ON-OFF LIQUID LEVEL CONTROL
USING FLUIDIC CONTROL SYSTEM

by

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CHAPTER I.

INTRODUCTION

The main purpose of this master report is to analyze and experimentally verify the performance of a given system (Figure 1) in which the liquid level in a tank is to be maintained within a certain practical desired range using a fluidic on-off controller designed to have a hysteresis characteristic.

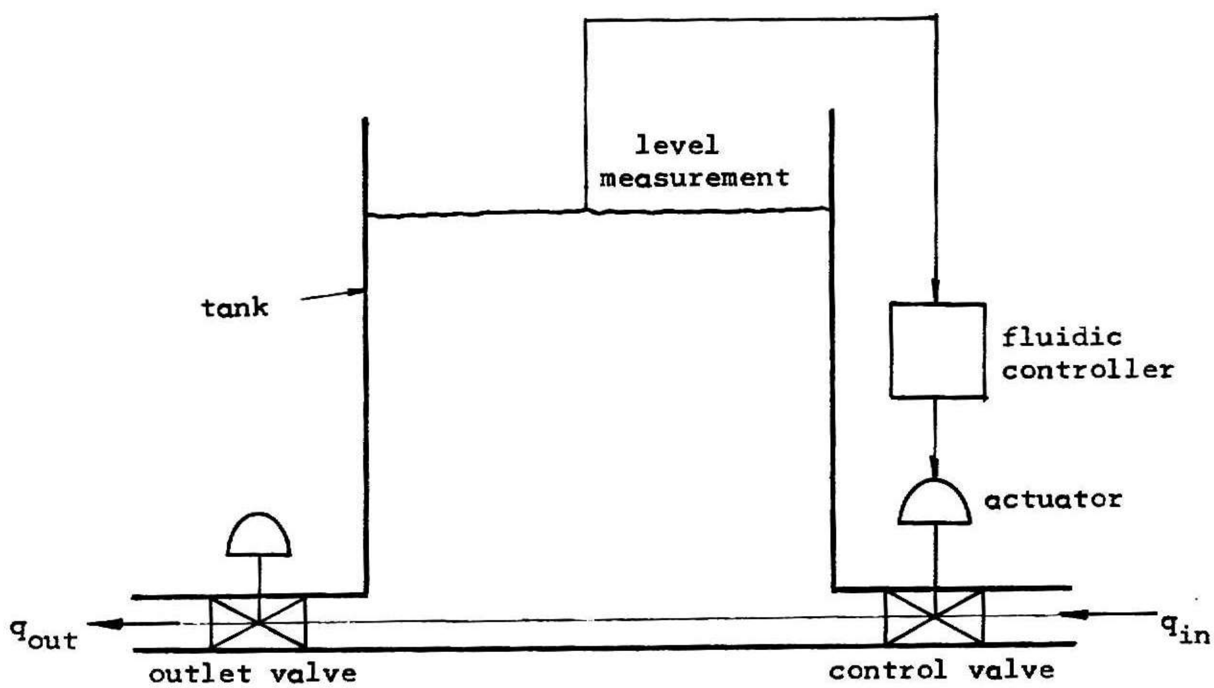

Fig. 1. A liquid level control system.

Fluidic level controllers have been sucessfully field tested in mining and paper industries. Systems are contemplated for chemical processing and marine fuel monitoring and control. Accurate measure and display of rapid refueling lend themselves to fluidic solutions.

In extracting atomic fuels, end product composition depends on flow rates and on the concentration of solutions in the extractors. Fluidic controllers, with no moving parts and no electronic circuits, are advantageous because reliability of electrical and electronic system is a problem in high radiation environments.

Fluidics, a general study of fluidic devices and systems, is an established and rapidly growing part of both military and industrial control system technology.

Devices, through which gases or liquids flow in intricate and precise channels, are called fluidic devices. Such devices can be used to perform sensing, logic, amplification and control functions.

The advantages of fluidic devices are that they are quite rugged because of the absence of mechanical moving parts and can perform control and computational funtions in adverse environments. Fluidic devices can withstand wide temperature changes, extremely high temperature, shock, vibration, acceleration force and radiation. In addition, the simplicity of basic fluidic devices assures high reliability, long life and very low maintenance.

As far as the speed of response of fluidic devices, they are comparable to conventional pneumatic and hydraulic devices or electromechanical relays.

In on-off or relay control systems, the actuator (see Figure 1) has only fixed positions (outputs), — in many cases, either on or off. This is in contrast with proportional or other control actions in which the actuator is capable of continuous variation of its output over a specific range. Since the actuator is switched abruptly between discrete conditions, and if the abrupt transient can appear at the closed loop system output, then this operation results in an unsatisfactory jerky response. However the low-pass filtering effect of the process being controlled will usually smooth the closed loop system response to within satisfactory bounds.

There are several advantages of using on-off control systems (from reference 2): First, on-off control represents the simplest possible amplifier. Second, on-off operation of the actuator represents the most economical use of the installed capacity. Third, with on-off control the design and construction, indeed the entire principle of operation of the output element may be simplified with consequent improvement in reliability, economy, and saving in weight. Finally on-off control is in general the time optional control.

In practice, no controller can be operated in such a fashion that a very small change in the error signal shifts the output signal from one extreme to the other. Consequently, in actual controllers there is a differential gap between the points at which the output signal changes from minimun to maximum or vice versa. Thus actual on-off controllers are often designed to exhibit a hysteresis characteristic in order to prevent too frequent operation of the on-off mechanism.

Systems using on-off type of controller have following disadvantages: First, being limited to two positions, the on-off controller either supplies too much or too little correction to the system. Thus the controlled variable must continuously fluctuate between the two limits required to cause the actuator to move from one fixed position to the other. Second, the wear of the system can be considerably large if the frequency of control operation is high.

CHAPTER II

SYSTEM DESCRIPTION

The system which is the subject of this report is depicted in Figure 2. It is located in the Mechanical Engineering Laboratory at Kansas State University and consists of the following components:

1) The process:

   A water tank of cross section area 112 square inches and about 30 inches in height.

2) Fluidic controller

   a) Bubble tube level sensor.(3/16 inches I.D.)

   b) Two Corning "NOR" gates. (No. 190815)

   c) Two Corning "AND" gates. (No. 190814)

   d) A Corning Flip-Flop gate (No. 190415)

   e) A Humphery interface valve (No. 250AL)

   f) Three Honeywell fluidic restrictors of size 0.021,0.030, 0.014 inches diameter.

   g) Plastic tubing (3/16 inches I.D.)

3) A Fisher positioner-actuator (3560-657/30)

4) A Fisher control valve (1-1/4", Design "A", single port)

5) A Fisher outlet valve (2", Design "G", V-port)

6) A water supply system regulated by a Fisher pressure control loop.

A detailed description of each system component as well as a description of system operation is made in this chapter.

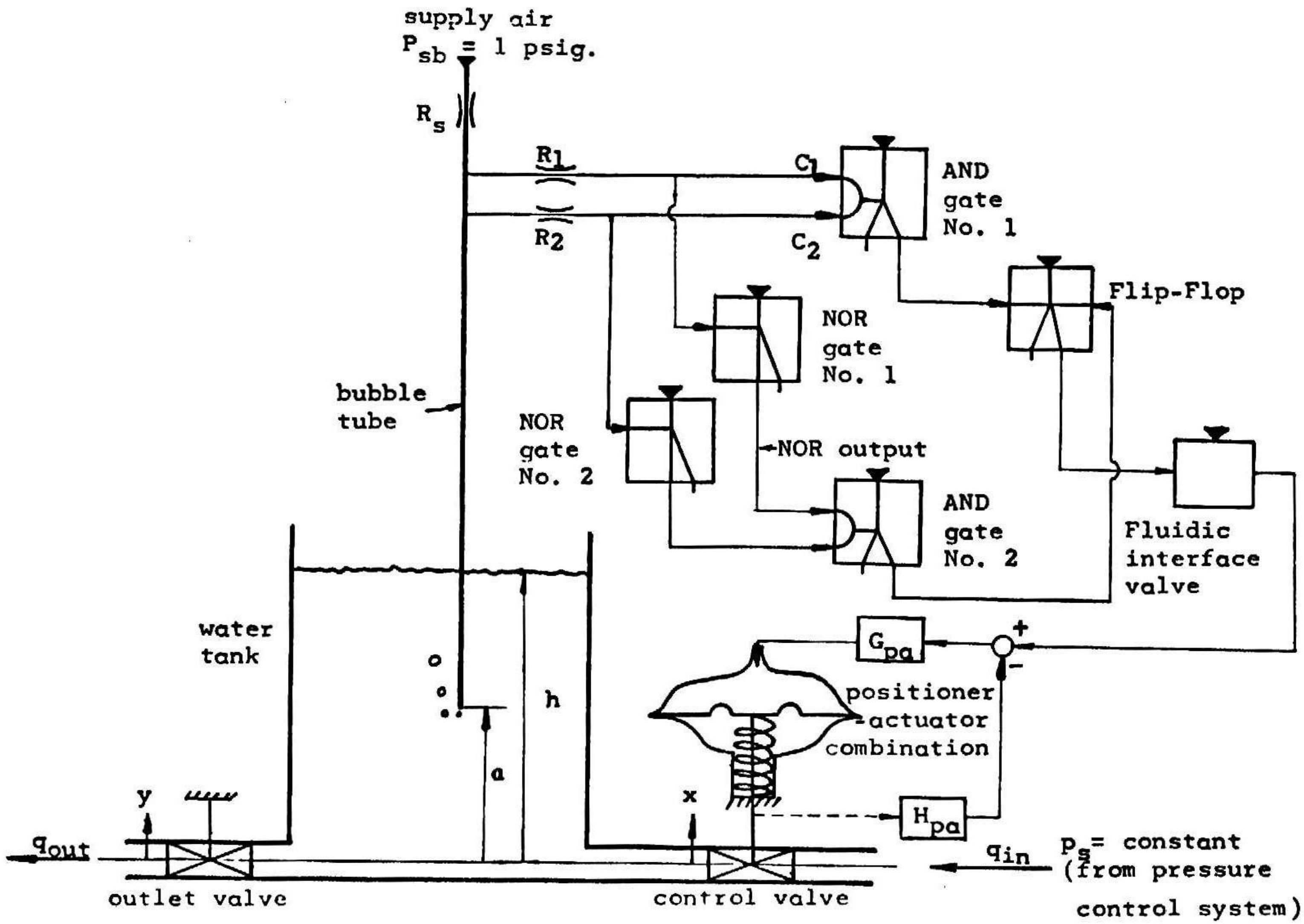

Fig. 2. A liquid level control system with fluidic controller.

Process

The process (Figure 3) uses water although in actual industrial systems liquid fuel or a chemical mixture might be used. It is desired that the water level be maintained at some nearly constant height h for variable flow out. Assume that the pressure ahead of the control valve is kept constant at P_s and that outlet flow discharges to atmosphere.

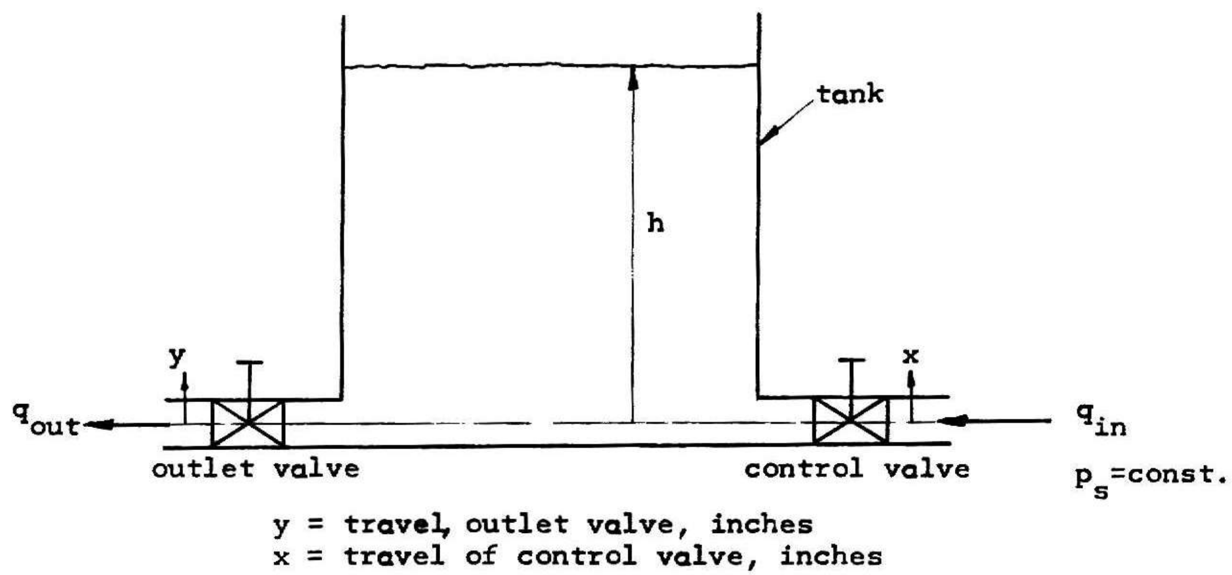

Fig. 3. Process to be controlled.

The linearized transfer function for this process is well known (see reference 6) and for small changes about a steady state operating point is :

$$F_p(s) = \frac{H(s)}{X(s)} = \frac{K_p}{1+\tau_p s} \qquad 2\text{-}1$$

where:

$H(s)$ = Laplace transform of water level.

$X(s)$ = Laplace transform of control valve stem travel.

K_p = process gain, in./in.

τ_p = process time constant, seconds.

1. Calculating process time constant and gain

The equations for the process time constant and process gain have been derived in the reference 6 as:

$$\tau_p = \frac{A_t}{\dfrac{dq_{out}}{dh} - \left.\dfrac{\partial q_{in}}{\partial h}\right|_x} \qquad 2\text{-}2$$

$$K_p = \frac{\left.\dfrac{\partial q_{in}}{\partial x}\right|_h}{\dfrac{dq_{out}}{dh} - \left.\dfrac{\partial q_{in}}{\partial h}\right|_x} \qquad 2\text{-}3$$

where:

A_t = tank area, in.^{2}

x = control valve stem travel, inches.

h = water level (head), inches.

$\dfrac{dq_{out}}{dh}$ = derivative for the outlet valve since $q_{out}=q_{out}(h)$ only for a fixed opening.

$\left.\frac{\partial q_{in}}{\partial h}\right|_x$, $\left.\frac{\partial q_{in}}{\partial x}\right|_h$ = partial derivatives for the control valve since $q_{in}=q_{in}(h,x)$, in.^{3}/in.-sec.

Since the outflow, q_{out} , is proportional to the square root of the pressure due to tank level, then in general

$$^*q_{out} = C\, h^{\frac{1}{2}} \qquad 2\text{-}4$$

and

$$\frac{dq_{out}}{dh} = \frac{C}{2h^{\frac{1}{2}}} \qquad 2\text{-}5$$

also at steady state operating point

$$q_{out_{ss}} = C\, h_{ss}^{\frac{1}{2}}$$

or

$$C = \frac{q_{out_{ss}}}{h_{ss}^{\frac{1}{2}}} \qquad 2\text{-}6$$

substituting equation 2-6 into equation 2-5 gives

$$\frac{dq_{out}}{dh} = \frac{q_{out_{ss}}}{2(hh_{ss})^{\frac{1}{2}}} \qquad 2\text{-}7$$

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*Also see Control Valve section, page 17

When $\frac{dq_{out}}{dh}$ is to be evaluated at a steady state operating point, $h=h_{ss}$, equation 2-7 becomes

$$\left.\frac{dq_{out}}{dh}\right|_{ss} = \frac{q_{out_{ss}}}{2\ h_{ss}} \qquad 2\text{-}8$$

Applying the same approach to obtain the partial derivatives for the control valve, then

$$\left.\frac{\partial q_{in}}{\partial h}\right|_{ss} = -\frac{q_{in_{ss}}}{2(p_s/0.0361 - h_{ss})} \qquad 2\text{-}9$$

Since the relation of flow to valve stem position is assumed linear for small changes about a steady state operating point, then

$$\left.\frac{\partial q_{in}}{\partial x}\right|_{ss} = \frac{q_{in_{ss}}}{x_{ss}} \qquad 2\text{-}10$$

For a geometrically fixed system the system time constant τ_p is a function of the operating point and the control valve supply pressure, i.e., $\tau_p=\tau_p(q_{ss},h_{ss},p_s)$. The qualitative effect of choosing different operating points or different supply pressures on the magnitude of the time constant is shown by the following table:

Table 1. Qualitative effect of q_{ss}, h_{ss}, and p_s on the magnitude of the process time constant

	$\left.\frac{\partial q_{in}}{\partial h}\right\vert_{ss}$	$\left.\frac{dq_{out}}{dh}\right\vert_{ss}$	τ
$^{*}q_{ss}$ ↑	↑	↑	↓
p_s ↓	↑	—	↓
h_{ss} ↑	↑	↓	?

↑ denotes increase
↓ denotes decrease
$^{*}q_{ss}=q_{in_{ss}}=q_{out_{ss}}$

For taking experimental data on the process it was considered undesirable to have a large process time constant (several mimutes) at least from the point of view of time required. Therefore it was possible to pick an operating poing with a relatively small process time constant by increasing the steady state inlet flow, reducing the supply pressure while keeping the water level unchanged at a convenient position for observation.

2. operating point selected

Using table 1 as a guide the following operating point was picked:

p_s = 4.5 psig. (125 inches of water)

q_{ss} = 49 $in.^3$/sec.

h_{ss} = 20 inches

x_{ss} = 0.248 inches

y_{ss} = 0.565 inches (determined experimentally to give h_{ss}=20 inches)

3. Evaluation of process time constant and gain at steady state operating point selected

Once the operating point had been selected, the process time constant and process gain could be evaluated. First, evaluation of the derivatives and partial derivatives gave:

$$\left.\frac{dq_{out}}{dh}\right|_{ss} = \frac{q_{ss}}{2h_{ss}} = \frac{49}{2(20)} = 1.225 \text{ in.}^2/\text{sec.}$$

$$\left.\frac{\partial q_{in}}{\partial h}\right|_{ss} = -\frac{q_{ss}}{2(p_s/0.0361-h_{ss})} = -\frac{49}{2(125-20)}$$
$$= -0.233 \text{ in.}^2/\text{sec.}$$

$$\left.\frac{\partial q_{in}}{\partial x}\right|_{ss} = \frac{q_{ss}}{x_{ss}} = \frac{49}{0.248} = 197.5 \text{ in.}^2/\text{sec.}$$

Then substituting these values into equations 2-2 and 2-3 as required gave:

$$\tau_p = \frac{A_t}{\dfrac{dq_{out}}{dh} - \left.\dfrac{\partial q_{in}}{\partial h}\right|_x} = \frac{112}{1.225-(-0.233)} = 77 \text{ sec.}$$

$$K_p = \frac{\left.\dfrac{\partial q_{in}}{\partial x}\right|_h}{\dfrac{dq_{out}}{dh} - \left.\dfrac{\partial q_{in}}{\partial h}\right|_x} = \frac{197.5}{1.225-(-0.233)} = 135.5 \text{ in./in.}$$

Fluidic controller

The controller was originally assembled by two mechanical engineering seniors at Kansas State University as a laboratory project. It was designed to be an on-off controller with hysteresis rather than just an on-off controller. The controller characteristic, ideally, is as shown in the Figure 4.

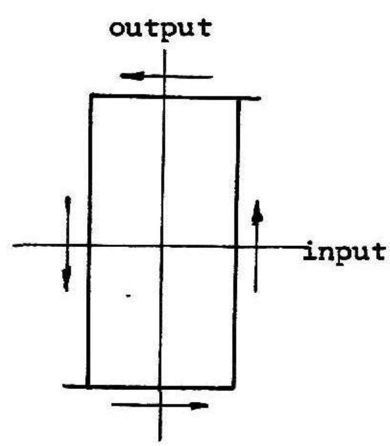


Fig. 4. On-off control characteristic with hysteresis.

The controller was assembled using fluidic hardware already available in the M.E. Department at K.S.U. and was intended to be an experimental model only.

1. Bubble tube level sensing

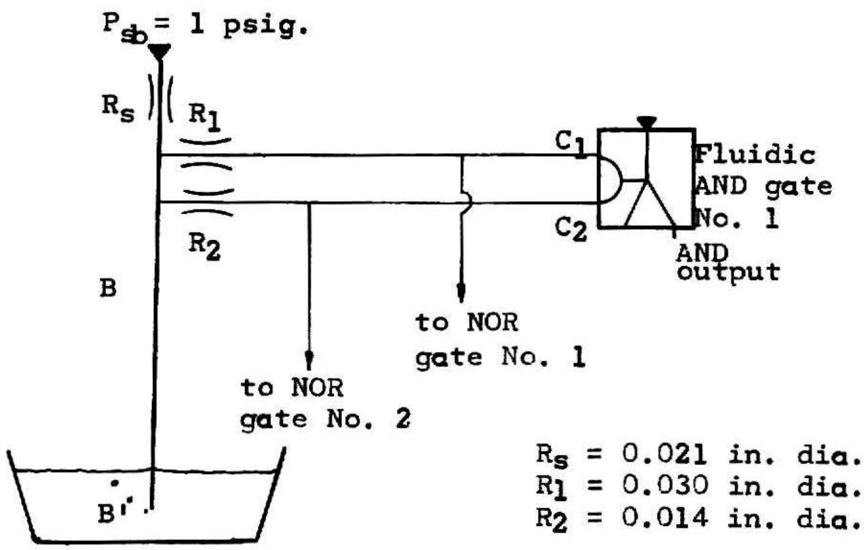


Fig. 5. Bubble tube level sensing.

Bubble tube level sensing (Figure 5) utilizes the variable restriction of air flow from B to B' to vary pressures (and flow) to both control ports of AND gate No. 1 and also to the control ports of NOR gates No. 1 and No. 2. The restriction at B' is directly proportional to the difference in head between the end of the sensor, point B', and the liquid level.

The change of pressures at C_1 and C_2, in relation to the change of level with respect to B', is linear (the proof is shown in the appendix). However, care must be exercised such that the tube is not submerged too deep, then linearity no longer holds.

The pressures at C_1 and C_2 may either be amplified by use of a proportional fluidic device or used directly to switch an AND gate to give an AND output (as was done in the controller defined herein). The bubble tube level sensing method can be used only in vented tanks.

2. The operation of the fluidic controller

Closing the control valve (level too high)

As the bubble tube is immersed by increasing level, the pressure at the end of the bubble tube (pressure increases with depth) increases. Since the fluidic restrictors R_1 and R_2 are different in size, the pressures at C_1 and C_2 are different. Also since R_1 R_2, the pressure at C_1 is always the greater. When the pressures at C_1 and C_2 both exceed the switching pressure of AND gate No. 1, it switches as indicated in Figure 6. When AND gate No. 1 switches, it in turn switches the flip-flop gate, which in turn causes the fluidic interface valve to receive a control signal. The interface valve, upon receiving the control signal, switches and supplies a preset pressure to the positioner-actuator causing it to close down the control valve to a minimum flow position. During the same series of events NOR gates No. 1 and No. 2 are both switched to their OR output ports, therefore there are no inputs to the control ports of AND gate No. 2.

Opening the control valve (level too low)

As the level decreases, the pressures at C_1 and C_2 both decrease.

Before the higher of the two, i.e., pressure at C_1, reaches the switching pressure, NOR gate No. 2 switches to its NOR output due to the insufficient pressure at C_2. By the time the pressure at C_1 reaches the switching pressure both NOR gates No. 1 and No. 2 have switched to their NOR outputs. The presence of these NOR outputs at the control ports of AND gate No. 2 cause it to switch and give an AND output. This causes the flip-flop to switch to its vented output. Now the fluidic interface valve closes and pressure is no longer supplied to the positioner-actuator. The positioner-actuator then opens the control valve until it reaches a preset stop which limits maximum control valve opening.

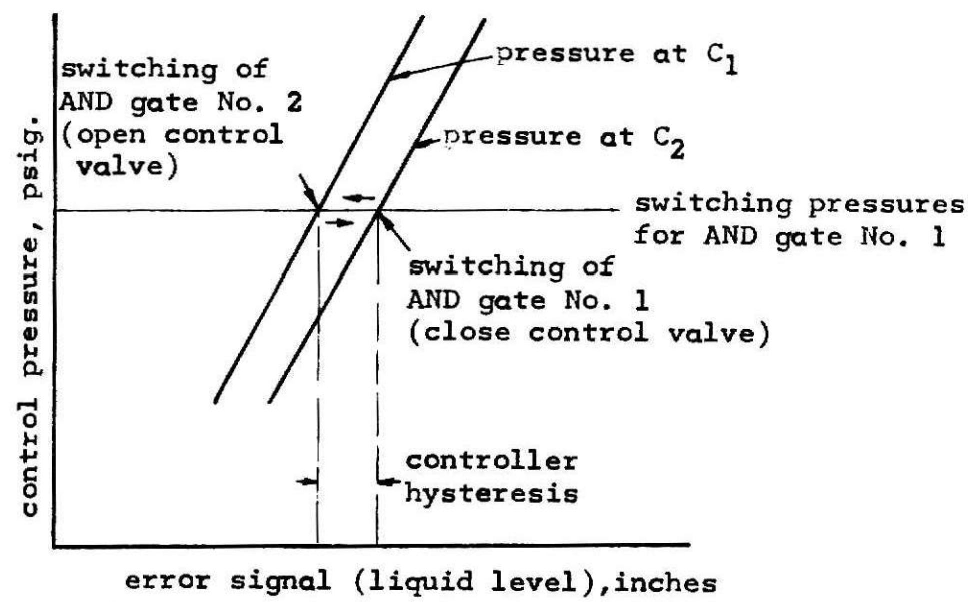


Fig. 6.* Switching of control circuit vs. liquid level.

*This graph is based on equations A-4a and A-4b derived in the appendix.

The Positioner-Actuator

The positioner-actuator for the control valve is a Fisher type 3560 positioner combined with a type 657 size 30 actuator. The input to the device is a pressure signal whose normal range is 3 to 15 psig. The output is a stroking range of 3/4 inches. The transfer function supplied by the manufacturer is:

$$F(s) = \frac{K_a}{\left(\frac{s}{\omega_a}\right)^2 + \left(\frac{2\zeta s}{\omega_a}\right) + 1} \qquad 2\text{-}11$$

where:

$K_a = 0.0625$ in./psi.

$\omega_a = 37.7$ rps

$\zeta = 0.5$

Control valve

The control valve is a Fisher, 1-1/4 inch size, design "A", single port valve with a linear characteristic. Its flow is defined by the manufacturer as:

$$Q = 22.7\ x\ (\Delta p)^{\frac{1}{2}}$$

where:

Q = flow, GPM

x = valve travel from the seat, inches. ($x_{max.} = 0.75$ inches)

Δp = pressure drop across valve, psi.

Outlet valve

The outlet valve is a Fisher, 2 inch size, design "G", single port valve. Its flow is defined by the manufacturer as:

$$Q = C_v(y)(\Delta p)^{\frac{1}{2}}$$

where:

Q = flow, GPM

y = valve travel from seat, inches. ($y_{max.}$ = 0.75 inches)

$C_v(y)$= valve sizing coefficient, GPM/ psi.$^{\frac{1}{2}}$
$C_v(0.565) = 28.6$

CHAPTER III

THEORETICAL ANALYSIS OF CLOSED LOOP SYSTEM

Describing function analysis

The describing function method, originally developed by Goldfarb and Kochenburger, is a frequency response technique for analyzing certain simple nonlinear systems and is useful in predicting the limit cycle behavior of such systems.

Systems which satisfy the following assumptions and considerations can be analyzed by the describing function method.

1. The system contains nonlinear elements n which can be separated from linear elements g. A typical form of such a system is shown in Figure7.

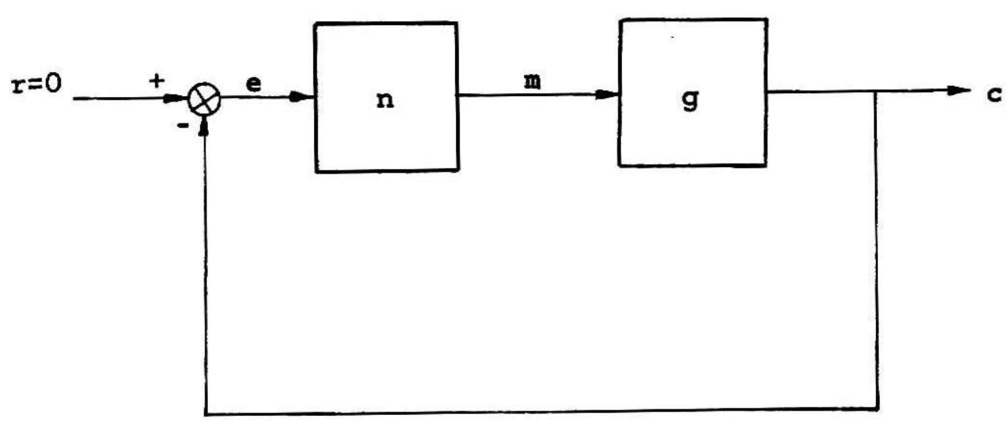


Fig. 7. Block diagram of a typical nonlinear control system.

2. The system is autonomous (i.e.,unforced and time invariant)

3. The linear transfer contains sufficient low-pass filtering to warrant excluding from consideration the higher harmonics in the output of n.

4. The input to the nonlinear element is assumed to be sinusoidal. The output of the nonlinear element m is a periodic function with zero D.C. level and can be represented by a Fourier Series.

The describing function is defined for nonlinearity n as

$$N=\frac{\text{Fundamental component of output from Fourier analysis}}{\text{Amplitude of the sinusoidal input signal}}$$

The system which is the subject of this report can be represented by the following generalized block diagram

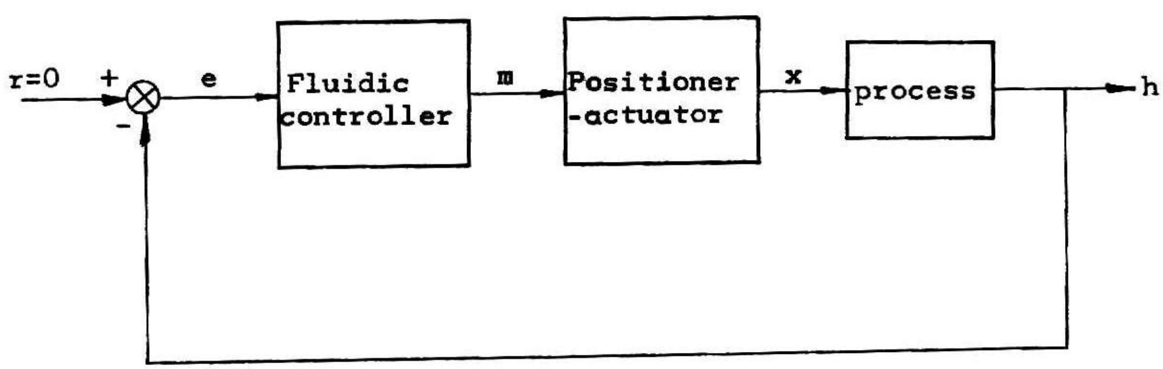
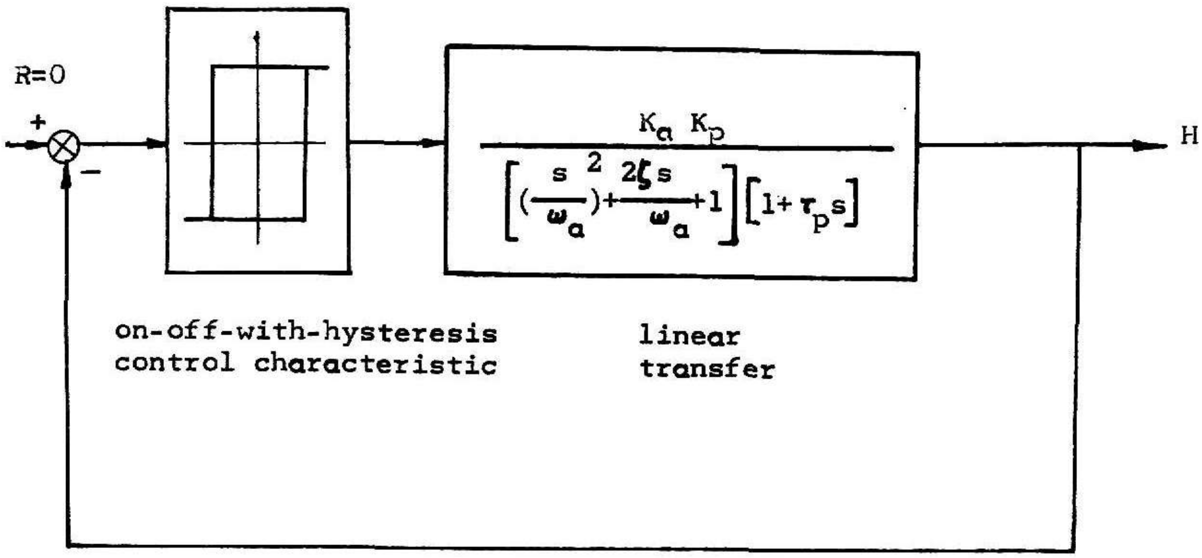


Fig. 8. Generalized block diagram for liquid level control system.

Transforming the linear components of the system from the time domain to the s domain and combining the linear transfer into one block, Figure 8 becomes



note: The time domain variables h(t) and m(t) must be restricted to small changes for linear transfer characteristic to exist.

Fig. 9. Block diagram for liquid level control system studied.

Note that the block diagram in Figure 9 is identical to the block diagram in Figure 7.

With two position on-off control the input pressure to the positioner-actuator is such that the control valve is either at its most open position or at its most closed position, varying about a steady state operating point. Thus the water rate is either a larger positive constant or a smaller positive constant. As shown in Figure 10, the water level in the tank continuously moves between the two limits of the controller differential gap causing the control valve to move from one fixed position to the other. The output curve follows one of two exponential-like curves, one corresponding to a "filling" curve and the other to an "emptying" curve. Such output oscillation between two limits is a typical response characteristic of a system under two position on-off control. This output oscillation, which is also the input to the nonlinear element, will be approximately sinusoidal if the linear elements are a good low-pass filter.

Since the natural frequency of the actuator, given by the manufacturer, is so high the predominant part of the linear transfer of the system is the first order lag of the process which has a long time constant. Thus the linear element will attenuate higher harmonics considerably and is therefore a good low-pass filter.

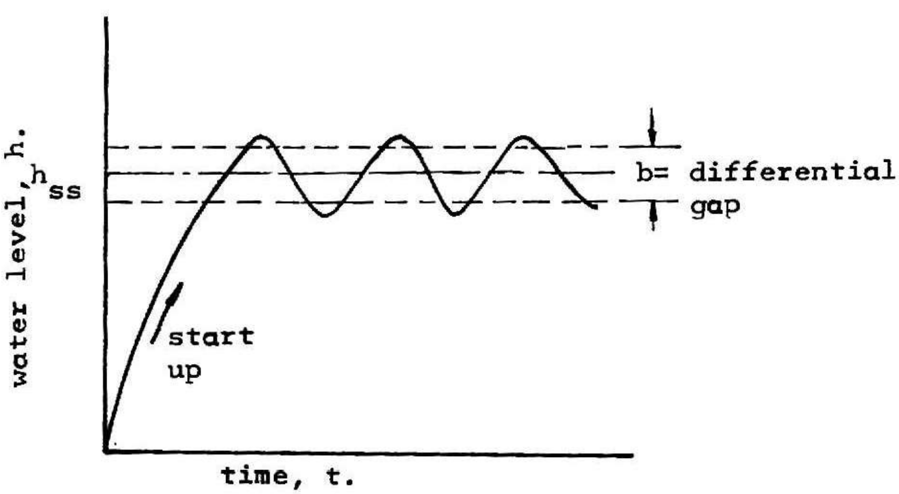


Fig. 10. The system output vs. time.

Since the system has no disturbances and is autonomous, all of the assumptions and considerations for application of the describing function method of analysis are apparently satisfied.

The describing function (from reference 2) for an on-off controller with hysteresis is

$$N = \frac{4M}{\pi E} \angle \sin^{-1}\frac{b}{2E} \qquad 3\text{-}1$$

where:

N = describing function of the nonlinear element n.

E = amplitude of the input sinusoid, inches.

M = output of controller, psig.

b = magnitude of hysteresis, inches. (differential gap)

Limit cycle analysis

The describing function (equation 3-1) obtained for the on-off with hysteresis nonlinearity can be used to determine the stability and, therefore, the limit cycle behavior of the system. The configuration of Figure 7 can be used to illustrate the technique of stability analysis.

The stability of this system is a function of the open loop gain, i.e. the gain from a given point (such as the input of n) around the loop and back to the same point. If the nonlinear element can be adequately characterized by the describing function N, the loop gain is simply $-NG(j\omega)$, where N depends only on the amplitude of the sinusoidal input to n (since b and M are fixed). Thus the study of stability involves an investigation of the zeros of $1+NG(j\omega)$, i.e.

the values of frequency and amplitude which satisfy the equation

$$1 + NG(j\omega) = 0$$

or

$$-\frac{1}{N} = G(j\omega)$$

this further implies

$$\left|-\frac{1}{N}\right| = \left|G(j\omega)\right| \qquad \text{3-2a}$$

$$\angle -\frac{1}{N} = \angle G(j\omega) \qquad \text{3-2b}$$

The stability is investigated by a modification of the Nyquist diagram. Two loci are drawn in the complex plane: the plot of the imaginary versus the real part of $G(j\omega)$, and the variation of $-1/N$ with the amplitude of the assumed sinusoid input to the nonlinear element n. The former is the frequency response locus, the latter the amplitude locus. An illustration of an arbitrary example of the modified Nyquist diagram is shown in Figure 11.

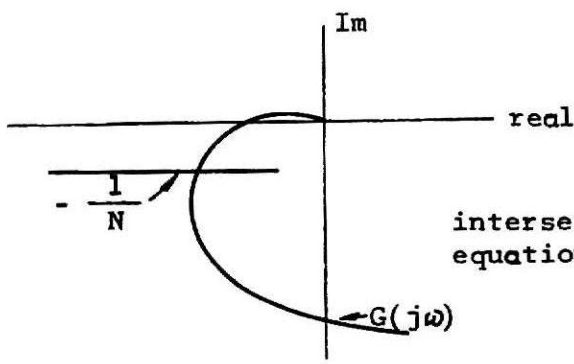


Fig. 11. Arbitrary example of modified Nyquist diagram.

Based on the manufacturer's data for the positioner-actuator the $G(j\omega)$ locus was plotted as shown in Figure 12. Since $\angle N = \angle -\sin b/2E$ and $E=b/2$, this implies that $0^\circ \geq \angle N \geq -90^\circ$ or $180^\circ \leq \angle -1/N \leq 270^\circ$ so the first possibility for $G(j\omega)$ locus and $-1/N$ locus to have an intersection is at $\angle G(j\omega)=-90^\circ$ where $|G(j\omega)| = 0.11$. Thus, from equation 3-2a, $-1/N = 0.11$ at $\angle -1/N = -90^\circ$ this leads to $b/M = 0.28$. So for $b=0.69$ inches, $M=2.46$ psi., $E=0.345$ inches and $=1$ rps. Thus a limit cycle of amplitude 0.345 inches and frequency 1 rps. is predicted. There are many possibilities for limit cycles of higher frequencies with different b/M ratios for $\angle G(j\omega)$ more negative than -90 .

To determine whether the theoretically predicted limit cycle of the system is stable or not it is assumed the $-1/N$ locus intersects the $G(j\omega)$ locus at point A_i (as shown in Figure 12), which has a frequency of no less than 1 rps., and a slight perturbation displaces the operating point slightly to the left of A . This represents an increase in the amplitude of limit cycle oscillation. However it also represents a decrease in the magnitude of N thus $|NG|$ is less than unity. Since the system is then stable, in the Nyquist sense, the amplitude of oscillation decreases until the system once more operates at the equilibrium point. A similar argument will show that a perturbation to the right on $-1/N$ will cause the system to become more unstable, which will encourage the amplitude of oscillation to increase, returning the system to its equilibrium point. Thus A_i is a convergent equilibrium point and the limit cycle of the system is stable.

However during preliminary testing of the complete system it was found that a limit cycle did exist for $b/M=0.64$ and $M=1.06$ psi. at

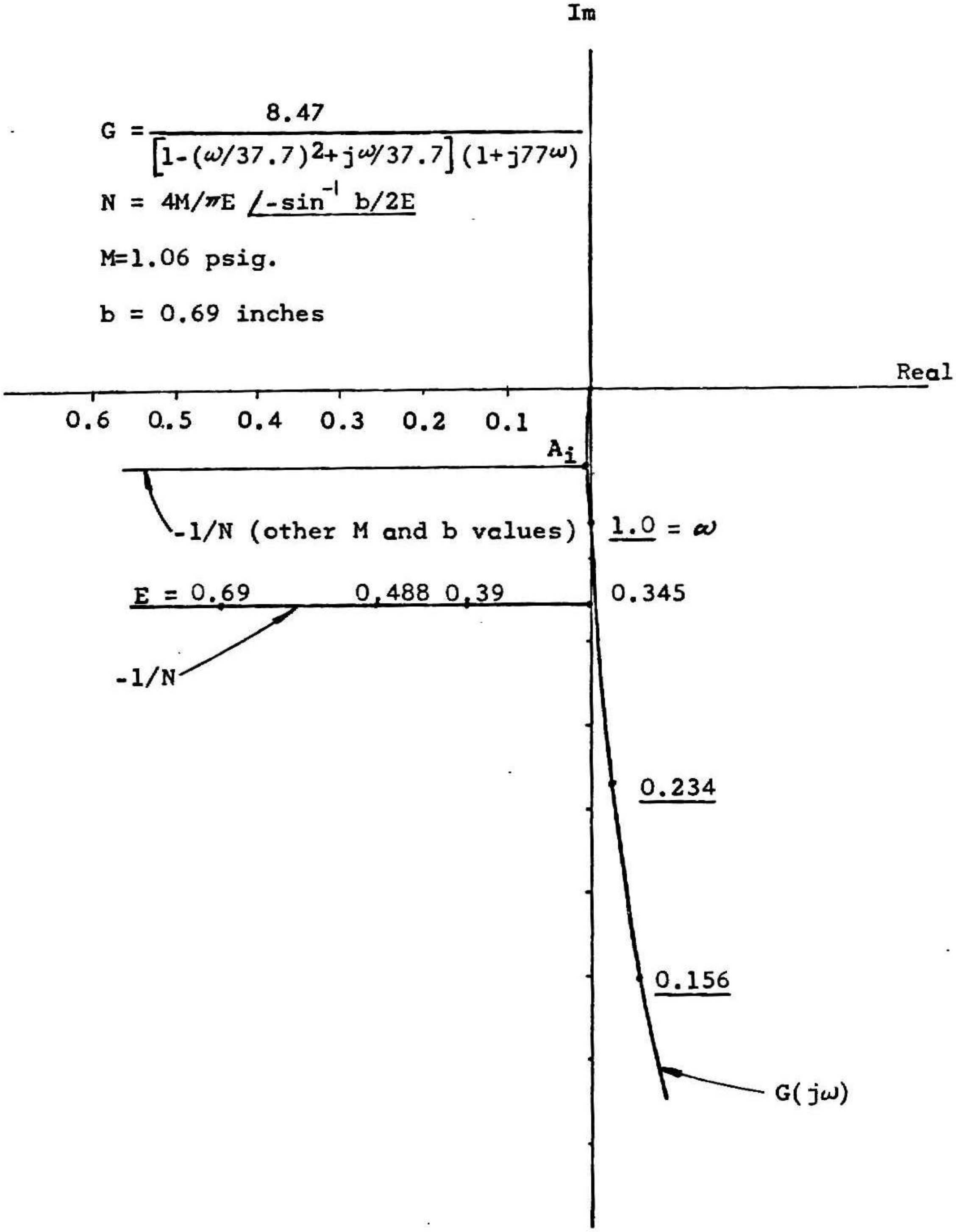


Fig. 12. Modified Nyquist plot for the system based on the manufacturer's data for the positioner-actuator.

ω=0.31 rps.. Since this frequency is less than half the minimum frequency predicted theoretically it was decided to experimentally verify the transfer functions for both the process and the positioner-actuator.

Experimental results indicted that the time constant for the process is τ_p=83 seconds and the transfer function for the positioner-actuator is:

$$F_{A1} = \frac{K_a e^{-Ts}}{1+\tau_{A1}s} \quad \text{, (valve closing)} \qquad 3\text{-}3a$$

$$F_{A2} = \frac{K_a}{1+\tau_{A2}s} \quad \text{, (valve opening)} \qquad 3\text{-}3b$$

where:

K_a= 0.0625 in./psi.

T = 1.2 seconds

$\tau_{A1} \simeq$ 0.66 seconds

$\tau_{A2} \simeq$ 0.11 seconds

Thus the positioner-actuator actually possesses different transfer functions for its two different paths of operation: valve closing and valve opening. Therefore one can not really apply the describing function method except in an attempt to get "ballpark" figures for amplitude and frequency of limit cycle. This was done first using $G_1(j\omega)$, the frequency response function for valve closing (which includes F_{A1}), and then using $G_2(j\omega)$, the frequency response function for valve opening (which includes F_{A2}).

Figure 13 shows the modified Nyquist diagram for the two cases just mentioned. The $-1/N$ locus, with b approximately equal to 0.69 inches and M=1.06 psi., intersects the $G_1(j\omega)$ locus at point A where $E \approx 0.42$ inches, $\omega \approx 0.33$ rps. The intersection with $G_2(j\omega)$ locus is at point B where $E \approx 0.346$ inches, $\omega \approx 0.39$ rps. It was hoped that these two points might indicate the upper and lower boundaries of actual limit cycle performance.

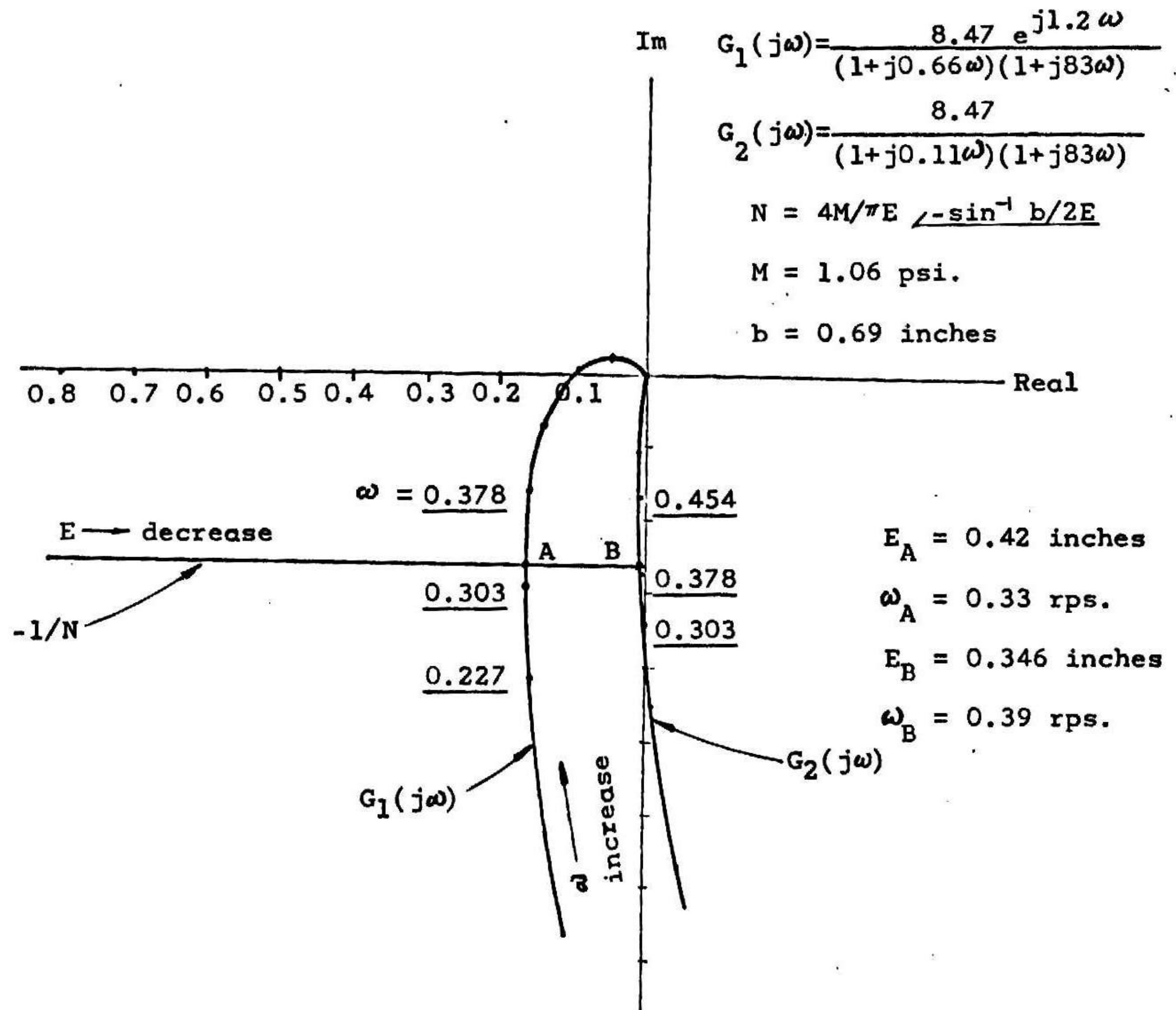


Fig. 13. Modified Nyquist plot for the system based on the tested result for the positioner-actuator.

CHAPTER IV

EXPERIMENTAL METHODS AND RESULTS

A list of instruments used in obtaining the experimental results presented in this chapter is included in the appendix.

Measurement of control valve travel

By applying a D.C. voltage across a linear potentiometer fixed to the movable control valve stem, the control valve stem travel could be detected and measured. The set up was as shown in Figure 14.

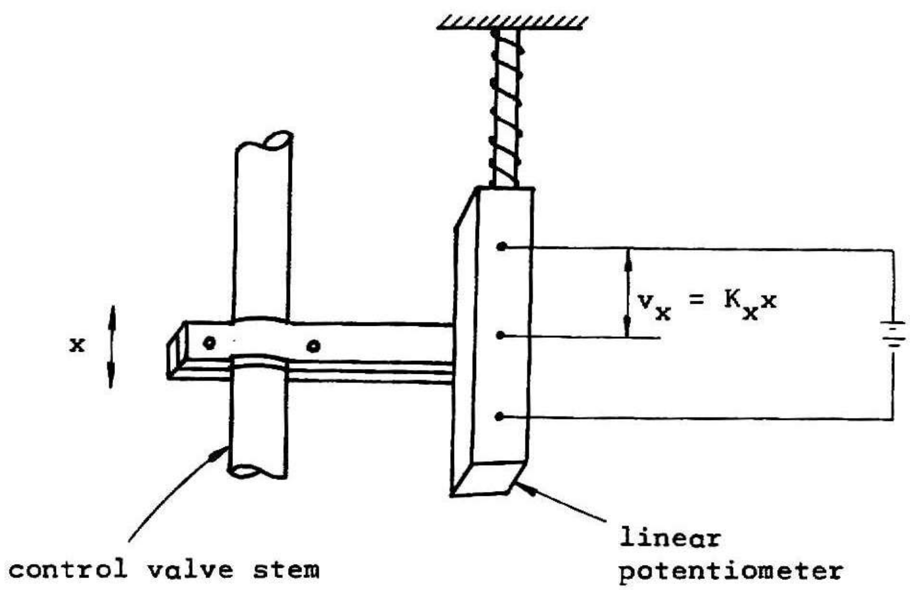


Fig. 14. Measurement of control valve travel with a linear potentiometer.

The change of the valve stem travel in inches was determined from the change of the voltage indicated by the voltmeter divided by a calibration factor K_x.

During dynamic testing the voltage output from the potentiometer was fed into a strip chart recorder and recorded.

Measurement of the water level

Referring to Figure 15, the change of water level was detected by a wooden float which in turn caused the pulley on a rotary potentiometer to turn. When a D.C. voltage was applied across the potentiometer, the change of voltage indicated by the voltmeter, after dividing by a calibration factor K_h, gave the change of the water level in inches.

During dynamic testing the voltage signal from the potentiometer was fed into a strip chart recorder to be recorded.

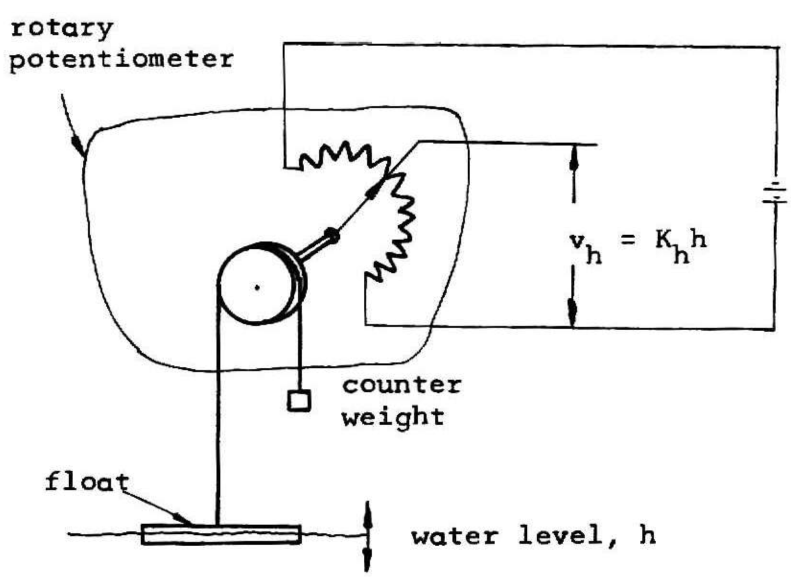


Fig. 15. Set up for measurement of the water level.

Adjusting system to steady state operation point

To adjust the system to the specified steady state operating point the following procedure was used:

1. adjust the supply water pressure ahead of control valve to be at 4.5 psig.
2. adjust the supply air pressure to the fluidic interface valve so that the control valve will allow 49 in.^{3}/sec. of flow.
3. adjust the outlet valve, 0.565 inches open, so that the steady state water level was stationed at 20 inches.

Measurement of process time constant

When the process was at its steady operation point the outlet valve was suddenly opened such that the water level would descend approximately one inch. The time required for the level to reach 63.2 % of its total change was the experimental process time constant.

Since the change of the water level versus time could be recorded by a strip chart recorder, the time constant could be easily determined and was found to be approximately 83 seconds. This is 8 % greater than the value of 77 seconds obtained by calculation.

Measuring the response of the control valve positioner-actuator

This was found necessary because use of the frequency response function for the positioner-actuator, given by the manufacturer, and the theoretical frequency response function for the process, in the describing function analysis, predicted limit cycle behavior which did not fit actual preliminary system performance as previously noted in Chapter III.

An approximate step pressure input (generated by supplying an input control signal to the fluidic interface valve) was introduced to obtain the transient response of the positioner-actuator. The step pressure change was such that control valve travel was only 0.143 inches. The input pressure, converted by a pressure transducer into a D.C. electrical signal, was fed into one channel of a strip chart recorder, while the control valve travel (detected by the linear potentiometer) was simultaneously fed into the other channel of the recorder. The response curve associated with valve closing, Figure 16, showed there was a transportation lag followed by an approximately 0.66 seconds. (see equation 3-3a for transfer function). On the other hand the response curve associated with valve opening, Figure 17, showed only an approximate first order exponential type growth with a time constant of approximately 0.11 seconds (see equation 3-3b for transfer function). Thus the response of the positioner-actuator was not symmetric as anticipated. Furthermore neither of the responses is that predicted by the manufacturer's transfer function, equation 2-11.

<u>Measurement of controller characteristics</u>

The nonlinear controller (on-off-with-hysteresis), as previously noted in Chapter II, includes the bubble tube level sensing element, fluidic NOR and AND gates and the fluidic interface valve. The change of the water level is the input while the pressure output of the interface valve is considered the output of the controller. Both input and output, after being converted into electrical signals as already described, were fed into an X-Y plotter with the level change as the

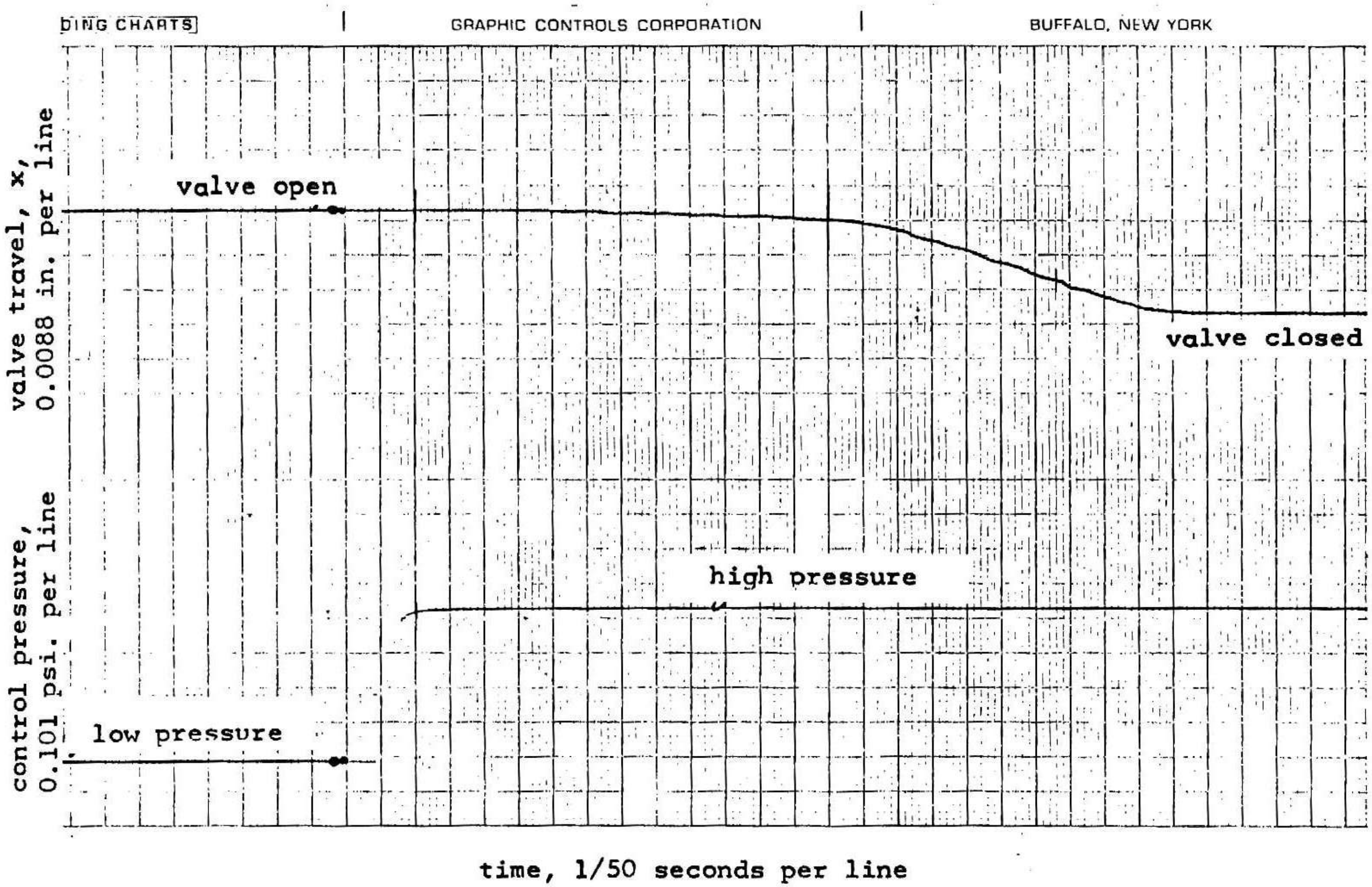


Fig. 16. Response curve for the Fisher positioner-actuator (valve closing).

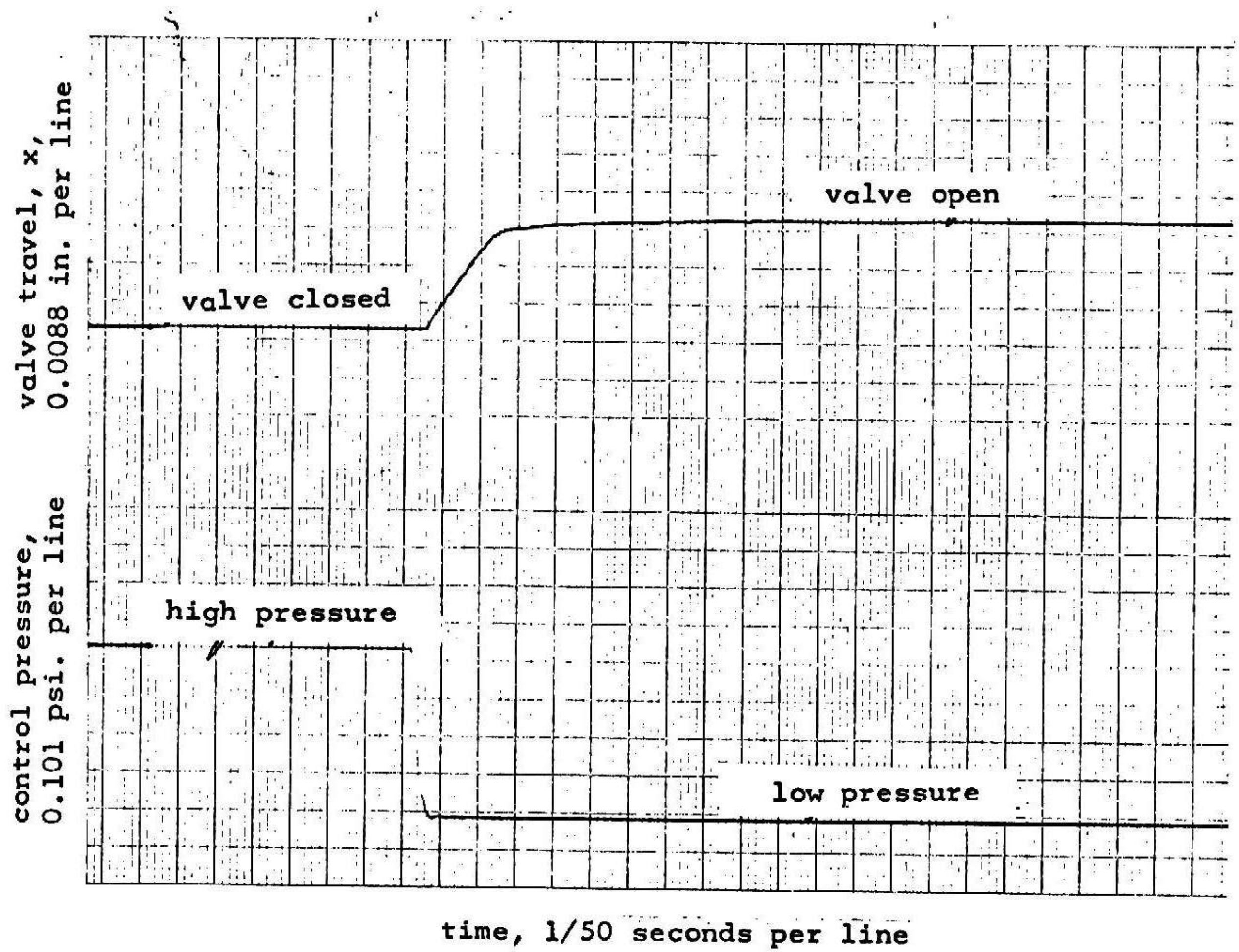


Fig. 17. Response curve for the Fisher positioner-actuator (valve opening).

independent variable. Both one cycle and several cycles of operation were recorded as shown in Figures 18 and 19 respectively. In Figure 18 the controller exhibits a reasonably good on-off-with-hysteresis control characteristic. However, Figure 19 shows the repeatability of the controller is very poor, at both upper and lower switch points, over several cycles of operation. This poor repeatability is more serious at the upper switch point as Figure 19 reveals and for some cycles this variation in switching is almost 50 % of the average differential gap.

Also a fact revealed from both figures is that the water level, while on its increasing path, keeps increasing even after the upper switch point, where the control pressure changes from minimum to maximum of its two extremities, is attained. After an overshoot of approximately 0.12 inches it then starts to decrease. On the other hand, the water level has little or no overshoot while on its decreasing path.

Measurement of complete system response

Figures 20 and 21 show the complete system response curve for one cycle and several cycles of limit cycle operation respectively. Shown in the upper half of both figures is the trace representing control valve travel (for maximum opening and minimum opening). The controlled variable (water level), shown as the lower trace in each figure, varies in a fashion which appears more like a trianglar wave rather than the sinusoidal wave assumed for describing function analysis. Both at the top and bottom of the water level response curve, where the level reverses direction, exist flat spots due to the effect

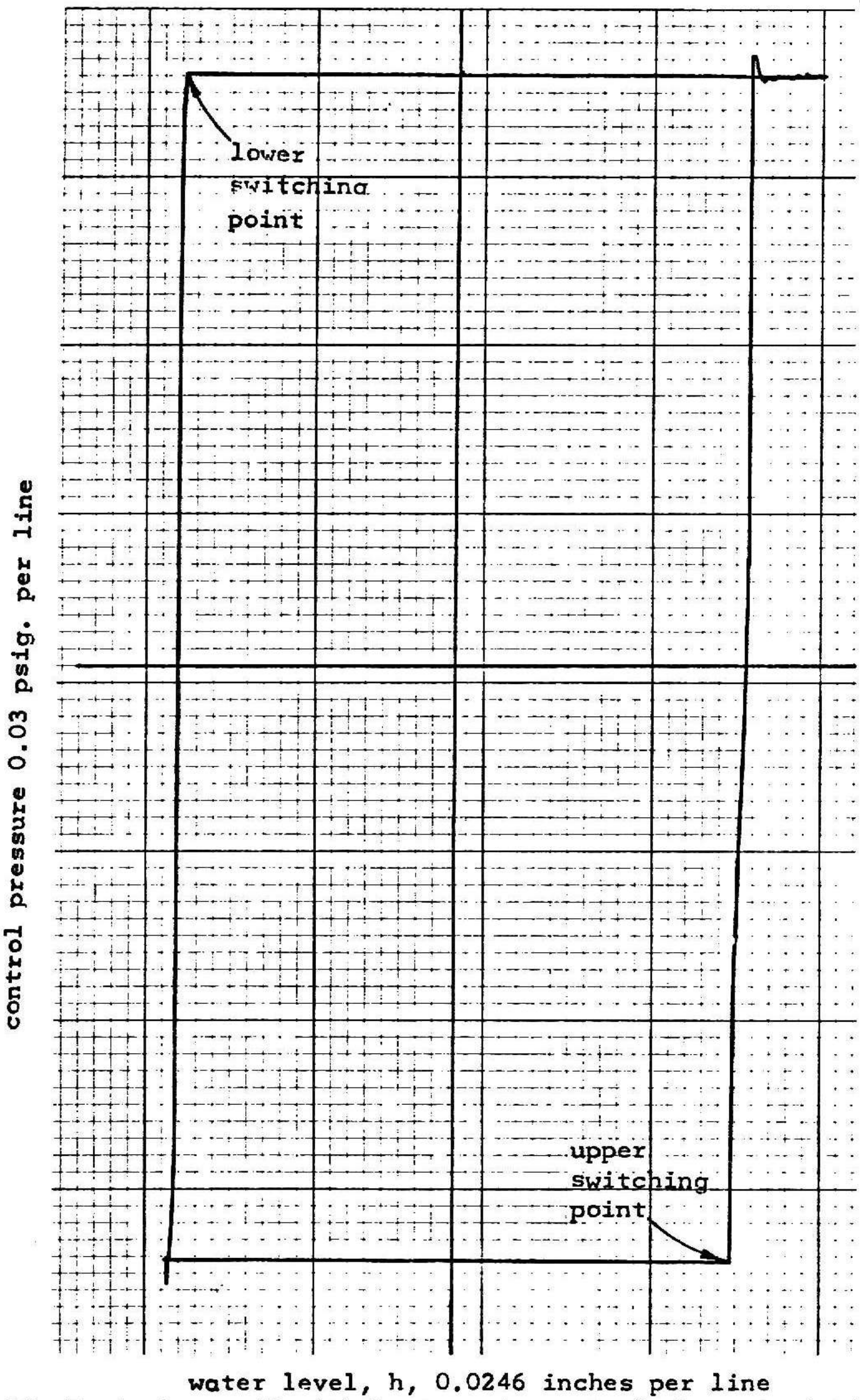


Fig. 18. Typical on-off-with-hysteresis control characteristic for the nonlinear fluidic controller (one cycle of operation).

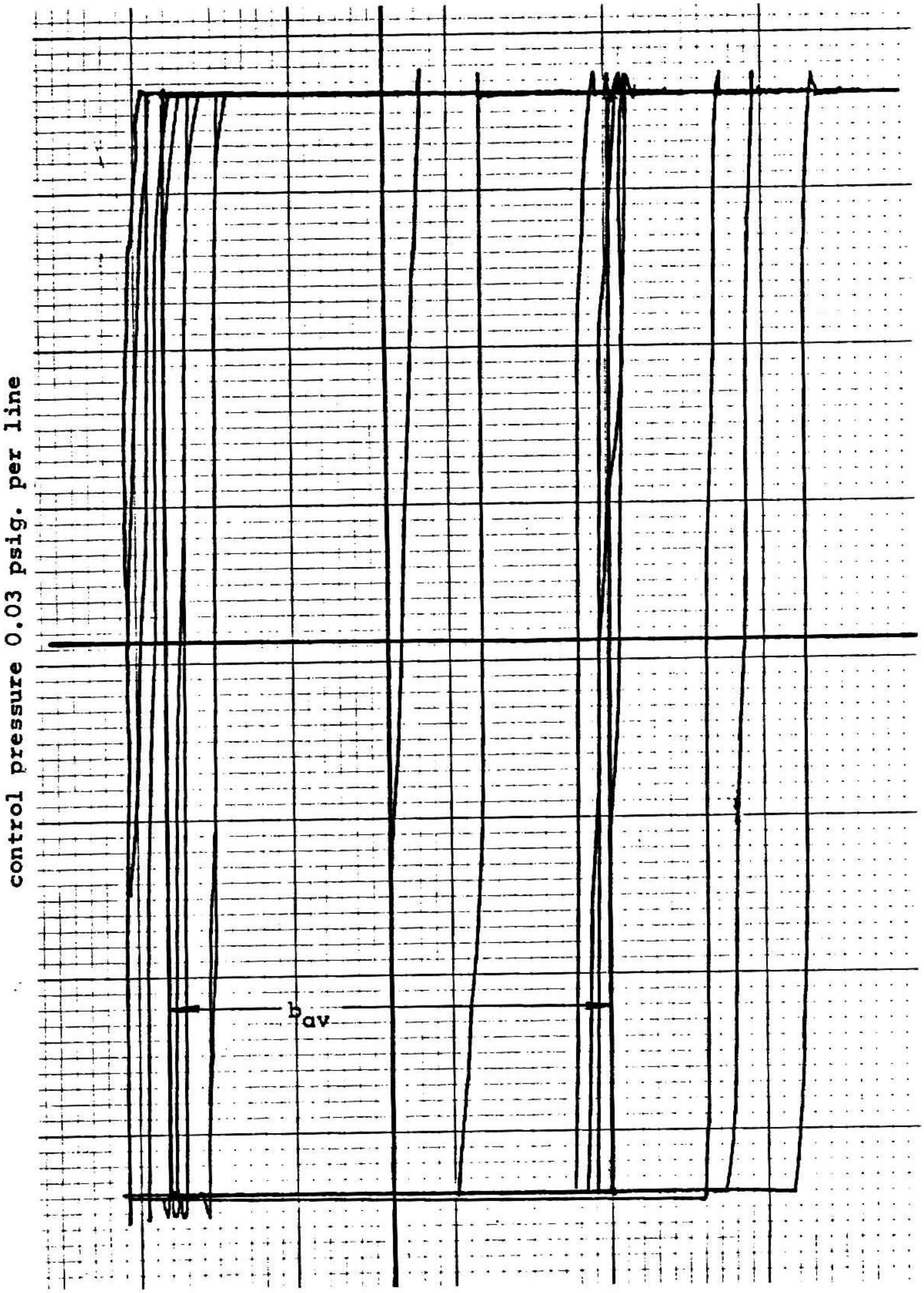


Fig. 19. On-off-with-hysteresis control characteristic for fluidic controller (several cycles of operation).

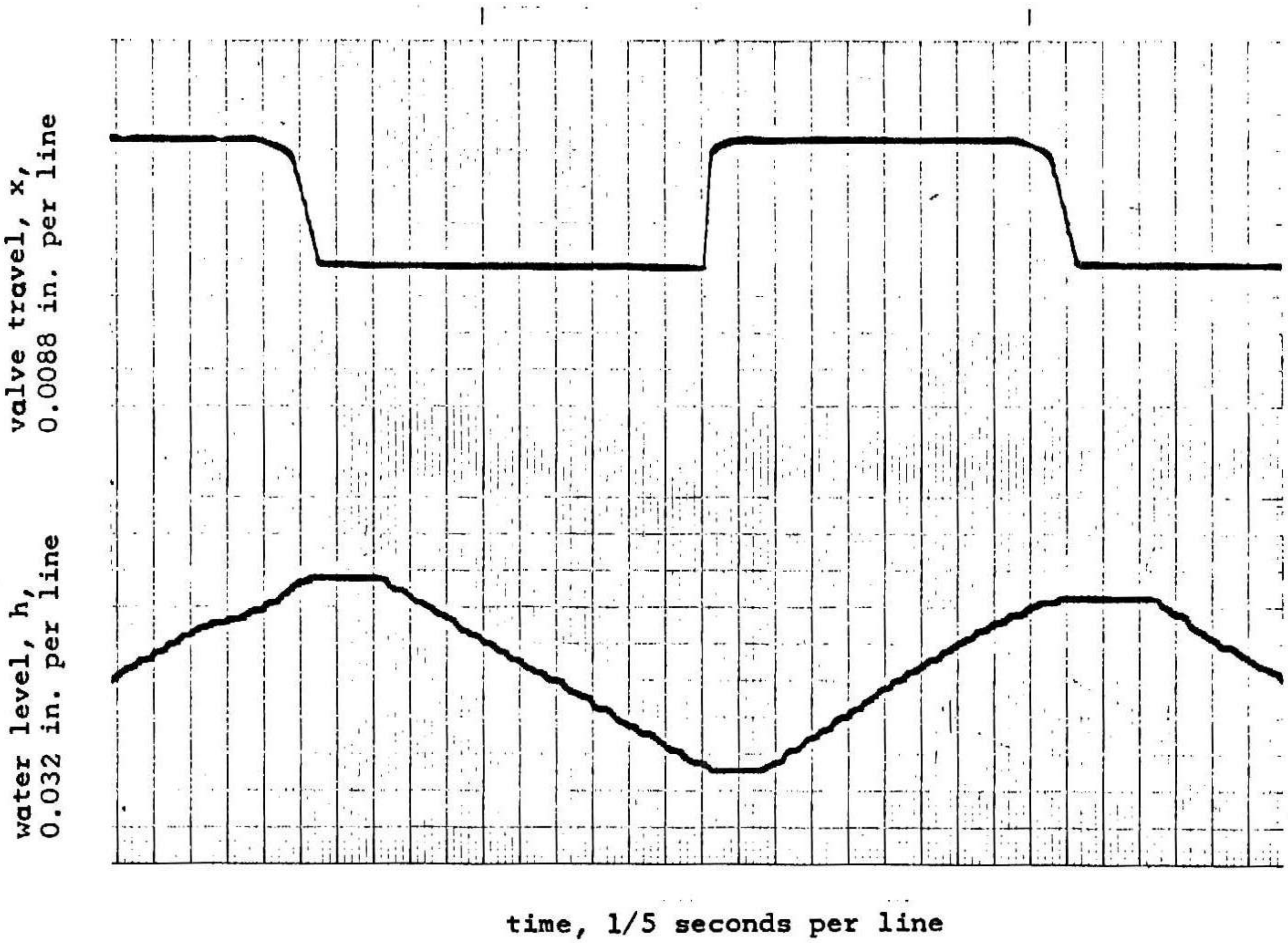


Fig. 20. Complete system response - water level control system (one cycle of operation).

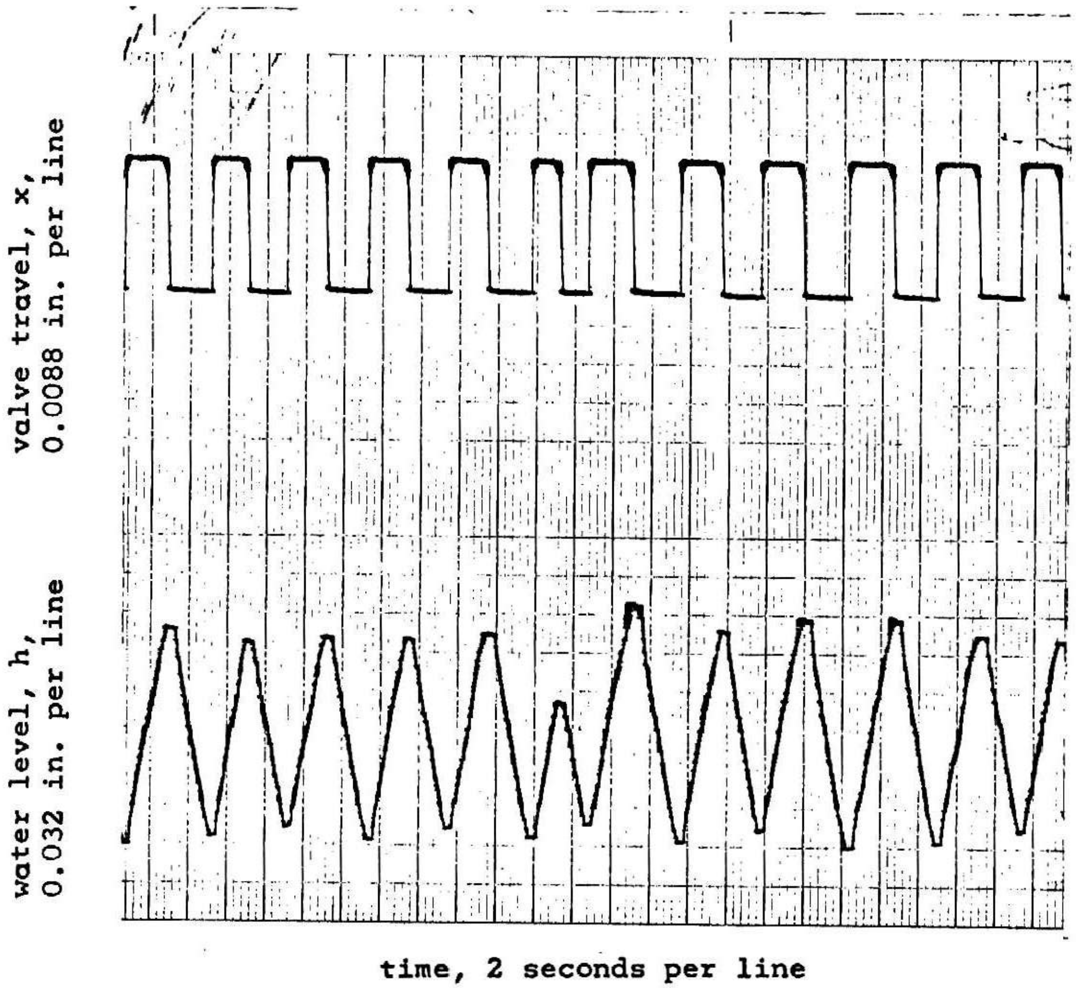


Fig. 21. Complete system response water level control system (several cycle of operation).

of friction in the rotary potentiometer. Figure 21 shows that the amplitude of the water level is usually different from cycle to cycle. This is due to the poor repeatability of the controller. The limit cycle of the system was experimentally determined from one cycle of operation (Figure 20) for b=0.7 inches, M=1.06 psi., as E=o.4 inches and ω=0.3 rps. or from several cycles of operation (Figure 21) for b_{av}=0.69 inches, M=1.06 psi., as E=0.375 inches and ω=0.31 rps.

CHAPTER V

DISCUDDION OF RESULTS

The experimentally determined limit cycle with $\omega \approx 0.3$ rps. and $E \approx 0.4$ inches (evaluated from one cycle of operation), or $\omega \approx 0.31$ rps. and $E \approx 0.375$ inches (evaluated from several cycles of operation) does not quite agree with the attempt made to predict theoretical figures for limit cycle behavior using G=G and G=G . These predictions were $\omega \approx 0.33$ rps. $E \approx 0.42$ inches, or $\omega \approx 0.39$ rps. $E \approx 0.346$ inches.

This disagreement (approximately 10 % for both frequency and amplitude E) may be due to fluctuations in water supply pressure (which was assumed to be constant at 4.5 psig. in the theoretical analysis, but which actually varied from approximately 4.0 psig. to 5.0 psig. while system was operating) and errors from evaluating the response time constants and the transportation lag of the positioner-actuator and the b balue which was taken approximately from the average of values of some several cycles.

The undesirable phenomenon, poor repeatability of the controller, did affect the magnitude of the differential gap and, therefore, did cause the frequency and amplitude E of the limit cycle to change from cycle to cycle of operation. This change for some cycles is approximately 25 % both in frequency and amplitude if Figure 21 is studied carefully. No study of this phenomenon was attempted therefore exactly why it occurred cannot be answered. However some possible reasons are as follows:

1. Hysteresis effects which are inherent in all fluidic elements.
2. Non-uniformity in switching pressures for the fluidic elements used.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. The limit cycle of the system was experimentally determined from one cycle of operation for M=1.06 psi., b=0.7 inches as E=0.4 inches and ω=0.3 rps. or from several cycles of operation for b=0.69 inches, M=1.06 psi. as E=0.375 inches and ω=0.31 rps.
2. Theoretical analysis can predict amplitude and frequency for the limit cycle for this water level control system within approximately 10 %.
3. The control operation of this system is in general satisfactory except that limit cycle frequency and amplitude tends to vary from cycle to cycle.
4. The positioner-actuator (used in this system) has different transfer functions depending on the path of operation i.e,, valve closing or valve opening.
5. Because of the poor repeatability of the nonlinear fluidic controller, both frequency and amplitude of the limit cycle change from cycle to cycle. For some cycles the deviation from the average value is 25 % for both frequency and amplitude.

Recommendations

If future research is done with Fluidic Liquid Level Control Systems, it is recommended that a well established commercially avai- fluidic controller be used, or that second or third generation fluidic elements be used.

ACKNOWLEDGEMENT

The author wishes to express a deepest appreciation to Dr. Ralph O. Turnquist, major professor, for his suggestion of this area of study and his guidance in the preparation of this report.

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APPENDIX

NOMENCLATURE

Symbol	Description
a	Distance from centerline of control valve to end of bubble tube, inches
A, B	Intersections between G's and -1/N loci. Figure 13.
A_i	Possible intersection for G and -1/N loci. Figure 12.
A_1, A_2, B_1, B_2	Constant associated with bubble tube sensor.
A_t	tank area, in.^{2}
b	differential gap, inches.
b_{av}	average differential gap, inches.
c	Controlled variable in a typical control system.
C_1, C_2	Inlet ports to AND gate No. 1.
C_v	Liquid sizing coefficient, GPM/psi$^{1/2}$
e	Error in control system.
E	Amplitude of the sinusoidal input, inches
$F_{A1}, (F_{A2})$	Transfer function for the positioner-actuator associated with valve closing (valve opening)
F_p	Process transfer function
g	Linear elements in a control system.
G	Transfer function for the linear elements in a control system.
$G_1, (G_2)$	Transfer function for the linear elements associated with the valve closing process (valve opening process).
$G_{pa}, (H_{pa})$	Feedforward (feedback) transfer function in positioner-actuator control loop.
h_{ss}	Steady state water level, inches.
h	water level, inches.
H	Transfer function for water level.

Symbol	Description
K_p	Process gain, in./in.
K_a	Actuator gain, in./psi.
m	output of the nonlinear element, psig.
M	maximum output of the nonlinear element, psig.
n	Nonlinear element in a control system.
N	Describing function for nonlinear elements.
P	Pressure in bubble tube, psig.
P_{c1}, P_{c2}	Pressure at control port of AND gate or NOR gate, psig.
s	Water pressure supply, psig.
P_{sb}	Bubble tube sensor supply pressure, psig.
q	liquid flow, $in.^3$/sec.
q_{ss}	Steady state liquid flow, $in.^3$/sec.
$Q_b, Q_{sb}, Q_{c1}, Q_{R1}, Q_{R2}$	Flow in bubble tube sensor, $in.^3$/sec.
r	Input signal to control system.
R_1, R_2, R_c, R_s	Linear flow resistance,
T	Time lag, seconds.
x	Control valve stem travel, inches
y	Outlet valve stem travel, inches
γ	Specific weight of water, $lb_f/in.^3$
$\rho's$	Density of water in bubble tube sensor, $lb_f\text{-}sec^2/in.^4$
τ	Process time constant, seconds.
$\tau_{A1}, (\tau_{A2})$	Time constant for the positioner-actuator associated with valve closing process (valve opening process), seconds.
τ_p	Process time constant, seconds.

Symbol	Description
ω	Frequency, rad./sec.
ω_a	Natural frequency of positioner-actuator, rad./sec.
ζ	Damping ratio.

<u>The linearity of bubble tube level sensing</u>

Figure 22 shows a schematic of a bubble tube sensor. The pressures P_{c1} and P_{c2} in relation to the change of level, with respect to B', is linear. The proof is as follows:

By conservation of mass, steady flow

$$\rho_{sb}Q_{sb} = \rho_{R1}Q_{R1} + \rho_{R2}\,Q_{R2} + \rho Q_b$$

assume constant ρ due to supply pressure less than 1 psig.

$$Q_{sb} = Q_{R1} + Q_{R2} + Q_b$$

since the Honeywell restrictors have linear flow-pressure characteristics

$$\frac{P_{sb}-P}{R_s} = \frac{P-P_{c1}}{R_1} + \frac{P-P_{c2}}{R_2} + \frac{P-\gamma(h-a)}{R}$$

or

$$P = \left[\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R}\right]^{-1}\left[\frac{P_s}{R_s} + \frac{P_{c1}}{R_1} + \frac{P_{c2}}{R_2} + \frac{\gamma(h-a)}{R}\right] \qquad \text{A-1}$$

again using conservation of mass, steady flow across R_1 and R_2

$$\rho_{R1}Q_{R1} = \rho_{c1}Q_{c1}\;; \quad \rho_{R2}Q_{R2} = \rho_{c2}Q_{c2}$$

where:

Q_{c1}, Q_{c2} = combined flow to control ports of one AND gate and one OR gate in parallel.

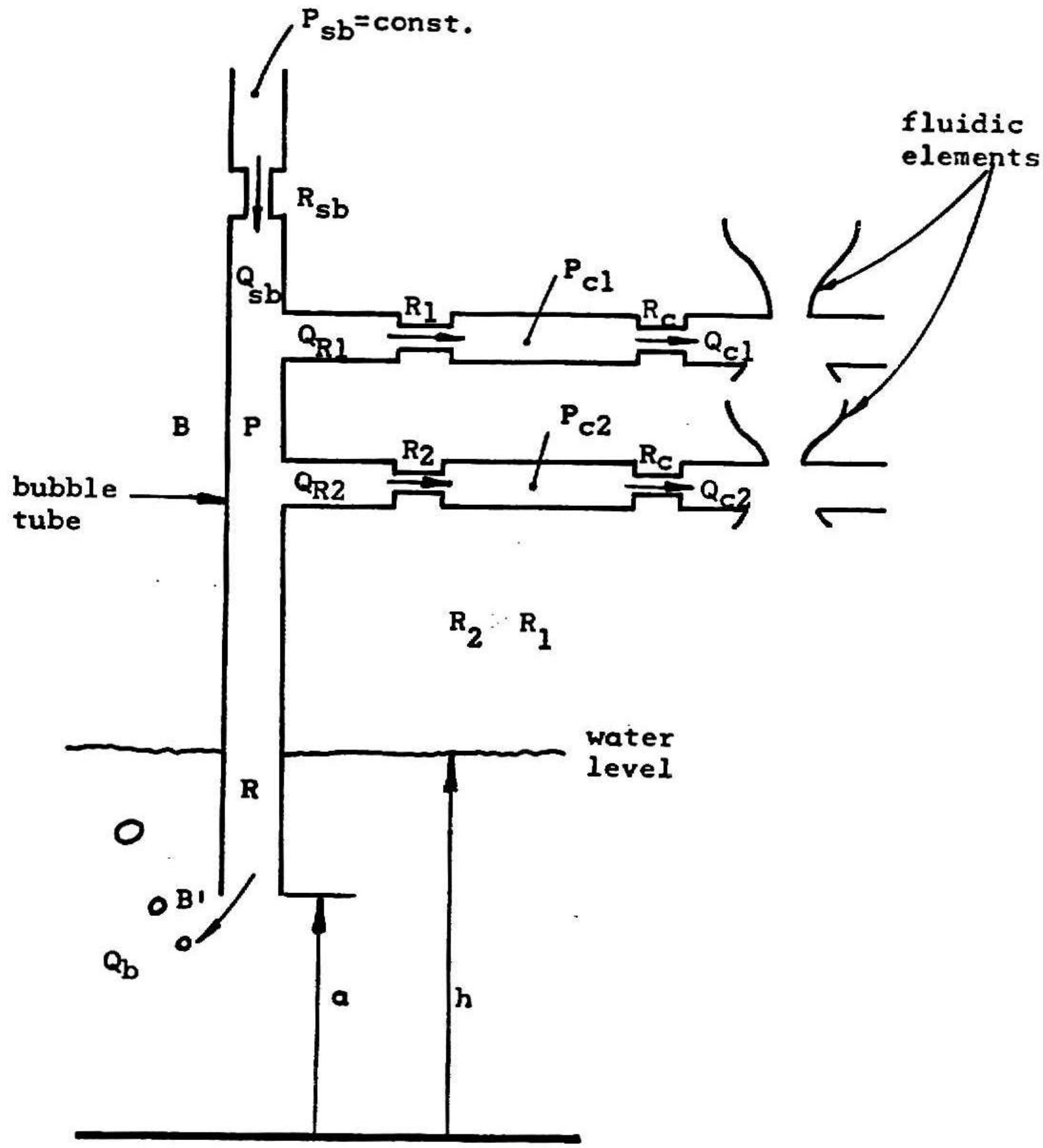

Fig. 22. Schematic of a bubble tube sensor.

assume constant , therefore

$$Q_{R1} = Q_{c1} \quad ; \quad Q_{R2} = Q_{c2}$$

since fluidic gate control ports have linear flow-pressure characteristic and the interaction region is at atmospheric pressure

$$\frac{P-P_{c1}}{R_1} = \frac{P_{c1}-0}{R_c} \quad ; \quad \frac{P-P_{c2}}{R_2} = \frac{P_{c2}-0}{R_c}$$

where:

R_c = combined resistance of control ports of one AND gate and one OR gate in parallel

or

$$P_{c1} = \left[\frac{1}{R_c} + \frac{1}{R_1}\right]^{-1} \frac{P}{R_1} \quad ; \quad P_{c2} = \left[\frac{1}{R_c} + \frac{1}{R_2}\right]^{-1} \frac{P}{R_2} \qquad \text{A-2}$$

substituting equation A-1 into A-2 and letting

$$K_1 = \left[\frac{1}{R_c} + \frac{1}{R_1}\right]^{-1} \frac{1}{R_1} \left[\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R}\right]^{-1}$$

$$K_2 = \left[\frac{1}{R_c} + \frac{1}{R_2}\right]^{-1} \frac{1}{R_2} \left[\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R}\right]^{-1}$$

then

$$P_{c1} = \left[1 - \frac{K_1}{R_1}\right]^{-1} K_1 \left[\frac{P_s}{R_s} + \frac{P_{c2}}{R_2} + \frac{\gamma(h-a)}{R}\right] \qquad \text{A-3a}$$

$$P_{c2} = \left[1- \frac{K_2}{R2}\right]^{-1} K_2 \left[\frac{P_s}{R_s} + \frac{P_{c1}}{R_1} + \frac{\tau(h-a)}{R}\right] \qquad \text{A-3b}$$

substituting equation A-3b into A-3a yields

$$P_{c1} = A_1 + B_1\ (h-a) \qquad \text{A-4a}$$

where:

$$A_1 = \left[1-(1- \frac{K_1}{R_1})^{-1}(1- \frac{K_2}{R_2})^{-1} K_1 K_2 R_1^{-1} R_2^{-1}\right]^{-1} (1- \frac{K_1}{R_1})^{-1} K_1 \left[(1- \frac{K_2}{R_2})^{-1} R_2^{-1} K_2 + 1\right] \frac{P_s b}{R_s}$$

$$B_1 = \left[1-(1- \frac{K_1}{R_1})^{-1}(1- \frac{K_2}{R_2})^{-1} K_1 K_2 R_1^{-1} R_2^{-1}\right]^{-1} (1- \frac{K_1}{R_1})^{-1} K_1 \left[(1- \frac{K_2}{R_2})^{-1} R_2^{-1} K_2 + 1\right] \frac{\tau}{R}$$

also substituting A-3a into A-3b, yields a similar results for P_{c2}:

$$P_{c2} = A_2 + B_2\ (h-a) \qquad \text{A-4b}$$

where

$$A_2 = \left[1-(1- \frac{K1}{R_1})^{-1}(1- \frac{K_2}{R_2})^{-1} K_1 K_2 R_1^{-1} R_2^{-1}\right]^{-1} (1- \frac{K_2}{R_2})^{-1} K_2 \left[(1- \frac{K_1}{R_1})^{-1} R_1^{-1} K_1 + 1\right] \frac{P_s b}{R_s}$$

$$B_2 = \left[1-(1- \frac{K_1}{R})^{-1}(1- \frac{K_2}{R})^{-1} K_1 K_2 R_1^{-1} R_2^{-1}\right]^{-1} (1- \frac{K_2}{R})^{-1} K_2 \left[(1- \frac{K_1}{R})^{-1} R_1^{-1} K_1 + 1\right] \frac{\tau}{R}$$

The intrumentation

The instruments used in the experiment include:

1. Sanborn Twin-Viso strip chart recorder model 60-1300
2. Honeywell X-Y recorder model 550
3. Pace Wiancko pressure transducer kit model KP15
4. Pace Wiancko transducer indicator model CD25
5. Hewlett Packard 427A voltmeter
6. KEPCO DC power supply model JQE-36-3M
7. Heathkit battary eliminator model BE-5
8. Bourns linear potentiometer model 108
9. BORG rotary potentiometer model 901 S 590 60
10. Fisher and Porter Co. rotameter
11. Bourdon tube pressure gage

ON-OFF LIQUID LEVEL CONTROL USING FLUIDIC CONTROL SYSTEM

by

FANG-LUNG LIAO

Diploma, Taipei Institute of Technology, 1967

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

The describing function analysis technique was employed to analyze the performance of a liquid level control system which used a fluidic on-off-with-hysteresis type controller. The performance was also verified experimentally.

The characteristics of the fluidic controller, the transfer function of the positioner-actuator (for the control valve), the liquid level process time constant and the complete system response were all determined experimentally in this report.

The repeatability of the controller (assembled by two mechanical engineering seniors at Kansas State University as a laboratory project) was found very poor and the positioner-actuator for the control valve was found exhibiting nonsymmetric response characteristics (disagreeing with the data given by the manufacturer). The expected limit cycle of the system was determined both analytically and experimentally. However, because of the above two reasons, the frequency and the amplitude of the experimentally determined limit cycle changes from cycle to cycle of operation and the describing function analysis can only estimate a rough "ballpark" figure for the limit cycle of the system.

The limit cycle determined experimentally from several cycles of operation for b_{av}=0.69 inches and M=1.06 psig. was E=0.375 inches and ω=0.31 rps.. However, for some cycles, these values change from 0.26 inches to 0.51 inches for amplitude E and from 0.26 rps. to 0.39 rps. for frequency ω.