

A METHOD OF PROJECT SELECTION FOR THE PRIVATE FIRM

by

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CHAPTER I

INTRODUCTION

1.0 Purpose

The "capital asset pricing model" is a linear mathematical model that relates the random rate of return on an individual security available in the market to the "risk" of the security. It was originally developed in the 1960's by Sharpe (17), Lintner (10), Mossin (14) and others as an extension of some of the earlier work by Markowitz (12) on the return from portfolios of investments. About 1970, it was discovered that the model can be applied to capital budgeting problems faced by the firm. Using this methodology, the rate of return for an individual project can be compared to the rate of return for the firm demanded at the risk level of the project. The model has not been applied to non-public firms; that is, those firms whose stock is not traded on a stock exchange. This thesis will develop a method of applying the model to the *non-public* firm. A numerical example is also used to illustrate the proposed method.

1.1 Problem

The problem to be investigated in this thesis is the application of capital asset "pricing" methodology to the capital budgeting, or project selection problem, faced by firms whose stocks are not actively traded on a stock market. The capital asset pricing model is the basis for a resource allocation method that can be applied to publicly owned companies, whose stock is actively traded on a public exchange.

Unfortunately, a large number of privately owned firms, some of sizable net worth, are precluded from using this valuable method of allocating scarce resources. The capital asset pricing model provides a viable resource allocation method for listed firms in dealing with risky investments. It appears, however, that this model can be applied to private, unlisted firms, and the purpose of this thesis is to extend the methodology of the capital asset pricing model so that it is a useful tool for closely held firms.

1.2 Literature Survey

Capital asset pricing theory had its beginning in 1952 when Harry Markowitz (12) first developed an analytical method for explicitly recognizing the uncertainty associated with the future returns on a portfolio of investments. Markowitz assumed that the variance of a security, which he demonstrated to be a measure of the "risk" or "uncertainty" of the security, included a correlational term to express the covariance of the security with all possible pairs of securities in a portfolio. He also suggested that the computational effort could be considerably reduced if one were to recognize only the covariance of the security with one particular market index. As extensions to Markowitz' work, Sharpe (17), Lintner (10), and Mossin (14) each formulated mathematical models which relate the risk premium of a given security to the risk premium required by the securities market itself. These models essentially followed Markowitz' suggestion, but differed in form from each other. Fama (4), however, showed that the three models were in fact essentially the same. Thus, there is now a single model that is generally accepted by authorities in the finance

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field to represent the capital market theory originally suggested by Markowitz.

The capital asset pricing model evolved from portfolio theory. Portfolio theory deals with securities and the stock market. The pricing model contains market parameters, and unlike many theoretical models, it is empirically testable by direct evidence. The capital asset pricing model, (CAPM), as understood and used in financial theory, is a simple, linear model of the form

$$E_i = R_f + \beta_i(E_M - R_f) \quad (1.1)$$

where E_i = expected return from security i

R_f = "risk-free" interest rate (generally taken to be a constant)

β_i = volatility of security i

E_M = expected return of the market portfolio.

It is especially useful and desirable because of its simplicity and capability of uniform application. All factors on the right-hand side of the equation are market-based parameters except the volatility constant, β_i , the relationship between the return of security i and the return of the market portfolio. β_i is the ratio of the covariance of the i th security and the market to the variance of the market portfolio. If the covariance term dominates the variance, the price of the security will possess a high volatility, fluctuating more widely than the market. If the variance of the market portfolio dominates, the security will possess a low volatility, its fluctuations less wide than the market. The risk-free interest rate, R_f , is often referred to as the "time value of money". It is approximated by short-term U. S. Treasury bills and is currently about 5½%. It remains fairly stable in the range 5-7%. It is assumed to be the same for all securities, as is of course, the expected

return on the market portfolio. The market portfolio is the portfolio which contains all securities available for purchase.

Standard & Poor, Value Line, and others, have compiled financial market information on virtually every major listed corporation, mutual fund, and public bond offering dating back for many years. With these data available, many authors have tested the Sharpe-Lintner-Mossin capital asset pricing model. Jensen has compiled summaries of most of the major empirical tests of the pricing model through 1972 in his comprehensive survey paper (7).

Most of the early tests were simply attempts to evaluate the performance of mutual funds using historical data. Tests by Sharpe (19) and Jensen (8) demonstrated the potential of the model by indicating a positive correlation between the return rates for mutual funds and the return rate of the market, as represented by a market index.

Since the model showed promise, many direct tests have been made later. These direct tests were of a cross-sectional nature. The returns of a cross-section of securities were regressed against the covariances of each security with a market index over a specified time period. The results of these tests have been conflicting and there is still a controversy over whether this is the fault of the model or of the data used in the tests. Some say that the model has too many assumptions and is over-simplified and that capital asset pricing theory would be better represented by a two-factor or four-factor coefficient model instead of the one-factor model (1) (5). In any event, this controversy is not of consequence in this thesis, since our purpose here is to demonstrate a capital allocation methodology, given that a capital market line exists.

Applications of the pricing model to capital budgeting and project selection problems have been less extensively reported in the literature. Tuttle and Litzenberger (22) proposed a risk-equalized cost of capital method of applying pricing methodology to capital budgeting. This method utilizes the device of leverage via borrowing or lending to equalize the risk inherent in individual projects with the current risk position of the firm.

Logue and Merville (11) report a detailed study of the characteristics of the volatility coefficient (β) of a firm, and propose methods of calculating an approximate β which can be used in the CAPM. They show that the β of a firm can be expressed as a function of the variables that represent the financial, marketing, production, and corporate policies of that firm. However, since all policies are effectively "encapsulated" by the financial policies of the firm, they confine their attention to that portion. The functional form used for calculations allows for the effects of liquidity, leverage, dividends and several other financial parameters.

Weston (25) uses the capital asset pricing method as a project selection criterion and contrasts it with the weighted average cost of capital (WACC) method, which is frequently used in industry today. He notes the possibility that the firm can arrive at conflicting decisions concerning the acceptance of projects when the WACC and CAPM methods are used to evaluate investment alternatives. He then develops another "asset pricing" decision criterion for the selection of projects which differs somewhat from that used by Tuttle and Litzenberger. This difference is only of a minor nature, however. No conflicting decisions

between Weston's criterion and that of Tuttle and Litzenberger will result. The principal difference is that while Tuttle and Litzenberger equalize the "risk" involved by borrowing or lending (in order to prevent a theoretical change in the market price of the firm's stock), Weston does not equalize the risk of the project with that of the firm. Thus, under Weston's model the risk of the firm would continually change as new projects are accepted and the firm's return rate and risk parameters would require constant revision. The Tuttle and Litzenberger criterion would require changes over long time periods only, since the risk of the firm changes as the uncertain future affects the expected return of the firm. Weston also gives a numerical example of the application of pricing methodology to the project selection problem. In this example, he assumes different possible "states of the world", their associated probabilities of occurrence, and then calculates expected returns and variances of returns for the individual projects under consideration by the firm. The calculated project parameters are then compared with the characteristic return and variance of that firm. By this comparison, an investment decision for the acceptance or rejection of a project can be made.

1.3 Thesis Organization

This thesis contains four chapters. This, the first chapter, is the introduction and contains the purpose of the thesis, the problem to be dealt with, and a brief survey of the more important literature concerning the topic of capital asset pricing theory and its application to capital budgeting. The second chapter contains development of the theoretical basis for applying capital asset pricing theory to the

project selection problem. It includes definitions of the required terminology and the fundamental assumptions of capital asset pricing theory. The concepts of indifference curves and the market portfolio are discussed. The concepts of the "capital market line" and the "security line" are introduced. The "company" security line is established as the project selection criterion. Lastly, an explanation of the linear regression analysis used to establish the "company" security line from empirical data, will be introduced. Chapter III applies capital asset pricing theory to project selection, introducing the reward-to-variability and reward-to-volatility criteria. The application is then extended to include a method of project selection for the private firm by using a "surrogate" security, a combination of the firm's publicly owned competitors. The brewing industry is selected for illustration. Four prospective projects are considered for analysis. The results of the reward-to-volatility criterion are compared with the results using the weighted average cost of capital method. A statistical analysis and discussion of the results is then included. The final chapter is the conclusion of the thesis. The significant results of the thesis are discussed and areas for possible further study are cited.

CHAPTER II

THE CAPITAL ASSET PRICING MODEL

✓ 2.0 Introduction ✓

The problem of how to cope with the "risk" associated with capital expenditures has continually puzzled and harassed business decision makers. For many years, companies routinely have used decision models for investigating investment opportunities. These models have often assumed that the return on investment, by whatever measure, is known with certainty. But models using this assumption do not fully describe the "price" of an asset that is actually paid by the firm. When analyzing a prospective capital asset for purchase, management wants to know not only what it can expect in the way of an outcome (a return on the asset), but it also wants an idea of the probable divergence of outcomes. This is the "risk" involved in the purchase of the asset and is part of the "price" paid. It seems perfectly rational for a company, therefore, to demand a greater return on a project having a large range of possible returns than for a project on which the return seems fairly certain.

✓ 2.1 Current Economic Evaluation Methods

There are two principal methodologies for evaluating project alternatives (investment opportunities) that are recognized at the present time. These are based, respectively, on deterministic and utilitarian assumptions. The deterministic approach, which is by far the most common in use by industry today, has one major fault. This fault is that the deterministic approach is based on the obviously false assumption that

all future events are known with certainty. Under this approach all future cash flows, equipment lives, salvage values and all other estimated values are assumed to occur in the future exactly as predicted at the time the decision is made. There is no margin for error. Obviously this is a major drawback of the deterministic approach. The future cannot be known with certainty.

The utilitarian approach is one that is based on the assumption that a decision maker maximizes some function of the return -- say a "utility of money" function -- rather than simply the maximization of the return itself. The utilitarian approach is based on the axioms developed by Von Neumann and Morgenstern (15), and assumes that investors' choices can be expressed by preference-ordering functions which measure preferential utility instead of monetary value. But there are also problems with the utilitarian approach. Utility functions for decision makers are difficult to specify. They may vary widely in shape from individual to individual. Utility functions also change over time. As a person ages he often becomes more conservative in his thinking, causing a change in his perceptions of the utility of money. Another problem with the utilitarian approach is caused by the normal method of business decision making. Decisions regarding projects are often made in group meetings. There is good evidence to indicate that a "group utility function" is a theoretical impossibility (26). Thus, the utilitarian approach has some serious defects when one attempts to apply it in real-life decision making situations.

2.2 Assumptions of Capital Asset Pricing Theory

Some of the shortcomings of both the deterministic and utilitarian approaches can be avoided by application of "capital asset pricing" theory.

The foundations of capital asset pricing theory were laid in 1952 when Harry Markowitz (12) developed an analytic method for dealing with the divergence of returns on a portfolio of investments. From Markowitz' initial work, others in the field, principally Sharpe (17), Lintner (10), and Mossin (14), have derived equilibrium market-based models that require the simultaneous consideration of both the yield from and the risk connected with the purchase of a marketed security. One of the functions of the market is to adjust the "price" of an asset so as to reflect not only what is expected as a return from the asset, but also to account for any possible "riskiness" arising from holding the asset. Jensen (7) has synthesized most of the earlier work, and states that these asset-pricing models either explicitly or implicitly contain the following assumptions:

1. All investors are single-period expected utility-of-terminal-wealth maximizers who choose among alternative portfolios on the basis of the mean and standard deviation of return. This means that there is no compounding of any of the factors involved.
2. All investors can borrow or lend an unlimited amount of money at a given risk-free rate of interest with no restrictions on the short sales of any assets.
3. All investors have identical subjective estimates of the means, variances and covariances of return among all assets.
4. All assets are perfectly divisible and perfectly liquid. That is, any amount of an asset can be held and it can be sold and converted into money at will.
5. There are no taxes.
6. All investors are price takers. That is, there is no one investor who can purchase enough of an asset or have enough influence in the market to control or significantly affect the price of an asset.
7. The quantities of all assets are assumed given.

2.3 Portfolio Terminology

Capital asset pricing theory is based on the concept of the investor "holding", or purchasing, a portfolio of investments, the return from which is a random variable. The portfolio is assumed to have a mean return and a finite variance of return. By way of explanation, a portfolio is composed of stocks, bonds, or other assets, each of which is called a security. The return of a portfolio (e.g., in dollars) is just the increase (or decrease) in monetary value of the portfolio over one time period. Normally a "return" is expressed in monetary units (e.g., dollars); whereas, when one speaks of a return relative to an investment, the proper terminology shifts to the words "return rate", which implies a decimal or fractional form (dimensionless). For example, if a portfolio were purchased for \$100 and had a value of \$110 at the end of one period, its return would be \$10, whereas its return rate would be 10%. In general the return (either in monetary units or as a rate) may be calculated in this manner:

$$R_p = \sum_{i=1}^n X_i R_i \quad (2.1)$$

where R_p = return of the portfolio (dollars, or decimal rate)

R_i = return of security i (dollars, or decimal rate)

X_i = proportion of portfolio p held in security i in terms of dollar amount in the total portfolio (a decimal value); in general $\sum X_i = 1$.

Since the return of a portfolio is conceived of as a random variable, the mean or expected return of a portfolio can be calculated as follows:

$$E_p = \sum_{i=1}^n X_i E_i \quad (2.2)$$

where E_p = expected return rate of the portfolio per unit time

E_i = expected return rate of security i per unit time

X_i = proportion of portfolio p held in security i in terms of dollar amount in the total portfolio.

The variance of a portfolio return rate is not found by the usual means of adding variances of independent random variables. Instead, each security in a portfolio is assumed to be correlated with every other security in the portfolio, so the variance of the portfolio return rate is defined as

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n X_i X_j \rho_{ij} \sigma_i \sigma_j \quad (2.3)$$

where σ_p^2 = variance of return rate of portfolio p

X_i = proportion held in security i

X_j = proportion held in security j

ρ_{ij} = correlation of return rates of securities i and j ($0 \leq \rho_{ij} \leq 1$)

σ_i = standard deviation of return rate of security i

σ_j = standard deviation of return rate of security j .

2.4 Utility Theory

The variance of a return is taken to be a measure of the uncertainty or risk associated with owning a portfolio. There is an economic basis for this measure. The concept can easily be demonstrated by the use of utility theory. Consider a decision maker whose utility-of-money function is of the form

$$U(X) = A + BX - CX^2 \quad (2.4)$$

where X = a money return

$U(X)$ = utility of the money return

A, B, C = non-negative constants.

The shape of the function $U(X)$ on Cartesian coordinates of $(U(X), X)$ is concave downward. If the decision maker is one who maximizes expected utility instead of expected monetary value, then he will maximize the expectation of Equation (2.4) above, or

$$\text{MAX: } E(U(X)) = A + B \cdot E(X) - C \cdot E(X^2) \quad . \quad (2.5)$$

Now, $E(X)$ is simply the expectation of the return, or μ_X . However $E(X^2)$ is the second moment of the random variable X , so that we have

$$E(X^2) = V(X) + (E(X))^2 \quad , \quad (2.6)$$

and if we let $V(X) = \sigma_X^2$ = the variance of X , then Equation (2.6) becomes

$$E(X^2) = \sigma_X^2 + \mu_X^2 \quad (2.7)$$

and finally Equation (2.5) then becomes

$$E(U(X)) = A + B\mu_X - C(\sigma_X^2 + \mu_X^2) \quad . \quad (2.8)$$

It is then easy to show that the directional partial derivatives of Equation (2.8), respectively, are

$$\partial E(U(X)) / \partial \mu_X = B - 2C\mu_X \quad (2.9)$$

$$\partial E(U(X)) / \partial \sigma_X = -2C\sigma_X \quad . \quad (2.10)$$

Risk attitudes can be inferred from Equations (2.9) and (2.10).

For example, since B and C are taken to be both non-negative constants, and $\mu_X > 0$ is assumed (this is necessary for the project to be "attractive" to the decision maker), then the quantity $B - 2C\mu_X > 0$ in the range where $B > 2C\mu_X$. This is generally taken as the "valid" range of the parent utility Equation (2.4). Hence, in this range ($B > 2C\mu_X$) the function $E(U(X))$ slopes positively with respect to X , indicating an increase in expected utility with an increase in the project expected return.

Similarly, with respect to Equation (2.10), since both C and σ are non-negative constants, then $\partial E(U(X))/\partial \sigma_X < 0$ always, which indicates that the decision maker's expected utility decreases with an increase in σ_X . Such a decision maker is called a "risk avoider", since he suffers a reduction in expected utility if he makes an investment in which the standard deviation of return, σ_X (or the variance, σ_X^2) increases. Hence, σ_X (or σ_X^2) is taken as a measure of the decision-maker's exposure to risk for that particular project.

2.5 Indifference Curve Analysis

To predict which particular portfolio a typical risk-avoiding investor would logically select, the concept of expected utility is again invoked. A rational decision maker, who maximizes the expected utility of return, will do so in accordance with Equation (2.8). By setting $E(U(X))$ in Equation (2.8) equal to a constant and differentiating twice we have

$$E(U(X)) = A + B\mu_X - C(\sigma_X^2 + \mu_X^2) = \text{constant}$$

$$B(d\mu_X/d\mu_X) - 2C\sigma_X(d\sigma_X/d\mu_X) - 2C\mu_X(d\mu_X/d\mu_X) = 0$$

$$\text{or} \quad d\sigma_X/d\mu_X = (B - 2C\mu_X)/2C\sigma_X \quad (2.11)$$

$$\text{and} \quad d^2\sigma_X/d\mu_X^2 = (-4C\sigma_X^2 - (B - 2C\mu_X)2Cd\sigma_X/d\mu_X)/4C^2\sigma_X^2;$$

and upon substituting (2.11) we have

$$d^2\sigma_X/d\mu_X^2 = -\frac{1}{\sigma} - \frac{1}{\sigma}(d\sigma_X/d\mu_X)^2 \quad (2.12)$$

Now, for the valid region of Equation (2.8), we have that $B > 2C$ always, and since $\mu_X > 0$, the numerator of (2.11) is positive always. Also, the variance $\sigma_X^2 > 0$ always, so that $\sigma_X > 0$ is taken as necessary. Hence, (2.11) is always positive. By similar reasoning, and since $d\sigma_X/d\mu_X > 0$, then (2.12) is always negative. Thus the parent expected utility function $E(U(X)) = \text{constant}$, Equation (2.8), will plot on (σ_P, E_P) coordinates $(\sigma_P = \sigma_X, E_P = \mu_X)$ as an

upward sloping, concave downward curve; or on (E_p, σ_p) coordinates as an upward sloping ($d\mu_X, d\sigma_X > 0$), concave upward ($d^2\mu_X/d\sigma_X^2 > 0$) function.

Hence, the decision maker's "indifference" among (E_p, σ_p) combinations, is an upward sloping, concave upward function, as illustrated in Figure 2.1. Along a particular indifference curve, an investor is indifferent between one portfolio and another. However, expected utility increases as one moves from one indifference curve to another, that is, upward and to the left. (This is inferred from Equations (2.9) and (2.10)). Thus, as the expected return increases and the risk decreases, expected utility increases.

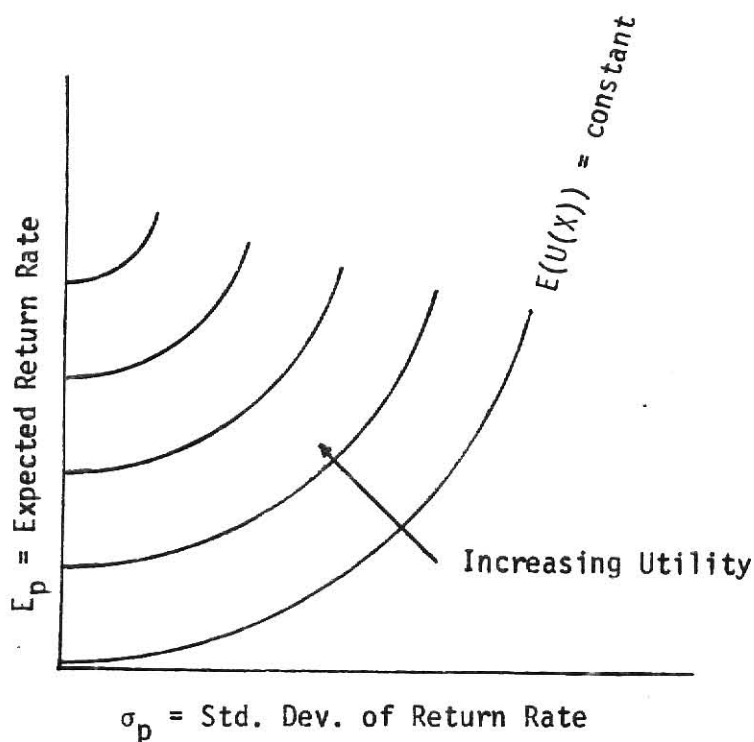


Figure 2.1. Typical decision maker's indifference curves

2.6 Dominance Among Securities and Portfolios: The Efficient Portfolio

Since a portfolio can be measured in terms of its mean and standard deviation, it can be plotted as a point on E_p, σ_p coordinates, as in Figure 2.2. In this figure, consider three points which describe the expected values and standard deviations of three possible portfolios, A, B, and C. In order to determine which of these points will be preferred by a typical decision maker over the other two, the investor will be assumed to be a risk avoider as above. Because he is assumed to maximize expected utility, he will choose between portfolios on the basis of the risk-avoiding indifference curves described previously. The choices can be summarized as follows:

1. If two portfolios have the same standard deviation of return, the one with the larger expected return is preferred by an investor who avoids risks, since he attaches greater expected utility to the portfolio with the higher return.
2. If two portfolios have the same expected return, the one with the smaller standard deviation of return is preferred, since the same investor will tend to avoid risk (have greater disutility for risk).

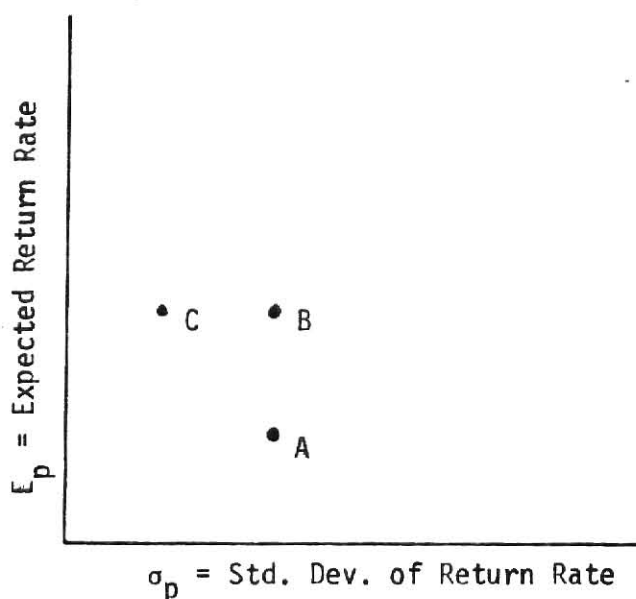


Figure 2.2. Graphical representation of three portfolios

Thus, in Figure 2.2, portfolio B is preferred to portfolio A since B has a greater expected return, even though the risk is equal. Hence, B is said to "dominate" A. Likewise, portfolio C would be preferred to B since, although the expected return is the same, its risk is less. Thus, C dominates B. Also the commutative property applies, so that if B dominates A and C dominates B, then C dominates A. In general, points lying upwards and to the left on E_p, σ_p coordinates will dominate portfolios represented by points lying downward and to the right.

The calculation of the standard deviation of a portfolio, using Equation (2.3), becomes a lengthy process as the number of securities in the portfolio becomes large. According to Equation (2.3) the risk of a portfolio is a function of the variances of each of its component securities and their covariances. Each security is affected to some extent by various factors, some of which are general, such as the state of the economy, and others that are specific, such as the particular earnings of each security. Also, securities tend to be affected by each other and this correlative relationship is measured by the correlation coefficients of the securities. The correlation coefficient, ρ_{jk} , is defined as

$$\rho_{jk} = \sum P(d_j, d_k)(d_j d_k) \quad (2.13)$$

where $d_j = (R_j - E_j)/\sigma_j$
 $d_k = (R_k - E_k)/\sigma_k$

$P(d_j, d_k)$ = probability that a pair of deviations d_j, d_k will occur. Note that d_j is simply the relative deviation of the actual random return R_j from the expected return, expressed in terms of the standard deviation for security j . The correlation coefficient is then taken as the sum of all the possible event pairs times their probabilities.

The covariance C_{jk} between two securities is defined in terms of the correlation coefficient as

$$C_{jk} = \rho_{jk}\sigma_j\sigma_k \quad . \quad (2.14)$$

The covariance is stated in terms of the square of the units used for the random variable (the return). Calculation of the standard deviation thus can become a tedious process when the correlation coefficients of each pair of securities in a large portfolio is required. For large portfolios it becomes necessary to find another method for calculating the standard deviation of portfolio return in a more economical fashion.

Markowitz proposed a simplifying method of calculating portfolio variance. To illustrate this method, consider a portfolio composed of only two securities, say securities 1 and 2. Let X_i be the proportion of each security in the portfolio, so that $\sum X_i = 1$, where $i = 1, 2$. Let the two securities have expected returns of E_1 and E_2 . The expected return of the portfolio then is

$$E_p = \sum X_i E_i = X_1 E_1 + X_2 E_2 \quad . \quad (2.15)$$

The variance of the return for the portfolio is

$$\begin{aligned} \sigma_p^2 &= \sum \sum X_i X_j \rho_{ij} \sigma_i \sigma_j \\ \sigma_p^2 &= X_1^2 \sigma_1^2 + 2X_1 X_2 \rho_{12} \sigma_1 \sigma_2 + X_2^2 \sigma_2^2 \quad . \end{aligned} \quad (2.16)$$

If the correlation coefficient $\rho_{jk} = +1$, indicating that the securities are perfectly and positively correlated, then the variance reduces to

$$\sigma_p^2 = (X_1 \sigma_1 + X_2 \sigma_2)^2 \quad (2.17)$$

$$\text{or} \quad \sigma_p = X_1 \sigma_1 + X_2 \sigma_2 \quad (2.18)$$

which is simply a linear relationship. Note also that the expected value of return, E_p , is given by Equation (2.15), so that as the values of X_1 and X_2 change, the standard deviation of the portfolio will change

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linearly with respect to the expected return. This is illustrated in Figure 2.3.

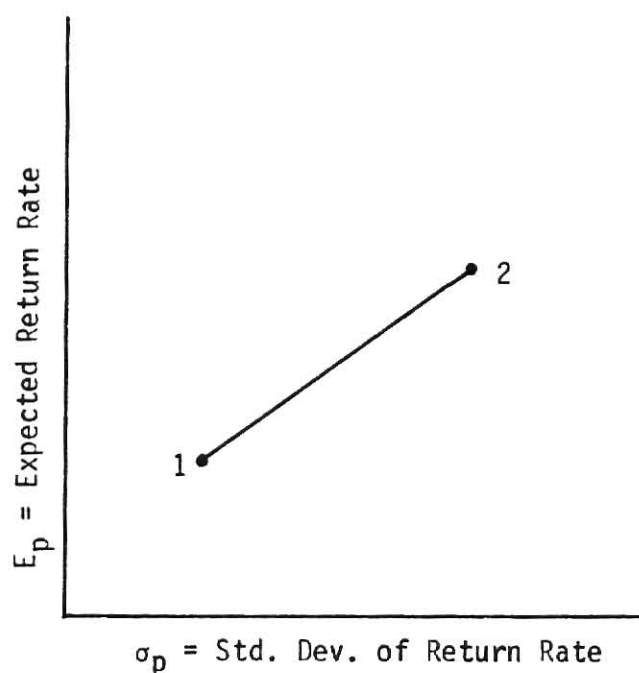
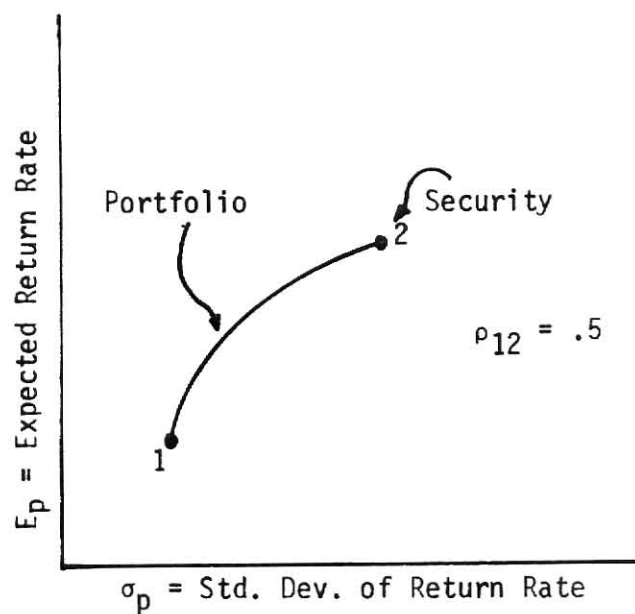
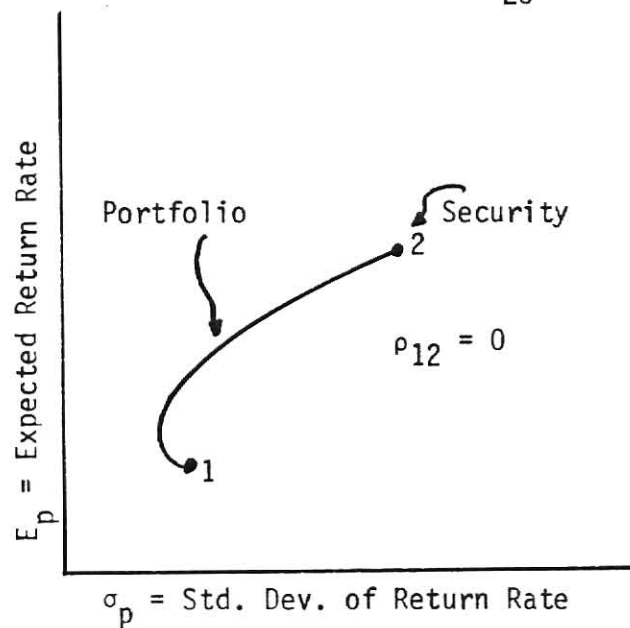


Figure 2.3. Linear combination of 2 perfectly correlated securities

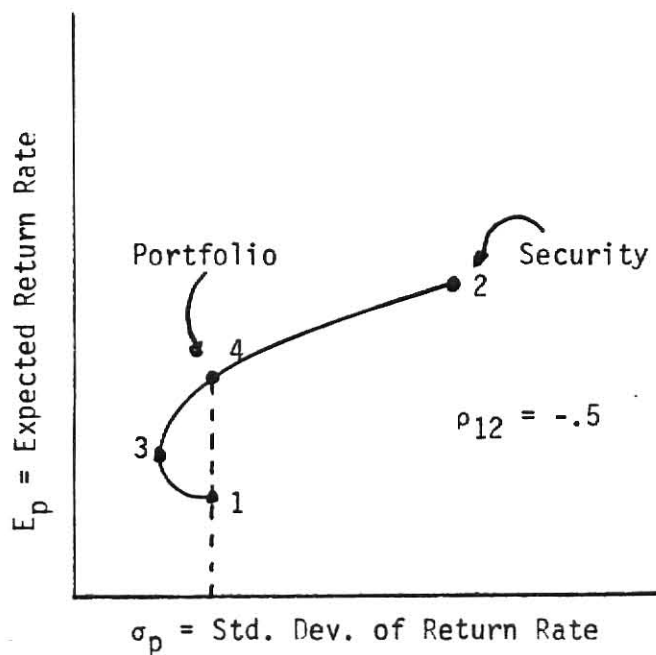
However this linear relationship only holds for the particular $\rho_{ij} = +1$. As the correlation coefficient becomes less than unity the relationship between σ_p and E_p becomes non-linear. This functional relationship will then look like the curves in Figure 2.4. Notice from Figure 2.4b and 2.4c that there is a portion of the curve in which there can be two expected returns for the same standard deviation. An important concept can be implied from this.



(a)



(b)



(c)

Figure 2.4. Combination of 2 securities with varying degrees of correlation

Consider two expected returns with the same risk (points 1, 4 in Figure 2.4c). The investor will always prefer the greater of the two expected values. The portfolio which results in the greater expected return will be referred to as an "efficient" portfolio. The other portfolio will be referred to as an "inefficient" portfolio.

The properties of an efficient portfolio need to be considered further, in order to develop a basis for understanding the concept of portfolio analysis. In Figure 2.4a, any point along the mean-standard deviation "mix" line between portfolios 1 and 2 can be obtained by considering the proportion of each security held. Depending on the values of X_1 and X_2 , any point on the line $\overline{12}$ comprises another portfolio. Also, a third security, say 3, could be added to the set, giving the relationship shown in Figure 2.5. Any point along the lines $\overline{13}$ and $\overline{23}$ can be obtained by the relationships given in Equations (2.15) and (2.18) above.

Now consider some portfolio B, which is a combination of securities 2 and 3, as illustrated in Figure 2.6. A mean-standard deviation locus will connect point B with point 1 also, thus indicating (by prior reasoning) that any point on this locus can be obtained by holding different proportions of security 1 and portfolio B. This idea can be extended until there is an infinity of portfolios among points 1, 2 and 3, which is represented by a region of portfolios bounded by the loci previously developed (see Figure 2.7). Hence, the investor is free to choose any point as a portfolio in this feasible region.

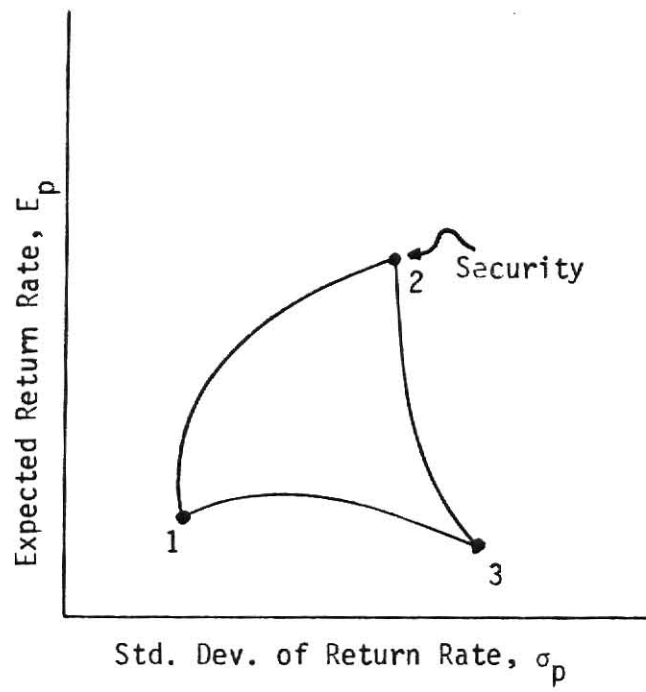


Figure 2.5. Addition of a third security

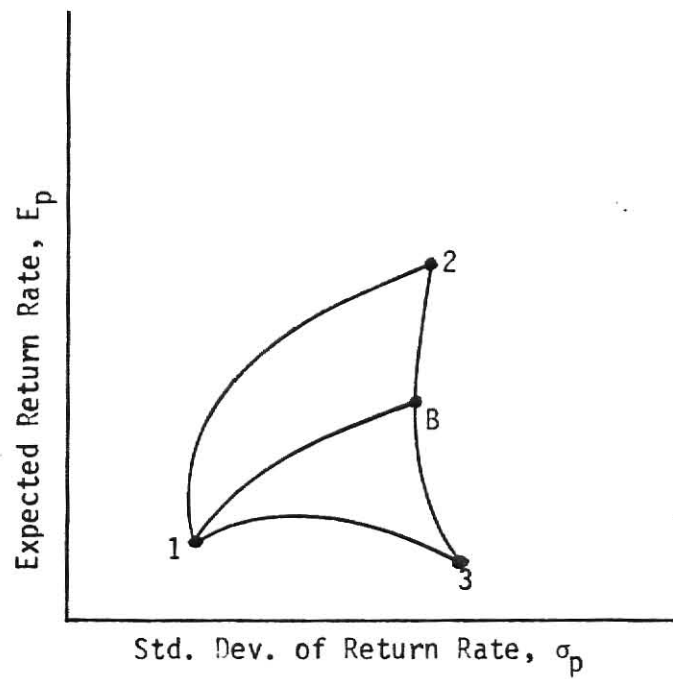


Figure 2.6. Combination of more than two securities

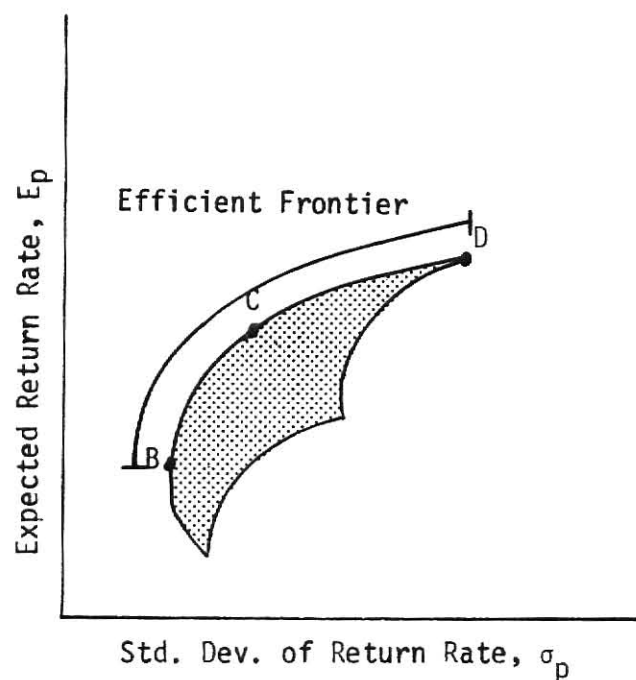


Figure 2.7. The efficient frontier

In this region, however, there are only a few efficient portfolios. If a portfolio is dominated by another it is inefficient. Therefore only portfolios represented by points along the upper left-hand boundary of the region, between points BCD (Figure 2.7), are considered efficient. All other portfolio possibilities are dominated by one of the portfolios lying on this arc, and hence are inefficient. The curve BCD is referred to as the "efficient border" or the "efficient frontier". An investor may select any portfolio corresponding to the infinity of points between B and D, and always have an efficient portfolio.

Indifference curves can be used to determine the portfolio that will be selected by a particular risk-avoiding investor. Such a portfolio will

lie on the "efficient frontier", and the one selected will be the one that is tangent to the highest level of indifference curve for the individual, such as point B in Figure 2.8. However, because each particular investor possesses his own set of indifference curves (depending on the values of the coefficients A, B and C in the utility function), the point B (Figure 2.8) varies from investor to investor. If the investor places all of his funds in some (E_p, σ_p) portfolio, he would do so at some typical point B, since this maximizes his expected utility.

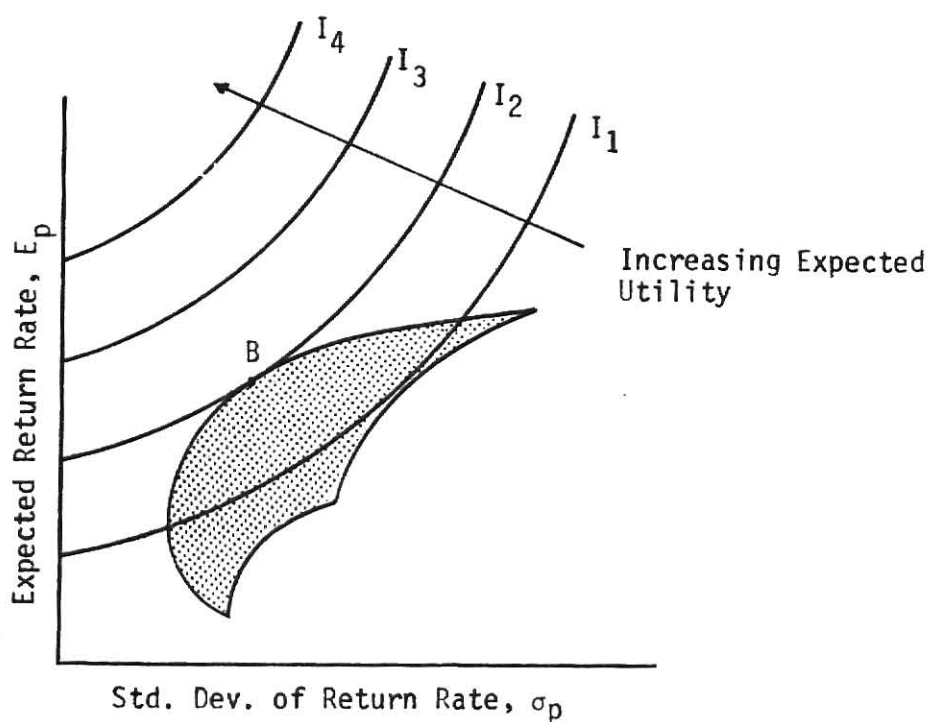


Figure 2.8. Maximization of investor's utility

2.7 Borrowing and Lending in Connection with Portfolios

The second major assumption of capital market theory permits all investors to borrow or lend an unlimited amount of money in the market at a given "risk-free" rate of interest. The term "risk-free" is not in actuality correct. There is no such thing as a riskless asset. A preferable term is "default-free" asset. For example, short term U. S. Treasury bills, usually of no more than one year maturity, are considered virtually default free. This is due to the government's power to raise money by taxes and to print currency. Hence, when a "risk-free" rate is used, the term "virtually default free" is implied. The relaxation of this assumption causes no insurmountable problems. The current risk-free rate is assumed to be around 5-6%, but whether an exact (constant) value exists is arguable. The risk-free interest rate, since it is assumed to have "zero" risk ($\sigma=0$), can be plotted on E_p, σ_p coordinates, and is a point on the expected return axis, since it is assumed to have zero variance. The risk-free rate is denoted by R_f .

Fama (4) points out that borrowing or lending at the risk-free rate, R_f , can be coupled with investment in any risky security or portfolio. Consider portfolio C involving combinations of R_f and an arbitrary security A, as in Figure 2.9. The expected return and standard deviation of return for the combined portfolio C are:

$$E_C = XR_f + (1-X)E_A \quad X \leq 1 \quad (2.19)$$

$$\sigma_C = (1-X)\sigma_A + \cancel{X\sigma_{R_f}} = (1-X)\sigma_A \quad (2.20)$$

where X is the proportion of available funds invested in the risk-free security at the risk-free rate, R_f , and $(1-X)$ is the proportion invested in Security A. Applying the chain rule to Equations (2.19) and (2.20),

$$d\sigma_C/dE_C = (d\sigma_C/dX) \cdot (dX/dE_C) = \sigma_A/(E_A - R_f) \quad (2.21)$$

This implies that the combinations of expected return and standard deviation involving the riskless asset and security A must lie along the straight line through R_f and A as in Figure 2.9. Portfolios lying between R_f and A involve the lending of funds ($0 < X \leq 1$). That is, an investor invests a portion of his money in security A and lends the rest of it by purchasing riskless assets (i.e., default-free bonds). Points lying beyond A involve the borrowing of funds ($X < 0$). An investor will invest all his available funds in security A and borrow more funds at the risk-free rate by selling default free bonds and investing these funds in security A. For $X = 0$, the investor invests all of his available funds in security A, without borrowing or lending outside funds. For $X = 1$, the investor uses only his own funds to purchase the riskless asset, yielding the return rate R_f . This case is the one in which all funds are "lent" at the riskless rate R_f .

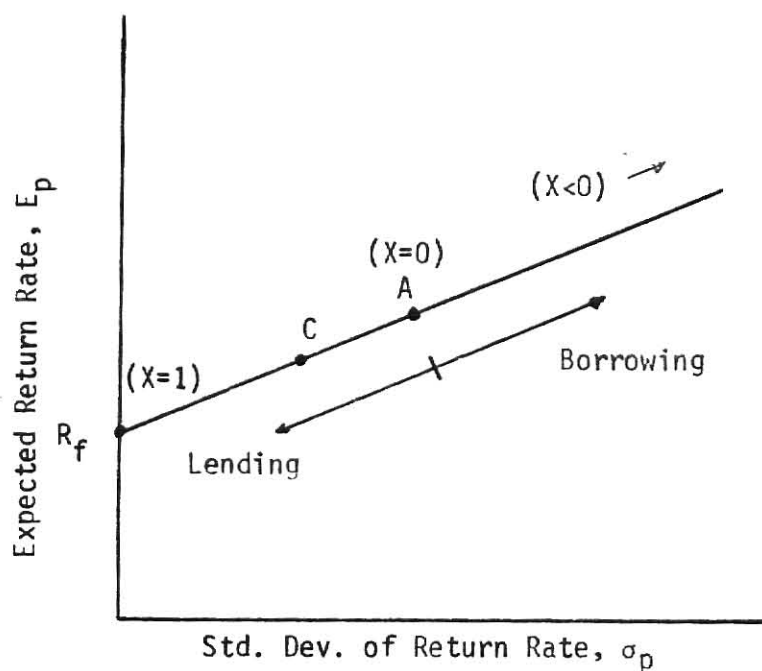


Figure 2.9. Combination of any portfolio and the riskless asset

2.8 The Capital Market Line

It is possible to connect the point $(R_f, 0)$ with the efficient frontier of risky portfolios by means of a straight line, as is shown in Figure 2.10. The line is tangent to the efficient frontier at some point (E_M, σ_M) , denoted by M in Figure 2.10. The point of tangency, M, describes an "optimal" portfolio of "risky" securities, since by the application of indifference curve analysis any combination of the risky portfolio, M, plus borrowing or lending at the riskless rate, R_f , will dominate all other possible portfolios lying along the efficient frontier. The particular optimal investment for an investor will lie somewhere along the line $\overline{R_f M}$, as dictated by the investor's indifference curves (see Figure 2.10). His optimal portfolio, consisting of a fraction X_M of portfolio M and a fraction $(1 - X_M)$ of the riskless asset, would be at, say, point A in the diagram. If $X_M = 0$, corresponding to no investment in M, then his optimum would be at R_f . If $X_M = 1$, however, then his optimum portfolio would be at (E_M, σ_M) , or at point M. The concept of interest here is that the proportion of funds invested in the risky portfolio (X_M) versus the proportion invested in the riskless asset, $(1 - X_M)$, is determined by the investor's indifference function that is tangent to the line $\overline{R_f M}$.

The point M has a special interpretation. It is called the "market portfolio", whose mean return rate and standard deviation are (E_M, σ_M) . It needs to be demonstrated that M is, in fact, the market portfolio; that is, a portfolio composed of all securities in the market, in which the i th security is included in the portfolio at a relative level X_i , so that when all securities are included, $\sum_i X_i = 1$.

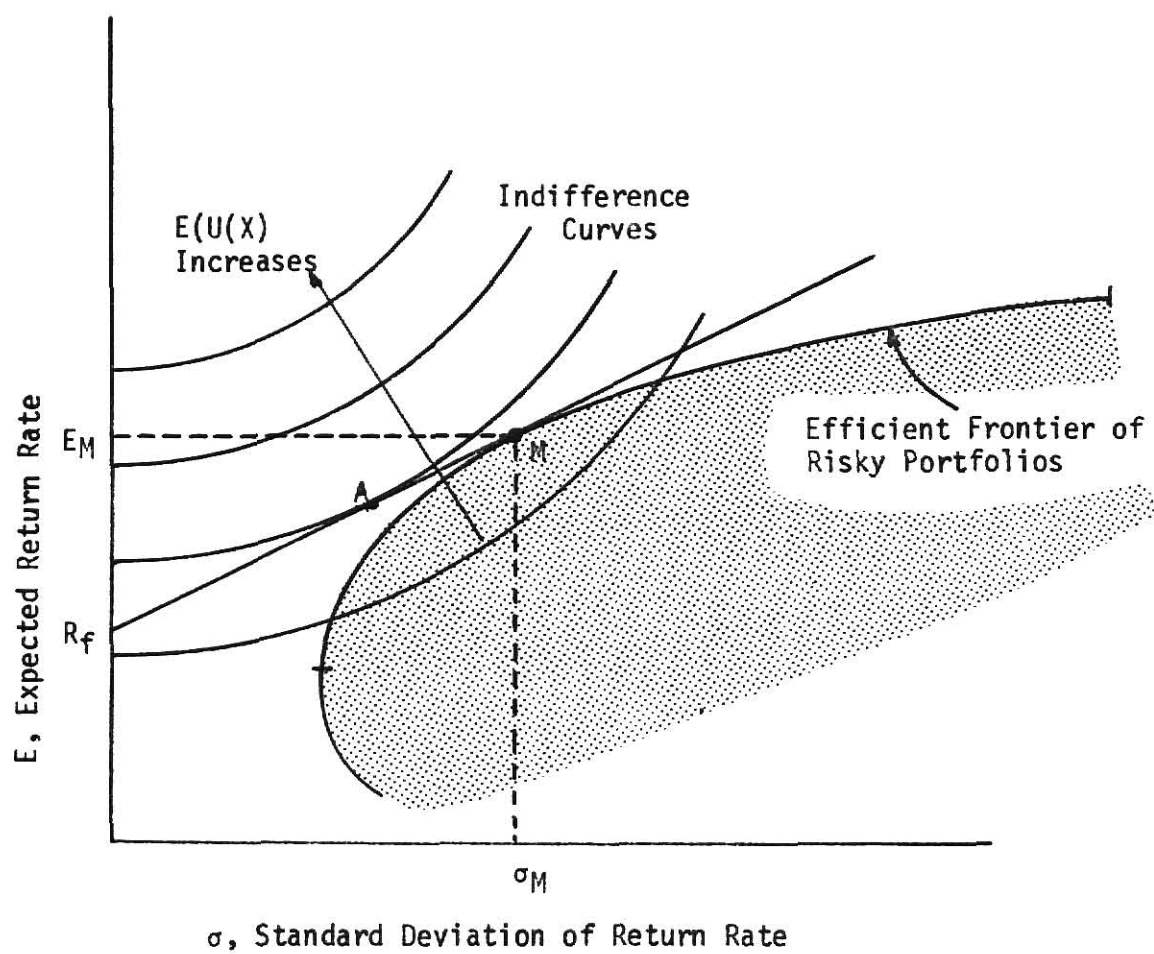


Figure 2.10. The Capital Market Line

Recalling from the third assumption of capital asset pricing theory that all investors view all securities in the same way, we can make further inferences. As a consequence of this assumption, all investors will hold equal proportions of all securities in an equilibrium market. This proposition can be demonstrated by assumption and contradiction. If this assumption were false, so that the optimal combination of securities contained a different proportion of securities than the portfolio representing the entire market, then all investors would try to hold different proportions of securities than those available in the market. Equilibrium in the market makes it impossible for this to be done. Therefore, portfolio M will be the optimal combination of risky securities that is available in the market, and is called the "market portfolio". The line intersecting the risk-free rate, R_f , and the market portfolio, (E_M, σ_M) , is called the "capital market line", and represents all linear combinations of portfolios composed of the market portfolio, M, and the risk-free asset R_f .

The equation of the capital market line can be written in linear slope-intercept form:

$$E_M = R_f + \lambda \sigma_M \quad (2.22)$$

where λ = a proportionality constant, which can be interpreted as the premium required for incurring additional risk for a finite-variance portfolio; i.e., one having an assessable risk, σ_p .

E_M = expected return of the market portfolio

σ_M = standard deviation or risk of the market portfolio.

This equation simply says that the expected return on the market portfolio is a linear function of the market portfolio standard deviation (risk).

2.9 Individual Securities and Portfolios

While a theoretical "market line" -- i.e., one that hypothesizes a straight line increase in the expected return of the market portfolio as market uncertainty increases -- is certainly a simplifying concept in finance theory, it tells us nothing about the prospective behavior of the firm vis-a-vis its decisions concerning prospective investment projects. The reason is that the firm does not invest in the market portfolio, or in combinations of the market portfolio and riskless assets. On the contrary, the firm invests in discrete projects, probably far removed from the leveling effects of the market. Hence, we need to know something about individual investments and specific combinations of individual investments in individual portfolios.

To investigate the behavior of individual securities, consider the following development. Suppose an investor wished to divide his funds in some way between some security j and the market portfolio, so as to form a new portfolio "A" (see Figure 2.11). The composition of this new portfolio "A" would be represented by some point on a line connecting the (E_j, σ_j) point for the security and the (E_M, σ_M) point representing the market portfolio. The shape of the connecting curve would again depend on the value of the correlation coefficient between the security and the market portfolio, ρ_{jM} ; and the position of the point value (E_p, σ_p) of the portfolio would depend upon the relative amounts invested.

Equations (2.15) and (2.18) can again be used as functions to define the implicit slope $dE/d\sigma$ of the capital market line. The portfolio p would be a combination of the market portfolio, M , and security j . Let X_j = the amount of resources invested in security j and $(1-X_j)$ = the

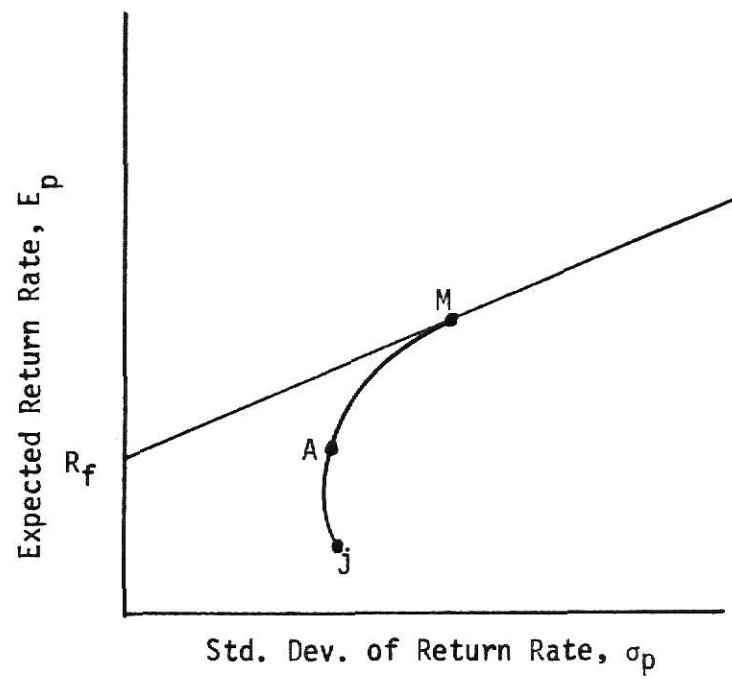


Figure 2.11. Combination of any security and the market portfolio

amount invested in the market portfolio, M . Using the chain rule the slope of the curve, jM , can be calculated, thus:

$$dE_p/d\sigma_j = (dE_p/dX_j) \cdot (dX_j/d\sigma_p) \quad (2.23)$$

where, from Equation (2.15):

$$dE_p/dX_j = E_j - E_M \quad (2.24)$$

and from Equation (2.18):

$$dX_j/d\sigma_p = \sigma_p / (X_j(\sigma_j^2 + \sigma_M^2 - 2C_{jM}) + C_{jM} - \sigma_M^2) \quad (2.25)$$

Substitution of (2.24) and (2.25) into (2.23) yields

$$dE_p/d\sigma_p = (E_j - E_M) / ((X_j(\sigma_j^2 + \sigma_M^2 - 2C_{jM}) + C_{jM} - \sigma_M^2) / \sigma_p) \quad (2.26)$$

Now, at point M , $X_j = 0$ and $\sigma_p = \sigma_M$. Thus,

$$dE_p/d\sigma_p = (E_j - E_M)\sigma_M / (C_{jM} - \sigma_M^2) \quad (2.27)$$

However, at point M in Figure 2.11, the slope of curve jM is required to be tangent to the capital market line for the following reasons. If the slope of curve jM were steeper or flatter than the slope of the capital market line, at point M , there would be points on jM which would represent portfolios that were "better" than the market portfolio and plot above the capital market line, or "worse" than the market and plot below the market portfolio. The only possibility left for curve jM , since the market is required to be in equilibrium, is that it be tangent to the capital market line at point M . When curve jM is tangent to the capital market line then its slope must equal the slope of the capital market line. Thus, the trade-off between expected return and risk for small changes in the amount of security j included in the market portfolio must equal the trade-off in the capital market as a whole. From Equation (2.22), the slope of the capital market line, λ , is

$$\lambda = (E_M - R_f) / \sigma_M \quad (2.28)$$

Equating the slopes of the capital market line (Equation (2.28)) and curve jM (Equation (2.27)) gives

$$E_j - R_f = (E_M - R_f) \cdot (C_{jM} / \sigma_M^2) \quad (2.29)$$

where now the left-hand side is the expected risk premium (above the risk-free rate) required for the j th security and $(E_M - R_f)$ is the expected risk premium of the market portfolio itself. Thus, the expectation of the risk premium required for investing in the j th security is simply a multiple of the expected market risk premium, where the multiplier is the factor, C_{jM} / σ_M^2 .

This multiplier, which relates an individual security's expected risk premium to the expected risk premium of the market itself, is called the "volatility" of security j . It is taken to be a constant, denoted β_j ; or $\beta_j = C_{jM} / \sigma_M^2$. On substituting β_j into Equation (2.29), we have

$$E_j - R_f = \beta_j (E_M - R_f) \quad (2.30)$$

Equation (2.30) is called the "Capital Asset Pricing Model".

β_j , the volatility constant for the j th security, is also the appropriate measure of risk for an individual security. This is because of the fact that the individual security is assumed to be correlated only with the market portfolio, M . β_j relates each security to the variance of the market portfolio. Thus, the problem of an appropriate measure of risk for an individual security is approached through the use of the volatility concept.

Since it is an important concept, more should be said of the nature of β_j , the volatility. The market return, R_M , is taken to be a random variable with mean E_M and variance σ_M^2 . Hence the market risk premium, $E_M - R_f$, is also a random variable. If a security exhibits a β_j greater

than 1, an increase in the random market risk premium will mean an even larger increase in the expected premium for the security. Similarly, a larger decrease would occur in the expected return of the security in the event of a decrease in the expected random market premium. When $\beta_j < 1$, the increase or decrease in the risk premium for a particular security is less than that of the expected market premium.

2.10 Linear Regression

Equation (2.30), the equation of the security line is unfortunately never observable because of the deviations away from the security line that occur in actuality. Using regression techniques, the relationship can still be examined, however.

The volatility, β_j , can be estimated from the relationship of the return of security j , R_j , and the market return, R_M . β_j can be estimated if

- 1) both R_j and R_M are assumed to be bivariate random variables with finite means and variances and are correlated with a covariance C_{jM} . The returns occur in a time series $t = 1, 2, \dots, n$ with observable values, R_{jt} and R_{Mt} .
- 2) the nature of the correlative relationship can be examined using regression techniques and taking R_{jt} as being conditionally distributed upon fixed values of R_{Mt} .

If this is done, Equation (2.30) can be estimated by the time-series regression

$$(R_{jt} - R_f) = \alpha_j + \beta_j(R_{Mt} - R_f) + \epsilon_{jt} \quad (2.31)$$

where R_{jt} = random return on security j at time t

R_{Mt} = return rate on market portfolio M at time t

R_f = the risk-free or default-free interest rate

α_j = an unknown intercept parameter

β_j = an unknown proportionality factor, or the "volatility" of security j with respect to the market

ϵ_{jt} = normally distributed random error term representing the deviation away from the security line.

By assumption, ϵ_{jt} , the random error term, has the following three properties:

1. $E(\epsilon_{jt}) = 0$;
2. $C(\epsilon_{jt}, \epsilon_{jt-1}) = 0$; i.e., there is no timewise correlation between error terms;
3. For two securities, i and j , $C(\epsilon_{it}, \epsilon_{jt}) = 0 \quad i \neq j$.

The third property is based on the assumption by Markowitz (12) that the covariance between two securities is zero. Covariance is assumed to exist only between each security and the market.

Letting $R_{j,t} = R_{jt} - R_f$ and $R_{M,t} = R_{Mt} - R_f$ in (2.31), then

$$R_{j,t} = \alpha_j + \beta_j \cdot R_{M,t} + \epsilon_{jt} \quad , \quad (2.32)$$

where $R_{j,t}$ = random return rate on security j above the risk-free rate for period t

$R_{M,t}$ = return rate on the market portfolio M above the risk-free rate for period t .

Equation (2.32) is a linear regression model with parameters α_j and β_j . These parameters are not directly observable, and, as such, must be estimated. The linear regression model can then be estimated by a linear regression function based on least squares methods. This estimate of (2.32) is

$$\hat{R}_{j,t} = \hat{\alpha}_j + \hat{\beta}_j R_{M,t} \quad (2.33)$$

where $\hat{R}_{j,t}$, $\hat{\alpha}_j$, and $\hat{\beta}_j$ are estimates of the corresponding parameters in (2.32).

This method gives point estimates of α and β for a security, j , using the normal equations. These estimates are

$$\hat{\beta}_j = (\sum (R_{M,t} - E_{M,t})(R_{j,t} - E_{j,t})) / \sum (R_{M,t} - E_{M,t})^2 \quad (2.34)$$

and

$$\hat{\alpha}_j = E_{j,t} - \beta_j E_{M,t} \quad (2.35)$$

Using the definitions of covariance and variance, Equation (2.34) can be rewritten as

$$\hat{\beta}_j = C_{j,M} / \sigma_M^2 \quad (2.36)$$

which is a point estimate of the slope, β_j , in Equation (2.30). From Equation (2.35), we have a point estimate, $\hat{\alpha}_j$, of the true intercept, α_j , which by analogy with Equation (2.30) becomes a test of the estimate of R_f . If the correct value for R_f has been chosen, $\hat{\alpha}_j$ should be indistinguishable from zero. Hence, the Capital Asset Pricing Model (Equation 2.30)) is empirically testable, and the parameters R_f and β_j can be estimated by linear regression techniques.

CHAPTER III

THE CAPITAL ASSET PRICING MODEL APPLIED TO CAPITAL BUDGETING

3.0 Introduction

The Capital Asset Pricing Model appears to be a very useful tool, based on its simplistic assumption of a linear relationship between risk and expected return for individual securities. It is this simplicity which is appealing in its application to industrial capital budgeting. In their study of seven industrial firms, Bower and Lessard (3) discovered that all seven firms wanted simple, intuitive measures which could be used at the project decision level. Six of the seven companies used internal rate of return as the principal screening device. None of the firms was satisfied with its current methods for incorporating risk into the decision model. The Capital Asset Pricing Model meets the requirements of simplicity and theoretical correctness. It should then be possible for management to use this model directly as a decision criterion. This is a very important point. It is of little importance how effective or efficient a new idea may be if its originator cannot get its implementation approved by management. This chapter will detail the application of the Capital Asset Pricing Model to industrial capital budgeting, and specifically to privately owned firms.

3.1 Risk Equivalency

One should recall that all securities can be plotted on (E_j, σ_j) coordinates in which E_j is the expected return rate on the market value of the j th firm's equity and σ_j is the estimated standard deviation on

the market value of the firm's equity. A line could be drawn extending from the risk-free rate of interest, R_f , through the firm's "market trade-off point", I , as in Figure 3.1. This line would represent the possible combinations of the risk-free asset and the "risky" asset which an investor could opt by investing a portion of his funds in security j and lending the rest by purchasing riskless assets or borrowing at this risk-free rate and investing the money in security j . This concept has previously been demonstrated. This line could theoretically then be used in evaluating individual project alternatives available to the firm. All projects (i) having (E_i, σ_i) values lying above the line would yield expected returns higher than those required by the firm's common stock in the market, and thereby increase the market value of the firm. Hence, all projects whose (E_i, σ_i) plot above this line should be accepted by the firm and all projects below the line should be rejected. The firm would be indifferent about projects lying on the line. From a practical view, this criterion is acceptable, but there are theoretical problems associated with it.

Each firm has a current risk level, σ_0 , on its equity. Individual projects have varying risk levels. As each project is accepted and invested, the "riskiness" and equity price structure of the firm would be altered, thereby changing the risk level of the firm and hence, the firm's market trade-off point. This difference between project risk and the firm's risk also forces the need for moving from one indifference function to another in order to determine the desirability of one return-risk combination against another. Tuttle and Litzenberger (22) have proposed a method of risk adjustment to achieve a risk-equivalency of the proposed project and the firm's residual return to equity. The

purpose of this adjustment is to leave the market price of the firm's shares unchanged. This is accomplished by combining borrowing or lending with equity capital to finance the proposed project.

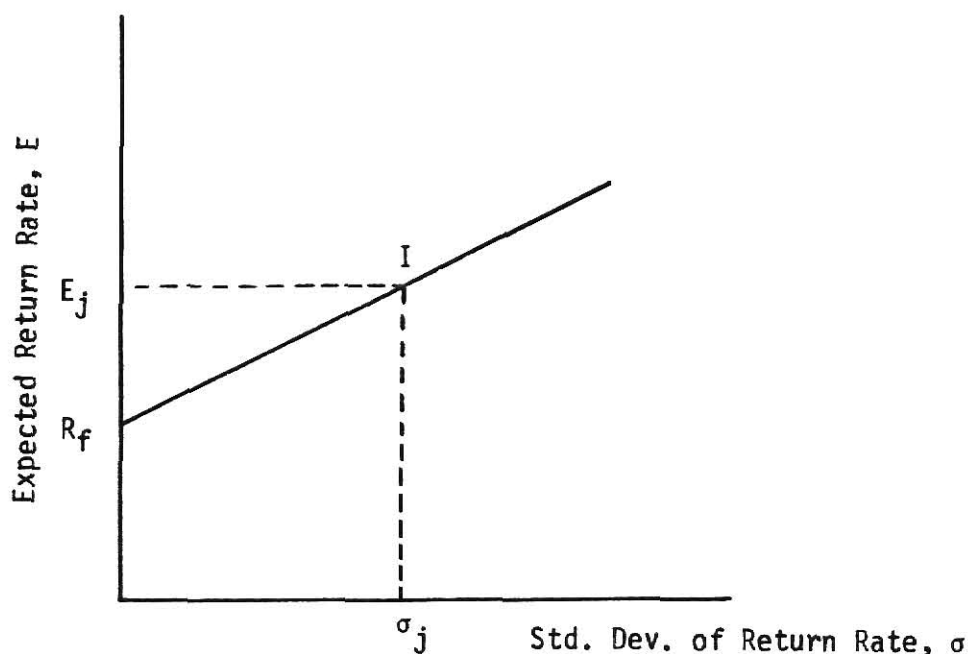


Figure 3.1. Combination of any security and the riskless asset

To illustrate this approach, consider a project alternative, I, for a firm, with an expected return rate and estimated standard deviation of return rate. Let:

- R_z = the expected return rate to equity from the project;
- σ_z = the estimated standard deviation of the return rate to equity;
- R_f = the risk-free borrowing and lending rate;
- α = the financing ratio of the project; unity plus the project's debt-equity ratio $(1+D/E)$; where debt is either borrowed or loaned;

R_i = expected return rate from project i ;

σ_i = estimated standard deviation of return from project i .

The estimate of the risk on return to equity since $\sigma_f = 0$, is

$$\sigma_z = \alpha \sigma_i \quad . \quad (3.1)$$

When α dollars per dollar of equity are invested in a project and $(\alpha-1)$ dollars are borrowed per dollar of equity, the expected return on equity from the project is αR_i and the cost of borrowing is $(\alpha-1)R_f$. From the fact that the total return is composed of the return to equity plus the return to borrowed capital, we find that the expected return rate to equity is

$$R_z = R_i + (\alpha-1)R_i - (\alpha-1)R_f \quad (3.2)$$

which simplifies to

$$R_z = R_f + \alpha(R_i - R_f) \quad . \quad (3.3)$$

Recalling the form of Equation (3.1), we can write Equation (3.3) in the form

$$R_z = R_f + (R_i - R_f)(\sigma_z/\sigma_i) \quad . \quad (3.4)$$

Differentiating return rate with respect to its standard deviation,

$$dR_z/d\sigma_z = (R_i - R_f)/\sigma_i \quad . \quad (3.5)$$

Now, there exists some factor (α') which, when multiplied by the risk of the investment project, σ_i , will equate $\alpha'\sigma_i$ with the current risk of the firm, σ_0 . Thus,

$$\sigma_0 = \alpha' \sigma_i \quad (3.6)$$

$$\text{or} \quad \alpha' = \sigma_0/\sigma_i \quad . \quad (3.7)$$

The risk effect of a project on return to equity may be equalized either through long-term lending of an amount equal to $((1/\alpha') - 1)$ of the cost of the investment project if $\sigma_i > \sigma_0$, or the long term borrowing of $(1 - (1/\alpha'))$ of the cost of the project if $\sigma_i < \sigma_0$. The risk-adjusted return rate on an investment project, R_i , is

$$R_i' = R_f + \alpha'(R_i - R_f) \quad (3.8)$$

or
$$R_i' = R_f + (\sigma_0/\sigma_i)(R_i - R_f) \quad (3.9)$$

Figure 3.2 illustrates the risk adjustment procedure of Tuttle and Litzenberger. Point I is the return-risk combination of project I. Point O is the return-risk level of the firm. Through borrowing or lending (in this example, borrowing) at the risk-free rate, R_f , the return-risk level of project I can be equalized with the risk level of the firm. This equalization occurs at point I'. Thus in this case, project I would be accepted as it yields a greater return than the firm is currently receiving on its investments at the firm's current risk level.

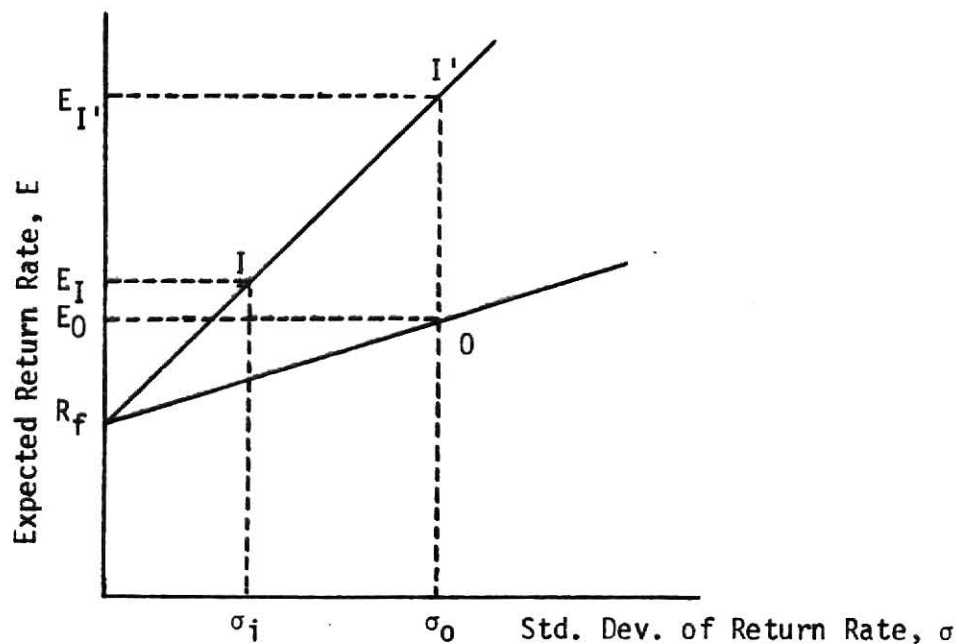


Figure 3.2. Risk adjustment of a project

3.2 Capital Budgeting Criteria

As illustrated in Figure 3.2, all projects will be accepted if the slope of the line connecting the (R_i, σ_i) coordinates of the project and the riskless asset exceeds the slope of the firm's security line. These projects will be preferred by the firm whether or not the projects are risk equalized, since all points on the project line will lie on a higher indifference curve than the firm's shares themselves. The slopes of these lines can be calculated by Equation (3.5). They can then be compared and used in a decision criterion for the firm. The resulting criterion is:

$$\text{Accept the project if } (R_i - R_f)/\sigma_i > (R_0 - R_f)/\sigma_0 \quad . \quad (3.10)$$

If the firm wishes to risk equalize, the criterion becomes

$$(R_i' - R_f)/\sigma_i' > (R_0 - R_f)/\sigma_0 \quad . \quad (3.11)$$

Recalling that $\sigma_i' = \sigma_0$ from risk equalization Equation (3.11) reduces to:

$$\text{Accept if } R_i' > R_0 \quad . \quad (3.12)$$

These relationships are known as "reward-to-variability" ratios. The ratio was developed by Sharpe (18) and means simply that a reward above the riskless rate exists for accepting the risk of such investments; in order for a project to be accepted by a firm, its reward-to-variability ratio must be greater than the ratio the firm is currently achieving from its existing projects.

Sharpe (18) questions the application of the reward-to-variability ratio as being a legitimate measure in evaluating a single security or single project. He states:

The reward-to-variability ratio is designed to measure the performance of a portfolio. The investor is presumed to have placed a substantial portion of his wealth in the portfolio in question. Variability is thus the relevant measure of the amount of risk actually borne. To evaluate the performance of a single security, or that of a portfolio constituting only

part of an investor's holdings, a different measure is needed. Variability will not adequately represent the risk actually borne. A more appropriate choice is volatility.

This objection is a theoretical one. Sharpe demonstrates that the total variability of the project in Equations (3.10) and (3.11), σ_i , can be separated into two parts; i.e., into what he calls the "systematic" risk -- risk associated with market (or "system") fluctuations -- and the "nonsystematic" risk -- the portion of the risk which can be eliminated by combining the project into a diversified portfolio of projects. He then points out that for a portfolio the nonsystematic risk becomes zero, since it can be diversified away, but for a single project (or single stock) "portfolio" it cannot be set to zero. Hence, Sharpe says, the correct measure of variability for a single project (or security) is the reward-to-volatility ratio, $(R_i - R_f)/\beta_i$, which requires no premium $(R_i - R_f)$ being expected for that portion of the firm's risk that can be diversified away (the nonsystematic risk).

The resulting reward-to-volatility criterion is of similar form to the reward-to-variability criterion, except that the volatility, β_i , replaces the standard deviation, σ_i , in the denominator. As first proposed by Treynor (21), the reward-to-volatility criterion is:

$$\text{Accept the project if } (R_i - R_f)/\beta_i > (R_0 - R_f)/\beta_0 \quad (3.13)$$

where β_i is the volatility of the project and β_0 is the volatility of the firm. Risk equalization can again be achieved using a similar analysis as that of Tuttle and Litzenberger. The result is:

$$\text{Accept the project if } (R_i' - R_f)/\beta_i' > (R_0 - R_f)/\beta_0 \quad (3.14)$$

$$\text{or } R_i' > R_0 \quad (3.15)$$

The concept could be illustrated in a manner similar to Figure 3.2, by replacing standard deviation with volatility along the horizontal axis.

The return from a project is actually a random variable because the future is uncertain. Thus, the reward to volatility criterion for future investments is more correctly stated using an expected return, or

$$\text{Accept the project if } (E_i - R_f)/\beta_i > (E_0 - R_f)/\beta_0 \quad . \quad (3.16)$$

The expected return then could be risk-equalized in a manner similar to Equation (3.9); thus:

$$E_i' = R_f + (\beta_0/\beta_i)(E_i - R_f) \quad . \quad (3.17)$$

3.3 The Reward-to-Volatility Ratio Applied to the Privately Owned Firm

The reward-to-volatility ratio can be used by publicly held firms for use in capital budgeting decisions. However, since the reward-to-volatility ratio requires a relationship between the return of the firm and the market, the ratio cannot be used directly for the many privately owned and financed firms that operate in the industrial sector. The concept of volatility with the market for a private firm is meaningless, since there is no market for the firm's stock. This does not however, eliminate the possibility of indirectly applying the reward-to-volatility ratio to the private firm.

There are numerous industries in which private firms are engaged in competition. Often they are competing chiefly against publicly owned corporations. In any type of industry, one way of evaluating the success of any firm would be by making a comparison against its competitors. If the firm had a return on its investment that was as good or better than its competitors it is reasonable to assume that the firm is performing well in its investment decisions. Thus, an average of the competing, publicly owned firms could be calculated and used as a comparative "standard". This

average could serve as a "surrogate" or pseudo firm, against which the private firm could be compared. The private firm could evaluate all investment alternatives against a security line constructed from the surrogate firm. A reward-to-volatility ratio could then be used for any private firm which was competing in the same field of endeavor against publicly owned firms.

A weighted average of the returns of selected individual public firms could be calculated. Thus

$$R_i = \sum X_j R_j \quad (3.18)$$

where X_j is the proportion invested in firm j using net worth of the firm as the weighting factor, X_j . The larger firms would then be weighted greater than the smaller firms, acknowledging their importance in the industry. The value for R_i in Equation (3.18) could then be calculated for a number of years and these values could be used in a time series regression model, such as that developed in Chapter II. The estimates for R_i and β_i could be used in a reward-to-volatility ratio criterion for the private firm. This results in a "surrogate security line" on (E, β) coordinates which the private firm could use in its capital budgeting analysis. All projects plotting above the line would be accepted as before with projects plotting below the line being rejected. That is, all projects with (E_i, β_i) coordinates above the line are projects which would improve the firms expected position relative to its competition. This is a true test of the survival of a firm, how it performs in relation to its competition.

3.3.1 Example. The use of a surrogate security in application of capital asset pricing theory to capital budgeting for the private firm

cannot be considered all inclusive. In industries which are dominated by conglomerate firms which are diversified in several or many different industries, the concept of a surrogate firm is unrealistic. The return of the surrogate would be distorted by the return of divisions of these firms whose product or service is entirely unrelated to the industry for which the surrogate is being developed. This is not necessarily a severe limitation, but an important one. In selecting an industry for purposes of this thesis further requirements were specified. The industry should have approximately five to twenty firms which are publicly owned and should be easily identifiable and familiar to the general public. These requirements kept the example simple for illustrative purposes while still constituting a measure of the industry. Under these requirements, the brewing industry was selected for this thesis.

The Standard Industrial Classification (S.I.C.) Manual (23) categorizes types of industry by function, giving each particular category a unique S.I.C. number. Thus, 2082 is the S.I.C. number for Malt Beverages. Standard & Poor's publishes a list of corporations by S.I.C. number. From this list, the publicly held firms with listings on the New York and American stock exchanges, or "over-the-counter" price information were selected. These six firms are Anheuser-Busch, Carling O'Keefe Ltd., Falstaff Brewing, Pabst Brewing, F&M Schaefer, and Joseph Schlitz Brewing. Closing prices, dividends paid, and net worth for the years 1961-1975 were obtained using issues of the Wall Street Journal and Value Line (24). For the market, Standard & Poor's 500 Composite Index was selected as the indicator of market return. This is a composite of 500 common stocks traded on the New York exchange. The dividend yield of the market was obtained from various issues of the Federal Reserve Bulletin.

The return rate for each of the breweries was then calculated according to the following relationship:

$$R_i = (P_t + D_t - P_{t-1})/P_{t-1} \quad (3.19)$$

where R_i = return rate for security i for year t

P_t = closing price for security i at year t (dollars)

P_{t-1} = closing price for security i at year $t-1$ (dollars)

D_t = dollar amount of dividends paid by firm during year t .

These returns were weighted by net worth and summed using Equation (3.18) to obtain a surrogate return of the brewing industry. (This procedure is consistent with other investigative works on the Sharpe-Lintner model).

The return of the market for each year was calculated in a similar manner:

$$R_M = (P_t - P_{t-1})/P_{t-1} + (D_t/P_t) \cdot (P_t/P_{t-1}) \quad (3.20)$$

where R_M = return rate for the market for year t

P_t = closing market price (relative) of Standard & Poor's 500 Composite Index at year t

P_{t-1} = closing market price (relative) of Standard & Poor's 500 Composite Index at year $t-1$

D_t/P_t = current dividend-to-price ratio for the market at close of year t .

The results of this calculation are shown in Table 3.1. The raw data used in obtaining these results can be found in the Appendix.

Using the information from Table 3.1, the linear regression analysis developed in Chapter II was performed. The market premium ($R_M - R_f$) is the independent variable, with the surrogate security premium ($R_i - R_f$) the dependent variable of the analysis. The analysis was performed using the computer system, AARDVARK, developed by the Statistics Department at K.S.U. (6). The results of the regression analysis are found in Table 3.2.

TABLE 3.1
RETURN RATES FOR INDIVIDUAL SECURITIES, BREWING SURROGATE SECURITY AND THE MARKET

	1961	1962	1963	1964	1965	1966	1967	1968
Anheuser-Busch	44.27	-13.64	1.79	49.37	47.06	26.30	43.93	56.00
Carling O'Keefe	31.49	- 8.90	- 5.25	11.23	-15.13	-12.38	13.85	68.29
Falstaff	13.91	-16.97	12.48	34.68	- 5.84	-30.47	4.98	32.24
Pabst	91.43	-20.15	64.08	60.00	28.08	- 9.40	78.08	51.56
F&M Schaefer								
Schlitz			70.08	11.49	4.70	-58.51	90.95	42.98
Surrogate	44.39	-13.33	22.47	27.00	12.66	-12.44	48.85	53.24
Market (S&P 500)	26.66	- 8.82	22.64	16.42	12.40	- 9.93	23.79	12.10
	1969	1970	1971	1972	1973	1974	1975	
Anheuser-Busch	15.31	5.81	48.02	- 0.30	-39.40	-25.18	43.29	
Carling O'Keefe	-27.24	- 4.44	2.10	-15.99	-33.09	-46.29	32.98	
Falstaff	-37.82	-41.54	52.75	-39.65	-56.14	-44.44	--	
Pabst	- 4.00	10.51	54.00	0.26	-64.85	-31.72	35.70	
F&M Schaefer								
Schlitz	47.54	- 6.12	60.22	62.21	- 3.03	-72.02	32.07	
Surrogate	6.55	- 0.47	40.62	7.43	-35.81	-41.36	37.03	
Market (S&P 500)	-10.35	4.82	14.10	17.80	-13.83	-25.79	36.96	

TABLE 3.2

RESULTS OF REGRESSION ANALYSIS USING AARDVARK SUBROUTINE

<u>Variable</u>	<u>Mean</u>	<u>Std. Dev.</u>	<u>Parameter</u>	<u>Estimate</u>	<u>Std. Dev.</u>
$R_i - R_f$	8.122	29.818	Alpha	3.900	4.252
$R_M - R_f$	2.965	17.814	Beta	1.425	.243

ANALYSIS OF VARIANCE

	<u>DF</u>	<u>Sum of Squares</u>	<u>Mean Squares</u>	<u>F</u>
Total (Uncorrected)	15	13,437.143		
Corrections:				
Mean	1	989.503	989.503	
Total (Corrected)	14	12,447.640	889.117	
Due to Regression	1	9,023.363	9,023.363	34.256
Deviations from Regression	13	3,424.277	263.406	

$$r^2 = .7249$$

$$r = .8514$$

The risk-free rate of return, R_f , was assumed to be 5%. If this value is correct, the capital asset pricing model predicts that the regression line will pass through the origin in equilibrium. Using information from the AARDVARK analysis a statistical test can be performed to test whether or not a risk-free rate of 5% is a reasonable assumption. A t-test was performed as follows:

$$H_0: \alpha = 0$$

$$H_a: \alpha \neq 0 \quad \text{with } \alpha = .05 \text{ significance, } t_{.05}(13) = 2.16$$

$$t = (\alpha - 0)/\sigma_\alpha = 3.90/4.25 = .92$$

$$\text{Since } .92 < 2.16, \text{ fail to reject } H_0$$

The decision, based on the t-test was to accept the null hypothesis. A value of 5% would be considered acceptable to use as the risk-free rate.

The slope parameter, β_j , is the volatility of the brewing industry. The estimate of the volatility, $\beta_j = 1.43$, implies that the mean return rate of the brewing industry is expected to fluctuate nearly $1\frac{1}{2}$ times that of the market itself. A test was made to see if this estimate of β_j was significantly different from zero using an F-test as follows:

$$H_0: \beta = 0 \quad \text{with } \alpha = .05, F(1,13) = 4.69$$

$$H_a: \beta \neq 0$$

$$F_{\text{test}} = \text{MSR}/\text{MSE}$$

where MSR = sum of squares due to the regression divided by its associated degrees of freedom. (See Table 3.2)

MSE = sum of squares of deviations from the regression divided by its associated degrees of freedom. (See Table 3.2)

$$F = 9023.363/263.406 = 34.256$$

$$\text{Since } 34.256 > 4.69, \text{ reject } H_0.$$

The decision based on the F-test is to reject H_0 . The volatility of the brewing industry is significantly different from zero. Since it is different from zero, the best estimate available for the volatility is used, which is the value obtained from the regression analysis, $\hat{\beta}_j = 1.43$.

Estimating the return for the market or individual projects in the future is at best a speculative procedure. Weston (25) is one of many who have proposed reasonable methods for estimating future events. He suggests assuming various states of the world and the associated returns of the market and individual projects with those states. A subjective probability of occurrence is attached to each of these states.

Table 3.3 shows the returns for the market and four projects given 4 states of the world along with the probabilities for these 4 states. Using this information expected returns and variances of return can be calculated as in Tables 3.4 and 3.5. Using the same information the covariance of the projects with the market is also estimated in Table 3.5. Since the volatility of a project is defined as the covariance of that project with the market divided by the variance of the market, an estimate of the volatility for each project can be made. The volatility for each project is shown in Table 3.6. The reward-to-volatility ratio for the projects can then be compared with the reward-to-volatility ratio of the surrogate, or the risk equalized return of the projects compared with the current return level of the surrogate.

TABLE 3.3

RETURN OF THE MARKET AND FOUR PROJECTS GIVEN FOUR STATES OF THE WORLD

State of the World (S)	Subjective Probability (π)	Market Return (R_M)	Project Return			
			R_1	R_2	R_3	R_4
1. Serious recession	.05	-.30	-1.00	-.20	-.05	-.48
2. Mild recession	.20	-.10	-.40	.07	.07	-.12
3. Mild recovery	.45	.10	.25	.16	.10	.13
4. Strong recovery	.30	.30	.50	.25	.17	.33

TABLE 3.4

EXPECTED RETURN AND STANDARD DEVIATION OF RETURN FOR THE MARKET

<u>S</u>	<u>π</u>	<u>R_M</u>	<u>πR_M</u>	<u>$R_M - E_M$</u>	<u>$\pi(R_M - E_M)^2$</u>
1	.05	-.30	-.015	-.40	.008
2	.20	-.10	.02	-.20	.008
3	.45	.10	.045	.00	.000
4	.30	.30	.09	.20	.012

$$E_M = .10$$

$$\sigma_M^2 = .028$$

TABLE 3.5

EXPECTED RETURN AND COVARIANCE OF RETURN WITH THE MARKET FOR FOUR PROJECTS

Project	S	π	R_i	πR_i	$R_i - E_i$	$R_M - E_M$	$\pi(R_i - E_i)(R_M - E_M)$
1	1	.05	-1.00	-.05	-1.133	-.40	.0223
	2	.20	-.40	-.08	-.533	-.20	.0214
	3	.45	.25	.1125	.118	0	
	4	.30	.50	.15	.368	.20	.0147
			$E_1 = .133$				$\text{Cov}(R_1, R_M) = .0584$
2	1	.05	-.20	-.01	-.351	-.40	.007
	2	.20	.07	.014	-.081	.20	.0032
	3	.45	.16	.072	.009	0	
	4	.30	.25	.075	.099	.20	.0059
			$E_2 = .151$				$\text{Cov}(R_2, R_M) = .0161$
3	1	.05	-.05	.0025	-1.55	-.40	.0031
	2	.20	.03	.006	-.075	-.20	.0030
	3	.45	.10	.045	0	0	
	4	.30	.17	.051	.065	.20	.0026
			$E_3 = .105$				$\text{Cov}(R_3, R_M) = .0087$
4	1	.05	-.48	-.024	-.59	-.40	.0118
	2	.20	-.12	-.024	-.23	-.20	.0092
	3	.45	.13	.0585	.02	0	
	4	.30	.33	.099	.22	.20	.0132
			$E_4 = .110$				$\text{Cov}(R_4, R_M) = .0342$

TABLE 3.6
VOLATILITY FOR FOUR PROJECTS

Project	$\text{Cov}(R_i, R_M)$	β
1	.0584	2.08
2	.0161	.57
3	.0087	.31
4	.0342	1.22

The reward-to-volatility ratio for the surrogate is

$$(R_0 - R_f)/\beta_0 = (13.12 - 5)/1.43 = 5.68$$

The reward-to-volatility ratio for each of the projects is

$$(E_1 - R_f)/\beta_1 = (13.3 - 5)/2.08 = 3.99 \quad \text{For project 1}$$

$$(E_2 - R_f)/\beta_2 = (15.1 - 5)/.57 = 17.72 \quad \text{For project 2}$$

$$(E_3 - R_f)/\beta_3 = (10.5 - 5)/.31 = 17.74 \quad \text{For project 3}$$

$$(E_4 - R_f)/\beta_4 = (11.0 - 5)/1.22 = 4.92 \quad \text{For project 4}$$

Comparing these values with the reward-to-volatility ratio of the surrogate results in the acceptance of projects 2 and 3 and the rejection of projects 1 and 4.

Figure 3.3 illustrates graphically the reward-to-volatility criterion as a project selection tool. The (E_i, β_i) coordinates for each project are plotted and compared with the security line of the surrogate. This line connects the risk-free rate for $\beta = 0$ with the (E_0, β_0) coordinates for the surrogate. The reward for accepting additional risk for projects 2 and 3 is greater than the reward for accepting risk currently in effect for the surrogate, resulting in (E_i, β_i) coordinates above the firm security line. Since the reward-

to-volatility ratio for projects 1 and 4 are less than the ratio of the surrogate, those projects plot below the security line and are rejected.

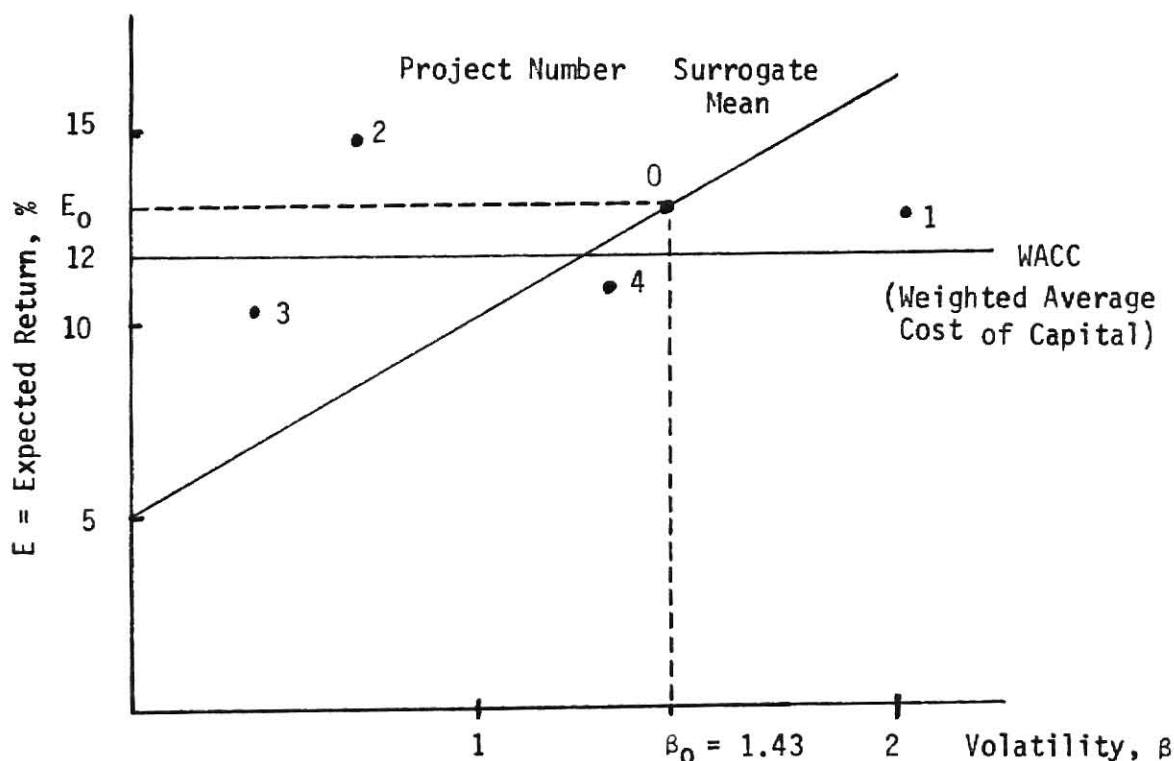


Figure 3.3. The reward-to-volatility and weighted average cost of capital criteria

To point out the possibility of conflicting decisions between the reward-to-volatility criterion and the weighted average cost of capital (WACC) criterion, the WACC was also included in Figure 3.3. A WACC of 12% is assumed for the surrogate. This is a typical value. There are many books and articles explaining the weighted average cost of capital and how it is calculated. One such book is written by Quirin (16). Since the volatility of the surrogate with the market has no effect on the WACC, it is represented by a horizontal line at $E_p = 12\%$. There are

two areas in which the WACC criterion and the reward-to-volatility ratio give conflicting decisions. These are the triangular shaped areas containing points 1 and 3. Under the reward-to-volatility ratio, project 1 would be rejected while accepted by the WACC criterion. Project 3 would be accepted by the reward-to-volatility criterion and rejected by the WACC criterion. The remaining areas would not result in conflicting decisions. Project 2 and any other project plotting above both lines would be accepted by both criteria. Project 4 and any project with (E_p, β_p) coordinates below both lines would be rejected by both criteria.

In order to maintain the current risk level of the surrogate, all projects could be risk equalized using the method of Tuttle and Litzenberger introduced earlier. From Equation (3.16)

$$R_1' = 5 + (1.43/2.08)(13.3 - 5) = 10.8$$

$$R_2' = 5 + (1.43/.57)(15.1 - 5) = 30.3$$

$$R_3' = 5 + (1.43/.31)(10.5 - 5) = 30.4$$

$$R_4' = 5 + (1.43/1.22)(11 - 5) = 12.0$$

These risk-equalized rates are then compared directly with the rate of return for the surrogate, $R_0 = 13.12\%$. The decision regarding investment in each project is not changed since movement occurs along the line representing the slope or reward-to-volatility ratio for each surrogate. This risk-equalization process is shown in Figure 3.4. For projects 2, 3, and 4 this movement corresponds to borrowing at the risk-free rate and investing in the project. Funds from project 1 would be lent to another investment alternative at the risk-free rate to achieve risk-equalization.

3.4 Statistical Analysis and Discussion

Capital asset pricing theory assumes a linear relationship between the return of an individual security and the return of the market. The

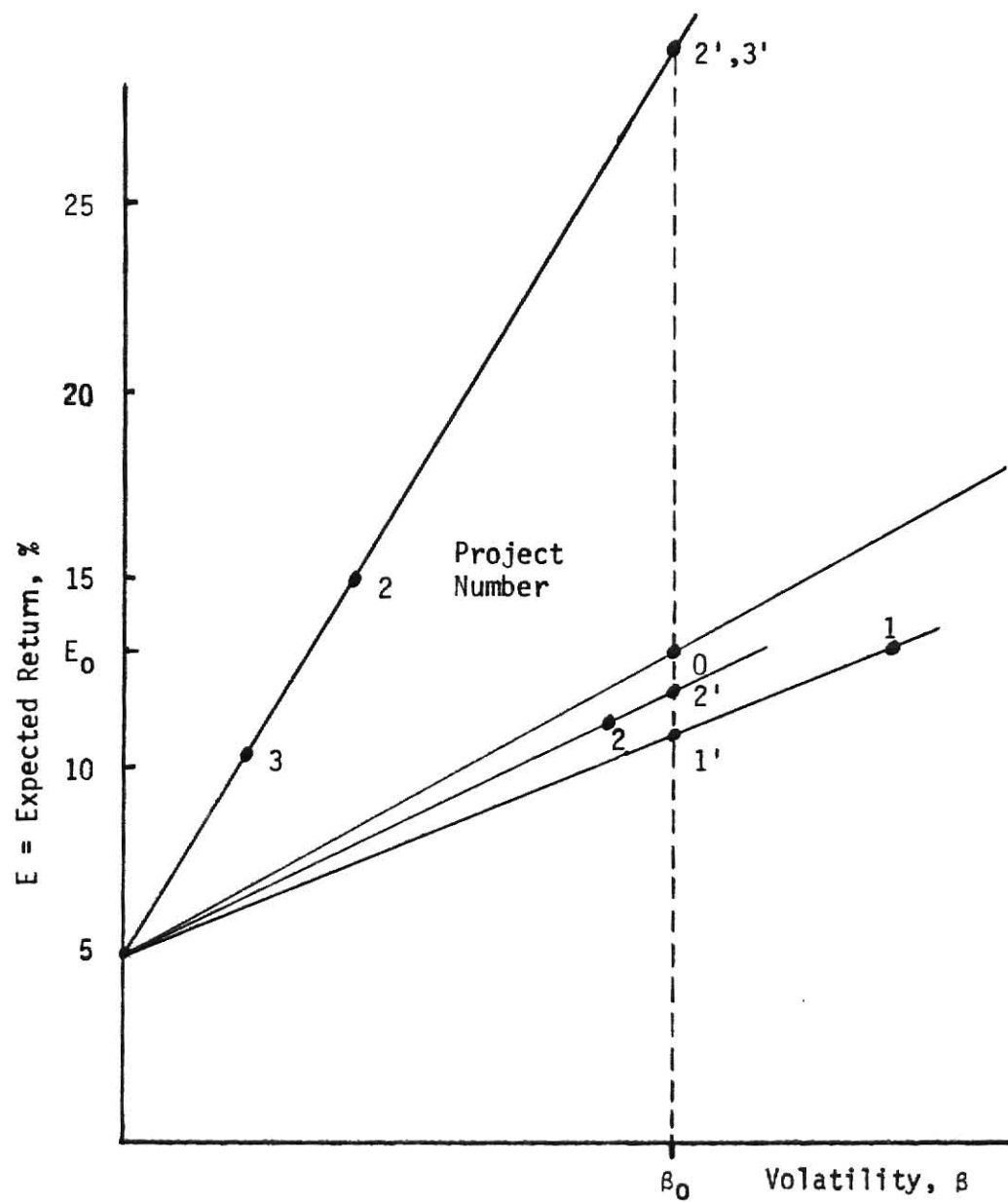


Figure 3.4. Risk-equalization of 4 potential projects

relationship was tested by regression analysis to determine whether or not this linear relationship was true for the brewing industry. The r^2 value, the square of the sample correlation coefficient, is the proportion of the variance of the return of the surrogate security attributed to its linear regression on the return of the market. This value was .725, indicating that nearly three quarters of the variance associated with R_i , the return of the security, was associated with the return of the market, R_M .

Unfortunately, the reward-to-volatility criterion uses only point estimates in its evaluation of projects. Ideally, the variance associated with the random nature of the return of the project and the surrogate security should be incorporated into the decision criterion. No attempt has been made here to develop a statistical model which uses interval estimates in comparing a project against the surrogate firm for an investment decision. A statistical criterion is not used because of the nature of the estimate of the return for the project. Unlike the surrogate security, no sample is involved in determining the expected return and covariance of return with the market for the proposed project. These values are truly only estimated values. To compare the project with the surrogate for acceptance of the project at a specified confidence level, it becomes necessary to pseudo-sample the future project via a simulation technique, generating a number of points for return rates, which would comprise a sample. Unless the project is sampled it is not possible to use a statistical model for capital budgeting.

A possible problem with a statistical model of capital budgeting might occur if the variances of the samples that determine the estimated parameters of the regression analysis, α and β , are large. The confidence

bands about these parameters then would be quite wide. The wider these confidence bands, the less capable the model is in distinguishing a significant difference between possible projects and the surrogate firm.

CHAPTER IV

CONCLUSION

The most significant result of the thesis is the demonstration of the application of capital asset pricing theory to the project selection problem for the private firm. This result was accomplished by using the concept of a surrogate security for the private firm. This surrogate was composed of the publicly owned competitors of the firm. Using the reward-to-volatility criterion projects were compared against the performance of the surrogate security for acceptance. The industry selected for the example was the brewing industry. Four possible projects were compared against the surrogate security as a demonstration of the application of the model. Thus by example, it has been demonstrated that capital asset pricing theory can be applied as a criterion to the capital budgeting selection procedures of the firm.

The capital asset pricing model is applicable to homogeneous industries in which there are publicly owned firms competing with a private firm. The only restriction occurs in industries which are dominated by diversified, conglomerate corporations whose return rate is from varied divisions of the corporation. Inclusion of these conglomerate firms would distort the return rate of the surrogate security.

The capital asset pricing model was demonstrated as superior to the common weighted average cost of capital method. The WACC method of capital budgeting assumes that the future cost of capital is known with certainty and requires all investments, regardless of risk, to

generate a certain specified minimum return rate. The reward-to-volatility criterion confronts both of these problems, acknowledging that the return rates of investments are random variables, and requiring an increased return premium for increased risk. Conflicting decisions result between the WACC criterion and the reward-to-volatility ratio; for example, the WACC criterion often giving an incorrect investment decision, as illustrated in the example of Chapter III.

Only point estimates are used in the reward-to-volatility criterion of capital asset pricing theory. This satisfies the constraint Bower and Lessard (3) posed concerning management's desire for simple, intuitive measures to be used as capital budgeting criteria. However, the reward to volatility criterion implies that in accepting a project, all that can be stated concerning the project is that, on the average, it will perform as an investment superior to the level of return commanded by the firm's competitors from their past and present investments. This is only a survival criterion for the firm. If the firm wants to perform in a manner superior to its competitors with a certain degree of confidence, a statistical model that permits interval estimates must be used. In order to apply the statistical model, it is necessary to pseudo-sample the future project via a simulation technique since no sample is involved in calculation of the expected return for the project.

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APPENDIX

DATA OF PUBLICLY OWNED FIRMS IN BREWING INDUSTRY

Firm	Price	1961			1962			1963			1964			1965		
		P	D	NW	P	D	NW	P	D	NW	P	D	NW	P	D	NW
Anheuser-Busch	4.97	7.00	.17	153.4	5.88	.17	163.1	5.59	.18	170.9	8.16	.19	182.6	11.78	.22	197.6
Carling O'Keefe	8.88	11.35	.32	164.4	10.00	.34	190.9	9.13	.35	216.1	9.75	.40	221.6	7.88	.40	221.8
Falstaff	17.25	19.00	.65	43.9	15.13	.65	47.2	16.31	.70	49.7	21.25	.72	53.7	19.25	.76	57.9
Pabst	4.38	8.38	---	75.5	6.44	.25	79.9	10.31	.25	83.9	16.25	.25	92.1	20.56	.25	100.4
F&M Schaefer	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
Schlitz	-----	-----	---	-----	8.79	.20	-----	14.71	.24	144.9	16.17	.23	147.2	16.67	.26	163.6
		1966			1967			1968			1969			1970		
		P	D	NW	P	D	NW	P	D	NW	P	D	NW	P	D	NW
Anheuser-Busch		14.63	.25	220.5	20.75	.30	255.4	32.00	.37	285.3	36.50	.40	314.1	38.19	.43	358.5
Carling O'Keefe		6.50	.40	221.8	7.00	.40	222.0	11.38	.40	191.9	7.88	.40	195.2	7.13	.40	172.6
Falstaff		12.63	.76	58.6	12.50	.76	54.5	16.13	.40	51.8	9.63	.40	51.6	5.63	---	52.8
Pabst		18.25	.38	112.2	32.00	.50	125.7	48.00	.50	141.3	45.50	.58	158.3	49.63	.65	175.4
F&M Schaefer		-----	---	-----	-----	---	-----	-----	---	-----	51.50	---	43.9	27.00	---	50.7
Schlitz		6.63	.29	172.7	12.33	.33	186.9	17.25	.38	186.7	25.00	.45	194.6	23.00	.47	208.1

P = price in dollars

D = dividend per share in dollars

NW = net worth (dollars x 10⁶)

Firm	1971			1972			1973			1974			1975		
	P	D	NW	P	D	NW	P	D	NW	P	D	NW	P	D	NW
Anheuser-Busch	56.00	.53	414.0	55.25	.58	462.0	32.88	.60	500.8	24.00	.60	537.8	33.75	.64	590.0
Carling O'Keefe	6.88	.40	167.6	5.38	.40	163.2	3.50	.10	166.8	1.88	---	162.3	2.50	---	158.4
Falstaff	8.50	.10	53.7	5.13	---	47.7	2.25	---	41.8	1.25	---	37.9	---	---	---
Pabst	75.63	.80	193.1	75.00	.83	213.1	25.50	.86	226.1	16.50	.91	223.9	21.38	1.01	236.0
F&M Schaefer	16.50	---	55.0	8.88	---	48.6	3.75	---	49.6	2.38	---	50.3	2.88	---	49.0
Schlitz	36.33	.52	230.3	58.38	.55	252.5	56.00	.61	285.5	15.00	.67	315.3	19.13	.68	330.0

P = price in dollars

D = dividend per share in dollars

NW = net worth (dollars X 10⁶)

INDEX AND DIVIDEND-PRICE RATIO OF STANDARD POOR'S 500 COMPOSITE INDEX

I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P	I	D/P
1960		1961		1962		1963		1964		1965		1966		1967	
58.11	3.37	71.55	2.86	63.10	3.39	75.02	3.15	84.75	3.05	92.43	3.06	80.33	3.64	96.47	3.08
1968		1969		1970		1971		1972		1973		1974		1975	
105.04	2.96	90.97	3.51	92.15	3.47	102.09	2.99	117.09	2.71	97.35	3.64	68.56	5.37	90.19	4.11

I = index

D/P = dividend-price

A METHOD OF PROJECT SELECTION FOR THE PRIVATE FIRM

by

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AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

This thesis develops a method for using the "capital asset pricing model", originally proposed by Sharpe, Lintner and Mossin, as a capital budgeting criterion for the private firm. The theory underlying the capital asset pricing model is introduced. The capital asset pricing model is then applied as a capital budgeting criterion to the privately owned firm using the "reward-to-volatility" criterion. Previous investigative work has been limited to the application of capital asset pricing theory to publicly owned firms. The theory is applied here to the private firm's capital budgeting decision by constructing a surrogate "security" composed of the private firm's publicly owned competitors. All projects being considered for investment are then compared to the surrogate "security" for evaluation. The brewing industry is selected to demonstrate this application. Four hypothetical projects are compared to the surrogate "security" resulting in decisions to accept or reject the projects. The reward-to-volatility criterion is then demonstrated as one that is superior to the traditional weighted average cost of capital method for making the proper investment decision. Problems associated with the statistical testing of the capital asset pricing model are discussed with further study being recommended in this area.