIES ARE...'//(' ',10F10.2))
       75 TPDT=6.2831E*CT
C         CCMMENT-TPCT IS TWC PI CELTA T
          RCMZ=SCRT(1.0-ZETA**2)
C         REAC THE INPUT DATA
       80 REAC (5,20C,END=1CCC) (X(I),I=1,N)
      200 FCRMAT(10F8.4)
          PRINT 25C,(X(I),I=1,N)
      250 FORMAT('C','INPUT SAMPLED DATA:'/(' ',10F1C.2))
          CC 5CC K=1,KK
          ARG=TPCT*FREC(K)
          EXX=EXP(-ZETA*ARG)
          CC=EXX*COS(RCMZ*ARG)
          SS=EXX*SIN(RCMZ*ARG)
          L=N+1
C         INITIALISE THE FILTERS
          CC 3CC M=1,L
          Z1(M)=C.
          Z2(M)=C.
      3C0 CCNTINUE
          PRINT 85,K,FREQ(K)
       85 FORMAT('C','FILTER #=',I3,1CX,'FRECUENCY =',F1C.3)
          IF(INT.EC.C) GO TO 95
          PRINT 9C
       9C FCRMAT('C',1CX,'INSTANT CF TIME',12X,'INSTANTANECUS PWCER')
       95 CCNTINUE
          CC 4CC M=1,N
C         CCMPUTE THE TIME-VARYING PCWER
          Z1(M+1)=CC*Z1(M)-SS*Z2(M)+X(N+1-M)
          Z2(M+1)=SS*Z1(M)+CC*Z2(M)
```

```
          PCWER(K)=Z1(M+1)**2+Z2(M+1)**2
          IF(INT.EQ.C) GO TC 4CC
          IF(INT.EQ.1) GO TO 11C
          AM=M
          AINT=INT
          RINT=AM/AINT
          KR=RINT
          RKR=KR
          IF(RINT.NE.RKR) GC TC 400
      110 PRINT 115,M,POWER(K)
      115 FORMAT(' ',15X,I3,20X,F12.6)
      4CC CCNTINLE
          PRINT 120,PCWER(K)
      120 FCRMAT(' ',10X ,'TCTAL POWER=',F12.4)
      5CC CCNTINLE
C         RETURN TO READ NEXT SET OF INPLT CATA
          GC TC 80
     1CCO CCNTINLE
          STOP
          END

$ENTRY
```

```
        FOURIER POWER SPECTRUM AT THE CHOSEN FREQUENCIES

THE CHOSEN FREQUENCIES ARE...

      1.38      1.55      1.74      1.96      2.20      2.48      2.79      3.14      3.50      3.97
      4.47      5.03      5.65      6.36      7.15      8.05      9.05     10.19     11.46     12.89

INPUT SAMPLED DATA:   S(m)
      2.77      2.66      0.58     -1.35     -1.71     -0.68      0.58      1.04      0.59     -0.18
     -0.59     -0.44      0.00      0.32      0.31      0.07     -0.15     -0.19     -0.08      0.07
      0.12      0.07     -0.02     -0.07     -0.05      0.00

FILTER #=  1          FREQUENCY =     1.380
        TOTAL POWER=     17.5316

FILTER #=  2          FREQUENCY =     1.550
        TOTAL POWER=     18.8941

FILTER #=  3          FREQUENCY =     1.740
        TOTAL POWER=     20.1759

FILTER #=  4          FREQUENCY =     1.960
        TOTAL POWER=     22.1253

FILTER #=  5          FREQUENCY =     2.200
        TOTAL POWER=     26.2697

FILTER #=  6          FREQUENCY =     2.480
        TOTAL POWER=     33.6680

FILTER #=  7          FREQUENCY =     2.790
        TOTAL POWER=     43.3884

FILTER #=  8          FREQUENCY =     3.140
        TOTAL POWER=     63.3715

FILTER #=  9          FREQUENCY =     3.500
        TOTAL POWER=    103.0351

FILTER #= 10          FREQUENCY =     3.970
        TOTAL POWER=    136.8965

FILTER #= 11          FREQUENCY =     4.470
        TOTAL POWER=     91.2525

FILTER #= 12          FREQUENCY =     5.030
        TOTAL POWER=     36.6051

FILTER #= 13          FREQUENCY =     5.650
        TOTAL POWER=     17.2301

FILTER #= 14          FREQUENCY =     6.360
        TOTAL POWER=      8.6547

FILTER #= 15          FREQUENCY =     7.150
        TOTAL POWER=      4.7345

FILTER #= 16          FREQUENCY =     8.050
        TOTAL POWER=      2.8977
```

```
FILTER #= 17          FREQUENCY =     9.C5C
          TOTAL POWER=       1.9288

FILTER #= 18          FREQUENCY =    10.19C
          TCTAL POWER=       1.4204

FILTER #= 19          FREQUENCY =    11.46C
          TCTAL POWER=       1.1871

FILTER #= 20          FREQUENCY =    12.890
          TOTAL POWER=       1.0336

CCRE USAGE      OBJECT CODE=    5368 BYTES,ARRAY AREA=    2CCO BYTES,TCTAL AREA AVAILABLE=   49504  BYTES

DIAGNOSTICS       NUMBER CF ERRORS=      G, NUMBER CF WARNINGS=      C, NUMBER OF EXTENSICNS=      0

CCMPILE TIME=     5.41 SEC,EXECUTION TIME=     2.64 SEC,  WATFIV - VERSICN 1 LEVEL 3 MARCH  1971      DATE=  72/094
```

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                 FOURIER POWER SPECTRUM AT THE CHOSEN FREQUENCIES

THE CHOSEN FREQUENCIES ARE...

       1.38      1.55      1.74      1.96      2.20      2.48      2.79      3.14      3.50      3.97
       4.47      5.03      5.65      6.36      7.15      8.05      9.05     10.19     11.46     12.89

INPUT SAMPLED DATA:  P1(m)
       1.83      3.62      1.49     -2.54     -2.41     -0.84     -0.08      2.27      0.45     -0.69
      -1.05     -0.96      0.98     -1.07      0.24      1.04     -1.85      0.19     -2.67     -0.34
       0.07      1.43      0.24      0.55     -0.64      0.56

FILTER #=  1           FREQUENCY =      1.380
          TOTAL POWER=      10.1119

FILTER #=  2           FREQUENCY =      1.550
          TOTAL POWER=       0.8515

FILTER #=  3           FREQUENCY =      1.740
          TOTAL POWER=      14.0882

FILTER #=  4           FREQUENCY =      1.960
          TOTAL POWER=      53.6911

FILTER #=  5           FREQUENCY =      2.200
          TOTAL POWER=      88.3181

FILTER #=  6           FREQUENCY =      2.480
          TOTAL POWER=      71.1964

FILTER #=  7           FREQUENCY =      2.790
          TOTAL POWER=      19.3315

FILTER #=  8           FREQUENCY =      3.140
          TOTAL POWER=      68.6336

FILTER #=  9           FREQUENCY =      3.500
          TOTAL POWER=     248.6265

FILTER #= 10           FREQUENCY =      3.970
          TOTAL POWER=     307.3279

FILTER #= 11           FREQUENCY =      4.470
          TOTAL POWER=      97.4597

FILTER #= 12           FREQUENCY =      5.030
          TOTAL POWER=     122.1950

FILTER #= 13           FREQUENCY =      5.650
          TOTAL POWER=      72.0854

FILTER #= 14           FREQUENCY =      6.360
          TOTAL POWER=       7.0922

FILTER #= 15           FREQUENCY =      7.150
          TOTAL POWER=      16.0353

FILTER #= 16           FREQUENCY =      8.050
          TOTAL POWER=      22.4882
```

```
FILTER #= 17          FREQUENCY =      9.050
          TOTAL POWER=       2.3833

FILTER #= 18          FREQUENCY =     10.190
          TOTAL POWER=      56.5992

FILTER #= 19          FREQUENCY =     11.460
          TOTAL POWER=      28.1444

FILTER #= 20          FREQUENCY =     12.890
          TOTAL POWER=      43.7750

CORE USAGE      OBJECT CODE=    5368 BYTES,ARRAY AREA=    2000 BYTES,TOTAL AREA AVAILABLE=   49488  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=    3.37 SEC,EXECUTION TIME=     2.94 SEC,  WATFIV - VERSION 1 LEVEL 3 MARCH  1971       DATE=  72/095
```

SMOOTHED POWER SPECTRUM

DELTA T =0.0395 ZETA =0.0580 FREQUENCY RANGE = 1. [illegible]

THE COMPUTED FREQUENCIES ARE....

BASE= 1.300

NYQUIST= 13.000

TABLE OF FILTER CENTER FREQUENCIES

COUNT	CENTER FREQUENCY	BANDWIDTH	3 DB CROSS [illegible] PT.
1	1.376457	0.1619	1.4577
2	1.548439	0.1621	1.64[illegible]
3	1.741909	0.2048	1.8475
4	1.959553	0.2304	2.0764
5	2.204390	0.2592	2.3[illegible]
6	2.479818	0.2916	2.6302
7	2.789660	0.3281	2.95[illegible]
8	3.138216	0.3691	3.3265
9	3.530322	0.4152	3.7444
10	3.971420	0.4670	4.2122
11	4.467631	0.5254	4.7[illegible]
12	5.025842	0.5910	5.3306
13	5.653798	0.6649	5.99[illegible]
14	6.360215	0.7480	6.7459
15	7.154896	0.8414	7.5[illegible]7
16	8.048868	0.9465	8.5369
17	9.054539	1.0648	9.6036
18	10.185860	1.1979	1[illegible]
19	11.458540	1.3475	12.[illegible]
20	12.890230	1.5159	13.67[illegible]

INPUT SAMPLED DATA: $P_1(m)$

1.83	3.62	1.49	-2.54	-2.41	-0.84	-0.08	2.27	0.45	-0.69
-1.05	-0.96	0.98	-1.07	0.24	1.04	-1.85	0.19	-2.67	-0.34
0.07	1.43	0.24	0.55	-0.64	0.56				

FILTER #= 1 FREQUENCY = 1.376
TOTAL POWER= 10.7119

FILTER #= 2 FREQUENCY = 1.548
TOTAL POWER= 4.7482

FILTER #= 3 FREQUENCY = 1.742
TOTAL POWER= 13.9064

FILTER #= 4 FREQUENCY = 1.960
TOTAL POWER= 37.5840

FILTER #= 5 FREQUENCY = 2.204
TOTAL POWER= 58.1155

FILTER #= 6 FREQUENCY = 2.480
TOTAL POWER= 52.4280

```
FILTER #=  7          FREQUENCY =      2.790
        TOTAL POWER=      33.6061

FILTER #=  8          FREQUENCY =      3.138
        TOTAL POWER=      60.7156

FILTER #=  9          FREQUENCY =      3.530
        TOTAL POWER=     140.1672

FILTER #= 10          FREQUENCY =      3.971
        TOTAL POWER=     156.6906

FILTER #= 11          FREQUENCY =      4.468
        TOTAL POWER=      90.0711

FILTER #= 12          FREQUENCY =      5.026
        TOTAL POWER=      71.8861

FILTER #= 13          FREQUENCY =      5.654
        TOTAL POWER=      35.7446

FILTER #= 14          FREQUENCY =      6.360
        TOTAL POWER=      10.8121

FILTER #= 15          FREQUENCY =      7.155
        TOTAL POWER=      12.1541

FILTER #= 16          FREQUENCY =      8.049
        TOTAL POWER=       8.7351

FILTER #= 17          FREQUENCY =      9.055
        TOTAL POWER=       2.1489

FILTER #= 18          FREQUENCY =     10.186
        TOTAL POWER=       1.7850

FILTER #= 19          FREQUENCY =     11.459
        TOTAL POWER=       0.5198

FILTER #= 20          FREQUENCY =     12.890
        TOTAL POWER=       0.0222

CORE USAGE      OBJECT CODE=    5268 BYTES,ARRAY AREA=     2000 BYTES,TOTAL AREA AVAILABLE=   49488  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=       0, NUMBER OF WARNINGS=       0, NUMBER OF EXTENSIONS=       0

COMPILE TIME=    3.39 SEC,EXECUTION TIME=     3.17 SEC,  WATFIV - VERSION 1 LEVEL 3 MARCH  1971      DATE=  72/095
```

REFERENCES

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2. C. R. Arnold, "Spectral Estimation for Transient Waveforms", IEEE Trans. Audio Electracoust., Vol. AU-18, pp. 248-257, Sept. 1970.

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SPECTRAL ANALYSIS OF APERIODIC DIGITAL SIGNALS

by

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B. E., 1970, Annamalai University, Tamilnadu, India

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

ABSTRACT

This report can be divided into two parts. First, the time-varying Fourier transform which was recently introduced for noise-free discrete signals is extended to the case when noise is present. Next, a study of Arnold's on-line computational algorithm is presented.

It is demonstrated that Arnold's algorithm can be effectively used for the signal representation of noisy transient waveforms, prior to their classification. In such cases since the transients are usually well below the noise level, their epoch and duration are not known. Therefore, standard techniques available such as fast Fourier transform fail to yield adequate signal representation of the transients.

Again, it is shown that the time-varying Fourier transform can be used for the effective estimation of Fourier power and phase spectra of transients in the presence of noise, with any desired frequency resolution. It is also demonstrated that the spectral estimates so obtained are superior to those obtained by standard smoothing techniques.