EFFECTS OF CRACKING AND WARPING ON THE RESPONSE OF REINFORCED CONCRETE PLATES SUBJECTED TO LATERAL AND ECCENTRIC LOADS

by

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CHAPTER 1

INTRODUCTION

Reinforced concrete plates form component parts of complex structural systems such as shear walls in multistory buildings, components of shell and folded plate roofs, and box girders. In most of these applications the reinforced concrete plate element is subjected to compressive forces which may be applied eccentrically.

Eccentricity in the planar element can result from any of the following: line of load application, support details, variation in flatness, unsymmetrical cross-section of the planar element with respect to the supports, and rotation of floor and roof slabs supported on the planar element.

In some designs of reinforced concrete shear walls and other load bearing walls, the walls are analyzed as compression members with flexure (1)[1]. The required reinforcement is designed for the internal forces resulting from the elastic analysis. Other types of analyses (for shells, folded plate roofs and walls) consider the planar element in a general plane stress state and involve the solution of a two-dimensional continuum problem (1). In these analyses the reinforced concrete has been considered to be an elastic, homogeneous and isotropic material.

However, a realistic analysis of reinforced concrete for internal stress should include several complexities such as the non-linearity in the behavior of materials and the influence of progressive cracking of concrete under increasing load. Cracks cause a redistribution of stress in the concrete. Also, the effect on the response of such planar elements due to

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[1]Numbers in parentheses refer to references listed in the Bibliography.

the eccentricity of the applied loads should be considered. In addition, plate bending is characterized by the warping of the plate into a non-developable surface.

The purpose of this paper is to present an analytical approach to predict the response of a reinforced concrete plate subjected to normal loads and to eccentrically applied, uni-directional compressive loads taking into account the effects of the cracking of the concrete. The effects of different warping stiffnesses on the plate response will also be presented.

Initially, an analytical approach for predicting the response of a reinforced concrete plate with a uniformly distributed lateral load is presented. This approach is then extended to the case of a simply supported reinforced concrete plate with an eccentrically applied uni-directional compressive loading. The study is limited to the service load range. The analysis involves the use of a two-dimensional continuum problem. With the non-homogeneous nature of reinforced concrete and the cracking of concrete, a classical solution of the problem in closed form is not thought to be possible. However, a solution can be obtained by numerical methods such as the finite difference or finite element.

The finite difference method is used in this presentation. Computer programs are developed for the solution of the problems. Numerical results are presented and comparison made with published solutions of equivalent problems.

CHAPTER 2

LITERATURE REVIEW

Literature on reinforced concrete subjected to in-plane compressive loads is very much limited. However, some work has been reported on reinforced concrete plates subjected to lateral loading with effects of cracking and the non-linearity of reinforced concrete taken into account.

In 1923 Huber (2), working with concrete as an orthotropic material, first formulated the linear differential equation which governs the small deflection behavior of such an orthotropic plate. His equation was:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H_o \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q(x,y) \quad \text{- - - - - - - - - - - - - -} \quad (2.1)$$

where $q(x,y)$ is the lateral load function. He recommended the following rigidities:

$$D_x = \frac{E_c I_x}{1 - v_c^2} \qquad D_y = \frac{E_c I_y}{1 - v_c^2} \qquad H_o = \sqrt{D_x D_y}$$

where v_c is Poisson's ratio for concrete, I_x, I_y are the transformed moments of inertia in the x and y coordinate directions respectively, E_c is the elastic modulus for concrete, and H_o is the warping stiffness.

Timoshenko and Winowsky-Krieger (2) have done theoretical work on the small deflection behavior of elastic isotropic plates under lateral and in-plane loads. The differential equation for the deflection of a plate under lateral and in-plane loads is given as:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + q\right) \quad \text{- - -} \quad (2.2)$$

where N_x,N_y are the in-plane loads per unit length in the x and y directions respectively, N_{xy} is the shearing force in the y direction per unit length of section of plate perpendicular to the x axis, q is the intensity of the uniformly distributed lateral load, and D is the flexural rigidity of the plate (see Figure 2.1 for positive signs of these forces).

Joeffriet and McNeice (3) reported a finite element analysis on the response of reinforced concrete slabs subjected to lateral loads in the service load range. Their analysis included the effects of cracked regions. They gave suggested procedures for the following: (1) a means of taking account of the orientation of the cracks with respect to the coordinate system of the slab; (2) a reasonably good estimate of the effect of the rigidity of a cracked region at moment levels greater than the cracking moment; and, (3) an estimate of the effect on rigidity of steel orientation with respect to the crack direction. For flexural plate rigidities mention was made of the suggestion by Huber (2) in the case where the crack direction coincides with either direction of reinforcement. Joeffriet and McNeice suggested that since for low steel percentages (which is the case for reinforced concrete plates) the reinforcement contributes little to the resisting moments of the uncracked section, an uncracked region may be treated as isotropic regardless of the orientation and orthotropy of reinforcement. In their finite element analysis a unit load was applied and scaled until only one element cracked governed by the magnitude of the maximum principal moment. The stiffness of the cracked element was appropriately changed. The unit load was applied again and scaled until the next crack observation. The procedure was repeated until the desired load level was reached. The analysis allowed for cracking in two orthogonal directions if the minor principal moment in an element reached the cracking moment value. However, they did not discuss the significance of the warping stiffness.

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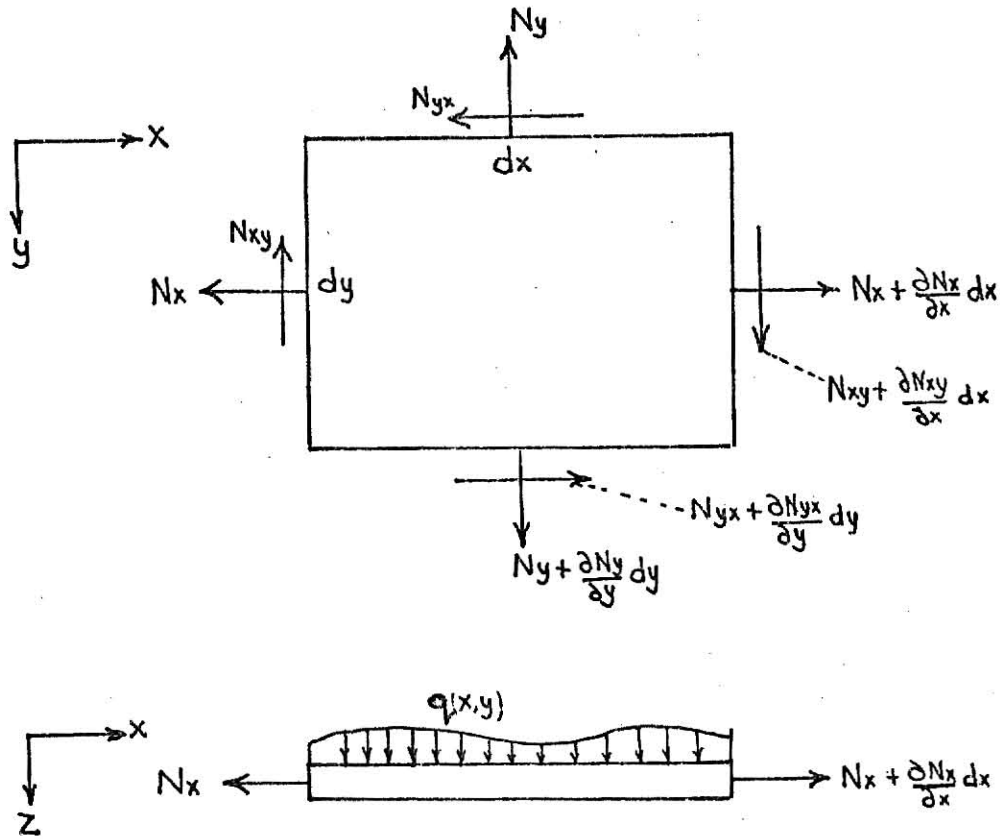

FIG. 2.1 FORCES ACTING ON RECTANGULAR PLATE ELEMENT.

Several non-linear stress analyses of reinforced concrete slabs with lateral loading have been reported, e.g. (4,5). In each of these cases the stress-strain relationship for concrete was assumed to be elastic-perfectly plastic in compression and elastic and brittle in tension. The stress-strain relationship for reinforcing steel was assumed to be elastic-perfectly plastic.

Vanderbilt, Sozen and Siess (6) reported the deflections of multi-panel reinforced concrete floor slabs in which finite difference solutions were compared to experimental results.

Cervenka and Gestle (1,7) at the University of Colorado made studies on reinforced concrete panels. The studies were on the inelastic analysis of reinforced concrete planar elements subjected to in-plane loading. However, the eccentricity of the loading was not considered. The following assumptions were made for an analytical model of reinforced concrete capable of describing the interaction of uncracked or cracked concrete and reinforcing steel in any number of arbitrary directions: (1) the uncracked concrete is an isotropic, homogeneous material in plane stress state; (2) the cracked concrete is anisotropic and capable of resisting only stresses normal to the crack direction; (3) the crack direction is perpendicular to the principal tension in the concrete just prior to the crack formation; and, (4) both cracked and uncracked concrete are in a uniaxial stress state. Different component material stiffness matrices were obtained for (1) elastic uncracked concrete, (2) plastic uncracked concrete, (3) elastic cracked concrete, (4) plastic cracked concrete, (5) elastic reinforcement, and (6) plastic reinforcement. A numerical analysis was performed by load increment using the finite element method. After comparing numerical solutions to experimental results, they made the following conclusions: (1) crack propagation and plasticity of materials are the most important non-linear effects in the problems with monotonically increasing loads; (2) beam analysis of shear walls and other load bearing walls over estimates the stiffness of the uncracked panel; and, (3) beam analysis of the cracked panel based on elastic transformed cross-section excluding tension concrete well represents the average stiffness of the cracked panel.

Athavichitjanyaraks (8) presented the development of an analytical approach for predicting the behavior of a simply supported rectangular plate made of a linearly elastic, homogeneous and isotropic material and subjected to uniformly applied unidirectional compressive load with equal end eccentricities. He found that for the given boundary conditions there were three different classes of solution to the governing differential equation:

$$\nabla^4 w = - N_x \frac{\partial^2 w}{\partial x^2} \qquad (2.3)$$

where N_x is the compressive load per unit length in the x direction. These three cases depended on the value of N_x and its relationship to the plate buckling load. Numerical solutions to the equation indicated the following effects of the plate aspect ratio, $\frac{a}{b}$, on the lateral deflections of the plate: (1) the maximum deflection for a given load occurs for an aspect ratio, $\frac{a}{b} = 1$; (2) the curvature (for bending about the y-axis) is single valued for $\frac{a}{b} = 0.5$ or 1, but triple valued for $\frac{a}{b} \geq 2$; and, (3) for $\frac{a}{b} \geq 2$ the maximum deflection does not occur at the midpoint. Results of experiments on plates with various aspect ratios generally agreed with the theoretical results. He concluded that the primary geometric parameter affecting the behavior of simply supported rectangular plates under eccentric compressive loads is the length to width ratio and that the eccentricity of the loading has an effect on the plate behavior.

CHAPTER 3

DEVELOPMENT OF EQUATIONS

3.1 - Differential Equation For Bending of a Laterally Loaded Reinforced Concrete Plate

The following assumptions have been made in the development of the equations:

1. Deflections are small compared to the thickness of the plate;
2. The stress-strain relationship for concrete is assumed to be elastic-perfectly plastic in compression and elastic and brittle in tension (See Figure 3.1);
3. The stress-strain relationship for reinforcing steel is assumed elastic-perfectly plastic in both tension and compression;
4. The reinforcing steel may be in one or two layers and has the same amount in either coordinate direction;
5. The cracking direction of the concrete coincides with the longitudinal or transverse coordinate directions;
6. Depth of cracking is the same in both coordinate directions; and,
7. Dowel action of the reinforcement is neglected.

Equilibrium Equation

Consider a plate that is subjected to a uniformly distributed load, q, applied perpendicular to the plane of the plate. The plate is of uniform thickness, h. The x-y plane is taken as the middle plane of the plate with the z-axis perpendicular to the plate. The forces and moments acting on an element of size dxdy in the middle plane of the plate are shown in Figure 3.2. The forces and moments are per unit length.

For the element shown, three equilibrium equations may be written (2): (1) summing up forces in the z-direction and equating them to zero,

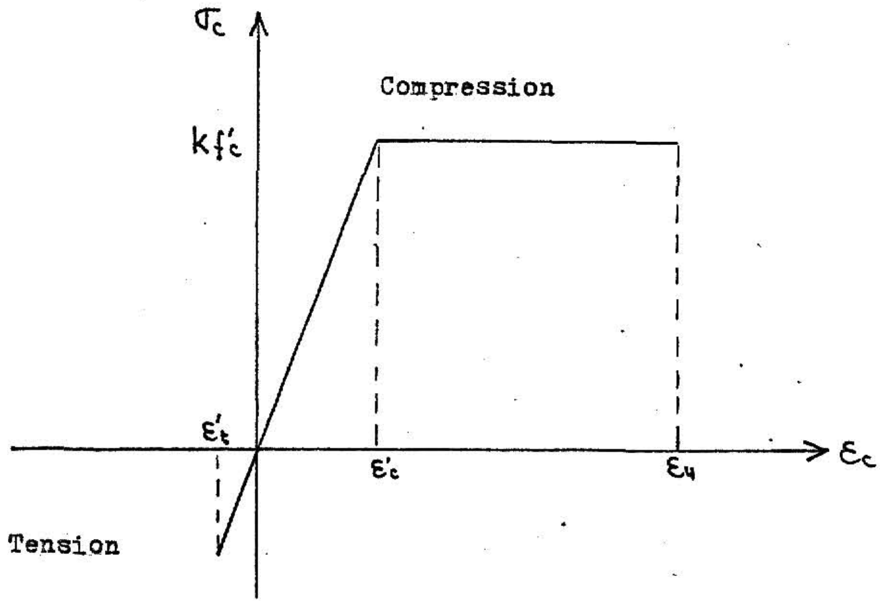

FIG.3.1 STRESS-STRAIN RELATIONSHIP FOR CONCRETE

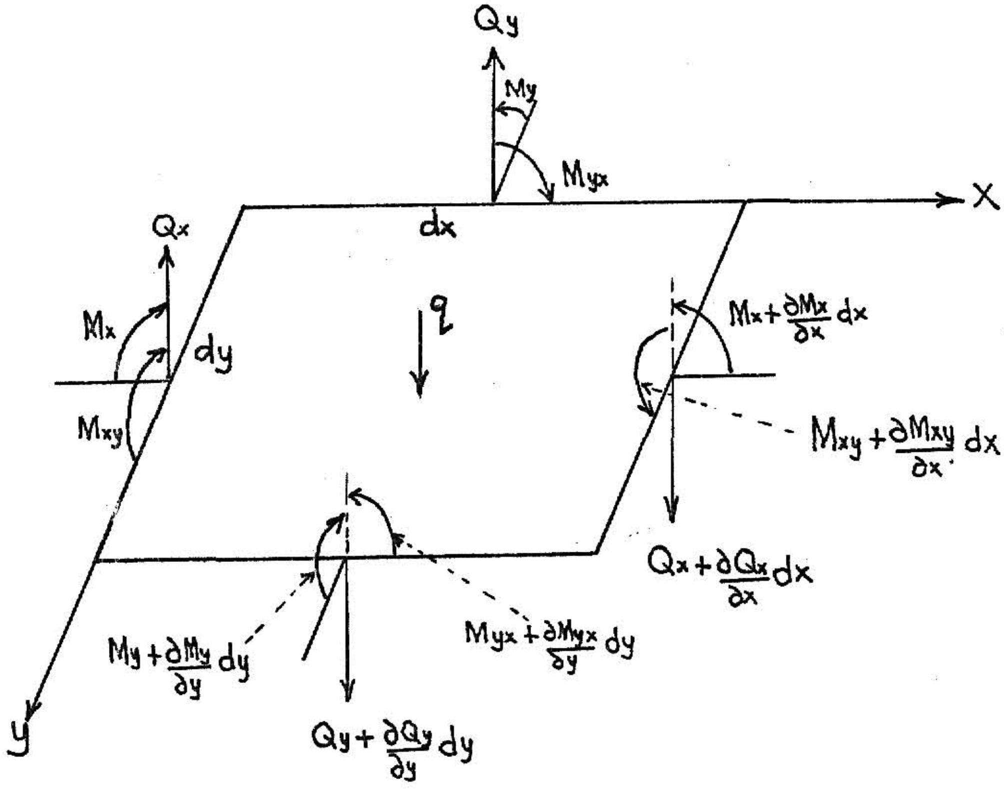

FIG.3.2 FORCES ACTING ON AN ELEMENT IN THE MIDDLE PLANE OF PLATE

$$Q_x dy - (Q_x + \frac{\partial Q_x}{\partial x} dx)\, dy + Q_y dx - (Q_y + \frac{\partial Q_y}{\partial y} dy)\, dx - q dx dy = 0$$

Simplifying, $\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$ - (3.1)

(2) Taking moments about the x-axis and neglecting the moment of the load, q, and the moment due to the change in the force Q_y since these are small quantities of higher order, we obtain after simplifying:

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \quad (3.2)$$

(3) Similarly, taking moments about the y-axis, we get:

$$\frac{\partial M_{yx}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x = 0 \quad (3.3)$$

Substituting equations (3.2) and (3.3) into equation (3.1) and eliminating Q_x and Q_y, we obtain:

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q \quad (3.4)$$

Since $T_{xy} = T_{yx}$, $M_{yx} = -M_{xy}$, equation (3.4) can be represented as:

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - \frac{2\partial^2 M_{xy}}{\partial x \partial y} = -q \quad (3.5)$$

Equation (3.5) is the equilibrium equation.

Uncracked Section

To represent equation (3.5) in terms of w, the deflection of the plate in the z-direction, consider the element cut out of the plate by two pairs of planes parallel to the xz and yz planes as shown in Figure 3.3. NN is the neutral surface. Let $\frac{1}{r_x}$, $\frac{1}{r_y}$ denote the bending curvatures and $\frac{1}{r_{xy}}$ the

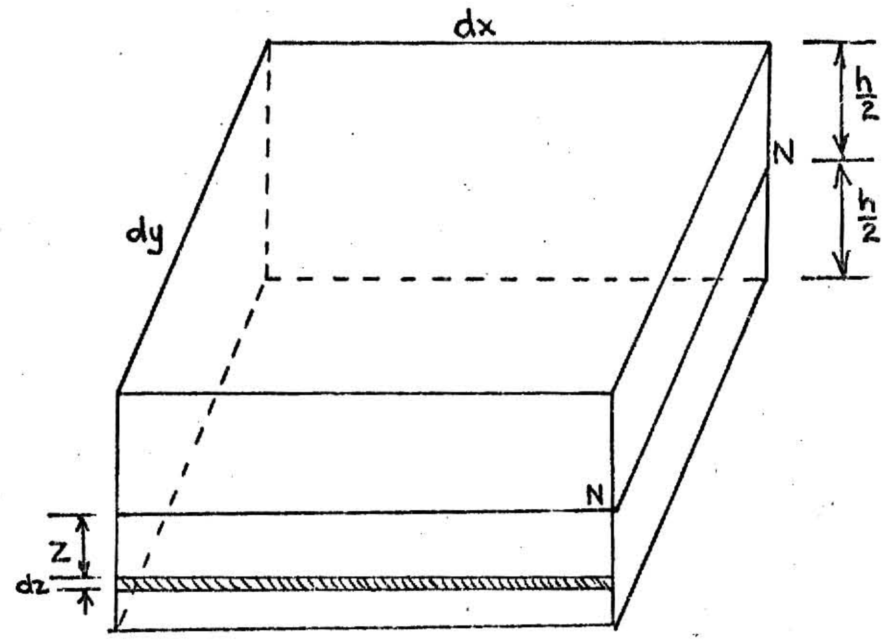

FIG. 3.3 ELEMENT OF PLATE SHOWING NEUTRAL SURFACE.

warping curvature of this neutral surface. Then the unit elongations, ε_x, ε_y of a fiber at a point distant Z from NN are given by $\varepsilon_x = \frac{Z}{r_x}$, $\varepsilon_y = \frac{Z}{r_y}$.

For small deflections, we can write (2):

$$\frac{1}{r_x} = -\frac{\partial^2 w}{\partial x^2} \qquad \frac{1}{r_y} = -\frac{\partial^2 w}{\partial y^2} \qquad \frac{1}{r_{xy}} = \frac{\partial^2 w}{\partial x \partial y}$$

Therefore, the unit elongations are given by (2):

$$\varepsilon_x = -Z \frac{\partial^2 w}{\partial x^2} \tag{3.6a}$$

$$\varepsilon_y = -Z \frac{\partial^2 w}{\partial y^2} \tag{3.6b}$$

$$\gamma_{xy} = -2Z \frac{\partial^2 w}{\partial x \partial y} \tag{3.6c}$$

The bending moments per unit length in the elemental strip are given by:

$$M_x = \int_{-h/2}^{h/2} \sigma_x Z dz \tag{3.7a}$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y Z dz \tag{3.7b}$$

$$M_{xy} = -M_{yx} = -\int_{-h/2}^{h/2} \tau_{xy} Z dz \tag{3.7c}$$

To determine the stresses in the reinforced concrete consider the effects of the steel and concrete separately and superimpose these effects.

<u>Steel Stresses</u>

For the steel, let

$$\sigma_{xsi} = E_s \varepsilon_{xsi} \tag{3.8a}$$

$$\varepsilon_{xsi} = -\bar{Z}_i \frac{\partial^2 w}{\partial x^2} \tag{3.8b}$$

$$\sigma_{ysi} = E_s \varepsilon_{ysi} \qquad (3.8c)$$

$$\varepsilon_{ysi} = -\bar{Z}_i \frac{\partial^2 w}{\partial y^2} \qquad (3.8d)$$

where $i = i^{th}$ layer of reinforcement; $i = 1$ or 2; $\bar{Z}_i$ = distance from i^{th} layer of reinforcement to the neutral surface of the plate; σ_{xsi}, σ_{ysi} are the steel stresses in the i^{th} layer of reinforcement in the x and y directions respectively, and ε_{xsi}, ε_{ysi} are the steel strains in the i^{th} layer of reinforcement in the x and y directions respectively.

Concrete Stresses

Assuming the linear stress-strain relationship for the concrete as shown in Figure 3.1, the concrete stress, f_c, is given by:

$f_c = E_c \varepsilon_c \qquad \varepsilon_c < \varepsilon'_c \qquad$ (compression)

$f_c = kf'_c \qquad \varepsilon_c \geq \varepsilon'_c \qquad$ (compression)

$f_c = E_c \varepsilon_t \qquad \varepsilon_t < \varepsilon'_t \qquad$ (tension)

where E_c = Young's modulus for concrete

ε_c = compressive concrete strain

ε_t = tensile concrete strain

$\varepsilon'_c, \varepsilon'_t$ = limiting compressive and tensile strains respectively

f'_c = cylindrical strength of concrete

k = a factor giving the upper limit of the working range

For the concrete stresses, let (2):

$\sigma_{xc} = E'_x \varepsilon_{xc} + E'' \varepsilon_{yc}$

$\sigma_{yc} = E'_y \varepsilon_{yc} + E'' \varepsilon_{xc}$

$\tau_{xy} = G \gamma_{xy}$

where $\varepsilon_{xc}, \varepsilon_{yc}$ are the concrete strains in the x and y directions respectively

γ_{xy} = concrete shear strain

Hence:

$$\sigma_{xc} = -Z\left(E'_x \frac{\partial^2 w}{\partial x^2} + E'' \frac{\partial^2 w}{\partial y^2}\right) \quad (3.9a)$$

$$\sigma_{yc} = -Z\left(E'_y \frac{\partial^2 w}{\partial y^2} + E'' \frac{\partial^2 w}{\partial x^2}\right) \quad (3.9b)$$

$$T_{xy} = -2GZ \frac{\partial^2 w}{\partial x \partial y} \quad (3.9c)$$

Superimposing the effects of the steel and concrete, Equation 3.7a becomes:

$$M_x = \int_{-h/2}^{h/2} \sigma_{xc}\, Z dz + \sum_{i=1}^{2} (\rho_i h)\, E_s\, \epsilon_{xsi}\, \overline{Z}_i \quad (3.10a)$$

$$= -\frac{h^3}{12}\left[E'_x \frac{\partial^2 w}{\partial x^2} + E'' \frac{\partial^2 w}{\partial y^2}\right] - E_s h \frac{\partial^2 w}{\partial x^2} \sum_{i=1}^{2} \rho_i \overline{Z}_i^{\,2}$$

where $(\rho_i h)$ is the area of steel per unit length in the i^{th} layer,

$\rho_i = \frac{A_{sti}}{bh}$; A_{sti} = total amount of steel in i^{th} layer.

Similarly, equation 3.7b becomes:

$$M_y = -\frac{h^3}{12}\left[E'_y \frac{\partial^2 w}{\partial y^2} + E'' \frac{\partial^2 w}{\partial x^2}\right] - E_s h \frac{\partial^2 w}{\partial y^2} \sum_{i=1}^{2} \rho_i \overline{Z}_i^{\,2} \quad (3.10b)$$

Neglecting the dowel action of the steel, Equation 3.7c becomes:

$$M_{xy} = 2G \frac{h^3}{12} \frac{\partial^2 w}{\partial x \partial y} \quad (3.10c)$$

Now let:

$$E'_x = \frac{E_{xc}}{1 - v^2} \qquad E'_y = \frac{E_{yc}}{1 - v^2} \qquad E'' = \frac{\lambda v}{1 - v^2} \sqrt{E_{xc} E_{yc}}$$

$$G = \frac{\lambda \sqrt{E_{xc} E_{yc}}}{2(1+v)}$$

These are Huber's assumptions (2) with the addition of the warping parameter, λ, where

v = Poisson's ratio for concrete

E_{xc}, E_{yc} = Young's elastic moduli for concrete in the x and y directions respectively

λ = parameter for the influence of warping on the torsional stiffness, $0 \leq \lambda \leq 1$ (Huber assumed $\lambda = 1$)

Therefore:

$$M_x = -(D_{xu} \frac{\partial^2 w}{\partial x^2} + D_{1u} \frac{\partial^2 w}{\partial y^2}) \text{ - - - - - - - - - - - - - - - - - - - } (3.11a)$$

$$M_y = -(D_{yu} \frac{\partial^2 w}{\partial y^2} + D_{1u} \frac{\partial^2 w}{\partial x^2}) \text{ - - - - - - - - - - - - - - - - - - - } (3.11b)$$

$$M_{xy} = 2D_{xyu} \frac{\partial^2 w}{\partial x \partial y} \text{ - } (3.11c)$$

where[2] $D_{xu} = D_{x1u} + D_s$

$D_{yu} = D_{y1u} + D_s$

$D_{xyu} = \lambda \left(\frac{1-v}{2}\right) \sqrt{D_{x1u} D_{y1u}}$

$D_{1u} = \lambda v \sqrt{D_{x1u} D_{y1u}}$

$D_{x1u} = \frac{E_{xc}}{1 - v^2} \frac{h^3}{12}$

$D_{y1u} = \frac{E_{yc}}{1 - v^2} \frac{h^3}{12}$

$D_s = E_s h \sum_{i=1}^{n} \rho_i \bar{Z}_i^2 \qquad n = 1 \text{ or } 2$

Hence, for the uncracked section, the equilibrium equation (3.5) becomes:

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[2]The subscript "u" denotes uncracked section.

$$D_{xu} \frac{\partial^4 w}{\partial x^4} + D_{yu} \frac{\partial^4 w}{\partial y^4} + 2\lambda \sqrt{D_{xlu} D_{ylu}} \frac{\partial^4 w}{\partial x^2 \partial y^2} = q \qquad (3.12)$$

Cracked Section

Referring to Figure 3.4, C_n is the depth of the compression face to the neutral surface of the cracked section, $\bar{c}_i$ is the distance from the i^{th} layer of reinforcement to the neutral surface.

As shown in the section Equilibrium Equation, the bending moments per unit length in an elemental strip of the plate are given by:

$$M_x = \int \sigma_x Z dz$$

$$M_y = \int \sigma_y Z dz$$

$$M_{xy} = -M_{yx} = -\int T_{xy} Z dz$$

Superimposing the effects of steel and concrete:

$$M_x = \int_0^{C_n} (E'_x \epsilon_{xc} + E'' \epsilon_{yc})\, Zdz + \sum_{i=1}^{2} (\rho_i h)\, E_s \epsilon_{xsi} \bar{c}_i$$

$$= -\frac{C_n^3}{3(1-v^2)} (E_{xc} \frac{\partial^2 w}{\partial x^2} + \lambda v \sqrt{E_{xc} E_{yc}} \frac{\partial^2 w}{\partial y^2} - E_s h \frac{\partial^2 w}{\partial x^2} \sum_{i=1}^{2} \rho_i \bar{c}_i^2 \qquad (3.13a)$$

Similarly,

$$M_y = -\frac{C_n^3}{3(1-v^2)} (E_{yc} \frac{\partial^2 w}{\partial y^2} + \lambda v \sqrt{E_{xc} E_{yc}} \frac{\partial^2 w}{\partial x^2} - E_s h \frac{\partial^2 w}{\partial y^2} \sum_{i=1}^{2} \rho_i \bar{c}_i^2 \qquad (3.13b)$$

$$M_{xy} = 2G \frac{C_n^3}{3} \frac{\partial^2 w}{\partial x \partial y} = \frac{\lambda \sqrt{E_{xc} E_{yc}}}{1 + v} \frac{C_n^3}{3} \frac{\partial^2 w}{\partial x \partial y} \qquad (3.13c)$$

Hence,[3]

$$M_x = -(D_{xcr} \frac{\partial^2 w}{\partial x^2} + D_{1cr} \frac{\partial^2 w}{\partial y^2}) \qquad (3.14a)$$

$$M_y = -(D_{ycr} \frac{\partial^2 w}{\partial y^2} + D_{1cr} \frac{\partial^2 w}{\partial x^2}) \qquad (3.14b)$$

[3]The subscript "cr" denotes a cracked section.

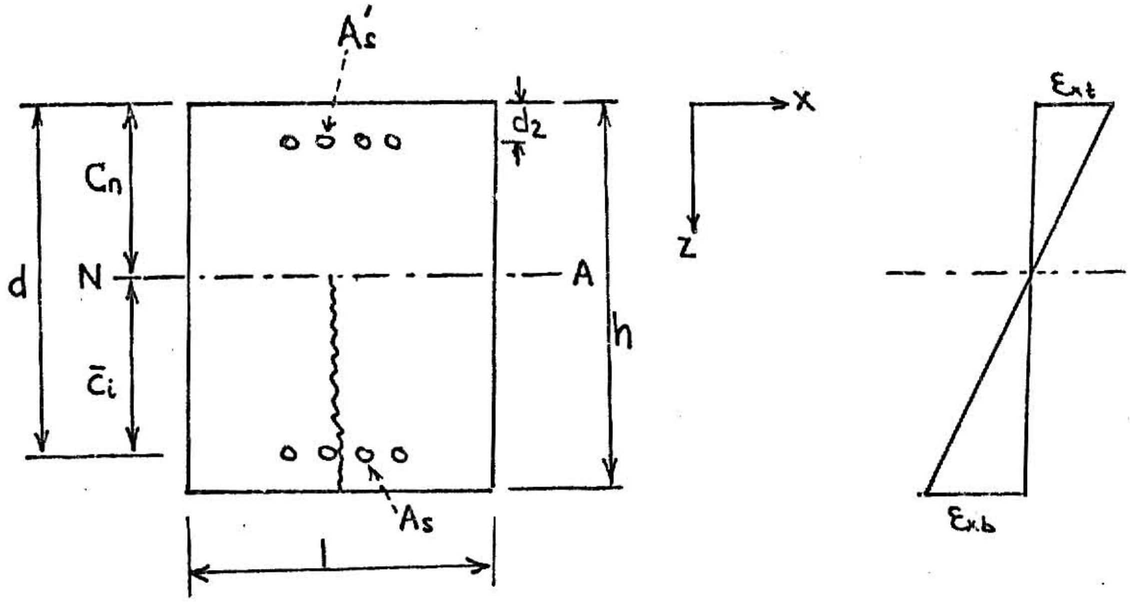

FIG. 3.4 CRACKED SECTION SHOWING NEUTRAL AXIS.

$$M_{xy} = 2D_{xycr} \frac{\partial^2 w}{\partial x \partial y} \quad \text{- -} \quad (3.14c)$$

where $D_{xcr} = D_{x1cr} + D_s$

$$D_{ycr} = D_{y1cr} + D_s$$

$$D_{xycr} = \lambda \frac{(1-v)}{2} \sqrt{D_{x1cr} D_{y1cr}}$$

$$D_{1cr} = \lambda v \sqrt{D_{x1cr} D_{y1cr}}$$

$$D_{x1cr} = \frac{E_{xc}}{1-v^2} \frac{C_n^3}{3}$$

$$D_{y1cr} = \frac{E_{yc}}{1-v^2} \frac{C_n^3}{3}$$

$$D_s = E_s h \sum_{i=1}^{n} \rho_i \bar{c}_i^2$$

Hence, for a cracked section the equilibrium equation (3.5) becomes:

$$D_{xcr} \frac{\partial^4 w}{\partial x^4} + D_{ycr} \frac{\partial^4 w}{\partial y^4} + 2\lambda \sqrt{D_{x1cr} D_{y1cr}} \frac{\partial^4 w}{\partial x^2 \partial y^2} = q \quad \text{- - - - - - - -} \quad (3.15)$$

<u>Depth of Compression Face to Neutral Axis of Cracked Section, C_n</u>

Figure 3.4 shows a cracked section. The concrete has been reinforced in two layers. A_s, A_s' are the areas of steel reinforcement per unit length in the two layers. Using the transformed area method:

$$C_n \cdot 1 \cdot \frac{C_n}{2} + (\underline{n}-1)\, A_s' \, (C_n - d_2) = \underline{n}\, (A_s)\, (d - C_n)$$

$$\frac{C_n^2}{2} + [(\underline{n}-1)\, A_s' + \underline{n} A_s]\, C_n - (\underline{n}-1)\, A_s' d_2 - \underline{n} A_s d = 0$$

For $A_s = A_s'$

$$\frac{C_n^2}{2} + (2\underline{n}-1)\, A_s C_n - A_s[(\underline{n}-1) d_2 + \underline{n} d] = 0$$

Hence,

$$C_n = -(2\underline{n}-1)p_i d + \frac{1}{2}\sqrt{4(2\underline{n}-1)^2 p_i{}^2 d^2 + 8[(\underline{n}-1)d_2 + \underline{n}d]p_i d} \quad (3.16)$$

where $\underline{n} = \dfrac{E_s}{E_c}$

$$p_i = \frac{A_{si}}{d}$$

3.2 - Finite Difference Representation of Bending Equation

Using the central difference operator with an error of order $(\Delta x)^2$ or $(\Delta y)^2$ the following derivatives may be expressed in finite difference for a general point (i,j) as follows (9):

$$\frac{\partial w}{\partial x} = \frac{1}{2\Delta x}(w_{i+1,j} - w_{i-1,j}) \quad (3.17a)$$

$$\frac{\partial w}{\partial y} = \frac{1}{2\Delta y}(w_{i,j+1} - w_{i,j-1}) \quad (3.17b)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{(\Delta x)^2}(w_{i+1,j} - 2w_{i,j} + w_{i-1,j}) \quad (3.17c)$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{(\Delta y)^2}(w_{i,j+1} - 2w_{i,j} + w_{i,j-1}) \quad (3.17d)$$

$$\frac{\partial^4 w}{\partial x^4} = \frac{1}{(\Delta x)^4}(w_{i+2,j} - 4w_{i+1,j} + 6w_{i,j} - 4w_{i-1,j} + w_{i-2,j}) \quad (3.17e)$$

$$\frac{\partial^4 w}{\partial y^4} = \frac{1}{(\Delta y)^4}(w_{i,j+2} - 4w_{i,j+1} + 6w_{i,j} - 4w_{i,j-1} - w_{i,j-2}) \quad (3.17f)$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \frac{1}{(\Delta y)^2(\Delta x)^2}(w_{i+1,j+1} - 2w_{i+1,j} + w_{i+1,j-1} - 2w_{i,j+1} + 4w_{i,j} - 2w_{i,j-1} + w_{i-1,j+1} - 2w_{i-1,j} + w_{i-1,j-1}) \quad (3.17g)$$

Substituting the last three of Equations 3.17 in Equation 3.12 and setting $\Delta x = \Delta y = H$ we get:

$$n w_{i,j+2} - 4(n+p) w_{i,j+1} + (6m+6n+8p) w_{i,j} - 4(n+p) w_{i,j-1} +$$
$$n w_{i,j-2} + m w_{i+2,j} - 4(m+p) w_{i+1,j} - 4(m+p) w_{i-1,j} +$$
$$m w_{i-2,j} + 2p(w_{i+1,j+1} + w_{i+1,j-1} + w_{i-1,j-1} + w_{i-1,j+1}) = q \quad - - (3.18)$$

where $m = \frac{D_{xu}}{H^4}$ $\quad n = \frac{D_{yu}}{H^4}$ $\quad p = \frac{\lambda}{H^4} \sqrt{D_{x1u} D_{y1u}}$

The left hand side of Equation 3.18 may be represented by the module diagram shown in Figure 3.5 for any point (i,j) in the slab.

For a cracked section the values of m, n, and p in Equation 3.18 and in the module diagram are replaced by m', n', and p', where:

$$m' = \frac{D_{xcr}}{H^4} \qquad n' = \frac{D_{ycr}}{H^4} \qquad p' = \frac{\lambda}{H^4} \sqrt{D_{x1cr} D_{y1cr}}$$

3.3 - Boundary Conditions

<u>Simply Supported Plate</u>

For a plate of size a x b (see Figure 3.6) simply supported along all edges on rigid supports and subjected to uniformly distributed lateral load there will be no deflections and no bending moments along the edges.

Hence, for $x = 0,a$ $\qquad w = 0$ and $M_x = 0 = \frac{\partial^2 w}{\partial x^2}$

for $y = 0,b$ $\qquad w = 0$ and $M_y = 0 = \frac{\partial^2 w}{\partial y^2}$

These can be represented in finite differences for the point (i,j) as follows:

$$\frac{\partial^2 w}{\partial x^2} = 0 = \frac{w_{i-1,j} - 2w_{i,j} + w_{i+1,j}}{H^2} \text{ when } x = 0,a$$

$$\frac{\partial^2 w}{\partial y^2} = 0 = \frac{w_{i,j-1} - 2w_{i,j} + w_{i,j+1}}{H^2} \text{ when } y = 0,b$$

Therefore, $w_{i+1,j} = -w_{i-1,j}$ when $x = 0,a$ - - - - - - - - - - - - - - (3.19a)

$w_{i,j+1} = -w_{i,j-1}$ when $y = 0,b$ - - - - - - - - - - - - - - (3.19b)

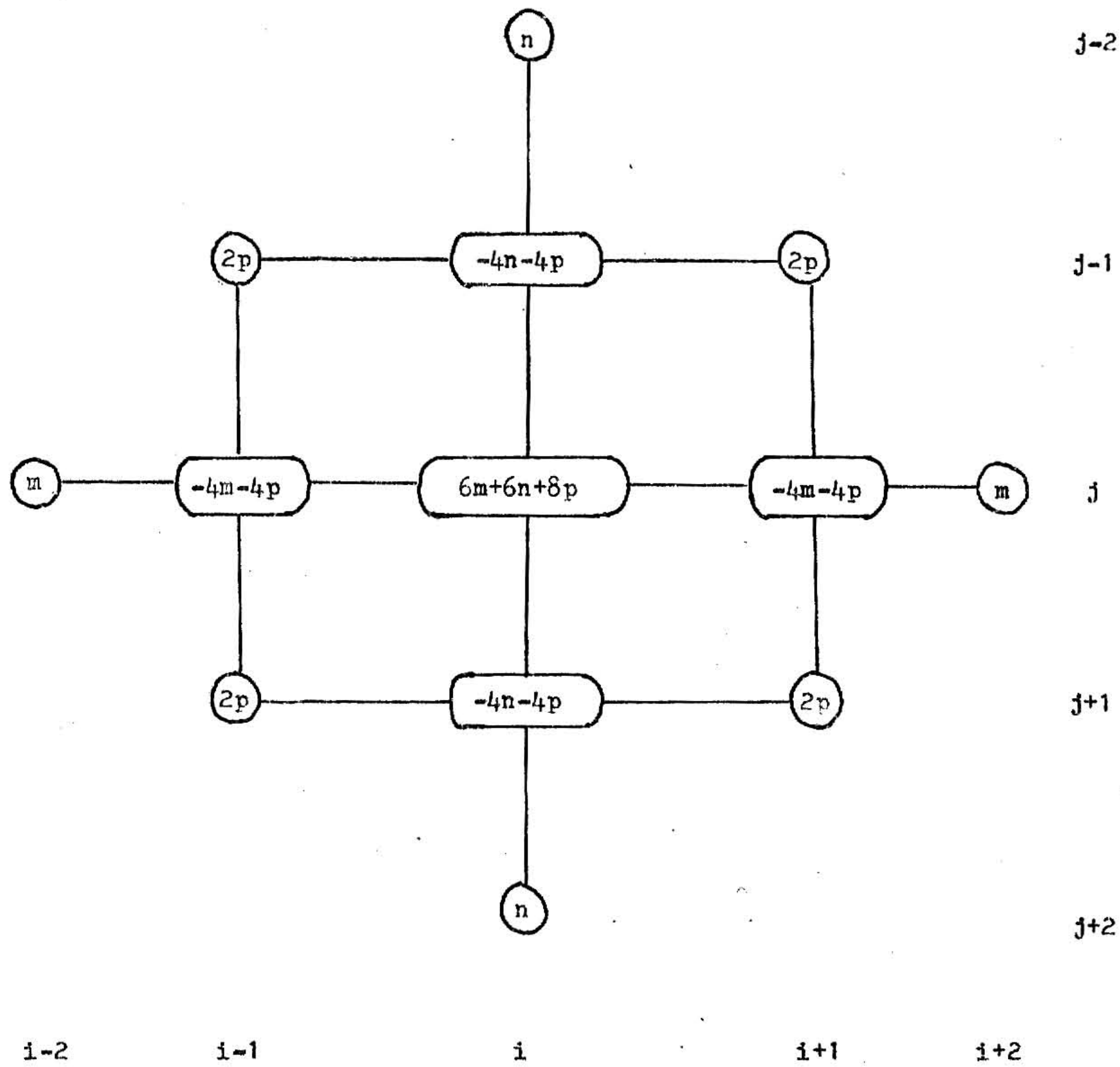

FIG. 3.5 MODULE DIAGRAM OF EQUATION 3.18

Fixed Edge Plate

For a plate fixed along all edges against rotation and supported on rigid supports there will be zero deflection and zero slope along the edges.

Hence, for $x = 0,a$ $\quad w = 0$ and $\frac{\partial w}{\partial x} = 0$

for $y = 0,b$ $\quad w = 0$ and $\frac{\partial w}{\partial y} = 0$

Expressed in finite differences for the point (i,j):

$$\frac{\partial w}{\partial x} = 0 = \frac{w_{i+1,j} - w_{i-1,j}}{2H} \quad \text{when } x = 0,a$$

$$\frac{\partial w}{\partial y} = 0 = \frac{w_{i,j+1} - w_{i,j-1}}{2H} \quad \text{when } y = 0,b$$

Therefore, $w_{i+1,j} = w_{i-1,j}$ when $x = 0,a$ - - - - - - - - - - - - - - - (3.20a)

$w_{i,j+1} = w_{i,j-1}$ when $y = o,b$ - - - - - - - - - - - - - - - (3.20b)

Due to symmetry of loading and support conditions, one quarter of the plate can be analyzed so as to reduce the number of simultaneous equations to be solved. The numbering order of points in the finite difference network for one quarter of the plate used in the analysis is shown in Figure 3.6. An equation similar to Equation 3.18 should be written for each of the points.

For a general point, TM + J, in the network where $2 \leq T \leq N-2$ and $3 \leq J \leq M-2$ the equation is as follows for an uncracked section:

$$(6m+6n+8p)w_I - (4m+4p)w_{I-1} - (4m+4p)w_{I+1} - (4n+4p)w_{I-M} - (4n+4p)w_{I+M} +$$
$$mw_{I-2} + mw_{I+2} + n(w_{I-2M}+w_{I+2M}) + 2p(w_{I-M-1}+w_{I-M+1}+w_{I+M-1}+w_{I+M+1}) = q \quad \text{- (3.21)}$$

where $I = TM + J$.

For the other points in the network, the equation will be modified due to the boundary conditions (Equations 3.19 and 3.20) and the symmetry conditions. For example, for a simply supported plate the equation for the point M (see Figure 3.6) is as follows:

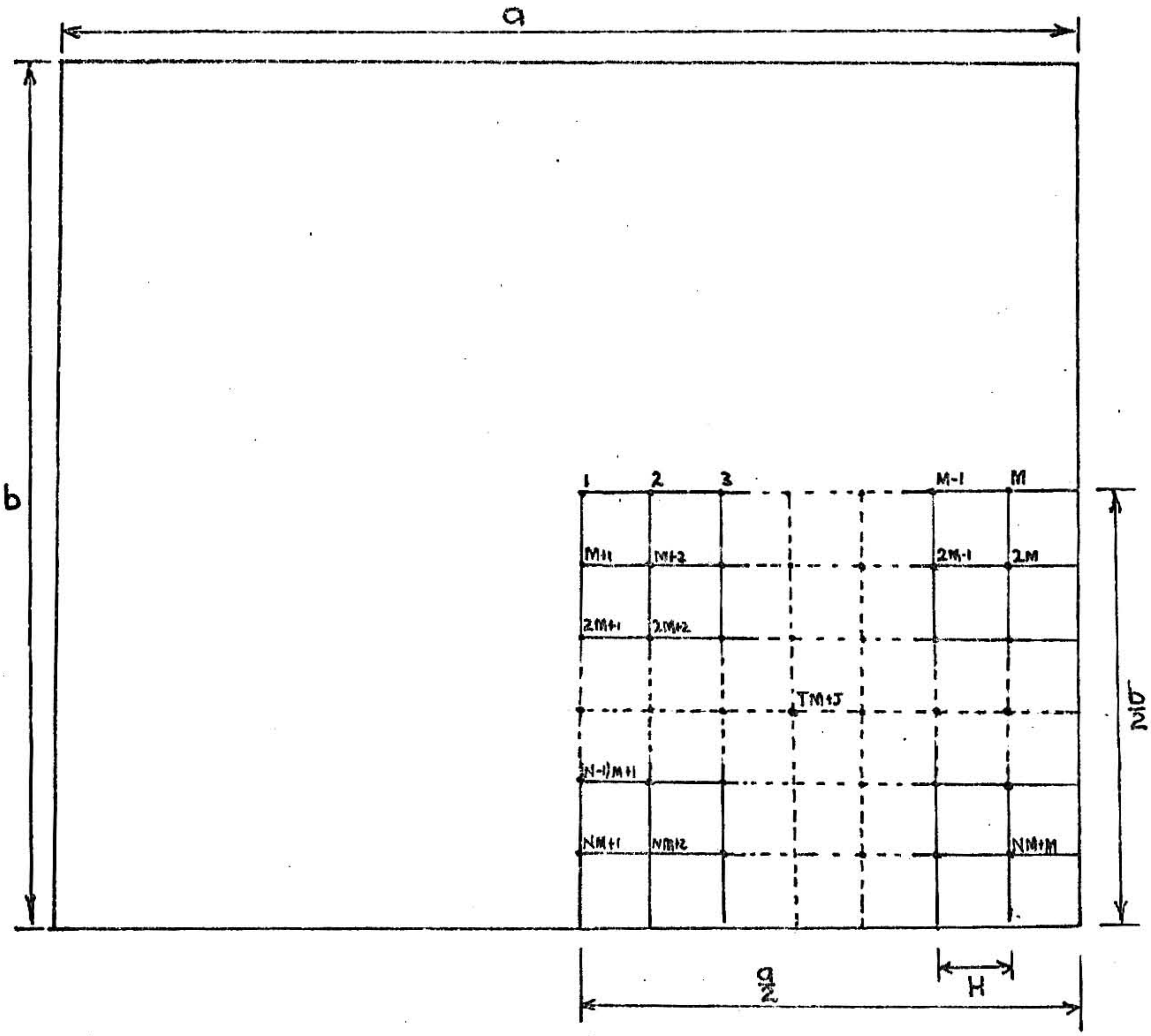

FIG. 3.6 NUMBERING SYSTEM FOR FINITE DIFFERENCE ANALYSIS

$$(5m+6n+8p)w_M - (4m+4p)w_{M-1} + mw_{M-2} - (8n+8p)w_{M+M} +$$
$$2nw_{M+2M} + 4pw_{M+M-1} = q \quad \text{------------------------} \quad (3.22)$$

These equations are solved simultaneously to determine the deflections at the points.

3.4 - Equations for Bending of Reinforced Concrete Plate Under Eccentric In-Plane Compressive Loading

Differential Equations for Bending of Plate

Figure 3.7 shows a plate that is subjected to the action of compressive forces in one coordinate direction. The forces are uniformly distributed along the edges x = 0 and x = a with an eccentricity, e. Let the magnitude of the compressive force be N_x per unit length of the edge. By analogy with Equations 2.2 and 3.12, the differential equation of the deflected surface for an uncracked section is given by Equation 3.23a:

$$D_{xu}\frac{\partial^4 w}{\partial x^4} + D_{yu}\frac{\partial^4 w}{\partial y^4} + 2\lambda\sqrt{D_{x1u}D_{y1u}}\frac{\partial^4 w}{\partial x^2 \partial y^2} = -N_x\frac{\partial^2 w}{\partial x^2} \quad \text{--------} \quad (3.23a)$$

For a cracked section the equation becomes:

$$D_{xcr}\frac{\partial^4 w}{\partial x^4} + D_{ycr}\frac{\partial^4 w}{\partial y^4} + 2\lambda\sqrt{D_{x1cr}D_{y1cr}}\frac{\partial^4 w}{\partial x^2 \partial y^2} = -N_x\frac{\partial^2 w}{\partial x^2} \quad \text{------} \quad (3.23b)$$

where D_{xu}, D_{yu}, D_{xcr}, D_{ycr}, D_{x1u}, D_{y1u}, D_{x1cr}, and D_{y1cr} are as defined in Section 3.1.

Expressed in finite differences, for a point (i,j) in the slab, Equation 3.23a becomes:

$$nw_{i,j+2} - 4(n+p)w_{i,j+1} + (6m+6n+8p-2s)w_{i,j} - 4(n+p)w_{i,j-1} +$$
$$nw_{i,j-2} + mw_{i+2,j} - (4m+4p-s)w_{i+1,j} - (4m+4p-s)w_{i-1,j} + mw_{i-2,j} +$$
$$2p(w_{i+1,j+1}+w_{i+1,j-1}+w_{i-1,j-1}+w_{i-1,j+1}) = 0 \qquad (3.24)$$

where $s = \dfrac{N_x}{H^2}$

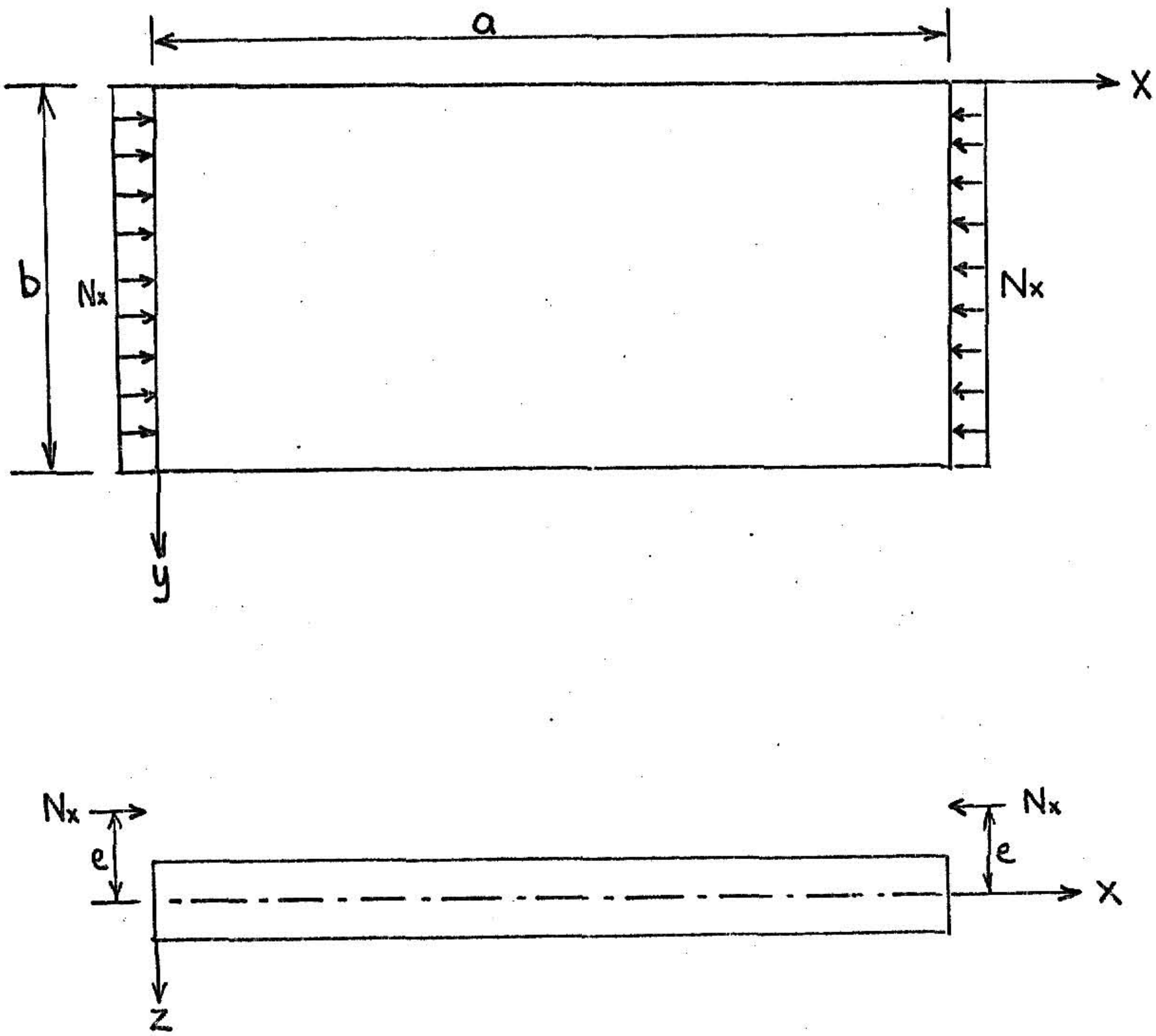

FIG. 3.7 SIMPLY-SUPPORTED PLATE
SUBJECTED TO ECCENTRICALLY-APPLIED LOAD

Boundary Conditions

For a plate simply supported along all edges on rigid supports and loaded as shown in Figure 3.7, there will be no deflections along the edges. Also, there will be no bending moments along the edges parallel to the x-axis. Along the edges parallel to the y-axis the bending will be equal to $N_x e$ per unit length.

Hence, for $y = 0,b$ $\quad w = 0$ and $M_y = 0 = \dfrac{\partial^2 w}{\partial y^2}$

for $x = 0,a$ $\quad w = 0$ and $M_x = N_x e = -D_x \dfrac{\partial^2 w}{\partial x^2}$

Expressed in finite differences for a point (i,j)

$$\frac{\partial^2 w}{\partial y^2} = 0 = \frac{1}{H^2}(w_{i,j-1} - 2w_{i,j} + w_{i,j+1}) \text{ when } y = 0,b$$

$$-D_x \frac{\partial^2 w}{\partial x^2} = N_x e = -\frac{D_x}{H^2}(w_{i-1,j} - 2w_{i,j} + w_{i+1,j}) \text{ when } x = 0,a$$

For an uncracked section $D_x = D_{xu}$; for a cracked section $D_x = D_{xcr}$.

Therefore, $w_{i,j+1} = -w_{i,j-1}$ when $y = 0,b$ - - - - - - - - - - - - - - (3.25a)

$$w_{i+1,j} = -(N_x e)\frac{H^2}{D_x} - w_{i-1,j} \text{ when } x = 0,a \quad \text{- - - - - - - -} \quad (3.25b)$$

Due to symmetry of loading and support conditions, one quarter of the plate can be analyzed with the numbering order shown in Figure 3.6. Equation 3.24 is rewritten for each point in the network taking into account the boundary conditions (Equations 3.25) and also the symmetry conditions.

For a general point, TM + J, in the network where $2 \leq T \leq N-2$ and $3 \leq J \leq M-2$ the equation is as follows for an uncracked section:

$$(6m+6n+8p-2s)w_I - (4m+4p-s)w_{I-1} - (4m+4p-s)w_{I+1} - (4n+4p)w_{I-M} -$$
$$(4n+4p)w_{I+M} + m(w_{I-2}+w_{I+2}) + n(w_{I-2M}+w_{I+2M}) +$$
$$2p(w_{I-M-1}+w_{I-M+1}+w_{I+M+1}+w_{I+M-1}) = 0 \quad (3.26)$$

where $I = TM + J$.

For points adjacent to the boundary parallel to the y-axis, i.e., the points M, 2M, 3M, . . ., (N+1)M (see Figure 3.6), due to the boundary condition (Equation 3.25b) the right hand side of Equation 3.24 will not be zero. Thus, there will be a non-homogeneous set of equations to be solved. For example, the equation for the point M is obtained as follows for an uncracked section:

$$(6m+6n+8p-2s)w_M - (4m+4p-s)w_{M-1} - (8n+8p)w_{M+M} + 2nw_{M+2M} +$$
$$4pw_{M+M-1} + mw_{M-2} + m(-w_M-N_x e\cdot\frac{H^2}{D_{xu}}) = 0$$

Simplifying,

$$(5m+6n+8p-2s)w_M - (4m+4p-s)w_{M-1} - (8n+8p)w_{M+M} + 2nw_{M+2M} +$$
$$4pw_{M+M-1} + mw_{M-2} = \frac{N_x e}{H^2} \quad (3.27)$$

since $mN_x e \dfrac{H^2}{D_{xu}} = \dfrac{D_{xu}}{H^4} \cdot \dfrac{N_x eH^2}{D_{xu}} = \dfrac{N_x e}{H^2}$

For a cracked section, m, n and p in Equation 3.27 are replaced by m', n' and p' respectively.

3.5 - Plate With Equal End Moments Applied to Two Opposite Edges

Consider the plate shown in Figure 3.7 to be bent by moments, M_o, distributed along the edges x=0 and x=a instead of the compressive load, N_x. The deflection of the plate is given by the following homogeneous equation:

$$D_{xu} \frac{\partial^4 w}{\partial x^4} + D_{yu} \frac{\partial^4 w}{\partial y^4} + 2\lambda \sqrt{D_{xlu} D_{ylu}} \frac{\partial^4 w}{\partial x^2 \partial y^2} = 0 \quad \text{------------} \quad (3.28)$$

Expressed in finite differences, the equation is similar to Equation 3.18 except that the right hand side of the equation is equal to zero.

The deflections must satisfy the following boundary conditions:

For $x = 0$ and $x = a$, $w = 0$ and $-D_x \frac{\partial^2 w}{\partial x^2} = M_o$

For $y = 0$ and $y = b$, $w = 0$ and $\frac{\partial^2 w}{\partial y^2} = 0$

Therefore,

$$w_{i,j+1} = -w_{i,j-1} \quad \text{------------------------} \quad (3.29a)$$

$$w_{i+1,j} = -w_{i,j-1} - \frac{M_o H^2}{D_x} \quad \text{--------------------} \quad (3.29b)$$

For a general point, TM + J, in the network where $2 \le T \le N-2$ and $3 \le J \le M-2$ the bending equation in finite differences is given by Equation 3.30:

$$(6m+6n+8p)w_I - (4m+4p)w_{I-1} - (4m+4p)w_{I+1} - (4n+4p)w_{I-M} - (4n+4p)w_{I+M} +$$
$$m(w_{I-2}+w_{I+2}) + n(w_{I-2M}+w_{I+2M}) + 2p(w_{I-M-1}+w_{I-M+1}+w_{I+M-1}+w_{I+M+1}) = 0 \quad - \quad (3.30)$$

where $I = TM + J$.

Equations for other points in the network are modified due to the boundary conditions (Equations 3.29) and the symmetry conditions. For the point, M, in the network, the bending equation is given by Equation 3.22 with $\frac{M_o}{H^2}$ substituted for q in the right hand side of the equation.

CHAPTER 4

SOLUTION PROCEDURE

4.1 - Step-by-Step Solution

The analysis of the reinforced concrete plate incorporated the cracking of the concrete. Cracking of the concrete reduces the plate stiffness and makes the plate deflect more for the same loading. Hence, a step-by-step solution is adopted.

Initially a unit load is applied to the plate. For the unit load a finite difference equation of bending is written for each point in the finite difference network using the uncracked stiffness factors, m, n and p. The simultaneous equations are solved to determine the deflections of the points. Strains in both coordinate directions are computed for both the top and bottom fibers of the concrete. The unit load is scaled to a value, Q, where only one point cracks. This is represented by point 1 in Figure 4.1. (A point is assumed to be cracked if the tensile strain at that point in either coordinate direction exceeds a limiting value for concrete tensile strain.) The uncracked stiffness factors, m, n and p are then replaced by the cracked stiffness factors, m', n' and p' in the equation for the point which cracked. New deflections are computed for the same load, Q. Strains are computed again and examined for new cracked points. If any new points cracked, their stiffness factors are changed to m', n' and p' and the process repeated until no new points crack for the load, Q. This stage is represented by the horizontal line 1-2 in Figure 4.1.

A load increment, ΔQ, specified as a fraction of Q, is then applied and new deflections computed. This is represented by point 3 in Figure 4.1.

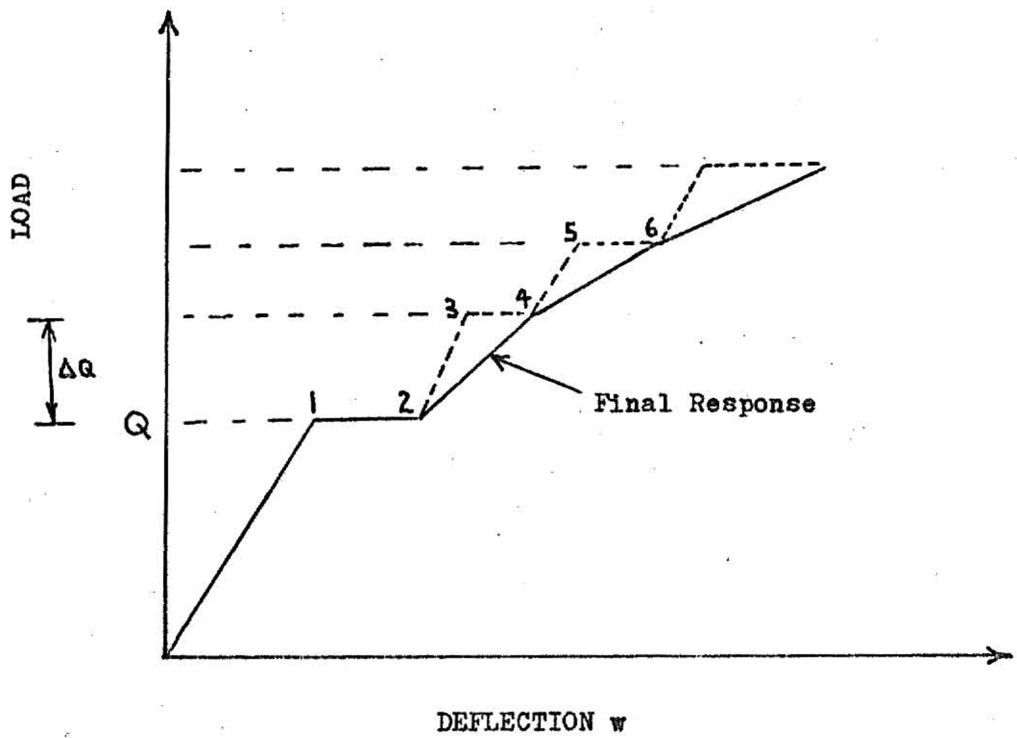

FIG. 4.1 LOAD-DEFLECTION DIAGRAM SHOWING STEP-BY-STEP ANALYSIS

The plate is then examined for new cracked points. The stiffness factors for these new cracked points are then modified and new deflections computed for the same loading. This cycle is repeated until no more new points crack for the same load and equilibrium is attained. Point 4 in Figure 4.1 represents this stage. A new load increment is applied and the process repeated.

The loading is increased until the compressive strain in the concrete exceeds a specified limit for concrete. After equilibrium has been attained for a load increment, the compressive strains in the concrete are examined to find if the specified amount has been exceeded.

In the analysis when a crack occurs at a point in one coordinate direction (as determined by the tensile strain in that direction) the concrete is assumed cracked in the other direction.

The analysis was performed using a computer program which is described in the next section.

4.2 - Computer Program

A computer program was written in the Fortran IV Language to perform the solution procedure described in the previous section. The program was written to handle different support conditions and types of loading: simply supported or fixed edge plate and lateral loading or plate with eccentric in-plane loading or end moments.

Input data include concrete and steel material properties EC, ES, PR, EPSM, EPCM (see Appendix B for definition of notation), slab thickness T, reinforcing steel ratio for one layer SR1, warping parameter WP, the size of squares in the finite difference grid H, and the values of M and N which specify the number of points being used in the finite difference grid. The total number of points is given by IA = (N + 1)M. In addition, control data NBC, IPROB, IND (or ECC) are given to specify the support condition and type of loading. NBC = 1 for a fixed edge plate and equal to zero for a simply supported plate. IPROB = 1 for eccentric in-plane loading and equal to zero for both lateral loading and end moments. IND = 1 for lateral loading and equal to zero for end moment. If IPROB = 1 the eccentricity of the load, ECC, is input as data instead of IND. For IPROB = 0, IND is input as data.

The following subroutines are used in the program: MATRIX, ROWMAT, GELB and EPSMAX. The subroutine MATRIX computes matrix A which constitutes the coefficients of the left hand side of the simultaneous equations obtained from Equation 3.18 or 3.24, depending on the type of loading. The appropriate stiffness factors (uncracked or cracked) are used in computing the coefficients for each point by means of the control code IC(I). The subroutine MATRIX can work only if the number of points in the finite difference network is at least five in the direction of the shorter side of the plate, i.e., $N \geq 4$.

The subroutine ROWMAT changes matrix A into a vector so that it can be fed into IBM Library subroutine GELB for the solution of the simultaneous equations.

After deflections are computed for a load increment, strains (in both coordinate directions) in both top and bottom fibers of the plate are computed for each point. EPSMAX determines the points which cracked during the load increment by comparing the strains with the specified maximum allowable tensile strain for concrete EPSM.

Output data after equilibrium is attained for each load increment are: the number of iterations before equilibrium was attained, the points which cracked during the load increment, deflections and strains for all the points in the network.

A listing of the computer program is given in Appendix B.

CHAPTER 5

NUMERICAL RESULTS

5.1 - Sample Calculation of Stiffness Factors and Terms in Matrix A

Given: Reinforced concrete square plate, 8' x 8' x 1"

$E_{xc} = E_{yc} = E_c = 3.0 \times 10^6$ psi, $v_c = 0.15$, $\lambda = 0.8$

Reinforcement: Two layers in both longitudinal and transverse directions; steel ratio, $\rho_i = 0.01$

$E_s = 30 \times 10^6$ psi, $\overline{Z}_i = 0.375"$

<u>Stiffness Factors for Case of No Cracking</u>

$$D_{x1u} = D_{y1u} = \frac{E_c}{(1-v^2)} \frac{h^3}{12} = \frac{3.0 \times 10^6 \times 1^3}{12(1-.15^2)} = 2.55754 \times 10^5 \text{ lb-in}^2$$

$$D_s = E_s h \sum_{i=1}^{2} \rho_i \overline{Z}_i^2 = 30 \times 10^6 \times 2\,(.375)^2\,(.01) = 8.4375 \times 10^4 \text{ lb-in}^2$$

$$D_{xu} = D_{yu} = D_{x1u} + D_s = 2.55754 \times 10^5 + 8.4375 \times 10^4 = 3.40129 \times 10^5 \text{ lb-in}^2$$

<u>Terms in Matrix A</u>

A sample calculation for terms in the Matrix A used in the computer analysis is shown below. The calculation is made for three points in the finite difference network.

Using a quarter of the plate for the analysis and 25 points in the finite difference network, matrix A will be a 25 x 25 matrix. The input data M and N in the computer program (Appendix B) will be 5 and 4 respectively.

H, the size of square in the network $= \frac{1}{2}\,\frac{(96")}{5} = 9.6"$

For the case of no cracking,

$$m = \frac{D_{xu}}{H^4} = \frac{3.40129 \times 10^5}{9.6^4} = 40.04596 \text{ lb/in}^2$$

$$n = \frac{D_{yu}}{H^4} = \frac{3.40129 \times 10^5}{9.6^4} = 40.04596 \text{ lb/in}^2$$

$$p = \frac{\lambda}{H^4} \sqrt{D_{xlu} D_{ylu}} = \frac{0.8 \times 2.55754 \times 10^5}{9.6^4} = 24.0894832 \text{ lb/in}^2$$

For point 1 (see Figure 3.6) Equation 3.18 will become:

$$(6m + 6n + 8p)w_1 - (8m + 8p)w_2 + (2m)w_3 - (8n + 8p)w_6 + (8p)w_7 + (2n)w_{11} = q \quad \text{-------------------------} \quad (5.1)$$

Substituting the values of m, n and p computed above into Equation 5.1 we get:

$$673.27\ w_1 - 513.08\ w_2 + 80.092\ w_3 - 513.08\ w_6 + 192.72\ w_7 + 80.092\ w_{11} = q$$

For point 5, using Equation 3.22 (M=5):

$$(5m + 6n + 8p)w_5 - (4m + 4p)w_4 + (m)w_3 - (8n + 8p)w_{10} + (2n)w_{15} + (4p)w_9 = q$$

Hence,

$$40.046\ w_3 - 256.54\ w_4 + 633.22\ w_5 + 96.358\ w_9 - 513.08\ w_{10} + 80.092\ w_{15} = q$$

General Point, TM + J

For a network with 25 points, there is only one general point where T = 2 and J = 3. Thus, the general point is 13. For point 13, using Equation 3.21, we get:

$$(6m+6n+8p)w_{13} - (4m+4p)w_{12} - (4m+4p)w_{14} - (4n+4p)w_8 - (4n+4p)w_{18} + (m)w_{11} + n(w_3+w_{23}) + 2p(w_7+w_9+w_{17}+w_{19}) + (m)w_{15} = q$$

Substituting in the values of m, n and p, we obtain:

$40.046\ w_3 + 48.179\ w_7 - 256.54\ w_8 + 48.179\ w_9 + 40.046\ w_{11} - 256.54\ w_{12} +$
$673.27\ w_{13} - 256.54\ w_{14} + 40.046\ w_{15} + 48.179\ w_{17} - 256.54\ w_{18} +$
$48.179\ w_{19} + 40.048\ w_{23} = q$

See Appendix C for the 25 x 25 Matrix output from the computer program.

5.2 - Effect of Warping Parameter, λ, and Steel Ratio, ρ_i, on Response of a Laterally Loaded Concrete Plate Using Timoshenko's (2) Solution

The differential equation for the solution of a rectangular reinforced concrete plate with lateral loading is given by Equation 3.12 for the case of no cracking.

$$D_{xu} \frac{\partial^4 w}{\partial x^4} + 2\lambda \sqrt{D_{xlu} D_{ylu}} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{yu} \frac{\partial^4 w}{\partial y^4} = q \quad (3.12)$$

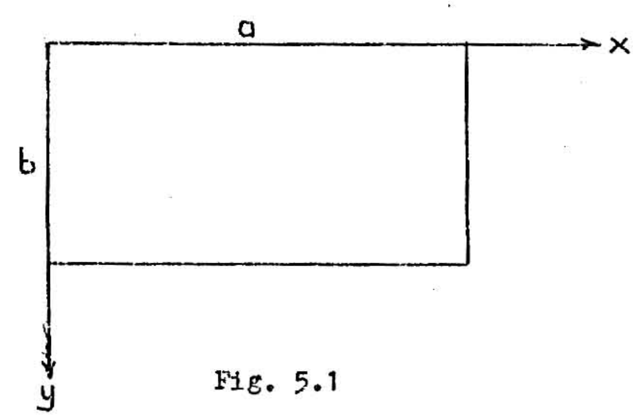

Fig. 5.1

For a simply supported plate with the coordinate system shown in Figure 5.1, the load, q, in Equation 3.12 may be expressed in the form of a double trigonometric series as follows (2):

$$q = \frac{16}{\pi^2} q_o \sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} \frac{1}{\bar{m}\bar{n}} \sin \frac{\bar{m}\pi x}{a} \sin \frac{\bar{n}\pi y}{b} \quad (5.1)$$

A solution of Equation 3.12 that satisfies the boundary conditions may be expressed in the form shown below:

$$w = \sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} a_{\bar{m}\bar{n}} \sin \frac{\bar{m}\pi x}{a} \sin \frac{\bar{n}\pi y}{b} \quad (5.2)$$

From Equations 3.12, 5.1 and 5.2, we obtain:

$$a_{\bar{m}\bar{n}} = \frac{16q_o}{\pi^6} \frac{1}{\bar{m}\bar{n}\left(\frac{\bar{m}^4}{a^4} D_{xu} + \frac{2\bar{m}^2\bar{n}^2}{a^2b^2} \lambda \sqrt{D_{xlu}D_{ylu}} + \frac{\bar{n}^4}{b^4} D_{yu}\right)} \qquad (5.3)$$

Hence,

$$w = \frac{16q_o}{\pi^6} \sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} \frac{\sin \frac{\bar{m}\pi x}{a} \sin \frac{\bar{n}\pi y}{b}}{\bar{m}\bar{n}\left(\frac{\bar{m}^4}{a^4} D_{xu} + \frac{2\bar{m}^2\bar{n}^2}{a^2b^2} \lambda \sqrt{D_{xlu}D_{ylu}} + \frac{\bar{n}^4}{b^4} D_{yu}\right)} \qquad (5.4)$$

For a square plate of side, a, the maximum deflection, w_{max}, is at the point where $x = a/2$, $y = a/2$.

$$w_{max} = \frac{16q_o}{\pi^6} \sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} \frac{\sin \frac{\bar{m}\pi}{2} \sin \frac{\bar{n}\pi}{2}}{\bar{m}\bar{n}\left(\frac{\bar{m}^4}{a^4} D_{xu} + 2\frac{\bar{m}^2\bar{n}^2}{a^4} \lambda \sqrt{D_{xlu}D_{ylu}} + \frac{\bar{n}^4}{a^4} D_{yu}\right)} \qquad (5.5)$$

Example:

Reinforced concrete square plate, 8' x 8' x 1"

$E_{xc} = E_{yc} = E_c = 3.0 \times 10^6$ psi, $v = 0.15$

Reinforcement: Two layers in both longitudinal and transverse directions. Steel ratio, ρ_i, to vary from 0.005 to 0.4

$E_s = 30 \times 10^6$ psi, $\bar{Z}_i = 0.375"$

$$D_{xlu} = D_{ylu} = \frac{E_c}{1-v^2} \frac{h^3}{12} = \frac{3 \times 10^6 \times 1^3}{(1-0.15^2)12} = 2.55754 \times 10^5 \text{ lb-in}^2$$

$$D_s = E_s h \sum_{i=1}^{2} \rho_i \bar{Z}_i^{\,2} = 30 \times 10^6 \times 2 \times (0.375)^2 \rho_i$$

$$= 8.4375 \times 10^6 \rho_i \text{ lb-in}^2$$

$$D_{xu} = D_{yu} = D_{xlu} + D_s = 2.55754 \times 10^5 + 8.4375 \times 10^6 \rho_i$$

For $\rho_i = 0.01$, λ variable

$D_{xu} = D_{yu} = 3.40129 \times 10^5$ lb-in^2

From Equation 5.5 for $\bar{m}=1$, $\bar{n}=1$

$$w_{max} = \frac{16q_o}{\pi^6} \frac{(8 \times 12)^4}{2\ (3.40129 \times 10^5) + 2\lambda\ (2.55754 \times 10^5)}$$

$$= \frac{1.413532 \times 10^6\ q_o}{6.80258 \times 10^5 + \lambda\ (5.11508 \times 10^5)}$$

Table 1 shows the variation of w_{max} with λ for $\rho_i = 0.01$ as obtained from Timoshenko's solution (using $\bar{m} = 1,3$ and $\bar{n} = 1,3$) and from the computer program for the case of no cracking. These results are plotted in Figure 5-2a.

	Trig. Series	Finite Difference (M=6, N=5)
λ	w_{max}/q_o in^3/lb	w_{max}/q_o in^3/lb
0	2.047	2.043
0.2	1.776	1.772
0.4	1.568	1.566
0.6	1.403	1.401
0.8	1.269	1.268
1.0	1.159	1.157

Table 1 - Variation of Maximum Deflection of Square Plate
With Warping Parameter

For $\lambda = 1.0$, ρ_i variable

$$D_{yu} = D_{xu} = D_{x1u} + D_s = 2.55754 \times 10^5 + (8.4375 \times 10^6)\ \rho_i$$

From Equation 5.5 for $\bar{m} = 1$, $\bar{n} = 1$

$$w_{max} = \frac{16q_o\ (8 \times 12)^4}{\pi^6\ [4(2.55754 \times 10^5) + 2(8.4375 \times 10^6)\ \rho_i]}$$

$$= \frac{1.413532 \times 10^6\ q_o}{1.023016 \times 10^6 + (1.6875 \times 10^7)\ \rho_i}$$

Table 2 shows the variation of w_{max} with steel ratio, ρ_i, for $\lambda = 1.0$ as obtained from Timoshenko's solution with $\bar{m} = 1,3$ and $\bar{n} = 1,3$. Also see Figure 5.2b.

ρ_i	w_{max}/q_o in^3/lb
.005	1.246
.01	1.159
.015	1.083
.02	1.016
.025	0.958
.03	0.905
.035	0.858
.04	0.825

Table 2 - Variation of Maximum Deflection of Square Plate With Steel Ratio, ρ_i

Bending Moments

For the case of no cracking,

$$M_x = -\left(D_{xu} \frac{\partial^2 w}{\partial x^2} + D_{1u} \frac{\partial^2 w}{\partial y^2}\right)$$

Differentiating Equation 5.4 with respect to x and y,

$$\frac{\partial^2 w}{\partial x^2} = -\sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} K \frac{\bar{m}^2\pi^2}{a^2} \sin\frac{\bar{m}\pi x}{a} \sin\frac{\bar{n}\pi y}{b}$$

$$\frac{\partial^2 w}{\partial y^2} = -\sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} K \frac{\bar{n}^2\pi^2}{b^2} \sin\frac{\bar{m}\pi x}{a} \sin\frac{\bar{n}\pi y}{b}$$

where

$$K = \frac{16q_o}{\pi^6} \frac{1}{\bar{m}\bar{n}\left(\frac{\bar{m}^4}{a^4} D_{xu} + \frac{2\bar{m}^2\bar{n}^2}{a^2b^2} \lambda \sqrt{D_{x1u}D_{y1u}} + \frac{\bar{n}^4}{b^4} D_{yu}\right)} \quad \text{-------} \quad (5.6)$$

For a square plate with side, a, maximum moment is at the point where $x = \frac{a}{2}$ and $y = \frac{b}{2}$.

$$M_{max} = \frac{\pi^2}{a^2} \sum_{\bar{m}=1,3,5}^{\infty} \sum_{\bar{n}=1,3,5}^{\infty} \left(K \sin\frac{\bar{m}\pi}{2} \sin\frac{\bar{n}\pi}{2}\right) \left(\bar{m}^2 D_{xu} + \bar{n}^2 D_{yu}\right) \quad - (5.7)$$

For $\rho_i = 0.01$, λ variable

$$D_{1u} = \lambda v \sqrt{D_{x1u}D_{y1u}} = 0.15\,\lambda\,(2.55754 \times 10^5)$$

$$= \lambda\,(3.83632 \times 10^4) \text{ lb-in}^2$$

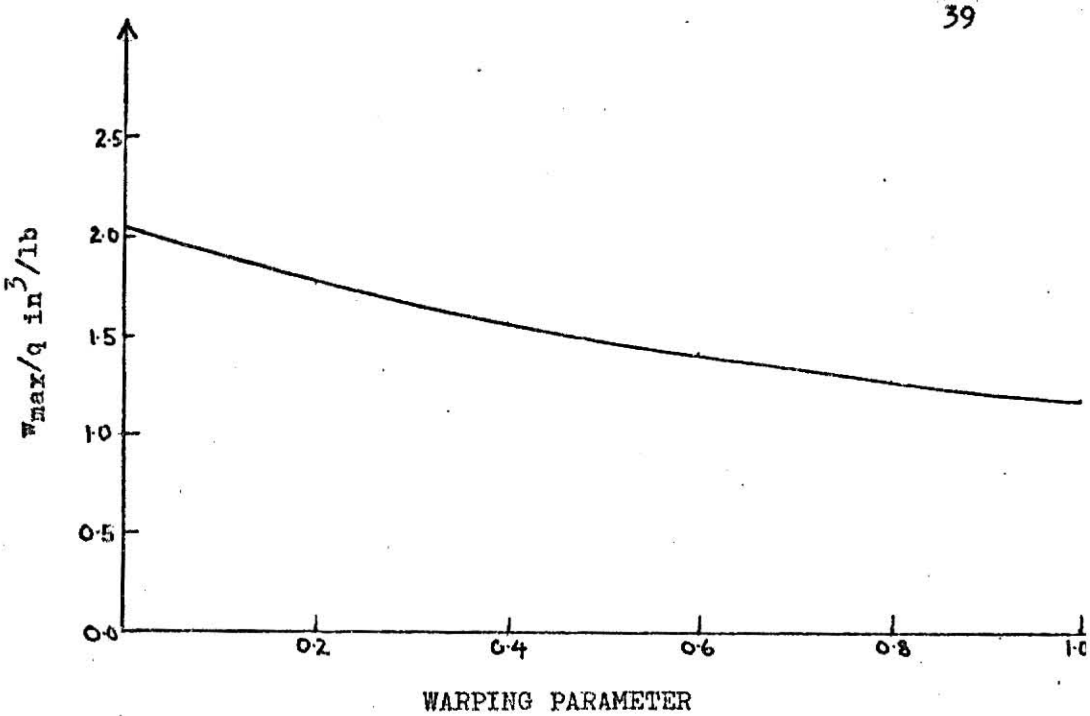

FIG. 5.2a MAXIMUM DEFLECTION OF SQUARE PLATE vs WARPING PARAMETER
(PLATE UNCRACKED, LATERALLY-LOADED)

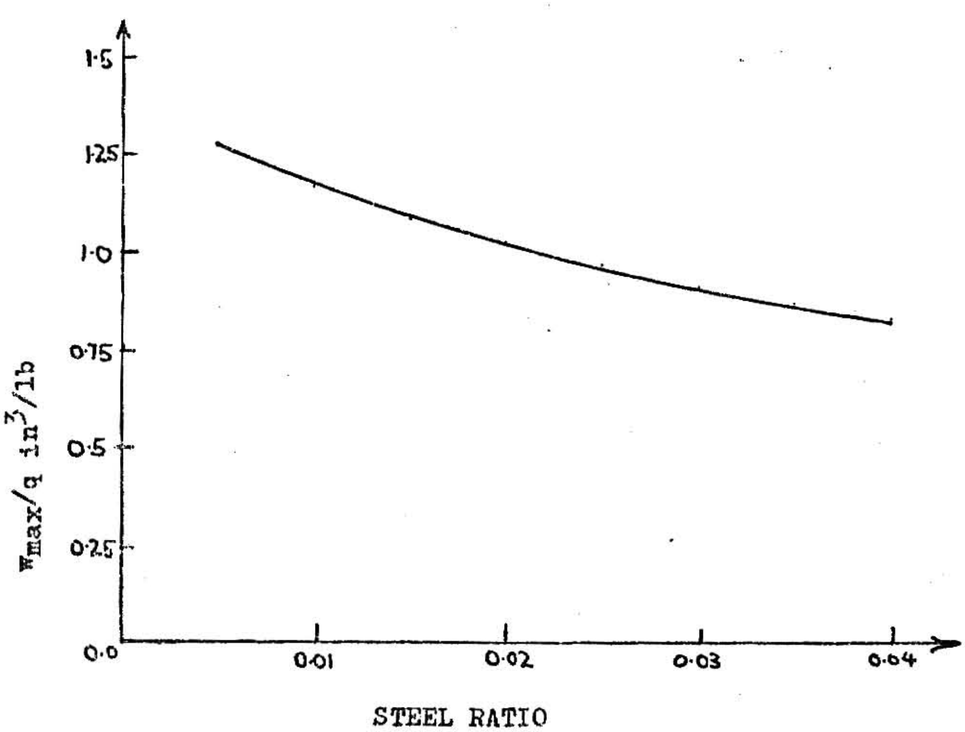

FIG. 5.2b MAXIMUM DEFLECTION vs STEEL RATIO
(PLATE UNCRACKED, LATERALLY-LOADED)

$D_{xu} = 3.40129 \times 10^5$ lb-in^2

From Equation 5.6, for $\bar{m} = 1$, $\bar{n} = 1$

$$K = \frac{16q_o}{\pi^6} \frac{(96)^4}{2(3.40129 \times 10^5) + 2\lambda(2.55754 \times 10^5)}$$

From Equation 5.7,

$$M_{max} = \frac{\pi^2}{(96)^2} K \left[3.40129 \times 10^5 + \lambda\ (3.83632 \times 10^5)\right] \quad \text{lb-in}$$

Table 3 shows the variation of the maximum bending moments with the warping parameter, λ. Values obtained from both Timoshenko's solution and the computer program are given.

	Trig. Series	Finite Difference (M=6, N=5)
λ	M_{max}/q_o in^3	M_{max}/q_o in^3
0	704.73	705.17
0.2	620.18	621.75
0.4	555.32	557.75
0.6	503.96	506.95
0.8	462.36	466.24
1.0	427.93	432.23

Table 3 - Variation of Maximum Bending Moments in Square Plate With Warping Parameter, λ.

Tables 1, 2 and 3 show that the results for deflections and bending moments obtained from the finite difference solution are in agreement with those obtained using the trigonometric series solution. The results show that for a given load, the maximum deflection increases by 77% and the maximum bending moment increases by 63% when the warping parameter, λ, decreases from 1 to 0. Also the deflection increases as the steel ratio, ρ_i, decreases.

5.3 - Response of Simply-Supported Reinforced Concrete Plate With Lateral Loading Including the Effects of Cracking

The computer program which was developed was used to analyze the simply-supported reinforced concrete plate whose details were given in the example in Section 5.2. The effects of cracking were included. A steel ratio of 0.01 was used.

Thirty-six points were used in the finite difference network of one quarter of the plate (see Figure 5.3a). Table 4 shows the maximum deflection and bending moments for various loads as obtained from the computer program. The results shown are for $\lambda = 0.8$. The table also indicates the points which cracked during each load increment. It should be noted that there are two values of deflection (and bending moment) for the load of 0.239 lb/in^2. The first value is the deflection just before the first crack and the second is the deflection after cracking starts.

Figure 5.3b shows plots of load versus maximum deflection for various values of λ, the warping parameter. Figure 5.4 shows plots of load versus maximum bending moments in the plate. The plots indicate that the load at which cracking starts in the plate decreases as the warping parameter decreases. Also the deflection (and bending moments) for a given load increases as the warping parameter decreases. For the case of no cracking the maximum plate deflection increased by 77% and the maximum bending moments increased by 63% when the warping parameter decreased from 1 to 0. For the cracked case the maximum deflection increased by 18% and the maximum bending moments increased by 16% for a given load when the warping parameter decreased from 1 to 0.

Figure 5.5 shows the progression of cracked zones in one quarter of the plate as the loading increases. The crack progression was similar for all values of λ. The cracking started from the center of the plate and extended

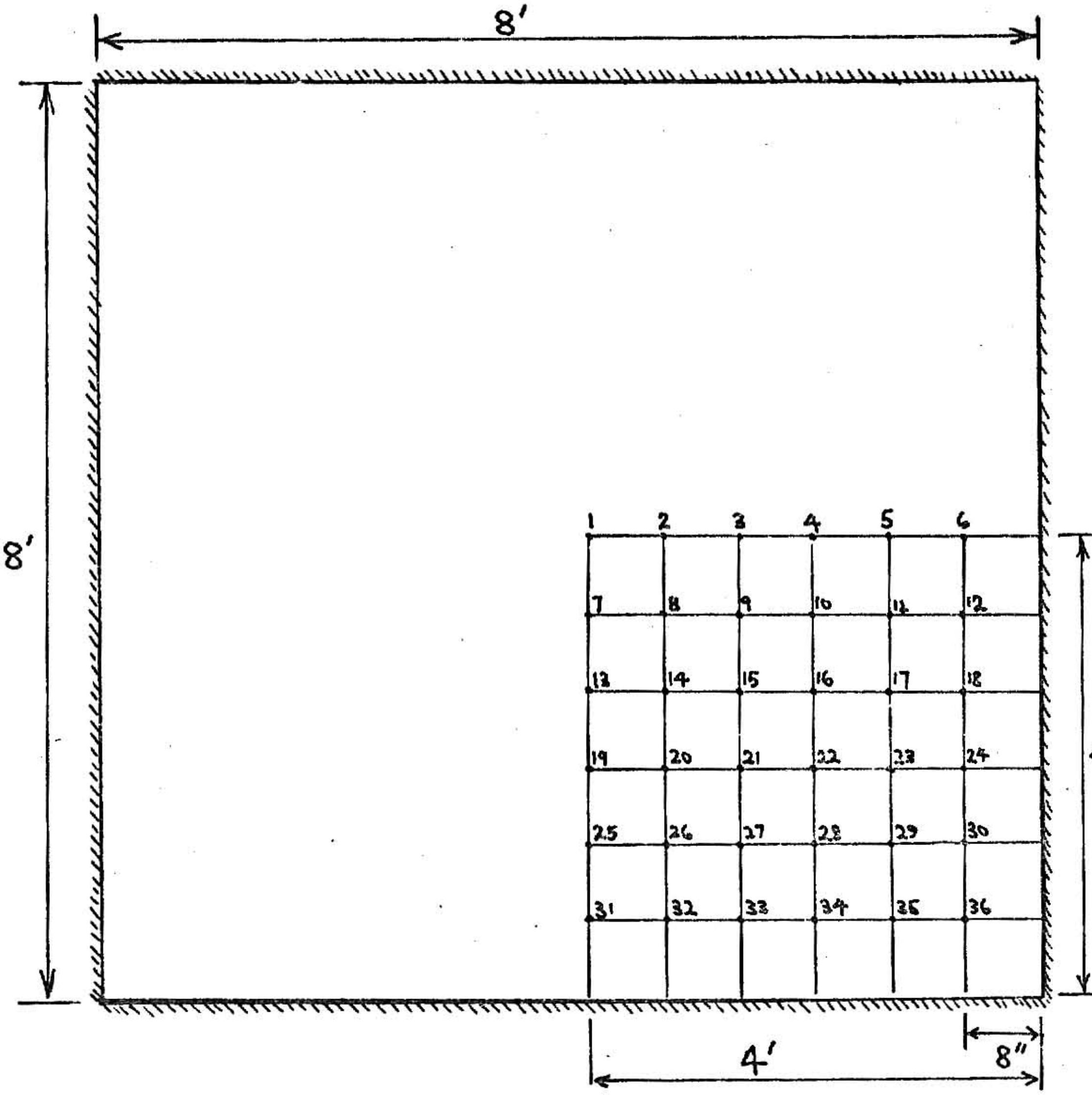

FIG. 5.3a NUMBERING ORDER OF 36 POINTS USED IN FINITE DIFFERENCE SOLUTION

toward the edges. About half of the plate cracked as soon as cracking set in. This result may be due to the coarseness of the finite difference grid used in the solution. In the analysis when a point cracks the stiffness factors for the cracked section are used in the finite difference equation (Equation 3.18) for that point in subsequent computations. This means that the cracking of one point affects other points as can be seen from the module diagram in Figure 3.5. For a coarse grid the effect is significant.

Load lb/in^2	Maximum Deflection ins.	Maximum Moments lb-in/in	Points Cracked By Load Increment
0.239	.303	111.25	(No cracking)
0.239	.954	135.78	1, 2, 7, 8, 3, 4, 9, 13, 14, 15, 19, 5, 10, 11, 16, 20, 21, 22, 25, 26, 17, 23, 27, 28
0.263	1.134	159.28	6, 12, 29, 31, 32, 18, 33
0.286	1.237	173.77	- - - - -
0.334	1.468	205.69	24, 34
0.382	1.677	235.10	- - - - -
0.430	1.907	267.12	30, 35
0.477	2.119	296.76	- - - - -

Table 4 - Maximum Deflection and Bending Moments for $\lambda = 0.8$

5.4 - Fixed-Edge Reinforced Concrete Plate Subjected to Lateral Loading

Vanderbilt, Sozen and Siess (6) developed an approximate method to predict deflections of floor slab systems with or without beams on the basis of finite difference solution and test data from five multiple-panel structures. They obtained the following coefficients for mid-panel deflection of a panel supported on beams and surrounded on all sides by other panels:

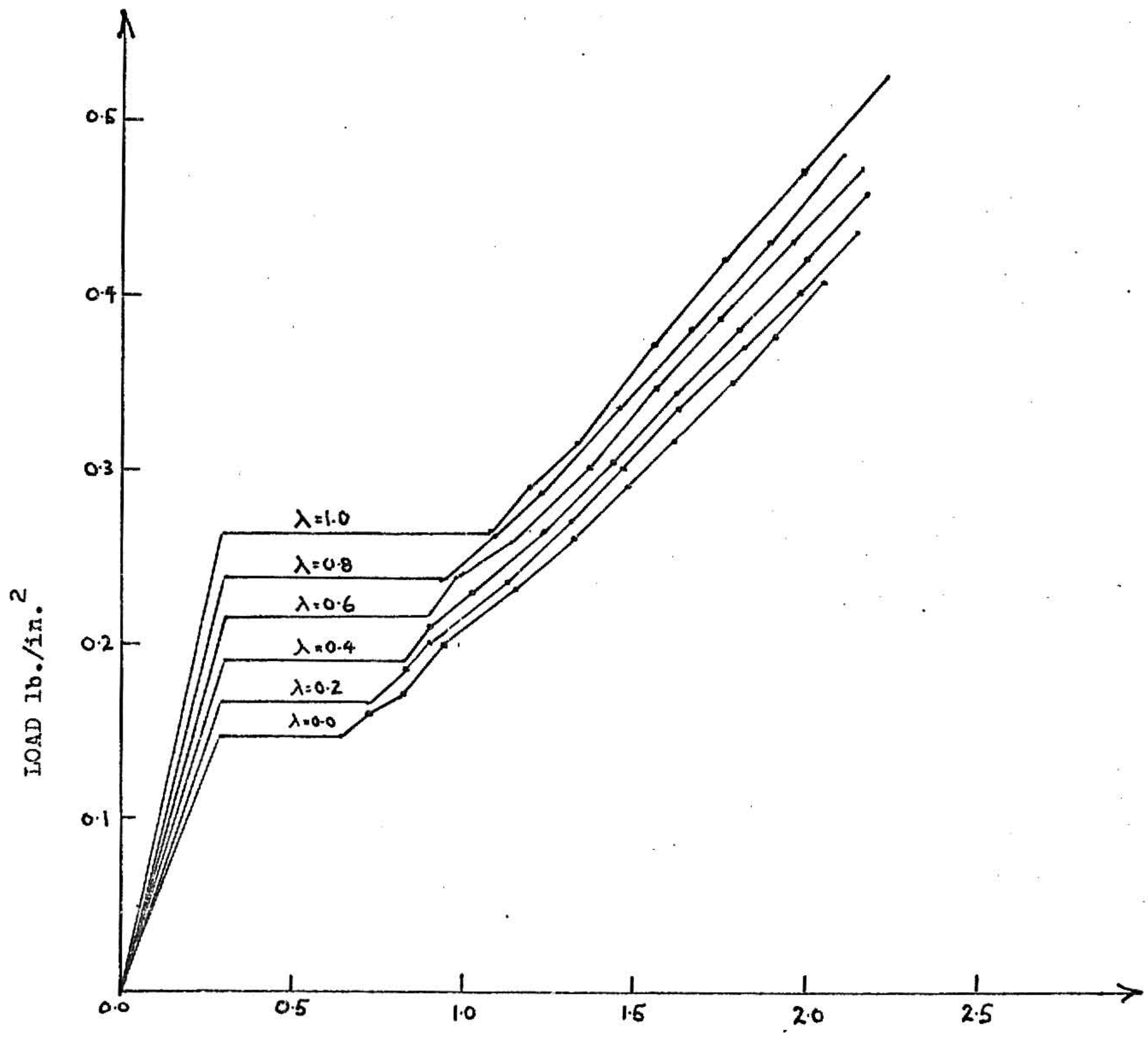

FIG. 5.3b LOAD vs MAXIMUM DEFLECTION
FOR REINFORCED CONCRETE SQUARE PLATE

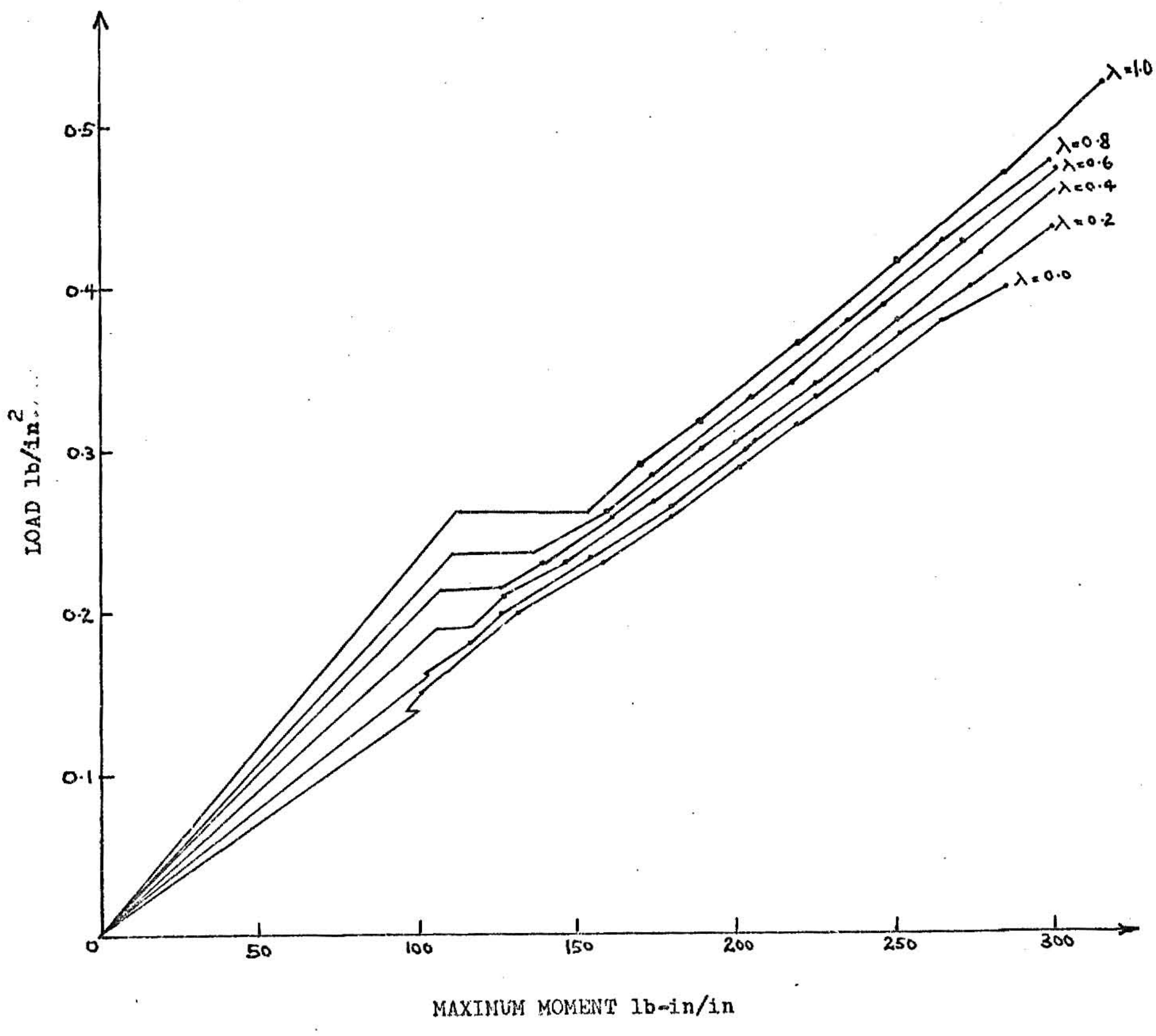

FIG. 5.4 LOAD vs MAXIMUM MOMENT
IN LATERALLY-LOADED REINFORCED CONCRETE SQUARE PLATE

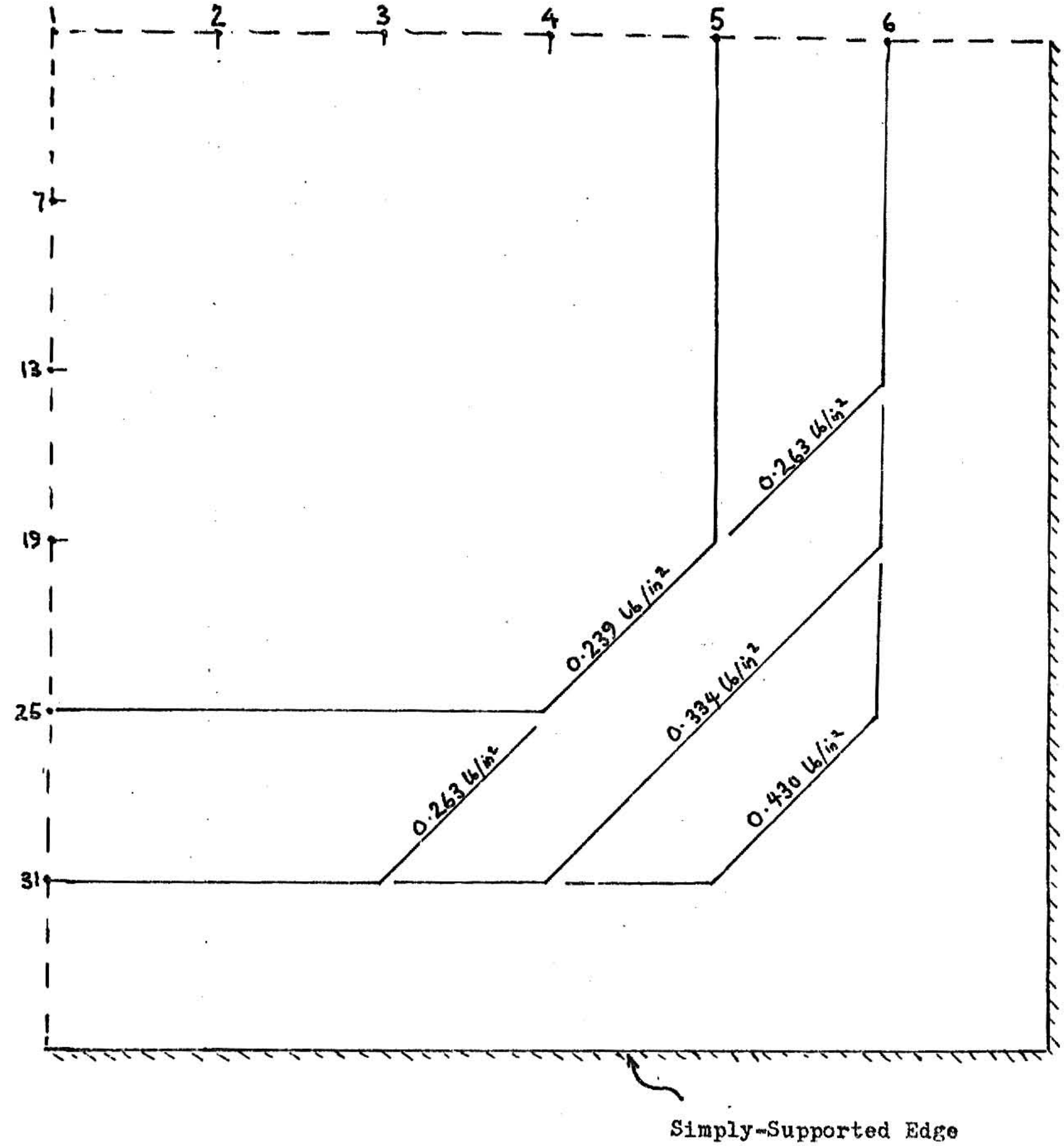

FIG. 5.5 PROGRESSION OF CRACKED ZONES
IN ONE QUARTER OF LATERALLY LOADED SQUARE PLATE

Coefficient (uncracked slab) = 0.00146

Coefficient (cracked slab) = 0.00906

The deflection is given by:

$w = \frac{qL^4}{D}$ where L = long span, D is the flexural rigidity of uncracked slab and is given by $D = \frac{E_c h^3}{12(1-v^2)}$

Example

Reinforced concrete slab, 60" x 60" x 1.5", $E_c = 3 \times 10^6$ psi, $E_s = 30 \times 10^6$ psi, steel ratio $\rho_i = .01$, $\varepsilon'_t = 0.133 \times 10^{-3}$ where ε'_t is the limiting tensile strain in concrete, $v_c = 0.15$.

Using the method of Vanderbilt, Sozen and Siess:

$$D = \frac{E_c h^3}{(1-v^2)12} = 8.6317 \times 10^5 \text{ lb/in}$$

$$\frac{L^4}{D} = 15.0144 \text{ in}^3/\text{lb}$$

For $q = 1$ lb/in^2

mid-panel deflection (uncracked slab) = 15.0144 x 0.00146 = 0.0219"

For $q = 8.546$ lb/in^2

mid-panel deflection (cracked slab) = 0.00906 x 8.546 x 15.0144

= 1.162"

The method of solution presented in this paper was used to solve the same problem assuming the edges of the panel are fixed against rotation. The following results were obtained:

For $q = 1$ lb/in^2

mid-panel deflection (uncracked slab) = 0.0153"

For $q = 8.546$ lb/in^2

mid-panel deflection (cracked slab) = 0.349"

Vanderbilt, Sozen and Siess ignored the effect of reinforcement in the calculation of the flexural rigidity, D. If this effect is taken into account the deflection for the uncracked slab obtained from their solution is 0.0155" for $q = 1$ lb/in^2. This agrees well with the result obtained from the solution presented in this paper. However, the mid-panel deflection of the cracked slab for $q = 8.546$ lb/in^2 if the reinforcement effect is taken into account is 0.826" which is about twice the value obtained with the solution presented in this paper. Vanderbilt, Sozen and Siess allowed some rotation of the edges of the panels. Also, they obtained the deflection coefficient for the cracked slab by multiplying the deflection coefficient of the uncracked slab by the ratio of the moment of inertia for the uncracked section to the average of the negative and positive moments of inertia for the fully cracked section.

5.5 - Response of Plate With Eccentric Compressive In-Plane Loading

Aluminum Plate (Elastic)

To test the accuracy of the method of solution used for a plate with in-plane compressive loading, the finite difference solution was used to analyze the aluminum plates with the following data used by Athavitchijanyaraks (8):

Plate 1: Aluminum Plate 8" x 8" x 1/8"

Plate 2: Aluminum Plate 16" x 8" x 1/8"

$E = 10.5 \times 10^6$ psi, $\nu = 1/3$, Eccentricity of loading = 1/2"

Deflection profiles for the two plates as obtained from the finite difference solution were compared with the results of Athavitchijanyaraks. The curvature of the plate deflection is single-valued for Plate 1 and triple-valued for Plate 2. The results from the solution used in this study agreed well with those obtained by Athavichitjanyaraks. These are shown in Figure 5.6.

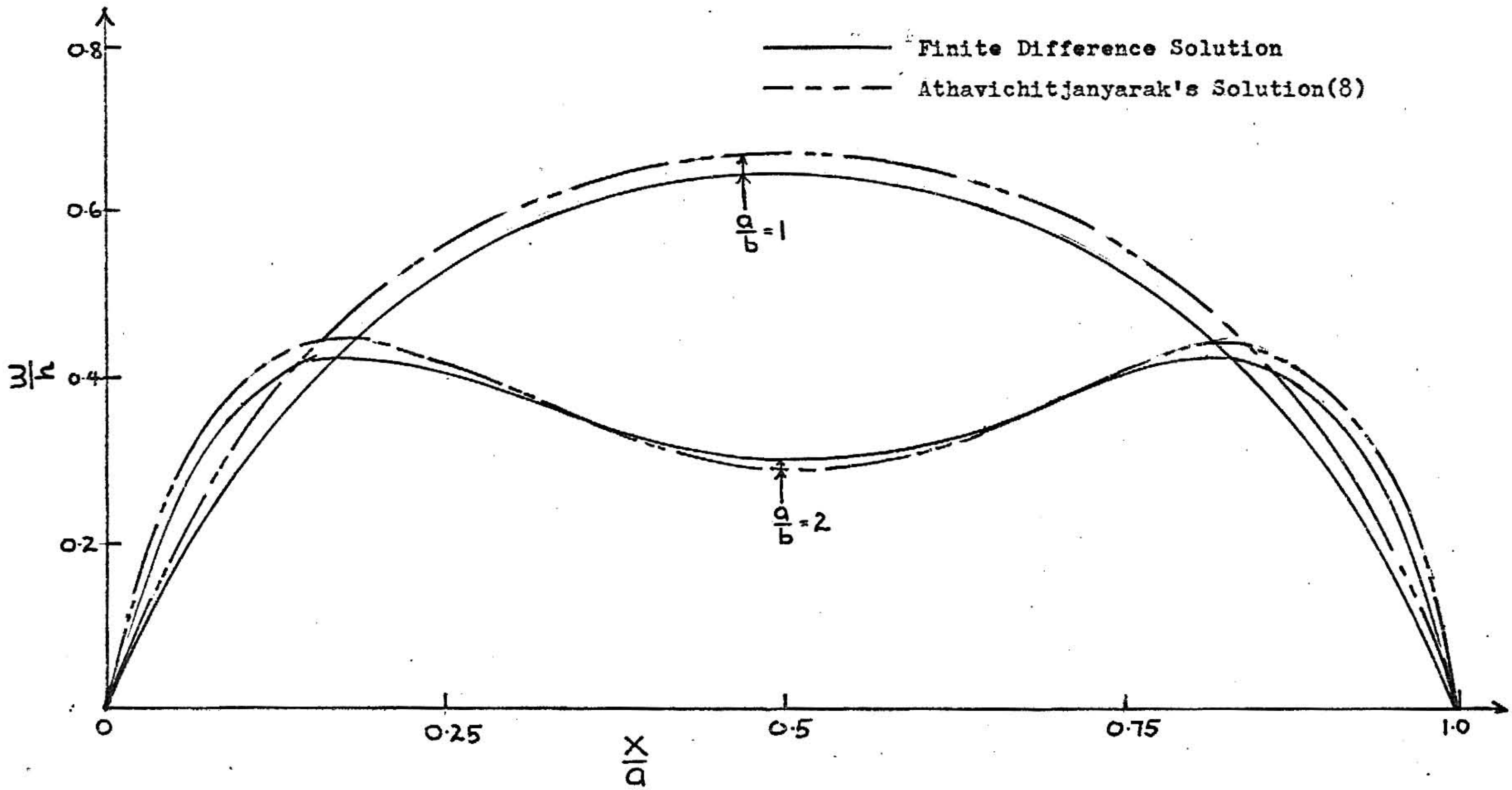

FIG. 5.6 DEFLECTION PROFILES FOR ECCENTRICALLY-LOADED ALUMINUM PLATES WITH ASPECT RATIOS a/b=1,2.

Reinforced Concrete Plate

Two reinforced concrete plates were analyzed using the finite difference solution for their response under eccentric compressive loading. The data for the plates are given below:

Plate 3: 8' x 8' x 1"

Plate 4: 16' x 8' x 1"

Eccentricity of loading = 0.5", $\nu = 0.15$

$E_{xc} = E_{yc} = E_c = 3.0 \times 10^6$ psi

Reinforcement: Two layers in both longitudinal and transverse directions

Steel ratio, $\rho_i = 0.01$

$E_s = 30 \times 10^6$ psi, $\overline{Z}_i = 0.375"$

The plates were loaded as shown in Figure 3.7. Plate 3 was analyzed using thirty-six points in the finite difference network for one quarter of the plate. For plate 4, fifty points were used. The load-deflection curves for plates 3 and 4 for the case of no cracking are shown in Figures 5.7 and 5.8 respectively. They show that the deflection of the plate is not directly proportional to the loading. For a given load, the deflection of the plate increases as the warping parameter decreases. Figure 5.9 shows the load-deflection curves for Plate 4 for both cases when the plate is uncracked and when cracked. The deflection increased considerably when cracking started.

The deflection profile for Plate 3 was different from that of Plate 4. In the x-direction, Plate 3 deflected into a single-valued curvature while the curvature of Plate 4 was triple-valued. These are shown in Figure 5.10. In the y-direction both plates deflected into a single-valued curvature. For Plate 4, the deflection of the middle part of the plate reversed direction when cracking started and the loading increased. This is shown in Figure 5.11.

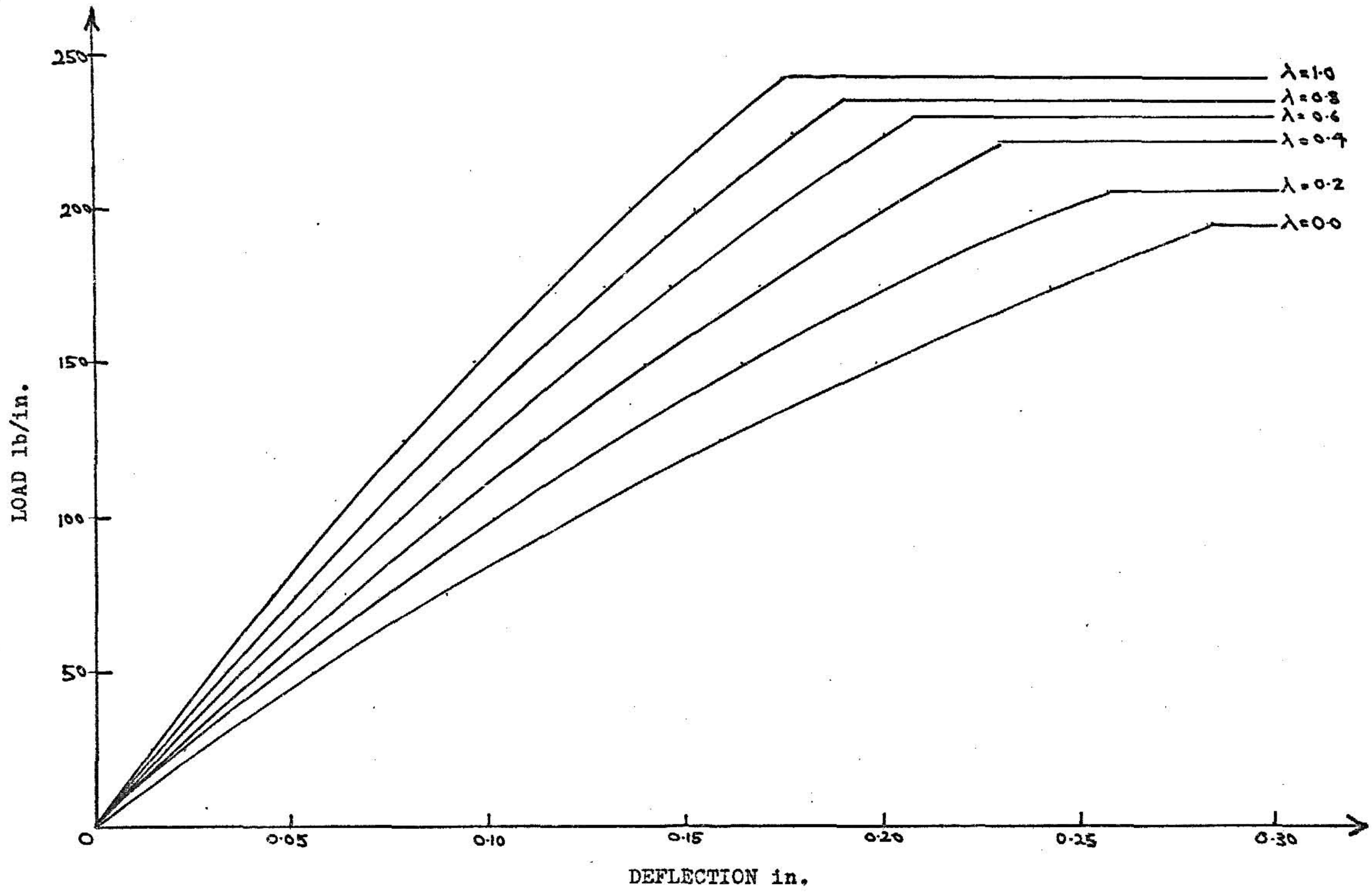

FIG. 5.7 LOAD-DEFLECTION CURVES FOR ECCENTRICALLY-LOADED UNCRACKED REINFORCED CONCRETE SQUARE PLATE

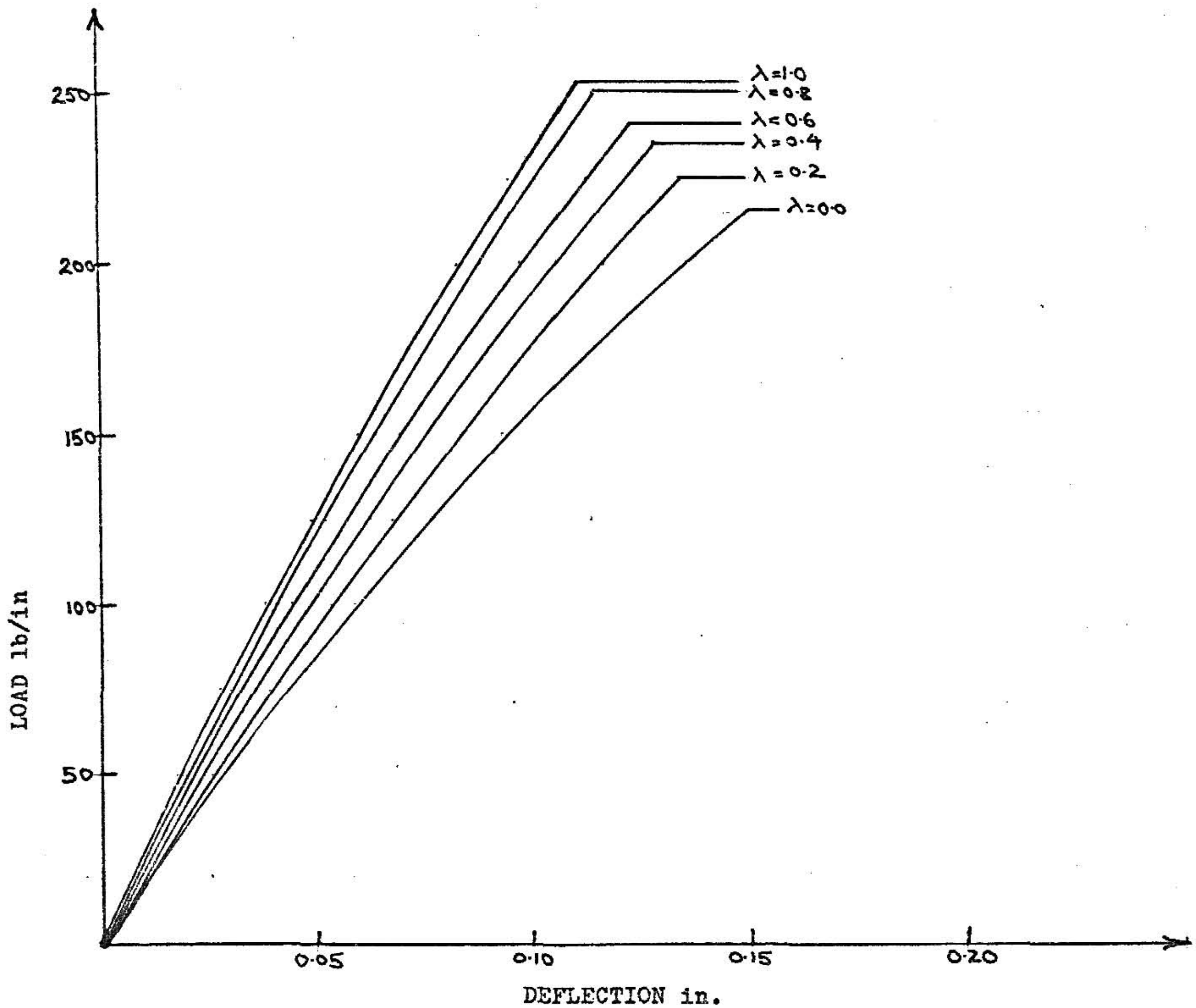

FIG. 5.8 LOAD-DEFLECTION CURVES FOR ECCENTRICALLY-LOADED REINFORCED CONCRETE PLATE
(PLATE UNCRACKED, ASPECT RATIO OF 2)

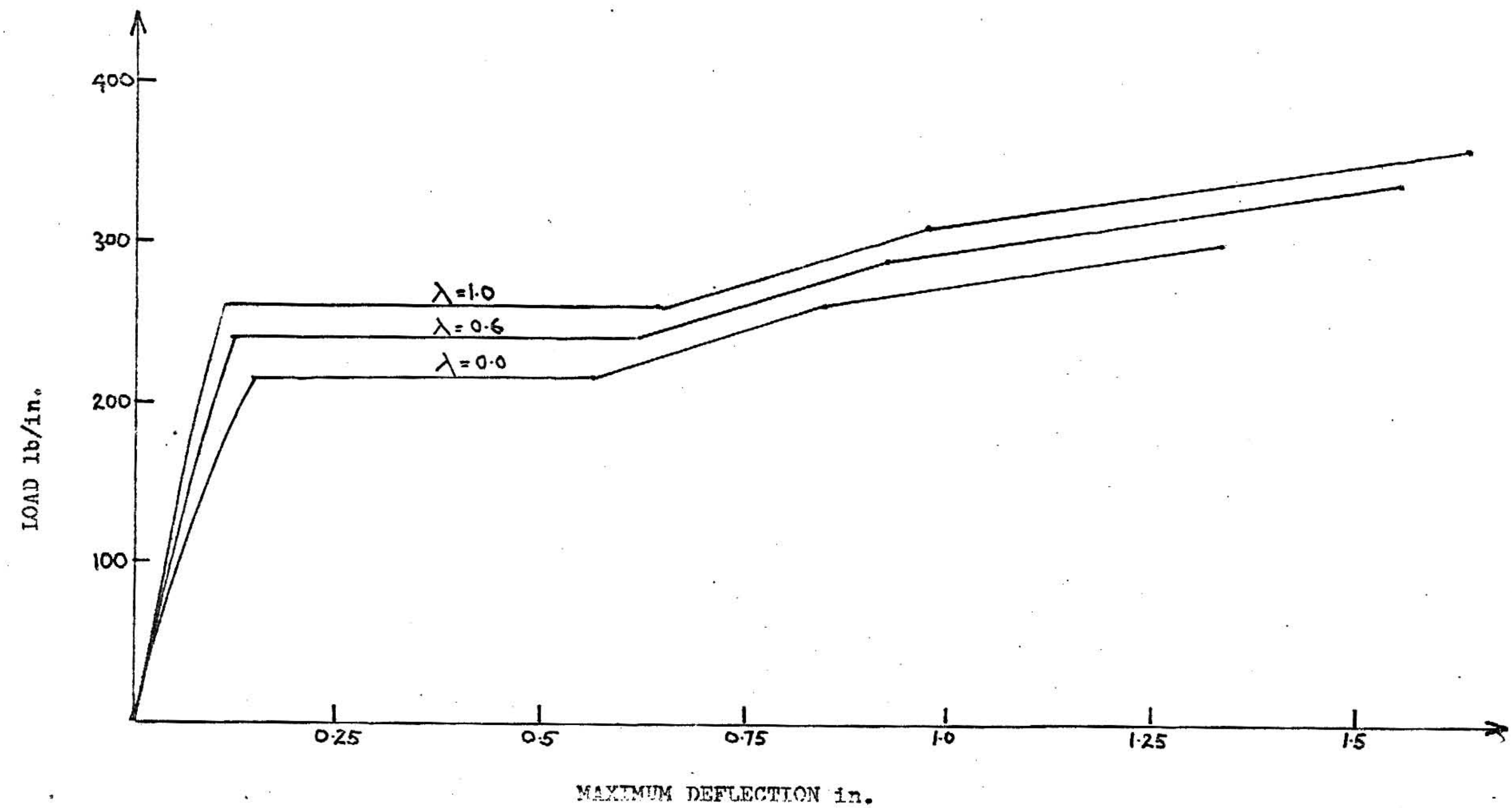

FIG. 5.9. LOAD vs MAXIMUM DEFLECTION OF REINFORCED CONCRETE PLATE WITH ASPECT RATIO OF 2
(ECCENTRIC LOAD)

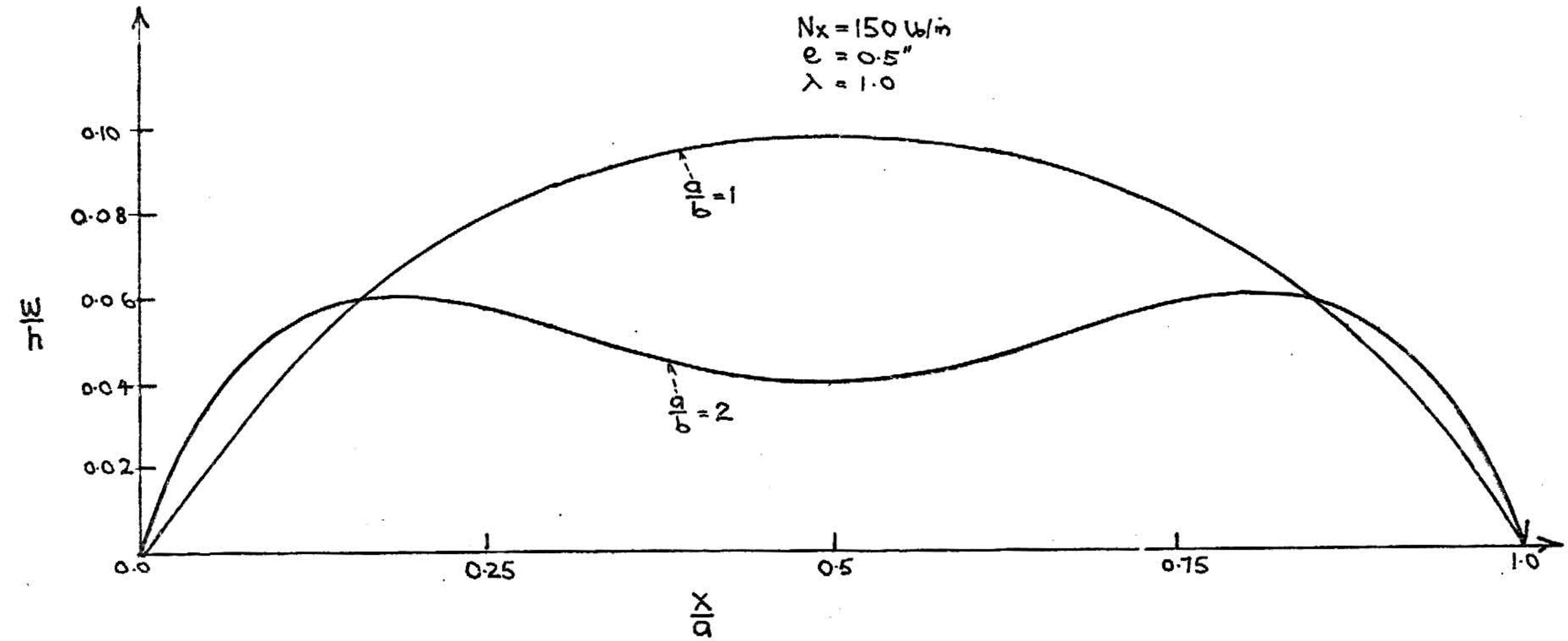

FIG. 5.10 DEFLECTION PROFILES FOR REINFORCED CONCRETE PLATES WITH ASPECT RATIOS a/b=1,2 - PLATE UNCRACKED, ECCENTRIC LOAD

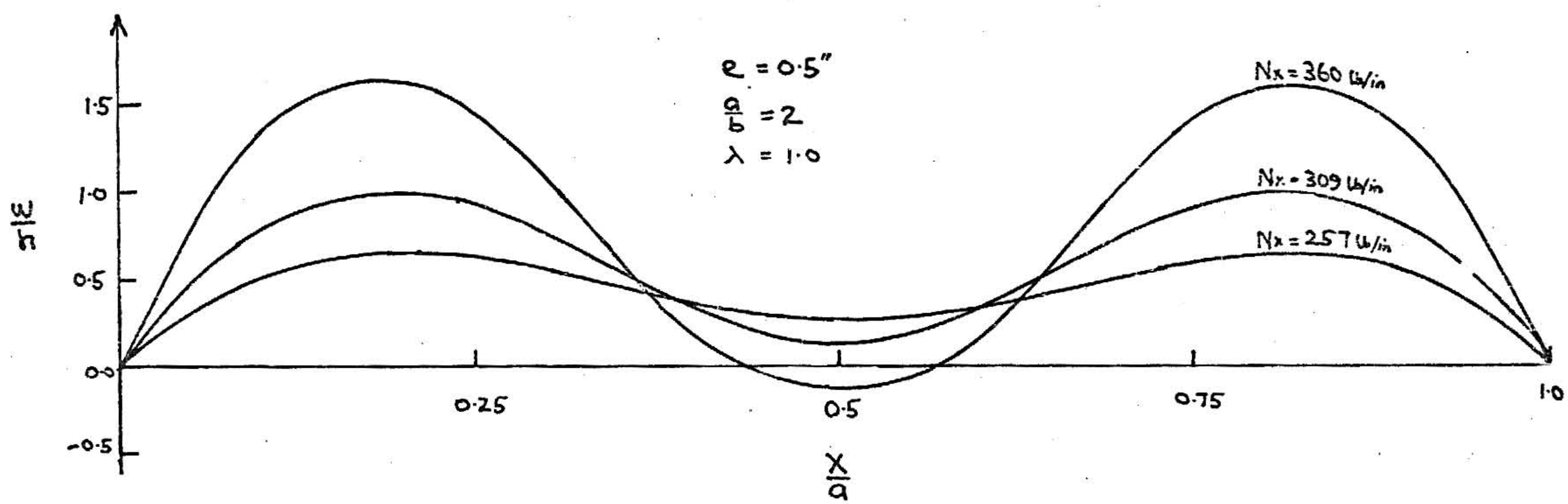

FIG. 5.11 DEFLECTION PROFILES FOR REINFORCED CONCRETE PLATE WITH ASPECT RATIO OF 2 - PLATE CRACKED, ECCENTRIC LOAD.

CHAPTER 6

CONCLUSIONS

On the basis of the numerical results obtained from this study, the following conclusions can be drawn.

The average slope of the load versus maximum plate deflection plot is lower for the cracked plate than for the uncracked plate. The same can be said of the plot of load versus maximum moments. Thus, the cracking of the concrete has an effect on the plate behavior and should be taken into account in the analysis of reinforced concrete plates.

The maximum plate deflection and bending moments for a given load on the uncracked plate change considerably when the warping parameter, λ, changes from 1 to 0. The corresponding percent changes for the cracked plate are less than those for the uncracked plate. Hence, the plate torsional stiffness of the plate affects the response of the reinforced concrete plate. However, the response of the cracked plate is less sensitive to the plate torsional stiffness than the response of the uncracked plate.

The behavior of a simply-supported plate subjected to an eccentric in-plane compressive load is dependent on the length-width ratio of the plate. For a square plate the plate bends into a single curvature with maximum plate deflection at the center of the plate while for a plate with a length-width ratio of two the curvature of the deflected plate is tri-valued and the maximum deflection is near the shorter edge of the plate. The load-deflection relationship for the plate with eccentric in-plane loading is not linear. This behavior, which was observed for elastic, isotropic plates (8) is apparently not affected by cracking.

The finite difference analysis can adequately predict the crack propagation of the reinforced concrete plate subjected to load.

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APPENDIX A

Notation

NOTATION

a	plate length
A_s	area of tension reinforcement per unit length
A_s'	area of compression reinforcement per unit length
b	plate width
$\bar{c}_i$	distance from i^{th} layer of reinforcement to neutral axis of cracked section
C_n	depth from compression face to neutral axis of cracked section
d	depth from compression face to bottom layer of reinforcement
d_2	depth from compression face to top layer of reinforcement
D	flexural rigidity of plate
D_x, D_y	stiffness factors in x and y coordinate directions
D_{xu}, D_{yu}	stiffness factors for uncracked section
D_{xcr}, D_{ycr}	stiffness factors for cracked section
e	eccentricity of in-plane load
E_c	modulus of elasticity for concrete
E_s	modulus of elasticity for steel
E_{xc}, E_{yc}	modulus of elasticity for concrete in x and y coordinate directions
f'_c	cylinder strength of concrete
h	plate thickness
H	size of square in finite difference grid
H_o	warping stiffness
I	point in finite difference grid
I_x, I_y	transformed moments of inertia in x and y directions respectively
M_o	moment per unit length

M, N	numbers used to specify the number of points in the finite difference grid
M_x, M_y, M_{xy}	bending and twisting moments per unit distance in plate
$\underline{n}$	modular ratio
N_x, N_y, N_{xy}	normal and shearing forces per unit distance in the plane of plate
p_i	steel ratio per layer
q	transverse load per unit area
Q_x, Q_y	shear forces per unit distance in plate or shell
w	deflection normal to plate
$\overline{Z}_i$	depth from i^{th} steel layer to neutral axis of uncracked section
λ	warping parameter
v	Poisson's ratio for concrete
ϵ_x, ϵ_y	unit normal strains in x and y directions
ϵ_{xsi}, ϵ_{ysi}	unit strains in i^{th} layer of reinforcement in x and y directions
ϵ_{xc}, ϵ_{yc}	unit strains in concrete in x and y directions
γ_{xy}	unit shearing strain
σ_x, σ_y	unit normal stresses in x and y directions
τ_{xy}	unit shearing stress on plane perpendicular to x-axis and parallel to y-axis
σ_{xsi}, σ_{ysi}	unit stresses in i^{th} layer of reinforcement in x and y directions
σ_{xc}, σ_{yc}	unit stresses in concrete in x and y directions
ρ_i	steel ratio

APPENDIX B

Computer Program

THIS BOOK CONTAINS NUMEROUS PAGES WITH ILLEGIBLE PAGE NUMBERS THAT ARE CUT OFF, MISSING OR OF POOR QUALITY TEXT.

THIS IS AS RECEIVED FROM THE CUSTOMER.

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      DIMENSION A(50,50),B(50),S(2500),EPSB(50),EPST(50),IC(50),IP(50),
     1PSTY(50),EPSBY(50),ICR(50),IR(50)
      REAL PR,T,H,SR1,SR2,Z1U,Z2U,WP,D,D2,NR,ES,EC,DQ,Q,EPSM,EPCM,TEMP,
     11C,Z2C,NX,ECC
      INTEGER M,N,ICOUNT,JCOUNT,KCOUNT,JJ,NBC,LCOUNT,IPROB,IND,NNN
   95 FORMAT('0',57X,'W-DEFLECTIONS FOR Q=',E11.5)
  102 FORMAT(10(2X,E10.4))
 1207 FORMAT(1H0,20X,'ERROR IN DATA OR MATRIX IS SINGULAR')
 1208 FORMAT(1H0,20X,'WARNING-POSSIBLE LOSS OF SIGNIFICANCE INDICATED')
      READ(5,5) NBC
    5 FORMAT(I2)
      READ(5,90) M,N,ES,EC,PR,EPSM,EPCM
   90 FORMAT(2I4,2E13.6,F6.2,2E10.3)
   91 READ(5,96) T,H,D,D2,Z1U,Z2U,SR1
   96 FORMAT(7F10.4)
C*********************************************************************
C     NBC=0 MEANS FIXED EDGES
C     NBC=1 MEANS SIMPLY SUPPORTED EDGES
C     IPROB=0 MEANS LATERAL LOADING OR END MOMENTS
C     IPROB=1 MEANS IN-PLANE LOADING
C     IND=0 MEANS PLATE IS LOADED BY END MOMENTS
C     IND=1 MEANS LATERAL LOADING
C     NX IS THE IN-PLANE COMPRESSIVE LOAD PER UNIT LENGTH
C     ECC IS THE ECCENTRICIYY OF THE IN-PLANE LOAD
C*********************************************************************
      READ(5,5) IPROB
C*********************************************************************
C     IF PLATE IS LOADED LATERALLY OR BY END MOMENTS,
C     INPUT DATA IS VALUE OF IND
C   IF PLATE IS LOADED IN ITS PLANE, INPUT DATA IS VALUE OF ECCENTRICIT
C*********************************************************************
      IF(IPROB) 1401,1401,76
 1401 READ(5,5) IND
      GO TO 1
   76 READ(5,1315) ECC
 1315 FORMAT(F6.2)
    1 READ(5,2,END=999) WP
    2 FORMAT(F6.2)
C*********************************************************************
C     DEFINITION OF NOTATIONS
C*********************************************************************
      WRITE(6,85)
   85 FORMAT(55X,'NOTATIONS')
      WRITE(6,86)
   86 FORMAT('0',8X,'M,N,GIVE THE NUMBER OF SQUARES IN THE FINITE
     1DIFFERENCE GRID',3X,'ES=MODULUS OF ELASTICITY OF STEEL',/,4X,'EC=
     2MODULUS OF ELASTICITY OF CONCRETE',3X,'WP=WARPING PARAMETER',3X,'
     3R=POISSONS RATIO',3X,'EPSM=MAX. TENSILE STRAIN IN CONCRETE',/,'
     4EPCM=MAX. ALLOWABLE COMPRESSIVE STRAIN IN CONCRETE')
      WRITE(6,87)
   87 FORMAT('0',4X,'T=PLATE THICKNESS',3X,'H=SIZE OF FINITE DIFFERENCE
     1SQUARE',3X,'D=DEPTH OF LOWER STEEL LAYER FROM COMPRESSION FACE',/
     14X,'D2=DEPTH OF UPPER STEEL LAYER FROM COMPRESSION FACE',3X,'SR1=
     2STEEL RATIO, AS/T',3X,'SR2=STEEL RATIO, AS/D')
      WRITE(6,88)
   88 FORMAT('0',4X,'Z1U,Z2U=DEPTHS OF UPPER AND LOWER STEEL LAYERS
     1RESPECTIVELY FROM NEUTRAL AXIS OF UNCRACKED SECTION',/,4X,'Z1C,Z2
     2=DEPTHS OF UPPER AND LOWER STEEL LAYERS RESPECTIVELY FROM NEUTRAL
     3AXIS OF CRACKED SECTION')
      IF(NBC) 6,6,8
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      DIMENSION A(50,50),B(50),S(2500),EPSB(50),EPST(50),IC(50),IP(50),
     1PSTY(50),EPSBY(50),ICR(50),IR(50)
      REAL PR,T,H,SR1,SR2,Z1U,Z2U,WP,D,D2,NR,ES,EC,DQ,Q,EPSM,EPCM,TEMP,
     11C,Z2C,NX,ECC
      INTEGER M,N,ICOUNT,JCOUNT,KCOUNT,JJ,NBC,LCOUNT,IPROB,IND,NNN
   95 FORMAT('0',57X,'W-DEFLECTIONS FOR Q=',E11.5)
  102 FORMAT(10(2X,E10.4))
 1207 FORMAT(1HO,20X,'ERROR IN DATA OR MATRIX IS SINGULAR')
 1208 FORMAT(1HO,20X,'WARNING-POSSIBLE LOSS OF SIGNIFICANCE INDICATED')
      READ(5,5) NBC
    5 FORMAT(I2)
      READ(5,90) M,N,ES,EC,PR,EPSM,EPCM
   90 FORMAT(2I4,2E13.6,F6.2,2E10.3)
   91 READ(5,96) T,H,D,D2,Z1U,Z2U,SR1
   96 FORMAT(7F10.4)
C***********************************************************************
C      NBC=0 MEANS FIXED EDGES
C      NBC=1 MEANS SIMPLY SUPPORTED EDGES
C      IPROB=0 MEANS LATERAL LOADING OR END MOMENTS
C      IPROB=1 MEANS IN-PLANE LOADING
C      IND=0 MEANS PLATE IS LOADED BY END MOMENTS
C      IND=1 MEANS LATERAL LOADING
C      NX IS THE IN-PLANE COMPRESSIVE LOAD PER UNIT LENGTH
C      ECC IS THE ECCENTRICIYY OF THE IN-PLANE LOAD
C***********************************************************************
      READ(5,5) IPROB
C***********************************************************************
C      IF PLATE IS LOADED LATERALLY OR BY END MOMENTS,
C      INPUT DATA IS VALUE OF IND
C   IF PLATE IS LOADED IN ITS PLANE, INPUT DATA IS VALUE OF ECCENTRICIT
C***********************************************************************
      IF(IPROB) 1401,1401,76
 1401 READ(5,5) IND
      GO TO 1
   76 READ(5,1315) ECC
 1315 FORMAT(F6.2)
    1 READ(5,2,END=999) WP
    2 FORMAT(F6.2)
C***********************************************************************
C      DEFINITION OF NOTATIONS
C***********************************************************************
      WRITE(6,85)
   85 FORMAT(55X,'NOTATIONS')
      WRITE(6,86)
   86 FORMAT('0',8X,'M,N,GIVE THE NUMBER OF SQUARES IN THE FINITE
     1DIFFERENCE GRID',3X,'ES=MODULUS OF ELASTICITY OF STEEL',/,4X,'EC=
     2MODULUS OF ELASTICITY OF CONCRETE',3X,'WP=WARPING PARAMETER',3X,'
     3R=POISSONS RATIO',3X,'EPSM=MAX. TENSILE STRAIN IN CONCRETE',/,'
     4EPCM=MAX. ALLOWABLE COMPRESSIVE STRAIN IN CONCRETE')
      WRITE(6,87)
   87 FORMAT('0',4X,'T=PLATE THICKNESS',3X,'H=SIZE OF FINITE DIFFERENCE
     1SQUARE',3X,'D=DEPTH OF LOWER STEEL LAYER FROM COMPRESSION FACE',/,
     14X,'D2=DEPTH OF UPPER STEEL LAYER FROM COMPRESSION FACE',3X,'SR1=
     2STEEL RATIO, AS/T',3X,'SR2=STEEL RATIO, AS/D')
      WRITE(6,88)
   88 FORMAT('0',4X,'Z1U,Z2U=DEPTHS OF UPPER AND LOWER STEEL LAYERS
     1RESPECTIVELY FROM NEUTRAL AXIS OF UNCRACKED SECTION',/,4X,'Z1C,Z2(
     2=DEPTHS OF UPPER AND LOWER STEEL LAYERS RESPECTIVELY FROM NEUTRAL
     3AXIS OF CRACKED SECTION')
      IF(NBC) 6,6,8
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    6 WRITE(6,7)
    7 FORMAT('0',50X,'SUPPORT CONDITION-FIXED EDGES')
      GO TO 1300
    8 WRITE(6,9)
    9 FORMAT('0',50X,'SUPPORT CONDI ION-SIMPLY SUPPORTED')
 1300 IF(IPROB) 1301,1301,1302
 1301 IF(IND) 1307,1307,1308
 1307 WRITE(6,1304)
 1304 FORMAT('0',50X,'LOADING--END MOMENTS')
      GO TO 1305
 1308 WRITE(6,1410)
 1410 FORMAT('0',50X,'LATERAL LOADING')
      GO TO 1305
 1302 WRITE(6,1303)
 1303 FORMAT('0',50X,'IN-PLANE COMPRESSIVE LOADING')
 1305 WRITE(6,89)
   89 FORMAT('0',57X,'DATA')
      WRITE(6,97) M,N,ES,EC,PR,WP,EPSM,EPCM
   97 FORMAT('0',3X,'M=',I4,3X,'N=',I4,3X,'ES=',E13.6,3X,'EC=',E13.6,3X,
     1'PR=',F6.2,3X,'WP=',F6.2,3X,'EPSM=',E10.3,3X,'EPCM=',E10.3)
      WRITE(6,3)
    3 FORMAT('+',71X,'__________')
      WRITE(6,93) T,H,D,D2,Z1U,Z2U,SR1
   93 FORMAT(3X,'T=',F10.4,3X,'H=',F10.4,3X,'D=',F10.4,3X,'D2=',F10.4,/
     13X,'Z1U=',F10.4,3X,'Z2U=',F10.4,3X,'SR1=',F10.4)
      ICOUNT=0
      JCOUNT=0
      KCOUNT=0
      JJ=0
      LCOUNT=0
      NNN=0
C***********************************************************************
C     COMPUTE MATERIAL STIFFNESS FACTORS
C***********************************************************************
      NR=ES/EC
      DX1U=(EC*T**3)/(12.*(1.-PR**2))
      DY1U=(EC*T**3)/(12.*(1.-PR**2))
      DSU=ES*T*((SR1*Z1U**2)+(SR1*Z2U**2))
      DXU=DX1U+DSU
      DYU=DY1U+DSU
      DXYU=(WP*(1.-PR)/2.)*SQRT(DX1U*DY1U)
      D1U=WP*PR*SQRT(DX1U*DY1U)
      SR2=SR1*T/D
      CM=-2.*(2.*NR-1.)*SR2*D+SQRT(4.*(2.*NR-1.)**2*SR2**2*D**2+8.*((NR-
     11.)*D2+NR*D)*SR2*D)
      CN=CM/2.
      DX1C=(EC*CN**3)/(3.*(1-PR**2))
      DY1C=(EC*CN**3)/(3.*(1-PR**2))
      Z1C=D-CN
      Z2C=CN-D2
      DSC=ES*T*((SR1*Z1C**2)+(SR1*Z2C**2))
      DXC=DX1C+DSC
      DYC=DY1C+DSC
      D1C=WP*PR*SQRT(DX1C*DY1C)
      DXYC=(WP*(1.-PR)/2.)*SQRT(DX1C+DY1C)
      BMU=DXU/H**4
      BNU=DYU/H**4
      BPU=(WP/H**4)*SQRT(DX1U*DY1U)
      BMC=DXC/H**4
      BNC=DYC/H**4
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      BPC=(WP/H**4)*SQRT(DX1C*DY1C)
C*********************************************************************
C     USE UNCRACKED STIFFNESS FACRORS IN COMPUTING MATRIX COEFFICIENTS
C*********************************************************************
      BMI=BMU
      BNI=BNU
      BPI=BPU
      WRITE(6,79) DXU,D1U,DXC,D1C,CN
   79 FORMAT(10X,'DXU=',E13.6,4X,'D1U=',E13.6,/,10X,'DXC=',E13.6,4X,'D1C
     1=',E13.6,4X,'CN=',E13.6)
C*********************************************************************
C     IA IS NUMBER OF POINTS IN FINITE DIFFERENCE MESH
C     IC(I) DETERMINES WHETHER POINT IS CRACKED OR NOT
C*********************************************************************
      IA=N*M+M
      DO 15 I=1,IA
   15 IC(I)=1
      DO 150 I=1,IA
      DO 150 J=1,IA
      A(I,J)=0.
  150 CONTINUE
      NX=3.
      IF(IPROB.NE.1) GO TO 603
      WRITE(6,1380) ECC
 1380 FORMAT('0',50X,'ECCENTRICITY OF LOADING=',F6.2)
  603 BQ=NX/(H**2)
      CALL MATRIX(A,M,N,BMI,BNI,BPI,BMU,BNJ,BPU,BQ,IC,NBC,IPROB)
      IF(NNN.EQ.1) GO TO 1309
      WRITE(6,100)
  100 FORMAT(50X,'MATRIX A')
      WRITE(6,101)((A(I,J),J=1,IA),I=1,IA)
  101 FORMAT(9(2X,E12.5))
      MUD=2*M
      MLD=MUD
      MM=M*(N+1)
      EPS=1.E-65
 1309 CALL ROWMAT(A,S,MM,MUD,MLD)
      DO 1310 I=1,IA
 1310 B(I)=0.
      IF(IPROB) 1323,1323,1320
 1320 DO 1311 I=M,IA,M
 1311 B(I)=NX*ECC/(H**2)
      Q=NX
      GO TO 1326
 1323 IF(IND) 1321,1321,1324
 1321 DO 1322 I=M,IA,M
 1322 B(I)=1.
      Q=1.
      GO TO 1326
 1324 DO 1325 I=1,IA
 1325 B(I)=1.
      Q=1.
C*********************************************************************
C     COMPUTATION OF DEFLECTIONS
C*********************************************************************
 1326 CALL GELB(B,S,MM,1,MUD,MLD,EPS,KS)
      IF(KS) 1203,1204,1205
 1203 WRITE(6,1207)
      GO TO 1000
 1205 WRITE(6,1208)
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 1204 WRITE(6,95) Q
      WRITE(6,77)
   77 FORMAT('+',57X,'______________________________')
      WRITE(6,102) (B(I),I=1,IA)
  105 DO 106 I=1,IA
  106 IC(I)=0
      NNN=1
C***********************************************************************
C     COMPUTATION OF STRAINS
C     EPSB IS STRAIN IN BOTTOM FIBER IN X-DIRECTION
C     EPST IS STRAIN IN TOP FIBER IN X-DIRECTION
C     EPSBY IS STRAIN IN BOTTOM FIBER IN Y-DIRECTION
C     EPSTY IS STRAIN IN TOP FIBER IN Y-DIRECTION
C***********************************************************************
      NV=N*M+1
      NCOL=M-1
      NROW=N+1
      NU=(N-1)*M+M
      NZ=M+1
      NY=N*M+1
  500 DO 107 I=1,NV,M
C***********************************************************************
C     IF POINT IS ALREADY CRACKED ONLY COMPRESSIVE STRAIN IS COMPUTED
C***********************************************************************
      IF(IC(I)-1) 40,41,41
   40 EPSB(I)=-T/(2.*H**2)*(2.*B(I+1)-2.*B(I))
      EPST(I)=-EPSB(I)
      GO TO 107
   41 IF(EPST(I)-EPSB(I)) 42,42,43
   42 EPST(I)=CN/(H**2)*(2.*B(I+1)-2.*B(I))
      GO TO 107
   43 EPSB(I)=-CN/(H**2)*(2.*B(I+1)-2.*B(I))
  107 CONTINUE
      DO 108 I=M,IA,M
      IF(IC(I)-1) 35,36,36
   35 EPSB(I)=-T/(2.*H**2)*(B(I-1)-2.*B(I))
      EPST(I)=-EPSB(I)
      GO TO 108
   36 IF(EPST(I)-EPSB(I)) 37,37,38
   37 EPST(I)=CN/(H**2)*(B(I-1)-2.*B(I))
      GO TO 108
   38 EPSB(I)=-CN/(H**2)*(B(I-1)-2.*B(I))
  108 CONTINUE
      DO 109 JK=1,NROW
      ISTART=JK+1+(JK-1)*NCOL
      IEND=JK*M-1
      DO 110 I=ISTART,IEND
      IF(IC(I)-1) 30,31,31
   30 EPSB(I)=-T/(2.*H**2)*(B(I+1)-2.*B(I)+B(I-1))
      EPST(I)=-EPSB(I)
      GO TO 110
   31 IF(EPST(I)-EPSB(I)) 32,32,33
   32 EPST(I)=CN/(H**2)*(B(I+1)-2.*B(I)+B(I-1))
      GO TO 110
   33 EPSB(I)=-CN/(H**2)*(B(I+1)-2.*B(I)+B(I-1))
  110 CONTINUE
  109 CONTINUE
      DO 300 I=1,M
      IF(IC(I)-1) 301,302,302
  301 EPSBY(I)=-T/(2.*H**2)*(2.*B(I+M)-2.*B(I))
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      EPSTY(I)=-EPSBY(I)
      GO TO 300
  302 EPSTY(I)=CN/(H**2)*(2.*B(I+M)-2.*B(I))
  300 CONTINUE
      DO 305 I=NY,IA
      IF(IC(I)-1) 306,307,307
  306 EPSBY(I)=-T/(2.*H**2)*(B(I-M)-2.*B(I))
      EPSTY(I)=-EPSBY(I)
      GO TO 305
  307 EPSTY(I)=CN/(H**2)*(B(I-M)-2.*B(I))
  305 CONTINUE
      DO 310 I=NZ,NU
      IF(IC(I)-1) 311,312,312
  311 EPSBY(I)=-T/(2.*H**2)*(B(I-M)-2.*B(I)+B(I+M))
      EPSTY(I)=-EPSBY(I)
      GO TO 310
  312 EPSTY(I)=CN/(H**2)*(B(I-M)-2.*B(I)+B(I+M))
  310 CONTINUE
C************************************************************************
C     SUBROUTINE EPSMAX DETERMINES CRACKED POINTS
C************************************************************************
  161 CALL EPSMAX(IA,IC,EPST,EPSB,TEMP,ICOUNT,EPSM,EPSBY,EPSTY,IR)
      IF(ICOUNT-1) 160,70,70
  160 IF(IPROB) 162,162,163
  162 DO 170 I=1,IA
      B(I)=B(I)*EPSM/TEMP
      EPST(I)=EPST(I)*EPSM/TEMP
      EPSB(I)=-EPST(I)
      EPSTY(I)=EPSTY(I)*EPSM/TEMP
  170 EPSBY(I)=-EPSTY(I)
      Q=Q*EPSM/TEMP
      GO TO 165
  163 QN=Q*EPSM/TEMP
      WRITE(6,599) TEMP,QN
  599 FORMAT('0',30X,'TEMP=',E11.5,10X,'QN=',E11.4)
      TRY=ABS(QN-Q)/Q
      IF(TRY-.001) 600,600,601
  601 DO 602 I=1,IA
  602 IC(I)=1
      NX=QN
      GO TO 603
  600 Q=QN
  165 WRITE(6,80) Q
      DQ=Q/10.
   80 FORMAT(45X,'W-DEFLECTIONS JUST BEFORE FIRST CRACK,   Q=',E11.4)
      WRITE(6,78)
   78 FORMAT('+',45X,'______________________________________________
     1___')
      WRITE(6,75) (B(I),I=1,IA)
   75 FORMAT(10(2X,E10.4))
  200 WRITE(6,210)
  210 FORMAT('0',57X,'STRAINS IN TOP FIBERS')
      WRITE(6,215)
  215 FORMAT(57X,'X-DIRECTION')
      WRITE(6,220) (EPST(I),I=1,IA)
  220 FORMAT(9(2X,E11.4))
      WRITE(6,225)
  225 FORMAT(57X,'Y-DIRECTION')
      WRITE(6,220) (EPSTY(I),I=1,IA)
      WRITE(6,230)
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  230 FORMAT('0',57X,'STRAINS IN BOTTOM FIBERS')
      WRITE(6,215)
      WRITE(6,220) (EPSB(I),I=1,IA)
      WRITE(6,225)
      WRITE(6,220) (EPSBY(I),I=1,IA)
      ICOUNT=1
C***********************************************************************
C     USE CRACKED STIFFNESS FACTORS IN COMPUTING MATRIX COEFFICIENTS
C     KCOUNT COUNTS NUMBER  OF NEW CRACKED POINTS
C***********************************************************************
      BMI=BMC
      BNI=BNC
      BPI=BPC
   70 KCOUNT=0
   71 DO 65 I=1,IA
      IF(IC(I)-1) 65,60,65
   60 JJ=JJ+1
      IP(JJ)=I
      ICR(JJ)=IR(I)
      KCOUNT=KCOUNT+1
   65 CONTINUE
      IF(KCOUNT-1) 55,50,50
C***********************************************************************
C     JCOUNT COUNTS NUMBER OF CYCLES OF COMPUTATION FOR EACH LOAD
C     INCREMENT BEFORE EQUILIBRIUM IS REACHED
C***********************************************************************
   50 JCOUNT=JCOUNT+1
      WRITE(6,1381) Q
 1381 FORMAT('0',57X,'DEFLECTIONS      Q=',E11.5)
      WRITE(6,75) (B(I),I=1,IA)
      DO 48 I=1,IA
   48 B(I)=0.
      IF(IPROB) 1327,1327,1328
 1328 NX=Q
      BQ=NX/(H**2)
      DO 1329 I=M,IA,M
 1329 B(I)=NX*ECC/(H**2)
      GO TO 1334
 1327 IF(IND) 1330,1330,1332
 1330 DO 1331 I=M,IA,M
 1331 B(I)=Q
      GO TO 1334
 1332 DO 1333 I=1,IA
 1333 B(I)=Q
 1334 CALL MATRIX(A,M,N,BMI,BNI,BPI,BMU,BNU,BPU,BQ,IC,NBC,IPROB)
      CALL ROWMAT(A,S,MM,MUD,MLD)
      CALL GELB(B,S,MM,1,MUD,MLD,EPS,KS)
      IF(KS) 1210,116,1213
 1210 WRITE(6,1207)
      GO TO 1000
 1213 WRITE(6,1208)
  116 DO 117 I=1,IA
      IF(IC(I).NE.1) GO TO 117
      IC(I)=2
  117 CONTINUE
      GO TO 500
C***********************************************************************
C     OUTPUT DATA
C***********************************************************************
   55 WRITE(6,54) JCOUNT
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   54 FORMAT('0',50X,'NUMBER OF CYCLES BEFORE EQUILIBRIUM=',I4)
      WRITE(6,53)
   53 FORMAT('0',50X,'POINTS CRACKED BY LOAD INCREMENT')
      WRITE(6,52) (IP(K),K=1,JJ)
   52 FORMAT('0',18(3X,I4))
      WRITE(6,1342)
 1342 FORMAT('0',44X,'CRACK DIRECTIONS----- X-DIRN(1)     Y-DIRN(2)')
      WRITE(6,52) (ICR(K),K=1,JJ)
      WRITE(6,140) Q
  140 FORMAT('0',57X,'W-DEFLECTIONS(2),    Q=',E11.4)
      WRITE(6,77)
      WRITE(6,141) (B(I),I=1,IA)
  141 FORMAT(10(2X,E10.4))
      WRITE(6,210)
      WRITE(6,215)
      WRITE(6,220) (EPST(I),I=1,IA)
      WRITE(6,225)
      WRITE(6,220) (EPSTY(I),I=1,IA)
      WRITE(6,230)
      WRITE(6,215)
      WRITE(6,220) (EPSB(I),I=1,IA)
      WRITE(6,225)
      WRITE(6,220) (EPSBY(I),I=1,IA)
      JCOUNT=0
      DO 120 K=1,JJ
      ICR(K)=0
  120 IP(K)=0
      JJ=0
C
C     DETERMINE IF ALLOWABLE COMPRESSIVE STRAIN HAS BEEN EXCEEDED
C
      DO 121 I=1,IA
      IF(EPST(I)-EPSB(I)) 250,250,251
  250 IF(EPST(I)-EPCM) 137,135,159
  251 IF(EPSB(I)-EPCM) 252,253,159
  159 IF(EPSTY(I)-EPCM) 151,152,121
  121 CONTINUE
C***********************************************************************
C     INCREASE LOAD
C     DDQ,DQ GIVE LOAD INCREMENT VALUES
C***********************************************************************
      LCOUNT=LCOUNT+1
      QOLD=Q
      IF(LCOUNT-2) 401,401,402
  401 DDQ=DQ
      GO TO 403
  402 DDQ=2.*DQ
  403 Q=Q+DDQ
      IF(IPROB) 1335,1335,1336
 1336 DO 1337 I=1,IA
 1337 B(I)=0.
      NX=0
      BQ=NX/(H**2)
      DO 1338 I=M,IA,M
 1338 B(I)=NX*ECC/(H**2)
      CALL MATRIX(A,M,N,BMI,BNI,BPI,BMU,BNJ,BPU,BQ,IC,NBC,IPROB)
      CALL ROWMAT(A,S,MM,MUD,MLD)
      CALL GELB(B,S,MM,1,MUD,MLD,EPS,KS)
      GO TO 1215
 1335 DO 125 I=1,IA
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      B(I)=B(I)*Q/QOLD
      EPST(I)=EPST(I)*Q/QOLD
      EPSB(I)=EPSB(I)*Q/QOLD
      EPSTY(I)=EPSTY(I)*Q/QOLD
      EPSBY(I)=EPSBY(I)*Q/QOLD
  125 CONTINUE
 1215 WRITE(6,126) Q
  126 FORMAT('0',57X,'W-DEFLECTIONS(I),     Q=',E11.4)
      WRITE(6,77)
      WRITE(6,127) (B(I),I=1,IA)
  127 FORMAT(10(2X,E10.4))
      IF(IPROB) 1339,1339,1340
 1339 GO TO 161
 1340 GO TO 500
  135 WRITE(6,136) I
  136 FORMAT('0',57X,'EPST(',I3,') EQUAL TO EPCM')
      GO TO 1000
  137 WRITE(6,138) I
  138 FORMAT('0',57X,'EPST(',I3,') GREATER THAN EPCM')
      GO TO 1000
  253 WRITE(6,255) I
  255 FORMAT('0',57X,'EPSB(',I3,') EQUAL TO EPCM')
      GO TO 1000
  252 WRITE(6,254) I
  254 FORMAT('0',57X,'EPSB(',I3,') GREATER THAN EPCM')
      GO TO 1000
  151 WRITE(6,155) I
  155 FORMAT('0',57X,'EPSTY(',I3,') GREATER THAN EPCM')
      GO TO 1000
  152 WRITE(6,156) I
  156 FORMAT('0',57X,'EPSTY(',I3,') EQUAL TO EPCM')
C***********************************************************************
C     READ ANOTHER VALUE OF WARPING PARAMETER
C***********************************************************************
 1000 GO TO 1
  999 STOP
      END
```

```
C     SUBROUTINE FOR COMPUTING MATRIX A
C*****************************************************************************
      SUBROUTINE MATRIX(A,M,N,BMI,BNI,BPI,BMU,BNU,BPU,BQ,IC,NBC,IPROB)
      DIMENSION A(50,50),IC(50)
      REAL BM,BN,BP,BMI,BNI,BPI,BMU,BNU,BPU,BQ
      INTEGER M,N,NBC,IPROB
      L=N*M+M
      BM=BMI
      BN=BNI
      BP=BPI
   10 I=1
      IF(IPROB.NE.0) GO TO 12
      IF(IC(I)-1) 20,15,20
   12 IF(IC(I)-1) 14,11,11
   14 BM=BMU
      BN=BNU
      BP=BPU
   11 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,2)=-(8.*BM+8.*BP)+2.*BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 13
   15 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,2)=-(8.*BM+8.*BP)
   13 IF(IC(I).NE.1) GO TO 20
      A(I,3)=2.*BM
      A(I,M+1)=-(8.*BN+8.*BP)
      A(I,2*M+1)=2.*BN
      A(I,M+2)=8.*BP
   20 I=2
      IF(IPROB.NE.0) GO TO 22
      IF(IC(I)-1) 30,25,30
   22 IF(IC(I)-1) 24,21,21
   24 BM=BMU
      BN=BNU
      BP=BPU
   21 A(I,I)=7.*BM+6.*BN+8.*BP-2.*BQ
      A(I,1)=-(4.*BM+4.*BP)+BQ
      A(I,3)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 23
   25 A(I,I)=7.*BM+6.*BN+8.*BP
      A(I,1)=-(4.*BM+4.*BP)
      A(I,3)=-(4.*BM+4.*BP)
   23 IF(IC(I).NE.1) GO TO 30
      A(I,4)=BM
      A(I,M+1)=4.*BP
      A(I,M+2)=-(8.*BN+8.*BP)
      A(I,M+3)=4.*BP
      A(I,2*M+2)=2.*BN
   30 LL=M-2
      DO 101 I=3,LL
      IF(IPROB.NE.0) GO TO 32
      IF(IC(I)-1) 101,35,101
   32 IF(IC(I)-1) 34,31,31
   34 BM=BMU
      BN=BNU
```

```
      BP=BPU
   31 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 33
   35 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
   33 IF(IC(I).NE.1) GO TO 101
      LLA=I+M
      LLB=I+2*M
      A(I,I-2)=BM
      A(I,I+2)=BM
      A(I,LLA)=-(8.*BN+8.*BP)
      A(I,LLB)=2.*BN
      A(I,LLA-1)=4.*BP
      A(I,LLA+1)=4.*BP
  101 CONTINUE
   40 I=M-1
      IF(IPROB.NE.0) GO TO 42
      IF(IC(I)-1) 50,45,50
   42 IF(IC(I)-1) 44,41,41
   44 BM=BMU
      BN=BNU
      BP=BPU
   41 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,M)=-(4.*BM+4.*BP)+BQ
      A(I,M-2)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 43
   45 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,M)=-(4.*BM+4.*BP)
      A(I,M-2)=-(4.*BM+4.*BP)
   43 IF(IC(I).NE.1) GO TO 50
      A(I,M-3)=BM
      A(I,2*M-1)=-(8.*BN+8.*BP)
      A(I,3*M-1)=2.*BN
      A(I,2*M-2)=4.*BP
      A(I,2*M)=4.*BP
   50 I=M
      IF(IPROB.NE.0) GO TO 52
      IF(IC(I)-1) 60,55,60
   52 IF(IC(I)-1) 54,51,51
   54 BM=BMU
      BN=BNU
      BP=BPU
   51 A(I,I)=5.*BM+6.*BN+8.*BP-2.*BQ
      A(I,M-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 53
C************************************************************************
C     NBC=0 MEANS FIXED EDGES
C     NBC=1 MEANS SIMPLY SUPPORTED EDGES
```

```
C*****************************************************************************
   55 IF(NBC) 56,56,57
   56 A(I,I)=7.*BM+6.*BN+8.*BP
      GO TO 58
   57 A(I,I)=5.*BM+6.*BN+8.*BP
   58 A(I,M-1)=-(4.*BM+4.*BP)
   53 IF(IC(I).NE.1) GO TO 60
      A(I,M-2)=BM
      A(I,2*M)=-(8.*BN+8.*BP)
      A(I,3*M)=2.*BN
      A(I,2*M-1)=4.*BP
   60 I=M+1
      IF(IPROB.NE.0) GO TO 62
      IF(IC(I)-1) 70,65,70
   62 IF(IC(I)-1) 64,61,61
   64 BM=BMU
      BN=BNU
      BP=BPU
   61 A(I,I)=6.*BM+7.*BN+8.*BP-2.*BQ
      A(I,M+2)=-(8.*BM+8.*BP)+2.*BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 63
   65 A(I,I)=6.*BM+7.*BN+8.*BP
      A(I,M+2)=-(8.*BM+8.*BP)
   63 IF(IC(I).NE.1) GO TO 70
      A(I,M+3)=2.*BM
      A(I,1)=-(4.*BN+4.*BP)
      A(I,2*M+1)=-(4.*BN+4.*BP)
      A(I,3*M+1)=BN
      A(I,2)=4.*BP
      A(I,2*M+2)=4.*BP
   70 I=M+2
      IF(IPROB.NE.0) GO TO 72
      IF(IC(I)-1) 80,75,80
   72 IF(IC(I)-1) 74,71,71
   74 BM=BMU
      BN=BNU
      BP=BPU
   71 A(I,I)=7.*BM+7.*BN+8.*BP-2.*BQ
      A(I,M+1)=-(4.*BM+4.*BP)+BQ
      A(I,M+3)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 73
   75 A(I,I)=7.*BM+7.*BN+8.*BP
      A(I,M+1)=-(4.*BM+4.*BP)
      A(I,M+3)=-(4.*BM+4.*BP)
   73 IF(IC(I).NE.1) GO TO 80
      A(I,M+4)=BM
      A(I,2)=-(4.*BN+4.*BP)
      A(I,2*M+2)=-(4.*BN+4.*BP)
      A(I,3*M+2)=BN
      A(I,1)=2.*BP
      A(I,3)=2.*BP
      A(I,2*M+1)=2.*BP
      A(I,2*M+3)=2.*BP
   80 LM=M+3
```

```
      LN=2*M-2
      DO 102 I=LM,LN
      IF(IPROB.NE.0) GO TO 82
      IF(IC(I)-1) 102,85,102
   82 IF(IC(I)-1) 84,81,81
   84 BM=BMU
      BN=BNU
      BP=BPU
   81 A(I,I)=6.*BM+7.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 83
   85 A(I,I)=6.*BM+7.*BN+8.*BP
      A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
   83 IF(IC(I).NE.1) GO TO 102
      LLA=I+M
      LLB=I+2*M
      LLC=I-M
      A(I,I+2)=BM
      A(I,I-2)=BM
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLB)=BN
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
  102 CONTINUE
   90 MM=2*M+1
      NN=(N-2)*M+1
      DO 103 I=MM,NN,M
      IF(IPROB.NE.0) GO TO 92
      IF(IC(I)-1) 103,95,103
   92 IF(IC(I)-1) 94,91,91
   94 BM=BMU
      BN=BNU
      BP=BPU
   91 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(8.*BM+8.*BP)+2.*BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 93
   95 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I+1)=-(8.*BM+8.*BP)
   93 IF(IC(I).NE.1) GO TO 103
      LLA=I+M
      LLB=I+2*M
      LLC=I-M
      LLD=I-2*M
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLB)=BN
      A(I,I+2)=2.*BM
      A(I,LLC+1)=4.*BP
```

```
      A(I,LLA+1)=4.*BP
  103 CONTINUE
  270 I=2*M-1
      IF(IPROB.NE.0) GO TO 272
      IF(IC(I)-1) 120,275,120
  272 IF(IC(I)-1) 274,271,271
  274 BM=BMU
      BN=BNU
      BP=BPU
  271 A(I,I)=6.*BM+7.*BN+8.*BP-2.*BQ
      A(I,2*M)=-(4.*BM+4.*BP)+BQ
      A(I,2*M-2)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 273
  275 A(I,I)=6.*BM+7.*BN+8.*BP
      A(I,2*M)=-(4.*BM+4.*BP)
      A(I,2*M-2)=-(4.*BM+4.*BP)
  273 IF(IC(I).NE.1) GO TO 120
      A(I,2*M-3)=BM
      A(I,3*M-1)=-(4.*BN+4.*BP)
      A(I,M-1)=-(4.*BN+4.*BP)
      A(I,4*M-1)=BN
      A(I,M-2)=2.*BP
      A(I,M)=2.*BP
      A(I,3*M)=2.*BP
      A(I,3*M-2)=2.*BP
  120 I=2*M
      IF(IPROB.NE.0) GO TO 122
      IF(IC(I)-1) 130,125,130
  122 IF(IC(I)-1) 124,121,121
  124 BM=BMU
      BN=BNU
      BP=BPU
  121 A(I,I)=5.*BM+7.*BN+8.*BP-2.*BQ
      A(I,2*M-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 123
  125 IF(NBC) 126,126,127
  126 A(I,I)=7.*BM+7.*BN+8.*BP
      GO TO 128
  127 A(I,I)=5.*BM+7.*BN+8.*BP
  128 A(I,2*M-1)=-(4.*BM+4.*BP)
  123 IF(IC(I).NE.1) GO TO 130
      A(I,2*M-2)=BM
      A(I,M)=-(4.*BN+4.*BP)
      A(I,3*M)=-(4.*BN+4.*BP)
      A(I,4*M)=BN
      A(I,M-1)=2.*BP
      A(I,3*M-1)=2.*BP
  130 JN=2*M+2
      JM=(N-2)*M+2
      DO 104 I=JN,JM,M
      IF(IPROB.NE.0) GO TO 132
      IF(IC(I)-1) 104,135,104
  132 IF(IC(I)-1) 134,131,131
  134 BM=BMU
```

```
      BN=BNU
      BP=BPU
  131 A(I,I)=7.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 133
  135 A(I,I)=7.*BM+6.*BN+8.*BP
      A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
  133 IF(IC(I).NE.1) GO TO 104
      LLA=I+M
      LLB=I+2*M
      LLC=I-M
      LLD=I-2*M
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLB)=BN
      A(I,I+2)=BM
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
  104 CONTINUE
  140 NJ=3*M-1
      MJ=(N-1)*M-1
      DO 105 I=NJ,MJ,M
      IF(IPROB.NE.0) GO TO 142
      IF(IC(I)-1) 105,145,105
  142 IF(IC(I)-1) 144,141,141
  144 BM=BMU
      BN=BNU
      BP=BPU
  141 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 143
  145 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I+1)=-(4.*BM+4.*BP)
      A(I,I-1)=-(4.*BM+4.*BP)
  143 IF(IC(I).NE.1) GO TO 105
      LLA=I+M
      LLB=I+2*M
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLB)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
```

```
105 CCNTINUE
150 NK=3*M
    MK=(N-1)*M
    DO 106 I=NK,MK,M
    IF(IPROB.NE.0) GO TO 152
    IF(IC(I)-1) 106,155,106
152 IF(IC(I)-1) 154,151,151
154 BM=BMU
    BN=BNU
    BP=BPU
151 A(I,I)=5.*BM+6.*BN+8.*BP-2.*BQ
    A(I,I-1)=-(4.*BM+4.*BP)+BQ
    BM=BMI
    BN=BNI
    BP=BPI
    GO TO 153
155 IF(NBC) 156,156,157
156 A(I,I)=7.*BM+6.*BN+8.*BP
    GO TO 158
157 A(I,I)=5.*BM+6.*BN+8.*BP
158 A(I,I-1)=-(4.*BM+4.*BP)
153 IF(IC(I).NE.1) GO TO 106
    LLA=I+M
    LLB=I+2*M
    LLC=I-M
    LLD=I-2*M
    A(I,I-2)=BM
    A(I,LLC)=-(4.*BN+4.*BP)
    A(I,LLD)=BN
    A(I,LLA)=-(4.*BN+4.*BP)
    A(I,LLB)=BN
    A(I,LLC-1)=2.*BP
    A(I,LLA-1)=2.*BP
106 CCNTINUE
160 K=N-2
    DO 107 IT=2,K
    KK=IT*M+3
    KM=(IT+1)*M-2
    DO 108 I=KK,KM
    IF(IPROB.NE.0) GO TO 162
    IF(IC(I)-1) 108,165,108
162 IF(IC(I)-1) 164,161,161
164 BM=BMU
    BN=BNU
    BP=BPU
161 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
    A(I,I-1)=-(4.*BM+4.*BP)+BQ
    A(I,I+1)=-(4.*BM+4.*BP)+BQ
    BM=BMI
    BN=BNI
    BP=BPI
    GO TO 163
165 A(I,I)=6.*BM+6.*BN+8.*BP
    A(I,I-1)=-(4.*BM+4.*BP)
    A(I,I+1)=-(4.*BM+4.*BP)
163 IF(IC(I).NE.1) GO TO 108
    LLA=I+M
    LLB=I+2*M
    LLC=I-M
    LLD=I-2*M
```

```
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,I-2)=BM
      A(I,I+2)=BM
      A(I,LLD)=BN
      A(I,LLB)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
  108 CONTINUE
  107 CONTINUE
  170 NA=(N-1)*M+3
      NB=N*M-2
      DO 109 I=NA,NB
      IF(IPROB.NE.0) GO TO 172
      IF(IC(I)-1) 109,175,109
  172 IF(IC(I)-1) 174,171,171
  174 BM=BMU
      BN=BNU
      BP=BPU
  171 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 173
  175 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
  173 IF(IC(I).NE.1) GO TO 109
      LLA=I+M
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,I+2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
  109 CONTINUE
  180 NC=N*M+3
      ND=(N+1)*M-2
      DO 110 I=NC,ND
      IF(IPROB.NE.0) GO TO 182
      IF(IC(I)-1) 110,185,110
  182 IF(IC(I)-1) 184,181,181
  184 BM=BMU
      BN=BNU
      BP=BPU
  181 A(I,I)=6.*BM+5.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
```

```
      GO TO 183
  185 IF(NBC) 186,186,187
  186 A(I,I)=6.*BM+7.*BN+8.*BP
      GO TO 188
  187 A(I,I)=6.*BM+5.*BN+8.*BP
  188 A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
  183 IF(IC(I).NE.1) GO TO 110
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,I+2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
  110 CONTINUE
  190 I=(N-1)*M+1
      IF(IPROB.NE.0) GO TO 192
      IF(IC(I)-1) 200,195,200
  192 IF(IC(I)-1) 194,191,191
  194 BM=BMU
      BN=BNU
      BP=BPU
  191 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(8.*BM+8.*BP)+2.*BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 193
  195 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I+1)=-(8.*BM+8.*BP)
  193 IF(IC(I).NE.1) GO TO 200
      LLA=I+M
      LLC=I-M
      LLD=I-2*M
      A(I,I+2)=2.*BM
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLC+1)=4.*BP
      A(I,LLA+1)=4.*BP
      A(I,LLD)=BN
  200 I=(N-1)*M+2
      IF(IPROB.NE.0) GO TO 202
      IF(IC(I)-1) 210,205,210
  202 IF(IC(I)-1) 204,201,201
  204 BM=BMU
      BN=BNU
      BP=BPU
  201 A(I,I)=7.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 203
  205 A(I,I)=7.*BM+6.*BN+8.*BP
      A(I,I+1)=-(4.*BM+4.*BP)
      A(I,I-1)=-(4.*BM+4.*BP)
  203 IF(IC(I).NE.1) GO TO 210
```

```
      LLA=I+M
      LLC=I-M
      LLD=I-2*M
      A(I,I+2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
  210 I=(N-1)*M+M-1
      IF(IPROB.NE.0) GO TO 212
      IF(IC(I)-1) 220,215,220
  212 IF(IC(I)-1) 214,211,211
  214 BM=BMU
      BN=BNU
      BP=BPU
  211 A(I,I)=6.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 213
  215 A(I,I)=6.*BM+6.*BN+8.*BP
      A(I,I+1)=-(4.*BM+4.*BP)
      A(I,I-1)=-(4.*BM+4.*BP)
  213 IF(IC(I).NE.1) GO TO 220
      LLA=I+M
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
      A(I,LLA-1)=2.*BP
      A(I,LLA+1)=2.*BP
  220 I=N*M+1
      IF(IPROB.NE.0) GO TO 222
      IF(IC(I)-1) 230,225,230
  222 IF(IC(I)-1) 224,221,221
  224 BM=BMU
      BN=BNU
      BP=BPU
  221 A(I,I)=6.*BM+5.*BN+8.*BP-2.*BQ
      A(I,I+1)=-(8.*BM+8.*BP)+2.*BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 223
  225 IF(NBC) 226,226,227
  226 A(I,I)=6.*BM+7.*BN+8.*BP
      GO TO 228
  227 A(I,I)=6.*BM+5.*BN+8.*BP
  228 A(I,I+1)=-(8.*BM+8.*BP)
  223 IF(IC(I).NE.1) GO TO 230
      LLC=I-M
```

```
      LLD=I-2*M
      A(I,I+2)=2.*BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC+1)=4.*BP
  230 I=N*M+M
      IF(IPROB.NE.0) GO TO 232
      IF(IC(I)-1) 240,235,240
  232 IF(IC(I)-1) 234,231,231
  234 BM=BMU
      BN=BNU
      BP=BPU
  231 A(I,I)=5.*BM+5.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 233
  235 IF(NBC) 236,236,237
  236 A(I,I)=7.*BM+7.*BN+8.*BP
      GO TO 238
  237 A(I,I)=5.*BM+5.*BN+8.*BP
  238 A(I,I-1)=-(4.*BM+4.*BP)
  233 IF(IC(I).NE.1) GO TO 240
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
  240 I=(N-1)*M+M
      IF(IPROB.NE.0) GO TO 242
      IF(IC(I)-1) 250,245,250
  242 IF(IC(I)-1) 244,241,241
  244 BM=BMU
      BN=BNU
      BP=BPU
  241 A(I,I)=5.*BM+6.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 243
  245 IF(NBC) 246,246,247
  246 A(I,I)=7.*BM+6.*BN+8.*BP
      GO TO 248
  247 A(I,I)=5.*BM+6.*BN+8.*BP
  248 A(I,I-1)=-(4.*BM+4.*BP)
  243 IF(IC(I).NE.1) GO TO 250
      LLA=I+M
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,LLA)=-(4.*BN+4.*BP)
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLA-1)=2.*BP
  250 I=N*M+2
      IF(IPROB.NE.0) GO TO 252
```

```
      IF(IC(I)-1) 260,255,260
  252 IF(IC(I)-1) 254,251,251
  254 BM=BMU
      BN=BNU
      BP=BPU
  251 A(I,I)=7.*BM+5.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 253
  255 IF(NBC) 256,256,257
  256 A(I,I)=7.*BM+7.*BN+8.*BP
      GO TO 258
  257 A(I,I)=7.*BM+5.*BN+8.*BP
  258 A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
  253 IF(IC(I).NE.1) GO TO 260
      LLC=I-M
      LLD=I-2*M
      A(I,I+2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
  260 I=N*M+M-1
      IF(IPRCB.NE.0) GO TO 262
      IF(IC(I)-1) 300,265,300
  262 IF(IC(I)-1) 264,261,261
  264 BM=BMU
      BN=BNU
      BP=BPU
  261 A(I,I)=6.*BM+5.*BN+8.*BP-2.*BQ
      A(I,I-1)=-(4.*BM+4.*BP)+BQ
      A(I,I+1)=-(4.*BM+4.*BP)+BQ
      BM=BMI
      BN=BNI
      BP=BPI
      GO TO 263
  265 IF(NBC) 266,266,267
  266 A(I,I)=6.*BM+7.*BN+8.*BP
      GO TO 268
  267 A(I,I)=6.*BM+5.*BN+8.*BP
  268 A(I,I-1)=-(4.*BM+4.*BP)
      A(I,I+1)=-(4.*BM+4.*BP)
  263 IF(IC(I).NE.1) GO TO 300
      LLC=I-M
      LLD=I-2*M
      A(I,I-2)=BM
      A(I,LLC)=-(4.*BN+4.*BP)
      A(I,LLD)=BN
      A(I,LLC-1)=2.*BP
      A(I,LLC+1)=2.*BP
  300 RETURN
      END
C     SUBROUTINE FOR TRANSFORMING MATRIX A INTO A VECTOR
C*************************************************************************
      SUBROUTINE ROWMAT(XK,A,M,MUD,MLD)
      DIMENSION XK(50,50),A(2500)
```

```
      CO 201 I=1,2500
  201 A(I)=0.
      KL=1+MLD
      KU=M-MUD
      DO500 I=1,M
      IF(I-2)501,502,502
  501 LL=1
      LAL=1
      LU=1+MUD
      LAU=LU
      GO TO 550
  502 IF(MLD)503,503,504
C*****************************************************************************
C     UPPER TRIANGULAR OR DIAGONAL MATRIX
C*****************************************************************************
  503 IF(I-KU)505,505,506
  505 LL=I
      LU=I+MUD
      LAU=LAU+MUD+1
      GO TO 550
  506 LL=I
      LU=M
      LAU=LAU+1+M-I
      GO TO 550
  504 IF(I-KU)5C7,507,508
  507 IF(I-KL)509,510,510
  510 LL=I-MLD
      LU=I+MUD
      LAU=LAU+1+MLD+MUD
      GO TO 550
  508 LL=I-MLD
      LU=M
      LAU=LAU+1+M-I+MLD
      GO TO 550
  509 LL=1
      LU=I+MUD
      LAU=LAU+I+MUD
  550 LB=LL-1
      DO551 LA=LAL,LAU
      LB=LB+1
  551 A(LA)=XK(I,LB)
  500 LAL=LAU+1
      RETURN
      END
C     SUBROUTINE FOR DETERMINING POINTS WHICH HAVE CRACKED
C*****************************************************************************
      SUBROUTINE EPSMAX(IA,IC,EPST,EPSB,TEMP,ICOUNT,EPSM,EPSBY,EPSTY,IR
      DIMENSION IC(50),EPST(50),EPSB(50),TEMP2(50),EPSBY(50),EPSTY(50),
     1PSTM(50),EPSBM(50),IR(50)
      INTEGER ICOUNT
      REAL TEMP,TEMP1
      TOL=1.E-8
      TEMP=0.
      DO 35 I=1,IA
      IF(IC(I)-2) 5,35,35
    5 IF(EPST(I)-EPSTY(I)) 31,30,30
   30 EPSTM(I)=EPST(I)
      GO TO 32
   31 EPSTM(I)=EPSTY(I)
   32 IF(EPSB(I)-EPSBY(I)) 34,33,33
```

```
   33 EPSBM(I)=EPSB(I)
      GO TO 35
   34 EPSBM(I)=EPSBY(I)
   35 CONTINUE
      IF(ICCLNT.EQ.1) GO TO 122
C*************************************************************************
C     DETERMINE POINT WITH MAXIMUM TENSILE STRAIN
C*************************************************************************
      DO 20 I=1,IA
   20 TEMP2(I)=0.
      DO 100 I=1,IA
      IF(IC(I)-2) 1,100,100
    1 IF(EPSTM(I)) 2,3,3
    2 IF(EPSBM(I)) 100,8,8
    3 IF(EPSBM(I)) 7,6,6
    6 IF(EPSBM(I)-EPSTM(I)) 7,7,8
    7 TEMP1=EPSTM(I)
      IF(EPST(I)-EPSTY(I)) 145,145,146
  145 IR(I)=1
      GO TO 9
  146 IR(I)=2
      GO TO 9
    8 TEMP1=EPSBM(I)
      IF(EPSB(I)-EPSBY(I)) 147,147,148
  147 IR(I)=1
      GO TO 9
  148 IR(I)=2
    9 TEMP2(I)=TEMP1
      IF(TEMP1-TEMP) 100,10,10
   10 TEMP=TEMP1
  100 CONTINUE
      DO 120 I=1,IA
      TEMP3=ABS(TEMP2(I)-TEMP)
      IF(TEMP3-TOL) 121,121,120
  121 IC(I)=1
  120 CONTINUE
      GO TO 135
C*************************************************************************
C     DETERMINE POINTS WHICH HAVE CRACKED
C*************************************************************************
  122 DO 123 I=1,IA
      IF(IC(I)-2) 124,123,123
  124 IF(EPSTM(I)) 125,126,126
  125 IF(EPSBM(I)) 123,131,131
  126 IF(EPSBM(I)) 128,127,127
  127 IF(EPSBM(I)-EPSTM(I)) 128,128,131
  128 TEMP1=EPSTM(I)
      IF(EPST(I)-EPSTY(I)) 140,140,141
C*************************************************************************
C     VALUE OF IR(I) INDICATES THE DIRECTION IN WHICH
C     CRACK OCCURED AT POINT I
C*************************************************************************
  140 IR(I)=1
      GO TO 129
  141 IR(I)=2
      GO TO 129
  131 TEMP1=EPSBM(I)
      IF(EPSB(I)-EPSBY(I)) 142,142,143
  142 IR(I)=1
      GO TO 129
```

```
143 IR(I)=2
129 IF(TEMP1-EPSM) 123,130,130
130 IC(I)=1
123 CONTINUE
135 RETURN
    END
```

APPENDIX C

Computer Output of 25 x 25 Matrix

SUPPORT CONDITION-SIMPLY SUPPORTED

LATERAL LOADING

DATA

M= 5 N= 4 ES= 0.300000E 08 EC= 0.300000E 07 PR= 0.15
WP= 0.80 EPSM= 0.150E-03 EPCM=-0.600E-03
T= 1.0000 H= 9.6000 D= 0.8750 D2= 0.1250
Z1U= 0.3750 Z2U= 0.3750 SR1= 0.0100
DXU= 0.340129E 06 D1U= 0.306905E 05
DXC= 0.135822E 06 D1C= 0.309811E 04 CN= 0.293322E 00

MATRIX A

0.67327E 03	-0.51308E 03	0.80092E 02	0.0	0.0
-0.51308E 03	0.19272E 03	0.0	0.0	0.0
0.80092E 02	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
-0.25654E 03	0.71331E 03	-0.25654E 03	0.40046E 02	0.0
0.96358E 02	-0.51308E 03	0.96358E 02	0.0	0.0
0.0	0.80092E 02	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03	0.40046E 02
0.0	0.96358E 02	-0.51308E 03	0.96358E 02	0.0
0.0	0.0	0.80092E 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03
0.0	0.0	0.96358E 02	-0.51308E 03	0.96358E 02
0.0	0.0	0.0	0.80092E 02	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.40046E 02	-0.25654E 03	0.63322E 03
0.0	0.0	0.0	0.96358E 02	-0.51308E 03
0.0	0.0	0.0	0.0	0.80092E 02
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.71331E 03	-0.51308E 03	0.80092E 02	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.40046E 02	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
-0.25654E 03	0.75336E 03	-0.25654E 03	0.40046E 02	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
0.0	0.40046E 02	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.40046E 02	-0.25654E 03	0.71331E 03	-0.25654E 03	0.40046E 02
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.0	0.0	0.40046E 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.40046E 02	-0.25654E 03	0.71331E 03	-0.25654E 03
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02

0.0	0.0	0.0	0.40046E 02	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.40046E 02	-0.25654E 03	0.67327E 03
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.0	0.0	0.40046E 02
0.0	0.0	0.0	0.0	0.0
0.40046E 02	0.0	0.0	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.67327E 03	-0.51308E 03	0.80092E 02	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.40046E 02	0.0	0.0	0.0	0.0
0.0	0.40046E 02	0.0	0.0	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
-0.25654E 03	0.71331E 03	-0.25654E 03	0.40046E 02	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
0.0	0.40046E 02	0.0	0.0	0.0
0.0	0.0	0.40046E 02	0.0	0.0
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03	0.40046E 02
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.0	0.0	0.40046E 02	0.0	0.0
0.0	0.0	0.0	0.40046E 02	0.0
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.0	0.0	0.40046E 02	0.0
0.0	0.0	0.0	0.0	0.40046E 02
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.40046E 02	-0.25654E 03	0.63322E 03
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.0	0.0	0.40046E 02
0.0	0.0	0.0	0.0	0.0
0.40046E 02	0.0	0.0	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.67327E 03	-0.51308E 03	0.80092E 02	0.0	0.0
-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.40046E 02	0.0	0.0	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
-0.25654E 03	0.71331E 03	-0.25654E 03	0.40046E 02	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.40046E 02	0.0	0.0
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03	0.40046E 02
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.40046E 02	0.0
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.40046E 02	-0.25654E 03	0.67327E 03	-0.25654E 03
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.40046E 02
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.40046E 02	-0.25654E 03	0.63322E 03
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.40046E 02	0.0	0.0	0.0	0.0

-0.25654E 03	0.96358E 02	0.0	0.0	0.0
0.63322E 03	-0.51308E 03	0.80092E 02	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.40046E 02	0.0	0.0	0.0
0.48179E 02	-0.25654E 03	0.48179E 02	0.0	0.0
-0.25654E 03	0.67327E 03	-0.25654E 03	0.40046E 02	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.40046E 02	0.0	0.0
0.0	0.48179E 02	-0.25654E 03	0.48179E 02	0.0
0.40046E 02	-0.25654E 03	0.63322E 03	-0.25654E 03	0.40046E 02
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.40046E 02	0.0
0.0	0.0	0.48179E 02	-0.25654E 03	0.48179E 02
0.0	0.40046E 02	-0.25654E 03	0.63322E 03	-0.25654E 03
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.40046E 02
0.0	0.0	0.0	0.48179E 02	-0.25654E 03
0.0	0.0	0.40046E 02	-0.25654E 03	0.59318E 03

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EFFECTS OF CRACKING AND WARPING ON THE RESPONSE OF REINFORCED CONCRETE PLATES SUBJECTED TO LATERAL AND ECCENTRIC LOADS

by

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AN ABSTRACT OF A MASTER'S THESIS

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1975

ABSTRACT

The behavior of reinforced concrete plates is characterized by the progressive cracking of the concrete under increasing load. These cracks cause a redistribution of stresses in the concrete. In addition, the plate bends into a warped surface. This warping of the plate is dependent on the plate torsional stiffness which has been difficult to evaluate for reinforced concrete plates.

This paper presents an analytical approach using the method of finite differences to predict the effects of cracking and plate torsional stiffness on the response of reinforced concrete plates subjected to normal loads and to eccentrically applied in-plane compressive loads. A computer program was developed to perform the analysis. Laterally loaded, rectangular, reinforced concrete plates with simple supports along all edges and also a plate with fixed-edge supports were examined. The torsional stiffness was varied by means of the warping parameter, λ, whose value varied from 1 to 0. Also simply-supported rectangular plates under eccentric in-plane compressive loads were examined. Two plate length-width ratios were considered.

In general for all the types of loading that were considered, the load-maximum deflection curve and the load maximum moments curves had lower slopes for the cracked plate compared to those for the uncracked plate. This shows that the cracking of the concrete affects the plate response.

For a given load the maximum plate deflection for a simply-supported square plate under lateral load changed by about 75% for λ ranging from 1 to 0 for the case of no cracking. For the cracked case the corresponding

change was 18%. The maximum moment changed by 63% for the uncracked case and by 16% for the cracked case. Similar results were obtained for the plate under eccentric in-plane compressive load. The results indicate that the plate torsional stiffness has a significant effect on the plate response but that the effect is less pronounced for the cracked plate.