

AN IN-SERVICE TRAINING COURSE IN
MODERN MATHEMATICS FOR ELEMENTARY SCHOOL TEACHERS

by 680

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CHAPTER I

INTRODUCTION

The need for in-service training in new or modern mathematics arose when these new programs began to become a part of the elementary school curriculum. Not many elementary school teachers have a thorough background in mathematics. Too, during a school year they do not always have time to pursue college hours to make up a deficiency of the moment. An in-service training course in modern mathematics seemed the best way to give the teachers an introduction to this mathematics, acquaint them with the philosophy of such a curriculum, and to orient them to new terminology and different methods of presentation of the materials to be taught.

During the winter of the school term 1964-1965 each elementary school in Manhattan held an in-service training course in modern mathematics for elementary schools. There had been three principals and two classroom teachers from the Manhattan Public Schools who had taken an NDEA Institute in modern mathematics the previous year. They were the instructors or leaders for the in-service course. All elementary school teachers were required to attend. The seventeen teachers at Marlatt and Marlatt Annex met at the Marlatt School one hour after school every other Tuesday for instruction and discussion of modern mathematics. The

instructor was Bill E. McArthur, principal.

The course was developed in some twelve sessions. While all of the teachers said they benefited from it, some of them resented being forced to take any in-service training course even though they might voluntarily have taken the same course had it been given on a volunteer basis. They preferred to decide for themselves how to spend their time and to choose their own method of self-improvement.

No data was kept on this course to show whether or not the participants did learn or make any progress. Therefore to be able to show whether or not an in-service course in modern mathematics was beneficial to teachers, the course was again offered on a volunteer basis to anyone in the Manhattan Unified School District #383 who wished to take it.

In late September, 1966, invitations were sent to all elementary school teachers in the Manhattan Unified School District #383. At that time there were 123 teachers in the elementary schools. Parents of the Marlatt area were invited through the school bulletin. A copy of the invitation is in Appendix A.

Thirteen individuals responded to the invitation. Ten were Marlatt teachers, two were teachers from other elementary schools in the district, and one was a resident near the school. Of this group of teachers one was a kindergarten teacher, two taught grade three, three taught

grade four, two taught grade five, and four taught grade six. One sixth grade teacher attended only two sessions, one fourth grade teacher attended six of the sessions, and one sixth grade teacher attended irregularly. However, all except the one sixth grade teacher who dropped the course took the concepts test both at the beginning and the close of the sessions.

The body of the report contains a description of the content of each of the sessions of the in-service training course as held during the fall semester of 1966. A modern mathematics concepts test was given at the beginning and at the close of the sessions. The comparison of the results of these two testings and an item analysis of the test are given in Chapter IV. The author has added what he considers to be implications for teaching made on the basis of the number of times any particular type of item was missed by the persons taking the test. The appendix has samples of the hand-out materials distributed during the course. These materials were samples of different ideas for construction of test or drill materials, ideas for presentation of modern mathematics concepts, and original materials for the teachers to use in the classroom for the enrichment of the mathematics program then being taught.

CHAPTER II

REVIEW OF RELATED LITERATURE

There has been much research in the field of mathematics education concerning pupil performance, the grade or age level when various concepts should be presented, the content of the curriculum, methodology, materials, and teacher effectiveness. The genesis of the "modern" mathematics in the latter 1950's further pointed up the need to take a probing look again at mathematics programs. Weaver stated it this way:

Every elementary school situation today faces the challenge of improving its program of mathematics instruction. This cannot be accomplished without appropriate provision for the in-service education of teachers.

Improvement cannot take place over-night, nor can it be effected in any one way. The plans must be long-range ones; they must embrace a variety of approaches to meet a variety of individual needs; and they must provide recurring efforts in the face of an everchanging teacher population.¹

There is no disagreement with this statement in any of the literature concerning mathematics education.

Another phase studied is the program for prospective teachers, both undergraduate and graduate, and the background of teachers in the field.

¹J. Fred Weaver, "Curriculum Development and In-Service Education in Cincinnati," The Arithmetic Teacher, 10 (March, 1963), 154-158.

Two early programs developed in modern mathematics were those developed by the Greater Cleveland Mathematics Program for kindergarten through grade three, published by Science Research Associates, Inc., 1962, and by the School Mathematics Study Group, published finally by Leland Stanford University and distributed through Yale University, 1961 (4-6)-1963 (K-3). Methods, terminology, and concepts presented in these early programs and in their development and in their being used with children proved to be so different from the traditional arithmetic it was soon discovered that teachers, indeed, needed training and work with these specific understandings to be able to teach them to children. This became especially true when other publishing companies began to develop "modern" programs in mathematics and tried to get them adopted by various schools and states.

The universities and colleges soon began offering courses in modern mathematics as requirements for elementary school majors and for teachers already in the field. The Federal Government assisted the universities and colleges in this endeavor by funding institutes for teachers and principals under the National Science Foundation Act. The author, two other principals, and two classroom teachers from the Manhattan Public Schools attended such an institute at Kansas State University on Saturday mornings through the fall of 1963 and the spring of 1964. Summer institutes were

held later. In general, the people accepted for these institutes were accepted with the understanding that each would return to his respective district and conduct an in-service program. Since the author and the others from the Manhattan Schools who attended the NSF Institute were never asked if they had conducted any in-service programs, he assumes there is no record as to how many in-service courses were held as a result of these institutes over the United States. The five participants from the Manhattan Public Schools did hold their institutes as described in the introduction of this paper and the author conducted, in addition, the one on which this report is based. The number of such in-service programs could have been quite large as the NSF Institutes were held in all areas of the United States and are still being held during summers. The enrollment for each institute was usually limited to about thirty.

Dr. Joseph A. Izzo and Mrs. Ruth Izzo who conducted some summer NSF-sponsored institutes did some follow-up studies. They surveyed twenty-three in-service programs conducted during the first half of the school year of 1963-1964. Most of the participants had been from northeast United States, but one was from California. Their survey indicated that more than 800 elementary school personnel were directly involved in the in-service programs. These 800 people were teaching mathematics to about 20,000 school children. These statistics show how wide-spread the effects

of one institute could be. The conclusions in the Izzo study pointed up the need for teacher re-education and suggested in-service courses, college and university courses, with elementary teachers, supervisors, and principals taking the leadership role.¹

A description of an in-service course for elementary school arithmetic teachers in Omaha tells of this type of cooperation.² The College of Education of the Omaha University and the Omaha Public Schools worked together to set up a course to help teachers learn about new developments in mathematics, newer materials, and new methods of presentation. College credit was given for the course. While it was intended to be a one-time course, it was so successful it was to be repeated. The pattern of classes was full group instruction, then breakdown into primary or intermediate, or grade level groups for more specific instruction and material demonstrations.

An NDEA financed program during 1963-1964 in California was reported by Gerald W. Brown, University of

¹Joseph A. Izzo and Ruth Kelley Izzo, "Re-education in Mathematics for Elementary School Personnel: In-Service Programs--One Way to Solve the Problem," The Arithmetic Teacher, 11 (Oct., 1964), 413-417.

²Dan Tredway, "An In-Service Course for Elementary Arithmetic Teachers," The Arithmetic Teacher, 10 (Oct., 1963), 344-346.

California, Riverside.¹ This program included only about 75 teachers from five elementary schools. A consultant was hired to conduct the program. Enrollment was voluntary and attendance was not kept. Evaluative data were obtained from a five-page questionnaire. Teachers helped formulate plans and arranged for twelve biweekly presentations by a university professor. Every other meeting was a discussion meeting. Implied conclusions were that teachers would recognize their own needs or deficiencies and attend such an in-service program and that the biggest share of them did gain from the experience. The statements by teachers in this program were much the same as those given by the teachers in the in-service course used as a basis for this report.

Schools which had Educational TV available often used that source for both in-service training for the teachers and conducting classes directly for the children while the teachers monitored the pupils. In-service films were and are available also.

In other instances when a state or city adopted a modern mathematics text, consultants from the publishing company would come in and conduct in-service courses for the teachers who would be teaching the texts. The

¹Gerald W. Brown, "An In-Service Program in Arithmetic," The Elementary School Journal, 66 (Nov., 1965), 75-80.

organization was set up to accommodate the school involved and helped the teachers become acquainted with the philosophy, terminology, methodology, and materials for that particular text.

As to the efficacy of in-service courses in mathematics for elementary school teachers, most of the literature reviewed reported positive gains in teacher attitude and knowledge and a corresponding gain in their pupils' achievement. That last is quite difficult to measure because of the near impossibility of maintaining a stable control group. However, Dr. John L. Creswell, University of Houston, Houston, Texas, had considerable doubts as to the effectiveness of modern mathematics workshops. This is indicated in his report published in March, 1967.¹ Dr. Creswell related that Robert Bradford who had conducted a workshop in Texas in the summer of 1965 also found different reactions.

Bradford planned to use a pretest-posttest approach. When the teachers were administered the pretest, they objected so strenuously that this method of evaluation had to be discontinued, even though each teacher had been assured that the scores were to be kept strictly confidential.

The author (Creswell) has had similar reactions from teachers enrolled in workshops conducted by

¹John L. Creswell, "How Effective Are Modern Mathematics Workshops?" The Arithmetic Teacher, 14 (March, 1967), 205-208.

him. These teachers seemed very insecure in their knowledge of the subject matter, and many became quite agitated when the subject of their being tested was discussed.¹

As a result of testing 1,075 school teachers of grades 1-6 in eastern Texas with a 120-item test, Dr. Creswell concludes:

The foregoing test results seem to indicate that college courses are far more effective in preparing teachers for teaching the new mathematics than are the present type of in-service workshop programs, as far as content is concerned. It is recommended that more effective in-service training techniques be devised, oriented more toward subject matter than has previously been the case, especially in the area of the new mathematics.²

In contrast, Hand made an evaluation of the effectiveness of in-service instruction for elementary teachers conducted in Georgia during the year 1965-1966, at 21 centers. Her conclusions were favorable to the workshop in-service programs. Especially noticeable was the one, "Type of Instructor - the achievement of the group taught by public school instructors was significantly higher than groups taught by college instructors or graduate students."³

A bibliography of books published during 1960-1964

¹Ibid., p. 206.

²Ibid.

³Edith Frances Hand, "Evaluation of a Large Scale Mathematics In-Service Institute for Elementary Teachers," Dissertation Abstracts, XXVII (Dec., 1967), 2118-2119A.

as listed in The Arithmetic Teacher¹, January, 1965, would be an excellent help for anyone wishing to set up in-service education courses in modern mathematics. While some of the listing would be outdated by now, they would provide a place to begin.

Indications are that in-service programs will continue to be used by schools to help teachers become acquainted with the teaching procedures that any school might have adopted, to help teachers in self-improvement, and to help them become more effective teachers.

¹J. Fred Weaver, "The Mathematics Education of Elementary School Teachers: Pre-Service and In-Service," The Arithmetic Teacher, 12 (Jan., 1965), 71-75.

CHAPTER III

CONTENT OF THE TRAINING SESSIONS

Session 1, October 4, 1966

As stated in the invitation the first session was to be an organizational meeting. At this time it was explained there would be no professional growth or college credits involved. The participants were also made aware that any data coming out of the course would be used in a report as part of the work of the instructor towards a Specialist Degree.

The content of the course was discussed and the participants suggested specific areas in which they would like information and explanation. The time for meeting was set to be each Tuesday from 4:00 to 5:00 p.m. No specific number of sessions was decided upon. The group did indicate it preferred not to run into the Christmas season.

Since time did not permit the reading and discussion of "The Story of Numbers" written by the instructor, it was requested that the story be dittoed so each participant could have a copy and so it would be available for use in his own classroom. This was agreeable, and copies of the story were prepared and ready for distribution the following week. A copy of "The Story of Numbers" is in Appendix A.

Session 2, October 11, 1966

"Sets" was the topic of discussion of this session. This included the definition of a set as a collection of things. Hence, before any set could be identified a description of its members or elements was necessary. This led into a discussion of the notation of sets, one-to-one correspondence, number or cardinality of sets, comparison of sets, union of sets, intersection of sets, venn diagrams, and other exercises with sets.

The differentiation between equal and equivalent sets helped the group better understand the preciseness necessary in modern mathematics, particularly in discussing number and numeral. A short discussion of the null set and its various notations and universal sets concluded the discussion.

The hand-out for this session was a sheet giving some mathematical definitions which included the most common symbols used with sets. A sample is in Appendix B.

Session 3, October 18, 1966

Because some members of the "set of persons" in the class were uncertain about various aspects of sets--especially null sets and universal sets--a review of these was in order. The hand-out of the previous week served as a basis of discussion.

The rest of the period was used for the group to take

the Stanford Achievement Test, Intermediate II, Modern Mathematics Concepts Test, published by Harcourt, Brace, World, Inc., New York, 1965. This was the instrument selected to make the check as to whether or not this in-service training course was effective in increasing the modern mathematics concepts of the group involved. An advanced form of the test was available but the one chosen seemed to be more representative of the concepts the teachers in the group would be using in their own classrooms. A copy of the test is in Appendix B. A separate chapter deals with gains or losses made by the participants on this test.

Session 4, October 25, 1966

Scored answer sheets on the mathematics concepts test were returned to the group.

The discussion for this session centered on the abacus and its value for use in elementary school classrooms. Mr. Mutsugi Ohno who was educated in Japan and who currently is a glass blower at Kansas State University made the presentation.

Mr. Ohno had brought several abacuses and the teachers had brought some. For demonstration he used a soroban which is a modification of the Chinese suan pan. Modern ones have only five beads--one above and four below a metal strip marker. Mr. Ohno could perform all mathematical processes on his soroban and with speed. In

verbalizing how to do a process he often lapsed into Japanese in search of words to explain his thoughts.

This was probably the most interesting session of the whole course. In visiting about it afterwards with the teachers, the instructor sensed they realized they had participated in a real teaching-learning situation of a kind they should be using in their classrooms.

Session 5, November 8, 1966

There had been no meeting on November 1, 1966, as that was the time for parent-teacher conference for reporting pupil progress.

The hand-out sheet giving a summary of sets was reviewed and an attempt was made to clarify questions from the group. There was further discussion on universal sets and explanation showing that $\emptyset$ is always a subset of any set even though it is an improper subset. More terminology was reviewed. The group worked out tables for intersection and union of a group of four sets.

The rest of the period was devoted to a discussion of natural or counting numbers, number and numeral, and order of numbers as a property. Base was defined as the collection point of any number system to go into the next place value. After further discussion of what base any number system is and digits, the group was assigned the project

of making a base twelve matrix for multiplication to have ready for the next session.

Session 6, November 15, 1966

Natural numbers were reviewed briefly and it was pointed out that by including zero with them the set of whole numbers was obtained. This led into a discussion of how zero was different in its operational effects from the natural numbers. It was agreed that one and zero were the most important numbers because one is an entity and the beginning, and zero is what makes our place value system really function. In fact, if we use the one's place as the beginning place for counting place value, the place values either way from the one's place correspond. For example tens are one place to the left of one's place and tenths are one place to the right; thousands are three places to the left of one's place and thousandths are three places to the right of one's place.

After the participants' base twelve multiplication and addition charts were checked, a summary of the needs of any number system was made. These include:

- number names,
- an order of numbers,
- a number base, and
- definition of operations or tables.

To confirm this idea the following artificial number system

was made up and addition and multiplication tables were made for it.

☆ skunk | eenie ∠ meenie Δ miney □ moe

☆ skunk	∠Δ meenty-miney
eenie	∠□ meenty-moe
∠ meenie	Δ☆ minety
Δ miney	Δ minety-eenie
□ moe	Δ∠ minety-meenie
☆ tees	ΔΔ minety-minety
tee eenie	Δ□ minety-moe
∠ tee meenie	□☆ moety
Δ tee miney	□ moety-eenie
□ tee moe	□∠ moety-meenie
∠☆ meenty	□Δ moety-miney
∠ meenty-eenie	□□ moety-moe
∠∠ meenty-meenie	☆☆ eenieless

+	☆		∠	Δ	□
☆	☆		∠	Δ	□
		∠	Δ	□	☆
∠	∠	Δ	□	☆	
Δ	Δ	□	☆		∠
□	□	☆		∠	Δ

X	☆		∠	Δ	□
☆	☆	☆	☆	☆	☆
	☆		∠	Δ	□
∠	☆	∠	□		Δ
Δ	☆	Δ		□	∠∠
□	☆	□	Δ	∠∠	Δ

The building of this number system and the tables gave the group a better insight into number systems. This in turn suggested to them that having children construct and use other number "bases" would help them to understand their own decimal system. A closer scrutiny of the tables reveals the difference between one and zero when used as identity numbers in addition and multiplication. It was about this time that Scott-Foresman Co. distributed 1967 calendars with numeration for two months each in the following bases: five, seven, two, six, eight, and twelve. Each member of the group was able to get such a calendar.

If 10 is used for the collection point of a base, then by using $\leftarrow 10^2, 10^1, 10^0, 10^{-1}, 10^{-2} \rightarrow$ and so on as the basis of construction of this system, any base can be converted to base ten very easily. This is accomplished by multiplying the number in a place position by the base to the power of that position and then adding them all together. A decimal base may be converted to any other base by dividing the number to be converted by the number of the base desired and using the remainders of the successive divisions as the number of each place in order. The first remainder would be the ones, the next would be the base first power (tens), and the next would be the base squared (100's), etc.

Session 7, November 22, 1966

Since whole numbers and the decimal system were discussed the previous session, the discussion of this session naturally considered the operation of these numbers. The best conclusion the group could make for a definition of addition was that "addition is the union of disjoint sets." Subtraction is the inverse of addition. Identity element, closure property, commutative property, and associative property were discussed particularly as items the participants had been using in their teaching but perhaps not using that terminology.

If multiplication is to be considered an operation in itself and not just repeated addition of like numbers, then this operation must be described and its algorithms formed. This topic created a lively discussion. Multiplication was approached by the use of the Cartesian Product of two sets--a one to many pairing or matching instead of one-to-one. Further discussion as to the difference between the identity element of addition and multiplication emphasized the difference between zero and the counting numbers. Too, multiplication has commutative and associative properties besides the distributive property of multiplication over addition. This last is the basis for the algorithm for the use of more than one digit multipliers.

The discussion of division as the inverse of

multiplication precipitated a comparison of the properties of addition and subtraction, and multiplication and division, and why zero cannot be a divisor.

Some ideas were given on how to help children better learn the addition and multiplication facts and how to use an "array" to teach multiplication. Hand-out sheets of examples of different types of drill to use in teaching these processes were given to the group before dismissal.

Session 8, November 29, 1966

The discussion of numbers and their factors was continued from the previous week. There was much discussion about "operation" on numbers as well as reporting on the outcome of the trials of various ideas with the group's own pupils. Composite and prime numbers became the topic of interest. Charts on the order of the "Sieve of Eratosthenes" were handed out. These charts showed arrangements of the numbers from one to 100 for use in developing a pattern or patterns for eliminating non-prime numbers. The group readily saw the relationship of the use of this chart for use later in teaching the finding of the greatest common factor, the lowest common multiple, and in helping make rules of divisibility. Factor trees were demonstrated to show another interesting way to find the prime factorization of a number.

After practicing factoring in several ways, the group was in agreement with the statement of THE FUNDAMENTAL THEOREM OF ARITHMETIC: Except for the order in which the factors are written, a composite number can be expressed as a product of prime numbers in only one way.

The hand-out sheet dealt with some problems concerning the distributive property of multiplication over addition and a group of definitions from the materials discussed.

Session 9, December 6, 1966

"Is there a greatest prime number?" was the question leading to a review of prime numbers. It was agreed that there is not and a statement of the Postulate of Bertran was given: Between any counting number, other than one and its double, there is at least one prime. Perfect numbers--those numbers which equal the sum of its factors including 1 but not the number itself--were harder to find. Six $(1+2+3)$ and twenty-eight $(1+2+4+7+14)$ are the smallest ones. While it is not known how many there are, it is known that they all end in 6 or 28. Prime and perfect numbers are intriguing to intermediate age children and lead them into much enrichment and drill work.

The set of rational numbers as a topic of discussion easily followed the discussion concerning whole numbers. Use of a number line helped explain how whole numbers and counting numbers are subsets of rational numbers and how

they complement one another. Fractions were defined as the set of numbers whose elements are obtained by dividing a whole number by a counting (natural) number. If one thinks of fractions as another ordered pair, the meaning of "ordered" is reemphasized as they have notation order and counting order. The etymology of the word fraction shows it to come from frangere meaning to break. Yet, to try to define a fraction as a part of something is misleading. While part of a whole can be represented by a fraction, so can a subset of a set or a ratio be so represented. Some older systems of notations using fractions were mentioned. The Egyptians used only unit fractions so our fraction $5/8$ would have been $1/2 + 1/8$. The Babylonians who used a sexagesimal system of numbers had better use of fractions because sixty is divisible by so many numbers.

Use of shaded congruent regions of geometric plane figures is probably the most common way to introduce fractions to children. The making of a multiple fraction chart and number lines helped demonstrate that any fraction can have many names; that every whole number is an element in the set of fractional (rational) numbers, but that not every fraction is a whole number. Such a chart showed equivalency of fractions and led into a discussion of how to change any fraction of a given denominator to another equivalency. This is the time when the previous study of factoring came in to use. This was another situation in which the

instructor could stress the difference between multiplication and addition. Children can easily see that both the numerator and denominator of a fraction may be multiplied by the same number without changing its value, but that adding the same number to both denominator and numerator does make a new fraction of a different value.

As well as knowing the algorithms for computing with fractions, children must understand that the properties for use of whole numbers are the same for fractions or rational numbers. The simplest device to illustrate multiplication of fractions is the use of shaded regions. Division is the inverse of multiplication and is defined as: $p/q \div r/s = t/u$ when $p/q = t/u \times r/s$ which is the same as $a \div b = c$ when $a = b \times c$. Then zero cannot be a denominator for the same reason that zero cannot be used as a divisor. ($a \div b = c$, b cannot be 0 for then $a = 0 \times c$ which is 0).

This was much too large an area for one discussion period--in fact all the topics were--and only the high spots were touched. Further discussion would be planned for at future faculty meetings. Hand-outs were given for extra types of drill to arouse thinking.

Session 10, December 13, 1966

Questions concerning rational numbers were answered

and further explanation was given about any phase in question. Some "short cuts" for computing were illustrated. Some of these were using aliquot parts for multiplying, multiplying by 11, gelosia or lattice multiplying, adding to or subtracting from subtrahend and minuend, subtracting mentally and others. It was mentioned that copies of The Trachtenberg Speed System of Basic Mathematics were in the Marlatt Professional Library. It was stressed that any "short cut" was useful only to the extent that any person understood the number system and why the "short cut" worked.

Multiplication and division by 10, 100, 1000 and other multiples of ten were discussed. This went into the idea that decimals are an extension of whole numbers, as special fractions, and their relationship to ordinary fractions. An astonishing thing that evolved from the discussion was that several in the group had not realized that the zeros on the end of a divisor could be removed (divide by 10, 100, etc.) if the decimal in the dividend were set over to the left as many places as there were zeros on the end of the divisor (divide the dividend by the same number of tens as had been taken into the divisor), whether or not there were zeros on the end of the dividend.

Since this was to be the last discussion session, some of the new terminology on geometric forms was discussed. These included line segments, point, set of points, ray, closed curved lines, closed surface, and others. Had time

permitted, topology would have been an interesting and enriching subject to discuss to show how more advanced pupils would gain by working with topology on their own.

The session closed with a short demonstration using a "small modular" system and clock counting. More sample drill sheets were handed out.

Session 11, January 10, 1967

This last session was merely taking the mathematics concepts test for the second time. The comparison of the two testings is included as a separate chapter.

CHAPTER IV

RESULTS OF THE TESTS

The instrument used to measure the gain or loss in the understanding of mathematical concepts as the result of the members of the group attending the sessions of the in-service course was the Stanford Achievement Test, Intermediate II, Modern Mathematics Concepts Test, published by Harcourt, Brace, World, Inc., New York, 1965. This particular test was selected because the instructor believed it contained questions and problem situations covering what he anticipated the content of the course would be. While there was an advanced form available, he did not wish mathematical computation to become too involved as it might overshadow the concept idea. Too, as stated earlier, this intermediate test did cover the materials the participants could use to advantage in the grade levels they taught at this time.

Figure I shows a comparison of the score made by each participant at the beginning of the course when he first took the test and at the close of the course when he took the test the second time. Only one regressed. Three gained one point, and the rest gained four or more points. The greatest gain was made by the person who was not a teacher in Unified School District #383. She had taught on the secondary school level, but she had never taught mathematics. She gained 11 points. The class median was 46 at the first

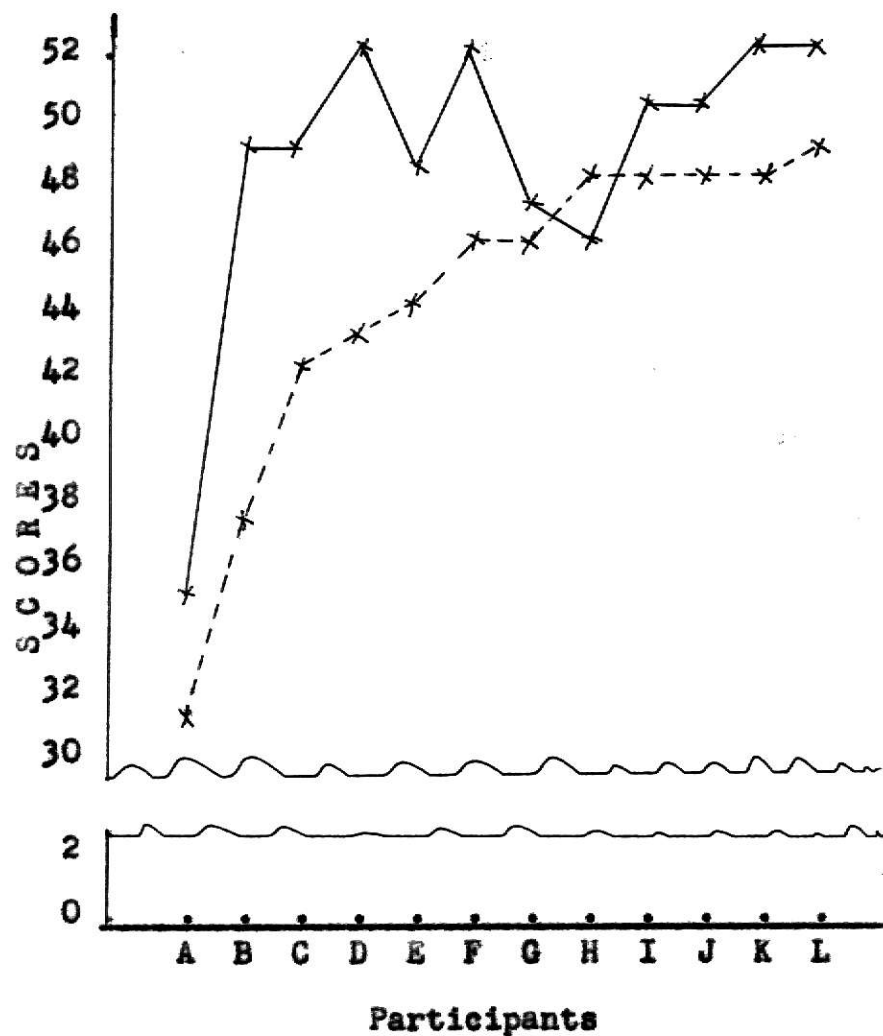


FIGURE I

DISTRIBUTION OF SCORES ON THE STANFORD MODERN
MATHEMATICS CONCEPT TESTS FOR OCTOBER, 1966,
AND JANUARY, 1967

October, 1966 ----- January, 1967 _____

testing and rose to 49 at the second testing.

Following are tables giving an item analysis report. Each item is categorized according to the concept that particular item was supposed to test. A statement is made describing the most frequently missed items of the testings. Blank spaces indicate zero.

TABLE I
NUMBER SYSTEMS AND NUMERATION

Test Item	2	3	4	6	7	8	20	30	44	45	49	51
Times missed: Oct.				2	1	3	1	3	6	3		3
Jan.							1	2	3	4		1

Item 30 had to do with changing Roman Numerals to base ten Arabic numbers. Item 44 was the selection of a set of stars which were equal to a specified base two number. Item 45 was an addition in base four. It would appear that more work in different bases and a better understanding as to what a "collection point" really is were needed.

TABLE II
GEOMETRY AND MEASUREMENT

Test Item	9	10	11	15	16	18	27	29	31	32	37	39	43	50	53
Times missed: Oct.			1	3	2		1	1		3		5	7	4	2
Jan.			1	5	1				1			1	7	3	1

Items 15 and 43 which were missed the most times in the January testing had to do with visual judgment of measure in comparing the size of regions or line segments. Little time had been spent in discussing geometry and measures in this course. However, at least one session had been given to this topic during the 1964-1965 course. There is much to teach children about measurement.

TABLE III
OPERATIONS AND NUMBER PROPERTIES

Test Item	1	24	26	34	35	40
Times missed: Oct.			2	2	2	1
Jan.					1	

Item 35 had to do with the closure property. It would seem this section had been fairly well understood by the participants.

TABLE IV
MATHEMATICAL SENTENCES

Test Item	5	12	19	28	52
Times missed: Oct.				3	1
Jan.					1

Both items 28 and 52 had to do with interpreting a number

sentence. The inference of this section had to do with making a number sentence to fit the problem and then solving the problem more as an algebraic equation than by what is ordinarily thought of as arithmetic.

TABLE V
FACTORS AND PRIMES

Test Item	17	46	54
Times missed: Oct.	2	5	7
Jan.		2	5

Item 46 was the selection of a set of prime numbers from a series of sets of mixed composite and prime numbers. Item 54 was the selection of a factored common denominator. A point which should have been emphasized was that any number in a factored common denominator must appear as many times as that factor appears in any of the prime factorization of the denominators being converted.

TABLE VI
SETS

Test Item	23	25	33	47	48
Times missed: Oct.	5		1	1	7
Jan.	2		1	1	3

Item 23 had to do with the translation of a described situation into set language. This particular set was a null set. Item 33 concerned a set intersection, item 47 equal sets, and item 48 concerned intersection of sets but using the symbol instead of the word. The participants still needed more practice with sets and in the understanding of sets.

TABLE VII

LOGIC

Test Item	21	22	36	38
Times missed: Oct.	2	2	3	6
Jan.			1	4

Both items 36 and 38 had to do with the "If . . . then" type of question. They also involved a precise understanding of some mathematical symbols. This is an interesting area on which very little time was spent. Had time been available the group, no doubt, would have enjoyed making "truth tables."

TABLE VIII

SYMBOLS AND DEFINITIONS

Test Item	13	14
Times missed: Oct.	2	1
Jan.	1	

Item 13 involved reading mathematical sentences meaningfully while item 14 asked for a mathematical symbol to be placed in a frame to make a sentence true. There were not enough items in this area to actually test knowledge of symbols as there are many modern mathematical symbols to be learned.

TABLE IX
GRAPHS AND TABLES

Test Item	41	42
Times missed: Oct.	2	5
Jan.	2	6

Both of these items involved the halving or doubling of one or both of the dimensions of rectangles, then rereading from a table the quantity which could be placed in a region affected by the halving or doubling of dimensions. This is a difficult concept to teach. Nothing had been done with geometry in this session other than to note that the sides of angles were called "rays" in modern mathematics.

The two testings showed the participants did make some gains. They also showed the group could have used more discussion and understanding of some of the areas if they were to be effective teachers of modern mathematics. Perhaps as they teach these concepts to children they will approach them more moderately and will obtain a greater grasp

of the logic of mathematics, realizing that one concept evolves from the understanding of a previous concept.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

In addition to the Modern Mathematics Concepts Test, each participant was given a self and course evaluation check sheet to complete. A sample of this is in Appendix A. This check sheet was mailed to each of them but one. This was the non-teacher who had moved away and left no forwarding address. This mailing was done in August, 1968, and eleven returned the form. The delay in distributing the evaluation check sheet gave the participants time to employ the information and concepts learned in the course, as the local school district had adopted a modern mathematics text in the elementary grades that next term. It also gave them time to forget the details of the course!

For the Self Evaluation part of the check sheet the responses tabulate:

Self Evaluation

1. The purpose of the study was clear to me.
no-0 somewhat-1 yes-10
2. My interest at the beginning of the study
would best be described as
low-0 medium-6 high-5
3. My interest level at the end of the study
would best be described as
low-0 medium-2 high-9

4. In comparison with other group members the quality of the contributions I made were

poor-1 average-9 good-1

Item 5 was: The two greatest benefits to me of working on this study were:

Four participants stated that the introduction to and learning modern mathematics terminology was the greatest benefit.

Three believed that the reinforcement of concepts learned in an earlier college mathematics course was most beneficial.

Two others stated much the same idea saying it was a boost to a poor mathematics background. Other responses included:

"Mr. Ohno's teaching us how to use the abacus."

"It helped evaluate my own progress."

"Gave me a better historical background of mathematics."

"A gain in new approach in teaching math."

"Ideas for practical classroom adaptation."

"Discovering the many facets of working with mathematical problems."

"Good math review and learning about square root and the abacus."

"Learning more about modern mathematics."

"Realizing more fully the benefits that the children

using the program receive."

"Helping me really see the 'why' of modern math, for until this time it has seemed 'much ado about nothing.'"

"Adding 'spice' to the regular math lessons in my own classroom."

"This was an introduction to modern math for me."

Item 6 asked for ways the course could have been of more benefit to the participant. The responses can be summarized:

Two would like to have been using a modern math text in their teaching at the time so they could have used the materials given out along with it. Two others would have liked greater correlation to the scope of their particular grade levels. Others believed that required assignments of study sheets or problems in a book would have been beneficial. And, finally, some would have had more sessions. At least it was gratifying to know they wanted more.

The section of the check sheet asking for course evaluation contained these responses:

1. Were the topics covered suitable for your own classroom use?

no-0 partly-5 usually-6

2. Were the hand-out materials adaptable for your own classroom use?

no-0 partly-4 usually-7

3. Did the concepts covered in this course help you feel more at ease in your classroom when the arithmetic text was changed to a modern one?

no-0 somewhat-2 yes-4

Five said they could not give a valid answer to the last question as they had not yet taught the material to any class.

Question 4 of this section asked, "What would you have deleted or added to the course?"

No participant said he would have deleted anything. A few suggested some additions:

"A comparison of modern math books." (These were available and mention was made of them but no comparison was done in class.)

"More study sheets for practice."

"Make some aids in the course." (Quite a number of such aids were demonstrated, but the participants were not required to make any. Several did, and used them with their pupils.)

"More work in the bases."

"More time spent in learning the 'why' of math. i.e. why the divisor in fractions is inverted." (The instructor felt the 'whys' were the basis of the whole idea of modern mathematics.)

"More work in the field of geometry."

Response to item 5, "What is your opinion of

in-service courses such as this?" could be briefly summarized as: They are beneficial. They should be held at an hour different from 4:00 p.m.--preferably on released school time. They should not be compulsory. One participant suggested more of them should be held on topics of interest and need to the teachers. Another stated that in-service courses should be held any time new texts or other materials were being adopted. Thus none of the group disavowed the benefits of a well organized and presented in-service training course. Their objections centered on the time of day, required attendance (the first session), and not using school time for it. Some teachers would liked to have had each grade level separate with the hand-outs for a specified level. This was particularly true in the first course. The instructor believes all-over concepts are more important with general suggestions. From this each participant could adapt such ideas and materials according to the requirements and capacities of any group he is teaching. Too, when the group consists of teachers of several grade levels, the participants are able to see better the vertical structure of the whole program rather than being limited to one small horizontal area. The instructor had made his position clear on this point at the beginning of the second session. This might explain why no first or second grade teachers enrolled for the course. The instructor realized some of the group took the course because of loyalty to him and to help him

with his project.

The instructor felt some definite benefits did occur. The testings showed the participants did gain in knowledge of mathematical concepts in modern mathematics as a result of their taking the in-service course in this subject. Some gained more than others and apparently one was more confused than before.

The participants did try out the ideas in their classrooms and many used the hand-out materials as new ways of constructing both learning and drill sheets as well as tests. These and their discussions may have been the greatest value of the course. More teachers used the three levels of Enrichment Mathematics, Programs, A, B, and C, published by Harcourt, Brace and World, Inc. to supplement their basic texts than had been done previously. These were a programmed type materials on modern mathematics for use by intermediate grade students. Each pupil could progress as fast and as far as he wished in these books. This past year Unified School District #383 went entirely into a modern mathematics program published by Harcourt, Brace, and World, Inc. The first and second grades had used the "modern" edition the previous year. Because the intermediate grade teachers had been superimposing the new math on the traditional math program of the year before, the Marlatt teachers and pupils made the transition with little difficulty.

Another benefit gained from this specific in-service training course was the greater amount of supplementary materials purchased for enrichment of the mathematics program. As the members of the group obtained information about materials and types of materials, they requested more for use in their classrooms. The Marlatt Elementary School Library now contains sixteen titles of books dealing directly with some phase of mathematics in addition to numerous ones which have to do with counting or number rhymes and similar beginning mathematical concepts. A number of film strips on mathematics were also purchased. The faculty professional library now has several books and pamphlets on mathematics. Among these were sets of elementary school texts such as the Mathematics for the Elementary School by the School Mathematics Study Group published by the Yale University Press, the Addison Wesley series, and others. The school also keeps a subscription to the magazine, "The Arithmetic Teacher."

When only 13 of 126 teachers start a voluntary in-service course, it would seem evident that teachers do not willingly exert themselves to take such a course for self-improvement only. Credit of some kind given might have provided a greater incentive for them to enroll. For example, "school board credit" to be counted towards their professional growth requirements might have been arranged. The fact that only one teacher from another attendance center

of the district completed the course, and he with irregular attendance, implies teachers are unwilling to exert real effort to get to such a course even though several teachers from other buildings had evinced an interest in modern mathematics courses. Twenty-five of the 126 teachers were listed in the Unified School District #383 Administrative Handbook for 1965-1966 as being new to the district. It is possible all 25 of them had had college courses in modern mathematics.

In spite of the evidence above, this is not meant as a blanket indictment saying teachers have negative attitudes towards in-service programs. It is the belief of the writer and apparently the conclusion of most writers in the field that teachers are anxious to improve themselves and will sacrifice their own time and family to do it.

The main complaint of teachers concerning in-service programs is the time--both in hours and the time of day--needed to work in in-service programs. Preparation for their classroom work and the teaching itself often have them physically and mentally exhausted by the time of the afternoon dismissal. "By four o'clock, I could care less about self-improvement!" one teacher expressed it. Some of the training might be done on released school time during the regular school day when substitute teachers take over in the classroom or pupils are dismissed early. Finances often prohibit the first, and bus schedules the latter. Before

school convenes mornings is not good for in-service programs. Programs at this time of day take the teachers' minds off their teaching plans for the day and the edge off their energy, cutting down their efficiency with their pupils.

Teachers do not object to an occasional evening or Saturday meeting, but do object to any required series of such timed meetings. These are the times in addition to summers when they obtain college credit for certificate renewals, to meet professional growth requirements, or to advance a step on the salary schedule. These are their times to build a personal life, pursue hobbies, and in general regenerate themselves for the next day and the press of the days to follow.

One solution to the time problem would be to extend the contractual time of teachers perhaps four to six weeks above the time they are to be teaching in the classroom. They would be on salary for this time and it could be devoted to in-service programs, curriculum work, and other phases of instruction to which the teacher could give his full attention and working time. So much of this business of teaching must be done outside the regular classroom that this would seem to be a fair way to care for some of it. Part of these days might be assigned for programs before the beginning of the school term in preparation for the new term, and part might be assigned for the closing out and evaluating the preceding term at its close. Each district

would have to decide how it could best use the days and the type of work which would be most beneficial to its pupils and the progress of reaching its planned goals. This type of plan would permit the teacher to devote practically all his energy and thinking to the children under his instruction during the days of classroom sessions, without interrupting and dividing his attention during the year.

The influx of "multi-media" and electronically controlled programs, computerized enrollment, and similar devices assures the continued need for in-service training for teachers already in the field. Since each school needs to select the items in terms of its own constituency, the programs must vary from school to school. Thus, no matter how much training a teacher may have had before he assumes duties in any school, he will need further instruction in the aims, methods, aspirations, and use of materials for that specific school. It would seem that in-service programs are here to stay. Perhaps use of more efficient curriculums, programs, and materials will lighten the load of teachers while actually with the pupils, so that much of their time will be devoted to the planning and preparation of tailoring programs for each pupil. This could be, then, just part of the regular school day and be effected during regular school hours. Increased use of "para-professionals" can help relieve teachers of many non-teaching duties and let them really become professionals.

Regardless of time or place, in-service programs must be purposeful and practical. They must be presented as well as school administrators and education departments of colleges and universities insist teachers should perform in the classroom. When teachers see practical results from their efforts they do not offer complaints.

Some other activities in which the author has participated concerning modern mathematics included talks and demonstrations at Parent Teacher Association meetings, being a panelist at a local teachers' institute, and working directly with various classes of elementary school students at Marlatt School when requested to do so by the teachers. He has taught in a mathematics workshop and a regular summer school session in mathematics for the elementary school teacher on the Kansas State University Campus, and he has taught the mathematics interest groups for grades 3-4 and 5-6 for the public schools' summer school.

Actually the author was not discouraged by the seemingly small enrollment in the in-service training course described in this report. He gained considerable knowledge from teaching the course a second time as well as enjoyment from working with mathematics and a group of interested adults. In fact, he would be willing to teach another series of lessons should enough teachers and/or parents desire them.

Recommendations for any in-service training course:

1. In-service training inauguration and organization should involve both teachers and administration.
2. Specific goals or ways in which the course would improve instruction should be evident.
3. The format and calendar of meetings should be decided upon from the beginning of the course.
4. If the number of sessions needed to cover the topic for in-service training needs to be extensive, the teachers attending should receive some kind of credit to advance them on the scale of promotion or salary, or they should be relieved of part of their regular classroom duties so they may concentrate better on the study at hand. This release might be via substitutes, para-professionals, or early dismissal. The teacher might simply receive extra pay for this extra work, perhaps through an extended contract.
5. Qualified and capable instructors should be in charge of the course. Instructors could be a classroom teacher, an administrator, an outsider, or a combination of the three mentioned. The participants should gain some methods of teaching, as well as knowledge of what to teach.
6. The participants should expect to spend some time and energy to make the course successful.

7. Some method of evaluation of the in-service course should be devised to measure its efficacy and to help plan future in-service training courses.

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APPENDIX A

INTRODUCTORY AND EVALUATIVE MATERIALS

A Story of Numbers
by
Bill E. McArthur

A STORY OF NUMBERS

Introduction

Numbers are fascinating, not only because of their intrinsic value, but from the viewpoint of sheer pleasure one can derive from their use in puzzles and in manipulating them. It is easy to understand why many people of the Middle Ages thought them to be magic--especially the zero, which they could not or did not grasp as a place holder but as something to signify nothing. How could something be used to signify nothing!

Using various sources this comprehensive story of numbers is written on the level of the understanding of intermediate grade children. It is hoped that those reading or hearing this story will have created in them a greater interest in numbers and in all mathematics, and gain a better basic understanding of numbers.

In this age of electronics, electronic computers have become the heart of business, industry and professions. Anything which can be reduced to a mathematical equation or formula can be fed into such a computer which has been programmed to deliver a solution. In considering the prodigious amount of calculation these machines can do, it still must be remembered that it was the mind of man that conceived the machine and that it is still man who programs the machine to make it work. So—

"One for the money,
Two for the show,
Three to make ready,
And four to go."

A STORY OF NUMBERS

Had you ever thought of a time when there were no numbers? Most of us take numbers as a matter of course and never stop to think we have not always had them as ours are today. No doubt, you know Roman numbers and realize the Chinese and Japanese do not use our symbols for numbers. But, had you realized that man has not always been able even to count?

Counting is a man-made device. Animals do not count. (Some scientists believe some animals have an instinct for knowing groups up to four or five.) For instance, if a mother cat wanted to move her six kittens to a new home, how many trips would she make? One who can count would say, "Six." Not so a cat. She would make seven trips to be sure none was left, for she could not count to check them to be sure.

Ogg, the cave-man, did not need to count. If he ran out of food he merely went out and hunted more. He did not collect things so he had no use for counting. No doubt he could match pieces of food with members in his family, but he did not total items in the way we think of counting or check to see if there were enough pieces when he was away from the ones who would use them. When people began to live in larger groups such as tribes, clans, and such, they began to collect things. That was a long time after the world began. Then, too, began the need to count.

In China, Mount Yu may have had three turtles, but he did not know "three," so he just used a word that meant "a lot."

Over in Mesopotamia, An-am had some sheep. He could say, "One, two, three," but after that he used a word for "many." Menes lived in Egypt. His father had six palm trees. Menes could count, "One, two, three, four, many."

Hundreds of years passed by. Finally in China Mount Yu's great, great, ever-so-great grandchild could count. "One, two, two and one, two twos, two twos and one." But over that all he could say was merely, "A lot." In Babylon descendants of An-am had learned to count, "One, two, three, three and one, three and two, two threes, two threes and one, two threes and two, three threes, three threes and one, three threes and two--many." That would be only to our eleven. Meanwhile, in Egypt Mene's descendant could count further because he could count to five, and, by repeating, he could go as far as 29. You see he used only one hand. Thus he would count, "One, two, three, four, five, five and one, five and two, five and three, five and four, two fives, two fives and one, two fives and two, two fives and three, two fives and four, three fives, three fives and one, three fives and two, three fives and three, three fives and four, four fives, four fives and one, four fives and two, four fives and three, four fives and four, five fives, five fives and one, five fives and two, five fives and three, five fives and four -- many."¹

It is easy to see that counting developed very gradually. In comparing counting and calculating systems of savage tribes

¹Ernest Horn, Mabel Snedaker, and Bess Goodykoontz, "How We Came to Have Numbers," Learn to Study Readers, Book V, Grade Six (Boston: Ginn and Company, 1926), p. 72.

of today with those of prehistoric times, anthropologists find them to be very similar. Curiously enough, they also believe man had numbers up to three or four before he began using his fingers to help him count. The reason for this is that the names these early people used for numbers up to three or four have no connection with fingers, but rather were named for some familiar object which usually appeared in groups of that many. "Thus the 2 in Chinese is the word for ears; in Tibetan it is the word for wing; in Hottentot for hand; 4 with the Abipones is ostrich-toes; with Marquesans, 4 is a bunch of four fruits; in Hindustan, the word for 1 is moon."¹

On the other hand, in nearly every language the word for 5 is "hand." From there it was easy to go on,--one hand and one for six, one hand and two for 7, and so on to "two hands" for 10. "In Greenland and Australia alike, 6 means 'one on the other hand,' 10 is 'two hands,' 11 is 'two hands and a toe;' and 20 is 'one man.' The Tamanacs, a tribe in South America say 11 is 'one to the foot,' 15 a 'whole foot,' 16 'one to the other foot.' 20 is 'one Indian.' 21 is 'one to the hand of the other Indian.' 40 is 'two Indians.' 60, three Indians, 100, five Indians."² In some tribes 10 was "half man," while 20 was "man finished."

The Zulus started with the little finger on the left hand, on to the thumb which is "finish hand." The thumb on the right

¹Lena Mural Beck, "Enumeration Systems," (an unpublished Thesis, University of Texas, 1933), p. 2.

²Ibid., p. 3.

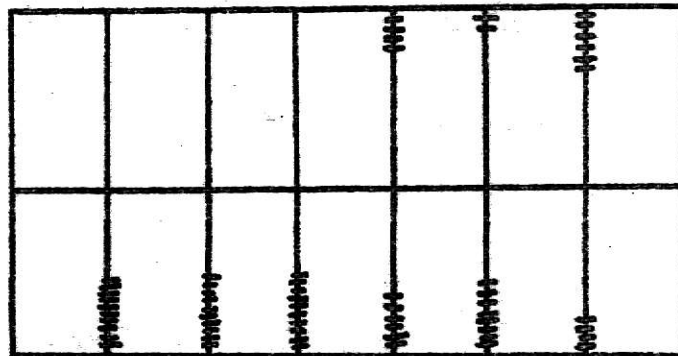
hand is six, and 7 is their word for "to point," as it is the finger on the right hand used for pointing. The Zulus used a system of subtracting for 8, as it was "keep two fingers back," and 9 was "keep one finger back." Ten was a clap of the open hand. You are acquainted with the Roman XL for forty or XC for 90 which is the same idea. (The Romans seldom used IV for four as they preferred IIII.) At this early time, however, men were not writing their numbers. In fact many savage tribes had no words even for higher numbers but they could count quite far in sign language. We still use some of the words our ancestors used to indicate a number greater than they could count. Some of these are: a "flock" of sheep, a "school" of fish, a "herd" of deer. Perhaps you have said there were a "zillion" something or other when you thought there were an uncountable number.

By this time man was counting far enough that he found the need of some system to help him keep track of his numbers. Perhaps some notched a stick or put a pebble in a pile and let each notch or each pebble represent so many. In fact our score, a word for twenty, actually means a mark. Others made marks in the dust and from that developed dust tables, swan pans, and the abacus.

The dust table was just what the name says--a table or area of dust. It was divided into parts. As things were counted a mark would be made in the dust in one part. When so many, usually 10, were made in one part, they were rubbed out and a mark representing that whole group was put in the next part. Sometimes deeper grooves were made in sand. Then pebbles were put in the grooves. When so many got in one groove, they were

they were taken out and one to represent all of them was put ⁶⁰ into the next groove. Finally people realized they could draw permanent lines on the table to divide it into part and use discs, beads, or other counters to move from part to part. These were the forerunners of the abacus. The Chinese and other orientals used counting rods, but they developed a "swan pan" which was a kind of abacus they could carry around.

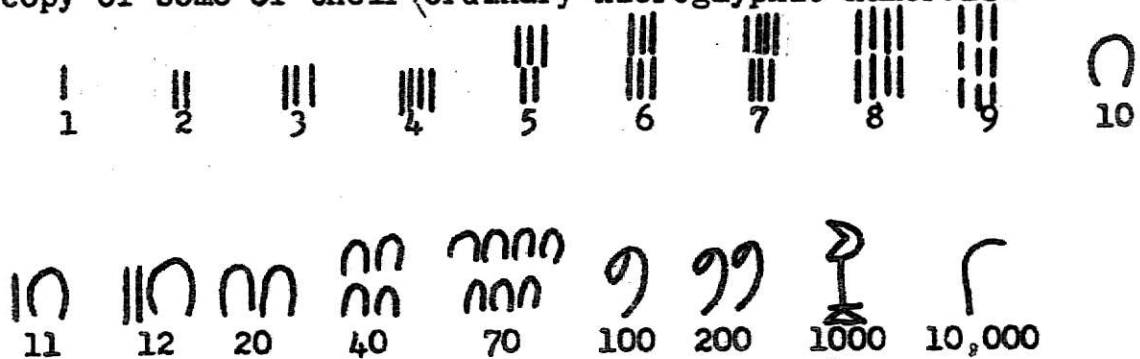
The abacus has been used for a long, long time. The dust table was really one. The idea of the instrument was to let an area or line separate groups of numbers. The numbers were represented by discs, beads, or other counters, and placed on them. The first line was units, the next tens, the next hundreds, etc. The form usually thought of is a frame with wires stretched across it, each wire strung with ten beads. The number of beads pushed up on each wire tells the number of that group counted. Nearly all parts of the world has used them. The drawing below represents the number 426.



As man began to develop a written language, he also began to develop a written system of symbols for numbers. He did this so he could keep a record of his flocks, if he were a herdsman, accounts, if he were a merchant, etc. Since men lived in all parts of the world, different ways of writing numbers

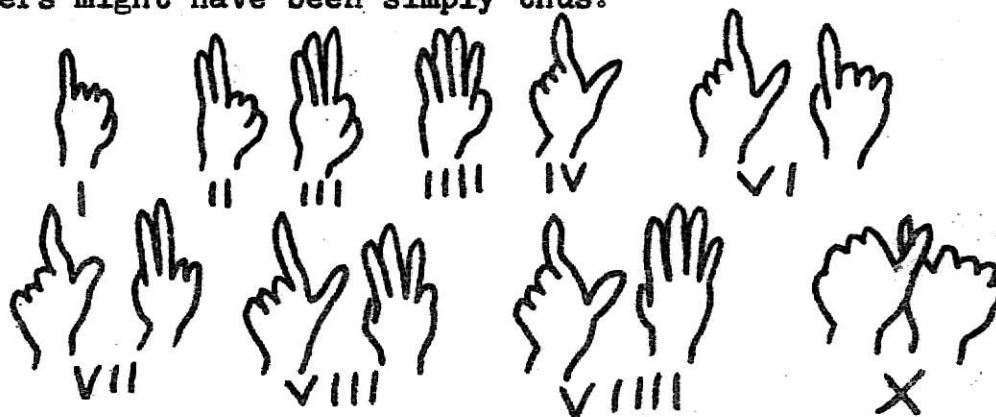
arose. The Egyptians used hieroglyphics, the Sumerians and the Babylonians used cuneiform writing. They wrote on clay tablets and wax tablets, and finally papyrus, parchment, and paper. In many cases one was represented by a vertical line just about like the little finger when it is held up to count. The two was often represented by two vertical lines, $\parallel$, representing two raised fingers, or two horizontal lines, $=$, to represent two sticks laid down. Three and four were often represented in the same way. You can easily see how the $=$, written rapidly could become our 2 (2), or $\equiv$, become our 3 (3).

The Egyptians and Greeks had written number systems. The Egyptians used hieroglyphics and there are quite a few samples still in existence, since they were carved in stone. Here is a copy of some of their ordinary hieroglyphic numerals:¹



It is believed the Phoenicians actually did not do a lot to develop numbers of their own. They did use the systems of other peoples. The most important thing the Phoenicians did was to carry these systems of counting to other places in the world and to teach the people in these parts to use them. The Phoenicians were traders and sailed over the known world to trade and had to have some method of keeping accounts. To be really useful to them they had to teach the people who traded with them how to use the same number systems.

Although nearly every civilization produced a number system, the Roman system lasted the longest. We still use it today for numbering chapters in books, parts of outlines, numbers on some clocks or watches, dates on buildings and motion picture films, and many others. The Romans used a combination of marks and letters of their alphabet to represent numbers. They did not use successive letters of their alphabet for this representation as did the Syrians, Hebrews, and Greeks. While various theories have been developed by writers as to how the Roman system of numbers evolved, it seems very plausible their numbers might have been simply thus:



No doubt the C for one hundred came from the Latin word centum which means hundred, and M for a thousand from their mille. Some writers believe our plus (+) sign was from the Latin word et which means and. As it was written rapidly and more rapidly it was finally reduced to a simple cross.

As mentioned before, the Romans, as well as others, used the rule that smaller numbers to the left of a greater one should be subtracted from the greater. However, the Romans did not do this a great deal until after the Middle Ages. The Romans used several methods to write large numbers. One was the

placing of a bar above a number to increase it 1,000 times. Such as $\overline{CXX}$ would be 120,000 or $\overline{X}$ would be 10,000. Sometimes they put the strokes at the side, so $IX \cdot DC \cdot XC$ was 1690.¹ Another symbol used to represent 1,000 was OO . Sometimes it had a vertical line through it to keep it from being mixed up with X. This led to its being written CID . When half of it was taken, ID , it became 500. If joined, it makes the D which became standard after printing was invented and is still used for the Roman 500.

The Romans could use their written system to add, subtract, and multiply, and in making fractions. They probably used an abacus or some similar counting device more often, because they could figure more quickly with it. They did not do division with their numbers until after 1000 A.D. and then they mixed their numbers with the Arabic numerals to do it more easily. The Romans had a pretty good number system but it lacked two things to make it easy to use. It was based on ten, but it did not have place value for numbers or a zero. People already had the idea of place value as they used it with their counting boards or abacus.

There is no more proof where our number system came from, than how the Roman system developed. Most authorities of today believe it had its beginnings in India with the Hindus. A man, Severus Sebokht, about 650 A.D. wrote:

¹Florian Cajori, A History of Mathematical Notations, Volume II, Notations Mainly in Higher Mathematics (Chicago: The Open Court Publishing Company, 1928), p. 32.

I will omit all discussion of the science of the Hindus, a people not the same as the Syrians; their subtle discoveries in this science of astronomy, discoveries that are more ingenious than those of the Greeks and the Babylonians; their valuable method of calculation; and their computing that surpasses description. I wish only to say that this computation is done by means of nine signs.¹

The symbols developed in India went on west through Arabia and finally into Europe. The only forms which have significance are the first three, |, ||, |||, or —, =, ==, which are similar to many other beginning systems which indicated fingers or counting sticks. To both the Egyptians and Hindus, numbers were sort of magic. They mixed their symbols for numbers so much with symbols used in their religious and mystic rites, it is practically impossible to tell why a certain symbol was used to represent a certain number.

People got the idea of place value of numbers through their use of the abacus in counting. No doubt this idea developed gradually. The zero is what makes our number system so superior to any developed. Dantzig wrote: "In the history of culture the discovery of zero will always stand out as one of the greatest single achievements of the human race."² Zero is usually thought of as a device of the Hindus. However, a study of the hieroglyphics of the Mayas of Mexico and Central America indicates those people used a symbol for zero centuries before the Hindus did. The Mayas also used a local place value. Their

¹David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1925), pp. 64-65.

²Beck, op. cit., p. 41.

numbers were based on 20. A bar represented 5.









 . By

adding one of their oval symbols below a number that number was multiplied by 20 just as annexing a zero to a number in our system multiplies it by 10:




 .¹

Also before the time of the Hindu zero, the Babylonians had developed a symbol for zero. Their numbers were based on 60. This was a convenient base and we still use traces of it in our system of measure of time: 360 days (roughly) in a year, 60 minutes in an hour, 60 seconds in a minute.

If one considers that even today there is not one definite name for our symbol "0," but that it is called cipher, zero, nought, ought, or even the letter O, then it is easier to understand that it took hundreds of years for a number system to develop. The Arabic-Hindu system probably did not make its way into Europe until the ninth or tenth century A.D. Its number forms were not kept the same even then, and often they were mixed with other systems such as the Roman. A book, The Development of Arabic Numerals in Europe,² written by G. F. Hill, presents the gradual development of numbers in Europe up to

¹Florian Cajori, A History of Mathematical Notations, Volume I, Notations in Elementary Mathematics. (Chicago: The Open Court Publishing Company, 1928), p. 43.

²G. F. Hill, The Development of Arabic Numerals in Europe (London: Oxford University Press, 1915) 125 pp.

about 1500 A.D. Mr. Hill got the samples for his numbers⁶⁶ from old manuscripts, sculpturing, churches, other buildings, brasses, coins, pottery, paintings, woodcuts, metal engravings, medals, anything of a durable nature which he could prove was authentic. He has set up these number forms in 64 tables. Besides showing the form of the number he tells the date and the place where he found it. With all of that, Mr. Hill does not claim to have told all. He says, in effect, that the tables are there, one can make his own conclusions.

The forms for the number symbols did not tend to become standardized until after the invention of printing. That invention did for numbers what it did for written language. The use of numbers became more universal and as it spread to all people there had to develop a common form for each symbol. The following is taken from a thesis written by Lena Beck.¹ Miss Beck summarized her notations from Mr. Hill's book.

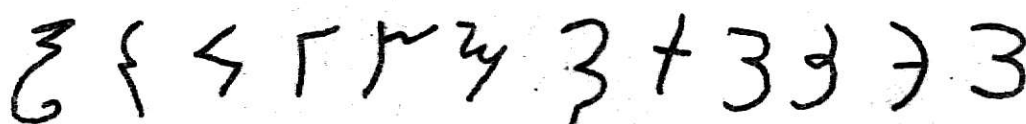
"The symbol one has varied varied little. It was sometimes written I, but for the most part it has been a single vertical stroke.

"Two has varied much. We find the following forms:

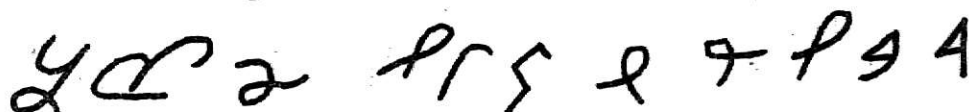
u 6 7 8 9 1 2 3 4 5 6 7 8 9

"Three has changed less than most of the digits and is generally traceable to the cursive writing of the three original strokes. Some of the old forms are:

¹Beck, op. cit., pp. 93-94.



"Four has gone through many changes, and students of old manuscripts are said to base their judgment of the age of a manuscript on the type of four to be found within its pages. A few of these changes follow.



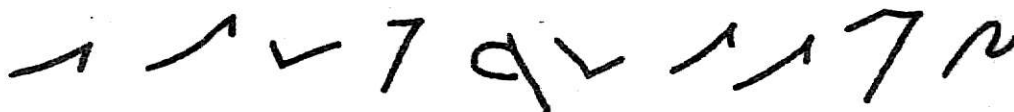
"Five has assumed many shapes. Here are a few.



"Six is another one of the digits that has shown few variations in its history. Below are some of its older appearances.



"Seven has been almost uniform in contour throughout the years. In the earlier writings the digit was laid flat; and it has been only since the fifteenth century that it has become erect. Here are some samples of its early positions.



"Eight has seen relatively few variations. Below are some of them.



"Nine has assumed only a few different shapes. Those following will illustrate.

2 9 6 9 6 9 9 9 9

"The zero, being younger, has probably varied less than its companions. Here are some of the forms found:

⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙ ⊙

Our number system as we know it today has been in use only about 400 years. That is not a very long time in the history of man. Our system has a single symbol for each digit and a "space holder," zero. You may prefer to think of zero as any other digit and meaning "not any." However, that is what makes it a space holder. It is the zero which makes our system so simple that the study of mathematics is begun in primary grades instead of being a subject so difficult it was taught only in a university. That was the case some 400 years ago. The zero translated "not any" and combined with place value of a number enables us to write any size number using no digit larger than 9. Your arithmetic book will explain to you about the place value of numbers if you do not understand it. If you like large numbers, you can easily write and read one up to 66 places. Start with the ones, progress to the left one place to the tens, another place to hundreds and you have the basis for reading and writing any number. You never have to read more than three at a time

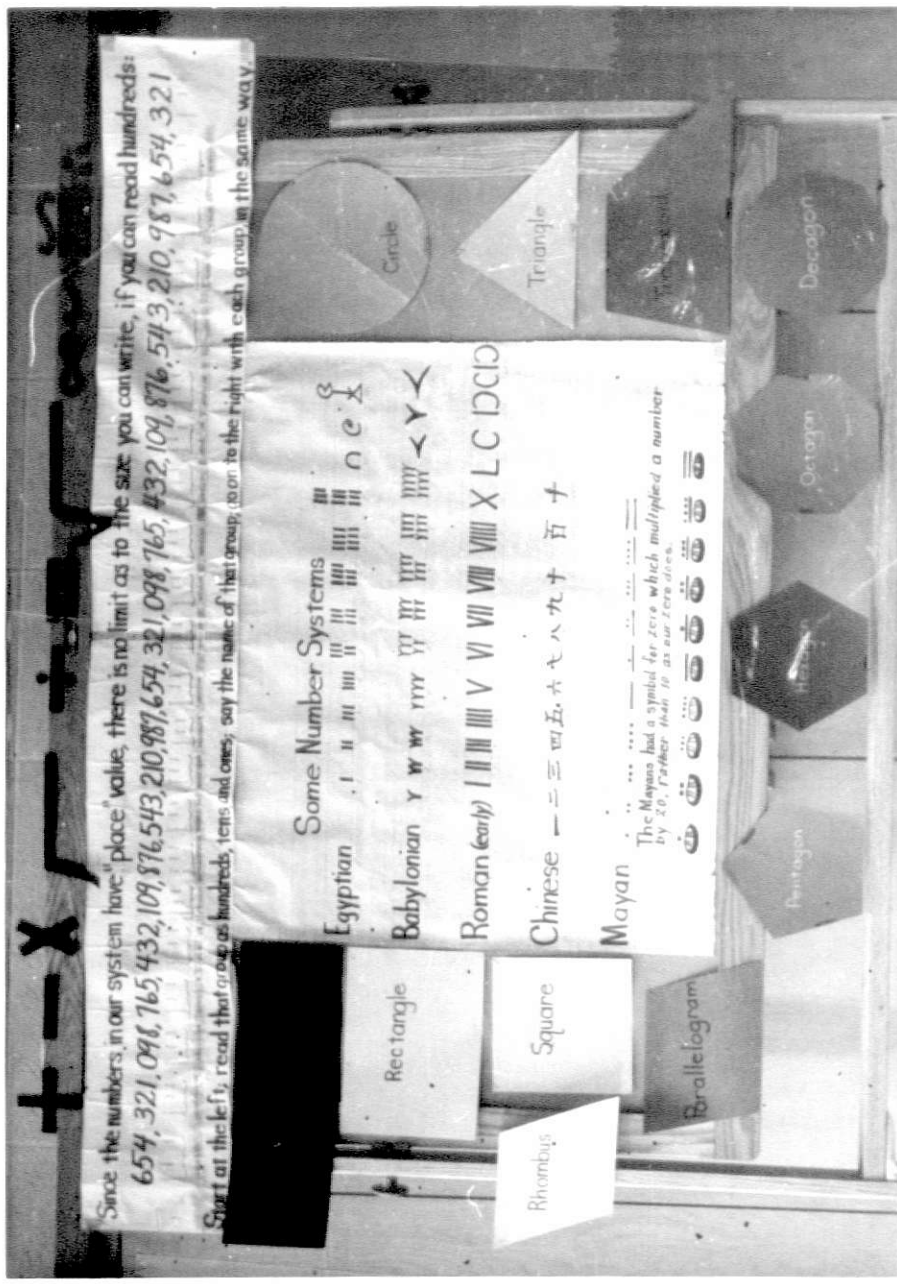
and then tell the name of the place that group is: thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quattuordecillions, quindecillions, sexdecillions, septendecillions, octodecillions, novemdecillions, vigintillions. You can write decimal fractions in much the same way by simply moving to the right of the ones place, making a decimal point, write the digits telling the amount in each place, but call them fractional names: tenths, hundredths, thousandths, etc. The fractional names correspond to the whole number names the same number of places to the right of the ones place as the whole number is to the left of the ones place.

The digits all progress in the same order of size, zero to nine, and since they are used to tell the amount in any place, you cannot go wrong. The tens are the collection point in counting, thus shifting you to another place. That is why we call our number system a decimal system. Decimal comes from the Latin word decem which means ten. The digits are so called in reference to fingers which were early used in counting, as digit is a word meaning finger. Strictly speaking, zero is not a digit, although some consider it to be.

So, numbers are fun. Learn to use them well and they will work for you and give you much pleasure.

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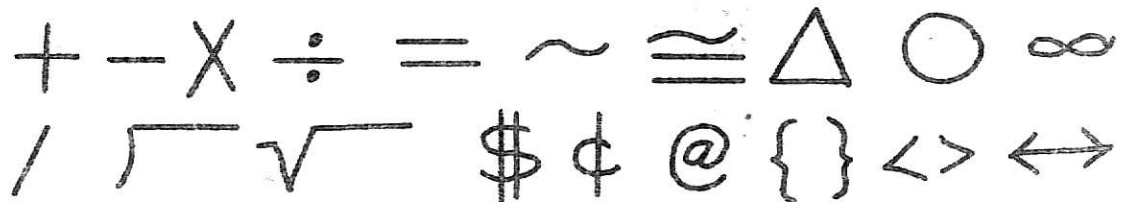


A MATHEMATICS BULLETIN BOARD

SOME SUGGESTIONS FOR USE OF ARITHMETIC
IN ROOM DECORATIONS AND BULLETIN BOARDS

72

1. Make border decorations of mathematical symbols. Make them quite large and use celophane tape to hold the parts together for tacking them along borders or sticking them to window panes. Usually there will be some children in the class who will do some research to find more symbols that are used in higher or "modern" mathematics.



2. Make border decorations of geometric figures. Make them of colored construction paper and place them in the border above the chalkboards or elsewhere in the room. The children will want to know their names and how to construct them.



3. Use numerals, themselves, for designs. Overlap the same numerals or use more than one numeral to make an all-over design on a 12" by 18" piece of plain construction paper. The pupils will soon make up their own designs after they are given the initial idea.



4. Have the pupils use a compass to draw circles and using the same radius, make designs within the circle by placing the point of the compass on the circumference of the circle to make arcs within the circle. They may color the design to be used for room decorations.
5. Select about a nine-inch square and have the pupils design "quilt" block patterns and color them.
6. Make mobiles using numerals and mathematical symbols for the suspended parts that move.
7. Use well-arranged arithmetic papers as bulletin board displays.
8. Make inch cubes from construction paper for building three-dimensional designs.
9. Make three-dimensional five or six pointed stars for Christmas decorations.
10. Make a bulletin board display showing various units of measures of long ago in comparison with the standard unit of that measure used today.

These are just a few suggestions to give the idea of the real beauty in mathematics as well as its practical values.

Stanford Achievement Test

TRUMAN L. KELLEY • RICHARD MADDEN • ERIC F. GARDNER • HERBERT C. RUDMAN

If a separate answer sheet is being used, do not make any marks on this test booklet.

NAME _____
last first initial

NAME _____
last first initial
BOY ☐ GIRL ☐ GRADE _____ TEACHER _____

SCHOOL _____ DATE OF TESTING _____
year month day

CITY OR TOWN _____ DATE OF BIRTH _____
year month day

STATE _____ **AGE** _____
years months

	SCORE	PERCENTILE RANK*	STANINE*								
Modern Mathematics			1	2	3	4	5	6	7	8	9

*Percentile Ranks and Stanines based on tables for Beginning ☐ Middle ☐ End ☐ of grade (check one)

Modern Mathematics Concepts Test

DIRECTIONS: Read each question. Decide which of the answers given below is correct. Look at the answer spaces at the right or on your separate answer sheet (if you have one). Fill in the space which has the same letter as the answer you have chosen.

SAMPLE

A Six is what part of twelve?

- a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{1}{2}$ d $\frac{1}{12}$

A ☐ a ☐ b ☒ c ☐ d

1 Any number minus zero is always —

- a one more
b one less
c that number
d an even number

☐ a ☐ b ☐ c ☐ d

2  All but one of these is a correct name for the number of objects shown at the left. Which name is incorrect?

- e $12 \div 2 + 1$ g VII
f seven h $2 \times 2 + 1$

☐ e ☐ f ☐ g ☐ h

3 Which of these numerals is twelve thousand, thirty-five?

- a 1235 c 120035
b 12035 d 1200035

☐ a ☐ b ☐ c ☐ d

4 In which numeral does the 6 have the greatest value?

- e 84.63 g 463.8
f 86.34 h 836.4

☐ e ☐ f ☐ g ☐ h

5 Which sentence can be used to find out how far our car will go in 3 hours at an average speed of 50 miles per hour? (*d* means distance.)

- a $50 \div 3 = d$ c $d \times 50 = 3$
b $3 \times 50 = d$ d $3 \times d = 50$

☐ a ☐ b ☐ c ☐ d

6 In the set of odd numbers, how many numbers are greater than 6 and less than 12?

- e 3 g 7
f 4 h 11

☐ e ☐ f ☐ g ☐ h

7 Which numeral means 9 hundreds + 4 thousands + 5 ones?

- a 9401 c 945
b 495 d 4905

☐ a ☐ b ☐ c ☐ d

8 What number comes next when counting in base three: 1, 2, 10, 11, 12, ...?

- e 13 g 21
f 20 h 22

☐ e ☐ f ☐ g ☐ h

9 Which of these figures is *not* a quadrilateral?



☐ a ☐ b ☐ c ☐ d

10 A straight piece of copper tubing used for a water pipe is usually in the shape of a —

- e prism g cylinder
f cube h sphere

☐ e ☐ f ☐ g ☐ h

11  This figure is a prism. Its faces are —

- a 1 triangle and 2 quadrilaterals
b 2 triangles and 2 quadrilaterals
c 2 triangles and 3 quadrilaterals
d 3 triangles and 4 quadrilaterals

☐ a ☐ b ☐ c ☐ d

12 A boy counted for a store owner the people who passed his store. In 15 minutes, 30 people passed. At that rate, how many people will pass from 12:00 noon to 5:00 p.m.?

- e 600 g 250
f 480 h 150

☐ e ☐ f ☐ g ☐ h

13 Which sentence below is *not* true?

- a $6 + 4 \neq 12$ c $3 + 5 < 7$
b $4 + 2 > 5$ d $5 + 2 = 7$

☐ a ☐ b ☐ c ☐ d

14 Which sign, if put where the box is, will make this sentence true? $4 + 3 \square 6$

☐ e = ☐ f - ☐ g < ☐ h >

15  The triangle at the left appears to be congruent with which triangle below?



☐ a ☐ b ☐ c ☐ d

16 In which group are all four figures polygons?



☐ e ☐ f ☐ g ☐ h

17 2^3 is another name for —

- a 3×3 c $3 \times 3 \times 3$
b 2×3 d $2 \times 2 \times 2$

☐ a ☐ b ☐ c ☐ d

- 18 To draw a line segment to measure $\frac{1}{4}$ inch plus $\frac{3}{8}$ inch, you should have a ruler that is marked in a scale of —

e one fourth inch
f one half inch
g one eighth inch
h one inch

18 ☐ e ☐ f ☐ g ☐ h

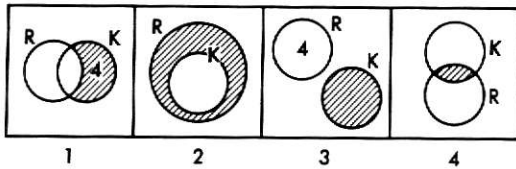
- 19 What is the value of n in the sentence $\frac{3}{n} = \frac{6}{18}$?

a 2 b 6 c 9 d 12 19 ☐ a ☐ b ☐ c ☐ d

- 20 On a number line, the point that represents 432 would be —

e to the right of the point for 440
f to the left of the point for 478
g the same as the point for 234
h to the left of the point for 396

20 ☐ e ☐ f ☐ g ☐ h



- 21 Which drawing above shows that all members of set K are members of set R?

a 1 b 2 c 3 d 4 21 ☐ a ☐ b ☐ c ☐ d

- 22 Which shaded area above shows those that are members of both set K and set R?

e 1 f 2 g 3 h 4 22 ☐ e ☐ f ☐ g ☐ h

- 23 $n\{\text{bricks that have no weight}\} =$

a $n\{\text{bricks over 8 inches long}\}$
b $n\{\text{bricks that have only one side}\}$
c $n\{\text{bricks that have been broken}\}$
d $n\{\text{bricks that are red}\}$

23 ☐ a ☐ b ☐ c ☐ d

- 24 Which is another way to multiply $a \times b \times c$?

e $c \times a \times b$
f $(a \times b) + (a \times c)$
g $b \times (a + c)$
h $(a + b) \times c$

24 ☐ e ☐ f ☐ g ☐ h

- 25 Which one of these sets of objects does not match one-to-one with any of the other three sets?



a ☐ a ☐ b ☐ c ☐ d 25 ☐ a ☐ b ☐ c ☐ d

- 26 Which sentence is true if r and s are positive, whole numbers?

e $r + s < r - s$ g $r + s > r - s$
f $r - s = s - r$ h $r + s = s + r$

26 ☐ e ☐ f ☐ g ☐ h

- 27 The best estimation of the distance between the door knob and the floor in your classroom is —

a 2 ft. c 3 ft.
b $2\frac{1}{2}$ ft. d 4 ft.

27 ☐ a ☐ b ☐ c ☐ d

- 28 In which of these sentences will you *add* to find the value of w ?

e $w = 5 \div 3$ g $4 = w - 6$
f $9 = w + 3$ h $9 - 7 = w$

28 ☐ e ☐ f ☐ g ☐ h

- 29 When two rays have the same end point, they form —

a an angle c a straight line
b a triangle d a diameter

29 ☐ a ☐ b ☐ c ☐ d

- 30 The Roman numeral XCIX =

e 49 f 99 g 109 h 119 30 ☐ e ☐ f ☐ g ☐ h

- 31 One half of 1 hr. 40 min. is —

a 20 min. c 50 min.
b 45 min. d 70 min.

31 ☐ a ☐ b ☐ c ☐ d

- 32 A simple, closed curve with all of its points the same distance from another point in the same plane is a —

e circle g cylinder
f prism h sphere

32 ☐ e ☐ f ☐ g ☐ h

- 33 The *intersection* of $\{3, 6, 9, 12, 15, 18\}$ and $\{6, 8, 10, 12, 14, 16, 18\}$ is —

a $\{3, 6, 8, 10, 12, 14, 15, 16, 18\}$
b $\{2 \text{ and } 3\}$
c $\{3, 8, 9, 10, 14, 16\}$
d $\{6, 12, 18\}$

33 ☐ a ☐ b ☐ c ☐ d

- 34 The fact that the order of two factors does not affect their product in multiplication means that the operation is —

e commutative
f the inverse of division
g distributive
h a product of an ordered pair

34 ☐ e ☐ f ☐ g ☐ h

- 35 A set of numbers is "closed" for an operation when —

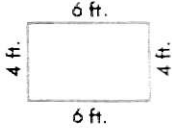
a no one can do the operation
b no more numbers are needed for the operation
c there are no rules for the operation
d there is no remainder

35 ☐ a ☐ b ☐ c ☐ d

- 36 If K is a positive whole number, and $K + 4 < 10$, then K may be —

e $\{1, 2, 3, 4, 5\}$
f $\{14\}$
g $\{1, 2, 3, 4, 5, 6\}$
h $\{6\}$

36 ☐ e ☐ f ☐ g ☐ h

37  What is the area of this rectangular region?

a 12 sq. ft. c 24 sq. ft. a b c d
b 20 sq. ft. d 36 sq. ft. 37

- 38 If $K > 2$ and $R = 6$, you know that —
- e K could equal $2 \times R$
f $K < R$
g $K > R$ e f g h
h $K \neq R$ 38

- 39 A piece of paper is 1 ft. $1\frac{1}{4}$ in. long. The unit of measure used is —
- a 1 foot
b 1 inch
c a half inch a b c d
d a quarter inch 39

- 40 The product of two factors and one of the factors are known. To find the other factor you would —
- e multiply g add e f g h
f subtract h divide 40

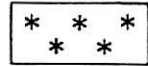
Number of Plants per Acre of Ground

Planted feet apart	Number of plants
1 by 1	43,560
2 by 1	21,780
2 by 2	10,890
4 by 1	10,890
4 by 3	3,630
4 by 4	2,722
5 by 5	1742
6 by 6	1210
8 by 8	680
10 by 10	435
$16\frac{1}{2}$ by $16\frac{1}{2}$	160
33 by 33	40

- 41 See the table at the left. How many plants are there per acre when planted 2 by 3 feet apart?
- a 1815
b 7260
c 32,670 a b c d
d 63,340 41

- 42 According to the table, how many trees would there be per acre if planted 12 by 12 feet apart?
- e 302
f 400
g 605 e f g h
h 3302 42

- 43  A pupil has drawn this figure. It appears to be an effort to make —
- a an equilateral triangle
b a triangle congruent to another triangle
c a triangle similar to another a b c d
d an isosceles triangle 43

- 44 In the binary base, in which set are there 101_{two} stars?
- e  g  e f g h
f  h  44

- 45 Add these numbers in base four: $13_{\text{four}} + 12_{\text{four}} =$

a 21_{four} c 30_{four} a b c d
b 25_{four} d 31_{four} 45

- 46 Which is a set of prime numbers?

e {2, 3, 5, 7, 11}
f {0, 1, 2, 3, 5}
g {2, 4, 6, 9, 10} e f g h
h {2, 3, 5, 7, 9} 46

- 47 $\{4, 3, 6\} =$

a {436} c {3, 6, 4} a b c d
b {13} d {7, 6} 47

- 48 $\{2, 4, 6, 8\} \cap \{3, 4, 5, 6, 7\} =$

e {4, 6}
f {4, 5, 6}
g {2, 3, 4, 5, 6, 7, 8} e f g h
h {2, 3, 4, 4, 5, 6, 6, 7, 8} 48

- 49 $4 + (-3) + (-5) =$

a 4 c -4 a b c d
b -2 d -12 49

- 50 The most precise of these measurements of length is —

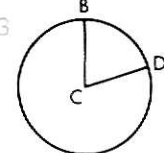
e $4\frac{1}{2}$ ft. g 2 ft. e f g h
f 29 in. h 3.25 ft. 50

- 51 3148.69 may be rounded to —

a 3.1×10^3 c 3.1×10^4 a b c d
b 3.1×10^5 d 3.1×10^6 51

- 52 Six pencils cost 25c. Which sentence can *not* be used to find the cost of each?

e $6 \times c = 25$ g $\frac{25}{6} = c$
f $\frac{c}{6} = 25$ h $\frac{25}{c} = 6$ e f g h
52

- 53  Arc BD is one fifth of a circle. What is the measurement of $\angle BCD$ if C is the center of the circle?

a 20° c 72° a b c d
b 60° d 90° 53

- 54 What is the least common denominator of two fractions whose denominators are $2 \times 2 \times 3 \times 3$ and $3 \times 3 \times 5$?

e $2 \times 2 \times 3 \times 3 \times 5$
f $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$
g $2 \times 3 \times 5$ e f g h
h $2 \times 3 \times 3 \times 5$ 54

EVALUATION OF THE IN SERVICE TRAINING COURSE IN MODERN
MATHEMATICS TAKEN IN THE FALL SEMESTER, 1966-67, AT
MARLATT SCHOOL

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Each participant in the Modern Mathematics In-Service Training Course
is requested to complete the following questionnaire:

Self Evaluation

1. The purpose of the study was clear to me. no somewhat yes
2. My interest at the beginning of the study
 would best be described as low medium high
3. My interest level at the end of the study
 would best be described as low medium high
4. In comparison with other group members the
 quality of the contributions I made were poor average good
5. The two greatest benefits to me of working on this study were:
 (a)
 (b)
6. On the reverse side please list ways the course could have been
 of more benefit to you.

Course Evaluation

1. Were the topics covered suitable for your own
 classroom use? no partly usually
2. Were the hand-out materials adaptable for your
 own classroom use? no partly usually
3. Did the concepts covered in this course help you
 feel more at ease in your classroom when the
 arithmetic text was changed to a modern one? no somewhat yes
4. What would you have deleted or added to the course? .
5. What is your opinion of in-service courses such as this?

APPENDIX B

SETS

SOME MATHEMATICAL DEFINITIONS

$=$ equals
 $\neq$ does not equal
 $\wedge$ and
 $\vee$ or
 $\rightarrow$ if....., then.... (implies)
 $\leftrightarrow$ if and only if
 $>$ is greater than
 $<$ is less than
 $\geq$ is greater than or equal to
 $\leq$ is less than or equal to
 $|a|$ absolute value of a
 $\subseteq$ is included in, inclusion
 $\subset$ is proper subset of
 $\equiv$ is congruent to
 $\cap$ intersection
 $\cup$ union
 $\bar{A}$ complement of set A
 $\emptyset$ nul set, or empty set
 $\in$ is an element of
 $\notin$ is not an element of

set notation $\{x\}$
 empty set $\{\}$ or $\emptyset$
 member of a set $\{x\}$
 equality symbol $=$ or $\equiv$
 equal sets $\{\theta\} = \{\theta\}$
 union of sets $\{x\} \cup \{y\} = \{xy\}$
 N notation N
 number 5
 number property of a set $N\{xy\} = 2$
 variable or unknown $m + 5 = 8, m = \underline{\hspace{1cm}}$
 ordinal number = fifth, fourth, etc.

78

Open sentences or "Frame" Arithmetic

The sentence, "He has been President of the United States. is neither true nor false. It The sentence, "He has been President of the United States." is neither true nor false. It is an "open sentence." "He" is holding the place for a whole set of frames. As names are substituted, the sentence becomes true or false depending upon the name replacing the pronoun "he." In frame arithmetic the square, triangle, or circle are representing the variables the same as x or y does in algebra. The idea is to find the set of numbers which will make the arithmetic sentence true. The variable may hold the place of a set of numbers or for only one number. There may be no numbers that make it true.

gr. 1 $5 + \square = 9$
 gr. 2 & 3 $\square + \square + \triangle = 11$
 gr. 4, 5, 6 $6 \times 3 > (\square \times 5) + 1$
 gr. 8 $-4 + \square = 7$
 gr. 9 $2x + y = .6$

A set is a collection of things which is defined as to what it includes. Equal sets have the same elements and are in one-to one correspondence. Equivalent sets are sets with the same cardinality.

Cardinality of sets is really the number of items (different items) in the set. This will range from the null set to infinity.

The null set or empty set is one that has no elements. Hence, this is always a subset of any set. All null sets are equal. $\{\} = \phi$

Subset of a given set is a set that contains all, some, or none of the elements of the given set, and contains only elements of the given set. Two subsets of every given set are the set itself and the null set.

Improper subsets. For any given set, the set itself and the empty set are called improper subsets.

Proper subsets. Proper subsets are subsets of a given set that contain only elements of the given set and contain some, but not all, of the elements of the given set.

Universal set. All the elements that can be discussed in a given situation compose the universal set for that situation. For example, the set $\{a, e, i\}$ is a subset of the set of all vowels in the English alphabet, and a subset of all written letters. The universal set is generally represented by the capital letter U

Operations of sets:

Intersection: the overlapping area of sets--the elements common to the sets in discussion.

Union: joining together--all elements belonging to either or both of two sets.

The structure of set systems is not the same as the structure of arithmetic.

Complement - elements in set (universal set) use symbol $\bar{A}$ (read A bar) or A' set not in A (named one).

Equality of sets = $A = B$ (elements are identical so

$$A \subset B \text{ or } B \subset A$$

Summary of operations of universal sets:

$$A \cap \phi = \phi \quad A \cup \phi = A \quad A \cap U = A \quad A \cup U = U$$

$$\bar{U} = \phi \quad \bar{\phi} = U \quad A \cup \bar{A} = U \quad A \cap \bar{A} = \phi$$

$$A = \{0, 1\}$$

$$B = \{0\}$$

$$C = \{1\}$$

	A	B	C	D
A				
B				
C				
D				

Complete the tables of operation indicated.

	A	B	C	D
A				
B				
C				
D				

Specifying a set:

1. Tabulation
2. Description or rule

Universe: finite and infinite sets

The universal set includes all the possible members of any type of set.

finite sets are those which are empty or if it can be counted by a natural number.

A set is infinite if it is not finite.

Every set has subsets. The empty set is always a subset of a set.

Operations of sets.

Two of the operations on sets are operations on two sets to produce one new set. Such operations are called binary operations. Addition and multiplication are binary operations on numbers since they assign single numbers to pairs of numbers.

Intersection

Union

(Addition is sometimes defined as the union of disjoint sets.

Complement (not a binary operation since the action is on only one set)

Sentences

Closed sentences express statements. They may be true or false.

Open sentences express conditions that, when satisfied, produce true statements.

Condition: the requirement made by an open sentence. $x + 3 = 7$, $2x > 9$.

Solution sets - the set whose members make true statements form a condition upon replacement for the variable. (Might be the Universe itself or a proper subset of the Universe - this would include the empty set.)

APPENDIX C

**NATURAL NUMBERS,
ADDITION AND SUBTRACTION**

Write the correct words, numerals and mathematical sentences to complete this chart. **82**
 The first one is done for you.

	Numbers operated on	Result	Operation Used	Mathematical Sentence
1.	12, 9	3	Subtraction	$12 - 9 = 3$
2.	18, 9	9	_____	$18 - 9 = 9$
3.	6, 3	9	_____	$6 + 3 = 9$
4.	11, 0	11	_____	$11 - 0 = 11$
5.	0, 11	11	_____	$0 + 11 = 11$
6.	6, 7	13	Addition	_____
7.	8, 2	6	Subtraction	_____
8.	3, 7	_____	Addition	_____
9.	14, 6	8	Subtraction	_____
10.	12, _____	5	Subtraction	_____
11.	3, 14	17	Addition	_____
12.	15, 9	6	_____	_____
13.	12, 4	_____	Subtraction	_____
14.	5, 6, 6	11	_____	_____
15.	6, 2	_____	Subtraction	_____

Count and write to "100"
in base 3.

Fill in this addition chart in base 3.

+	0	1	2
0			
1			
2			

Complete this subtraction chart.

-	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

The result of the operation of subtraction on a pair of numbers is 1. Write 5 of these pairs

Using only the numbers 12, 5 and 1, write two additions and two subtractions.

If you subtract 1 from any counting number, what is the result?

1
 2
 3
 4
 5
 6
 7
 8
 9
 5
 E
 10
 11
 12
 13
 14
 15
 16
 17
 18
 19
 1E
 1E
 20

+	1	2	3	4	5	6	7	8	9	E	10
1	2	3	4	5	6	7	8	9	6	E	10
2	3	4	5	6	7	8	9	6	E	10	11
3	4	5	6	7	8	9	6	E	10	11	12
4	5	6	7	8	9	6	E	10	11	12	13
5	6	7	8	9	6	E	10	11	12	13	14
6	7	8	9	6	E	10	11	12	13	14	15
7	8	9	6	E	10	11	12	13	14	15	16
8	9	6	E	10	11	12	13	14	15	16	17
9	6	E	10	11	12	13	14	15	16	17	18
5	6	E	10	11	12	13	14	15	16	17	18
E	10	11	12	13	14	15	16	17	18	19	1E
10	11	12	13	14	15	16	17	18	19	1E	20

X	1	2	3	4	5	6	7	8	9	E	E	10
1	1	2	3	4	5	6	7	8	9	6	E	10
2	2	4	6	8	6	10	12	14	16	18	16	20
3	3	6	9	10	13	16	19	20	23	26	29	30
4	4	8	10	14	18	20	24	28	30	34	38	40
5	5	5	13	18	21	26	2E	34	39	42	47	50
6	6	10	16	20	26	30	36	40	46	50	56	60
7	7	12	19	24	2E	36	41	48	53	56	65	70
8	8	14	20	28	34	40	46	54	60	68	74	80
9	9	16	23	30	39	46	53	60	69	76	83	90
6	5	18	26	34	42	50	55	68	78	84	92	EO
E	E	15	29	38	47	56	65	74	83	92	E1	EO
10	10	20	30	40	50	60	70	80	90	EO	EO	100

Zero times zero will get you nowhere,
And two times nothing will not make a pair.
Double or triple or times it by one,
Your product is zero, your answer is none.
Multiply by six or even by ten,
A zero stays zero. Let's say it again!
A zero times a number will just leave it blank;
A number times zero will not change its rank.
You can't make something out of what isn't there,
So---remember that goose egg! Put that zero there.

- ① 2, 4, 6, —, —, —, —, —, —, —
- ② 25, 22, 19, —, —, —, —, —, —, —
- ③ 9, 18, 27, —, —, —, —, —, —, 90
- ④ 10, 8, 13, 11, 16, 14, 19, 17, 22, —, —

Sequences.

$$1 + 2 = \underline{\quad}$$

$$1 + 2 + 3 = \underline{\quad}$$

$$1 + 2 + 3 + 4 = \underline{\quad}$$

$$1 + 2 + 3 + 4 + 5 = \underline{\quad}$$

$$9 + 8 = \underline{\quad}$$

$$9 + 8 - 1 = \underline{\quad}$$

$$9 + 8 - 1 - 6 = \underline{\quad}$$

$$9 + 8 - 1 - 6 + 5 = \underline{\quad}$$

$$9 + 8 - 1 - 6 + 5 + 4 = \underline{\quad}$$

$$9 + 8 - 1 - 6 + 5 + 4 - 3 = \underline{\quad}$$

$$9 + 8 - 1 - 6 + 5 + 4 - 3 - 2 = \underline{\quad}$$

There are 2 planets closer to the sun than the earth. There are 6 planets farther from the sun than the earth. How many planets are there altogether?

a. Because $16 - 8 = 8$, $16 - 7 = \underline{9}$

b. Because $13 - 6 = 7$, $13 - 5 = \underline{8}$

c. Because $15 - 8 = 7$, $15 - 9 = \underline{6}$

d. Because $11 - 7 = 4$, $11 - 8 = \underline{3}$

e. If $17 - 5 = 12$, Then $20 - 8 = \underline{12}$
why?

f. If $37 - 14 = 23$, then $30 - 7 = \underline{23}$
why?

g. If $40 - 31 = 9$, then $47 - 39 = \underline{18}$
why?

h. If $40 - 31 = 9$, then $40 - 36 = \underline{4}$
why?

i. If $20 - 6 = 14$, then $17 - 9 = \underline{8}$

j. If $10 - 4 = 6$, then $20 - 8 = \underline{12}$
why?

k. If $10 - 4 = 6$, then $5 - 2 = \underline{3}$
why?

APPENDIX D

BASES

Name _____

Add or subtract according to the base indicated

(a) $\begin{array}{r} 25 \\ + 31 \\ \hline \end{array}$

seven

(b) $\begin{array}{r} 44 \\ - 35 \\ \hline \end{array}$

seven

(c) $\begin{array}{r} 24 \\ + 26 \\ \hline \end{array}$

seven

89

Construct an addition table using these four consecutive symbols: 0, 1, Δ , $\square$

+	0	1	Δ	$\square$
0	0		Δ	
1	1	Δ		
Δ	Δ		10	11
$\square$				

Use the table to solve these problems.

- (a) $\begin{array}{r} \Delta \\ + \square \\ \hline \end{array}$ (b) $\begin{array}{r} 10 \\ - 1 \\ \hline \end{array}$ (c) $\begin{array}{r} \square \\ + 1 \\ \hline \end{array}$ (d) $\begin{array}{r} \square \\ - 1 \\ \hline \end{array}$
- (e) $\begin{array}{r} 1 \\ + \Delta \\ \hline \end{array}$ (f) $\begin{array}{r} \Delta \Delta \\ + \square \square \\ \hline \end{array}$ (g) $\begin{array}{r} 1 \\ + 1 \\ \hline \end{array}$ (h) $\begin{array}{r} \square \\ + \square \\ \hline \end{array}$

Arrange in order of size going from the smallest to the largest:

- (a) 3, 7, 0, 1, 9
- (b) four tens, three zeros, fifteen tens, fifty ones
- (c) $7_{(ten)}$ $8_{(ten)}$ $3_{(four)}$ $11_{(two)}$ $12_{(three)}$

Numbers Operated on	Result	Operation Used	Mathematical Sentence
(a) $(7+4)$, 8	3	_____	_____
(b) 20, $(40+30)$	_____	Addition	_____
(c) $(30-30)$, $(40-40)$	_____	Subtraction	_____
(d) $(88-61)$, $(199+1)$	_____	Addition	_____

	Numbers Operated On	Result	Operation Used	Mathematical Sentence.
1.	12, 9	3	Subtraction	$12 - 9 = 3$
2.	18, 9	9	_____	$18 - 9 = 9$
3.	6, 3	9	_____	$6 + 3 = 9$
4.	5, 8	40	_____	$5 \times 8 = 40$
5.	18, 3	_____	_____	$18 \div 3 = 6$
6.	6, m	13	Addition	_____
7.	p, 7	6	Subtraction	_____
8.	3, 7	_____	Addition	_____
9.	14, t	8	subtraction	_____
10.	12, _____	5	subtraction	_____
11.	15, 9	6	_____	_____
12.	3, 14	17	_____	_____
13.	12, 4	_____	Division	_____
14.	5, 3	_____	Multiplication	_____
15.	6, 2	_____	Subtraction	_____

Is this true?

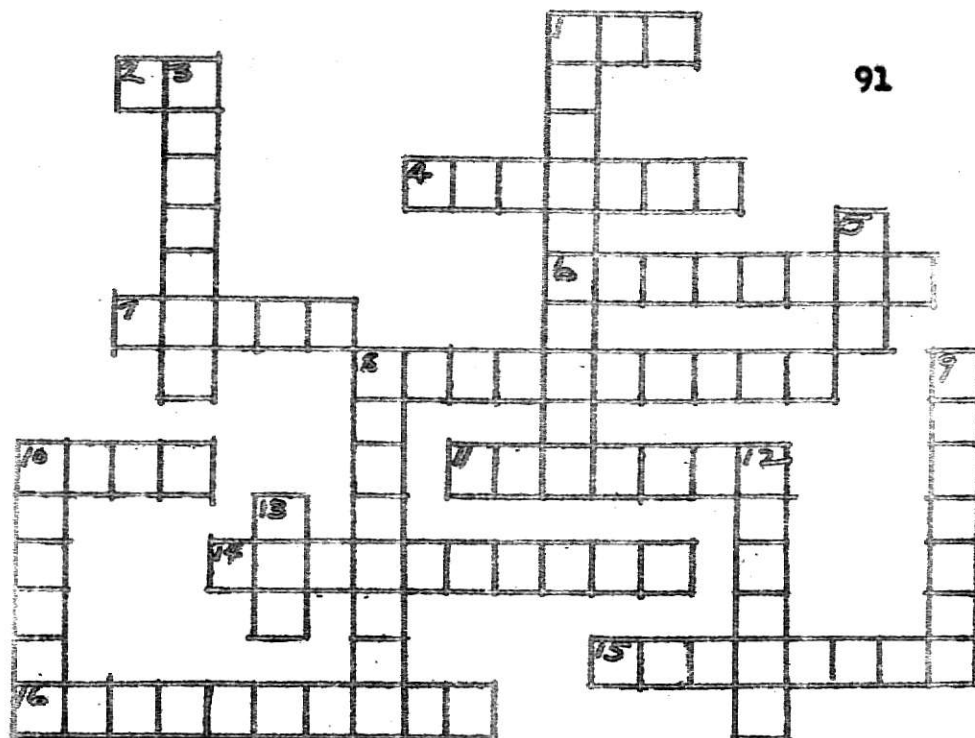
The names of the parts in a subtraction

sentence are: $9 - 4 = 5$
 sum addend addend

When two whole numbers are added, the sum is always larger than either addend.

ARITHMETIC TERMS PUZZLE

Bill E. McArthur



ACROSS

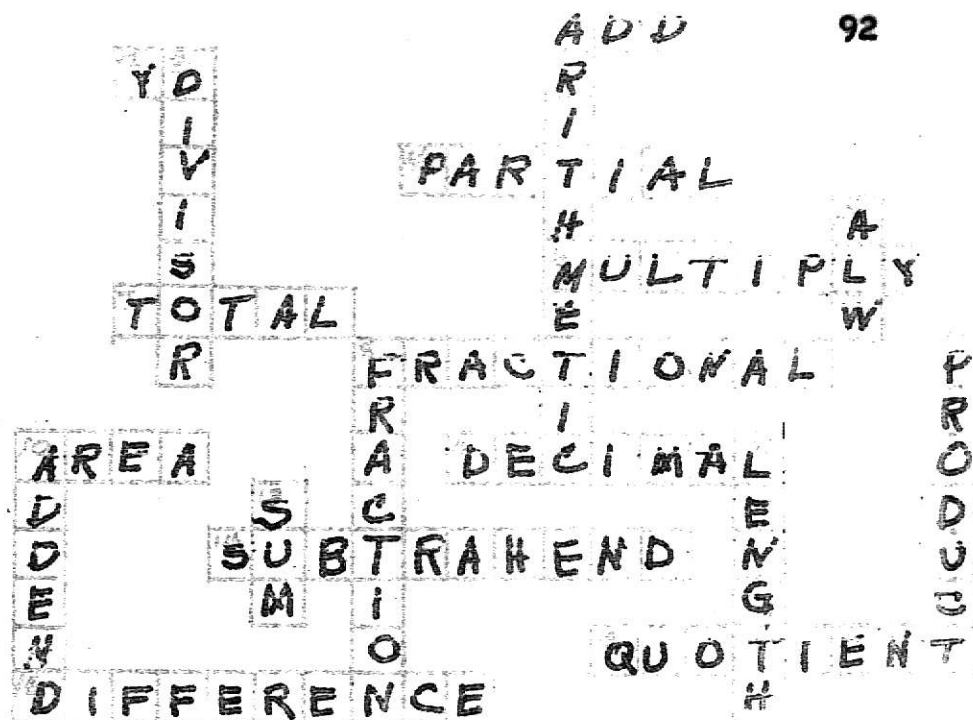
1. To combine two or more numbers into one sum.
2. The abbreviation for yard.
4. The ones' product, tens' product, hundreds' products, and so on, in a multiplication problem are called _____ products.
6. To find the total number in a stated number of equal groups.
7. The whole sum or amount.
8. In the example, $\frac{1}{4}$ of $20 = 5$, 5 is a _____ part.
10. The number of square units in a flat surface is its _____.
11. .5 is a _____ fraction.
14. In the example, $8 - 6 = 2$, 6 is the _____.
15. The answer to a division problem.
16. The answer to a subtraction problem.

DOWN

1. The subject of this puzzle.
3. In the example, $6 \div 2 = 3$, 2 is the _____.
5. The formula for finding the area of a rectangle is: _____ = _____ X _____.
8. A part of a number or object.
9. The answer to a multiplication problem.
10. A number to be added to another number to make a sum.
12. The opposite of width.
13. The answer to an addition problem.

ARITHMETIC TERMS
PUZZLE

Bill E. McArthur



Write a mathematical sentence which goes with each problem. Draw an array if you need one. Beginning with problem 4, be sure you answer each question in a complete sentence.

1. Write the mathematical sentence which shows the matching of a set of 2 things with a set of 9 things.
2. Write the mathematical sentence which shows the matchings of a set of 2 objects and a set of 3 objects.
3. In how many arrays can 12 dots be arranged?
Write the mathematical sentences.
4. The calendar is arranged in 5 rows of squares. Each row is divided into 7 squares. How many squares are shown on the calendar?
5. Some Christmas ornaments were packed in boxes of 4 rows. There were 3 ornaments in each row. How many ornaments were there in the box?
6. A bar of chocolate candy was divided into 2 rows of 4 squares each. How many squares of chocolate were in the bar?
7. There are 3 rows of windows in our room. Each row has 5 panes. How many window panes are in the room?
8. Candy was arranged in a box in 5 rows with 9 pieces of candy in each row. How many pieces of candy were in the box?
9. For our class picture, the children were grouped in 4 rows. There were 8 children in each row. How many children were there in the picture?
10. In a box, there were 2 rows of erasers with 2 erasers in each row. How many erasers were there in the box?

BRAINTWISTER: How many possible arrays of 24 dots could you make?
Describe each array by writing a mathematical sentence.
(Draw the arrays, if necessary.)

Write the correct words or numerals to complete this chart.

Number Pair	Result	Operation	Result	Operation
a) 40, 25	65	Addition	15	Subtraction
b) 72, 8	9		576	
c) 96, 8		Subtraction	12	
d) 84, 23	1932			Addition
e) 369, 9	378			Division
f) 80, 12		Addition		Subtraction
g) 45, 5	225		9	
h) 90, 9	81			Multiplication

Place parentheses in these sentences to make them true.

Example $4 \times 2 - 1 = 4$, $4 \times (2 - 1) = 4$

a) $23 + 2 \times 5 = 125$

b) $14 \div 2 \times 3 = 21$

c) $30 - 7 + 3 \neq 20$

d) $6 \times 2 - 5 = 7$

e) $5 + 3 \times 5 \neq 20$

f) $6 + 2 \times 3 \neq 12$

g) $16 \div 2 \times 4 \neq 2$

h) $135 \div 5 \times 3 = 30$

i) $323 \times 6 - 5 = 232$

j) $123 \times 3 - 3 = 0$

Write each of these sentences using numerals and the symbols for "less than" and "greater than."

a). Three is less than five.

b). Three hundred thousand is greater than three thousand.

APPENDIX E

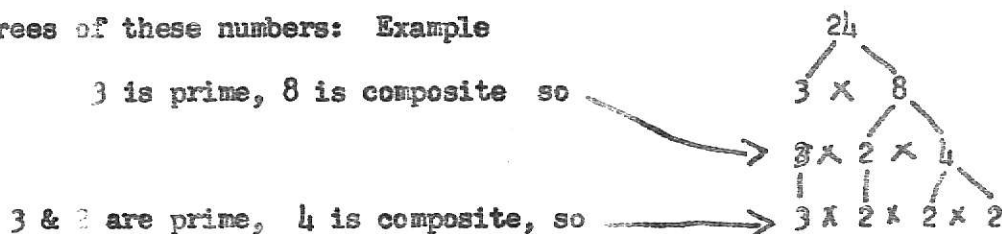
PRIME AND COMPOSITE NUMBERS,
FACTORIZATION AND RATIONAL NUMBERS

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108

Every composite number is the product of smaller numbers. If one of these numbers is composite, then it also is the product of smaller numbers. If we continue this, we must come to a product expression in which no number is composite and every factor is a prime. Doing this is called complete factorization of a composite number.

Make factor trees of these numbers: Example



54

18

28

35

If we know how to express a number as a product of primes, then we can find the set of all factors of the number. Check from your above factor trees.

Then by combining these primes in sets of 2, 3, etc. we can find all the factors of the numbers.

prime
If the set of factors are: $2 \times 2 \times 3 \times 7 \times 11 \times 11$

Could these numbers be factors of the set of all factors of that number?

6

14

28

210

242

What is the difference between factors and multiples?

Multiply these numbers:

$$\begin{array}{r} 27 \\ \times 7 \\ \hline \end{array}$$

$$\begin{array}{r} 207 \\ \times 17 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \times 17 \\ \hline \end{array}$$

$$\begin{array}{r} 27 \\ \times 107 \\ \hline \end{array}$$

$$\begin{array}{r} 207 \\ \times 107 \\ \hline \end{array}$$

$$\begin{array}{r} 3589 \\ \times 78 \\ \hline \end{array}$$

$$\begin{array}{r} 391866 \\ \times 5781 \\ \hline \end{array}$$

For each number try to make an array with more than one row and more than one object in each row. If you can do it, draw the array. If you can't do it, put an X in the blank.

2 _____ 3 _____ 4 _____ 5 _____ 6 _____ 7 _____ 8 _____ 9 _____ 10 _____
 11 _____ 12 _____ 13 _____ 14 _____ 15 _____ 16 _____ 17 _____
 18 _____ 19 _____ 20 _____

What is the name given to the kind of numbers you have marked with the X?

These numbers are called multiples of 2: list the next five of them.

2, 4, 6, 8, 10, 12,

These numbers are called multiples of 3: list the next five of them.

3, 6, 9, 12, 15, 18,

Write the first ten multiples of 5.

What is the special name for multiples of 2?

In the list below put a 1 next to every multiple of 1, put a 2 next to every multiple of 2, put a 3 next to every multiple of 3, and so forth as far as you can go.

2 _____	8 _____	15 _____
3 _____	9 _____	16 _____
4 _____	10 _____	17 _____
5 _____	11 _____	18 _____
6 _____	12 _____	19 _____
7 _____	14 _____	20 _____

How many multiples is 8 a multiple of? What are they?

What is the smallest number that is a multiple of six numbers?

What is the smallest number that is a multiple of exactly 5 numbers?

Write a number sentence (equation) to show that you know how to find the product of each of these pairs of numbers.

2, 5 6, 7 8, 7 7, 9 10, 8 12, 3

1. Complete each statement so that each number is a product of 3 factors.

a) $24 = 2 \times 3 \times 4$

c) $36 =$

e) $8 =$

b) $18 =$

d) $12 =$

f) $30 =$

2. For each set of 3 numbers write the smallest number which has each of the numbers as a factor.

a) 2, 5, 7

b) 2, 3, 4

c) 5, 7, 1

d) 4, 6, 8

e) 2, 4, 8

f) 3, 6, 9

3. Some of the following numbers are prime numbers. Encircle them.

27

31

55

53

310

143

37

101

4. Find two different prime factors of each of these numbers.

a) 785

b) 3,042

c) ~~505~~ 5,055

d) 6,060

e) 4,314

5. Write the set of all factors of each number.

96

225

363

189

7. Find the lowest common multiples of these sets of numbers.

- a) 6, 8, 12
 b) 6, 15
 c) 3, 5, 9, 15
 d) 3, 4, 8, 12
 e) 3, 6, 5
 f) 3, 4, 5, 10, 12

6. Find the greatest common factor of the following pairs of numbers.

90, 84

90, 50

72, 60

48, 30

12, 9

1. Can you find any names for whole numbers to replace the frames in these sentences so that the sentences are false? Names for the same whole number must replace all frames of the same kind.

$$\begin{aligned} \square \times (\triangle + \nabla) &= \square \times \triangle + \square \times \nabla \\ (\nabla + \triangle) \times \square &= \nabla \times \square + \triangle \times \square \end{aligned}$$

$$2. 6 \times 7 = 6 \times (5 + 2) = (6 \times 5) + (6 \times 2) = 30 + 12 = 42$$

$$8 \times 7 = (6 + 2) \times 7 = (6 \times 7) + (2 \times 7) = 42 + 14 = 56$$

3.

$$a. \quad 7 \times 5 = (3 + \square) \times 5 = (3 \times 5) + (\square \times 5) = \triangle + \nabla = \bigcirc$$

[illegible]

Product Expression	A product expression is a number composed of two or more factors expressed as a multiplication operation. For example, 2×3 is a product expression for 6.
Factors	Factors are two or more numbers that, under the operation of multiplication, result in a single, unique number (called the product).
Product	A product is the single, unique number that results from the operation of multiplication on two or more numbers (called factors).
Prime number	A prime number is a whole number greater than 1 that cannot be expressed as the product of two smaller whole numbers (each greater than 1). A prime number has only two factors: itself and 1. The number 1 is by definition neither prime nor composite.
Composite Number	A composite number is a whole number greater than 1 that has whole-number factors other than 1 and itself.
Fundamental Theorem of Arithmetic	The fundamental theorem of arithmetic states: A composite number can be expressed as a product of prime numbers in only one way.
Factorization (or factoring)	Factorization is the process of expressing a whole number as the product of two or more whole numbers.
Prime Factorization	Prime factorization is the process of expressing a composite number as the product of prime numbers only.
Greatest Common Factor (GCF)	The greatest common factor (GCF) of two whole numbers is the greatest whole number that is a factor of each of the two numbers.

Complete the blanks so that each mathematical sentence is true.

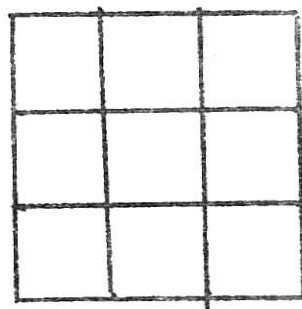
Use the largest whole number.

Use the largest multiple of 10

Use the largest multiple of 100

1. $\underline{\quad} \times 6 < 25$ $\underline{\quad} \times 6 < 252$ $\underline{\quad} \times 6 < 2526$
2. $\underline{\quad} \times 4 < 31$ $\underline{\quad} \times 4 < 315$ $\underline{\quad} \times 4 < 3158$
3. $\underline{\quad} \times 9 < 28$ $\underline{\quad} \times 9 < 283$ $\underline{\quad} \times 9 < 2834$
4. $\underline{\quad} \times 8 < 44$ $\underline{\quad} \times 8 < 446$ $\underline{\quad} \times 8 < 4465$
5. $\underline{\quad} \times 3 < 26$ $\underline{\quad} \times 3 < 263$ $\underline{\quad} \times 8 < 2639$
6. $\underline{\quad} \times 8 < 76$ $\underline{\quad} \times 8 < 765$ $\underline{\quad} \times 8 < 7657$
7. $\underline{\quad} \times 8 < 60$ $\underline{\quad} \times 8 < 600$ $\underline{\quad} \times 8 < 6000$
8. $\underline{\quad} \times 7 < 45$ $\underline{\quad} \times 7 < 456$ $\underline{\quad} \times 7 < 4560$

Using the digits 1 through 9 arrange them in the square so that each column or row or diagonal will add up to 15.



$$8 \times 216 = 1728$$

$$5 \times 216 = 1080$$

$$\text{then } 13 \times 216 = \underline{\quad\quad\quad}$$

TESTS FOR DIVISIBILITY

102

If the divisor is	Dividend is divisible	Test these examples
2	if dividend is an even number	$9,135,713,578 \div 2 = n$
3	if the sum of digits in dividend is divisible by 3	$13,572 \div 3 = n$
4	if number formed by the two right-hand digits in dividend is divisible by 4	$42,367 \div 4 = n$
5	if dividend ends in ____ or ____.	$98,765 \div 5 = n$
6	if dividend is divisible by 2 and by 3.	$13,572 \div 6 = n$
7	(There are rules but it is just as easy to divide by 7)	$29,687 \div 7 = n$
8	if the number formed by the three right-hand digits of the dividend is divisible by 8.	$72,865,128 \div 8 = n$
9	if the excess of nines in dividend is 0.	$54,637,281 \div 9 = n$
10	if one's digit in dividend is ____.	$7,684,320 \div 10 = n$
11	if the excess of elevens in dividend is 0	$92,847,249 \div 11 = n$
12	if dividend is divisible by 3 and 4.	$13,572 \div 12 = n$

1. Tell by what numbers from 2 through 12 each of the following numbers is divisible.

a) 7,680 b) 440 c) 495 d) 3,960 e) 145 f) 540

For each of Ex. 24, write in the digit that will make the number divisible by the numbers indicated:

2. 76, _ 28 is divisible by 3 and by 8.

3. 587,35 _ is divisible by 2 and 9.

4. 128, 5 _ 6 is divisible by 3 and by 4.

5. a) If 531 is divisible by 3, is 3 a factor of 531?

b) If 531 is divisible by 3, then 531 is a _____ of 3.

6. How many classrooms are needed to hold 232 pupils if each room seats 29. Work 2 ways.

7. Is division a binary operation?

8. In the division of whole numbers, which must be larger, the divisor or the dividend, in order for the quotient to be a whole number?

9. Encircle the divisor $\frac{26}{24}$ $116 \overline{) 812}$ $639 \div 113 = n$

10. Find the missing term (Use back for figuring)

2) $673 \times n = 234,877$

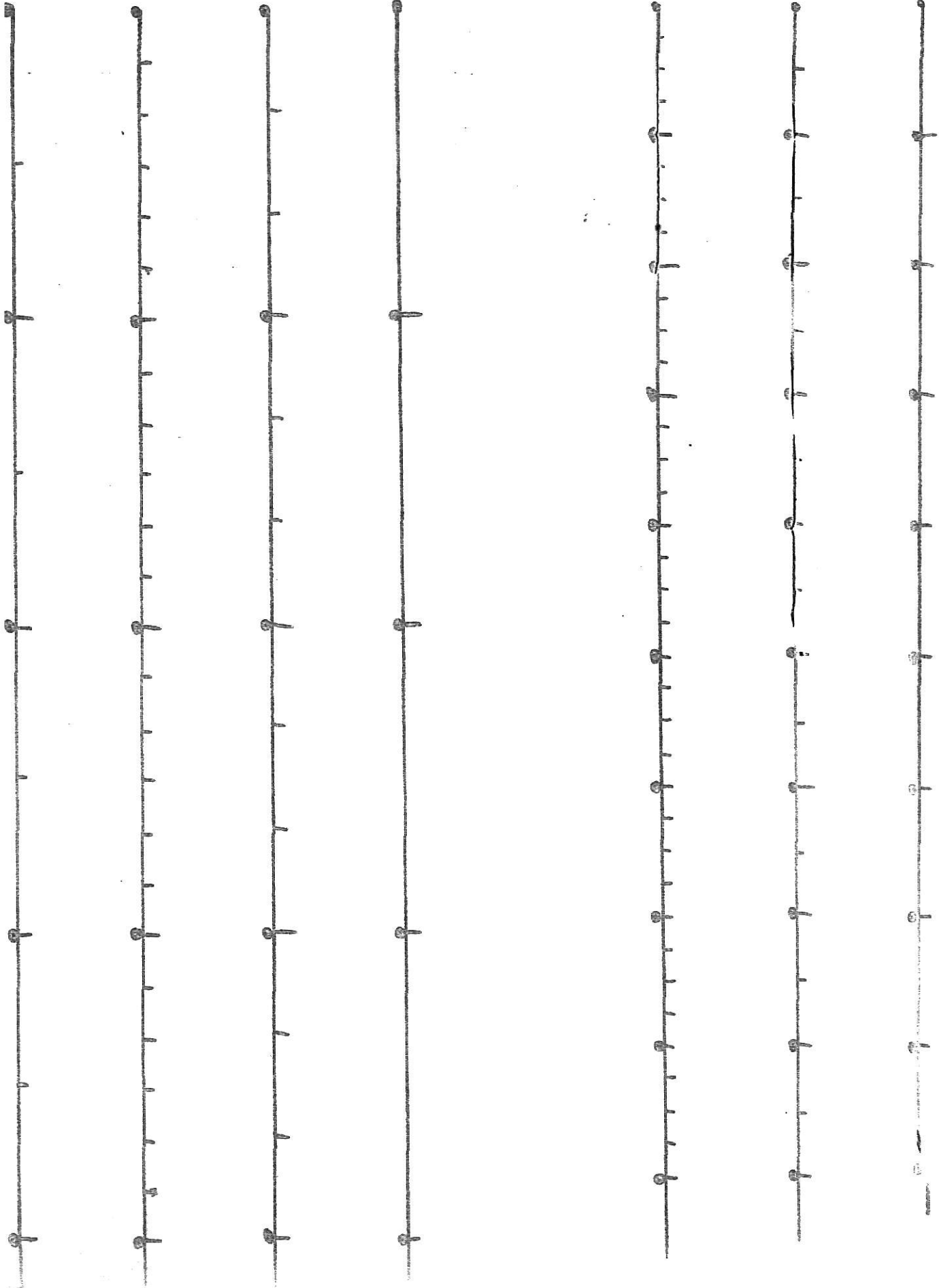
c) $n \div 729 = 653$

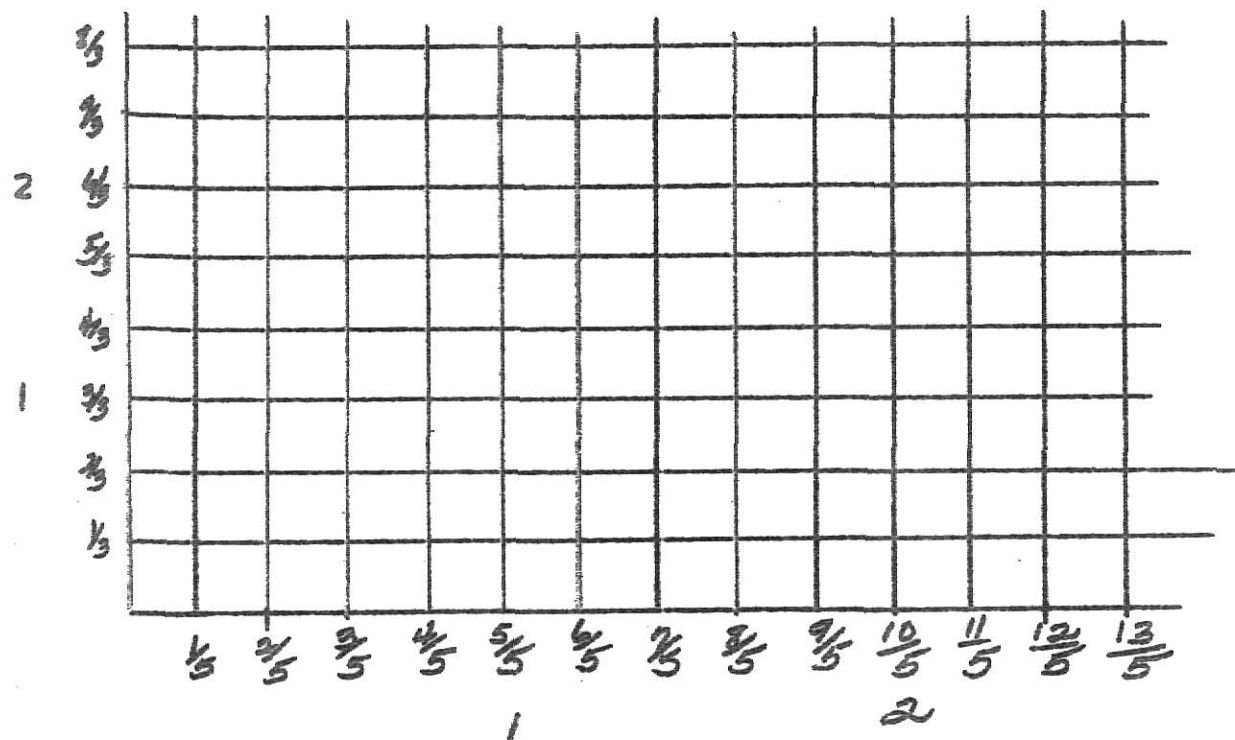
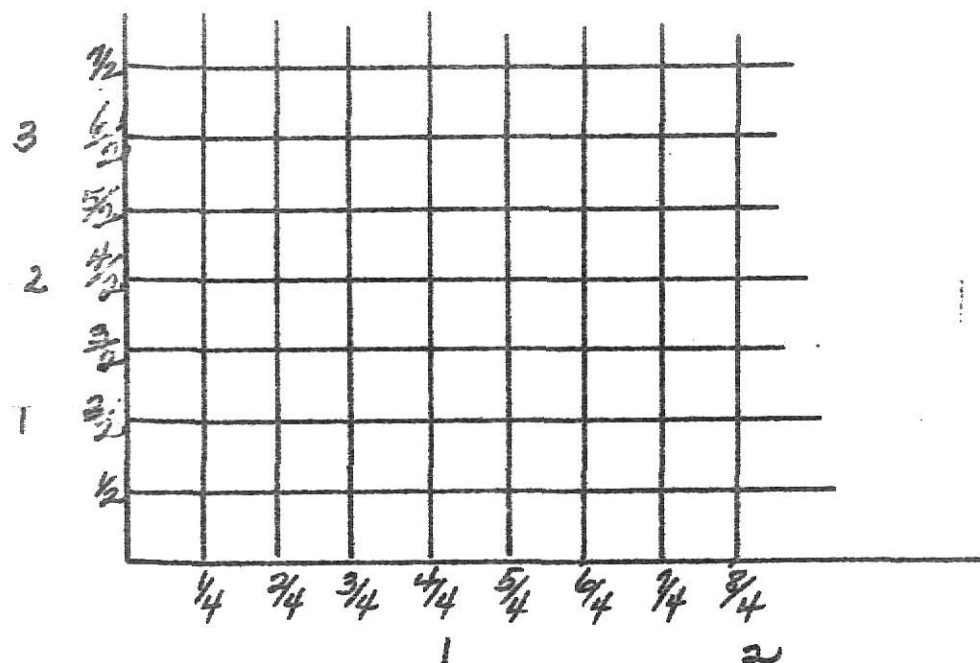
3) $413,913 \div n = 843$

d) $81,503 = n \times 462$

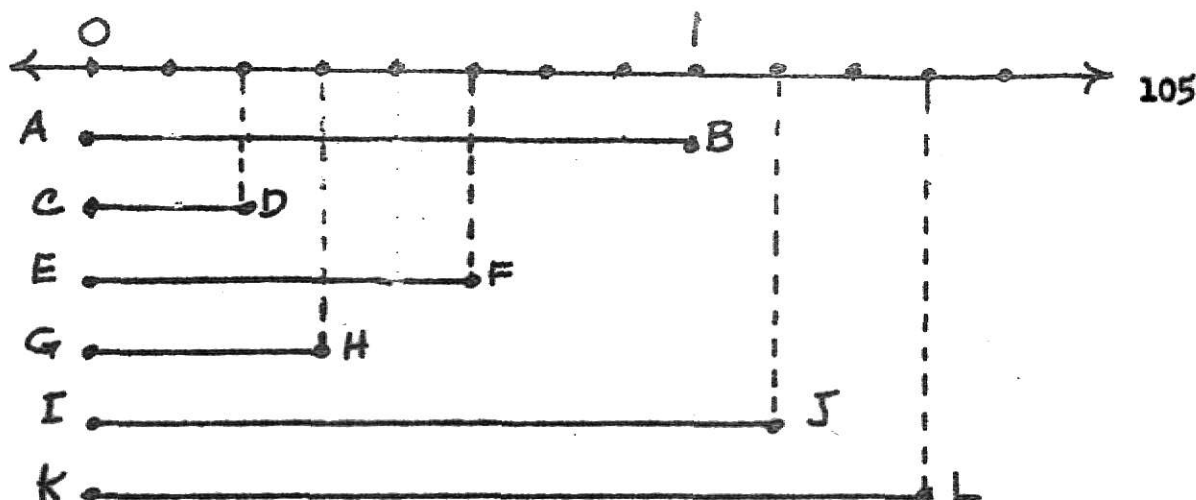
Charts to show equality of fractions

103





Example	Unknown factor is between	Unknown factor is between	Unknown factor is closer to
$4 \div 3\frac{5}{6}$	$24 \div 3$ and $24 \div 4$	8 and 6	6
$6 \div 2\frac{2}{3}$			
$8 \div 3\frac{1}{3}$			
$9 \div 2\frac{3}{4}$			
$8 \div 4\frac{7}{8}$			



In the picture above, the line segment AB is 1 unit long.

1. (a) on the number line the unit segment is separable into _____ congruent segments.
- (b) Use a fraction: Each small segment is _____ of the unit segment.
- (c) The measure of $\overline{AB}$ is 1. The measure of $\overline{AB}$ is also _____. (fraction)
- (d) What is the measure of line segment AB? _____
- (e) What is the measure of line segment CD? _____
- (f) what is the measure of the line segment EF? _____
- (g) what is the measure of line segment GH? _____
- (h) What is the measure of line segment IJ? _____
- (i) What is the measure of line segment KL? _____

2. Each unit segment of the number line below has been separated into 3 congruent segments. $\overline{AR}$ is the same length as the unit segment.



Use the number line to answer the questions.

- (a) what fraction names the measure of $\overline{AR}$? _____
- (b) What fraction names the measure of $\overline{AB}$? _____, $\overline{AC}$? _____, $\overline{AD}$? _____.

Some samples for making creative math questions and drills.

What number is a factor of every number?

106

Every prime number has exactly _____ factors.

Every prime number is odd except _____.

What is the only prime number between 61 and 71? 113 and 131?

What pairs of primes will result in each of the following products?

a) 10 b) 33 c) 9 d) 26 e) 14 f) 25 g) 4 h) 21 i) 22

Circle each number below that is the product of at least one pair of factors. Do not use 1 or the number itself.

5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
* * * * *

Place the correct relation symbol ($=$, $>$ or $<$) between each pair of fractions to make true statements.

a) $\frac{3}{4} \bigcirc \frac{6}{8}$ b) $\frac{1}{4} \bigcirc \frac{3}{4}$ c) $\frac{2}{4} \bigcirc \frac{1}{4}$ d) $\frac{3}{4} \bigcirc \frac{1}{8}$ e) $\frac{1}{8} \bigcirc \frac{1}{3}$

f) $\frac{7}{8} \bigcirc \frac{5}{6}$ g) $\frac{2}{3} \bigcirc \frac{5}{6}$ h) $\frac{2}{3} \bigcirc \frac{4}{6}$ i) $\frac{1}{8} \bigcirc \frac{1}{7}$ j) $\frac{1}{8} \bigcirc \frac{2}{16}$

Complete the chart. Use the example to help you.

Fraction	Factors in the numerator	Factors in the denominator	GCF	Simplest form
$\frac{6}{8}$	2, 3, 6, 1	2, 4, 8, 1	2	$\frac{3}{4}$
$\frac{8}{12}$				
$\frac{5}{15}$				
$\frac{14}{10}$				
$\frac{4}{3}$				
$\frac{16}{9}$				

Write the correct sign of operation (+ or -) in each Δ .

a) $\frac{2}{3} \Delta \frac{3}{8} = \frac{5}{8}$ b) $\frac{11}{11} = \frac{11}{11} \Delta \frac{0}{11}$ c) $\frac{1}{2} \Delta \frac{1}{3} = \frac{1}{6}$ d) $\frac{2}{3} = \frac{1}{3} \Delta \frac{3}{9}$

Complete each sentence by writing the correct symbol ($<$, $>$, or $=$) in each $\bigcirc$.

a) $\frac{1}{2} + \frac{1}{4} \bigcirc \frac{1}{8} + \frac{2}{4}$ b) $\frac{11}{12} - \frac{1}{4} \bigcirc 1 - \frac{1}{3}$ c) $\frac{1}{3} + \frac{1}{6} \bigcirc \frac{1}{2} - \frac{1}{3}$
d) $\left(\frac{2}{3} + \frac{1}{6}\right) - \frac{3}{4} \bigcirc \left(\frac{3}{8} + \frac{3}{6}\right) - \frac{3}{4}$

Factor the denominators and multiply.

a) $\frac{5}{6} \times \frac{2}{3}$

b) $\frac{6}{10} \times \frac{9}{15} \times \frac{11}{30}$

Example: $\frac{7}{10} \times \frac{2}{3} = \frac{7 \times 2}{2 \times 5 \times 3}$
 $= \frac{2 \times 7}{2 \times 5 \times 3} = \frac{2}{2} \times \frac{7}{5 \times 3} = 1 \times \frac{7}{15}$
 $= \frac{7}{15}$

Write the next five members of each set:

A = $\{1, 3, 5, \dots\}$ B = $\{\frac{2}{1}, \frac{4}{1}, 6, \dots\}$ C = $\{\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots\}$ D = $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots\}$

Which product is not another name for 1?

a) $1\frac{1}{2} \times \frac{2}{3}$ b) $\frac{7}{8} \times \frac{8}{7}$ c) $\frac{1}{10} \times 1$ d) $1\frac{5}{6} \times \frac{6}{11}$ e) $\frac{16}{4} \times \frac{1}{4}$

Write the reciprocal of each fraction:

a) $\frac{5}{8}$ b) $\frac{1}{2}$ c) $\frac{9}{8}$ d) $\frac{4}{9}$ e) $\frac{12}{10}$ f) $9\frac{1}{2}$

Write $>$, $<$ or $=$ in each circle

a) $\frac{5}{8} \bigcirc \frac{8}{5}$ b) $\frac{2}{1} \bigcirc \frac{1}{2}$ c) $\frac{5}{6} \bigcirc \frac{6}{5}$ d) $\frac{9}{8} \bigcirc \frac{8}{9}$ e) $\frac{4}{9} \bigcirc \frac{9}{4}$

True or False?

a) If $\frac{2}{3} \times \square = \frac{4}{9}$, then $\frac{2}{3} \times 2 \times \square = 2 \times \frac{4}{9}$

b) If $\frac{4}{3} \times \square = \frac{3}{4}$, then $\square < 1$

c) If $\frac{5}{6} \times \square = 1\frac{1}{6}$, then $\square > 1$

d) If $\square \times 1\frac{3}{5} = 4\frac{4}{5}$, then $\square \times 1\frac{3}{5} \times 10 = \frac{240}{5}$

e) If $2\frac{1}{2} \times \square = 25$, then $\frac{1}{4} \times \square = 2\frac{1}{2}$

Example	Unknown factor is between:	Unknown factor is between:	Unknown factor is closer to:
$24 \div 3\frac{5}{6}$	$24 \div 3$ and $24 \div 4$	8 and 6	6
$16 \div 2\frac{2}{3}$			
$18 \div 3\frac{1}{3}$			

AN IN-SERVICE TRAINING COURSE IN
MODERN MATHEMATICS FOR ELEMENTARY SCHOOL TEACHERS

by

BILL E. MC ARTHUR

B. S. E., Fort Hays Kansas State College, 1946

M. S., Fort Hays Kansas State College, 1950

AN ABSTRACT OF A SPECIALIST'S REPORT

submitted in partial fulfillment of the

requirements for the degree

SPECIALIST IN EDUCATION

College of Education

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1969

The purpose of the report was to show whether or not an in-service training course in modern mathematics would be beneficial to teachers. The course was offered on an entirely voluntary basis to the teachers of Unified School District #383 and to lay persons in the Marlatt area. Thirteen persons responded to the invitation; eleven completed the course.

The course consisted of eleven sessions conducted on Tuesdays from 4:00 to 5:00 p.m. The materials covered included "The Story of Numbers," sets, terminology, the use of the abacus, natural or counting numbers, order as a property, base, place value, whole numbers, identity numbers, needs of a number system, addition, subtraction, closure, commutative, associative, and distributive property, multiplication and division, factors and prime numbers, rational numbers, a little geometry, and a small modular system. Many hand-out sheets were distributed to the group to present ideas for their own use and ideas for drills and tests.

Evaluation was partially made on the basis of a pre-test and a posttest. The instrument used was the Stanford Achievement Test, Intermediate II, Modern Mathematics Concepts Test, published by Harcourt, Brace, World, Inc., New York, 1965. Each participant also completed a Self and Course Evaluation Check Sheet following the close of the course.

Conclusions were that the group did gain mathematics

concepts knowledge from this in-service course in modern mathematics. Attitudes as shown by the Self Evaluation part of the check sheet were positive. They enjoyed the course and considered it personally beneficial. The Course Evaluation section showed that while they would not have deleted anything from the course, they would have added to it, even to the extent of desiring more sessions. The objections to in-service courses centered around the time of day those courses were given. The group believed in-service courses should be conducted on released school time.

Other benefits were that the participants did try out the materials with their students. More books, pamphlets, film strips, and other mathematical materials were obtained for the school. The teachers involved made the transition from traditional mathematics to modern mathematics very easily the next year when the modern mathematics was adopted as a basic text for the school.

With the influx of multi-media, machine teaching, computerized programming, electronically controlled records, instant communications, and similar devices, it would seem that in-service training will become a feature of every school in order to help new staff members become functioning members of a team with a philosophy, curriculum, methods, and materials tailored to fit that specific school.

Recommendations for any in-service training course:

1. In-service training inauguration and organization should involve both teachers and administration.
2. Specific goals or ways in which the course would improve instruction should be evident.
3. The format and calendar of meetings should be decided upon from the beginning of the course.
4. If the number of sessions needed to cover the topic for in-service training needs to be extensive, the teachers attending should receive some kind of credit to advance them on the scale of promotion or salary, or they should be relieved of part of their regular classroom duties so they may concentrate better on the study at hand. This release might be via substitutes, para-professionals, or early dismissal. The teacher might simply receive extra pay for this extra work, perhaps through an extended contract.
5. Qualified and capable instructors should be in charge of the course. Instructors could be a classroom teacher, an administrator, an outsider, or a combination of the three mentioned. The participants should gain some methods of teaching, as well as knowledge of what to teach.
6. The participants should expect to spend some time and energy to make the course successful.

7. Some method of evaluation of the in-service course should be devised to measure its efficacy and to help plan future in-service training courses.