

EFFECT OF END SUPPORT DIAPHRAGMS
ON FOLDED PLATE MODELS

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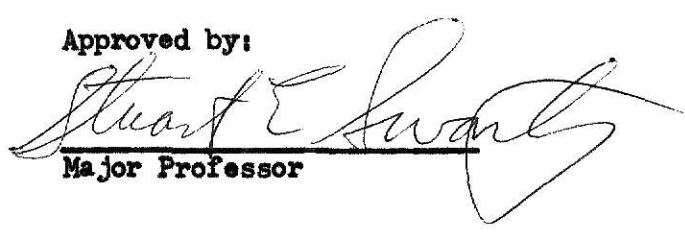
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SYNOPSIS

This thesis presents an investigation of the effects of end diaphragms on internal forces and deformations in single span, single bay folded plate models subjected to vertical loads.

Five different end diaphragms were used to support a three plate and a five plate model. The designs of the end diaphragms were intended either to copy those commonly in experimental use and actual practice, or to satisfy theoretical support conditions.

The experimental results indicate that end diaphragm types have little effect on stresses, moments, and deflections away from the supports but greatly affect these parameters at locations near the support.

It was also seen that the longitudinal moment was much larger than expected and may not be small enough to neglect in design.

INTRODUCTION

Folded plate structures are those prismatic shells consisting of flat plates which are mutually supported along their longitudinal edges and supported transversely on end diaphragms. All acceptable methods of analysis for simply-supported folded plates assume that the end diaphragms have infinite stiffness when loaded in their planes and complete flexibility when subjected to forces and moments not in their planes (1).

Various transverse diaphragms have been used in experimental work on folded plates in an attempt to satisfy the theoretical support assumptions. The most widely used diaphragm is a monolithic, transverse sheet which is welded, bolted, or otherwise rigidly connected to the folded plate (2,4,6,7). Other designs employ hinges in various configurations in an attempt to release moments in the diaphragms (5,8).

The ASCE Task Committee on Folded Plate Construction in their Phase I Report on Folded Plate Construction (1) reviewed the method of analysis and reported that one basic assumption common to all methods is that "each supporting diaphragm is infinitely stiff parallel to its own plane but is perfectly flexible normal to its plane." Later in the report it was stated that this assumption should be included in the analysis for simply-supported prismatic folded plates.

Very little has been published concerning particular transverse diaphragm effects on stresses and deformations in folded plates. Shahrokhizadeh (5) did an experimental study of different types of end supports on folded plate models which varied in thickness and length to depth ratios in which he attempted to simulate the theoretical support

conditions. He concluded that plate thickness was not an important factor when considering end diaphragm effects and that longitudinal stresses at mid-span are not significantly influenced by the type of end diaphragm. He also found that end effects on deflections are very small and can ordinarily be neglected except for short span structures.

Beaufait (2) used one inch (2.54 cm.) fir pieces fastened to the folded plate with wood screws. The diaphragms were then bolted to pairs of angle irons which in turn were connected to a framework. These diaphragms caused two-way slab action in the vicinity of the supports thus giving a smaller transverse moment than that calculated by assuming one-way slab action.

Scordelis (6) ran an experimental and analytical study of a folded plate and obtained a difference between the experimental results and those obtained by the elasticity method. He blamed this difference on an error in one or more of the assumptions made in the theoretical analysis. He checked the assumed end conditions and found that some restraint existed at the end diaphragm and thus the diaphragms were not perfectly flexible normal to their own plane. His diaphragms were 0.190 in. (0.483 cm.) thick 2024 T3 sheet aluminum welded to one end of the folded plate and fastened to the other by closely spaced machine screws.

Mark (4), in a study of photoelastic analysis of folded plate structures, analyzed a five-plate structure by the method of stress freezing. He concluded that boundary conditions along the end diaphragms assumed in the theoretical solutions were not perfectly satisfied in the model, but the perturbations thus introduced were confined

to areas adjacent to the diaphragms.

A later Scordelis experiment dealt with 0.50 in. (1.27 cm.) thick concrete models with 1.50 in. (3.81 cm.) thick concrete end diaphragms (7). These were tested with cycled loading and then loaded to failure. "Ultimate failure occurred at four and one-half times the full design load in both models and was caused by diagonal tension cracking in the shell near the supports and cracking in the supporting diaphragms that was produced by warping of the diaphragms induced by longitudinal strains in the folded plate elements." This led Scordelis to conclude that "special consideration should be given to reinforcing the supporting diaphragms for bending out of their own plane." This conclusion violates the support condition assumption of the diaphragm being flexible with respect to tractions not in its plane. But in light of evidence that supports have little effect on stresses and deflections away from the ends and due to the practical impossibility of achieving the ideal support conditions, his conclusion is justifiable.

Swartz (8), in a study of buckling of folded plate structures, suggests the difference between observed and theoretical buckling may be in part due to "the practical impossibility of achieving the ideal boundary conditions presumed in the development of the usual elastic analysis of folded plate structures."

In all cases reviewed, where inconsistencies between theoretical and experimental stresses at the supports existed the investigator cited the fact that the supports were not ideal.

This paper presents the results of load tests run on two folded plate

models, as shown in Fig. 1, supported by five different end diaphragm types. These results, in the form of stresses and moments versus load, are plotted and compared with theoretical results. The method used to obtain the theoretical results is presented in the Theoretical Considerations section.

The Experimental Program section describes in some detail the fabrication of models and diaphragms, strain and dial gage locations and equipment, the method of testing, and the reduction of strains to stresses and moments.

The Experimental Results section discusses longitudinal stress, shear stress, and longitudinal and transverse moments. Deflection results are then examined.

THEORETICAL CONSIDERATIONS

The method of analysis used in this paper for obtaining theoretical stresses and deformations in the models is that of Goldberg and Leve (3). The theoretical model results which are used herein were given by Swartz and Rosebraugh (9) who used a matrix approach (10) to the solution of the Goldberg formulation of the folded plate equations. Referring to Fig. 2 the loading is assumed to be uniform over the entire plate surface, and may be expressed as a Fourier Series in the x-direction and a constant in the y-direction for each plate. For a given mode in the longitudinal Fourier Series, the final matrix form of the folded plate equations may be expressed as (10):

$$K\Delta = F_F \quad [1]$$

where K is the structure stiffness matrix of order $4(n_p+1) \times 4(n_p+1)$; Δ is a column matrix of the unknown maximum displacements for the mode along the longitudinal edges; and F_F is a column matrix of the maximum edge tractions for the mode created by the applied loads acting on the plate surface with the longitudinal edges assumed fixed; finally n_p is the number of plates in the structure. The forces and deformations throughout the structure for a given mode of the Fourier Series may be expressed in terms of the applied loads and maximum edge deformations (3). The latter may be determined from

$$\Delta = K^{-1} F_F. \quad [2]$$

Thus the problem becomes one of calculating K^{-1} (the structure flexibility matrix), determining F_F for the given load case, and calculating Δ for a given mode of the series. Finally the forces and deformations throughout the structure are calculated for each mode and summed up.

One assumption implicit in this method is that the plate elements are assumed to be supported on end diaphragms which are perfectly rigid parallel to the plane of the diaphragm and are perfectly flexible normal to the plane of the diaphragm.

This method of analysis was implemented on the IBM 360-50 computer at Kansas State University and the results given in reference (9). The theoretical results given were obtained using three nonzero modes of the Fourier Series solution. The available results included deflections, longitudinal stresses, and transverse moments, but not longitudinal moments. Therefore no theoretical longitudinal moments are plotted. Theoretical results are presented later along with those experimentally obtained.

EXPERIMENTAL PROGRAM

FABRICATION OF MODELS

The folded plate models used in this study were fabricated from a sheet of 2024 T3 aluminum, 0.041 in. (0.104 cm.) thick, by shearing to size and then bending in a press to the proper shape (see Fig. 1). The length of the models were 54 in. (137 cm.) which gave a length to depth ratio of 21 for the three-plate model and 16 for the five-plate model. The material properties were determined as (9): Young's Modulus = 10.6×10^6 psi (7.31×10^{10} N/m²); Poisson's Ratio = 1/3; proportional limit stress = 30,000 psi (20.7×10^7 N/m²); yield stress at 0.2% off-set = 43,000 psi (2.96×10^8 N/m²).

The models were loaded by the use of a whiffle tree to distribute loads from two low capacity 800 lb. (363 kgf.) tension rams (Tom Thumb "IIV" series). The hydraulic pressure was measured using a Heise pressure gage. The load points on the model surface were spaced 1 11/16 in. (4.28 cm.) longitudinally and transversely on the horizontal projection.

FABRICATION OF END DIAPHRAGMS

Type 1 end diaphragm (Figs. 3a & 3b) was a 1/2 in. (1.27 cm.) thick plexiglas sheet, bolted to the folded plate with 1/4 in. (0.64 cm.) bolts at two in. (5.08 cm.) intervals. This type diaphragm is the simplest to construct and is the one used most in actual practice.

Type 2 end diaphragm was identical to type 1 except for a set of 1 in. x 1 in. (2.54 cm. x 2.54 cm.) brass hinges located at mid-diaphragm as shown in Figs. 4a & 4b.

Diaphragm type 3, shown in Figs. 5a & 5b, was identical to type 2 but was hinge-connected to the folded plate with 1 in. x 1 in. brass hinges in an attempt to produce flexibility with respect to the moments in the longitudinal direction at the diaphragm-folded plate interface.

It should be noted that although diaphragm types 2 and 3 were hinged in the plane of the diaphragm, rotation about the center of gravity of the model cross section was not permitted by this type hinge. Type 1 diaphragm restricted all rotation.

Figures 6a & 6b show diaphragm type 4 which approximates a roller support for the folded plate acting as a simple beam. The diaphragm was cut from a sheet of 28 gage sheet steel (thickness of 0.0149 in.) (0.0378 cm.) and was connected to the folded plate through 1 in. x 1 in. brass hinges. The diaphragm was supported at the centroid of the cross-section of the folded plate through a 1/2 in. (1.27 cm.) plexiglas roller which in turn rested on a 1/2 in. (1.27 cm.) sheet of plexiglas supported by two 1/4 in. (0.64 cm.) steel rollers. To limit buckling and thus gross deformation of the sheet steel a 1 in. x 4 in. x 1/2 in. (2.54 cm. x 10.16 cm. x 1.27 cm.) plexiglas stiffener was bolted beneath the opening in the sheet steel.

This diaphragm deflected vertically under load therefore a dial gage was placed at the center of the top plate at the support so relative deflections at midspan could be calculated.

Diaphragm type 5 shown in Figs. 7a & 7b consisted of an 18 in. x 30 in. (45.7 cm. x 76.2 cm.) sheet of 28 gage sheet steel from which the folded plate models were suspended through 1 in. x 1 in. brass

hinges. The sheet steel was supported by 1 1/2 in. x 1 1/2 in. (3.81 cm. x 3.81 cm.) angles resting on 2 1/2 in. (6.35 cm.) pipe columns at four corners. It was expected that this type of diaphragm would more closely satisfy the theoretical boundary conditions than the others.

TESTING

The overall test setup is given in Fig. 8. The equipment used to monitor thirty channels of strain gage output consisted of three Budd-Datran C-10 switch and balance units, a Budd-Datran A-110 digital strain indicator connected to a Budd E-140 printer control unit and a Victor Digit Matic printer. An additional four channels were monitored manually with a Budd SB1 switch and balance unit and a Budd HW1 strain indicator.

The locations of strain and dial gages for the three-plate model are shown in Fig. 9. Deflections at midspan were measured with Soil Test LC10 dial gages which have a least reading of 0.001 in. (.00254 cm.) and a travel of two inches (5.08 cm.). These gages were in all cases positioned to measure directly deflections perpendicular to the plate. Metalfilm C6-141-R2TC electrical, resistance, strain rosettes were used to determine longitudinal stresses at the free edges at midspan. These two-element strain rosettes were also used at strain gage locations 5,9,13,17,21, and 25 to give strains in both the longitudinal and transverse directions from which longitudinal stresses and moments and transverse moments were derived. Metalfilm C6-124D-R3Y three-element rosettes were used at location 29 to give longitudinal stress and moment, transverse moment, and shear stress at the center of the inclined plate near

the support. This location was as far from loadpoint stress concentrations as was feasible. At all strain gage locations gages were placed on both the inside and outside surfaces of the folded plate and all stress results averaged.

The strain gages for the five-plate model are in corresponding locations with the exception of gages at locations 1 and 3. These are at the free edges of the vertical plates on the five-plate model at a distance of 25 1/2 in. from the end.

The model was exercised to remove hysteresis from the gages by cycling the loading between 100 psi (gage) and 600 psi several times. These hydraulic pressures correspond to 0.1396 psi (963 N/m²) and 0.840 psi (5.78 x 10³ N/m²) respectively actual distributed load on the horizontal projection. Strain and deflection readings were taken at 50 psi (gage) increments from 150 psi to 600 psi. Each 50 psi gage increment represented 0.0698 psi (482 N/m²) actual distributed load. To ensure proper whiffle tree action 150 psi (gage) was taken as zero load in all cases.

The longitudinal and transverse normal membrane stress, σ_x and σ_y , were calculated using:

$$\sigma_x = \frac{E}{1-\mu^2} (\epsilon_x + \epsilon\mu_y) \quad [3]$$

and

$$\sigma_y = \frac{E}{1-\mu^2} (\epsilon_y + \epsilon\mu_x) \quad [4]$$

for stresses at the outside and inside surfaces where E is Young's modulus; μ is Poisson's Ratio; and ϵ_x and ϵ_y are measured or computed

strains on the corresponding surfaces. The stresses were then averaged to produce the final stresses presented in the results.

The longitudinal and transverse moments were calculated from surface stresses assuming a straight linear variation with depth using:

$$M_x = (\sigma_x \text{ inside} - \sigma_x \text{ outside}) \frac{t^2}{12} \quad [5]$$

and

$$M_y = (\sigma_y \text{ inside} - \sigma_y \text{ outside}) \frac{t^2}{12} \quad [6]$$

where t is the plate thickness.

The strain gages at location 29, at the middle of the inclined plate near the support, were the only three-element rosettes used. The gage was orientated with one element along the longitudinal x-axis, one element at 60° to the x-axis, and one element at 120° to the x-axis. When orientated in this manner the longitudinal, transverse, and shear strains were found to be:

$$\epsilon_x = \epsilon_{0^\circ} \quad [7]$$

$$\epsilon_y = \frac{2(\epsilon_{60^\circ} + \epsilon_{120^\circ}) - \epsilon_{0^\circ}}{3} \quad [8]$$

and

$$\gamma_{xy} = \frac{2(\epsilon_{120^\circ} - \epsilon_{60^\circ})}{\sqrt{3}} \quad [9]$$

The longitudinal stresses and longitudinal and transverse moments were calculated as described above. The shear stress, τ_{xy} , at location 29 was calculated from:

$$\tau_{xy} = G\gamma_{xy}$$

where G is the modulus of rigidity.

The strain data was reduced to stresses and moments using the IBM 1620 electronic computer.

EXPERIMENTAL RESULTS

For the purpose of comparing experimental results for the different types of end diaphragm with theoretical results, average stresses and moments at various points on the folded plate have been plotted in Figs. 10 through 41.

LONGITUDINAL STRESSES

Examination of results substantiates the findings of others that end conditions have little effect on longitudinal stresses away from the supports.

Figures 10 and 11 show longitudinal stress versus applied load at the free edges at midspan of the three-plate and five-plate models respectively. Quite good agreement between experimental and theoretical longitudinal stresses is shown.

Longitudinal stress versus load at the edge of the top plate at midspan is plotted in Figs. 12 and 13. Again good agreement with the theoretical is shown by both models.

It should be pointed out that diaphragm type 3 with the five-plate model appeared to produce longitudinal stresses three times the theoretical at gage location 21 (Fig. 15). This apparent discrepancy is due to a large zero offset and is demonstrated to lesser degrees in other graphs. Diaphragms type three and four consistently produced higher longitudinal stresses at this location.

The longitudinal stress at gage location 25 (top plate, center near support) is plotted in Figs. 16 and 17. It is apparent that the

longitudinal stress at this location in the five-plate model was greater when hinge-connected diaphragms were used than when bolt-connected ones were employed.

The longitudinal stress at gage location 29 (inclined plate, middle near support) showed good agreement with the theoretical when the models were supported by diaphragm type 5 (Figs. 18 & 19).

The stress profile along the edge of the top plate in the three-plate model for a surface load of .209 psi ($1.44 \times 10^3 \text{ N/m}^2$) is plotted in Fig. 20. A definite trend which conforms with the theoretical curve was evident for all diaphragm types. The hinged types produced consistently better results away from the ends but poorer results near the ends. The five-plate model results were similar to the three-plate results.

SHEAR STRESS

The shear stress was calculated from strain data at gage location 29 (inclined plate, middle near support) and is plotted in Figs. 22 and 23. All plotted results agree closely with theory in both models.

LONGITUDINAL MOMENT

The plot of longitudinal moment, M_x , at the edge of the top plate near the support (Figs. 24 & 25) indicates that bolt-connected diaphragms produced smaller moments than hinge-connected ones. This effect was not expected in light of the normal definition of a hinge and may have been due to localized stress concentrations created at load points of the whiffle tree or hinge connection points. Another possibility is

that the hinge connection may have allowed greater relative displacement between the center and longitudinal edges of the top plate than the bolt connection. This would lead to a larger negative transverse moment at the edge and hence a larger negative longitudinal moment at the same location.

M_x results at the middle of the top and the inclined plates near the diaphragms (gage locations 25 and 29) shown in Figs. 26 through 29 support the preceding theory. In all cases the bolt-connected diaphragms produced negative moments whereas the hinge-connected diaphragms produced positive moments.

For a distributed load of 0.209 psi ($1.44 \times 10^3 \text{ N/m}^2$) the M_x profile along the edge of the top plate from mid-span to support is plotted in Figs. 30 and 31. In both the three-plate and five-plate cases the longitudinal moment is seen to be fairly constant with x away from the ends.

TRANSVERSE MOMENT

Figures 34 and 35 show the variation of transverse moment with load at the center of the top plate near the support (gage location 25). The theoretical values for moments in both models are computed from a straight line interpolation between zero moment at the support and the moment at a point 5.4 in. (13.7 cm.) from the support. This interpolation assumes a straight line variation of transverse moment between points. The data from the hinge-connected diaphragm types on the three-plate model and all data on the five-plate model indicate that this assumption is in error. In fact the experimental results indicate not only a change in value but

also a change in sign of the transverse bending moment. Similar trends are shown in the transverse moments at the middle of the inclined plate near the support (gage location 29 Figs. 36 and 37).

The transverse moment profiles along the edge of the top plate for a distributed load of 0.209 psi ($1.44 \times 10^3 \text{ N/m}^2$) are shown in Figs. 38 and 39. Although the experimental and theoretical results do not agree in value, the experimental results showed good agreement with each other. The variation with x is similar to that predicted by theory. At the supports the transverse moments in the models supported by hinge-connected diaphragms were in general larger than those in models supported by bolt-connected diaphragms. This again was probably due to the fact that the hinge connections allowed larger relative deflections near the supports and therefore larger transverse moments.

Most folded plate theory (1) assumes one-way slab action and thus neglects the effects of longitudinal moments, M_x . Figures 40 and 41 show the M_x/M_y variation with x . It is evident that in general M_x has a value of approximately one-third M_y along the edge of the top plate and therefore may be significant with regard to design considerations.

DEFLECTIONS

The deflections at midspan of the center of the top plate were plotted in Figs. 42 and 43. These indicate that the structure deflected at midspan in a linear manner and that this deflection was independent of end conditions.

In neither the three-plate nor the five-plate model did the experimental data show good agreement with the theoretical solution, the actual

deflections being larger than the theoretical in both cases. That there existed good agreement among the experimental results is encouraging evidence that end conditions have little effect on deflections away from the ends.

CONCLUSIONS

The results of an experimental study on folded plate models subjected to uniform vertical loads to determine the effects of different end support configurations on induced stresses, moments, and deformations are presented in this report. Five types of end diaphragm were considered in an attempt to simulate the theoretical and practical boundary conditions at support locations.

The type 1 diaphragm used in this study was similar to those commonly found in practice. The other four types of diaphragm each were designed in some way to satisfy a part of the theoretical support conditions of the diaphragm being perfectly rigid in its plane and perfectly flexible normal to its plane. Type 5 diaphragm, the sheet steel support, perhaps satisfied these criteria better than the others but the major cause of differences in stress and moment results near the ends was the method of connection, whether bolted or hinged.

On the basis of results obtained in this investigation the following conclusions are made:

(1) It is concluded that the type of end diaphragm does in fact have little effect on stresses and deformations away from the support. The great differences in observed stresses all occurred within 0.1 span of the supports with only minor differences within the middle 0.8 span. No general advantage was found in the hinge connection over the bolt connection of the diaphragms with respect to stresses and deformations in this middle 0.8 length. Therefore if all data is to be taken within this area the simplest diaphragm to construct produces acceptable

results. In the author's case, the bolt-connected diaphragm was by far the simplest to construct and use. When values of stresses and moments near the support are important the support should be modeled as closely as possible to the prototype and measurements made near the support. In practice this support will generally be the bolted type.

(2) The fact that the roller support and the "fixed" support both produced identical deflections and stresses at midspan rules out the possibility of analyzing these types of folded plates as beams. Beam theory gives different results for these deflections and stresses.

(3) In practice, M_x , the longitudinal moment, may not be small enough to be neglected in design.

APPENDIX I: NOTATION

E	= Young's modulus
F_F	= Matrix of edge tractions
G	= Modulus of rigidity
K	= Structure stiffness matrix
M_x	= Longitudinal moment
M_y	= Transverse moment
n_p	= Number of plates
q	= Distributed load on horizontal projection of structure
q_o	= 1 psi (0.6895 N/cm ²)
t	= Plate thickness
x	= Longitudinal direction
y	= Transverse direction
γ_{xy}	= Shear strain
Δ	= Displacement matrix
ϵ_x	= Longitudinal strain
ϵ_y	= Transverse strain
μ	= Poisson's ratio
σ_x	= Longitudinal, normal, membrane stress
σ_y	= Transverse, normal, membrane stress
τ_{xy}	= Shear membrane stress

APPENDIX II: FIGURES

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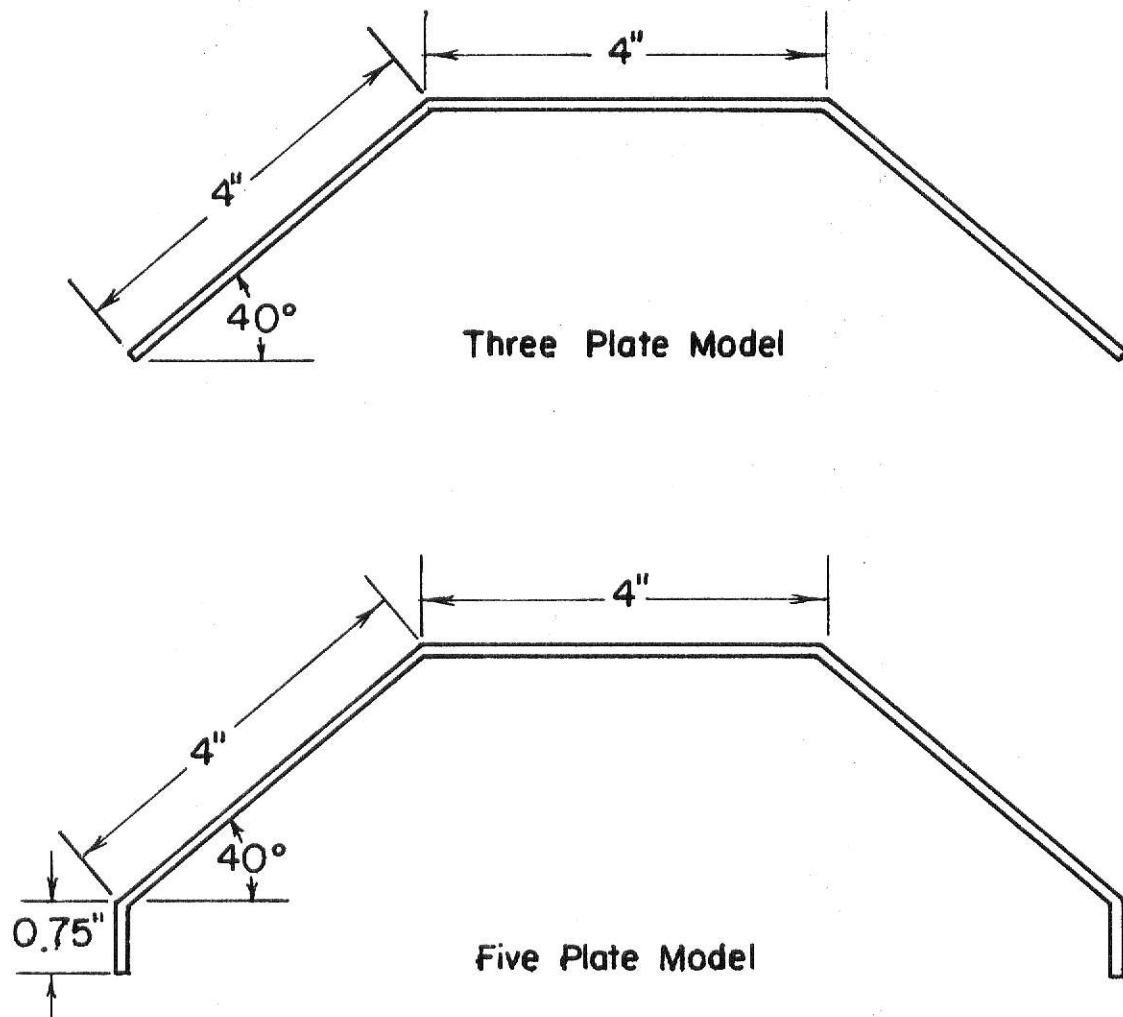


Figure 1 Model Cross-sections

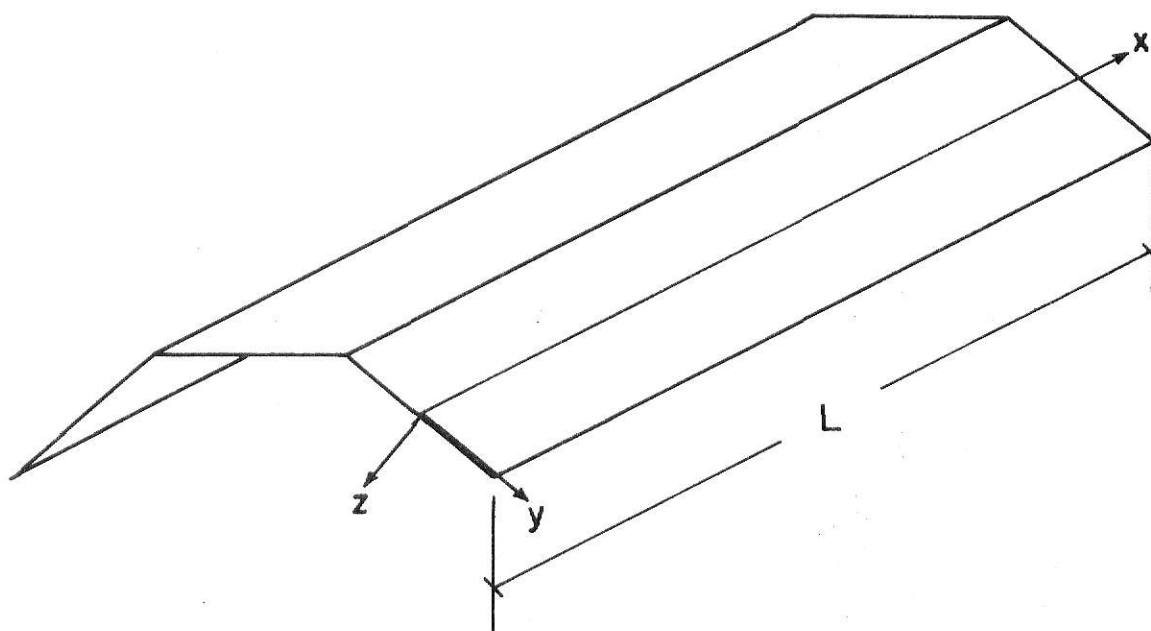


Figure 2 Three Plate Model with Coordinate System

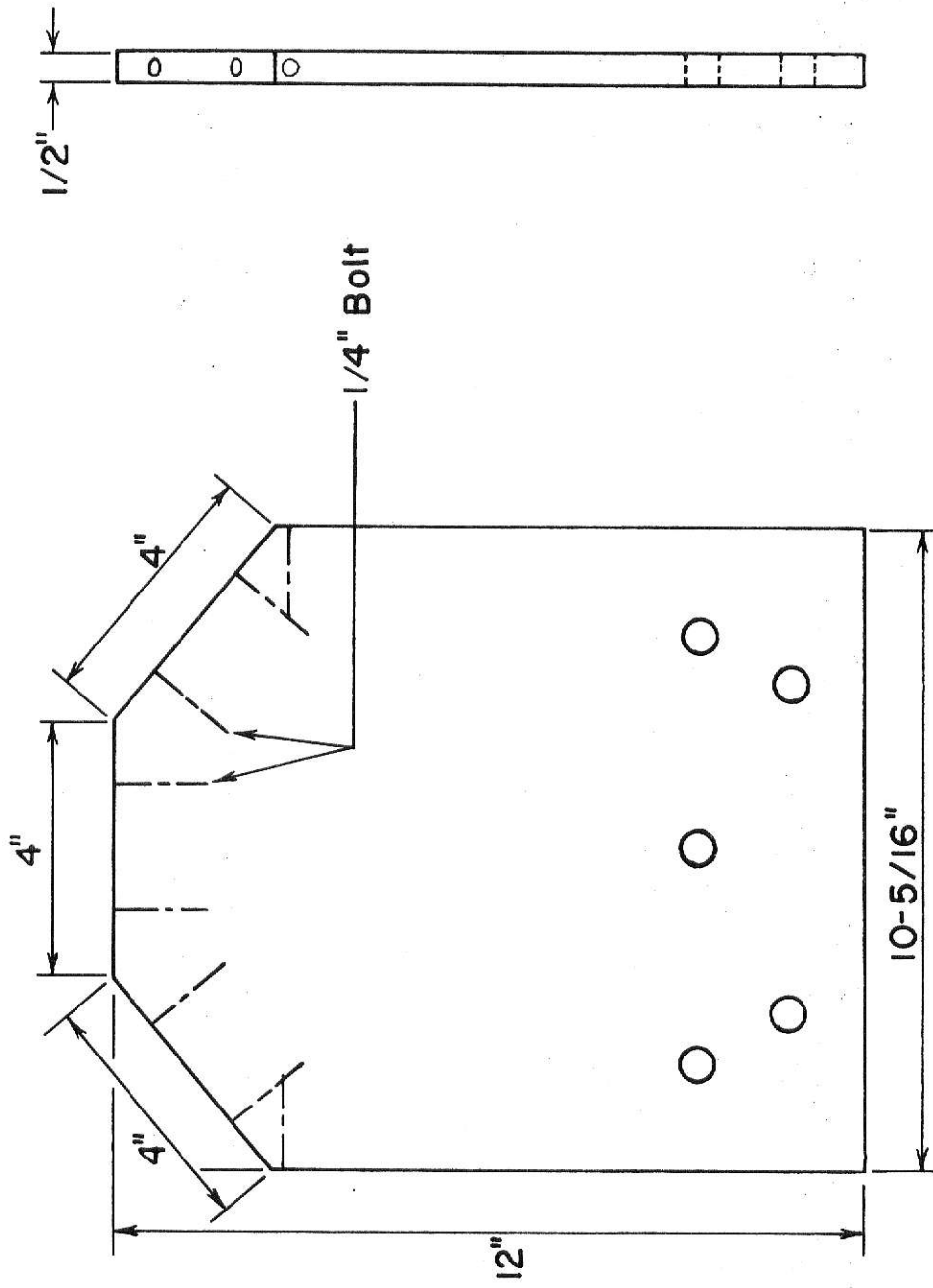


Figure 3a Type One End Diaphragm

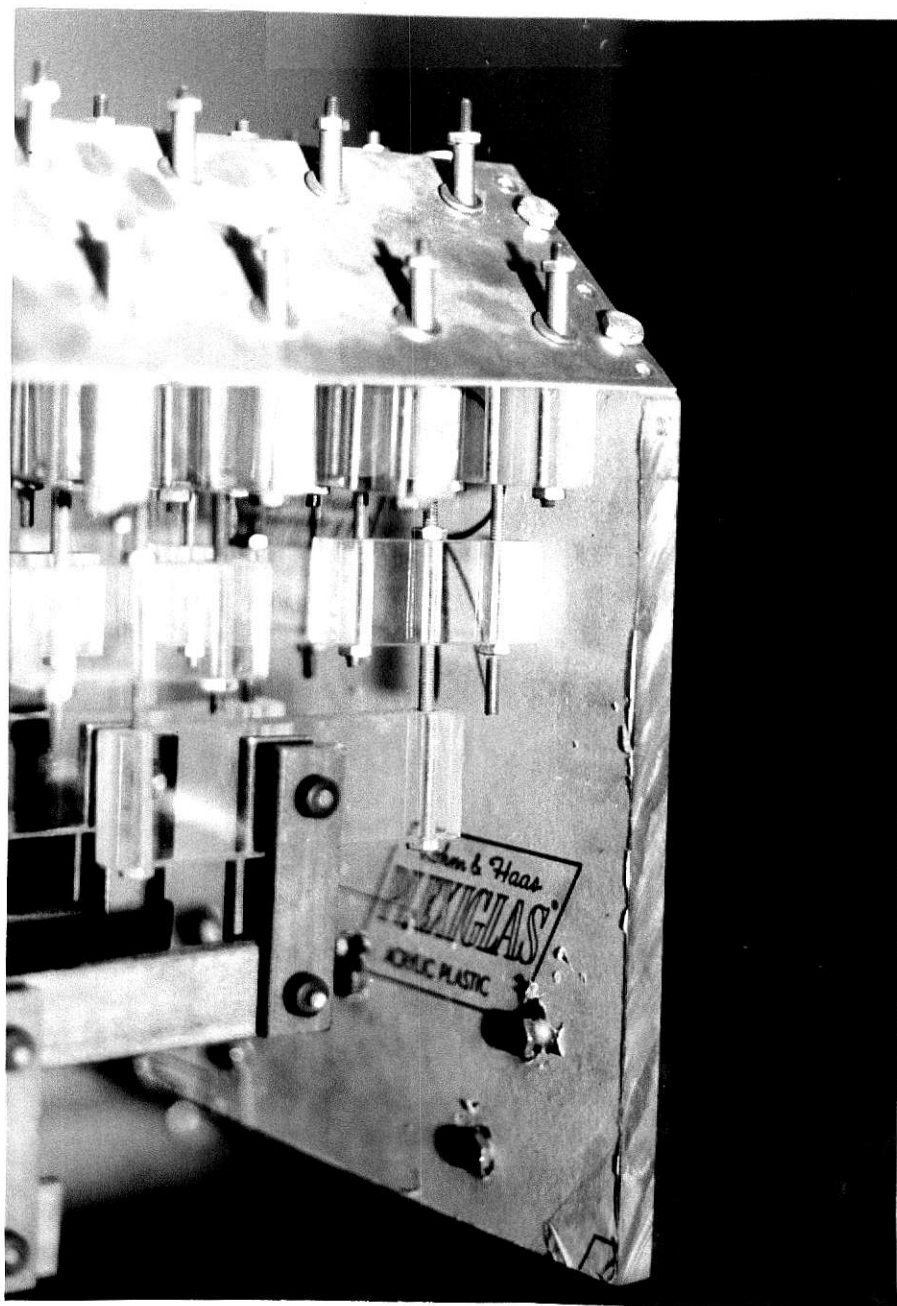


Figure 3b Type One End Diaphragm

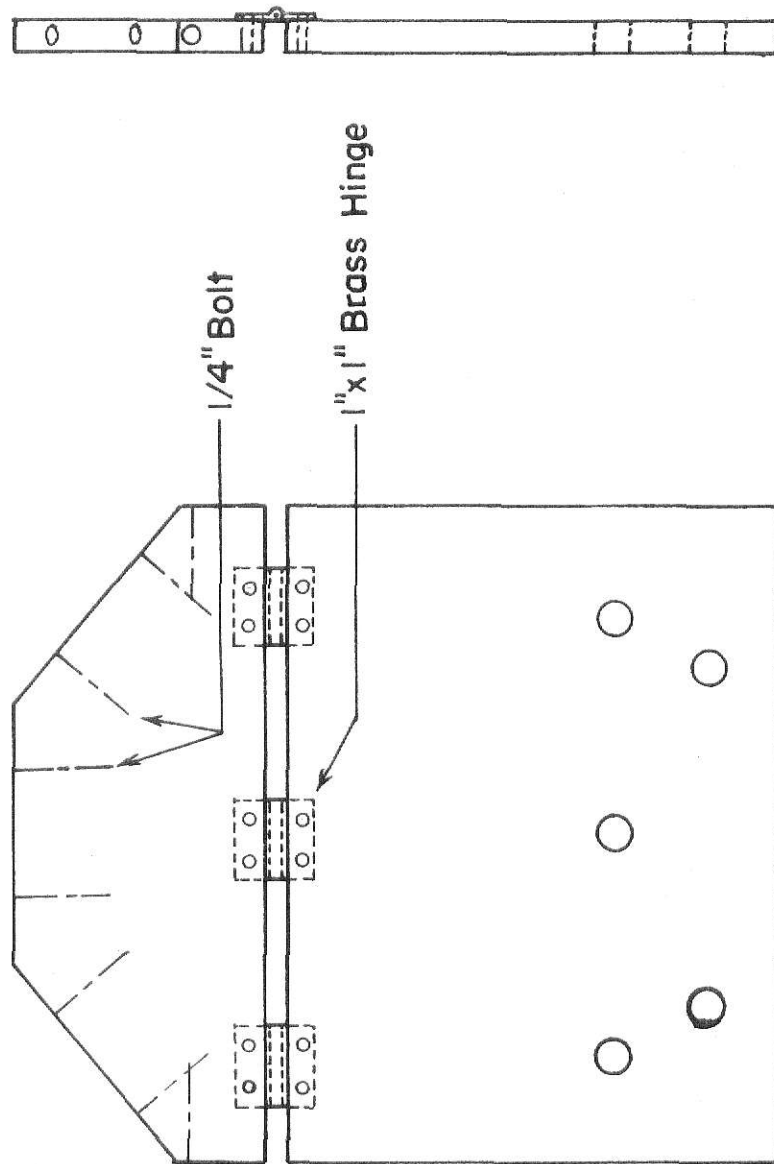


Figure 4a Type Two End Diaphragm

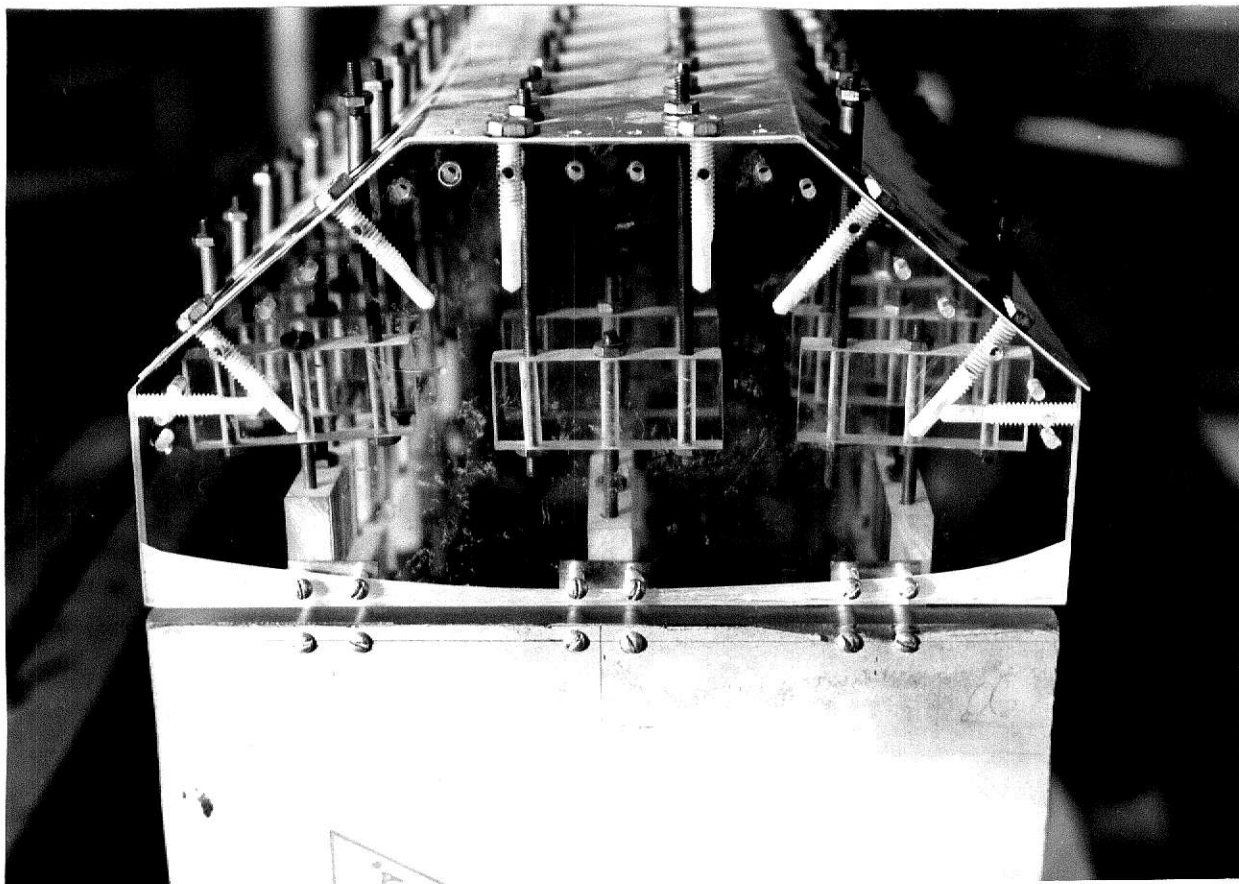


Figure 4b Type Two End Diaphragm

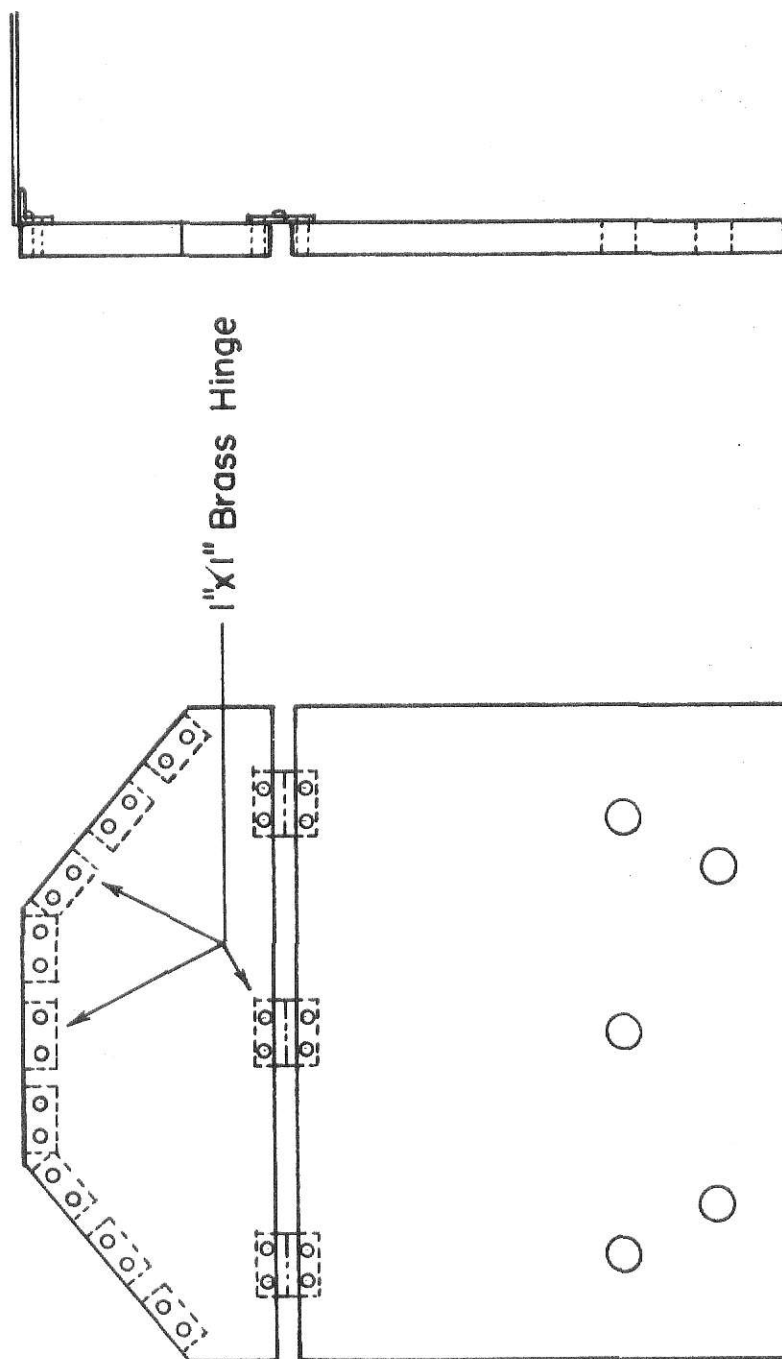


Figure 5a Type Three End Diaphragm

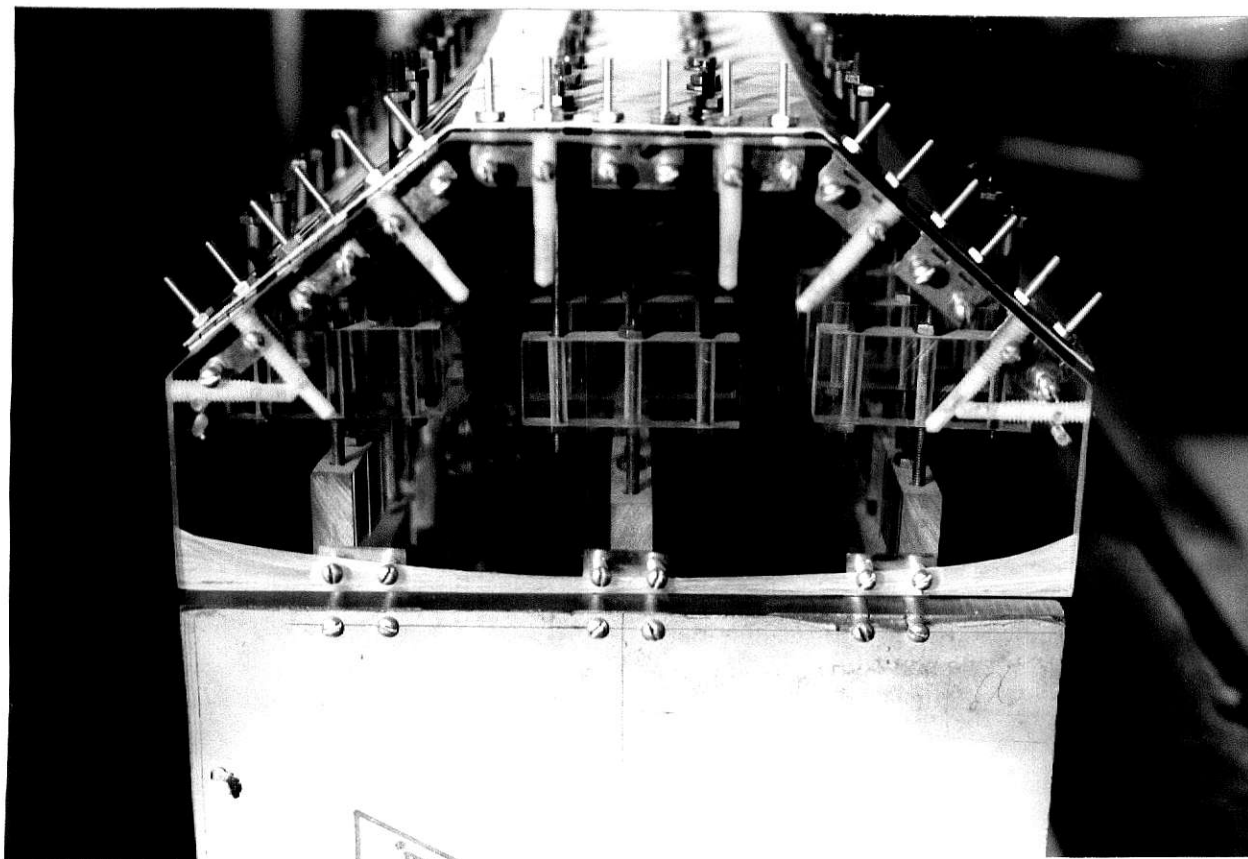


Figure 5b Type Three End Diaphragm

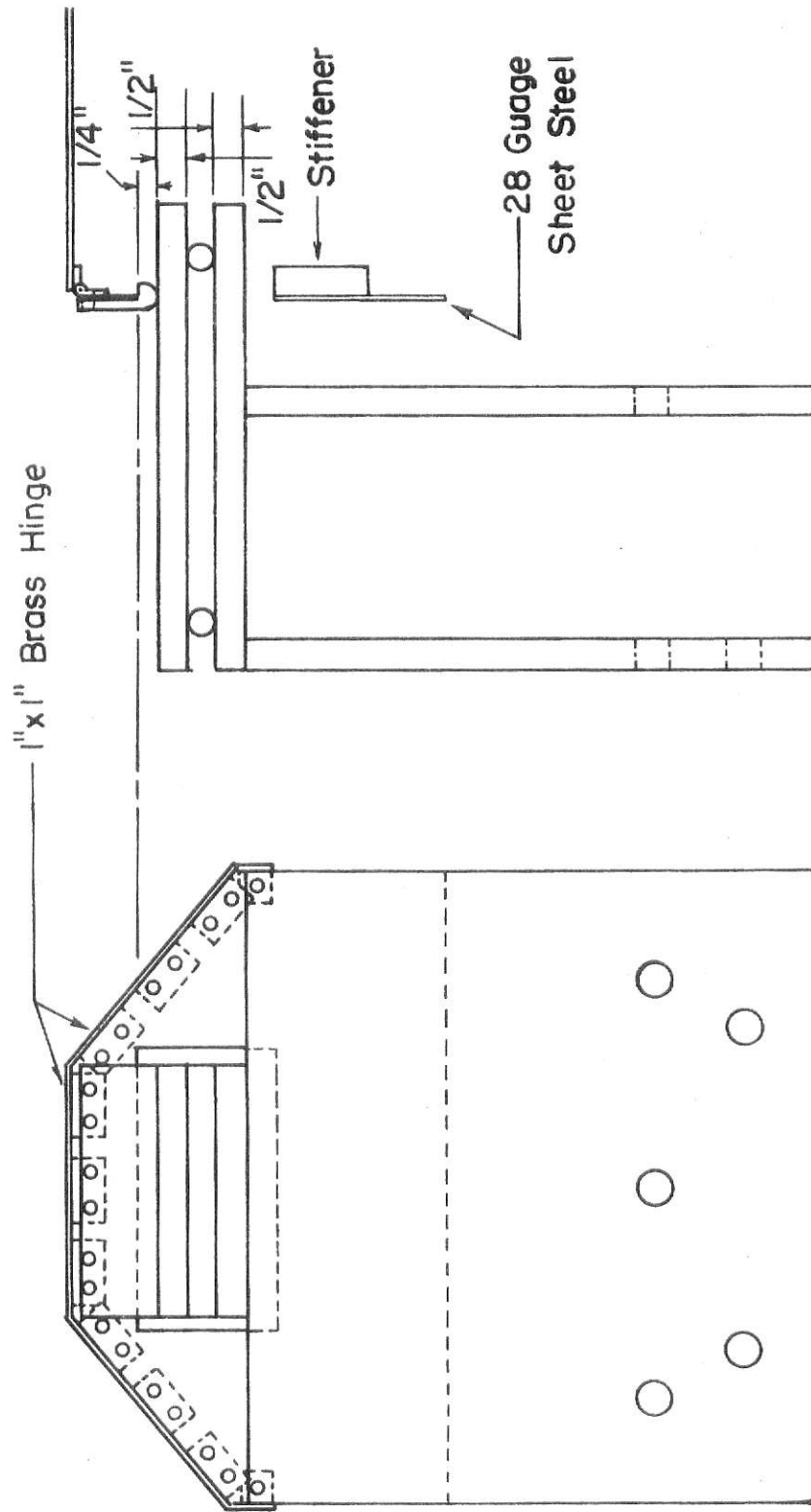


Figure 6a Type Four End Diaphragm

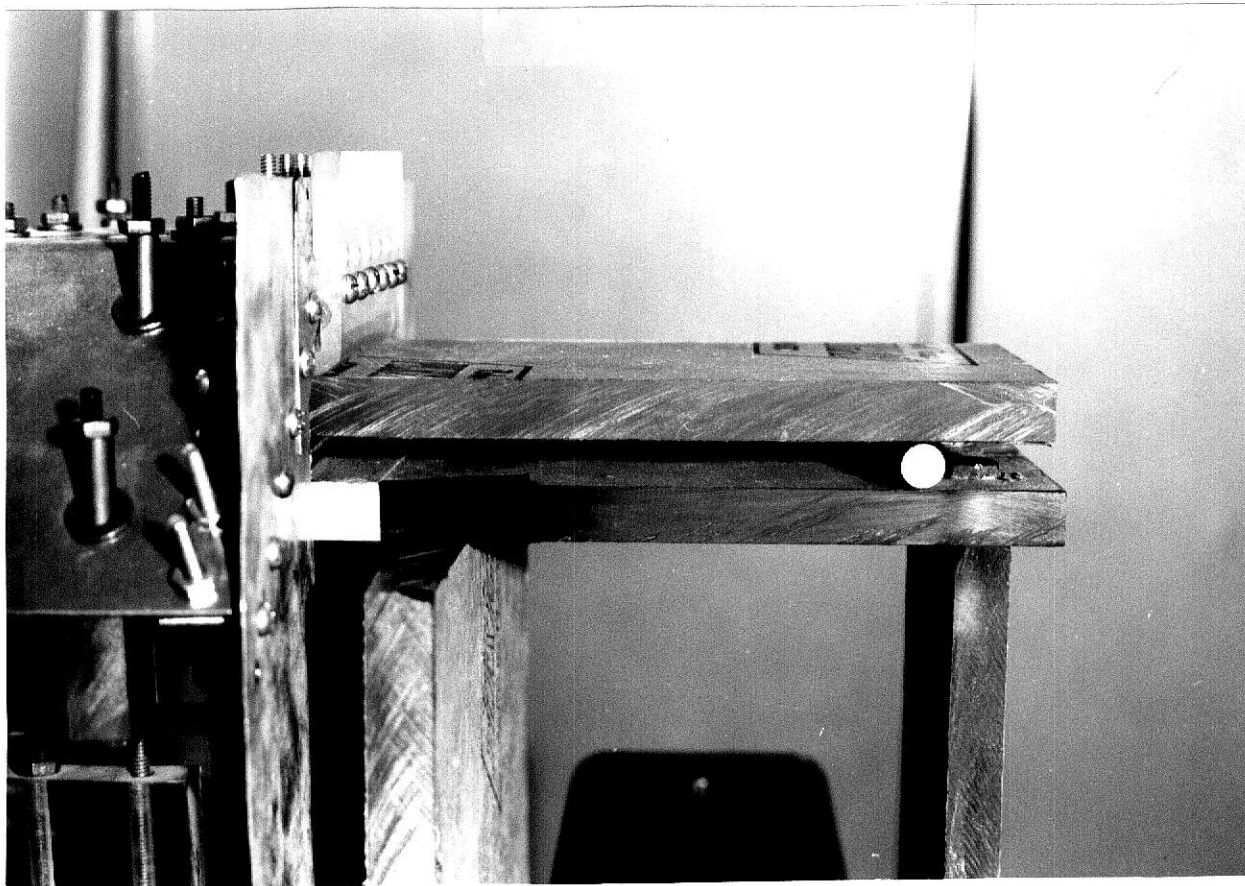


Figure 6b Type Four End Diaphragm

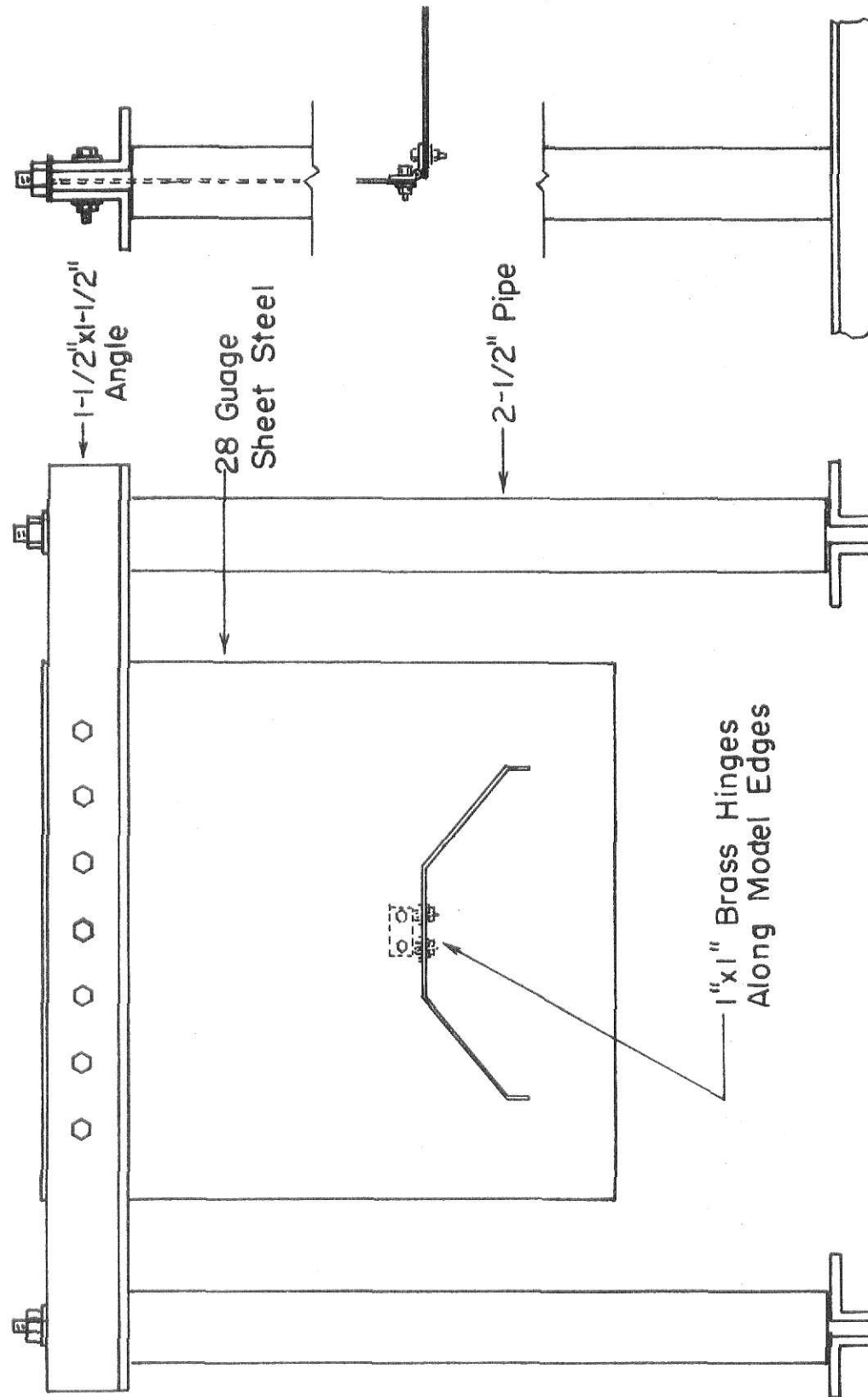


Figure 7a Type Five End Diaphragm

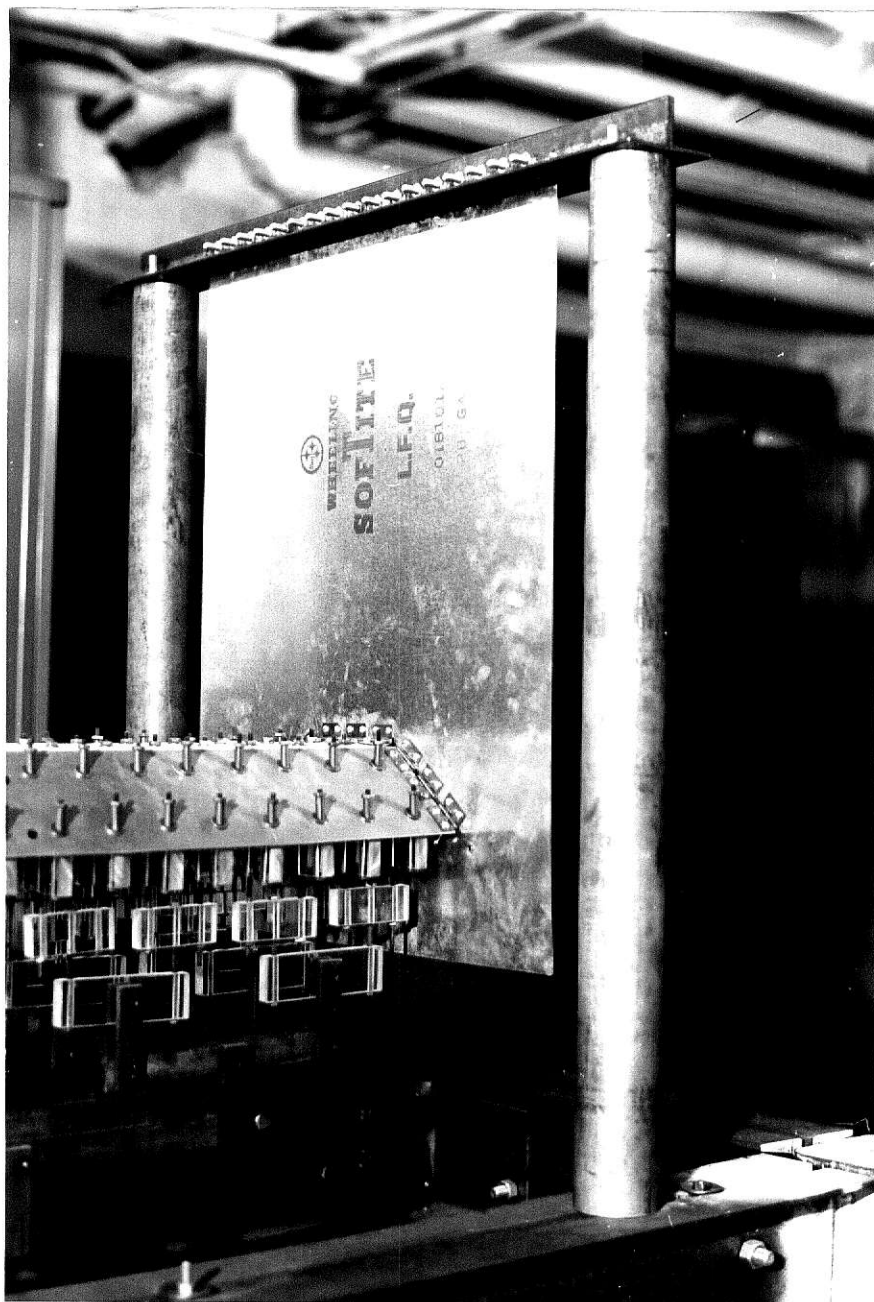


Figure 7b Type Five End Diaphragm

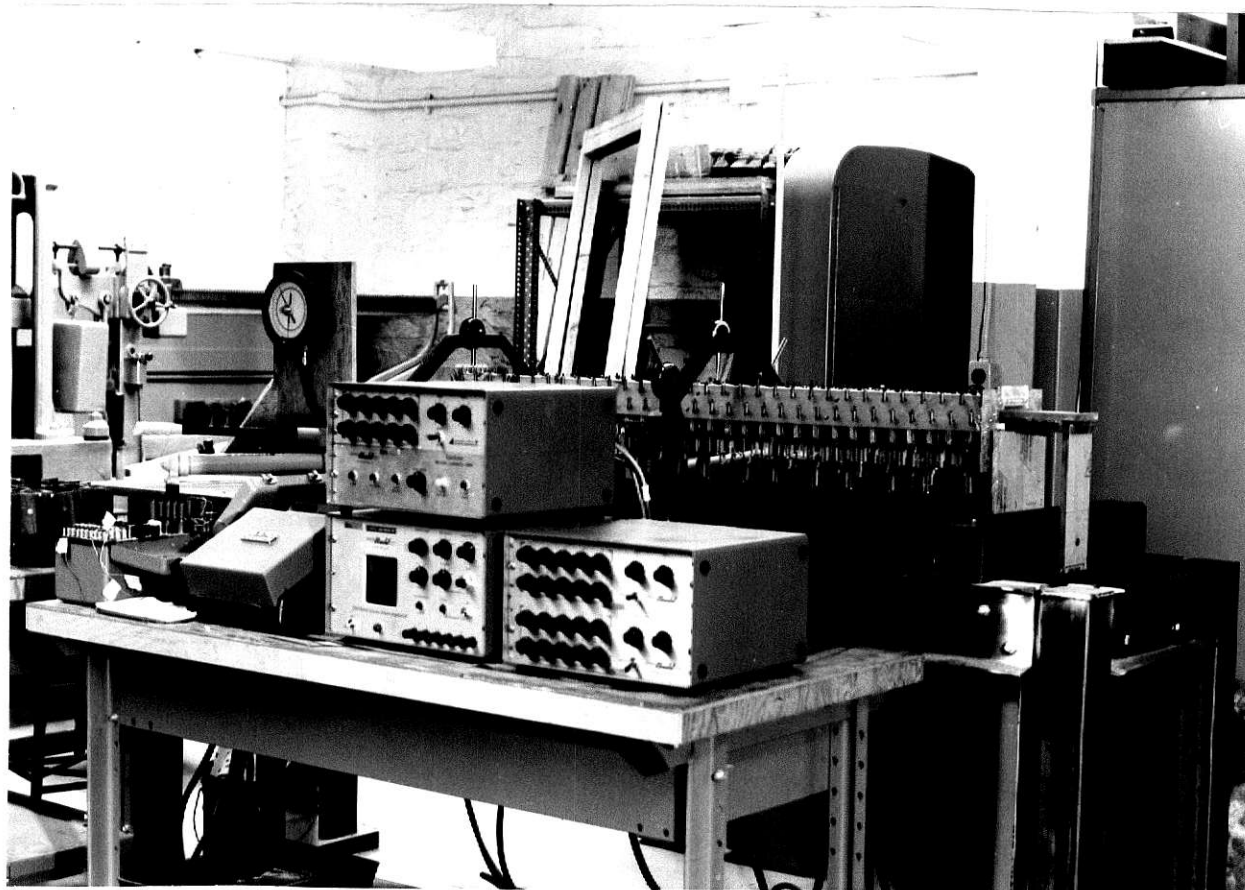


Figure 8 Support Frame and Loading System

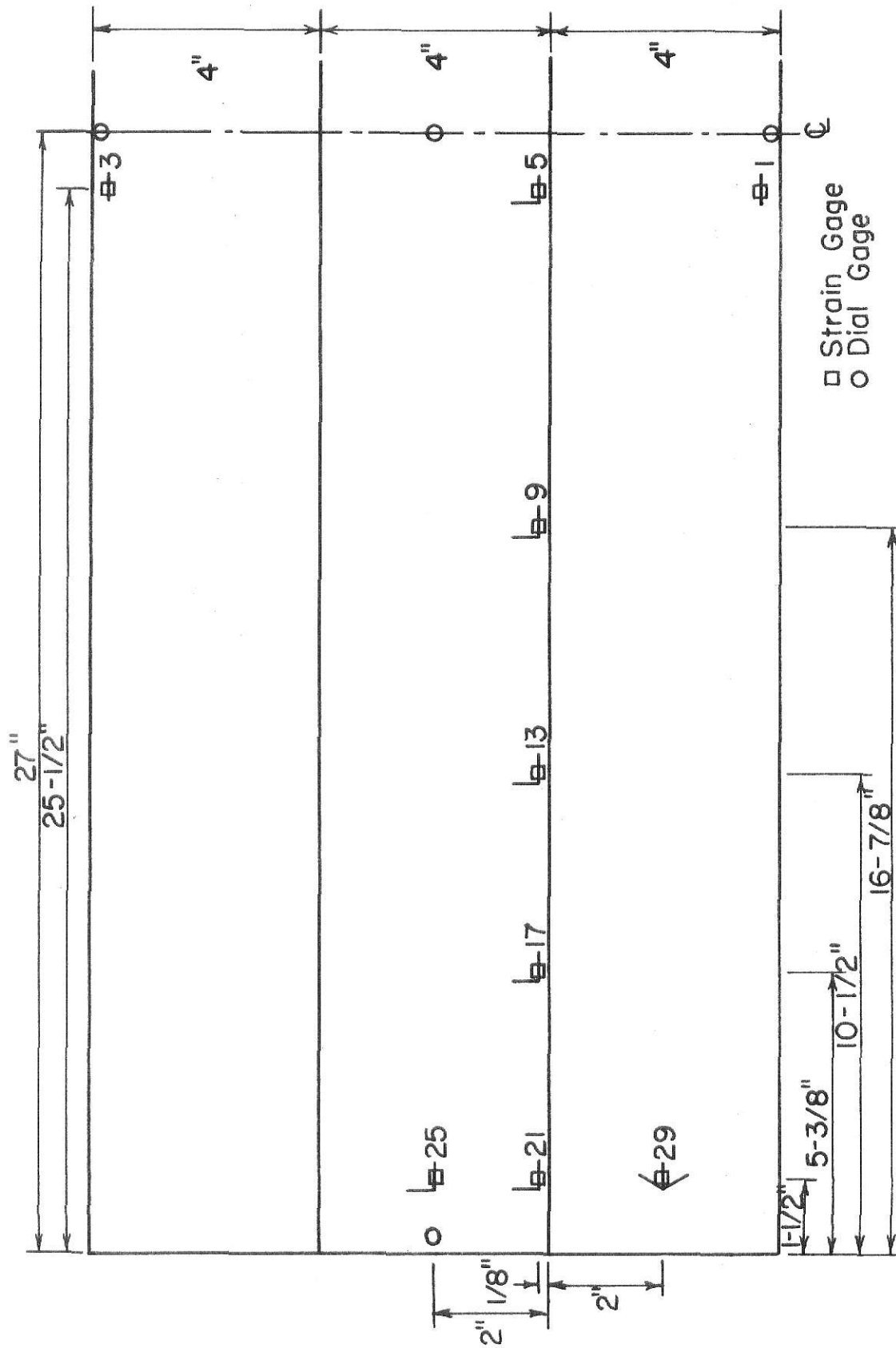


Figure 9 Strain and Dial Gage Locations

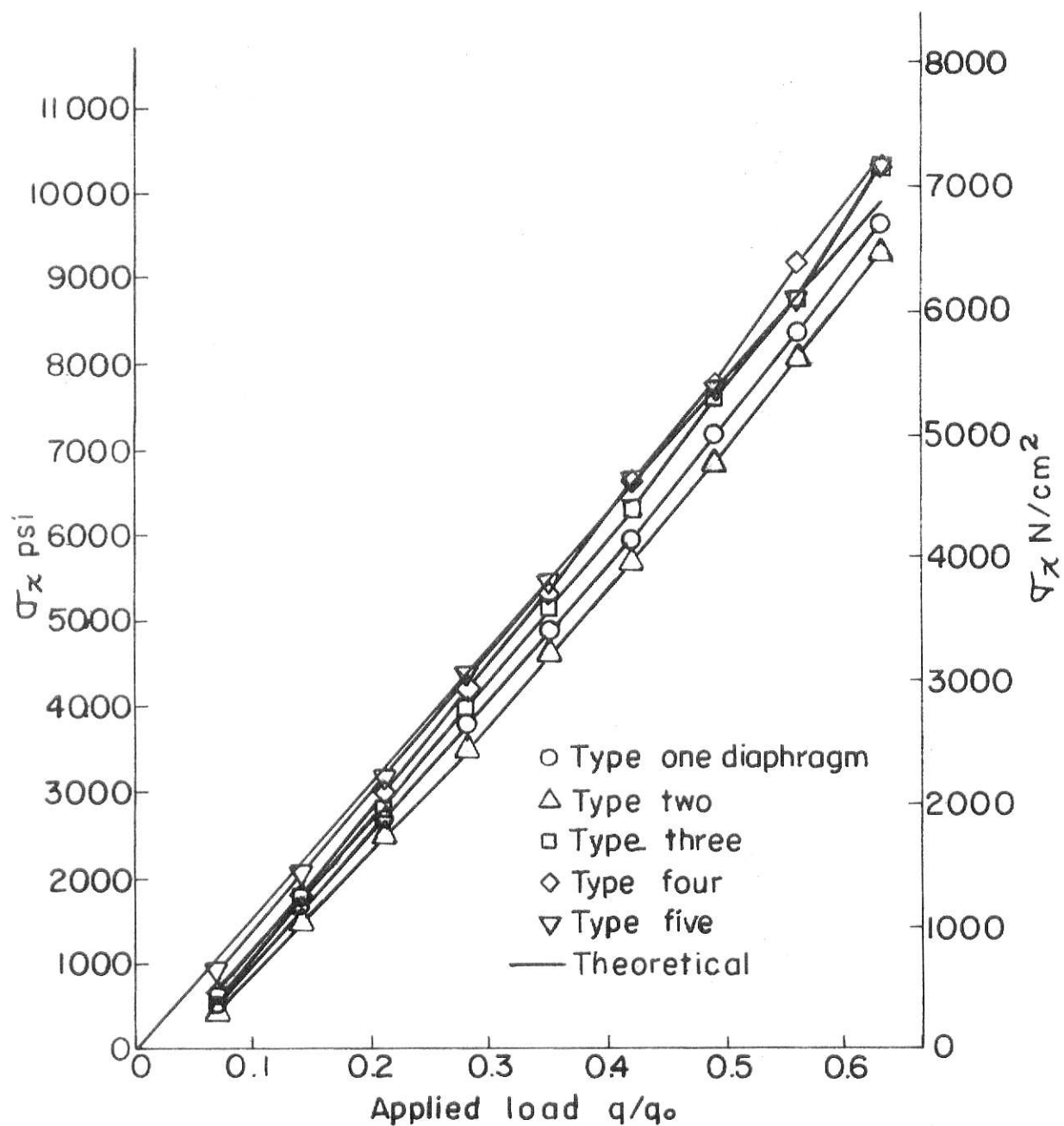


Figure 10 Longitudinal Membrane Stress, σ_x , vs Load, Free Edge at Midspan, 3-plate Model

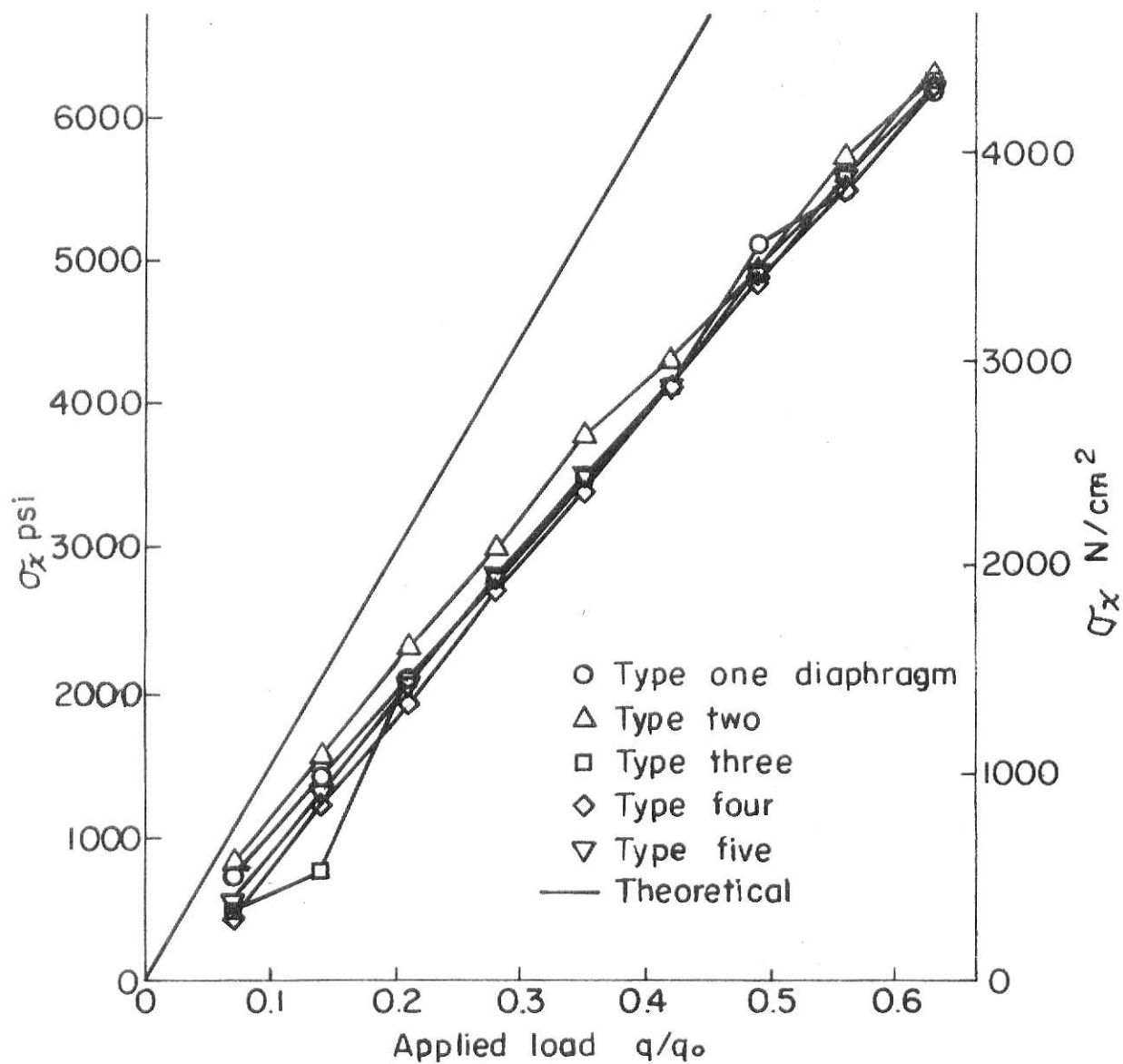


Figure II Longitudinal Membrane Stress, σ_x , vs Load, Free Edge at Midspan, 5-plate Model

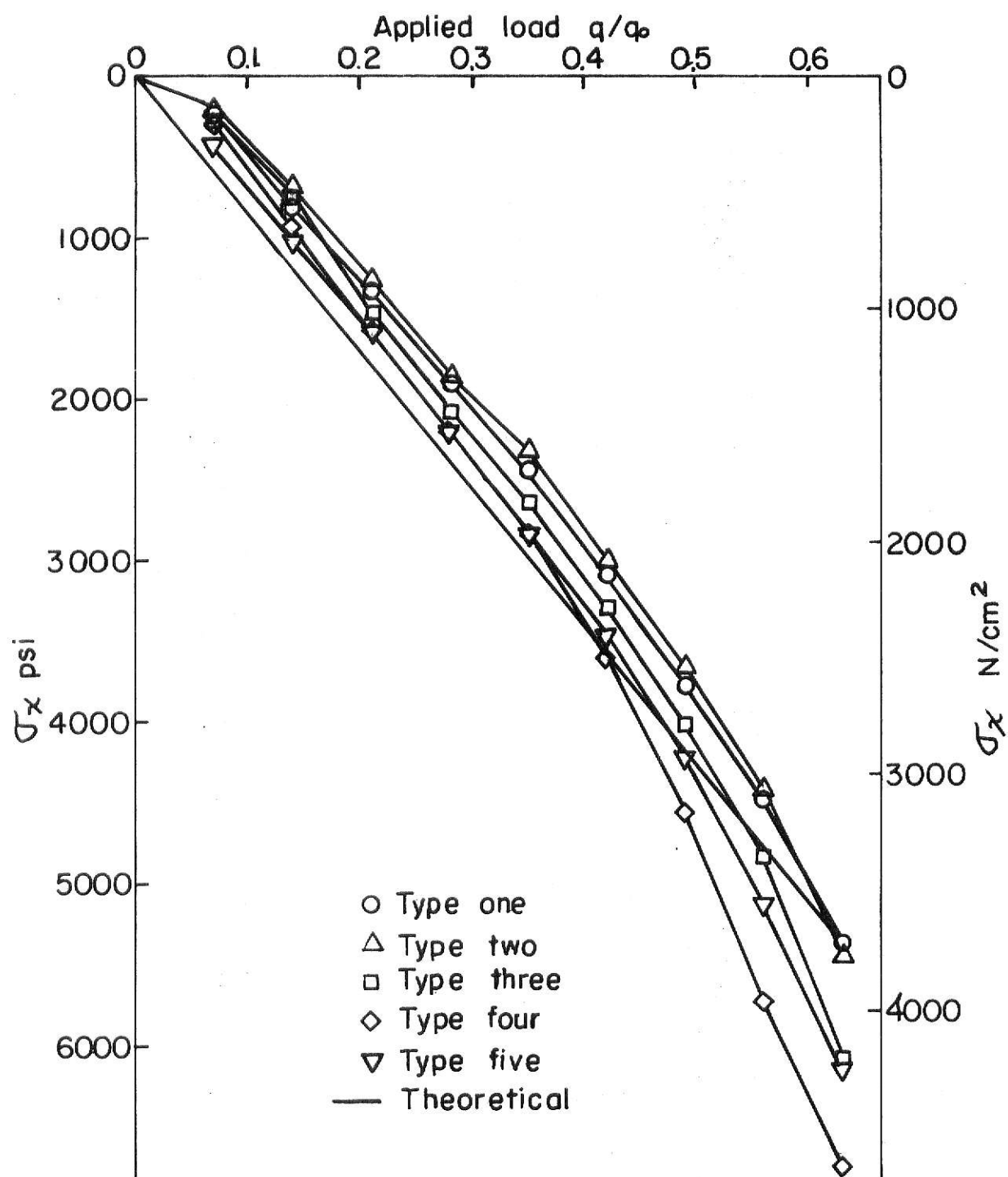


Figure 12 Longitudinal Membrane Stress, σ_x , vs. Load, Edge of Top Plate at Midspan, 3-plate Model

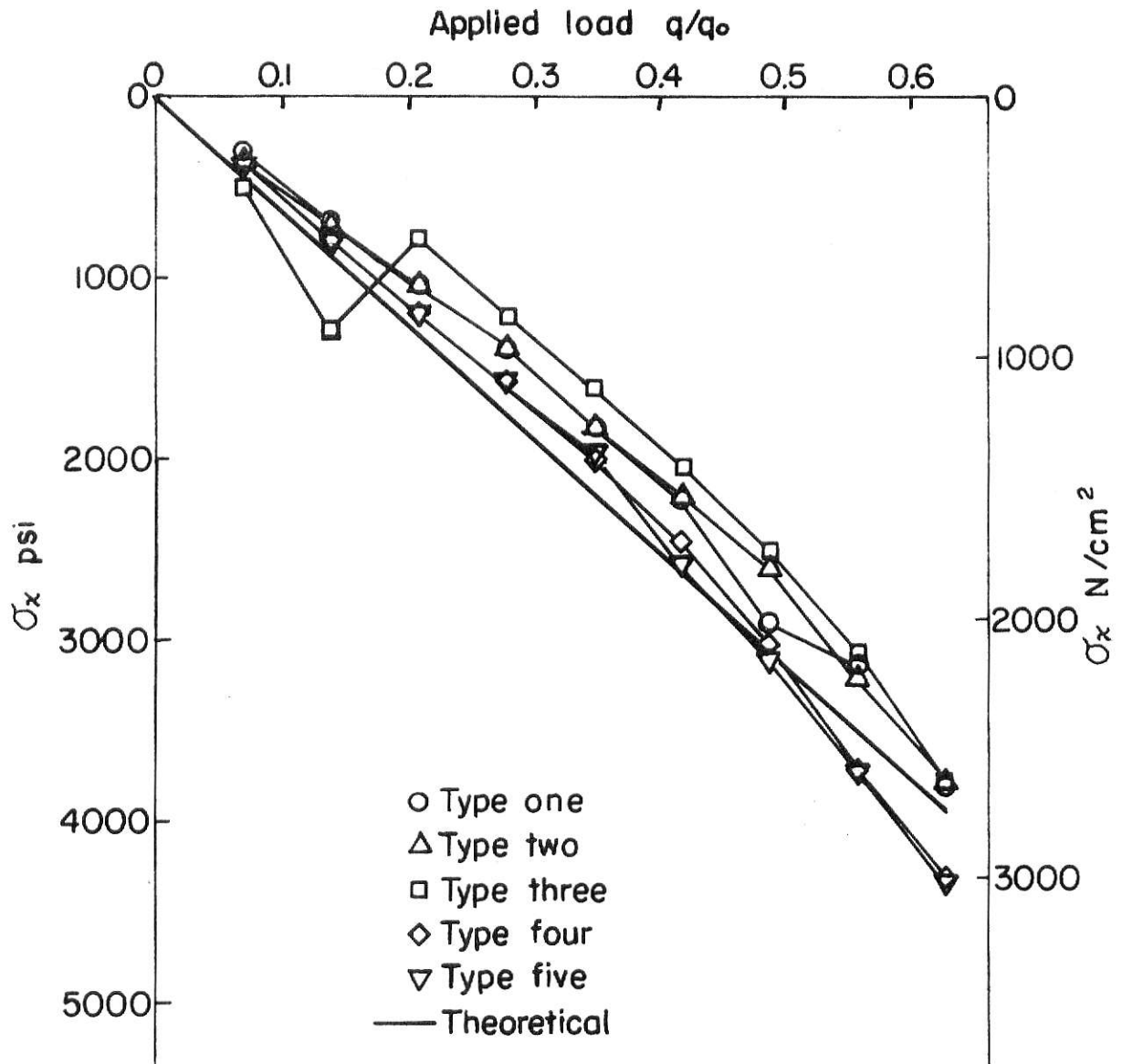


Figure 13 Longitudinal Membrane Stress, σ_x , vs Load, Edge of Top Plate at Midspan, 5-plate Model

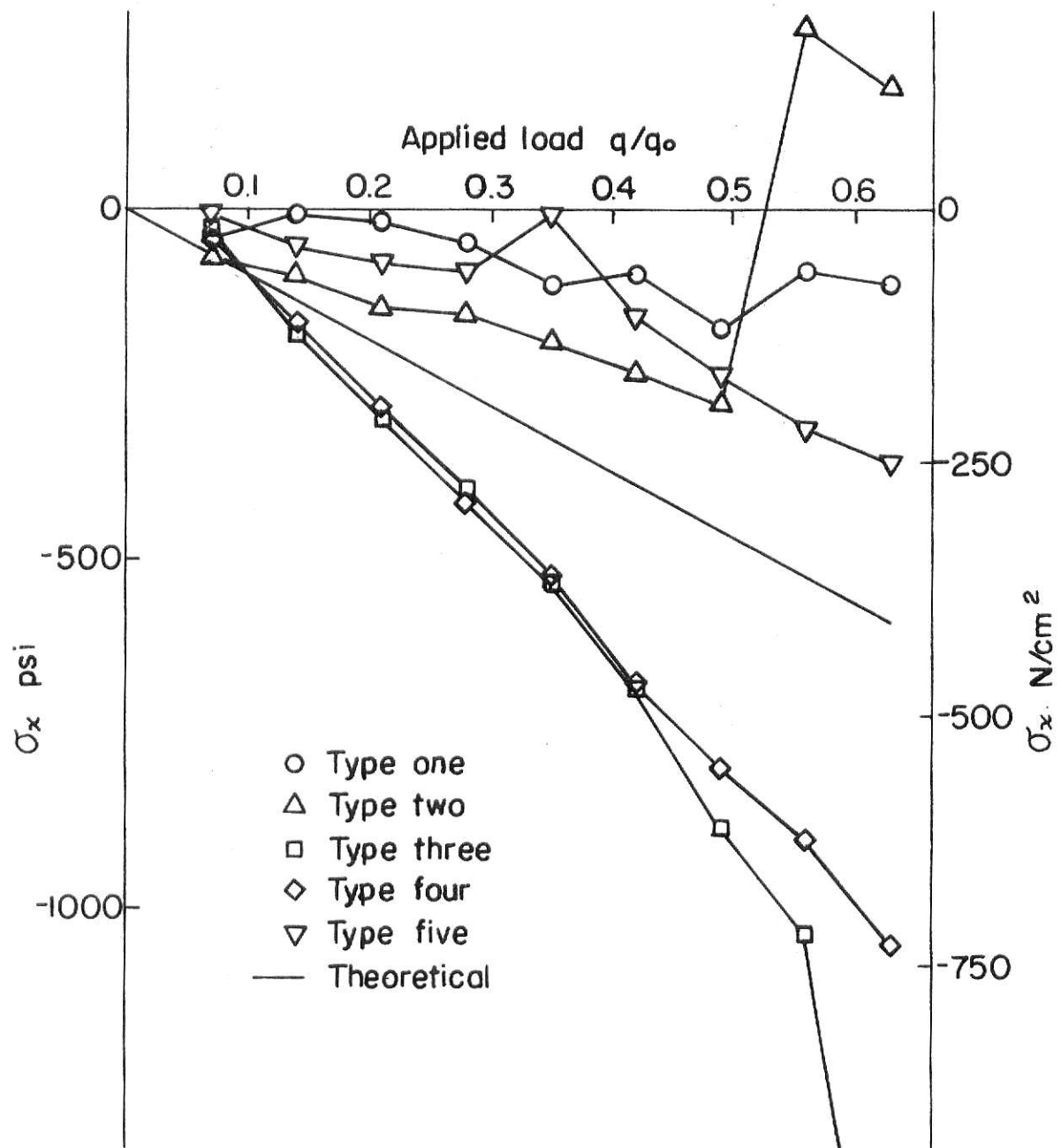


Figure 14 Longitudinal Membrane Stress, σ_x , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

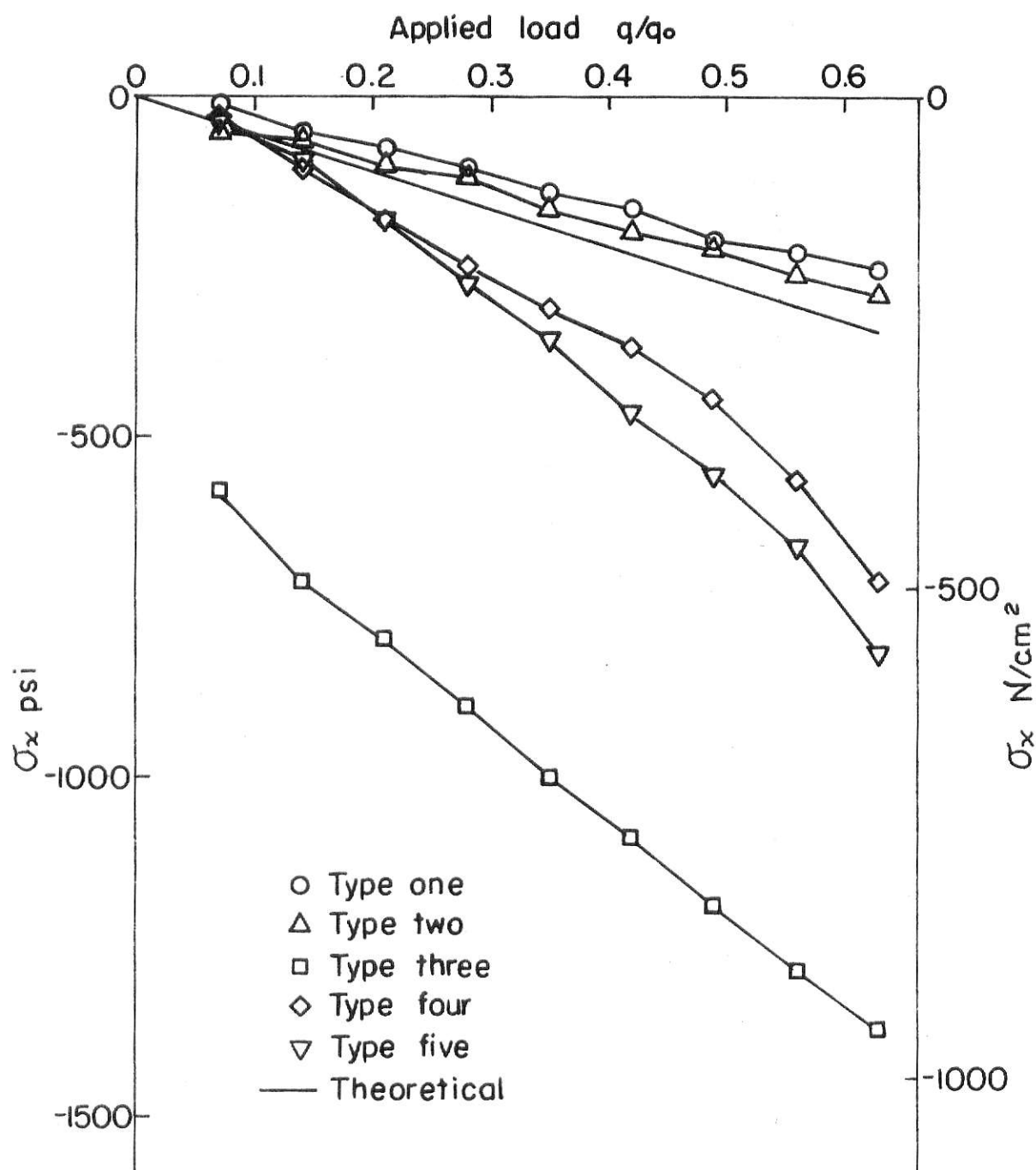


Figure 15 Longitudinal Membrane Stress, σ_x , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

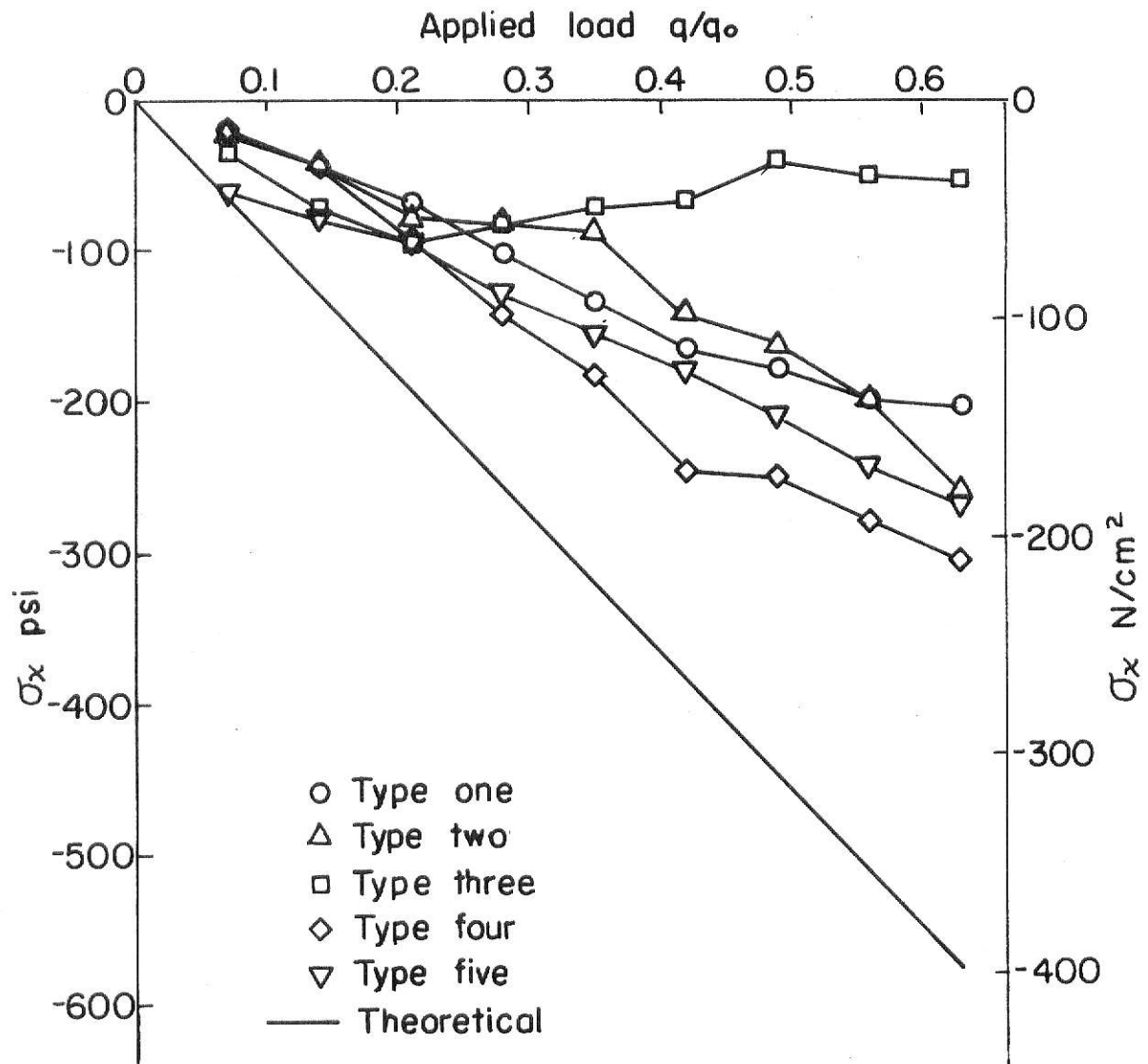


Figure 16 Longitudinal Membrane Stress, σ_x , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81cm.) from Support, 3-plate Model

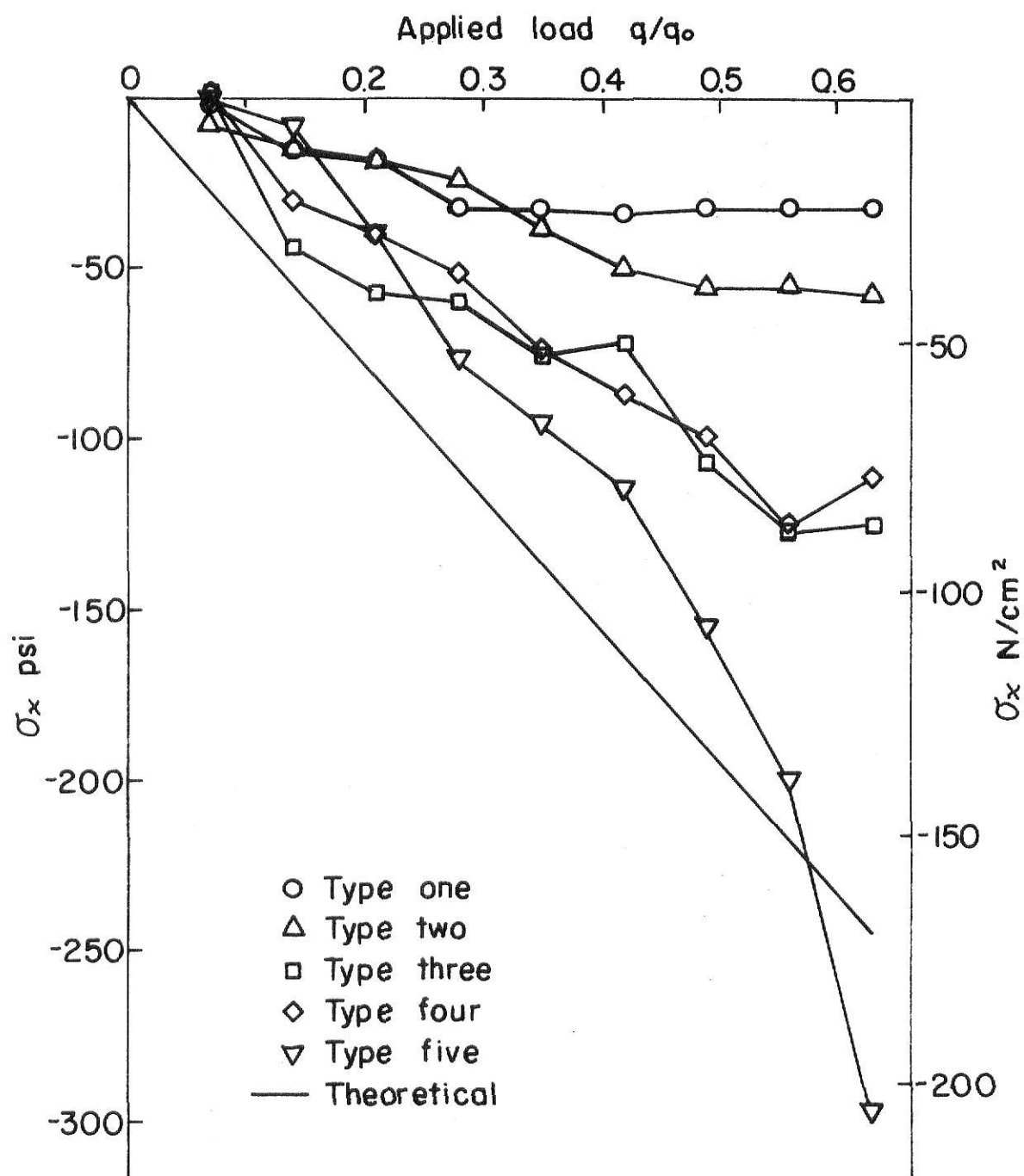


Figure 17 Longitudinal Membrane Stress, σ_x , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

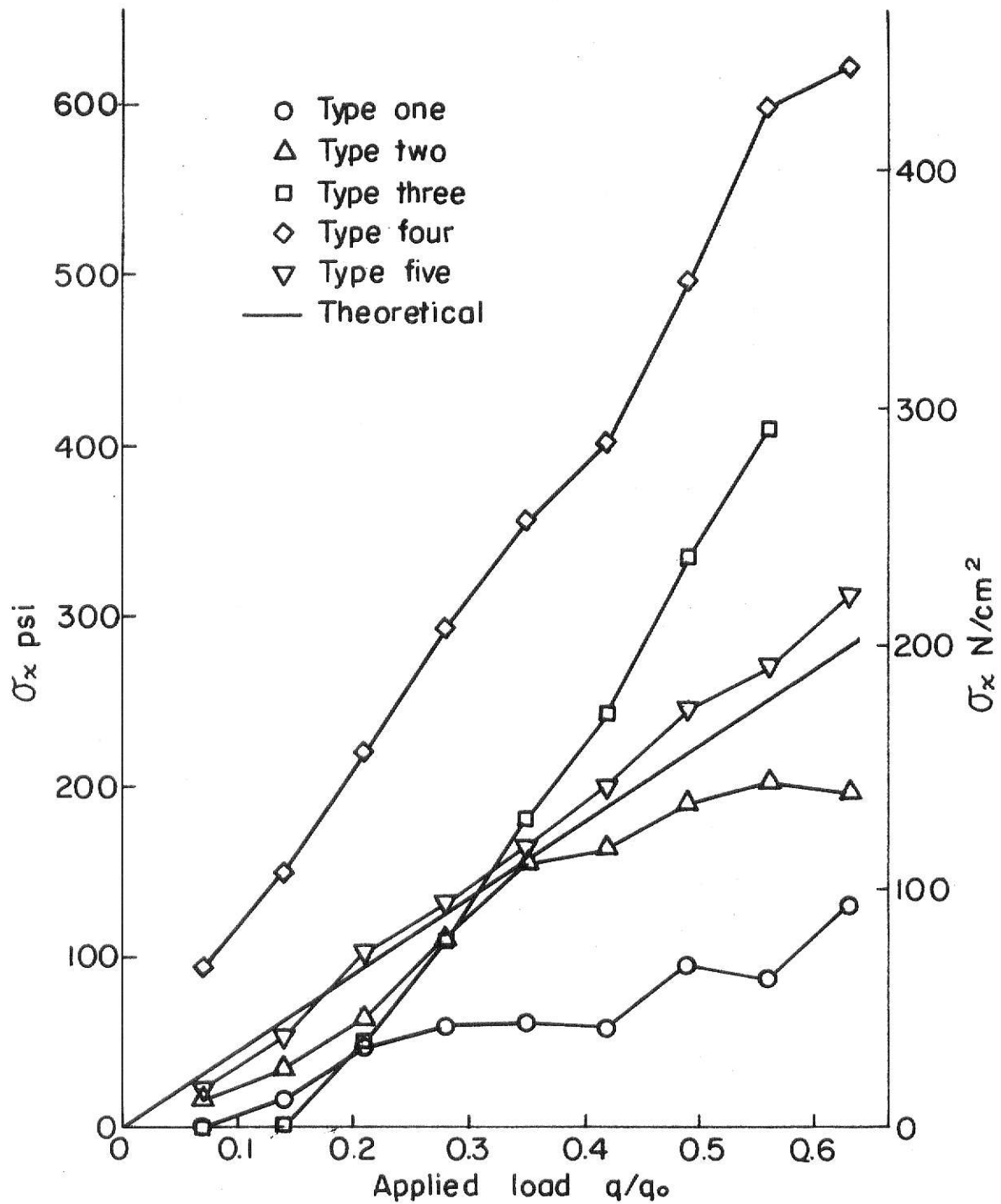


Figure 18 Longitudinal Membrane Stress, σ_x , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

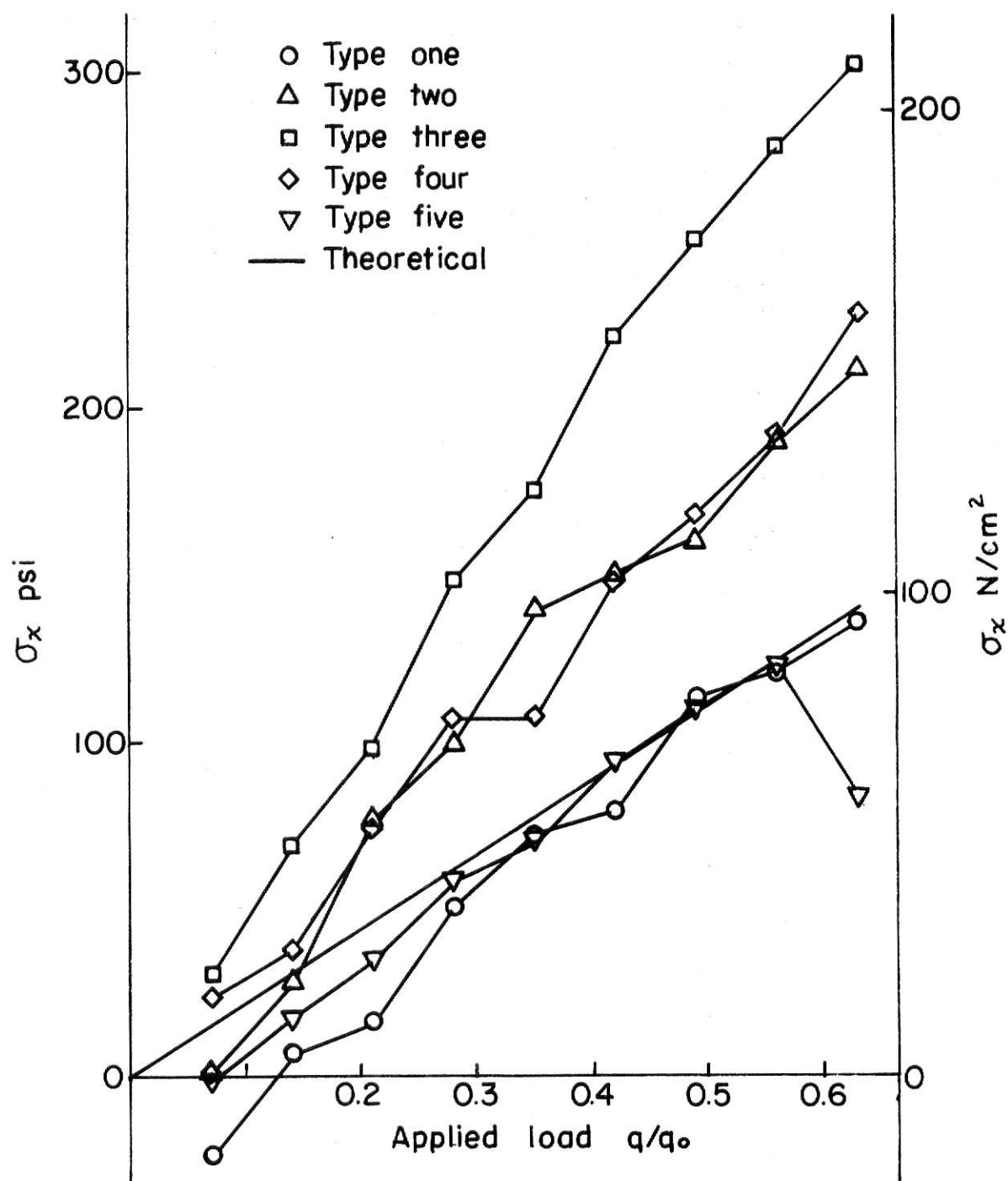


Figure 19 Longitudinal Membrane Stress, σ_x , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

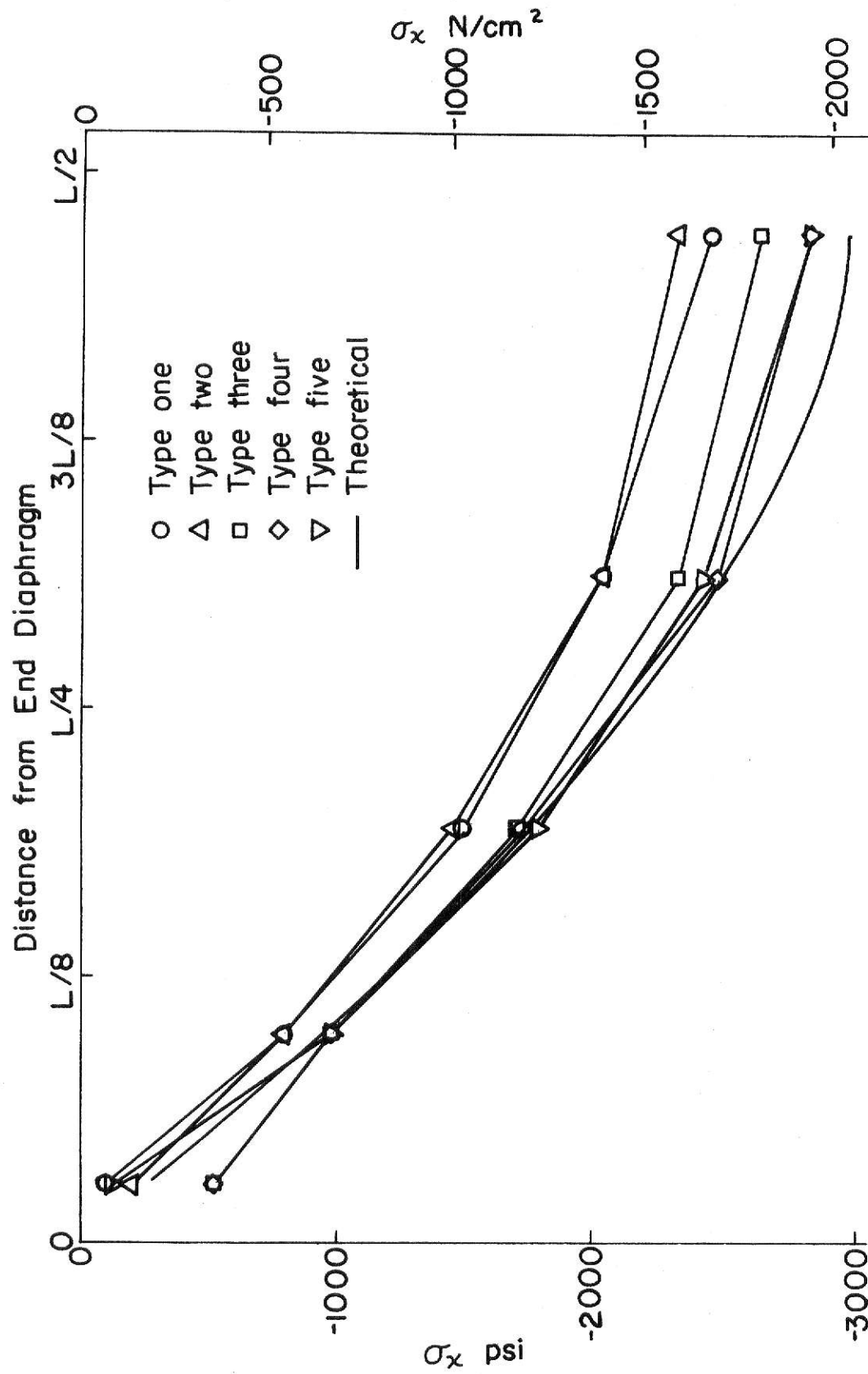


Figure 20 Longitudinal Stress Profile, Top Edge, 3-plate Model

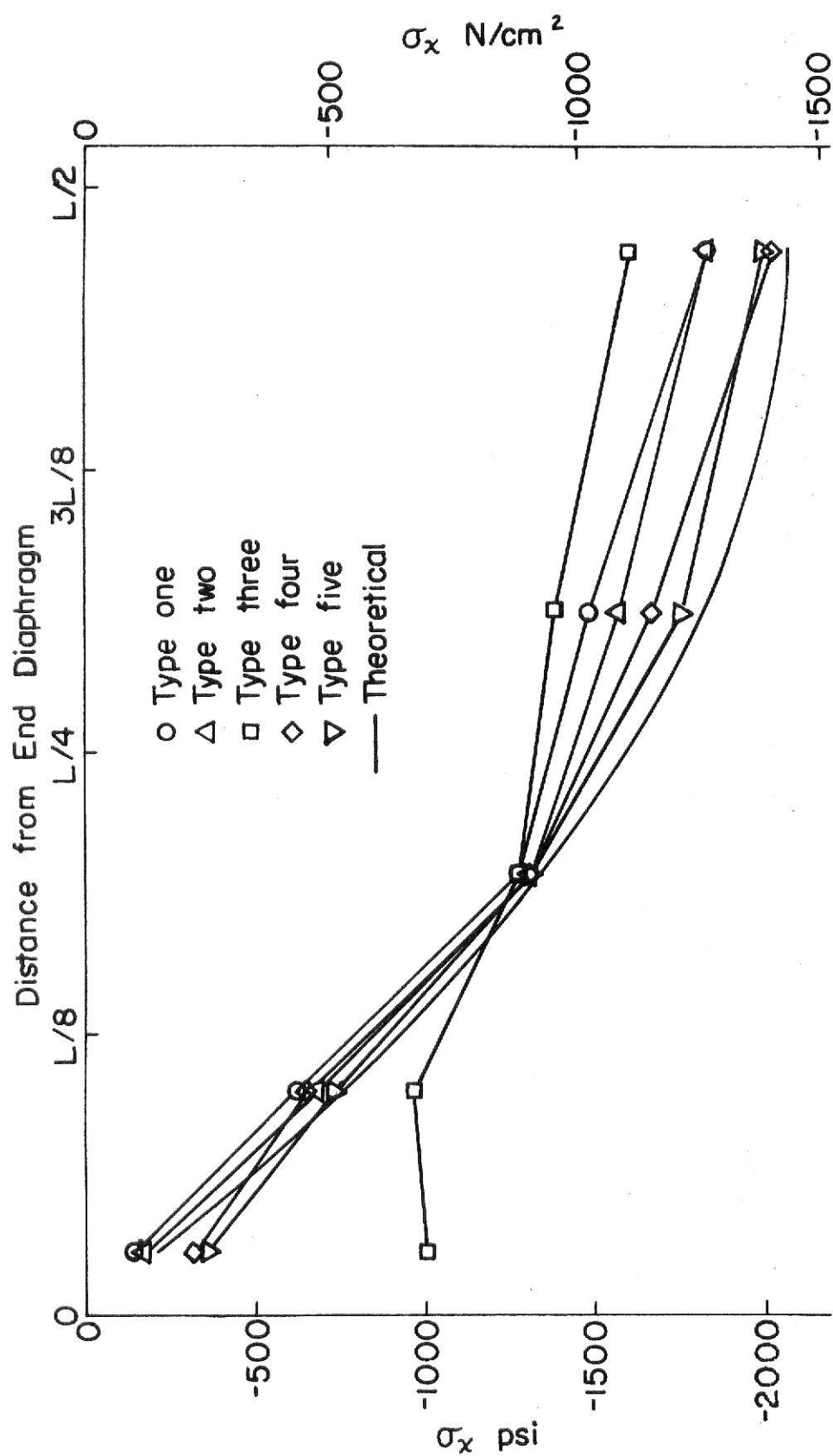


Figure 21 Longitudinal Stress Profile, Top Edge, 5-plate Model

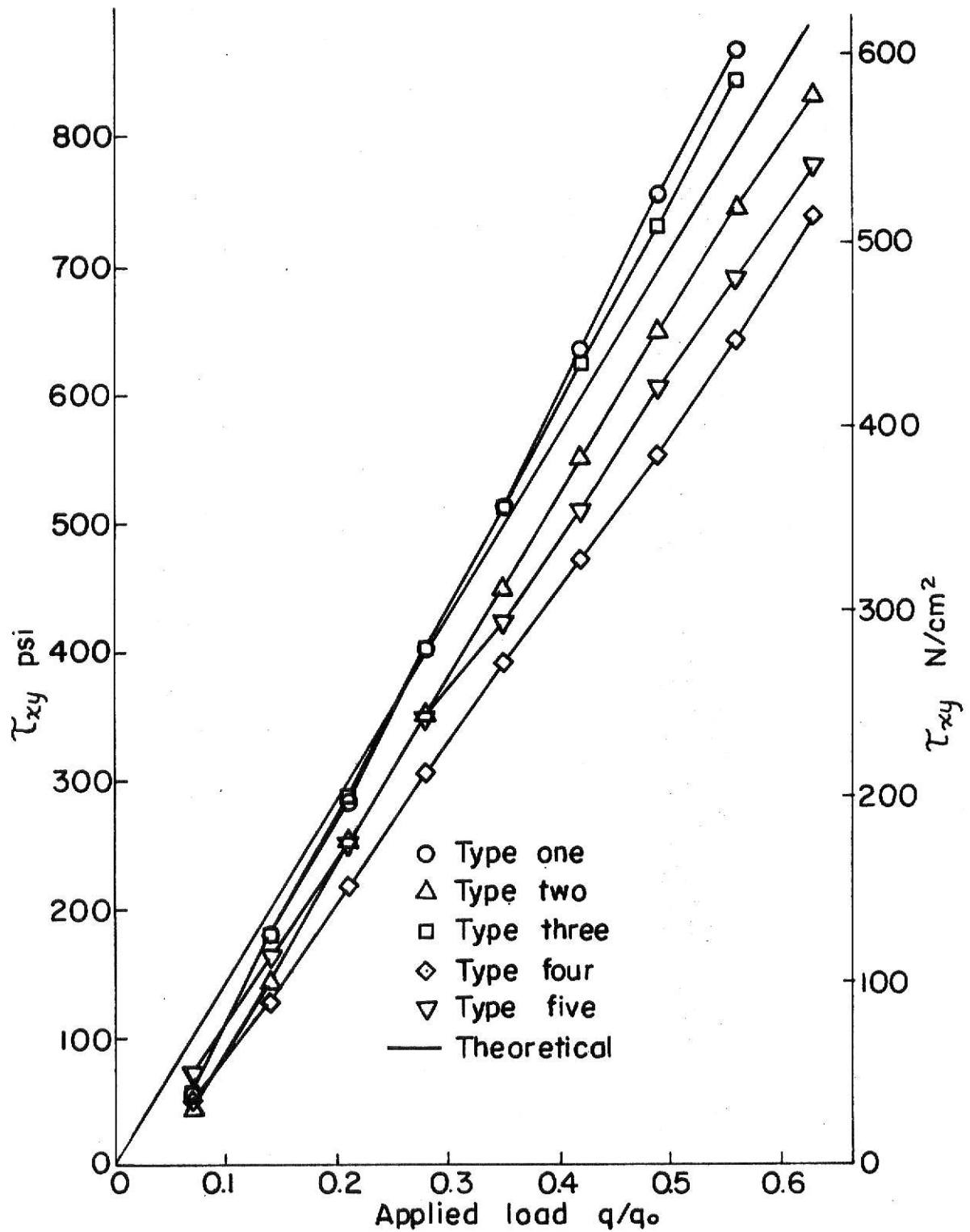


Figure 22 Shearing Membrane Stress, τ_{xy} , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

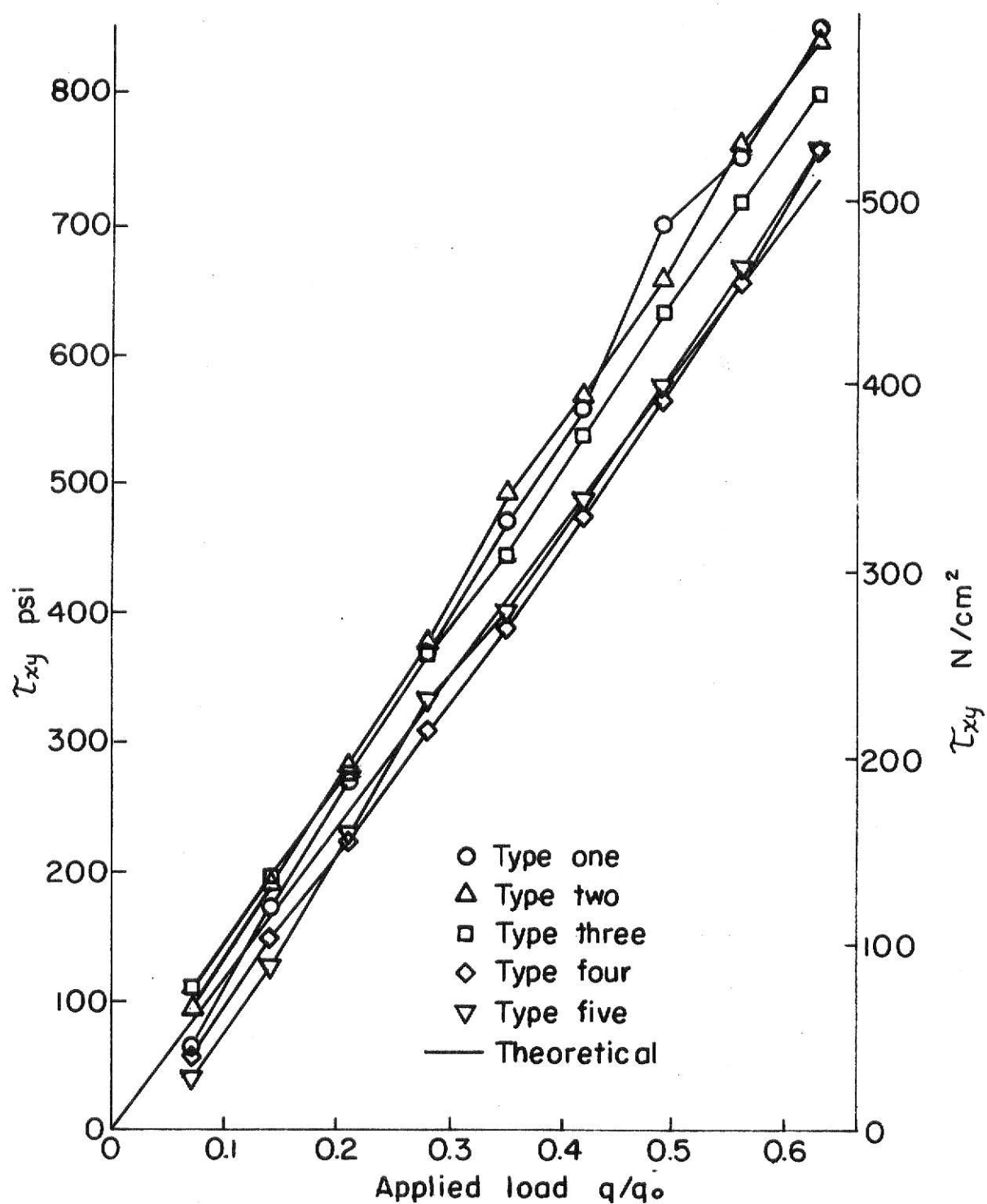


Figure 23 Shearing Membrane Stress, τ_{xy} , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

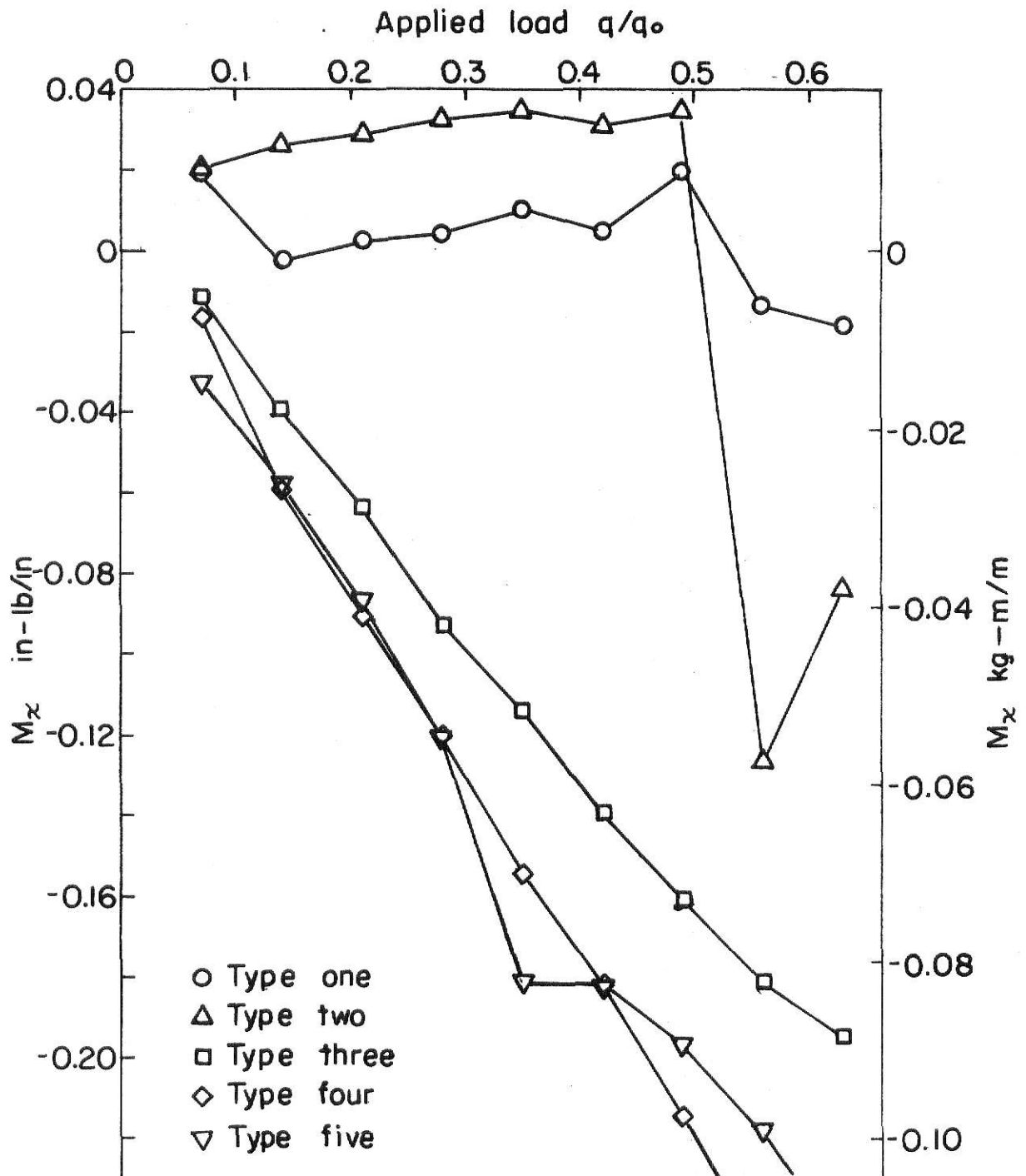


Figure 24 Longitudinal Moment, M_x , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

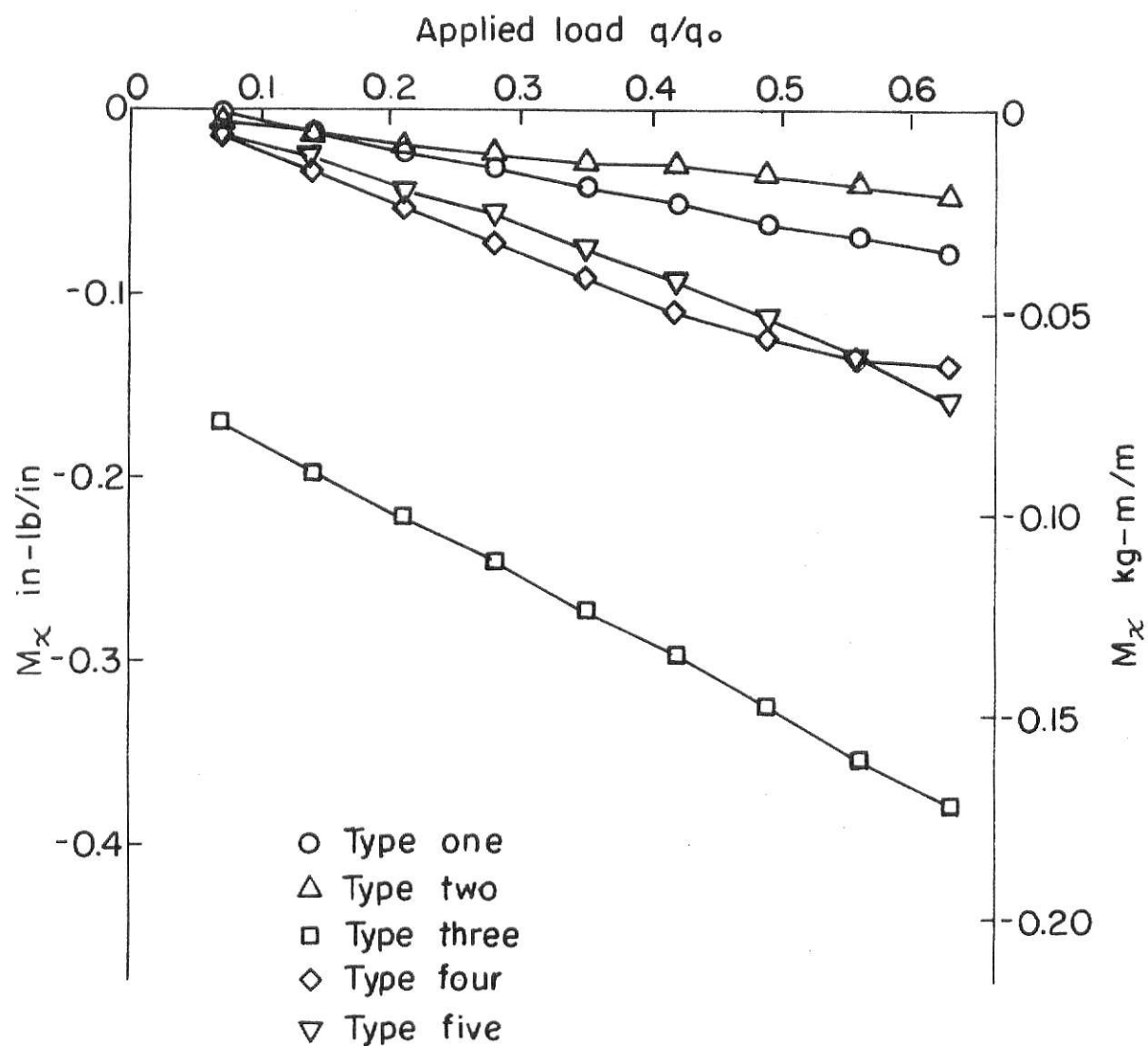


Figure 25 Longitudinal Moment, M_x , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

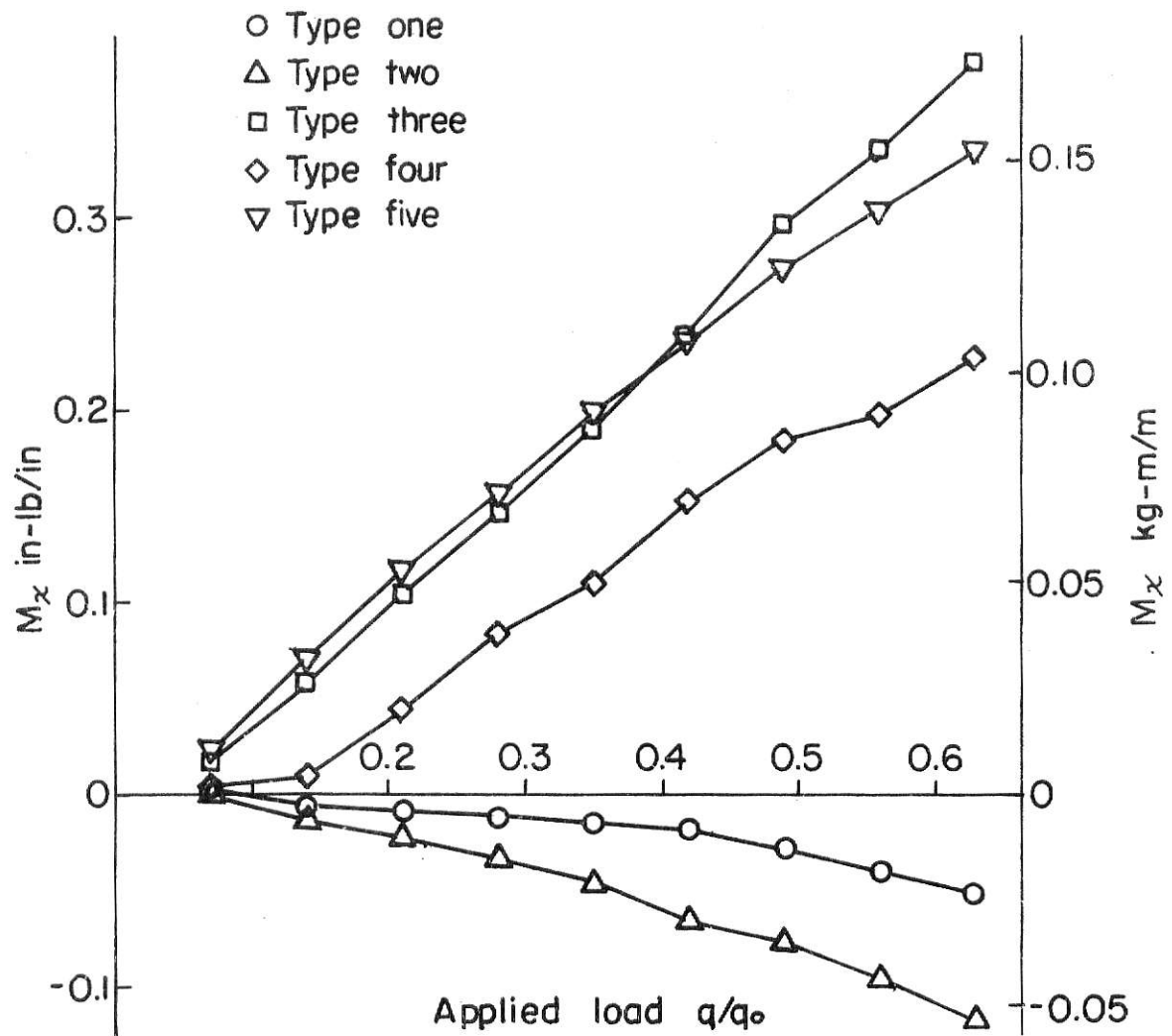


Figure 26 Longitudinal Moment, M_x , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

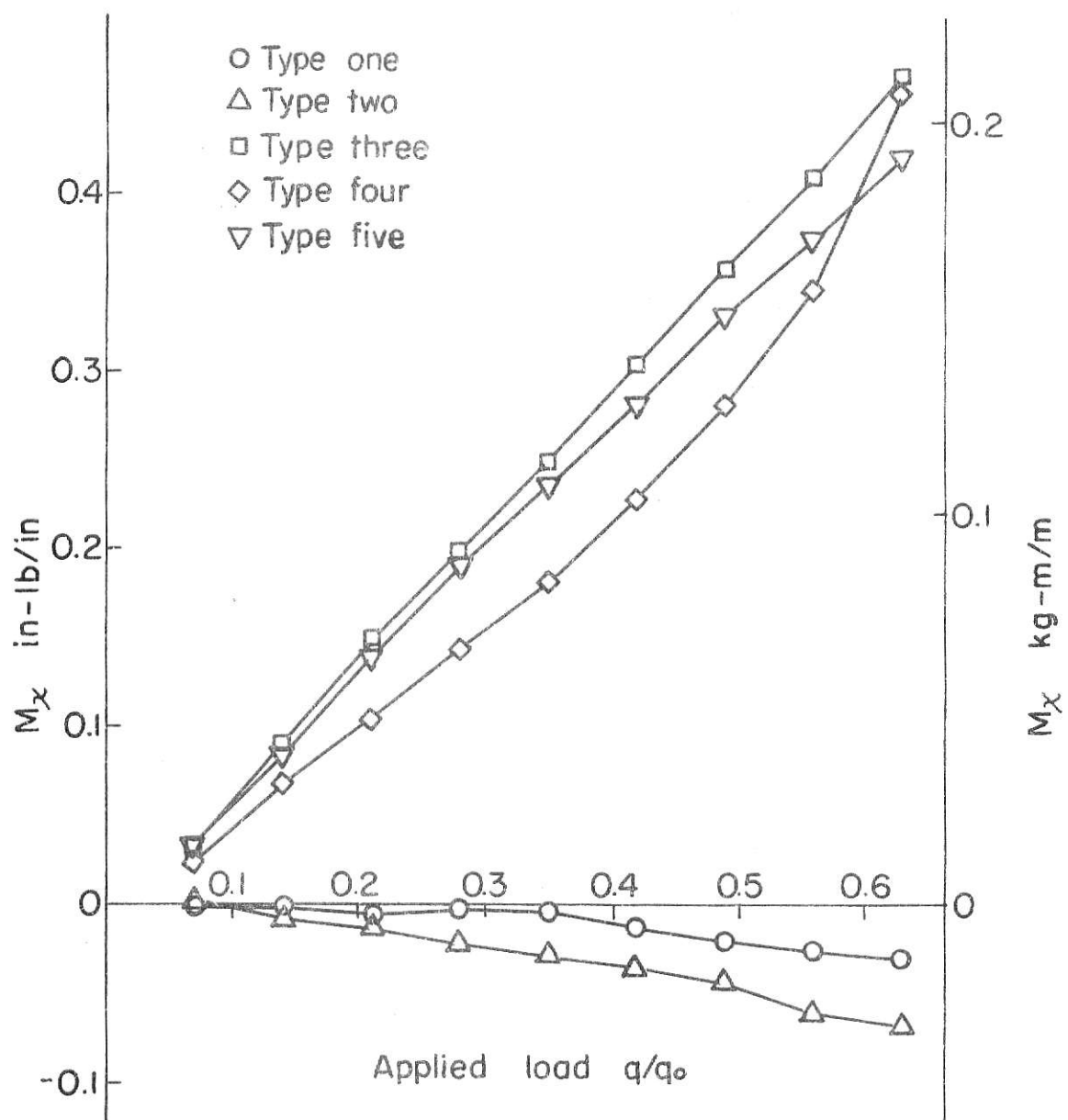


Figure 27 Longitudinal Moment, M_x , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

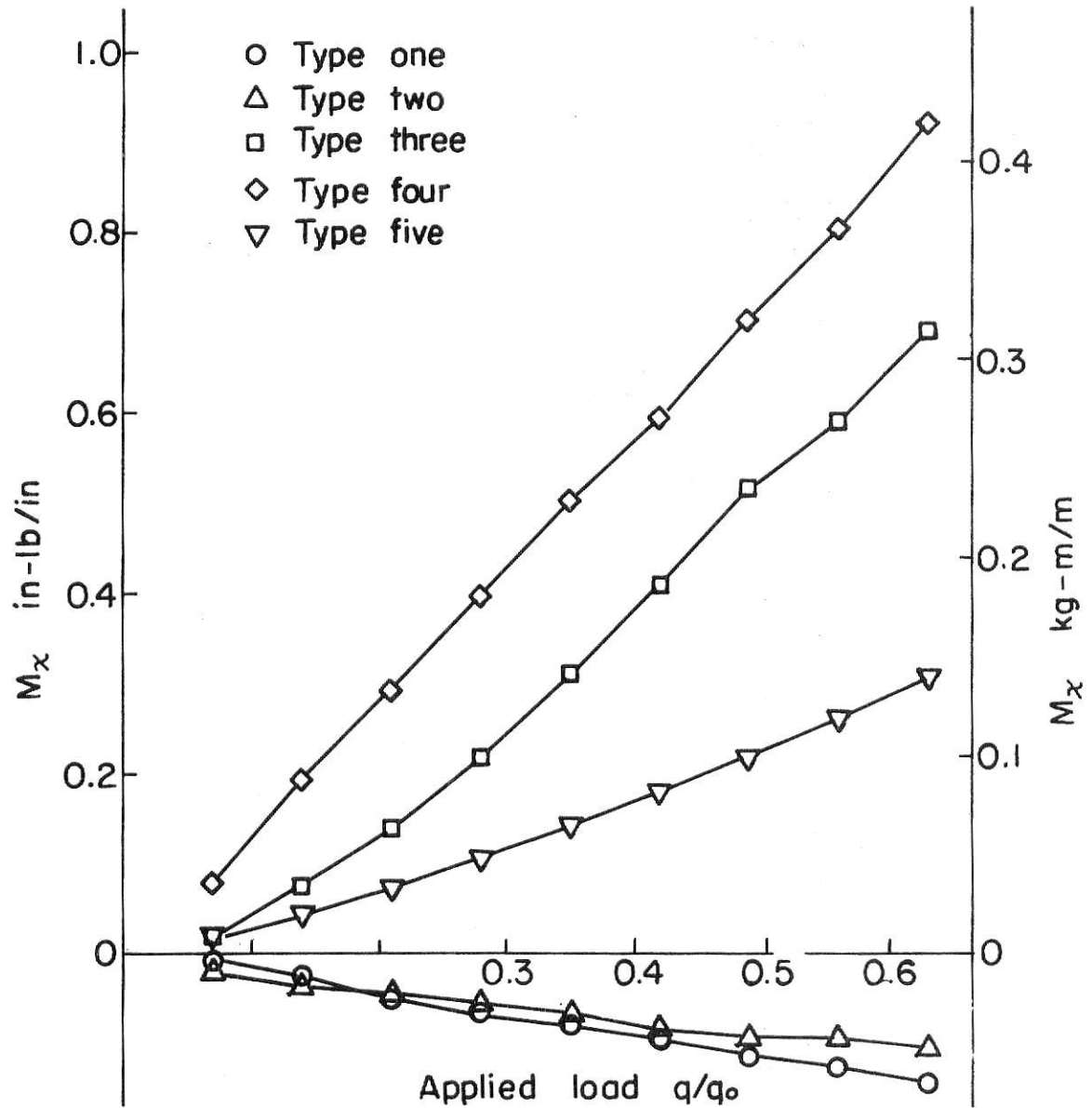


Figure 28 Longitudinal Moment, M_x , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

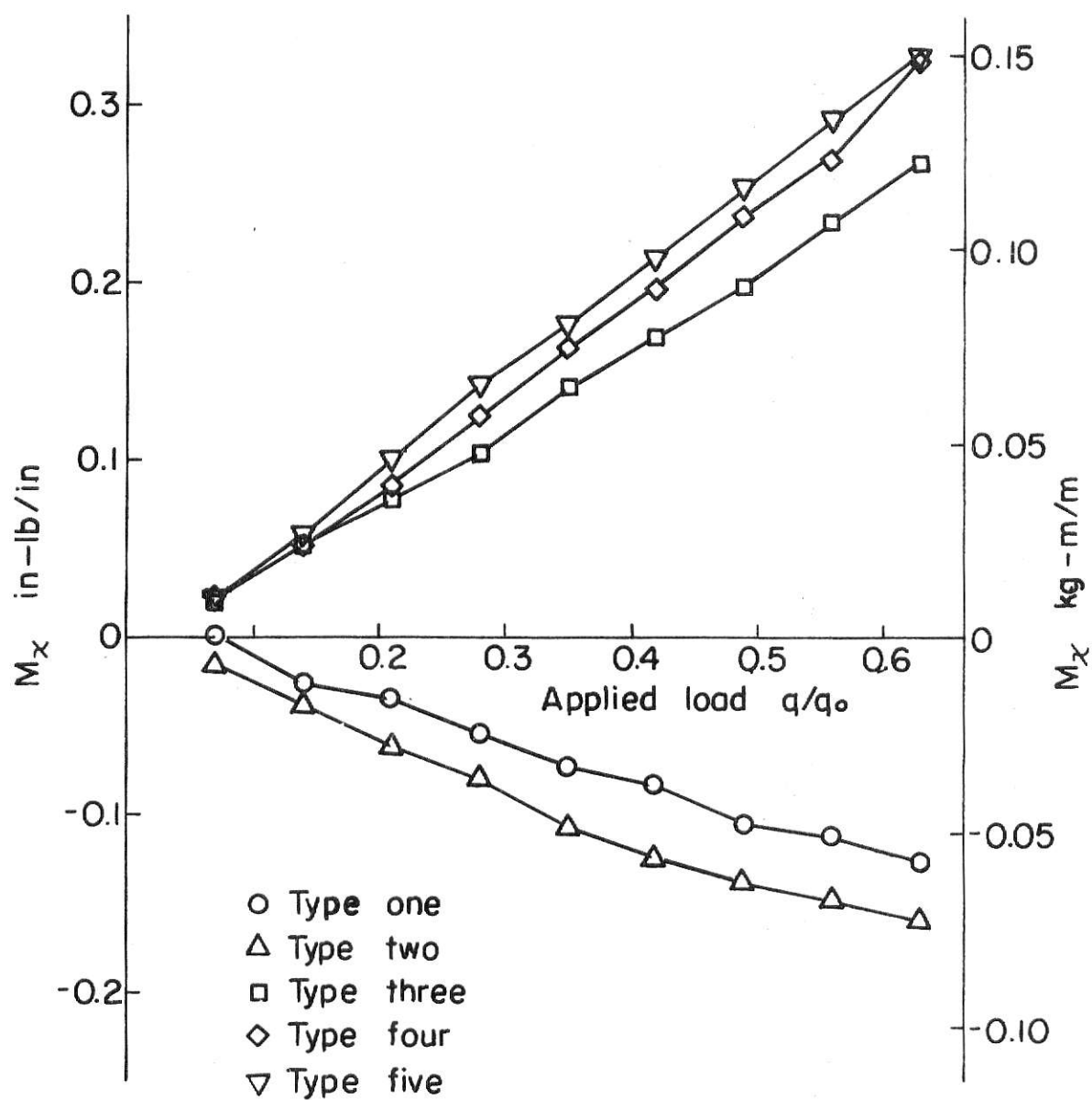


Figure 29 Longitudinal Moment, M_x , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

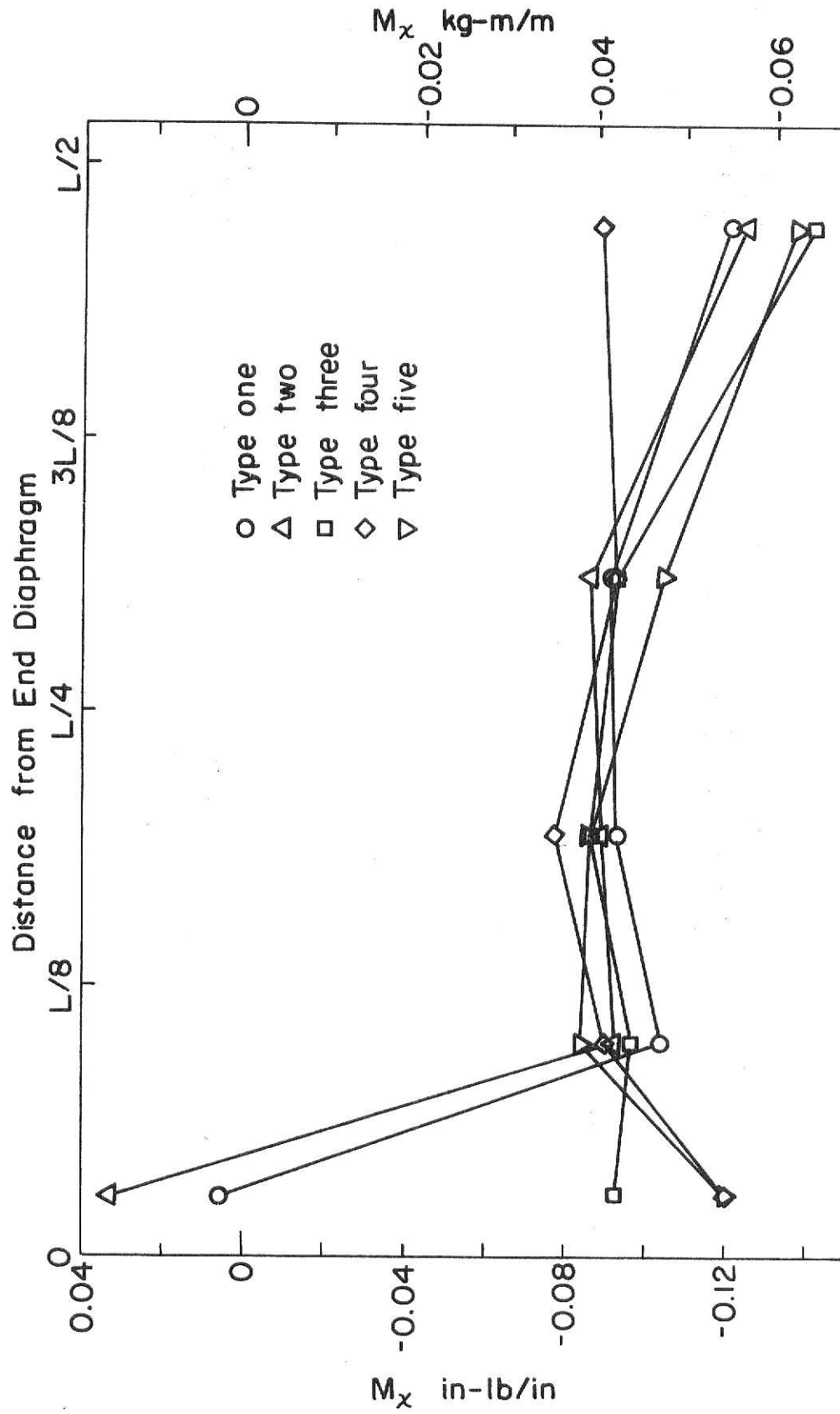


Figure 30 Longitudinal Moment Profile, Top Edge, 3-plate Model

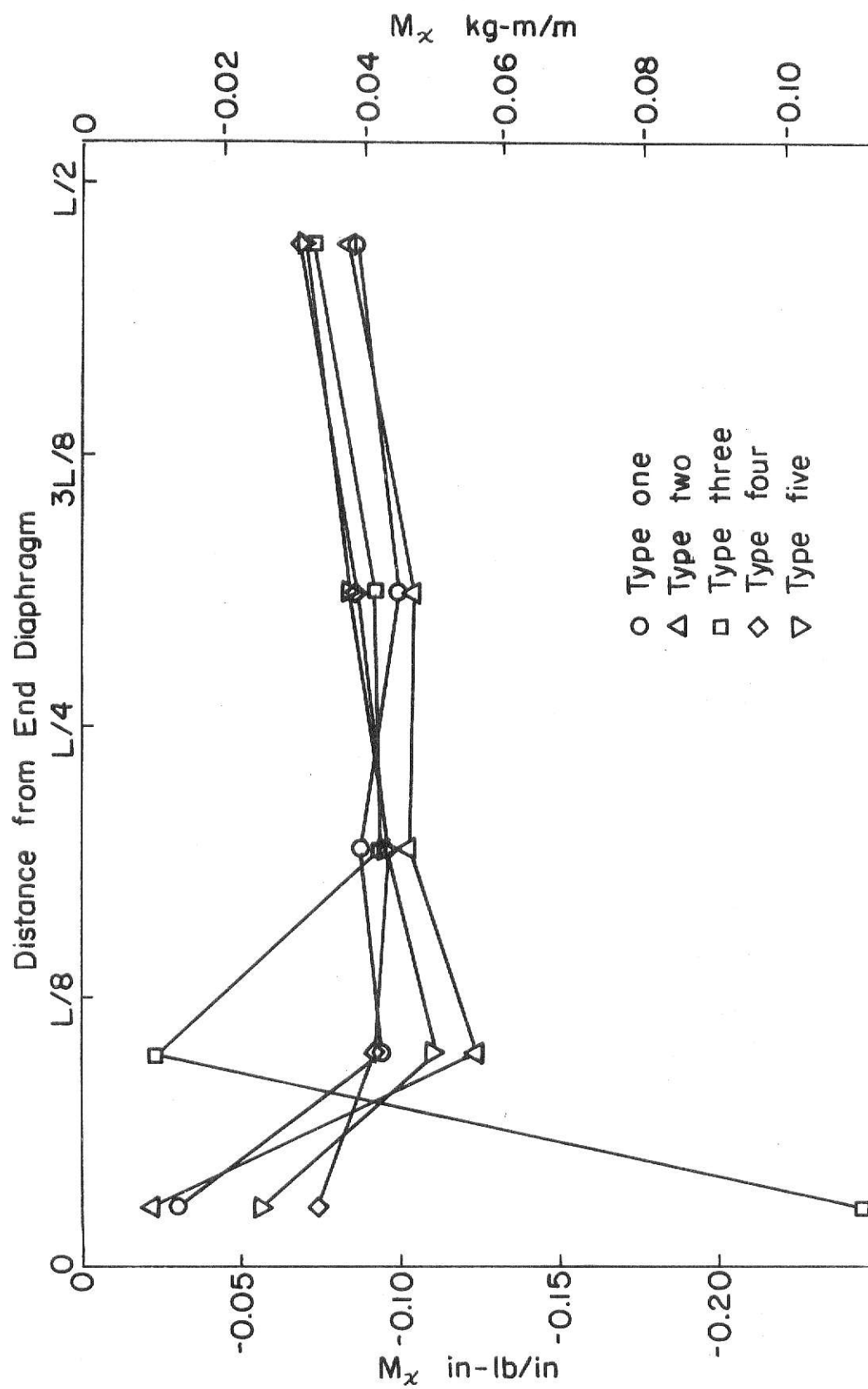


Figure 31 Longitudinal Moment Profile, Top Edge, 5-plate Model

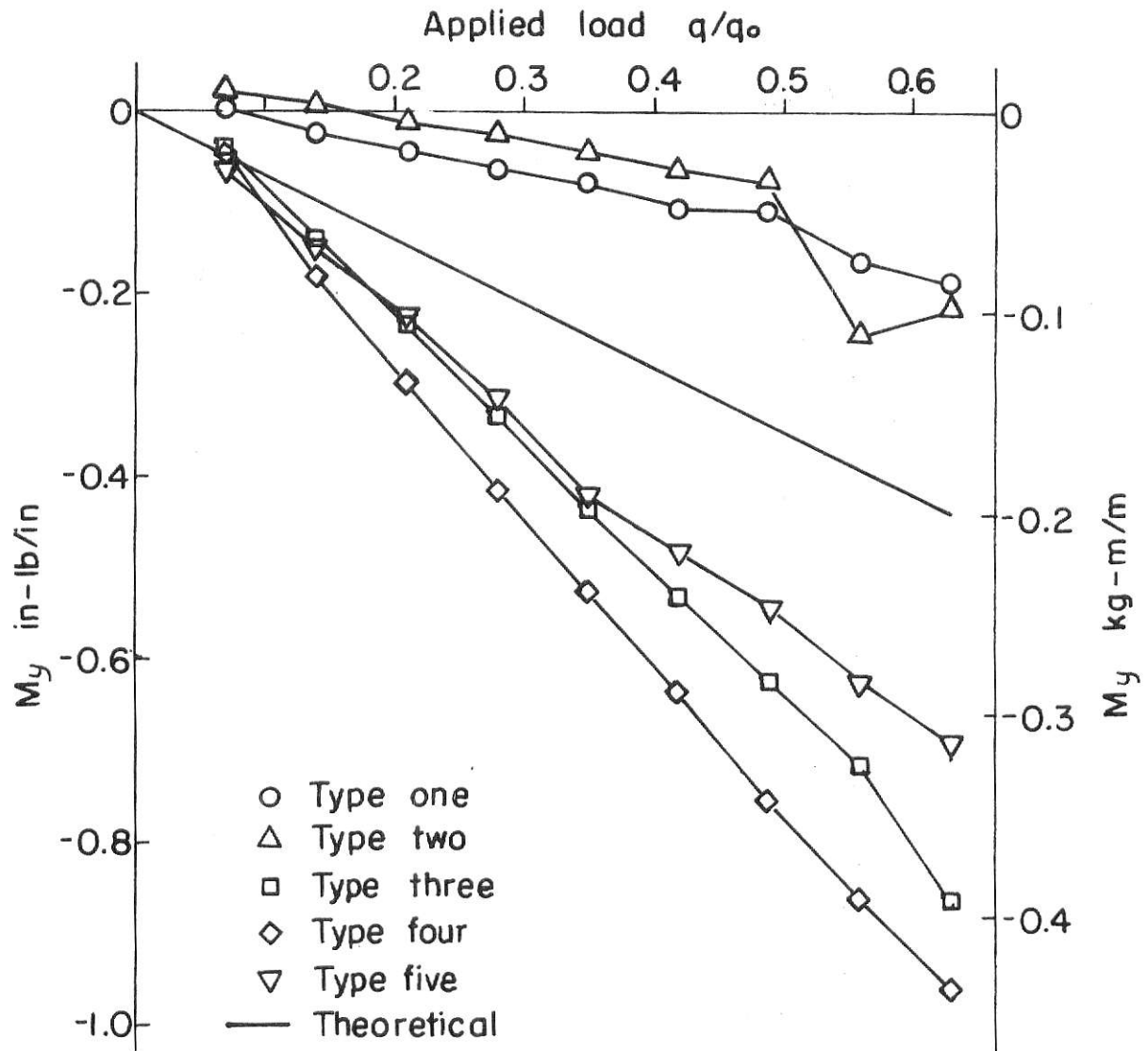


Figure 32 Transverse Moment, M_y , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

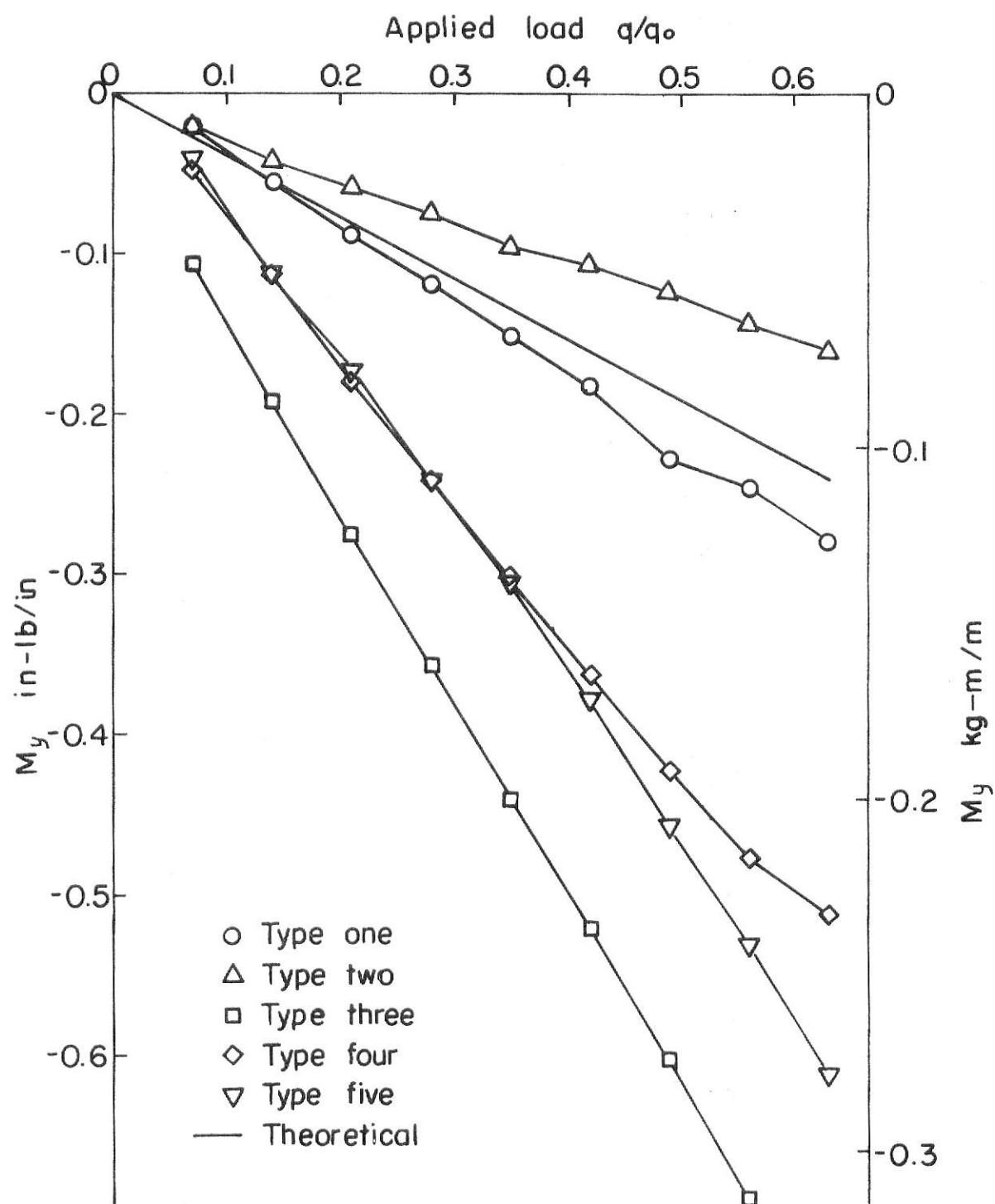


Figure 33 Transverse Moment, M_y , vs Load, Edge of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

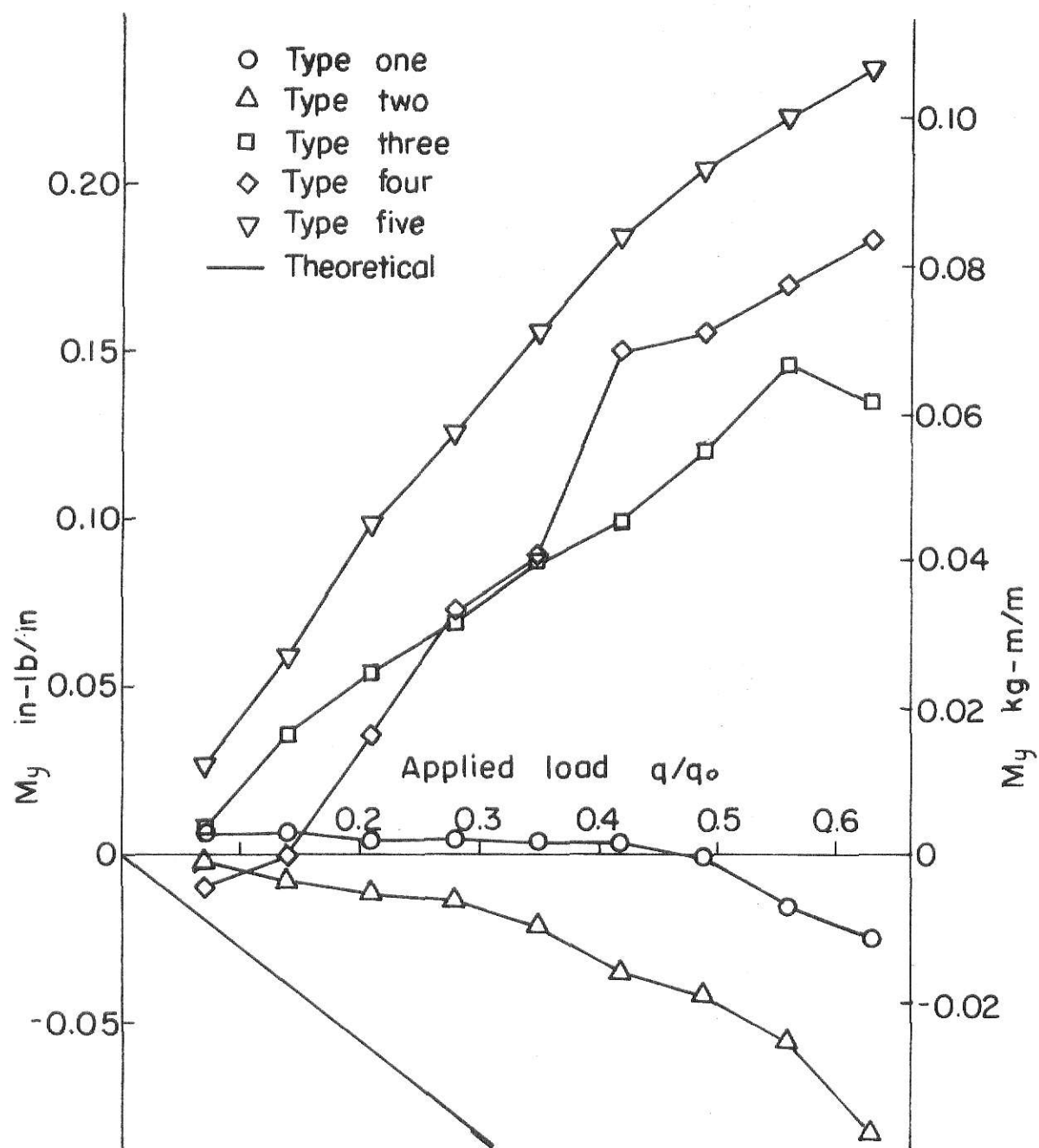


Figure 34 Transverse Moment, M_y , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

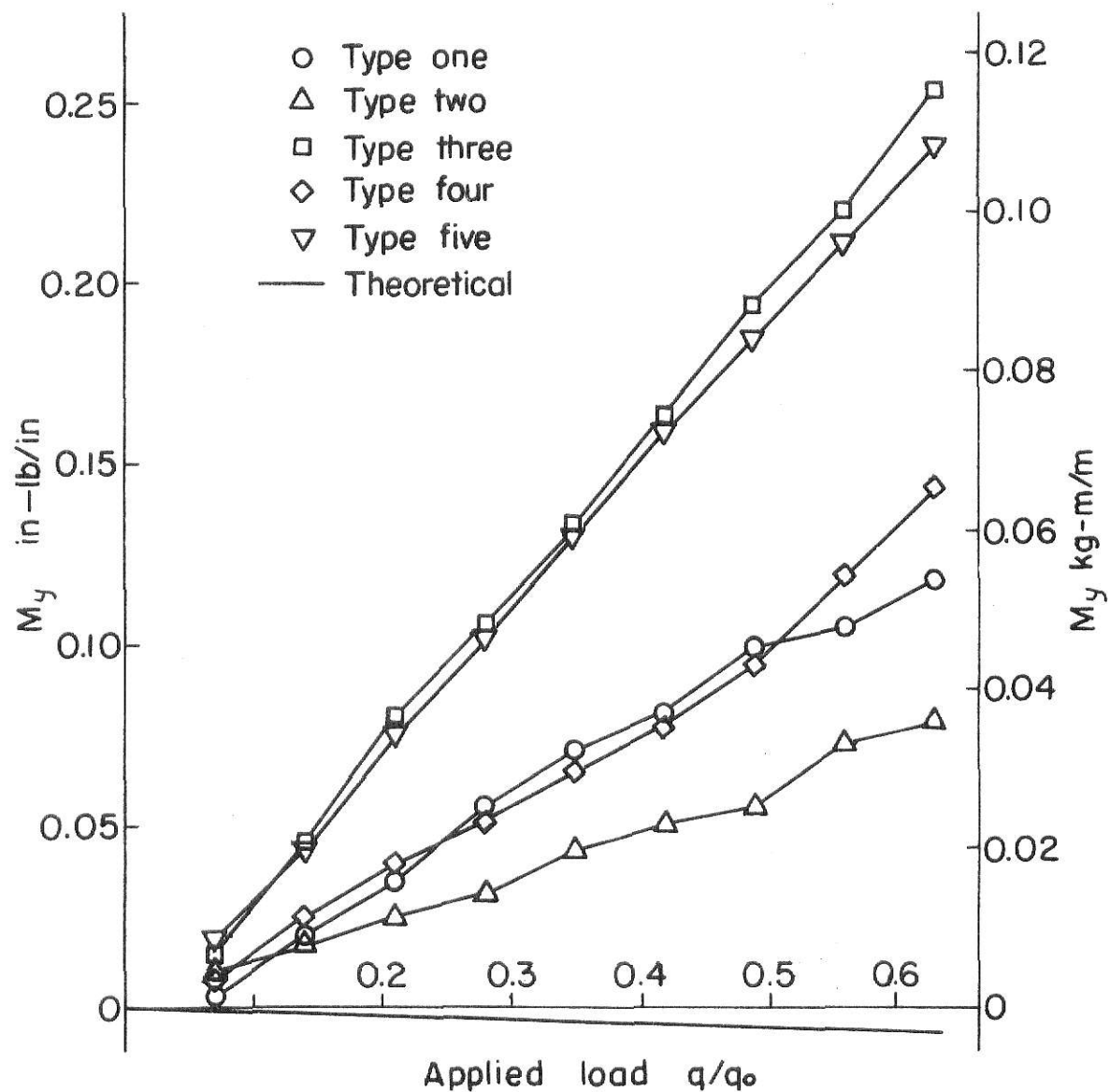


Figure 35 Transverse Moment, M_y , vs Load, Middle of Top Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

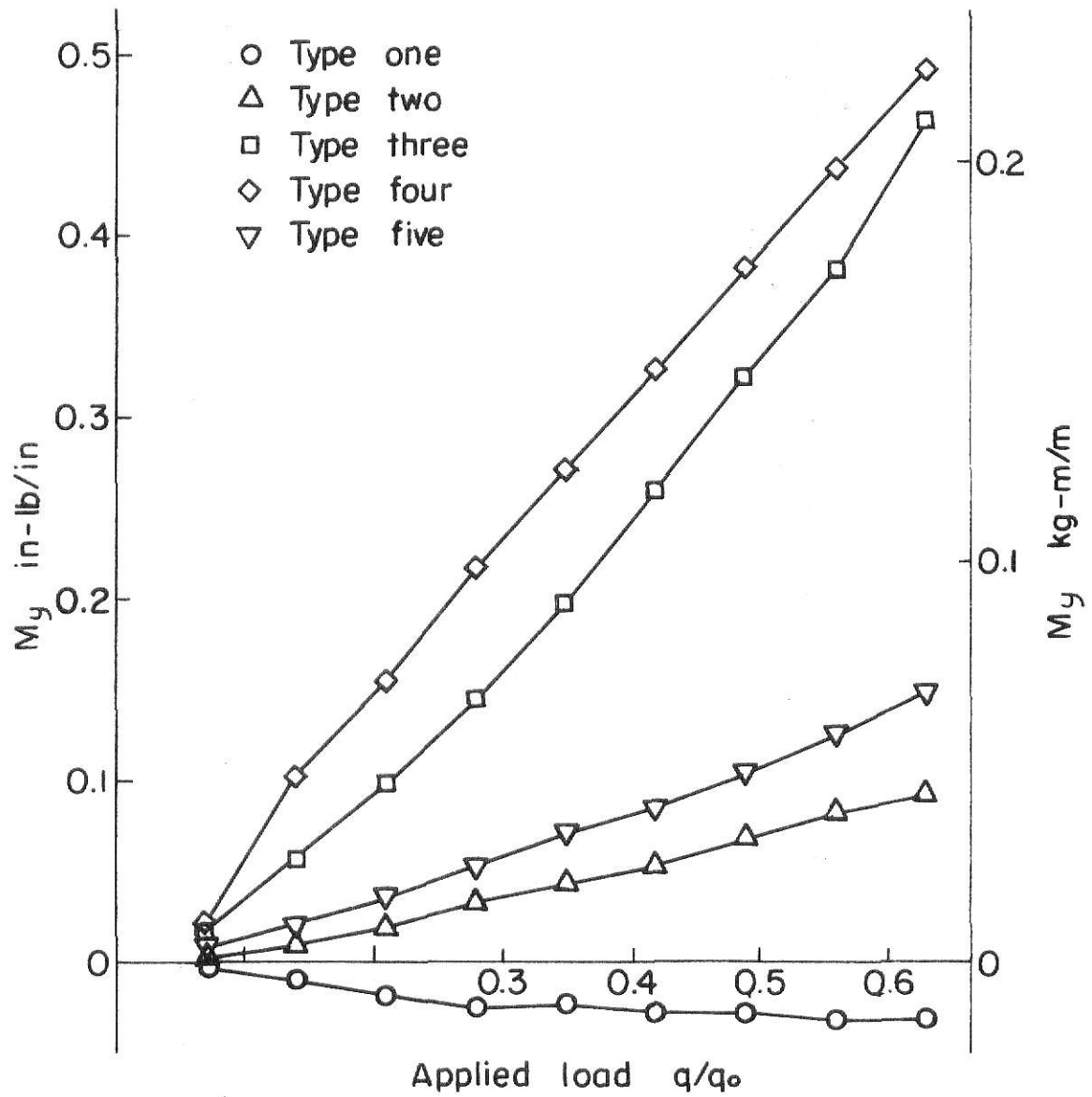


Figure 36 Transverse Moment, M_y , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 3-plate Model

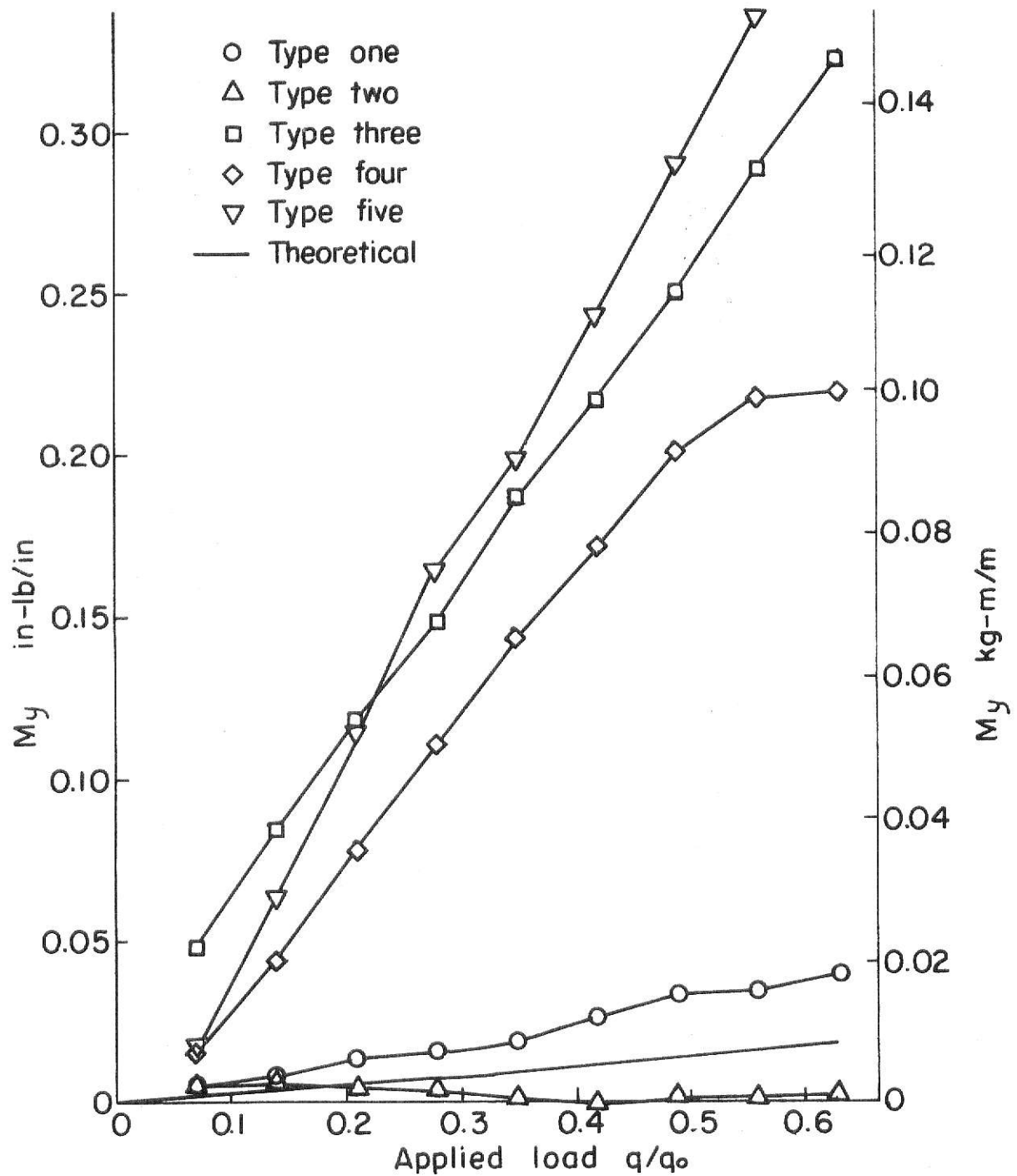


Figure 37 Transverse Moment, M_y , vs Load, Middle of Inclined Plate at $x=1.5$ in. (3.81 cm.) from Support, 5-plate Model

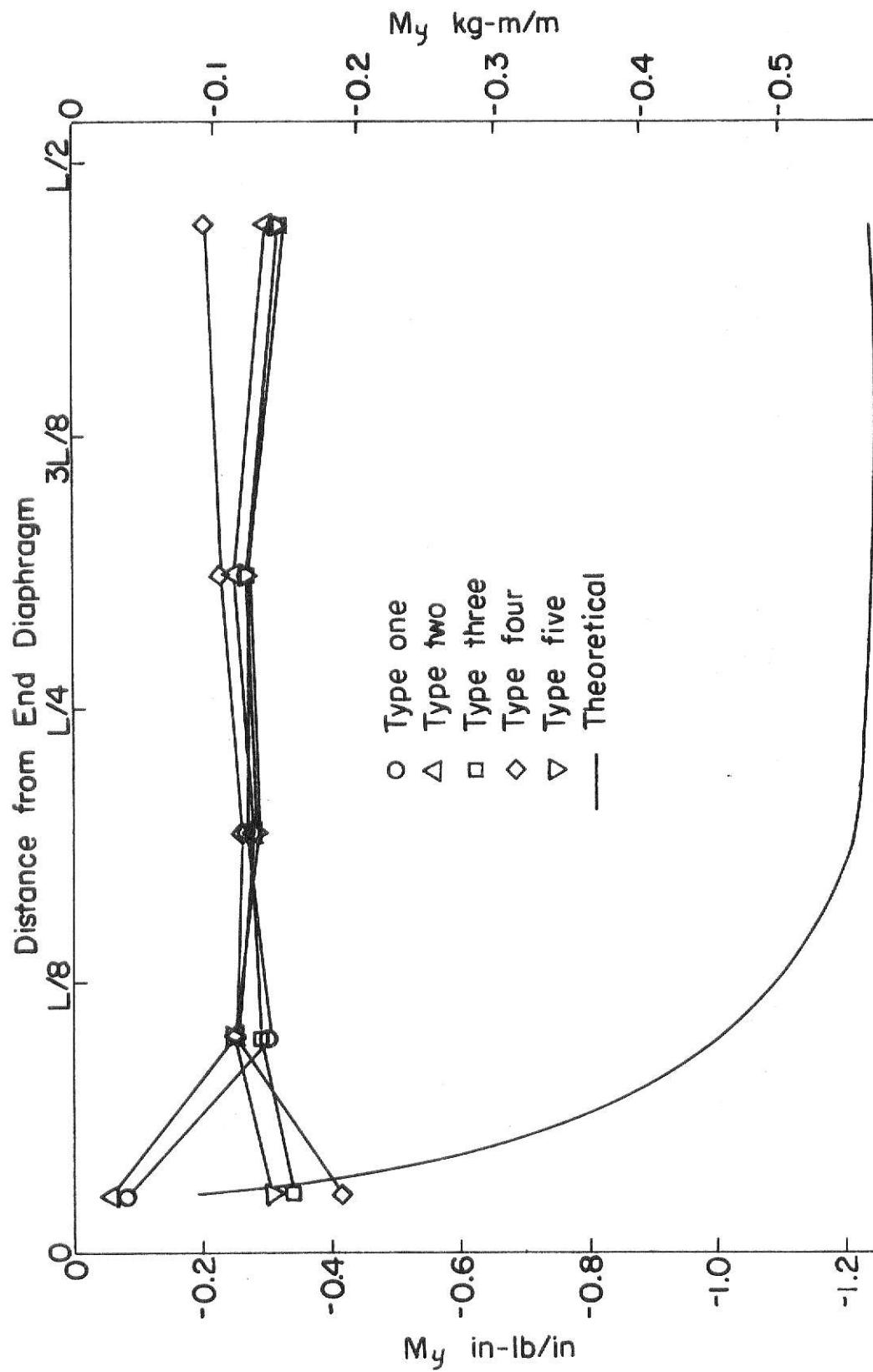


Figure 38 Transverse Moment Profile, Top Edge, 3-plate Model

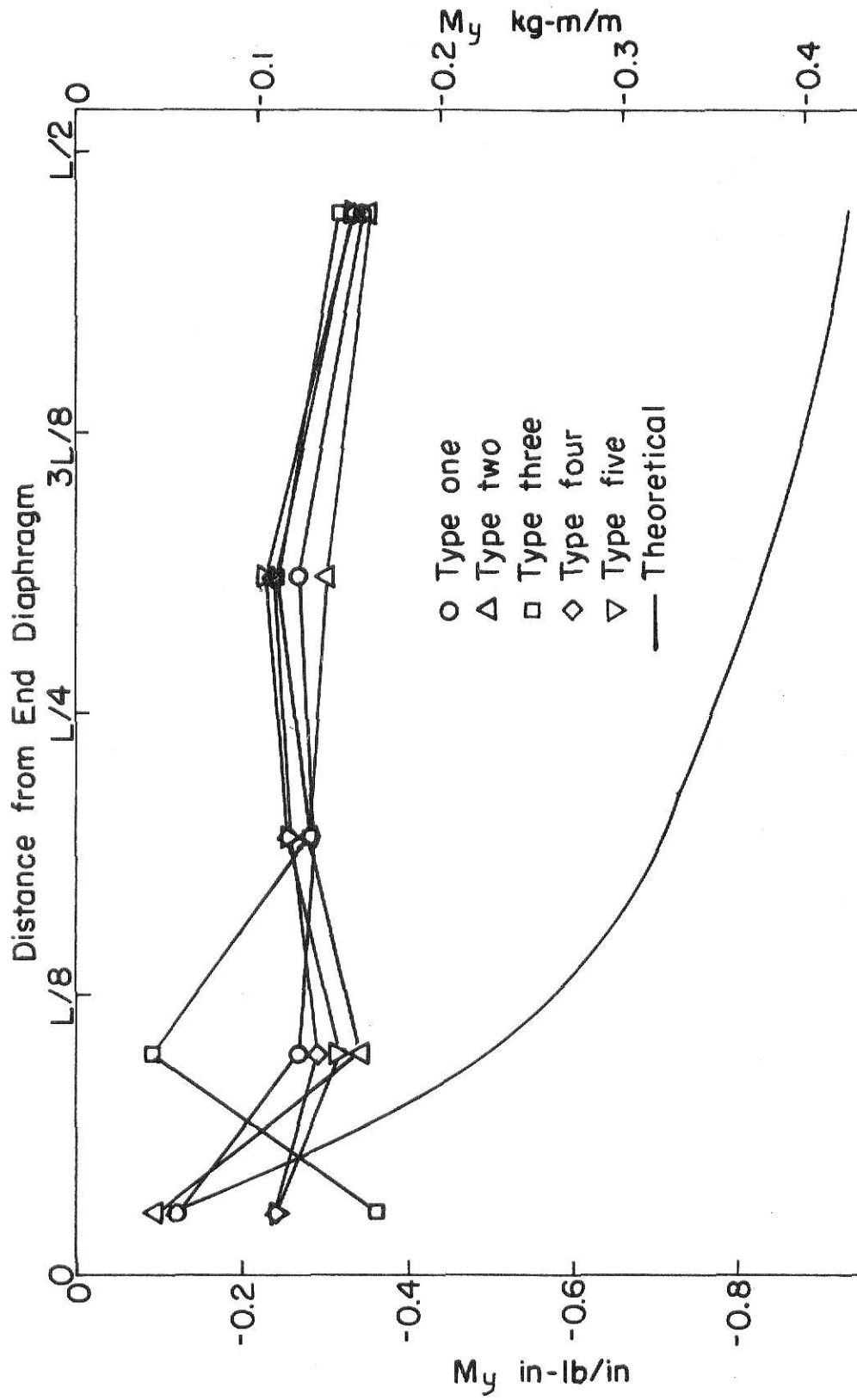


Figure 39 Transverse Moment Profile, Top Edge, 5-plate Model

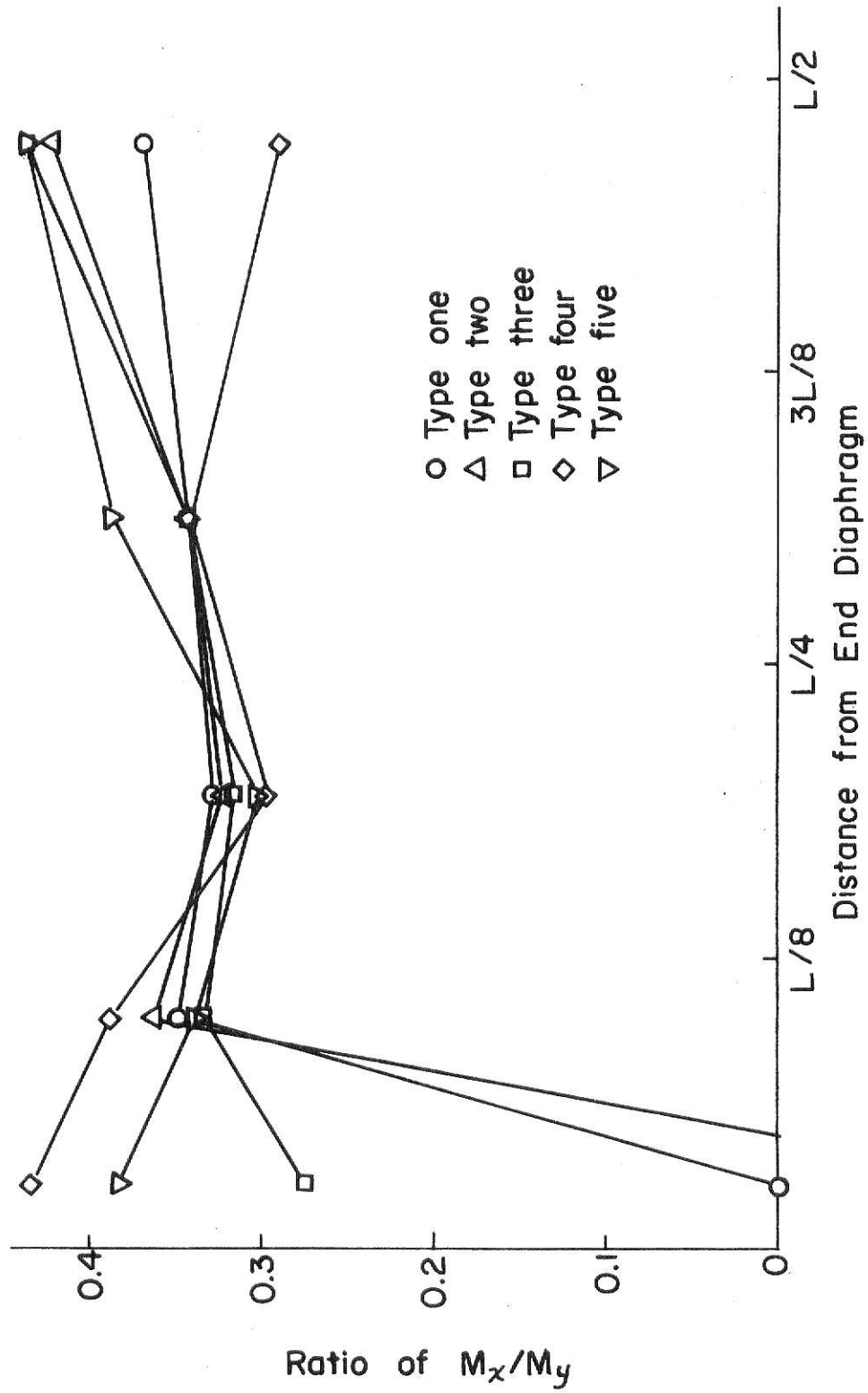


Figure 40 Ratios of Longitudinal to Transverse Moments along Top Edge, 3-plate Model

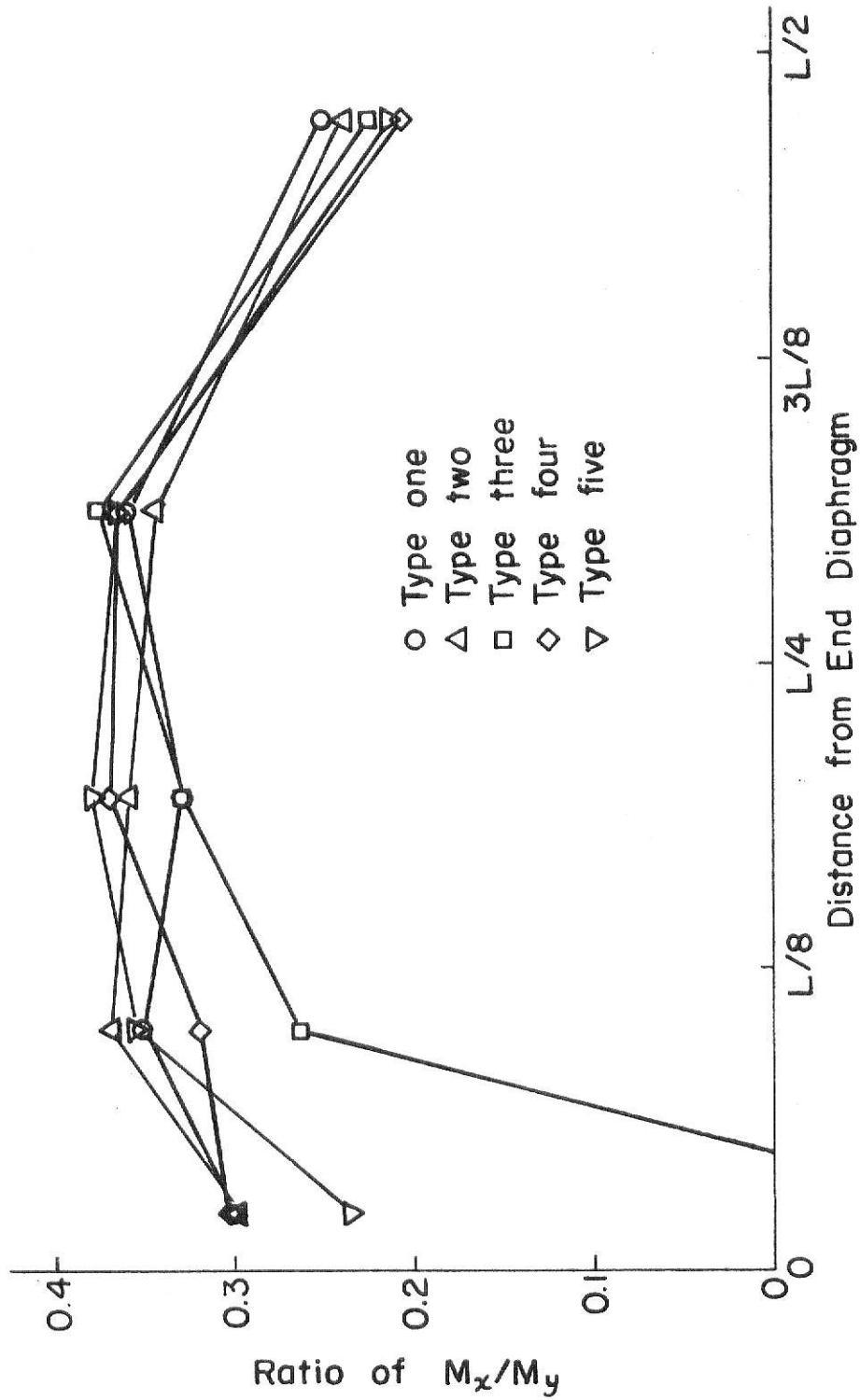


Figure 41 Ratios of Longitudinal to Transverse Moments along Top Edge, 5-plate Model

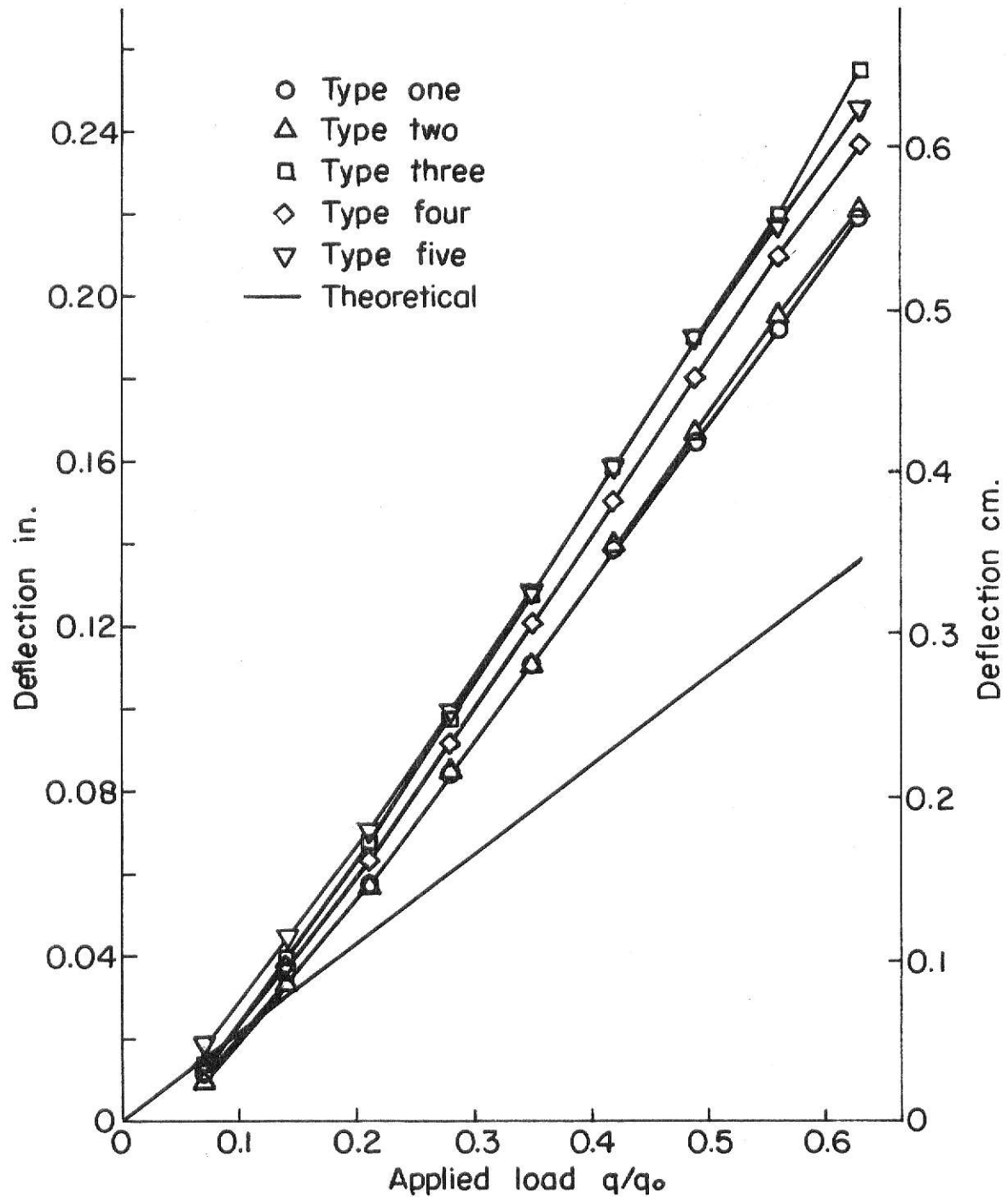


Figure 42 Vertical Deflection vs Load, Middle of Top Plate at Midspan, 3-plate Model

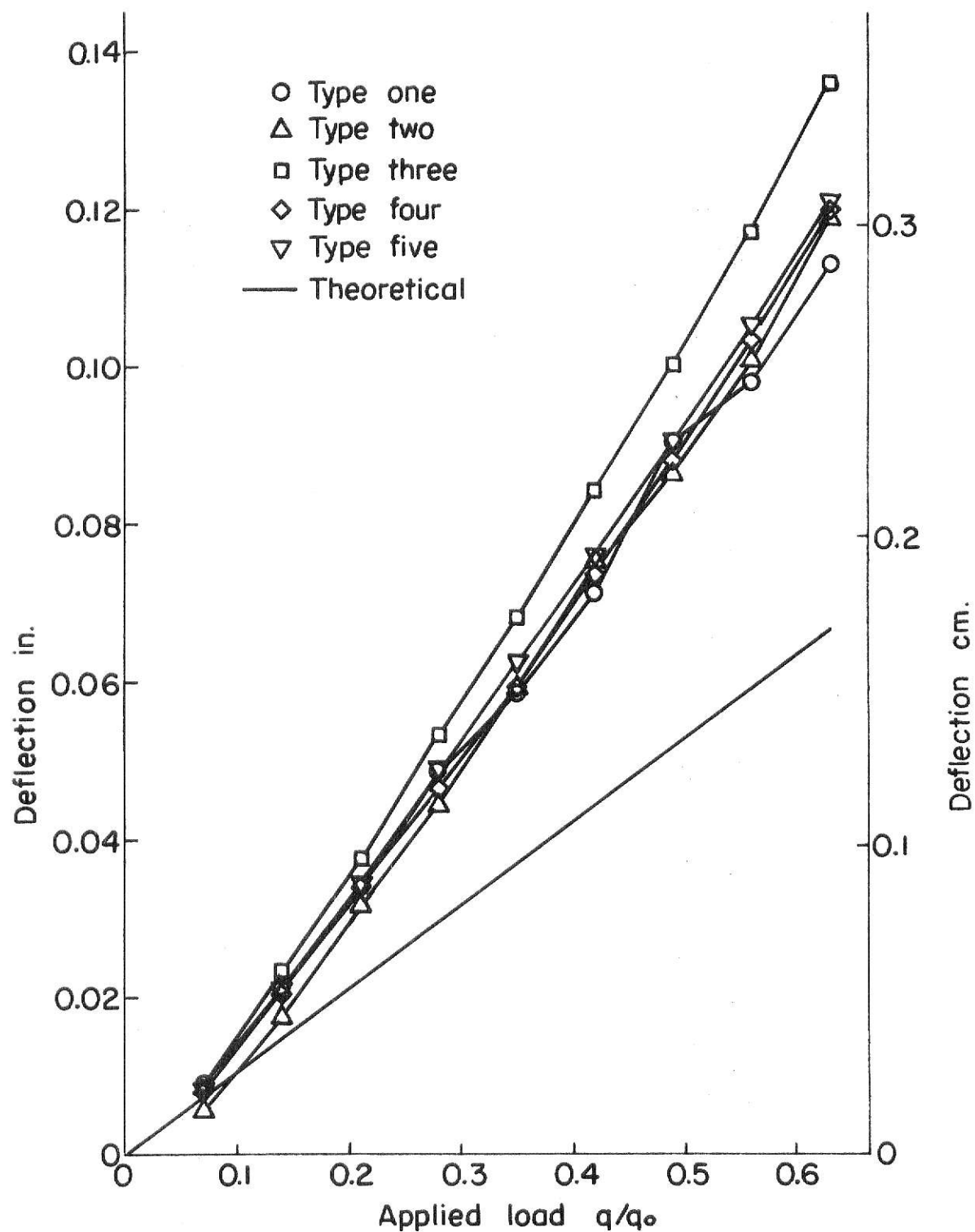


Figure 43 Vertical Deflection vs Load, Middle of Top Plate at Midspan, 5-plate Model

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EFFECT OF END SUPPORT DIAPHRAGMS
ON FOLDED PLATE MODELS

by

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B. S. Civil Engineering, 1970
Kansas State University

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering
KANSAS STATE UNIVERSITY
Manhattan, Kansas

1973

ABSTRACT

This thesis presents the results of an experimental study of the effects of five different types of end diaphragms on stresses and deformations on two single span, single bay, folded plate models. The attempt was made in designing the diaphragms either to simulate those used in practice or to satisfy theoretical support conditions of the diaphragms being perfectly flexible normal to their planes and perfectly rigid parallel to their planes.

The three-plate and five-plate models had a length of 54 in. (137 cm.) and were loaded through the use of a whiffle tree to produce a distributed load from 0 to 0.629 psi ($4.34 \times 10^3 \text{ N/m}^2$) in 0.0698 psi (482 N/m^2) increments.

Five different types of diaphragm were used. Two were rigidly connected to the models by bolts. The other three were connected by hinges in an attempt to produce flexibility at the supports.

Deflections were measured at midspan of the folded plate. Strains were measured across midspan, along the longitudinal edge of the top plate, and across the models at the supports.

The experimental results were compared with those calculated from accepted theory. The results indicated that end supports had little effect on stresses and deformations in the middle 0.8 length of the models but had great effect on stresses near the supports.

It was also found that the longitudinal moment may not be small enough to neglect in design, its experimental value being approximately 1/3 that of the transverse moment.