A STUDY OF THE PRESSURE DROP ACCOMPANYING FLOW OF A FLASHING FLUID IN A CIRCULAR PIPE

by

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INTRODUCTION

The flow of a liquid past the stationary walls of a pipe occasions a frictional loss in mechanical energy which manifests itself by a decrease in the static pressure of the flowing fluid. A liquid flowing through a long pipe reaches some point where the static pressure is reduced to the vapor pressure of the liquid, and vaporization takes place. From the outset of vaporization, the nature of the fluid flow problem changes. The line now handles a vaporizing or "flashing" liquid, and conventional flow equations are not applicable.

The purpose of this investigation was to determine if, and in what manner, conventional flow equations should be altered to the new conditions to predict pressure drops under these unusual flow conditions.

A successful method for accurately predicting the pressure drop of a flashing fluid should also be capable of application to the study of two-phase, two-component flow problems.

Numerous industrial applications are evident: calculation of pressure drops associated with simultaneous flow and vaporization in tubular heaters, pressure relief lines serving vessels containing vaporizable contents, reboilers, process lines carrying liquids at or near the saturation point, vertical condensers and the expansion coils of refrigerating systems. Some possible applications in the area of two-phase, two-component flow deal with separation of lower hydrocarbons in transporting petroleum crudes and fractions, gas formation via dissociation in the cooling coils of a nuclear reactor, liquid air lines, and conceivably many others (8).

PHYSICAL ANALYSIS

One-Phase Flow

Bernoulli's theorem is an application of the conservation of energy principle to the flow of fluids. Included in this energy balance are: a potential energy term, which accounts for changes in elevation of the fluid stream between the inlet and outlet; a kinetic energy term, which accounts for changes in the kinetic energy of the fluid; a pressure-volume term, which is the flow work done on the fluid; a friction term, which accounts for the loss of energy due to friction against the pipe wall; and a work term which measures the work input to or useful work done by the system.

Evaluation of the friction term depends upon the flow mechanism. The classic experiments of Osborne Reynolds showed that two distinct types of flow were associated with the movement of fluids in a closed channel. One type, the laminar or viscous flow, was characterized by flow of the fluid in concentric stream tubes. The other, turbulent flow, moved in an erratic, churning manner, with constant mixing of the fluid stream. When the velocity of a stream in viscous flow was increased, a velocity was finally reached at which laminar type flow vanished. This velocity was definite for dynamical similar systems and was termed the "critical velocity".

Reynolds showed that the critical velocity depended upon the diameter of the pipe, and the velocity, density and viscosity of the fluid. He also showed that these four factors must be combined in the form of the dimensionless ratio, $Dv\rho/\mu$, where D is the inside diameter of the pipe, v, the average velocity of the fluid, ρ the fluid density and μ fluid viscosity. The function $Dv\rho/\mu$ is known as the "Reynolds number".

Later investigators showed that for a straight circular pipe, the flow was viscous if the Reynolds number was less 2100, and if the Reynolds number was greater than 4000, the flow was turbulent. Between the values of 2100 and 4000, the flow was of either type, or a combination, depending upon construction of the apparatus under study (Badger and McCabe, 1, p. 29).

The friction term may be evaluated by using the Fanning equation which relates the flow characteristics, properties of the fluid, pipe dimensions and a friction factor. This friction factor has been correlated against Reynolds number with good results. The use of the Fanning equation in combination with the friction factor-Reynolds number correlation is the accepted means for obtaining the frictional loss in static pressure due to flow of single-phase fluids through circular pipes.

Two-Phase Flow

Flow of a flashing fluid constitutes a two-phase flow problem. Possible application of the single phase flow relationships to the two-phase problem is desirable. The remainder of this paper is devoted to the possible application of the above-described one-phase flow relationships to a two-phase flow system.

The mechanism of flashing must be differentiated from that of boiling. When a liquid is said to "flash", the heat of vaporization required for vapor formation is obtained at the expense of the sensible heat of the liquid. The process of boiling implies that the heat of vaporization is obtained from an external source.

A qualitative picture of the flashing mechanism for adiabatic flow indicates clearly the fundamental nature of the changing flow conditions:

1. If equilibrium conditions are to hold, a decrease in static pressure is accompanied by a decrease in the temperature.

2. A decrease in the saturation temperature decreases the enthalpy of the liquid, and makes available heat which is utilized as latent heat of vaporization for a portion of the liquid, in order that the total enthalpy of the fluid stream remain constant.

3. Absorption of latent heat of vaporization causes vapor formation.

4. The total volume of the fluid stream is increased by virtue of the large specific volume of the vapor.

5. In order that there be no accumulation of material within the system, or that the overall mass rate of flow remain constant, an increase takes place in the linear velocity of the stream.

6. An increase in velocity increases the numerical value of the pressure drop and causes a further decrease in the static pressure with additional vapor formation.

There is no information available concerning the transverse velocity gradient in a two-phase flow system, and it is assumed that the picture is similar to that of a one-phase flow system. The local velocity of a single-phase flowing stream varies across the pipe diameter, rising from a value of zero velocity at the pipe wall to a maximum at the center line, as shown in Fig. 1.

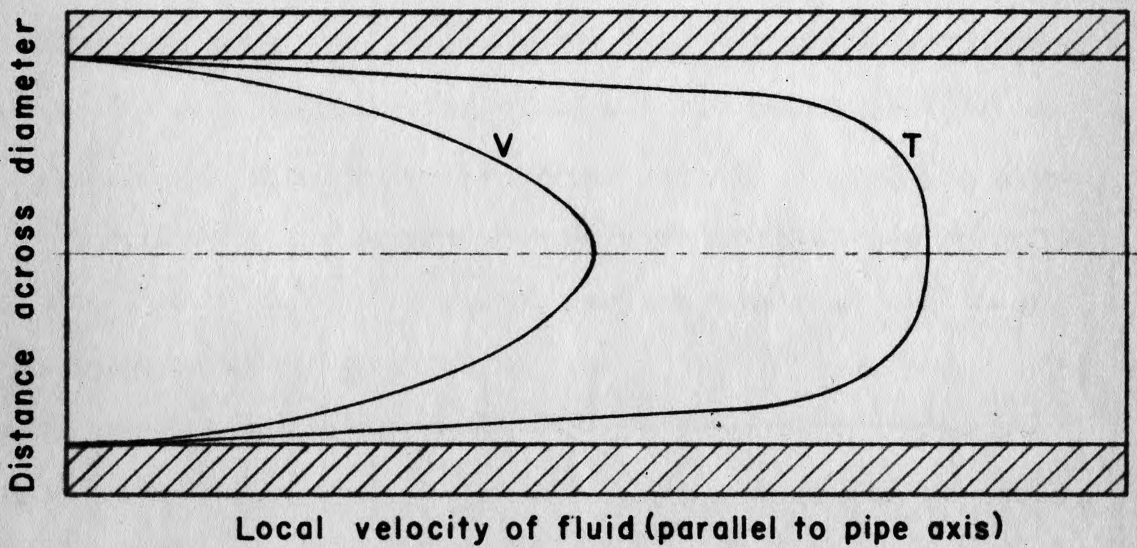

Fig. I. Relative velocity distribution. (5)

V: viscous flow

T: turbulent flow

The static pressure distribution across the pipe diameter is the inverse of the above curve, decreasing from a maximum at the pipe wall to a minimum at the center line. If the flowing fluid is saturated liquid, the initial vapor formation takes place at the point of lowest static pressure, the center line of the pipe. This occurrence constitutes an annular flow condition. If vapor formation were to take place at some other point along the diameter, as for example, at the wall in a boiler tube, there is still a tendency for these vapor bubbles to accumulate at the center line of the pipe. This may be explained in terms of the velocity differential existing on the top and bottom surfaces of each vapor bubble. A velocity head differential causes movement of the bubble toward the point of lowest pressure, in this case the pipe center line. This effect in analogous to the "lift" effect experienced by an airfoil in a moving air stream.

The picture of annular flow, in which initial vapor formation takes place, can similarly be extended to cover the entire flow picture from initial to any stage of vaporization. Investigations of two-phase, two-component flow conducted at the University of California by Martinelli et al. (14) show that this is one of the possible types of flow conditions. High speed photographs through a glass pipe handling water-air mixtures for various flow conditions led to the following observations:

1. When gas and liquid streams both entered the pipe in the highly turbulent region, a frothy homogeneous mixture existed along the entire length of pipe. Reduction of the liquid rate caused an accumulation of air bubbles in the upper portion of the pipe.

2. "Slug" flow occurred at very low liquid and gas rates. Alternate

slugs or charges of liquid and gas flowed through the pipe, each gas slug separated from another by a liquid membrane, and each liquid slug separated from the next by a gaseous interval.

3. At low turbulent liquid and high turbulent air flow rates, annular flow occurred. The liquid flowed in the form of an annulus, even though the pipe was horizontal. The liquid surface was covered with small capillary waves the frequency of which was lowered when the liquid flow took place in the viscous region.

4. An increase of quality[1] (reduced liquid rate at a medium air rate) led to the formation of exaggerated waves on the annular liquid surface (Fig. 2a). Further increase in the air to liquid ratio flattened down the crests (Fig. 2b):

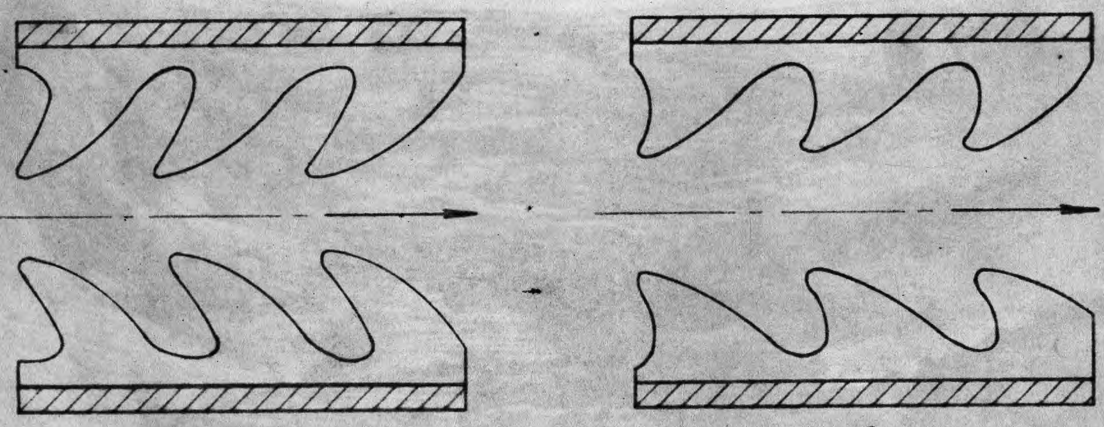

Fig. 2a. Fig. 2b.

Fig. 2. Annular two-phase flow.

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[1] Quality: the ratio of gas to total fluid flowing, on a percentage by weight basis.

In the California experiments, relative flow rates and velocities for each stream were controlled by means of adjusted inlet conditions. Inlet velocities of each stream were externally controlled and bore no dependence upon one another. The paper was valuable from the standpoint of the picture presented.

Evidences obtained support the annular flow idea for flashing fluids. It is believed that cases 3 and 4 above describe in detail the flow picture.

Owing to the nature of the experimental injection apparatus used in obtaining data for this paper, the linear inlet velocities of both streams were assumed to be the same. A further assumption regarding the nature of the relative downstream velocities of both streams is made here:

The accelerating fluid is the vapor, the volume of which is continually increased as a result of the dynamic equilibrium between the vapor and saturated liquid at the existing static pressure any point downstream in the pipe. If the total volume of the system is the sum of the liquid and vapor volumes for any finite differential section, the velocity of the stream is increased, as previously pointed out. The point of greatest speculation surrounds that concerning the motion of the liquid, which is being accelerated by the force applied to the liquid surface by the accelerating vapor. Two possibilities exist: 1) that the liquid flows along the length of pipe as a tube the inside surface of which consists of wave forms (Fig. 3a), or 2) that the liquid passes through the pipe in such a manner that the waves themselves flow along the length as a series of "rings" riding on a liquid skin which remains in contact with the pipe walls and is in itself stationary (Fig. 3b).

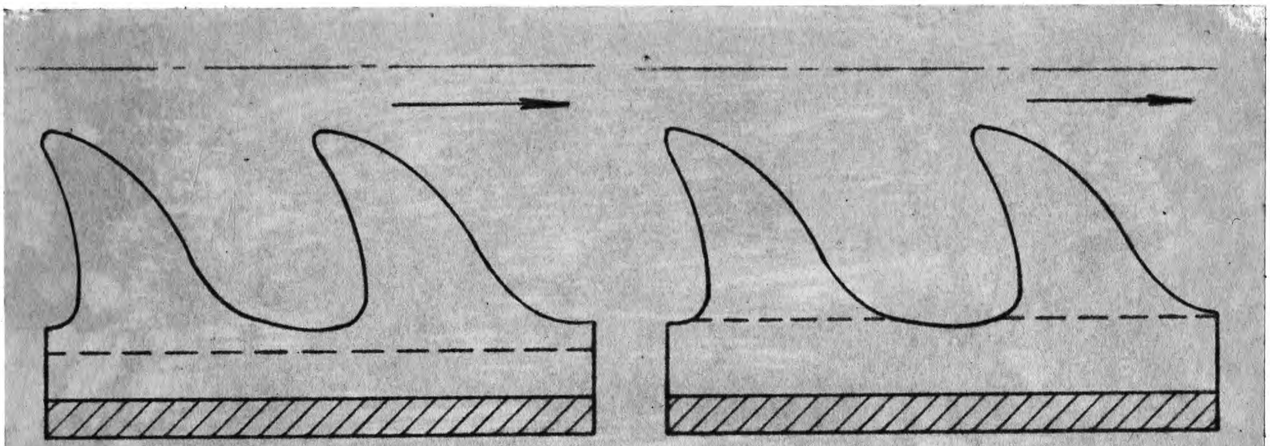

Fig. 3a. Tube flow.   Fig. 3b. Ring flow.

Fig. 3. Tubular and ring flows.

The ring type flow is believed to represent best the flow picture. The basis for this belief is consideration of the greatly reduced liquid shear area (Fig. 3), and the simplified treatment of the velocity and diameter terms which result.

Several inferences may be drawn from the qualitative picture of case two:

1. The liquid velocity approaches the gas velocity. The wave forms expose a large liquid area to the accelerating vapor and in effect form pockets similar to a condition of slug flow. Further expansion of the vapor in this pocket further accelerates the liquid. Resisting vapor expansion and liquid acceleration is the drag of the liquid against the stationary liquid film. The net effect is that the gas stream velocity is lowered and the liquid stream velocity increased until an equilibrium velocity distribution takes place, at which point both streams move at the same linear velocity. The point velocity of both streams can be based upon the average density of the fluid at that point.

2. The diameter of gas flow approaches the inside diameter of the

pipe since the largest portion of mass transfer of liquid takes place in liquid rings. Past the initial stage of vapor formation the gas occupies by far the larger portion of the total volume. For example, at a pressure of 26.8 psia. and 1.2 percent quality, the volume occupied by the gas is 92 percent of the total volume.

3. The character of the liquid film is the same as that of the pipe with regard to surface irregularities.

4. Both liquid and gas flow over the same liquid film surface. Since this surface corresponds to that of the inside pipe surface, a correlation relating frictional effects as a function of pipe surface characteristics, is applied to both streams.

Figure 4 is a more detailed sketch of the assumed nature of flow, from which the deductions are more readily observed:

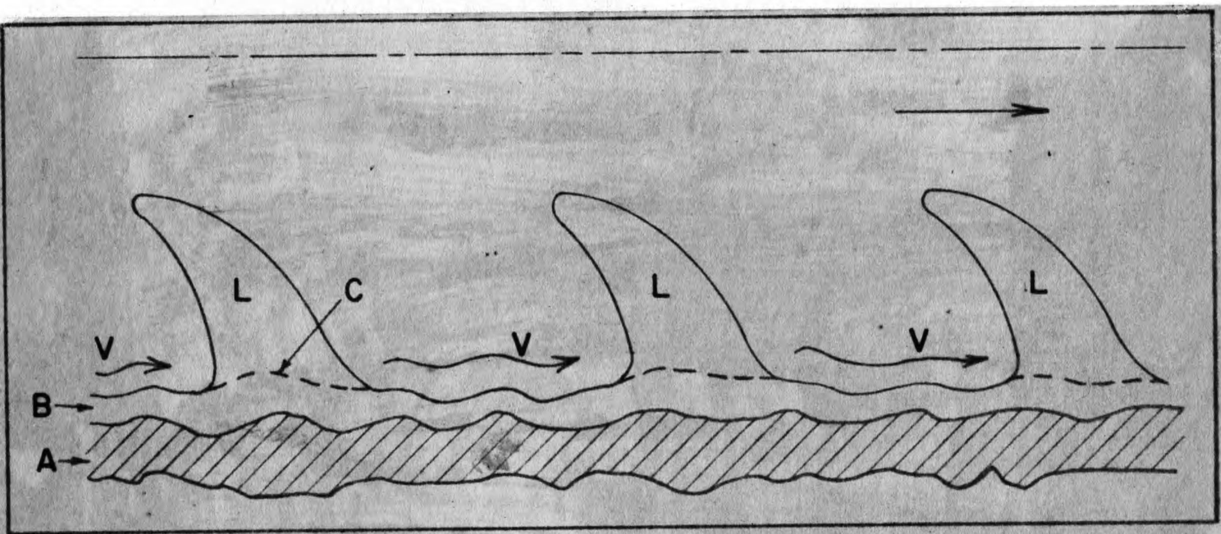

Fig. 4. Detailed sketch of ring flow.

A: pipe wall, showing irregularities
B: stationary liquid film
C: line of liquid shear
L: liquid rings
V: vapor flow

MATHEMATICAL ANALYSIS

The Mechanical Energy Equation

If an energy balance around the flow system of Fig. 5 from point (A) to (B) is taken, for steady state flow:[1]

(1) $U_2 + (KE)_2 + mZ_2 + P_2V_2 = U_1 + (KE)_1 + mZ_1 + q - w$

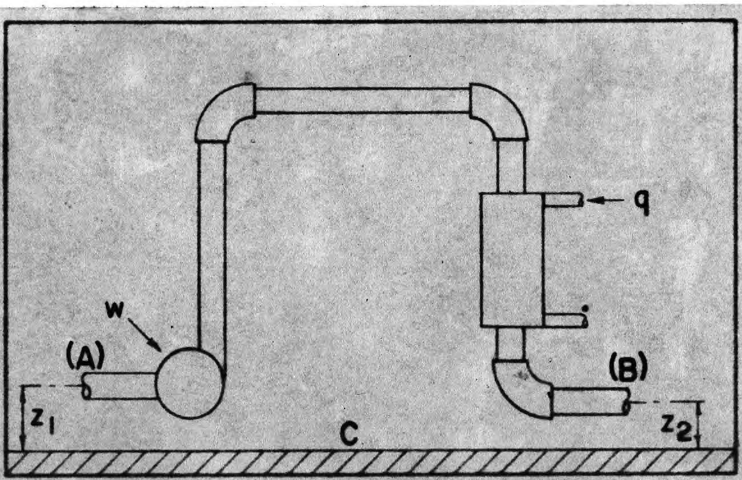

Fig. 5. Piping system.
C: base plane w: work to pump
q: heat input z: height above base

Equation (1) may also be expressed in differential form:

(2) $d(PV) + d(KE) + d(mZ) + dU + dw = dq$,

where the terms represent in order: flow work; external kinetic energy; potential energy; internal energy; work done upon the surroundings; and heat added from the surroundings. Changes in surface energy have been

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[1] Steady-state flow: applying the conservation of mass principle to the system in order that no accumulation or depletion of material take place. Therefore, for steady-state flow, one pound of fluid entering at point (A) displaces one pound of fluid from the system at point (B).

neglected in equations (1) and (2) since insufficient information is available regarding the subject. When the elevation of points A and B are the same above a reference plane, as is the case of a horizontal pipe, the potential energy term may be dropped, simplifying the equation to:

(3) $d(PV) + d(KE) + dU + dw = dq$

The kinetic energy is defined by $KE = mv^2/\theta g$. The value of θ is dependent upon the mechanism of fluid flow, and will be evaluated in a later discussion.

The useful work, dw, done upon the surroundings is the maximum total mechanical work obtainable from the system. Frictional effects in the system, however, reduce the net obtainable work recoverable because of the dissipation of mechanical work into energy associated with fluid friction. In the form of an equation:

(4) $dw = d'w + dF$

where dw represents the total possible mechanical work without friction; d'w the available mechanical work when friction is present; and dF, the work made unavailable as a result of the irreversible friction processes which take place in the system. Combination of equations (3) and (4) yields an energy balance for a flow process with friction:

(5) $d(PV) + d(KE) + dU + d'w + dF = dq$

The units of the mechanical energy equation terms are those of ft.-lb.(force)/lb.(mass) of flowing fluid. Based upon a mass of one pound of fluid, the units of the equation become feet of fluid. These terms expressed in length units are called "heads" (e.g., the "friction head"), and express the height of a column of the fluid. Multiplying terms of the equation by the fluid density converts the head into pressure units. An example problem can perhaps best illustrate.

Example: The friction head observed for the flow of an oil (sp. gr. = 0.8) through a piping system is 2.0 feet. What is the decrease in static pressure caused by frictional effects?

$$P = 2.0 \text{ ft.} \times 0.8 \times \frac{62.5 \text{ lb.}}{\text{cu.ft.}} \times \frac{1 \text{ sq.ft.}}{144 \text{ sq.in.}} = 0.7 \text{ psi.}$$

One-Phase Flow

The frictional loss of mechanical energy may be evaluated for a single-phase system by means of the Fanning equation, in consistent units:

$$(6) \quad F = \frac{4fLv^2}{2g_cD} = \frac{4fLG^2}{2g_cD\rho^2}$$

where F is the friction head, in feet, of the flowing fluid as observed over a finite duct length, L; D is the inside diameter of the duct; ρ, the density of the flowing fluid; v, the linear velocity of the fluid stream; g_c, a conversion factor; G, the mass velocity;[1] and f, the Fanning friction factor which varies with the Reynolds number $(Dv\rho/\mu) = (DG/\mu)$.

The friction head, F, may be expressed in terms of the pressure drop due to friction over the length of pipe, L:

$$(7) \quad P = F \times \rho = \frac{4fLG^2}{2g_cD\rho}$$

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[1] Mass velocity: the product of density and velocity, convenient to use in place of linear velocity and density, especially in the case of gases; since this quantity is independent of pressure and temperature, whereas both linear velocity and density change with these variables (Badger and McCabe, 1, p. 135).

Equations (6) and (7) apply only to steady flow conditions in circular pipes running full of fluid of constant density.

When the density varies over the length of pipe, as in the flow of gases or the non-isothermal flow of liquids, the Fanning equation must be used in differential form:

$$(8) \quad \frac{d(\Delta P)}{dx} = \frac{d(F\rho)}{dx} = \frac{4fG^2}{2g_c D\rho}$$

Equation (8) applies to a differential length of pipe, dx, over which the density may be considered constant.

The dimensionless Fanning friction factor has been shown (Perry, 16, p.381) to be a function of the Reynolds number for pipes and ducts of all cross-sectional shapes. Experiments conducted with artificially roughened pipes showed that f is a function of pipe wall roughness at sufficiently large Reynolds numbers (Colebrook and White, 6).

Figure 6 shows the friction factor, f, as ordinates, plotted vs. the Reynolds number, $Dv\rho/\mu$, as abscissas, for ordinary types of pipe.

Curve A of Fig. 6 holds for all pipes and is theoretically deducible for considerations of viscous flow. The equation of the curve is:

$$(9) \quad f = \frac{16\mu}{Dv\rho} = \frac{16}{Re}$$

Curve C of Fig. 6 represents consistent data obtained for smooth copper, lead and glass tubes. According to Drew, Koo and McAdams (7), the equation of this curve is:

$$(10) \quad f = 0.00140 + 0.125 \left(\frac{1}{Re}\right)^{0.32}$$

Curve D of Fig. 6 represents practically all known data for clean commercial iron and steel pipe. The data lies within a band of ± 10 percent of the curve. According to Wilson, McAdams and Seltzer (18) the equation of this curve is:

(11) $f = 0.0035 + 0.264 \left(\frac{1}{Re}\right)^{0.42}$

When $Dv\rho/\mu$ increases past the Reynolds criterion ($Re = 2100$), f rises rapidly as $Dv\rho/\mu$ increases (e.g., curve B, Fig. 6) and then falls off along a curve of gradually decreasing slope (curve C or D, Fig. 6). The location of curve B depends to a large extent upon the nature of the wetted pipe surface (Perry, 16, p. 383).

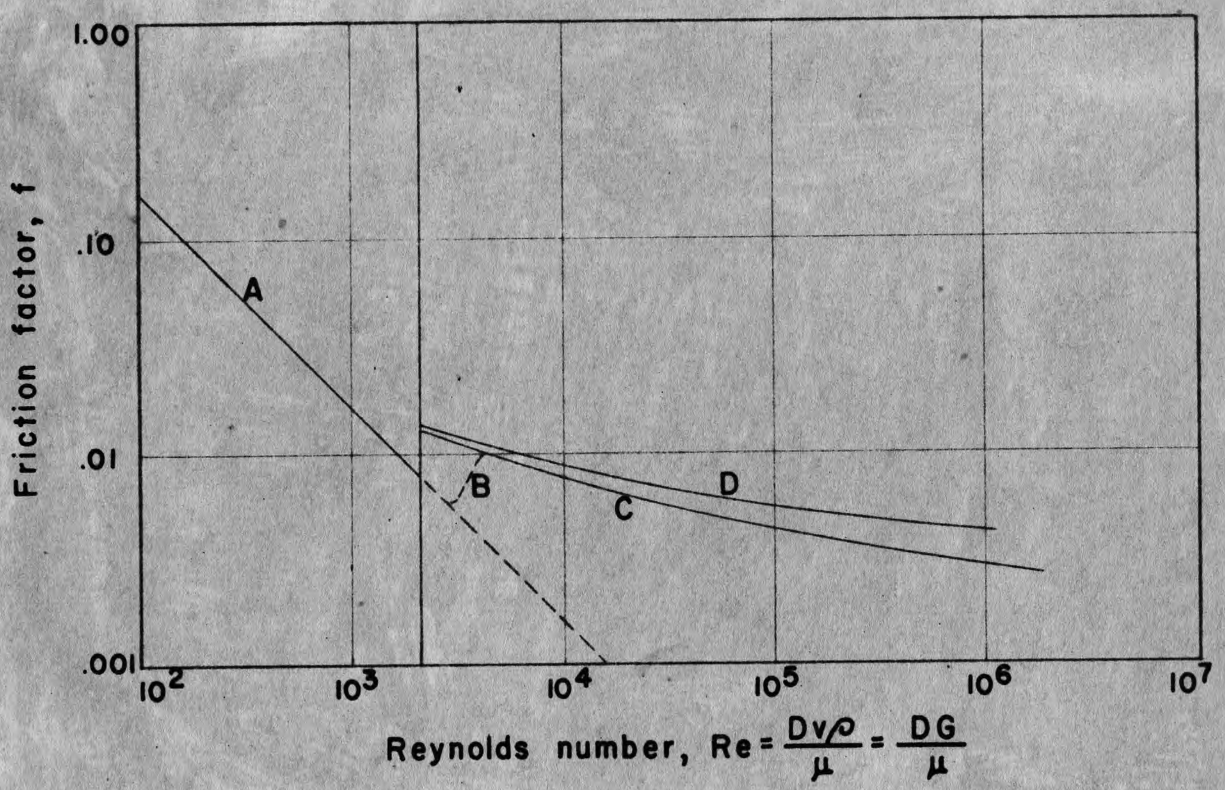

Fig. 6. Fanning friction factors, f, for straight ducts. (16)

A: viscous flow C: smooth tubes
B: critical region D: commercial pipe

Moody (15) has correlated the Fanning friction factor vs. Reynolds number with relative pipe roughness as a parameter. The relative roughness is obtained from an auxiliary plot of relative roughness vs. pipe diameter, for various types of commercial pipe (drawn tubing, steel, cast iron, concrete, etc.).

Two-Phase Flow

The previously stated purpose of this investigation was to determine if, and in what manner, the conventional one-phase flow equations could be altered to make them applicable for predicting frictional pressure drops due to the flow of flashing liquids. The basis for the following analysis was the picture of ring flow developed in the Physical Analysis section of this paper.

The combined frictional pressure drop at a cross-section (1) in a piping system conveying a flashing fluid is assumed to be the sum of the pressure drops due to vapor and liquid flow, respectively:

$$(12) \quad \left[\frac{d(\Delta P)}{dx}\right]_{com._1} = \left[\frac{d(\Delta P)}{dx}\right]_{V_1} + \left[\frac{d(\Delta P)}{dx}\right]_{L_1}$$

The individual pressure drop contribution of each phase is calculated from the Fanning equation:

$$(13) \quad \left[\frac{d(\Delta P)}{dx}\right]_{V_1} = \alpha\left[\frac{4fG^2}{2g_c D\rho}\right]_{V_1}$$

$$(14) \quad \left[\frac{d(\Delta P)}{dx}\right]_{L_1} = \beta\left[\frac{4fG^2}{2g_c D\rho}\right]_{L_1}$$

where α and β are coefficients peculiar to the two-phase system.

Martinelli, Putnum and Lockhart (13) utilize the same approach to a two-phase, two-component flow problem in which both phases flow in the viscous region.

The friction factors of equations (13) and (14) are based upon modified Reynolds numbers:

(15) $f_{V_1} = \varphi\left(\frac{aDv\rho}{\mu}\right)_{V_1} = \varphi\left(\frac{aDG}{\mu}\right)_{V_1}$

(16) $f_{L_1} = \varphi\left(\frac{bDv\rho}{\mu}\right)_{L_1} = \varphi\left(\frac{bDG}{\mu}\right)_{L_1}$

Based upon inferences drawn in the section entitled "Physical Analysis", the terms of equations (13) through (16) have the following identity:

1. The velocity terms of equations (15) and (16) are both equal to the average velocity of the fluid stream at cross-section (1). The average velocity is determined by the average density at this cross-section.

2. The diameter terms of equations (13) through (16) are all equal to the inside diameter of the pipe under consideration.

3. The density terms of equations (13) and (15) refer to the vapor density at cross-section (1), and the density terms of equations (14) and (16) refer to the liquid density at the same cross-section.

4. The viscosity terms in equations (15) and (16) refer to the viscosity of the vapor and liquid, respectively, at cross-section (1).

5. The functions in equations (15) and (16) are identical to those functions describing the variation of friction factor vs. Reynolds number for one-phase flow in the pipe under consideration (Fig. 6).

The coefficients α, β, a and b, in equations (13) through (16) were unknown. Experimental data compiled in this investigation were used to evaluate these unknowns.

Analysis of the Test Apparatus

Data used in this paper were obtained in an insulated horizontal test pipe. Calculations showed that the heat loss from the test pipe was small and could be neglected (see Sample Calculations). The experimental data are therefore for an adiabatic process.[1]

The enthalpy change for this flow process can be shown to equal the change in kinetic energy, and for small values of kinetic energy the process approaches an isenthalpic process, which greatly simplifies the mathematical treatment.[2]

(17)

a) $d(PV) + d(KE) + dU + dF = dq - d'w$ (equation 5)

b) $d'w = PdV$ (the useful mechanical expansion work)

c) $dU = dq - PdV$ (definition of internal energy)

d) $d(PV) + d(KE) + dF = 0$ (combining a, b and c)

e) $dH = Tds + VdP$ (definition)

f) $d(PV) = PdV + VdP = PdV + dH - Tds$ (substituting e)

g) $PdV + dH - Tds + d(KE) + dF = 0$ (combining d and f)

h) $dq < Tds$ (for an irreversible process)

i) $dq + dF = Tds$ (for an irreversible frictional process)

j) $dq = 0$ (for an adiabatic process)

k) $dF = Tds$

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[1] An adiabatic process is carried out in such a manner that no heat is absorbed or evolved by a system.

[2] Isenthalpic processes take place without change in the enthalpy, or heat content of the system.

l) $PdV + dH + d(KE) = 0$ (combining g and e)

m) $PdV = d'w = 0$ (since no useful expansion work is done)

n) $dH = -d(KE)$ (combining m and n)

o) $\Delta H = -\Delta KE$ (in an incremental form)

Therefore, the change in enthalpy of the flow process studied equals the kinetic energy change of the fluid, since no machines for introducing or withdrawing mechanical energy were present in the system, and the flow was adiabatic. This kinetic energy change equals $(mv^2/\theta g_c)_2 - (mv^2/\theta g_c)_1$. The coefficient, θ, depends upon the type of flow, either viscous or turbulent, and is equal to 1.0 for viscous flow and 2.0 for turbulent flow (Perry, 16, p. 376). On a numerical basis, the heat equivalent of the kinetic energy change was so small that it could be neglected, for practical purposes (see Sample Calculations).

LITERATURE SURVEY

A large portion of the published material concerning two-phase, fluid flow in pipes resulted from studies conducted at the University of California Agricultural Experiment Station (3, 12, 14). The primary objective of the study was to arrive at a satisfactory method for estimating the pipe size of fuel lines serving orchard heaters, and various liquid-air mixtures, pipe sizes and fluid temperatures were used in obtaining the experimental measurements. The California work was concerned with two-component flow, and the proposed method of calculation utilized a friction factor based on liquid properties alone, used in conjunction with a "flow modulus" which was correlated in terms of liquid properties and the cross-sectional area of flow occupied by the liquid. Deviations of calculated friction heads from experimental measurements were within thirty percent. The flow modulus correlations limited application of the method to two-phase, two-component flow problems for the systems studied.

Martinelli et al. (13) attempted to extend the method and relationships developed in the above studies to predict the pressure drops during flow of flashing water-steam mixtures, but the limitations of the two-phase, two-component flow analysis remained. No data appeared which could be used in this paper, and as before, no attempt was made to correlate results in terms of the standard frictional pressure drop equations.

The first published data for the pressure drop due to the flow of a flashing mixture was presented by Bottomley (4). His observations centered about one run performed on a marine boiler within a narrow range of conditions, and necessarily limited the conclusions to the observation that lines carrying flashing fluids should be much larger than those carrying the equivalent flow rates of a single liquid phase. The data for the marine boiler test run were presented in conjunction with a thorough discussion of the flow of boiling water through a converging nozzle. Information concerning the flow of a flashing water-steam mixture through an orifice may be useful for the design of experimental facilities for further study of the flow of flashing water-steam mixtures through pipes. For this reason, reference was made to the same article in the Future Recommendations section of this paper.

Benjamin and Miller (2) have published more extensive data concerning the flow of flashing water and steam mixtures through pipes. Their observations and data resulted from erosion studies conducted for the bends of boiler drain lines, since it was believed that increased erosion resulted from acceleration of the moving fluid due to additional vapor formation during flow. The desired result of the investigation was the evolution of a successful method for designing the drain lines which would compensate for velocity increases and thereby reduce erosion.

The method proposed consisted of a graphical integration of the mechanical energy equation (equation 1). The friction factor term used by the authors was an average of all observed friction factors for the test lines and existing flow conditions. Use of this average value was justified since flow conditions encountered in the test system, the drain lines of a Detroit, Michigan power plant, were substantially constant, yielding friction factors which varied within a twenty percent band. Since no attempt was made to correlate friction factors in terms of physical properties or flow conditions, the method of calculation was limited to drain line design over the same range of operating conditions and to the water-steam system only. The method, however, is of value to designers of similar stationary equipment.

In addition to the foregoing, Benjamin and Miller noted the existence of a critical ratio of inlet and outlet pressures for several experimental runs, similar to critical pressure and acoustic velocity relationships known to exist for the expansion of single phase fluids flowing through pipes, orifices, and convergent nozzles.[1]

A stepwise calculation of pressure drop for the design of tubular heaters handling flashing hydrocarbons has been proposed by Kraft (11). The friction factor term used in the Fanning equation was based on liquid properties only, while the velocity term was determined by combining properties of the two phases on a weight basis. The method appeared to approach actual conditions for the type of heaters investigated.

Conclusions drawn from the published work include:

1. The flow mechanism for flashing mixtures was unknown, and the approach used was to attempt correlation of frictional pressure drops in terms of gross flow properties.

2. Friction factors based on liquid properties have been used almost exclusively, despite their failure to reproduce experimental observations within the usual range of error of the Fanning equation-Reynolds number-friction factor relationships for one phase fluid flow.

3. The need for further investigation of the flow of flashing fluids exists, especially for the development of a satisfactory flow picture and associated mathematical analysis. Using as a basis the widely accepted

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[1] Acoustic velocity: the velocity of sound in the fluid medium.

When the linear velocity of a fluid is equal to its acoustic velocity, the maximum flow rate possible through the pipe or orifice for the given upstream conditions is obtained, and no further increase in flow can result from any change in downstream conditions.

Consideration of acoustic velocity and critical pressure ratios is unavoidable in the treatment of high velocity fluid flow problems, but has been disregarded in this paper since the linear velocities here encountered ($v_{max.}$ = 150 ft./sec.) fall far short of the acoustic velocity of either fluid stream ($>$ 1000 ft./sec.). Relationships which permit calculation of acoustic velocities based on fluid properties are available in the literature (Brown, 5, p. 200) (Perry, 16, p. 375).

flow equations and correlations would enable prediction of pressure drops for the flow of any flashing mixture in commercial pipes.

EXPERIMENTAL EQUIPMENT

The advantages of using the steam-water system for this investigation were: availability in the laboratory, ease of handling and abundance of physical property data in the literature. The experimental apparatus (Plate I), however, is not limited to this system.

The sequence of events which took place in the test apparatus were as follows:

1. Steam was injected into cold water in a mixer, and the mixture allowed to come to saturation equilibrium conditions.

2. Excess steam was separated from the saturated liquid in an entrainment separator.

3. Saturated liquid and excess vapor were injected through a second mixer into the test line.

4. The discharge of the test line was condensed and cooled in a heat exchanger and collected.

5. Flow conditions were recorded from gauges and meters strategically located throughout the system.

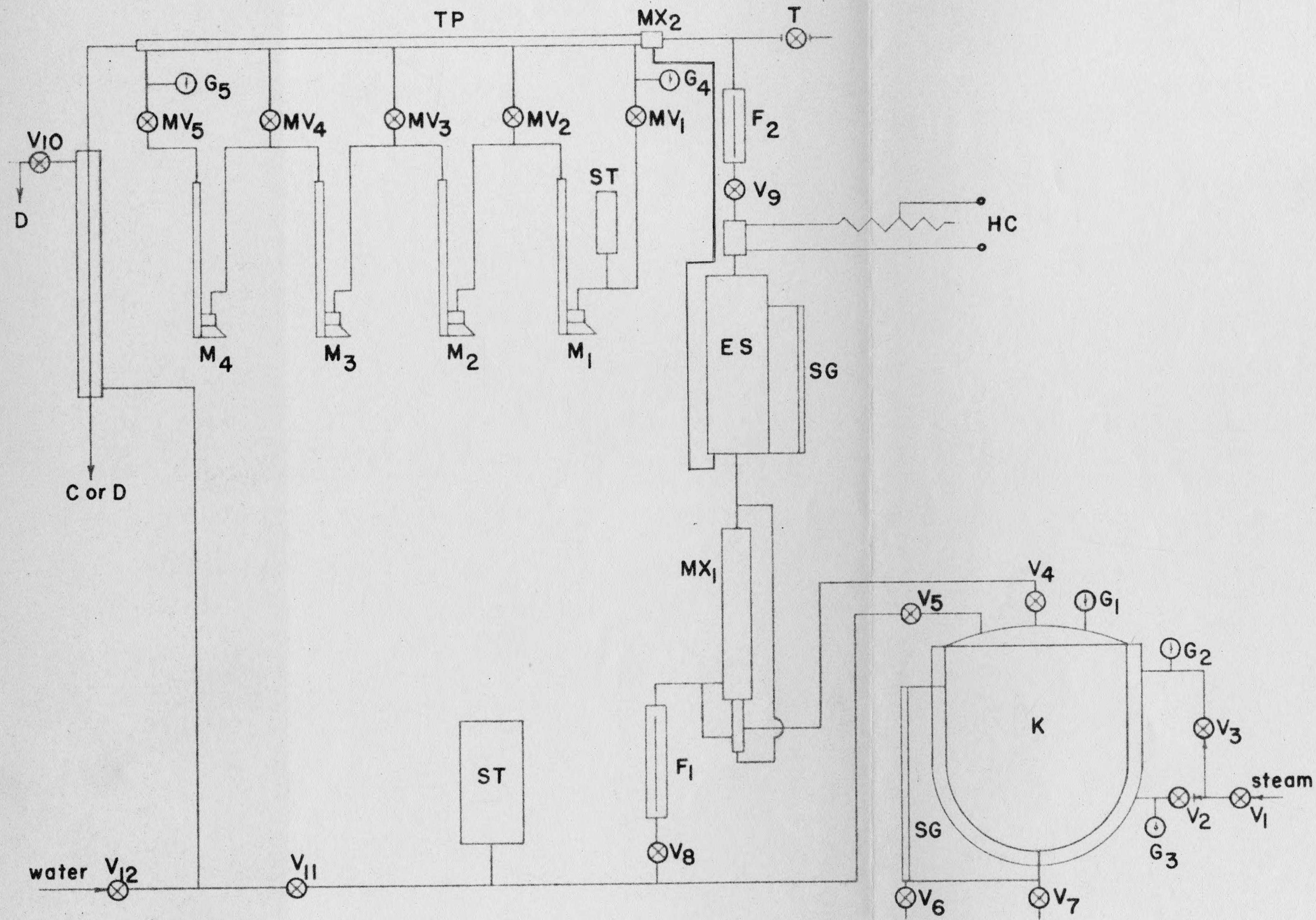
Steam was supplied to the system from a steam-heated, jacketed kettle. The steam generated was fed to a vertical mixer in which it was injected into a controlled stream of cold water from the laboratory mains. Plate II is a detailed sketch of this first mixer. Pressure differentials distributed by lines (F) caused piston (C) to adjust its position, mixing

EXPLANATION OF PLATE I

Schematic diagram of experimental apparatus

C : tared container
D : drain
ES: entrainment separator
F : Flowrator
G : gauge
HC: electric heating coil
K : steam-generating kettle
M : Merriam mercury well manometer
MV: manometer valve
MX: mixer
SG: sight glass
ST: surge tank
T : steam trap
TP: test pipe
V : valve

PLATE I



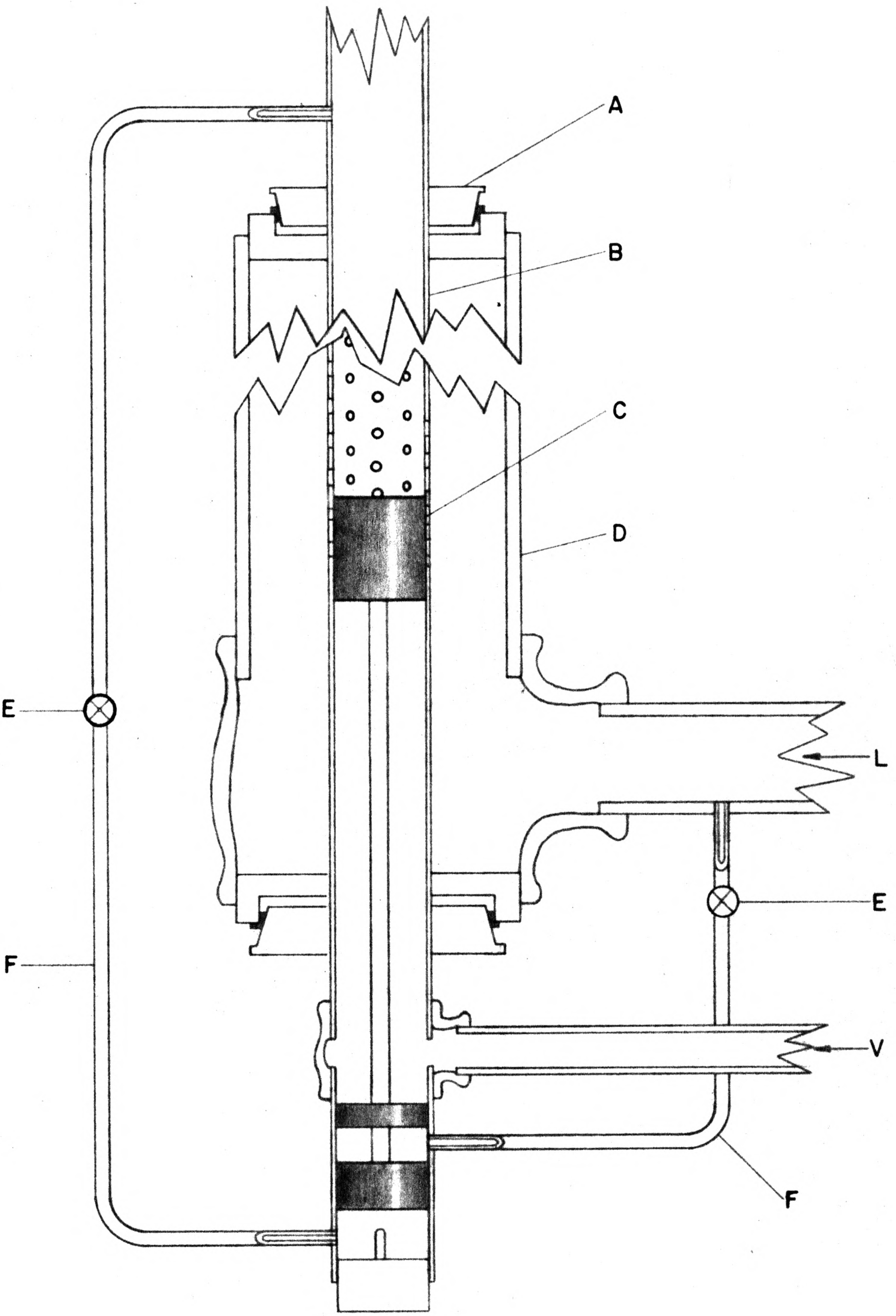
TP
MX2
T
G5
G4
MV5
MV4
MV3
MV2
MV1
V10
F2
D
ST
V9
HC
M4
M3
M2
M1
ES
SG
C or D
MX1
V4
V5
G1
G2
K
V3
steam
F1
V2
V1
ST
SG
G3
V8
water
V12
V11
V6
V7

EXPLANATION OF PLATE II

Details of first mixer

A: stainless steel end fitting
B: one-half inch brass pipe
C: piston
D: one and one-half inch iron pipe jacket
E: adjustment valves
F: one-eighth inch copper tubing
L: cold water inlet
V: steam inlet

PLATE II



the inlet vapor and liquid with a minimum of bumping and knocking which normally accompanies the mixing of steam and cold water.

Mounted above the first mixer was a combination entrainment separator and collection space, which consisted of a six inch diameter iron cylinder packed with copper turnings to facilitate removal of the last portions of entrained liquid from the vapor. The steam-water mixture from the first mixer reached saturation equilibrium conditions in this vessel and was separated into two streams. One, the saturated vapor stream, was forced from the top of the chamber through a Fischer-Porter Flowrator for metering, while the other, the saturated liquid stream, was forced from the bottom of the vessel and passed directly to the test line.

From the Flowrator the vapor passed into a second mixer placed at the inlet of the test line, where it once again contacted the saturated liquid. The second mixer consisted of a short length of perforated copper tubing (Plate III, Fig. 1).

The test line consisted of forty feet of straight, horizontal three-eighths inch schedule 40 galvanized iron pipe, fully lagged with one and one-half inches of Owens-Corning Foamglas insulation. This test pipe was the largest sized pipe which could be adapted to laboratory facilities. Pipes of smaller diameter were less likely to yield data of commercial value. Pressure taps were located at intervals of ten feet in the pipe line, and the pressure drop over each of the intervals measured by using a Merriam mercury well manometer. Plate III, Fig. 2 shows the manometer connections used.

The test line discharged into a double-pipe, water-cooled, counter-

EXPLANATION OF PLATE III

Fig. 1. Sectional view of second mixer

V: vapor inlet
L: liquid inlet

Fig. 2. Manometer connections

A: test pipe
B: bleeding petcocks
C: Merriam mercury well manometer
D: valves for dampening line pulsations

PLATE III

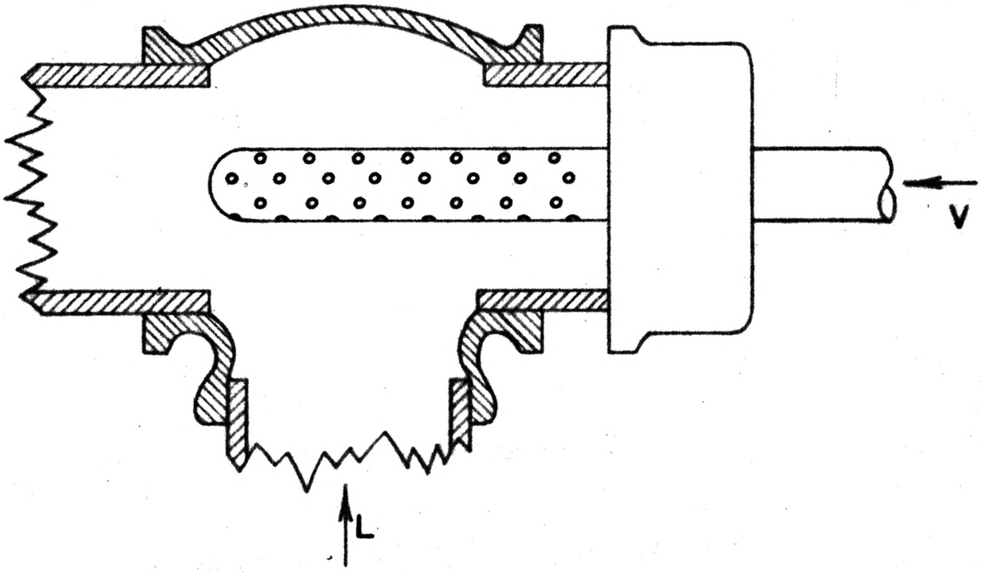


Fig. 1.

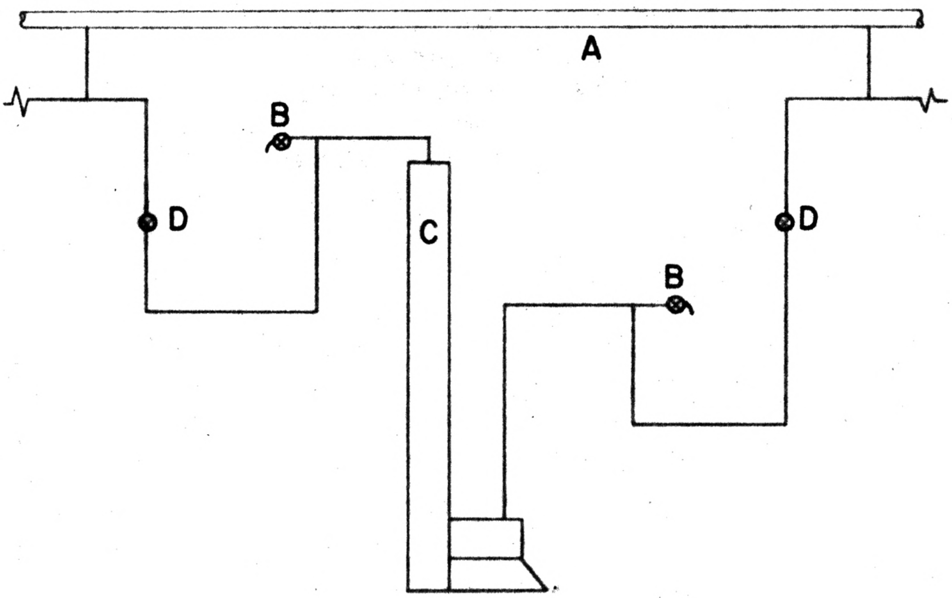


Fig. 2.

current heat exchanger, which in turn discharged into a collection vessel or drain, as was desired.

Auxiliary equipment included a large surge tank placed in the laboratory water line to dampen pulsations which are characteristic of the College water system; a small surge tank in the first manometer line, to dampen pulsations caused by the second mixer; a series of valves on the manometer lines to dampen pulsations transmitted from the test pipe; an inlet and outlet pressure gauge, to ascertain inlet fluid characteristics and check on the overall pressure drop shown on the mercury manometers, respectively; valves and pressure gauges used in conjunction with the steam generating facilities; and an electrical heater placed between the entrainment separator and vapor Flowrator. Adjustment of the electrical input to the heater enabled condensation in the Flowrator tube to be reduced to a minimum, thereby reducing the "liquid drag"[1] effect on the stainless steel float.

The operating range of the test line was limited by the maximum internal working pressure of the kettle (40 psia.), and the line water pressure at the discharge of the first mixer (45 psia.).

Attempts to feed laboratory steam directly into the experimental apparatus resulted in difficulty in maintaining constant conditions.

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[1] Liquid drag: the Fischer-Porter Flowrator operates on the principle that a different orifice area exists for each rate of flow, and as a consequence, the differential pressure is constant. The float remains in a fixed (but rotating) position when the differential pressure just balances the weight of the float. Liquid drag is caused by the accumulation of condensate in the tapered Flowrator tube and manifests itself by 1) reducing the orifice area for a particular position of the plummet, thereby altering the differential pressure, and 2) increasing the apparent weight of the calibrated plummet.

EXPERIMENTAL PROCEDURE

Calibration of the test equipment preceded the actual trial runs. Calibrated were: 1) the vapor Flowrator and 2) the test line, using water only. The test line showed favorable results on the basis of a Fanning equation correlation.

The operating procedure for an actual test run follows. The symbols refer to notations in Plate I:

1. The manometer lines were bled by opening valves $V_{8,11,12}$, MV_{1-5} and the pair of petcocks on each manometer for this purpose. Bleeding continued for about fifteen minutes in order to assure complete removal of air from the manometer system. Bleeding petcocks were closed and flow was stopped by closing valve V_{12}. Zero manometer readings were then taken.

2. Valve V_{12} was opened and valve V_8 adjusted to give a flow rate sufficient to keep the manometer lines filled with water. Valves $V_{1 \text{ and } 3}$ were opened to begin steam generation. When gauge G_1 indicated sufficient steam pressure, valves V_4 and V_{10} were opened, admitting steam to the mixer and cooling water to the heat exchanger, respectively.

3. When the level of liquid in the entrainment separator sight glass indicated saturation of the contents, valve V_9 was opened, admitting saturated water vapor to the vapor Flowrator, thence to the test line. Electrical power supplied to the auxiliary heating coil was adjusted so that condensation in the Flowrator was reduced to a minimum.

4. Valves $V_{4,\ 8 \text{ and } 9}$ were adjusted to vary inlet conditions.

5. Valves MV_{1-4} were adjusted to dampen pulsations in the manometer line.

6. When the apparatus had adjusted to a dynamic equilibrium condition as indicated by a constant level of liquid in the entrainment separator, readings of F_2; $M_{1, 2, 3, 4}$; $G_{4, 5}$ and the total weight rate of flow were obtained. The total weight rate of discharge was obtained by collecting the cooled discharge from the test line in a tared container, over a measured time interval.

7. Valves $V_{4, 8, 9}$ were adjusted to give new inlet conditions, and the procedure of steps 3 to 7 repeated.

Care had to be exercised when recording data, in order to insure that no change took place in the inlet conditions over the time interval required for reading instruments and recording those readings.

THE DATA

The experimental data obtained in this investigation are tabulated in Tables 1, 2 and 3. Table 2 is the steam Flowrator calibration; Table 3, a comparison of observed pressure drops and pressure drops calculated with the Fanning equation, for the flow of water only; and Table 1, pressure drop data for the flow of flashing steam-water mixtures for various inlet conditions.

Weight rates of discharge were obtained by collecting the cooled effluent from the test line over a measured time interval in a tared container.

Calibration of the steam Flowrator was made on a volumetric rate of discharge basis, using the weight rate of discharge and known physical properties of the vapor in the meter. The steam Flowrator calibration curve is shown in Fig. 7.

Experimental values of the friction factor, f, for the one-phase liquid run were based upon observed pressure drops and physical property data. The experimental results are compared graphically in Fig. 8 to curve D of Fig. 6.

The range of operating variables used for obtaining pressure drop data for flow of the flashing fluid was:

1. mass velocity, 73.5 to 133 lb./(sec.)(sq.ft.)
2. inlet pressure, 27.7 to 42 psia.
3. inlet quality, 0.026 to 1.95 percent

Precision of the pressure drop data was limited by the accuracy with which manometer readings could be taken, except in the case of very low inlet qualities, where the limiting factor was the reading of the steam Flowrator. Fluctuations in the Flowrator reading were in the order of three-tenths of a scale division, which would account for an error of ± 15 percent for small scale readings such as noted in run number 5. Pulsations in the manometer lines varied mercury manometer readings by a consistent ± 5 percent. Variation of the inlet pressure gauge was within ± 2 percent. This last deviation falls far short of the already-mentioned experimental errors.

Runs were made covering the obtainable range of the steam Flowrator. Eleven full runs were conducted, yielding forty-four measurements of pressure drop for individual ten-foot increments. Of the forty-four measurements taken, forty were retained for mathematical evaluation, and the four obtained from run number seven omitted, due to a fluctuation in inlet conditions during the course of recording data for that run.

ANALYSIS OF THE DATA

Evaluation of Terms

The measurements taken for the flow of a flashing mixture through the test line were directed toward obtaining data which could be utilized in some form of the Fanning equation and friction factor vs. Reynolds number correlations.

Experimentally observed pressure drops consisted of two parts: 1) that which was due to friction (friction head), and 2) that which was due to an increase in the kinetic energy of the flowing stream (velocity head). Since the properties of the fluid streams were fixed by the inlet conditions and the combined pressure drop, these properties were used to describe the dynamic conditions of the two-phase flow. However, when the merit of the proposed method for calculating friction head was to be evaluated, results calculated could only be compared to the experimental friction heads.

Inlet qualities were determined using the steam Flowrator calibration and weight rate of discharge, w, combined with the experimentally determined static pressure:

(18) a) $w_V = \rho_V \times (V/t)$

b) $$Q = \frac{w_V}{w_V - w_L} = \frac{w_V}{w}$$

where w_V = weight rate of discharge of vapor; ρ_V, the vapor density at the inlet pressure; and (V/t) the volumetric rate of discharge obtained from the steam Flowrator calibration. The mass velocity for each run was calculated by dividing the total weight rate of discharge by the inside cross-sectional area of the test pipe since $G = w/A$.

The pressure was known for the cross-section at ten-foot intervals along the pipe, and qualities at these cross-sections were evaluated by drawing an isenthalpic (vertical) line, originating at the inlet conditions, on the Pressure-Enthalpy-Quality curve (Fig. 14)[1] and reading off qualities for the observed static pressures. Since the static pressure at a given cross-section was the saturation pressure, the temperature at this cross-section was known (Fig. 9). Knowing the temperature, vapor and liquid densities were determined (Figs. 10 and 11). The average density of the two-phase fluid stream was calculated using the relationship:

$$(19) \quad \left[\frac{1}{\rho_{av}}\right]_{T_1} = \left[\frac{Q}{\rho_V} + \frac{(1 - Q)}{\rho_L}\right]_{T_1}$$

where the subscript T_1 refers to the temperature existing at cross-section (1) of the pipe.

The average linear velocity of the fluid stream was then calculated using the following relationship:

$$(20) \quad v_{av} = G/\rho_{av} \,.$$

[1] The Pressure-Enthalpy-Quality curve was constructed from published thermodynamic data and the relationship:

$$(21) \quad H_{Q,P} = (QH_V)_P + \left[(1 - Q)H_L\right]_P$$

where $H_{Q,P}$ refers to the total enthalpy of the fluid at the pressure P and the quality Q; H_V is the enthalpy of the vapor at pressure P; and H_L is the enthalpy of the liquid at pressure P.

Methods Used to Determine Pressure Drops for the Flow of Flashing Fluids

A large portion of the early work in this investigation was conducted on a "cut-and-try" basis, since little or no information was available in the literature to serve as a guidepost to procedure.

Tried first were a series of successive approximations based upon a friction factor defined by liquid properties, and liquid velocity adjusted for vapor formation. Results obtained were unsatisfactory.

Next tried was a method in which gas and liquid properties were combined on a weight basis to yield "pseudo" properties which were then used to determine the value of the Reynolds number and the friction factor for the Fanning equation. This method was unsatisfactory since combination of vapor and liquid viscosity on a weight basis yielded a pseudo viscosity of numerical value much the same as that of the liquid, in the range of qualities observed. The friction factor, therefore, corresponded to a liquid friction factor, as in the first method tried.

The third method tried utilized only gas properties in the Fanning equation and for the calculation of Reynolds number. A friction factor was calculated by substituting experimental pressure drops and known gas properties of the fluid stream. This friction factor was then compared to the friction factor of curve D, Fig. 6. The calculated values showed a marked resemblance to curve D when plotted on the same co-ordinates, but showed wide scattering and a general tendency to fall below the D curve. All attempts to rationalize the deviation from curve D as a function of physical properties or flow conditions failed. The inference drawn from this method was that gas properties are most important in determining the pressure drop for the flow of a flashing liquid.

On the basis of the foregoing observation, another method utilizing only gas properties was tried. The relationships employed are expressed in equation form:

(22) $D_V = (D^2 \times S)^{\frac{1}{2}}$

(23) $Re = \dfrac{D_V G Q}{\mu_V}$

(24) $\dfrac{d(\Delta P)}{dx} = \dfrac{4fG^2Q}{2g_c D \rho_V}$

where D_V refers to a diameter based upon the cross-sectional area occupied by the vapor; and S, the fractional part of the cross-section occupied by the vapor.

An overall average error of less than plus one percent was obtained, with an average deviation of ten percent for the data consisting of pressure drops for the ten runs over the pipe line composed of four ten-foot sections, or a total of forty ten-foot sections.

Despite the excellent results, however, there were evident short-comings of the method: 1) where the quality is zero, the pressure drop was indeterminate; this was unsatisfactory since the saturated liquid of zero percent quality does experience a pressure drop, and 2) there was no justification for the use of a diameter term based on cross-sectional area occupied by the gas to calculate the Reynolds number, nor for the use of the total inside diameter in the Fanning equation.

The weaknesses of the above method were absent when the mathematical analysis represented by equations (12) through (16) and the ring flow picture of the section entitled "Physical Analysis" were used. Substitution of Q_1, the quality at cross-section (1) for α_1, and a_1, in equations (13) and (15), and $(1-Q)_1$ for the coefficients β_1, and b_1, in equations (14) and (16), succeeded in reproducing the experimental friction heads with an overall average error of -0.75 percent and an overall average deviation of ten percent based on the forty measurements. Equations (13) through (16) are rewritten in terms of the quality as follows:

$$(25)\quad \left[\frac{d(\Delta P)}{dx}\right]_{V_1} = \left[\frac{4fG^2Q}{2g_cD\rho}\right]_{V_1}$$

$$(26)\quad \left[\frac{d(\Delta P)}{dx}\right]_{L_1} = \left[\frac{4fG^2(1-Q)}{2g_cD\rho}\right]_{L_1}$$

$$(27)\quad f_{V_1} = \varphi\left[\frac{DGQ}{\mu_V}\right]_1$$

$$(28)\quad f_{L_1} = \varphi\left[\frac{DG(1-Q)}{\mu_L}\right]_1$$

$$(29)\quad \left[\frac{d(\Delta P)}{dx}\right]_{com.1} = \frac{4G^2}{2g_cD}\left[\frac{f_VQ}{\rho_V} + \frac{f_L(1-Q)}{\rho_L}\right]_1$$

The experimental data were evaluated as shown in the Sample Calculations section of the Appendix. The method briefly was as follows:

1. The inlet condition was used to fix the inlet point on the Pressure-Enthalpy-Quality diagram (Fig. 14).

2. An isenthalpic (vertical) line was drawn on the diagram, originating at the inlet point.

3. Qualities at cross-sections whose static pressures were known were read from the diagram.

4. Qualities and static pressures were used to determine the vapor density and viscosity, the liquid density and viscosity, and the average

velocity at the given cross-sections.

5. Reynolds numbers for the vapor and liquid phases were calculated and from these the respective Fanning friction factors obtained.

6. The combined rate of change of static pressure, $d(\Delta P)/dx$, was calculated at the given cross-sections by using equation 29.

7. The calculated frictional pressure drops, ΔP(calc.) were obtained for each ten-foot section by graphically integrating $[d(\Delta P)/dx]_{com.}$ versus L.

8. The pressure drops resulting from the kinetic energy increase of the flowing fluid were calculated and subtracted from the experimentally observed combined pressure drops, leaving experimental friction heads only.

9. Calculated friction heads were compared to the experimental friction heads on a percentage basis to determine the relative agreement between the two.

Experimental data and calculations of the frictional pressure drops for the flow of the flashing water-steam mixtures are presented in Table 1.

Table 1. Experimental data and calculations for flashing steam-water mixtures.

Run no.	H_2O, lbs.	Time, min.	w, lb./sec.	G, lb./sec.-sq.ft.	Overall experimental pressure drop, inches of mercury, for section A	B	C	D	Static pressure, psia. at L = 0 ft.	10 ft.	20 ft.	30 ft.	40 ft.
1	44.5	5	0.148	112	4.35	5.20	7.5	12.9	31.7	29.7	27.4	24.0	18.1
2	48.5	5	0.153	116	3.80	4.85	6.3	11.7	30.7	29.0	26.8	23.9	18.6
3	39.8	5	0.133	101	5.50	6.80	10.3	17.1	34.2	31.7	28.6	23.9	16.2
4	68.5	7	0.164	124	4.80	6.35	8.8	15.9	36.7	34.5	31.6	27.6	20.4
5	52.5	5	0.175	133	2.40	3.35	5.4	9.5	27.7	26.6	25.1	22.6	18.3
6	43.0	5	0.143	108	4.90	6.40	9.0	15.7	35.2	33.0	30.1	26.0	18.9
8	29.0	5	0.097	73.5	4.35	4.80	6.4	8.9	28.2	26.2	24.0	21.1	17.1
9	76.0	9	0.141	107	3.90	6.30	8.8	17.2	31.2	29.4	26.6	22.6	14.8
10	43.0	4	0.180	136	5.30	7.20	11.3	23.2	42.1	39.7	36.6	31.5	21.0
11	44.0	4	0.183	139	5.30	8.40	11.7	22.7	40.7	38.3	34.5	29.2	18.9

Run no.	Vapor Flow-rator	Inlet vapor, lb./sec.	Percent quality, Q 0 ft.	10 ft.	20 ft.	30 ft.	40 ft.	Average density, ρ_{av}, lb./cu.ft. 0 ft.	10 ft.	20 ft.	30 ft.	40 ft.
1	9.8	0.00121	0.82	1.23	1.71	2.48	4.03	7.50	5.36	3.69	2.32	1.092
2	6.0	0.000535	0.35	0.70	1.20	1.80	3.22	15.90	8.78	5.11	3.15	1.425
3	13.2	0.00213	1.60	2.08	2.69	3.75	5.80	4.75	3.49	2.50	1.554	0.692
4	9.3	0.001295	0.79	1.18	1.73	2.55	4.27	9.20	6.27	4.12	2.555	1.165
5	2.3	0.0000455	0.026	0.27	0.62	1.20	2.30	48.1	16.90	9.10	4.37	1.860
6	11.6	0.00168	1.14	1.57	2.12	3.00	4.80	6.55	4.66	3.26	2.078	0.955
8	13.5	0.00189	1.95	2.35	2.90	3.62	4.74	3.35	2.67	1.997	1.420	0.819
9	8.8	0.00103	0.73	1.10	1.70	2.63	4.90	8.30	5.85	3.67	2.077	0.759
10	7.4	0.00101	0.56	0.95	1.48	2.41	4.78	13.60	8.43	5.35	2.983	1.060
11	7.4	0.00101	0.55	0.90	1.57	2.60	5.10	13.30	8.63	4.80	2.630	0.899

Table 1. (cont'd.)

Run no.	Average velocity, $= \frac{G}{\rho_{av}}$, ft./sec.					Reynolds number for the vapor $= DGQ/\mu_V$				
	0 ft.	10 ft.	20 ft.	30 ft.	40 ft.	0 ft.	10 ft.	20 ft.	30 ft.	40 ft.
1	14.90	20.90	30.3	48.2	102.5	4,170	6,240	8,770	12,850	22,000
2	7.29	13.20	22.7	36.8	81.4	1,840	3,700	6,380	9,650	17,600
3	21.25	28.95	40.4	65.0	146.0	7,230	9,380	12,320	17,500	27,900
4	13.48	19.80	30.1	48.6	106.5	4,470	6,530	9,650	14,400	24,700
5	2.77	7.87	14.6	30.4	71.5	158	1,640	3,540	7,400	14,750
6	16.50	23.17	33.1	51.9	113.0	5,500	7,630	10,350	14,900	24,400
8	21.95	27.55	36.8	51.8	89.7	6,530	7,930	9,850	12,450	16,600
9	12.89	18.28	29.2	51.6	141.0	3,520	5,350	8,330	13,050	25,200
10	10.00	16.12	25.4	45.6	128.2	3,380	5,720	9,000	14,900	30,600
11	10.45	16.10	29.0	52.8	154.7	3,380	5,870	9,850	16,400	33,400

Run no.	Fanning friction factor for the vapor, f_V					Reynolds number for the liquid				
	0 ft.	10 ft.	20 ft.	30 ft.	40 ft.	0 ft.	10 ft.	20 ft.	30 ft.	40 ft.
1	0.0110	0.00967	0.0088	0.0079	0.0070	29,800	29,200	28,400	27,100	24,600
2	0.0087	0.0114	0.0094	0.00855	0.00735	30,700	30,300	29,300	28,400	26,000
3	0.00928	0.00862	0.00785	0.00735	0.00672	27,200	26,600	25,500	24,100	21,100
4	0.0107	0.00955	0.00856	0.00770	0.00688	34,700	33,700	32,800	31,200	28,100
5	0.1013	0.00975	0.0116	0.00920	0.00765	34,400	34,000	34,700	32,100	29,600
6	0.0100	0.00910	0.00840	0.00760	0.00688	29,400	28,400	27,800	26,700	23,800
8	0.00955	0.00900	0.00851	0.00800	0.00743	18,800	18,300	17,800	17,000	15,900
9	0.0116	0.0102	0.00890	0.00790	0.00685	27,500	27,700	27,200	25,500	22,000
10	0.01175	0.00990	0.00872	0.00760	0.00662	39,700	38,600	37,500	35,600	31,000
11	0.01175	0.00985	0.00852	0.00745	0.00652	38,700	39,200	37,700	35,700	30,700

Table 1. (cont'd.)

Run no.	Fanning friction factor for the liquid, f_L: 0 ft.	10 ft.	20 ft.	30 ft.	40 ft.	$\left[\frac{d(\Delta P)}{dx}\right]_{com.}$: 0 ft.	10 ft.	20 ft.	30 ft.	40 ft.
1	0.00664	0.00668	0.00670	0.00675	0.00687	0.176	0.231	0.309	0.444	0.842
2	0.00662	0.00662	0.00667	0.00670	0.00735	0.073	0.173	0.253	0.379	0.727
3	0.00675	0.00678	0.00682	0.00690	0.00708	0.205	0.262	0.345	0.509	1.046
4	0.00650	0.00650	0.00655	0.00660	0.00672	0.176	0.237	0.329	0.484	0.952
5	0.00650	0.00650	0.00650	0.00658	0.00667	0.093	0.097	0.220	0.383	0.760
6	0.00668	0.00670	0.00672	0.00678	0.00692	0.182	0.234	0.311	0.444	0.881
8	0.00725	0.00730	0.00730	0.00740	0.00750	0.159	0.189	0.241	0.318	0.513
9	0.00674	0.00675	0.00676	0.00683	0.00700	0.158	0.203	0.287	0.453	1.031
10	0.00635	0.00638	0.00640	0.00647	0.00660	0.149	0.216	0.309	0.489	1.211
11	0.00640	0.00640	0.00640	0.00647	0.00660	0.161	0.220	0.352	0.571	1.472

Run no.	ΔP_f (com.)(calc.), psi. by graphical integration for section: A	B	C	D	Observed overall pressure drop, $\Delta P_{(ov.)}$(obs.), psi.: A	B	C	D	ΔP_{KE} (cum.), psi.: A	B	C	D
1	2.00	2.65	3.65	6.00	1.97	2.36	3.40	5.85	0.07	0.19	0.40	1.06
2	1.34	2.10	3.08	5.13	1.72	2.20	2.83	5.30	0.07	0.19	0.37	0.93
3	2.33	3.00	4.20	7.25	2.49	3.08	4.68	7.77	0.09	0.21	0.50	1.36
4	2.05	2.80	3.92	6.82	2.18	2.88	4.00	7.23	0.08	0.22	0.47	1.24
5	0.93	1.52	2.90	5.50	1.09	1.52	2.45	4.30	0.07	0.17	0.40	0.99
6	2.07	2.70	3.80	6.08	2.22	2.90	4.05	7.13	0.08	0.19	0.41	1.13
8	1.73	2.12	2.77	4.05	1.97	2.18	2.90	4.03	0.05	0.12	0.24	0.53
9	1.81	2.41	3.57	6.58	1.77	2.86	4.00	7.81	0.06	0.19	0.45	1.47
10	1.82	2.57	3.88	7.58	2.40	3.27	5.13	10.50	0.09	0.23	0.52	1.74
11	1.88	2.80	4.50	9.00	2.40	3.81	5.31	10.30	0.08	0.28	0.64	2.20

Table 1. (concl.)

Run no.	Observed frictional pressure drop, ΔP_f(obs.), psi. A	B	C	D	Percentage error $= \left[\frac{\text{exp.-calc.}}{\text{exp.}}\right] \times 100$ A	B	C	D	Average percent error per run	Average percent deviation per run
1	1.90	2.28	3.29	5.19	+ 5.27	+16.23	+10.94	+15.60	+12.04	12.01
2	1.65	2.08	2.65	4.84	-18.85	+ 0.96	+16.25	+ 6.00	+ 0.61	10.52
3	2.40	2.96	4.39	6.91	- 2.91	+ 1.35	- 4.33	+ 4.92	- 0.24	3.38
4	2.10	2.74	3.75	6.46	- 2.38	+ 2.19	+ 4.33	+ 5.57	+ 2.43	3.62
5	1.02	1.42	2.23	3.71	- 8.81	+ 7.04	+30.10	+32.50	+15.21	19.11
6	2.14	2.79	3.83	6.41	- 3.27	- 3.22	- 0.26	- 5.15	- 2.98	2.98
8	1.92	2.11	2.78	3.74	- 9.89	+ 0.47	- 0.36	+ 8.28	- 0.38	4.75
9	1.71	2.73	3.74	6.83	+ 5.85	-11.72	- 4.53	- 3.66	- 3.52	6.42
10	2.31	3.13	4.84	9.28	-21.20	-17.90	-19.85	-18.29	-19.31	19.31
11	2.32	3.61	4.95	8.74	-18.97	-22.42	- 9.11	+ 2.97	-11.88	13.37
Average percent error per section					- 7.51	- 2.70	+ 2.32	+ 4.87	- 0.75	———
Average deviation per section					9.94	8.32	10.00	10.29	———	9.65

APPLICATION OF THE METHOD TO FLOW PROBLEMS

The most common fluid flow problem consists of predicting the pressure drop suffered by the flow of a known weight of fluid through a duct of known length for a finite period of time. Many variations of the problem are possible, but the discussion in this section will be confined to the solution of this type.

When analyzing the experimental measurements collected in this investigation, attempts were made to reproduce experimental pressure drops by using physical property data for each of the phases, based on the experimental pressure drops. Since the results obtained showed good agreement, the same type of analysis may be applied to flow problems in which the pressure drop is the desired unknown. The method is as follows:

1. The inlet point is located on the Pressure-Enthalpy-Quality diagram corresponding to fluid properties at the inlet of the pipe.

2. The path of the flashing flow process is established (e.g., for an isenthalpic process a vertical line is drawn).

3. Pressure drops are selected, and the quality and physical properties evaluated at the static pressures which these pressure drops determine. For example, if the inlet pressure is 40 psia., and pressure drops of 5 psi. are selected, physical properties and qualities are determined at 40, 35, 30, 25, etc., psia.

4. $[d(\Delta P)/dx]_{com.}$ is calculated at each static pressure corresponding to a selected pressure drop by using equation (29).

5. Graphical integration of $[dx/d(\Delta P)]_{com.}$ vs. ΔP, the selected

pressure drop, yields the length of pipe required to produce each frictional pressure drop.

6. A smooth curve is drawn between the selected pressure drops on a plot of ΔP_f vs. L, and the value of the friction head corresponding to the length of pipe being evaluated is picked off. In order to determine the overall pressure drop for this pipe length, a velocity head term must be added to the friction head.

CONCLUSIONS

Use of the ring flow picture and the associated mathematical analysis yielded satisfactory results for the prediction of frictional pressure drops due to the adiabatic flow of a flashing water-steam mixture in a horizontal three-eighths inch I.P.S. schedule 40 galvanized iron pipe, within the range of experimental data. The range of data observed were: mass velocity, G, from 73.5 to 133 lb./(sec.)(sq.ft.); inlet pressure, 27.7 to 42 psia.; and inlet quality, Q, 0.026 to 1.95 percent.

More detailed conclusions resulting from acceptance of the presented analysis are as follows:

1. A flashing fluid exhibits an annular, ring type flow.

2. Linear velocities of the vapor and liquid streams increase together and are the same at any cross-section downstream in the pipe.

3. The diameter of flow for both phases approaches closely the inside diameter of the pipe, even at low values of quality.

4. The overall head loss measured during flow of a flashing fluid consists of velocity and friction head terms. There is little difficulty

associated with approximating velocity heads, which are based on kinetic energy changes only, for both one and two-phase flow problems. Combined friction heads are the sum of vapor and liquid flow contributions.

5. The coefficients, α, β, a and b introduced to alter the Fanning equation and Reynolds number relationship to account for the unusual flow conditions studied, serve merely to correct mass velocity terms to actual, individual phase, mass velocities (e.g., mass velocity of the flowing vapor = Q x G).

6. Except for conditions of very low inlet quality, the gas phase flow frictional contribution is most important in determining the overall friction head.

The preceding conclusions were drawn from observations of the flow of water-steam mixtures in an experimental apparatus and results obtained after mathematical treatment. However, since the relationships and physical picture presented are not inherently limited to the water-steam system or the experimental apparatus used, the method should be applicable to the prediction of friction heads for the flow of other liquid-vapor mixtures in any sized duct. Application to any system is limited only by the availability of physical property data. Pipe size and type limitations are those associated with the Fanning equation and friction factor correlation; namely, long, straight, circular pipes running full of fluid.

Although the purpose of this investigation concerned flow of a flashing fluid, or a two-phase, one-component system, the picture and mathematical analysis evolved may well serve as an approach to the question of two-phase, two-component flow problems as well.

RECOMMENDATIONS FOR FUTURE WORK

In view of the satisfactory results obtained by applying the ring flow picture and associated mathematical analysis to the flow of flashing water-steam mixtures, further investigation of the method is justified.

The possible paths of future studies include:

1. Critical investigation of the flow mechanism.

2. Measurement of the pressure drops for the flow of other two-phase, one-component systems in different sized pipes and over extended operating conditions.

The second path is the more preferable, since useful data capable of application will be made immediately available. If the results of these studies compare favorably with those obtained by the author and presented in this paper, the method would be available for more general application, and the inference may be drawn that the analysis presented satisfactorily describes the flow picture.

Conduct of Future Study

The possible manner in which continued measurements could proceed is two-fold:

1. By using other systems and different pipe sizes in the existing experimental facilities.

2. By using other systems and different pipe sizes in a completely revised experimental set-up.

Use of the existing facilities for further investigations necessarily limits the extent of study to much the same operating ranges of mass velocity, inlet pressure and inlet quality used in preparing this paper. At the same time, this procedure would fail to establish whether or not the method of calculation used is peculiar to the experimental apparatus in which the measurements were taken.

Major revision of the existing facilities would enable extension of the range of operating conditions beyond those attainable at present; would ascertain the dependence of calculated results upon the apparatus used; and would provide an excellent opportunity for correction of some of the more troublesome features characteristic of the present facilities. Several worthwhile innovations would be:

1. Improvement of the method whereby inlet qualities are estimated, since considerable error is possible at present when measuring low values of quality.

2. Removal of the uncertainty regarding the existence of saturation equilibrium conditions in the liquid and vapor inlets, the result of mixing saturated vapor and cold liquid.

3. A method whereby a more thorough dispersal of vapor bubbles in the saturated liquid is obtained at the pipe inlet.

4. Introduction of the mixed liquid and vapor phases without transmission of pulsations to the manometer system.

The author feels that the advantages offered by a complete revision of present facilities seriously outweigh the convenience of using the existing apparatus without change.

Additional Systems

Investigation of other two-phase, one-component systems is emphasized in this section, since a primary consideration regarding the usefulness of the analysis presented for predicting pressure drops accompanying the flow of flashing fluids is whether or not the method is confined only to the system studied (water-steam).

Choice of other systems depend upon the availability of thermodynamic and physical property data, such as saturation properties, enthalpies, densities and viscosities of the saturated vapor and liquid, each as a function either of saturation pressures or temperatures; commercial availability with regard to plentiful supply, uniform purity and low unit price; and the relative ease of handling.

Toxicity, flammability, normal boiling point, chemical inertness to air and moisture, and possible corrosion of materials employed in the test equipment and storage facilities, all comprise the ease with which a proposed substance is handled. The final criterion applied to the choice of new systems is their frequency of occurrence in industrial flow problems.

The author believes that investigation of the flow of flashing trichloroethylene, a purified hydrocarbon such as hexane, and acetone, would provide adequate proof regarding the generality of the method presented, and at the same time supply data useful in rectifying many industrial flow situations.

Modifications of the Experimental Apparatus

When working with systems other than water-steam, economy of operation dictates "closing" the piping system to allow continuous recirculation of the fluid used. Work required for circulation may be provided by a suitable pump strategically located in the apparatus. In addition, the installation of storage facilities such as a tank would even flow through the system and act as a convenient reservoir, permitting changes in flow rates without adding or removing material from the closed system.

A series of nozzles or orifice plates designed to produce mixtures of saturated vapor and liquid phases suitable for injection to the test pipe would appear to be more effective than the kettle-first mixer-entrainment separator-vapor Flowrator-second mixer combination used for the same purpose in this investigation. Varying the pressure and flow rates of a near saturated or saturated liquid fed to the upstream side of the nozzles or orifices would yield a wide range of possible saturated vapor-liquid mixtures of different qualities at the downstream side of the constrictions. An example is indicative of the process:

Example: Saturated trichloroethylene at 140 psia. is throttled isenthalpically through a nozzle. Estimate the exit quality if the downstream pressure is 100 psia.; 50 psia.

pressure, psia.	140	100	50
enthalpy of the saturated liquid, Btu./lb.	42.00	26.60	11.10
enthalpy of the saturated vapor, Btu./lb.		135.41	122.15

(Perry, 16, p. 281)

$H_{Q,P} = (QH_V)_P + [(1 - Q)H_L]_P$ (equation 21)
since the process is isenthalpic,
$H_{P_1} = (QH_V)_{P_2} + [(1 - Q)H_L]_{P_2}$

at $P_2 = 100$ psia.:

$$42.00 = Q(135.41) + (1 - Q)26.60$$
$$Q = 14.1\%$$

at $P_2 = 50$ psia.:

$$42.00 = Q(122.15) + (1 - Q)11.10$$
$$Q = 27.8\%$$

By calibrating the nozzles or orifices used, downstream conditions may be accurately predicted, and upstream conditions adjusted to yield desired exit qualities and pressures.

One of the most desirable attributes of the recommended system is the high degree of mixing of the liquid and vapor phases obtained without bumping or pulsation, as the result of extreme turbulence in the fluid stream which accompanies flow through orifices or nozzles.

Further suggested improvements include:

1. Using a pressure-boosting device and heater prior to throttling in the nozzles or orifices, thereby increasing the range of discharge pressures from the constrictions. A jet pump used in conjunction with the circulating pump should supply an adequate pressure range.

2. Decreasing the length of test pipe over which each individual pressure drop measurement is made, thereby increasing the accuracy of the graphical integration and rendering the integration more sensitive to minor fluctuations (e.g., the variation taking place in the critical region, $Re > 2100, < 4000$). Use of pressure taps placed at five-foot intervals would yield more satisfactory pressure drop measurements.

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APPENDIX

Nomenclature

A	cross-sectional area, sq.ft.
a	a constant, no units
b	a constant, no units
D	inside diameter, ft.
F	force due to friction, (ft. lb. force)/(lb. mass) of flowing fluid
f	Fanning friction factor, no units
G	mass velocity, lb./(sec.)(sq.ft.)
g	local acceleration due to gravity, ft./sec.^{2}
g_c	= 32.174 (lb. mass)(ft.)/(lb. force)(sec.^{2}), dimensional constant
H	enthalpy, Btu./lb.
L	length, ft.
m	mass, slugs
P	pressure, lb./sq.in.
Q	quality, no units
q	heat input, Btu.
Re	Reynolds number, no units
S	fraction of cross-sectional area occupied by the vapor, no units
s	entropy, Btu./lb.-$^{\circ}$F.
T	temperature, $^{\circ}$F.
t	time, min. or sec.
U	internal energy, Btu./lb.

V	volume, cu.ft.
v	velocity, ft./sec.
w	weight rate of discharge, lb./sec.
x	distance along the pipe length, ft.
Z	elevation above datum plane, ft.
α	a constant, no units
β	a constant, no units
Δ	denotes an incremental value
φ	denotes some function
μ	viscosity, lb./(ft.)(sec.)
ρ	density, lb./cu.ft.
θ	a constant, no units

<u>Subscripts</u>

1	refers to cross-section 1 in the apparatus
2	refers to cross-section 2 in the apparatus
av.	average
com.	combined vapor and liquid contributions
F	frictional
KE	refers to kinetic energy term
L	refers to the liquid phase
ov.	overall, consisting of friction and kinetic energy contributions
P	refers to a given pressure
Q	refers to a given quality
T	refers to a given temperature
V	refers to the vapor phase

<u>Abbreviations</u>

Btu.	British thermal units
calc.	calculated
cu.ft.	cubic feet
cum.	cumulative
ft.	feet
Hg	mercury
in.	inches
Kg.	kilograms
lb.	pounds
max.	maximum
min.	minutes
obs.	observed
psi.	pounds per square inch
psia.	pounds per square inch, absolute
sec.	seconds
sq.ft.	square feet
vel.	velocity
wt.	weight

Table 2. Vapor Flowrator calibration.

Run no.	Flow-rator reading	Time, min.	Wt., kg.	Wt., lb.	w_V, $\frac{lb.}{min.}$	P, psia.	$1/\rho_V$, $\frac{cu.ft.}{lb.}$	Flow, $\frac{cu.ft.}{min.}$
0	1.8							
1	21.6	20	2.93	6.45	0.323	41.7	10.5	3.39
2	21.3	10	1.53	3.37	0.337	40.2	10.7	3.60
3	16.6	21	2.05	4.52	0.215	43.7	10.2	2.19
4	12.1	22	1.38	3.03	0.138	43.2	10.3	1.42
5	19.1	20	2.35	5.18	0.259	39.2	10.8	2.80
6	5.3	30	0.40	0.88	0.029	39.5	10.8	0.31
7	9.0	20	0.80	1.76	0.088	40.7	10.4	0.84
8	13.9	15	1.20	2.64	0.176	42.0	10.2	1.73

Table 3. Calibration of the test pipe using water only.

Run no.	Time, min.	Wt., lb.	w_L, $\frac{\text{lb.}}{\text{sec.}}$	Vel., $\frac{\text{ft.}}{\text{sec.}}$	Re	$\Delta P_{obs.}$, in.Hg.	$f_{calc.}$
1	5.0	21.3	0.0708	0.86	3,340	0.12	0.0120
2	5.0	35.0	0.1168	1.41	5,470	0.27	0.0102
3	5.0	59.3	0.1975	2.39	9,300	0.65	0.0086
4	5.0	84.5	0.282	3.42	13,300	1.25	0.0080
5	4.0	88.3	0.368	4.46	17,350	1.99	0.0075
6	3.5	93.5	0.446	5.42	21,100	2.78	0.0071
7	3.0	95.5	0.532	6.45	25,100	4.16	0.0075
8	2.53	94.5	0.623	7.54	29,300	5.42	0.0071
9	2.0	82.6	0.688	8.35	32,500	6.81	0.0073
10	5.0	84.0	0.280	3.38	13,200	1.16	0.0076

Sample Calculations

Run number four will be used to demonstrate the manner in which the experimental data was used to determine the validity of equations (25) through (29).

<u>The Data</u>

Steam Flowrator:		9.3
Inlet pressure gauge:		22.0 psi.
Manometer readings:	M_1:	4.80 in. Hg
	M_2:	6.35 in. Hg
	M_3:	8.8 in. Hg
	M_4:	15.9 in. Hg
Weight of discharge:		68.5 lb.
Time interval:		7.0 min.
Room temperature:		70 °F.

<u>Calculations at the Inlet (L = 0)</u>

1. Weight Rate of Discharge.

$$w = \frac{68.5 \text{ lb.} \times \text{min.}}{7.0 \text{ min.} \times 60 \text{ sec.}} = .164 \text{ lb./sec.}$$

2. Quality.

volumetric discharge of vapor Flowrator, $(V/t) = 0.88$ cu.ft./min. (Fig. 7)

inlet temperature at $(22.0 + 14.7) = 36.7$ psia., $= 262°$ F. (Fig. 9)

specific volume of the vapor at 262° F. = 11.50 cu.ft./lb. (Fig. 10)

weight rate of discharge through vapor Flowrator

$$w_V = \frac{0.88 \text{ cu.ft.}}{\text{min.}} \times \frac{1 \text{ min.}}{60 \text{ sec.}} \times \frac{1 \text{ lb.}}{11.50 \text{ cu.ft.}} = .00128 \frac{\text{lb.}}{\text{sec.}} \text{ (equation 18a)}$$

quality, $Q = w_V/w = (.00128/.164) \times 100 = 0.79\%$ (equation 18b)

3. Average Density.

specific volume of the liquid at 262° F. = .0171 cu.ft./lb. (Fig. 11)

average specific volume
$= (.79 \times 10^{-2} \times 11.50 \text{ cu.ft./lb.}) + (.9921 \times .0171 \text{ cu.ft./lb.})$
$= 0.109$ cu.ft./lb. (equation 20)
average density, $\rho_{av.} = 1$ lb./0.109 cu.ft. $= 9.20$ lb./cu.ft.

4. Mass Velocity.
cross-sectional area of nominal 3/8" schedule 40 pipe,
A = .00133 sq.ft. (Perry, 16, p. 415)
mass velocity, $G = \dfrac{0.164 \text{ lb.}}{.00133(\text{sq.ft.})(\text{sec.})} = \dfrac{124 \text{ lb.}}{(\text{sec.})(\text{sq.ft.})}$

5. Average Velocity.
average velocity, $v_{av.} = \dfrac{124 \text{ lb.}}{(\text{sec.})(\text{sq.ft.})} \times \dfrac{\text{cu.ft.}}{9.20 \text{ lb.}} = 13.48$ ft./sec. (equation 21)

6. Reynolds Number for the Vapor.
viscosity of the vapor at 262° F.,
$\mu_V = .01372$ centipoises (Fig. 12)
$= .01372/1488 = \dfrac{.923 \times 10^{-5} \text{ lb.}}{(\text{ft.})(\text{sec.})}$
diameter of the pipe, D = .0411 ft. (Perry, 16, p. 415)
Reynolds number,
$Re = \dfrac{.0411 \text{ ft.} \times 124 \text{ lb.} \times .79 \times 10^{-2} \times \text{ft.} \times \text{sec.}}{\text{sec.} \times \text{ft.}^2 \times .923 \times 10^{-5} \text{ lb.}} = 4400$ (equation 27)

7. Frictional Pressure Drop Due to Vapor Flow.
Fanning friction factor, f, at Re = 4400, = .0107 (curve D, Fig. 6)
conversion factor, $g_c = 32.17$ ft./sec.^{2}
Fanning equation $= \left[\dfrac{d(\Delta P)}{dx}\right]_V = \dfrac{4fG^2Q}{2g_cD\rho_V}$ (equation 25)
frictional pressure drop due to vapor flow, $\left[\dfrac{d(\Delta P)}{dx}\right]_V$,
$= \dfrac{4 \times .0107 \times \text{sec.}^2 \times (124)^2 \times \text{lb.}^2 \times .79 \times 10^{-2} \times 11.50 \text{ ft.}^3 \times \text{ft.}^2}{2 \times 32.17 \times \text{ft.} \times \text{sec.}^2 \times \text{ft.}^4 \times .0411 \text{ ft.} \times \text{lb.} \times 144 \text{ in.}^2}$
= .157 (lb./sq.in.)/ft.

8. Reynolds Number for the Liquid.
viscosity of the liquid at 262° F., $\mu_L = .217$ centipoises (Fig. 13)
$= .217/1488 = \dfrac{1.46 \times 10^{-4} \text{ lb.}}{(\text{ft.})(\text{sec.})}$
Reynolds number, $Re = \dfrac{.0411 \text{ ft.} \times 124 \text{ lb.} \times .9921 \times \text{ft.} \times \text{sec.}}{\text{sec.} \times \text{sq.ft.} \times 1.46 \times 10^{-4} \text{ lb.}}$
= 35,000 (equation 28)

9. Frictional Pressure Drop Due to Liquid Flow.
Fanning friction factor, f, at Re = 35,000, = .0065 (curve D, Fig. 6)
Fanning equation $= \left[\dfrac{d(\Delta P)}{dx}\right]_L = \dfrac{4fG^2(1-Q)}{2g_cD\rho_L}$ (equation 26)

Frictional pressure drop due to liquid flow, $\left[\frac{d(\Delta P)}{dx}\right]_L$,

$$= \frac{4 \times .0065 \times (124)^2 \times .9921 \times .0171}{2 \times 32.17 \times .0411 \times 144} = .019 \text{ (lb./sq.in.)/ft.}$$

10. Overall Frictional Pressure Drop.

$$\left[\frac{d(\Delta P)}{dx}\right]_{com.} = .157 + .019 = 0.176 \text{ (lb./sq.in.)/ft. (equation 12)}$$

<u>Calculations for First Ten Feet of Flow (L = 10 feet)</u>

1. Quality.
an isenthalpic (vertical) line is drawn on the Pressure-Enthalpy-Quality curve (Fig. 14) originating at the inlet conditions of P = 36.7 psia. and Q = 0.79 percent. This line is the locus of point conditions existing in the pipe line.

manometer reading at L = 10 ft. = 4.80 in. Hg
density of mercury at room temperature, 70° F. = 13.54 g./cc. (Perry, 16, p. 176)
density of water at room temperature, 70° F. = 0.998 g./cc. (Perry, 16, p. 175)
pressure drop, inches of water = 4.80 x (13.54 - 1.00)
= 60.2 in. water
pressure drop, engineering units

$$= 60.2 \text{ in.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \times \frac{62.2 \text{ lb.}}{\text{cu.ft.}} \times \frac{1 \text{ cu.ft.}}{144 \text{ cu.in.}} = 2.18 \text{ lb./sq.in.}$$

static pressure at L = 10 ft., = 36.7 - 2.2 = 34.5 psia.
quality, read off at intersection of P = 34.5 psia. and established isenthalpic line, is Q = 1.18% (Fig. 14)

2. Average Velocity.
temperature at 34.5 psia. = 259° F. (Fig. 9)
specific volume of the vapor = 12.10 cu.ft./lb. (Fig. 10)
specific volume of the liquid = .01707 cu.ft./lb. (Fig. 11)
average density = 6.27 lb./cu.ft. (equation 20)

$$\text{average velocity} = \frac{124 \text{ lb.}}{(\text{sec.})(\text{sq.ft.})} \times \frac{1 \text{ cu.ft.}}{6.27 \text{ lb.}} = 19.80 \text{ ft./sec. (equation 21)}$$

3. Frictional Pressure Drop Due to Vapor Flow.
viscosity of the vapor at 259° F. = .01367 centipoises (Fig. 12)
$= .919 \times 10^{-5}$ lb./(ft.)(sec.)

$$\text{Reynolds number, Re} = \frac{.0411 \times 124 \times 1.18 \times 10^{-2}}{.919 \times 10^{-5}} = 6500 \text{ (equation 27)}$$

Fanning friction factor, f = .00955 (curve D, Fig. 6)

$$\left[\frac{d(\Delta P)}{dx}\right]_V = \frac{4 \times .00955 \times (124)^2 \times 1.18 \times 10^{-2} \times 12.10}{2 \times 32.17 \times .0411 \times 144}$$

$$= .219 \text{ (lb./sq.in.)/ft. (equation 25)}$$

4. Frictional Pressure Drop Due to Liquid Flow.
viscosity of the vapor at 259° F. = .222 centipoises (Fig. 13)
$= 1.49 \times 10^{-4}$ lb./(ft.)(sec.)
Reynolds number, $Re = \dfrac{.0411 \times 124 \times .9882}{1.49 \times 10^{-4}} = 34{,}000$ (equation 28)
Fanning friction factor, f = .0065 (curve D, Fig. 6)

$$\left[\frac{d(\Delta P)}{dx}\right]_L = \frac{4 \times .0065 \times (124)^2 \times .9882 \times .01707}{2 \times 32.17 \times .0411 \times 144}$$

= .018 (lb./sq.in.)/ft. (equation 26)

5. Overall Frictional Pressure Drop.

$$\left[\frac{d(\Delta P)}{dx}\right]_{com.} = 0.219 + .018 = 0.237 \text{ (lb./sq.in.)/ft.}$$

6. Pressure Drop Due to Kinetic Energy Increase.
inlet velocity = 13.48 ft./sec.
velocity at L = 10 feet, = 19.80 ft./sec.
$\Delta KE = (v_2^2/2g_c) - (v_1^2/2g_c)$
pressure drop due to change in kinetic energy, ΔP_{KE}

$$= (v_2^2 \rho_2/2g_c) - (v_1^2 \rho_1/2g_c)$$

$$= \frac{(19.80)^2 \times \text{ft.}^2 \times \text{sec.}^2 \times 6.27 \text{ lb.} \times \text{ft.}^2}{2 \times \text{sec.}^2 \times 32.17 \text{ ft.} \times \text{ft.}^3 \times 144 \times \text{in.}^2} - \frac{(13.48)^2 \times 9.20}{2 \times 32.17 \times 144}$$

= .265 - .181 = .084 psi.

<u>Calculations for the Second Ten Feet of Flow (L = 20 feet)</u>

1. Quality.
experimental pressure drop = 6.35 in. Hg = 2.88 psi.
quality, Q, at P = (34.5 - 2.9) = 31.6 psia., = 1.73% (Fig. 14)

2. Average Velocity.
average density = 4.12 lb./cu.ft. (equation 20)
average velocity = 30.1 ft./sec. (equation 21)

3. Overall Frictional Pressure Drop.
Reynolds number for the vapor, Re = 9650 (equation 27)
Fanning friction factor for the vapor, f_V = .00856 (curve D, Fig. 6)
Reynolds number for the liquid, Re = 33,000 (equation 28)
Fanning friction factor for the liquid, f_L = .00655 (curve D, Fig. 6)

$$\left[\frac{d(\Delta P)}{dx}\right]_{com.} = \frac{4G^2}{2g_cD}\left[\frac{f_VQ}{\rho_V} + \frac{f_L(1 - Q)}{\rho_L}\right]$$

= (0.312 + .017) = 0.329 (lb./sq.in.)/ft. (equation 29)

4. Pressure Drop Due to Kinetic Energy Increase.

$$\Delta P_{KE} = \frac{(30.1)^2 \times 4.12}{2 \times 32.17 \times 144} - (.181)$$

= .403 - .181 = .222 psi.

Calculations for the Third Ten Foot Section (L = 30 feet)

1. Quality.
   experimental pressure drop = 8.8 in. Hg = 4.00 psi.
   quality, Q, at P = (31.6 - 4.0) = 27.6 psia., = 2.55% (Fig. 14)

2. Average Velocity.
   average density = 2.555 lb./cu.ft. (equation 20)
   average velocity = 48.6 ft./sec. (equation 21)

3. Overall Frictional Pressure Drop.
   Reynolds number for the vapor, Re = 14,400 (equation 27)
   Fanning friction factor for the vapor, f_V = .0077 (curve D, Fig. 6)
   Reynolds number for the liquid, Re = 31,200 (equation 28)
   Fanning friction factor for the liquid, f_L = .0066 (curve D, Fig. 6)
   $\left[\frac{d(\Delta P)}{dx}\right]_{com.} = (0.467 + .017) = 0.484$ (lb./sq.in.)/ft. (equation 29)

4. Pressure Drop Due to Kinetic Energy Change.
   $\Delta P_{KE} = \frac{(48.6)^2 \times 2.555}{2 \times 32.17 \times 144} - .181 = .652 - .181 = .471$ psi.

Calculations for the Fourth Ten Foot Section (L = 40 feet)

1. Quality.
   experimental pressure drop = 15.9 in. Hg = 7.23 psi.
   quality, Q, at P = (27.6 - 7.2) = 20.4 psia., = 4.27% (Fig. 14)

2. Average Velocity.
   average density = 1.165 lb./cu.ft. (equation 20)
   average velocity = 106.5 ft./sec. (equation 21)

3. Overall Frictional Pressure Drop.
   Reynolds number for the vapor, Re = 24,700 (equation 27)
   Fanning friction factor for the vapor, f_V = .00688 (curve D, Fig. 6)
   Reynolds number for the liquid, Re = 28,100 (equation 28)
   Fanning friction factor of the liquid, f_L = .00672 (curve D, Fig. 6)
   $\left[\frac{d(\Delta P)}{dx}\right]_{com.} = (0.936 + .016) = 0.952$ (lb./sq.in.)/ft. (equation 29)

4. Pressure Drop Due to Kinetic Energy Change.
   $\Delta P_{KE} = \frac{(106.5)^2 \times 1.165}{2 \times 32.17 \times 144} - .181 = 1.24$ psi.

Comparison of Experimental and Calculated Friction Heads

1. Calculated Friction Heads.
   graphical integration of $\left[\frac{d(\Delta P)}{dx}\right]_{com.}$ vs. L (Fig. 15) yields the following frictional pressure drops:

section	0 – 10'	10 – 20'	20 – 30'	30 – 40'
section	A	B	C	D
P_f	2.05	2.80	3.92	6.82

2. Comparison of Calculated Results and Experimental Values.

section	A	B	C	D
P(calc.)	2.05	2.80	3.92	6.82
$P_{com.}$(obs.)	2.18	2.88	4.00	7.23
P_{KE}(cum.)	.08	.22	.47	1.24
P_{KE}	.08	.14	.25	.77
P_f(obs.)	2.10	2.74	3.75	6.46
error, psi.	– .05	+ .06	+ .17	+ .36
% error	–2.38%	+2.19%	+4.53%	+5.47%

average error = +2.45%
average deviation = 3.86%

Calculation of Enthalpy Change Due to Kinetic Energy Increase

The enthalpy change is calculated at L = 40 feet, since the kinetic energy is the largest for run number four at this cross-section of the pipe.

Basis: one pound of flowing fluid.

$$\Delta KE = \frac{wv_2^2}{2g_c} - \frac{wv_1^2}{2g_c} = \frac{1 \text{ lb.} \times (106.5)^2 \times \text{ft.}^2 \times \text{sec.}^2}{2 \times 32.17 \text{ ft.} \times \text{sec.}^2} - \frac{(13.48)^2}{2 \times 32.17}$$

$= (176 - 2.83)(\text{ft.})(\text{lb.})/\text{lb.} = 173(\text{ft.})(\text{lb.})/\text{lb.}$

1 Btu. = 778(ft.)(lb.) (Brown, 5, p. 133)

change in enthalpy due to kinetic energy increase

$$= \frac{173(\text{ft.})(\text{lb.})}{\text{lb.}} \times \frac{\text{Btu.}}{778(\text{ft.})(\text{lb.})} = \frac{.22 \text{ Btu.}}{\text{lb.}} \text{ (equation 17o)}$$

percent change in enthalpy caused by kinetic energy increase

$$= \frac{.22 \times 100}{236} = +.093\% \text{ (Fig. 14)}$$

Calculation of Heat Losses from the Test Pipe to the Air

1. Relationships. (Walker, Lewis, McAdams and Gilliland, 17, Chapt. IV) Heat loss = $dq = \Sigma\Delta/\Sigma R$ where $\Sigma\Delta$ = the temperature difference of the outside and inside surfaces; and ΣR = the sum of the resistances to heat flow offered by the inside liquid film, pipe wall, insulation and outside air film. The resistance to heat flow may be expressed in terms of the individual resistances:

$$\Sigma R = \frac{1}{h_l A_l} + \frac{L'_m}{k_m A_m} + \frac{L'_i}{k_i A_i} + \frac{1}{(h_c + h_r)A_a}$$

where h = heat transfer coefficient; A = heat transfer area; L' = thickness of conductor; k = thermal conductivity; subscripts l refer to liquid film, m to metal pipe wall, i to insulation, a to air film, c to conduction - convection, and r to radiation.

2. Calculation of Liquid Film Resistance.

$h_l = 160\ (1 + .01\ t)(V_s)^{0.8}/(D')^{0.2}$

where t = average water temperature; V_s, average water velocity; and D' = the inside diameter of the pipe.

All average conditions will be selected at L = 20 feet in the test pipe:

t, water temperature = 254° F.
V_s, velocity = 30.1 ft./sec.
D', inside diameter = .493 in.

$h_l = 160\ \ 1 + .01(254)\ \ x\ (30.1)^{0.8}/(.493)^{0.2}$
= 9900 Btu./(hr.)(sq.ft.)(°F.)
A_l = 40 ft. x 2 x 3.14 x .0206 ft. = 5.17 sq.ft.
$1/h_l A_l$ = 1/9900 x 5.17 = 1.96 x 10^{-5}(°F.)(hr.)/Btu.

3. Calculation of Pipe Wall Resistance.

k_m = 26 Btu./(hr.)(sq.ft.)(°F./ft.)
$A_m = 40 \times 3.14\ \frac{(.675 + .493)}{2 \times 12} = 6.12$ sq.ft.
L_m = (.675 - .493)/2 x 12 = .00675 ft.
$L_m/k_m A_m$ = .00675/26 x 6.12 = 4.25 x 10^{-5}(°F.)(hr.)/Btu.

4. Calculation of Insulation Resistance.

k_i = .042 Btu./(hr.)(sq.ft.)(°F./ft.) (Brown,5, p. 584)
$A_i = 40 \times 3.14 \times \frac{(3.675 + .675)}{2 \times 12} = 22.8$ sq.ft.
L_i = 1.50/12 = .125 ft.
$L_i/k_i A_i$ = .125/(.042 x 22.8) = .131(°F.)(hr.)/Btu.

5. Calculation of Air Film Resistance.

$A_a = 40 \times 3.14 \times \frac{(3.675)}{12} = 38.5$ sq.ft.

assuming the external temperature of the insulation to be 100°F., the temperature difference between the outside air and the insulation surface is (100 - 70) = 30°F., and

$(h_c + h_r)$ = 2.0 Btu./(hr.)(sq.ft.)(°F.) (Fig. 65, W. L. M and G., 17, p. 162)

$1/(h_c h_r)(A_a)$ = 1/2.0 x 38.5 = .0130(°F.)(hr.)/Btu.

6. Calculation of Heat Loss from the Test Pipe.

$\Sigma R = 1.96 \times 10^{-5} + 4.25 \times 10^{-5} + .131 + .0130$

$= .144(^{\circ}F.)(hr.)/Btu.$

overall temperature difference = 254 - 70 = 184°F.

$dq = \dfrac{184^{\circ}F. \times Btu.}{.144(^{\circ}F.)(hr.)} = \dfrac{1280\ Btu.}{hr.}$

weight rate of flow $= \dfrac{.164\ lb.}{sec.} \times \dfrac{3600\ sec.}{hr.} = 590\ \dfrac{lb.}{hr.}$

heat loss from the test pipe $= \dfrac{1280\ Btu.}{hr.} \times \dfrac{hr.}{590\ lb.} = \dfrac{2.16\ Btu.}{lb.}$

percent change in enthalpy of flowing fluid caused by heat loss to the surroundings $= \dfrac{2.16 \times 100}{236} = -0.95\%$

The assumption that changes in the enthalpy of the flowing fluid caused by a change in kinetic energy and heat losses is negligible is justified by the preceding calculations.

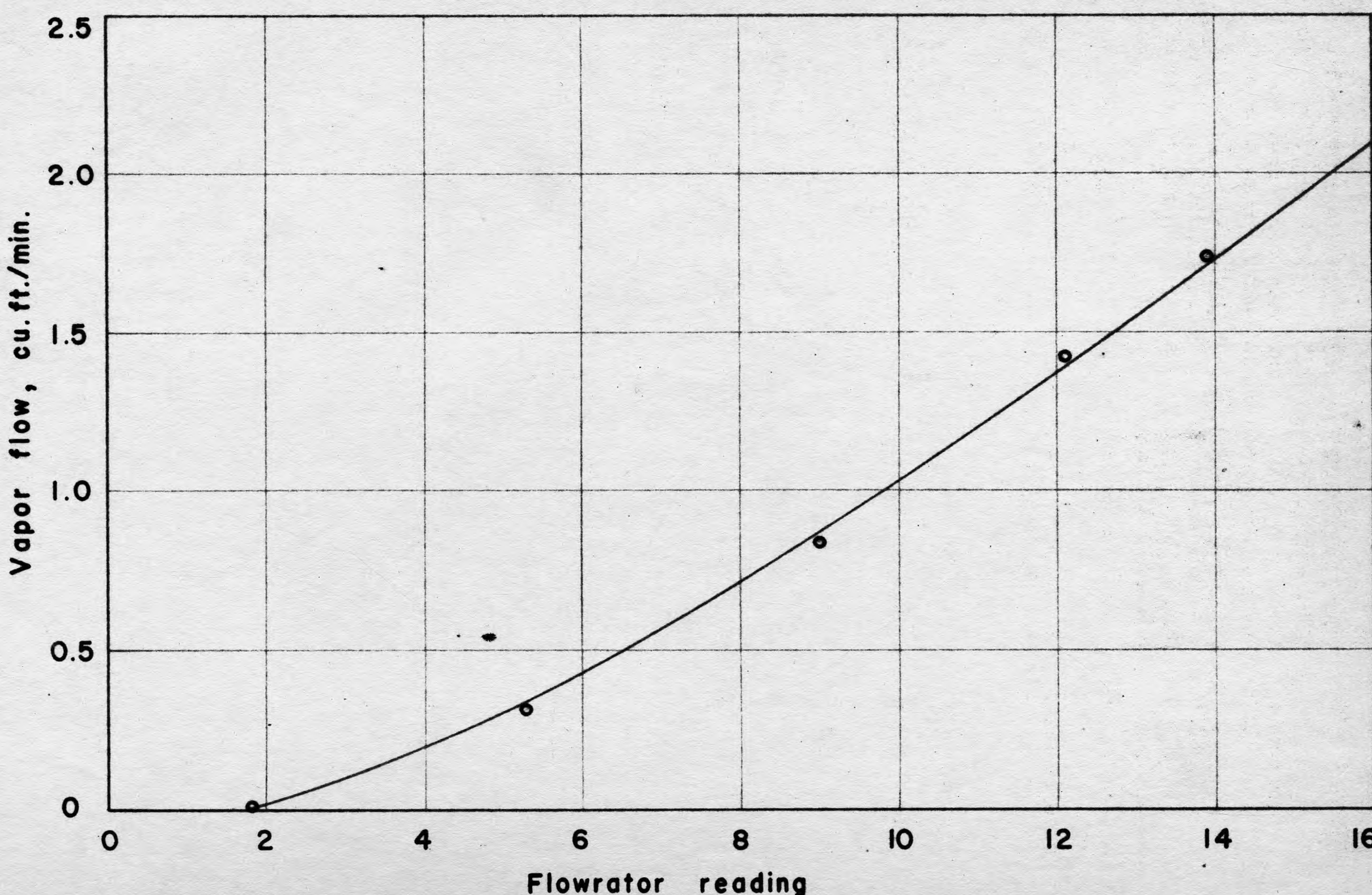

Fig. 7. Vapor Flowrator calibration.

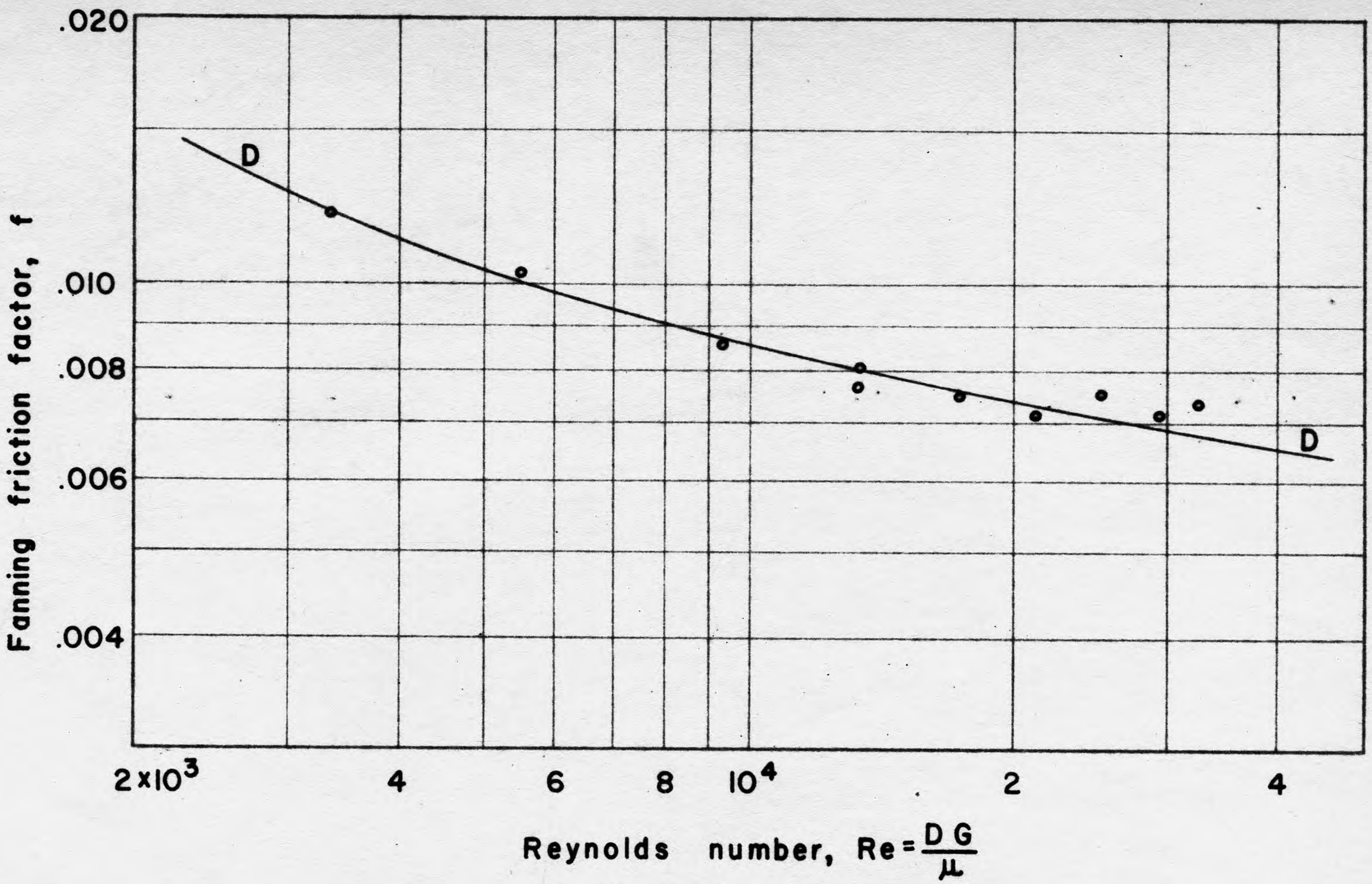

Fig. 8. Friction factors for the test pipe compared to curve D.

o : experimental values

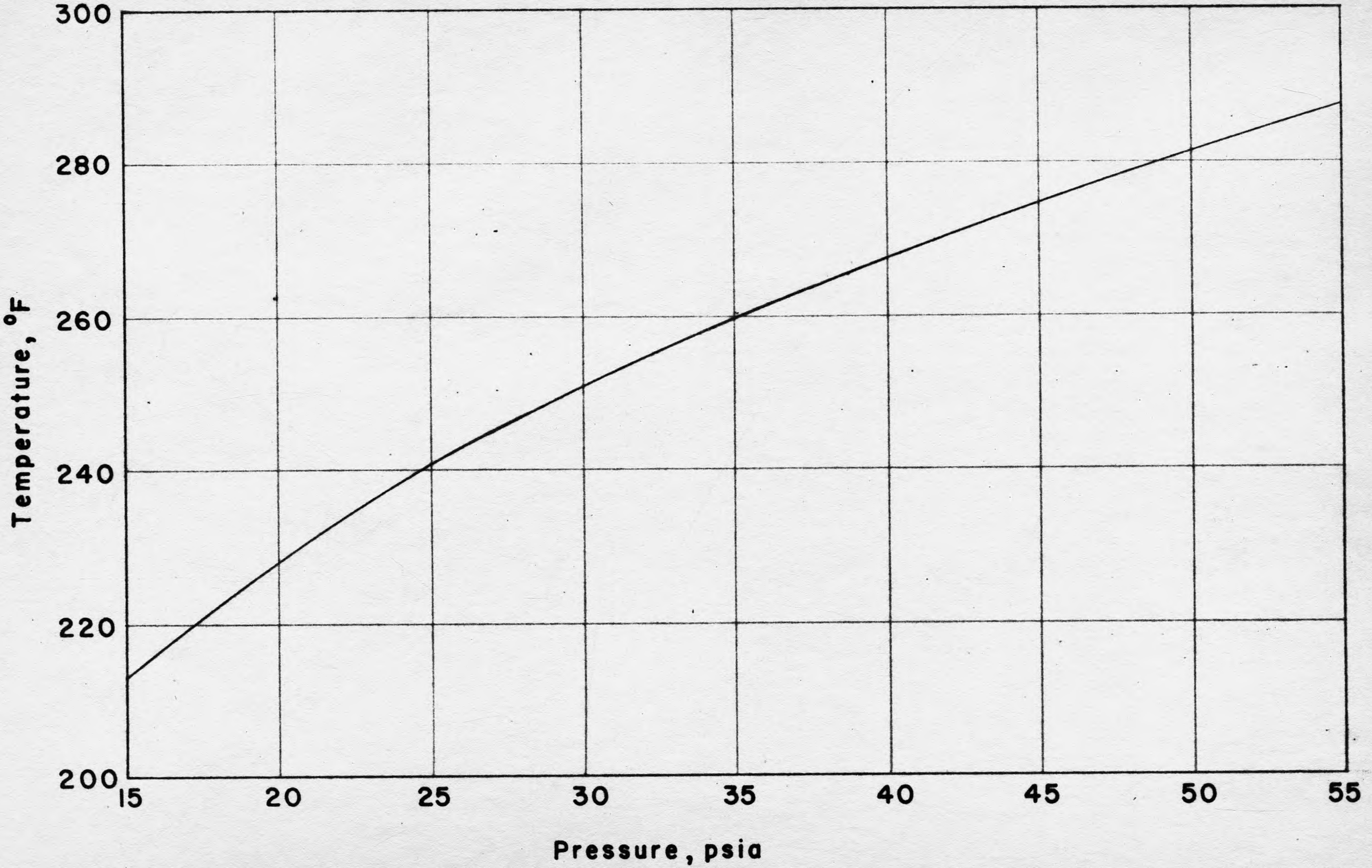

Fig. 9. Saturation properties of water. (10)

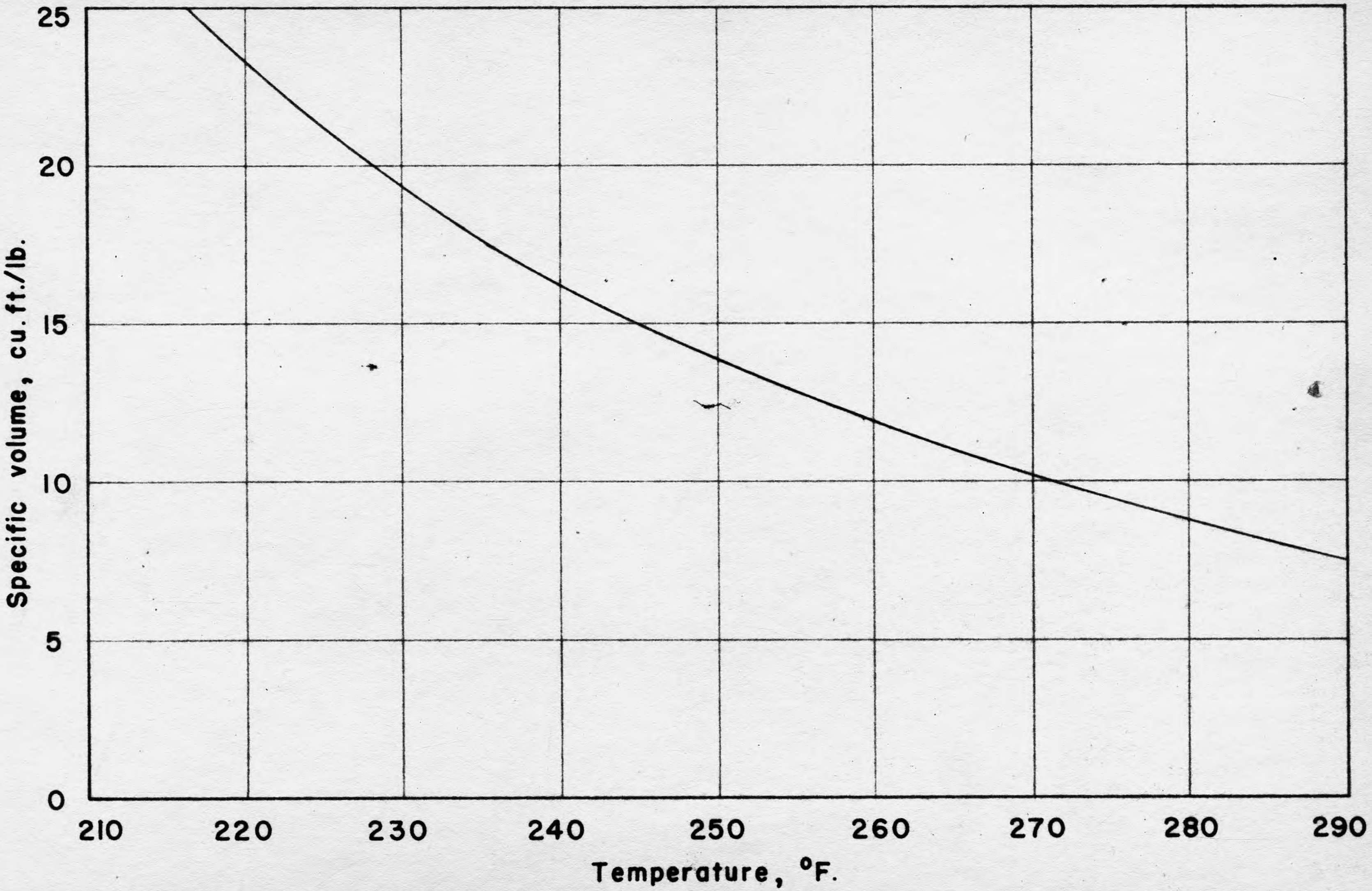

Fig. 10. Specific volume of saturated steam. (10)

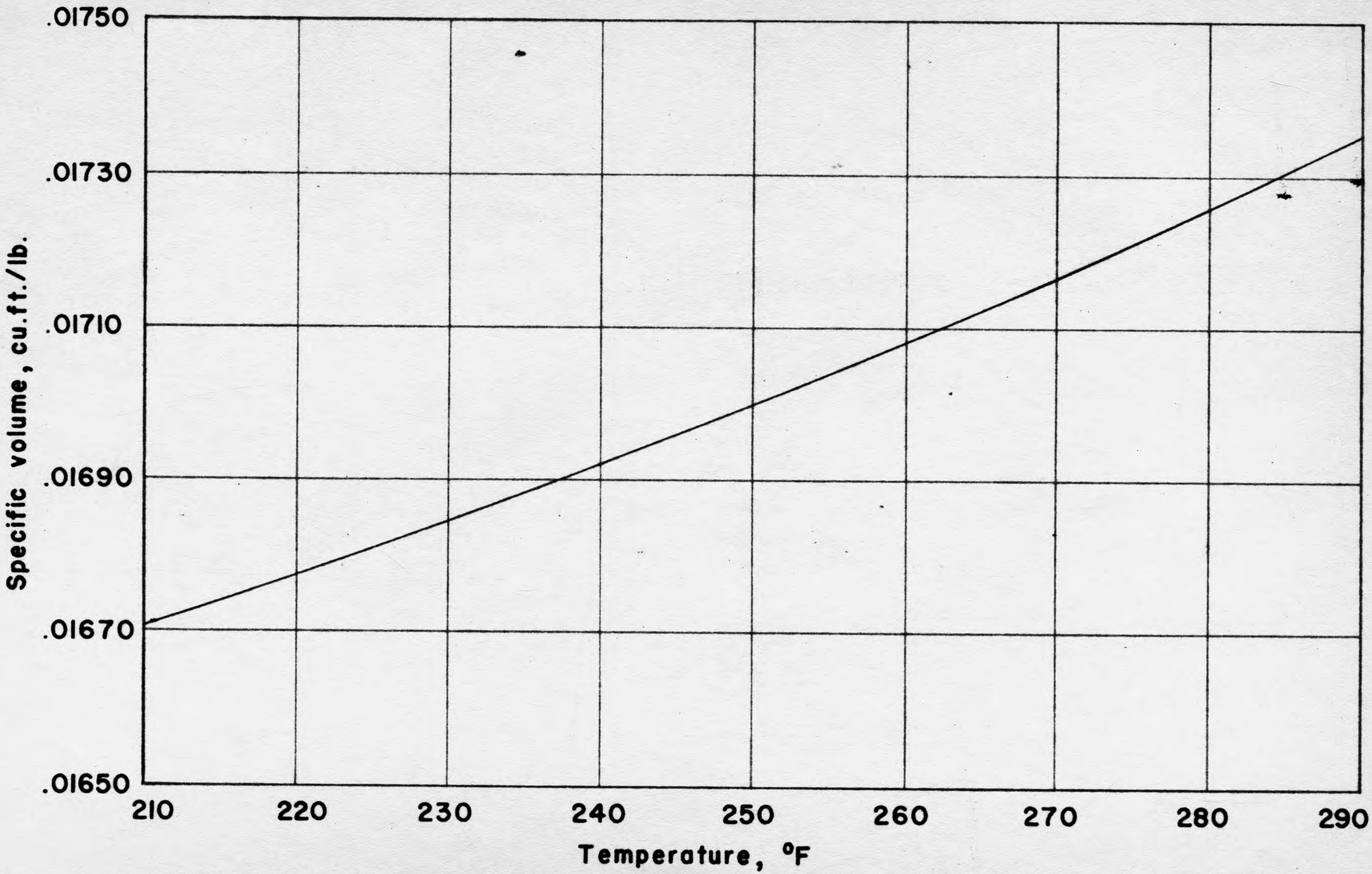

Fig. 11. Specific volume of saturated water. (10)

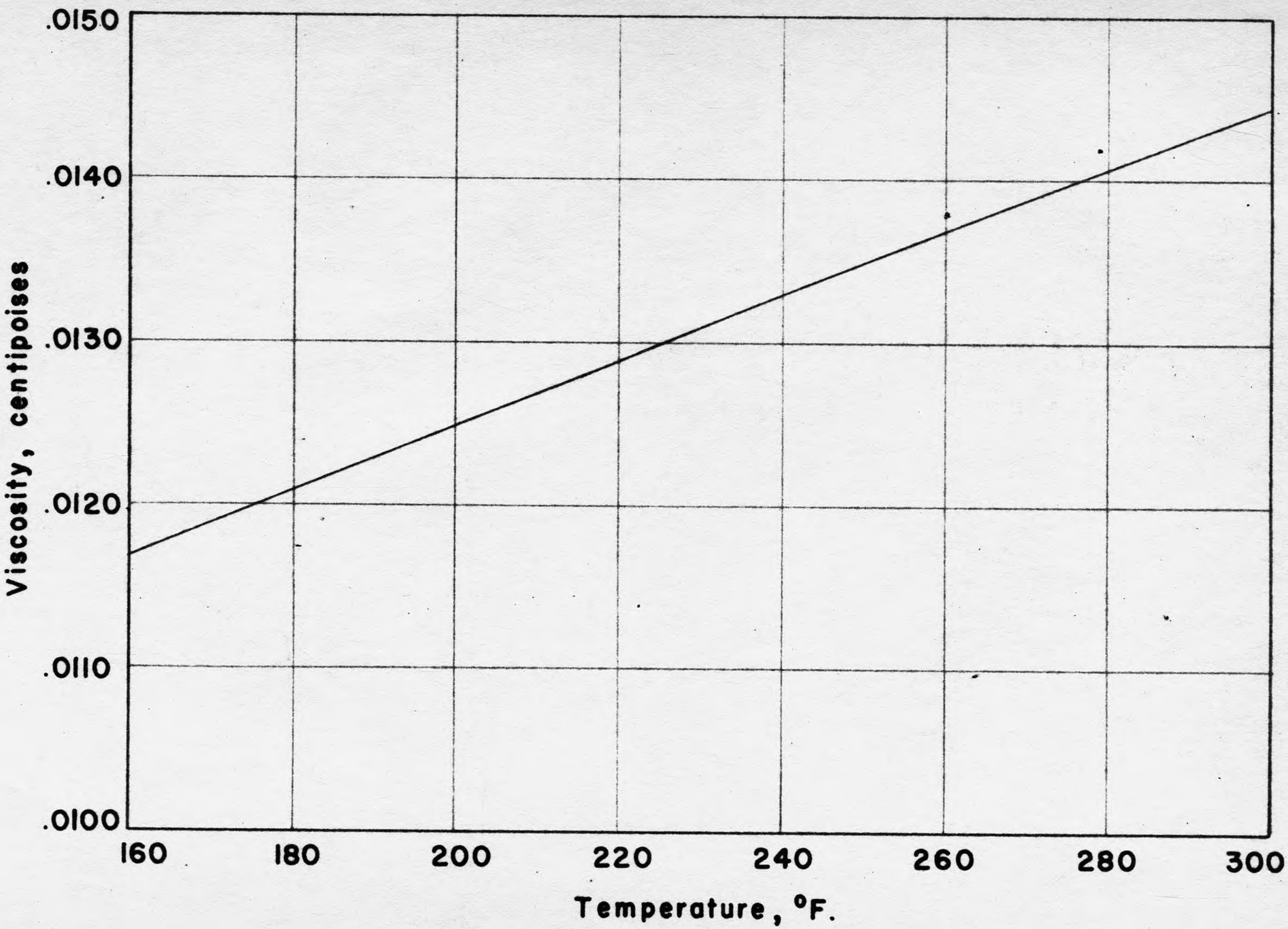

Fig. 12. Viscosity of saturated water vapor. (9)

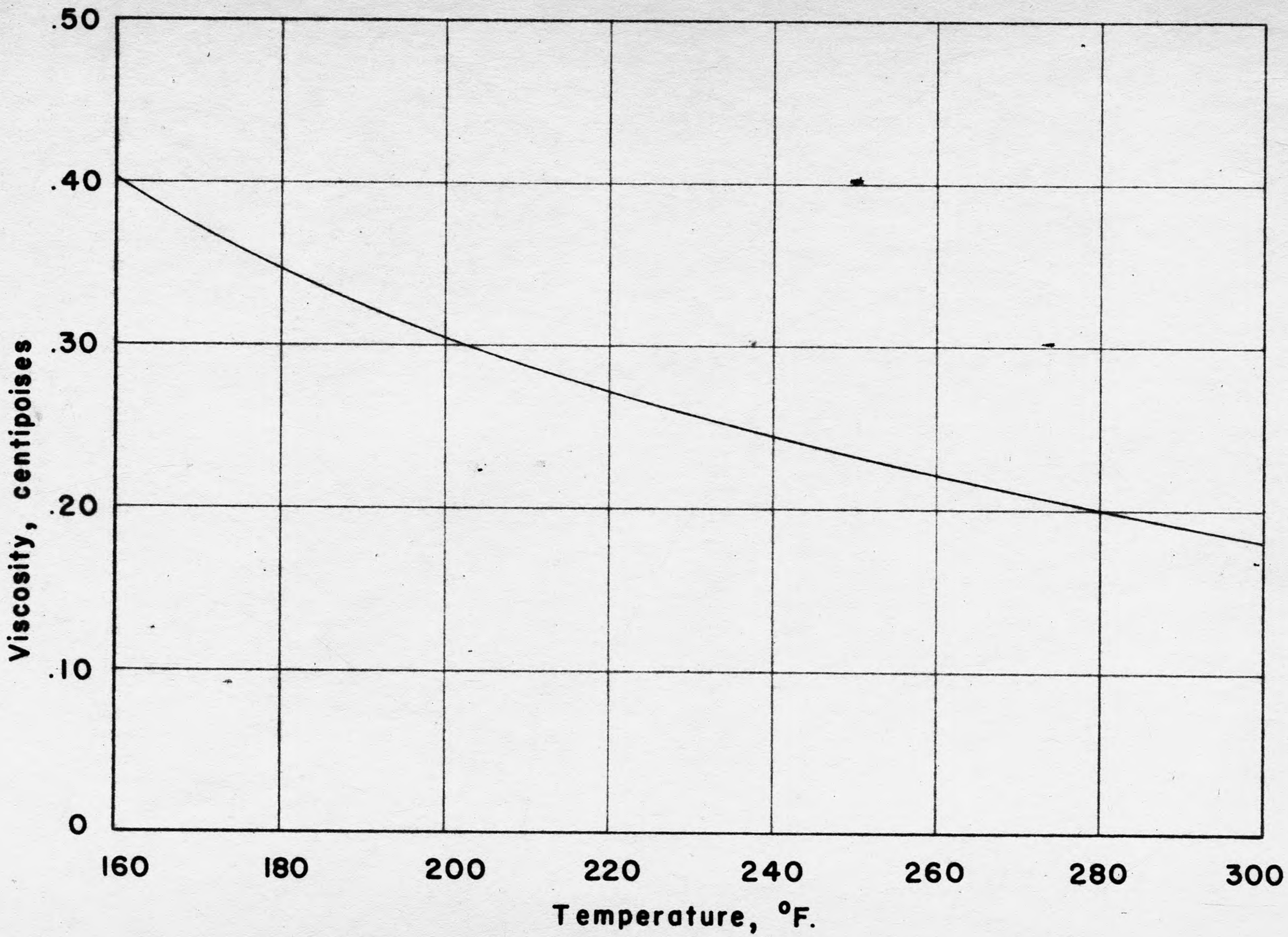

Fig. 13. Viscosity of saturated water. (9)

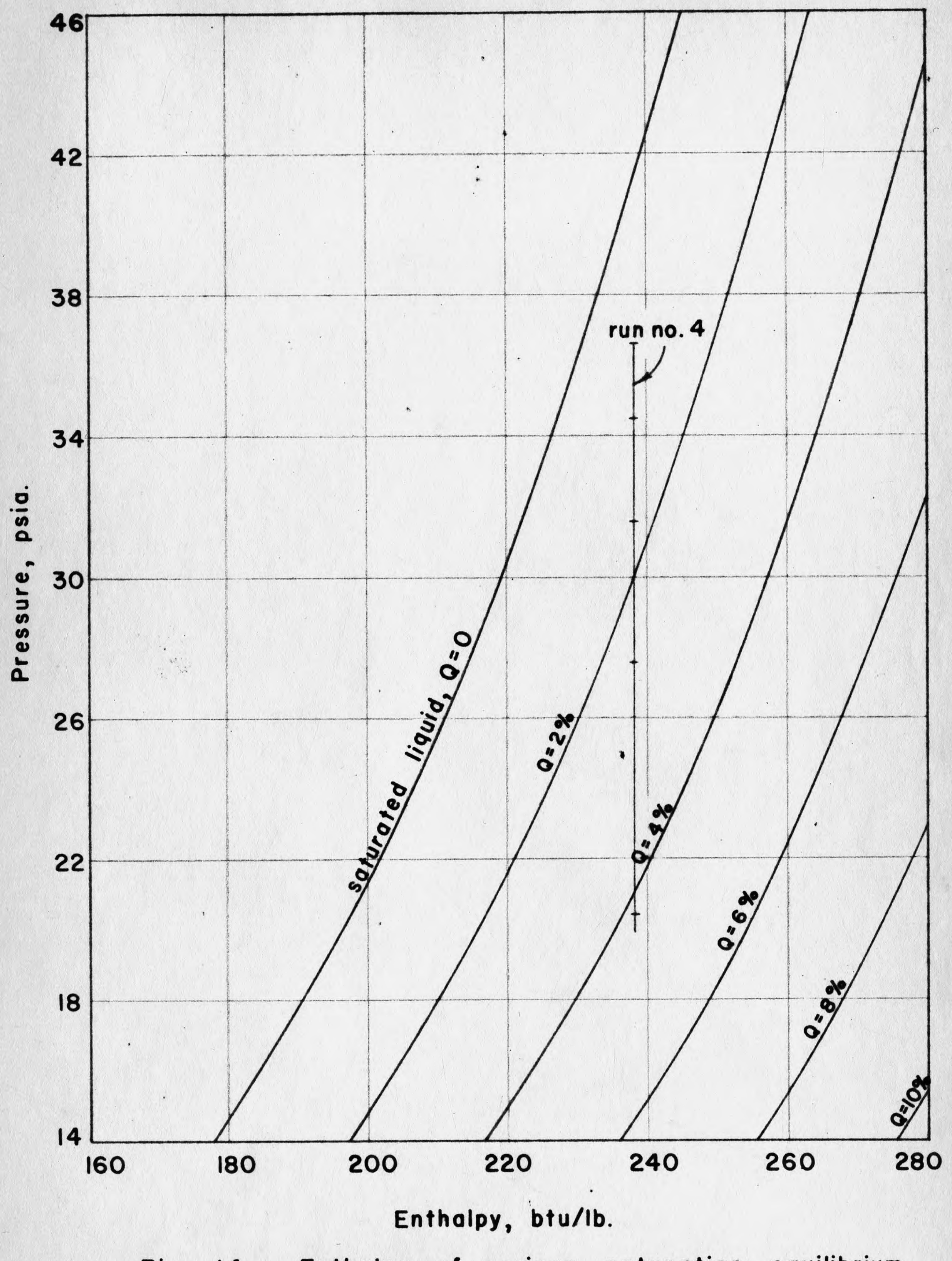

Fig. 14. Enthalpy of various saturation equilibrium mixtures of steam and water. (10)

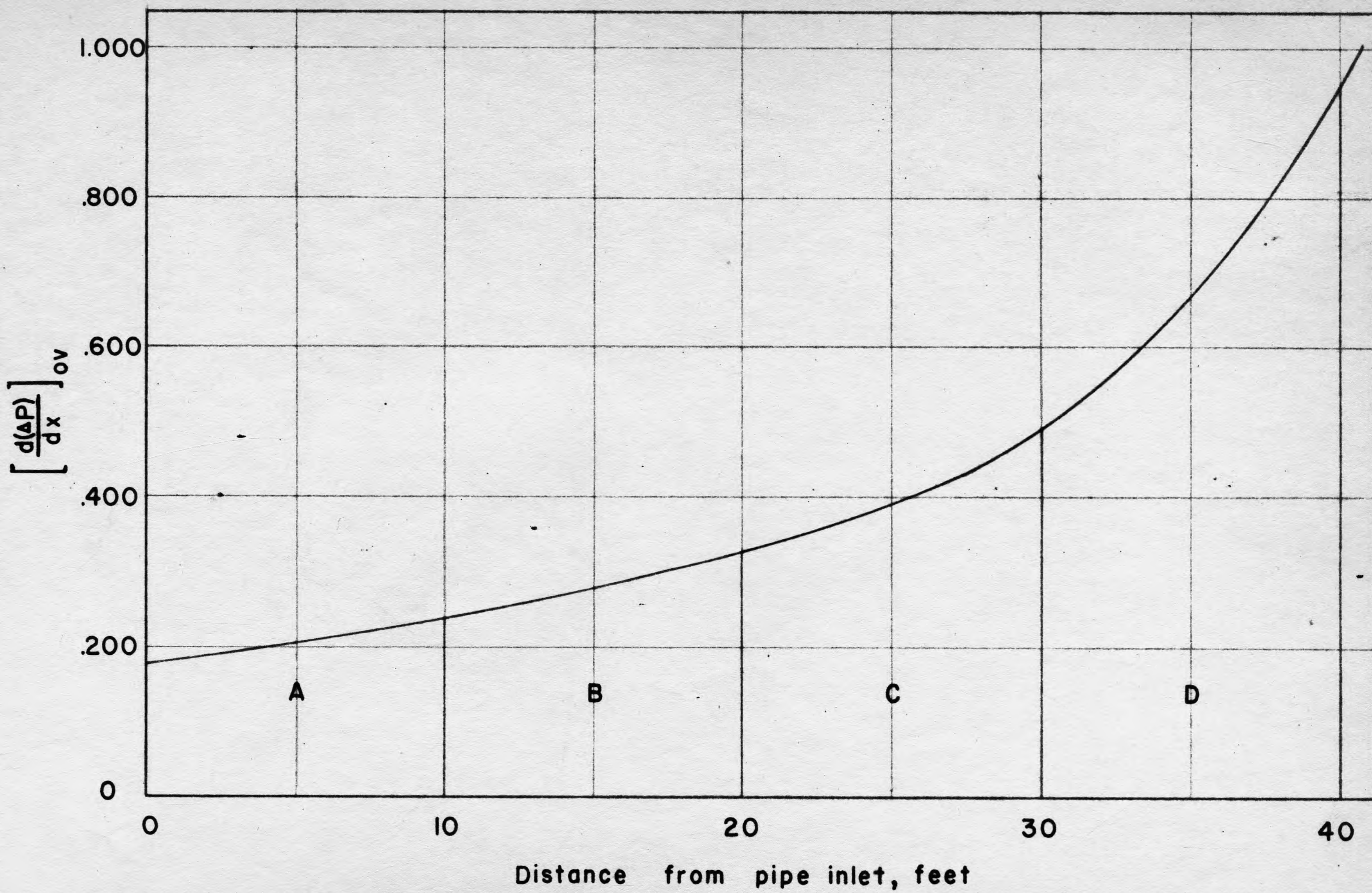

Fig. 15. Graphical integration of test run number four.

A STUDY OF THE PRESSURE DROP ACCOMPANYING FLOW OF A FLASHING FLUID IN A CIRCULAR PIPE

by

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ABSTRACT OF THESIS

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ABSTRACT

An insulated pipe line which carries a vaporizing liquid and receives no heat from an outside source is said to transport a flashing fluid. Vaporization is the result of a decrease in the static pressure of the saturated liquid due to frictional effects normally associated with flow through pipes, and is accompanied by a marked increase in the linear velocity of the fluid stream because of the large volume of vapor formed. Conventional fluid flow equations are not directly applicable to the prediction of frictional pressure drops for this unusual type of flow, and the purpose of this investigation was to determine if, and in what manner, these conventional equations could be altered to fit the new conditions.

An energy balance taken about any piping system must account for changes in the flow work term, kinetic energy, internal energy, and potential energy of the flowing fluid, in addition to the work input to or work done by the fluid, head input to or loss from the system, and mechanical energy lost as a result of irreversible frictional processes. Individual terms of the mechanical energy balance are expressed in terms of friction head, or pressure. The Fanning equation and friction factor-Reynolds number correlation are commonly used to estimate the frictional head, or pressure drop due to frictional effects. The Fanning equation relates friction head to physical properties of the flowing fluid and flow conditions existing in the pipe. When the fluid properties or flow conditions vary along the pipe length, the equation

must be used in differential form:

(1) $$\frac{d(\Delta P)}{dx} = \frac{4fG^2}{2g_c D\rho}$$

where ΔP is the frictional pressure drop; dx, a differential section of pipe; f, the Fanning friction factor; G, the mass velocity; g_c, a conversion constant; D, the inside diameter of the pipe; and ρ, the density of the fluid, all terms being expressed in consistent units. The Fanning friction factor, f, is expressed mathematically as follows:

(2) $$f = \varphi(Re) = \varphi\left(\frac{DG}{\mu}\right)$$

The Fanning friction factor, f, is a function of the Reynolds number, Re, defined by $\frac{DG}{\mu}$, where D is the inside diameter of the pipe; G, the mass velocity; and μ, the viscosity of the fluid. Equation (2) has been found to yield consistent results for commercial pipes and tubing. The only limitations surrounding application of the Fanning equation and friction factor correlation are that steady state flow exist and that the pipe be horizontal, round and running full of fluid. The author has attempted to alter equations (1) and (2) to make them applicable to the solution of flashing flow problems.

On the basis of published work for two-phase, two-component flow, a physical picture of the flow of a flashing fluid has been presented. This picture postulates that the liquid phase flows along the pipe length in a series of "rings" and that the vapor phase flows as an annulus, both phases riding on a liquid film which remains in contact with the pipe wall and is in itself stationary.

Experimental measurements of the flow of flashing water-steam mixtures were made in an insulated, horizontal three-eighths inch I.P.S. schedule 40 galvanized iron pipe. Various methods of treating the data were attempted in an effort to yield calculated friction heads compatible with those observed. The most satisfactory results were obtained by modifying the Fanning equation to account for vapor and liquid flow contributions, to yield a combined friction head. The relationships are expressed as follows:

$$(3) \quad \left[\frac{d(\Delta P)}{dx}\right]_{com.} = \frac{4G^2}{2g_c D}\left[\frac{f_V Q}{\rho_V} + \frac{f_L(1 - Q)}{\rho_L}\right]$$

$$(4) \quad f_V = \varphi\left[\frac{DGQ}{\mu_V}\right]$$

$$(5) \quad f_L = \varphi\left[\frac{DG(1 - Q)}{\mu_L}\right]$$

where the left-hand term of equation (3) represents the combined liquid and vapor friction head contributions; Q, the quality or weight percent of vapor in the flowing fluid; and the subscripts V and L refer to vapor and liquid phases respectively.

The limitations of the method presented for analyzing flashing flow problems are that:

1. The investigation was limited to the water-steam system.

2. The range of operating conditions observed was mass velocity, G, from 73.5 to 133 lb./(sec.)(sq.ft.); inlet pressure, 27.7 to 42 psia.; and inlet quality, Q, 0.026 to 1.95 percent.

3. The limitations associated with the Fanning equation and friction factor correlations apply; namely, horizontal, round commercial pipe or tubing, running full of fluid.