MATRIX ANALYSIS OF MULTIPLE HIGH TOWER FRAME WITH SIDESWAY

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Matrix analysis in structures is convenient since it can be performed by a computer. To solve a structure with sidesway, first the degrees of freedom of sidesway must be determined, then the total degrees of freedom of the structure must be calculated, that is, including the degrees of freedom of rotation (1). The displacements of all joints due to sidesway can be figured out easily by sketching a joint-displacement diagram (2). An example is shown in Fig. 1.

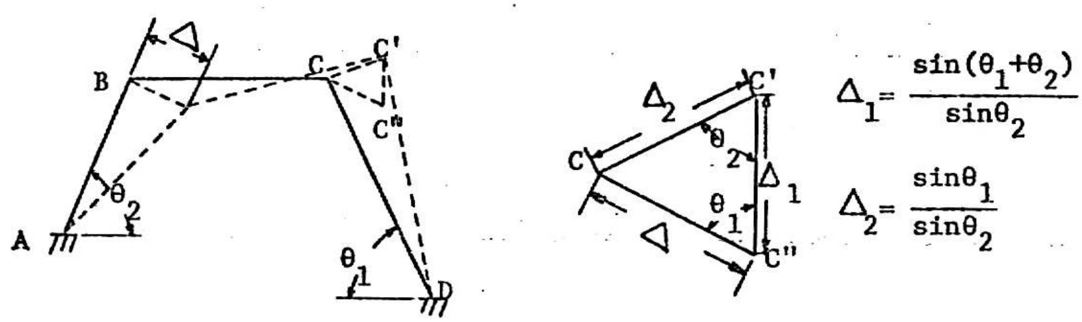

Fig. 1. Joint-displacement diagram

Since in matrix analysis, the loads are applied on nodes only, distributed loads cannot be handled directly.

> "When distributed loads are involved, the alternative is to fix the loaded beam and to apply to its two ends the reverse of the fixed-end moment and shears (Fig. 2). The final moments and shears in the loaded member must be obtained by adding the internal forces of the fixed-end beam to those resulting from the nodal force analysis (3)."

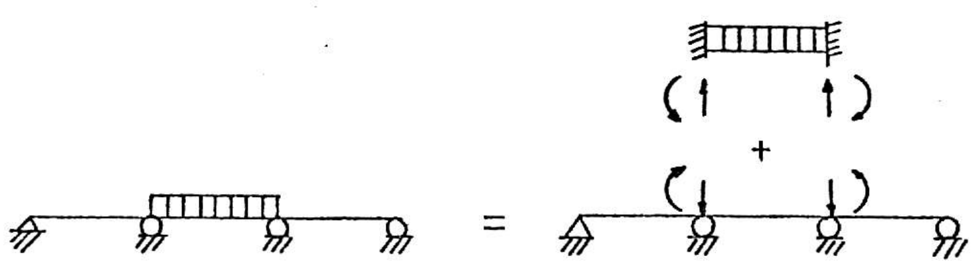

Fig. 2. Convert the distributed loads to nodal loads

After the deformation matrix and applied loads matrix have been established, the displacements and internal forces at every joint can be determined.

Solving a multiple sidesway high tower is based on analyzing a rigid frame. The most important step is to calculate the sidesway at each joint and establish the deformation matrix. Although the displacement method is tedious in computation, the basic theory of this method is simple and can be easily used to solve any complicated structure.

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(1). Chu-Kia Wang, " Matrix Methods of Structural Analysis ", International Textbook Company, 1966.

(2). Charles Head Norris and John Benson Wilbur, " Elementary Structural Analysis ", McGraw-Hill, New York, 1960.

(3). Yuan-Yu Hsieh, " Elementary Theory of Structures ", McGraw-Hill, New York, 1970.

Derivations of Formulae

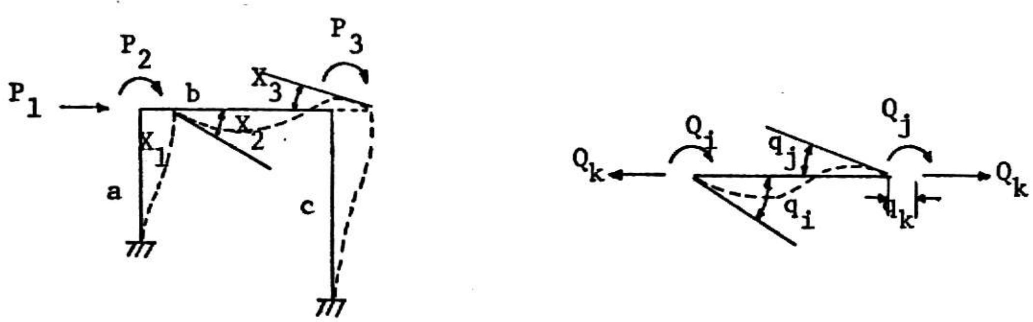
Any complicated structure can be cut into simpler components. For example, a truss may be considered as composed of many two-force members connected at their ends. A rigid frame may be taken as composed of a number of three-force members. The behavior of an individual member, as well as a whole structure, must satisfy the following conditions:

(1) Equilibrium of forces

(2) Compatibility of displacements

(3) Force-displacement relationships specified by the geometric and elastic properties of the elements.

Consider the rigid frame shown in Fig. 3-(a) which is composed of three members subjected to external forces denoted by P_1, P_2, P_3. We neglect the effect of shear deformations and denote the nodal displacements due to these loads by X_1, X_2, X_3, respectively.



(a) The whole structure (b) Member b

Fig. 3. Force-displacement diagram

In Fig. 3-(b) Q_i, Q_j are internal end moments, Q_k is the internal axial force and q_i, q_j, q_k are their corresponding deformations. Taking clockwise and rightward forces as positive, if we use a column matrix P, where

$$P = \begin{Bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_n \end{Bmatrix}$$

to denote all the external forces and a column matrix X, where

$$X = \begin{Bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ X_n \end{Bmatrix}$$

to denote the corresponding nodal displacements, then the external work done can be expressed as

$$W_E = (P)^T(X) = (X)^T(P).$$

Similarly, if we use column matrix Q, where

$$Q = \begin{Bmatrix} Q_i^a \\ Q_j^a \\ \cdot \\ \cdot \\ \cdot \\ Q_i^b \\ Q_j^b \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}$$

to denote all the internal end forces of members a, b, c ... etc., and a column matrix q, where

$$q = \begin{Bmatrix} q_i^a \\ q_j^a \\ \cdot \\ \cdot \\ \cdot \\ q_i^b \\ q_j^b \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix}$$

to denote the corresponding internal displacements, then the internal work done can be expressed as

$$W_I = Q^T q = q^T Q,$$

According to the principle of virtual work " If a system in equilibrium under the action of a set of external forces is given a small virtual displacement compatible with the constraint imposed on the system, then the work done by the external forces equals the increase in strain energy stored in the system (3)." Equating W_E and W_I, we have

$$P^T X = Q^T q$$

or

$$X^T P = q^T Q.$$

In the displacement method, several basic matrices should be established.

(A) Deformation matrix a

The compatibility of displacements is that the member deformations q should be consistently related to the nodal displacement X. Let a_{ij} represent the value of the member deformation q_i due to a nodal displacement X_j. The total value of the member deformation caused by all the nodal displacements can be written as

$$q_1 = a_{11}X_1 + a_{12}X_2 + \dots\dots + a_{1n}X_n$$

$$q_2 = a_{21}X_1 + a_{22}X_2 + \dots\dots + a_{2n}X_n$$

$$\vdots$$

$$q_m = a_{m1}X_1 + a_{m2}X_2 + \dots\dots + a_{mn}X_n$$

where $q_1, q_2, \dots . q_n$ represent the total value of the member deformations and $X_1, X_2, \dots . X_n$ are the nodal displacements.

In matrix form

$$\begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{Bmatrix}$$

where

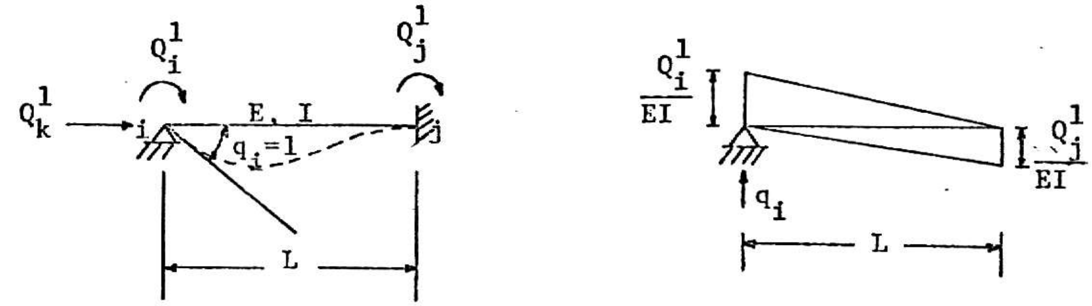
$$a = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

is called the deformation matrix.

(B) Diagonal stiffness matrix S

The stiffness coefficient S_{ij} is defined as the force developed at point i due to a unit displacement at joint j, while all other points are fixed. From this definition we can write the stiffness matrix S^a for an individual member a. Then applying the principle of superposition, we can determine the diagonal stiffness matrix S.

Consider a prismatic member with length L, cross-section area A, moment of inertia I, and modulus of elasticity E in Fig. 4-(a).



(a) Prismatic beam (b) Conjugate beam

Fig. 4. End forces by the conjugate beam method

By definition, the forces caused by a unit rotation q_i at joint i are (Fig. (b).)

$$\Sigma M_i=0, \quad \left(\frac{Q_i^1 L}{2EI}\right)\left(\frac{L}{3}\right) - \left(\frac{Q_j^1 L}{2EI}\right)\left(\frac{2L}{3}\right) = 0$$

$$\Sigma M_j=0, \quad (q_i L) - \left(\frac{Q_i^1 L}{2EI}\right)\left(\frac{2L}{3}\right) + \left(\frac{Q_j^1 L}{2EI}\right)\left(\frac{L}{3}\right) = 0.$$

Solving the above two equations and substituting $q_i=1$, gives

$$Q_i^1 = \frac{4EI}{L}$$

$$Q_j^1 = \frac{2EI}{L}$$

$$Q_k^1 = 0 ,$$

Similarly, if a unit rotation q_j occurs at joint j we get

$$Q_i^2 = \frac{2EI}{L}$$

$$Q_j^2 = \frac{4EI}{L}$$

$$Q_k^2 = 0.$$

Next, consider a unit axial displacement $q_k=1$ as shown in Fig. 5.

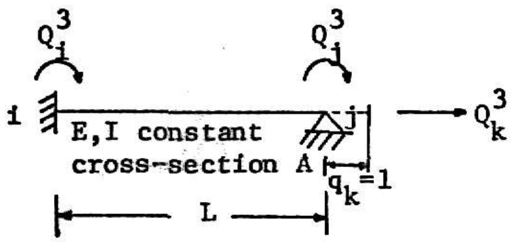


Fig. 5. End forces due to a unit axial displacement

According to the properties of elastic deformation

$$q_k = \frac{Q_k^3 \, L}{AE} .$$

Substitute $q_k=1$, which gives

$$Q_k^3 = \frac{AE}{L}$$

and

$$Q_i^3 = Q_j^3 = 0.$$

The end forces due to unit axial displacement and end rotations are shown in Fig. 6.

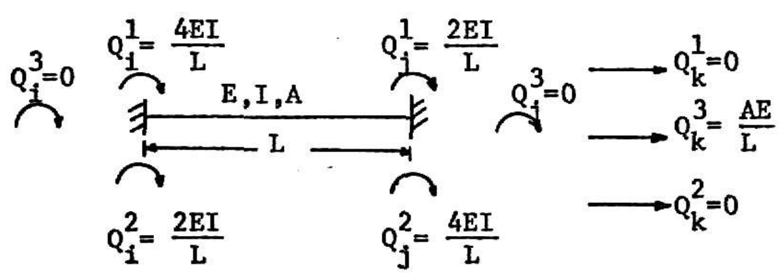


Fig. 6. End forces due to a unit angular displacement

Combining the corresponding end forces in terms of end displacements, we have

$$Q_i = \frac{4EI}{L} q_i + \frac{2EI}{L} q_j + (0) q_k$$

$$Q_j = \frac{2EI}{L} q_i + \frac{4EI}{L} q_j + (0) q_k$$

$$Q_k = (0)\ q_i + (0)\ q_j + \frac{AE}{L} q_k\ .$$

In matrix form

$$Q^a = [S^a]\ q^a$$

in which

$$S^a = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{AE}{L} \end{bmatrix}$$

is defined as the stiffness matrix for an individual member a.

For the entire assemblage of members a, b, c, ... etc., we have

$$\begin{aligned} Q^a &= [\,S^a\,]\ q^a \\ Q^b &= [\,S^b\,]\ q^b \\ &\ \ \vdots \\ Q^n &= [\,S^n\,]\ q^n \end{aligned}$$

The above equations may be expressed as

$$Q = [\,S\,]\ q$$

where

$$Q = \begin{Bmatrix} Q^a \\ Q^b \\ \vdots \\ Q^n \end{Bmatrix}, \qquad S = \begin{bmatrix} S^a & & & \\ & S^b & & \\ & & \ddots & \\ & & & S^n \end{bmatrix}, \qquad q = \begin{Bmatrix} q^a \\ q^b \\ \vdots \\ q^n \end{Bmatrix},$$

where S is the banded stiffness matrix.

Note that in the rigid frame, since the effect of axial forces is disregarded, the stiffness matrix for an individual member can be expressed as

$$S^a = \begin{pmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{pmatrix}.$$

Using this form can simplify the computation of the structure.

(C) Total stiffness matrix K

The following relationships can be obtained from the previous discussion:

$$X^T P = q^T Q \quad \ldots\ldots\ldots\ldots\ldots \text{(a)}$$

$$q = [\ a\ ]\ X \quad \ldots\ldots\ldots\ldots\ldots \text{(b)}$$

and

$$Q = [\ S\ ]\ q \quad \ldots\ldots\ldots\ldots\ldots \text{(c)}$$

Substituting Eq.(b) in Eq.(c), gives

$$Q = [\ S\ ][\ a\ ]\ X \quad \ldots\ldots\ldots \text{(d)}$$

From Eq.(b), we obtain

$$q^T = ([\ a\ ]\ X\ )^T = X^T[\ a\ ]^T \ldots \text{(e)}$$

Substituting Eqs.(d) and (e) in Eq.(a) gives

$$X^T P = X^T[\ a\ ]^T[\ S\ ][\ a\ ]\ X$$

from which

$$P = [\ a\ ]^T[\ S\ ][\ a\ ]\ X\ .$$

Let

$$P = [\ K\ ]\ X \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(f)}$$

where

$$[\ K\ ] = [\ a\ ]^T[\ S\ ][\ a\ ]$$

is called the total stiffness matrix.

(D) The displacement matrix X

From part (C)-(f), we get

$$P = [K]\ X$$

$$[K]^{-1}\ P = [K]^{-1}[K]\ X$$

$$X = [K]^{-1}\ P\ .$$

(E) The internal forces matrix Q

From part (C)-(d), we get

$$Q = [S][a]\ X\ .$$

THEORY OF MATRIX METHOD OF ANALYSIS

A two-story tower frame is shown in Fig. 7. The procedure in solving this problem is stated in the following steps.

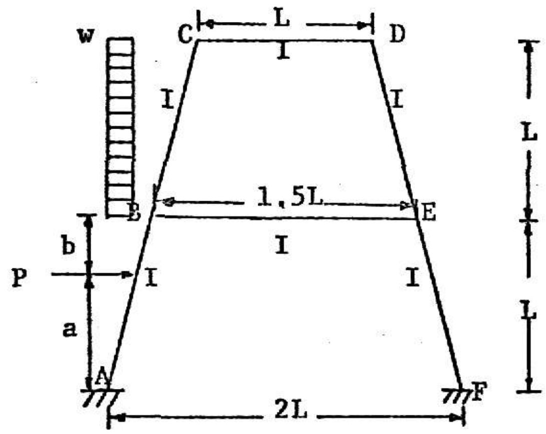


Fig. 7. A two-story tower frame with sidesway

Step 1. Determine the degrees of freedom

(a) The degrees of freedom in sidesway

The degrees of freedom in sidesway of a rigid frame are equal to the number of unknown linear displacements. Let

j= number of joints including supports

f= number of fixed supports

h= number of hinged supports

r= number of roller supports

m= number of members

s= degrees of freedom in sidesway

then

$$s = 2j - (2f + 2h + r + m) .$$

Two conditions are needed to determine the position of each joint. Every fixed or hinged support furnishs two conditions, each roller provides one condition and each connecting member gives one condition. Therefore, the total number of available conditions is $2f + 2h + r + m$. The number of sidesway degrees is the difference between required and available conditions.

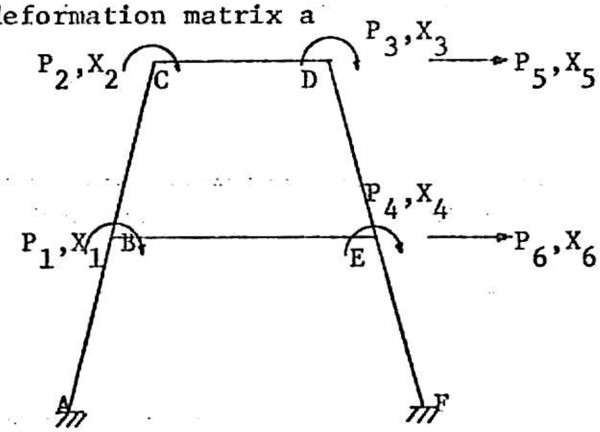
(b) The degrees of freedom in rotation

At each free joint there is a rotation, therefore, the degrees of freedom in rotation equal the number of free joints. Thus for this two-story tower frame, the degrees of freedom in rotation are 4 and

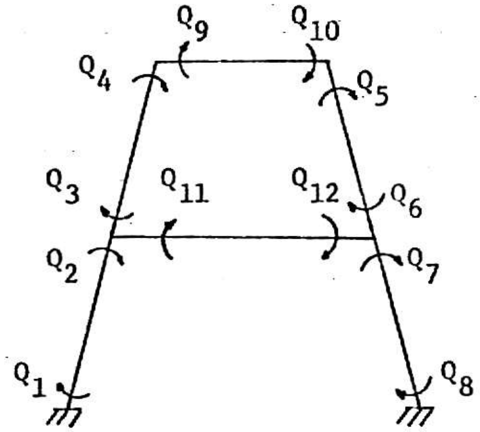
$$s = 2(6) - [\ 2(2) + 0 + 0 + 6\ ] = 2\ .$$

The total degrees of freedom are $2 + 4 = 6$.

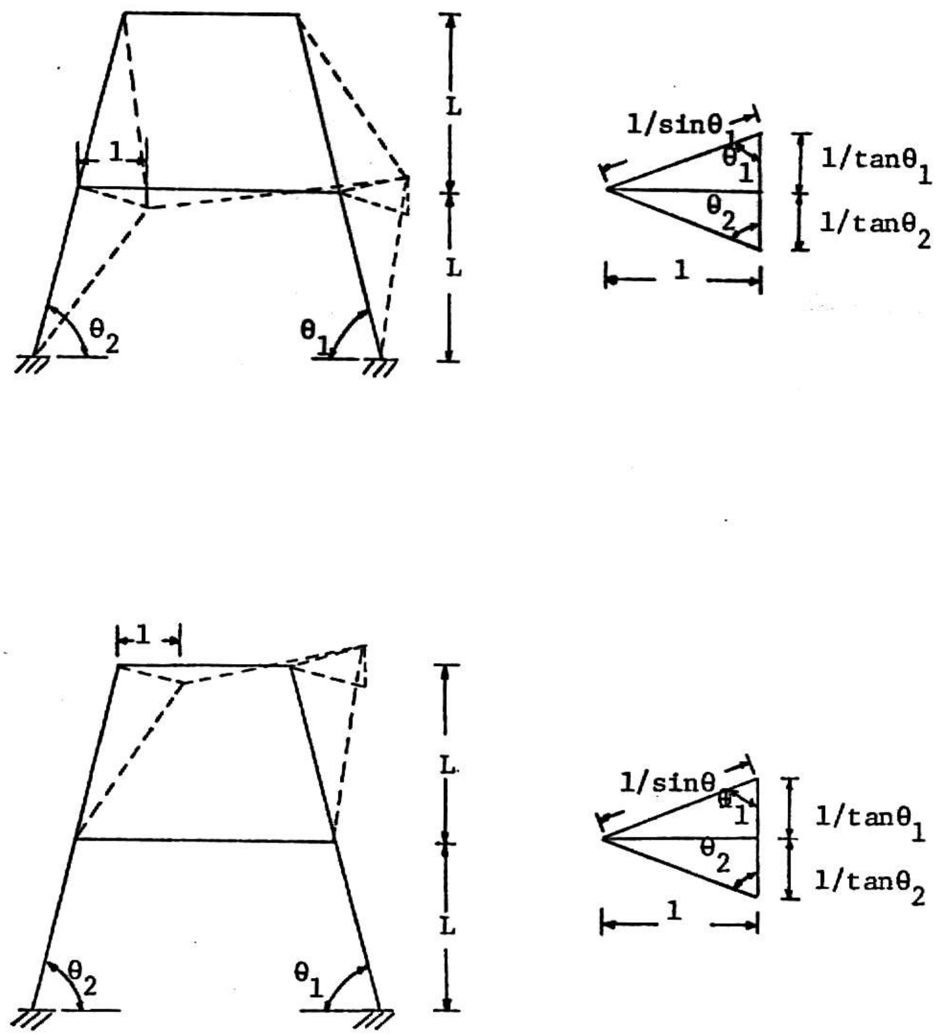
Step 2. Determine the deformation matrix a



(a) The applied loads and displacements diagram (P-X)



(b) The internal forces and displacements diagram (Q-q)



(c) The joint-displacement diagram

Fig. 8. Unit displacements at free joints

The deformation matrix is shown in Table 1. Note that clockwise and rightward directions are taken as positive.

Table 1. The deformation matrix a

q \ X	1	2	3	4	5	6
1	0	0	0	0	0	$-\frac{1}{L}$
2	1	0	0	0	0	$-\frac{1}{L}$
3	1	0	0	0	$-\frac{1}{L}$	$\frac{1}{L}$
4	0	1	0	0	$-\frac{1}{L}$	$\frac{1}{L}$
5	0	0	1	0	$-\frac{1}{L}$	$\frac{1}{L}$
6	0	0	0	1	$-\frac{1}{L}$	$\frac{1}{L}$
7	0	0	0	1	0	$-\frac{1}{L}$
8	0	0	0	0	0	$-\frac{1}{L}$
9	0	1	0	0	$\frac{1}{2L}$	0
10	0	0	1	0	$\frac{1}{2L}$	0
11	1	0	0	0	0	$\frac{1}{2L}$
12	0	0	0	1	0	$\frac{1}{2L}$

<u>Step 3.</u> Determine the diagonal stiffness matrix S

For every individual member we have

$$[S^{a-b}] = \begin{pmatrix} \frac{4EI}{1.031L} & \frac{2EI}{1.031L} \\ \\ \frac{2EI}{1.031L} & \frac{4EI}{1.031L} \end{pmatrix}, \qquad [S^{b-c}] = \begin{pmatrix} \frac{4EI}{1.031L} & \frac{2EI}{1.031L} \\ \\ \frac{2EI}{1.031L} & \frac{4EI}{1.031L} \end{pmatrix}$$

$$[S^{c-d}] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}, \qquad [S^{d-e}] = \begin{bmatrix} \frac{4EI}{1.031L} & \frac{2EI}{1.031L} \\ \frac{2EI}{1.031L} & \frac{4EI}{1.031L} \end{bmatrix}$$

$$[S^{e-f}] = \begin{bmatrix} \frac{4EI}{1.031L} & \frac{2EI}{1.031L} \\ \frac{2EI}{1.031L} & \frac{4EI}{1.031L} \end{bmatrix}, \qquad [S^{b-e}] = \begin{bmatrix} \frac{4EI}{1.5L} & \frac{2EI}{1.5L} \\ \frac{2EI}{1.5L} & \frac{4EI}{1.5L} \end{bmatrix}$$

Thus, the banded stiffness matrix S is

$$[\ S\ ] = \begin{bmatrix} S^{a-b} & & & & & \\ & S^{b-c} & & & & \\ & & S^{d-e} & & & \\ & & & S^{e-f} & & \\ & & & & S^{c-d} & \\ & & & & & S^{b-e} \end{bmatrix}$$

Step 4. Determine the total stiffness matrix K

Use the formula

$$[\ K\ ] = [\ a\ ]^T[\ S\ ][\ a\ ]$$

Therefore,

$$[K] = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} & 0 & 0 & \frac{1}{2L} & \frac{1}{2L} & 0 & 0 \\ -\frac{1}{L} & -\frac{1}{L} & \frac{1}{L} & \frac{1}{L} & \frac{1}{L} & \frac{1}{L} & -\frac{1}{L} & -\frac{1}{L} & 0 & 0 & \frac{1}{2L} & \frac{1}{2L} \end{bmatrix}$$

$$\frac{EI}{L}\begin{bmatrix} 3.88 & 1.94 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.94 & 3.88 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.88 & 1.94 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.94 & 3.88 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.88 & 1.94 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.94 & 3.88 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3.88 & 1.94 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.94 & 3.88 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.67 & 1.33 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.33 & 2.67 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 1 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 1 & 0 & 0 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 1 & 0 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 1 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 & 1 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{L} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 0 & 1 & 0 & 0 & \frac{1}{2L} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2L} & 0 \\ 1 & 0 & 0 & 0 & 0 & \frac{1}{2L} \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{2L} \end{bmatrix}$$

$$[K] = \frac{EI}{L}\begin{bmatrix} 10.43 & 1.94 & 0 & 1.33 & -\frac{5.82}{L} & \frac{2}{L} \\ 1.94 & 7.88 & 2 & 0 & -\frac{2.82}{L} & \frac{5.82}{L} \\ 0 & 2 & 7.88 & 1.94 & -\frac{2.82}{L} & \frac{5.82}{L} \\ 1.33 & 0 & 1.94 & 3.15 & -\frac{5.82}{L} & \frac{2}{L} \\ -\frac{5.82}{L} & -\frac{2.82}{L} & -\frac{2.82}{L} & -\frac{5.82}{L} & \frac{24.78}{L^2} & -\frac{23.28}{L^2} \\ \frac{2}{L} & \frac{5.82}{L} & \frac{5.82}{L} & \frac{2}{L} & -\frac{23.28}{L^2} & \frac{48.56}{L^2} \end{bmatrix}$$

We can see that matrix K is a symmetrical matrix. This can give us a check on the stiffness matrix K.

Step 5. Determine the applied forces matrix P

The applied forces acting on the structure in Fig. 7. can be converted as shown in Fig. 9.

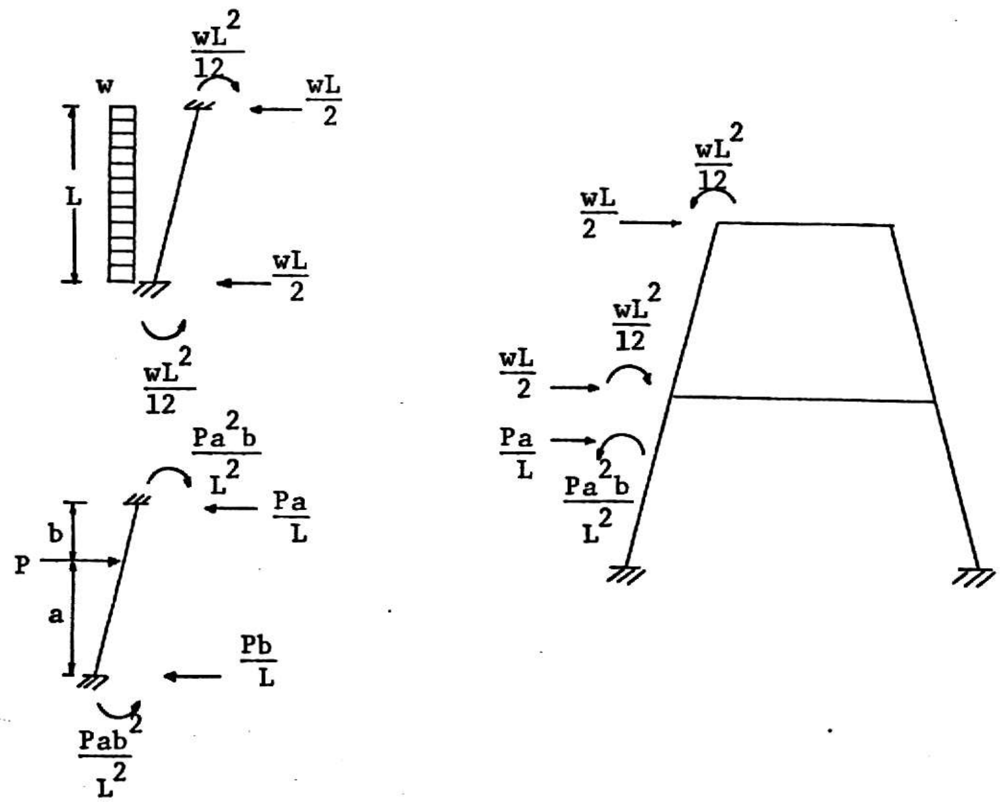


Fig. 9. Conversion of the applied loads to nodal loads

From Fig. 9., we can write the external loads matrix. Refer to Fig. 8.-(a), thus

$$P = \begin{Bmatrix} P_1 = \dfrac{wL^2}{12} - \dfrac{Pa^2b}{L^2} \\ P_2 = -\dfrac{wL^2}{12} \\ P_3 = 0 \\ P_4 = 0 \\ P_5 = \dfrac{wL}{2} \\ P_6 = \dfrac{wL}{2} + \dfrac{Pa}{L} \end{Bmatrix}$$

Note again that clockwise and rightward directions are taken as positive.

Step 6. Calculate the displacement matrix X

Use the formula

$$X = [\ K\ ]^{-1}\ P$$

Therefore,

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{Bmatrix} = [K]^{-1} \begin{Bmatrix} \dfrac{wL^2}{12} - \dfrac{Pa^2b}{L^2} \\ -\dfrac{wL^2}{12} \\ 0 \\ 0 \\ \dfrac{wL}{2} \\ \dfrac{wL}{2} + \dfrac{Pa}{L} \end{Bmatrix}$$

We can write a computer program to compute K^{-1} and then get displacements.

Step 7. Calculate the internal forces matrix Q

Use the formula

$$Q = [\,S\,][\,a\,]\,X\,,$$

Therefore,

$$Q_1 = \begin{bmatrix} S^{a-b} & & & & & \\ & S^{b-c} & & & & \\ & & S^{d-e} & & & \\ & & & S^{e-f} & & \\ & & & & S^{c-d} & \\ & & & & & S^{b-e} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 1 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 1 & 0 & 0 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 1 & 0 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 1 & 0 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 & 1 & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{L} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{L} \\ 0 & 1 & 0 & 0 & \frac{1}{2L} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2L} & 0 \\ 1 & 0 & 0 & 0 & 0 & \frac{1}{2L} \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{2L} \end{bmatrix}$$

$$[K]^{-1} \begin{Bmatrix} \frac{wL^2}{12} - \frac{Pa^2b}{L^2} \\ -\frac{wL^2}{12} \\ 0 \\ 0 \\ \frac{wL}{2} \\ \frac{wL}{2} + \frac{Pa}{L} \end{Bmatrix}$$

From Fig. 9. we know the fixed-end moments are

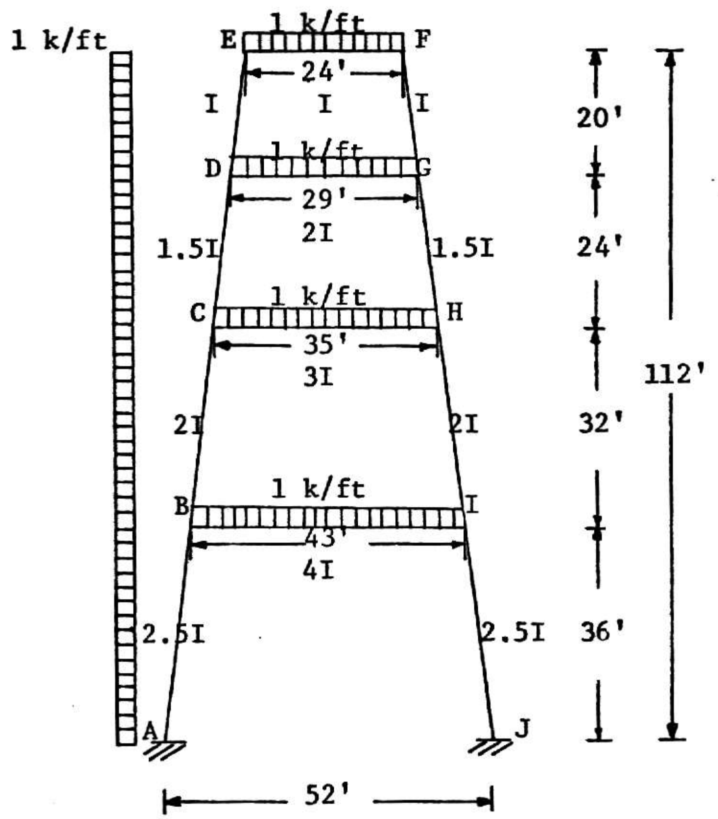
$$Q_0 = \begin{Bmatrix} Q_1^0 = -\frac{Pab^2}{L^2} \\ Q_2^0 = \frac{Pa^2b}{L^2} \\ Q_3^0 = -\frac{wL^2}{12} \\ Q_4^0 = \frac{wL^2}{12} \\ Q_5^0 = 0 \\ Q_6^0 = 0 \\ Q_7^0 = 0 \\ Q_8^0 = 0 \\ Q_9^0 = 0 \\ Q_{10}^0 = 0 \\ Q_{11}^0 = 0 \\ Q_{12}^0 = 0 \end{Bmatrix}$$

The total internal forces matrix Q is

$$Q = Q_1 + Q_0$$

APPLICATION OF MATRIX METHOD OF ANALYSIS TO A MULTIPLE HIGH TOWER FRAME WITH SIDESWAY

Given a four-story tower frame under the loads as shown in Fig. 10. Find the moments and displacements at each joint.



E=30000000 psi

I=2000 in^4

Fig. 10. Four-story tower frame

Solution:

Step 1. Determine the degrees of freedom

(a) The degrees of freedom in sidesway

$$s = 2j - (2f + 2h + r + m)$$

$$= 2(10) - [2(2) + 2(0) + 0 + 12]$$

$$= 20 - 16$$

$$= 4 .$$

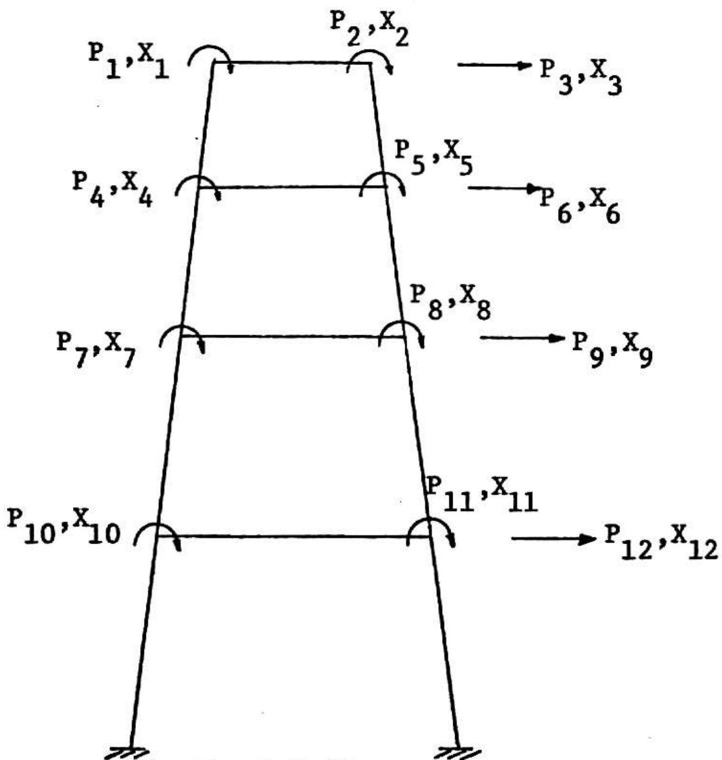
(b) The degrees of freedom in rotation

Each free joint has a rotation, therefore, the degrees of freedom in rotation are 8.

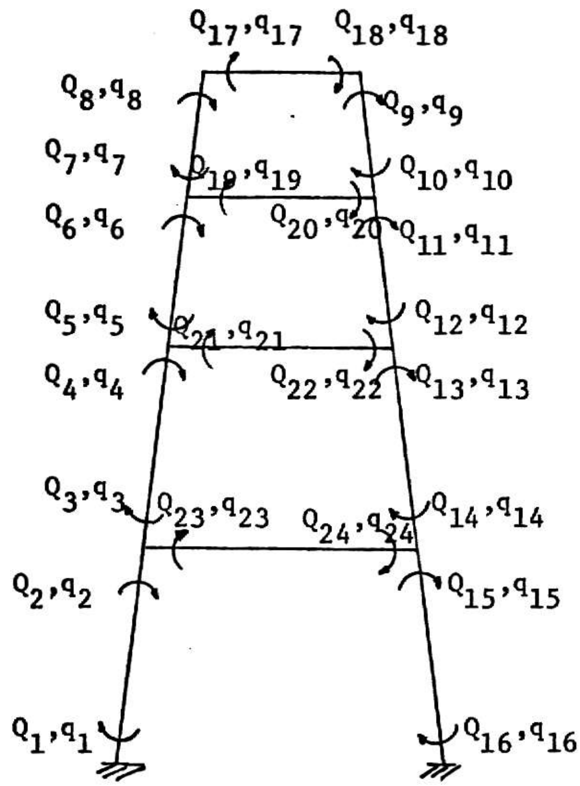
The total degrees of freedom of this four-story tower frame are

$$4 + 8 = 12 .$$

Step 2. Determine the deformation matrix a



(a) The P-X diagram



(b) The Q-q diagram

Fig. 11. Forces and displacements diagram

The applied loads and their corresponding displacements diagram are shown in Fig. 11.-(a). The internal end moments and their corresponding displacement diagrams are shown in Fig. 11.-(b).

To set up the deformation matrix a, we first consider that a unit rotation occurs at joint E as shown in Fig. 12. The numbers in the circle indicate the order of the members.

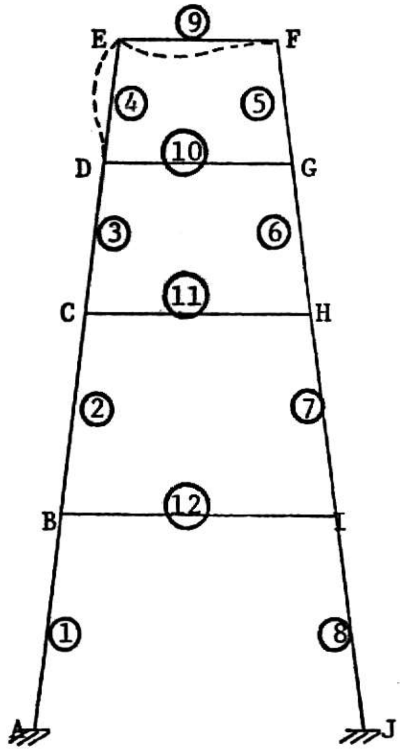


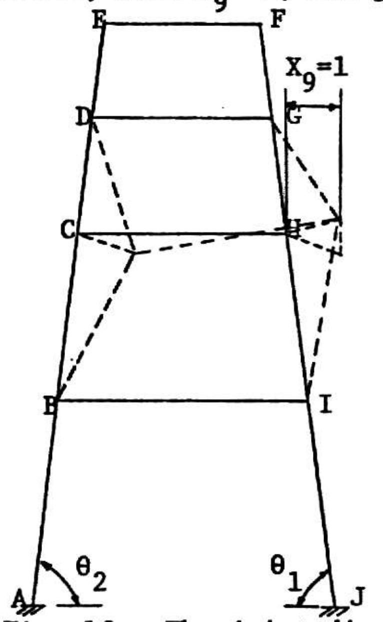
Fig. 12. A unit rotation occurs at joint E
(Number in the circle is the order of the members)

When $X_1=1$, $X_2=X_3=X_4=X_5=X_6=X_7=X_8=X_9=X_{10}=X_{11}=X_{12}=0$,

the deformation matrix is

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

In the same manner, we can obtain the deformation matrix when $X_2=1$, $X_3=1$, ... $X_{12}=1$. Next, in order to determine the deformation matrix due to a unit sidesway displacement, we must draw a joint-displacement diagram. For instance, when $X_9=1$, the joint-displacement diagram is shown in Fig. 13.



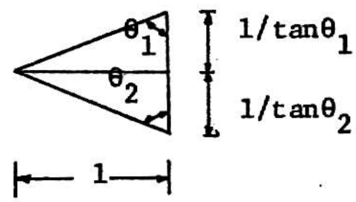


Fig. 13. The joint-displacement diagram when $X_9=1$.

Therefore, the deformation matrix is

$$\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{32} \\ -\frac{1}{32} \\ \frac{1}{24} \\ \frac{1}{24} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left\{ \begin{array}{c} \frac{1}{24} \\ \frac{1}{24} \\ -\frac{1}{32} \\ -\frac{1}{32} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{140} \\ \frac{1}{140} \\ 0 \\ 0 \end{array} \right\}$$

The computed deformation matrix a is shown in Table 2.

Table 2. The deformation matrix a for multiple high tower frame

X / q	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{36}$
2	0	0	0	0	0	0	0	0	0	1	0	$-\frac{1}{36}$
3	0	0	0	0	0	0	0	0	$-\frac{1}{32}$	1	0	$\frac{1}{32}$
4	0	0	0	0	0	0	1	0	$-\frac{1}{32}$	0	0	$\frac{1}{32}$
5	0	0	0	0	0	$-\frac{1}{24}$	1	0	$\frac{1}{24}$	0	0	0
6	0	0	0	1	0	$-\frac{1}{24}$	0	0	$\frac{1}{24}$	0	0	0

Table 2. (continued)

7	0	0	$-\frac{1}{20}$	1	0	$\frac{1}{20}$	0	0	0	0	0	0
8	1	0	$-\frac{1}{20}$	0	0	$\frac{1}{20}$	0	0	0	0	0	0
9	0	1	$-\frac{1}{20}$	0	0	$\frac{1}{20}$	0	0	0	0	0	0
10	0	0	$-\frac{1}{20}$	0	1	$\frac{1}{20}$	0	0	0	0	0	0
11	0	0	0	0	1	$-\frac{1}{24}$	0	0	$\frac{1}{24}$	0	0	0
12	0	0	0	0	0	$-\frac{1}{24}$	0	1	$\frac{1}{24}$	0	0	0
13	0	0	0	0	0	0	0	1	$-\frac{1}{32}$	0	0	$\frac{1}{32}$
14	0	0	0	0	0	0	0	0	$-\frac{1}{32}$	0	1	$\frac{1}{32}$
15	0	0	0	0	0	0	0	0	0	0	1	$-\frac{1}{36}$
16	0	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{36}$
17	1	0	$\frac{1}{96}$	0	0	0	0	0	0	0	0	0
18	0	1	$\frac{1}{96}$	0	0	0	0	0	0	0	0	0
19	0	0	0	1	0	$\frac{1}{116}$	0	0	0	0	0	0
20	0	0	0	0	1	$\frac{1}{116}$	0	0	0	0	0	0
21	0	0	0	0	0	0	1	0	$\frac{1}{140}$	0	0	0
22	0	0	0	0	0	0	0	1	$\frac{1}{140}$	0	0	0
23	0	0	0	0	0	0	0	0	0	1	0	$\frac{1}{172}$
24	0	0	0	0	0	0	0	0	0	0	1	$\frac{1}{172}$

Step 3. Determine the diagonal stiffness matrix S

For members AB and IJ

$$\frac{4EI}{L} = \frac{4E(2.5I)}{36.28} = \frac{4(30000)(2.5)(2000)}{(36.28)(144)} = 114847.482^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(2.5)(2000)}{(36.28)(144)} = 57423.741^{\text{kips-ft}}$$

$$[S^{a-b}] = [S^{i-j}] = \begin{bmatrix} 114847.482 & 57423.741 \\ 57423.741 & 114847.482 \end{bmatrix}$$

For members BC and HI

$$\frac{4EI}{L} = \frac{4(30000)(2)(2000)}{(32.249)(144)} = 103362.378^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(2)(2000)}{(32.249)(144)} = 51681.189^{\text{kips-ft}}$$

$$[S^{b-c}] = [S^{h-i}] = \begin{bmatrix} 103362.378 & 51681.189 \\ 51681.189 & 103362.378 \end{bmatrix}$$

For members CD and GH

$$\frac{4EI}{L} = \frac{4(30000)(1.5)(2000)}{(24.187)(144)} = 103361.31^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(1.5)(2000)}{(24.187)(144)} = 51680.655^{\text{kips-ft}}$$

$$[S^{c-d}] = [S^{g-h}] = \begin{bmatrix} 103361.31 & 51680.655 \\ 51680.655 & 103361.31 \end{bmatrix}$$

For members DE and FG

$$\frac{4EI}{L} = \frac{4(30000)(2000)}{(20.1556)(144)} = 82690.005^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(2000)}{(20.1556)(144)} = 41345.003^{\text{kips-ft}}$$

$$[S^{d-e}] = [S^{f-g}] = \begin{bmatrix} 82690.005 & 41345.003 \\ 41345.003 & 82690.005 \end{bmatrix}$$

For member EF

$$\frac{4EI}{L} = \frac{4(30000)(2000)}{(24)(144)} = 69444.44^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(2000)}{(24)(144)} = 34722.22^{\text{kips-ft}}$$

$$[S^{e-f}] = \begin{bmatrix} 69444.44 & 34722.22 \\ 34722.22 & 69444.44 \end{bmatrix}$$

For member DG

$$\frac{4EI}{L} = \frac{4(30000)(2000)2}{(29)(144)} = 114942.529^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(2)(2000)}{(29)(144)} = 57471.264^{\text{kips-ft}}$$

$$[S^{d-g}] = \begin{bmatrix} 114942.529 & 57471.264 \\ 57471.264 & 114942.529 \end{bmatrix}$$

For member CH

$$\frac{4EI}{L} = \frac{4(30000)(3)(2000)}{(35)(144)} = 142857.143^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(3)(2000)}{(35)(144)} = 71428.571^{\text{kips-ft}}$$

$$[S^{c-h}] = \begin{bmatrix} 142857.143 & 71428.571 \\ 71428.571 & 142857.143 \end{bmatrix}$$

For member BI

$$\frac{4EI}{L} = \frac{4(30000)(4)(2000)}{(43)(144)} = 155038.76^{\text{kips-ft}}$$

$$\frac{2EI}{L} = \frac{2(30000)(4)(2000)}{(43)(144)} = 77519.38^{\text{kips-ft}}$$

$$[S^{b-i}] = \begin{bmatrix} 155038.76 & 77519.38 \\ 77519.38 & 155038.76 \end{bmatrix}$$

The banded stiffness matrix S is

$$[S] = \begin{bmatrix} S^{a-b} & & & & & & & & & & & \\ & S^{b-c} & & & & & & & & & & \\ & & S^{c-d} & & & & & & & & & \\ & & & S^{d-e} & & & & & & & & \\ & & & & S^{f-g} & & & & & & & \\ & & & & & S^{g-h} & & & & & & \\ & & & & & & S^{h-i} & & & & & \\ & & & & & & & S^{i-j} & & & & \\ & & & & & & & & S^{e-f} & & & \\ & & & & & & & & & S^{d-g} & & \\ & & & & & & & & & & S^{c-h} & \\ & & & & & & & & & & & S^{b-i} \end{bmatrix}$$

Step 4. Determine the total stiffness matrix K

From the equation $[K] = [a]^T[S][a]$, we can determine the total stiffness matrix K. For matrices having large orders, a computer program can be written to calculate that equation. (see appendix A) The stiffness matrix K is shown in Table 3.

Table 3. The stiffness matrix K of high tower frame (kips-ft)

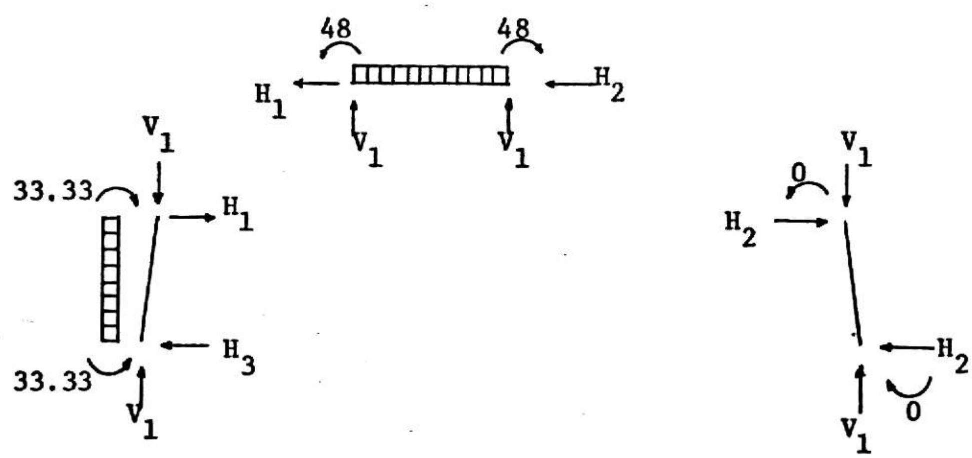
col. / row	1	2	3	4	5	6
1	152134.2	34722.21	-5116.66	41344.91	0	6201.73
2	34722.21	152134.2	-5116.66	0	41344.91	6201.73
3	-5116.66	-5116.66	1262.951	-6201.73	-6201.73	-1240.345
4	41344.91	0	-6201.73	300994.5	57471.25	1227.93
5	0	41344.91	-6201.73	57471.25	300994.5	1227.93
6	6201.73	6201.73	-1240.345	1227.934	1227.928	2342.657
7	0	0	0	51681.13	0	-6460.125
8	0	0	0	0	51681.13	-6460.125
9	0	0	0	6460.136	6460.136	-1076.686
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0

Table 3. (continued)

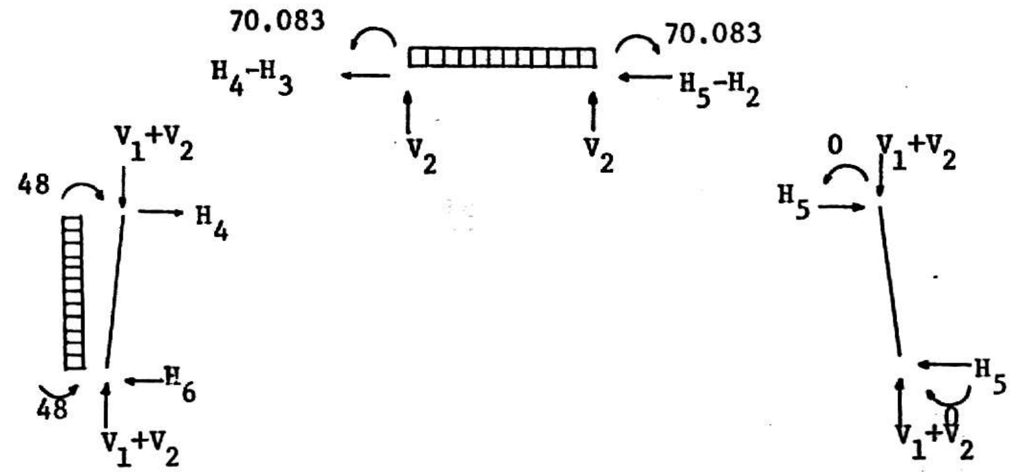
col. / row	7	8	9	10	11	12
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	51681.13	0	6460.136	0	0	0
5	0	51681.13	6460.136	0	0	0
6	-6460.125	-6460.125	-1076.687	0	0	0
7	349581.6	71428.56	3145.643	51681.13	0	4845.105
8	71428 56	349581.6	3145.643	0	51681.13	4845.105
9	3145.645	3145.643	1704.192	-4845.105	-4845.105	-605.6379
10	51681.13	0	-4845.105	373247.8	77519.31	1411.912
11	0	51681.13	-4845.105	77519.31	373247.8	1411.912
12	4845.105	4845.105	-605.6379	1411.912	1411.912	1153.054

Step 5. Determine the applied loads matrix P

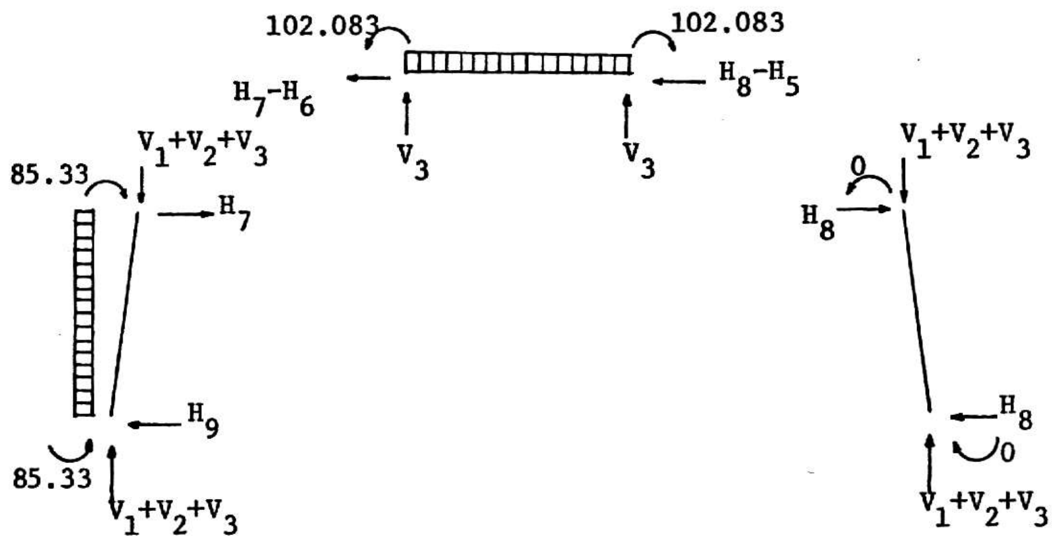
Under the applied loads, the fixed-end moments and sidesway forces are computed from the free body diagrams shown in Fig. 14. (a), (b), (c) and (d).



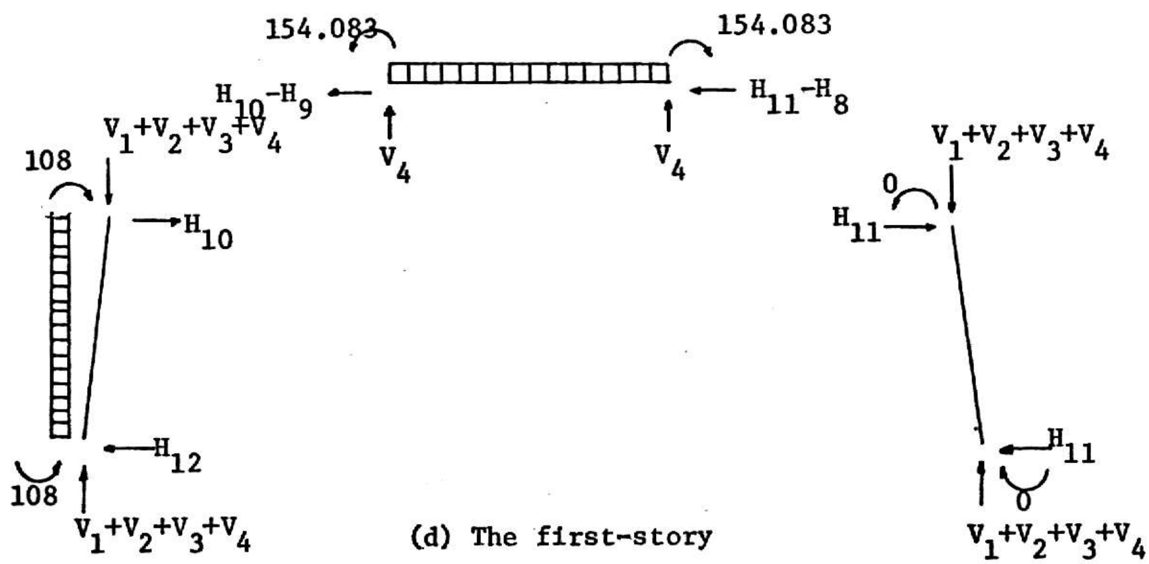
(a) The fourth-story



(b) The third-story



(c) The second-story



(d) The first-story

Fig. 14. Free body diagrams of the tower frame

From Fig. 14. (a)

$\Sigma M_F = 0$, $\quad 24V_1 - 24(12) = 0$,

$V_1 = 12$ kips (up)

$\Sigma M_D = 0$, $\quad 20H_1 + 2.5(12) + 20(10) = 0$,

$H_1 = -11.5$ kips (left)

$20 + H_1 = H_3$,

$H_3 = 8.5$ kips (left)

$\Sigma M_G = 0$, $\quad 12(2.5) - 20H_2 = 0$,

$H_2 = 1.5$ kips (right)

The diagram of applied forces P_1, P_2, P_3 are shown in Fig. 15.

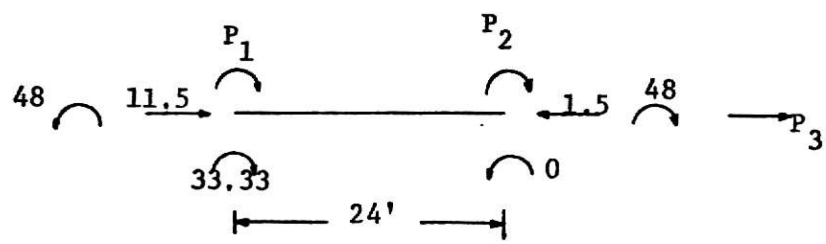


Fig. 15. The diagram of P_1, P_2, P_3

Therefore,

$$P_1 = -(-48 + 33.33) = 14.67 \text{ kips-ft}$$

$$P_2 = -48 \text{ kips-ft}$$

$$P_3 = 11.5 - 1.5 = 10 \text{ kips}$$

From Fig. 14. (b)

$$\Sigma M_G = 0 , \qquad 29V_2 - 29(14.5) = 0 ,$$

$$V_2 = 14.5 \text{ kips} \qquad \text{(up)}$$

$$\Sigma M_C = 0 , \qquad 24H_4 + (12 + 14.5)(3) + 24(12) = 0 ,$$

$$H_4 = -15.3125 \text{ kips} \qquad \text{(left)}$$

$$H_6 = 24 + H_4 ,$$

$$H_6 = 8.6875 \text{ kips} \qquad \text{(left)}$$

$$\Sigma M_H = 0 , \qquad (12 + 14.5)(3) - 24H_5 = 0 ,$$

$$H_5 = 3.3125 \text{ kips} \qquad \text{(right)}$$

The diagram of applied forces P_4, P_5, P_6 are shown in Fig. 16.

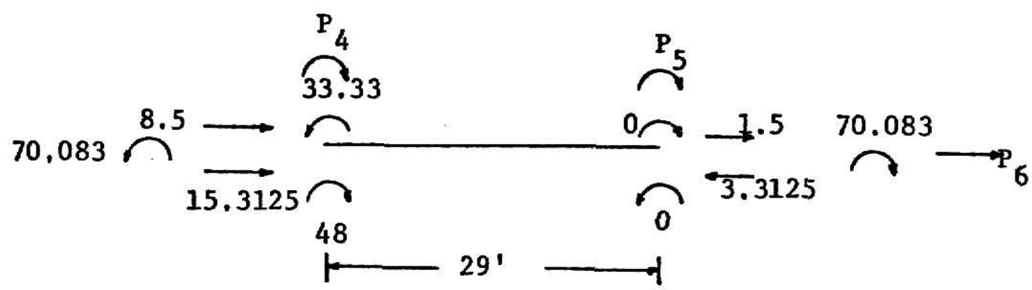


Fig. 16. The diagram of P_4, P_5, P_6

Therefore,

$$P_4 = -(-70.083 - 33.33 + 48) = 55.413 \text{ kips-ft}$$

$$P_5 = -70.083 \text{ kips-ft}$$

$$P_6 = 8.5 + 15.3125 + 1.5 - 3.3125 = 22 \text{ kips}$$

From Fig. 14. (c)

$$\Sigma M_H = 0 , \quad 35V_3 - 35(17.5) = 0 ,$$
$$V_3 = 17.5 \text{ kips} \quad \text{(up)}$$

$$\Sigma M_B = 0 , \quad 32H_7 + (12 + 14.5 + 17.5)(4) + 32(16) = 0 ,$$
$$H_7 = -21.5 \text{ kips} \quad \text{(left)}$$
$$H_9 = 32 + H_7 ,$$
$$H_9 = 10.5 \text{ kips} \quad \text{(left)}$$

$$\Sigma M_I = 0 , \quad 32H_8 - (12 + 14.5 + 17.5)(4) = 0 ,$$
$$H_8 = 5.5 \text{ kips} \quad \text{(right)}$$

The diagram of applied forces P_7, P_8, P_9 are shown in Fig. 17.

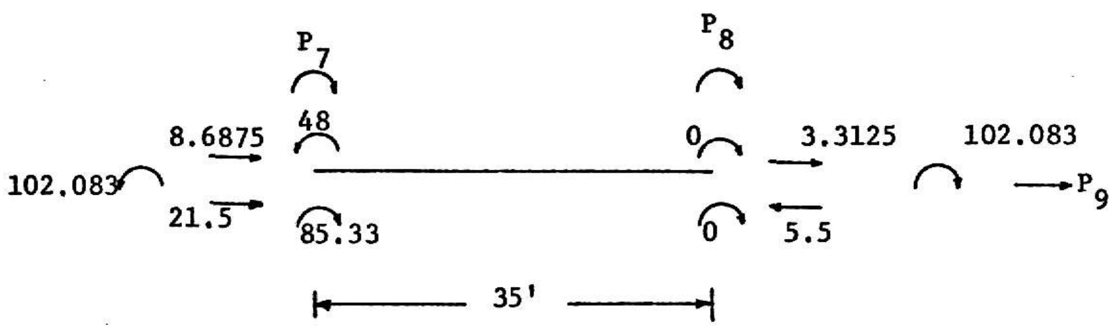


Fig. 17. The diagram of P_7, P_8, P_9

Therefore,

$$P_7 = -(-48 - 102.083 + 85.33) = 64.753 \text{ kips-ft}$$

$$P_8 = -102.083 \text{ kips-ft}$$

$$P_9 = 8.6875 + 21.5 + 3.3125 - 5.5 = 28 \text{ kips}$$

From Fig. 14. (d)

$$\Sigma M_I = 0 , \qquad 43V_4 - 43(21.5) = 0 ,$$

$$V_4 = 21.5 \text{ kips} \qquad \text{(up)}$$

$$\Sigma M_A = 0 , \qquad 36H_{10} + (12 + 14.5 + 17.5 + 21.5)(4.5) + 36(18) = 0 ,$$

$$H_{10} = -26.1875 \text{ kips} \qquad \text{(left)}$$

$$H_{12} = 36 + H_{10} ,$$

$$H_{12} = 9.8125 \text{ kips} \qquad \text{(left)}$$

$$\Sigma M_J = 0 , \qquad 36H_{11} - (12 + 14.5 + 17.5 + 21.5)(4.5) = 0 ,$$

$$H_{11} = 8.1875 \text{ kips} \qquad \text{(right)}$$

The diagram of applied forces are shown in Fig. 18.

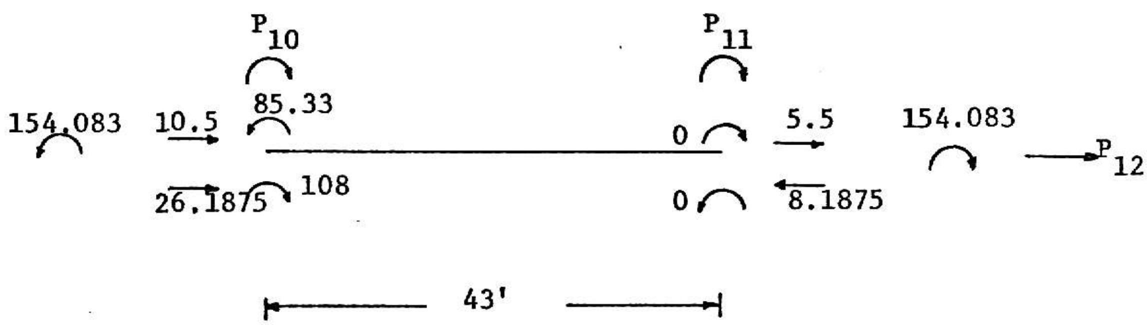


Fig. 18. The diagram of P_{10}, P_{11}, P_{12}

Therefore,

$$P_{10} = -(-85.33 - 154.083 + 108) = 131.413 \text{ kips-ft}$$

$$P_{11} = -154.083 \text{ kips-ft}$$

$$P_{12} = 10.5 + 26.1875 + 5.5 - 8.1875 = 34 \text{ kips}$$

Therefore, the applied loads matrix P is

$$P = \begin{Bmatrix} 14.67 \\ -48 \\ 10 \\ 55.413 \\ -70.083 \\ 22 \\ 64.753 \\ -102.083 \\ 28 \\ 131.413 \\ -154.083 \\ 34 \end{Bmatrix}$$

Step 6. Calculate the displacement matrix X

We can obtain the displacement matrix X by using the formula $X = [K]^{-1} P$. To find the inverse of a large order matrix is laborious, therefore, a computer program can be written and let the machine compute the inversion(see appendix A). The inverse of the matrix K is shown in Table 4.

Table 4. The inverse of the stiffness matrix K

col. / row	1	2	3	4	5	6
1	0.000008	-0.000001	-0.000007	-0.000001	0.000001	-0.00004
2	-0.000001	0.000008	-0.000007	0.000001	-0.000001	-0.00004
3	-0.000007	-0.000007	0.00515	0.000013	0.000013	0.00436
4	-0.000001	0.000001	0.000013	0.000004	0	-0.00001
5	0.000001	-0 000001	0.000013	0	0.000004	-0.00001
6	-0.00004	-0.00004	0.00436	-0.00001	-0.00001	0.00487
7	0	0	0.000021	0	0	0.000022
8	0	0	0.000021	0	0	0.000022
9	-0.000033	-0.000033	0.003262	-0.00003	-0.00003	0.003892
10	0	0	0.000028	0	0	0.000034
11	0	0	0.000028	0	0	0.000034
12	-0.000015	-0.000015	0.001472	-0.000015	-0.000015	0.001774

Table 4. (continued)

col. / row	7	8	9	10	11	12
1	0	0	-0.000033	0	0	-0.000015
2	0	0	-0.000033	0	0	-0.000015
3	0.000021	0.000021	0.003262	0.000028	0.000028	0.001472
4	0	0	-0.00003	0	0	-0.000015
5	0	0	-0.00003	0	0	-0.000015
6	0.000022	0.000022	0.003892	0.000034	0.000034	0.001774
7	0.00004	0	0.000003	0	0	-0.000012
8	0	0.000004	0.000003	0	0	-0.000012
9	0.000003	0.000003	0.004225	0.000039	0.000039	0.002101
10	0	0	0.000039	0.000003	0	0.000018
11	0	0	0.000039	0	0.000003	0.000018
12	-0.000012	-0.000012	0.002101	0.000018	0.000018	0.00203

Therefore, the displacement matrix X is

$$X = [K]^{-1} \begin{Bmatrix} 14.67 \\ -48 \\ 10 \\ 55.413 \\ -70.083 \\ 22 \\ 64.753 \\ -102.083 \\ 28 \\ 131.413 \\ -154.083 \\ 34 \end{Bmatrix} = \begin{Bmatrix} -0.00231274 \\ -0.00271632 \\ 0.287427 \\ -0.00127435 \\ -0.00164402 \\ 0.319941 \\ 0.000492024 \\ 0.000128472 \\ 0.308560 \\ 0.00313555 \\ 0.00223369 \\ 0.182375 \end{Bmatrix}$$

That is,

θ_B	$=0.00313555$ rad.	(clockwise)
θ_C	$=0.00223369$ rad.	(clockwise)
θ_D	$=0.000492024$ rad.	(clockwise)
θ_E	$=0.000128472$ rad.	(clockwise)
θ_F	$= -0.00127435$ rad.	(countclockwise)
θ_G	$= -0.00164402$ rad.	(countclockwise)
θ_H	$= -0.00231274$ rad.	(countclockwise)
θ_I	$= -0.00271632$ rad.	(countclockwise)
Δ_{BI}	$=0.182375$ ft.	(rightward)
Δ_{CH}	$=0.308560$ ft.	(rightward)
Δ_{DG}	$=0.319941$ ft.	(rightward)
Δ_{EF}	$=0.287427$ ft.	(rightward)

<u>Step 7.</u> Determine the end moments matrix Q

From Step 5., we know that the fixed-end moments due to the applied loads are

$$
Q_0 = \begin{Bmatrix} M_{FAB} \\ M_{FBA} \\ M_{FBC} \\ M_{FCB} \\ M_{FCD} \\ M_{FDC} \\ M_{FDE} \\ M_{FED} \\ M_{FFG} \\ M_{FGF} \\ M_{FGH} \\ M_{FHG} \\ M_{FHI} \\ M_{FIH} \\ M_{FIJ} \\ M_{FJI} \\ M_{FEF} \\ M_{FFE} \\ M_{FDG} \\ M_{FGD} \\ M_{FCH} \\ M_{FHC} \\ M_{FBI} \\ M_{FIB} \end{Bmatrix} = \begin{Bmatrix} -108 \\ 108 \\ -85.33 \\ 85.33 \\ -48 \\ 48 \\ -33.33 \\ 33.33 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -48 \\ 48 \\ -70.083 \\ 70.083 \\ -102.083 \\ 102.083 \\ -154.083 \\ 154.083 \end{Bmatrix} \text{kips-ft}
$$

We can determine the internal moments by using the formula $Q_1 = [S][a] X$, and the data obtained in Step 2., 3. and 6.

$$Q_1 = \begin{Bmatrix} -692.6591 \\ -512.6052 \\ -261.8525 \\ -398.4729 \\ -88.52416 \\ -179.812 \\ 0.648193 \\ -42.28417 \\ -90.93994 \\ -46.60546 \\ -236.8105 \\ -145.2062 \\ -482.6596 \\ -373.8598 \\ -616.1818 \\ -744.4475 \\ 56.95367 \\ 42.94067 \\ 234.5761 \\ 213.3308 \\ 551.7507 \\ 525.7827 \\ 905.8706 \\ 835.9587 \end{Bmatrix} \text{kips-ft}$$

Note that the final end moment is the sum of fixed-end moment Q_0 and internal moment Q_1. Therefore, the final end moments are

$$Q = Q_0 + Q_1 = \left\{ \begin{array}{r} -800.6591 \\ -404.6052 \\ -347.1823 \\ -313.1428 \\ -136.5241 \\ -131.8120 \\ -32.6818 \\ -8.954178 \\ -90.93994 \\ -46.60546 \\ -236.8105 \\ -145.2062 \\ -482.6596 \\ -373.8598 \\ -616.1818 \\ -744.4475 \\ 8.953674 \\ 90.940670 \\ 164.4931 \\ 283.4135 \\ 449.6677 \\ 627.8654 \\ 751.7875 \\ 990.0415 \end{array} \right\} \text{kips-ft}$$

Table 5. shows a comparison of the results obtained by the displacement method and the slope-deflection method. It can be seen that they

show good agreement.

Table 5. The comparison of the displacement method and the slope-deflection method

	The displacement method	The slope-deflection method
θ_B	0.00313555	0.00309532
θ_C	0.000492024	0.000468611
θ_D	-0.00127435	-0.0011642
θ_E	-0.00231274	-0.002373
θ_F	-0.00271632	-0.0027046
θ_G	-0.00164402	-0.00168819
θ_H	0.000128472	0.000134696
θ_I	0.00223369	0.00218827
Δ_1	0.182375	0.179291
Δ_2	0.30856	0.303584
Δ_3	0.319941	0.314566
Δ_4	0.287427	0.285519
M_{AB}	-800.66	-788.22
M_{BA}	-404.61	-394.48
M_{BC}	-347.18	-343.39
M_{CB}	-313.14	-308.48
M_{CD}	-136.52	-130.68
M_{DC}	-131.81	-120
M_{DE}	-32.68	-37.57
M_{ED}	-8.95	-7.11
M_{FG}	-90.94	-96.38
M_{GF}	-46.61	-38.20
M_{GH}	-236.81	-238.48
M_{HG}	-145.21	-144.27
M_{HI}	-482.66	-475.2
M_{IH}	-373.86	-369.07
M_{IJ}	-616.18	-606.65
M_{JI}	-744.45	-732.31
M_{EF}	8.95	6.74
M_{FE}	90.94	87.6
M_{DG}	164.49	166.63
M_{GD}	283.41	276.68
M_{CH}	449.67	439.2
M_{HC}	627.86	619.47
M_{BI}	751.79	737.86
M_{IB}	990.04	975.72

ANALYSIS OF MULTIPLE HIGH TOWER FRAME WITH SIDESWAY BY THE SLOPE-DEFLECTION METHOD

In order to check the answers obtained by the stiffness method, we also solve the problem by the slope-deflection method and compare the answers obtained by these two different methods.

<u>Sign conventions</u>

Moment —— clockwise is positive.

Rotation —— clockwise is positive.

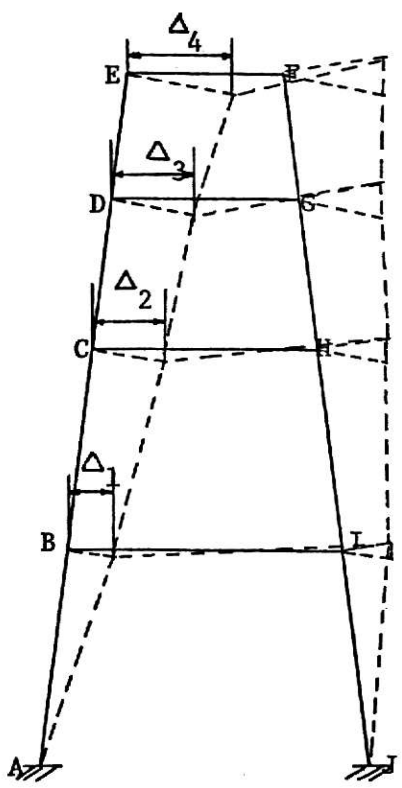


Fig. 19. The sidesway diagram of the tower frame.

The stiffness and angle of rotation are shown in Table 6.

Table 6. The stiffness and angle of rotation

Member	Stiffness (kips-ft)	angle of rotation
AB, IJ	$\frac{30000(2.5)(2000)}{36.28(144)} = 28711.871$	$\frac{\Delta_1}{36}$
BC, HI	$\frac{30000(2)(2000)}{32.249(144)} = 25840.594$	$\frac{\Delta_2 - \Delta_1}{32}$
CD, GH	$\frac{30000(1.5)(2000)}{24.187(144)} = 25840.327$	$\frac{\Delta_3 - \Delta_2}{24}$
DE, FG	$\frac{30000(2000)}{20.1556(144)} = 20672.501$	$\frac{\Delta_4 - \Delta_3}{20}$
EF	$\frac{30000(2000)}{24(144)} = 17361.111$	$-\frac{\Delta_4}{96}$
DG	$\frac{30000(2)(2000)}{29(144)} = 28735.632$	$-\frac{\Delta_3}{116}$
CH	$\frac{30000(3)(2000)}{35(144)} = 35714.286$	$-\frac{\Delta_2}{140}$
BI	$\frac{30000(4)(2000)}{43(144)} = 38759.69$	$-\frac{\Delta_1}{172}$

The fixed-end moments are

$$M_{BA} = -M_{AB} = 108 \text{ kips-ft}$$

$$M_{BC} = -M_{CB} = 85.33 \text{ kips-ft}$$

$$M_{DC} = -M_{CD} = 48 \text{ kips-ft}$$

$$M_{ED} = -M_{DE} = 33.33 \text{ kips-ft}$$

$$M_{FE} = -M_{EF} = 48 \text{ kips-ft}$$

$$M_{GD} = -M_{DG} = 70.083 \text{ kips-ft}$$

$$M_{HC} = -M_{CH} = 102.083 \text{ kips-ft}$$

$$M_{IB} = -M_{BI} = 154.083 \text{ kips-ft}$$

Therefore, the slope-deflection equations are

$$M_{AB} = 2(28711.871)[\theta_B - 3(\frac{\Delta_1}{36})] - 108$$

$$M_{BA} = 2(28711.871)[2\theta_B - 3(\frac{\Delta_1}{36})] + 108$$

$$M_{BC} = 2(25840.594)[2\theta_B + \theta_C - 3(\frac{\Delta_2 - \Delta_1}{32})] - 85.33$$

$$M_{CB} = 2(25840.594)[2\theta_C + \theta_B - 3(\frac{\Delta_2 - \Delta_1}{32})] + 85.33$$

$$M_{CD} = 2(25840.327)[2\theta_C + \theta_D - 3(\frac{\Delta_3 - \Delta_2}{24})] - 48$$

$$M_{DC} = 2(25840.327)[2\theta_D + \theta_C - 3(\frac{\Delta_3 - \Delta_2}{24})] + 48$$

$$M_{DE} = 2(20672.501)[2\theta_D + \theta_E - 3(\frac{\Delta_4 - \Delta_3}{20})] - 33.33$$

$$M_{ED} = 2(20672.501)[2\theta_E + \theta_D - 3(\frac{\Delta_4 - \Delta_3}{20})] + 33.33$$

$$M_{FG} = 2(20672.501)[2\theta_F + \theta_G - 3(\frac{\Delta_4 - \Delta_3}{20})]$$

$$M_{GF} = 2(20672.501)[2\theta_G + \theta_F - 3(\frac{\Delta_4 - \Delta_3}{20})]$$

$$M_{GH} = 2(25840.327)[2\theta_G + \theta_H - 3(\frac{\Delta_3 - \Delta_2}{24})]$$

$$M_{HG} = 2(25840.327)[2\theta_H + \theta_G - 3(\frac{\Delta_3 - \Delta_2}{24})]$$

$$M_{HI} = 2(25840.594)[2\theta_H + \theta_I - 3(\frac{\Delta_2 - \Delta_1}{32})]$$

$$M_{IH} = 2(25840.594)[2\theta_I + \theta_H - 3(\frac{\Delta_2 - \Delta_1}{32})]$$

$$M_{IJ} = 2(28711.871)[2\theta_I - 3(\frac{\Delta_1}{36})]$$

$$M_{JI} = 2(28711.871)[\theta_I - 3(\frac{\Delta_1}{36})]$$

$$M_{EF} = 2(17361.111)[2\theta_E + \theta_F - 3(- \frac{\Delta_4}{96})] - 48$$

$$M_{FE} = 2(17361.111)[2\theta_F + \theta_E - 3(- \frac{\Delta_4}{96})] + 48$$

$$M_{DG} = 2(28735.632)[2\theta_D + \theta_G - 3(- \frac{\Delta_3}{116})] - 70.083$$

$$M_{GD} = 2(28735.632)[2\theta_G + \theta_D - 3(- \frac{\Delta_3}{116})] + 70.083$$

$$M_{CH} = 2(35714.286)[2\theta_C + \theta_H - 3(- \frac{\Delta_2}{140})] - 102.083$$

$$M_{HC} = 2(35714.286)[2\theta_H + \theta_C - 3(- \frac{\Delta_2}{140})] + 102.083$$

$$M_{BI} = 2(38759.69)[2\theta_B + \theta_I - 3(- \frac{\Delta_1}{172})] - 154.083$$

$$M_{IB} = 2(38759.69)[2\theta_I + \theta_B - 3(- \frac{\Delta_1}{172})] + 154.083$$

From joint condictions, we get

$$\Sigma M_B = 0\ ,\quad 373248.62\theta_B + 51681.188\theta_C + 77519.38\theta_I + 1411.882\Delta_1 - 4845.111\Delta_2 - 131.413 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

$$\Sigma M_C = 0\ ,\quad 51681.188\theta_B + 349580.828\theta_C + 51680.654\theta_D + 71428.572\theta_H + 4845.111\Delta_1 + 3145.583\Delta_2 - 6460.082\Delta_3 - 64.753 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (2)$$

$$\Sigma M_D = 0\ ,\quad 51680.654\theta_C + 300993.84\theta_D + 41345.002\theta_E + 57471.264\theta_G + 6460.082\Delta_2 + 1227.994\Delta_3 - 6201.75\Delta_4 - 55.413 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (3)$$

$$\Sigma M_E = 0\ ,\quad 41345.002\theta_D + 152134.448\theta_E + 34722.222\theta_F + 6201.75\Delta_3 - 5116.681\Delta_4 - 14.67 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (4)$$

$$\Sigma M_F = 0\ ,\quad 34722.222\theta_E + 152134.448\theta_F + 41345.002\theta_G + 6201.75\Delta_3 - 5116.681\Delta_4 + 48 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (5)$$

$$\Sigma M_G = 0\ ,\quad 57471.264\theta_D + 41345.002\theta_F + 300993.84\theta_G + 51680.654\theta_H + 6460.082\Delta_2 + 1227.994\Delta_3 - 6201.75\Delta_4 + 70.083 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (6)$$

$$\Sigma M_H = 0\ ,\quad 71428.572\theta_C + 51680.654\theta_G + 349580.828\theta_H + 51681.188\theta_I + 4845.111\Delta_1 + 3145.583\Delta_2 - 6460.082\Delta_3 + 102.083 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (7)$$

$$\Sigma M_I = 0\ ,\quad 77519.38\theta_B + 51681.188\theta_H + 373248.62\theta_I - 3373.431\Delta_1 - 4845.111\Delta_2 + 154.083 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (8)$$

Now we have to find out the shear equations,

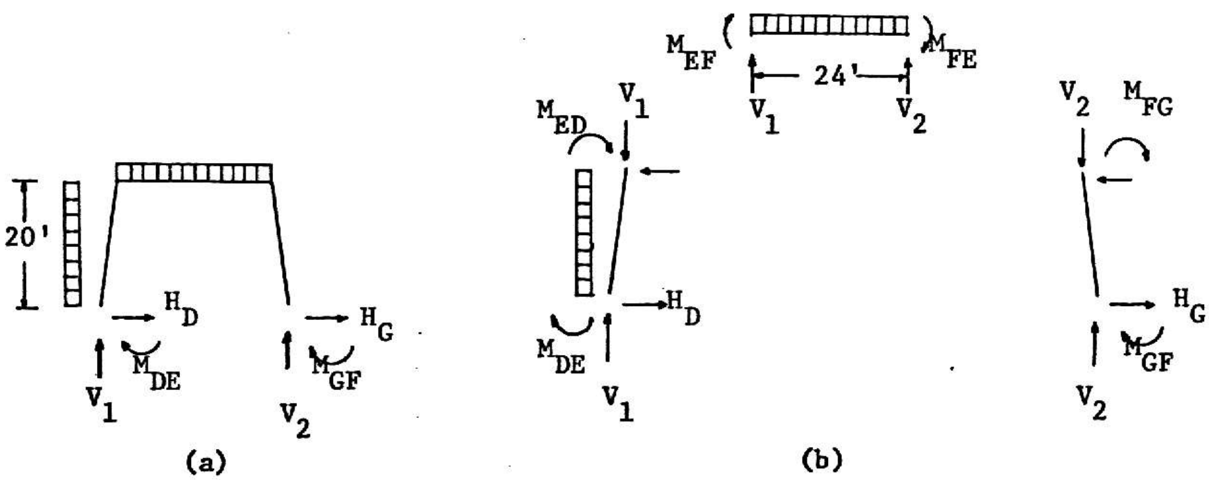


Fig. 20. The free body diagram of the fourth-story

From Fig. 20. (b)

$$M_{EF} + M_{FE} + 24V_1 - 24(12) = 0 \quad ,$$

$$V_1 = \frac{288 - M_{EF} - M_{FE}}{24} \quad ,$$

$$V_2 = 24 - \frac{288 - M_{EF} - M_{FE}}{24} \quad ,$$

$$\Sigma M_E = 0 \ , \quad M_{DE} + M_{ED} + 2.5V_1 - 20(10) - 20H_D = 0 \quad ,$$

$$H_D = \frac{M_{DE} + M_{ED} + 2.5V_1 - 200}{20} \quad .$$

$$\Sigma M_F = 0 \ , \quad M_{FG} + M_{GF} - 2.5V_2 - 20H_G = 0 \quad ,$$

$$H_G = \frac{M_{FG} + M_{GF} - 2.5V_2}{20} \quad .$$

From Fig. 20. (a)

$$\Sigma F_X = 0 \ , \quad \frac{M_{DE} + M_{ED} + M_{FG} + M_{GF}}{20} + (\frac{1}{8})(\frac{288 - M_{EF} - M_{FE}}{24})$$

$$- (\frac{1}{8})(24 - \frac{288 - M_{EF} - M_{FE}}{24}) + 10 = 0$$

Therefore,

$$6201.75\theta_D + 5118.42\theta_E + 5118.42\theta_F + 6201.75\theta_G + 1240.35\Delta_3$$
$$- 1262.92\Delta_4 + 10 = 0 \qquad \ldots\ldots\ldots\ldots\ldots(9)$$

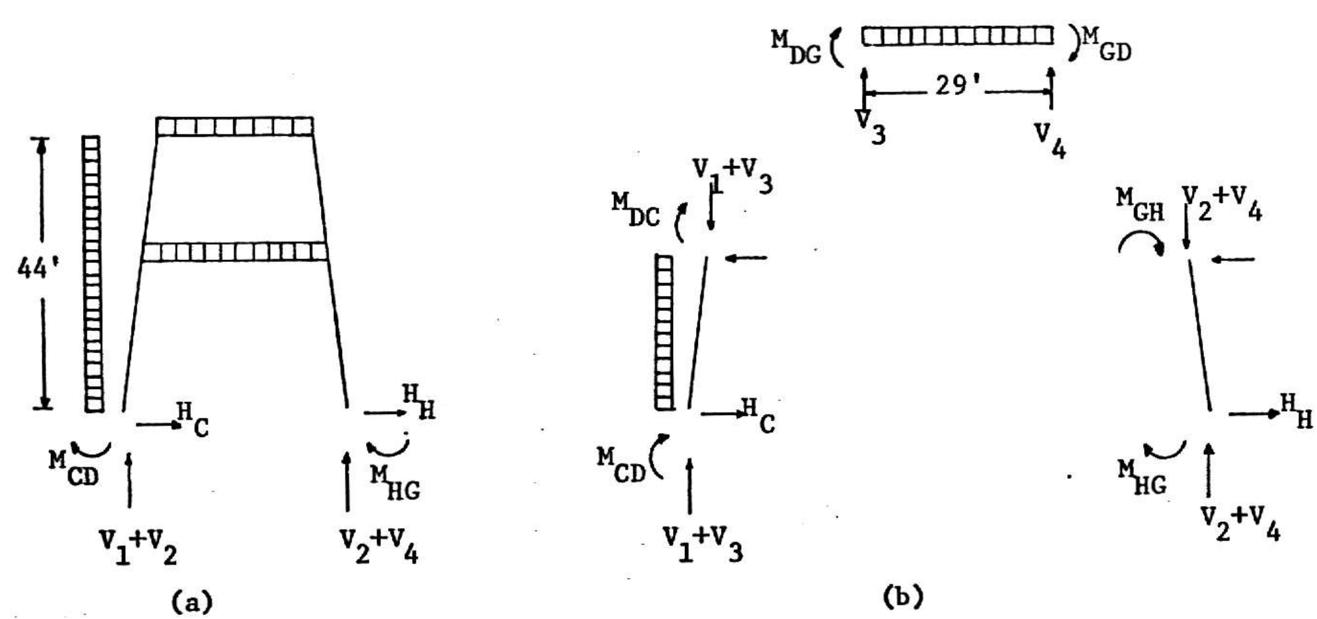


Fig. 21, The free body diagram of the third-story

From Fig. 21. (b)

$$M_{DG} + M_{GD} + 29V_3 - 29(14.5) = 0 \quad ,$$

$$V_3 = \frac{420.5 - M_{DG} - M_{GD}}{29} \quad ,$$

$$V_4 = 29 - \frac{420.5 - M_{DG} - M_{GD}}{29} \quad .$$

$$\Sigma M_D = 0 \quad , \quad M_{CD} + M_{DC} + 3(\, V_1 + V_3 \,) - 24(12) - 24H_c = 0 \quad ,$$

$$H_C = \frac{M_{CD} + M_{DC} + 3(\, V_1 + V_3 \,) - 288}{24} \quad .$$

$$\Sigma M_G = 0 \quad , \quad M_{HG} + M_{GH} - 3(\, V_2 + V_4 \,) - 24H_H = 0 \quad ,$$

$$H_H = \frac{M_{HG} + M_{GH} - 3(\, V_2 + V_4 \,)}{24} \quad .$$

From Fig. 21. (a)

$$\Sigma F_X = 0 \quad , \quad 32 + \frac{M_{CD} + M_{DC} + M_{GH} + M_{HG}}{24} + (\tfrac{1}{8})\left(\frac{288 - M_{EF} - M_{FE}}{24} + \frac{420.5 - M_{DG} - M_{GD}}{29}\right) - (\tfrac{1}{8})\left(5 - \frac{288 - M_{EF} - M_{FE}}{24} - \frac{420.5 - M_{DG} - M_{GD}}{29} \right) = 0$$

Therefore,

$$6460.082\theta_C + 4977.732\theta_D - 1085.07\theta_E - 1085.07\theta_F + 4977.732\theta_G + 6460.082\theta_H + 1076.68\Delta_2 - 1102.307\Delta_3 - 22.61\Delta_4 + 32 = 0 \quad \ldots\ldots\ldots(10)$$

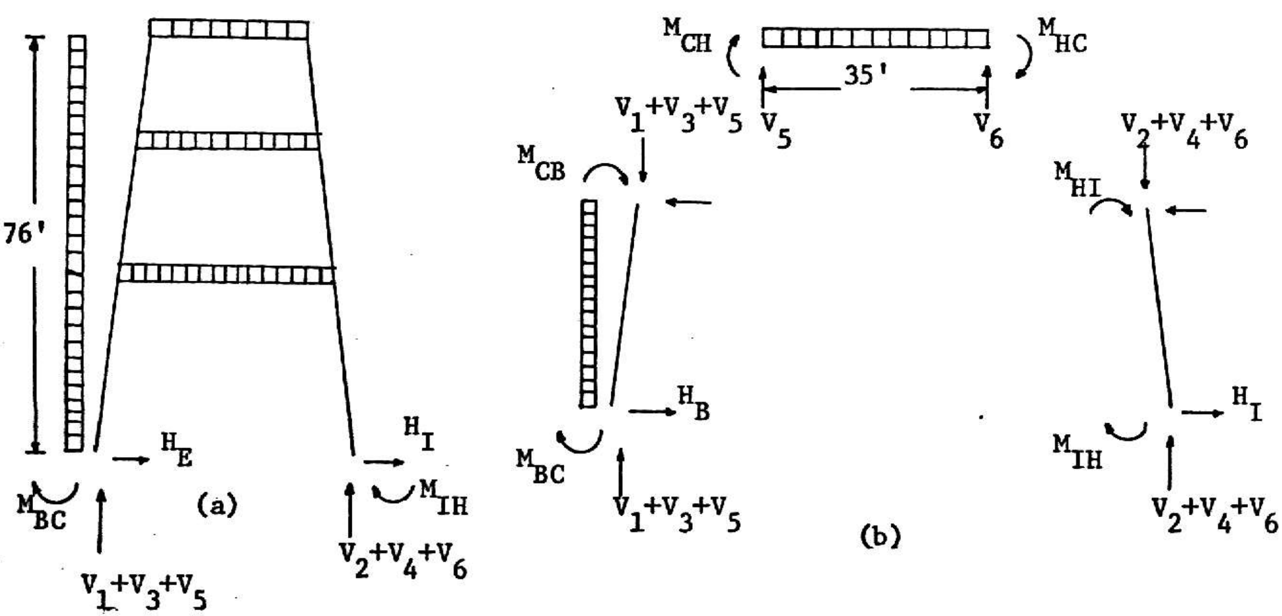


Fig. 22 The free body diagram of the second story

From Fig. 22. (b)

$$M_{CH} + M_{HC} + 35V_5 - 35(17.5) = 0 \ ,$$

$$V_5 = \frac{612.5 - M_{CH} - M_{HC}}{35} \ .$$

$$V_6 = 35 - \frac{612.5 - M_{CH} - M_{HC}}{35} \ .$$

$$M_C = 0 \ , \quad M_{BC} + M_{CB} + 4(V_1 + V_3 + V_5) - 32(16) - 32H_B = 0 \ ,$$

$$H_B = \frac{M_{BC} + M_{CB} + 4(V_1 + V_3 + V_5) - 512}{32} \ .$$

$$M_H = 0 \ , \quad M_{IH} + M_{HI} - 4(V_2 + V_4 + V_6) - 32H_I = 0 \ ,$$

$$H_I = \frac{M_{IH} + M_{HI} - 4(V_2 + V_4 + V_6)}{32} \ .$$

From Fig. 22. (a)

$$F_X = 0 \ , \quad 60 + \frac{M_{BC} + M_{CB} + M_{IH} + M_{HI}}{32} + (\tfrac{1}{8})(\frac{-2M_{EF} - 2M_{FE}}{24}) + (\tfrac{1}{8})(\frac{-2M_{DG} - 2M_{GD}}{29}) + (\tfrac{1}{8})(\frac{-2M_{CH} - 2M_{HC}}{35}) = 0$$

Therefore, we obtain

$$4845.111\theta_B + 3314.499\theta_C - 1468.326\theta_D - 1085.69\theta_E - 1085.69\theta_F - 1468.326\theta_G - 3314.499\theta_H + 4845.111\theta_I + 605.639\Delta_1 - 627.505\Delta_2 - 25.626\Delta_3 - 22.61\Delta_4 + 60 = 0 \quad \text{.....................(11)}$$

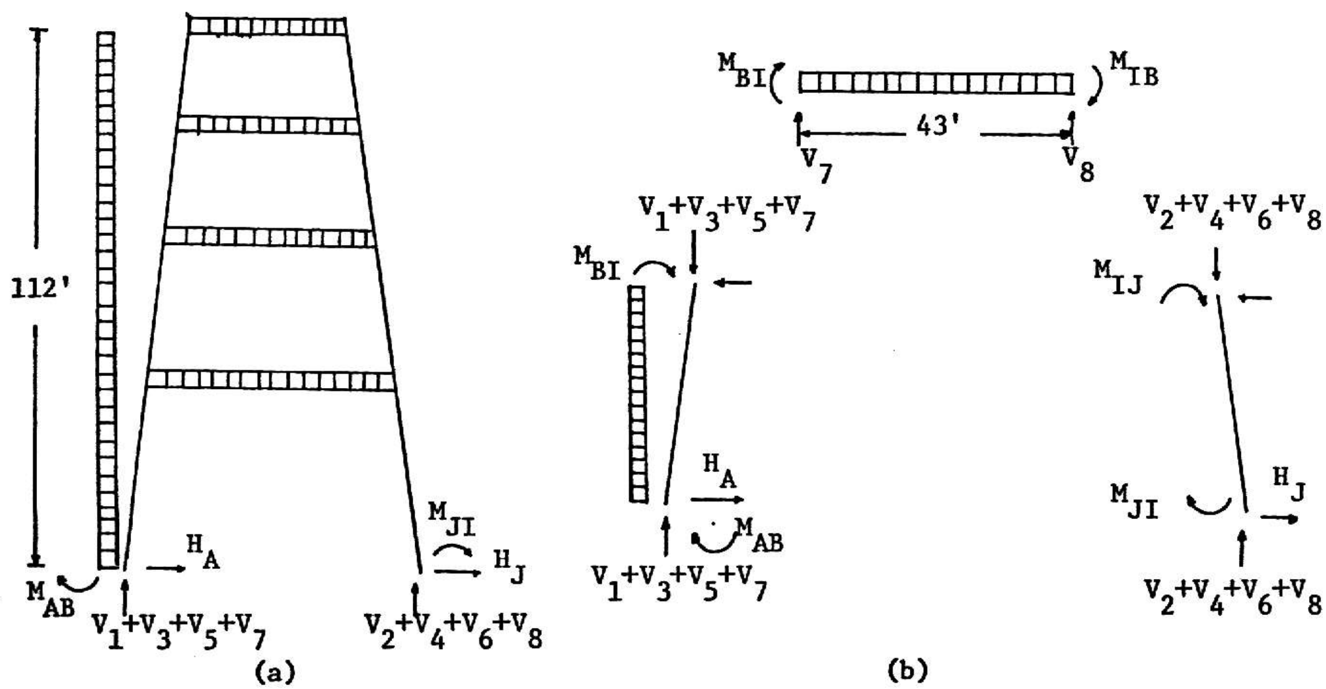


Fig. 23. The free body diagram of the first story

From Fig. 23. (b)

$$M_{BI} + M_{IB} + 43V_7 - 43(21.5) = 0 \quad ,$$

$$V_7 = \frac{924.5 - M_{BI} - M_{IB}}{43}$$

$$V_8 = 43 - \frac{924.5 - M_{BI} - M_{IB}}{43} \quad .$$

$$\Sigma M_B = 0 \; , \quad M_{AB} + M_{BA} + 4.5(\, V_1 + V_3 + V_5 + V_7 \,) - 36(18) - 36H_A = 0$$

$$H_A = \frac{M_{AB} + M_{BA} + 4.5(\, V_1 + V_3 + V_5 + V_7 \,) - 648}{36} \quad .$$

$$\Sigma M_I = 0 \; , \quad M_{IJ} + M_{JI} - 4.5(\, V_2 + V_4 + V_6 + V_8 - 36H_J = 0 \quad ,$$

$$H_J = \frac{M_{IJ} + M_{JI} - 4.5(\, V_2 + V_4 + V_6 + V_8 \,)}{36} \quad .$$

From Fig. 23. (a)

$$\Sigma F_X = 0 \; , \quad 94 + \frac{M_{AB} + M_{BA} + M_{IJ} + M_{JI}}{36} + (\tfrac{1}{8})(\frac{-2M_{EF} - 2M_{FE}}{24}) + (\tfrac{1}{8})$$

$$(\frac{-2M_{DG} - 2M_{GD}}{29}) + (\tfrac{1}{8})(\frac{-2M_{CH} - 2M_{HC}}{35}) + (\tfrac{1}{8})(\frac{-2M_{BI} - 2M_{IB}}{43}) = 0 \quad ,$$

Therefore, we obtain

$$3433.23\theta_B - 1530.612\theta_C - 1795.977\theta_D - 1085.07\theta_E - 1085.07\theta_F - 1795.977\theta_G - 1530.612\theta_H + 3433.23\theta_I - 547.422\Delta_1 - 21.866\Delta_2 - 30.965\Delta_3 - 22.606\Delta_4 + 94 = 0 \quad \ldots\ldots (12)$$

Solve the above twelve simultaneous equations, we get

θ_B = 0.00309532 rad.

θ_C = 0.000468611 rad.

θ_D =-0.0011642 rad.

θ_E =-0.002373 rad.

θ_F =-0.0027046 rad.

θ_G =-0.00168819 rad.

θ_H = 0.000134696 rad.

θ_I = 0.00218827 rad.

Δ_1 =0.179291 ft.

Δ_2 =0.303584 ft.

Δ_3 =0.314566 ft.

Δ_4 =0.285519 ft.

and

$$M_{AB} = -788.22 \text{ kips-ft}$$
$$M_{BA} = -394.48 \text{ kips-ft}$$
$$M_{BC} = -343.39 \text{ kips-ft}$$
$$M_{CB} = -308.48 \text{ kips-ft}$$
$$M_{CD} = -130.68 \text{ kips-ft}$$
$$M_{DC} = -120.0 \text{ kips-ft}$$
$$M_{DE} = -37.57 \text{ kips-ft}$$
$$M_{ED} = -7.11 \text{ kips-ft}$$
$$M_{FG} = -96.38 \text{ kips-ft}$$
$$M_{GF} = -38.2 \text{ kips-ft}$$
$$M_{GH} = -238.48 \text{ kips-ft}$$
$$M_{HG} = -144.27 \text{ kips-ft}$$
$$M_{HI} = -475.2 \text{ kips-ft}$$
$$M_{IH} = -369.07 \text{ kips-ft}$$
$$M_{IJ} = -606.65 \text{ kips-ft}$$
$$M_{JI} = -732.31 \text{ kips-ft}$$
$$M_{EF} = 6.74 \text{ kips-ft}$$
$$M_{FE} = 87.60 \text{ kips-ft}$$
$$M_{DG} = 166.63 \text{ kips-ft}$$
$$M_{GD} = 276.68 \text{ kips-ft}$$
$$M_{CH} = 439.2 \text{ kips-ft}$$
$$M_{HC} = 619.47 \text{ kips-ft}$$
$$M_{BI} = 737.86 \text{ kips-ft}$$
$$M_{IB} = 975.72 \text{ kips-ft}$$

CONCLUSIONS

The matrix method of structural analysis, in general, is a very powerful and convenient method which is applicable to almost all types of structural frameworks. The basic procedures of this method are:

(1) Select nodal displacement as basic unknowns

(2) Establish the displacement transformation matrix

(3) Evaluate the diagonal stiffness matrix

(4) Obtain the total stiffness matrix

(5) Express the nodal forces in terms of the nodal displacements

The advantages of the displacement method are: (1) It enables the use of the same procedures for analyzing statically determinate structures and statically indeterminate structures; and (2) It is easy to construct the displacement deformation matrix.

The comparison of the results of the matrix method and slope-deflection method indicates that the results which are obtained by these two methods are identical, as shown in Table 5. It is much easier to solve the problem by using the displacement method than the slope-deflection method, especially using the computer to solve the problem.

ACKNOWLEDGMENT

The writer wishes to express his great gratitude and deep appreciation to his advisor, Professor Vernon H. Rosebraugh, for his advice, guidance and suggestions during the preparation of this report.

NOTATIONS

P = The applied joint force.

X = The nodal displacement.

Q = The internal joint force.

q = The internal displacement.

P^T = The transpose of the matrix P.

a = The deformation matrix.

S = The diagonal stiffness matrix.

E = The Young's modulus.

I = The moment of inertia.

L = The length of the span.

A = The cross-section of a member.

K = The total stiffness matrix.

a^T = The transpose of matrix a.

w = The uniform load.

K^{-1} = The inverse of the matrix K.

Q_1 = The internal forces due to the applied loads.

Q_0 = The fixed-end moments due to the applied loads.

Q = The total internal forces.

S^{a-b} = The stiffness matrix of member AB.

V = The internal vertical forces due to the loads.

H = The internal horizontal forces due to the loads.

θ = The angular displacement.

Δ = The horizontal displacement of a member.

M = The end moment.

Q_i^1, Q_j^1, Q_k^1 = The forces caused by angular displacement occurs at joint i.

Q_i^2, Q_j^2, Q_k^2 = The forces caused by angilar displacement occurs at joint j.

Q_i^3, Q_j^3, Q_k^3 = The forces caused by a unit horizontal displacement of the member.

Q_i, Q_j, Q_k = The total forces caused by joint displacement.

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APPENDIX A

COMPUTER PROGRAM AND SOLUTION OF MULTIPLE HIGH TOWER FRAME WITH SIDESWAY USING MATRIX METHOD

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

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C***********************************************************************
C*                                                                     *
C*     THIS IS THE PROGRAM TO SOLVE MULTIPLE TOWER FRAME              *
C*     WITH SIDESWAY USING THE DISPLACEMENT METHOD                    *
C*                                                                     *
C*                              ---PRESENTED BY YUNG-TSENG CHUNG      *
C***********************************************************************
      DIMENSION A(30,30),S(60),P(20,5),ATSA(20,20),B(20,20),
     1C(20),D(20),BI(20),BL(20),X(20,5),Q(30,5),STIFF(60),
     2QQ(30,5),QI(30,5)
C     NQ=THE NUMBER OF INTERNAL MOMENTS
C     NP=THE NUMBER OF APPLIED LOADS
C     NLC=THE NUMBER OF LOADING CONDITIONS
C     NOM=THE NUMBER OF MEMBERS
      READ(5,1) NQ,NP,NLC,NOM
    1 FORMAT(4I5)
C     READ THE DEFORMATION MATRIX A
      READ(5,2) ((A(I,J),J=1,NP),I=1,NQ)
    2 FORMAT(6F10.2)
      DO 3 I=1,NOM
C     READ THE DIMENSION OF THE MEMBERS
C     BI=THE MOMENT OF INERTIA
C     BL=THE LENGTH OF THE MEMBER
    3 READ(5,4) BI(I),BL(I)
    4 FORMAT(2F10.2)
      E=30000.
      NS=NQ*2
      DO 5 M=1,NOM
      M1=4*M-3
      M2=4*M-2
      M3=4*M-1
      M4=4*M
      STIFF(M)=E*BI(M)/BL(M)
      S(M1)=4./144.*STIFF(M)
      S(M2)=2./144.*STIFF(M)
      S(M3)=2./144.*STIFF(M)
      S(M4)=4./144.*STIFF(M)
    5 CONTINUE
C     READ THE FIXED-END MOMENTS QQ
      READ(5,2) ((QQ(I,J),I=1,NQ),J=1,NLC)
C     READ THE APPLIED LOADS P
      READ(5,2) ((P(I,J),I=1,NP),J=1,NLC)
      WRITE(6,6)
    6 FORMAT(1H1,17X,'THE DISPLACEMENT METHOD OF MULTIPLE'/18X,
     1'HIGH TOWER FRAME WITH SIDESWAY'///)
      WRITE(6,100)
  100 FORMAT(18X,'NOTATIONS:',/,18X,
     1'NQ=THE NUMBER OF INTERNAL MOMENTS',/,18X,
     2'NP=THE NUMBER OF POSSIBLE EXTERNAL JOINT FORCES',/,18X,
     3'NLC=THE NUMBER OF LOADING CONDITIONS',/,18X,
     4'NOM=THE NUMBER OF MEMBERS',/,18X,
     5'A=THE DEFORMATION MATRIX',/,18X,
     6'BI=THE MOMENT INERTIA OF AN INDIVIDUAL MEMBER',/,18X,
     7'BL=THE LENGTH OF THE MEMBER',/,18X,
     8'E=THE MODULUS OF ELASTICITY',/,18X,
     9'S=THE DIAGONAL STIFFNESS MATRIX')
      WRITE(6,99)
   99 FORMAT(18X,'P=THE APPLIED LOADS MATRIX',/,18X,
```

```
     1'ATSA=THE TOTAL STIFFNESS MATRIX',/,18X,
     2'X=THE JOINT DISPLACEMENTS MATRIX',/,18X,
     3'QO=THE FIXED END MOMENTS',/,18X,
     4'QI=THE INTERNAL END MOMENTS DUE TO APPLIED LOADS',/,18X,
     5'Q=THE TOTAL END MOMENTS MATRIX')
      WRITE(6,101)
  101 FORMAT(1H1,17X,'THE INPUT DATA',/)
      WRITE(6,7)
    7 FORMAT(18X,'THE DEFORMATION MATRIX A')
      DO 8 I=1,NQ
    8 WRITE(6,9) I,(A(I,J),J=1,NP)
    9 FORMAT(22X,'ROW',I3,1X,2F16.6/(29X,2F16.6))
      WRITE(6,102)
  102 FORMAT(18X,'THE DIMENSIONS OF BARS')
      DO 103 I=1,NOM
  103 WRITE(6,104) I,BL(I),BI(I)
  104 FORMAT(22X,'BAR',I3,11X,'LENGTH=',F10.4,'FT',/,
     128X,'MOMENT OF INERTIA=',F10.4,'INCH TO THE FOURTH POWER')
      WRITE(6,10)
   10 FORMAT(18X,'THE DIAGONAL STIFFNESS MATRIX S',
     15X,'(IN KIPS-FT)')
      DO 11 L=1,NQ
      L1=(L-1)/2*2+1
      L2=2*L-1
      L3=(L+1)/2*2
      L4=2*L
   11 WRITE(6,12) L,L1,S(L2),L3,S(L4)
   12 FORMAT(22X,'ROW',I3,5X,'COL',I3,F16.6,/,
     133X,'COL',I3,F16.6)
      WRITE(6,106)
  106 FORMAT(18X,'THE FIXED END MOMENT MATRIX QO',
     15X,'(IN KIPS-FT)')
      DO 107 J=1,NLC
  107 WRITE(6,108) J,(QO(I,J),I=1,NQ)
  108 FORMAT(22X,'COL',I3,1X,F16.6/(29X,F16.6))
      WRITE(6,13)
   13 FORMAT(18X,'THE APPLIED LOADS MATRIX P',5X,
     1'(IN KIPS-FT OR KIPS)')
      DO 14 J=1,NLC
   14 WRITE(6,15) J,(P(I,J),I=1,NP)
   15 FORMAT(22X,'COL',I3,1X,F16.6/(29X,F16.6))
C     COMPUTE THE STIFFNESS MATRIX K, K=AT*S*A
      DO 16 I=1,NP
      DO 16 J=1,NP
      ATSA(I,J)=0.
      DO 16 K=1,NQ
      K1=(K-1)/2*2+1
      K2=(K+1)/2*2
      K3=2*K-1
      K4=2*K
   16 ATSA(I,J)=ATSA(I,J)+A(K,I)*(S(K3)*A(K1,J)+S(K4)*A(K2,J))
      WRITE(6,17)
   17 FORMAT(18X,'THE STIFFNESS MATRIX K',5X,'(IN KIPS-FT)')
      DO 18 I=1,NP
   18 WRITE(6,19) I, (ATSA(I,J),J=1,NP)
   19 FORMAT(22X,'ROW',I3,1X,2F16.6/(29X,2F16.6))
      DO 20 I=1,NP
      DO 20 J=1,NP
```

```
   20 B(I,J)=ATSA(I,J)
      NA=NP+1
      DO 32 NF=1,NLC
      DO 21 I=1,NP
      DO 21 J=NA,NA
      K=NA-NP+NF-1
   21 B(I,J)=P(I,K)
      NB=NP-1
C*    USE GAUSSIAN PIVOTAL CONDENSATION METHOD TO SOLVE
C*    SIMULTANEOUS LINEAR EQUATIONS
C*    DETERMINE THE ROW POSITION OF THE DESIRED PIVOT
      DO 29 NC=1,NB
      ND=NC+1
      DO 22 NE=NC,NP
   22 C(NE)=ABS(B(NE,NC))
      KK=0
      TRY=C(NC)
      DO 24 NE=ND,NP
      IF(C(NE)-TRY) 24,24,23
   23 TRY=C(NE)
      KK=NE
   24 CONTINUE
C*    INTERCHANGE ROWS TO PROPERLY PLACE THE PIVOT
      IF(KK) 25,28,25
   25 DO 26 NE=NC,NA
      D(NE)=B(KK,NE)
   26 B(KK,NE)=B(NC,NE)
      DO 27 NE=NC,NA
   27 B(NC,NE)=D(NE)
C*    PERFORM THE ELIMINATION PROCESS
   28 K=NC+1
      DO 29 I=K,NP
      DO 29 J=K,NA
   29 B(I,J)=B(I,J)-B(I,K-1)*B(K-1,J)/B(K-1,K-1)
C*    BACK SUBSTITUTION TO DETERMINE THE SOLUTION SET
      X(NP,NF)=B(NP,NA)/B(NP,NP)
      DO 31 K=2,NP
      L=NA-K
      M=L+1
      X(L,NF)=0
      DO 30 I=M,NP
   30 X(L,NF)=X(L,NF)+B(L,I)*X(I,NF)
   31 X(L,NF)=(B(L,NA)-X(L,NF))/B(L,L)
   32 CONTINUE
      WRITE(6,105)
  105 FORMAT(//,18X,'ANSWERS'/)
      WRITE(6,33)
   33 FORMAT(18X,'THE JOINT DISPLACEMENTS MATRIX X',
     15X,'(IN RAD. OR FT)')
      DO 34 NF=1,NLC
   34 WRITE(6,35) NF, (X(I,NF),I=1,NP)
   35 FORMAT(22X,'COL',I3,1X,F16.6/(29X,F16.6))
C     COMPUTE THE INTERNAL MOMENTS Q1
      DO 36 J=1,NLC
      DO 36 I=1,NQ
      I1=(I-1)/2*2+1
      I2=(I+1)/2*2
      I3=2*I-1
```

```
      I4=2*I
      OI(I,J)=0.
      DO 36 K=1,NP
   36 OI(I,J)=OI(I,J)+X(K,J)*(A(I1,K)*S(I3)+A(I2,K)*S(I4))
      WRITE(6,110)
  110 FORMAT(18X,'THE INTERNAL END MOMENT MATRIX OI',
     15X,'(IN KIPS-FT)')
      DO 111 J=1,NLC
  111 WRITE(6,112) J,(OI(I,J),I=1,NO)
  112 FORMAT(22X,'COL',I3,1X,F16.6/(29X,F16.6))
C     COMPUTE THE TOTAL END MOMENTS Q, Q=QO+OI
      DO 109 J=1,NLC
      DO 109 I=1,NO
  109 Q(I,J)=QO(I,J)+OI(I,J)
      WRITE(6,37)
   37 FORMAT(18X,'THE TOTAL END MOMENT MATRIX Q',
     15X,'(IN KIPS-FT)')
      DO 38 J=1,NLC
   38 WRITE(6,39) J, (Q(I,J),I=1,NO)
   39 FORMAT(22X,'COL',I3,1X,F16.6/(29X,F16.6))
      STOP
      END
```

THE DISPLACEMENT METHOD OF MULTIPLE
HIGH TOWER FRAME WITH SIDESWAY

NOTATIONS:
NQ=THE NUMBER OF INTERNAL MOMENTS
NP=THE NUMBER OF POSSIBLE EXTERNAL JOINT FORCES
NLC=THE NUMBER OF LOADING CONDITIONS
NOM=THE NUMBER OF MEMBERS
A=THE DEFORMATION MATRIX
BI=THE MOMENT INERTIA OF AN INDIVIDUAL MEMBER
BL=THE LENGTH OF THE MEMBER
E=THE MODULUS OF ELASTICITY
S=THE DIAGONAL STIFFNESS MATRIX
P=THE APPLIED LOADS MATRIX
ATSA=THE TOTAL STIFFNESS MATRIX
X=THE JOINT DISPLACEMENTS MATRIX
QQ=THE FIXED END MOMENTS
QI=THE INTERNAL END MOMENTS DUE TO APPLIED LOADS
Q=THE TOTAL END MOMENTS MATRIX

```
THE INPUT DATA

THE DEFORMATION MATRIX A
    ROW  1          0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                -0.027778
    ROW  2          0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 1.000000
                    0.0                -0.027778
    ROW  3          0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                   -0.031250            1.000000
                    0.0                 0.031250
    ROW  4          0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
                    1.000000            0.0
                   -0.031250            0.0
                    0.0                 0.031250
    ROW  5          0.0                 0.0
                    0.0                 0.0
                    0.0                -0.041667
                    1.000000            0.0
                    0.041667            0.0
                    0.0                 0.0
    ROW  6          0.0                 0.0
                    0.0                 1.000000
                    0.0                -0.041667
                    0.0                 0.0
                    0.041667            0.0
                    0.0                 0.0
    ROW  7          0.0                 0.0
                   -0.050000            1.000000
                    0.0                 0.050000
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
    ROW  8          1.000000            0.0
                   -0.050000            0.0
                    0.0                 0.050000
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
    ROW  9          0.0                 1.000000
                   -0.050000            0.0
                    0.0                 0.050000
                    0.0                 0.0
                    0.0                 0.0
                    0.0                 0.0
    ROW 10          0.0                 0.0
                   -0.050000            0.0
                    1.000000            0.050000
```

	0.0	0.0
	0.0	0.0
	0.0	0.0
ROW 11	0.0	0.0
	0.0	0.0
	1.000000	-0.041667
	0.0	0.0
	0.041667	0.0
	0.0	0.0
ROW 12	0.0	0.0
	0.0	0.0
	0.0	-0.041667
	0.0	1.000000
	0.041667	0.0
	0.0	0.0
ROW 13	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	1.000000
	-0.031250	0.0
	0.0	0.031250
ROW 14	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	-0.031250	0.0
	1.000000	0.031250
ROW 15	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	1.000000	-0.027778
ROW 16	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	-0.027778
ROW 17	1.000000	0.0
	0.010417	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
ROW 18	0.0	1.000000
	0.010417	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
	0.0	0.0
ROW 19	0.0	0.0
	0.0	1.000000
	0.0	0.008621
	0.0	0.0
	0.0	0.0
	0.0	0.0
ROW 20	0.0	0.0
	0.0	0.0
	1.000000	0.008621

```
                    0.0              0.0
                    0.0              0.0
                    0.0              0.0
    ROW 21          0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    1.000000         0.0
                    0.007143         0.0
                    0.0              0.0
    ROW 22          0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    0.0              1.000000
                    0.007143         0.0
                    0.0              0.0
    ROW 23          0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    0.0              1.000000
                    0.0              0.005814
    ROW 24          0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    0.0              0.0
                    1.000000         0.005814
THE DIMENSIONS OF BARS
    BAR  1              LENGTH=    36.2802FT
         MOMENT OF INERTIA= 5000.0000INCH TO THE FOURTH POWER
    BAR  2              LENGTH=    32.2490FT
         MOMENT OF INERTIA= 4000.0000INCH TO THE FOURTH POWER
    BAR  3              LENGTH=    24.1863FT
         MOMENT OF INERTIA= 3000.0000INCH TO THE FOURTH POWER
    BAR  4              LENGTH=    20.1556FT
         MOMENT OF INERTIA= 2000.0000INCH TO THE FOURTH POWER
    BAR  5              LENGTH=    20.1556FT
         MOMENT OF INERTIA= 2000.0000INCH TO THE FOURTH POWER
    BAR  6              LENGTH=    24.1863FT
         MOMENT OF INERTIA= 3000.0000INCH TO THE FOURTH POWER
    BAR  7              LENGTH=    32.2490FT
         MOMENT OF INERTIA= 4000.0000INCH TO THE FOURTH POWER
    BAR  8              LENGTH=    36.2802FT
         MOMENT OF INERTIA= 5000.0000INCH TO THE FOURTH POWER
    BAR  9              LENGTH=    24.0000FT
         MOMENT OF INERTIA= 2000.0000INCH TO THE FOURTH POWER
    BAR 10              LENGTH=    29.0000FT
         MOMENT OF INERTIA= 4000.0000INCH TO THE FOURTH POWER
    BAR 11              LENGTH=    35.0000FT
         MOMENT OF INERTIA= 6000.0000INCH TO THE FOURTH POWER
    BAR 12              LENGTH=    43.0000FT
         MOMENT OF INERTIA= 8000.0000INCH TO THE FOURTH POWER
THE DIAGONAL STIFFNESS MATRIX S      (IN KIPS-FT)
    ROW  1       COL  1    114846.937500
                 COL  2     57423.496094
    ROW  2       COL  1     57423.496094
                 COL  2    114846.937500
    ROW  3       COL  3    103362.250000
                 COL  4     51681.132813
    ROW  4       COL  3     51681.132813
```

```
                   COL  4     103362.250000
     ROW  5        COL  5     103362.250000
                   COL  6      51681.132813
     ROW  6        COL  5      51681.132813
                   COL  6     103362.250000
     ROW  7        COL  7      82689.812500
                   COL  8      41344.914063
     ROW  8        COL  7      41344.914063
                   COL  8      82689.812500
     ROW  9        COL  9      82689.812500
                   COL 10      41344.914063
     ROW 10        COL  9      41344.914063
                   COL 10      82689.812500
     ROW 11        COL 11     103362.250000
                   COL 12      51681.132813
     ROW 12        COL 11      51681.132813
                   COL 12     103362.250000
     ROW 13        COL 13     103362.250000
                   COL 14      51681.132813
     ROW 14        COL 13      51681.132813
                   COL 14     103362.250000
     ROW 15        COL 15     114846.937500
                   COL 16      57423.496094
     ROW 16        COL 15      57423.496094
                   COL 16     114846.937500
     ROW 17        COL 17      69444.437500
                   COL 18      34722.218750
     ROW 18        COL 17      34722.218750
                   COL 18      69444.437500
     ROW 19        COL 19     114942.500000
                   COL 20      57471.257813
     ROW 20        COL 19      57471.257813
                   COL 20     114942.500000
     ROW 21        COL 21     142857.125000
                   COL 22      71428.562500
     ROW 22        COL 21      71428.562500
                   COL 22     142857.125000
     ROW 23        COL 23     155038.687500
                   COL 24      77519.312500
     ROW 24        COL 23      77519.312500
                   COL 24     155038.687500
THE FIXED END MOMENT MATRIX DD       (IN KIPS-FT)
     COL  1         -108.000000
                     108.000000
                     -85.329987
                      85.329987
                     -48.000000
                      48.000000
                     -33.329987
                      33.329987
                       0.0
                       0.0
                       0.0
                       0.0
                       0.0
                       0.0
                       0.0
                       0.0
                     -48.000000
                      48.000000
```

```
                 -70.082993
                  70.082993
                -102.082993
                 102.082993
                -154.082993
                 154.082993
THE APPLIED LOADS MATRIX P      (IN KIPS-FT OR KIPS)
    COL  1        14.660000
                 -48.000000
                  10.000000
                  55.412994
                 -70.082993
                  22.000000
                  64.752991
                -102.082993
                  28.000000
                 121.412994
                -154.082993
                  34.000000
THE STIFFNESS MATRIX K        (IN KIPS-FT)
    ROW  1     152134.250000    34722.218750
               -5116.660156    41344.914063
                   0.0          6201.730469
                   0.0             0.0
                   0.0             0.0
                   0.0             0.0
    ROW  2      34722.218750   152134.250000
               -5116.660156        0.0
               41344.914063     6201.730469
                   0.0             0.0
                   0.0             0.0
                   0.0             0.0
    ROW  3     -5116.660156    -5116.660156
                1262.951172    -6201.730469
               -6201.730469    -1240.345703
                   0.0             0.0
                   0.0             0.0
                   0.0             0.0
    ROW  4     41344.914063        0.0
               -6201.730469   300994.562500
               57471.257813     1227.930176
               51681.132813        0.0
                6460.136719        0.0
                   0.0             0.0
    ROW  5         0.0         41344.914063
               -6201.730469    57471.257813
              300994.562500     1227.930176
                   0.0         51681.132813
                6460.136719        0.0
                   0.0             0.0
    ROW  6      6201.730469     6201.730469
               -1240.345703     1227.933350
                1227.928467     2342.657227
               -6460.125000    -6460.125000
               -1076.687500        0.0
                   0.0             0.0
    ROW  7         0.0             0.0
                   0.0         51681.132813
                   0.0         -6460.125000
              349581.625000    71428.562500
```

```
                3145.643066     51681.132813
                   0.0           4845.105469
     ROW  8        0.0              0.0
                   0.0              0.0
               51681.132813     -6460.125000
               71428.562500    349531.625000
                3145.643066         0.0
               51681.132813      4845.105469
     ROW  9        0.0              0.0
                   0.0           6460.136719
                6460.136719     -1076.686523
                3145.644043      3145.642822
                1704.192139     -4845.105469
               -4845.105469      -605.637939
     ROW 10        0.0              0.0
                   0.0              0.0
                   0.0              0.0
               51681.132813         0.0
               -4845.105469    373247.375000
               77519.312500      1411.912109
     ROW 11        0.0              0.0
                   0.0              0.0
                   0.0              0.0
                   0.0          51681.132813
               -4845.105469     77519.312500
              373247.375000      1411.912109
     ROW 12        0.0              0.0
                   0.0              0.0
                   0.0              0.0
                4845.105469      4845.105469
                -605.637939      1411.912354
                1411.909912      1153.054688


ANSWERS

THE JOINT DISPLACEMENTS MATRIX X      (IN RAD. OR FT)
     COL   1      -0.231274E-02
                  -0.271632E-02
                   0.287427E 00
                  -0.127434E-02
                  -0.164101E-02
                   0.219941E 00
                   0.492024E-03
                   0.128473E-03
                   0.308559E 00
                   0.313555E-02
                   0.223368E-02
                   0.182375E 00
THE INTERNAL END MOMENT MATRIX MI      (IN KIPS-FT)
     COL   1      -692.658936
                  -512.604980
                  -261.851807
                  -399.471924
                   -88.524414
                  -179.812500
                     0.648193
                   -42.284130
                   -90.240186
                   -46.605469
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```
                    -236.811035
                    -145.207031
                    -482.653936
                    -373.859375
                    -616.181396
                    -744.447021
                      56.953506
                      42.940430
                     234.576218
                     213.331055
                     551.750244
                     525.782471
                     905.370117
                     835.958008
THE TOTAL END MOMENT MATRIX Q        (IN KIPS-FT)
   COL   1          -800.658936
                    -404.604980
                    -347.131641
                    -313.141846
                    -126.524414
                    -131.812500
                     -32.681793
                      -8.954193
                     -90.940186
                     -46.505469
                    -236.811035
                    -145.207031
                    -482.658936
                    -373.859375
                    -616.181396
                    -744.447021
                       8.953506
                      90.940430
                     164.493225
                     232.413818
                     449.667236
                     627.355234
                     751.787109
                     990.040771
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APPENDIX B

GIVEN COMPUTER PROGRAM AND SOLUTION OF MULTIPLE HIGH TOWER FRAME WITH SIDESWAY USING MATRIX METHOD

```
C      MATRIX SOLUTION OF STRUCTURAL PROBLEMS      JUNE 1974
C      FORTRAN 4    DISPLACEMENT METHOD   ROSEBRAUGH V H
C      PROGRAM MAY BE USED FOR TRUSSES, BEAMS AND FRAMES
C      INA=ROWS IN "A" MATRIX   IMA=COLS IN "A"   IMP=COLS IN "RR"
C      AT=TRANSPOSED "A" MATRIX   RR=VALUES IN "R" MATRIX
    1 FORMAT (3I3)
    2 FORMAT (6F12.5)
    3 FORMAT(/,14X,12H INPUT DATA   )
    4 FORMAT(//,14X,25H VALUES OF INA, IMA, IMP   )
    5 FORMAT(//,14X,30H VALUES OF RR FOR LOADING NO. ,I2/)
    6 FORMAT(//,14X,18H VALUES OF A(I,J) )
    7 FORMAT(//,14X,21H VALUES OF STIF(I,J) )
    8 FORMAT(2H   //)
    9 FORMAT(//14X,' MEMBER FORCES OR MOMENTS ',
     1'FOR LOADING NO. ',I2/)
   10 FORMAT(//,14X,34H INVERTED MATRIX -- ATKA TO ATKAI  /)
   11 FORMAT(//,14X,32H TRANSPOSED A MATRIX = B MATRIX  /)
   12 FORMAT(//,14X,27H MATRIX MULTIPLY -- AT X K  /)
   13 FORMAT(//,14X,28H MATRIX MULTIPLY -- ATK X A  /)
   14 FORMAT(//14X,' MATRIX MULTIPLY -- ATKAI *RR ',
     1'FOR LOADING NO. ',I2/)
   15 FORMAT(//,14X,26H MATRIX MULTIPLY -- K X A  /)
      DIMENSION A(36,36),STIF(36,36),S(36, 1),B(36,36)
      DIMENSION ATKA(36,36),R(36, 1),RR(36, 1),ATKAI(36,36)
      DIMENSION RKA(36,36),S1(36,1),ATK(36,36)
  100 READ(5,1,END=99)INA,IMA,IMP
      READ(5,2)((A(I,J),I=1,INA),J=1,IMA)
      READ(5,2)((STIF(I,J),I=1,INA),J=1,INA)
  200 FORMAT(14X,4F14.5)
      WRITE(6,8)
      WRITE(6,3)
      WRITE(6,4)
      WRITE(6,101)INA,IMA,IMP
  101 FORMAT(14X,3I8)
      WRITE(6,6)
      WRITE(6,200)((A(I,J),I=1,INA),J=1,IMA)
      WRITE(6,7)
      WRITE(6,200)((STIF(I,J),I=1,INA),J=1,INA)
      WRITE(6,8)
C      MATRIX TRANSPOSE        B = AT
      DO 18 I=1,INA
      DO 18 J=1,IMA
   18 B(J,I)=A(I,J)
      WRITE(6,11)
      WRITE(6,200)((B(J,I),J=1,IMA),I=1,INA)
C      PROGRAM FOR MATRIX MULTIPLY       B X K =ATK
      DO 17 I=1,IMA
      DO 17 J=1,INA
      SUM=0.0
      DO 16 K=1,INA
   16 SUM=B(I,K)*STIF(K,J)+SUM
      ATK(I,J)=SUM
   17 CONTINUE
      WRITE(6,12)
      WRITE(6,200)((ATK(I,J),I=1,IMA),J=1,INA)
C      MATRIX MULTIPLY     ATK X A = ATKA
      DO 19 I=1,IMA
      DO 19 J=1,IMA
```

```
      SUM1=0.0
      DO 20 K=1,IMA
   20 SUM1=ATK(I,K)*A(K,J)+SUM1
      ATKA(I,J)=SUM1
   10 CONTINUE
      WRITE(6,13)
      WRITE(6,200)((ATKA(I,J),I=1,IMA),J=1,IMA)
C     MATRIX INVERSION     ATKA TO ATKAI
      DO 21 I=1,IMA
      DO 21 J=1,IMA
   21 ATKAI(J,I)=ATKA(I,J)
      DO 22 K=1,IMA
      SUM2=0.0
      DO 23 I=1,IMA
   23 SUM2=SUM2+ATKAI(K,I)*ATKA(I,K)
      DO 24 I=1,IMA
   24 ATKAI(K,I)=ATKAI(K,I)/SUM2
      DO 44 J=1,IMA
      L=J-K
      IF(L) 25,44,25
   25 SUM3=0.0
      DO 26 I=1,IMA
   26 SUM3=SUM3+ATKAI(J,I)*ATKA(I,K)
      DO 46 I=1,IMA
   46 ATKAI(J,I)=ATKAI(J,I)-SUM3*ATKAI(K,I)
   44 CONTINUE
   22 CONTINUE
      WRITE (6,10)
      WRITE(6,200)((ATKAI(I,J),I=1,IMA),J=1,IMA)
C     MATRIX MULTIPLY  K X A = PKA  (P ADDED FOR FLOATING POINT)
      DO 30 I=1,INA
      DO 30 J=1,IMA
      SUM5=0.0
      DO 31 K=1,INA
   31 SUM5=STIF(I,K)*A(K,J)+SUM5
      PKA(I,J)=SUM5
   30 CONTINUE
      WRITE(6,15)
      WRITE(6,200)((PKA(I,J),I=1,INA),J=1,IMA)
C     MATRIX MULTIPLY      ATKAI X RR = R
      IE=0
   36 IE=IE+1
      READ(5,2)(RR(I,1),I=1,IMA)
      DO 28 I=1,IMA
      J=1
      SUM4=0.0
      DO 29 K=1,IMA
   29 SUM4=ATKAI(I,K)*RR(K,J)+SUM4
      R(I,J)=SUM4
   28 CONTINUE
      WRITE(6,14)IE
      WRITE(6,200)(R(I,1),I=1,IMA)
C     MATRIX MULTIPLY      PKA X R = S
      DO 32 I=1,INA
      J=1
      SUM6=0.0
      DO 33 K=1,IMA
   33 SUM6=PKA(I,K)*R(K,J)+SUM6
```

```
      S(I,J)=SUM6
   22 CONTINUE
      READ(5,2) (S1(I,1),I=1,INA)
      DO 39 I=1,INA
   39 S(I,1)=S(I,1)+S1(I,1)
      WRITE (6,5)IE
      WRITE(6,200)(PR(I,1),I=1,INA)
      WRITE (6,9)IE
      WRITE(6,200)(S(I,1),I=1,INA)
      IF(IMP-IE)37,37,36
   37 WRITE(6,8)
   38 GO TO 100
   99 STOP
      END
```

INPUT DATA

VALUES OF INA, IMA, IMP

24	12	1

VALUES OF A(I,J)

0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	-0.05000	-0.05000
-0.05000	-0.05000	0.0	0.0
0.0	0.0	0.0	0.0
0.01042	0.01042	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-0.04167	-0.04167	0.05000	0.05000
0.05000	0.05000	-0.04167	-0.04167
0.0	0.0	0.0	0.0
0.0	0.0	0.00862	0.00862
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
1.00000	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
1.00000	0.0	0.0	0.0
0.0	0.0	0.0	0.0

0.0	1.00000	0.0	0.0
0.0	0.0	-0.03125	-0.03125
0.04167	0.04167	0.0	0.0
0.0	0.0	0.04167	0.04167
-0.03125	-0.03125	0.0	0.0
0.0	0.0	0.0	0.0
0.00714	0.00714	0.0	0.0
0.0	1.00000	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	1.00000	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
-0.02778	-0.02778	0.03125	0.03125
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.03125	0.03125	-0.02778	-0.02778
0.0	0.0	0.0	0.0
0.0	0.0	0.00581	0.00581

VALUES OF STIF(I,J)

114847.43750	57423.73828	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
57423.73828	114847.43750	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	103362.37500	51631.18750
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	51631.18750	103362.37500
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
103361.25000	51680.65234	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
51680.65234	103361.25000	0.0	0.0

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	82690.00000	41345.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	41345.00000	82690.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
82690.00000	41345.00000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
41345.00000	82690.00000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	103361.25000	51680.65234
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	51680.65234	103361.25000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
103362.37500	51681.18750	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
51681.18750	103362.37500	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	114947.43750	57423.73828
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

0.0	0.0	0.0	0.0
0.0	0.0	57423.73828	114847.43750
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
69444.43750	34722.21875	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
34722.21875	69444.43750	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	114942.50000	57471.26172
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	57471.26172	114942.50000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
142857.12500	71428.56250	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
71428.56250	142857.12500	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	155038.75000	77519.37500
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	77519.37500	155038.75000

TRANSPOSED A MATRIX = B MATRIX

0.0	0.0	0.0	0.0

0.0	0.0	0.0	0.0
0.0	0.0	0.0	-0.02778
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	0.0	-0.02778
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-0.03125	1.00000	0.0	0.03125
0.0	0.0	0.0	0.0
0.0	0.0	1.00000	0.0
-0.03125	0.0	0.0	0.03125
0.0	0.0	0.0	0.0
0.0	-0.04167	1.00000	0.0
0.04167	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
0.0	-0.04167	0.0	0.0
0.04167	0.0	0.0	0.0
0.0	0.0	-0.05000	1.00000
0.0	0.05000	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	0.0	-0.05000	0.0
0.0	0.05000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	-0.05000	0.0
0.0	0.05000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	-0.05000	0.0
1.00000	0.05000	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	-0.04167	0.0	0.0
0.04167	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	-0.04167	0.0	1.00000
0.04167	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
-0.03125	0.0	0.0	0.03125
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-0.03125	0.0	1.00000	0.03125
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	1.00000	-0.02778
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	-0.02778
1.00000	0.0	0.01042	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	0.01042	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
0.0	0.00362	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
1.00000	0.00362	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

0.0	0.0	1.00000	0.0
-0.00714	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.00000
0.00714	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	1.00000	0.0	0.00581
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	1.00000	0.00581

MATRIX MULTIPLY -- AT X K

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	57423.73828	0.0	-4785.69141
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	114847.43750	0.0	-4785.69141
0.0	0.0	0.0	0.0
0.0	0.0	51681.18750	0.0
-4845.10938	103362.37500	0.0	4845.10938
0.0	0.0	0.0	0.0
0.0	0.0	103362.37500	0.0
-4845.10938	51681.18750	0.0	4845.10938
0.0	0.0	0.0	51680.65234
0.0	-6460.59375	103361.25000	0.0
6460.59375	0.0	0.0	0.0
0.0	0.0	0.0	103361.25000
0.0	-6460.59375	51680.65234	0.0
6460.59375	0.0	0.0	0.0
41345.00000	0.0	-6201.74219	82690.00000
0.0	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
82690.00000	0.0	-6201.74219	41345.00000
0.0	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
0.0	82690.00000	-6201.74219	0.0
41345.00000	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
0.0	41345.00000	-6201.74219	0.0
82690.00000	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
103361.25000	-6460.59375	0.0	51680.65234
6460.59375	0.0	0.0	0.0
0.0	0.0	0.0	0.0
51680.65234	-6460.59375	0.0	103361.25000
6460.59375	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	103362.37500
-4845.10938	0.0	51681.18750	4845.10938
0.0	0.0	0.0	0.0
0.0	0.0	0.0	51681.18750
-4845.10938	0.0	103362.37500	4845.10938
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	114847.43750	-4785.69141

0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	57423.73828	-4795.69141
69444.43750	34722.21875	1085.07227	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
34722.21875	69444.43750	1085.07227	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	114942.50000
57471.26172	1486.32690	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	57471.26172
114942.50000	1486.32690	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	142857.12500	71428.56250
1530.61182	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	71428.56250	142857.12500
1530.61182	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	155038.75000	77519.37500	1352.09204
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	77519.37500	155038.75000	1352.09204

MATRIX MULTIPLY -- ATK X A

152134.43750	34722.21875	-5116.66797	41345.00000
0.0	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
34722.21875	152134.43750	-5116.66797	0.0
41345.00000	6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
-5116.66797	-5116.66797	1262.95313	-6201.74219
-6201.74219	-1240.34766	0.0	0.0
0.0	0.0	0.0	0.0
41345.00000	0.0	-6201.74219	300993.75000
57471.26172	1227.47534	51680.65234	0.0
6460.59375	0.0	0.0	0.0
0.0	41345.00000	-6201.74219	57471.26172
300993.75000	1227.47534	0.0	51680.65234
6460.59375	0.0	0.0	0.0
6201.74219	6201.74219	-1240.34766	1227.47900
1227.47412	2342.82520	-6460.59375	-6460.59375
-1076.85156	0.0	0.0	0.0
0.0	0.0	0.0	51680.65234
0.0	-6460.59375	349590.75000	71428.56250
3146.09619	51631.13750	0.0	4845.10938
0.0	0.0	0.0	0.0
51680.65234	-6460.59375	71428.56250	349530.75000
3146.09619	0.0	51681.13750	4845.10938
0.0	0.0	0.0	6460.59375
6460.59375	-1076.85156	3146.09741	3146.09424
1704.35596	-4845.10938	-4845.10938	-605.63867
0.0	0.0	0.0	0.0
0.0	0.0	51631.13750	0.0

-4345.10938	373248.56250	77519.37500	1411.51001
0.0	0.0	0.0	0.0
0.0	0.0	0.0	51681.18750
-4845.10938	77519.37500	373248.56250	1411.51001
0.0	0.0	0.0	0.0
-0.0	0.0	4845.10938	4845.10938
-605.63367	1411.51196	1411.50903	1153.14575

INVERTED MATRIX -- ATKA TO ATKAI

0.00001	-0.00000	-0.00001	-0.00000
0.00000	-0.00004	-0.00000	-0.00000
-0.00003	-0.00000	-0.00000	-0.00002
-0.00000	0.00001	-0.00001	0.00000
-0.00000	-0.00004	-0.00000	-0.00000
-0.00003	-0.00000	-0.00000	-0.00002
-0.00001	-0.00001	0.00515	0.00001
0.00001	0.00436	0.00002	0.00002
0.00726	0.00003	0.00003	0.00147
-0.00000	0.00000	0.00001	0.00000
-0.00000	-0.00001	-0.00000	0.00000
-0.00003	-0.00000	-0.00000	-0.00001
0.00000	-0.00000	0.00001	-0.00000
0.00000	-0.00001	0.00000	-0.00000
-0.00003	-0.00000	-0.00000	-0.00001
-0.00004	-0.00004	0.00436	-0.00001
-0.00001	0.00487	0.00002	0.00002
0.00339	0.00003	0.00003	0.00177
-0.00000	-0.00000	0.00002	-0.00000
0.00000	0.00002	0.00000	-0.00000
0.00000	-0.00000	0.00000	-0.00001
-0.00000	-0.00000	0.00002	0.00000
-0.00000	0.00002	-0.00000	0.00000
0.00000	0.00000	-0.00000	-0.00001
-0.00003	-0.00003	0.00326	-0.00002
-0.00003	0.00389	0.00000	0.00000
0.00423	0.00004	0.00004	0.00210
-0.00000	-0.00000	0.00003	-0.00000
-0.00000	0.00003	-0.00000	0.00000
0.00004	0.00000	-0.00000	0.00002
-0.00000	-0.00000	0.00003	-0.00000
-0.00000	0.00003	0.00000	-0.00000
0.00004	-0.00000	0.00000	0.00002
-0.00002	-0.00002	0.00147	-0.00001
-0.00001	0.00178	-0.00001	-0.00001
0.00210	0.00002	0.00002	0.00203

MATRIX MULTIPLY -- K X A

0.0	0.0	0.0	0.0
0.0	0.0	41345.00000	82690.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
69444.43750	34722.21875	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
82690.00000	41345.00000	0.0	0.0

0.0	0.0	0.0	0.0
34722.21875	69444.43750	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	-6201.74219	-6201.74219
-6201.74219	-6201.74219	0.0	0.0
0.0	0.0	0.0	0.0
1085.07227	1085.07227	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
51680.65234	103361.25000	82690.00000	41345.00000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	114942.50000	57471.26172
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
41345.00000	82690.00000	103361.25000	51680.65234
0.0	0.0	0.0	0.0
0.0	0.0	57471.26172	114942.50000
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
-6460.59375	-6460.59375	6201.74219	6201.74219
6201.74219	6201.74219	-6460.59375	-6460.59375
0.0	0.0	0.0	0.0
0.0	0.0	1486.32690	1486.32690
0.0	0.0	0.0	0.0
0.0	0.0	51681.13750	103362.37500
103361.25000	51680.65234	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
142857.12500	71428.56250	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	51680.65234	103361.25000
103362.37500	51681.13750	0.0	0.0
0.0	0.0	0.0	0.0
71428.56250	142857.12500	0.0	0.0
0.0	0.0	-4845.10938	-4845.10938
6460.59375	6460.59375	0.0	0.0
0.0	0.0	6460.59375	6460.59375
-4845.10938	-4845.10938	0.0	0.0
0.0	0.0	0.0	0.0
1530.61182	1530.61182	0.0	0.0
57423.73828	114847.43750	103362.37500	51681.18750
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	155038.75000	77519.37500
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0
51681.18750	103362.37500	114847.43750	57423.73828
0.0	0.0	0.0	0.0
0.0	0.0	77519.37500	155038.75000
-4785.69141	-4785.69141	4845.10938	4845.10938
0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0

4845.10938	4845.10938	-4785.69141	-4785.69141
0.0	0.0	0.0	0.0
0.0	0.0	1352.09204	1352.09204

MATRIX MULTIPLY -- ATKAI * PP FOR LOADING NO. 1

-0.00231	-0.00272	0.28751	-0.00127
-0.00164	0.22002	0.00049	0.00013
0.30861	0.00314	0.00223	0.18238

VALUES OF RR FOR LOADING NO. 1

14.67000	-48.00000	10.00000	55.41299
-70.08299	22.00000	64.75299	-102.08299
28.00000	131.41299	-154.08299	34.00000

MEMBER FORCES OR MOMENTS FOR LOADING NO. 1

-300.71313	-404.62134	-347.32227	-313.28174
-136.60425	-131.91577	-32.72061	-9.01279
-90.99976	-46.64575	-236.91479	-145.23637
-492.79907	-373.93951	-616.13775	-744.50146
9.01540	91.00195	164.63921	233.55933
449.89038	628.08739	751.94507	990.19922

MATRIX ANALYSIS OF MULTIPLE HIGH TOWER FRAME WITH SIDESWAY

by

YUNG-TSENG CHUNG

Diploma, Taipei Institute of Technology, 1972

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1976

ABSTRACT

The purpose of this report is to show how to solve a multiple high tower sidesway frame with different cross sections using the matrix method of analysis. This method is based on the following assumptions: (1) Structures, such as beams, trusses and rigid frames, are defined as assemblages of structural elements joined together at a finite number of discrete points, called nodes, and loaded only at these points; (2) In analyzing rigid frames the effect of the change in the length of any member on the joint displacement, due to the presence of an internal axial force in it, is negligible.

In a rigid frame with n degrees of freedom in sidesway, n displacements at n joints need to be assumed and the displacements of all joints may then be determined in terms of these n unknown displacements. This may be conveniently done by making a sketch of the joint-displacement diagram.

The advantages of the displacement method are that it permits the use of the same procedures for analyzing statically determinate structures and statically indeterminate structures and it is much easier to form the displacement-transformation matrix of the structure by the direct stiffness method for complicated structures. The high tower frame is first solved by the matrix method, then checked by the slope-deflection method. Also the solution is worked out with the help of a computer program which was written as a part of this report to check the results. This computer program is shown in Appendix A. Finally, the solution obtained by this computer program agreed with the results computed by the slope-deflection method, the matrix method and another computer program (Appendix B) furnished by Professor Rosebraugh.