SHEAR STRENGTH OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

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INTRODUCTION

1. Problem

Reinforced concrete beams, more than those of metal, have a potential weakness in the region where loads are transferred to the supports through the web. This is a complex problem and it is evident that the value of the diagonal tension is generally indeterminate. No accurate working formulas are available. There seems to be little doubt that existing permissible safe values are much too low, and that in some kinds of design, particularly in deep beams, this restriction presents a considerable hardship to the designer.

2. Purpose

In this report, shear and diagonal tension in a simply supported reinforced concrete beam without web reinforcement are studied. The purpose of the study is to summarize the results of published research work in this area and to suggest topics for additional research.

3. Scope

Several representative research papers on shear and diagonal tension published since 1950 are studied, covering beams without web reinforcement.

Experimental investigations included in this study are those of the report of ACl-ASCE Committee 326 (1), Clark (2), Mathey and Watstein (3), Rajagopalan and Ferguson (4), Hsieh (5), and Zsutty (6). Tests series involving variables such as (a) the compressive strength of concrete, (b) the size, number and yield strength of bars used as tensile reinforcement, and (c) the ratio of effective depth of beam to shear span, are discussed.

A summary of the theories resulting from each investigation, a discussion of available empirical formulas and a comparison between the theories and model beam test results are also included to derive the conclusions and recommendations for further research.

LITERATURE SURVEY

1. General Review of Literature

In 1909 Talbot first pointed out that shear and diagonal tension in concrete beams is a complex problem involving many variables (7). He also demonstrated that the percentage of reinforcement and the length-to-depth ratio play an important role in the shear and diagonal tension strength of beams without web reinforcement. Unfortunately, he did not express his findings in mathematical terms. Slater stressed this point ten years later in comments published in Engineering News-Record (1).

The early design specifications in the United States accepted Morsch's concept (1) that shear failure in reinforced concrete beams is a tensile phenomenon. By considering the normal shearing stress, $v=V/bjd$, to be a measure of diagonal tension and relating it to the strength of concrete cylinder tests (f'_c), the allowable shear stress was expressed as a function of f'_c. This approach neglected the other factors affecting the strength of reinforced concrete beams in diagonal tension and considered the compressive strength as the only principle variable. The conservative advice of Talbot and other pioneers was forgotten.

In 1950, Clark identified the span to depth ratio a/d as an important variable in the determination of shear strength, where a is the length of the shear span and d is the effective depth of the beam (2). Research work returned to the forgotten fundamentals, and the principle variables affecting the shear and diagonal tensile strength of reinforced concrete beams without web reinforcement were defined as

(a) Compressive strength of concrete, f'_c.

(b) Percentage of longitudinal reinforcement, p.

(c) Ratio of shear span to effective depth of bean, a/d.

In the early 1950's, researchers at the University of Illinois proposed a modification to item (c), using M/Vd instead of a/d (1). The use of M/Vd ratios retains the physical significance for all kinds of loading conditions, and may be considered a breakthrough toward an empirical solution of shear and diagonal tension as a design problem.

Many experimental investigations in this field based on the variables stated above have been carried out, but up to now an exact analytical means of evaluating the maximum shear and diagonal tensile stress which a reinforced concrete beam can resist has not been developed.

2. Experimental Investigations

(a) *Report of ACI-ASCE Committee 326* (1)

The committee developed a design formula based on four criteria:

(1) Diagonal tension is a combined stress problem in which horizontal tensile stresses due to bending as well as shearing stresses must be considered.

(2) Failure due to shear may occur with the formation of the critical diagonal crack or, if redistribution of internal forces is accomplished, failure may occur by shear-compression destruction of the compression zone at a higher load.

(3) The load causing the formation of the critical diagonal tension crack must ordinarily be considered in design as the ultimate load carrying capacity of a reinforced concrete member without web reinforcement.

(4) Distributions of shear and flexural stress over a cross section of reinforced concrete are not known.

By a systematic study of data from more than 440 recent tests, it was determined that the shear capacity depends primarily on three variables: the percentage of longitudinal reinforcement p, the dimensionless quantity M/Vd, and the quality of concrete as expressed by the compressive strength f'_c. An empirical formula for evaluating the shear capacity was derived from the principle stress formula

$$f_t\ (\max) = 1/2\ f_t + \sqrt{(1/2\ f_t)^2 + v^2} \qquad (1)$$

where f_t is the tensile bending stress and v is the shearing stress. Replacing f_t and v in Eq. 1 by a constant times the steel tensile stress f_s and another constant times the average shearing stress on the cross section V/bd, respectively, and evaluating the constants from test data, the following design formula was obtained:

$$V/bd\sqrt{f'_c} = 1.9 + (2500\ \text{psi})\ PVd/M\sqrt{f'_c}$$

$$\text{but not greater than } 3.5 \qquad (2)$$

V = external shear at ultimate load of the section considered.

b,d = dimensions of the cross section.

p = ratio of longitudinal reinforcement, As/bd.

V/M = ratio of shear to moment at section considered.

The committee's choice of Eq. 2 was based on two major considerations. First, the equation should be simple to facilitate every-day design work, and second, the equation should be such that the ultimate strength of beams resulting from practical design will be governed by flexure rather than by shear. Therefore, a simple straight-line relationship was chosen and the line was placed near the lower extremes of observed shear strengths rather than through the average values.

(b) Clark's Investigation (2)

Clark provided the first mathematical expression of Talbot's concept. He also summarized four factors that determine the capacity of a reinforced concrete beam to resist failure by diagonal tension: (1) compressive strength of concrete; (2) amount, distribution and yield strength of web reinforcement; (3) size, number and yield strength of bars used as tensile reinforcement; (4) ratio of effective depth of beam to shear span. Included in his experiments were tests on full-size beams of two cross sections (8" x 18", 6" x 15"), four span lengths (6', 8', 9'-7", 10'), and concrete strengths ranging from 2,000 to 6,000 psi. Five different positions of two symmetrically positioned concentrated loads were studied. The results showed that the shear capacity of a beam increases with the strength of the concrete when other factors are held constant. For the same concrete strength the resistance to failure in diagonal tension increased as the loads were shifted from the center of the span toward the supports. The strength in shear varied as the compressive strength multiplied by a factor representing the ratio of the depth of beam to the distance from the plane of the load to the plane of the support. The resistance to shear was also found to vary as the square root of the ratio of web reinforcement and the first power of the ratio of tensile reinforcement.

An empirical formula was derived based on the ultimate loads observed from test beams:

$$v_c = 7000\ p + (0.12\ f_c')\ d/a + 2500\sqrt{r} \qquad (3)'$$

v_c = calculated shearing stress at maximum load.

r = ratio of web reinforcement.

For beams without web reinforcement Eq. 3' reduces to

$$v_c = 7000\ p + (0.12\ f_c')\ d/a \qquad (3)$$

It was concluded that "the results of this study, with tests under five different loading conditions, show definitely that the loading condition is an important factor. More data than available at present are needed to evaluate some of the factors which determine the diagonal tension resistance of a beam."

(c) Mathey and Watstein's Tests (3)

In this investigation it was emphasized that the use of high yield strength steel reinforcing bars has an important effect on the shear strength. Deformed bars having six different yield strengths ranging from 40,000 psi. to 100,000 psi. were used in an experimental investigation of the behavior of reinforced concrete beams failing in shear. In these tests the shear span-to-depth ratio and the ratio of reinforcement were also varied.

It was found that a linear relationship appeared to exist between the terms V_c/bdp and $(f_c'/p)(d/a)$ for the shear strength of beams without web reinforcement and subjected to two equal concentrated loads symmetrically placed. The following empirical formula was suggested:

$$v_{cr} = V_{cr}/bd = 3.1\sqrt{f_c'}\ d/a + 4000\ p$$

$$= 3.1\sqrt{f_c'}\ V_{cr}\ d/M_{max.} + 4000\ p \qquad (4)$$

for simply supported beams, $1/a = V_{cr}/M_{max}$.

v_{cr} = calculated shear strength at diagonal tension cracking load.

V_{cr} = external shear force at diagonal tension cracking load of the section considered.

It was also suggested that the diagonal tension cracking load of a beam without web reinforcement be considered as the "ultimate" load for design purposes since the margin of safety above the cracking load is highly erratic and unpredictable.

(d) Rajagopalan and Ferguson's Tests (4)

Rajagopalan and Ferguson tested ten rectangular beams without web reinforcement and having p between 0.0173 and 0.0025. Twenty-seven other tests on beams with p less than 0.012 from various other investigations were also analyzed. All the beams considered had a/d ratios greater than 2.75. The aim of this investigation was to check the unconservative nature of the ACI shear strength formula for a reinforced concrete beam when the ratio of the longitudinal reinforcement p is small.

It was concluded that when p is smaller than 0.01 the value v from the ACI formula should be reduced according to Eq. 5 for the sake of safety.

$$v_u = (0.8 + 100\ p)\ \sqrt{f'_c} \qquad (5)$$

v_u = calculated shear strength at ultimate load where $p \leq 0.012$ and Max.

$v_u = 2\ \sqrt{f'_c}$

(e) Tests by Hsieh (5)

Hsieh investigated the shear strength of reinforced concrete beams with rectangular cross section by using model beams instead of full size beams, "Ultracal 30" as a substitute for cement and threaded rods for tensile reinforcement. The main test variables were the amount of tensile reinforcement and the length of the shear span.

The tests indicated that small scale model beam tests can yield results which are comparable to full size beam tests provided proper precautions are taken in preparing and testing the specimens. The model beam test results sgreed very closely with Clark's formula for the shear strength of beams without web reinforcement, and indicated that the steel reinforcement percentage has a considerable influence on the ultimate shear strength.

(f) Zsutty's Investigation (6)

Zsutty used a combination of dimensional analysis and statistical regression analysis to summarize and analyze the existing test data on reinforced concrete beams without web reinforcement. First, the test beam behavior was divided into the arch action of short beams, and the beam action of slender beams. Second, accurate prediction equations for the shear strength of slender beams along with a lower bound strength prediction for short beams were determined. The study also distinguished the relative magnitudes of cracking shear, sudden diagonal tension shear, and ultimate shear for slender beams, the variability of the high strength of "arch action" in short beams, and the effect of beam support conditions on this strength.

By an analysis of data above successive a/d values, it was possible to separate the "arch action" and "slender beam action" with a/d smaller than 2.5 as "arch action" and a/d larger than 2.5 as "slender beam action". The analysis showed that the poor correlation of all test data with the ACI formula is to a large extent due to the wide dispersion of the arch action shear stress values. The results of Zsutty's analysis can be summarized as follows:

(1) For beams with a/d above 2.5 the method produced failure stress prediction equations of the form

$$v = K\,(f_c'\,p\,d/a)^{1/3} \qquad (6)$$

where K = 59 for diagonal tension cracking shear.

K = 61 for sudden diagonal tension shear or ultimate shear.

Equation (6) also gives a lower bound for short beams with $a/d < 2.5$.

(2) Short lateral stub beams, without top and bottom load and support block pressures, appear to exhibit slender beam behavior.

COMPARISON OF THEORIES

1. Basic Theories

A basic difference exists between the theoretical investigations of the shear strength of reinforced concrete beams carried out by various researchers in that some considered that the ultimate shear strength of beams should be governed by flexure rather than pure shear, while others considered pure shear failures only.

The ACI-ASCE Committee 326 (1) apparently concluded that, from a practical design viewpoint, the ultimate shear strength of reinforced concrete beams should be controlled by flexure. The empirical formula which they proposed was based on a lower bound instead of the average value which provided a bending failure. The investigation by Rajagopalan and Ferguson (4) is a modification of the report of ACI-ASCE Committee 326.

Clark's investigation (2) of this problem is a study of pure shear failures; he suggested a better concept for studying the shear strength problem in reinforced concrete beams. The investigations by Mathey and Watstein (3), Hsieh (5), and Zsutty (6) are modifications of Clark's investigation.

2. Variables Considered

From the point of view of the variables which affect the shear strength of reinforced concrete beams, it is seen that all of the experimental investigations are based on some of the variables stated in the scope of this report.

The ACI-ASCE Committee 326 (1) developed an empirical formula by considering recent test data in which the major variables were p, M/Vd and f'_c. The test data also included several different loading and supporting test conditions; therefore, from the viewpoint of the scope of the test data, it is more general and more suitable for developing an empirical formula for design work.

Rajagopalan and Ferguson (4) studied experimentally the shear and diagonal tension strength of reinforced concrete beams when the longitudinal reinforcement ratio p is small.

Clark's test investigation (2) demonstrated that the loading position as expressed by a/d is an important variable.

Mathey and Watstein's tests (3) emphasized two variables: the shear span-to-depth ratio and the ratio of reinforcement. The tests also included high yield strength steel reinforcing bars.

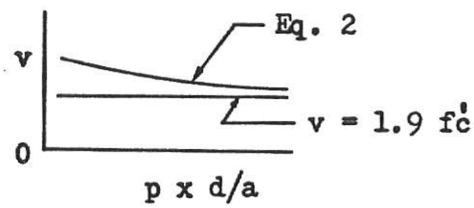
Hsieh (5) studied the problem by small scale model beam tests. The main variables were the amount of tensile reinforcement and the length of the shear span.

Zsutty analyzed the available test data for reinforced concrete beams without web reinforcement. His most important contribution was to separate the beam behavior into "arch action" for short beams and "beam action" for slender beams, depending on the variable a/d.

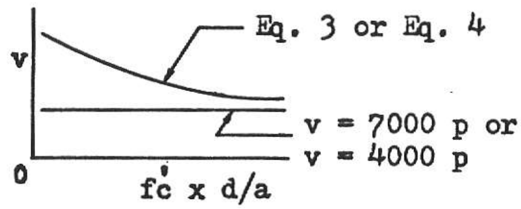
3. Comparison of Empirical Formulas

Five empirical formulas for evaluating the shear strength of reinforcement concrete beams without web reinforcement have been studied in this report. These are given by Eqs. 2 through 6. Table 1 and Fig. 1 present a comparison of the calculated shear stresses from these five equations with f_c' constant, that is, $f_c' = 3000$ psi. For this comparison, p varies from 0.5% to 3.5%, and a/d from 1.0 to 5.0. The blank columns for Eq. 5 in Table 1 means that this equation is only used when $a/d > 2.75$ and $p \leq 0.012$; and the blank columns for Eq. 6 indicates that this equation is applicable only when $a/d > 2.5$. Fig. 1 is a plot of v against a/d, and the various values of p are indicated on the curves. From Table 1 and Fig. 1, it is seen that the five equations yield drastically different results for the same conditions.

Examine the form of Eq. 2, which was derived from the principle stress formula, Eq. 1. The latter is only true for a homogeneous beam, but the beams investigated consist of two different materials, concrete and steel; therefore, it could be said that it is a mistake to use Eq. 1 as a basis for deriving the empirical formula given by Eq. 2. By making f_c' equal to a constant, Eq. 2 approaches the straight line $v = 1.9\sqrt{f_c'}$ as a limit. This shows that Eq. 2 weighs f_c' much more than p and d/a.



Eqs. 3 and 4 have the same form. By making p constant, they approach $v = 7000\ p$ and $v = 4000\ p$ as a limit, respectively.



This means that p is much more important than f_c' and d/a in Eqs. 3 and 4.

Eq. 5 gives the same values of shear strength for any value of d/a. Since this formula does not include the variable d/a, it obviously has a limited usefulness.

The form of Eq. 6 may be considered the best one among the six formulas, since it gives the three factors f_c', p and d/a equal importance.

It should be noted that Eqs. 2, 3 and 5 evaluate the ultimate loads, P_u; Eq. 4 evaluates the diagonal tension cracking loads, P_{cr}; and Eq. 6 applies to both P_u and P_{cr}, depending on the value of K which is employed.

In spite of its shortcomings, it appears that the ACI formula (Eq. 2) is adequate for design purposes until additional research leads to the development of a new design formula. The ACI formula is conservative in most cases and is based on a large number of beam tests covering a wide range of values of the variables important in the determination of the shear strength of reinforced concrete beams.

COMPARISON BETWEEN THEORIES AND MODEL TEST RESULTS

As stated before, Hsieh (5) investigated the shear strength of reinforced concrete beams by small scale model beam tests using "Ultracal 30" as a substitute for cement and threaded rods for tensile reinforcement. If this method provides reasonable results compared with full size reinforced concrete beam tests, it suggests a relatively easy way to investigate the shear strength of reinforced concrete beams experimentally.

The details of Hsieh's test program are summarized in Table 2, and Table 3 and Fig. 2 present a comparison of Hsieh's test results with the formulas of the ACI (Eq. 2), Clark (Eq. 3), Mathey and Watstein (Eq. 4), Rajagopalan and Ferguson (Eq. 5) and Zsutty (Eq. 6). As Hsieh indicated in his thesis, from these comparisons it is seen that Clark's formula provides a much better predication of the model test results than the other formulas. In Fig. 2a, Eqs. 2, 5 and 6 are inclined lines corresponding to the different steel reinforcement ratios. There is an important fact which was neglected by Hsieh, that is, Eq. 4 cannot be compared with the other formulas as Hsieh did, because Eq. 4 gives the diagonal tension cracking load while the other equations give ultimate loads. Therefore, Eq. 4 should be compared with Hsieh's results separately, i.e., as in Fig. 2b. From Fig. 2b, the correlation ratio of Eq. 4 with Hsieh's results is about 1.8.

Tables 4 and 5 are comparisons of the ultimate loads with the diagonal tension cracking loads of Hsieh's test results, Mathey and Watstein's test results and Zsutty's empirical formula, Eq. 6. These comparisons are also plotted in Fig. 3 and 4. From Fig. 3, it is seen that the difference between the curves of Pu and Pcr is not very large, but Hsieh's results shows when $a/d \geq 6.0$, Pu approaches and equals to Pcr, while Zsutty's formula gives a straight line. From Fig. 4, Pcr is much smaller than Pu when a/d is small, and the curve approaches 1.0 when a/d is large. The comparison of the two test results and one empirical formula shows the ultimate loads are usually not stable, but they approach the diagonal tension cracking loads when a/d is large. This behavior could be due to the different laboratory test conditions and the different material characteristics used for the test beams.

CONCLUSIONS

From the papers studied in this report, the following conclusions can be formulated:

1. It is evident that the shear strength of simply supported reinforced concrete beam without web reinforcement is controlled by the following variables:
   (a) The compressive strength of concrete.
   (b) The size, number and yield strength of bars used as tensile reinforcement.
   (c) The effective depth of beam-to-shear span ratio.
2. Up to now there has not been sufficient test data to evaluate the variables stated above; therefore, no accurate shear strength formula is available.
3. Model beam tests provide an easier way of investigating the shear strength problem than tests on full size beams.
4. The close agreement between Hsieh's test results and Clark's empirical formula is due to the fact that both of their test programs emphasized the variable d/a.
5. Arching action is a very important factor affecting the shear strength of short reinforced concrete beams.
6. From Figs. 3 and 4 it can be seen that ultimate test loads are usually unstable. Therefore, the shear strength formulas should be evaluated based on diagonal tension cracking loads.
7. Design equations should be such that the computed shear strength of reinforced concrete beams will be governed by flexure rather than by shear.
8. The ACI formula (Eq. 2) is the most suitable one for design purposes until additional research leads to the development of a new design formula.

RECOMMENDATIONS FOR FURTHER RESEARCH

After studying the problem of the shear strength of simply supported reinforced concrete beams without web reinforcement, it is felt that the following topics could be profitably investigated in future research projects:

1. Genevate additional test data to evaluate the three principle variables f_c', p and d/a, which affect the shear strength of reinforced concrete beams without web reinforcement.
2. Conduct further research work on "arching action" in short beams.
3. Study experimentally the difference between the diagonal tension cracking load and ultimate load.

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NOTATION

a = distance from plane of the nearest concentrated load point to plane of the support, in.

As = area of longitudinal tension reinforcement, in^2.

Av = area of two legs of a stirrup, in^2.

b = width of rectangular beam, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

f'_c = compressive strength of concrete, psi.

fy = yield stress of longitudinal reinforcement, psi.

M = bending moment, in-lbs.

P = concentrated load on beam, lbs.

p = As/bd = steel ratio of longitudinal tension reinforcement.

Pcr = diagonal tension cracking load, lbs.

Pu = ultimate load, lbs.

r = Av/bs = ratio of web reinforcement.

v = shear stress, psi.

V = total shear at section, lbs.

v_c = calculated shear stress, psi.

v_{cr} = shear stress corresponding to the diagonal tension cracking load, psi.

v_u = shear stress corresponding to the ultimate load, psi.

V_c = calculated shear force, lbs.

V_{cr} = shear force corresponding to the diagonal tension cracking load, lbs.

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Beam No.	P (%)	a/d	Shear Stresses (v = V/bd) (psi) ACI Eq. (2)	Clark Eq. (3)	M & W Eq. (4)	R & F Eq. (5)	Zsutty Eq. (6)
B_{11}	0.5		116.7	395.0	190	-	-
B_{12}	1.0		129.2	430.0	210	-	-
B_{13}	1.5		141.7	465.0	230	-	-
B_{14}	2.0	1.0	154.2	500.0	250	-	-
B_{15}	2.5		166.7	535.0	270	-	-
B_{16}	3.0		189.2	570.0	290	-	-
B_{17}	3.5		201.7	605.0	310	-	-
B_{21}	0.5		110.5	215	105	-	-
B_{22}	1.0		116.7	250	125	-	-
B_{23}	1.5		123.0	285	145	-	-
B_{24}	2.0	2.0	129.2	320	165	-	-
B_{25}	2.5		135.5	355	185	-	-
B_{26}	3.0		141.7	390	205	-	-
B_{27}	3.5		148.0	425	225	-	-

Table 1a. Comparison of Calculating Shear Stresses.

* ACI, Clark, R & F evaluating v by ultimate load; M & W by diagonal tension cracking load.

* The K value of Zsutty's formula used here is K = 60.

* $f_c' = 3000$ psi for all beams.

Beam No.	P (%)	a/d	(2)	(3)	(4)	(5)	(6)
B_{31}	0.5		109.2	179	88	-	109.1
B_{32}	1.0		114.2	214	108	-	137.5
B_{33}	1.5		119.2	249	128	-	157.3
B_{34}	2.0	2.5	124.2	284	148	-	173.2
B_{35}	2.5		129.2	319	168	-	186.6
B_{36}	3.0		134.2	354	188	-	198.3
B_{37}	3.5		139.2	389	208	-	208.7
B_{41}	0.5		108.4	155	76.7	71.4	102.7
B_{42}	1.0		112.5	190	96.7	98.8	129.4
B_{43}	1.5		116.7	225	116.7	-	148.1
B_{44}	2.0	3.0	120.9	260	136.7	-	163.0
B_{45}	2.5		125.0	295	156.7	-	175.5
B_{46}	3.0		129.2	330	176.7	-	186.7
B_{47}	3.5		133.4	365	196.7	-	196.3

Table 1b

Beam No.	P (%)	a/d	(2)	(3)	(4)	(5)	(6)
B_{51}	0.5		107.8	137.9	68.7	71.4	97.5
B_{52}	1.0		111.4	172.9	88.7	98.8	122.8
B_{53}	1.5		114.9	207.9	108.7	-	140.8
B_{54}	2.0	3.5	118.5	242.9	128.7	-	154.9
B_{55}	2.5		122.1	277.9	148.7	-	166.8
B_{56}	3.0		125.6	312.9	168.7	-	177.3
B_{57}	3.5		129.2	347.9	188.7	-	186.8
B_{61}	0.5		107.3	125	62.5	71.4	93.4
B_{62}	1.0		110.5	160	82.5	98.8	117.5
B_{63}	1.5		113.6	195	102.5	-	134.5
B_{64}	2.0		116.7	230	122.5	-	148.0
B_{65}	2.5	4.0	119.8	265	142.5	-	159.5
B_{66}	3.0		123.0	300	162.5	-	169.5
B_{67}	3.5		126.1	335	182.5	-	178.5

Table 1c

Beam No.	P (%)	a/d	(2)	(3)	(4)	(5)	(6)
B_{71}	0.5		106.7	107	54	71.4	86.6
B_{72}	1.0		109.2	142	74	98.8	109.1
B_{73}	1.5		111.7	177	94	-	124.8
B_{74}	2.0	5.0	114.2	212	114	-	137.3
B_{75}	2.5		116.7	247	134	-	148.0
B_{76}	3.0		119.2	282	154	-	157.3
B_{77}	3.5		121.7	317	174	-	165.5

Table 1d

Beam No.	Average f_c' (psi.)	Shear Arm Ratio a/d	Percentage of reinforcement p (%)	fy (ksi.)
A_1P_1	3110	1.0	0.92	97.0
A_2P_1	2760	2.5	0.92	97.0
A_3P_1	3033	3.0	0.92	97.0
A_4P_1	2886	5.5	0.92	97.0
A_5P_1	2780	6.5	0.92	97.0
A_1P_2	3060	1.0	1.46	80.5
A_2P_2	3030	2.5	1.46	80.5
A_3P_2	2970	3.0	1.46	80.5
A_4P_2	2750	5.5	1.46	80.5
A_5P_2	3010	6.5	1.46	80.5
A_1P_3	3080	1.0	2.19	80.5
A_2P_3	2970	2.5	2.19	80.5
A_3P_3	3050	3.0	2.19	80.5
A_4P_3	2660	5.5	2.19	80.5
A_5P_3	2880	6.5	2.19	80.5

Table 2. Details of Hsieh's Test Program.

Beam No.	Msieh P_H	Clark P_C	$\frac{P_H}{P_C}$	ACI P_A	$\frac{P_H}{P_A}$	R & F P_R	$\frac{P_H}{P_R}$	Zsutty P_Z	$\frac{P_H}{P_Z}$
A_1P_1	818	876.0	0.93	258.0	3.17	-	-	-	-
A_2P_1	504	393.8	1.28	218.4	2.30	-	-	264	1.91
A_3P_1	352	371.0	0.95	224.6	1.57	189.4	1.86	257	1.37
A_4P_1	270	254.6	1.06	212.4	1.27	184.6	1.46	206	1.31
A_5P_1	218	231.6	0.93	207.4	1.05	181.6	1.2	193	1.13
A_1P_2	992	938.2	1.06	383.2	2.59	-	-	-	-
A_2P_2	515	495.0	1.04	238.4	2.16	-	-	318.5	1.62
A_3P_2	446	442.0	1.01	231.7	1.91	-	-	298.0	1.49
A_4P_2	318	324.0	0.98	212.7	1.49	-	-	237.0	1.34
A_1P_3	1037	1046.4	1.01	320.0	3.30	-	-	-	-
A_2P_3	593	591.6	1.00	251.4	2.35	-	-	362	1.64
A_3P_3	507	550.4	0.92	246.4	2.06	-	-	343	1.48
A_4P_3	389	422.4	0.92	215.5	1.81	-	-	268	1.45
A_5P_3	303	412.8	0.73	220.8	1.37	-	-	260	1.16

Table 3a

Hsieh P_H	M & W P_M	$\frac{P_H}{P_M}$
773	419.6	1.84
410	204.1	2.1
342	187.2	1.83
266	134.2	1.98
218	124.0	1.76
948	460.0	2.06
473	253.4	1.87
416	229.6	1.82
318	176.0	1.80
1018	519.4	1.96
555	310.6	1.79
455	289.4	1.57
389	233.3	1.67
303	226.4	1.34

Table 3b

Table 3. Comparison of Hsieh's Test Results with Other Shear Strength Formulas.

* The values of P used in Table 3a are Pu and in Table 3b are Pcr.

* The K value used in Zsutty's formula is K = 61.

Beam No.	f'_c (psi)	P%	a/d	Hsieh's Test Results			Zsutty's Empirical Formula (Eq. 6)		
				Pcr (lb)	Pu (lb)	Pcr/Pu	Pcr (lb)	Pu (lb)	Pcr/Pu
A_1P_1	3110		1	773	818	0.94	-	-	
A_2P_1	2760		2.5	410	504	0.81	255.6	264	
A_3P_1	3030	0.92	3.0	342	352	0.96	249.0	257	0.97
A_4P_1	2890		5.5	266	270	0.98	199.6	206	
A_5P_1	2780		6.5	218	218	1.00	186.6	193	
A_1P_2	3060		1	948	992	0.95	-	-	
A_2P_2	3030		2.5	473	515	0.92	307	318.5	
A_3P_2	2970	1.46	3.0	416	446	0.93	288	298.0	0.97
A_4P_2	2750		5.5	318	318	1.00	229	237.0	
A_5P_2	3010		6.5	291	291	1.00	-	-	
A_1P_3	3080		1	1018	1057	0.96	-	-	
A_2P_3	2970		2.5	555	593	0.93	350	362	
A_3P_3	3050	2.19	3.0	455	507	0.89	332	343	0.97
A_4P_3	2660		5.5	389	389	1.00	259	268	
A_5P_3	2880		6.5	303	303	1.00	252	260	

Table 4. Comparison of Pcr with Pu of Hsieh's Test Results and Zsutty's Empirical Formula.

* In Zsutty's Empirical Formula K = 61 for Pu and K = 59 for Pcr; $a/d \geq 2.5$.

Beam No.	a/d	P	f_c'	Pcr (lb)	Pu (lb)	$\frac{Pcr}{Pu}$
I -1	1.51	0.0305	3680	55,000	140,700	0.391
I -2	1.51	0.0305	3330	55,000	139,700	0.394
II -3	1.51	0.0188	3170	45,000	117,700	0.382
II -4	1.51	0.0188	3830	50,000	140,700	0.332
III -5	1.51	0.0185	3730	50,000	129,700	0.386
III -6	1.51	0.0185	3710	50,000	130,700	0.383
IV -7	1.51	0.0186	3500	50,000	130,750	0.382
IV -8	1.51	0.0186	3610	55,000	136,700	0.402
V -9	1.51	0.0116	3350	45,000	100,700	0.447
V -10	1.51	0.0116	3910	48,000	120,700	0.398
VI -11	1.51	0.0117	3680	40,000	100,700	0.397
VI -12	1.51	0.0117	3720	45,000	120,700	0.373
V -13	1.51	0.0075	3250	35,000	100,000	0.350
V -14	1.51	0.0075	3870	40,000	100,700	0.397
VI -15	1.51	0.0075	3700	40,000	80,700	0.496
VI -16	1.51	0.0075	3310	40,000	84,800	0.472
IIIa-17	3.78	0.0254	4240	35,000	39,600	0.884
IIIa-18	3.78	0.0254	3650	35,000	36,300	0.909
Va-19	3.78	0.0093	3410	24,000	28,450	0.844
Va-20	3.78	0.0093	3710	25,000	29,650	0.843
VIb-21	2.84	0.0084	3790	25,000	32,100	0.779
VIb-22	2.84	0.0084	3740	24,000	28,050	0.856
VIb-23	2.84	0.0084	4430	27,000	33,750	0.800
VIa-24	3.78	0.0047	3820	21,000	24,500	0.857
VIa-25	3.78	0.0047	3740	18,000	22,450	0.802

Table 5. Comparison of Pcr to Pu of Mathey and Watstein's Test Results.

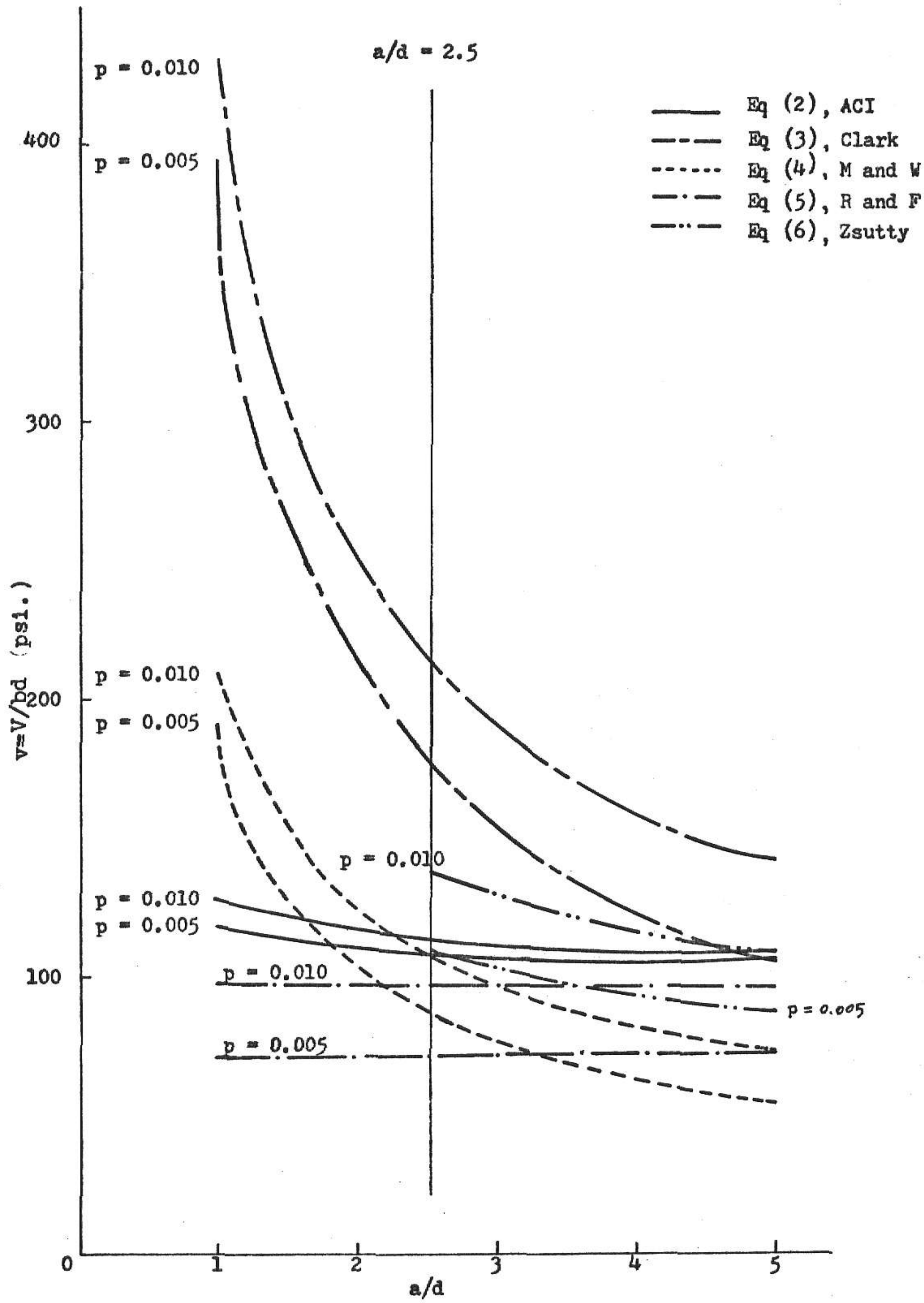

Fig. 1a. Comparison of empirical formulas, $0 < p \leq 0.01$

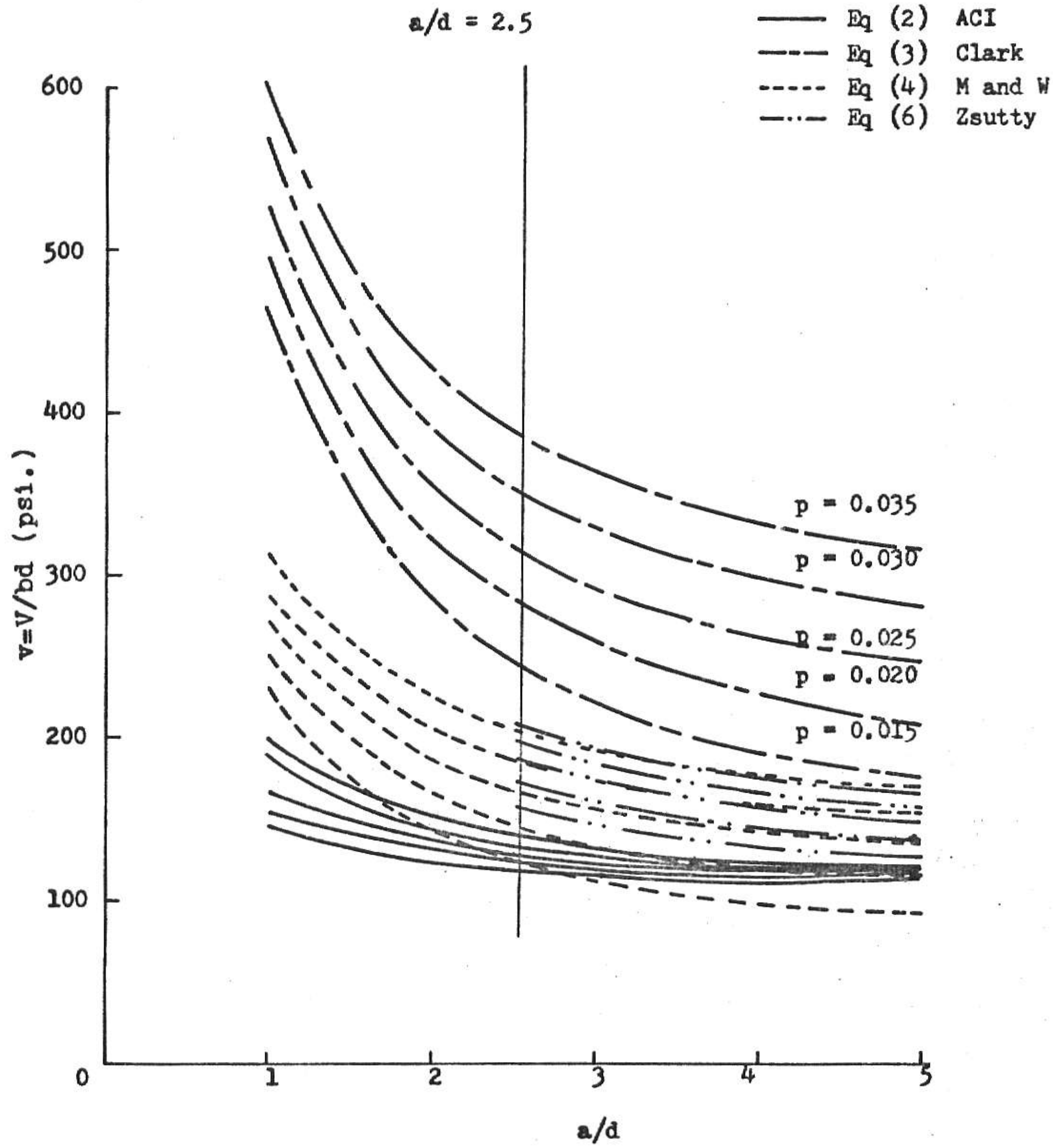
a/d = 2.5
Eq (2) ACI
Eq (3) Clark
Eq (4) M and W
Eq (6) Zsutty
600
500
400
300
200
100
0
v=V/bd (psi.)
p = 0.035
p = 0.030
p = 0.025
p = 0.020
p = 0.015
1
2
3
4
5
a/d

Fig. 1b. $p > 0.01$

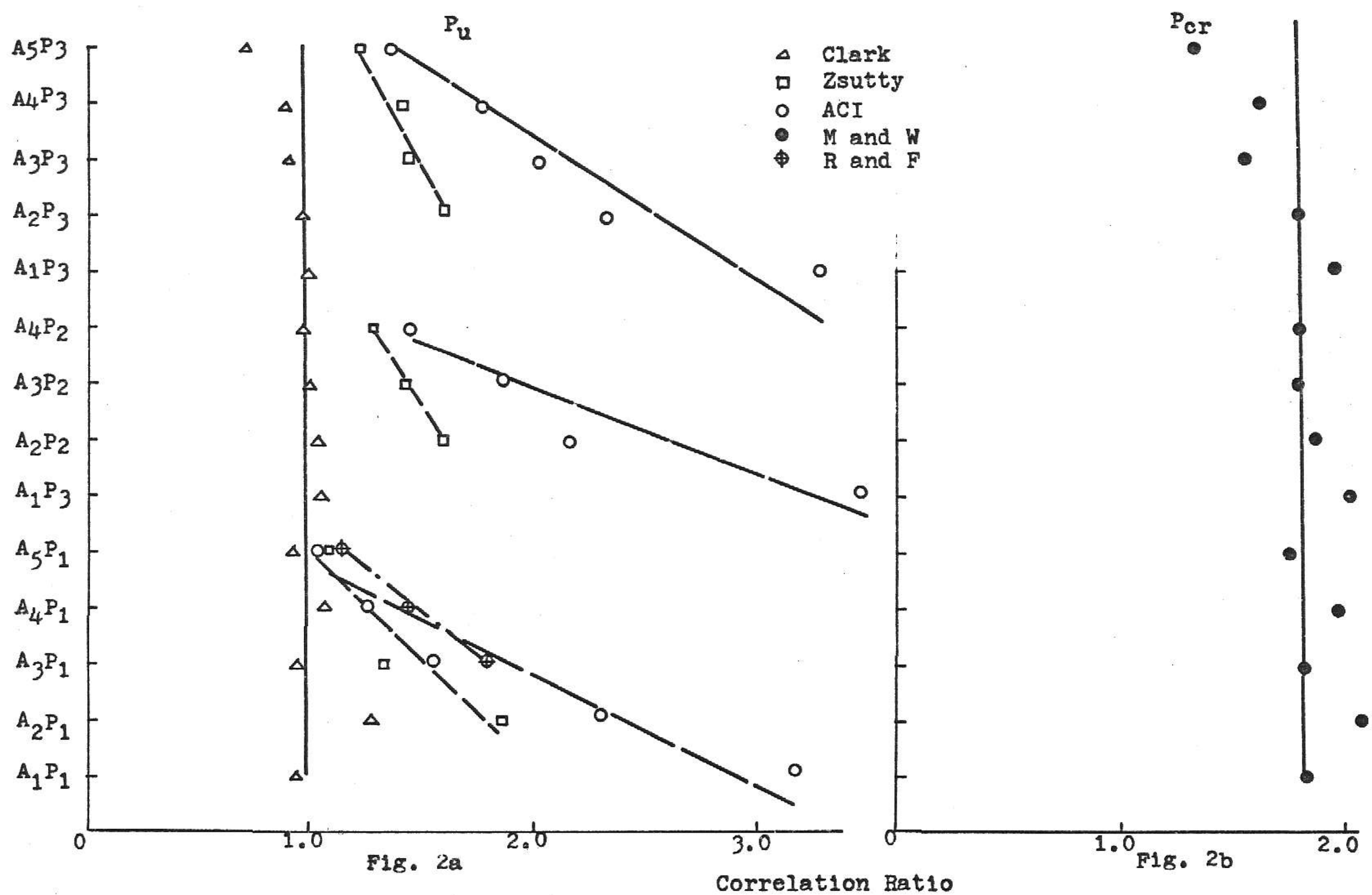

Fig. 2a

Fig. 2b

Comparison of Formulas with Hsieh' Test Results

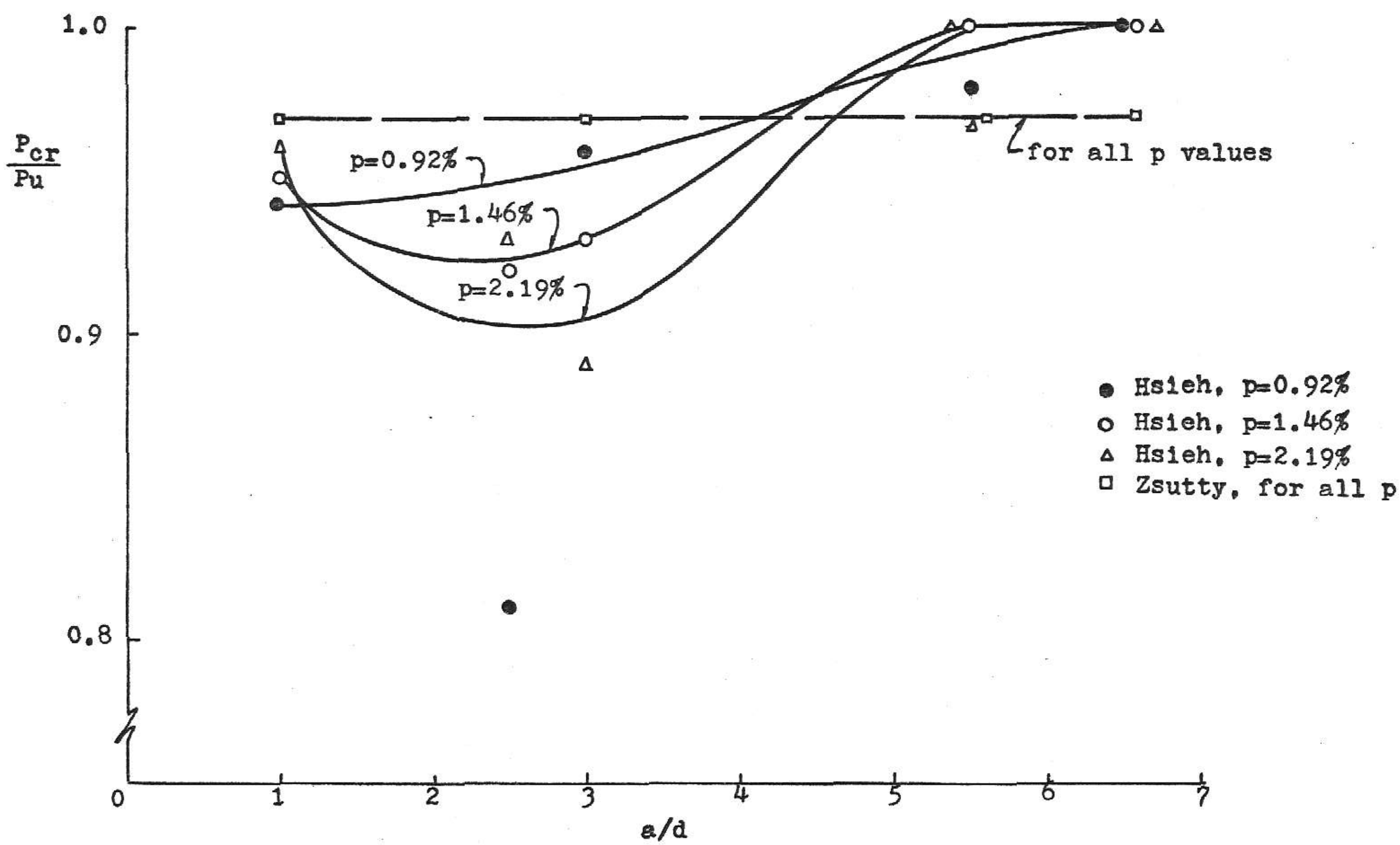

Fig. 3 Comparison of P_{cr} with P_u of Hsieh's Test Results and Zsutty's Empirical Formulas.

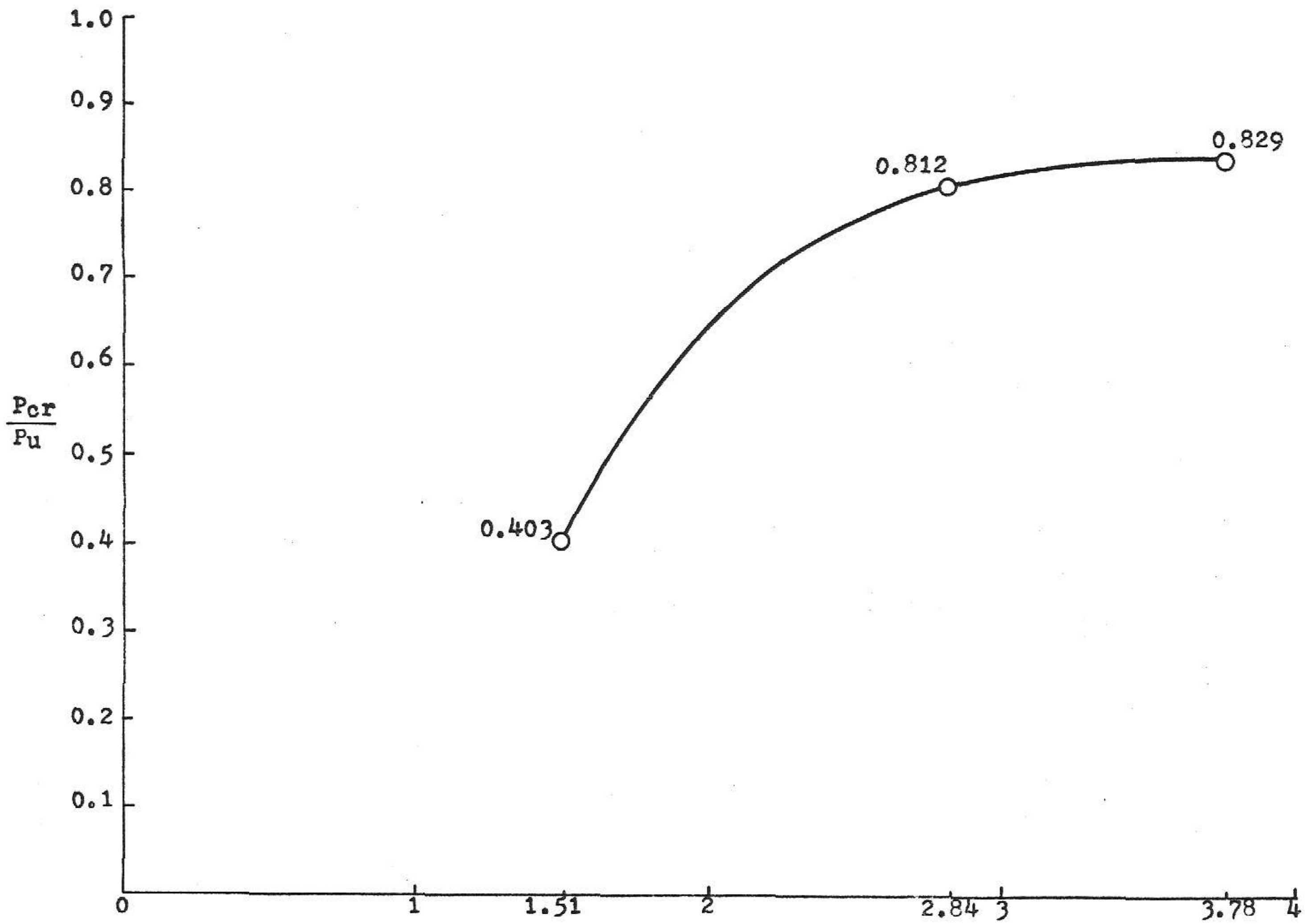

Fig. 4

Comparison of P_{cr} and P_u of M and W's Test Results

* Values of P_{cr}/P_u are average values of different values of a/d

SHEAR STRENGTH OF SIMPLY SUPPORTED REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

by

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B.S., Taiwan Provincial Taipei Institute of Technology, 1964

AN ABSTRACT OF A MASTER'S REPORT

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requirements for the degree

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Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1971

ABSTRACT

In this report the shear strength of simply supported reinforced concrete beams without web reinforcement was investigated.

Six representative research papers on the problem were studied. The results were summarized and the basic theories and empirical formulas compared. It was found that the basic theories are not much different from one another, but the test investigations and analytical research work emphasized different variables. The results obtained from empirical formulas are different from one another because they were all derived from differenttest data.

It is concluded that, at the present, there are no accurate empirical or working formulas available to predict the shear strength of reinforced concrete beams. More test data are necessary to evaluate the variables which affect the shear strength, and to derive a more accurate empirical shear strength formula.