OPTIMUM PLANNING OF CENTRALIZED WASTE TREATMENT SYSTEMS IN REGIONAL WATER QUALITY MANAGEMENT

by

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Taiwan, Rep. of China, 1972

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1976

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ACKNOWLEDGMENTS

The writer wishes to express his appreciation to Dr. C. L. Hwang, for the constructive criticism and guidance given in the preparation of this paper. He also wishes to express appreciation to Dr. Doris Grosh and Dr. Dean Eckhoff for their valuable advice and suggestions. Finally, the writer would like to extend particular appreciation to his mother, sisters and wife Ne Sa for the endless love and moral support provided to make this paper possible.

THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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TABLE OF CONTENTS

	Page
LIST OF FIGURES	v
LIST OF TABLES	vi
1. INTRODUCTION	1
2. DESCRIPTION OF THE PROBLEM	3
2.1 Statement of the Problem	3
2.2 General Model	5
2.3 Literature Review	6
3. OPTIMUM APPROACH BY AN ENUMERATION METHOD	8
3.1 Problem Formulation	8
3.2 Methodology	10
3.3 Illustrative Example	10
4. OPTIMUM APPROACH BY DYNAMIC PROGRAMMING	19
4.1 Introduction	19
4.2 Methodology	19
4.3 Illustrative Example	28
5. OPTIMUM APPROACH BY INTEGER PROGRAMMING	34
5.1 Introduction	34
5.2 Methodology	34
5.3 Illustrative Example	41
6. DISCUSSION AND CONCLUSION	51
6.1 Advantages of Each Method	51
6.2 Limits of Each Method	51
6.3 Conclusions	53

Page

REFERENCES 54

APPENDIX 55

LIST OF FIGURES

Figure Page

1. Relative construction costs of activated sludge plants. 4

2. A schematic sketch of the regional wastewater treatment system 9

3. Sketch of a watershed. 20

4. Illustration of nomenclature used in Converse's model 23

5. Dynamic programming interpretation of Converse's model 25

6. Fitted total pipe cost curve for a given link. 35

7. Piecewise linearized cost curve. 37

8. Schematic of Little Econ system. 43

9. Final network of pipe interceptors for Example 5.3. 48

LIST OF TABLES

Table		Page
1.	List of all possible cases and corresponding sewage allocations	11
2.	Wastewater loads, distances among communities and number of pump stations for example 3.3	13
3.	List of total cost and order of each case in example 3.3	18
4.	Wastewater loads, distances among communities and number of pump stations for example 4.3	29
5.	Results for single plant system in example 4.3	31
6.	Results of multi-plant systems in example 4.3	33
7.	Design flows and corresponding linearization segment for example 5.3	44
8.	Calculated coefficients for piecewise linearization of cost curves in example 5.3	45
9.	Optimum allocation for example 5.3	49

1. INTRODUCTION

Nowadays, people are more concerned about living in a clean environment. Ecological and other problems caused by water pollution have been noticed and watched almost by everybody. It is realized that even the largest rivers or lakes can no longer be treated as infinite sinks for human and industrial wastes. For preserving water quality, all kinds of wastewater should be treated to certain degree before dumped into rivers or lakes.

Numerous wastewater treatment plants have been built-up to prevent water resources from pollution. The ever increasing emphasis on and costs of water pollution control have focused attention on potential cost savings in alternative institutional arrangements and regional approaches toward that end. Regional wastewater treatment systems have been studied in recent years. The economies of scale favor installation of a few large treatment plants. This tends to be offset by the greater distance that wastewater must be piped. "The goal is to find an optimum system which has the minimum overall cost and satisfies the requirements of the sewage treatment of the whole region, based on the trade-off between economies of scale inherent in sewage treatment plants and added pipe network collection costs."

Converse (1972), Joeres et al. (1974), Naito et al. (1973) and Wanielista and Bauer (1972) have studied and published their works in this field respectively. Wanielista and Bauer applied integer programming to solve a quite general mathematical model of this problem. Joeres et al. adopted the same method and applied it to a regional wastewater system in Wisconsin. Naito et al. proposed an enumeration method of finding optimum number and location of plants of a regional sewage system. Converse used dynamic programming to get the optimum number and location of treatment plants systematically.

Objectives of this study are:

(1) to show how the methods of operations research may be used for planning of regional wastewater treatment systems.

(2) to find advantages and limits of those three methods mentioned above.

(3) to compare each method and modify them if possible.

Three methodologies: integer programming, dynamic programming and an enumeration method have been studied in detail. The advantages and limits of application of each method were found by means of comparison. Example problems were solved to illustrate the application of each methodology. The enumeration method is actually an exhaustive search methodology. The dynamic programming method is a more general and systematical methodology. The integer programming model is the most generalized model, however, it is the most difficult one in application.

2. DESCRIPTION OF THE PROBLEM

2.1 Statement of the Problem

The degradation of water quality becomes more serious as industry and community grow rapidly and continuously. With the public demand for a clean environment the cost of wastewater treatment will increase exponentially to meet higher requirements of water quality. For instance, primary treatment removes only 30% of BOD (biological oxygen demand) in wastewater. The primary and secondary treatments are capable of removing 90% of BOD. If the standard is 96% removal of BOD, then a tertiary treatment is required in addition to the primary and secondary.

As the standard of water quality increases, the cost of wastewater treatment increases also. One of the hopes of lowering the costs is the joining of industries and communities in a few treatment facilities to take advantage of the economies of scale. The economies of scale inherent in wastewater treatment plants is illustrated in Fig. 1 (Deininger and Su, 1973). The figure shows the relative construction costs of activated sludge plants (secondary treatment) in the U.S., England and Germany. The relative operation costs of activated sludge plants have similar characteristics as shown in Fig. 1. For other types of treatment plants similar economies of scale exist. The graph shows that the per capita costs of waste treatment are three times higher in a small town of 1,000 compared to the per capita costs in a city of 100,000. Since it is quite difficult to compare absolute costs in this world of changing monetary systems, the construction cost per capita in a city of 10,000 was arbitrarily assumed to be unity. From the standpoint of regional wastewater system, substantial savings may be achieved

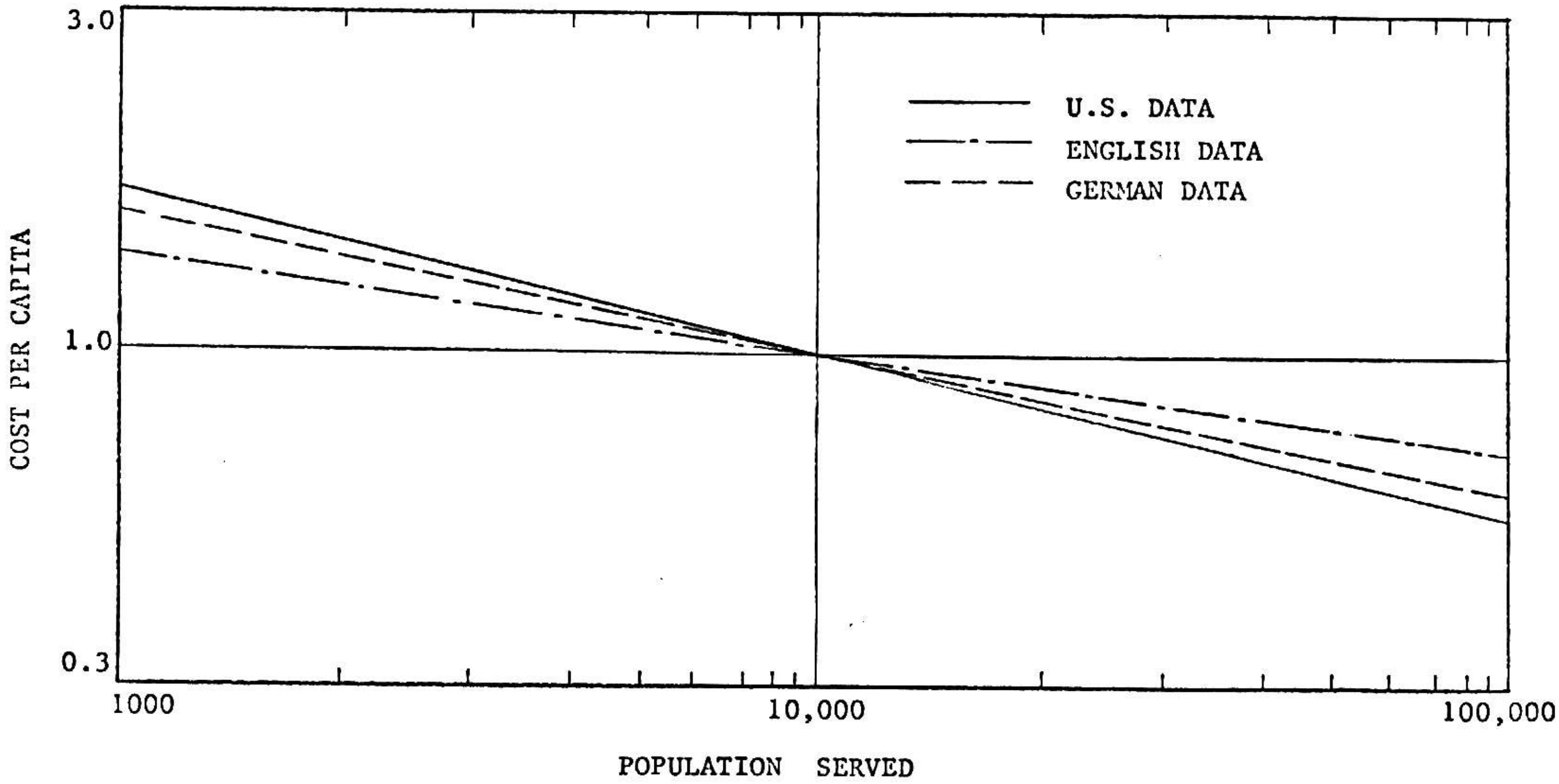


Figure 1. Relative construction costs of activated sludge plants

by building a few large plants to treat total sewage generated in the whole region, instead of many small plants, one for each community. Wastewater generated at each community is transported by pipes to the big regional treatment plants.

There is a certain amount of cost associated with sewer networks of the region. These include: pipe construction cost, maintenance and operating cost and pump station cost if pumping is needed. There comes out the very fundamental trade-off, that of cost savings due to economies of scale inherent in sewage treatment plants versus the added collection cost of the pipe interceptor networks.

The major question investigated is: given a number of communities and industries in a geographical region, where should treatment plants be built, how many, and which intercepting sewers are necessary to connect the municipalities and industries to these plants, such that the total cost of wastewater treatment is a minimum.

2.2 General Model

There are many models in existence. But basically their objective is to minimize the sum of treatment and piping costs of the system, subject to the requirement that all wastewater generated in the system must be treated to a specified water quality standard. The general model may be written as:

$$\text{minimize} \left(\sum_{j=1}^{N} T_j(X_j) + \sum_{j=1}^{N} \sum_{i=1}^{N} P_{ij}(Y_{ij}) \right)$$

subject to:

$$L_j + \sum_{i=1}^{N} Y_{ij} - \sum_{i=1}^{N} Y_{ji} - X_j = 0 \qquad \forall j$$

$$X_j \geqslant 0 \qquad \forall\ j$$

$$Y_{ij} \geqslant 0 \qquad \forall\ i\text{-}j \text{ pipe interceptor}$$

where

i, j = location indeces

T_j = cost function of wastewater treatment at location j

X_j = amount of wastewater treated at location j

P_{ij} = cost function of wastewater transportation

Y_{ij} = amount of wastewater transported from location i to location j

N = total number of communities in the system

L_j = amount of wastewater generated at location j

As to cost calculation, various cost functions were used for different illustrative examples in the following sections.

2.3 Literature Review

Wanielista and Bauer (1972) investigated the centralization of waste treatment facilities. A general model that can be used for planning the location and size of wastewater treatment plants and sewer lines was presented. An integer programming algorithm was used for finding the solution. The application of their model was illustrated by examining an existing sewer system, the Little Econlockahatchee (Econ) River basin in Orlando, Florida.

Converse (1972) published a paper entitled "Optimum number and location of treatment plants." His method is based on an extension of dynamic programming. The method was illustrated by planning a new wastewater treatment system for the Merrimack River Valley in New Hampshire and Massachusetts.

Naito and Takamatsu (1973) published their investigation on the "Optimum Planning of Sewage Treatment Systems for Preserving Stream Quality." They published (1971, 1973) other two Japanese papers which discuss similar problems. In these papers, an enumeration method was adopted to find the optimum number and location of treatment plants for a regional sewage system. They applied this method in planning an optimum sewage treatment system of six communities along a river in Japan.

Deininger and Su (1973) published the "Modelling Regional Waste Water Treatment Systems." They used a nonconvex quadratic minimization algorithm. Joeres et al. (1974) studied "The Planning Methodology for the Design of Regional Waste Water Treatment Systems." They used a model which is very similar to that used by Wanielista and Bauer (1972). A new wastewater treatment system was designed for the central portion of Dane County, Wisconsin. An integer programming algorithm was adopted to solve that problem.

A general review of decision models on regional water quality management can be seen in Budgaard-Nielsen and Hwang (1976).

3. OPTIMUM APPROACH BY AN ENUMERATION METHOD

3.1 Problem Formulation

A region-wide wastewater treatment system may be composed of several communities located closely along a river as shown in Fig. 2. The communities line up in series roughly. In each sub-region (area), the amount of wastewater generated locally plus that transported from upstream areas should equal the sum of the waste water treated there and that transported to downstream areas. When there are not many communities in the region, the overall cost of each possible treatment system configuration can be obtained without difficulty.

The mathematical model is:

minimize $\sum_{i} J(i)$

subject to:

$$Y_i = Y_{i-1} + Q_i - q_i \qquad \forall\, i \qquad (1)$$

where

$J(i)$ = total cost function for area i

= treatment plant cost + pipe cost + pump station cost

= $f(q_i, S_i) + g(\ell_i, P_i) + h(m_i, Y_i)$

i = area index

q_i = amount of sewage treated at the plant located in the ith area

Q_i = amount of sewage generated at the ith area

Y_{i-1} = amount of sewage transported from the (i-1)th area to ith area

S_i = amount of sewage sludge generated at the ith area

ℓ_i = distance between the ith and (i+1)th plants

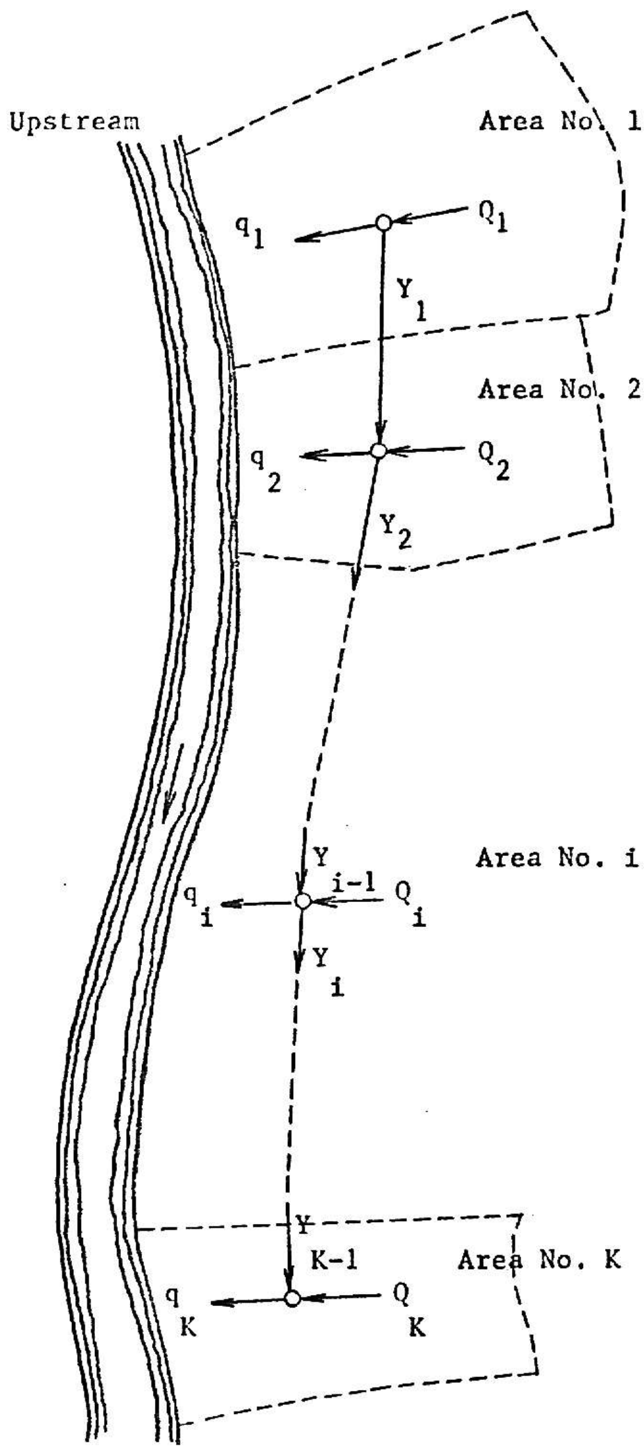


Figure 2. A schematic sketch of the regional wastewater treatment system

P_i = pipe diameter

m_i = number of pump stations

An optimum configuration associated with a minimum overall cost can be found by an exhaustive search through all cases.

3.2 Methodology

It is proved that the present problem of choosing economical sizes and locations of treatment plants may be reduced to a simple combinatorial problem. Consider, for example, the case of K communities located along a river. There are K-1 branches among these communities. Starting from the most upstream community, no sewer is required to connect the next community if one treatment plant is built here. But one sewer connecting the next community is necessary if a treatment plant is built there to handle the total sewage generated by these two communities. Hence for each branch between two communities, there are two alternatives: build or not build a sewer. This gives $2^{(K-1)}$ ways in total. It is better to put all possible cases in a table like Table 1. In that table, "0" represents that there is no pipe connection between those two adjacent areas, and "1" represents that there is a pipe connection. With the aid of that table, the amount of wastewater treated, q_i, and the amount of wastewater transported, Y_i, are obtained step by step. Then it is possible to calculate the total cost of each case through the cost functions.

3.3 Illustrative Example

A regional wastewater treatment system around B river in Japan is considered. In the region, six areas line up along the river. The estimation of wastewater generated in each area, Q_i, in 1985 is given. The

Table 1. List of all possible cases and corresponding sewage allocations

Case \ Area no.	1	2	3	4	5	6	Amount of sewage transported (10^5t/d) Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Amount of sewage treated (10^5t/d) q_1	q_2	q_3	q_4	q_5	q_6
1	1	1	1	1	1		1.57	4.77	7.37	13.19	14.63	0	0	0	0	0	0	15.08
2	0	1	1	1	1		0	3.2	5.8	11.62	13.06	0	1.57	0	0	0	0	13.51
3	1	0	1	1	1		1.57	0	2.6	8.42	9.86	0	0	4.77	0	0	0	10.31
4	0	0	1	1	1		0	0	2.6	8.42	9.86	0	1.57	3.2	0	0	0	10.31
5	1	1	0	1	1		1.57	4.77	0	5.82	7.26	0	0	0	7.37	0	0	7.71
6	0	1	0	1	1		0	3.2	0	5.82	7.26	0	1.57	0	5.8	0	0	7.71
7	1	0	0	1	1		1.57	0	0	5.82	7.26	0	0	4.77	2.6	0	0	7.71
8	0	0	0	1	1		0	0	0	5.82	7.26	0	1.57	3.2	2.6	0	0	7.71
9	1	1	1	0	1		1.57	4.77	7.37	0	1.44	0	0	0	0	13.19	0	1.89
10	0	1	1	0	1		0	3.2	5.8	0	1.44	0	1.57	0	0	11.62	0	1.89
11	1	0	1	0	1		1.57	0	2.6	0	1.44	0	0	4.77	0	8.42	0	1.89
12	0	0	1	0	1		0	0	2.6	0	1.44	0	1.57	3.2	0	8.42	0	1.89
13	1	1	0	0	1		1.57	4.77	0	0	1.44	0	0	0	7.37	5.82	0	1.89
14	0	1	0	0	1		0	3.2	0	0	1.44	0	1.57	0	5.8	5.82	0	1.89
15	1	0	0	0	1		1.57	0	0	0	1.44	0	0	4.77	2.6	5.82	0	1.89
16	0	0	0	0	1		0	0	0	0	1.44	0	1.57	3.2	2.6	5.82	0	1.89
17	1	1	1	1	0		1.57	4.77	7.37	13.19	0	0	0	0	0	0	14.63	0.45
18	0	1	1	1	0		0	3.2	5.8	11.62	0	0	1.57	0	0	0	13.06	0.45
19	1	0	1	1	0		1.57	0	2.6	11.62	0	0	0	4.77	0	0	9.86	0.45
20	0	0	1	1	0		0	0	2.6	11.62	0	0	1.57	3.2	0	0	9.86	0.45
21	1	1	0	1	0		1.57	4.77	0	5.82	0	0	0	0	7.37	0	7.26	0.45
22	0	1	0	1	0		0	3.2	0	5.82	0	0	1.57	0	5.8	0	7.26	0.45
23	1	0	0	1	0		1.57	0	0	5.82	0	0	0	4.77	2.6	0	7.26	0.45
24	0	0	0	1	0		0	0	0	5.82	0	0	1.57	3.2	2.6	0	7.26	0.45
25	1	1	1	0	0		1.57	4.77	7.37	0	0	0	0	0	0	13.19	1.44	0.45
26	0	1	1	0	0		0	3.2	2.6	0	0	0	1.57	0	0	11.62	1.44	0.45
27	1	0	1	0	0		1.57	0	2.6	0	0	0	0	4.77	0	8.42	1.44	0.45
28	0	0	1	0	0		0	0	2.6	0	0	0	1.57	3.2	0	8.42	1.44	0.45
29	1	1	0	0	0		1.57	4.77	0	0	0	0	0	0	7.37	5.82	1.44	0.45
30	0	1	0	0	0		0	3.2	0	0	0	0	1.57	0	5.8	5.82	1.44	0.45
31	1	0	0	0	0		1.57	0	0	0	0	0	0	4.77	2.6	5.82	1.44	0.45
32	0	0	0	0	0		0	0	0	0	0	0	1.57	3.2	2.6	5.82	1.44	0.45

distances between each area, ℓ_i, and the number of pump stations, m_i, needed in each pipe connection are also given in Table 2.

There are $2^{(6-1)} = 32$ possible ways to locate treatment plants and pipes in this region. The q_i and Y_i values for each of the 32 cases are given in Table 1. The cost function of area i for case j, $J_j(i)$, is given as:

$$
\begin{aligned}
J_j(i) = {} & \left.\begin{aligned}
& 5900\, q_i^{0.742} \text{........construction cost} \\
& + 401.5Nq_i^{0.798} \text{....operating cost excluding sludge treatment} \\
& + 30.0Nq_i \text{.....maintenance cost for sludge treatment}
\end{aligned}\right\} \text{Treatment plant cost} \\
& + 150(3.3)q_i^{0.7} \text{...........land cost} \\
& + \left\{\begin{aligned}
& 0.55(\ell_i)P_i \text{.........if } P_i \leqslant 1.8 \text{ meter} \\
& 1.15(\ell_i)(P_i - 0.7) \text{...if } P_i \geqslant 1.8 \text{ meter}
\end{aligned}\right\} \text{Pipe cost} \quad (2) \\
& \left.\begin{aligned}
& + 600m_iY_i^{0.6} \text{............construction cost} \\
& + (6.7Y_i + 200)Nm_i \text{........operating cost} \\
& + 0.793(3.3)Y_i^{0.5}m_i \text{........land cost}
\end{aligned}\right\} \text{Pump station cost}
\end{aligned}
$$

where

j = case suffix

i = area suffix

q_i = amount of wastewater treated at area i(10^3 ton/day)

N = project life

ℓ_i = distance between the ith and (i+1)th areas (10^2 meter)

P_i = diameter of the ith trunk sewer connecting the ith and the (i+1)th areas (meter)

m_i = number of pump stations needed in the ith trunk sewer

Y_i = amount of wastewater transported from the ith to the (i+1)th area (10^3 ton/day)

$J_j(i)$ = total cost at the ith area for case j (10^4¥, Japanese dollar)

Table 2. Wastewater loads, distances among communities and number of pump stations for example 3.3

Area	1	2	3	4	5	6
Q_i (10^3t/d)	157	320	260	582	144	45
ℓ_i (10^2m)	61.5	265	225	50	87.5	
m_i	0	3	1	0	4	

A formula for calculating the pipe diameter is given as:

$$P = \left[\frac{Y_i(10)}{4.32\ (86.4)} \right]^{1/2\cdot 38} \times 10^{-3} \qquad (3)$$

$$= 0.19084 \cdot Y_i^{0.42017} \qquad \text{(meter)}$$

The pipe diameter formula and the maintenance cost for sludge treatment were obtained through private correspondence with Naito (1976). For consistency, there should be a m_i in the last term (land cost of pump station) of the cost function.

The total cost of case j is $J_j = \sum_i J_j(i)$, and there are 32 J_j's, since 32 cases exist in this problem. The minimum total cost is chosen among these J_j's. The optimum configuration of this regional wastewater treatment system is the case which has the minimum total cost. In case 8, three plants serve areas 1, 2 and 3 respectively and the fourth plant serves areas 4,5 and 6. In this case:

$Y_1 = 0(10^3 t/d)$, $q_1 = 157(10^3 t/d)$, $\ell_1 = 6150(m)$, $m_1 = 0$, $P_1 = 0$ (m)
$Y_2 = 0$, $q_2 = 320$, $\ell_2 = 26500$, $m_2 = 3$, $P_2 = 0$
$Y_3 = 0$, $q_3 = 260$, $\ell_3 = 22500$, $m_3 = 1$, $P_3 = 0$
$Y_4 = 582$, $q_4 = 0$, $\ell_4 = 50000$, $m_4 = 0$, $P_4 = 2.7695$
$Y_5 = 726$, $q_5 = 0$, $\ell_5 = 8750$, $m_5 = 4$, $P_5 = 3.3091$
$Y_6 = 0$, $q_6 = 771$, $\ell_6 = 0$, $m_6 = 0$, $P_6 = 0$

area 1	area 2	area 3
$J_8(1)$=.25131 E10	$J_8(2)$=.42626 E10	$J_8(3)$=.36539 E10
+.34049 E10	+.60102 E10	+.50924 E10
+.70650 E 9	+.14400 E10	+.11700 E10
+.17050 E 9	+.28068 E 9	+.24271 E 9
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
=.67950 E10	=.11993 E11	=.10159 E11

area 4	area 5	area 6
$J_8(4)$= 0	$J_8(5)$= 0	$J_8(6)$=.81855 E10
+ 0	+ 0	+.12124 E11
+ 0	+ 0	+.34695 E10
+ 0	+ 0	+.51945 E 9
+.11900 E 9	+.23537 E 9	+ 0
+ 0	+.12496 E10	+ 0
+ 0	+.30385 E10	+ 0
+ 0	+.28204 E 7	+ 0
=.11900 E 9	=.45263 E10	=.24298 E11

Total cost for this case is:

$$J_8 = \sum_{i=1}^{6} J_8(i) = 578.91 \times 10^8 \text{ ¥}$$

For case 17, the cost is calculated as follows:

Y_1=157 (10^3t/d), q_1= 0(10^3t/d), ℓ_1=6150 (m), m_1= 0, P_1=1.5971(m)

Y_2=477 , q_2= 0 , ℓ_2=26500 , m_2= 3, P_2=2.5474

$Y_3=737(10^3 t/d)$, $q_3= 0\ (10^3 t/d)$, $\ell_3=22500(m)$, $m_3= 1$, $P_3=3.0584(m)$

$Y_4=1319$, $q_4= 0$, $\ell_4=5000$, $m_4= 0$, $P_4=3.9057$

$Y_5= 0$, $q_5=1463$, $\ell_5=8750$, $m_5= 4$, $P_5= 0$

$Y_6= 0$, $q_6= 45$, $\ell_6= 0$, $m_6= 0$, $P_6= 0$

The Y_i and q_i are obtained step by step according to the configuration of this case. The configuration which determines pipe connections among areas is (1 1 1 1 0). This means: Wastewater generated at areas 1, 2, 3 and 4 is transported to area 5 and one plant at area 5 treats the total amount of $1463\times10^3 t/d$. Another plant at area 6 handles only the wastewater generated there, i.e., $45\times10^3 t/d$.

Cost at each area is:

area 1	area 2	area 3
$J_{17}(1)=$ 0	$J_{17}(2)=$ 0	$J_{17}(3)=$ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
+.54020 E 8	+.56300 E 9	+.61023 E 9
+ 0	+.72842 E 9	+.31523 E 9
+ 0	+.15282 E10	+.77068 E 9
+ 0	+.17146 E 7	+.71043 E 6
=.54020 E 8	=.28213 E10	=.16969 E10

area 4	area 5	area 6
$J_{17}(4) = 0$	$J_{17}(5) = .13166\text{ E11}$	$J_{17}(6) = .99434\text{ E 9}$
+ 0	+.20213 E11	+.12561 E10
+ 0	+.65835 E10	+.20250 E 9
+ 0	+.81335 E 9	+.71097 E 8
+.18433 E 9	+ 0	+ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
+ 0	+ 0	+ 0
=.18433 E 9	=.40777 E11	=.25241 E10

The total cost for case 17 is

$$J_{17} = \sum_{i=1}^{6} J_{17}(i) = 480.57 \times 10^8 \text{ ¥}$$

In a similar way, costs for the other cases can be obtained as shown in Table 3. Among those 32 cases, the optimum configuration is case 17. In that case, two plants are built, one serves areas 1 through 5 and another plant serves area 6 only. This method assumes the first plant is located at area 5. The corresponding minimum total cost is 480.57×10^8 ¥. A computer program written in FORTRAN IV was used to calculate: pipe diameter, individual cost and the total cost. The program and sample input coding are given in Appendix 1. Because the formula for pipe diameter and the calculations for maintenance cost of sludge treatment and land cost of pump station have been modified, the results shown in Table 3 are not the same as those presented in Naito's paper (1973).

Table 3. List of total cost and order of each case in example 3.3

Case \ Area No.	1 2 3 4 5 6	Total cost (x 10^8 ¥)*	Order
1	1 1 1 1 1	547.95	19
2	0 1 1 1 1	561.62	25
3	1 0 1 1 1	540.90	14
4	0 0 1 1 1	563.10	26
5	1 1 0 1 1	552.50	23
6	0 1 0 1 1	571.34	29
7	1 0 0 1 1	556.71	24
8	0 0 0 1 1	578.91	32
9	1 1 1 0 1	511.90	5
10	0 1 1 0 1	532.51	12
11	1 0 1 0 1	525.70	10
12	0 0 1 0 1	547.90	18
13	1 1 0 0 1	548.07	20
14	0 1 0 0 1	566.91	28
15	1 0 0 0 1	552.29	22
16	0 0 0 0 1	574.48	31
17	1 1 1 1 0	480.57	1
18	0 1 1 1 0	501.79	3
19	1 0 1 1 0	496.87	2
20	0 0 1 1 0	519.07	6
21	1 1 0 1 0	520.99	7
22	0 1 0 1 0	539.83	13
23	1 0 0 1 0	525.20	9
24	0 0 0 1 0	547.40	17
25	1 1 1 0 0	509.33	4
26	0 1 1 0 0	529.95	11
27	1 0 1 0 0	523.13	8
28	0 0 1 0 0	545.33	15
29	1 1 0 0 0	545.51	16
30	0 1 0 0 0	564.34	27
31	1 0 0 0 0	549.72	21
32	0 0 0 0 0	571.91	30

* ¥ is Japanese dollars (320 ¥ = 1 U.S. dollar)

4. OPTIMUM APPROACH BY DYNAMIC PROGRAMMING

4.1 Introduction

Consider a watershed such as the one shown in Fig. 3. The wastewater sources all lie along the main stem of the river. Presumably there is a finite number of locations at which sewage treatment plants could be located. The economies of scale favor installation of a few large treatment plants as discussed before. However, this will be offset by the piping cost. Hence, a trade-off exists between the number of treatment plants and the extent of trunk sewers. Converse (1972) presented a method for determining the optimum allocation of sewage treatment facilities. His method is based on an extension of dynamic programming in which the solution is embedded in both the upstream and downstream conditions.

4.2 Methodology

Consider K communities lined up in sequence in a watershed. The amount of sewage generated at each location is known. The distances among these K locations are given. It is also assumed that the number of pump stations needed in each pipe interceptor is known. Based on the above information, we can calculate the treatment plant cost and piping cost by cost functions which will be presented in the next paragraph. Since the communities are in sequence, it is easy to order them from 1 to K. Then we may divide the whole region into subregions to find optimum locations of treatment plants. For instance, one subregion may start at location 1 (the first community) and end up at location 3 (the third community). In this subregion we may have one treatment plant at location 2 and two pipe

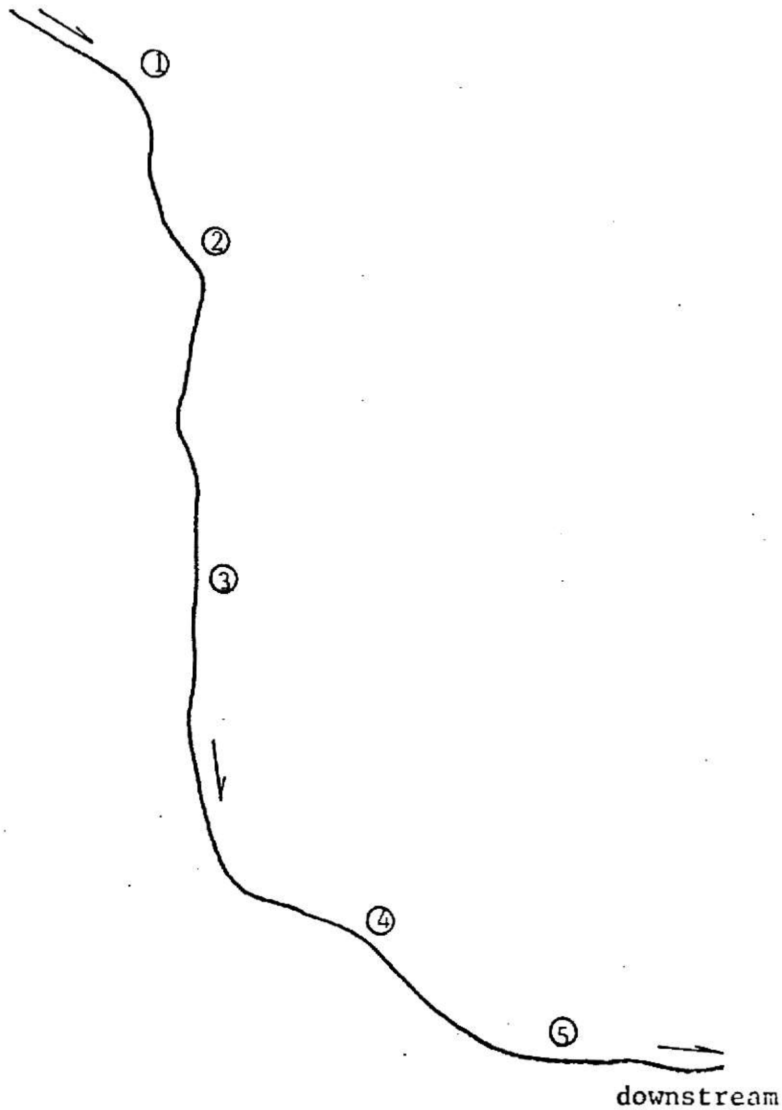


Figure 3. Sketch of a watershed

interceptors and four pump stations to transport sewage from locations 1 and 3 to that treatment plant. The total cost including treatment cost and piping cost for above allocation can be calculated. There are many ways to group communities into subregions. It is better to put costs of subregions into mathematical notations, so that the minimum cost can be found systematically.

Firstly, three triangular matrices, $\overline{\overline{V}}$, $\overline{\overline{L}}$ and $\overline{\overline{H}}$ are evaluated. The $\overline{\overline{V}}$ matrix is given as:

$$\overline{\overline{V}} = \begin{pmatrix} V(1,1) & V(1,2) & V(1,3) & . & . & . & V(1,K) \\ 0 & V(2,2) & V(2,3) & . & . & . & V(2,K) \\ 0 & 0 & V(3,3) & . & . & . & V(3,K) \\ . & . & . & . & . & . & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 0 & 0 & 0 & . & . & . & V(K,K) \end{pmatrix}$$

The matrices $\overline{\overline{L}}$ and $\overline{\overline{H}}$ have the same form as $\overline{\overline{V}}$'s. The entities of those matrices are respectively:

i,j = location indexes of community (sewage source)

$V(i,j)$ = the minimum cost of a single plant system serving the subregion from location i to location j (i,j included)

$L(i,j)$ = the corresponding optimum location of that single plant

$H(i,j)$ = the corresponding sewage flow to that plant

There is no need to have $V(j,i)$, since it has the same meaning as $V(i,j)$ has. So the matrices $\overline{\overline{V}}$, $\overline{\overline{L}}$, $\overline{\overline{H}}$ are triangular. These entity values are found by an exhaustive search.

Next, two rectangular matrices $\bar{\bar{B}}$ and $\bar{\bar{N}}$ are evaluated. The $\bar{\bar{B}}$ matrix is given as :

$$\bar{\bar{B}} = \begin{bmatrix} B(1,1) & B(1,2) & . & . & . & B(1,K) \\ B(2,1) & B(2,2) & . & . & . & B(2,K) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ B(K,1) & B(K,2) & . & . & . & B(K,K) \end{bmatrix}$$

Matrix $\bar{\bar{N}}$ has the same form as given by $\bar{\bar{B}}$. The entities of $\bar{\bar{B}}$ and $\bar{\bar{N}}$ are respectively:

B(i,s) = minimum cost of a system of s plants serving the region from location i through the right end of the set of sources

N(i,s) = optimum location of the right end of the left-most subregion which is bounded on one side by i

Figure 4 illustrates the above nomenclature. For the case of a single plant,

$$B(i,1) = V(i,K) \tag{4}$$

$$N(i,1) = K \tag{5}$$

For two plants,

$$B(i,2) = \underset{j}{\text{minimum}} \left[V(i,j) + B(j+1,1) \right] \tag{6}$$

$$N(i,2) = j, \text{ optimum as determined above} \tag{7}$$

where,

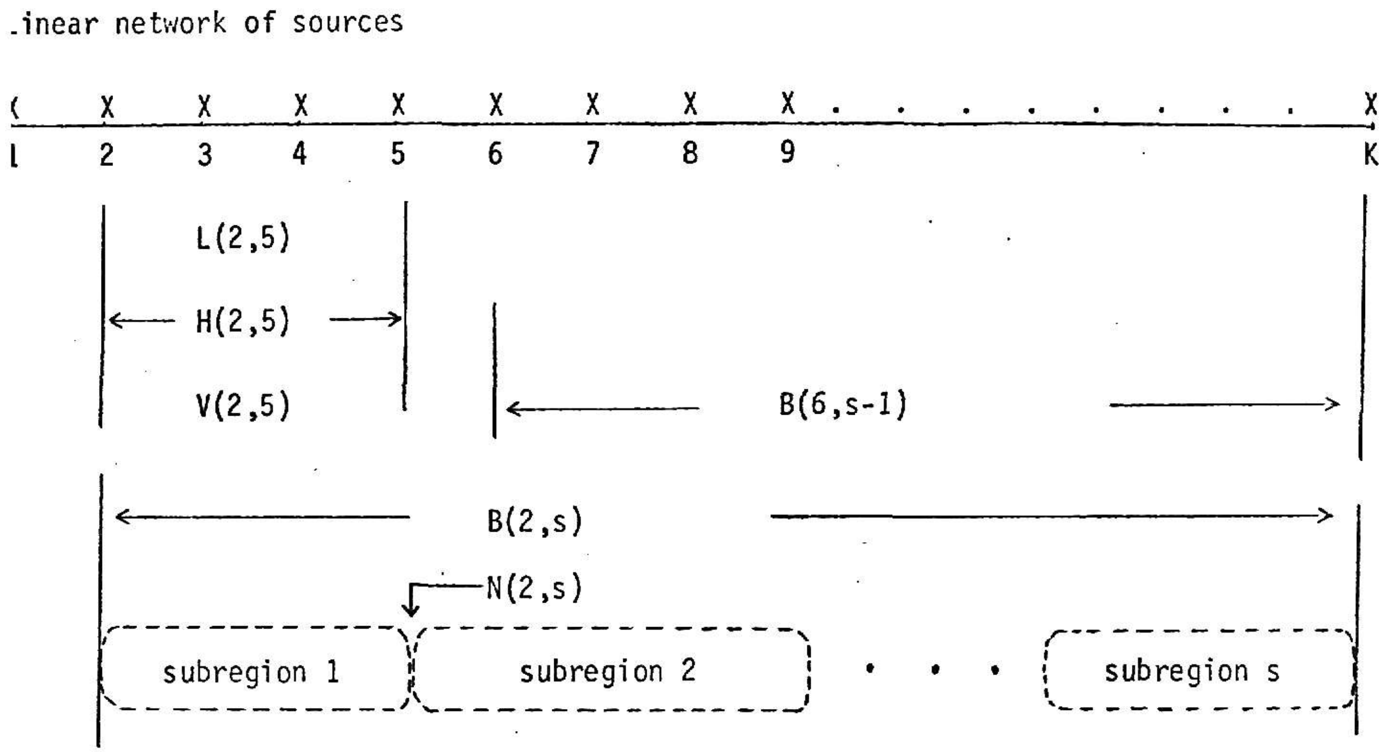
$$i = 1,2,. . . , K$$

$$j = i, i+1, . . . , K$$

For s plants,

$$B(i,s) = \underset{j}{\text{minimum}} \left[V(i,j) + B(j+1,s-1) \right] \tag{8}$$

$$N(i,s) = j, \text{ optimum as determined above} \tag{9}$$



(2,5) = minimum cost of single-plant system between 2 and 5

(2,5) = location of treatment plant in the above system

(2,5) = capacity of the above plant

(6,s-1) = minimum cost of (s-1)-plant system extending from 6 to K

(2,s) = minimum cost of s-plant system extending from 2 to K

(2,s) = location of right end of left most subregion in the above s-plant system

Figure 4. Illustration of nomenclature used in Converse's model

where,

$i = 1,2, \ldots, K$ $\qquad$ $j = i, i+1, \ldots, K$

$s = 1,2, \ldots, K-i+1$

Equations (4) and (5) serve as boundary conditions, and the matrices $\bar{\bar{B}}$ and $\bar{\bar{N}}$ are built up in a recursive manner indicated by equations (8) and (9). The optimal number of plants is the value for which $B(i,s)$ is a minimum. This is basically the procedure of dynamic programming.

Once the optimum number of plants, s', has been determined, the system design is given as follows: the first plant serves the subregion from 1 to $N(1,s')$ is located at $L(1,N(1,s'))$ and treats a flow of $H(1,N(1,s'))$ for a cost of $V(1,N(1,s'))$. The next plant serves a region from $N(1,s') + 1$ to $N(N(1,s')+1, s'-1)$, and so forth. The kth plant serves the region from the right boundary of the (k-1)th plant, to $i = N(\text{left boundary}, s'-k+1)$.

Dynamic programming interpretation

This method is interpreted in dynamic programming terminology as follows:

A multi-stage system is shown in Fig. 5. The notations are:

i_s = community location index (starting point of subregion)

= a parameter which depends on s

$j_s(i_s)$ = decision variable

= location index (end point of subregion starting at i)

s = number of treatment plants = stage index

Y_i = sewage input to location i ($i = 1,2,\ldots, K$) = state variable

Q_i = sewage generated at location i (given constant)

q_i = sewage treated at location i (variable depends on Y_i, Q_i)

K = total number of communities in the whole region

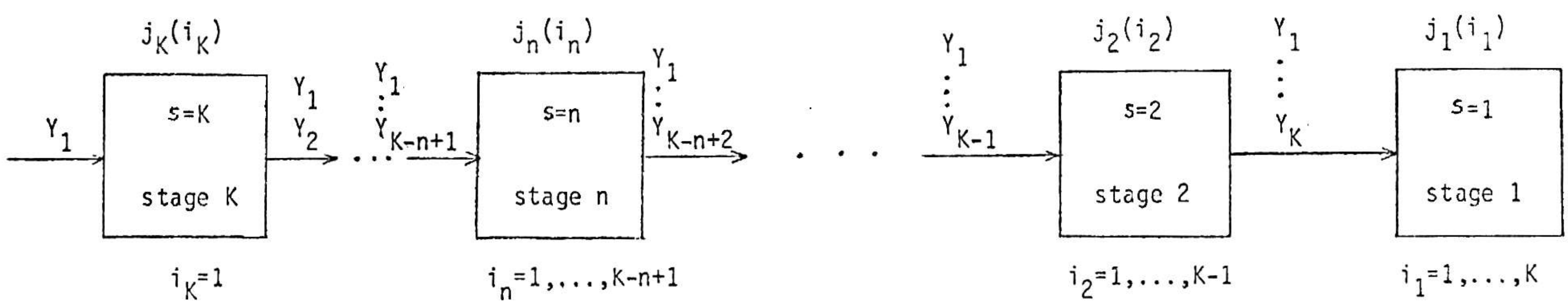


Figure 5. Dynamic programming interpretation of Converse's model

The transformation function is:

$$Y_{i+1} = Q_i + Y_i - q_i \qquad i = 1,2,\dots,K-1 \qquad Y_1 = 0$$

The return functions for each stage are cost functions in terms of Y_i, q_i and Q_i. They are:

$$TV(i,j(i)) \text{ and } B(i,s)$$

The detail of these functions will be given in the next paragraph.

The objective function is:

$$\underset{j}{\text{minimize}} \left[TV(i,j(i)) + B(j(i)+1,\ s-1) \right] \qquad \forall\, i,\ \forall\, s$$

subject to

$$\sum_{n=1}^{K} Q_n = \sum_{n=1}^{K} q_n$$

Cost functions

The total cost function for a single plant system serving the subregion from location i to location j is given (Joeres et al., 1974, Converse, 1972) as:

$$\begin{aligned} TV(i,j) = {} & 38.06 \left(\sum_{n=i}^{j} Q_n\right)^{0.690} + 10.8 \left(\sum_{n=i}^{j} Q_n\right)^{0.754} && \text{....treatment plant cost} \\ & + 0.08 \left[NP_{ij}(3.58)\left(\sum_{n=i+1}^{j} Y_n\right)(365) + 6500NP_{ij} \right] && \text{...pump station cost} \\ & + 0.69924\, \ell_{ij} && \text{...........pipe maintenance cost} \\ & + a_i\, 10^{b_i}\, \ell_{ij} && \text{..........pipe construction cost} \end{aligned}$$

$$i = 1,2,\dots,K$$

$$j = 1,2,\dots,K$$

where,

i,j,n = community location indexes

Q_n = sewage generated at location n

NP_{ij} = number of pump stations in pipe link connecting i and j

Y_n = sewage input to location n

ℓ_{ij} = distance between locations i and j

a_i, b_i = empirical constants, given in Appendix 2

Calculation Procedures

$$\text{Let } V(i,j) = \underset{j}{\text{minimum}} \left[TV(i,j),\ TV(j,i) \right] \qquad i,j = 1,2,\ldots,K$$

Notice for specific i and j, $V(i,j) = V(j,i)$, since the minimum is unique. For instance, if $TV(1,3) = 650$ and $TV(3,1) = 480$, then $V(1,3) = V(3,1) = 480$. We may confine the notation $V(i,j)$ to $i \leqslant j$ only, this is the reason we get a triangular matrix $\overline{\overline{V}}$.

The first step of this method is to find $V(i,j)$ for each $i = 1,2,\ldots,K$ and $j = i,i+1,\ldots,K$ by an exhaustive search. The second step of this method applies the basic idea of dynamic programming. The cost function $B(i,s)$ is defined as the minimum cost of a s-plant system serving the subregion from i to K. $B(i,s)$ is obtained recursively from $V(i,j)$. For example, when $s = 1$, i.e., stage 1 (single plant for whole region)

$$B(i,1) = V(i,K) \qquad i = 1,2,\ldots,K$$

When $s = 2$, i.e., stage 2 (two plants for whole region)

$$B(i,2) = \underset{j}{\text{minimum}} \left[V(i,j) + B(j+1,1) \right] \qquad \begin{array}{l} i = 1,2,\ldots,K-1 \\ j = i,i+1,\ldots,K-1 \end{array}$$

where $B(j+1,1)$ is the minimum cost obtained from stage 1. So at this stage, we may take advantage of the optimum results which have already been found in the preceding stage. When $s = n$, i.e., stage n (n plants for whole region)

$$B(i,n) = \underset{j}{\text{minimum}} \left[V(i,j) + B(j+1,n-1) \right] \qquad \begin{array}{l} i = 1,2,\ldots,K-n+1 \\ j = i,i+1,\ldots,K-n+1 \end{array}$$

where B(j+1,n-1) is the optimum value obtained from the previous stage. This is the characteristic of dynamic programming. Actually indexes i and j are parameters which depend on the stage index s.

The final optimal policy shall be obtained by minimum [B(1,1), B(1,2),..., B(1,K)]

4.3 Illustrative Example

A simple numerical example is considered here to illustrate the method of optimization. Table 4 gives the necessary data such as Q_i, wastewater generated at location i, ℓ_{ij}, the distance between locations i and j, NP_{ij}, the number of pump stations in pipe route i – j, a_i and b_i, empirical constants for pipe cost calculation. A computer program written by the present author was used to solve this problem. The program and sample input coding is given in Appendix 3. In this problem, the five communities are located in sequence along a river as shown in Figure 3.

Step 1 Find V(i,j) by exhaustive search

For obtaining V(1,2), the minimum cost of a single plant system serving the area from location 1 to location 2, two costs which are TV(1,2) and TV(2,1) must be calculated first. TV(1,2) is the cost associated with a system in which one plant is built at location 2. TV(2,1) is that of a system in which one plant is built at location 1. Firstly, TV(1,2) is calculated as follows:

$$\begin{aligned} TV(1,2) = {} & 38.06x(1.0x10^6+2.5x10^6)^{0.69}+10.8x(1.0x10^6+2.5x10^6)^{0.754} \\ & +0.08(0x3.58x10^6x365+6500x0) \\ & +0.69924x79200 \\ & +3.885x10^{0.0442}x79200 \\ = {} & \$2.57 \times 10^6 \end{aligned}$$

Table 4. Wastewater loads, distances among communities and number of pump stations for example 4.3

Area	1		2		3		4		5
Q_i (MGD)	1.0		2.5		6.0		3.0		5.0
ℓ_{ij} (ft)		79200		105600		184800		132000	
NP_{ij}				$\xrightarrow{2}$				$\xrightarrow{1}$	
a_i	3.885^a		4.072^b		4.4151^c		4.072		4.4151
b_i	0.0442^a		0.0394^b		0.0389^c		0.0394		0.0389

[a]pipe diameter = 60 inches

[b]pipe diameter = 66 inches

[c]pipe diameter = 78 inches

$$TV(2,1) = 38.06x(1.0x10^6+2.5x10^6)^{0.69}+10.8x(1.0x10^6+2.5x10^6)^{0.754}$$
$$+0.08(0x3.58x2.5x10^6x365+6500x0)$$
$$+0.69924x79200$$
$$+4.072x10^{0.0394}x79200$$
$$= \$2.584 \times 10^6$$

Hence, $V(1,2) = \text{minimum}\left[TV(1,2),\ TV(2,1)\right]$

$$= TV(1,2) = \$2.57 \times 10^6$$

This single plant locates at location 2, i.e., L(1,2) = 2. It has a capacity H(1,2) = 3.5 MGD. The other V(i,j)'s are obtained by similar calculations. The results are shown in Table 5. Notice that B(i,1) for each i are also known now, because B(i,1) = V(i,K) and N(i,1) = K. In this problem K equals 5.

Step 2 Application of dynamic programming

At stage 1 (s=1), we find B(i,1) which represents the minimum cost of a single-plant system serving the subregion from location i to location K (the last community). All B(i,1) and N(i,1) are found in step 1.

At stage 2 (s=2), the way of finding B(i,2) and N(i,2) is shown by calculation of B(1,2) which represents the minimum cost associated with a two-plant system serving the region from location 1 to location 5.

$$B(1,2) = \text{minimum}\left[V(1,j) + B(j+1,1)\right] \qquad j = 1,2,3,4$$
$$= \text{minimum}\left[V(1,1) + B(2,1),\ V(1,2) + B(3,1),\ V(1,3) + B(4,1),\ V(1,4) + B(5,1)\right]$$
$$= \text{minimum}\left[V(1,1) + V(2,5),\ V(1,2) + V(3,5),\ V(1,3) + V(4,5),\ V(1,4) + V(5,5)\right]$$
$$= \text{minimum}\left[0.89+8.85,\ 2.57+7.57,\ 5.40+4.62,\ 7.32+2.81\right]$$
$$= \text{minimum}\left[9.74,\ 10.14,\ 10.02,\ 10.13\right]$$
$$= \$9.74 \times 10^6$$

Table 5. Results for single plant system in example 4.3

I	J	V(I,J) \$ 10^6	L(I,J)	H(I,J)
1	1	0.89	1	1.00
1	2	2.57	2	3.50
1	3	5.40	3	9.50
1	4	7.32	3	12.50
1	5	9.54	3	17.50
2	1	2.57	2	3.50
2	2	1.71	2	2.50
2	3	4.66	3	8.50
2	4	6.61	3	11.50
2	5	8.85	3	16.50
3	1	5.40	3	9.50
3	2	4.66	3	8.50
3	3	3.20	3	6.00
3	4	5.24	3	9.00
3	5	7.57	3	14.00
4	1	7.32	3	12.50
4	2	6.61	3	11.50
4	3	5.24	3	9.00
4	4	1.95	4	3.00
4	5	4.62	5	8.00
5	1	9.54	3	17.50
5	2	8.85	3	16.50
5	3	7.57	3	14.00
5	4	4.62	5	8.00
5	5	2.81	5	5.00

$N(1,2) = j$ = optimum as determined above = 1

Hence the first plant is built at location 1. The location of the second plant is determined by $L(2,5)$, because the second plant serves the subregion from location 2 to location 5.

All $B(i,2)$ and $N(i,2)$ can be found similarly. The results of $B(i,s)$ and $N(i,s)$ for $s = 1,2,3,4,5$ and plant locations are given in Table 6. As another illustration of the technique, we find $B(2,3)$ at stage 3 ($s=3$).

$$
\begin{aligned}
B(2,3) &= \underset{j}{\text{minimum}} \left[V(2,j) + B(j+1,2) \right] \qquad j = 2,3 \\
&= \text{minimum} \left[V(2,2) + B(3,2),\ V(2,3) + B(4,2) \right] \\
&= \text{minimum} \left[1.71 + 7.82,\ 4.66 + 4.76 \right] \\
&= \text{minimum} \left[9.53,\ 9.42 \right] \\
&= \$9.42 \times 10^6
\end{aligned}
$$

$N(2,3)$ = optimum j determined above = 3

where:

$B(3,2)$ and $B(4,2)$ are the optimum results found in previous stage ($s=2$). In dynamic programming, the optimum results found in previous stages can be used to find the optimum policy for the present stage, so that the burden of exhaustive search is eliminated.

As shown in Table 6, if a single plant is desired to serve those five communities, the optimum location of that plant is at location 3. The minimum total cost, $B(1,1)$, of this single plant system is $\$9.54 \times 10^6$. If a two-plant system is preferred, the minimum total cost, $B(1,2)$, is $\$9.74 \times 10^6$. The first plant is at location 1, and the second plant is at location 3 which is read from $L(2,5)$.

This method provides alternative minimum cost policies for various i and s values.

Table 6. Results of multi-plant systems in example 4.3

I	S	B(I,S) $\$ 10^6$	N(I,S)	Plant location
1	1	9.54	5	3
2	1	8.85	5	3
3	1	7.57	5	3
4	1	4.62	5	5
5	1	2.81	5	5
1	2	9.74	1	1 3
2	2	9.28	2	2 3
3	2	7.82	3	3 5
4	2	4.76	4	4 5
1	3	10.15	3	3 4 5
2	3	9.42	3	3 4 5
3	3	7.96	3	3 4 5
1	4	10.30	1	1 3 4 5
2	4	9.67	2	2 3 4 5
1	5	10.56	1	1 2 3 4 5

5. OPTIMUM APPROACH BY INTEGER PROGRAMMING

5.1 Introduction

The economies of scale favor installation of a few large treatment plants. This, however, tends to be offset by the greater distance that sewage must be piped. The fundamental trade-off inherent in regional wastewater treatment systems has been discussed in those papers cited in literature review. In those papers, water quality standards are not explicitly specified - instead the level of treatment (primary, secondary or tertiary) is given - and thus for constant loads to the treatment plants water quality is indirectly stated. The problem becomes one of allocating treatment plants and pipe interceptors, such that the entire regional demand for wastewater service is satisfied at the lowest overall cost to the region, without violating any of the environmental capacity limits. The economies of scale associated with wastewater treatment systems has been discussed in section 2 of this report. Wanielista and Bauer presented an integer programming model to solve this problem in 1972. Two years later, Joeres et al. also presented a similar integer programming model.

5.2 Methodology

Smith and Eilers (1970) developed plant construction cost function and annual maintenance and operating cost function. Applying a present value factor corresponding to an amortization of 7% per year over 25 years, the total treatment plant cost as a function of flow is

$$C_T = 38.06\ Q^{0.690} + 10.8\ Q^{0.754} \qquad (10)$$

where Q is in gallons per day. Figure 6 (Joeres et al., 1974) shows the

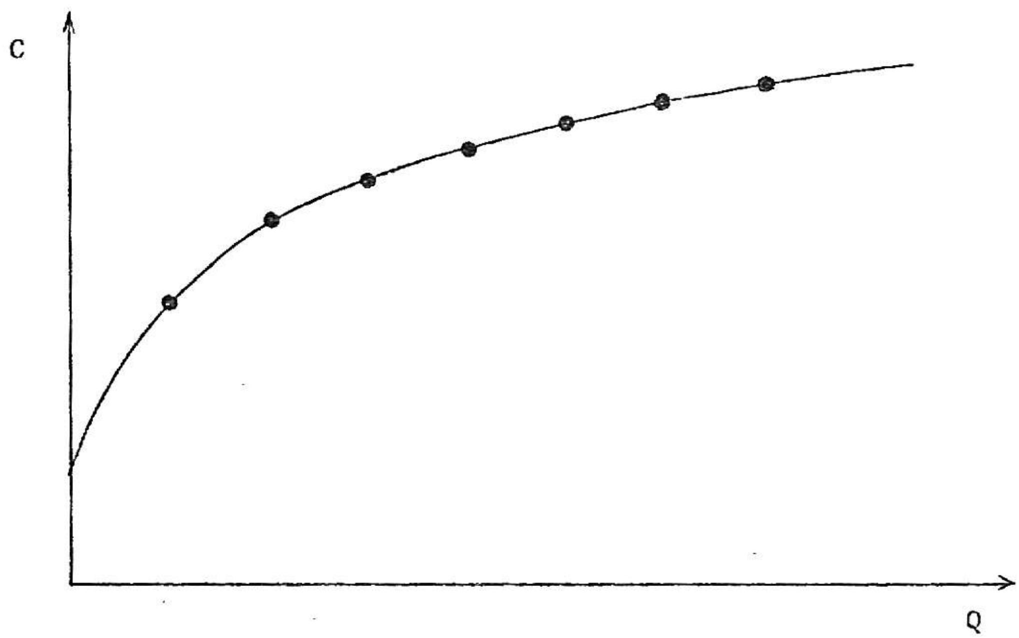


Figure 6. Fitted total pipe cost curve for a given link

resulting treatment cost computed from (10) for discrete flow values. Pipe cost functions comprise construction, maintenance, and operating costs, the latter comprising energy costs for those alternative connections where pumping would be required. The resulting cost function has a curve which is similar to that in Fig. 6.

The concave cost curve for treatment plant or pipe can be approximated by a piecewise linear approximation consisting of linearized segments as shown in Fig. 7 (Joeres et al., 1974), where the Y_{ijk} is the nonnegative pipe flow from i to j corresponding to the linearized segment k, f_{ijk} is the associated fixed cost, and C_{ijk} is the slope of the kth cost curve segment up to break-point capacity B_{ijk}. For a flow Y_{ij2} between B_{ij1} and B_{ij2} the cost of piping becomes

$$C_P = f_{ij2} + Y_{ij2}\, C_{ij2} \tag{11}$$

Similarly, the concave cost curve for the treatment plant can also be approximated by a piecewise linear approximation consisting of linearized segments.

Mathematical Model

The mathematical model presented here is based on that of Joeres et al. (1974), and Wanielista and Bauer (1972). The objective is to minimize the total present value of system expenditures subject to restrictions on the treatment and use of the water resources. The objective function

$$\text{Minimize} \left\{ \sum_i \sum_j \sum_k C_{ijk}\, Y_{ijk} + \sum_i \sum_j \sum_k f_{ijk}\, \delta_{ijk} + \sum_j \sum_k d_{jk}\, X_{jk} + \sum_j \sum_k g_{jk}\, \gamma_{jk} \right\} \tag{12}$$

which is the sum of variable and fixed transmission costs, and variable and

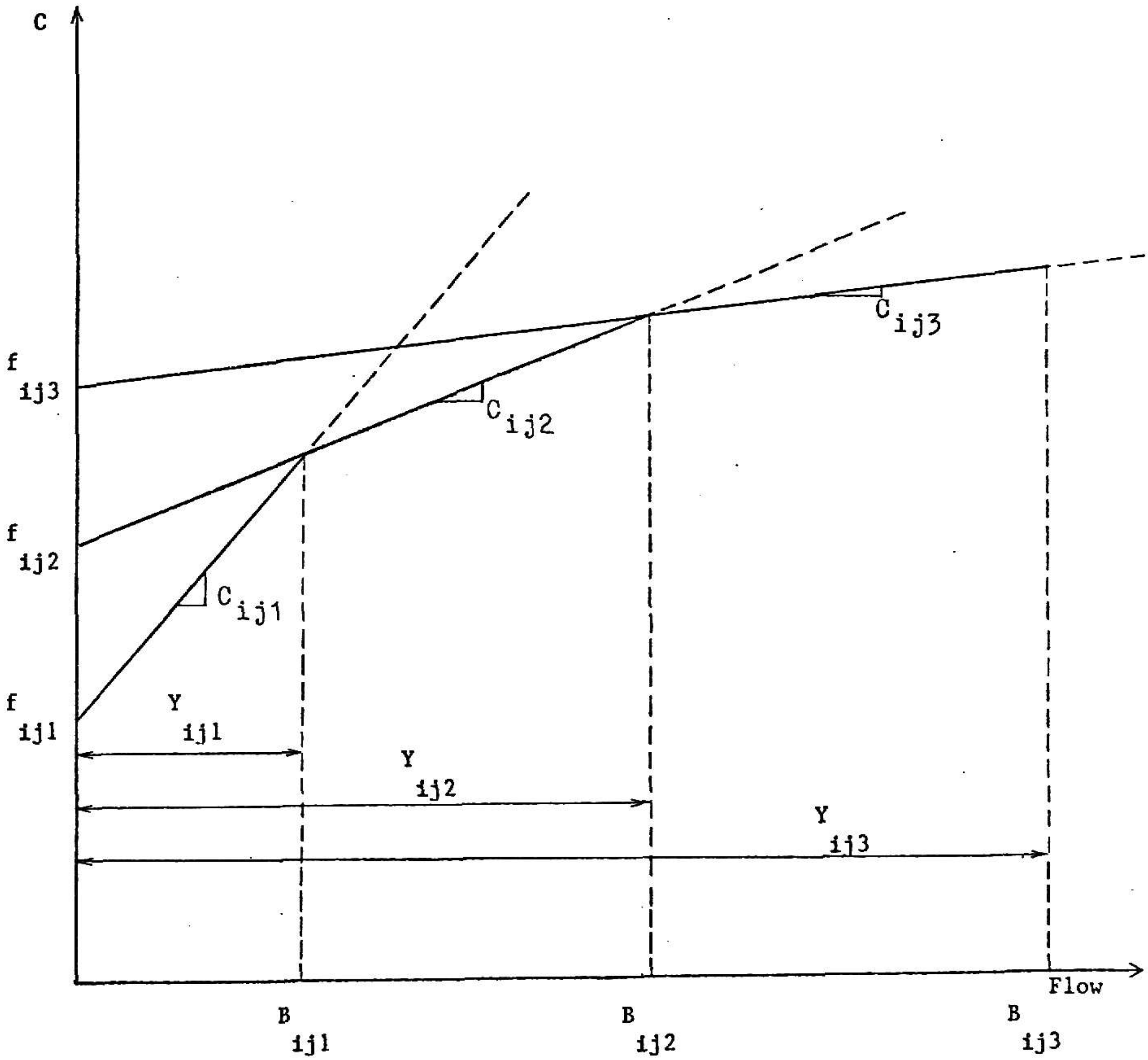


Figure 7. Piecewise linearized cost curve

fixed treatment costs. The summation indexes are taken as:

$$i \in I,\ j \in J,\ k \in K$$

where

I,J = variable feasibility descriptor sets (certain transmission line paths are geographically desirable, others are not possible, such as $Y_{11} = 0$)

K = a set describing input data availabilities in terms of segmentations of the linearized cost curves of the (ij)th system component

C_{ijk} = variable cost of transmitting 1 MGD of sewage from location i to location j associated with the kth segment of the linearized cost curve in dollars/MGD

Y_{ijk} = quantity of sewage transported from location i to location j associated with the kth segment of the linearized cost curve, MGD

f_{ijk} = fixed cost of route i-j associated with the kth segment of the linearized cost curve, dollars

d_{jk} = variable cost of treating 1 MGD of sewage at location j associated with the kth segment of the linearized cost curve, \$/MGD

X_{jk} = size of facility at location j associated with the kth segment of the linearized cost curve, MGD

g_{jk} = fixed cost of treatment plant located at j associated with the kth segment of the linearized cost curve, dollars

$$\delta_{ijk} = \begin{cases} 0 & \text{if } Y_{ijk} = 0 \\ 1 & \text{if } Y_{ijk} > 0 \end{cases} \tag{13}$$

$$Y_{jk} = \begin{cases} 0 & \text{if } X_{jk} = 0 \\ 1 & \text{if } X_{jk} > 0 \end{cases} \tag{14}$$

The minimization of the objective function, (12), is subject to the following constraints:

(1) The quantity of wastewater to a location, L_j, must be either treated or removed from that location, that is,

$$L_j + \sum_i \sum_k Y_{ijk} - \sum_i \sum_k Y_{jik} - \sum_k X_{jk} = 0 \qquad \forall\, j \tag{15}$$

where the first term is the total generated waste load L_j at node j, the second term is all flow Y_{ijk} entering node j from all nodes i from which an interceptor exists, and the third term represents all of the flow leaving node j.

(2) The variable pipe flow values can exist over only their specified range or be forced to equal zero:

$$Y_{ijk} \leqslant B_{ijk}\, \delta_{ijk} \qquad \text{for all pipes i-j, all segment k} \tag{16}$$

(3) The integer zero-one variable δ_{ijk} must be constrainted to be equal to 1 at most once for any interceptor between points i and j to force all but at most one flow value in (16) to zero. Equation (17) guarantees that at most one flow can exist between i and j: thus

$$\sum_k \delta_{ijk} + \sum_k \delta_{jik} \leqslant 1 \qquad \text{for all pipes i-j} \tag{17}$$

There are two summations to cover flow in either direction; and should the terms on the left sum to zero, the interceptor is not built. If they do not sum to zero, then at most one variable equals one, identifying the appropriate k segment of the cost curve and associated Y_{ijk} flow value.

(4) Like equation (16) for pipes, a constraint must be stated either to limit the maximum capacity of treatment plant j or not to allow any treatment at site j at all:

thus

$$x_{jk} \leq U_{jk} \quad \gamma_{jk} \qquad \text{for all plants j, all segments k} \qquad (18)$$

Here U_{jk} are the capacity limits of the linear segments of the treatment plant cost curve, and γ_{jk} are the zero-one variables. For k corresponding to the last linear segment of the plant cost curve, U_{jk} will equal the maximum plant capacity possible at j.

(5) Equation (19) insures that at most one plant may exist at site j:

$$\sum_{k} \gamma_{jk} \leq 1 \qquad \text{for all sites j} \qquad (19)$$

(6) Finally, all pipe flow and treatment plant capacity variables are specified to be nonnegative:

$$Y_{ijk} \geq 0 \qquad \text{for all i,j,k}$$

$$X_{jk} \geq 0 \qquad \text{for all j,k}$$

$$\delta_{ijk}, \ \gamma_{jk} = (0,1) \qquad \text{for all i,j,k}$$

The above problem has been solved by Joeres et al. (1974) and Wanielista and Bauer (1972) using integer programming.

To obtain the objective function, coefficients C_{ijk}, f_{ijk}, d_{jk} and g_{jk} must be found through linearization of cost curves. Fitted cost curves shown in Fig. 6 may come from various cost functions. Those presented by Deininger and Su (1973) are:

(1) Total treatment plant cost as a function of flow

$$T = 0.067\ Q^{0.78} \qquad \text{in millions of dollars} \qquad (20)$$

where Q is the design flow in MGD (million gallons per day)

(2) Pipe construction cost as a function of flow

$$C_s = 0.046\, Q^{0.55} L \quad \text{in million dollars} \tag{21}$$

where Q is the amount of sewage conveyed in MGD, L is the length of the pipe in mile. Cost curves obtained from equations (20) and (21) are concave curves. They are linearized to find those coefficients.

5.3 Illustrative Example

Joeres et al. (1974) reported that a rather large mixed integer optimization problem resulted when the model was applied to their problem. Through the use of pipe cost functions approximated by two linearized segment and treatment plant cost functions, a total of 136 constraints result, with 96 continuous and 96 integer program variables. They found that mixed integer optimization problems have in general unpredictable but long computer running time. By that time, the only algorithm available to them capable of handling such a large problem is a large mathematical programming system for the Univac 1108 computer identified as FMPS. It was only able to generate a feasible solution after 3 hours of computation on the Univac 1108. Since that computer program package is not available to us, we tried another program called BBMIP (Branch and Bound Mixed Integer Programming) prepared by IBM. After trials, we found that the BBMIP program could not solve this problem, because constraints (13) and (14) of Joeres' model can not be built in the BBMIP program. Wanielista and Bauer did not present the detail of calculation of their solution. So far we can not solve a practical problem to illustrate the application of this method, but the example problem presented in Wanielista and Bauer's paper is checked to show some degree of application of this method.

Figure 8 shows a regional sewage system. There are eleven communities scattered around a river and its branch. Wanielista and Bauer's solution specified that there are only two treatment plants, one at location T for centralized treatment and another at location 10. The distances among communities are not given in their paper, therefore they are estimated. The pipe costs are then calculated because these costs are functions of pipe length. Their optimum solution is checked as follows:

<u>Step 1</u>

First of all, the distance between each community was assumed and given in Fig. 8 for preparation of pipe cost calculation. Then costs were calculated by equations (20) and (21). Three Q values: 1, 15 and 45 MGD were used for fitting those cost curves. Two segments (breakpoints: B_{ij1} = 15MGD, B_{ij2} = 45MGD) were taken for piecewise linearization. The design flow given by Wanielista and Bauer (1972) and the corresponding linearization segment are given in Table 7. Twenty two pipe cost curves and two treatment cost curves were plotted. The C_{ijk}, f_{ijk}, d_{jk} and g_{jk} values are given in Table 8. For example, C_{122} and f_{122} are calculated as follows:

The second segment (k=2) is used to approximate the cost curve because the design flow for location 1 is 21 MGD which is greater than the first breakpoint B_{121}. The distance between location 1 and location 2 is 1.3 miles.

$$C_s = 0.460 \times Q^{0.55} \times 1.3 = 0.598 \times Q^{0.55} \quad \text{in million dollars}$$

C_s values are calculated for Q = 15 MGD and 45 MGD. They are 0.265 and 0.485. Then the equation of the line segment is found by:

$$\frac{C_s - 0.265}{Q - 15} = \frac{0.485 - 0.265}{45 - 15}$$

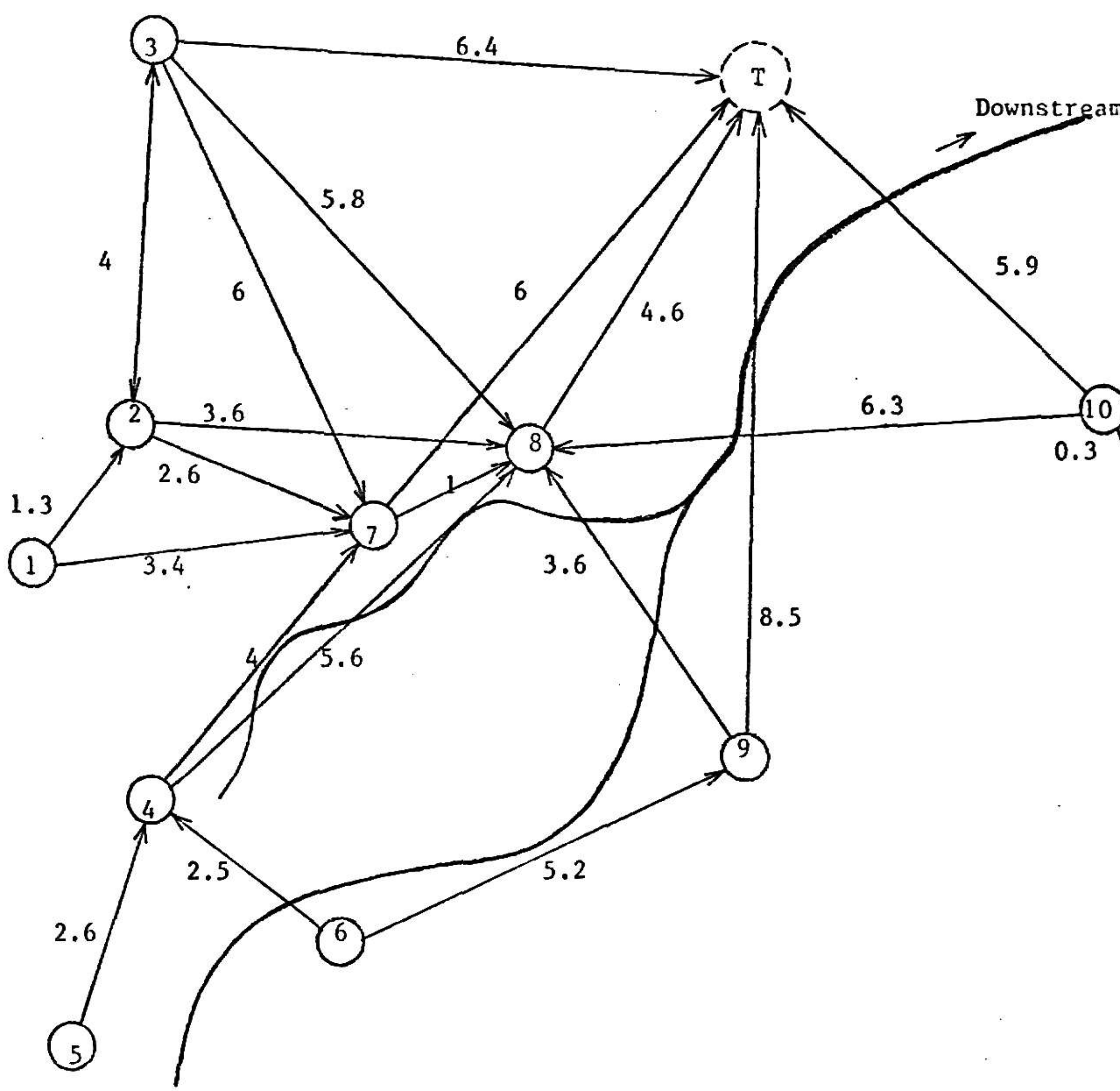


Figure 8. Schematic of Little Econ system

Table 7. Design flows and corresponding linearization segment for example 5.3

Community (area)	Design flow, Q (MGD)	Line segment (k)
1	21.0	2
2	1.0	1
3	5.0	1
4	6.0	1
5	3.0	1
6	1.0	1
7	0.5	1
8	1.5	1
9	2.5	1
10	1.0	1
11	1.5	1
Total	44.0	2

$Q \leq 15$ MGD $\quad k = 1$

$Q > 15$ MGD $\quad k = 2$

Table 8. Calculated coefficients for piecewise linearization of cost curves in example 5.3

c_{122}	.007	f_{122}	.160
c_{172}	.019	f_{172}	.407
c_{232}	.023	f_{232}	.477
c_{272}	.015	f_{272}	.308
c_{282}	.020	f_{282}	.427
c_{321}	.045	f_{321}	.139
c_{371}	.068	f_{371}	.208
c_{381}	.065	f_{381}	.202
c_{3T1}	.072	f_{3T1}	.222
c_{471}	.045	f_{471}	.139
c_{481}	.063	f_{481}	.200
c_{541}	.029	f_{541}	.091
c_{641}	.028	f_{641}	.090
c_{691}	.059	f_{691}	.180
c_{782}	.006	f_{782}	.119
c_{7T2}	.034	f_{7T2}	.716
c_{8T2}	.026	f_{8T2}	.548
c_{981}	.020	f_{981}	.430
c_{9T1}	.096	f_{9T1}	.295
c_{1081}	.072	f_{1081}	.221
c_{10T1}	.067	f_{10T1}	.213
c_{11101}	.003	f_{11101}	.010
d_{101}	.035	g_{101}	.033
d_{T2}	.026	g_{T2}	.179

Hence,

$$C_s = 0.007 \times Q + 0.160$$

and

$$C_{122} = \text{slope} = 0.007$$

$$f_{122} = \text{intercept} = 0.160$$

The problem has 24 continuous variables, 24 integer variables and 13 constraints.

Step 2

The objective function is the summation over the product of those coefficients given in Table 8 and their corresponding variables, namely Y_{ijk} and X_{jk}.

$$\begin{aligned} CT = {} & .007Y_{12} + .019Y_{17} + .023Y_{23} + .015Y_{27} + .020Y_{28} + .045Y_{32} + .068Y_{37} \\ & + .065Y_{38} + .072Y_{3T} + .045Y_{47} + .063Y_{48} + .029Y_{54} + .028Y_{64} + .059Y_{69} \\ & + .006Y_{78} + .034Y_{7T} + .026Y_{8T} + .020Y_{98} + .096Y_{9T} + .072Y_{108} + .067Y_{10T} \\ & + .003Y_{1110} + .035X_{10} + .026X_{T} + .160\delta_{12} + .407\delta_{17} + .477\delta_{23} + .308\delta_{27} \\ & + .427\delta_{28} + .139\delta_{32} + .208\delta_{37} + .202\delta_{38} + .222\delta_{3T} + .139\delta_{47} + .200\delta_{48} \\ & + .091\delta_{54} + .090\delta_{64} + .180\delta_{69} + .119\delta_{78} + .716\delta_{7T} + .548\delta_{8T} + .430\delta_{98} \\ & + .295\delta_{9T} + .221\delta_{108} + .213\delta_{10T} + .010\delta_{1110} + .033\gamma_{10} + .179\gamma_{T} \end{aligned}$$

Actually, this objective function can be used as the total cost function. The constraints are mainly the node equations:

(1) $Y_{12} + Y_{17} = 21$

(2) $Y_{12} - Y_{23} - Y_{27} - Y_{28} + Y_{32} = -1$

(3) $-Y_{32} - Y_{37} - Y_{38} - Y_{3T} = -5$

(4) $-Y_{47} - Y_{48} + Y_{54} + Y_{64} = -6$

(5) $Y_{54} = 3$

(6) $Y_{64} + Y_{69} = 1$

(7) $Y_{17} + Y_{27} + Y_{37} + Y_{47} - Y_{78} - Y_{7T} = -0.5$

(8) $Y_{28} + Y_{38} + Y_{48} + Y_{78} - Y_{8T} + Y_{98} + Y_{108} = -1.5$

(9) $Y_{69} - Y_{98} = -2.5$

(10) $-Y_{108} - Y_{10T} + Y_{1110} - X_{10} = -1.0$

(11) $Y_{1110} = 1.5$

(12) $Y_{3T} + Y_{7T} + Y_{8T} + Y_{9T} + Y_{10T} - X_T = 0$

(13) $\delta_{23} + \delta_{32} \leqslant 1$

In the optimum solution $\delta_{32} = 1$ and $\delta_{23} = 0$.

The optimum solution given in Wanielista and Bauer's paper is shown in Fig. 9 and Table 9. The minimum cost for this allocation of sewage treatment facilities is $\$9.872 \times 10^6$ according to our cost functions and calculation. Two plants are suggested to be built at location T and location 10. The minimum cost is obtained by substituting the optimum Y_{ijk} and X_{jk} given in Table 9 into the objective function.

Step 3

One alternative allocation of those wastewater treatment facilities is checked. If only one plant at location T is built, then $Y_{10T} = 2.5$, $\delta_{10T} = 1$, $\gamma_{101} = 0$, $X_{10} = 0$ and $X_T = 44$. The total cost in this case is $\$10.197 \times 10^6$ which is higher than the optimum value. According to Wanielista and Bauer's paper, location T has already been chosen as the site for the prospective centralization wastewater treatment plant for the Little Econ River system.

Step 4

An alternative wastewater shipment is checked. If we transport the sewage generated at location 3 directly to location T instead of passing by

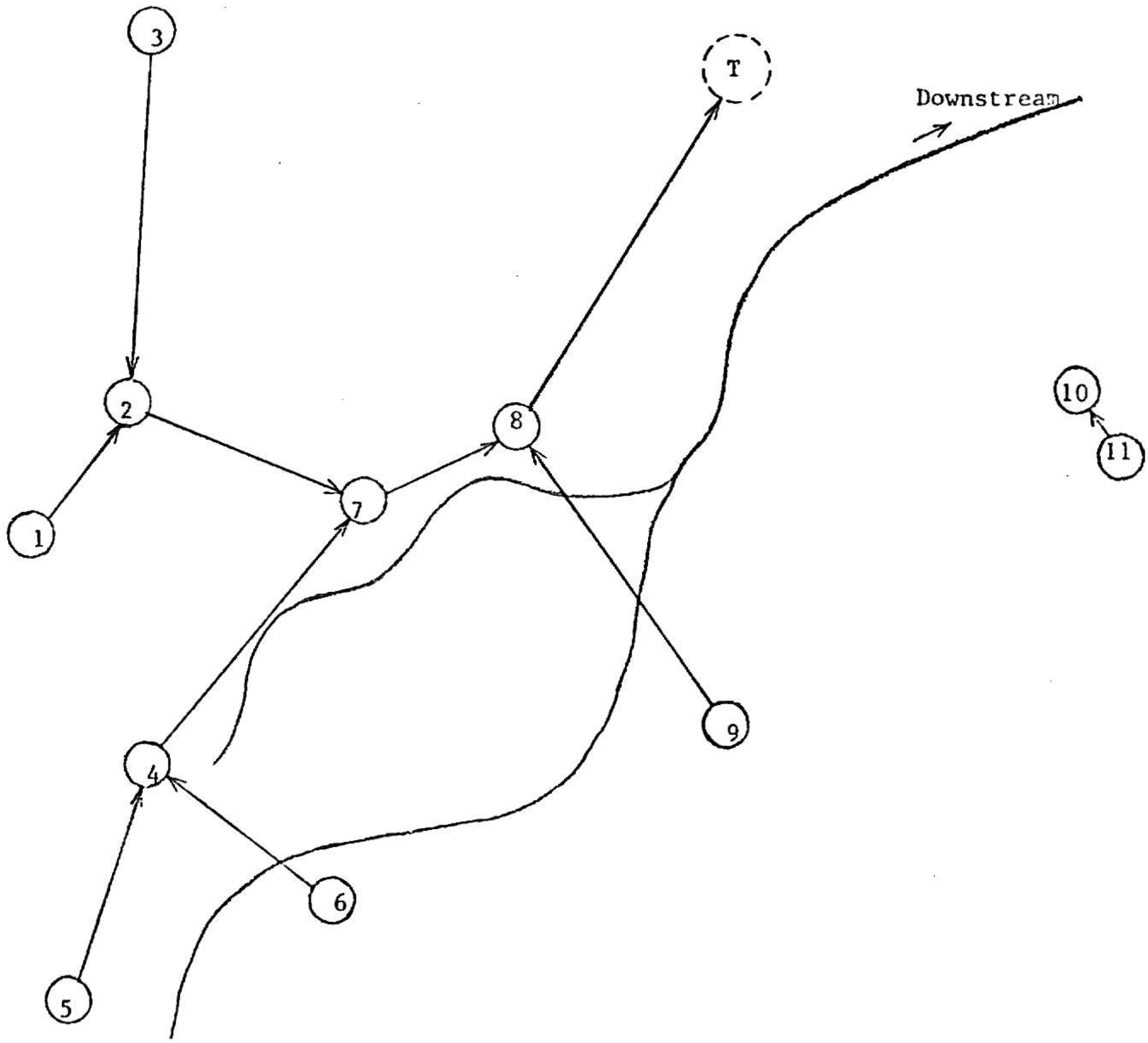


Figure 9. Final network of pipe interceptors for example 5.3

Table 9. Optimum allocation for example 5.3

Variable Name	From Location	To Location	Flow (MGD)
Y_{32}	3	2	5.0
Y_{27}	2	7	27.0
Y_{12}	1	2	21.0
Y_{54}	5	4	3.0
Y_{47}	4	7	10.0
Y_{64}	6	4	1.0
Y_{98}	9	8	2.5
Y_{78}	7	8	37.5
Y_{1110}	11	10	1.5
Y_{8T}	8	T	41.5
X_T	Treatment Plant		41.5
X_{10}	Treatment Plant		2.5

location 2. Then, $Y_{32} = 0$, $Y_{3T} = 5$, $Y_{27} = 22$, $Y_{78} = 32.5$, $Y_{8T} = 36.5$, $\delta_{32} = 0$ and $\delta_{3T} = 1$. The total cost in this case is obtained as \$9.985 x 10^6 which is again higher than the optimum value. This checking supports the optimum solution given by Wanielista and Bauer (1972).

6. DISCUSSION AND CONCLUSION

6.1 Advantages of Each Method

Naito's enumeration method is very straightforward and easy to be understood. Actually it is an exhaustive search method. Converse's method is more systematical. The recursive manner of the optimum cost function makes it easy to expand the size of the problem. Owing to the characteristic of dynamic programming, this method takes less time and effort because it takes advantages of the optimum results obtained in previous stages. Besides, this method has flexibility on the number of treatment plants. It provides information about systems of single plant, two plants, three plants, . . . etc. The minimum cost of each system and the exact location of each plant are specified in this method. The corresponding wastewater flow to each plant can be determined also.

The method presented by Joeres et al. is almost the same as Wanielista and Bauer's. Their methods built up a very general mathematical model which is capable of handling the situation when communities in a region are not in sequence. The piecewise linear approximation of cost curves eliminates the nonlinearity, so that linear optimization techniques can be applied. After the optimum has been found by the linear optimization technique, integer constraints are taken into consideration to find the optimum solution for the mixed integer programming problem. Integer programming is a developing technique. Their works showed a good application of this technique.

6.2 Limits of Each Method

The enumeration method applies only when communities are in sequence. Secondly, it fits small size (total number of communities less than 7) problems only, because it is uneconomical in time and effort to apply exhaustive search on large size problems. When there are more than eight communities in a region, there will be more than one hundred and twenty eight possible cases to be searched. Thirdly, it does not specify in which direction of flow the pump stations are needed. Fourthly, this method does not specify the exact location of each plant; instead it intuitively assumes that the most downstream community in each subregion is the site for treatment plant. Fifthly, the cost function does not take the present value factor into account.

Converse's method as well as Naito's method applies only when communities line up in sequence. When communities group together closely, it is hard to order them in sequence. Then these two methods can not be applied. When communities are located on both sides of a river, cross-river pipe network may be necessary. Under this circumstance, pipe construction costs increase and offset the cost saving due to economies of scale. Essentially pipe construction costs depend on the geographic situation (width, depth of the river) and the construction technology. In those published papers, none of them mentioned the difficulty of building pipes across rivers.

The recursive equation Converse used is:

$$B(i,s) = \underset{j}{\text{minimum}}\left[V(i,j) + B(j,s-1)\right]$$

Evidently, there is an overlap of the end point of $V(i,j)$ and the starting point of $B(j,s-1)$. To eliminate this overlap, $B(j,s-1)$ should be replaced by $B(j+1,s-1)$.

The major drawback of the integer programming method is that the size of problem increases rapidly under formulation. Because there are triple summations in the objective function and the same number of integer variables and continuous variables is required. Joeres formulated a practical model which consists of 12 communities with two-segment linearization. It resulted in 136 constraints with 96 continuous and 96 integer program variables. So this model requires large memory space and long execution time in computer computations. On the other hand, relationships between continuous variables and integer variables (equations (13) and (14)) are hard to express in the linear constraints which are necessary if linear programming techniques are applied. This is the main reason of that the BBMIP program could not solve example problem 5.3. That program applies the dual simplex method (a linear programming technique) first, then imposes integer constraints to solve the mixed integer programming problem by the branch and bound method.

6.3 Conclusions

(1) Operations research techniques can well be applied to find optimum planning for centralized regional wastewater treatment systems.

(2) The enumeration method and dynamic programming method are applicable to find optimum number of treatment plants to serve the whole region, so that the total system cost is a minimum. The dynamic programming method determines locations of plant also.

(3) The integer programming method fits problems in which communities are not in sequence, but it requires large memory space and long execution time in computer.

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Appendix 1

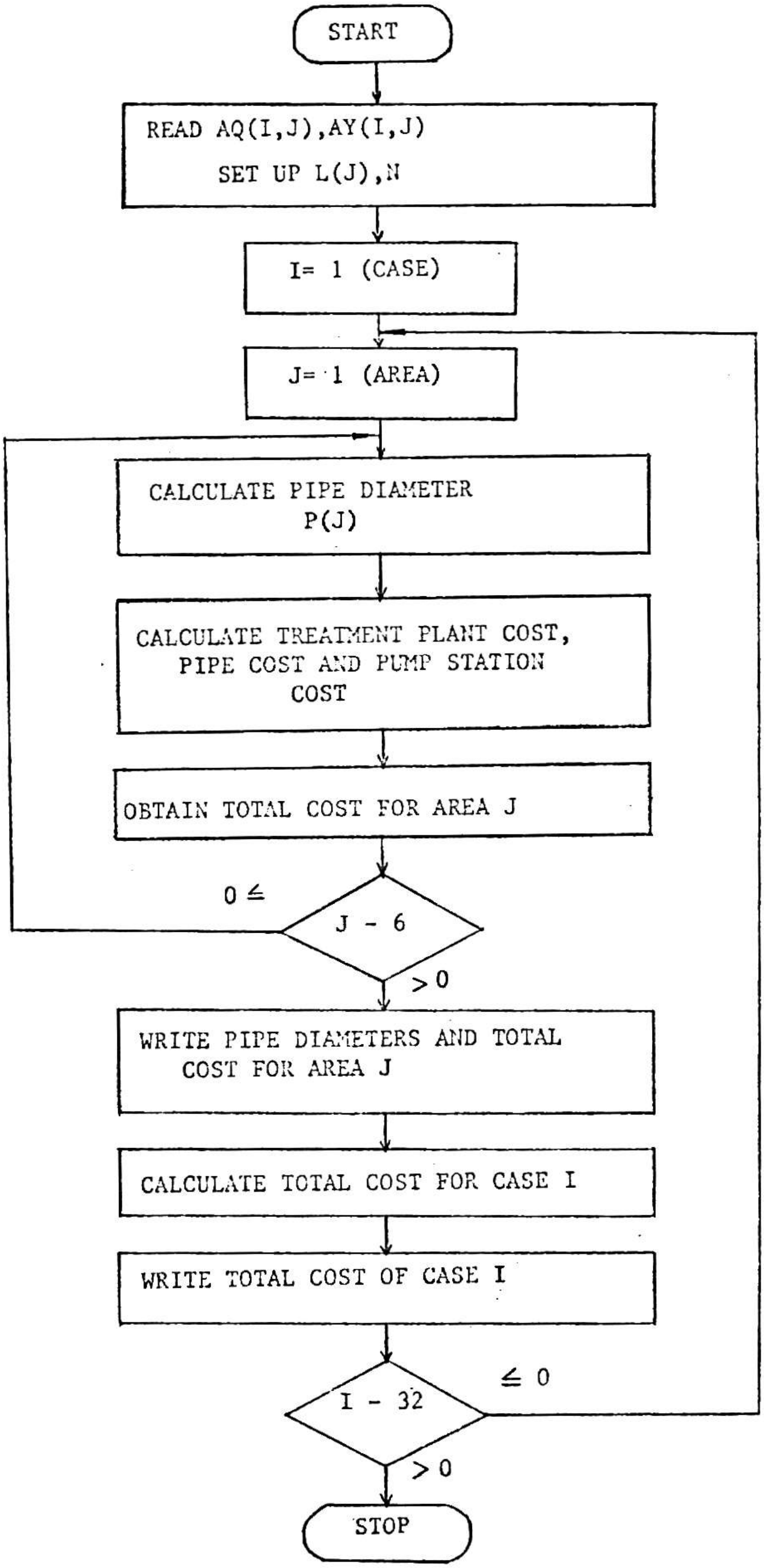
Enumeration Method Program

and

Sample Input Coding

Program symbols and explanation

Program symbols	Explanation	Mathematical symbols
AQ(I,J)	amount of wastewater treated at location J in case I	
AY(I,J)	amount of wastewater transported into location J in case I	
L(J)	distance between location J and location (J+1)	ℓ_j
N	durable year of facility	N
Q(J)	amount of sewage treated at area J	q_j
Y(J)	amount of sewage transported from Jth area to (J+1)th area	Y_j
P(J)	diameter of the pipe transmitting Y(J)	P_j
M(J)	number of pump stations in the Jth trunk sewer	m_j
C1(J)	treatment plant construction cost	
C2(J)	treatment plant operating cost (exclude that for sludge treatment)	
C3(J)	maintenance cost for sludge treatment	
C4(J)	land cost for treatment plant	
C5(J)	pump station construction cost	
C6(J)	pump station operating cost	
C7(J)	land cost for pump station	
C8(J)	total pipe cost	
CJ(J)	total cost at location J	$J_i(j)$
TC(I)	total cost for case I	J_i



Descriptive flow diagram for enumeration method program

```
C    ******************************
C    *                            *
C    *    ENUMERATION  METHOD     *
C    *                            *
C    ******************************
      DIMENSION C2(6),C3(6),C4(6),C5(6),C6(6),C7(6),C8(6),TC(32)
      DIMENSION AQ(32,6),AY(32,6),Q(6),Y(6),M(6),CJ(9),P(6),C1(6)
      REAL L(6)
      DATA M(1),M(2),M(3),M(4),M(5),M(6)/0,3,1,0,4,0/
    2 FORMAT(16F5.2)
   21 FORMAT(1X,I2,5X,E15.8)
   31 FORMAT(1X,I2,5X,6E15.8)
      READ(5,2) ((AQ(I,J),J=1,6),I=1,32)
      READ(5,2) ((AY(I,J),J=1,5),I=1,32)
      DO 66 I=1,32
   66 AY(I,6)=0.
C     DISTANCE IN METER
      L(1)=61.5
      L(2)=265.
      L(3)=225.
      L(4)=50.
      L(5)=87.5
      L(6)=0.
C     DURABLE YEAR
      N=15
      DO 99 I=1,32
      SUM=0.
      DO 88 J=1,6
C     AMOUNT OF WASTEWATER IN 10**3 TON/DAY
      Q(J)=AQ(I,J)*100
      Y(J)=AY(I,J)*100
C     PIPE DIAMETER IN METER
      P(J)=0.19084*Y(J)**(0.42017)
C    INDIVIDUAL COST FUNCTION J
      C1(J)=5900*Q(J)**0.742
      C2(J)=401.5*N*Q(J)**0.798
      C3(J)=30*N*Q(J)
      C4(J)=150*3.3*Q(J)**0.7
      C5(J)=600*M(J)*Y(J)**0.6
      C6(J)=N*M(J)*(6.7*Y(J)+200)
      C7(J)=0.793*3.3*Y(J)**0.5*M(J)
      IF(P(J)-1.8) 22,22,23
   22 C8(J)=0.55*L(J)*P(J)
      GO TO 24
   23 C8(J)=1.15*L(J)*(P(J)-0.7)
   24 CJ(J)=C1(J)+C2(J)+C3(J)+C4(J)+C5(J)+C6(J)+C7(J)+C8(J)
      CJ(J)=CJ(J)*10000
   88 CONTINUE
      WRITE(6,31) I,(P(J),J=1,6)
      WRITE(6,31) I,(CJ(J),J=1,6)
      DO 77 K=1,6
   77 SUM=SUM+CJ(K)
C    TOTAL COST
      TC(I)=SUM
      WRITE(6,21) I,TC(I)
   99 CONTINUE
      STOP
      END
```

Sample input coding for example problem 3.3

```
Col
1    6                   26    32                      56         67          80

 $ENTRY
                         15.08 1.57                     13.51      4.77
     10.31 1.57  3.2               10.31           7.37            7.71 1.57
  5.8            7.71       4.77  2.6             7.71 1.57  3.2  2.6               7.71
                13.19       1.89 1.57            11.62       1.89       4.77        8.42
      1.89 1.57  3.2        8.42       1.89            7.37 5.82       1.89 1.57
  5.8 5.82       1.89       4.77  2.6 5.82        1.89 1.57  3.2  2.6 5.82        1.89
                     14.63 0.45 1.57                13.06 0.45       4.77
 9.86 0.45 1.57  3.2             9.86 0.45             7.37       7.26 0.45 1.57
  5.8      7.26 0.45        4.77  2.6       7.26 0.45 1.57  3.2  2.6       7.26 0.45
                13.19 1.44 0.45 1.57            11.62 1.44 0.45       4.77        8.42
 1.44 0.45 1.57  3.2        8.42 1.44 C.45             7.37 5.82 1.44 0.45 1.57
  5.8 5.82 1.44 0.45        4.77  2.6 5.82 1.44 0.45 1.57  3.2  2.6 5.82 1.44 0.45
 1.57  3.2  2.6 5.82 1.44        3.2  2.6 5.82 1.44 1.57       2.6 5.82 1.44
       2.6 5.82 1.44 1.57  3.2      5.82 1.44       3.2      5.82 1.44 1.57
      5.82 1.44                5.82 1.44 1.57  3.2  2.6      1.44       3.2  2.6
      1.44 1.57       2.6       1.44            2.6      1.44 1.57  3.2
 1.44       3.2            1.44 1.57                1.44                          1.44
 1.57  3.2  2.6 5.82             3.2  2.6 5.82      1.57       2.6 5.82
       2.6 5.82      1.57  3.2      5.82             3.2      5.82      1.57
      5.82                     5.82      1.57  3.2  2.6                  3.2  2.6
           1.57       2.6                       2.6           1.57  3.2
            3.2                1.57
```

Appendix 2

Empirical Constants for Pipe Construction Cost Formula

Empirical Constants for Pipe Construction Cost Formula

Pipe Diameter (in.)	Value of Constants	
	a	b
8	1.89209	0.0657358
10	2.00568	0.0626208
12	2.07986	0.0607989
15	2.13563	0.0624380
18	2.22945	0.0621260
21	2.34403	0.0605660
24	2.45129	0.0607415
27	2.57817	0.0591515
30	2.74892	0.0550614
33	2.88386	0.0523984
36	2.99481	0.0519715
42	3.25583	0.0481838
48	3.48162	0.0450201
54	3.67929	0.0436656
60	3.88504	0.0442245
66	4.072	0.0393904
72	4.28404	0.036992
75	4.35491	0.0372479
78	4.41509	0.0388936

Appendix 3

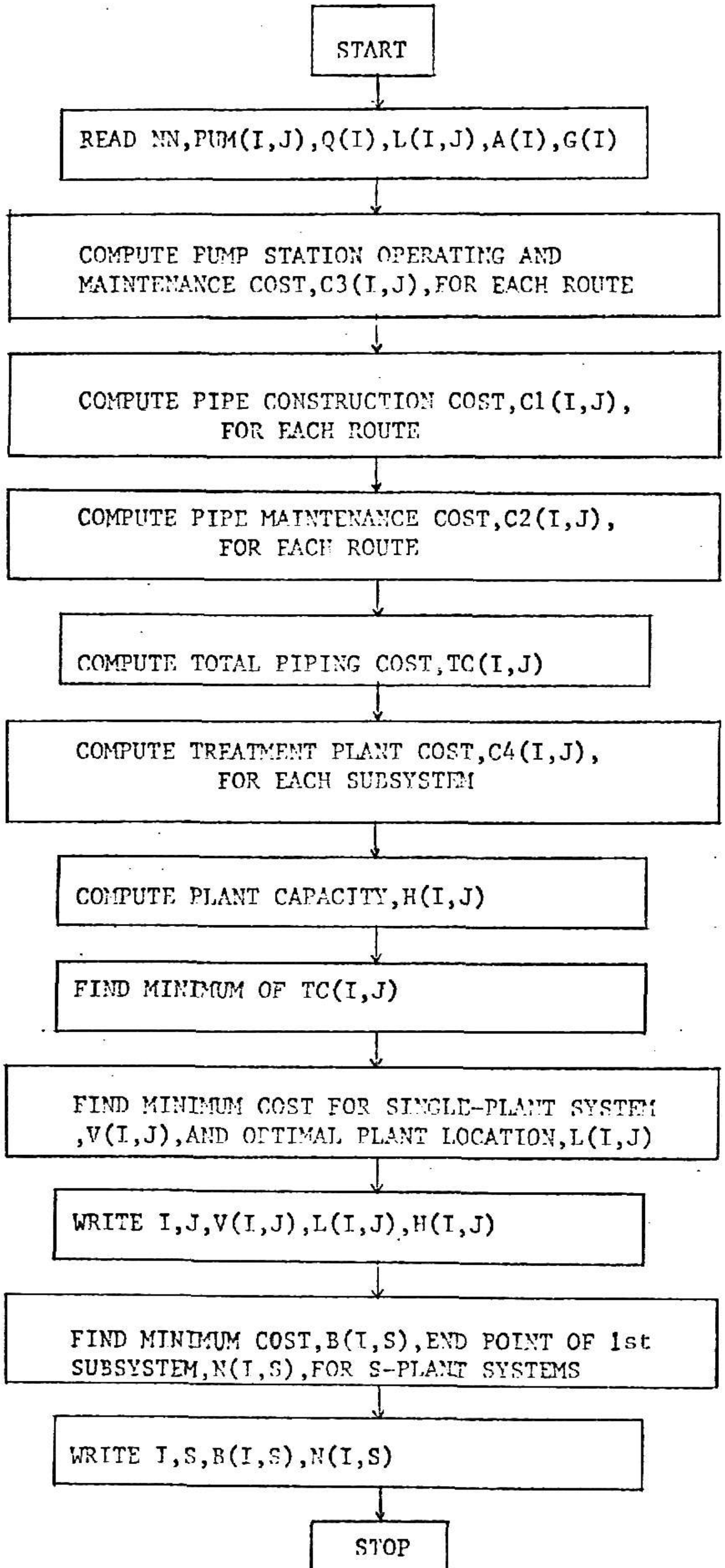
Dynamic Programming Program

and

Sample Input Coding

Program symbols and explanation

Program symbols	Explanation	Mathematical symbols
S	number of treatment plants in the system	s
PUM(I,J)	number of pump stations in the sewer connecting location I and location J	NP_{ij}
NN	total number of communities	K
Q(I)	amount of sewage generated at location I	Q_i
L(I,J)	distance between location I and location J	ℓ_{ij}
A(I)	empirical constant for the Ith sewer	a_i
G(I)	empirical constant for the Ith sewer	b_i
C3(I,J)	pump station operating and maintenance cost for each sewer route I-J	
QQ	cumulative amount of sewage for the system	
C1(I,J)	pipe construction cost for route I-J	
C2(I,J)	pipe maintenance cost for route I-J	
TC(I,J)	total piping cost for sewer route I-J	
C4(I,J)	treatment plant cost for the subregion from location I to location J	
H(I,J)	capacity for above treatment plant	H(i,j)
PTC(I,J)	minimum piping cost for a subregion from location I to location J	
L(I,J)	location of treatment plant for a single-plant system covering the region from I to J	L(i,j)
V(I,J)	minimum cost for a single-plant system covering the region from I to J	V(i,j)
B(I,S)	minimum cost for a S-plant system starting at location I	B(i,S)
N(I,S)	location of right end of the leftmost subregion in above system	N(i,S)



Descriptive flow diagram for dynamic programming program

```
      DIMENSION PTC(10,10),STC(10)
      DIMENSICN V(10,10),H(1C,10),L(10,10)
      DIMENSION Q(10),PO(1C),PCP(10,10)
      CIMENSICN TC(10,10),P(10,10),A(10)
      DIMENSION C1(10,10),C2(1C,10),C3(1C,10),C4(1C,10)
      DIMENSICN N(10,10),D(1C),G(10),PUM(1C,10),B(1C,10)
      INTEGER S,PUM
    7 FORMAT(8I10)
    8 FORMAT(8F10.4)
    9 FORMAT('1','     I     J      V(I,J)       L(I,J)           H(I,J)')
   19 FORMAT(' ',2X,I2,2X,I2,3X,F10.2,3X,I2,5X,F10.2)
   38 FORMAT(' ','     I     S                 B(I,S)      N(I,S)')
   39 FORMAT(' ',2X,I2,2X,I2,3X,F15.2,2X,I2)
   88 FORMAT(80I1)
   99 FORMAT(I3)
      READ(5,99) NN
      READ(5,88) ((PUM(I,J),J=1,NN),I=1,NN)
      READ(5,8) (Q(I),I=1,NN)

      READ(5,7) ((L(I,J),J=1,NN),I=1,NN)
      READ(5,8) (A(I),I=1,NN)
      READ(5,8) (G(I),I=1,NN)
C PUMPING STATION OPERATING & MAINTENANCE
                    DO 20 I=1,NN
                    DO 20 J=1,NN
      QQ=0
            IF(I-J) 21,22,23
   21 K=I+1
      Y=0
      DO 24 LP=K,J
      LL=LP-1
      QQ=QQ+Q(LL)
   24 Y=Y+((PUM(LL,LP)*3.58*QQ*365)+6500*PUM(LL,LP))*0.08
      C3(I,J)=Y
      GO TO 28
   22 C3(I,J)=0
      GO TO 28
   23 LL=I-1
      Y=0
      DO 25 K=J,LL
      LP=K+1
   25 QQ=QQ+Q(LP)
      DO 26 LR=J,LL
      LS=LR+1
   26 Y=Y+((PUM(LS,LR)*3.58*QQ*365)+6500*PUM(LS,LR))*0.08
      C3(I,J)=Y
   28 WRITE(6,27) I,J,QQ
   27 FORMAT(' ',2I2,F6.2)
   20               CONTINUE
C   PIPE CONSTRUCTION COST
      DO 30 I=1,NN
      DO 30 J=1,NN
      IF(I-J) 31,32,33
   31 K=I+1
      X=0
      DO 34 LP=K,J
      LL=LP-1
   34 X=A(LL)*10**G(LL)*L(LL,LP)+X
      C1(I,J)=X
      GO TO 30
```

```
   32 C1(I,J)=0
      GO TO 30
   33 K=J+1
      X=0
      DO 35 LP=K,I
      LL=LP-1
   35 X=A(LP)*10**G(LP)*L(LP,LL)+X
      C1(I,J)=X
   30 CONTINUE
C           PIPE MAINTENANCE COST
                 DO 40 I=1,NN
                 DO 40 J=1,NN
      C2(I,J)=L(I,J)*0.69924
      TC(I,J)=C1(I,J)+C2(I,J)+C3(I,J)
      WRITE(6,333) I,J,C1(I,J),C2(I,J),C3(I,J),TC(I,J)
  333 FORMAT(' ',2I3,4E15.8)
   40 CONTINUE
C   TREATMENT PLANT TOTAL COST
      DO 59 I=1,NN
      DO 59 J=1,NN
      IF(I-J) 71,72,73
   71 R=0
      DO 81 K=I,J
   81 R=R+Q(K)*1000000
      GO TO 124
   72 R=Q(I)*1000000
      GO TO 124
   73 R=0
      DO 82 K=J,I
   82 R=R+Q(K)*1000000
  124 C4(I,J)=38.06*R**0.69+10.8*R**0.754
      WRITE(6,555) I,J,C4(I,J)
   59 CONTINUE
C                CALCULATION OF FLOW
      WRITE(6,9)
      DO 50 I=1,NN
      DO 50 J=1,NN
      IF(I-J) 74,75,76
   74 SUM1=0
      DO 84 K=I,J
   84 SUM1=SUM1+Q(K)
      H(I,J)=SUM1
      GO TO 50
   75 H(I,J)=Q(I)
      GO TO 50
   76 SUM2=0
      DO 85 K=J,I
   85 SUM2=SUM2+Q(K)
      H(I,J)=SUM2
   50 CONTINUE
C             FIND MINIMUM
         J=5
  456 DO 777 I=1,J
         DO 999 MX=I,J
  999 STC(MX)=TC(I,MX)+TC(J,MX)
      SMALL=STC(I)
               DO 888 MX=I,J
      IF(STC(MX)-SMALL) 201,202,202
  201 SMALL=STC(MX)
      GO TO 888
```

```
202 SMALL=SMALL
888         CONTINUE
    PTC(I,J)=SMALL
    WRITE(6,555) I,J,PTC(I,J)
555 FORMAT(' ',2I3,E15.8)
            DO 111 MZ=I,J
            IF(PTC(I,J).EQ.STC(MZ)) GO TO 112
111         CONTINUE
112         L(I,J)=MZ
       V(I,J)=PTC(I,J)+C4(I,J)
    WRITE(6,19) I,J,V(I,J),L(I,J),H(I,J)
777    CONTINUE
         J=J-1
    IF(J.GT.0) GO TO 456
C          MULTI-PLANTS SYSTEM
    WRITE(6,38)
    DO 123 S=1,NN
    IF(S.GT.1) GO TO 56
     DO 1 I=1,NN
     B(I,1)=V(I,NN)
    N(I,1)=NN
100 WRITE(6,39) I,S,B(I,S),N(I,S)
  1 CONTINUE
    GO TO 123
 56 K=NN+1-S
      DO 125 II=1,K
    M=S-1
    NX=NN-M
    DO 10 J=II,NX
    JY=J+1
 10 D(J)=V(II,J)+B(JY,M)
    SMALL=D(II)
    DO 60 J=II,NX
            IF(D(J)-SMALL) 61,62,62
 61         SMALL=D(J)
            GO TO 60
 62         SMALL=SMALL
 60         CONTINUE
    B(II,S)=SMALL
    DO 70 J=II,NX
    IF(B(II,S).EQ.D(J)) GO TO 41
 70 CONTINUE
 41 N(II,S)=J
    WRITE(6,39) II,S,B(II,S),N(II,S)
125   CONTINUE
123 CONTINUE
    STOP
    END
```

Sample input coding for example problem 4.3

```
Col
1     8          20          34        44           56        65        75    80
 $ENTRY
   5
      222          1
  1.          2.5        6.        3.         5.
  1000.      25000.     60000.    30000.     50000.
                79200     184800    369600     501600     79200               105600
    290400     422400     184800    105600                184800    316800    369600
    290400     184800               132000     501600     422400    316800    132000

    3.885      4.072      4.4151    4.072       4.4151
    0.0442     0.0394     0.0389    0.0394     0.0389
```

OPTIMUM PLANNING OF CENTRALIZED WASTE TREATMENT SYSTEMS IN REGIONAL WATER QUALITY MANAGEMENT

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Taiwan, Rep. of China, 1972

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

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1976

The planning of centralized waste water treatment systems that include the number and location of treatment plants, pipe networks and pump stations is studied with an aim of minimizing the system's total cost. The system has to satisfy the water quality requirement (for instance, secondary treatment or 90 percent BOD removal) imposed by law.

There is a fundamental trade-off between the economies of scale inherent to treatment plants and the increasing piping cost. The economies of scale favor installation of a few large treatment plants. This tends to be offset by the increasing piping cost due to the greater distance that wastewater must be transported. Based on this trade-off, the optimum plan which determines the number and location of treatment plants and the corresponding pipe networks is found, so that the total cost of the system is a minimum. Three optimization techniques are applied to find the optimum plan.

The enumeration method applies an exhaustive search through all possible cases. It finds the optimum plan for small size problems. The dynamic programming method applies to problems in which communities locate in sequence. It provides alternative plans with various number of treatment plants. The integer programming method can handle problems in which communities are not in sequence.

Three example problems are presented and solved to illustrate these three optimization approaches. The advantages and limits of each method are also found by comparison.