AN INTRODUCTION TO THE DESIGN OF REINFORCED EARTH RETAINING WALLS

by

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INTRODUCTION

Soil is the most abundant material in the world, and it has long been used as a construction material by human beings. The concept of strengthening soil by embedding reinforcing elements such as rods or fibers is not new. Chinese in ancient times were aware of the value of adding straw in the making of sun-dried clay bricks; this improved the strength of the brick which was used to build dwellings. For more than one thousand years, mattresses made of wood branches have been embedded in the soil to form dikes or revetments along the Yellow River in China. Bundles of brush wood called faggots were used in stabilizing the bank of the Mississippi River in the 1880's (7). However, not until the 1960's had this concept been developed theoretically. During that time a French engineer, Mr. Henri Vidal, developed a disciplined approach based on a reasonable design procedure for the use of reinforced earth in important engineering structures (14, 15). Since then, a variety of related studies have been undertaken, and many experiments have been conducted for the purpose of further understanding the nature and behavior of steel-reinforced earth as a construction material.

Reinforced earth mobilizes the friction between soil particles and the reinforcing strips which are placed at regular intervals horizontally and vertically to create an earth retaining structure.

Reinforced earth is advantageous wherever the conventional retaining walls have to be built under difficult conditions, e.g., poor foundation soil with low bearing capacity, or where very high walls are needed, or in areas where concrete construction is uneconomical.

The large surface area of a reinforced earth structure spreads the load over a large area. The cost of a reinforced earth wall does not increase with height as rapidly as that of a concrete wall when the height required exceeds a certain limit. Reinforced earth walls have almost no limitation in height as does sheet piling (15).

Reinforced earth is a new construction process, and it is gaining acceptance in the construction of high walls and similar structures because it costs less than conventional retaining walls. Up to the present time, some 700 reinforced earth structures have been in service all over the world (11).

Purpose of the Study

The concept of reinforced earth is simple, but the application of this concept to design leads to different approaches (5). The purpose of this study is to review the pertinent literature on the theoretical studies and experimental tests, and to develop an approach for the analysis and design of reinforced earth walls.

Scope of the Study

This study initially consisted of a careful and comprehensive review of the available literature pertaining to reinforced earth structures. Then, based upon the analyses, a computer program was developed for the design of reinforced earth retaining walls for any given wall height and soil properties. Finally, a numerical example and the writer's conclusions complete the results of this study.

LITERATURE REVIEW

Theory of Friction

In 1699 a French engineer, Mr. Guillaume Amontons (1), published the results of an extensive series of friction tests from which he developed the basic laws of friction, namely:

- 1) The frictional force is proportional to the normal force.
- The frictional force is independent of the surface area of contact.
 - 3) The frictional force is independent of the rate of movement.

As shown in Fig. 1, a stationary block having a weight, W, and being pulled by a force, p, will reach a condition called limiting friction. This occurs when, as p is increased, the block is just on the verge of impending motion. The friction equation can be formulated as follows:

F = fN

where

F = friction force

N = normal force which in this case is equal to the weight, W and

f = coefficient of static friction ($f = tan \phi$, where ϕ is called the angle of friction)

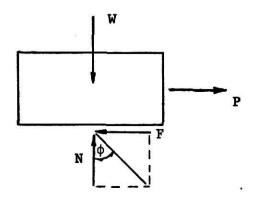


Fig. 1 - Friction Force at Impending Motion

Earth Pressure Theories

Coulomb's Earth Pressure Theory - In 1776 Coulomb (6) developed a classical theory of limiting earth pressure against a retaining wall. It was based on Amonton's laws of friction, and the assumption was made that the limiting pressure could be reached as the soil was just on the verge of sliding. The basic assumptions for the earth pressure theory proposed by Coulomb are as follows:

- 1) The soil behind the retaining wall is considered to be a dry, homogeneous, isotropic, elastically deformable but breakable, granular material. It possesses no cohesion but is capable of resisting compressive and shearing stresses.
- 2) The rupture surface is a plane. Coulomb realized that this is not so--rather, that it is a curved surface. This assumption, however, greatly simplifies the analysis and the computation.
- 3) The frictional force is distributed uniformly on the plane, ruptured surface.
 - 4) The wedge of soil which fails is a rigid body.
- 5) There is wall friction; i.e., the failure wedge moves along the back of the wall and develops a frictional force along the wall surface.
- 6) Failure is considered in a two-dimensional problem in which a unit length of an infinitely long wall is regarded.

The above assumptions limit the analysis to the ideal soil with many of the actual conditions simplified for ease of computation.

A. Active Earth Pressure

Let us consider Fig. 2. A retaining wall of height, H, is inclined at an angle α , and the backfill of unit weight, γ , is inclined at an angle β . The failure surface is assumed to

be inclined at angle θ . Between the wall and soil the friction angle is δ , and ϕ is the soil friction. The total active pressure against the wall is

$$P_{ac} = \frac{\gamma H^{2}}{2} \frac{\sin^{2}(\alpha + \phi)}{\sin^{2}\alpha \sin(\alpha - \delta) \left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\phi + \beta)}}\right]^{2}}$$
(1)

Note that when the active wall pressure is developed the wedge is on the verge of moving downward and toward the wall.

If the back of the retaining wall becomes vertical (α = 90°), equation (1) becomes

$$P_{ac} = \frac{\gamma H^2}{2} \frac{\cos^2 \phi}{\cos \delta \left[1 + \sqrt{\frac{\sin (\phi + \delta) \sin (\phi - \beta)}{\cos \delta \cos \beta}}\right]^2}$$
(2)

For this case the wall friction has been considered, and the backfill grading is upward; from equation (2) the coefficient of active earth pressure, $K_{\rm ac}$, is

$$K_{ac} = \frac{\cos^2 \phi}{\cos \delta \left[1 + \sqrt{\frac{\sin (\phi + \delta) \sin (\phi - \beta)}{\cos \delta \cos \beta}}\right]^2}$$
 (2a)

B. Passive Earth Pressure

When the passive wall pressure is developed, the failure wedge is on the verge of moving upward and to the right along the failure surface. Similar to the active earth pressure derivation from Fig. 2,

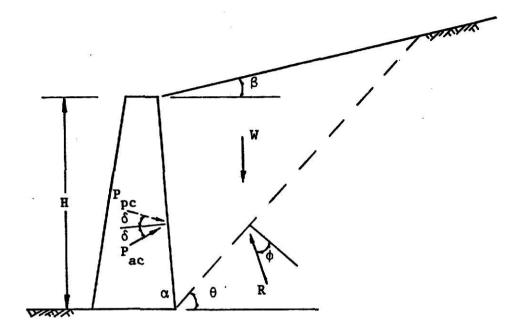


Fig. 2 - Failure Wedge and Acting Forces for the Coulomb's Theory

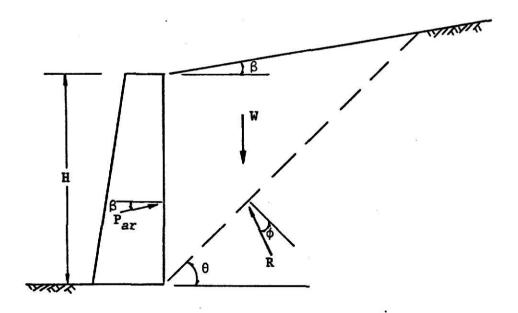


Fig. 3 - Failure Wedge and Acting Forces for the Rankine's Theory

we can derive the total passive earth pressure as

$$P_{pc} = \frac{\gamma H^{2}}{2} \frac{\sin^{2}(\alpha - \phi)}{\sin^{2}\alpha \sin(\alpha + \delta) \left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + \beta)}{\sin(\alpha + \delta)\sin(\alpha + \beta)}}\right]^{2}}$$
(3)

If the back of the retaining wall becomes vertical (α = 90°), equation (3) becomes

$$P_{pc} = \frac{\gamma H^2}{2} \frac{\cos^2 \phi}{\cos \delta \left[1 - \sqrt{\frac{\sin (\phi + \delta) \sin (\phi + \beta)}{\cos \delta \cos \beta}}\right]^2}$$
(4)

From equation (4) the coefficient of passive earth pressure, K_{pc} , is

$$K_{pc} = \frac{\cos^2 \phi}{\cos \delta \left[1 - \sqrt{\frac{\sin (\phi + \delta) \sin (\phi + \beta)}{\cos \delta \cos \beta}}\right]^2}$$
(4a)

For this case the wall friction has been considered, and the backfill is sloping upwards.

Rankine's Earth Pressure Theory - In 1857 Rankine (12) published a different and simpler method for calculating the earth pressure against walls. The basic assumptions made by Rankine are as follows:

- 1) The soil mass is semi-infinite, homogeneous, dry, and cohesionless.
- 2) The ground surface is plane which may be horizontal or inclined.
- 3) The back of the wall is vertical and smooth (no wall friction).
- 4) The soil mass is in a state of plastic equilibrium.

A. Active Earth Pressure

Fig. 3 shows the case of Rankine's earth pressure with the wall friction neglected. The total active earth pressure is

$$P_{ar} = \frac{1}{2} \gamma H^2 \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$
 (5)

From equation (5) the coefficient of active earth pressure, K_{ar} , is

$$K_{ar} = \cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}$$
 (5a)

B. Passive Earth Pressure

Similar to the active earth pressure derivation, the total passive earth pressure can be derived by analogy as

$$P_{pr} = \frac{\gamma H^2}{2} \cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$$
 (6)

From equation (6) the coefficient of passive earth pressure, K_{pr} , is

$$K_{pr} = \cos \beta \frac{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}$$
 (6a)

If a smooth, vertical wall with level backfill is considered (i.e., $\delta=\beta=0$ and $\alpha=90^{\circ}$) equations (2), (2a), (4), (4a), (5), (5a), (6), (6a) would be simplified to

$$P_{ac} = P_{ar} = \frac{1}{2} \gamma H^2 \tan^2 (45^\circ - \phi/2)$$
 (7)

$$K_{ac} = K_{ar} = \tan^2 (45^\circ - \phi/2)$$
 (7a)

$$P_{pc} = P_{pr} = \frac{1}{2} \gamma H^2 \tan^2 (45^\circ + \phi/2)$$
 (8)

$$K_{pc} = K_{pr} = \tan^2 (45^\circ + \phi/2)$$
 (8a)

In that case, according to the analysis made by Bowles (2), the failure surface is approximately a plane surface at angles of θ = 45° + ϕ /2 and θ = 45° - ϕ /2, with the horizontal plane, for active earth pressure and passive earth pressure, respectively, where ϕ is the internal friction angle of soil.

Geostatic Vertical Stress - Lambe (8) stated that when the ground surface is horizontal and when the nature of the soil varies only a very little in the horizontal direction, there are no shear stresses upon vertical and horizontal planes within the soil. Hence, the vertical stress at any depth can be computed simply by considering the weight of soil above that depth. The stress variation is shown in Fig. 4 and is formulated as follows:

$$\sigma_{v} = Z\gamma$$

where

Z = depth of overburden

and

γ = unit weight of soil

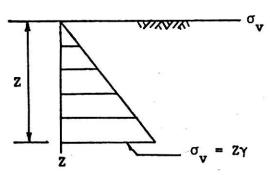


Fig. 4 - Vertical Stress Distribution Caused by Weight of Soil

Lambe also stated that "the unit weight of soil is seldom constant with depth. Usually a soil will become dense with depth because of the compression caused by the geostatic stress." If the unit weight of the soil varies continuously with depth, the vertical stress can be evaluated by means of the integral

$$\sigma_{\mathbf{v}} = \int_{\mathbf{o}}^{\mathbf{z}} \gamma \, d\mathbf{z}$$

where Y is variable.

As a matter of fact, engineers always use the average unit weight of the soil throughout the depth for the convenience of computation.

Reinforced Earth

Reinforced earth is a soil mass composed of fill strengthened by the inclusion of reinforcements such as bars, rods, fibers, or nets. The reinforcement, by means of frictional forces resulted from interaction with soil particles, aid in resisting tensile forces that the soil alone is unable to resist. According to Vidal (14), the term "earth" covers all types of soils found in nature, including both granular soils and earth which exhibits some cohesion. However, in order to avoid dealing with questionable pore pressure and possible cohesive bond, the free-draining backfill of non-cohesive soil is to be assumed for the study of this report.

Vidal (14) pointed out that when rectilinear reinforcements are horizontally embedded in non-cohesive soil, there is a transmission of forces by friction between the grains of soil and the reinforcements. This introduces some cohesion to the whole soil mass which then becomes capable of withstanding both internal and external forces, provided that there is no sliding between grains and the reinforcements. Therefore, the reinforcing members must be properly designed and arranged so that this no-sliding condition is always met.

Stresses in Reinforcement and Earth

To ascertain that the soil mass always remains in equilibrium, it is necessary to calculate the tension in the reinforcement and the stress in the earth. The inclusion of reinforcements in the earth insures that it has some characteristics of a heterogeneous material which is non-isotropic. Vidal (14) concluded, however, that this does not interfere with the validity of Mohr's circle to represent the stresses in the earth at a point. He pointed out that reinforced earth may be considered as a material having a certain elasticity.

Let us consider Fig. 5a. A non-cohesive soil element subjected to a compressive stress N_1 on its two faces cannot remain in equilibrium, because the corresponding Mohr's circle C_1 cuts the failure envelope as shown in Fig. 5b. The cube can only be stable when compressive stress N_2 acting on its other two faces are at least equal to K_aN_1 , as shown in Fig. 6a, where

$$K_a = \tan^2 (45^\circ - \phi/2)$$

As long as this condition is maintained, the Mohr's circle C_2 is tangential to the failure envelope, as seen in Fig. 6b. This is just at a critical condition for which the earth is on the imminent verge of failing, as explained by Coulomb (6). Also, N_2 is at a minimum value necessary to keep the earth in elastic equilibrium; the earth will remain in this state as long as N_2 is larger than K_aN_1 . This condition produces the Mohr's circle C_2 as shown in Fig. 6b.

By providing reinforcement to the soil elements in such a manner that the reinforcement is arranged perpendicular to the direction of the compressive force N_1 , as seen in Fig. 7a, the skin friction between reinforcement and earth particles will supply lateral resistance against

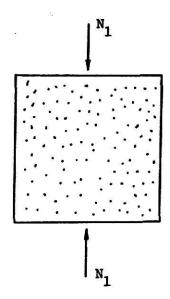


Fig. 5a - A Non-cohesive Soil Element Subjected to a Compressive Force N₁ on its Two Faces

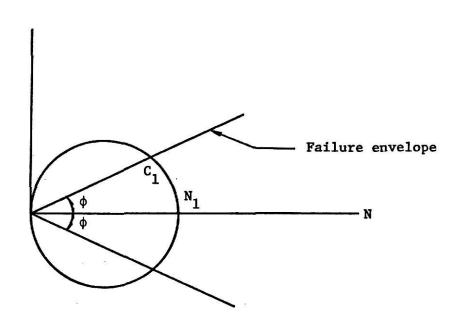


Fig. 5b - Mohr's Circle Cuts Through Failure Envelope Representing the State of the Stress in the Unbounded Earth

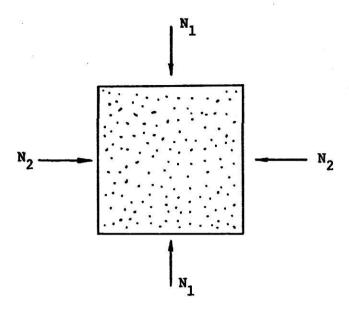


Fig. 6a - A Non-cohesive Soil Element Subjected to Compressive Forces N_1 and N_2 on its Two Axial Faces, Respectively

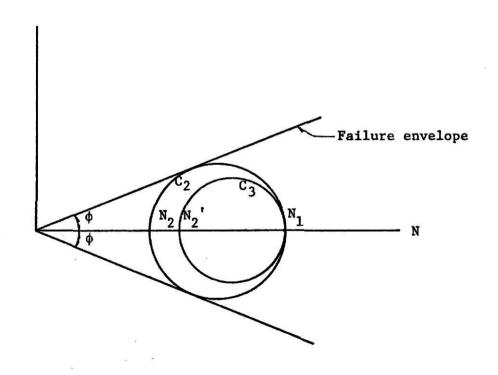
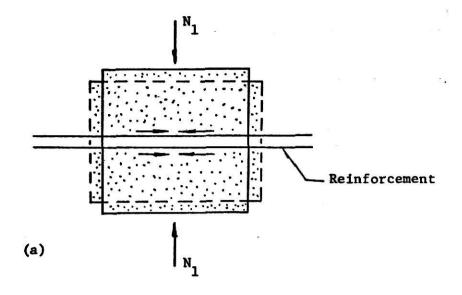


Fig. 6b - Mohr's Circle C₂ Corresponds to the Stress Just on Verge of Failure; C₃ Represents the Earth in Elastic Equilibrium



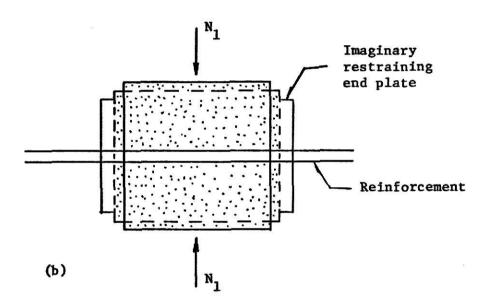


Fig. 7 - Reinforced Soil Element

- (a) Reinforcement supplies compressive force on soil element to prevent its expansion
- (b) Compressive force acts as an imaginary plate fixed at each end of soil element.

the expansion of earth. This lateral resistance acts as an imaginary plate fixed at each end of the soil elements and prevents the soil from continuing expansion (See Fig. 7b).

As a matter of fact, the sliding of the particles does occur within soil; however, it will be interrupted when the compressive force N_2 becomes equal to K_aN_1 in such a way that Mohr's circle becomes tangential to the failure envelope as described by Vidal (14).

Friction Between Reinforcement and Earth

In the theory of reinforced earth, friction force plays the whole role in resisting the earth pressure from backfill. Let us consider Fig. 8. When reinforcement is connected to the soil grains by tension forces T_1 and T_2 , a friction force equal to $T_1 - T_2$ (assuming that $T_1 > T_2$) must exist between reinforcement and soil particles to assure no sliding between them. If the stress in the earth perpendicular to the plane of the reinforcement has the value N, producing normal force Ndl to the reinforcement over the length dl (a value of 2Ndl on two faces of reinforcement), a relationship can be established as

$$\frac{\Delta T}{2Nd1} < f \tag{9}$$

where

$$\Delta T = T_1 - T_2$$

f = coefficient of friction between earth and reinforcement If K, the proportion of reinforcement per unit length, and S, the factor of safety, are introduced to the above formula, then it becomes

$$\Delta T = 2Nd1 K_r \frac{f}{S}$$
 (10)

This formula assumes that the reinforcing members are flat and that a layer of reinforcement forms a complete plane. It does not, however,

take into account the arching effect between adjoining reinforcing strips.

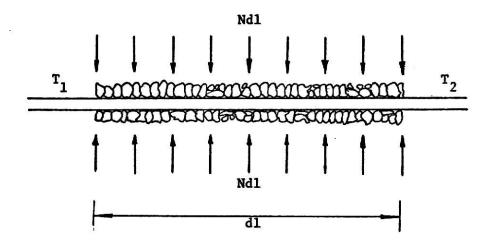


Fig. 8 - A Reinforcement is Introduced to the Soil
Grains with Tension Force T₁ - T₂ Resulting
from Friction Between Earth and Reinforcement

Arching in Soil (13)

Arching is one of the most common phenomena encountered in soils. There is no arching created without shearing stresses being mobilized. If only one part of the support of a soil mass yields, the soil adjoining the yielding part also tends to move out of its original position as shown in Fig. 9. The relative movement within the soil is resisted by the shearing resistance developed along the boundaries of contact between the yielding and unyielding portion of the soil. Since the shearing resistance tends to restrain the yielding part from moving out, it transfers part of the pressure on the yielding part to the adjoining soil, resulting in a redistribution of pressure by shear. Similarly, redistribution of pressure also occurs if different parts of a support yield by varying amounts.

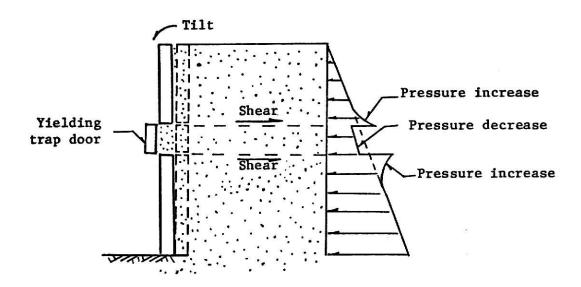


Fig. 9 - Arching Phenomenon: Shearing at One Part of the Support Causes Redistribution of Pressure

Analysis of Tension in Reinforcement

To make sure the reinforced earth is internally stable or in equilibrium, it is necessary to know the nature of the mechanism in the soil mass. Just as steel functions in tension and is bonded to reinforced concrete, so are the reinforcing strips in tension, and the bond prevents them from pulling out of the soil. Now let us first analyze the tension in the reinforcing strip. There are two ways employed for this analysis, i.e., Rankine's Method and Coulomb's Methods (9, 10):

Rankine's Method - This method is based on Rankine's theory of earth pressure in soil in which wall friction is ignored. Consider a reinforced earth structure, rectangular in shape, with a level backfill as shown in Fig. 10. It is assumed the reinforcing strips are long enough to prevent the possibility of failure due to strip pullout. At any depth, d, the vertical earth pressure is given by

$$\sigma_{ij} = \gamma d$$

where

 γ = unit weight of soil

The horizontal earth pressure is then related to the vertical earth pressure by an earth coefficient K such that

$$\sigma_h = K \sigma_v$$

As far as the value of K is concerned, it depends on the type and density of soil and on the amount of wall yield. Generally, it is necessary for the wall to yield about 0.1% and 0.2% of the wall height for loose and dense cohesionless-soils, respectively, to develop active pressure on the inner face of the wall (2).

At this moment, the coefficient of earth pressure becomes equal to $K_{\mathbf{a}}$ such that

$$\sigma_{\mathbf{h}} = \mathbf{K}_{\mathbf{a}} \, \sigma_{\mathbf{v}} \tag{11}$$

where

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 (45^\circ - \phi/2)$$
 (12)

 ϕ = friction angle of the soil

The force induced by horizontal earth pressure acting on the inner face of the skin plate will tend to either break or pull out the reinforcing strips. For simplicity, it is assumed that the horizontal and vertical spacings of strips are kept constant. At any depth, d, the horizontal pressure acting on the skin plate which is supported by a strip, is balanced by friction developed around the strip. The friction, at the same time, is resisted by tension in the strip. Thus we can conclude that the earth pressure is transferred through friction to develop tension in the strip. Therefore, at any depth, d, we can derive the following relationship as:

$$\sigma_{s} wt = K_{a} \gamma d \Delta H s$$

$$\sigma_{s} = \frac{K_{a} \gamma d \Delta H s}{wt}$$
(13)

or

where

 σ_{e} = tensile stress induced in reinforcing strip

w = width of strip

t = thickness of strip

s = horizontal spacings of strips

and

 ΔH = height of skin plate or the vertical spacings of strips

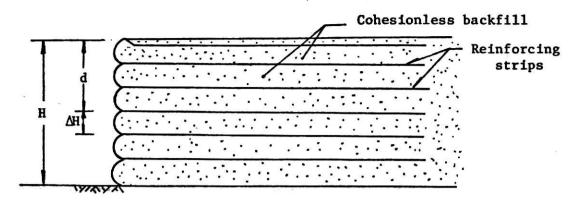


Fig. 10 - Rankine Method: Reinforcing Strips are Embedded in Soil Mass to Resist Earth Pressure

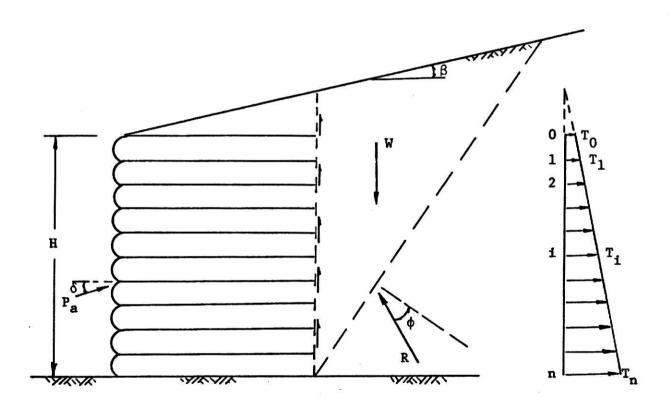


Fig. 11 - Forces and Tension Diagram in Coulomb's Analysis

Let the factor of safety be defined as the ratio of yielding strength to the stress induced in the reinforcing strip such that

$$FS = \frac{f_y}{\sigma_g} \tag{14}$$

Combining equations (13) and (14) gives

$$FS = \frac{f_y w t}{K_a \gamma d \Delta H s}$$
 (15)

From equation (15) we can calculate the required thickness of reinforcing strip.

Coulomb's Methods - As in Rankine's Method, the wall is assumed to yield enough to bring the backfill into a plastic equilibrium state.

Coulomb's approach for evaluation of earth pressure consists of studying the equilibrium of an earth wedge bounded on one side by a potential failure plane beginning from the toe of the wall. It is also assumed that within the wedge the earth around the strips is in a state of failure at any point. A reinforced earth structure of sloping backfill and with wall friction is considered here for analysis purpose (Fig. 11). It is essential that the tension in the reinforcing strips is assumed to be a linear variation with depth before proceeding with analysis. From equations (2) and (2a) we obtain the active earth pressure to be

$$P_{a} = \frac{1}{2} K_{a}^{\prime} \gamma H^{2} s \tag{16}$$

where

$$K_{a}' = \frac{\cos^{2} \phi}{\cos \delta \left[1 + \sqrt{\frac{\sin (\phi + \delta) \sin (\phi - \beta)}{\cos \delta \cos \beta}}\right]^{2}}$$
(17)

 ϕ = friction angle of soil

 β = angle of backfill

and

 δ = angle of wall friction

There are two approaches in Coulomb's Method; these are: (a) the sum of the tension forces in all strips is equated to horizontal earth pressure, and (b) the sum of moments about the toe of tension forces in every strip is equated to the moment produced by the lateral earth pressure about the toe.

a. Coulomb Force Method:

Equating the sum of the tension forces at every strip to the horizontal pressure gives:

where

and

i = index number for vertical strip layers

 T_i = tension in the ith strip below surface

 $\Delta T = T_i - T_{i-1}$, the difference of tension force between two adjacent strips

 $n = \frac{H}{\Delta H}$, the number of total layers of reinforcing strips

ΔH = height of skin plate or the vertical spacings of reinforcing strips

Therefore tension T_i in i^{th} layer is:

$$T_{i} = i\Delta T = \frac{i}{n(n+1)} K_{a}^{\dagger} \gamma H^{2} s \cos \delta \qquad (18)$$

which can be simplified to a form for calculating the tension at any depth, d, below the surface as:

$$T = \frac{n}{n+1} K_a \gamma \Delta Hd s \cos \delta$$
 (19)

b. Coulomb Moment Method:

The moment due to earth pressure about the toe is

$$M_{P_a} = P_a \frac{H}{3} \cos \delta = \frac{K_a' \gamma H^3 s}{6} \cos \delta \qquad (20)$$

It should be noted that the moment due to vertical component of lateral pressure is neglected, because its absence only leads to more conservative results.

The sum of moments due to tension forces in every layer of strip about the toe is

$$M_{t} = \sum_{i=0}^{n} i \Delta T (n-i) \Delta H = \frac{\Delta T \Delta H}{6} n (n^{2}-1)$$
 (21)

Equating M_{P_a} to M_t gives:

$$sn^3 \Delta H^3 K_a \gamma \cos \delta = \Delta T \Delta H n(n^2-1)$$

The tension in the ith strip below the surface is

$$T_{i} = i \Delta T = \frac{i n^{2}}{(n^{2}-1)} K_{a} \gamma \Delta H^{2} s \cos \delta$$
 (22)

Therefore at any depth, d, below the surface the tension can be expressed as:

$$T = \frac{n^2}{n^2 - 1} K_a \gamma \Delta H d s \cos \delta$$
 (23)

From the moment method it is clear that the wall is stable against overturning from earth pressure provided that the strips are long enough to exclude the failure by pullout of strip.

In the case where the wall friction is neglected and the backfill is level, resulting in K_a being equal to K_a and $\cos \delta = 1$, then all the above mentioned methods will lead to the same result as long as the wall has a large number of strip layers vertically. This is true because the terms $\frac{n}{n+1}$ and $\frac{n^2}{n^2-1}$ are close to unity for large values of n.

Analysis of Friction Around Reinforcement

Friction is necessary on the surface of the strips in the reinforced earth. As in reinforced concrete where the steel should have adequate length to develop bond to prevent slippage, so the reinforcing strips require enough length to develop friction to keep the strips from being pulled out. There are several approaches proposed to calculate friction and strip length, but different assumptions are required to derive formulae (3, 10, 11). Considering the reinforced earth to be a composite material, a corresponding approach will be derived. The friction force developed on both sides of a reinforcing strip (ignoring that developed along the thickness of the strip) at any depth, d, will be

$$F = 2 \text{ yd wl tan } \delta \tag{24}$$

where

 δ = friction angle between soil and reinforcing strip

1 = length of reinforcing strip

w = width of reinforcing strip

Because the friction is to resist the earth pressure acting on the skin plate supported by a strip, the factor of safety against slippage can be expressed as

$$FS = \frac{2 \text{ } \gamma \text{d } \text{w } 1 \text{ } \tan \delta}{K_a \text{ } \gamma \text{d } \Delta \text{H } \text{s}} = \frac{2 \text{ w } 1 \text{ } \tan \delta}{K_a \text{ } \Delta \text{H } \text{s}}$$
(25)

from which the width of strip is determined to be

$$W = \frac{K_a \Delta H s}{2 1 \tan \delta} FS$$
 (26)

It should be noted that the width of strips is independent of the height of overburden and surcharge (if any).

According to Chang, J. C. (4), the reinforcements must be closely spaced so that an arching action can be created within that soil which is not in direct contact with reinforcements. However, it is still uncertain as to what will be the maximum allowable horizontal and vertical spacings which can create an arching action and cause the soil particles to link together. The desirable and rational way to determine the maximum reinforcement spacings is through the large-scale model tests and field performance studies. Many experiments have come up with results which indicate that rather small size of reinforcements at relatively close spacing, ranging from one to five feet for horizontal spacing and 10 to 12 inches for vertical spacing, are satisfactory. Stability of Reinforced Earth Walls (2, 4, 11)

There are two possible failures involved in reinforced earth walls; these are failures due to internal and to external unstability. A reinforced earth wall, like any other structure, should be designed to satisfy internal stability and then checked for external stability. The previous two sections of this report analyzed tension and friction in strips and these are the main factors associated with internal stability.

As far as external stability is concerned, the reinforced earth embankment under expected loading condition is analyzed by regarding the reinforced earth mass as a solid block or a gravity type of concrete retaining wall (4, 5). In other words, sliding and overturning of the structure and bearing failure of the foundation soil are investigated in the same way as is ordinarily done in the design of retaining walls.

The lateral pressure due to backfill and surcharge (if any) tends to topple the reinforced earth mass over its toe. This overturning moment is resisted by the weight of the reinforced earth mass. Accordingly, the factor of safety against overturning is given by:

FS (overturning) =
$$\frac{M_r}{M_o}$$

where

Mr = moment available to resist overturning about toe

Mo = moment applied to the reinforced earth structure to topple over toe

A factor of safety against overturning of 1.5 is usually required for granular backfill.

The horizontal component of all the lateral pressures tends to push the wall and cause it to slide along the base of the reinforced earth mass. It is a common practice not to take into account the passive pressure in a sliding stability analysis. If the passive pressure is not considered, a minimum factor of safety of 1.5 is required. The sliding force along the bottom of the reinforced earth mass is resisted by horizontal friction force. For reinforced earth, it is earth sliding on earth, thus leading to results which are more conservative (11). The factor of safety against sliding is defined as

FS (sliding) =
$$\frac{\text{available resisting}}{\text{driving forces}} \frac{\text{forces}}{\text{driving forces}}$$

The bearing capacity of foundation soil also has to be checked to ensure a sufficient factor of safety against failure. The reinforced earth is so flexible that it will deform to follow the settlement of the earth mass. In addition, a large resting area is available to spread the load. Thus, reinforced earth structures can be considered where the foundation condition is 40-60 percent lower than that which would be

required by a conventional earth retaining structure or when the normal construction would give rise to large settlement (11).

Some rules of thumb suggested by the Reinforced Earth Company (4) for stability considerations are as follows (Fig. 12):

- The ratio of depth to clear height of reinforced earth walls should be at least 0.8.
- 2) The footing of reinforced earth wall should be embedded in an excavated or backfilled berm to one-fifth of the clear height of the reinforced earth wall.
 - 3) The width of berm should be at least five feet.
- 4) The outer slope of the berm should not be steeper than 1.5 feet horizontal to one foot vertical.

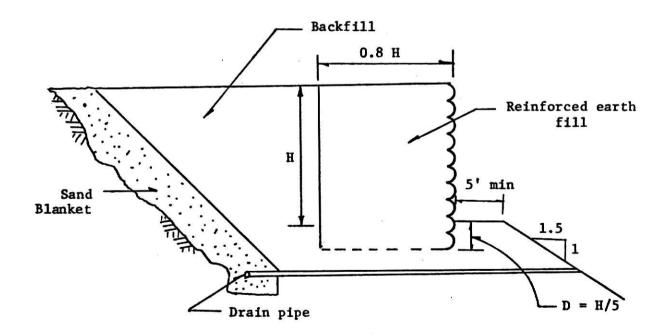


Fig. 12 - Tentative Dimension of Reinforced Earth
Wall for Stability Requirements

Analysis of Stress in Skin Plate

Skin plates which retain the particles of soil contained between individual rows of reinforcing strips should have sufficient flexibility to follow all deformations of the soil mass which result from settlement upon loading. When deforming, the skin plates will seek a curved shape which is a function of the maximum tensile strength of the skin plates. The semi-elliptical skin plates, as used by the Reinforced Earth Company are flexible enough so that they will deform into their lowest energy position.

Skin plates may fail either by loop tension or by bending as in a beam supported by two steel strips (9). For simplicity in the analysis, a semicircular section of skin plates is assumed in this report.

Based on the analysis of hoop tension as made in strength of materials and assuming a one-foot length of skin plate as shown in Fig. 13, the following relationship can be established for any depth, d, as

$$\sigma_{s}t = \sigma_{h}r = K_{a} \gamma d r$$

$$t = \frac{K_{a} \gamma d r}{\sigma_{s}}$$
(27)

or

where

 σ_s = tensile stress induced in the skin plate

r = radius of the skin plate

and

t = thickness of the skin plate

Let the factor of safety be defined as the ratio of yielding strength to the stress induced in the skin plate such that

$$FS = \frac{f_y}{\sigma_S} \tag{28}$$

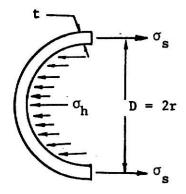


Fig. 13 - Semicircular Skin Plate with Acting Forces

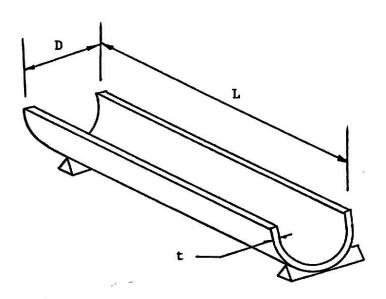


Fig. 14 - A Segment of the Skin Plate is Treated as a Simply-Supported Beam

Combining equations (27) and (28) gives:

$$t = \frac{K_a \gamma d r}{f_y} FS$$
 (29)

Let us consider the case of failure due to bending as in a beam simply supported by two reinforcing strips which can be reasonably replaced by two hinges at the outside face of the skin plates as shown in Fig. 14. The skin plates, in effect, go continuously across reinforcing strips horizontally; however, the assumption of simple support is based on the ground that the maximum bending moment induced in simply supported beam is larger than that induced in continuous beam, thus leading to results which are more conservative. In addition, the assumption simplifies the calculation.

According to the basic analysis of strength of materials, the following relation can be applied (See Fig. 14).

$$\sigma = \frac{M}{Z} = \frac{M}{T}$$
 (30)

The maximum bending moment induced in a simply supported beam of length, L, and carrying a uniformly distributed load, P, is

$$M_{\text{max}} = \frac{PL^2}{8} \tag{31}$$

At any depth, d, the skin plate is loaded by a uniformly distributed load which is

$$P = K_{a} \sigma_{W} D = K_{a} \gamma d D$$
 (32)

By combining equations (30), (31) and (32), we have

$$\sigma_{\text{max}} = \frac{K_{\text{a } \text{ } \text{ } \text{d } \text{D } \text{L}^2 \text{y}}}{8 \text{ I}} \tag{33}$$

where

σ = maximum bending stress induced at the outermost fiber

o, = vertical earth pressure at depth, d

D = diameter (or height) of the semicircular skin plate

L = horizontal spacing between two reinforcing strips

y = distance from neutral axis to outermost fiber

and

I = moment of inertia of the semicircular skin plate

When the moment of inertia, I, and the distance, y, which have the values of $0.012~\pi D^3 t$ and $0.57~D/\pi$, respectively, are substituted into equation (33), the thickness, t, may be determined to be

$$t = \frac{0.6 \, K_a \, \gamma \, d \, L^2}{\sigma_{max} \, D} \tag{34}$$

Let the safety factor be defined as the ratio of yielding strength to the induced maximum bending stress such that

$$FS = \frac{f_y}{\sigma_{max}}$$
 (35)

Substituting equation (35) into equation (34) gives:

$$t = \frac{0.6 \, K_a \, \gamma \, d \, L^2}{f_y \, D} \, FS$$
 (36)

The thickness of the skin plate based on equations (29) and (36) will be very small. It is even too thin to be handled or used for field installation. Also, in order to prevent failure resulting from corrosion impact, and abrasion, a greater thickness than that theoretically calculated should be used.

Both equations (29) and (36) are used to calculate the thickness of the skin plate. Whichever result is thicker will be used, plus an additional thickness as required to prevent failure resulted from those causes already mentioned.

NUMERICAL EXAMPLE

Design a reinforced earth wall which is to retain a 60-foot embankment under these given conditions:

- 1) The sand in the backfill is level and has an internal friction angle, ϕ , of 30° and a unit weight, γ , of 110 lb/cu ft.
- 2) The friction angle between the soil and the reinforcing strips is 25°.
 - 3) There is no surcharge.
- 4) The bearing capacity of the base is assumed to be large enough to sustain the weight of the soil mass.
- 5) The yielding stress of the strips and the skin plates is assumed to be 36,000 lb/sq in.

A computer program solution is employed here for the purpose of illustration. First of all, the width of wall (or the length of the strip) should be determined to satisfy the stability requirements of overturning and sliding. The spacings of strips have to be decided also. In order to cause arching action within the soil, the vertical spacings should be from 10 inches to 12 inches and the horizontal spacings from one foot to five feet. The spacings used in this solution are based on those considerations. However, here only one foot of horizontal spacing is used because the width of the strip is in linear variation with the spacing, and the thickness of the strips at every layer is constant for all kinds of spacings. The thickness of the skin plate is calculated for each case of strip spacings from one foot to five feet.

To meet the stability requirements, all strips should be at least 27 feet long in this example. After determining the length of the strip, there are additional considerations to be taken into account in the design. In this example, if 10 inches of skin plate and one foot of horizontal strip spacing are adopted, the width of each strip is 0.265 inches, and the thickness of the skin plate is 0.0073 inches. Similarly, if 12 inches of skin plate and two feet of horizontal strip spacing are used, the width of each strip is w = (2)(0.318) = 0.636 inches, and the thickness of the skin plate is 0.024 inches.

The thickness of strips may change from one layer to another layer.

In some cases, we may use the thickest strip in every layer for the

ease of construction. The thickness of skin plate is calculated to

withstand the most critical conditions; these occur at the lowest skin

plate.

As far as the factors of safety are concerned, it is assumed to be 1.5 for the stability requirement, and assumed to be 2 for reinforcing strip and skin plate design in this solution.

The use of this computer program is easy. The user just puts the wall height, the friction angle of the soil, the friction angle between the soil and the reinforcement, the unit weight of the soil, and the surcharge (if any) in one data card. If comparisons are needed for different wall heights and soil properties, then a set of data cards can be inserted for the design of different cases.

```
WALL: PPOC OPTIONS (MAIN):
14
                                                      #/
13
    THIS PROGRAM IS FOR DESIGNING REINFORCED EARTH WALLS
                                                      27
14:
    FOR ANY GIVEN WALL HEIGHT AND SOIL PROPERTIES
                                                      */
    THE OUTPUT INCLUDES: THE LENGTH AND WIDTH OF STRIP,
/*
                                                      */
1 **
    THE THICKNESS OF STRIPS AT EVERY VERTICAL LAYER. AND
                                                      */
/#
    THE THICKNESS OF SKIN PLATE FOR EACH CASE OF STRIP
                                                      4/
    SPACING FROM ONE FOOT TO FIVE FEET.
                                                      +/
/*
                                                      41
1 ...
DCL (FPW, FRW1)(80,2) FLOAT DEC(8), RW(3) FLOAT DEC,
        (WWR, HH, DH, SF2)(2) FLOAT DEC, (MR, MO, N) FLOAT DEC,
       (T,T2)(2,5) FLOAT DEC;
    II =0:
    GET LIST((DH(I)DO I=1 TO 2));
    ON ENDFILE(SYSIN) STOP:
1*
    TO GET DATA FOR REINFORCED EARTH WALL DESIGN
                                                      #/
/=
      IS THE HEIGHT OF WALL
                                                      #/
    FI IS THE FRICTION ANGLE OF SOIL
/*
                                                      * /
1%
    AL IS THE FRICTION ANGLE BETWEEN SOIL AND REINFORCEMENT
       IS THE UNIT WEIGHT OF SOIL
1 *
                                                      */
1=
      IS SURCHARGE
                                                      11
1*
    CA IS COEFFICIENT OF ACTIVE EARTH PRESSURE
                                                      41
    DE IS TOTAL DRIVING FORCE
                                                      */
1 4
    MO IS THE OVERTURNING MOMENT ABOUT TOE
12
                                                      3. 1
    MR IS THE RESISTING MOMENT ABOUT TOE
                                                      */
14.
/**********************
TAKE:GET LIST(H,FI,AL,R,Q);
    12=1:
    FF=FI/57.2957;
    AA=AL/57.2957;
    TAA=SIN(AA)/COS(AA);
    TFF=SIN(FF)/CDS(FF);
    CA=(1-SIN(FF))/(1+SIN(FF));
    DF1=0.5*R*CA*H**2;
    DF2=CA#Q#H:
    DF=DF1+DF2;
    MD=DF1*H/3+DF2*H/2;
    WT=R#H+Q:
    SF1=1;
【我来的我家庭家庭的出来去的我有谁我我我我的我的的去女女我的的的我的我的我的我的我的我的我们我们我们我们的一个女孩的不会的!
1 *
    TO DETERMINE THE LENGTH OF STRIP WHICH DEPENDS UPON THE
                                                      #/
    SAFETY FACTOR AGAINST OVERTURNING WHICH SHOULD BE AT
                                                      2:/
/*
                                                      */
/-
    LEAST 1.5
/*
    L IS THE LENGTH OF STRIP
                                                      #/
    SFI IS SAFETY FACTOR AGAINST OVERTURNING
                                                      4. /
1+
DO L=1 BY 1 WHILE(SF1<1.5);
      MR = R + H + L + L / 2 + 0 + L + L / 2 ;
       SF1=MR/MO:
    END;
    L=L-1;
TO CHECK THE LENGTH OF STRIP OBTAINED FROM OVERTURNING
1 50
    REQUIREMENT BY CHECKING THE SAFETY FACTOR AGAINST
1 4
```

THE FOLLOWING PAGE IS CUT OFF

THIS IS AS RECEIVED FROM THE CUSTOMER

```
SLIDING WHICH SHOULD BE AT LEAST 1.5. AND REDETERMINE
12.
                                                      #/
14
    THE LENGTH IF THE PREVIOUS LENGTH DOES NOT MEET SLIDING
/ *
    REQUIREMENT.
                                                      r /
1:
    THE WIDTH OF STRIP DESIGNATED BY RW(2) IS ALSO CALCULATED*/
    RE IS RESISTING FORCE
1.4
                                                      #/
1 #
    SF2 IS SAFETY FACTOR AGAINST SLIDING
                                                      t: /
DO I=1 TO 2:
       SF2(1)=1;
       HH(I) = DH(I)/12:
       SW=12*CA*HH(I);
       DO J=L BY 1 WHILE(SF2(I)<1.5);
         RW(2) = SW/(J + TAA):
         RF=WT*J*(TFF+RW(2)/12*(TAA-TFF));
         SF2(I)=RF/DF;
       END:
       IF L<(J-L) THEN
         DO:
            L=J-1:
            MR=R*H*L*L/2+C*L*L/2;
            SF1=MR/MD:
         END;
       WWR(I)=PW(2):
       BB=H/HH(I):
       N=TRUNC(BB):
       IF I=1 THEN N2=N:
         ELSF N1=N;
       NN=N+1:
12:
    TO CALCULATE THE THICKNESS OF STRIP WHICH IS PW(3)
                                                      */
                                                      */
/+
    AT EVERY VERTICAL LAYER
/****************************
       DO K=1 TO NN:
         IF K=NN & N<BB THEN RW(1)=H:
            ELSE IF K<NN THEN PW(1)=HH(1)*K;
         FRW(K,1)=RW(1);
         RN(3) = CA * (R*RN(1) + Q) * HH(1) / (RN(2) * 36000) * 2;
         FRW(K,?)=RW(3);
         IF I=1 THEN
            DO:
              FRW1(K,1)=FRW(K,1);
              FRWI(K,2) = FRW(K,2);
            END:
       END:
/ x
    TO CALCULATE THE THICKNESS OF SKIN PLATE WHICH IS
                                                      */
1*
    T(I,JJ) FOR DIFFERENT STRIP SPACINGS
                                                      */
1#
    HE IS HORIZONTAL FORCE DUE TO FARTH PRESSURE
                                                      */
HF = CA \times (P \times (H - HH(1)/2) + Q);
      T1=HF*DH(I)/(36000*144);
      DO JJ=1 TO 5;
         T2(1.JJ)=0.6*HF*JJ**2*2/(3600C*DH(I));
         IF T1>=T2(I,JJ) THEN T(I,JJ)=T1;
            ELSE T(1,JJ)=T2(1,JJ);
       END:
    FND;
```

```
I1=I1+1;
**************************************
    TO PRINT OUT THE RESULTS OF DESIGN
PUT PAGE EDIT('TABLE',11,'ALHA=',AL,'PHI=',FI,'GAMA=',R,
                 • PCF*)(SKIP(3),X(45),A,F(3),SKIP(2),X(3?),
                A,F(5,2),X(3),A,F(5,2),X(3),A,F(6,2),A);
    PUT EDIT('H=',H,' FT','L=',L,' FT','Q=',Q,' PSF','D= 1 FT',
            'H (FT)','B (IN)',']','H (FT)','B (IN)')(R(FORM));
FORM: FORMAT (CUL(33), A, F(5,2), A, X(2), A, F(6,2), A, X(2), A, F(7,2), A,
          SKIP,COL(47),A,SKIP,COL(23),A,F(5,2),A,X(12),A,X(10),
          A.F(5,2),A.SKIP,CUL(23),A.X(5),A,X(10),A,X(10),A,
          X(5),4);
    IF N<BB THEN
       DO:
          N1=NN;
          N2=N2+1:
       END;
    DO II=1 TC NI;
       IF II=49*12 THEN
          00:
             PUT PAGE EDIT('TABLE', II,' (CONT''D)', 'ALHA=',
                         AL, 'PHI=', FI, 'GAMA=', R, ' PCF')
                         (SKIP(3), COL(46), A, F(3), A, SKIP(2),
                         X(32), A, F(5,2), X(3), A, F(5,2), X(3),
                         A, F(6, 2), A);
             PUT EDIT('H=',H,' FT','L=',L,' FT','Q=',Q,' OSF',
                    'D= 1 FT', 'DH=', DH(1), ' INCHES', '|',
                    *DH=*,DH(2),' INCHES*, "H. (FT)*, *B (IN)*,
                    "|','H (FT)','B (IN)')(R(FORM));
          FND:
       IF II>N2 THEN
          PUT EDIT('1', FRW(II,1), FRW(II,2))(COL(50), A, X(8),
                 F(8,4),X(3),F(10,8));
          ELSE PUT EDIT(FRW1(II,1), FRW1(II,2), '| ', FRW(II,1),
                      FRW(II,2))(COL(21),F(3,4),X(3),F(10,8),
                      X(8), A, X(8), F(8, 4), X(3), F(10, 8));
    END;
    PUT EDIT('!','|','W=',WWR(1),' INCHES','|','W=',WWR(2),

    INCHES!)((2)(CCL(50),A),CGL(22),A,F(10,7),A,X(9),

            A.X(10),A,F(10,7),A);
    PUT EDIT('|','|','S.F. AGAINST SLIDING=',SF2(1),'|',
            'S.F. AGAINST SLIDING=', SF2(2), 'S.F. AGAINST OVER',
            'TURNING=', SF1)((2)(COL(50), A), COL(22), A, F(5,2),
            X(2),A,X(3),A,F(5,2),SKIP(2),X(34),(2)A,F(5,?));
    PUT EDIT('THICKNESS OF SKIN PLATF', 'HS (FT)', 'T (IN)', 'I',
            'HS (FT)','T (IN)')(SKIP(3),COL(39),A,SKIP,X(21),
            A_{1}X(5)_{1}A_{2}X(10)_{1}A_{1}X(9)_{1}A_{2}X(5)_{1}A);
    DO M=1 TO 5;
       PUT EDIT(M, T(1, M), '|', M, T(2, M))(COL(22), F(1), X(9),
              F(10,8),X(8),A,X(9),F(1),X(9),F(10,8));
    END;
TO SHIFT TO GET NEW DATA FOR ANOTHER WALL DESIGN
                                                          #/
GO TO TAKE:
    END WALL:
```

TABLE 1

| H=60.00 FT | | ALHA=25.00 | PHI=30.00 | GAMA=110.00 | PCF |
|--|----------|------------|-----------|-------------|--|
| DH=12.00 INCHES | | | | | |
| H (FT) | | | D= 1 FT | | |
| 1.000 | DH=12.00 | INCHES | i i | DH=10.00 | INCHES |
| 2.0000 | H (FT) | B (IN) | ı | H (FT) | B (14) |
| 3.0000 0.01923522 2.5000 0.01602934 4.0000 0.02564967 3.2323 0.02137246 6.0000 0.03847045 5.0000 0.03205870 7.0000 0.04488219 5.8333 0.3747183 8.0000 0.05770568 7.5000 0.04893805 10.0000 0.06411737 8.3333 0.05347115 11.0000 0.06411737 8.3333 0.05347115 12.0000 0.07652916 9.1667 0.05377428 12.0000 0.07652916 9.1667 0.05377428 12.0000 0.07652916 10.0000 0.06411737 13.0000 0.08335263 10.9333 0.06940045 14.0000 0.08335263 10.9333 0.06940045 14.0000 0.09617615 12.5000 0.09014667 16.0000 0.10258782 13.3333 0.08548987 17.0000 0.10258782 13.3333 0.08548987 17.0000 0.11541134 15.0000 0.09617603 19.0000 0.124823474 16.6667 0.09439295 21.0000 0.124823474 16.6667 0.1069025 22.0000 0.14107333 13.3333 0.1151917 20.0000 0.14747000 19.1667 0.1089025 22.0000 0.14747000 19.1667 0.12823462 23.0000 0.14747000 19.1667 0.1282362 25.0000 0.16029346 20.9333 0.13557782 26.0000 0.17311692 22.5000 0.14426410 28.0000 0.17952871 23.3333 0.195607 27.0000 0.17311692 22.5000 0.14426410 28.0000 0.17952871 23.3333 0.1966590 27.0000 0.17311692 22.5000 0.14426410 28.0000 0.19235229 25.0000 0.16029346 20.0000 0.19235229 25.0000 0.16029334 31.0000 0.19235229 25.0000 0.16029334 31.0000 0.2153773 25.3333 0.1966590 37.0000 0.22441089 29.1667 0.15895026 30.0000 0.22441089 29.1667 0.15895026 30.0000 0.22517576 26.6667 0.17077962 32.0000 0.22517576 26.6667 0.17077962 33.0000 0.27570486 31.83333 0.19166590 35.0000 0.27570486 31.83333 0.22375397 44.0000 0.27570486 35.8333 0.23775264 44.0000 0.26859140 34.1667 0.2030333 3.0000 0.27570486 35.8333 0.22757397 44.0000 0.28857844 37.5000 0.22441089 29.1667 0.2353144 44.0000 0.28857844 37.5000 0.22441089 29.1667 0.2353144 44.0000 0.28857844 37.5000 0.22441089 29.32512641 | 1.0000 | 0.00641174 | 1 | 0.8333 | 0.00534312 |
| 4,0000 0.02504697 1 3.3323 0.02137746 5,0000 0.03267745 1 5.0000 0.03226870 7,0000 0.04488219 1 5.8333 0.3741183 8,0000 0.05129391 1 6.6667 0.04274495 9,0000 0.05770568 1 7.5000 0.04493805 10,0000 0.06411737 1 8.3333 0.05347115 11,0000 0.07694083 1 10.0000 0.06411737 12,0000 0.07694083 1 10.0000 0.06411737 13,0000 0.08335263 1 10.9333 0.06446045 14,0000 0.08976436 1 11.6667 0.07489359 15,0000 0.09617615 1 22.5000 0.09014667 16,0000 0.10258782 1 3.3333 0.08548987 17,0000 0.10399755 1 14.1667 0.0933295 18,0000 0.1154134 1 15.0000 0.90417603 19,0000 0.12192307 1 15.3333 0.10151917 20,0000 < | 2.0000 | 0.01282348 | 1 | 1.5667 | 0.01063623 |
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| 그래 가는 경에 발생하는 그는 사람들이 되었다. 그는 경에 가는 가는 사람들이 되었다는 것이 되었다. 그는 사람들이 그는 사람들이 되었다. 그는 사람들이 그는 사람들이 되었다. 그는 사람들이 그는 | 46.0000 | | 1 | | |
| 48.0000 0.30776359 40.0000 0.25646937 | 47.0000 | 0.30135190 | 1 128 | 39.1667 | 0.25112641 |
| | 48.0000 | 0.30776358 | 1 | 40.0000 | 0.25646937 |

TABLE 1 (CONT'D)

| | ALH4=25.00 | PHI=30.00 | G4MA=110.00 | PCF | |
|---|---|-------------|--------------|-------------|--|
| | | L= 27.00 FT | | | |
| | | D= 1 FT | | | |
| DH=12.00 | INCHES | 1 | DH=10:00 | INCHES | |
| H (FT) | B (IN) | 1 | H (FT) | B (IN) | |
| 49.0000 | 0.31417537 | 1 | 40.8333 | 0.26131257 | |
| 50.0000 | 0.32059704 | 1 | 41.6667 | 0.26715577 | |
| 51.0000 | 0.32699883 | 1 | 42.5000 | 0.27740885 | |
| 52.0000 | 0.33341062 | | 43.3333 | 0.27784193 | |
| 53.0000 | 0.33982229 | 1 | 44.1667 | 0.28313512 | |
| 54.0000 | 0.34623409 | 1 | 45.0000 | 0.28852803 | |
| 55.0000 | 0.35264575 | 1 | 45.8333 | 0.29397123 | |
| 56.0000 | 0.35905755 | 1 | 46.6557 | 0.29921463 | |
| 57.0000 | 0.36546922 | İ | 47.5000 | 0.30455744 | |
| 58.0000 | 0.37188101 | j | 48.3333 | 0.30990076 | |
| 59.0000 | 0.37829280 | j | 49.1667 | 0.31524384 | |
| 60.2000 | 0.38470459 | Ĩ | 50.0000 | 0.32058680 | |
| | AND THE PERSON NAMED IN COLUMN TWO | ĺ | 50.8333 | 0.32593000 | |
| | | 1 | 51.6667 | 0.33127329 | |
| | | İ | 52.5000 | 0.33661623 | |
| | | ĺ | 53.3333 | 0.34195935 | |
| | | ì | 54.1667 | 0.34730256 | |
| | | Ī | 55.0000 | 0.35254552 | |
| | | 1 | 55.8333 | 0.35798883 | |
| | | i . | 56.6667 | 0.36333191 | |
| | | 1 | 57.5000 | 0.36867497 | |
| | | i | 58.3333 | 0.37401807 | |
| | | Ì | 59.1667 | 0.37936127 | |
| | | i | 60.0000 | 0.38470423 | |
| | | ì | | | |
| | | i | | | |
| W= 0.3177037 INCHES W= 0.2647530 INCHES | | | | | |
| name කො ජනතයක් සිටි ජ | nos rom 100 (DENESPECISTOR BRIDGES) (ABS) | j | | | |
| | | 1 | | | |
| S.F. AGAIN | VST SLIDING= | 1.55 S.F | . AGAINST SL | IDING= 1.55 | |
| | | - | | | |

S.F. AGAINST MOMENT= 1.82

| | THICKNESS | OF | SKIN | PLATE | | |
|---------|------------|-----|------|-------|------|------------|
| HS (FT) | T (IN) | 1 | | HS | (FT) | T (IN) |
| 1 | 0.00606018 | 1 | 81 | 1 | | 0.00723239 |
| 2 | 0.02424370 | 1 | | 2 | | 0.02912957 |
| 3 | 0.05454158 | - 1 | | 3 | | 0.06554151 |
| 4 | 0.09696281 | 1 | | 4 | | 0.11651826 |
| 5 | 0.15150440 | - 1 | | 5 | | 0.18205982 |

CONCLUSIONS

- 1) The Rankine and the Coulomb earth pressure theories are useful for the analysis of reinforced earth retaining walls.
- 2) In the theory of reinforced earth, the friction mobilized between soil grains and reinforcing strips plays the whole role that achieves the reinforcing action in the earth body. The lateral earth pressure is transferred through friction to the tensile forces induced in the reinforcing strips.
- 3) The vertical and horizontal spacings must be closely enough spaced so that an arching action can be created within those soil particles which are not in direct contact with the reinforcing strips. The arching action causes the soil particles to link together without separation.
- 4) To satisfy the minimum stability requirements, the ratio of depth to clear height of reinforced earth embankment is about 0.5.

 This is lower than that (0.8) proposed by the Reinforced Earth Company.
- 5) The width of reinforcing strip is independent of the height of overburden and surcharge (if any). But it is in proportion to the vertical and the horizontal spacings and is in inverse proportion to the length of the reinforcing strip.
- 6) The skin plate must be flexible enough to follow all deformation of the soil mass resulting from settlement. In order to prevent failures due to corrosion, impact, and abrasion, a greater thickness than theoretically required should be used.

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AN INTRODUCTION TO THE DESIGN OF REINFORCED EARTH RETAINING WALLS

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the requirements for the degree

MASTER OF SCIENCE

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Manhattan, Kansas

1976

ABSTRACT

Reinforced earth is a rather new construction process. It is a method of mobilizing the friction between soil particles and reinforcing strips placed at regular intervals, horizontally and vertically, in order to create an earth-retaining structure.

The essential requirement in a reinforced earth structure is that there must be an effective connection between the earth and the reinforcing strips. Therefore, the reinforcing strips must be properly designed and placed to insure that reinforcing action takes place in the earth mass.

Based on the principles of reinforced earth, equations for designing reinforcing strips and skin plates were developed. The reinforced earth retaining walls must be so designed as to satisfy both internal and external stability requirements.