LEAST-SQUARES ESTIMATES OF GENETIC AND ENVIROMENTAL PARAMETERS IN A BEEF CATTLE POPULATION

by

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INTRODUCTION AND LITERATURE REVIEW

Over the years the animal breeder and the experimental statistician have been faced with establishing adjustments for identifiable environmental factors in order to better estimate the true production of a beef cow from the characteristics found in her offspring. The factors which influence the weaning weights of bovine offspring and which serve to measure the producing ability of beef cattle have been the primary interest of numerous studies, and the factors which contribute to these final weights have been well established. From these studies the factors which have been seen to be predominant in determining the final weaning weight of a calf are the age of the dam, the age of the calf at the time of weaning, and the sex of the calf. With each of these factors playing an important role in the production of a cow, a method of analysis was needed that would give unbiased estimates of the effect of one category while holding constant the effects of the remaining categories, e.g. to estimate, say, the effects of the different ages of the dams on the weaning weights of their calves while eliminating any possible effect that the sex or the age of the calves might have on the same weight. This concept is especially essential when dealing with data composed of unequal subclass numbers; and, when this type analysis has been accomplished, the results are said to be efficient and unbiased estimates of the population parameters.

The age of the dam when her calf is dropped influences the calf's final weaning weight in that the younger the cow, the lighter will be the calf she produces. This effect, according to Knapp *et al*. (1942) and Koch *et al*. (1955a), appears to follow a curvilinear pattern; for the calves appear to reach a maximum weaning weight when the dam is 6 years of age. After that age, however, the weights decline somewhat. Knox *et al.* (1945) found that the optimum age of a producing cow was 7 years of age, after which its ability declines. However, Rollins *et al.* (1954), Brown (1958), and Marlowe *et al.* (1958), could find no differences in the production of cows of 6 years of age and older. Koch *et al.* (1955a) also pointed out that the largest change in weaning weights occur between the first and the second calf that the cow produces, and Sawyer *et al.* (1948) found a difference of 75 pounds in the weaned calves of 2 year-old dams when compared to those of mature cows.

The age of the calf at the time of weaning is reported to have a curvilinear effect by Johnson *et al.* (1951), who presented a regression of weight of calf on its age of 1.54 when the age of the calf is 154 days or less and a regression of 0.84 when the age of the calf is from 155 to 225 days of age, Rollins *et al.* (1952) also illustrates this with a regression of 1.90 for calves under 4 months of age and a regression of 1.81 for calves from 4 to 8 months of age. Linear relationships between age and weight were reported by Koch (1951), with a regression of 2.27 at 176 days, Koger *et al.* (1954b), with a regression of 1.33 at 205 days, and Botkin *et al.* (1953), with a regression of 1.46 at 210 days of age.

The sex of the calf also influences its final weaning weight in that the male offspring has been found to weigh more at birth and again at weaning than the female counterpart. Gregory *et al.* (1950) showed a difference of 3 to 14 pounds in favor of bull calves at birth, and Rollins *et al.* (1954) found a difference of 68 pounds between bull and heifer calves at 240 days, with the bull calves being the heavier. Knapp *et al.* (1941) also substantiated this by showing in their work that there is a highly significant difference in the rate of gain during the suckling period in favor of the male calves which accounts for the difference in the weights at weaning. Koger et al. (1945a), Knapp et al. (1942), Koch et al. (1955a), Koch (1951), Marlowe et al. (1958), and Botkin et al. (1953) arrived at 32, 22, 26, 23, 30, and 25 pounds difference, respectively, between male and female calves at the time of weaning.

The question of how well this characteristic of weaning weights can be transmitted from generation to generation cannot be answered from any one set of data. Each set of data seems to produce a different estimate of repeatability or heritability, depending upon how the data were procured and what relationships were used in determining these estimates. For instance, with the use of maternal half-sib correlations on first and second calves, Koger et al. (1947) arrived at a measure of repeatability equal to 0.66; but when they used more than two calf crops with the same maternal half-sib correlations, this figure dropped to between 0.51 and 0.53. Botkin et al. (1953) obtained a repeatability of 0.43, Koch (1941), a figure of 0.52, and Koch et al. (1955d) computed a measure equal to 0.34. Each of these works were based upon maternal half-sib correlations.

The measures of the heritability of weaning weight are even more variable than were those of repeatability; for Brown (1958), using data from a Hereford herd and an Aberdeen Angus herd, calculated estimate of heritabilities equal to 0.26 and 0.11 for each respective herd using paternal intraclass correlations and 0.52 and 1.10 for each respective herd using maternal intraclass correlations. This difference could probably be explained in part by a large, nongenetic, maternal effect. Carter et al. (1959) obtained estimates of heritability equal to 0.08 for male calves and 0.69 for heifer calves using paternal half-sib correlations; while Gregory et al. (1950), with a similar correlation, arrived at estimates of 0.26 to 0.52. Knapp et al. (1950) computed an estimate of 0.28 using half-sib correlations, but one of only 0.07 when using a sire-offspring regression. Koch et al. (1955b), using paternal half-sib correlation, arrived at an estimate of 0.24; but in later work, Koch et al. (1955c) obtained estimates of 0.11 and 0.25 with the use of offspring on dam and offspring on sire regressions, respectively. Shelby et al. (1955) also produced an estimate in this region with the use of paternal intraclass correlation from which wa obtained an estimate of heritability of 0.23. Thus it can be seen that not only additional estimates of this genetic parameter are needed, but also a standard statistical approach which will lead to reliable estimates.

As usual, when analyzing data of this type, the experimenter will be faced with arriving at unbiased estimators from unequal and disproportionate subclass numbers. Brandt (1933) was one of the first to elaborate on this "2 x s" table. With the method of applying the least squares procedure, Yates (1934) projected this analysis into the general "p x q" table with both proportionate and disproportionate subclass numbers and brought to light some of the assumptions that had to be made in order that the analysis would be valid. Hazel (1946) extended Yates's work and presented an example of an analysis of covariance of multiple classifications with unequal subclass numbers. Henderson (1953) presented a publication which points out the difference in estimates in the variance components when the same set of data is used but under different assumptions, i.e. assumptions used in a random, fixed, or mixed model experiments. Landblom (1954) also presented in detail an analysis similar to that of Henderson's, using what is known as the absorption method of fitting constants.

With the use of high-speed digital computers becoming more prominent and allowing more versatility and ease in handling large quantities of data, Harvey (1960) capitalized on Henderson's work and made use of matrix algebra to fit constants to least squares equations, thus enabling him to compile tables of adjustments for this type data. In an attempt to dispense with the method of fitting constants by the more tedious least squares method and to give the individual herdsman an easier method of adjusting his herd's production, Searle (1960) put forth a simplified herd-level correction factor which used a simple multiplicative factor based on mean production weights on a within year and a within herd basis.

Most of the analysis up to the present day have been performed on data compiled from calves which were kept on their dams until weaning and which had very little to supplement their diet other than the dam's milk production. Creep-feeding, while not a new method of bringing calves to weaning age, has not been used to any great extent by commercial breeders. Thus, a question could arise as to how well the parameters which were estimated from noncreep-fed calf data would estimate those found from data compiled from creep-fed calves. Marlowe et al. (1958) have shown that creep-fed calves gain more rapidly than do noncreep-fed claves, and under these conditions the steer calves gain better than 8% faster than do the heifer calves from 150 to 240 days of age. This advantage of gain in creep-feeding was also reported by Powell (1935) and by Hammes et al. (1959). The purpose of this paper is to estimate and present the parameters which arise from data taken from creep-fed calves and to present the methods of analysis from which these estimates are obtained.

METHODS AND MATERIALS

Materials

During the years 1957, 1958, and 1959, records were kept on the weaning weights of creep-fed calves dropped within the commercial Aberdeen Angus cow herd of the Ramsey Ranch located in Butler County, Kansas. Each cow had been tagged with a permanent number; thus each calf that was dropped could be identified not only with it's date of birth, but also with its dam and her age. The ranch's movement in these years to the use of an artificial insemination program facilitated records being kept on the sire of the calf. In some cases, where the cows could not be settled artificially, the cow herd was split into small herds and only one bull was allowed with the cows in any one particular herd. This method of herd management allowed records to be kept on the bull which settled any open cows within that specific herd and thus sired the calf dropped by any of these particular cows. Routinely kept records could thus be used to determine the sex of the calf, the age and weight of the calf at weaning, the age of the calf's dam, the year in which it was born, and, in some cases, the calf's sire.

A total of 1,861 usable records were obtained over the three year period 1957, 1958, and 1959 with 598, 589, and 674 calves being dropped in each respective year. The range of the weaning ages was from 128 to 317 days, and the range of weaning weights was from 210 to 750 pounds. Also 458, 405, and 471 paternal half-sib records were obtained for the years 1957, 1958, and 1959, respectively, along with 771 maternal half-sib and 332 maternal full-sib records.

Methods

Under the assumption that each weaning weight within eacy year fits the linear model

$$Y_{ijk} = u + s_i + d_j + b(X_{ijk} - \overline{X}) + E_{ijk}$$

where u = the general mean

s_i = the effect of the i^{th} sex

d_j = the effect of a j year old dam

b = the regression of weight on age

X_{ijk} = the age of the calf at weaning

$\overline{X}$ = the average age of calves at weaning

E_{ijk} = the individual error

the following assumptions were made: (1) the sex of calf effect and the age of dam effect are fixed effects with mean zero and "variance" estimated by $\sum s_i^2/n - 1$ and $\sum d_j^2/n - 1$ which will be denoted from now on as σ_s^2 and σ_d^2. (2) the E_{ijk} 's are uncorrelated random variables with mean zero and a constant variance σ_e^2. (3) the estimates of the regression coefficients are homogeneous between subclass groups. (4) even though the production of dams which were older than 8 years of age were used in this analysis, it was concluded after inspecting the data that, because they were few in number and the average weaning weights of these calves did not differ appreciably from those of the 8 year old dams, these cows produced with equal ability and thus these calves could be combined into one category. With these assumptions, each year's uncorrected weaning weights were broken down in subclasses by year, by sex of calf, and by age of dam as shown in Table 1. Sums of squares and cross-products were then obtained between the age of the calf and its unadjusted weaning weight within each subclass. This allowed the use of the analysis of covariance as presented by Snedecor (1956) or Li (1957). The analysis of covariance was computed to ascertain whether or not the assumption of homogeniety among the regression coefficients of the various subclasses was realized and also if there was a significant linear regression effect of weight at weaning on the age of calf.

With these two assumptions met, the weaning weights within each subclass could be adjusted to the mean age at weaning by use of the pooled regression coefficient obtained from the analysis of covariance. This adjustment and the assumption that all cows 8 years of age and older produced with equal ability allowed the sum of adjusted weights within each subclass to be placed in a "2 x 7" table from which constants and the analysis of variance could be obtained for each year by the use of the procedure described by Goulden (1952). These resulting constants are presented in Table 5, and the analyses of variance are presented in Table 8. If the assumption is made that no interaction exists, then the method of fitting constants by the method of least squares provides efficient estimates and efficient tests of significance. If the interaction were significant, the method of fitting constants would no longer provide a test of significance; and the method of weighted squares of means presented by Yates (1934) should be used. Using the same weaning weights adjusted for the age of the calf, a similar set of constants was obtained by using the method presented by Searle (1960). These constants are presented in Table 6.

When the three years' data were combined into one analysis, another component Y_k, was added to the linear model shown above. This component added another subscript to Y_{ijk}, X_{ijk}, and E_{ijk} making these Y_{ijkl}, X_{ijkl}, and E_{ijkl}, respectively, so that the effect of y_k represented the effect of the k^{th} year on the individual weaning weight. With the addition of this component to the model, another assumption was set forth. This assumption was that y_k was a random variable with mean zero and a constant variance σ_y^2. A set of linear nonorthogonal equations displayed in Table 2 was formed from these data, and the appropriate constants and analysis of variance were obtained by the method described by Harvey (1960). The resulting constants are presented in Table 5, and the analysis of variance in Table 9. Searle's simplified method was also used in estimating the adjustments based on this set of data, but only after all weights were adjusted for the age of the calf at weaning. These adjustments are presented in Table 6.

Kempthorne (1952) presents a method of accounting for variation within a set of experimental data by reducing the total sum of squares by amounts accounted for by fitting the estimate of each parameter to the data. With these reductions, which are presented in Table 16, was also a reduction of total unadjusted sum of squares due to fitting the sire effect after fitting the regression, the sex effect, and the age of dam effect, i.e. adjusting all calves to an equal age, sex and dam basis. This reduction was based on a paternal intraclass correlation and the sum of squares of the total adjusted weaning weights.

The estimate of the variance components obtained from the analysis of variance presented in Table 12 were used to compute the paternal intraclass correlation coefficients as described by Fisher (1954) within each year using the paternal half-sib records. A weighted paternal intraclass correlation was also found when combining the three years' data. These results are presented in Table 12. The three years' maternal half-sib records were used to obtain a maternal half-sib intraclass correlation; but, due to the large sample of maternal full-sib records during the years 1958 and 1959, only these two years' records were used to determine the maternal full-sib intraclass correlation. These resulting correlation coefficients are presented in Table 15 and were used to estimate the maternal effect of the dam on the weaning weights and to estimate heritability using the approach presented by Kempthorne (1957).

RESULTS AND DISCUSSION

The analyses of variance showed that the age of calf, the sex of the calf, and the age of the dam producing the calf have highly significant effects upon the final weaning weight of the calf. These significant effects lend meaning to the constants fitted to this data by the method of least squares. These constants, along with their respective standard errors, were as follows: for the s_i effect, $i = 1, 2$ (where s_1 denotes the steer calf while s_2 denotes the heifer calf,

$$s_1 = +20 \ (\pm 1.09)$$
$$s_2 = -20 \ (\pm 1.19);$$

for the dam effects, d_j, where $j = 2, 3, \ldots, 8$

$$d_2 = -64 \ (\pm 3.96)$$
$$d_3 = -22 \ (\pm 3.17)$$
$$d_4 = -1 \ (\pm 2.67)$$
$$d_5 = +4 \ (\pm 2.66)$$
$$d_6 = +23 \ (\pm 2.68)$$
$$d_7 = +27 \ (\pm 3.04)$$
$$d_8 = +32 \ (\pm 2.15);$$

for the year effects, y_k, where $k = 1957, 1958, 1959$

$$y_7 = -9 \ (\pm 1.65)$$
$$y_8 = +6 \ (\pm 1.71)$$
$$y_9 = +3 \ (\pm 1.56);$$

with a regression coefficient of 1.415 (± 0.044).

From these constants, the linear factor obtained to adjust a heifer calf's weight to that of a steer calf was 40 pounds, the factors obtained to adjust the weight of a calf dropped by 2, 3, 4, 5, 6, and 7 year old dams to those dropped by dams of 8 years of age and older were the following additions: 96 pounds for a calf of a 2 year old dam, 54 pounds for a calf of a 3 year old dam, 33 pounds for a calf of a 4 year old dam, 28 pounds for a calf of a 5 year old dam, 9 pounds for the calf of a 6 year old dam, and 5 pounds for the calf of a 7 year old dam. To adjust for differences in age among the calves, the addition or subtraction of 1.415 pounds for each day of age for which the calf was under or over the mean age of 238 days was found to be necessary.

The adjustments found using the multiplicative factor presented by Searle, after the unadjusted weights were corrected for the age of the calf, were the following additive constants: for a heifer calf, 44 pounds; for a calf from a 2 year old dam, 107 pounds; a 3 year old dam, 59 pounds; a 4 year old dam, 37 pounds; a 5 year old dam, 31 pounds; a 6 year old dam, 12 pounds; and a 7 year old dam, 9 pounds. Obtaining adjustments for the year effects were deemed inadvisable for, unlike the sex of calf effect and the age of dam effect where one can base the adjustments upon the means of the heaviest sex and the highest producing age of dam, one cannot be certain as to which year's mean to base these adjustments. A computation of this type was carried through, but the inclusion of the yearly means only made the adjustments of sex of calf and age of dam deviate further from those obtained from the method of least squares. Thus, it appears, not only from the comparisons of the within year adjustments, but from the above fact as well, that Searle's simplified method is suitable primarily for the within year analysis. Also because this method uses only means with which to arrive at adjustments and not a measure of dispersion, i.e. deviations about a mean, standard errors of the means cannot be obtained.

If one considers that the weight of the calf was taken only to the nearest 5 pounds, the adjustments obtained under these two different computational procedures deviate from one another by little more than the accuracy of the original data. This fact is pointed out even more clearly in Table 6 where the results of each year's adjustments were obtained and where it can be noted that the difference between the adjustments seldom exceed the allowance made when measuring the calf's weaning weight. In order to determine a measure of the relative efficiency that could be obtained by the use of Searle's method as compared to the method of least squares, the subclass sums were corrected with both sets of adjustments and a ratio formed from the resulting subclass sums of squares. These ratios produced percentages of 99.3, 97.9, and 99.1 for 1957, 1958, and 1959, respectively. Thus, it would appear that Searle's method would be beneficial to the individual herdsman in determining the production of his own herd on a within year basis, while the method of least squares would be more beneficial when trying to tabulate constants on an among year basis. The latter is especially true, for it has been noted previously that the standard errors of means can only be derived using the method of least squares.

As can be seen from Table 6, the linear adjustments needed when handling creep-fed calves are greater than those already set forth from previous experimentation using data collected from noncreep-fed calves. Powell (1935), Marlowe et al. (1958), and Hammes et al. (1959) have reported that creep-fed calves gain more rapidly than do the noncreep-fed calves. Marlowe further pointed out that the steer calves gain better than 8% faster than do the heifer calves. Although the results of the analysis of covariance, both within years and among years, led to the assumption that the regressions were homogeneous in nature, there is a noticable variability among these regressions. Moreover, two-thirds of the subclasses showed that the regression coefficient for the steer calf was greater than those for the female counterpart, and within each year the regression coefficient for steers was larger than the one for heifers. A study by Knapp et al. (1941) resulted in the conclusion that there exists a highly significant difference in rate of gain between sexes during the suckling period. This tendency for the steer calves to gain faster than the heifer calves might well result in obtaining this large difference between the two groups. The large difference between the 2 year old dam's production and that of the older dams might be attributed to a maternal effect, or it could also be suggested that the larger calves dropped by the older cows have a better start than do the lighter ones and thus can depend less upon their dam's milk production for their rate of gain. The last conjecture becomes more feasible; for, as one will note from the half-sib intraclass correlations in Tables 12 and 15, the nonheritable maternal effect existed but was found to be non-significant. The resulting adjustments needed for correcting for the age of the dam appear to uphold the results of experimentation accomplished by Rollins et al. (1954), Brown (1958), and Marlowe et al. (1958) in that the 9 and 5 pound adjustment needed for the 6 and 7 year old dams does not seem to be appreciatable when considering the accuracy with which the weaning weights were taken. Thus it could be said that there is little difference in the production of cows of 6 years of age and older. Also, as with the results of Koch et al. (1955a), the largest change in weaning weights occur between the first and second calf that the cow produced. The resulting regression coefficient, though smaller than most of those cited in the literature, might well be explained by pointing out that the average age of the weaning calves in this study was somewhat greater than those of the cited studies. Thus, if the daily rate of gain does follow a curvilinear pattern, one might expect that at a later weaning age the coefficient would be smaller or that the slope of the linear regression line would be less.

The analysis of variance for each of the years 1957, 1958, and 1959, as presented in Table 8, showed that the regression of weight on age, the sex of calf, and the age of the dam each had highly significant effects on the weaning weight of the calf. Due to the assumption that no interaction term existed in the linear model which had previously been set forth, and due to the inability of explaining such an interaction effect if it existed, each subclass mean was compared to the appropriate expected mean using the linear model and the constants that had been found from the analysis. None of these observed means were found to differ significantly from its corresponding expected mean when tested by the use of the t-statistic. Finding no significant deviations between the observed and expected means, plus the known fact that the F-test performed in the analyses of variance of non-orthogonal data are not exact tests, lead to the conclusion that the interation mean square was estimating the same variance component as was the error mean square. With this assumption, the interaction sum of squares was then combined with the error sum of squares to form the residual mean square.

In the analysis of variance among years, the mean square of the pooled interation effects was significant. However, a breakdown of this interaction into two parts, one of which could be identified as the within year sex of calf by age of dam interaction and the second of which could be identifiable as an among years sex of calf by age of dam interaction, showed the first interaction to be non-significant, thus lending support to pooling the above mentioned interaction with the error terms in the individual year analyses. The interaction mean square associated with the among year effects again proved to be significant. However, in order to make any inference from the results of these years' analyses to succeeding years, the year effect must be thought of as a random effect; and, thus, the interaction of these effects with those of the fixed main effects would have an expected value of zero. These interaction terms should then be thought of as independent, e.g. a part of the variability of the experimental units. Thus, according to Scheffé (1956), the interaction sum of squares could be combined with the error sum of squares to form the residual sum of squares. The corresponding mean square would represent the variance of the experimental units.

With the use of the analysis of variance technique, intraclass half-sib correlations were obtained from both the paternal and maternal half-sib records, while an intraclass full-sib correlation was obtained from the available maternal full-sib records. From the estimates of variance components presented in Table 12, the following paternal half-sib intraclass correlation coefficients were obtained: for the year 1957, r = 0.0556; for 1958, r = 0.1589; for 1959, r = 0.1634; and pooling the three years data, r = 0.1172. From the estimated variance components presented in Table 15, a maternal half-sib correlation of 0.2383 was obtained along with a maternal full-sib correlation of 0.3675. The difference between the paternal and maternal half-sib correlations was tested by using the z-transform and the t-statistic to discern whether there was a significant maternal effect on the weaning weights. This difference proved to be non-significant, but was recognized as a possible bias when computing the estimates of heritability; for, as might have happened with the cited results of Brown (1958), the heritabilities estimated from the naternal half-sib and full-sib correlations would result in much larger figures than those obtained from the paternal half-sib correlations. The estimates of heritability ranged from 0.47 using 4 times the paternal half-sib correlation to 0.49 using twice the difference between the maternal full-sib correlation and the maternal effect. These ae estimates of heritability in the broad sense; for the estimate of genetic variance contains not only the additive portion, but also some variance due to dominance and epistatic gene action. Since these calves were creep-fed, these estimates also contain the heritability of rate of daily gain which could not be taken into account with this data. Because daily rate of gain is highly heritable in beef cattle, the estimate of heritability for weaning weights of creep-fed cattle would be expected to be greater than that for calves receiving no supplementary feeding.

The reduction of the sum of squares of unadjusted weaning weights due to each identifiable source of variations, as presented in Table 16, shows both the sums of squares attributed to each source and the per cent of each sum of squares relative to the total variation. The sums of squares upon which these percentages were based are the same as those that appeared in Tables 8 and 9, which present the analyses of variance within each of the three years 1957, 1958, and 1959 and among these three years. The sum of squares for the sire of the calf was computed from the multiplication of the intraclass paternal half-sib correlation and the sum of squares of the adjusted weaning weights. The per cent reduction accounted for by the regression of weight on age of the calf is shown to be the greatest with a reduction of 25.64, 30.58, 20.50, and 24.97 percent for 1957, 1958, 1959, and Total, respectively, while the reduction accounted for by considering the age of the dam accounts for 12.19, 14.67, 21.47, and 14.75 per cent, respectively, for each of the forenamed categories. The reduction due to accounting for sex of calf is the most consistent over the three years with 8.92, 8.45, 8.54, and 8.49 per cent for each category; while the reduction accounted for by considering the sire of the calv was the smallest, these being 2.82, 7.48, 8.98, and 5.92 for 1957, 1958, 1959, and Total, respectively.

SUMMARY

Data from 1,861 beef calves raised during the three year period of 1957, 1958, and 1959 under creep-fed conditions on the Ramsey Ranch in Butler County, Kansas were analyzed. Under the assumption that each weaning weight could be expressed as a linear combination of a sex effect, an age of dam effect, an age of calf effect, and, in the among year analysis, a year effect, and that the production of cows eight years of age and older were essentially the same, least squares analyses were performed both on an among year and within year basis. With each of these effects proving to be highly significant, in these analyses the following constants and standard errors were obtained: +20 (± 1.09) and -20 (± 1.19) for the steer and heifer calves, respectively; -64 (± 3.96), -22 (± 3.17), -1 (± 2.67), +4 (± 2.66), +23 (± 2.68), +27 (± 3.04), and +32 (± 2.15) for the 2 through 8 years and older dams, respectively; -9 (± 1.65), +6 (± 1.71), and +3 (± 1.56) for the years 1957, 1958, and 1959, respectively; and a regression coefficient of weaning weight on age of calf of 1.415 (± 0.044) where the calves had an average weaning age of 238 days. From these constants the required adjustments were obtained by using steer calves as a basis to adjust for sex and 8 year old and older dams as the mature cows upon which to adjust for age of dam. With these adjustments, all calves could be compared

on an equal age, sex, and dam basis so that one could better evaluate the true production ability of each dam. The sex of calf and age of dam constants were much larger than those already published, based on data collected from noncreep-fed calves.

Searle's simplified method, involving a multiplicative factor based on subclass means, was also used in obtaining adjustments for this set of data on a within year analysis. These adjustments were found to be very close to those obtained from the least squares analysis; and, upon comparing the reduction in the sums of squares of the two methods, measures of 99.3, 97.9, and 99.1 per cent for 1957, 1958, and 1959, respectively, were found to be the relative efficiency of Searle's method. Being far simpler to compute this method, it would appear to be the most desirable to use for the individual herdsman on the within year basis, while the method of least squares analysis would seem to be the most beneficial for one who wants to tabulate large amounts of data either on a within year or among year basis. This conjecture is made because, due to the above mentioned fact that Searle's method is based on a multiplicative factor of means and not a measure of deviation from a mean, the standard errors of the means could only be obtained from the method of least squares.

A paternal half-sib correlation coefficient of 0.1172 was obtained by pooling the 458, 405, and 471 paternal half-sib records obtained in each of the three respective years. This coefficient was compared to one based on 771 maternal half-sib records taken over the same three years within the same herd to ascertain if there was a significant maternal effect upon the final weaning weight of the calf. This difference between paternal and maternal half-sib correlation coefficients existed, but proved to be nonsignificant. The estimate of heritability based upon the paternal half-sib records was 0.47. Three hundred and thirty two maternal full-sib records were 0.47. Three hundred and thirty two maternal full-sib records obtained during 1958 and 1959 produced an intraclass correlation coefficient of 0.37 and, after the maternal effect was deleted, gave an heritability estimate of 0.49.

Table 1. Numbers, weights, ages of calves, and regression coefficients classed according to age of dam, sex of calf, and calf crop year.

Age of dam		1957		1958		1959		Total		Total by age of dam
		Steer	Heifer	Steer	Heifer	Steer	Heifer	Steer	Heifer	
2 years	Number	28	24	9	22	26	36	64	82	145
	Weight(Y)	11,775	9,730	4,520	9,790	11,075	15,460	27,370	34,980	62,350
	Age(X)	6,795	5,639	2,579	6,091	6,398	8,960	15,772	20,690	36,462
	b	1.815	1.450	1.946	1.236	0.946	0.951			
3 years	Number	64	52	36	24	28	18	128	94	222
	Weight(Y)	32,670	24,620	16,870	10,150	13,445	7,860	62,985	42,630	105,615
	Age(X)	17,049	13,857	8,255	5,342	6,938	4,412	32,242	23,611	55,853
	b	1.549	0.992	1.990	1.472	1.287	2.285			
4 years	Number	66	50	58	49	37	27	161	126	287
	Weight(Y)	32,290	21,440	28,475	22,295	18,300	12,165	79,065	55,900	132,965
	Age(X)	15,603	11,489	13,283	11,520	8,729	6,044	37,615	29,053	66,668
	b	1.482	1.379	1.808	1.459	1.015	1.310			
5 years	Number	46	41	55	40	56	52	157	133	290
	Weight(Y)	21,185	17,410	27,815	18,780	27,460	24,340	76,460	60,530	136,990
	Age(X)	10,018	8,854	13,220	9,745	12,800	11,965	36,038	30,564	66,602
	b	1.332	1.190	2.056	0.978	0.920	1.199			
6 years	Number	48	36	46	43	49	58	143	137	280
	Weight(Y)	24,390	16,420	24,550	21,470	25,390	27,535	74,330	65,425	139,755
	Age(X)	11,139	8,401	10,890	10,369	11,619	13,723	33,648	32,493	66,141
	b	0.860	1.293	1.516	1.723	1.522	1.295			
7 years	Number	24	40	47	30	37	48	108	118	226
	Weight(Y)	11,855	18,440	25,345	14,720	19,830	23,410	57,030	56,570	113,600
	Age(X)	5,570	9,460	11,322	7,181	8,722	11,156	25,614	27,797	53,411
	b	1.381	1.401	1.550	1.570	1.341	1.208			
8 years & older	Number	39	40	69	61	113	89	221	190	411
	Weight(Y)	20,855	19,260	38,110	30,810	59,950	43,220	118,915	93,290	212,205
	Age(X)	9,311	9,728	17,024	14,938	26,439	21,173	52,774	45,839	98,613
	b	1.207	1.141	1.554	1.553	1.966	1.458			
Total	Number	315	283	320	269	346	328	981	880	1,861
	Weight(Y)	155,020	127,320	165,685	128,015	175,450	153,990	496,155	409,325	905,480
	Age(X)	75,485	67,428	76,573	56,186	81,645	77,433	233,703	210,047	443,750
	b	1.434	1.261	1.728	1.421	1.356	1.296			
Total by year	Number	598		589		674		1,861		
	Weight(Y)	282,340		293,700		329,440		905,480		
	Age(X)	142,913		141,759		159,078		443,750		
	b	1.352		1.577		1.328				

$\sum X^2 = 107{,}145{,}456$ $\sum XY = 217{,}588{,}865$ $\sum Y^2 = 531{,}418{,}503$

Table 2. System of equations for fitting of constants with the method of least square

	a	s_1	s_2	d_1	d_2	d_3	d_4	d_5	d_6	d_7	y_1	y_2	y_3	b	w
a	1,861	981	880	145	222	287	290	280	226	411	598	589	674	443,750	905,480
s_1	981	981	0	63	128	161	157	143	108	221	315	320	346	233,703	496,155
s_2	880	0	880	82	94	126	133	137	118	190	283	269	328	210,047	409,155
d_1	145	63	82	145	0	0	0	0	0	0	52	31	62	36,462	62,350
d_2	222	128	94	0	222	0	0	0	0	0	116	60	46	55,853	105,615
d_3	287	161	126	0	0	287	0	0	0	0	116	107	64	66,668	134,965
d_4	290	157	133	0	0	0	290	0	0	0	87	95	108	66,602	136,990
d_5	280	143	137	0	0	0	0	280	0	0	84	89	107	66,141	139,755
d_6	226	108	118	0	0	0	0	0	226	0	64	77	85	53,411	113,600
d_7	411	211	190	0	0	0	0	0	0	411	79	130	202	98,613	212,205
y_1	598	315	283	52	116	116	87	84	64	79	598	0	0	142,913	282,340
y_2	589	320	169	31	60	107	95	89	77	130	0	589	0	141,759	293,700
y_3	674	346	328	62	46	64	108	107	85	202	0	0	674	159,078	329,440
b	443,750	233,703	210,047	36,462	55,853	66,668	66,602	66,141	53,411	98,613	142,913	141,759	159,078	107,145,456	217,588,865

Table 3. System of reduced least-squares equations.

	a	s_1	d_1	d_2	d_3	d_4	d_5	d_6	y_1	y_2	b	w
a	1,861	101	-266	-189	-124	-121	-131	-185	- 76	- 85	443,750	905,480
s_1	101	1,861	- 50	3	4	- 7	- 25	- 41	14	33	23,656	86,830
d_1	-266	- 50	556	411	411	411	411	411	113	41	-62,151	-149,855
d_2	-189	3	411	633	411	411	411	411	193	86	-42,760	-106,590
d_3	-124	4	411	411	698	411	411	411	175	115	-31,945	- 77,240
d_4	-121	- 7	411	411	411	701	411	411	102	59	-32,011	- 75,215
d_5	-131	- 25	411	411	411	411	691	411	100	54	-32,472	- 72,450
d_6	-185	- 41	411	411	411	411	411	637	102	64	-45,202	- 98,605
y_1	- 76	14	113	193	175	102	100	102	1,272	674	-16,165	- 47,100
y_2	- 85	33	41	86	115	59	54	64	674	1,263	-17,319	- 35,740
b	443,750	23,656	-62,151	-42,760	-31,945	-32,011	-32,472	-45,202	-16,165	17,319	107,145,456	217,588,865

Table 4. Inverse of the variance-covariance matrix[1].

	a	s_1	d_1	d_2	d_3	d_4	d_5	d_6	y_1	y_2	b
a	47.495,49	-0.057,726,2	2.799,79	2.361,653	-1.625,770	-2.049,443	-0.741,767	-0.620,065	-0.085,069,4	-0.616,064	-0.195,623,1
s_1	-0.057,73	0.542,496,2	0.088,67	-0.063,243	-0.041,428	-0.021,461	0.010,591	0.046,704	0.005,191,0	0.015,418	0.000,147,9
d_1	2.799,79	0.088,672,5	5.675,50	-0.938,001	-1.004,181	-0.997,012	-0.940,429	-1.061,128	-0.042,681,1	0.183,331	-0.010,004,3
d_2	2.361,65	-0.063,243,4	-0.938,00	4.021,981	-0.576,478	-0.660,236	-0.619,785	-0.751,248	-0.321,644,5	0.104,925	-0.009,611,4
d_3	-1.625,77	-0.041,427,6	-1.004,18	-0.576,478	3.171,099	-0.343,818	-0.409,206	-0.532,326	-0.109,915,0	-0.118,432	0.006,388,2
d_4	-2.049,44	-0.021,461,3	-0.997,01	-0.660,236	-0,343,818	3.136,137	-0.384,972	-0.506,845	0.067,165,0	-0.057,105	0.008,155,7
d_5	-0.741,77	0.010,590,6	-0.940,43	-0.619,785	-0.409,206	-0.384,972	3.151,456	-0.540,040	0.065,027,6	-0.026,281	0.002,772,7
d_6	-0.620,07	0.046,704,3	-1.061,13	-0.751,248	-0.532,326	-0.506,845	-0.540,040	3.768,506	0.094,796,8	-0.065,788	0.002,762,1
y_1	-0.085,07	0.005,191,0	-0.042,68	-0.321,645	-0.109,915	0.067,165	0.065,028	0.094,797	1.148,219,8	-0.591,611	0.000,322,7
y_2	0.616,06	-0.015,518,0	-0.183,33	0.104,925	-0.118,432	-0.057,105	-0.026,281	-0.065,788	-0.591,610,6	1.121,301	-0.002,395,9
b	-0.195,62	0.000,147,9	-0.010,00	-0.009,611	0.006,388	0.008,156	0.002,773	0.002,762	0.000,322,7	-0.002,396	0.000,815,9

[1]All elements are presented times 10^{-3}.

Table 5. Constants obtained by using the least-squares analysis[1].

Factor	1957	1958	1959	Total[2]
Steer calf	21	21	18	20 ± 1.09
Heifer calf	-21	-21	-18	-20 ± 1.19
2 year old dam	-58	-77	-63	-64 ± 3.76
3 year old dam	-15	-18	-33	-22 ± 3.17
4 year old dam	- 2	0	2	- 1 ± 2.67
5 year old dam	3	- 1	10	4 ± 2.66
6 year old dam	22	33	17	23 ± 2.68
7 year old dam	15	30	36	27 ± 3.04
8 year old dam and older	36	34	31	32 ± 2.15
Regression of weight on age	1.354	1.577	1.328	1.415 ± 0.044

[1]In the total least-squares analysis, correction factors for years were also obtained. These being -9, 6, and 3 for 1957, 1958, and 1959, respectively.

[2]Constants with standard errors of least-squares means.

Table 6. Adjustments obtained from least-squares constants and by Searle's simplified method.

	1957		1958		1959		Total	
Factor	Least squares	Searle	Least squares	Searle	Least squares	Searle	Least squares	Searle
Steer calf								
Heifer calf	42	43	43	49	37	40	40	44
2 year old dam	94	96	111	127	94	103	96	107
3 year old dam	51	50	52	52	64	65	54	59
4 year old dam	38	36	34	35	29	29	33	37
5 year old dam	33	33	35	35	21	24	28	31
6 year old dam	14	11	1	2	14	18	9	12
7 year old dam	21	27	4	1	- 5	0	5	9
8 year old dam and older								
Mean weaning age	239		241		236		238	
Mean adjusted weights	526	527	542	545	527	531	532	536

Table 7. Division of the within subclass sum of squares.

Source of variation	Sum of squares	Degrees of freedom	Mean squares
Regression due to b	2,206,847	1	2,206,847
Variation among b's	112,324	41	2,740
Pooled residual	4,193,250	1,777	2,360
Within sample (Total)	6,512,421	1,819	

Table 8. Analysis of variance for weight of calves within years.

Source of variation	1957 Degrees of freedom	1957 Mean squares	1958 Degrees of freedom	1958 Mean squares	1959 Degrees of freedom	1959 Mean squares
Regression of weight on age	1	745,234	1	950,756	1	535,092
Sex of calf	1	259,269	1	262,879	1	222,954
Age of dam	6	59,053	6	76,040	6	94,566
Residual	589	2,481	580	2,528	665	2,157

Table 9. Analysis of variance for weight of calves between years.

Source of Variation	Degrees of freedom	Mean squares
Regression of weight on age	1	2,206,847
Sex of calf	1	750,867
Age of dam	6	217,233
Year of birth	2	38,396
Residual	1,850	2,403

Table 10. Expected values of mean squares.

Source of variation	Degrees of freedom	Expected mean square
Among sires	k - 1	${}^2_e + n_o\,{}^2_p$
Within sires	N - k	${}^2_e$

$$n_o = \frac{N - \quad (n_i^2)/N}{k - 1}$$

Table 11. Analysis of variance for paternal half-sib records within each year.

Source of variation	1957		1958		1959		Total	
	df	MS	df	MS	df	MS	df	MS
Among sires	9	7,942	10	10,169	10	15,738	29	11,398
Within sires	448	2,531	394	2,810	460	2,384	1,302	2,564

Table. 12. Intraclass correlations and estimates of variance components for 1957, 1958, and 1959.

Year	n_o	${}^2_e$	${}^2_p$	r
1957	36.34	2,531	148.9	0.0556
1958	13.86	2,810	530.9	0.1589
1959	28.69	2,384	465.5	0.1634
Total	25.95	2,564	340.4	0.1172

Table 13. Analysis of variance for maternal half-sib records for 1957, 1958, and 1959.

Source of variation	Degrees of freedom	Mean square	Expected mean square
Among years	2	17,335	$\sigma^2_e + 257\sigma^2_y$
Among dams	256	3,360	$\sigma^2_e + 3\sigma^2_d$
Residual	512	1,733	σ^2_e

Table 14. Analysis of variance for maternal full-sib records for 1958 and 1959.

Source of variation	Degrees of freedom	Mean square	Expected mean square
Among years	1	60,022	$\sigma^2_e + 166\sigma^2_y$
Among dams	165	2,735	$\sigma^2_e + 2\sigma^2_d$
Residual	165	1,265	σ^2_e

Table 15. Intraclass correlations and estimates of variance components for maternal half-sib and full-sib records.

Relationship	^2_e	^2_d	r
Half-sibs	1,733	542.3	0.2383
Full-sibs	1,265	735.0	0.3675

Table 16. Sums of squares and percent reduction of the total sums of squares of unadjusted weaning weights from each identifiable source.

Source of variation	1957		1958		1959		Total	
	SS	Percent	SS	Percent	SS	Percent	SS	Percent
Total	2,906,210		3,109,313		2,610,202		8,839,372	
Regression of weight on age	745,234	25.64	950,756	30.58	535,092	20.50	2,206,847	24.97
Sex of calf	259,269	8.92	262,879	8.45	222,954	8.54	750,857	8.49
Age of dam	354,316	12.19	456,240	14.67	567,398	21.74	1,303,396	14.75
Sire of calf[1]	81,836	2.82	232,478	7.48	234,346	8.98	522,933	5.92
Unexplained variation	1,465,555	50.43	1,206,960	38.82	1,050,412	40.24	4,055,339	45.87

[1]The reduction in the sum of squares due to the sire is based upon an intraclass half-sib correlation from the adjusted weaning weights.

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APPENDIX

APPENDIX

Harvey's technique for fitting constants and obtaining the analysis of variance.

Using the data compiled in Table 1, the system of least squares equations in Table 2 were formed. In order to have a unique solution, the restrictions $\sum s_i = 0$, $\sum d_j = 0$, and $\sum y_k = 0$ were imposed and the reduced least squares equations in Table 3 were formed. Imposing these restrictions was accomplished by subtracting the s_2 row vector from the s_1 row vector, the d_7 row vector from each of the remaining six rows vectors in the d submatrix, and the y_3 row vector from each of the remaining two rows vector in the y submatrix, then performing the same operation with the respective column vectors. The row and column coefficients for the b equations were treated in the same manner as the W values. These matrix elements were denoted by C_{ij} for the i^{th} row and the j^{th} column; thus C^{th} denoted the respective inverse element of the i^{th} row and the j^{th} column of the inverse matrix. The inverse of the reduced 11 x 11 variance-covariance matrix as shown in Table 4 was used to obtain the constants needed to solve this set equations. These constants were obtained by multiplying the row vectors of the inverse matrix by the corresponding right hand member (W) of the reduced matrix, i.e.

$$\sum C^{ij} W_j = \hat{c}_i, \text{ for each } i.$$

then

$$\hat{a} = (.047{,}49)(905{,}480) + (-.000{,}057{,}75)(86{,}830) + \ldots + (-.000{,}195{,}6)(217{,}588{,}865) = 141.1$$

$$\hat{s}_1 = (-.000{,}057{,}75)(905{,}480) + (.000{,}542{,}4)(86{,}830) + \ldots + (.000{,}000{,}147{,}9)(217{,}588{,}865) = 20.18$$

$\hat{s}_2 = -\hat{s}_1 = -20.18$

This same type of matrix multiplication was followed to obtain the remaining constants as presented in Table 5.

For the analysis of variance shown in Table 9, the residual sum of squares is obtained by finding the difference between the total unadjusted sum of squares and the reduction due to fitting the parameters to the data, if the assumption is made that no interaction between the parameters exists. This reduction $R(a, s_i, d_j, y_k, b)$ is obtained by multiplying the constants obtained from the reduced matrix by the right hand member (W) of this same matrix of

$$R(a, s_i, d_j, y_k, b) = (141.1)(905,480) + (20.18)(86,830) + \ldots + (1.415)(217,588,865) = 444,960,700.$$

Thus

$$\text{residual sum of squares} = \sum Y^2_{ijkl} - R(a, s_i, d_j, y_k, b)$$
$$= 449,405,705 - 444,960,700$$
$$= 4,445,050$$

If the assumption that no interaction exists is not met, then this residual term will also contain an interaction sum of squares, as well as the error sum of squares. To obtain this sum of squares, an analysis of covariance procedure must be used to obtain the within subclass sum of squares of the deviations due to regression. The pooled interaction sum of squares is then the total uncorrected subclass sum of squares plus the within subclass deviations due to regression minus the reduction due to fitting the parameters or

$$\text{interaction sum of squares} = \sum Y^2_{ijk\cdot}/n_{ijk\cdot} + b' \sum x_{ijkl}\, y_{ijkl} - R(a, s_i, d_j, y_k, b)$$

$= 442,893,329 + 2,206,807 - 444,960,700$

$= 139,436$

where $b'\sum x_{ijkl}y_{ijkl}$ is, by Snedicor (1956), the within subclass regression due to regression or, by Li (1957), the regression due to $\bar{b}$.

Therefore

error sum of squares $= 4,445,050 - 139,436$

$= 4,305,614$

The sums of squares due to the fitted parameters was then computed using the elements of the inverse matrix shown in Table 4. With this approach the sum of squares could be presented in matrix notation as

$$\text{sum of squares} = B'Z^{-1}B$$

where B' is the row vector of the constant estimates for a given set of constants; Z^{-1} is the inverse of the segment of the inverse of the variance-covariance matrix corresponding to this set of constants; and B is the column vector or the transpose of the row vector of the set of constants. Therefore,

sum of squares due to regression $= \left[1.415\right]\left[.000,000,815,9\right]^{-1}\left[1.415\right]$

$= 2,454,696,$

sex sum of squares $= \left[20.18\right]\left[.000,542,5\right]^{-1}\left[20.18\right]$

$= 750,857,$

year sum of squares =

$$\begin{bmatrix}-9.250 & 6.130\end{bmatrix}\begin{bmatrix}.001,148,2 & -.000,591,6 \\ -.000,591,6 & .001,121,3\end{bmatrix}^{-1}\begin{bmatrix}-9.250 \\ 6.130\end{bmatrix}$$

$= 76,794$

and

dam sum of squares = 1,303,396

where $B' = \begin{bmatrix} -63.65 & -21.98 & -1.289 & 4.192 & 22.93 & 27.35 \end{bmatrix}$

$$Z^{-1} = \begin{bmatrix} \underline{.005{,}675{,}5} & -.000{,}938{,}0 & -.001{,}004{,}2 & -.000{,}997{,}0 & -.000{,}940{,}4 & -.001{,}061{,}1 \\ -.000{,}038{,}0 & \underline{.004{,}022{,}0} & -.000{,}576{,}5 & -.000{,}660{,}2 & -.000{,}619{,}8 & -.000{,}751{,}2 \\ -.001{,}004{,}2 & -.000{,}576.5 & \underline{.003{,}171{,}1} & -.000{,}343{,}8 & -.000{,}409{,}2 & -.000{,}532{,}3 \\ -.000{,}997{,}0 & -.000{,}660{,}2 & -.000{,}343{,}8 & \underline{.003{,}136{,}1} & -.000{,}385{,}0 & -.000{,}506{,}8 \\ -.000{,}940{,}4 & -.000{,}619{,}8 & -.000{,}409{,}2 & -.000{,}385{,}0 & \underline{.003{,}151{,}5} & -.000{,}540{,}0 \\ -.001{,}061{,}1 & -.000{,}751{,}2 & -.000{,}532{,}3 & -.000{,}506{,}8 & -.000{,}540{,}0 & \underline{.003{,}768{,}5} \end{bmatrix}^{-1}$$

The mean squares of these effects are presented in Table 9.

The standard errors of the least squares means for each parameter or class were obtained from the inverse matrix in Table 4. For since

$$s_{\hat{u}} = \sqrt{C^{uu}\,\sigma_e^2}$$

where $C^{uu} = C^{aa} + 2\bar{x}\,C^{ab} + \bar{x}^2\,C^{bb}$

and σ_e^2 = the error means square

then $C^{uu} = .047,495 + 2(238.4)\,(-.000,195,6) + (238.4)^2(.000,000,815,9)$

$= .000,591,1$

so $s_{\hat{u}} = \sqrt{(.000,591,1)\quad 2,403}$

$= 1.19$

Now $C_u^{uc_i} = C^{ac_i} + \bar{x}\,C^{bc_i}$

where c_j is the i^{th} parameter or class so that

$$s_{\hat{u}} + \hat{c}_i = \sqrt{(C^{uu} + C^{cici} + 2C^{uc_i})\,\sigma_e^2}$$

Therefore $C^{us_i} = -.000,057,73 + (238.4)(.000,000,147,7)$

$= -.000,022,50$

and $s_{\hat{u}} + \hat{s}_1 = \sqrt{(.000,591,1) + (.000,542,5) + 2(-.000,022,50)\qquad 2,403}$

$= 1.62$

while $s_{\hat{u}} + \hat{s}_2 = \sqrt{(.000,591,1) + (.000,542,5) + 2(\ .000,022,50)\qquad 2,403}$

$= 1.68$

Then if the u and the s_i effects are independent

$$s_{\hat{s}_i} = \sqrt{s^2_{\hat{u}} + \hat{s}_i - s^2_{\hat{u}}}$$

or $s_{\hat{s}_1} = \sqrt{2.62 - 1.42} = 1.09$

and $s_{\hat{s}_2} = \sqrt{2.84 - 1.42} = 1.19$

In a similar manner

$$s_{\hat{u}} + \hat{d}_1 = \sqrt{(.000,591,1) + (.005,675) + 2(\ .000,414,3) \qquad 2,403} = 4.13$$

$$s_{\hat{u}} + \hat{d}_2 = \sqrt{(.000,591,1) + (.004,022) + 2(-.000,069,84) \qquad 2,403} = 3.38$$

$$s_{\hat{u}} + \hat{d}_3 = \sqrt{(.000,591,1) + (.003,171) + 2(-.000,102,5) \qquad 2,403} = 2.93$$

$$s_{\hat{u}} + \hat{d}_4 = \sqrt{(.000,591,1) + (.003,136) + 2(-.000,104,7) \qquad 2,403} = 2.91$$

$$s_{\hat{u}} + \hat{d}_5 = \sqrt{(.000,591,1) + (.003,151) + 2(-.000,080,6) \qquad 2,403} = 2.94$$

$$s_{\hat{u}} + \hat{d}_6 = \sqrt{(.000,591,1) + (.003,769) + 2(\ .000,038,55) \qquad 2,403} = 3.27$$

$$s_{\hat{u}} + \hat{d}_7 = \sqrt{(.000,591,1) + (.002,393) + 2(-.000,234,8) \qquad 2,403} = 2,46$$

Where the C^{ud_j} are found in the same manner as C^{us_i}, using the same assumption of independence between the u and the d_j effects, the standard errors for the individual d_j effects were computed and are shown in Table 5.

The same approach was used with the year effects in that

$$s_{\hat{u}} + \hat{y}_1 = \sqrt{(.000,591,1) + (.001,148) + 2(-.000,008,134) \qquad 2,403} = 2.04$$

$$s_{\hat{u}} + \hat{y}_2 = \sqrt{(.000,591,1) + (.001,121) + 2(\ .000,044,76) \qquad 2,403} = 2.08$$

$$s_{\hat{u}} + \hat{y}_3 = \sqrt{(.000,591,1) + (.001,086) + 2(-.000,036,62) \qquad 2,403} = 1.96$$

so that

$$s_{\hat{y}_1} = 1.65$$

$$s_{\hat{y}_2} = 1.71$$

$$s_{\hat{y}_3} = 1.56$$

Searle's simplified herd-level age - correction factors.

Due to the fact that this method of obtaining correction factors uses a multiplicative factor, the weights within each subclass must be adjusted for the age of calf at weaning. Using as an example the data collected in 1957, these adjustments can be made with relative ease with the formula

$$\sum Y_{aij} = \sum Y_{ij} - b(\sum X_{ij} - n_{ij}\bar{X}..)$$

or

$$\sum Y_{a12} = 11,775 - 1.354\left[6,795 - 28(238.98)\right]$$

$$= 11,635$$

$$\sum Y_{a22} = 9,861$$

Therefore, the sum of the weights of the calves dropped by 2 year old dams adjusted for the age of the calves at weaning are 11,635 + 9,861 or 21,496 with the mean equal to 21,496/52 or 413.38. Adjusted sums of 52,978, 54,582, 41,194, 41,533, 30,653, and 39,899 can be obtained in similar computations for the weights of the calves dropped by 3 through 8 year old dams, respectively, with means of 456.17, 471.53, 473.49, 494.45, 478.96, and 505.05.

If the 8 year old and older cows are considered to be the mature dams, the ratio of the mean yields of the mature dams to that of each of the younger dams produces an estimate of the regression coefficient f_j. That is, if $\bar{z}_m$ denotes the mean yield of the mature dams and $\bar{z}_j$ the mean yield of the j year old dams, then

$$\hat{f}_j = \bar{z}_m / \bar{z}_j$$

For example

$$\hat{f}_2 = 505.05/413.37 = 1.221,8$$

and if

$$\hat{P}_j = 1 - 1/\hat{f}_j$$

then

$$\hat{P}_2 = 1 - 1/1.221,8 = 0.181,52$$

In like manner, the regression coefficient f_s needed to raise the female calves to an equal basis with steer calves can be estimated using the mean weights of both sexes as computed from each appropriate sum of adjusted sub-class weights. During this year, the sum of the adjusted steer weights was equal to 154,740 with a mean of 491.24, while the sum of the adjusted heifer weights was 127,595 with mean of 450.87. Thus

$$\hat{f}_s = 491.24/450.87 = 1.089,5$$

and

$$\hat{P}_s = 0.082,182$$

Therefore, the correction needed to raise the production of a j year old dam to that of a mature dam is $\hat{p}_j h$, while the correction needed to raise the weights of the heifer calves to those of the steer calves is $\hat{p}_s h$ where

$$\hat{h} = \frac{\text{total actual production of herd}}{N - n_s\hat{p}_s - \sum n_j\hat{p}_j}$$

and is the average weaning weight for the herd corrected for sex of calf and age of dam. That is

$$\hat{h} = \frac{282,335}{598 - 23.258 - 38.975} = 526.98$$

Thus

$$\hat{p}_s\hat{h} = 43$$

$$\hat{p}_2\hat{h} = 96$$

etc.

LEAST-SQUARES ESTIMATES OF GENETIC AND ENVIRONMENTAL PARAMETERS IN A BEEF CATTLE POPULATION

by

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ABSTRACT

A study was made on records of 1,861 weaning weights from creep-fed calves taken over a three year period from the commercial Aberdeen Angus herd of the Ramsey Ranch located in Butler County, Kansas. The study was based on the linear model, weaning weight = overall average + sex of calf + age of dam + year + regression of weaning weight on age of calf + error, where sex of calf was steer and heifer, age of dams were categorized from 2 year old to 8 years and older where the assumption is made that cows of 8 years of age and older produced with equal ability, and the years were 1957, 1958, and 1959.

With the use of the least squares procedure, constants were fitted to the non-orthogonal data and the effects of each factor were estimated. Each of the factors had a highly significant effect upon the final weaning weight of the calf and resulted in the following constants and standard errors: +20 (± 1.09) and -20 (± 1.19) for the steer and heifer calves, respectively; -64 (± 3.96), -22 (± 3.17), -1 (± 2.67), +4 (± 2.66), +23 (± 2.68), +27 (± 3.04, and +32 (± 2.15) for the 2 through 8 years and older dams, respectively; -9 (± 1.65), +6 (± 1.71), and +3 (± 1.56) for the years 1957, 1958 and 1959, respectively; and a regression coefficient of weaning weight on age of calf of 1.415 (±0.044) where the calves had an average weaning age of 238 days. By pooling the 458, 405, and 471 paternal half-sib records for the three respective years, an estimated paternal half-sib correlation coefficient of 0.1172 was obtained. This was compared to a maternal half-sib correlation coefficient of 0.2383, which was derived from 771 records over the same three years, in order to determine the maternal effect of the dam on the calf's weaning weight. A difference in the two coefficients could

be observed, but proved to be non-significant. A maternal full-sib correlation coefficient of 0.3675 was also found from 332 records of 1958 and 1959. The estimate of heritability using the paternal half-sib records was 0.47, while the estimate arrived at by using the maternal full-sib records with the deletion of the maternal effect was 0.49.

Adjustments were obtained on a within year basis with Searle's simplified method, as well as with the method of least squares. The relative efficiency in the reduction of variation due to fitting the constants obtained in this manner compared to the reduction in variation due to fitting the constants derived from the method of least squares was 99.3, 97.9, and 99.1 per cent for 1957, 1958, and 1959, respectively. However, Searle's method, because it uses a multiplicative factor based upon the subclass means, does not enable one to obtain standard errors of the means, as does the method of least squares.