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DETERMINATION OF THE TRACKING SIGNAL CONTROL LIMITS

by

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CHAPTER I

INTRODUCTION

Forecasting is a technique of estimating future values if the underlying process continues as it has in the recent past. For any forecasting system the two essential requirements are past data and an adequate forecasting model.

A forecasting model is constructed on the basis of past data. Such a model will provide reasonably accurate forecasts as long as the actual process does not change. However, any process is not static and is liable to change with time. The forecasting system, by some means should be able to detect these changes so that the parameters of the model can be updated to meet the new situation.

A change in the process is immediately reflected in the magnitude of the forecast errors. A forecast error is defined as the difference between the actual observation and its prior forecast. Mathematically;

$$x(t) = \hat{x}_{\tau}(t) + e_{\tau}(t)$$

where

$x(t)$ = the observed value at time t

$\hat{x}_{\tau}(t)$ = the forecast value for time t, which has been made at time $(t-\tau)$, where τ is the forecast horizon.

and

$e_{\tau}(t)$ = error at time t with a horizon of τ.

Unless otherwise stated the forecast horizon τ is one. Whenever a change in the process occurs, the predicted values start lagging the observed values. This results in a change in the error magnitudes which can be used to signal a process change. For this purpose a statistic is necessary. The statistic should be expressed as a function of forecast errors.

The tracking signal is a function of forecast errors and is widely used as a means for detecting process changes. It is a ratio of two smoothed statistics derived from the forecast errors. If the forecasting system is adequate the magnitude of the tracking signal lies close to zero. When the process changes the forecast errors become large and the tracking signal attains large values. To determine whether the process has changed significantly one needs to specify an acceptable limiting value for the tracking signal.

Confidence limits for the tracking signal could be evaluated if the distribution of the tracking signal is known. The present work is an attempt to determine the distribution of the tracking signal and subsequently to propose confidence limits. Two approaches were adopted;

1. Approximating the tracking signal distribution with some known statistical distribution and then imposing confidence limits in terms of the parameters of the approximating distribution.
2. Evaluating confidence limits by computer simulation.

The above approaches deal with a constant model and use single exponential smoothing for predicting future values.

When the forecasting system is making satisfactory forecasts the forecast error magnitude is small and the tracking signal magnitude is close to zero. When a change takes place in process level, the tracking signal grows larger and sometimes exceeds specified limits depending on the magnitude of the change. At any such time two problems may arise:

1. When should one confirm a change in the process
2. If a change in the process is confirmed how should the forecast system be adjusted to recover rapidly from the change.

The solutions to these problems are not unique. However, a change in the process is usually confirmed when two successive values of the tracking signal exceed a specified limit.

Considering the above problems, the behavior of the tracking signal to a step increase in the process level was studied by simulation. A step increase in the process level was introduced, after the effect of the initial forecast conditions had been discounted to an acceptable value. The magnitude of the step increase varied from 2.5 percent to 25 percent of the process mean. By specifying the limiting value of the tracking signal and using various criteria a change in the process level was confirmed. Various detecting rules were simulated 1000 times and the probability of detecting a process change was obtained. These probabilities were evaluated for smoothing constants ranging from 0.05 to 0.25.

LITERATURE REVIEW

The concept of the tracking signal was first introduced by Brown [1]. If the forecasts are satisfactory, the average forecast error and the sum of forecast errors will be close to zero. When there is a change in the process, the sum of errors starts drifting away from zero. The problem is to determine when a drift is large enough to imply a process change. For this purpose Brown defined the tracking signal as:

$$\text{Tracking Signal} = \frac{\text{Sum of errors}}{\text{Mean absolute deviation of errors}}$$

$$= \frac{\text{S.E.}}{\text{M.A.D.}}$$

The S.E. is computed by adding the error arising from every new observation to the previous sum of errors. Mathematically

$$Y(t) = Y(t-1) + e(t)$$

where

$Y(t)$ = Sum of errors at time t

$Y(t-1)$ = Previous sum of errors

and

$e(t)$ = Error at time t.

The M.A.D. is a measure of the scatter of the data around the mean. If $e(t)$ represents the error having a statistical distribution $p(e)$ about the mean μ, then the M.A.D. Δ_e is defined as,

$$\Delta_e = \int_{-\infty}^{\infty} | e - \mu | \, p(e) \, de$$

In most cases $\mu = 0$ and the computation is considerably reduced. For a general case, a normalized variable Z may be defined such that

$$Z = \frac{e-\mu}{\sigma_e} ,$$

where σ_e is standard deviation of errors. The M.A.D. is then

$$\Delta_e = \int_{-\infty}^{\infty} |Z \sigma_e| \, p(Z) \, dZ$$

$$= \sigma_e \int_{-\infty}^{\infty} |Z| \, p(Z) \, dZ$$

Hence, the M.A.D. is proportional to the standard deviation of the errors. For normally distributed errors, the constant of proportionality is 0.79788. For detecting a process change, the control limits for the tracking signal are to be evaluated. For this purpose a constant model has been considered. A constant model is one in which the observations are random samples from some distribution whose mean does not change significantly with time. The observations x(t) from such a model can be represented by:

$$x(t) = \xi(t) + \varepsilon(t)$$

where

$\xi(t)$ = Process mean

$\varepsilon(t)$ = Random noise with $E[\varepsilon(t)] = 0$,

$$\mathrm{Var}\,(\varepsilon(t)) = \sigma_\varepsilon^2 \text{ and } \mathrm{cov}\,(\varepsilon_i \varepsilon_j) = 0, \quad \text{for } i \neq j$$

According to Brown pp. 289, the variance of S.E. say σ_Y^2, for the above model is given by:

$$\sigma_Y^2 = \frac{1}{1-\beta^2}\,\sigma_\varepsilon^2$$

where β = Discount factor = $(1-\alpha)$, α being smoothing constant. If the errors are normally and independently distributed with mean zero, and the noise samples are serially independent, the M.A.D. Δ_e evaluated by Brown pp. 289 is

$$\Delta_e = \sqrt{2/\pi}\,\sqrt{\frac{2}{2-\alpha}}\,\sigma_\varepsilon$$

or

$$\sigma_\varepsilon = \Delta_e\,\sqrt{\pi/2}\,\sqrt{\frac{2-\alpha}{2}} \qquad (1.1)$$

Expressing σ_ε in terms of Δ_e, the variance of S.E. for a constant model is:

$$\sigma_Y^2 = \frac{1}{1-\beta^2}\,\pi/2\,\frac{2-\alpha}{2}\,\Delta_e^2$$

$$= \frac{1}{1-(1-\alpha)^2}\,\pi/2\,\frac{2-\alpha}{2}\,\Delta_e^2$$

$$= \frac{1}{\alpha(2-\alpha)}\,\pi/2\,\frac{2-\alpha}{2}\,\Delta_e^2$$

$$= \pi/4\alpha\,\Delta_e^2$$

$$\therefore\ \sigma_Y = \sqrt{\pi/\alpha}\,\frac{\Delta_e}{2} \qquad (1.2)$$

Since the standard deviation σ_Y is proportional to M.A.D., the control limits for S.E. could be obtained in terms of Δ_e. The limiting values of the S.E., if divided by Δ_e, will be limiting values of the tracking signal. These limiting values will be pure numbers.

The definition of the tracking signal by Brown was modified by Trigg [2] of Eastman Kodak. Trigg found that there were two disadvantages with Brown's definition. To quote Trigg pp. 271, the disadvantages are:

"1) once the tracking signal has gone out of limits it will not necessarily return within limits even though the forecasting system itself comes back in control. Consequently, intervention is necessary to set the sum of forecast errors back to zero if future false alarms are to be avoided. Such interventions can be tedious and may tend to be neglected when several hundred items are being forecast.

2) Ironically if the system starts to give exceptionally accurate forecasts the tracking signal may go out of limits. For example, if perfect forecasts begin to occur, the M.A.D. will tend to zero whilst the sum of the errors will remain unaltered. This clearly leads to the tracking signal tending to infinity."

While considering these disadvantages Trigg modified the definition of the tracking signal to:

$$\text{Tracking Signal} = \frac{\text{Smoothed error}}{\text{Smoothed mean absolute dev. of errors}}$$

$$= \frac{\text{S.S.E.}}{\text{S.M.A.D.}}$$

where

$$S.S.E. = (1-\alpha) \text{ Previous } S.S.E. + \alpha \text{ Latest Error}$$

and

$$S.M.A.D. = (1-\alpha) \text{ Previous } S.M.A.D. + \alpha \text{ Latest absolute error.}$$

S.S.E. can attain positive or negative values but S.M.A.D. will always be a positive quantity. Further, S.M.A.D. is always greater than S.S.E. However, at times they may be equal. Hence the tracking signal magnitude is bounded by +1 and -1. With this modification the disadvantages were overcome. This modified definition allowed Trigg to find new control limits for the tracking signal. In proposing the control limits he assumed that S.S.E. and S.M.A.D. are independent and the S.M.A.D. a constant. Trigg's approach to control limits was as follows: Let the error i periods in the past be denoted by e_i. Assume they are serially independent and have a standard deviation of σ. The smoothed error is

$$S.S.E. = \sum_{i=0}^{\infty} \alpha(1-\alpha)^i \, e_i$$

The smoothed error variance will be

$$\text{Smoothed Error Variance} = \sum_{i=0}^{\infty} [\alpha \; \sigma \; (1-\alpha)^i \; e_i]^2$$

As $0 < \alpha < 1$, $(1-\alpha)$ will be less than unity and the above expression will sum to:

$$\text{Smoothed Error Variance} = \frac{\alpha^2\sigma^2}{[1 - (1-\alpha)^2]}$$

$$= \frac{\alpha}{2-\alpha}\sigma^2$$

Thus the two Sigma limits for S.S.E. are:

$$\pm 2\sqrt{\frac{\alpha}{2-\alpha}}\,\sigma$$

Trigg assumed S.M.A.D. to be constant for small values of α. For a wide range of distributions $\sigma \simeq 1.2 \cdot$ S.M.A.D. Hence S.M.A.D. is approximately equal to $\sigma/1.2$. The control limits for the tracking signal thus becomes:

$$\pm 2.4\sqrt{\frac{\alpha}{2-\alpha}}$$

However, Trigg mentions that these limits do not hold good for high values of α. Batty [3] shows these limits in error for $\alpha = 0.1$. For higher values of α, limits were obtained by simulation. For the simulation, the errors were drawn independently and randomly from a normal distribution. A cumulative probability table was obtained which gave the limiting values of the tracking signal. According to Trigg, this table should not be referred to when the forecasting system giving rise to the errors is an adaptive system since this introduces autocorrelation of the errors.

Further, work in this field was carried out by Batty [3]. The cumulative probability table obtained by Trigg [2] was based upon the assumption that the errors are serially independent. Batty obtained a similar cumulative probability table taking into account the correlation between the two smoothed statistics. Moreover, the forecast errors were also correlated since they came from an adaptive forecasting system. A series of data was generated as observations by taking noise samples from a normal distribution. Forecasts were calculated using an appropriate level of smoothing. For each forecast S.S.E. and S.M.A.D. were calculated. The cumulative probability table for the tracking signal was obtained. The values in this table differed from those in Trigg's [2] table by 20% for a smoothing constant equal to 0.1.

CHAPTER II

THE TRACKING SIGNAL DISTRIBUTION APPROXIMATED BY A BETA DISTRIBUTION

2.1 INTRODUCTION

In this approach the tracking signal distribution is approximated with a known statistical distribution. The selection of the approximating distribution depends on the shape of the tracking signal distribution for different values of the smoothing constant. By simulation, tracking signal frequency distributions were obtained for various values of the smoothing constant ranging from 0.05 to 0.9. For small values of α, the distribution is symmetrical and unimodal. For moderate values of α, it becomes flat at the mode and finally becomes bimodal. For high values of α, the distribution is U-shaped. This variability in shape is illustrated in figure (2.1).

The beta family of statistical distribution was considered for approximating the tracking signal distribution. It has a finite range of zero to one. The distributions from this family can adopt various shapes depending upon their parameters. For example, the shape of the beta distribution is:

a) symmetrical and unimodal when its parameters (m,n) are equal and greater than one. $(m = n > 1)$

b) rectangular when both parameters are equal to one. $(m = n = 1)$

c) U-shaped when both parameters are equal and less than one. $(m = n < 1)$

d) skewed when parameters are unequal. $(m \neq n)$

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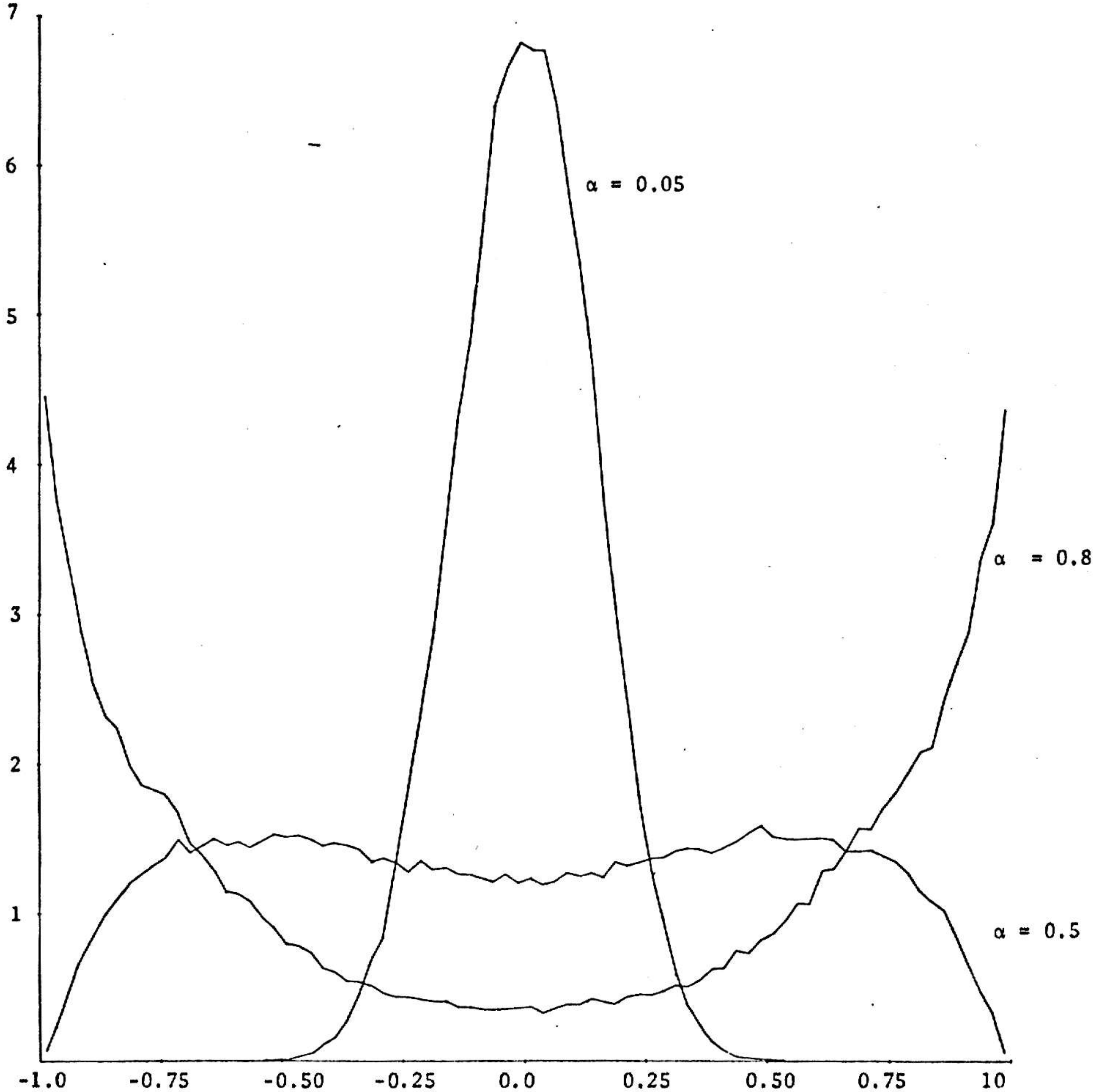


Figure (2.1) Variability in the Shape of the Tracking Signal Distribution

Further, a bimodal distribution could be obtained by combining two beta distributions with appropriate parameters.

2.2 DETAILS OF EXPERIMENTAL WORK

2.2.1 SIMULATION FOR TRACKING SIGNAL DISTRIBUTION

For simulating the tracking signal distribution a constant process has been considered with superimposed normal noise. The noise samples are assumed to be normally and independently distributed with mean zero and standard deviation of 1. Random normal deviates with a mean of 10 were generated as observations. Mathematically, the observations $x(t)$ could be represented as:

$$x(t) = \xi(t) + \varepsilon(t)$$

where

$\xi(t)$ = process mean

$\varepsilon(t)$ = noise samples.

These observations of the process were predicted by a forecasting system using single exponential smoothing. Let the predicted values be represented by $\hat{x}(t)$. The forecasting error at any time is given by:

$$ERR(t) = x(t) - \hat{x}(t)$$

Two smoothed statistics were obtained from this forecast error.

Smoothed Error: $S.S.E.(t) = (1-\alpha)\ S.S.E.(t-1) + \alpha\ ERR(t)$

Smoothed Mean Absolute Deviation

$$S.M.A.D.(t) = (1-\alpha)\ S.M.A.D.(t-1) + \alpha\ |ERR(t)|$$

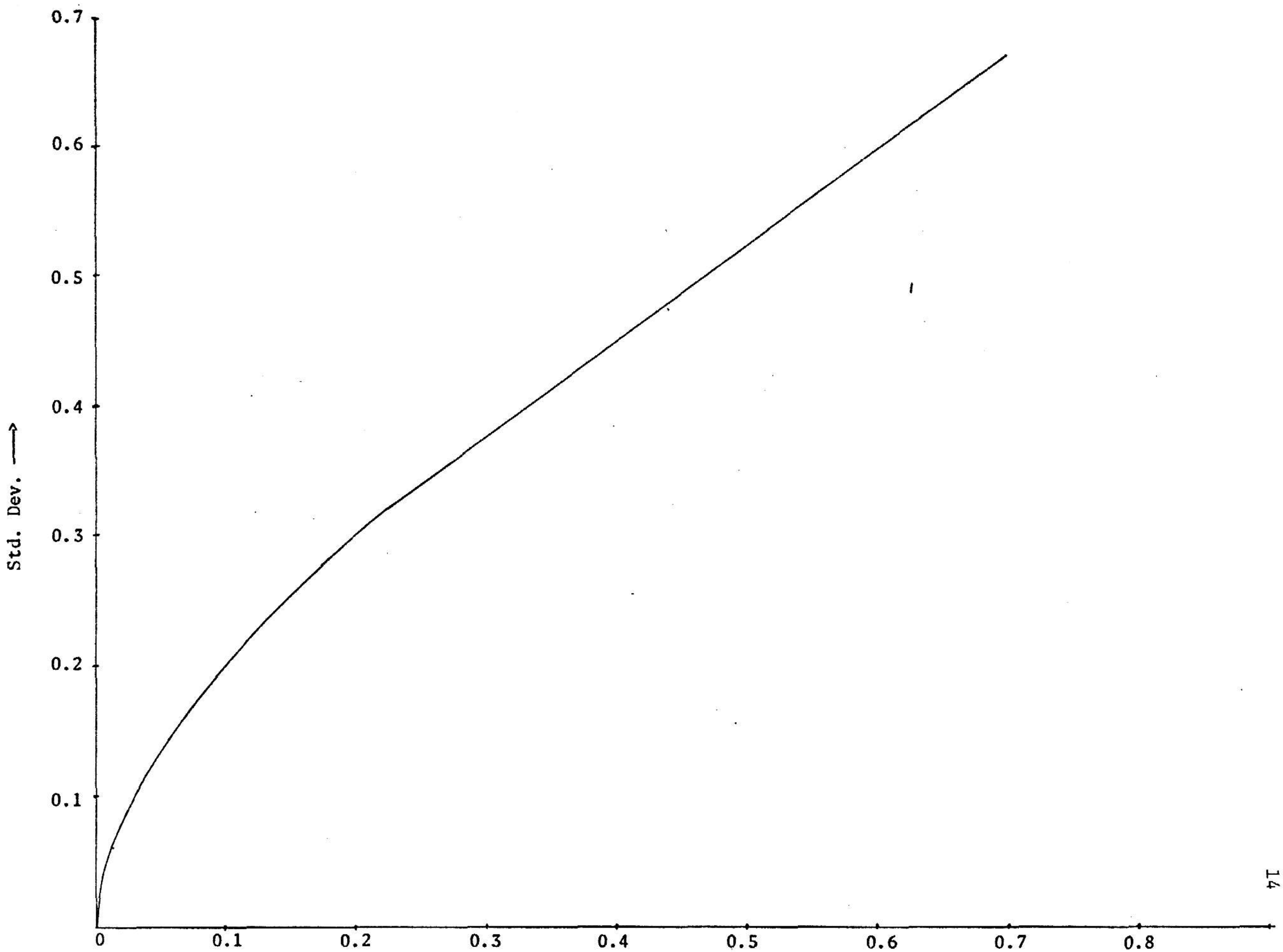


Figure (2.2) Variation of Standard Deviation of the Tracking Signal Distribution with α.

From these two statistics the tracking signal was evaluated.

$$\text{Tracking signal} = \frac{\text{S.S.E.}}{\text{S.M.A.D.}}$$

The tracking signal ranges from -1 to +1. This range was divided into 80 equal intervals. 20,000 values of the tracking signal were generated. The frequency plot of the tracking signal revealed a great fluctuation in frequency at the peak of the distribution. The fluctuation in the frequency was smoothed out to a great extent by increasing the number of simulations from 20,000 to 100,000. 20,000 simulations for each of 9 different values of α (a total of 180,000 simulations) took a system time of 0.083 hours in Fortran H level with an optimization level of 2. The cost was $17.14. For 100,000 simulations with each of 9 different smoothing constants (a total of 900,000) 0.342 hours and $70.24 were required. If the number of simulations is further increased. The system time will increase accordingly.

The tracking signal mean was approximately zero but the standard deviation varied with the smoothing constant. This variation is illutrated in figure (22). The range of the beta distribution is 0 to 1 and that of the tracking signal distribution is -1 to +1. To map one distribution on the other, a linear transformation is necessary. The transformation is given by:

$$X_\beta = \frac{X_T + 1}{2}$$

where X_β and X_T are points on the beta and the tracking signal distribution. Experiments were carried out with smoothing constants ranging from 0.1 to

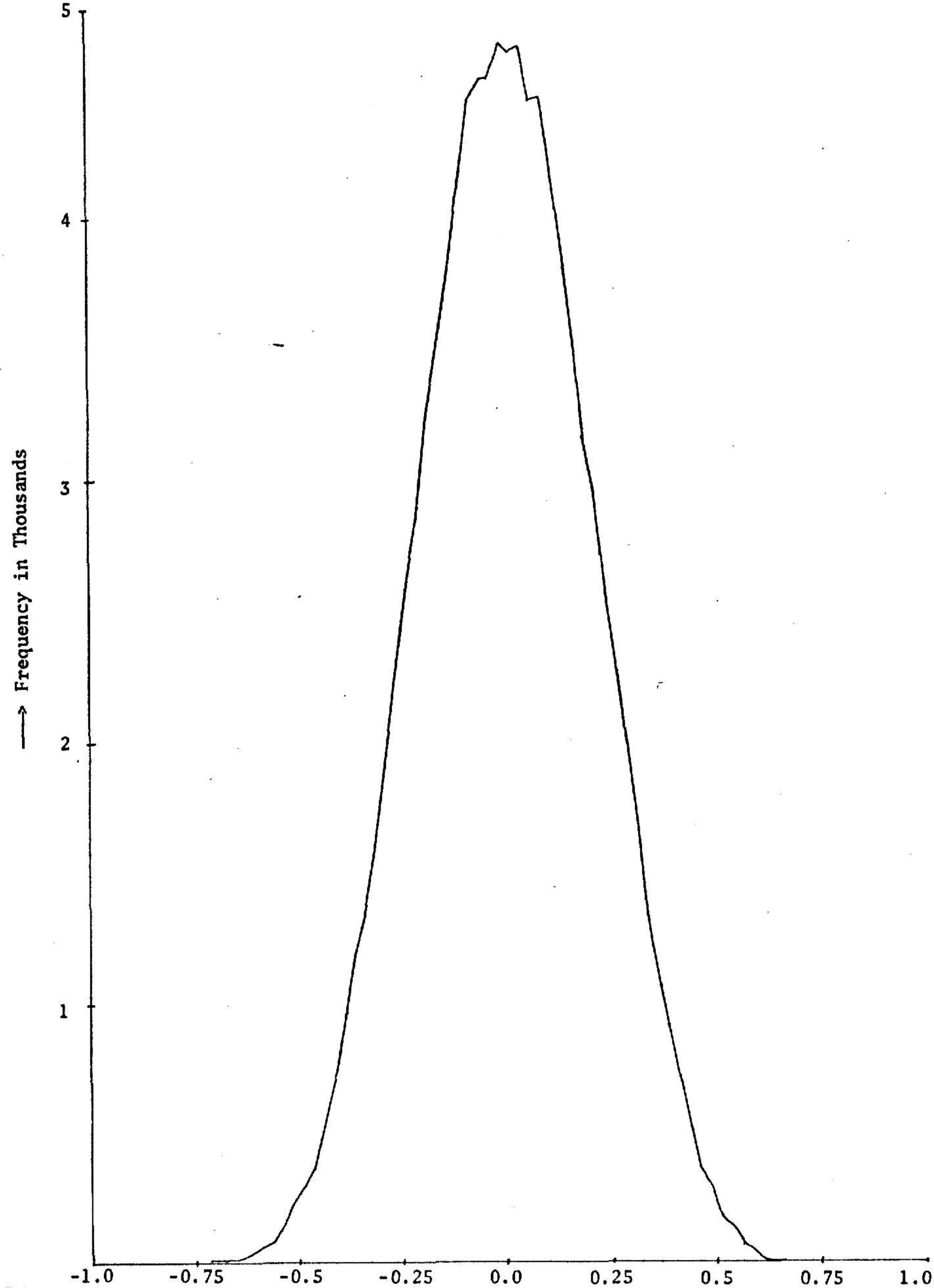


Figure (2.3) The tracking Signal Distribution for an α of 0.1

0.25. This is the usual range of the smoothing constant used in any normal forecasting system.

2.2.2 Details of Experiment with α of 0.1

The tracking signal distribution for α of 0.1 is unimodal and symmetrical. This is depicted in figure [23]. This indicates that for the purpose of approximation, a beta distribution with equal parameters greater than unity should be tried. For a closer approximation, the mean and the standard deviation of the tracking signal distribution should be equal to those of the approximating distribution. Consider a beta distribution B(m,n), with parameters m and n respectively. The mean and variance of this distribution in terms of its parameters is given by [4].

$$\text{Mean} = m/(m+n)$$

$$\text{Variance} = \frac{m \cdot n}{[(m+n+1) \cdot (m+n)^2]} \,.$$

For equal parameters (i.e. m=n) the mean of beta distribution is 0.5 and the variance is:

$$\text{Variance} = \frac{m^2}{[(2m+1) \cdot (2m)^2]} \qquad (2.1)$$

$$= \frac{1}{4(2m+1)}$$

As the mean of the tracking signal distribution is approximately 0.5 on the transformed scale, only the correspondence of the variance is to be

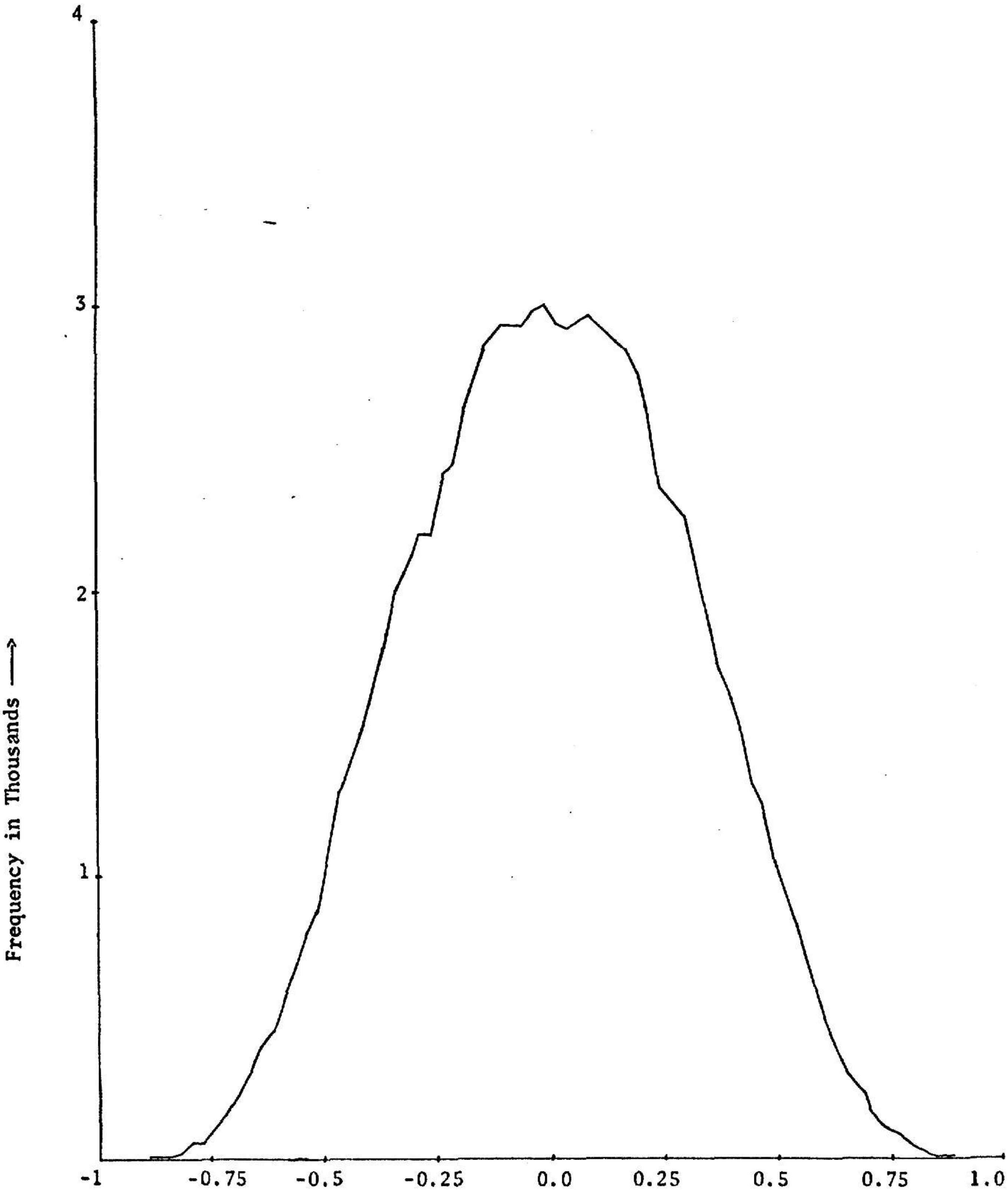


Figure (2.4) The Tracking Signal Distribution for an α of 0.2

considered. The standard deviation of the tracking signal is 0.1027 on the transformed scale. Equating this with the above expression for variance we have:

$$(0.1027)^2 = \frac{1}{4(2m+1)}$$

Solving the above equation, the parameters of the approximating distribution are $m = \bar{n} = 11.35$. The probabilities of observations falling in 80 equal intervals were obtained for the beta distribution for the I.B.M. scientific subroutine package program NDTR. From these probabilities a theoretical frequency distribution for the tracking signal was derived. Knowing the theoretical and the observed frequencies for the tracking signal, a chi-square test and a Kolmogorov-Smirnov test were applied for the goodness of fit. A chi-square value of 79.44 was obtained which is more than the critical value of 73.3. The fit was inadequate at the 5% significance level. However, the fit was adequate by the Kolmogorov-Smirnov test. The value obtained by this test was 0.00179 which is less than critical value of 0.00430 at the 5% significance level.

As the fit was inadequate by one of the tests, the tracking signal distribution could not be approximated satisfactorily with a single beta distribution. The next step was to investigate other values of α namely .2 and .01.

2.2.3 EXPERIMENT WITH α OF 0.2.

The tracking signal distribution for this value of α was a little flat at the peak as illustrated in figure(2.4). At this stage it was difficult

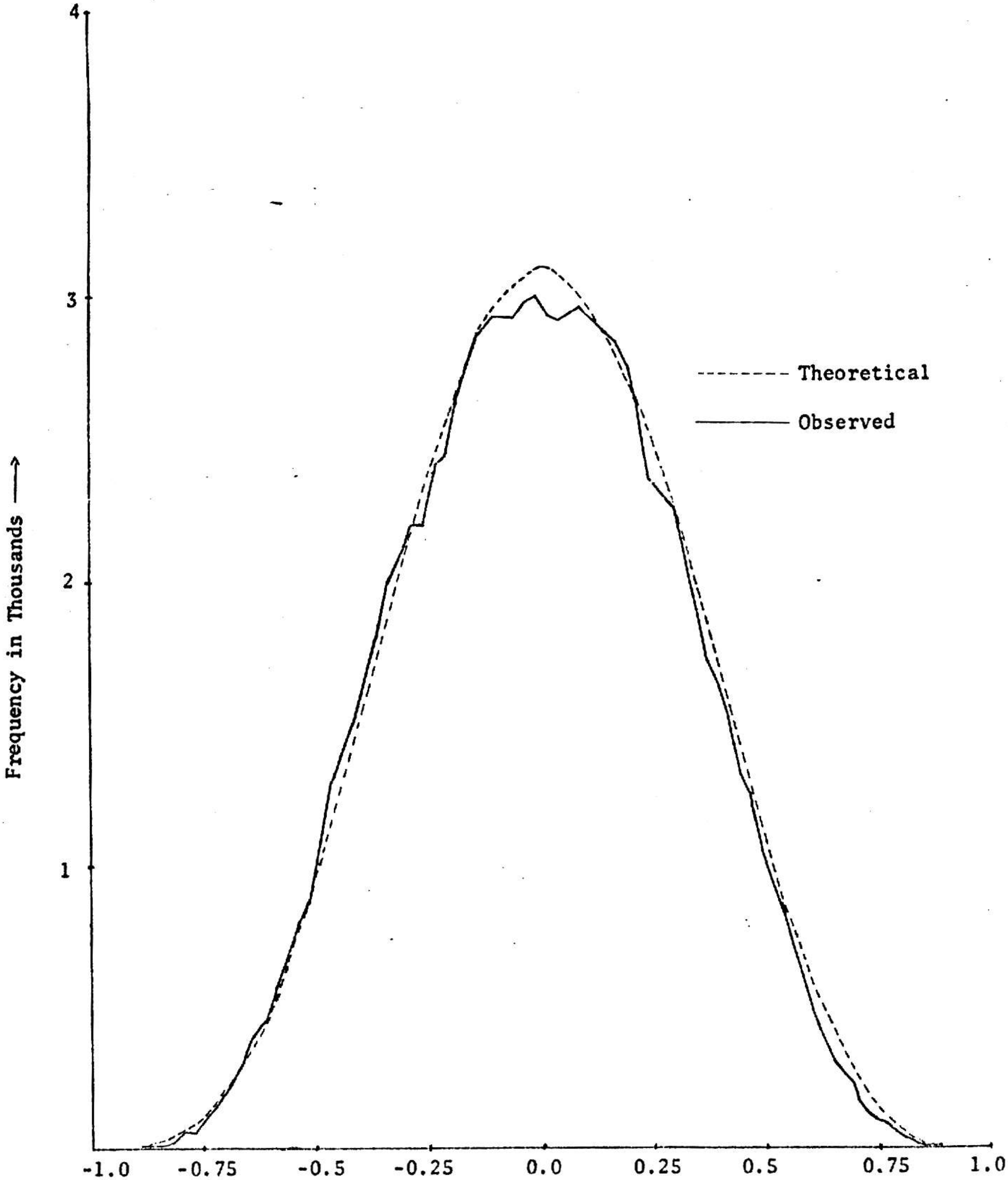


Figure (2.5) Comparison of Observed and Theoretical Tracking Signal Distribution to say whether this distribution is bimodal. So the first step was to approximate it with a single beta distribution. The frequencies in 80 of the cells were evaluated. These frequencies differed significantly at the peak, from the observed distribution as illustrated in figure (2.5). The next approach was to approximate this tracking signal distribution with some distribution which is flat at peak. This type of distribution could be obtained by combining two beta distributions as illustrated in figure [2.6]. Also, a bimodal distribution could be obtained by such combination of two beta distributions with appropriate parameters.

Consider two beta distributions B(m,n) and B(n,m) with p.d.f's given by:

$$\left.\begin{aligned} B(m,n) &= \frac{\Gamma(m+n)}{\Gamma(m)\ \Gamma(n)} \cdot X_1^{m-1}\ (1-X_1)^{n-1} \\ &= f_1(x_1) \end{aligned}\right\},\quad 0 < X_1 < 1$$

and

$$\left.\begin{aligned} B(n,m) &= \frac{\Gamma(n+m)}{\Gamma(n)\ \Gamma(m)}\ X_2^{n-1}\ (1-X_2)^{m-1} \\ &= f_2(X_2) \end{aligned}\right\},\quad 0 < X_2 < 1$$

At any point x on the range of these distributions, it is assumed that the ordinate f(x) of the resultant distribution is

$$f(x) = 1/2\ [f_1(x) + f_2(x)], \qquad 0 < x < 1 \tag{22}$$

or

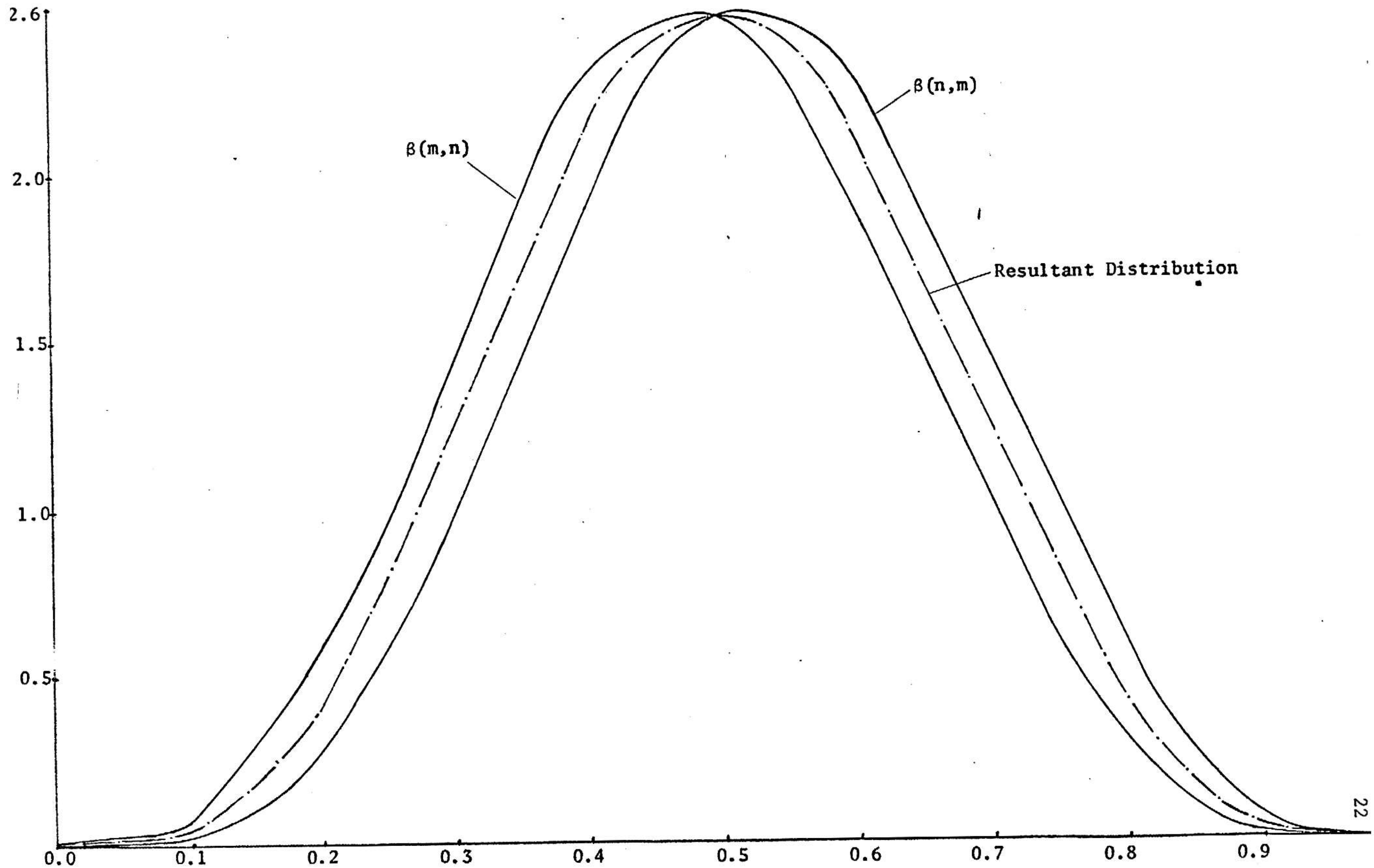


Figure (2.6) Resultant Distribution Obtained by Combining Two Beta Distributions.

$$\int_0^1 f(x)\,dx = 1/2\ [\int_0^1 f_1(x)\,dx + \int_0^1 f_2(x)\,dx], \qquad 0 < x < 1.$$

As $\int_0^1 f_1(x)\,dx = \int_0^1 f_2(x)\,dx = 1$ the area under the resultant distribution is also equal to one. Further, from equation (2.2),

$$\int_0^1 xf(x)\,dx = 1/2\,[\int_0^1 xf_1(x)\,dx + \int_0^1 xf_2(x)\,dx]$$

The above equation states that the mean of new distribution is half the sum of means of prior beta distributions. If the mean of the resultant distribution is represented by μ, then:

$$\mu = E(x) = 1/2\,[E_1(x) + E_2(x)].$$

Representing the means $E_1(x)$ and $E_2(x)$ in terms of parameters:

$$\mu = 1/2\ [\frac{m}{m+n} + \frac{n}{m+n}] = 0.5$$

The variance σ^2 of the resultant distribution is given by:

$$\sigma^2 = E(x^2) - E^2(x)$$

$$= 1/2\,[\int_0^1 x^2 f_1(x)\,dx + \int_0^1 x^2 f_2(x)\,dx] - \{1/2\,[\int_0^1 xf_1(x)\,dx + \int_0^1 xf_2(x)\,dx]^2$$

$$= 1/2\ E_1(x^2) + 1/2\ E_2(x^2) - \{1/2\ [E_1(x) + E_2(x)]\}^2$$

$$= 1/2\ E_1(x^2) + 1/2\ E_2(x^2) - 1/4\ E_1^2(x) - 1/4\ E_2^2(x) - 1/2\ E_1(x)\ E_2(x)$$

$$= 1/2\ E_1(x^2) - 1/2\ E_1^2(x) + 1/2\ E_2(x^2) - 1/2\ E_2^2(x) + 1/4\ E_1^2(x)$$

$$+ 1/4\ E_2^2\ (x) - 1/2\ E_1(x)\ E_2(x)$$

$$= 1/2\ E_1(x^2) - 1/2\ E_1^2(x) + 1/2\ E_2(x^2) - 1/2\ E_2^2(x)$$

$$+ \{1/2[E_1(x) - E_2(x)]\}^2$$

$$= 1/2\ \sigma_1^2 + 1/2\ \sigma_2^2 + [1/2(\mu_1 - \mu_2)]^2$$

where

μ_1 and σ_1^2 are the mean and variance of first beta distribution ($\beta(m,n)$),

and

μ_2, σ_2^2 are mean and variance of the second beta distribution. ($\beta(n,m)$).

Substituting the values of means and variances of the two beta distributions in terms of their parameters, the variance of the resultant distribution is given by:

$$\sigma^2 = 1/2\ \left[\frac{mn}{(m+n+1)(m+n)^2}\right] + 1/2\ \left[\frac{nm}{(n+m+1)(n+m)^2}\right] + 1/4\ \left[\frac{m}{n+m} - \frac{n}{n+m}\right]^2$$

$$= \frac{mn}{(m+n+1)(m+n)^2} + 1/4\ \left[\frac{m-n}{m+n}\right]^2$$

Rearranging,

$$\sigma^2 = \frac{(m+n) + (m-n)^2}{4\ (m+n)\ (m+n+1)} \tag{2.3}$$

After transforming, if the variance of the tracking signal distribution is equated with equation (2.3). The parameters of the approximating distribution could be found. But the above equation contains two unknowns m and n, as such the parameters of the approximating distribution could not be found easily. To obtain the parameters of the approximating distribution the following approach was adopted. Let the variance of the tracking signal distribution be denoted by Y^2. Then,

$$Y^2 = \frac{(m+n) + (m-n)^2}{4\ (m+n)\ (m+n+1)}$$

or

$$(4Y-1)n^2 + (4Y^2+8Y^2m+2m-1)n + 4Y^2m^2 - m^2 + 4mY^2 - m = 0 \tag{2.4}$$

The above equation has the form

$$An^2 + Bn + C = 0$$

where

$$A = 4Y^2-1$$

$$B = 4Y^2 + 8Y^2m + 2m - 1$$

and

$$C = 4Y^2m^2 - m^2 + 4mY^2 - m$$

For a particular value of Y and m, n will have two values n_1 and n_2. The roots of equation (2.4) were evaluated with different values of m and a

m	n_1	n_2	
4.939	5.416	5.416	<— Point c
5.000	5.191	5.787	
5.100	5.119	6.097	
5.200	5.099	6.357	
5.300	5.101	6.595	
5.400	5.116	6.820	
5.500	5.139	7.036	
5.600	5.168	7.247	
5.700	5.202	7.452	
5.800	5.239	7.655	
5.900	5.279	7.854	
6.000	5.322	8.050	
6.100	5.367	8.245	

Table (2.1) Two Values of n Corresponding to one Value of m by Solving Quadratic Equation

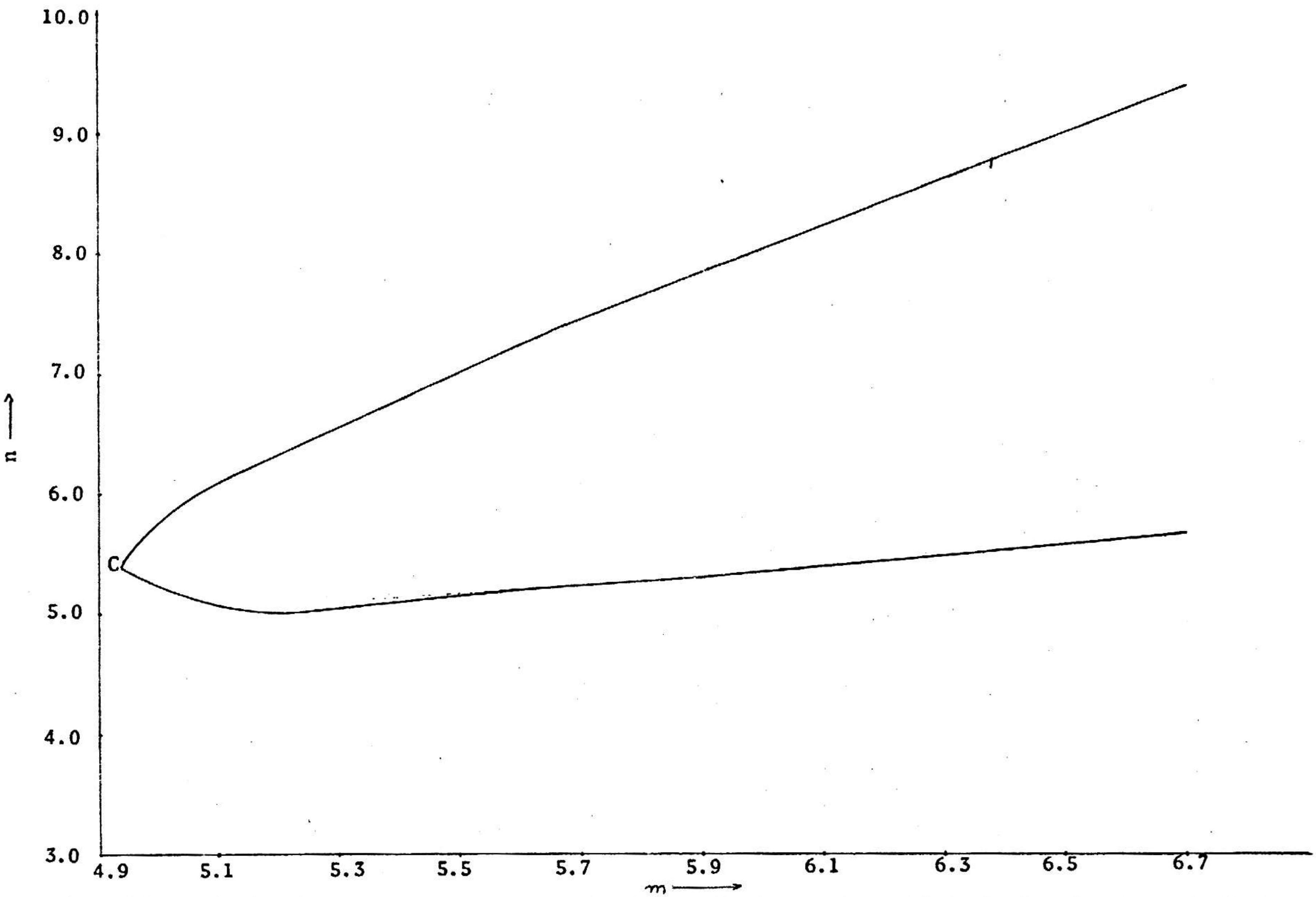


Figure (2.7) Parameters of Beta Distribution to Obtain Resultant Distribution for Approximation fixed value of Y, the standard deviation of the tracking signal distributon. They are tabulated in Table [2.1]. A graph was plotted between different values of m and the corresponding two values of n. This is illustrated in figure [2.7]. Equation [2.4] is quadratic in n and when $B^2 = 4AC$ the two values n_1 and n_2 for a particular value of m will be equal. This is illustrated by point C in figure (2.7). Theoretical frequency distributions were obtained for various pairs of parameters. These frequencies were compared with the actual frequencies of the tracking signal. A chi-square test was applied to test the goodness of fit. Table (2.2) shows the chi-square values with different pairs of parameters. It could be observed that the chi-square value is a minimum for point C (figure (2.7)). In other words the distribution obtained by combining appropriately two beta distributions corresponding to point C was closest approximating distribution. For the tracking signal distribution. Further, the standard duration of the distribution was 0.150045 as compared with 0.15005 of the observed tracking signal distribution. The standard deviation of the closest simple beta distribution was 0.150119. Even the minimum chi-square value exceeded the critical value of 100.70 at the 5% significance level. This shows that the fit is inadequate and the tracking signal distribution cannot be approximated with the resultant distribution obtained by combining two beta distribution. This was confirmed by the Kolmogorov Smirnov test carried out for point C in figure (2.7). The computed value was .00681 which exceeded the critical value of .0043 at 5% significance level. As this approach fialed with higher values of the smoothing constant too, it was tried with smoothing constants lower than 0.1.

m	n	Chi-Square Value	
4.939	5.416	205.9	<— Point c
5.00	5.191	213.3	
5.00	5.786	215.23	
5.10	5.119	233.5	
5.10	6.097	227.7	
5.20	5.099	259.1	
5.20	6.357	240.65	
5.30	5.101	290.8	
5.30	6.595	254.6	
5.40	5.116	327.0	
5.40	6.820	270.59	

Table (2.2) Parameters of the Beta Distribution and the Corresponding Chi-Square Values

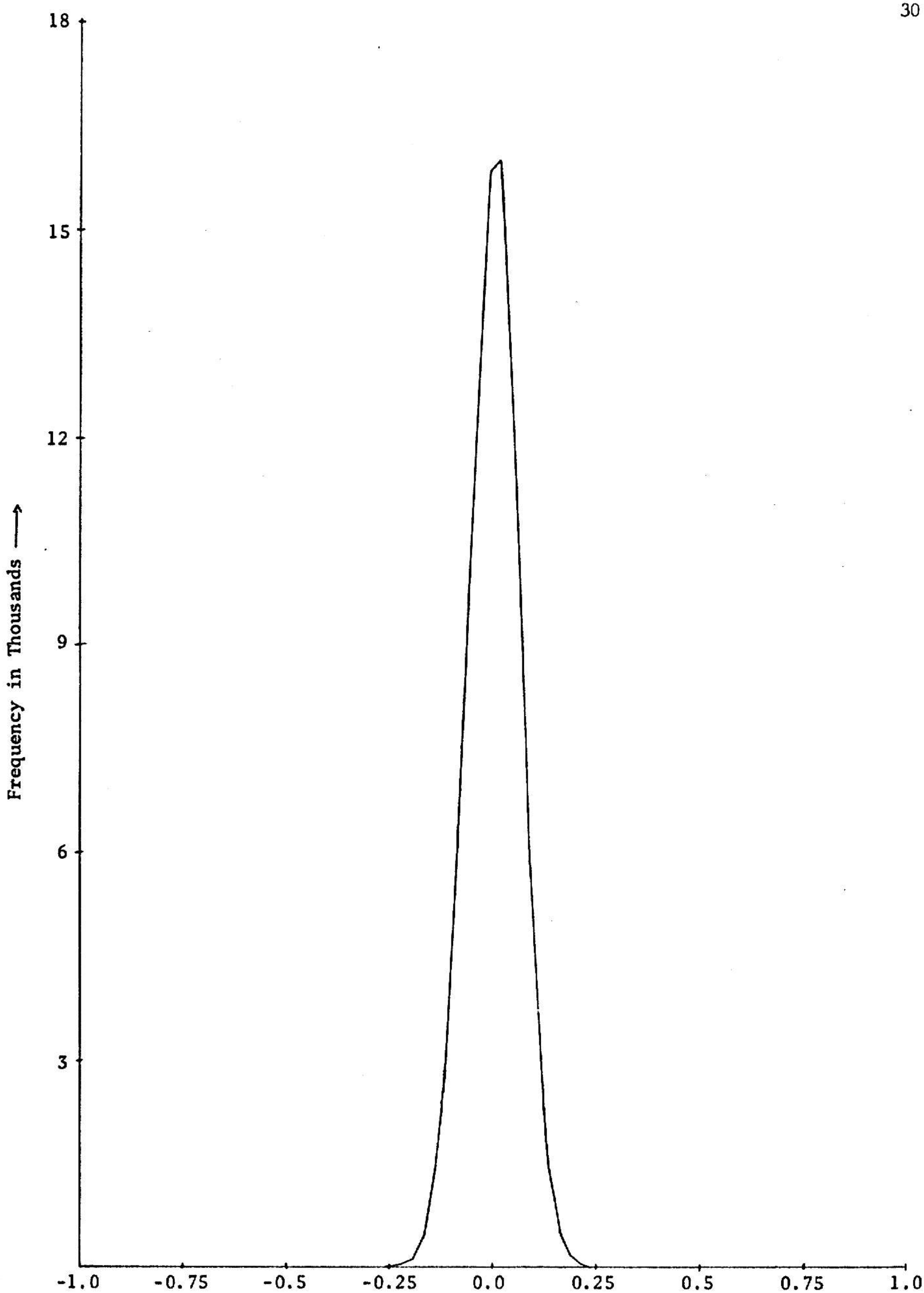


Figure (2.8) The Tracking Signal Distribution for an α of 0.01

2.2.4 EXPERIMENT WITH α OF 0.01

Figure [2.8] illustrates the tracking signal distribution for α of 0.01. It is unimodal and symmetrical with a standard deviation of 0.0306 on the transformed scale. The approach adopted for finding the parameters of the approximating beta distribution, was same as that for 0.1. The parameters of the approximating distribution were obtained from the equation

$$\sigma^2 = \frac{1}{4(2m+1)}$$

with the calculated parameters a theoretical frequency distribution was derived. These frequencies were compared with the observed frequencies for the goodness of fit. The fit was inadequate. The chi-square value was 162.27 as compared with critical value of 32.70 for a 5% significance level. The value obtained by the Kolmogorov Smirnov test was .00479 which exceeded critical value of 0.00473 at the 5% significance level.

2.3 CONCLUSION FROM THE EXPERIMENTAL WORK

From these three experiments it can be concluded that the tracking signal distribution for a normal range of the smoothing constant cannot be approximated adequately with a single beta distribution or a combined beta distribution.

CHAPTER III

THE TRACKING SIGNAL LIMITS BY COMPUTER SIMULATION

When the first approach failed to evaluate the limiting values of the tracking signal, limits were proposed by simulation. Figure [3.1] shows the frequency distribution of the tracking signal for α of 0.1. For illustration let the confidence interval for the tracking signal be 95%. Referring to figure [3.1], one is interested in evaluating values K_1 and K_2 such that 95 percent of the tracking signals lie between K_1 and K_2. Further, a 95% confidence limits on the tracking signal means that without any significant change in the process level, one value out of 20, may exceed the limiting values K_1 or K_2. These limiting values may be evaluated separately. To evaluate K_1 the cumulative frequency of the tracking signal for first 40 intervals, corresponding to range of -1 to 0 were plotted against the tracking signal magnitude. This is illustrated in figure [3.2]. 100,000 observations of the tracking signal were simulated. For 95% confidence, 2.5% of the tracking signals will lie between limiting value K_1 and -1. From figure [3.2], the limiting value K_1 is read directly on the X axis. Corresponding to a cumulative frequency of 2500 on Y axis. A similar procedure was carried out to obtain the limiting value K_2 of the tracking signal lying between 0 and 1 from figure [3.3]. It could be observed that the frequency distribution of the tracking signal is approximately symmetrical about zero. The limits K_1 and K_2, obtained above will be nearly equal in magnitude but opposite in sign. So instead of evaluating two limiting values K_1 and K_2 separately a value K could be evaluated such that the absolute

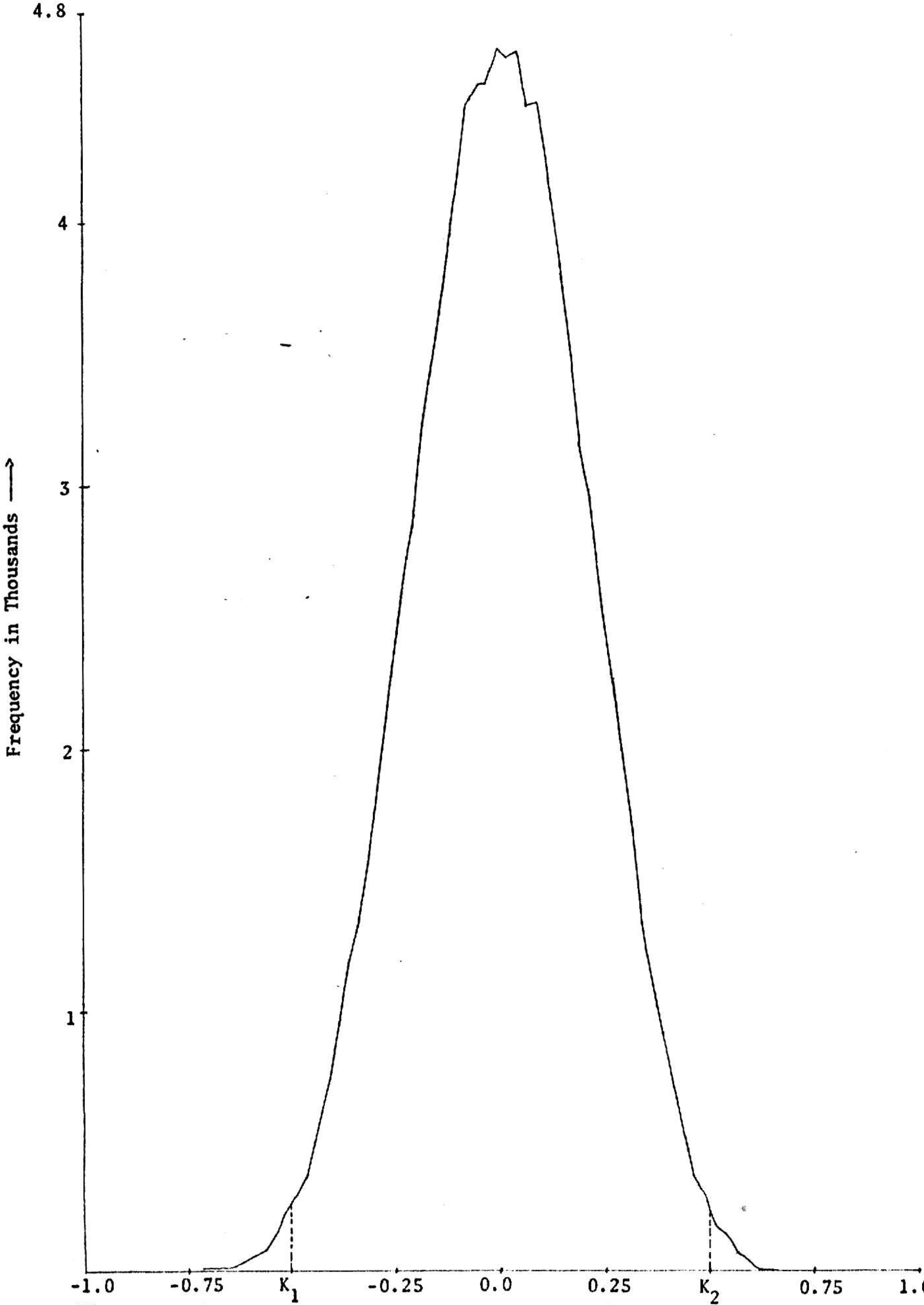


Figure (3.1) The Tracking Signal Distribution from α of 0.1, Depicting Limiting

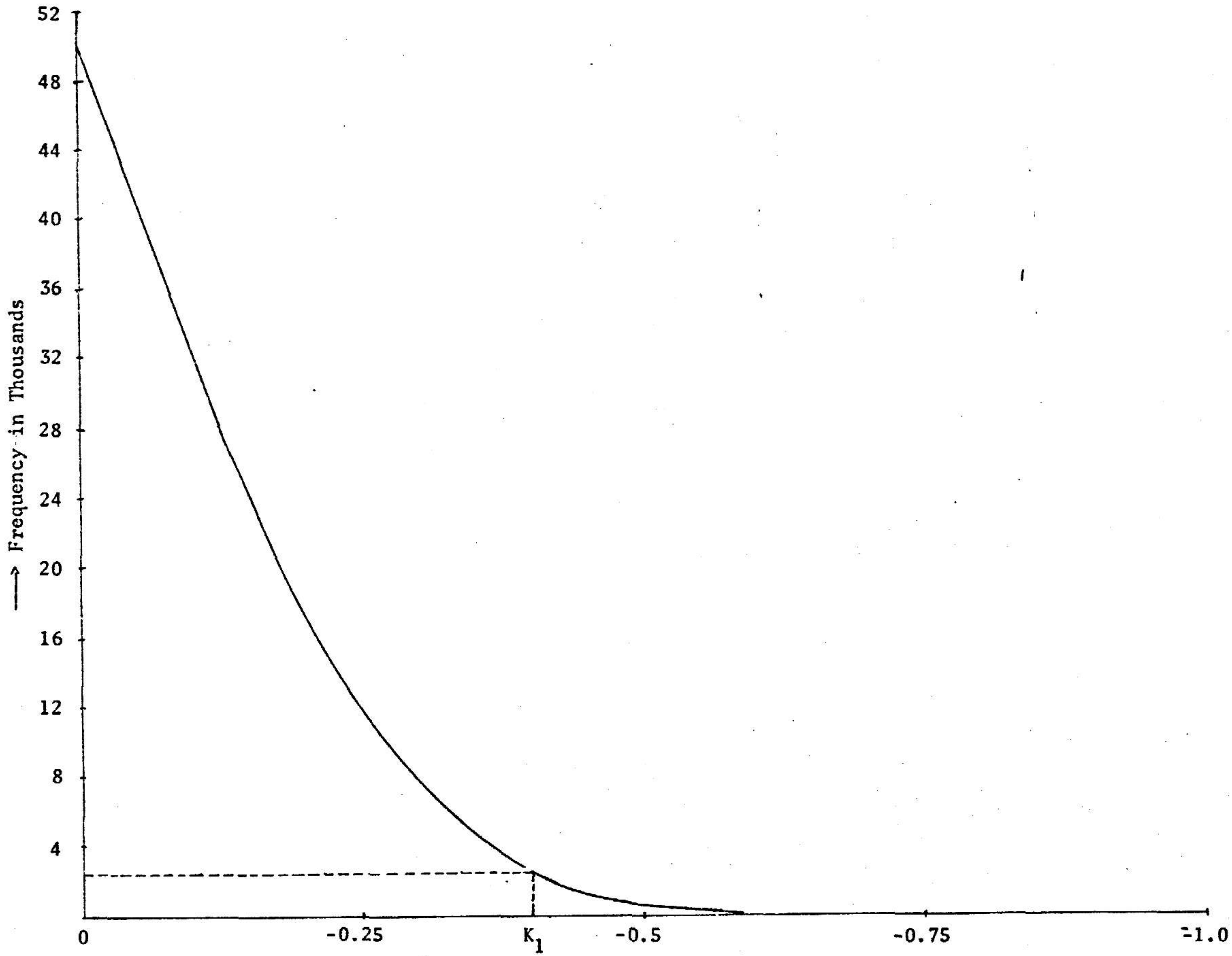


Figure (3.2) Cumulative Frequency Plot for First 40 Cells.

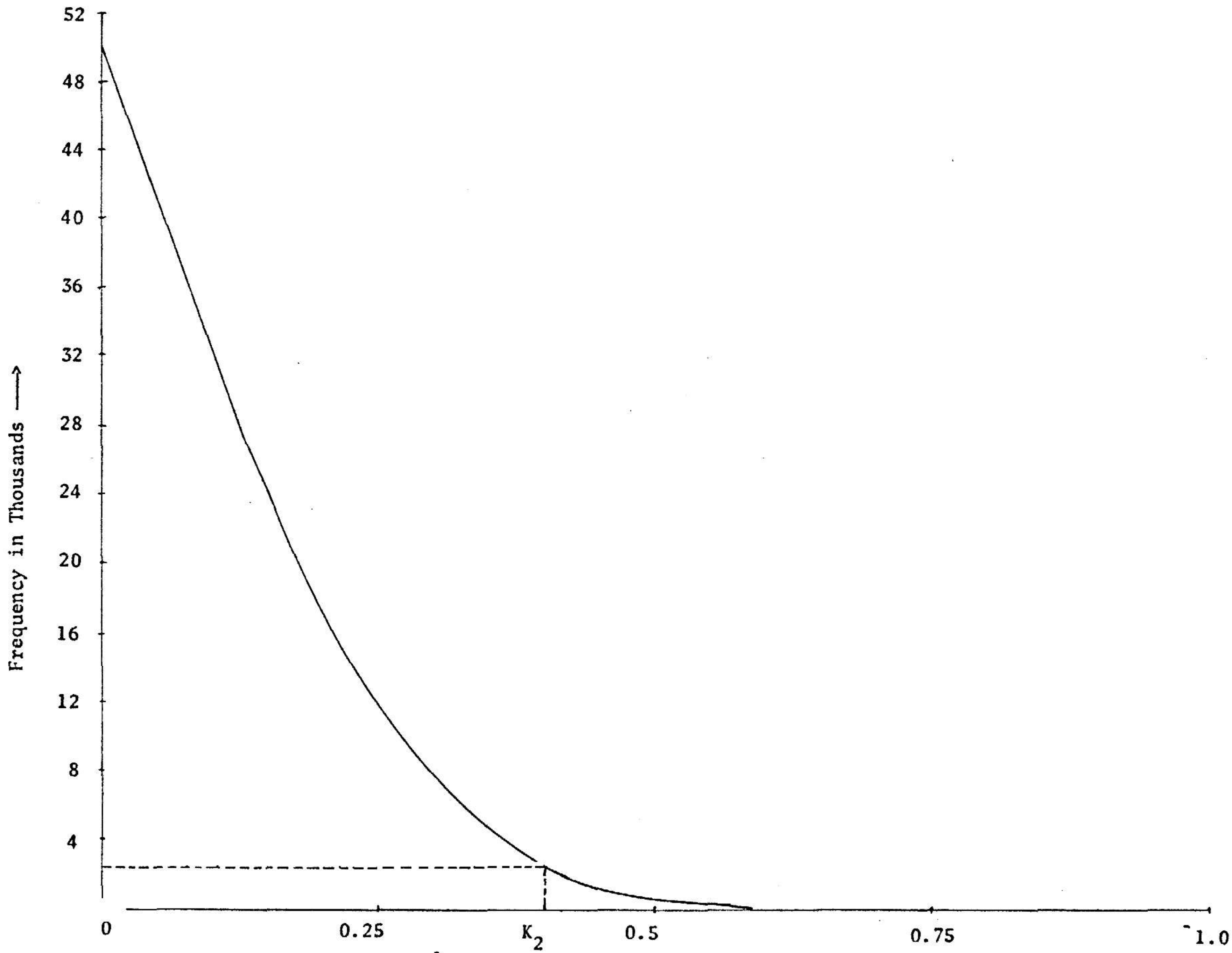


Figure (3.3) Cumulative Frequency Plot for Latter 40 Cells.

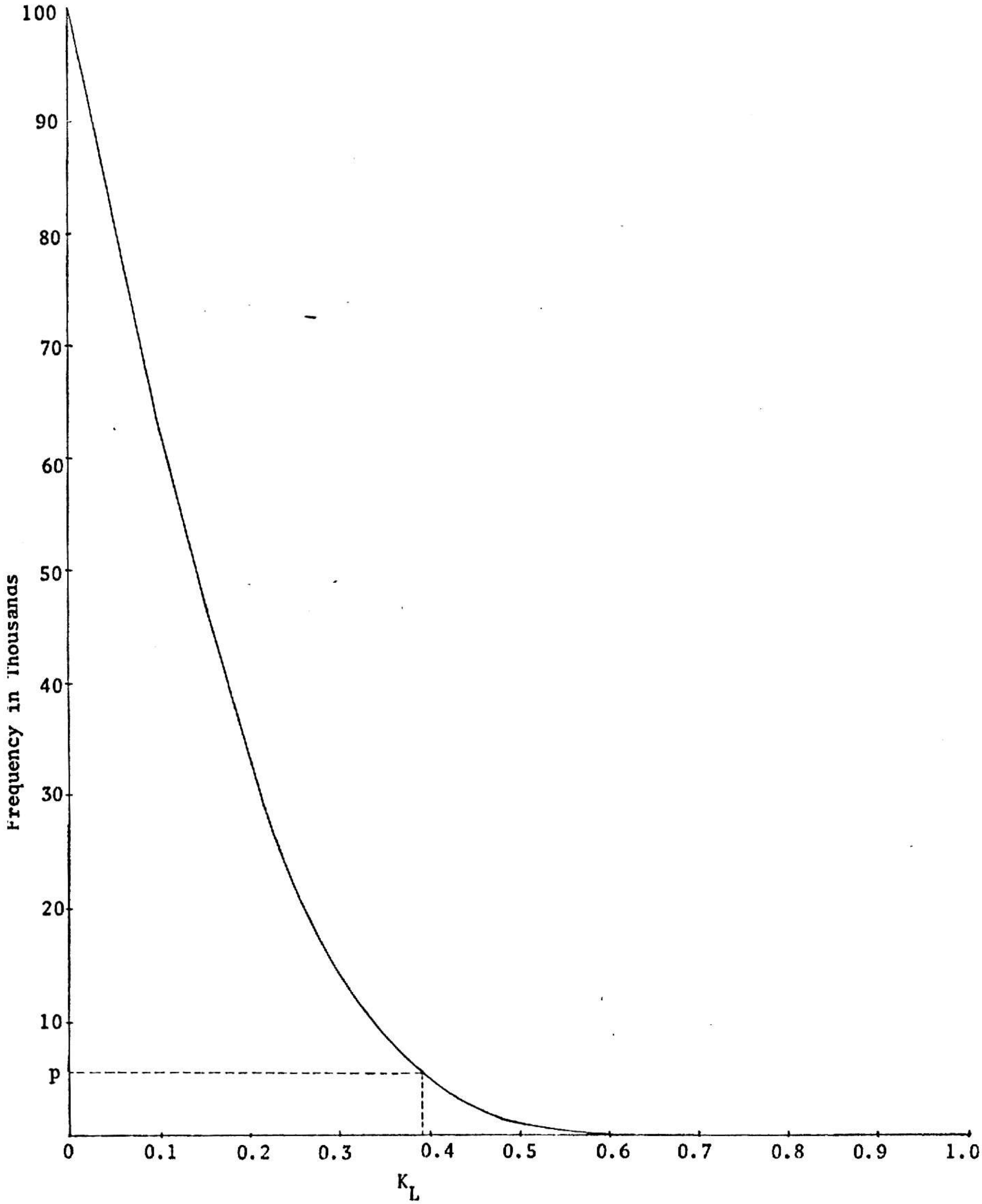


Figure (3.4) Cumulative Frequency Plot for Folded Tracking Signal Distribution

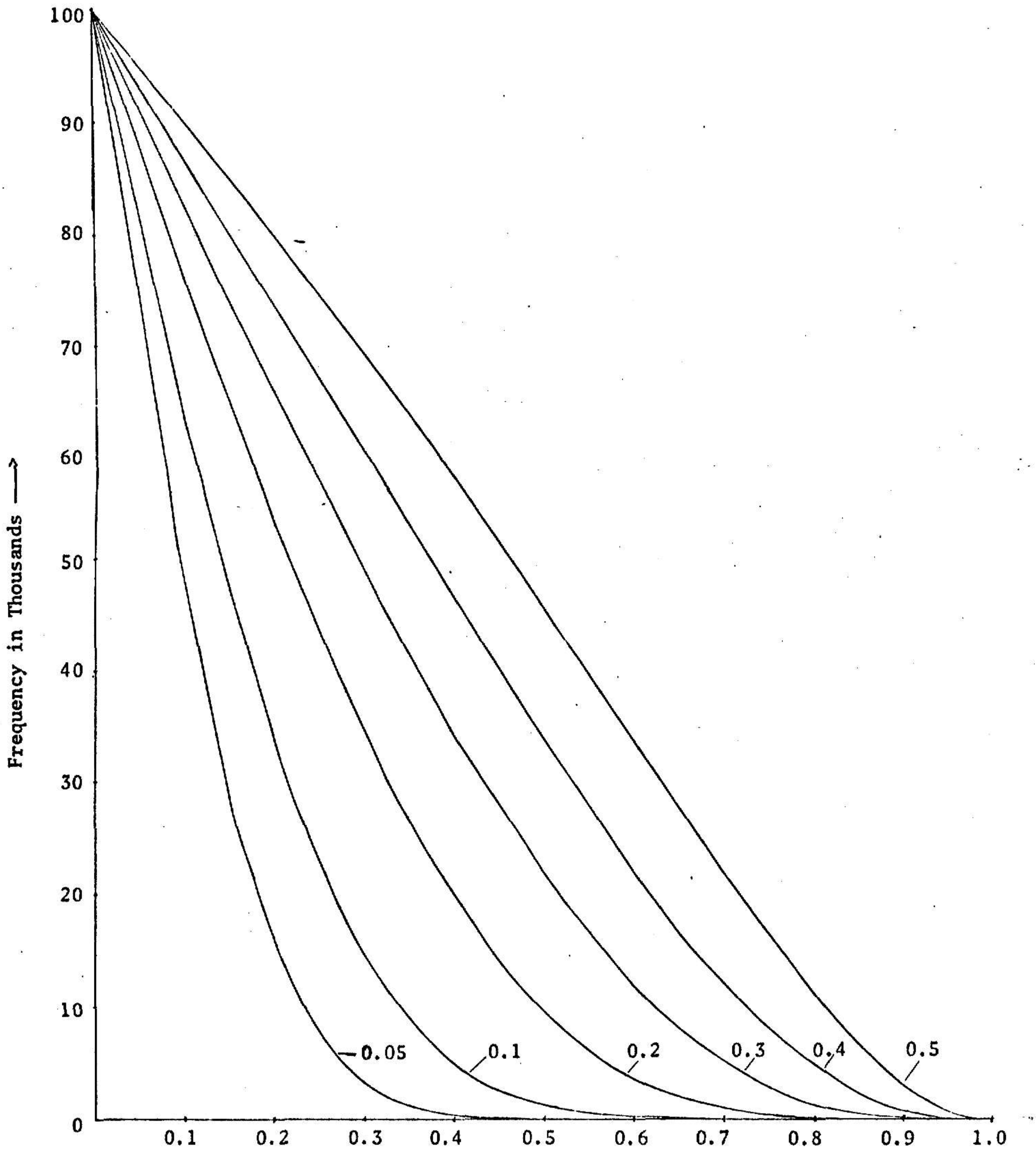


Figure (3.5) Cumulative Frequency Plots for Different Smoothing Constants.

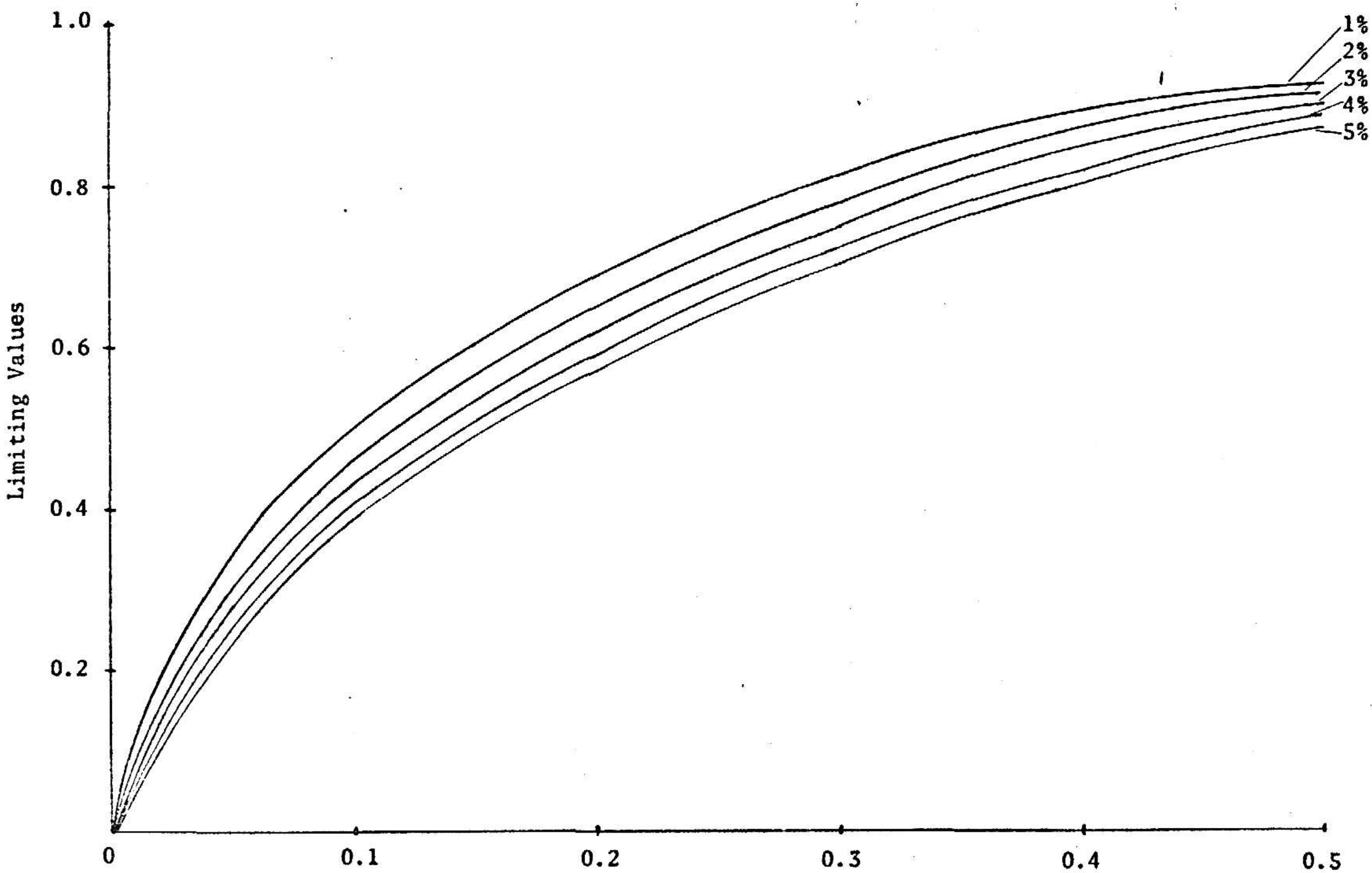


Figure (3.6) Limiting Values of the Tracking Signal for Various Smoothing Constants and Significance Levels.

Significance Level	Smoothing Constant α					
	0.05	0.1	0.2	0.3	0.4	0.5
1	0.360	0.508	0.690	0.810	0.890	0.925
2	0.325	0.468	0.647	0.770	0.864	0.918
3	0.306	0.438	0.618	0.746	0.835	0.900
4	0.290	0.415	0.590	0.724	0.820	0.885
5	0.278	0.400	0.570	0.703	0.800	0.872
10	0.235	0.342	0.497	0.625	0.725	0.815
15	0.205	0.300	0.444	0.570	0.670	0.765
20	0.185	0.270	0.402	0.520	0.620	0.720
30	0.148	0.219	0.330	0.436	0.534	0.634

Table (3.1) Limiting Values of the Tracking Signal value of K represents the limiting values of the tracking signal. For this purpose, the tracking signal distribution was folded about zero. The tracking signal which was classified into 80 equal intervals previously was reclassified into 40 equal intervals. The frequencies in these 40 intervals were approximately twice as much as those in the unfolded distribution. Cumulative frequencies in each interval were calculated. Effectively, the cumulative frequencies were plotted against the absolute values of the tracking signal. For α of 0.1, this curve is depicted in figure [3.4]. Such plots were obtained for the smoothing constants varying from 0.05 to 0.5. This is depicted in figure [3.5]. From figure [3.5], the limiting values of the tracking signal were evaluated for different smoothing constants and for various levels of significance. These limiting values are tabulated in table [3.1]. For illustration, the frequency corresponding to point P on the Y axis of figure [3.4] is five percent of the total frequency. The corresponding value K_L on the X axis of this graph gives the limiting value of the tracking signal for α of 0.1 and for 5% significance.

A graph was next plotted between the limiting values and the corresponding significance level, for different smoothing constants figure [3.6] from this figure one can find the limiting values of the tracking signal for any level of confidence and for any smoothing constant included in the graph.

The entries of this table are approximately the same as proposed by Batty. However, the details of simulation were not explained explicitly by Batty. In particular there was no indication as to the number of observations generated, hence the reliability of the limits.

CHAPTER IV

RESPONSE OF THE TRACKING SIGNAL TO PROCESS CHANGE

In this section, the behavior of the tracking signal to a process change has been investigated. Consider a constant process, which initially has a mean of a_1. At a certain time T_1, let the process level change to a new mean of a_2. Mathematically, the process can be represented as:

$$\xi_t = a_1 \qquad \text{for } t < T_1$$

and

$$\xi_t = a_2 \qquad \text{for } t \geq T_1$$

Effectively, at time T_1 a step increase of $(a_2 - a_1)$ has taken place in the process level. Such a situation is illustrated in figure (4.1).

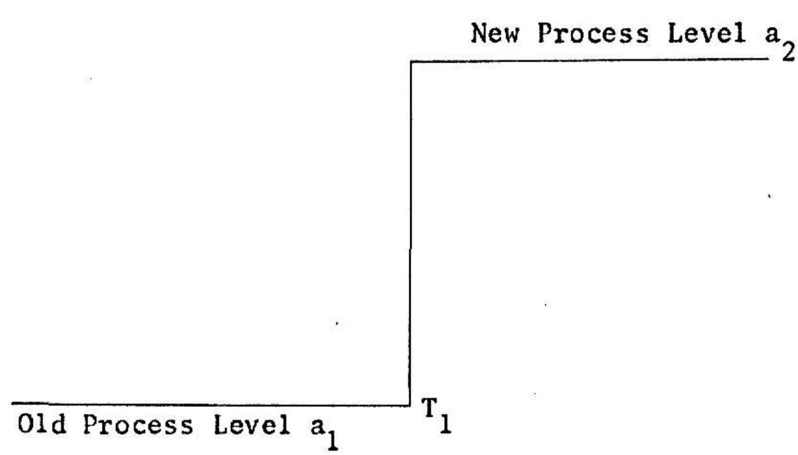


Figure (4.1) Step increase in the process level

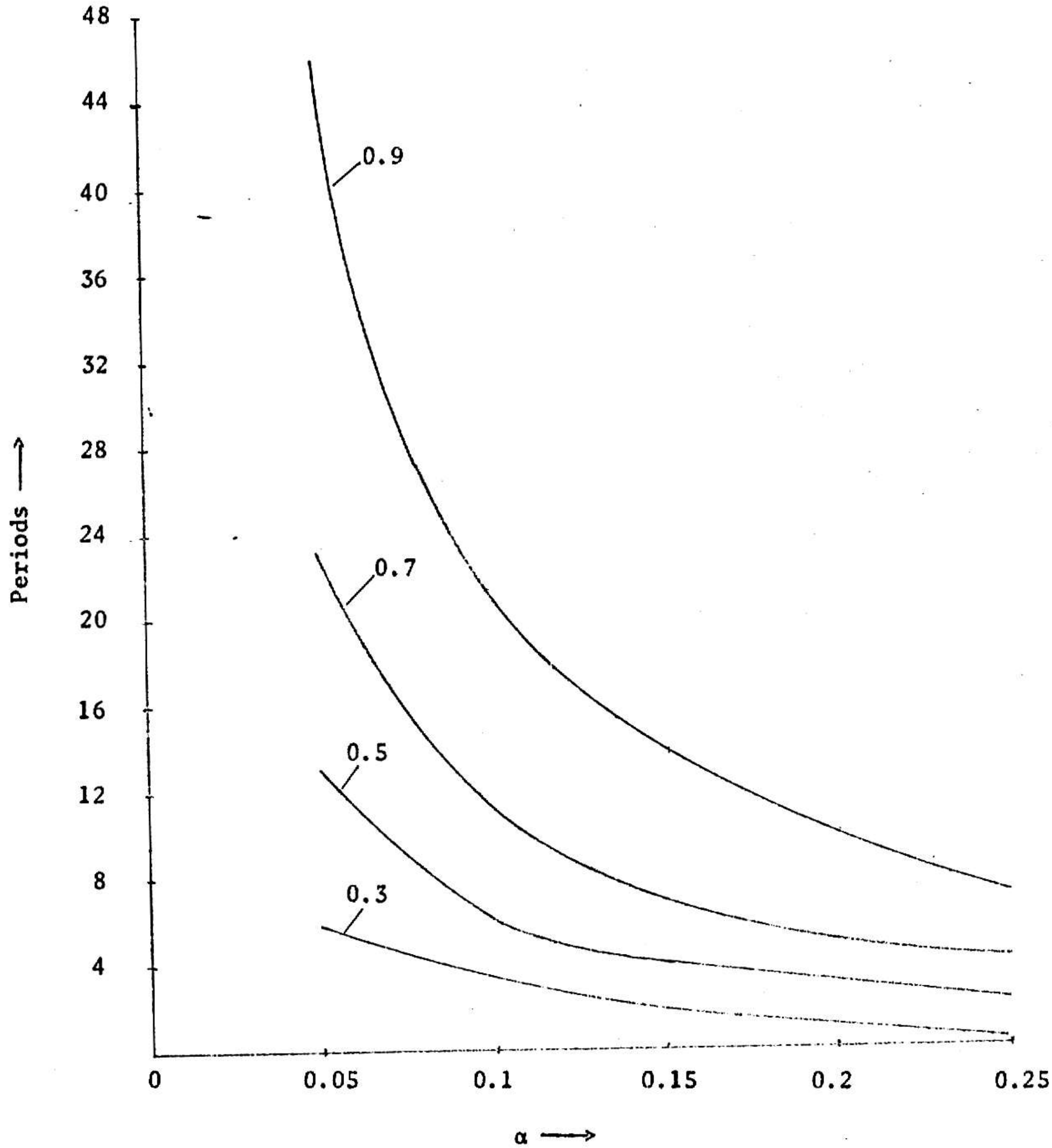
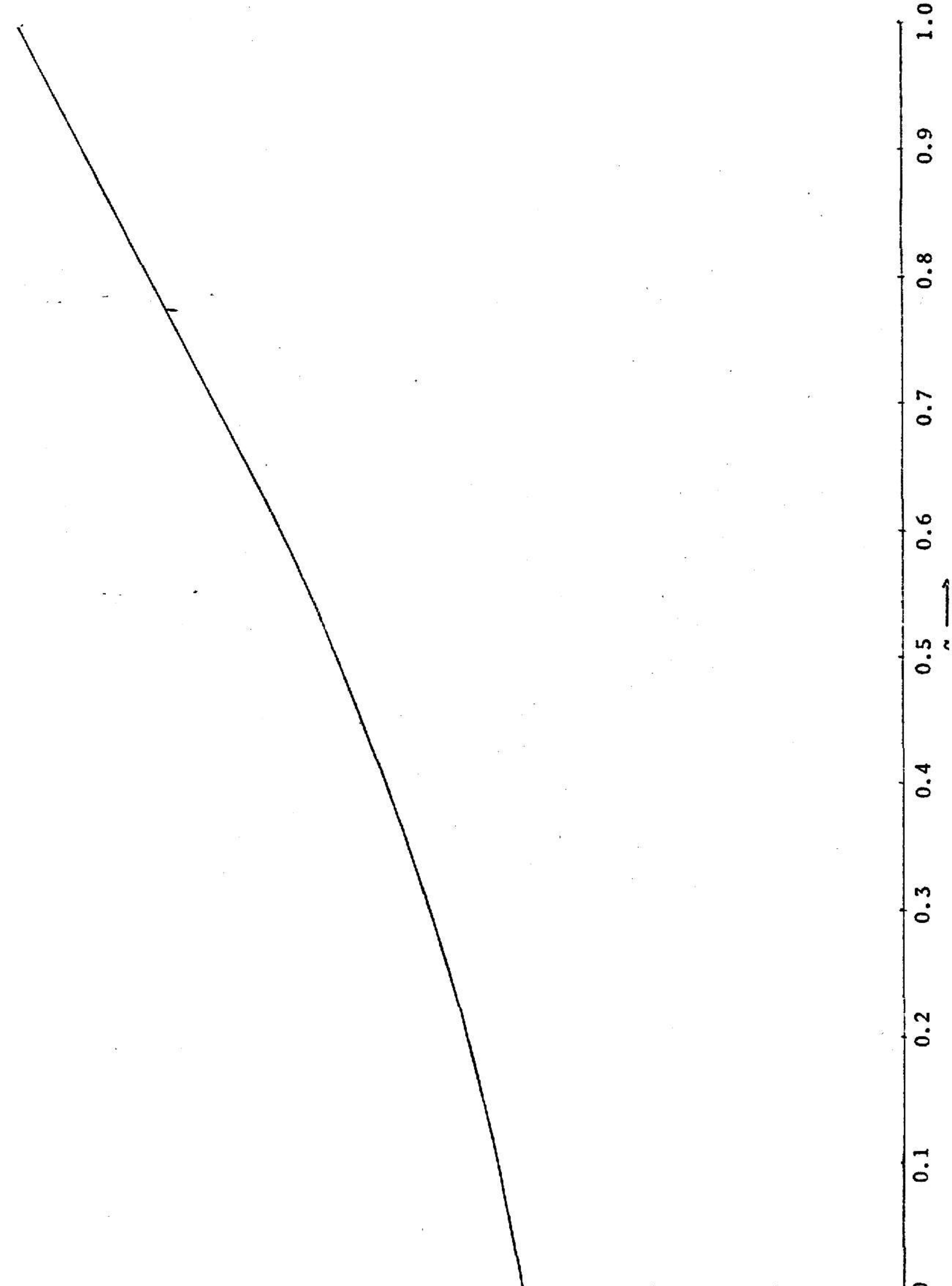


Figure (4.2) Effect of Smoothing Constant on the Time Required to Respond to a Fraction of Step Input.


1.4
1.3
1.2
1.1
1.0
0.9
0.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fraction of the Step 'f'	Smoothing Constant				
	0.05	0.1	0.15	0.2	0.25
0.1	2	1	-	-	-
0.3	6	3	2	-	-
0.5	13	6	4	3	2
0.7	23	11	7	5	4
0.9	46	21	14	10	8

Table (4.1) No. of Periods Required to Achieve a Fraction 'f' of Step for Various Values of the Smoothing Constants

As the forecasting system uses exponential smoothing to predict future observations, the forecasts will constantly lag the observations. According to Brown [1], using exponential smoothing, the predicted values can never attain the new level in a finite period. However, it can achieve a very high fraction of the step (a_2-a_1) in a finite period. Brown pp. 114, has developed a relation which evaluates the number of periods required to attain any fraction of the step, for various values of the smoothing constant.

$$n = \frac{\log (1-f)}{\log \beta} - 1$$

where n = number of periods required

f = the fraction of the step

β = discount factor = $(1-\alpha)$.

For various values of f and α, the number of periods required, were evaluated using the above relation. The results are tabulated in Table (4.1). Figure (4.2) illustrates relation between α, f and small periods required. From this figure it can be said that a high value of the smoothing constant is desirable for any forecasting system, for a rapid recovery from process changes. Higher values of the smoothing constant will lead to increased magnitude of the forecast errors. Figure (4.3) shows the effect of the smoothing constant on forecast accuracy. Small values of α lead to accurate forecasts but the forecasting system takes a longer time to recover from the process change. Thus, these relative advantages and disadvantages are to be suitably weighed while selecting a smoothing constant. Figure (4.2) also helps in investigating the sensitivity of

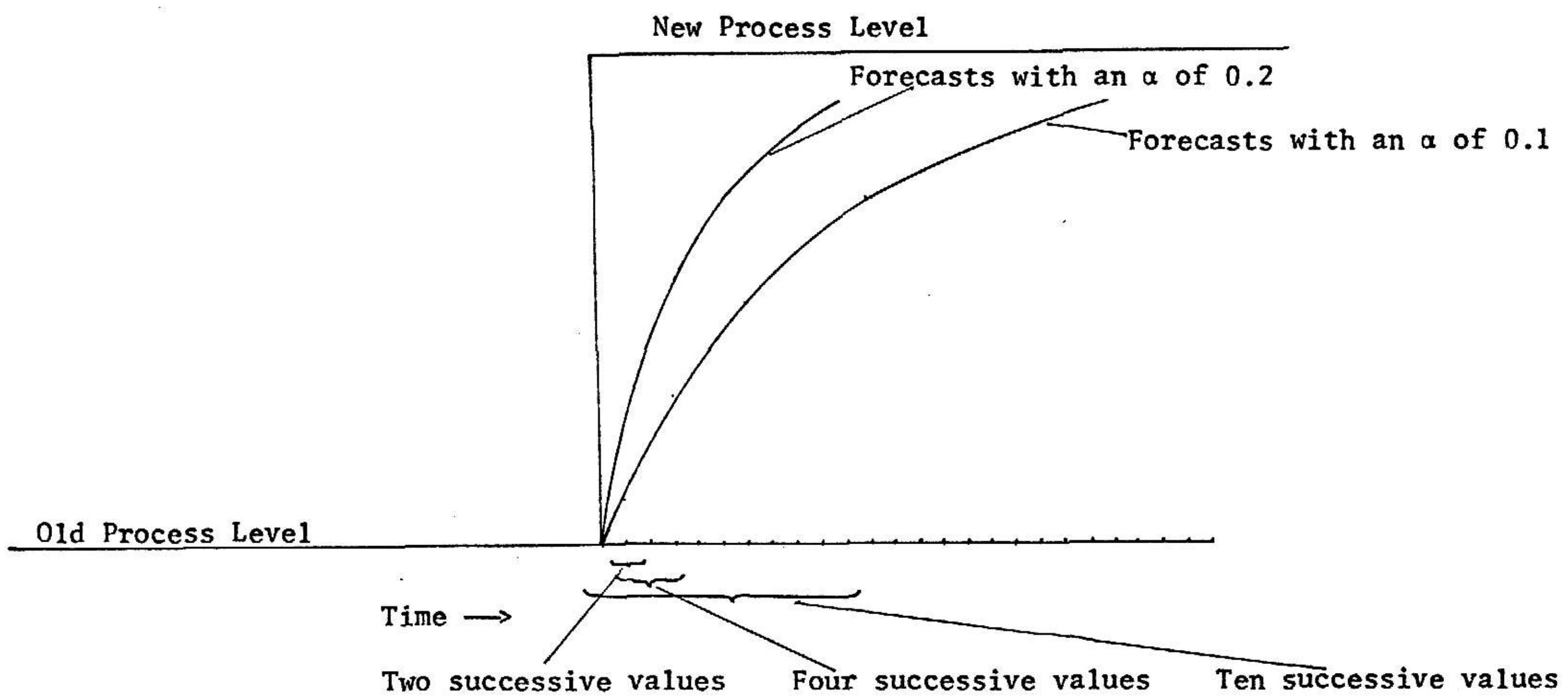


Figure (4.4) Illustration of detection criteria the tracking signal to process changes. It can be seen from this figure, for an α of 0.1, the number of periods required to achieve 90% of the step is 21. The tracking signal should give an indication of the process change at a very early stage. If it does not give an indication, the system will adjust to the new level by itself. For example after 21 periods the system will more or less achieve the new process level. From the sensitivity point of view this is the maximum period one can allow for the tracking signal to give an indication of the process change. It is desired to detect the process change before the system achieves 20 to 30 percent of the step. The sooner the detection, the more effective the tracking signal.

The sensitivity of the tracking signal was studied by simulation of a constant process. After the starting conditions were discounted, a step in the process level was introduced. The magnitude of the step varied from 2.5 to 25 percent of the process mean of 10. A change in the process was confirmed by formulating different detection criteria. For all criteria and all values of α, the control limits were set at a level corresponding to a 99 per cent confidence. The detection criteria were essentially based on examining a few tracking signals after incorporating a step in the process level. Referring to figure (4.4) the criteria for detecting process change were:

a) If one value of the tracking signal in two successive values exceeded the limiting value

b) If one value of the tracking signal in 4 successive values exceeded the limiting value

c) If one value of the tracking signal in 10 successive values exceeded the limiting values

d) If any two consective values out of ten successive values exceeded the limiting value. These detection systems were simulated 1000 times. The probability of detecting a process change was evaluated. The results are tabulated in Table (4.2 to 4.5). Figures (4.5 to 4.8) depict the relation between step size and probability of detection for various detection criteria.

RESULTS AND DISCUSSION

The results from the first detection system revealed that the probability of detecting a process change is low. Even with a 25 percent change in the process mean, this system gives a 0.171 probability of detecting a process change for an α of 0.05. The probability of detecting increases with the smoothing constant. The probabilities obtained with the second detection system are a little higher than those obtained by first detection system. For an α of 0.05 and for a 25 percent change in the process mean, a detection probability of 0.643 is attained. Based on the second detection system it can be concluded that the first detection system is less effective in detecting process changes.

The third and fourth detection systems have approximately the same probabilities of detection for higher magnitudes of the step. In both systems of an α of 0.05 and a 25 percent change in the process mean, the probability of detection is 1. However, for smaller magnitudes of the step, the third detection system has a higher probability of detection. As compared with first two detection systems, the probability of detection decreased with increased values of α. This reveals that with higher values of α, the forecasting system often adjusts to the new situation by itself. For all values of α and for all detection systems the probability of detection corresponding to a step of zero percent is referred to false alarms given by the tracking signal.

When the adopted detection criteria is one of these four, these probability tables (4.2 to 4.5) will help in deciding whether the system needs updating. However, the decision of updating the system is entirely left to the one in charge of the system. The decision will essentially depend upon considering the severity of penalties resulting from updating or not updating the system.

The further work in this field can be carried out by investigating various other detection criteria. Another effect which could be studied is the change in detection probabilities that occur when specifying different tracking signal limits at other confidence levels.

Smoothing Constant 'α'	Step Increase in the Process Mean (%)										
	0	2.5	7	7.5	10	1.25	15	17.5	20	22.5	25
0.05	0.012	0.011	0.016	0.025	0.035	0.050	0.057	0.078	0.096	0.126	0.171
0.10	0.019	0.019	0.025	0.025	0.042	0.056	0.081	0.104	0.140	0.189	0.217
0.15	0.015	0.015	0.027	0.036	0.047	0.066	0.103	0.129	0.160	0.211	0.252
0.2	0.022	0.029	0.034	0.046	0.068	0.082	0.112	0.144	0.184	0.236	0.293
0.25	0.021	0.023	0.030	0.043	0.064	0.083	0.117	0.153	0.196	0.242	0.288

Table (4.2) Probabilities of Detecting Process Change by Criteria (a)

Smoothing Constant 'α'	Step Increase in the Process Mean (%)										
	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0
0.05	0.015	0.023	0.035	0.062	0.115	0.189	0.271	0.344	0.429	0.537	0.643
0.10	0.026	0.043	0.058	0.098	0.158	0.228	0.338	0.446	0.550	0.666	0.753
0.15	0.031	0.033	0.055	0.093	0.159	0.253	0.358	0.464	0.594	0.704	0.786
0.20	0.040	0.042	0.064	0.112	0.168	0.251	0.353	0.460	0.555	0.651	0.739
0.25	0.026	0.030	0.064	0.102	0.157	0.238	0.321	0.419	0.521	0.614	0.695

Table (4.3) Probability of Detecting A Process Change by Criteria (b)

Smoothing Constant 'α'	Step Increase in the Process Mean (%)										
	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0
0.05	0.044	0.079	0.179	0.372	0.585	0.796	0.923	0.971	0.994	0.999	1.000
0.10	0.056	0.089	0.206	0.391	0.586	0.787	0.906	0.968	0.992	0.999	1.000
0.15	0.069	0.085	0.179	0.343	0.529	0.694	0.834	0.924	0.961	0.987	0.999
0.20	0.074	0.102	0.171	0.300	0.437	0.611	0.742	0.847	0.913	0.952	0.977
0.25	0.082	0.101	0.158	0.260	0.391	0.542	0.673	0.780	0.865	0.905	0.944

Table (4.4) Probability of Detecting a Process Change by Criteria (c)

Smoothing Constant 'α'	Step Increase in the Process Mean (%)										
	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5	25.0
0.05	0.019	0.042	0.105	0.238	0.462	0.663	0.841	0.935	0.978	0.994	1.000
0.10	0.018	0.040	0.093	0.226	0.408	0.607	0.797	0.914	0.966	0.991	0.998
0.15	0.014	0.043	0.088	0.185	0.337	0.506	0.683	0.806	0.899	0.946	0.983
0.20	0.023	0.039	0.067	0.139	0.248	0.378	0.512	0.662	0.783	0.868	0.923
0.25	0.025	0.027	0.054	0.098	0.179	0.282	0.403	0.542	0.652	0.756	0.827

Table (4.5) Probability of Detecting a Process Change by Criteria (d)

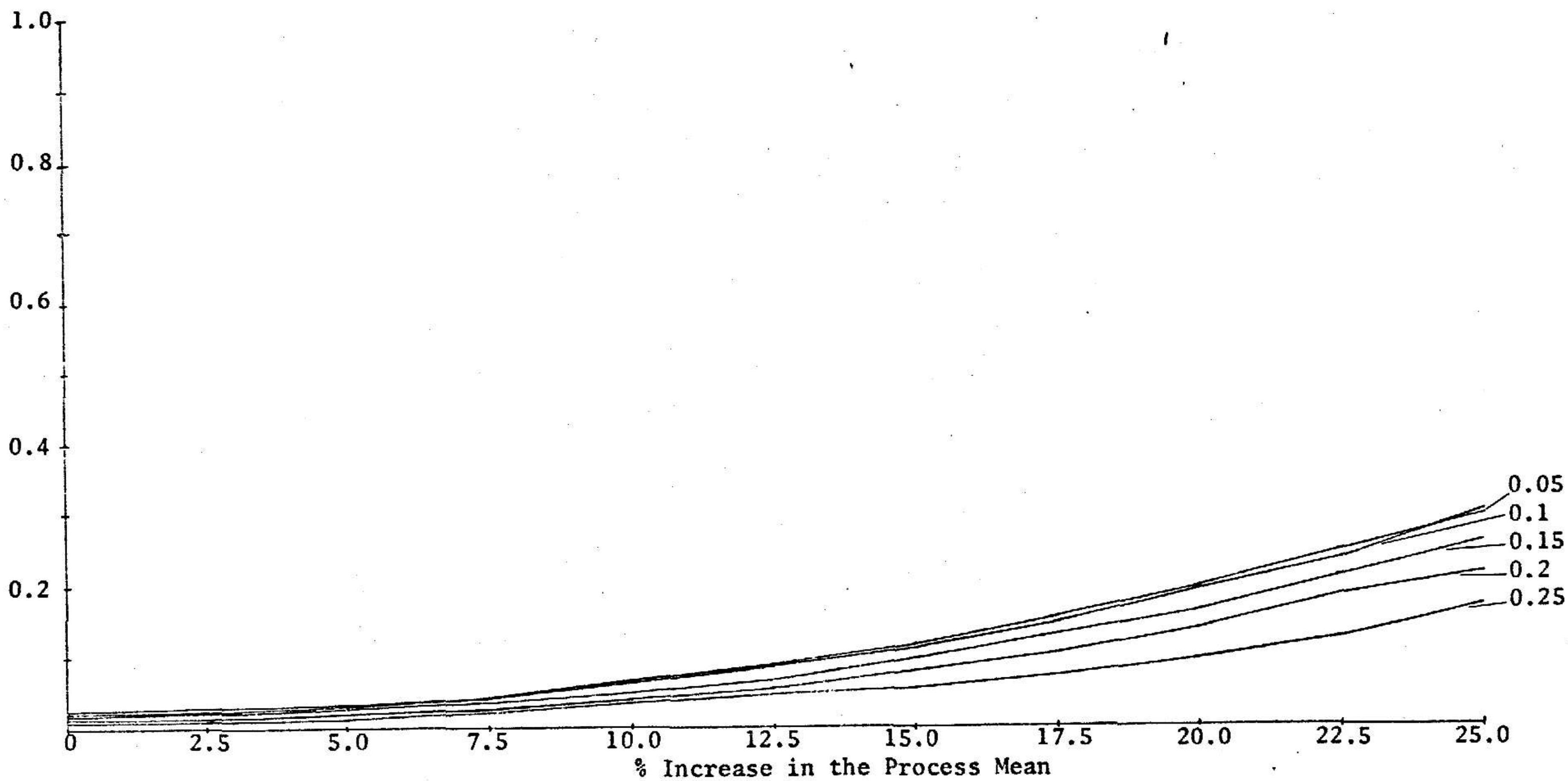


Figure (4.5) Probability Curve for Detection Criterion (a)

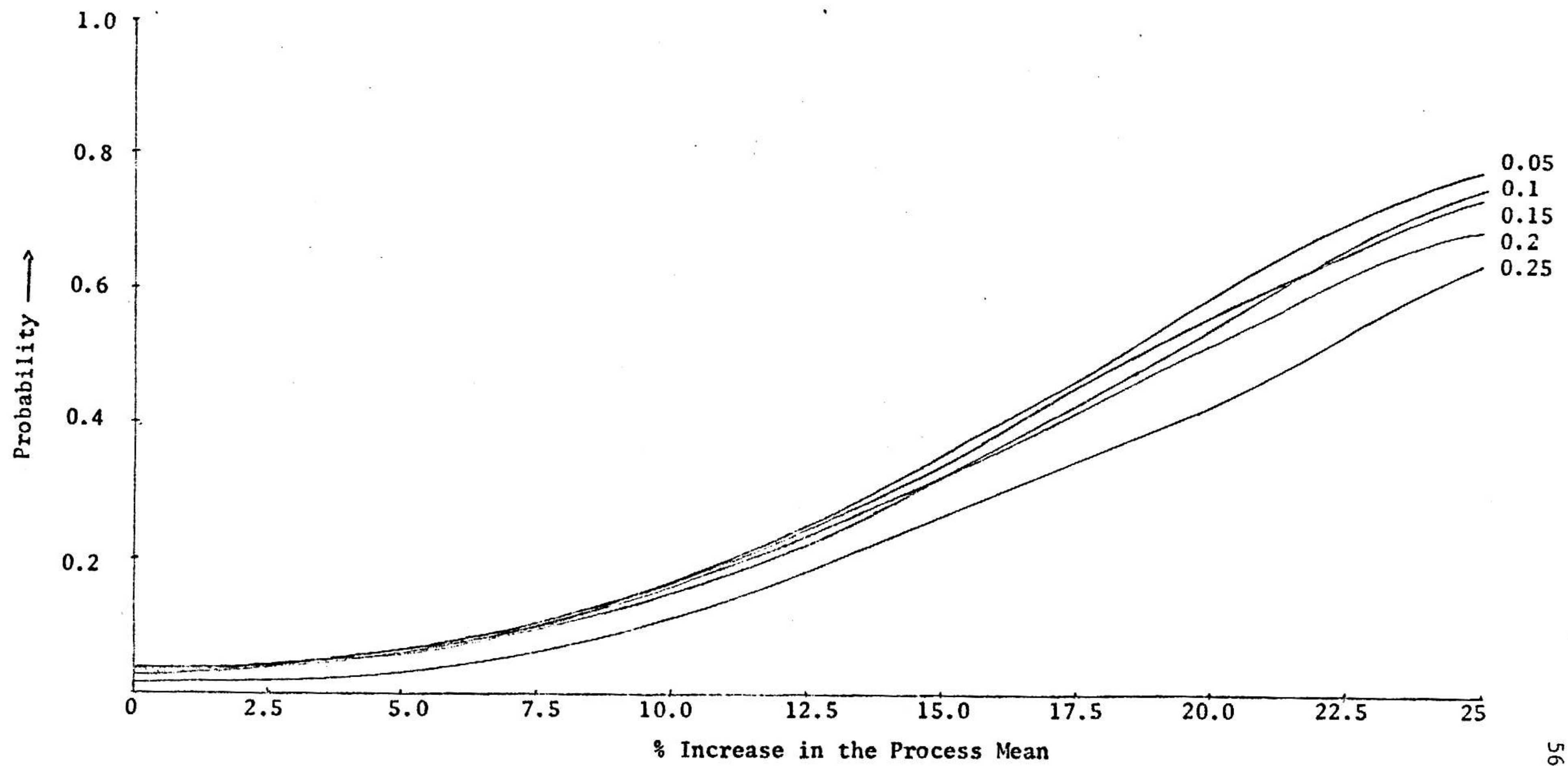


Figure (4.6) Probability Curve for Detection Criterion (b)

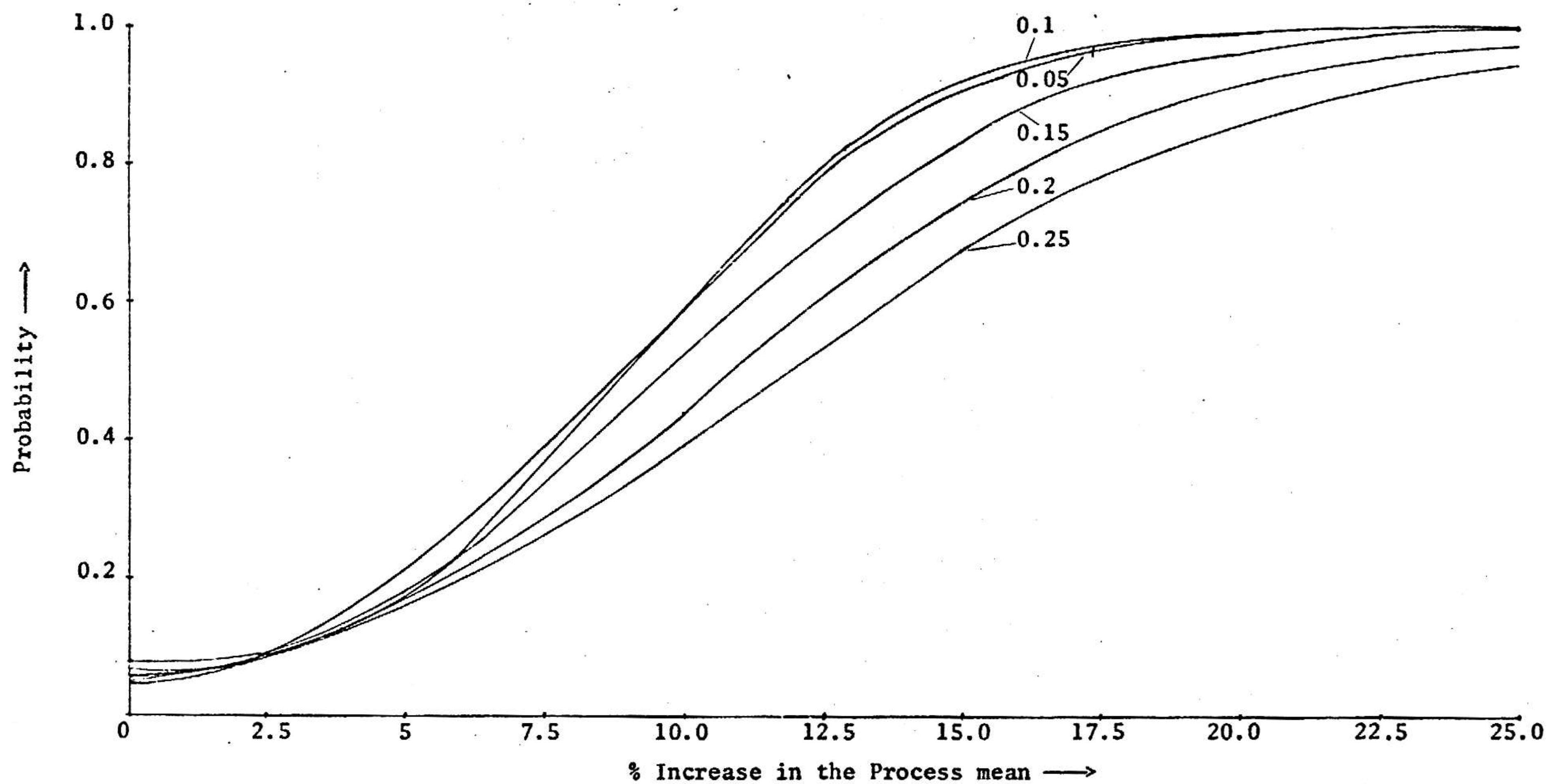
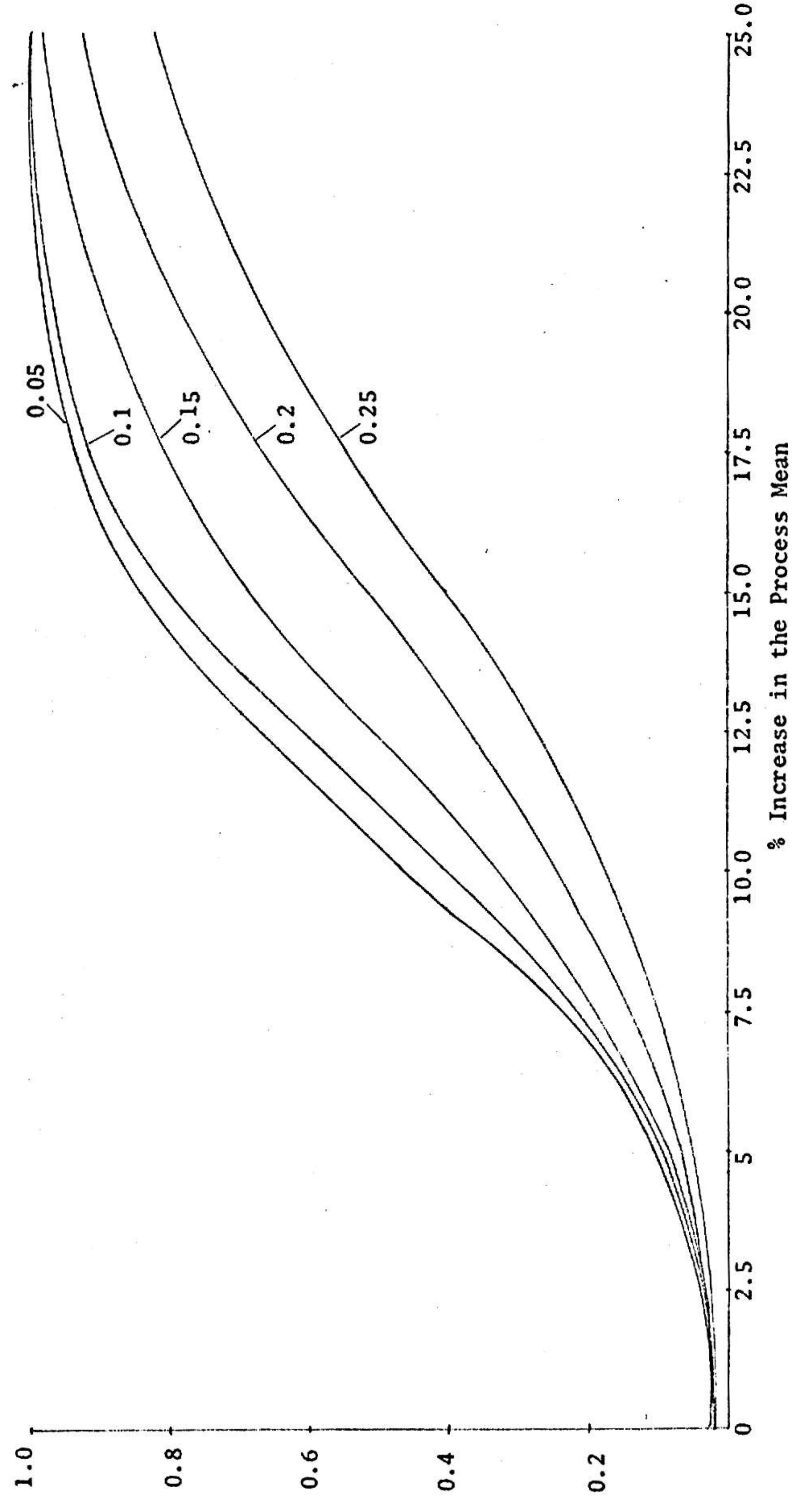


Figure (4.7) Probability Curves for Detection Criterion (c)


1.0
0.8
0.6
0.4
0.2
0
2.5
5
7.5
10.0
12.5
15.0
17.5
20.0
22.5
25.0
% Increase in the Process Mean
0.05
0.1
0.15
0.2
0.25

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APPENDIX A

Comparison of Frequencies for Chi-Square and
Kolmogorov Smirnov Tests for Goodness of Fit

Smoothing Constant = 0.1

Approximating Distribution = Beta Distribution with Parameters m = n = 11.35

Comparison of Frequencies for Chi-Square Test

CELL NO.	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NO.	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
1	0	0	21	304	284.33
2	0	0	22	372	390.35
3	0	0	23	531	522.50
4	0	0	24	685	683.11
5	0	0	25	919	873.73
6	0	0	26	1215	1094.78
7	0	0.02	27	1343	1345.45
8	0	0.07	28	1616	1623.48
9	0	0.22	29	1915	1925.15
10	0	0.59	30	2260	2245.22
11	0	1.43	31	2576	2577.13
12	2	3.13	32	2850	2913.08
13	3	6.35	33	3257	3244.40
14	10	12.03	34	3511	3561.83
15	17	21.49	35	3787	3855.96
16	40	36.49	36	4079	4117.64
17	52	59.19	37	4445	4338.43
18	80	92.23	38	4528	4510.98
19	137	138.58	39	4512	4629.43
20	230	201.50	40	4557	4689.70

CELL NO.	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NO.	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
41	4633	4689.70	61	183	201.50
42	4644	4629.43	62	150	138.58
43	4441	4510.98	63	76	92.23
44	4456	4338.43	64	56	59.19
45	4164	4117.64	65	35	36.49
46	3850	3855.96	66	11	21.49
47	3526	3561.83	67	10	12.03
48	3149	3244.40	68	4	9.48
49	2953	2913.08	69	7	2.02
50	2561	2577.13	70	4	0.22
51	2308	2245.22	71	2	0.07
52	1977	1925.15	72	0	0.02
53	1706	1623.48	73	0	0
54	1295	1345.45	74	0	0
55	1094	1094.78	75	0	0
56	893	873.73	76	0	0
57	696	683.11	77	0	0
58	524	522.50	78	0	0
59	368	390.35	79	0	0
60	291	284.33	80	0	0

The Obtained Chi-Square Value = 79.44

The Critical Chi-Square Value = 73.30, for 55 degrees of freedom and 5% significance level.

Smoothing Constant = 0.1

Approximating Distribution = Beta Distribution with Parameters m=n=11.35.

Comparison of Cumulative Frequencies for Kolmogorov Smirnov Test

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'
1	0	0.0	21	875	857.65
2	0	0.0	22	1247	1248.00
3	0	0.0	23	1778	1770.00
4	0	0.0	24	2463	2453.61
5	0	0.0	25	3382	3327.34
7	0	0.02	27	5940	5767.57
8	0	0.09	28	7556	7391.05
9	0	0.31	29	9471	9316.20
10	0	0.90	30	11731	11561.42
11	0	2.33	31	14307	14138.55
12	2	5.46	32	17157	17051.63
13	5	11.81	33	20414	20296.03
14	15	23.84	34	23925	23857.86
15	32	45.33	35	27712	27713.82
16	72	81.82	36	31791	31831.46
17	124	141.01	37	36236	36169.89
18	204	233.24	38	40764	40680.87
19	341	371.82	39	45276	45310.30
20	571	573.32	40	49933	50000.00

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'
41	54566	54689.70	61	99645	99628.18
42	59210	59319.13	62	99795	99766.76
43	63651	63830.11	63	99871	99858.99
44	68107	68168.54	64	99927	99918.18
45	72271	72286.18	65	99962	99954.67
47	79647	79703.97	67	99983	99988.19
48	82796	82948.37	68	99987	99994.54
49	85749	85861.45	69	99994	99997.67
50	88310	88438.58	70	99998	99999.10
51	90618	90683.80	71	100,000	99999.69
52	92595	92608.95	72	100,000	99999.91
53	94301	94232.43	73	100,000	99999.98
54	95596	95577.88	74	100,000	100,000.00
55	96690	96672.66	75	100,000	100,000.00
56	97583	97546.39	76	100,000	100,000.00
57	98279	98229.50	77	100,000	100,000.00
58	98803	98752.00	78	100,000	100,000.00
59	99171	99142.35	79	100,000	100,000.00
60	99462	99426.68	80	100,000	100,000.00

Maximum difference $|f_0 - f_t| = 179$

The obtained Kolmogorov Smirnov Value $= \frac{179}{100,000} = .00179$

The Critical Value" $= \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{100,000}} = .00430$ for 5% Significance level.

Smoothing Constant = 0.2

Approximating Distribution = Resultant distribution obtained by combining two beta distribution $\beta(4.939, 5.416)$ and $\beta(5.416, 4.939)$.

Comparison of Frequencies for Chi-Square Test

CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
1	0	0.01	21	1094	1028.86
2	0	0.10	22	1291	1165.12
3	0	0.67	23	1391	1306.37
4	0	2.35	24	1508	1451.31
5	2	6.03	25	1654	1598.53
6	5	12.71	26	1809	1746.53
7	14	23.48	27	1997	1893.80
8	24	39.46	28	2111	2038.77
9	61	61.71	29	2216	2179.90
10	69	91.24	30	2206	2315.63
11	115	128.92	31	2394	2444.48
12	174	175.51	32	2441	2565.02
13	233	231.59	33	2642	2675.88
14	311	297.55	34	2738	2775.82
15	398	373.61	35	2864	2863.71
16	459	459.78	36	2914	2938.55
17	574	555.86	37	2925	2999.47
18	675	661.47	38	2932	3045.78
19	814	776.04	39	2981	3076.96
20	876	898.81	40	2992	3092.64

CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
41	2960	3092.64	61	918	898.81
42	2939	3076.96	62	836	776.04
43	2968	3045.78	63	708	661.47
44	2860	2999.47	64	572	555.86
45	2896	2938.55	65	450	459.78
46	2809	2863.71	66	363	373.61
47	2851	2775.82	67	274	297.55
48	2764	2675.88	68	236	231.59
49	2609	2565.02	69	154	175.51
50	2364	2444.48	70	100	128.92
51	2322	2315.63	71	85	91.24
52	2277	2179.90	72	56	61.71
53	2131	2038.77	73	35	39.46
54	1968	1893.88	74	11	23.48
55	1745	1746.53	75	2	12.71
56	1655	1598.53	76	5	6.03
57	1528	1451.31	77	1	2.35
58	1334	1306.37	78	0	0.67
59	1260	1165.12	79	0	0.10
60	1050	1028.86	80	0	0.01

The obtained Chi-Square Value = 205.977

The critical chi-square value = 88.3, for 68 degrees of freedom and 5% significance level.

Smoothing Constant = 0.2

Approximating Distribution = Resultant Distribution Obtained by Combining Two Beta Distributions, β(4.939, 5.416) and β(5.416, 4.939).

Comparison of Cumulative Frequencies for Kolmogorov Smirnov Test

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t0
1	0	.02	21	5898	5825.77
2	0	.12	22	7189	6990.89
3	0	.79	23	8580	8297.26
4	0	3.14	24	10058	9748.57
5	2	9.17	25	11742	11347.10
6	7	21.88	26	13551	13093.63
7	21	45.36	27	15548	14987.43
8	45	84.82	28	17659	17026.20
9	106	146.53	29	19875	19206.10
10	175	237.77	30	22081	21521.73
11	290	366.69	31	24475	23966.21
12	464	542.20	32	26916	26531.23
13	697	773.79	33	29558	29207.11
14	1008	1073.04	34	32296	31982.93
15	1406	1444.95	35	35160	34846.64
16	1865	1904.73	36	38074	37785.16
17	2478	2460.59	37	409999	40784.66
18	3114	3122.06	38	43931	43830.44
19	3928	3898.10	39	46912	46907.40
20	4804	4796.91	40	49904	50000.00

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY $'f_0'$	THEORETICAL CUMULATIVE FREQUENCY $'f_t'$	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCEY $'f_0'$	THEORETICAL CUMULATIVE FREQUENCY $'f_t'$
41	52864	53092.68	61	96112	96101.98
42	55803	56169.64	62	96948	96878.02
43	58771	59215.42	63	97656	97539.49
44	61631	62214.89	64	98228	98095.35
45	64527	65153.44	65	98678	98555.13
46	67336	68017.15	66	99041	98928.74
47	70187	70792.97	67	99315	99226.29
48	72951	73468.85	68	99551	99457.88
49	75560	76033.87	69	99715	99633.39
50	77924	78478.35	70	99805	99762.31
51	80246	80793.98	71	99890	99853.55
52	82523	82973.88	72	99946	99915.26
53	84654	85012.65	73	99981	99954.72
54	86622	86906.45	74	99992	99978.20
55	88367	88652.98	75	99994	99990.91
56	90022	90251.51	76	99999	99996.94
57	91550	91702.82	77	100,000	99999.29
58	92884	93009.19	78	100,000	100,000.00
59	94144	94174.31	79	100,000	100,000.00
60	95194	95203.17	80	100,000	100,000.00

Maximum Difference $|f_0 - f_t| = 681$

The Obtained Kolmogorov Smirnov Value $= \dfrac{681}{100{,}000} = 0.00681$

The Critical Kolmogorov Smirnov Value $= \dfrac{1.36}{\sqrt{n}} = \dfrac{1.36}{\sqrt{100{,}000}} = 0.00430$
for 5% Significance Level.

Smoothing Constant = 0.01

Approximating Distribution = Beta distribution with parameter m=n=132.5.

Comparison of Frequencies for a Chi-Square Test.

CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
1	0	0.0	21	0	0.0
2	0	0.0	22	0	0.0
3	0	0.0	23	0	0.0
4	0	0.0	24	0	0.0
5	0	0.0	25	0	0.0
6	0	0.0	26	0	0.0
7	0	0.0	27	0	0.0
8	0	0.0	28	2.1	0.05
9	0	0.0	29	6.1	0.34
10	0	0.0	30	9	2.15
11	0	0.0	31	14	11.31
12	0	0.0	32	40	49.54
13	0	0.0	33	122	181.63
14	0	0.0	34	489	560.11
15	0	0.0	35	1340	1459.17
16	0	0.0	36	3138	3223.22
17	0	0.0	37	6101	6055.08
18	0	0.0	38	9656	9696.51
19	0	0.0	39	13146	13259.65
20	0	0.0	40	15801	15501.23

CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY	CELL NUMBER	OBSERVED FREQUENCY	THEORETICAL FREQUENCY
41	15977	15501.23	61	0	0.0
42	13394	13259.65	62	0	0.0
43	9604	9696.51	63	0	0.0
44	5891	6055.08	64	0	0.0
45	3082	3223.22	65	0	0.0
46	1459	1459.17	66	0	0.0
47	512	560.11	67	0	0.0
48	169	181.63	68	0	0.0
49	41	49.54	69	0	0.0
50	7	11.31	70	0	0.0
51	0	2.15	71	0	0.0
52	0	0.34	72	0	0.0
53	0	0.05	73	0	0.0
54	0	0.0	74	0	0.0
55	0	0.0	75	0	0.0
56	0	0.0	76	0	0.0
57	0	0.0	77	0	0.0
58	0	0.0	78	0	0.0
59	0	0.0	79	0	0.0
60	0	0.0	80	0	0.0

The obtained chi-square value = 162.27

The critical chi-square value = 327, for 21 degrees of freedom and at 5% significance level.

Smoothing Constant = 0.01

Approximating Distribution = Beta distribution with parameters m=n=132.5

Comparison of Cumulative Frequencies for Kolmogorov Smirnov Test.

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'
1	0	0	21	0	0
2	0	0	22	0	0
3	0	0	23	0	0
4	0	0	24	0	0
5	0	0	25	0	0
6	0	0	26	0	0
7	0	0	27	0	0
8	0	0	28	2	0
9	0	0	29	8	0
10	0	0	30	17	2
11	0	0	31	31	13
12	0	0	32	71	62
13	0	0	33	193	243
14	0	0	34	682	803
15	0	0	35	2022	2262
16	0	0	36	5160	5485
17	0	0	37	11261	11540
18	0	0	38	20917	21236
19	0	0	39	34063	34495
20	0	0	90	49864	49996

CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'	CELL NUMBER	OBSERVED CUMULATIVE FREQUENCY 'f_0'	THEORETICAL CUMULATIVE FREQUENCY 'f_t'
41	65841	65497	61	100,000	100,000
42	79235	78756	62	100,000	100,000
43	88839	88452	63	100,000	100,000
44	94730	94507	64	100,000	100,000
45	97812	97730	65	100,000	100,000
46	99271	99189	66	100,000	100,000
47	99783	99749	67	100,000	100,000
48	99952	99930	68	100,000	100,000
49	99993	99979	69	100,000	100,000
50	100,000	99989	70	100,000	100,000
51	100,000	100,000	71	100,000	100,000
52	100,000	100,000	72	100,000	100,000
53	100,000	100,000	73	100,000	100,000
54	100,000	100,000	74	100,000	100,000
55	100,000	100,000	75	100,000	100,000
56	100,000	100,000	76	100,000	100,000
57	100,000	100,000	77	100,000	100,000
58	100,000	100,000	78	100,000	100,000
59	100,000	100,000	79	100,000	100,000
60	100,000	100,000	80	100,000	100,000

Maximum Difference $|f_0 - f_t| = 479$

The obtained Kolmogorov Smirnov Value $= \frac{479}{100,000} = 0.00479$

The critical Kolmogorov Smirnov Value $= \frac{1.36}{\sqrt{n}} = \frac{1.36}{\sqrt{100,000}} = 0.00473$

APPENDIX B

COMPUTER PROGRAMS

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C      PURPOSE                                                         C
C                                                                      C
C      THE PURPOSE OF THIS PRCGRAMME IS TO SIMULATE TRACKING SIGNAL    C
C      DISTRIBUTION USING SINGLE EXPONENTIAL SMOOTHING                 C
C                                                                      C
C      DESCRIPTION OF PARAMETERS                                       C
C                                                                      C
C      ALPHA-THE VALUE OF SMOCTHING CONSTANT ,WHICH VARIES FROM 0 TO 1 C
C                                                                      C
C      STEP-THE DESIRED INCREMENT IN THE VALUE OF SMOOTHING CONSTANT.  C
C                                                                      C
C      LIMIT-STOPPING CRITERIA FOR THE PROGRAMME OR THE LAST VALUE OF  C
C      SMOOTHING CONSTANT FOR WHICH SIMULATION IS CARRIED OUT          C
C                                                                      C
C      XEAN-THE MEAN OF THE PROCESS                                    C
C                                                                      C
C      STDDEV-THE STANDARD DEVIATION OF ASSUMED NORMAL DISTRIBUTION    C
C      OF NOISE                                                        C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DIMENSION MT(80)
    1 FORMAT(3I10,2F10.4)
    2 FORMAT (' ',I10,' IY',I10,' IX',I10,' NCOUNT',F10.4,' MEAN',F10.4,
     1' STDDEV',F10.4,' ALPHA')
   10 FORMAT (3F8.4,I3)
  120 FORMAT(10X,'MEAN',10X,'STDEV')
  121 FORMAT( 5X,F8.4,10X,F8.4)
  123 FORMAT('1    I            TSIG')
  124 FORMAT(I3,10X,I6)
      READ 10,ALPHA,STEP,XLIMIT,N
      DO 200 NRUN =1,N
      READ(1,1) IY,IX,NCOUNT,XEAN,STDDEV
   13 WRITE(3,2) IY,IX,NCOUNT,XEAN,STDDEV,ALPHA
      DO 3 I=1,80
      MT(I)=0
    3 CONTINUE
      FACT=SQRT(2./(2.-ALPHA))
      SMAD=.79788*FACT*STDDEV
      XHAT=XEAN
      SSE=0.
      BETA=1.-ALPHA
      STRSIG=0.0
      SQTSIG=0.0
C
C     GENERATION OF UNIFORM RANDOM NUMBER
C
      DO 101 I=1,NCOUNT
      IY=IY*IX
      IF(IY.LT.0) IY=IY+2147483647+1
      RA=IY*.4656613E-9
      IY=IY*IX
      IF(IY.LT.0) IY=IY+2147483647+1
      RB=IY*.4656613E-9
C
C     GENERATION OF TWO RANDCM NORMAL DEVIATES
C
```

```
            UA=SQRT(-2.0*ALOG(RA))
            UB=6.2832*RB
            XA=UA*COS(UB)
            XB=UA*SIN(UB)
            X=XA*STDDEV+XEAN
            RD=XB*STDDEV+XEAN
      C
      C     EVALUATION OF TRACKING SIGNAL
      C
            DO 101 II=1,2
            ERR=X-XHAT
            SSE=ALPHA*ERR+BETA*SSE
            SMAD=ALPHA*ABS(ERR)+BETA*SMAD
            TRSIG=SSE/SMAD
            XHAT=ALPHA*X+BETA*XHAT
            STRSIG=STRSIG+TRSIG
            SQTSIG=SQTSIG+TRSIG*TRSIG
      C
      C     CLASSIFICATION OF TRACKING SIGNAL INTO 80 CELLS
      C
            NT=TRSIG*40.+41.
            IF (NT. GT. 80 ) NT=80
            IF(NT.LT.1 ) NT=1
            MT(NT)=MT(NT)+1
            X=RD
        101 CONTINUE
      C
      C     OBTAIN MEAN AND STANDARD DEVIATION OF TRACKING SIGNAL
      C
            CMEAN=STRSIG/(2*NCOUNT)
            STDEV=(SQTSIG/(2*NCOUNT)-(CMEAN)**2)**0.5
            DO 122 J=1,41,40
            JJ=J+39
            PRINT 123
            DO 122 I=J,JJ
            PRINT 124,I,MT(I)
        122 CONTINUE
            PRINT 120
            PRINT 121,CMEAN,STDEV
      C
      C     INCREMENT IN THE VALUE OF ALPHA FOR NEXT SIMULATION
      C
            ALPHA=ALPHA+STEP
            IF (ALPHA-XLIMIT) 11,12,12
         11 GO TO 13
      C
      C     RESET VALUE OF ALPHA FOR SIMULATION WITH A NEW SET OF DATA
      C
         12 ALPHA=0.1
        200 CONTINUE
            STOP
            END
```

```
      C
      C     ..................................................................
      C
      C        SUBROUTINE CDTR
      C
      C        PURPOSE
      C           COMPUTES P(X) = PROBABILITY THAT THE RANDOM VARIABLE U,
      C           DISTRIBUTED ACCORDING TO THE CHI-SQUARE DISTRIBUTION WITH G
      C           DEGREES OF FREEDOM, IS LESS THAN OR EQUAL TO X.  F(G,X), THE
      C           ORDINATE OF THE CHI-SQUARE DENSITY AT X, IS ALSO COMPUTED.
      C
      C        USAGE
      C           CALL CDTR(X,G,P,D,IER)
      C
      C        DESCRIPTION OF PARAMETERS
      C           X   - INPUT SCALAR FOR WHICH P(X) IS COMPUTED.
      C           G   - NUMBER OF DEGREES OF FREEDOM OF THE CHI-SQUARE
      C                 DISTRIBUTION.  G IS A CONTINUOUS PARAMETER.
      C           P   - OUTPUT PROBABILITY.
      C           D   - OUTPUT DENSITY.
      C           IER - RESULTANT ERROR CODE WHERE
      C               IER= 0 --- NO ERROR
      C               IER=-1 --- AN INPUT PARAMETER IS INVALID.  X IS LESS
      C                          THAN 0.0, OR G IS LESS THAN 0.5 OR GREATER
      C                          THAN 2*10**(+5).  P AND D ARE SET TO -1.E75.
      C               IER=+1 --- INVALID OUTPUT.  P IS LESS THAN ZERO OR
      C                          GREATER THAN ONE, OR SERIES FOR T1 (SEE
      C                          MATHEMATICAL DESCRIPTION) HAS FAILED TO
      C                          CONVERGE.  P IS SET TO 1.E75.
      C
      C        REMARKS
      C           SEE MATHEMATICAL DESCRIPTION.
      C
      C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
      C           DLGAM
      C           NDTR
      C
      C        METHOD
      C           REFER TO R.E. BARGMANN AND S.P. GHOSH, STATISTICAL
      C           DISTRIBUTION PROGRAMS FOR A COMPUTER LANGUAGE,
      C           IBM RESEARCH REPORT RC-1094, 1963.
      C
      C     ..................................................................
      C
0001        SUBROUTINE CDTR(X,G,P,D,IER)
0002        DOUBLE PRECISION XX,DLXX,X2,DLX2,GG,G2,DLT3,THETA,THP1,
           1DLG2,DD,T11,SER,CC,XI,FAC,TLOG,TERM,GTH,A2,A,B,C,DT2,DT3,THPI
      C
      C        TEST FOR VALID INPUT DATA
      C
0003        IF(G-(.5-1.E-5)) 590,10,10
0004     10 IF(G-2.E+5) 20,20,590
0005     20 IF(X) 590,30,30
      C
      C        TEST FOR X NEAR 0.0
      C
0006     30 IF(X-1.E-8) 40,40,80
0007     40 P=0.0
```

```
0008            IF(G-2.) 50,60,70
0009         50 D=1.E75
0010            GO TO 610
0011         60 D=0.5
0012            GO TO 610
0013         70 D=0.0
0014            GO TO 610
           C
           C       TEST FOR X GREATER THAN 1.E+6
           C
0015         80 IF(X-1.E+6) 100,100,90
0016         90 D=0.0
0017            P=1.0
0018            GO TO 610
           C
           C       SET PROGRAM PARAMETERS
           C
0019        100 XX=DBLE(X)
0020            DLXX=DLOG(XX)
0021            X2=XX/2.D0
0022            DLX2=DLOG(X2)
0023            GG=DBLE(G)
0024            G2=GG/2.D0
           C
           C       COMPUTE ORDINATE
           C
0025            CALL DLGAM(G2,GLG2,IOK)
0026            DD=(G2-1.D0)*DLXX-X2-G2*.6931471805599453 -GLG2
0027            IF(DD-1.68D02) 110,110,120
0028        110 IF(DD+1.68D02) 130,130,140
0029        120 D=1.E75
0030            GO TO 150
0031        130 D=0.0
0032            GO TO 150
0033        140 DD=DEXP(DD)
0034            D=SNGL(DD)
           C
           C       TEST FOR G GREATER THAN 1000.0
           C       TEST FOR X GREATER THAN 2000.0
           C
0035        150 IF(G-1000.) 160,160,180
0036        160 IF(X-2000.) 190,190,170
0037        170 P=1.0
0038            GO TO 610
0039        180 A=DLOG(XX/GG)/3.D0
0040            A=DEXP(A)
0041            B=2.D0/(9.D0*GG)
0042            C=(A-1.D0+B)/DSQRT(B)
0043            SC=SNGL(C)
0044            CALL NDTR(SC,P,DUMMY)
0045            GO TO 490
           C
           C       COMPUTE THETA
           C
0046        190 K= IDINT(G2)
0047            THETA=G2-DFLOAT(K)
0048            IF(THETA-1.D-8) 200,200,210
0049        200 THETA=0.D0
```

```
0050          210 THP1=THETA+1.D0
           C
           C         SELECT METHOD OF COMPUTING T1
           C
0051              IF(THETA) 230,230,220
0052          220 IF(XX-10.D0) 260,260,320
           C
           C         COMPUTE T1 FOR THETA EQUALS 0.0
           C
0053          230 IF(X2-1.68D02) 250,240,240
0054          240 T1=1.0
0055              GO TO 400
0056          250 T11=1.D0-DEXP(-X2)
0057              T1=SNGL(T11)
0058              GO TO 400
           C
           C         COMPUTE T1 FOR THETA GREATER THAN 0.0 AND
           C         X LESS THAN OR EQUAL TO 10.0
           C
0059          260 SER=X2*(1.D0/THP1 -X2/(THP1+1.D0))
0060              J=+1
0061              CC=DFLOAT(J)
0062              DO 270 IT1=3,30
0063              XI=DFLOAT(IT1)
0064              CALL DLGAM(XI,FAC,IOK)
0065              TLOG= XI*DLX2-FAC-DLOG(XI+THETA)
0066              TERM=DEXP(TLOG)
0067              TERM=DSIGN(TERM,CC)
0068              SER=SER+TERM
0069              CC=-CC
0070              IF(DABS(TERM)-1.D-9) 280,270,270
0071          270 CONTINUE
0072              GO TO 600
0073          280 IF(SER) 600,600,290
0074          290 CALL DLGAM(THP1,GTH,IOK)
0075              TLOG=THETA*DLX2+DLOG(SER)-GTH
0076              IF(TLOG+1.68D02) 300,300,310
0077          300 T1=0.0
0078              GO TO 400
0079          310 T11=DEXP(TLOG)
0080              T1=SNGL(T11)
0081              GO TO 400
           C
           C         COMPUTE T1 FOR THETA GREATER THAN 0.0 AND
           C         X GREATER THAN 10.0 AND LESS THAN 2000.0
           C
0082          320 A2=0.D0
0083              DO 340 I=1,25
0084              XI=DFLOAT(I)
0085              CALL DLGAM(THP1,GTH,IOK)
0086              T11=-(13.D0*XX)/XI +THP1*DLOG(13.D0*XX/XI) -GTH-DLOG(XI)
0087              IF(T11+1.68D02) 340,340,330
0088          330 T11=DEXP(T11)
0089              A2=A2+T11
0090          340 CONTINUE
0091              A=1.01282051+THETA/156.D0-XX/312.D0
0092              B=DABS(A)
0093              C= -X2+THP1*DLX2+DLOG(B)-GTH-3.951243718581427
```

```
0094             IF(C+1.68D02) 370,370,350                                          CDTR1750
0095         350 IF (A) 360,370,380                                                 CDTR1760
0096         360 C=-DEXP(C)                                                         CDTR1770
0097             GO TO 390                                                          CDTR1780
0098         370 C=0.D0                                                             CDTR1790
0099             GO TO 390                                                          CDTR1800
0100         380 C=DEXP(C)                                                          CDTR1810
0101         390 C=A2+C                                                             CDTR1820
0102             T11=1.D0-C                                                         CDTR1830
0103             T1=SNGL(T11)                                                       CDTR1840
           C                                                                        CDTR1850
           C        SELECT PROPER EXPRESSION FOR P                                  CDTR1860
           C                                                                        CDTR1870
0104         400 IF(G-2.) 420,410,410                                               CDTR1880
0105         410 IF(G-4.) 450,460,460                                               CDTR1890
           C                                                                        CDTR1900
           C        COMPUTE P FOR G GREATER THAN ZERO AND LESS THAN 2.0            CDTR1910
           C                                                                        CDTR1920
0106         420 CALL DLGAM(THP1,GTH,ICK)                                           CDTR1930
0107             DT2=THETA*DLXX-X2-THP1*.6931471805599453 -GTH                      CDTR1940
0108             IF(DT2+1.68D02) 430,430,440                                        CDTR1950
0109         430 P=T1                                                               CDTR1960
0110             GO TO 490                                                          CDTR1970
0111         440 DT2=DEXP(DT2)                                                      CDTR1980
0112             T2=SNGL(DT2)                                                       CDTR1990
0113             P=T1+T2+T2                                                         CDTR2000
0114             GO TO 490                                                          CDTR2010
           C                                                                        CDTR2020
           C        COMPUTE P FOR G GREATER THAN OR EQUAL TO 2.0                    CDTR2030
           C        AND LESS THAN 4.0                                               CDTR2040
           C                                                                        CDTR2050
0115         450 P=T1                                                               CDTR2060
0116             GO TO 490                                                          CDTR2070
           C                                                                        CDTR2080
           C        COMPUTE P FOR G GREATER THAN OR EQUAL TO 4.0                    CDTR2090
           C        AND LESS THAN OR EQUAL TO 1000.0                                CDTR2100
           C                                                                        CDTR2110
0117         460 DT3=0.D0                                                           CDTR2120
0118             DO 480 I3=2,K                                                      CDTR2130
0119             THPI=DFLOAT(I3)+THETA                                              CDTR2140
0120             CALL DLGAM(THPI,GTH,ICK)                                           CDTR2150
0121             DLT3=THPI*DLX2-DLXX-X2-GTH                                         CDTR2160
0122             IF(DLT3+1.68D02) 480,480,470                                       CDTR2170
0123         470 DT3=DT3+DEXP(DLT3)                                                 CDTR2180
0124         480 CONTINUE                                                           CDTR2190
0125             T3=SNGL(DT3)                                                       CDTR2200
0126             P=T1-T3-T3                                                         CDTR2210
           C                                                                        CDTR2220
           C        SET ERROR INDICATOR                                             CDTR2230
           C                                                                        CDTR2240
0127         490 IF(P) 500,520,520                                                  CDTR2250
0128         500 IF(ABS(P)-1.E-7) 510,510,600                                       CDTR2260
0129         510 P=0.0                                                              CDTR2270
0130             GO TO 610                                                          CDTR2280
0131         520 IF(1.-P) 530,550,550                                               CDTR2290
0132         530 IF(ABS(1.-P)-1.E-7) 540,540,600                                    CDTR2300
0133         540 P=1.0                                                              CDTR2310
0134             GO TO 610                                                          CDTR2320
```

```
      550 IF(P-1.E-8) 560,560,570                                          CDTR2330
      560 P=0.0                                                            CDTR2340
          GO TO 610                                                        CDTR2350
      570 IF((1.0-P)-1.E-8) 580,580,610                                    CDTR2360
      580 P=1.0                                                            CDTR2370
          GO TO 610                                                        CDTR2380
      590 IER=-1                                                           CDTR2390
          D=-1.E75                                                         CDTR2400
          P=-1.E75                                                         CDTR2410
          GO TO 620                                                        CDTR2420
      600 IER=+1                                                           CDTR2430
          P= 1.E75                                                         CDTR2440
          GO TO 620                                                        CDTR2450
      610 IER=0                                                            CDTR2460
      620 RETURN                                                           CDTR2470
          END                                                              CDTR2480
```

```
        C
        C...............................................................
        C
        C          SUBROUTINE NDTR
        C
        C          PURPOSE
        C             COMPUTES Y = P(X) = PROBABILITY THAT THE RANDOM VARIABLE  U,
        C             DISTRIBUTED NORMALLY(0,1), IS LESS THAN OR EQUAL TO X.
        C             F(X), THE ORDINATE OF THE NORMAL DENSITY AT X, IS ALSO
        C             COMPUTED.
        C
        C          USAGE
        C             CALL NDTR(X,P,D)
        C
        C          DESCRIPTION OF PARAMETERS
        C             X--INPUT SCALAR FOR WHICH P(X) IS COMPUTED.
        C             P--OUTPUT PROBABILITY.
        C             D--OUTPUT DENSITY.
        C
        C          REMARKS
        C             MAXIMUM ERROR IS 0.0000007.
        C
        C          SUBROUTINES AND SUBPROGRAMS REQUIRED
        C             NONE
        C
        C          METHOD
        C             BASED ON APPROXIMATIONS IN C. HASTINGS, APPROXIMATIONS FOR
        C             DIGITAL COMPUTERS, PRINCETON UNIV. PRESS, PRINCETON, N.J.,
        C             1955.  SEE EQUATION 26.2.17, HANDBOOK OF MATHEMATICAL
        C             FUNCTIONS, ABRAMOWITZ AND STEGUN, DOVER PUBLICATIONS, INC.,
        C             NEW YORK.
        C
        C...............................................................
        C
0001          SUBROUTINE NDTR(X,P,D)
        C
0002          AX=ABS(X)
0003          T=1.0/(1.0+.2316419*AX)
0004          D=0.3989423*EXP(-X*X/2.0)
0005          P = 1.0 - D*T*((((1.330274*T - 1.821256)*T + 1.781478)*T -
             1 0.3565638)*T + 0.3193815)
0006          IF(X)1,2,2
0007        1 P=1.0-P
0008        2 RETURN
0009          END
```

```
      C
      C     ..............................................................
      C
      C        SUBROUTINE DLGAM
      C
      C        PURPOSE
      C           COMPUTES THE DOUBLE PRECISION NATURAL LOGARITHM OF THE
      C           GAMMA FUNCTION OF A GIVEN DOUBLE PRECISION ARGUMENT.
      C
      C        USAGE
      C           CALL DLGAM(XX,DLNG,IER)
      C
      C        DESCRIPTION OF PARAMETERS
      C           XX   - THE DOUBLE PRECISION ARGUMENT FOR THE LOG GAMMA
      C                  FUNCTION.
      C           DLNG - THE RESULTANT DOUBLE PRECISION LOG GAMMA FUNCTION
      C                  VALUE.
      C           IER  - RESULTANT ERROR CODE WHERE
      C                  IER= 0----NO ERROR.
      C                  IER=-1----XX IS WITHIN 10**(-9) OF BEING ZERO OR XX
      C                            IS NEGATIVE.  DLNG IS SET TO -1.0D75.
      C                  IER=+1----XX IS GREATER THAN 10**70. DLNG IS SET TO
      C                            +1.0D75.
      C
      C        REMARKS
      C           NONE
      C
      C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
      C           NONE
      C
      C        METHOD
      C           THE EULER-MCLAURIN EXPANSION TO THE SEVENTH DERIVATIVE TERM
      C           IS USED, AS GIVEN BY M. ABRAMOWITZ AND I.A. STEGUN,
      C           'HANDBOOK OF MATHEMATICAL FUNCTIONS', U. S. DEPARTMENT OF
      C           COMMERCE, NATIONAL BUREAU OF STANDARDS APPLIED MATHEMATICS
      C           SERIES, 1966, EQUATION 6.1.41.
      C
      C     ..............................................................
      C
0001        SUBROUTINE DLGAM(XX,DLNG,IER)
0002        DOUBLE PRECISION XX,ZZ,TERM,RZ2,DLNG
0003        IER=0
0004        ZZ=XX
0005        IF(XX-1.D10) 2,2,1
0006      1 IF(XX-1.D70) 8,9,9
      C
      C        SEE IF XX IS NEAR ZERO OR NEGATIVE
      C
0007      2 IF(XX-1.D-9) 3,3,4
0008      3 IER=-1
0009        DLNG=-1.D75
0010        GO TO 10
      C
      C        XX GREATER THAN ZERO AND LESS THAN OR EQUAL TO 1.D+10
      C
0011      4 TERM=1.D0
0012      5 IF(ZZ-18.D0) 6,6,7
0013      6 TERM=TERM*ZZ
```

```
0014              ZZ=ZZ+1.D0
0015              GO TO 5
0016            7 RZ2=1.D0/ZZ**2
0017              CLNG =(ZZ-0.5D0)*DLOG(ZZ)-ZZ +0.9189385332046727 -DLOG(TERM)+
                 1(1.D0/ZZ)*(.8333333333333333D-1 -(RZ2*(.2777777777777777D-2 +(RZ2*
                 2(.7936507936507936D-3 -(RZ2*(.5952380952380952D-3)))))))
0018              GO TO 10
            C
            C        XX GREATER THAN 1.D+10 AND LESS THAN 1.D+70
            C
0019            8 CLNG=ZZ*(DLOG(ZZ)-1.D0)
0020              GO TO 10
            C
            C        XX GREATER THAN OR EQUAL TO 1.D+70
            C
0021            9 IER=+1
0022              CLNG=1.D75
0023           10 RETURN
0024              END
```

```
                C
                C     ...............................................................
                C
                C        SUBROUTINE BDTR
                C
                C        PURPOSE
                C           COMPUTES P(X) = PROBABILITY THAT THE RANDOM VARIABLE U,
                C           DISTRIBUTED ACCORDING TO THE BETA DISTRIBUTION WITH
                C           PARAMETERS A AND B, IS LESS THAN OR EQUAL TO X.  F(A,B,X),
                C           THE ORDINATE OF THE BETA DENSITY AT X, IS ALSO COMPUTED.
                C
                C        USAGE
                C           CALL BDTR(X,A,B,P,D,IER)
                C
                C        DESCRIPTION OF PARAMETERS
                C           X   - INPUT SCALAR FOR WHICH P(X) IS COMPUTED.
                C           A   - BETA DISTRIBUTION PARAMETER (CONTINUOUS).
                C           B   - BETA DISTRIBUTION PARAMETER (CONTINUOUS).
                C           P   - OUTPUT PROBABILITY.
                C           D   - OUTPUT DENSITY.
                C           IER - RESULTANT ERROR CODE WHERE
                C                 IER= 0 --- NO ERROR
                C                 IER=-1,+1  CDTR HAS BEEN CALLED AND AN ERROR HAS
                C                            OCCURRED.  SEE CDTR.
                C                 IER=-2 --- AN INPUT PARAMETER IS INVALID.  X IS LESS
                C                            THAN 0.0 OR GREATER THAN 1.0, OR EITHER A OR
                C                            B IS LESS THAN 0.5 OR GREATER THAN 10**(+5).
                C                            P AND D ARE SET TO -1.E75.
                C                 IER=+2 --- INVALID OUTPUT.  P IS LESS THAN ZERO OR
                C                            GREATER THAN ONE.  P IS SET TO 1.E75.
                C
                C        REMARKS
                C           SEE MATHEMATICAL DESCRIPTION.
                C
                C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                C           DLGAM
                C           NDTR
                C           CDTR
                C
                C        METHOD
                C           REFER TO R.E. BARGMANN AND S.P. GHOSH, STATISTICAL
                C           DISTRIBUTION PROGRAMS FOR A COMPUTER LANGUAGE,
                C           IBM RESEARCH REPORT RC-1094, 1963.
                C
                C     ...............................................................
                C
0001                  SUBROUTINE BDTR(X,A,B,P,D,IER)
0002                  DOUBLE PRECISION XX,DLXX,DL1X,AA,BB,G1,G2,G3,G4,DD,PP,X0,FF,FN,
                     1XI,SS,CC,RR,DLBETA
                C
                C        TEST FOR VALID INPUT DATA
                C
0003                  IF(A-(.5-1.E-5)) 640,10,10
0004               10 IF(B-(.5-1.E-5)) 640,20,20
0005               20 IF(A-1.E+5) 30,30,640
0006               30 IF(B-1.E+5) 40,40,640
0007               40 IF(X) 640,50,50
0008               50 IF(1.-X) 640,60,60
```

```
              C
              C         COMPUTE LOG(BETA(A,B))
              C
 0009            60 AA=DBLE(A)
 0010               BB=DBLE(B)
 0011               CALL DLGAM(AA,G1,IOK)
 0012               CALL DLGAM(BB,G2,IOK)
 0013               CALL DLGAM(AA+BB,G3,IOK)
 0014               DLBETA=G1+G2-G3
              C
              C         TEST FOR X NEAR 0.0 OR 1.0
              C
 0015               IF(X-1.E-8) 80,80,70
 0016            70 IF((1.-X)-1.E-8) 130,130,140
 0017            80 P=0.0
 0018               IF(A-1.) 90,100,120
 0019            90 D=1.E+75
 0020               GO TO 660
 0021           100 DD=-DLBETA
 0022               IF(DD+1.68D02)  120,120,110
 0023           110 DD=DEXP(DD)
 0024               D=SNGL(DD)
 0025               GO TO 660
 0026           120 D=0.0
 0027               GO TO 660
 0028           130 P=1.0
 0029               IF(B-1.) 90,100,120
              C
              C         SET PROGRAM PARAMETERS
              C
 0030           140 XX=DBLE(X)
 0031               DLXX=DLOG(XX)
 0032               DL1X=DLOG(1.D0-XX)
 0033               X0=XX/(1.D0-XX)
 0034               ID=0
              C
              C         COMPUTE ORDINATE
              C
 0035               DD=(AA-1.D0)*DLXX+(BB-1.D0)*DL1X-DLBETA
 0036               IF(DD-1.68D02) 150,150,160
 0037           150 IF(DD+1.68D02) 170,170,180
 0038           160 D=1.E75
 0039               GO TO 190
 0040           170 D=0.0
 0041               GO TO 190
 0042           180 DD=DEXP(DD)
 0043               D=SNGL(DD)
              C
              C         A OR B OR BOTH WITHIN 1.E-8 OF 1.0
              C
 0044           190 IF(ABS(A-1.)-1.E-8)  200,200,210
 0045           200 IF(ABS(B-1.)-1.E-8)  220,220,230
 0046           210 IF(ABS(B-1.)-1.E-8)  260,260,290
 0047           220 P=X
 0048               GO TO 660
 0049           230 PP=BB*DL1X
 0050               IF(PP+1.68D02) 240,240,250
 0051           240 P=1.0
```

```
0052            GO TO 660
0053        250 PP=DEXP(PP)
0054            PP=1.D0-PP
0055            P=SNGL(PP)
0056            GO TO 600
0057        260 PP=AA*DLXX
0058            IF(PP+1.68D02) 270,270,280
0059        270 P=0.0
0060            GO TO 660
0061        280 PP=DEXP(PP)
0062            P=SNGL(PP)
0063            GO TO 600
          C
          C        TEST FOR A OR B GREATER THAN 1000.0
          C
0064        290 IF(A-1000.) 300,300,310
0065        300 IF(B-1000.) 330,330,320
0066        310 XX=2.D0*AA/XO
0067            XS=SNGL(XX)
0068            AA=2.D0*BB
0069            DF=SNGL(AA)
0070            CALL CDTR(XS,DF,P,DUMMY,IER)
0071            P=1.0-P
0072            GO TO 670
0073        320 XX=2.D0*BB*XO
0074            XS=SNGL(XX)
0075            AA=2.D0*AA
0076            DF=SNGL(AA)
0077            CALL CDTR(XS,DF,P,DUMMY,IER)
0078            GO TO 670
          C
          C        SELECT PARAMETERS FOR CONTINUED FRACTION COMPUTATION
          C
0079        330 IF(X-.5) 340,340,380
0080        340 IF(AA-1.D0) 350,350,360
0081        350 RR=AA+1.D0
0082            GO TO 370
0083        360 RR=AA
0084        370 DD=DLXX/5.D0
0085            DD=DEXP(DD)
0086            DD=(RR-1.D0)-(RR+BB-1.D0)*XX*DD +2.D0
0087            IF(DD) 420,420,430
0088        380 IF(BB-1.D0) 390,390,400
0089        390 RR=BB+1.D0
0090            GO TO 410
0091        400 RR=BB
0092        410 DD=DL1X/5.D0
0093            DD=DEXP(DD)
0094            DD=(RR-1.D0)-(AA+RR-1.D0)*(1.D0-XX)*DD +2.D0
0095            IF(DD) 430,430,420
0096        420 ID=1
0097            FF=DL1X
0098            DL1X=DLXX
0099            DLXX=FF
0100            XO=1.D0/XO
0101            FF=AA
0102            AA=BB
0103            BB=FF
```

```
0104              G2=G1
            C
            C        TEST FOR A LESS THAN 1.0
            C
0105          430 FF=0.D0
0106              IF(AA-1.D0) 440,440,470
0107          440 CALL DLGAM(AA+1.D0,G4,IOK)
0108              DD=AA*DLXX+BB*DL1X+G3-G2-G4
0109              IF(DD+1.68D02) 460,460,450
0110          450 FF=FF+DEXP(DD)
0111          460 AA=AA+1.D0
            C
            C        COMPUTE P USING CONTINUED FRACTION EXPANSION
            C
0112          470 FN=AA+BB-1.D0
0113              RR=AA-1.D0
0114              II=80
0115              XI=DFLOAT(II)
0116              SS=((BB-XI)*(RR+XI))/((RR+2.D0*XI-1.D0)*(RR+2.D0*XI))
0117              SS=SS*X0
0118              DO 480 I=1,79
0119              II=80-I
0120              XI=DFLOAT(II)
0121              DD=(XI*(FN+XI))/((RR+2.D0*XI+1.D0)*(RR+2.D0*XI))
0122              DD=DD*X0
0123              CC=((BB-XI)*(RR+XI))/((RR+2.D0*XI-1.D0)*(RR+2.D0*XI))
0124              CC=CC*X0
0125              SS=CC/(1.D0+DD/(1.D0-SS))
0126          480 CONTINUE
0127              SS=1.D0/(1.D0-SS)
0128              IF(SS) 650,650,490
0129          490 CALL DLGAM(AA+BB,G1,IOK)
0130              CALL DLGAM(AA+1.D0,G4,IOK)
0131              CC=G1-G2-G4+AA*DLXX+(BB-1.D0)*DL1X
0132              PP=CC+DLOG(SS)
0133              IF(PP+1.68D02) 500,500,510
0134          500 PP=FF
0135              GO TO 520
0136          510 PP=DEXP(PP)+FF
0137          520 IF(ID) 540,540,530
0138          530 PP=1.D0-PP
0139          540 P=SNGL(PP)
            C
            C        SET ERROR INDICATOR
            C
0140              IF(P) 550,570,570
0141          550 IF(ABS(P)-1.E-7) 560,560,650
0142          560 P=0.0
0143              GO TO 660
0144          570 IF(1.-P) 580,600,600
0145          580 IF(ABS(1.-P)-1.E-7) 590,590,650
0146          590 P=1.0
0147              GO TO 660
0148          600 IF(P-1.E-8) 610,610,620
0149          610 P=0.0
0150              GO TO 660
0151          620 IF((1.0-P)-1.E-8) 630,630,660
0152          630 P=1.0
```

```
            GO TO 660                                                        BDTR2330
        640 IER=-2                                                           BDTR2340
            C=-1.E75                                                         BDTR2350
            P=-1.E75                                                         BDTR2360
            GO TO 670                                                        BDTR2370
        650 IER=+2                                                           BDTR2380
            P= 1.E75                                                         BDTR2390
            GO TO 670                                                        BDTR2400
        660 IER=0                                                            BDTR2410
        670 RETURN                                                           BDTR2420
            END                                                              BDTR2430
```

```
FORTRAN IV G LEVEL  18                MAIN              DATE = 71095          16/00/48

      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      C                                                                     C
      C     THE PURPOSE OF THIS PROGRAM IS TO OBTAIN THE PROBABILITIES       C
      C     UNDER A RESULTANT DISTRIBUTION BY COMBINING TWO BETAA
      C     UNDER A RESULTANT DISTRIBUTION BY COMBINING TWO BETA DISTRIBU-   C
      C     TIONS B(M,N) AND B(N,M),M AND N BEEING THE PARAMETERS.WHEN N AND C
      C     M ARE EQUAL THE RESULTANT DISTRIBUTION IS ALSO A BETA DISTRIBU-  C
      C     TION WITH PARAMETERS M AND N, BOTH BEEING EQUAL                  C
      C                                                                     C
      C     DESCRIPTION OF PARAMETERS---                                    C
      C                                                                     C
      C     RESP= PROBABILITY OF EQUAL INTERVALS ON THE RANGE OF BETA DISTRI-C
      C     BUTION                                                          C
      C                                                                     C
      C     FINP=PROBABILITY OF EQUAL INTERVALS ON THE RANGE OF THE RESUL-   C
      C     TANT DISTRIBUTION                                               C
      C                                                                     C
      C     FREQCY=THEORETICAL FREQUENCY OF THE TRACKING SIGNAL             C
      C                                                                     C
      C     NTRSIG=OBSERVED FREQUENCY OF THE TRACKING SIGNAL                C
      C                                                                     C
      C     NCELL= NUMBER OF POINTS ON THE RANGE OF THE BETA DISTRIBUTION    C
      C     AT WHICH THE PROBABILITIES ARE TO BE EVALUATED.THIS INCLUDES     C
      C     END POINT ZERO ALSO.                                            C
      C                                                                     C
      C     NCELLL=NUMBER OF INTERVALS ON THE RANGE OF THE BETA DISTRIBUTION.C
      C                                                                     C
      C     NTIME= NUMBER OF TIMES THE PROGRAM IS USED TO COMPARE THE TRACK- C
      C     ING SIGNAL FREQUENCIES WITH DIFFERENT RESULTANT DISTRIBUTIONS    C
      C                                                                     C
      C     NRUN=NUMBER OF RUNS TO BE MADE WITH DIFFERENT VALUES OF SMOOTHINGC
      C     CONSTANTS.                                                      C
      C                                                                     C
      C     STEP1= INCREMENT IN THE VALUE OF X.THIS INCREMENT DEPENDS ON THE C
      C     NUMBER OF INTERVALS.                                            C
      C                                                                     C
      C     NUMBER=MULTIPLYING FACTOR,WHICH IS THE SAME AS NUMBER OF TRACKINGC
      C     SIGNALS GENERATED TO OBTAIN THE ACTUAL FREQUENCY OF THE TRACKING C
      C     SIGNAL.WHEN THE PROBABILITY FINP(I) IS MULTIPLIED BY THIS NUMBER C
      C     THE THEORETICAL FREQUENCY OF THE TRACKING SIGNAL IS OBTAINED.    C
      C                                                                     C
      CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
            DIMENSION P(90),RESP(80),FINP(80),FREQCY(80),NTRSIG(80)
          1 FORMAT (4I8)
         11 FORMAT (4F9.5)
         12 FORMAT(5F9.6,I5)
         18 FORMAT(3X,'(X)',7X,'(M)',6X,'(N)',5X,'P(X)',6X,'D',5X,'IER')
         28 FORMAT (2F11.6,F11.2)
         50 FORMAT (1H-,16X,'MEAN',15X,'ST.DEV.')
         60 FORMAT (2(10X,E14.6))
         80 FORMAT (20I4)
         90 FORMAT (1H-,'THE CHI-SQUARE VALUE IS =',E14.6)
         98 FORMAT (1H-,3X,'RESP(I)',5X,'FINP(I)',3X,'FREQCY(I)')
         99 FORMAT ('1','OBSERVED FREQUENCY OF THE TRACKING SIGNAL')
        100 FORMAT(1H-,20I6)
            READ 1,NCELL,NTIME,NRUN,NUMBER
            NCELLL=NCELL-1
            DO 70 JJ=1,NRUN
```

```
              C
              C      READ THE OBSERVED FREQUENCIES OF THE TRACKING SIGNAL TO CALCULATE
              C      THE CHI-SQUARE VALUE.
              C
 0017                READ 80,(NTRSIG(I),I=1,NCELLL)
 0018                PRINT 99
 0019                PRINT 100,(NTRSIG(I),I=1,NCELLL)
 0020                DO 40 J=1,NTIME
 0021                READ 11,X,A,B,STEP1
 0022                PRINT 18
 0023                DO 26 I=1,NCELL
              C
              C      CALCULATE PROBABILITY AT THE EQUIDISTANT POINTS ON THE RANGE OF
              C      THE BETA DISTRIBUTION.THIS IS CUMULATIVE PROBABILITY.
              C
 0024                CALL BDTR(X,A,B,PR,D,IER)
 0025                P(I)=PR
 0026                PRINT 12,X,A,B,P(I),D,IER
              C
              C      CHECK WHETHER PROBABILITY AT THE LAST POINT ON THE RANGE IS
              C      EVALUATED.
 0027                IF (X-1.0) 13,14,14
              C
              C      IF NOT GIVE AN INCREMENT IN THE VALUE OF X.
              C
 0028             13 X=X+STEP1
 0029             26 CONTINUE
 0030             14 DO 10 I=2,NCELL
 0031                IF (I-2) 10,31,31
              C
              C      OBTAIN PROBABILITY OF INDIVIDUAL INTERVAL FROM CUMULATIVE
              C      PROBABILITY BY SUBTRACTION
 0032             31 RESP(I-1)=P(I)-P(I-1)
 0033             10 CONTINUE
 0034                CHISQ=0.0
 0035                DO 33 I=1,NCELLL
              C
              C      CALCULATE THE PROBABILITIES OF THESE INTERVALS UNDER THE RESULTANT
              C      DISTRIBUTION.
 0036                FINP(I)=(RESP(I)+RESP(NCELL-I))/2.0
              C
              C      OBTAIN THE THEORETICAL FREQUENCIES.
              C
 0037                FREQCY(I)=FINP(I)*NUMBER
              C
              C      CALCULATE THE CHI-SQUARE VALUE.
              C
 0038                CHISQ=CHISQ+((NTRSIG(I)-FREQCY(I))**2)/FREQCY(I)
 0039             33 CONTINUE
 0040                DO 27 I=1,NCELLL
 0041                PRINT 98
 0042                PRINT 28,RESP(I),FINP(I),FREQCY(I)
 0043             27 CONTINUE
 0044                SUMX=0.0
 0045                SSUMX=0.0
 0046                X=STEP1
 0047                DO 45 I=1,NCELLL
 0048                SUMX=SUMX+X*FINP(I)
```

```
      SSUMX=SSUMX+X**2*FINP(I)
      X=X+STEP1
   45 CONTINUE
C
C     CALCULATE THE MEAN AND STANDARD DEVIATION OF THE RESULTANT
C     DISTRIBUTION.
      XMEAN=SUMX
      XSTDEV=(SSUMX-SUMX**2)**0.5
      PRINT 50
      PRINT 60,XMEAN,XSTDEV
      PRINT 90,CHISQ
   40 CONTINUE
   70 CONTINUE
      STOP
      END
```

```
FORTRAN IV G LEVEL  18                 MAIN                 DATE = 71095          20/40/58

              CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
              C                                                                      C
              C      THIS PROGRAM CALCULATES THE PROBABILITY OF DETECTING A PROCESS  C
              C      CHANGE,WHEN THE PROCESS LEVEL IS SUBJECTED TO A STEP. THIS PROGRAM
              C      CAN BE USED ONLY WHEN THE DETECTION CRITERIA IS--ONE VALUE OF   C
              C      THE TRACKING SIGNAL AMONG ANY NUMBER OF SUCCEEDING VALUES ,     C      99
              C      EXCEEDS THE LIMITING VALUE AFTER STEP IS INCORPORATED.          C
              C                                                                      C
              C      DESCRIPTION OF THE PARAMETERS---                                C
              C                                                                      C
              C      NALPHA=NUMBER OF SMOOTHING CONSTANTS FOR WHICH SIMULATION IS TO C
              C      BE CARRIED OUT                                                  C
              C                                                                      C
              C      NSIMU=NUMBER OF SIMULATIONS TO BE CARRIED OUT.                  C
              C                                                                      C
              C      NSTEP=NUMBER OF STEPS OF DIFFERENT MAGNITUDE INCORPORATED IN THE C
              C      PROCESS LEVEL.                                                  C
              C                                                                      C
              C      NUMBER=PERIOD AT WHICH THE STEP IS TO BE INCORPORATED           C
              C                                                                      C
              C      ALPHA=SMOOTHING CONSTANT                                        C
              C                                                                      C
              C      XEAN=PROCESS MEAN                                               C
              C                                                                      C
              C      STDDEV=STANDARD DEVIATION OF RANDOM NOISE                       C
              C                                                                      C
              C      XINC=A MULTIPLIER WHICH ALTERS THE MAGNITUDE OF STEP TO BE      C
              C      INCORPORATED IN THE PROCESS MEAN                                C
              C                                                                      C
              C      XLIMIT=LIMITING VALUE OF THE TRACKING SIGNAL TO BE SPECIFIED BY C
              C      THE USER                                                        C
              C                                                                      C
              C      NCOUNT=NUMBER TO BE SPECIFIED WHICH IS HALF THE NUMBER OF THE   C
              C      TRACKING SIGNALS TO BE GENERATED.                               C
              C                                                                      C
              C      BY PROPERLY SELECTING VALUES FOR NCOUNT AND NUMBER THE CRITERIA C
              C      FOR DETECTING THE PROCESS CHANGE CAN BE VARIED.FOR EXAMPLE, WHEN C
              C      NCOUNT IS 30 AND NUMBER IS 26,10 SUCCEEDING VALUES OF THE TRACKING
              C      SIGNAL WILL BE TESTED FOR CONFIRMING A CHANGE IN THE PROCESS    C
              C      AFTER STEP IS INCORPORATED.                                     C
              C                                                                      C
  0001               DIMENSION ICOUNT(20),KCUNT(20),SMAD(20),SSE(20),XD(20),TRSIG(20),X
                    1HAT(20),PROB(20),ERR(20),XKOUNT(20)
  0002             1 FORMAT(4I4,5F8.4)
  0003             2 FORMAT('1','NSIMU',5X,'NSTEP',5X,'NCOUNT',5X,'NUMBER',6X,'ALPHA',
                    16X,'XEAN',5X,'STDDEV',6X,'XINC',6X,'XLIMIT')
  0004             3 FORMAT(   I6,7X,I2,8X,I2,9X,I2,8X,F5.3,5X,F5.2,7X,F3.1,7X,F5.3,6X
                    1,F5.3)
  0005             6 FORMAT (3I8)
  0006           140 FORMAT (1H-,11F8.4)
  0007               READ 6,IY,IX,NALPHA
  0008               DO 5 IJ =1,NALPHA
  0009               READ 1,NSIMU,NSTEP,NCOUNT,NUMBER,ALPHA,XEAN,STDDEV,XINC,XLIMIT
  0010               PRINT 2
  0011               PRINT 3,NSIMU,NSTEP,NCOUNT,NUMBER,ALPHA,XEAN,STDDEV,XINC,XLIMIT
  0012               FACT=2./(2.-ALPHA)
  0013               DO 8 I=1,NSTEP
  0014             8 KOUNT(I)=0
```

```
      DO 10 L=1,NSIMU
      DO 20 I=1,NSTEP
      ICOUNT(I)=0
      SMAD(I)=0.79788*FACT*STDDEV
      SSE(I)=0.0
      XHAT(I)=XEAN
   20 CONTINUE
      BETA=1.-ALPHA
C
C     GENERATE RANDOM NUMBER BETWEEN ZERO AND ONE
C
      DO 30 I=1,NCOUNT
      IY=IY*IX
      IF(IY.LT.0) IY=IY+2147483647+1
      RA=IY*.4656613E-9
      IY=IY*IX
      IF(IY.LT.0) IY=IY+2147483647+1
      RB=IY*.4656613E-9
      UA=SQRT(-2.0*ALOG(RA))
      UB=6.2832*RB
C
C     CALCULATE TWO RANDOM NORMAL DEVIATES
C
      XA=UA*COS(UB)
      XB=UA*SIN(UB)
C
C     CHECK IF TIME HAS REACHED TO INCORPORATE THE STEP
C
      IF (L-NUMBER) 40,50,50
   40 X=XA*STDDEV+XEAN
      RD=XB*STDDEV+XEAN
C
C     CARRY OUT SMOOTHING PROCESS
C
      DO 45 II=1,2
      DO 46 J=1,NSTEP
      XD(J)=X
      ERR(J)=XD(J)-XHAT(J)
      SSE(J)=ALPHA*ERR(J)+BETA*SSE(J)
      SMAD(J)=ALPHA*ABS(ERR(J))+BETA*SMAD(J)
      TRSIG(J)=SSE(J)/SMAD(J)
C
C     SMOOTHING CARRIED OUT ON THE OBSERVED VALUE OF THE PROCES TO
C     EVALUATE THE NEXT FORECAST
C
      XHAT(J)=ALPHA*XD(J)+BETA*XHAT(J)
   46 CONTINUE
      X=RD
   45 CONTINUE
      GO TO 30
   50 X=XA*STDDEV+XEAN
      RD=XB*STDDEV+XEAN
      DO 70 K=1,2
C
C     INCORPORATE STEP OF DIFFERENT MAGNITUDE IN THE OBSERVATIONS OF
C     THE PROCESS
C
      DO 80 J=1,NSTEP
```

```
            XD(J)=X+(J-1)*XINC
            ERR(J)=XD(J)-XHAT(J)
            SSE(J)=ALPHA*ERR(J)+BETA*SSE(J)
            SMAD(J)=ALPHA*ABS(ERR(J))+BETA*SMAD(J)
C
C     EVALUATE THE TRACKING SIGNAL
C
            TRSIG(J)=SSE(J)/SMAD(J)
C
C     CHECK IF THE TRACKING SIGNAL HAS EXCEEDED THE LIMITING VALUE
C
            IF (ABS(TRSIG(J))-XLIMIT) 65,75,75
C
C     IF IT HAS EXCEEDED DETECT A CHANGE IN THE PROCESS BY MOVING THE
C     DETECTION COUNTER ICOUNT.ICOUNT WILL HAVE A VALUE 0 OR 1.
C     0 FOR NO CHANGE IN THE PROCESS AND 1 FOR CHNAGE IN THE PROCESS
C
         75 ICOUNT(J)=1
         65 XHAT(J)=ALPHA*XD(J)+BETA*XHAT(J)
         80 CONTINUE
            X=RD
         70 CONTINUE
         30 CONTINUE
C
C     COUNT THE NUMBER OF TIMES A CHANGE HAS BEEN DETECTED IN THE TOTAL
C     NUMBER OF SIMULATIONS CARRIED OUT
C
            DO 90 J=1,NSTEP
            KOUNT(J)=KOUNT(J)+ICOUNT(J)
         90 CONTINUE
         10 CONTINUE
            XNSIMU=NSIMU
C
C     CALCULATE PROBABILITY OF DETECTING A CHANGE WITH DIFFERENT
C     MAGNITUDE OF STEP INCREASE IN THE PROCESS LEVEL
C
            DO 120 I=1,NSTEP
            XKOUNT(I)=KOUNT(I)
        120 PROB(I)=XKOUNT(I)/XNSIMU
            PRINT 140,(PROB(I),I=1,NSTEP)
          5 CONTINUE
            STOP
            END
```

```
FORTRAN IV G LEVEL  18                 MAIN              DATE = 71095           16/06/57

          CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
          C                                                                      C
          C      THIS PROGRAM CALCULATES THE PROBABILITY OF DETECTING A PROCESS  C
          C      CHANGE,WHEN THE PROCESS LEVEL IS SUBJECTED TO A STEP.THE PRO-   C
          C      GRAM CAN BE USED ONLY WHEN THE CRITERIA FOR DETECTION IS BASED  C
          C      ON A SUCCESSIVE NUMBER OF THE TRACKING SIGNALS EXCEEDING THE    C
          C      LIMITING VALUE AMONG ANY NUMBER OF SUCCEEDING VALUES,AFTER A    C
          C      STEP IN THE LEVEL OF THE PROCESS IS INCORPORATED.               C
          C                                                                      C
          C      DESCRIPTION OF THE PARAMETERS---                                C
          C                                                                      C
          C      NALPHA=NUMBER OF SMOOTHING CONSTANTS FOR WHICH SIMULATION IS TO C
          C      BE CARRIED OUT                                                  C
          C                                                                      C
          C      NSIMU=NUMBER OF SIMULATIONS TO BE CARRIED OUT.                  C
          C                                                                      C
          C      NSTEP=NUMBER OF STEPS OF DIFFERENT MAGNITUDE INCORPORATED IN THE C
          C      PROCESS LEVEL.                                                  C
          C                                                                      C
          C      NUMBER=PERIOD AT WHICH THE STEP IS TO BE INCORPORATED           C
          C                                                                      C
          C      ALPHA=SMOOTHING CONSTANT                                        C
          C                                                                      C
          C      XEAN=PROCESS MEAN                                               C
          C                                                                      C
          C      STDDEV=STANDARD DEVIATION OF RANDOM NOISE                       C
          C                                                                      C
          C      XINC=A MULTIPLIER WHICH ALTERS THE MAGNITUDE OF STEP TO BE      C
          C      INCORPORATED IN THE PROCESS MEAN                                C
          C                                                                      C
          C      XLIMIT=LIMITING VALUE OF THE TRACKING SIGNAL TO BE SPECIFIED BY C
          C      THE USER                                                        C
          C                                                                      C
          C      NCOUNT=NUMBER TO BE SPECIFIED WHICH IS HALF THE NUMBER OF THE   C
          C      TRACKING SIGNALS TO BE GENERATED.                               C
          C                                                                      C
          CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
0001             DIMENSION ICOUNT(20),KOUNT(20),SMAD(20),SSE(20),XD(20),TRSIG(20),X
                1HAT(20),PROB(20),ERR(20),XKOUNT(20),IDTECT(20)
0002           1 FORMAT(4I4,5F8.4)
0003           2 FORMAT('1','NSIMU',5X,'NSTEP',5X,'NCOUNT',5X,'NUMBER',6X,'ALPHA',
                16X,'XEAN',5X,'STDDEV',6X,'XINC',6X,'XLIMIT')
0004           3 FORMAT(   I6,7X,I2,8X,I2,9X,I2,8X,F5.3,5X,F5.2,7X,F3.1,7X,F5.3,6X
                1,F5.3)
0005           6 FORMAT (3I8)
0006         140 FORMAT (1H-,11F8.4)
0007             READ 6,IY,IX,NALPHA
0008             DO 5 IJ =1,NALPHA
0009             READ 1,NSIMU,NSTEP,NCOUNT,NUMBER,ALPHA,XEAN,STDDEV,XINC,XLIMIT
0010             PRINT 2
0011             PRINT 3,NSIMU,NSTEP,NCOUNT,NUMBER,ALPHA,XEAN,STDDEV,XINC,XLIMIT
0012             FACT=2./(2.-ALPHA)
0013             DO 8 I=1,NSTEP
0014           8 KOUNT(I)=0
0015             DO 10 L=1,NSIMU
0016             DO 20 I=1,NSTEP
0017             ICOUNT(I)=0
0018             SMAD(I)=0.79788*FACT*STDDEV
```

```
0019            SSE(I)=0.0
0020            XHAT(I)=XEAN
0021         20 CONTINUE
0022            BETA=1.-ALPHA
           C
           C    GENERATE RANDOM NUMBER BETWEEN ZERO AND ONE
           C
0023            DO 30 I=1,NCOUNT
0024            IY=IY*IX
0025            IF(IY.LT.0) IY=IY+2147483647+1
0026            RA=IY*.4656613E-9
0027            IY=IY*IX
0028            IF(IY.LT.0) IY=IY+2147483647+1
0029            RB=IY*.4656613E-9
0030            UA=SQRT(-2.0*ALOG(RA))
0031            UB=6.2832*RB
           C
           C    CALCULATE TWO RANDOM NORMAL DEVIATES
           C
0032            XA=UA*COS(UB)
0033            XB=UA*SIN(UB)
           C
           C    CHECK IF TIME HAS REACHED TO INCORPORATE THE STEP
           C
0034            IF (I-NUMBER) 40,50,50
0035         40 X=XA*STDDEV+XEAN
0036            RD=XB*STDDEV+XEAN
           C
           C    CARRY OUT SMOOTHING PROCESS
           C
0037            DO 45 II=1,2
0038            DO 46 J=1,NSTEP
0039            XD(J)=X
0040            ERR(J)=XD(J)-XHAT(J)
0041            SSE(J)=ALPHA*ERR(J)+BETA*SSE(J)
0042            SMAD(J)=ALPHA*ABS(ERR(J))+BETA*SMAD(J)
0043            TRSIG(J)=SSE(J)/SMAD(J)
           C
           C    SMOOTHING CARRIED OUT ON THE OBSERVED VALUE OF THE PROCES TO
           C    EVALUATE THE NEXT FORECAST
           C
0044            XHAT(J)=ALPHA*XD(J)+BETA*XHAT(J)
0045         46 CONTINUE
0046            X=RD
0047         45 CONTINUE
0048            GO TO 30
0049         50 X=XA*STDDEV+XEAN
0050            RD=XB*STDDEV+XEAN
0051            DO 70 K=1,2
           C
           C    INCORPORATE STEP OF DIFFERENT MAGNITUDE IN THE OBSERVATIONS OF
           C    THE PROCESS
           C
0052            DO 80 J=1,NSTEP
0053            XD(J)=X+(J-1)*XINC
0054            ERR(J)=XD(J)-XHAT(J)
0055            SSE(J)=ALPHA*ERR(J)+BETA*SSE(J)
0056            SMAD(J)=ALPHA*ABS(ERR(J))+BETA*SMAD(J)
```

```
              C
              C      EVALUATE THE TRACKING SIGNAL
              C
 0057                TRSIG(J)=SSE(J)/SMAD(J)
              C
              C      CHECK IF THE TRACKING SIGNAL HAS EXCEEDED THE LIMITING VALUE
              C
 0058                IF (ABS(TRSIG(J))-XLIMIT) 65,75,75
              C
              C      IF YES COUNT THE NUMBER OF TIMES THE TRACKING SIGNAL EXCEEDS THE
              C      LIMITING VALUE SUCCESSIVELY.
              C
 0059             75 ICOUNT(J)=ICOUNT(J)+1
              C
              C      DETECT ACHANGE IN THE PROCESS IF TWO OR MORE SUCCESSIVE VALUES
              C      OF THE TRACKING SIGNAL EXCEED THE LIMITING VALUE.
              C
 0060                IF (ICOUNT(J). GE. 2) IDTECT(J)=1
 0061                GO TO 200
 0062             65 ICOUNT(J)=0
 0063            200 XHAT(J)=ALPHA*XD(J)+BETA*XHAT(J)
 0064             80 CONTINUE
 0065                X=RD
 0066             70 CONTINUE
 0067             30 CONTINUE
              C
              C      COUNT THE NUMBER OF TIMES A CHANGE HAS BEEN DETECTED IN THE TOTAL
              C      NUMBER OF SIMULATIONS CARRIED OUT
              C
 0068                DO 90 J=1,NSTEP
 0069                KOUNT(J)=KOUNT(J)+IDTECT(J)
 0070             90 CONTINUE
 0071             10 CONTINUE
 0072                XNSIMU=NSIMU
              C
              C      CALCULATE PROBABILITY OF DETECTING A CHANGE WITH DIFFERENT
              C      MAGNITUDE OF STEP INCREASE IN THE PROCESS LEVEL
              C
 0073                DO 120 I=1,NSTEP
 0074                XKOUNT(I)=KOUNT(I)
 0075            120 PROB(I)=XKOUNT(I)/XNSIMU
 0076                PRINT 140,(PROB(I),I=1,NSTEP)
 0077              5 CONTINUE
 0078                STOP
 0079                END
```

DETERMINATION OF THE TRACKING SIGNAL CONTROL LIMITS

by

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B.E. (Mech.) University of Indore, India, 1968

AN ABSTRACT OF A MASTER'S THESIS

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MASTER OF SCIENCE

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ABSTRACT

The purpose of this thesis is twofold:

1) To approximate the tracking signal distribution with some known statistical distribution.

2) To evaluate the limiting values of the tracking signal.

Attempts were made to approximate the tracking signal distribution for three different values of α (0.01, 0.1, 0.2). For an α of 0.01 and 0.1, the approximating distribution was a single beta distribution. Whereas for an α of 0.2, the approximating distribution was a resultant distribution obtained by combining two beta distributions appropriately. The procedure essentially involved generation of the tracking signal distribution and the approximating distribution and their comparison. The goodness of fit was tested by a chi-square test and a Kolmogorov Smirnov test. The results obtained are tabulated below:

ALPHA	CHI-SQUARE TEST	KOLMOGOROV SMIRNOV TEST
0.01	Inadequate	Inadequate
0.1	Inadequate	Adequate
0.2	Inadequate	Inadequate

These results lead to a conclusion that the tracking signal distribution cannot be adequately approximated with a beta or combined beta distribution. Thus, it was not possible to evaluate the tracking signal limits by the above procedure. The limits were then evaluated by simulation.

In the latter phase of this thesis, the effectiveness of the tracking signal in detecting a process change was studied by simulation. For detecting a process change various criteria were formulated. Essentially, all the criteria were based on testing a few tracking signal values after a step increase is incorporated in the process mean. For the first three criteria, the number of the tracking signals tested were 2.4 and 10 respectively. A change in the process was detected when one value among these tested values exceeded the specified limit. In the fourth criterion , 10 values were tested but a change was detected when any two successive values among these 10, exceeded the specified limit. Theoretically, the first detection criterion is the best one as it gives an indication of a process change at a very early stage. However, the results revealed that the tracking signal is very insensitive to process changes, with first detection criterion. As compared with other criteria, the first criterion registered a low probability of detecting process changes. Based on the probabilities from third and fourth criterion, the tracking signal was comparatively effective in detecting process changes. However, with these two criteria, a corrective action to be undertaken is delayed.