

**THICKNESS EFFECTS ON THE CRITICAL CRACK TIP OPENING
ANGLE FRACTURE CRITERION IN 2024-T351 ALUMINUM**

by

SAMER HASSAN MAHMOUD

B.S., Kansas State University, 1992

M.S., Kansas State University, 1996

A DISSERTATION

submitted in partial fulfillment of the

requirements for the degree

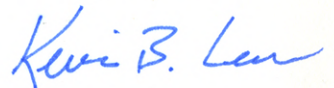
DOCTOR OF PHILOSOPHY

**Department of Mechanical and Nuclear Engineering
College of Engineering**

**KANSAS STATE UNIVERSITY
Manhattan, Kansas**

2000

Approved by:



**Major Professor
Kevin Lease**

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ABSTRACT

The critical crack tip opening angle (*CTOA*) is a promising fracture criterion for characterizing stable crack growth. However, material thickness effects on the critical *CTOA* have not been fully characterized. The goals of this investigation were to experimentally and computationally evaluate the effect of material thickness on the characterization of the *CTOA* parameter.

Fracture tests were conducted on 8 in. wide compact tension fracture specimens machined from 2024-T351 Aluminum alloy. This material was selected in order to allow for the direct comparison to, and extension of, previously evaluated thin-sheet form of the 2024-T3 alloy. The range of thickness chosen for this research included 0.090, 0.25, 0.5, and 1.0 inch. The experimental set-up allowed for real-time monitoring of the crack tip, acquisition of still-images, and the continuous recording of the entire stable tearing process. During the fracture tests, the load and load line displacement signals were continuously recorded. After test completion, the entire image history was reviewed to obtain *CTOA* and crack length measurements using the optical microscopy and digital image correlation methods. Both three- and two-dimensional elastic-plastic finite element fracture analyses were then used to simulate the stable tearing tests.

The experimentally measured *CTOA* (on the surface of the specimens) decreased with increasing specimen thickness and appeared to be asymptotically reaching a lower limiting value. As reported in earlier studies on thin-sheet 2024-T3 alloy, the experimentally determined *CTOA* became nearly constant after a small amount of crack extension and remained constant over a wide range in thickness. In addition, the non-

constant *CTOA* region and the scatter in measured *CTOA* values became smaller for the thicker specimens.

Three-dimensional computationally generated *CTOA* values (at the specimen center) were higher than the experimentally determined surface *CTOA* values for the three largest thicknesses. Using the *CTOA* values from the three-dimensional analyses, two-dimensional fracture analyses were performed to obtain the plain strain core height. Based on these analyses, the plain strain core height was seen to increase with increasing specimen thickness. Both of the two-dimensional and three-dimensional finite element based analyses produced good approximations of the measured crack extensions and load line displacements.

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Chapter 1

Introduction

Safety of engineering structures such as bridges and airplanes dictates avoiding structural failure at all costs. Designing a structural component requires knowing the type and magnitude of the applied load so dimensions and materials are selected properly to accomplish the designated task with minimal risk of failure. Most structural failures fall into two main categories, yielding-dominant and fracture-dominant [1][†]. Until the early 1900s, the design of structural elements relied solely on the classical strength of the material theory which is capable of predicting yielding failure but unable to handle fracture failure [2]. A combination between the need to produce lightweight structures and the existence of flaws in the structural materials instigated the necessity of damage-tolerant design. Defects and flaws in the structural materials influence both types of failure, but those influencing fracture are different from those affecting yielding and resistance to plastic flow [1].

Yielding failure is of special concern in engineering materials such as metals and ceramics which have crystalline grain microstructures. Ewalds [1] described the crucial defects for the yielding-dominant failure to be on the microscopic scale in size, thus, they tend to distort and discontinue the lattice planes and lattice sites of the crystal. Those defects may be classed as point defects, line defects, or surface defects [3]. Dowling [3] reported that the motion of line defects by shear stress is responsible for yielding and

[†] Numbers in brackets denote references given at the end of the document

plastic deformation which cause atoms in the material to change neighbors. On the other hand, the point defects and surface defects restrict and interfere with the dislocation motion, thus, providing resistance to plastic deformation which is essential to the strength of high strength metals.

Design against yielding failure requires selecting material and dimensions such that the applied stresses remain below the yield strength of the material. While this is achievable by employing the strength of material theory, the situation differs tremendously when designing against fracture failure. At the most basic level, fracture occurs when the applied stress on the atomic scale is sufficient to break the cohesive bonds that hold the material atoms together before the general yielding of the net section [4]. Thus, one might think knowing the strength of the atomic cohesion bonds is sufficient to determine the material resistance to fracture. According to Anderson [4], the material resistance to fracture evaluated based on the strength of the atomic cohesion bonds exceeds the resistance to fracture estimated from experimental mechanics by three to four times. This discrepancy is a result of the stress magnification caused by flaws and defects embedded in the material. Defects and flaws in the material increase the possibility of fracture failure because they increase the stress locally to adequately high levels that overcome the strength of the bonds between the atoms of the material. The defects that are of primary importance for the fracture-dominant failure are essentially macroscopic in size such as macro cracks and voids. Insuring structural integrity of a component demands addressing fracture-dominant failure during design. This requires

the determination of a critical relationship between flaw size, load, and material resistance to fracture. In order to establish such a design methodology, a designer must have a fracture parameter to accurately characterize the severity of material deformation at the location of the flaw or crack. Secondly, a critical value of the fracture parameter must be determined to allow predicting the onset of fracture and whether fracture will occur in a stable or unstable manner. According to Shih [5], an acceptable fracture parameter must provide a measure of the stress and deformation state in the vicinity of the crack tip. Also, the critical value of an admissible fracture parameter must be independent of the initial crack length and specimen geometry. Shih [5] also added that the fracture parameter must be effective for both instability analyses and handling three-dimensional crack geometries.

In general there are two types of fracture parameters, the single critical parameters and the resistance curve parameters [6]. The single parameter criteria assume that fracture occurs when the parameter reaches a critical value. On the other hand, the resistance curve parameters assume that there is a unique relationship between the parameter and the crack length [6]. Each fracture parameter has a range of applicability that is mainly influenced by the amount of plastic deformation encountered by the material around the fractured surface. Beside the extent of plastic deformation in the material, the size of the crack growth increments also has an impact on the validity range of the fracture parameters, specifically for the resistance curve parameters.

The earliest attempts to model fracture behavior were instigated by tragic

accidents such as the fracture of a molasses gas tanker in 1919 and the failure of 400 out of the 2700 Liberty ships during World War II [4]. This tragedy-motivated research resulted in the development of the linear elastic fracture mechanics theory (LEFM). The LEFM theory was developed to handle situations where the material fractures in a brittle manner and very little yielding accompanies fracture. The major developments of LEFM are credited to the efforts of Griffith [10] and Irwin [11]. According to the LEFM theory, the magnitude of the stress intensity around the tip of the crack is described by the stress intensity factor parameter, which is denoted by K . Wells [35,36] observed that the LEFM analysis and the K -criterion were not valid for characterizing the fracture toughness of the material when a substantial amount of plastic deformation precedes failure. Wells [35,36] proposed the crack tip opening displacement ($CTOD$) as an alternative fracture parameter, although no substitute fracture analysis method was introduced to account for the excessive plasticity at that time. A major milestone in the history of fracture mechanics was marked by the introduction of Rice's [22] J -integral for evaluating the intensity of stress around the tip of the crack under large scale yielding conditions. Introducing the J -integral parameter lead to the development of the theory of elastic-plastic fracture mechanics (EPFM). The EPFM theory becomes necessary to accurately model the stress and deformation fields around the tip of the crack when the excessive plasticity accompanying fracture invalidates the LEFM assumptions.

As reported by many researchers such as Han [2] and Wang [87], a certain amount of stable crack growth will accompany the fracture process when a substantial amount of

plastic deformation precedes fracture. Thus, for ductile behaving materials fracture failure occurs in three stages: (1) crack blunting/initiation, (2) stable crack growth, and (3) crack instability [2]. In ductile behaving materials, an initially sharp crack becomes blunt under increasing applied load. Crack blunting refers to increasing the crack tip radius, which makes the crack become somewhat dull. The crack tip radius continues to increase as the applied load increases until the crack starts to grow. The onset of crack growth is referred to as crack initiation. After crack initiation, the crack grows as the applied load increases, however, the crack growth occurs in a stable manner. Crack instability takes place after the maximum load is reached. Beyond this stage, the crack grows as the load decreases. The multi-staged fracture in ductile behaving materials adds to the complexity of damage tolerant design since it becomes necessary to have a criterion characterizing each stage of the fracture process.

While the J -integral was found to be appropriate for characterizing the crack initiation in ductile fracture, J -resistance curve was found to have shortcomings in characterizing stable crack growth. The J -integral resistance curve was found to appropriately characterize stable crack growth only for small amounts of crack growth. This shifted the focus of the fracture research to finding a new criterion for characterizing stable crack growth. Several fracture parameters have been introduced to characterize the material resistance to stable crack growth. Kanninen *et al.* [40] listed some of those parameters as the energy release rate, the J -integral, the generalized energy release rate, the crack opening angle (COA), the crack tip opening angle ($CTOA$), the finite element

crack tip nodal force, the elastic-plastic energy, and the computational process zone energy release rate. Also, Jeong *et al.* [88] used the strain energy density for characterizing the stable crack growth process. One of the most promising parameters was found to be the crack tip opening displacement (*CTOD*) evaluated at a small distance behind the crack tip or equivalently the crack tip opening angle (*CTOA*). The *CTOA* has many advantages since it can be measured experimentally with relative ease and it relates directly to the physics of the fracture process. Recent concerns regarding the worthiness of aging commercial and military aircraft has urged NASA to launch the Airframe Structural Integrity Program [85,86]. While this program has several components, a major focus was to develop fracture mechanics analysis tools for cracked bodies. This lead to the development and modification of several finite element based fracture mechanics programs with *CTOA* based fracture criterion. Researchers at NASA also developed an extensive experimental/computational methodology for measuring the critical *CTOA* of metallic materials [42,8].

The fracture mechanics parameters characterize the effects of loading on cracked bodies and components. The critical value of a fracture parameter is a material property that can be used to determine whether the applied load to a cracked component would cause incremental crack growth or catastrophic fracture. Most of the fracture mechanics criteria are dependant on the thickness of the material. For the stress intensity factor (*K*) and the *J*-integral, the thickness effects have been addressed in many studies. Most of the *CTOA* characterization testing has been primarily conducted on thin specimens which in

most cases resemble the aircraft skin such as the work of Newman *et al* [42-44] and Dawicke *et al.* [8, 20, 58, 64]. Based on conversation with Newman and Dawicke [94, 7], it appeared that recent interests have developed in extending the *CTOA* fracture criterion for designing thicker structural elements. Since the thickness dependencies have not been thoroughly investigated for the *CTOA* fracture criteria, the objective of the current research was to perform an experimental/computational investigation into the effect of thickness on the *CTOA* fracture parameter. The experimental portion of this program consisted of conducting fracture tests on specimens having different thicknesses. The selection of material and thickness has been coordinated with researchers at NASA Langley [7]. Since critical *CTOA* testing has been applied to characterize the material resistance to slow-stable crack growth in thin sheets of 2024-T3 Aluminum alloy of certain thicknesses, 2024-T351 Aluminum specimen was selected for this investigation. The fracture tests were conducted following the procedures outlined in [8] where experimental and computational critical *CTOA* characterizations were employed. The experimental aspect of this program involved using the optical microscopy (OM) and digital image correlation (DIC) techniques for measuring the *CTOA* on the surface of the specimen during the fracture tests. It also involved measuring the load, load line displacement, and crack extension during the tests. On the other hand, the computational portion involved using the two and three-dimensional finite element based fracture mechanics with *CTOA* criterion to simulate the stable tearing process in the test specimens. The three-dimensional finite element based fracture mechanics analyses were

used iteratively to predict the critical *CTOA* value (at the center of the specimen) that makes the analyses match the experimental measurements. Once the appropriate center *CTOA* value was found, the two-dimensional finite element based fracture mechanics analyses with plane strain core were used iteratively to determine the thickness effect on the plane strain core height. (See next chapter for definition of plastic strain core).

An outline of the thesis is as follow. Chapter 2 contains a review of the literature on fracture criteria. Chapter 3 provides an overview of the current research goals and methodology. Chapter 4 describes materials selection and specimen geometry. Chapter 5 gives a detailed description of the experimental procedures and employed measurement techniques. Chapter 6 describes the computational procedures employed in the current study. Chapter 7 presents the results and discussion pertaining to both the experimental and computational phases. Chapter 8 presents the conclusions of the current study and recommendations for future research. Appendix A provides computational procedures used to correct for load line displacement measurements. References are given at the end of text.

Chapter 2

Review of Fracture Criteria

As the yield stress of the material provides a critical value that is used to design against yielding failure, the critical value of a fracture parameter quantifies the material ability to resist fracture. A fracture criterion is used to determine whether an applied loading to a component will cause fracture failure. After crack initiation, the behavior of the crack growth and advancement depends on the ductility of the material. A material may behave in a brittle or ductile manner depending on several factors such as material type, loading rate, temperature, and environmental factors. In brittle behaving materials such as glass and low toughness high strength metals, occurrence of crack instability and structural failure is almost immediate after crack initiation. In ductile behaving materials, like polymers and high toughness low strength metals, the crack grows to a certain size before it becomes unstable [5]. Two types of fracture mechanics analysis are employed in engineering to assess the material resistance to fracture. When a component deforms globally in an elastic manner, the theory of linear elastic fracture mechanics (LEFM) may be adequate to characterize the material resistance to fracture. If large ductile deformation precedes failure, elastic-plastic fracture analysis is necessary to precisely model the fracture process. The following sections introduce some of the most common fracture criteria used in both of these situations.

2.1. Linear Elastic Fracture Mechanics (LEFM) Fracture Criteria

The theory of linear elasticity is a well-developed field that describes stress and strains in loaded continuum bodies. Extending the elasticity solution to treat cracked body problems resulted in the development of the LEFM branch of solid mechanics. LEFM uses the principles of linear elasticity to evaluate the severity of stress and strain around the tip of the crack. In LEFM, researchers adopted two analysis approaches, the energy based approach and the stress based approach [4,1]. Before proceeding in the introduction of the linear elastic fracture criteria, it is necessary to note that the load-to-crack orientation affects the fracture mode. As shown in Figure 2.1, three modes of fracture may exist. In mode I, the load is normal to the crack faces, so the load works to open the crack. Consequently, it is also known as the opening mode. When the load acts parallel to the crack plane and tangential to the crack faces, it tends to slide the crack faces relative to each other. This fracture mode is known as mode II or the sliding mode. Lastly, in mode III the load is tangential to the crack faces but acts normal to the plane of

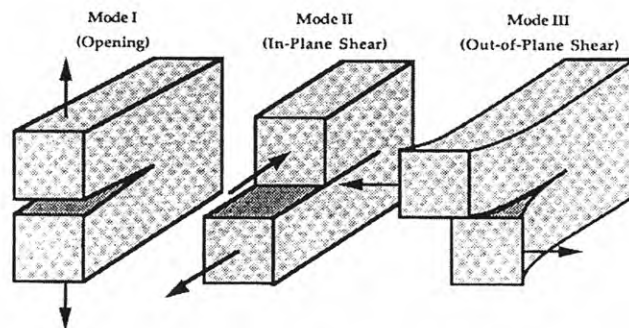
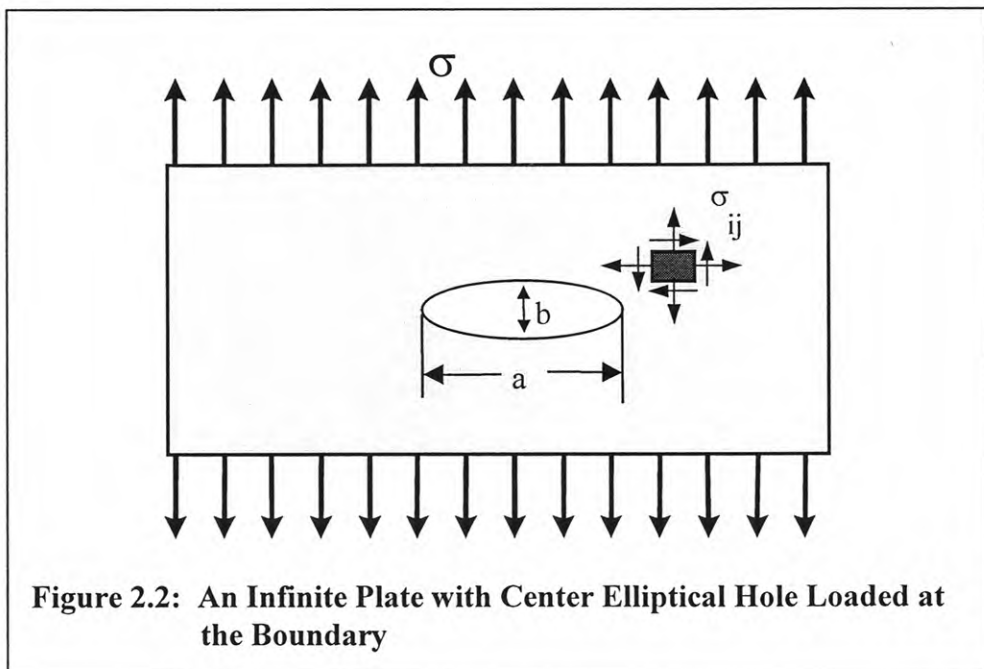


Figure 2.1: Illustration of Fracture Modes (from [4]).

the crack. Mode III is also known as the tearing mode. Mode I or the opening mode is the most commonly encountered mode of fracture in engineering structures, thus, the current research concentrates on investigating this type of fracture.

Griffith [10] was the first to apply energy-based formulation in fracture mechanics analysis. He combined an energy balance along with Inglis's [9] solution for the stress around an elliptical hole in an infinite plate loaded at the boundary with σ , as shown in Figure 2.2, to solve for the failure stress in term of the surface energy of the material. Griffith's results suggested that a crack might grow when the change in strain energy due to a crack growth increment overcomes the surface energy of the material. Griffith's theory is only applicable to ideally brittle materials such as glass and quartz, because he assumed that fracture occurs when the applied work becomes equal to the material surface energy [9]. Irwin [11] modified the Griffith model by correcting for the energy



dissipated by local plastic flow. Also, Irwin [12] introduced the strain energy release rate (G) concept which represents the rate of change in the strain energy with respect to the crack surface as defined by the following equation:

$$G = -\frac{dU}{dA} = -\frac{1}{B} \frac{dU}{da} \quad (2.1)$$

Where U is the strain energy, A is the crack surface area, a is the crack length, and B is the thickness of the cracked body. The energy release rate (G) provides a measure of the energy available in the material to initiate crack growth, thus, it is dependent on the applied stress and the crack size. Irwin identified the critical energy release rate (G_C) as the total energy release rate due to surface energy release and plastic flow dissipation, thus, the critical strain energy release rate is a characteristic of the material. According to Irwin, fracture and crack growth occur when the applied energy release rate (G) equals or exceeds the critical value of the strain energy release rate (G_C) for a given material, this became known as the G -criteria [2]. Although no standard procedures are available for measuring G , experimental determination of G may be accomplished by invoking its definition as given in Equation (2.1). Thus, measuring G requires determining the elastic potential energy stored in a number of specimens having similar dimensions but different crack lengths [4]. To determine the potential energy in each specimen, it is necessary to obtain measurements of the load and load line displacement, and then compute the area under the load-displacement curve using a numerical integration scheme. Once the potential energy is evaluated for each crack length, G can be determined by differentiating the potential energy with respect to crack size. Thus, measuring G

involves a complicated procedure that may be difficult to implement.

The LEFM stress based approach focused on establishing a relationship between the applied stress, the crack size, component geometry, and the local stress around the tip of the crack. Using elasticity theory, Westergaard [13], Williams [14], and Irwin [15] published solutions describing the stress at the vicinity of the crack tip in an infinite plate with center crack loaded at the boundary with σ as shown in Figure 2.3. In terms of the polar coordinates system with origin located at the tip of the crack as illustrated in Figure 2.4, the solution is given by:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + H.O.T \quad (2.2)$$

Where σ_{ij} is the stress tensor, K is a constant coefficient that depends on the applied load,

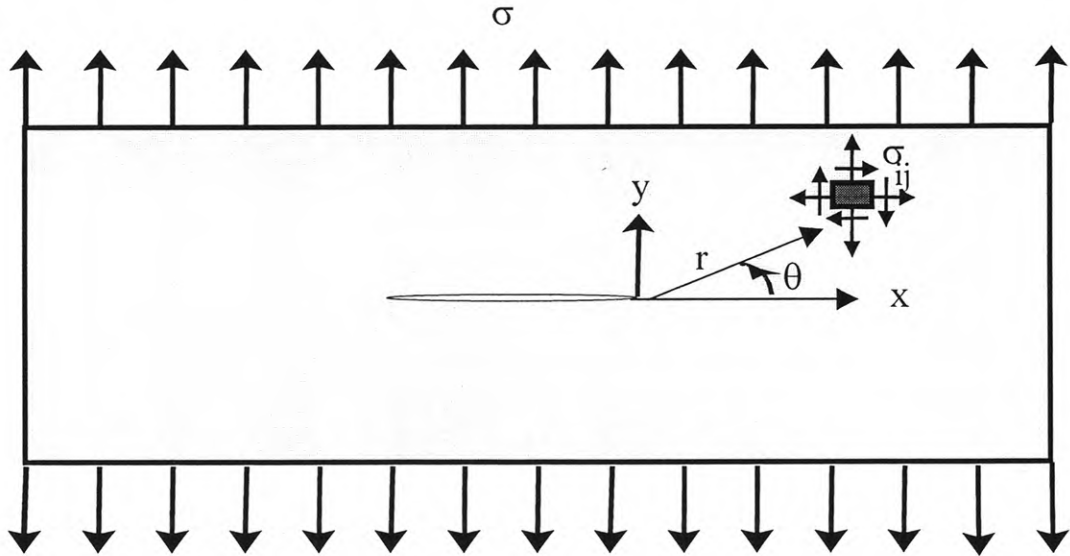


Figure 2.3: Infinite Plate with Center Crack Loaded at the Boundary

r and θ are the radial and angular coordinates with the origin at the crack tip as shown in Figure 2.3, $f_{ij}(\theta)$ is a function that is dependent on the angle θ , and the higher order terms are geometry dependent. Equation (2.3) shows that the stress is singular at the tip of the crack since the first term tends to infinity as r approaches 0, as illustrated in Figure 2.4. The K coefficient controls the magnitude of this stress singularity, thus it is termed as the *stress intensity factor*. In the above equation, if the crack size is chosen to represent the length dimension then the stress intensity factor (K) may be expressed in terms of the applied stress (σ) by the following expression:

$$K = \sigma \sqrt{\pi a} F \quad (2.3)$$

Where a is the crack length and F is a factor that depends on geometry. Equation (2.3) shows that the stress intensity factor is proportionally related to the product of the applied

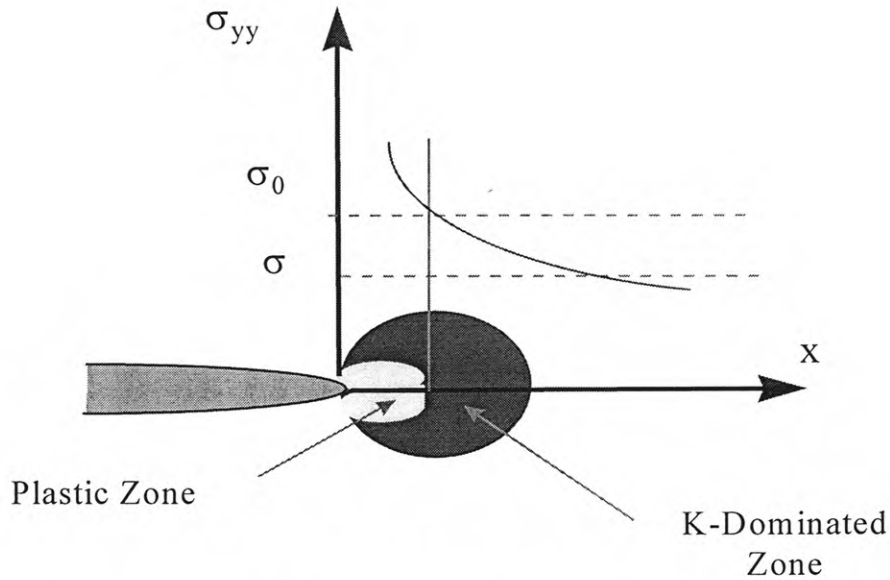


Figure 2.4: Schematic of LEFM Stress Distribution in the Vicinity of the Crack Tip

stress and square root of the crack length. Thus, a critical combination of stress and crack size may be described by a single critical stress intensity factor (K_C). The critical stress intensity factor (K_C) is dependent on the material, so it is a parameter that characterizes the fracture toughness of the material. In an analogy with the yielding criteria, i.e. a material yields when the stress reaches the yield strength of the material, K_C may be used as a critical parameter that quantifies the limiting stress intensity factor (K) which a material can sustain due to the existence of a crack. Hence, a fracture criterion was developed based on the fracture toughness (K_C). According to the K -criteria, the material continues to resist fracture until the stress intensity factor (K) reaches a critical value equal to the material fracture toughness (K_C). At this point the material fractures in a brittle manner. It has been shown that the critical stress intensity factor (K_C) is inversely proportional to the material thickness [1, 3, 4]. Figure 2.5 illustrates this variation where it can be seen that the fracture toughness (K_C) is a maximum for thin materials where fully plane stress condition would be expected. The fracture toughness then gradually decreases as the thickness increases until it reaches a minimal plateau at the thickness where plane strain condition exists. The critical plane strain stress intensity factor, denoted as (K_{IC}), designates the critical plane strain fracture toughness of the material, it is the lower limiting value of the fracture toughness (K_C). Thus, if the state of stress in a structural component is not fully plane strain, designs based on the critical plane strain fracture toughness (K_{IC}) are conservative.

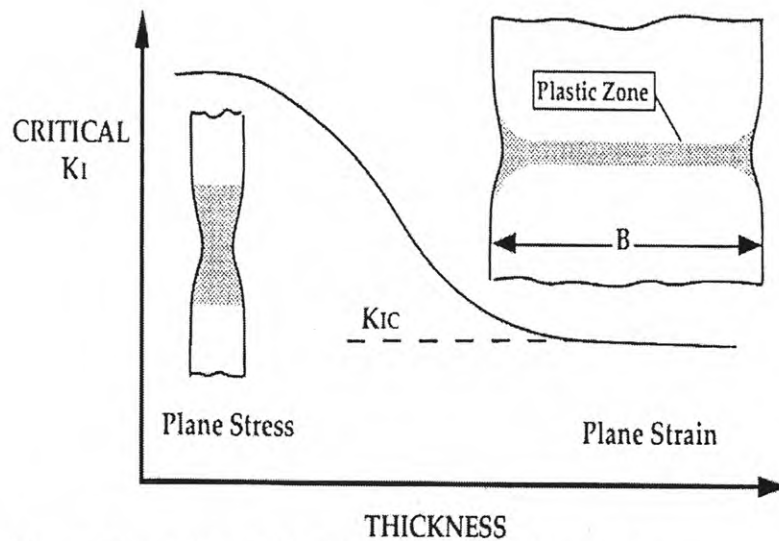


Figure 2.5: Effect of Thickness on K (from [4]).

ASTM Standard E399-83 [16] provides the procedure for experimentally determining K_{Ic} . During the test a record of the load versus load line displacement must be acquired. The load-displacement behavior, which must be predominantly linear, provides the ability to pick the proper load to be used in computing K . A variety of test specimens may be used such as tension or bending type specimens. A fatigue crack must be grown in the specimen to simulate the infinitely sharp crack assumption used in the LEFM theory. ASTM E399-83 recommends machining a fatigue-starting notch in the specimen to facilitate fatigue crack growth. A number of restrictions are placed on the pre-cracking process to guarantee the sharpness of the resulting crack. The test is repeated with increasing specimen thickness until the measured K_C has reached a minimum value and become independent of specimen thickness. To measure the critical stress intensity factor K_C for a material of any thickness, a specimen of the particular

thickness must be used then the steps in ASTM standard E399-83 [16] may be followed.

It can be shown that the stress intensity factor (K) is related to the square root of the strain energy release rate (G) [1,4]. Consequently, both of the K -criterion and the G -criterion produce similar predictions about the material resistance to fracture. Nevertheless, the K -criteria received more recognition than the G -criteria for several reasons. Some of which are that K relates to the local condition at the crack tip while G relates to the global behavior of a structure, the critical stress intensity factor is easier to measure than the critical strain energy release rate, and K provides a direct critical relation between the load and crack size [1,4].

Equation (2.2) shows that the stress has an infinite value at the tip of the crack. This is merely a mathematical abstraction since infinite stress does not exist, hence the point at which r is equal to zero should be excluded. However, in the neighborhood of the crack tip, for very small values of r , Equation (2.2) predicts a very high stress value that exceeds the yield strength of the material as illustrated in Figure 2.4. Thus, contradicting the original assumption of the linear elasticity theory since metallic materials yield and deform plastically when the applied stress reaches or exceeds the yield strength of the material. The region around the crack tip where the stress exceeds the yield strength is called the plastic zone, the LEFM solution is invalid within this zone. However, the LEFM solution is valid for an annular region outside the plastic zone, called the K -dominance zone, as shown in Figure 2.4. This K -dominance zone exists if the size of the plastic zone is small in comparison with the dimensions of the structure or

the crack size. Based on empirical studies [4], the plastic zone size is considered small in comparison to structural dimensions or crack size if one of the following inequalities are satisfied:

$$\begin{aligned} a, B, (W - a) &\geq 2.5 \left(\frac{K}{\sigma_o} \right)^2 \\ r_y &\leq \frac{1}{50} (a, B, (W - a)) \end{aligned} \tag{2.4}$$

Where a is the crack length, B is the specimen thickness, W is the specimen width, K is the stress intensity factor, σ_o is the yield strength of the material, and r_y is the plastic zone radius. If the LEFM validity requirements are met, then K continues to control the crack tip condition since it correctly describes the state of stress in the K -dominance zone within the cracked body.

In an attempt to correct for the plastic deformation in the plastic zone, Irwin [17] argued that the effective crack length is equivalent to the crack length plus the plastic zone size. Irwin [17] estimated the plastic zone size then he calculated an effective stress intensity factor K_{eff} based on the effective crack length. Dugdale [18] and Barenblatt [19] developed a more sophisticated model, based on a narrow strip of yielded material ahead of the crack tip, to estimate the plastic zone size, then they used their estimation of the plastic zone size to calculate an effective stress intensity factor K_{eff} . In summary, the critical stress intensity factor (K_C) is a useful fracture parameter to design against brittle fracture as long as the LEFM solution is valid. When the LEFM solution loses validity

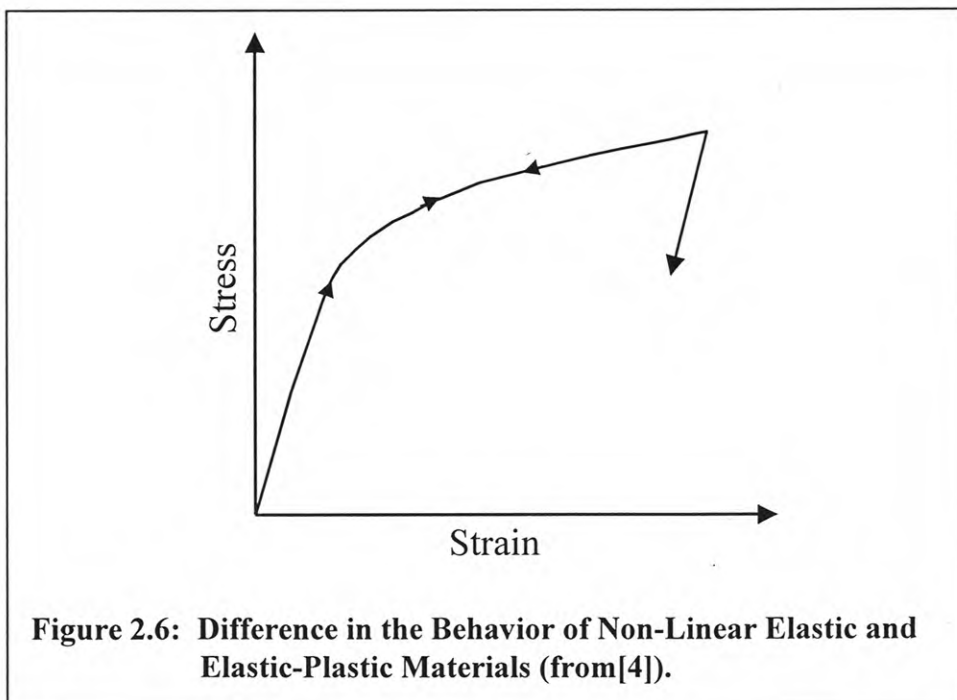
another analysis method must be employed to establish a valid fracture criterion. The next section introduces the elastic-plastic fracture mechanics criteria.

2.2. Elastic-Plastic Fracture Mechanics (EPFM) Criteria

The LEFM fracture criteria are appropriate for designing structures that undergo a limited amount of plastic deformation before they fracture in a brittle manner. However, many situations arise where large amounts of plasticity accompany the fracture process causing the LEFM validity conditions, discussed in the previous section, to be violated. Therefore, alternative fracture criteria are needed for characterizing ductile fracture where large plastic deformation takes place. Failure in ductile materials occurs in three stages (1) crack blunting and initiation, (2) stable crack growth, and (3) instability [5]. Thus, it is necessary to characterize the material resistance to fracture in each of these stages in order to allow for full potential utilization of the material while avoiding tragic failure. While many elastic-plastic fracture criteria have been proposed [20,21], the most commonly used elastic-plastic fracture criteria are the J -integral, the crack tip opening displacement, and the crack tip opening angle. The following sub-sections describe each of these criteria.

2.2.1. The J-Integral Fracture Criterion

Large strains and gross plasticity preceding fracture cause the plastic zone to be large in comparison with the K -dominated zone or the K -dominated zone does not exist, which invalidates the LEFM analysis. In this case, the LEFM fracture criteria do not properly characterize the fracture behavior of the material. To extend the linear fracture analysis beyond the validity limit of LEFM, it seems natural to expand on either the energy based or the stress based approaches of fracture mechanics. Rice [22] modified the evaluation of the energy release rate by idealizing the true elastic-plastic material behavior with a nonlinear elastic model. The nonlinear elastic material model has the ability to correctly describe the stress-strain relation for an elastic-plastic material during monotonic loading. However, the behavior of these two material models differs greatly during unloading. Figure 2.6 illustrates the difference between the nonlinear elastic and



the elastic-plastic material behaviors. When the elastic-plastic material unloads, it follows a straight unloading path having a slope equals to the material elastic modulus [4]. The nonlinear elastic material unloads in a nonlinear fashion along the loading path. Rice [22] showed that the energy release rate for the idealized nonlinear elastic material behavior could be expressed by a path-independent contour integral that he called the J -integral. For a given contour, Γ , as shown in Figure 2.7, the J -integral is defined as:

$$J = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} ds \right) \quad (2.5)$$

Where

$$w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad \text{and} \quad T_i = \sigma_{ij} n_j$$

Where w is the strain energy density, T_i is the traction vector normal to the contour, u_i is the displacement vector, and ds is an increment along the contour Γ . Although J was developed for approximating the energy release rate in an elastic-plastic material, J becomes equivalent to G in a pure elastic material. Hutchinson [23] and Rice and Rosengren [24] independently performed asymptotic analyses of problems involving two-dimensional cracked body to determine the stress and strain fields near the tip of the

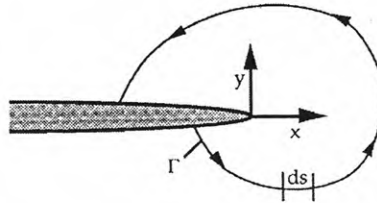


Figure 2.7: Arbitrary Contour for Evaluating the J -Integral (from [4]).

crack. In their formulation, they used the small strain theory and employed the Ramberg-Osgood stress-strain relation for representing the assumed nonlinear elastic material behavior. Their solution resulted in singular expressions for the stress and strain near the crack tip that vary as $1/r$, named the HRR singularity in recognition of their collective efforts [1,2,4], which takes the following form:

$$\begin{aligned}\sigma_{ij} &= \sigma_0 \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta) \\ \varepsilon_{ij} &= \frac{\alpha \sigma_0}{E} \left(\frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(n, \theta)\end{aligned}\tag{2.6}$$

where r and θ are the radial and angular dimensions as illustrated in Figure 2.3, α and n are the strain hardening coefficient and exponent of the Ramberg-Osgood material model, E is material modulus of elasticity, σ_0 is the yield stress, I_n is an integration constant that depends on n , and $\tilde{\sigma}_{ij}(n, \theta)$ and $\tilde{\varepsilon}_{ij}(n, \theta)$ are dimensionless functions that depend on n and θ [4]. In nonlinear elastic materials, the HRR singularity shows that J uniquely describes the stress and the strain fields around the tip of a “stationary” or non-growing crack. Thus J may be viewed as a stress parameter and an energy parameter as well.

McMeeking and Parks [25] analyzed the crack tip stress by performing a two-dimensional finite element analysis with large strains. Their results show that the HRR singularity overestimates the stress and strain near the tip of the crack. Such inaccuracy in the HRR singularity takes place because the theoretical development of the HRR singularity ignores crack blunting and incorporates small strain theory. Figure 2.8 shows

a comparison between the crack tip stress field obtained by both the HRR singularity and the finite element analysis by McMeeking and Parks [25]. Contrary to the HRR singularity, the large strain finite element analysis shows that the stress has a finite value at the blunted crack tip. Also, the HRR singularity overestimates the stress in the region extending between the crack tip and the location where the ratio $x\sigma_0/J$ (horizontal axis of Figure 2.8) is unity, σ_0 being the yield stress and x is the outward radial distance from the crack tip. The region in which the results of the HRR singularity deviate from the results of the analysis made by McMeeking and Parks [25] is dominated by large strains which causes small strain theory to be invalid in that region, thus, this region is named the large strain zone [4]. McMeeking and Parks [25] showed that for specimens with high constraint the large strain zone has a length equivalent to twice the crack tip opening

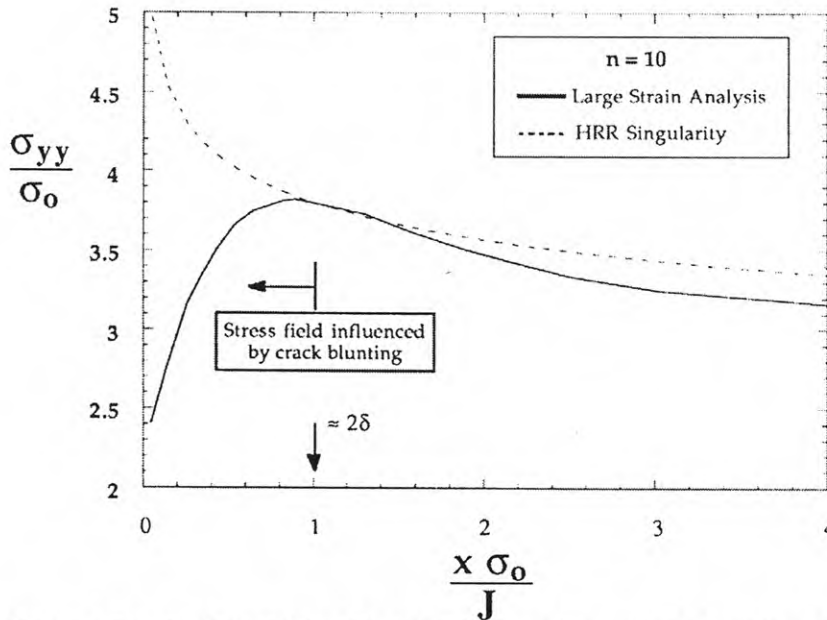


Figure 2.8: Large Strain Crack Tip Finite Element Results of McMeeking and Parks [25]. Blunting Causes the Stress to Deviate from the HRR Solution Close to the Crack Tip (from [4]).

displacement ($CTOD$) (displacement between crack faces), which will be discussed further in the next section.

For specimens with small constraint, they found that the large strain zone size can be several times the $CTOD$. Beyond the large strain zone, the stress predictions of the HRR singularity become in a good agreement with the large strain finite element predictions [25]. The validity region of the HRR singularity extends through the plastic zone but excludes the large strain zone as illustrated in Figure 2.9. The same argument used with the K -dominance approach may be applied to establish the J -dominance of the crack tip condition. J characterizes the crack tip condition since there is a region in the material where the HRR singularity is valid.

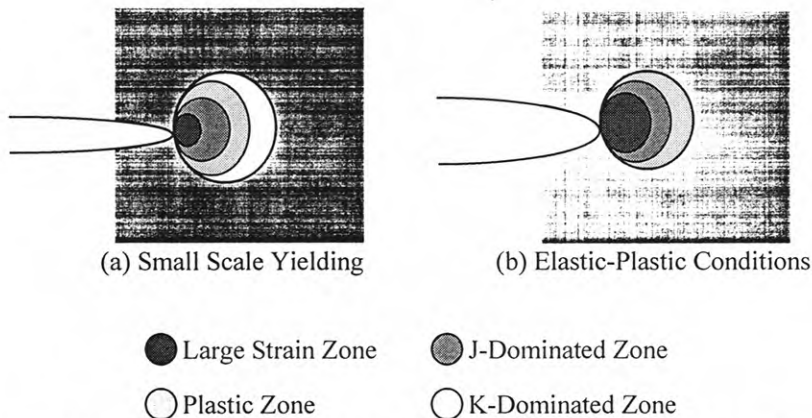


Figure 2.9: J -Dominance in Small Scale Yielding and Elastic Plastic Conditions.

Begley and Landes [26] were among the first to measure J experimentally by invoking the energy release rate definition of J which is given by the following equation:

$$J = -\frac{1}{B} \frac{dU}{da} = \frac{1}{B} \frac{dU}{db} \quad (2.7)$$

Where B is the specimen thickness, a is the crack length, and b is the uncracked ligament of the specimen. By determining the load displacement curves for several specimens having various crack lengths, the differential in Equation (2.7) can be evaluated to produce an estimate of J . Begley and Landes [26] also proposed the critical value of the J -integral (J_{IC}) as a mean for characterizing crack initiation under the plane strain condition. They argued that a fracture criterion based on J_{IC} has a wide range of applicability, since it may be used to characterize fracture for material behaviors that vary in ductility from essentially elastic to fully plastic. Also, they emphasized that in elastic materials a J_{IC} based fracture criterion becomes similar to the K_{IC} based fracture criterion. As mentioned earlier, in many high toughness materials crack initiation is followed by crack growth before crack instability [5]. Crack growth causes elastic-plastic materials to unload linearly in the wake of the crack, hence, contradicting the assumed nonlinear elastic material behavior used in the development of the HRR singularity. J_{IC} characterizes the material resistance to crack initiation, but when stable crack growth precedes instability, J_{IC} does not properly characterize the material resistance to fracture. However, a resistance curve of J against crack growth (J -resistance) may be used for

characterizing the material resistance to fracture as long as such a curve is independent of specimen geometry.

Since the original definition of J , introduced strictly for stationary cracks, becomes invalid for growing cracks, some procedures have been proposed to calculate J for growing cracks, including the far field $J(J_f)$, the deformation $J(J_D)$, and the modified $J(J_M)$ [4]. J_f involves determining J from Equation (2.5) using an integration that guarantees the crack tip to remain within the integration contour. This method is too difficult to implement experimentally, thus it is excluded from the standardized methods for determining J -resistance curves [4]. As the crack grows, a certain amount of energy will be released due to the elastic unloading of the material in the wake of the crack. The J_D procedure involves evaluating the strain energy of the specimen based on the current crack length and ignores the released energy during crack growth. Both J_D and J_f preserve the energy release rate definition of the J -integral, however, they do not guarantee the geometry independence of the J -resistance curve [4]. In an attempt to correct for the geometry dependence of the J -resistance curve, Ernst [27] proposed the J_M procedure for establishing the J -resistance curve by introducing the following modification to J_D .

$$J_M = J_D - \int_{a_0}^{a_1} \frac{\partial J}{\partial a} da \quad (2.8)$$

Where a_0 and a_1 are the previous and current crack lengths. J_M is dependent on the load

history, thus it does not correspond to the energy release rate definition of J . The difference between J_D and J_M is usually negligible for small crack growth and if such a difference becomes significant, the J -resistance curve characterization of fracture is invalid in most cases [4]. The condition that guarantees the existence of J -controlled crack growth is defined as follow:

$$\frac{b}{J} \frac{dJ}{da} \gg 1 \quad (2.9)$$

where b is the un-cracked ligament and a is the crack length [4]. This condition implies that the slope of the J -resistance curve must be very small. This is also termed as steady state crack growth. The slope of the J -resistance curve is quantified by the following dimensionless tearing modulus (T) [4]:

$$T = \frac{E}{\sigma_0^2} \frac{dJ}{da} \quad (2.10)$$

Where E is the modulus of elasticity, σ_0 is the yield stress, and a is the crack length. The tearing modulus evaluated from the slope of the J -resistance curve serves as a material property that is useful for determining the onset of unstable crack growth. The crack propagation remains stable as long as the applied load produces a tearing modulus that is less or equal to a limiting value. When the applied tearing modulus exceeds that limit,

unstable crack growth occurs [4].

ASTM standards E813-89 [28] and E1152-87 [29] outline the procedures for measuring J_{IC} and the J -resistance curve respectively. Although the purpose of ASTM E813-89 is to obtain a single value of J_{IC} , it requires determining the initial portion of the J -resistance curve. Both test procedures require measurements of the applied load, load line displacement, and crack extension. For monitoring crack extension, ASTM E1152-87 recommends the elastic compliance method for producing the J -resistance curve from a single specimen. ASTM E813-89 permits the use of either the elastic compliance method or optical methods for monitoring crack growth. Although using a single specimen may be adequate for measuring J_{IC} if the elastic compliance method is employed, ASTM E813 recommends using three or more specimens to account for material variability. If optical methods are used, ASTM E813-89 requires the testing of at least five identical specimens. Both procedures allow the use of compact tension and single edge bending type specimens. In preparation for the fracture test, the specimen must be fatigue pre-cracked to provide the test specimen with a sharp crack such that the ratio of the initial crack length to the specimen width is greater than 0.5 but less than 0.75. Experience indicates that a crack length to specimen width ratio of 0.6 is optimum. Each standard provides a formula for calculating J from the load and load-line displacement data obtained during testing in accordance with the deformation J procedure (J_D). ASTM E813-89 imposes a restriction on the collected J - Δa data points. To determine whether the collected J - Δa data points comply with this restriction, first a

blunting line having a slope equal to twice the material yield stress ($2\sigma_0$) is constructed from the origin of the plot, then four lines parallel to the blunting line and having horizontal offsets equal to 0.15-mm, 0.5-mm, 1-mm, and 1.5-mm are constructed. The appropriate J - Δa data set must have at least one data point between the blunting line and the 0.15-mm offset line, one data point between the 0.15-mm offset line and the 0.5-mm offset line, and one data point between the 1-mm offset line and the 1.5-mm offset line. The 0.15-mm and 1.5-mm-offset lines are called exclusion lines. The J - Δa data points falling between the two points where the exclusion lines intersects the J - Δa curve are selected for constructing a power law curve. The intersection of the J - Δa curve with a 0.2-mm offset line parallel to the blunting is selected as a candidate value J_Q . The J_Q is equivalent to J_{IC} if the specimen size and the measured portion of the J - Δa curve satisfy the following conditions:

- The thickness (B) $> 25 J_Q / \sigma_y$.
- The initial ligament length (b_0) $> 25 J_Q / \sigma_y$.
- The slope of the regression line evaluated at Δa_Q is less than the yield strength σ_y where Δa_Q refers to the crack extension at which J_Q is evaluated.

Determining the entire J - Δa resistance curve from standards E1152-87 is less restrictive, this standard only imposes limitations on the measuring capacity of the specimen and the number of data points needed to produce a valid J -resistance curve.

The measuring capacity of the specimen refers to the maximum measurable J (J_{max}) and crack extension (Δa_{max}). E1152-87 defines J_{max} as the smallest of the two ratios ($b\sigma_y/20$), ($B\sigma_y/20$) and Δa_{max} as one tenth of the ligament length ($0.1b$). If the measuring capacity of the specimen is exceeded then E1152-87 recommends performing the J calculation based on the modified J procedure J_M . Otherwise, the deformed J (J_D) is recommended. To determine the minimum number of points needed to produce a valid J -resistance curve, E1152-87 instructs constructing a secant line from the origin of the plot with a slope equals to $(4/3 \sigma_y \Delta a)$. A minimum of two data points should lie between the secant line and the ordinate of the plot and there should be a minimum of eight data points between the secant line and the Δa_{max} line.

J -controlled fracture may exist for a stationary or a growing crack if J characterizes the crack tip condition. For a stationary crack, J becomes invalid when the size of the large strain zone becomes significant with respect to the specimen dimensions or the J -dominance size. For a growing crack J characterization of fracture becomes invalid if the J -resistance curve becomes dependent on specimen geometry. Kobayashi and coworkers [30-34] and Han [2] measured surface displacement during fracture tests. Their results showed that the surface displacement and strain fields normal to the plane of the crack did not have the HRR trends. Also, they reported that for small crack growth increments, the J -resistance curve is reasonably independent of specimen geometry. However, the geometry dependence of the J -resistance curve decreases for large increments of crack growth. Moreover, J is only defined for two-dimensional crack

geometry, thus, violating another of Shih's[5] requirements that mandates a fracture parameter be applicable to three-dimensional crack geometry [26]. Therefore, an alternative fracture parameter for the J -integral must be found to better characterize the material resistance to fracture under the conditions of large-scale plasticity or large amounts of stable crack growth.

2.2.2 The Crack Tip Opening Displacement (CTOD) Fracture Criterion.

Unlike the development of the linear elastic stress intensity factor, K , and the J -integral in fracture mechanics, introducing the crack tip opening displacement ($CTOD$) as a fracture parameter resulted from experimental observations. While conducting fracture tests on high toughness materials, Wells [35,36] noticed that the crack faces had moved apart due to plastic deformation. Thus he was unable to explain the observed fracture behavior with the existing LEFM theory at the time. Therefore, Wells proposed the displacement between the crack faces near the tip of the crack ($CTOD$) for characterizing the fracture toughness in an elastic-plastic material undergoing large scale yielding.

According to a $CTOD$ based fracture criterion, fracture occurs when the $CTOD$ reaches a critical value. Schwalbe *et al.* [37] reported that the $CTOD$ definition is vague since the limiting value of $CTOD$ at the crack tip approaches zero as illustrated in Figure 2.10. Nevertheless, the $CTOD$ continues to be a useful fracture criterion because the intent of using the $CTOD$ is to define a characteristic displacement that controls the crack tip condition. Different researchers adopted many definitions of the $CTOD$ to quantify the height of the stretch zone caused by crack blunting. Some of those parameters, as

illustrated in Figure 2.10, are the displacement at the original crack tip, δ_0 , proposed by Shih [5,38] and the δ_{45} definition proposed by Rice [39]. The δ_{45} defines the height of the gap between the crack faces at the point where the lines extending from the crack tip and forming $\pm 45^\circ$ angles with the crack plane intercept the surfaces of the crack. Wells [35] used the LEFM solution to evaluate the *CTOD* in the limit of small scale yielding; his analysis produced an estimation of the *CTOD* that is equivalent to the δ_0 parameter proposed by Shih [5,38]. Thus under small scale yielding conditions, the *CTOD* can be related to K and G . Since J is equal to G in small scale yielding, J also can be related to the *CTOD*. Under small scale yielding and plane strain conditions Kanninen [40] has reported that the δ_0 parameter can be related to J , G , and K according to the following relation:

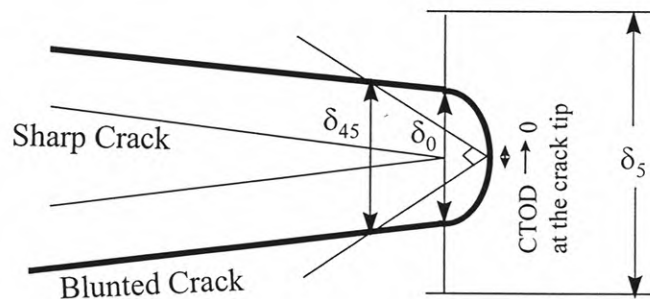


Figure 2.10: Definition of the Various Parameters Representing the CTOD .

$$J = G = \frac{1 - \nu^2}{E} K^2 = \sigma_0 \delta_0 \quad (2.11)$$

Where σ_0 , ν , and E are the material yield stress, Poission's ratio and modulus of elasticity respectively. For the situation where the HRR field describes the state of stress at the tip of a stationary crack, Shih [41] showed that J is related to the δ_{45} by the following relation:

$$\delta_{45} = \frac{d_n}{\sigma_0} J \quad (2.12)$$

Where d_n is a function of the material properties and σ_0 is the yield stress of the material.

The δ_0 and the δ_{45} parameters were originally proposed to characterize the initiation of crack growth in ductile behaving materials where ductility causes the crack tip to become blunt before it starts tearing. During stable crack growth the δ_0 parameter, which is defined at the location of the original crack tip, does not represent the displacement at the instantaneous tip of the crack, but it continues to give a measure of the crack opening displacement (*COD*). The difference between the *CTOD* and the *COD* is that the *CTOD* gives a measure of the crack opening displacement in the neighborhood of the crack tip while the *COD* gives a measure of the crack opening at a remote location from the crack tip. Experimental evidence [6, 8, 20] shows that tearing cracks have sharper profile than stationary blunt cracks. This in turn invalidates the δ_{45} definition of the *CTOD* for stable tearing fracture. An appropriate definition of the *CTOD* for a

growing crack can be taken as the crack opening displacement at a certain distance behind the current tip of the crack [42-44]. This definition is closely related to the *CTOA* parameter (crack tip opening angle) which will be discussed in the next section.

Many researchers have used *COD- Δa* resistance curves, known as δ_R resistance curves, to characterize the stable crack growth process. Shwalbe *et al.* [37] proposed the δ_5 parameter, which is measured at the crack tip of the fatigue pre-crack using a clip gage with 5 mm gage span as illustrated in Figure 2.10, for developing the δ_R resistance curve. Schwalbe *et al.* [37,45] concluded that the δ_R resistance curve was virtually independent of specimen geometry. Newman [46] proposed the V_R resistance curve concept for characterizing the material resistance to stable crack growth. The V_R parameter, as illustrated in Figure 2.11, sums the critical CTOD and the width of the plastic wake at the tip of the crack. The V_R resistance curve is advantageous over the δ_R curve since it relates

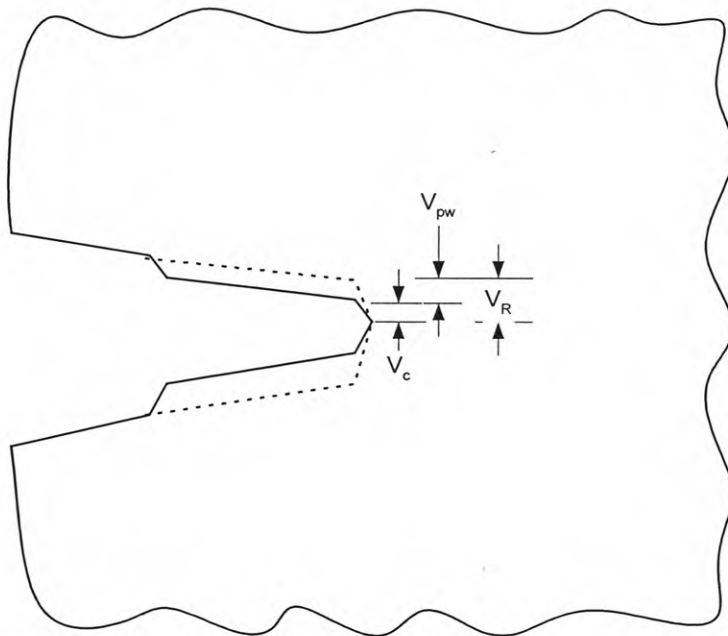


Figure 2.11: Illustration of the V_R resistance curve concept (from [46]).

to *CTOD* near the crack tip position [46]. However, Newman [46] reports that the V_R cannot be measured since it involves the difference in displacement between stationary and growing cracks. The V_R can be determined by making use of experimental load-load line displacement data and Dugdale solution for the loaded geometry. This is another disadvantage of the V_R curve method since the Dugdale solution is not available for all geometrical and loading situations. Newman [46] compiled the V_R curves for eleven combinations of material, initial crack length, and specimen thickness. He concluded that the V_R resistance curves were independent of initial crack length, specimen width, but dependent on material and specimen thickness. Thus, the resistance curves with using the *CTOD* as the crack driving force have an advantage over the J - Δa resistance curves since the δ_R/V_R resistance curves permit larger increments of crack growth without exhibiting geometry dependence [37].

Like the linear elastic stress intensity factor, K , and the J -integral, the *CTOD* depends on specimen thickness. Green and Knott [47] used the δ_0 parameter to measure the critical *CTOD* at the initiation of crack growth in a number of mild steel specimens ranging in thickness from 5 mm to 70 mm. They concluded that the *CTOD* at crack initiation decreases as the specimen thickness increases until the specimen thickness becomes sufficient to provide plane strain conditions, after that no further decrease of the *CTOD* at crack initiation was observed. Green and Knott [47] explained the observed behavior of the *CTOD* at crack initiation (decreasing with specimen thickness increase) by considering the effect of lateral constraint. Thickness increase leads to an increase in

the lateral constraint which tends to limit the extent of plastic deformation at the tip of the crack. This leads to smaller plastic deformation or strain which in turn causes less crack blunting and consequently a smaller critical *CTOD*. Paleebut [48] also ran a series of fracture tests on C(T) specimens having different thicknesses, ranging from 2.98 mm to 25.2 mm, in which he used the δ_0 parameter to measure the critical *CTOD* at the initiation of crack growth for 7075-T651 and 2024-T351 Aluminum alloys. For the 2024-T351 Aluminum alloy, Paleebut only reported the critical value of the δ_0 parameter for the 25.2-mm thick specimens. Thus he did not fully explore the thickness effect on the critical *CTOD* in the 2024-T351 Aluminum alloy. However, Paleebut measured the critical *CTOD* for all the 7075-T651 Aluminum alloy specimens. It is clear from his results, for the 7075-T651 Aluminum alloy specimens, that the critical *CTOD* decreases as the thickness increases with the lowest critical *CTOD* value observed for the thickest specimen which exhibited plane strain conditions. Paleebut [48] did not demonstrate the constancy of critical *CTOD* with specimen thickness when the plane strain condition exists as reported by Green and Knott [47]. Regarding the effect of specimen thickness on growing cracks, the V_R resistance curves for the eleven materials and thickness combinations in [46] show that the specimen thickness has an effect on the V_R resistance curves. However, no analysis or supporting interpretation was provided for this effect.

The British standard BS 5762:1979 [49] and the ASTM standard E1290-93 [50] outline the two available standard procedures for the critical *CTOD* fracture testing. The British standard allows testing of single edge (SENB) specimen only while the ASTM

standard E1290-93 allows testing of the compact tension (CT) specimen as well. The ASTM standard E1290-93 includes notation for the critical *CTOD* encountered in a variety of fracture behavior situations which are:

- δ_c : the critical CTOD at the onset of unstable fracture when no stable crack growth occurs.
- δ_u : the critical CTOD at the onset of unstable fracture when stable crack growth precedes unstable fracture.
- δ_i : the critical CTOD values near the initiation of stable crack growth. It must be determined from a δ - Δa resistance curve.
- δ_m : the CTOD at first attainment of maximum load plateau. It is used when the load vs. load-line displacement curve plateaus at a maximum load value before the load starts decreasing.

Both standard test methods require and provide instruction on fatigue pre-cracking of the test specimen. The BS 5762:1979 standard allows the ratio of the initial crack length to specimen width (a/W) to vary between 0.15 and 0.70 while the ASTM E1290-93 restricts the a/W ratio to a value between 0.45 and 0.55. In the BS 5762:1979, the only specimen size requirement on critical *CTOD* testing is that the specimen thickness must be equal to the width of the section used in the structural component. The ASTM E1290-93 standard adds another restriction which requires that the specimen width to thickness ratio be greater than 2 and less than 4. The test is conducted under displacement control such that

the cross-head displacement generates a rate of increase in the resulting stress intensity factor falling in the range between 0.55 and 2.75 MPa√m/s. During the test, a record of load against crack mouth opening displacement must be obtained. The crack mouth opening displacement is measured with a clip gauge mounted into the test specimen using knife-edges. After the test is completed, the specimen is broken and the initial and final crack lengths must be determined. The *CTOD* is estimated by adding the contributions of the elastic and plastic displacement components. The elastic displacement component is evaluated from the elastic stress intensity factor, K , which is computed based on the failure load, crack length, and specimen geometry. The plastic component is estimated from the measured crack mouth opening displacement by assuming that the specimen rotates about a plastic hinge. Combining both components of displacement, ASTM 1290-93 reports the measured *CTOD* by to the following equation:

$$\delta = \frac{K^2(1 - \nu^2)}{2\sigma_0 E} + \frac{r_p(W - a_0)V_p}{r_p(W - a_0) + a_0 + z} \quad (2.13)$$

Where K is the linear elastic stress intensity factor, ν is the Poisson's ratio, σ_0 is the yield strength of the material, E is the elastic modulus of the material, W is the specimen width, a_0 is the initial crack length, V_p is the measured crack mouth opening displacement, z is the height of the knife edges attached to the specimen for mounting the clip gauge, and r_p is the plastic rotational factor which lies between 0 and 1. The BS 5762:1979 standard recommends a value of 0.4 for r_p while the ASTM E1290-93 recommends a value of 0.44 for the SENB specimen. For the compact specimen, the ASTM E1290-93 provides

a formula for estimating the rotational factor based on the initial crack length and uncracked ligament of the specimen. An advantage of the critical $CTOD$ and J_{IC} fracture testing over the K_{IC} fracture testing is that the J_{IC} and critical $CTOD$ fracture testing apply to brittle as well as ductile fracture. Critical $CTOD$ fracture testing is advantageous over J_{IC} fracture testing because it involves less restriction and allows more plastic deformation.

As seen from this section, for a stationary crack the $CTOD$ parameter satisfies all of the essential requirements of a valid fracture criterion, outlined by Shih *et al.* [5]. The $CTOD$ provides a measure of the stress and deformation fields in the vicinity of the crack tip, and the critical value of the $CTOD$ is independent of specimen geometry and dimensions. For a growing crack, the δ_R resistance curve provides a valid characterization of the material resistance to crack growth. However, the various adopted parameters that was introduced earlier to characterize crack opening displacement vs crack growth resistance have shortfalls. The δ_5 and δ_0 parameters are measured a way from the tip of the crack tip thus they do not describe the local conditions at the crack tip. On the other hand the V_R parameter describe the local condition at the crack tip but it cannot be measured experimentally. A proper definition of $CTOD$ for growing cracks would be the $CTOD$ measured at a short distance behind the crack tip. As mentioned previously, this definition is closely related to the crack tip opening angle ($CTOA$) [518,42] and would be the subject of next section. Shih *et al.* [5] also suggested, for convenience, that a fracture parameter must have a constant critical value during crack

extension. The δ_R resistance curves obtained by different researchers [5,45] show that the critical *COD* value increases with increasing crack length during stable crack growth. This may introduce a difficulty in using the δ_R resistance curve to model the stable crack growth fracture since the entire δ_R resistance curve must be incorporated into the analysis instead of a single parameter.

2.2.3 The Crack Tip Opening Angle (CTOA) Fracture Criterion.

Stable crack growth adds to the complexity of modeling the stress and strain fields near the tip of the crack in fractured components. Thus, most of the fracture mechanics analyses for modeling stable crack growth are implemented using finite element simulation. Anderson [51] and de Koning [52] conducted finite element analysis of thin specimens from which they concluded that the angle between the crack faces (crack tip opening angle) remains constant during the stable crack growth. A fracture criterion based on the *CTOA* parameter assumes that the material will tear in a stable manner when the angle between the crack faces reaches a critical value. In a material undergoing stable crack growth, the critical value of *CTOA* is regarded as a material parameter that bounds the upper limit of the applied *CTOA* value that a material can sustain before the crack length increases in a slow-stable manner. de Koning [52] also ran experimental fracture tests in which he used an optical microscope to obtain the crack profile by measuring the crack opening displacement at various locations along the crack line. He obtained linear crack profiles that lead him to conclude that the *CTOA*, determined from the slope of the crack profile, remains essentially constant from the onset of crack initiation. Anderson

[51] argued that the *CTOA* does not become constant immediately after the initiation of stable crack growth. He explained that the *CTOA* will attain a maximum value just before the crack starts to grow, then it decreases to a constant value after a transition that takes place as soon as the tearing process takes place.

Rice *et al.* [53,54] were able to obtain an analytical expression for the displacement field in the vicinity of the momentary crack tip of a growing crack under small scale yielding and plane strain conditions. From their analysis, they concluded that the *CTOA* is undefined at the tip of the crack. However, they defined the *CTOA* as the ratio between the *CTOD* (δ) at a given distance behind the crack tip and the distance (r_m) at which the *CTOD* is measured. The resulting expression from their definition is given by:

$$CTOA = \frac{\delta}{r_m} = \frac{\alpha}{\sigma_0} \frac{dJ}{da} + \beta \frac{\sigma_0}{E} \ln \left(\frac{eR}{r_m} \right) \quad \text{for } r \rightarrow 0$$

where:

$$R = \frac{\lambda EJ}{\sigma_0^2}, \quad \beta = \frac{4(2 - \nu)}{\sqrt{3}} \quad (2.14)$$

Where α is a dimensionless coefficient, σ_0 is the yield stress, E is the material modulus of elasticity, a is the crack length, λ ($=0.2$) is a constant, and e ($=2.718$) is the natural logarithm base. Rice *et al.* [53,54] argued that if r_m is small enough such that a , R , and dJ/da remain sensibly constant during an increment of crack extension that is comparable to r_m in size, the *CTOA* stays constant during the stable crack growth. Equation (2.14)

also shows that the *CTOD* for a growing crack is also constant if it is defined at a small distance behind the current crack tip.

Kanninen [40] pointed out that two different definitions of the crack opening angle exist. The crack-tip opening angle (*CTOA*), which reflects the local slope of the crack profile near the tip of the crack, and the average crack opening angle (*COA*), which is the ratio of the crack opening displacement (*COD*) at the initial crack tip to the current crack length. In any case, the crack tip opening angle (*CTOA* or *COA*), ψ , can be evaluated from geometrical consideration based on the measured crack tip opening displacement, δ , by the following equation:

$$\psi = 2 \arctan\left(\frac{\delta}{2d}\right) \quad (2.15)$$

Where d is a given distance behind the current crack tip as illustrated in Figure 2.12. As long as the distance d is small and measured relative to the instantaneous tip of a growing crack, the *CTOD* and the *CTOA* will exhibit a similar behavior as the crack grows. Luxmoore *et al.* [55] and Green and Knott [47] conducted fracture tests on thin

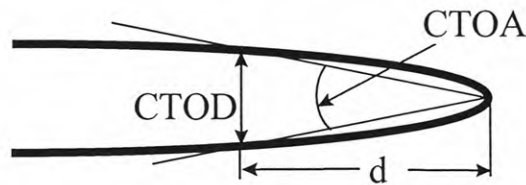


Figure 2.12:CTOA Definition

specimens from which they concluded that the *COA* parameter also remains constant

during crack extension. Luxmoore *et al.* [55] reported that the *COA* remains constant from the onset of crack growth, but it was dependent on crack configuration. The geometry dependence of the *COA* parameter may be due to the fact that the *COA* represents the average behavior of the specimen and not the local condition at the tip of the crack as does the *CTOA* parameter [40]. Brocks *et al.* [56] performed numerical simulation of stable crack growth to assess the viability of the *CTOA* based fracture criterion to model the stable crack growth process. They reported that during stable crack growth, the *CTOD* at the instantaneous crack tip becomes constant while the *COD* at the location of the original crack tip increases monotonically. Brocks *et al.* [56] also added that the *CTOA* becomes stable when the plastic zone extends through the uncracked ligament. The numerical simulation of stable crack growth conducted by Brocks *et al.* [56] lead them to conclude that the critical *CTOA* is independent of specimen type and geometry.

Newman *et al.* [42-44] used the constant critical *CTOD* evaluated at a small distance (1-mm) behind the instantaneous crack tip to analyze the behavior of fracture specimens. He obtained a good agreement between the numerical results and the experimental data, hence, adding support to the viability of the *CTOA* based fracture criterion in modeling the stable crack growth fracture. Regarding the transition crack growth (from blunt to stably tearing), experimental work by Demofonti and Rizzi [57], Newman *et al.* [42], and Dawicke and coworkers [8,58,59] show that in thin specimens the critical value of *CTOA* becomes constant after an amount of crack growth on the order

of the specimen thickness (one to two times the specimen thickness). Figure 2.13 shows a typical $CTOA$ - Δa resistance curve that was obtained by Dawicke and Sutton [8] for thin sheets of 2024-T3 Aluminum alloy. As Figure 2.13 shows, the $CTOA$ decreases rapidly during the early stage of crack growth, then it plateaus to a constant value after the short amount of transition crack growth. The extensive series of fracture tests performed by Newman *et al.* [42] and Dawicke *et al.* [8,58,59] lead them to experimentally conclude that the critical $CTOA$ is independent of specimen geometry, in-plane dimensions of the specimen, and the initial crack length.

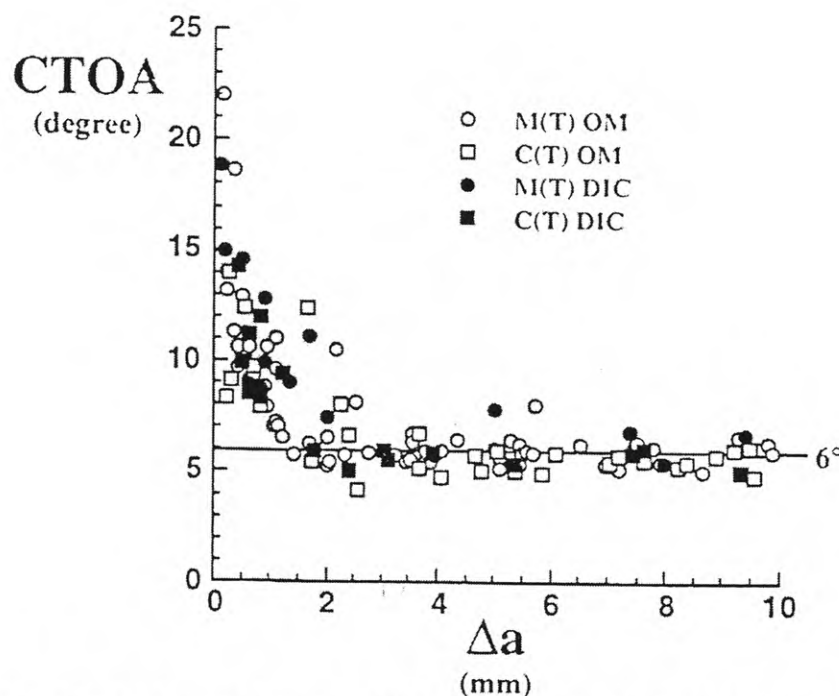


Figure 2.13: Typical Experimental $CTOA$ Resistance Curve For 2.4-mm Thick 2024-T3 Aluminum Specimen (from [8]).

Although no standardized procedure exists for critical $CTOA$ fracture testing, several researchers have published experimental methods for conducting such a test.

Demofonti and Rizzi [57] established a procedure for estimating the *CTOA* for a three-point-bending (3PB) specimen. Their technique was based on estimating the *CTOA* from the plastic rigid body rotation of the specimen during the test. They obtained photographs of the specimen during tests that allowed them to determine the *CTOA* within 5%. Branby *et al.* [60] imprinted a fine grid on the specimen surface and used magnifying imaging techniques for measuring the *CTOA* during the fracture test.

Newman [61,62] measured the load and load line displacement during fracture tests, then he used finite element analysis to simulate the behavior of the measured data. The value of the critical CTOD, at a certain distance behind the crack tip, producing the best match between the predicted and the measured maximum applied load was chosen to compute the proper critical *CTOA* value. Figure 2.14 shows a typical comparison between the experimental and finite element analysis of the load versus load-line displacement data obtained by Dawicke *et al.* [8]. The finite element analyses presented in Figure 2.14 are two-dimensional, one model uses plane stress assumption while the other assumes a mixed state of stress which consists of plane stress with plane strain core. The plane strain core assumes that plane strain condition exist around the tip of the crack. Dawicke and coworkers [8,58,59] ran an extensive series of *CTOA* fracture characterization tests on 2.3 mm thick specimens made out of 2024-T3 Aluminum alloy. They measured the *CTOA* using optical microscopy and digital image correlation techniques which produced a 1° scatter band in the measurements as shown in Figure 2.13. Since their measurements had about 1° scatter band, they used the finite element analysis method to find the proper

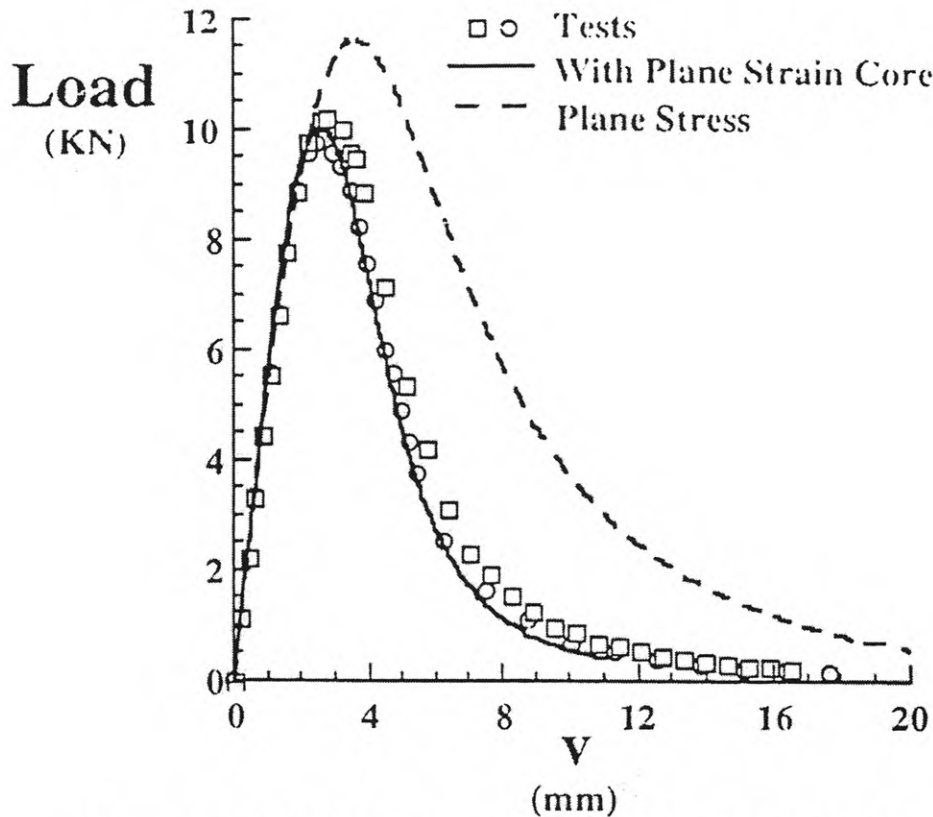


Figure 2.14: Comparison Between Load-Versus-Load-Line Displacement Behavior Obtained from Experiments and Finite Element Simulation Data (from[8]).

critical *CTOA* value that makes the finite element results best match the experimentally determined maximum load. The resulting critical *CTOA* value lies within the experimentally measured *CTOA*, but it was not equal to the average value of the measurement as seen from Figure 2.13. Hence their approach is similar to that of Newman's [61,62] except it had experimental verification for supporting the critical *CTOA* prediction.

For thin specimens of 2024-T3 Aluminum alloy, Dawicke and Sutton [8,59] observed tunneling in the crack front which refers to the phenomenon that includes more

crack growth in the interior of the specimen than on its surface. This indicates that the tearing process is three-dimensional even in thin specimens [8,59]. To account for the three-dimensional effects in the tearing process, Dawicke *et al.* [8] used finite element analysis with mixed state of stress (plane strain core: plane strain near the crack and plane stress else where). Figure 2.14 shows how the finite element analysis with plane strain core predicts the experimental load-deflection data more accurately than the finite element analysis with plane stress. Lloyd and Piascik [63] performed microtopographic examination of fractured surfaces in which they used optical triangulation to measure the relative surface height on the fracture surfaces. They concluded that the stable tearing initiates at mid-thickness, then it is followed by a crack tunneling region in which the $CTOA$ near the surface is larger than the $CTOA$ near mid-thickness. Also, they showed that the $CTOA$ behavior approaches a steady state where a constant single value is observed across the entire crack front. Dawicke *et al.* [64] incorporated the shape of the tunneled crack front into three-dimensional finite element analysis. The results of their analysis revealed that in the transition region the critical value of the $CTOA$ was lowest at the center of the specimen and highest at the specimen surface. As the crack grew, they found out that the $CTOA$ increased at the center and decreased at the surface until a uniform value of the $CTOA$ was attained across the specimen.

In the previous section, it was concluded that the $CTOD$ for a stationary crack and the COD for a growing crack are dependent on the thickness of the specimen. Also, Green and Knott [47] showed experimentally that the COA decreases with increasing

specimen thickness, thus, the *CTOA* is also expected to exhibit a similar behavior. Although the *CTOA* has been measured for various materials with different specimen thicknesses, comparison of the data reported in the literature is not straightforward; such comparison also may be misleading in some cases. One of the main difficulties involved in using the available critical *CTOA* data to quantify the effects of material thickness on the critical *CTOA* is that some researchers reported critical *CTOA* while others reported the critical *CTOD* at a given distance. Although the critical *CTOD* may be converted into a critical *CTOA* using Equation (2.15), other factors may be involved which complicate comparing the different data sets. Some of those factors may be due to differences in the fracture procedures followed by the different researchers, the measurement techniques used for obtaining critical *CTOA/CTOD*, or the distance between the tip of the crack and the measurement location of the *CTOD/CTOA*. Newman [61,62] reported the value of the critical *CTOD* evaluated at 0.4 mm behind the tip of the crack for 12.7 mm thick specimens made of three different metal alloys, 2024-T351 and 7075-T651 Aluminum alloys, and 304 Stainless Steel. Also, Dawicke and coworkers [42,8,58,59] experimentally measured the critical *CTOA* at approximately 1 mm behind the tip of the crack for 2.3 mm thick 2024-T3 Aluminum alloy specimens. A comparison between the critical *CTOA* values obtained from both references for the 2024-T351 and the 2024-T3 Aluminum alloys shows that the 2024-T351 Aluminum alloy has a slightly higher critical *CTOA* even for thicker specimen. However, such a comparison is inaccurate since the location of the critical *CTOD/CTOA* measurement used in both studies is different. Sutton *et al.* [65] and Han [2] obtained critical *CTOA* measurement for 0.38 mm thick

304 stainless steel specimens. Comparing both of the critical $CTOA$ values obtained from both studies for the 304 Stainless Steel shows that the critical $CTOA$ increases dramatically as the thickness changes from 0.38 mm to 12.7 mm. Again, this comparison is not accurate either since the location of the critical $CTOD/CTOA$ measurement used in both studies is different. Thus, the specimen thickness effects on critical $CTOA$ has not been fully explored.

Chapter 3

Overview of Research Plan

As mentioned earlier, the goal of the current research project was to perform an experimental/computational investigation into thickness effects on the critical crack tip opening angle (*CTOA*) as a suitable fracture parameter for characterizing material resistance to stable crack growth. In order to carry out this study in a comprehensive manner several steps were followed. These steps included material selection and specimen preparation, fracture testing, and computational simulation of the fracture experiments. The goal of this chapter is to provide a brief overview of these steps prior to covering the details of these activities in the later chapters.

With regards to material selection, it was desirable to choose a material having some data available in the literature so comparisons could be made and the results of the current research are tied to a solid previous research. Also, it was desired that the material have an industrial application such that the *CTOA* characterization is valuable in designing against fracture for those applications. Another aspect of the research plan that had to be considered carefully was specimen selection. It was necessary to select a standard fracture specimen that was appropriate for the *CTOA* measurements. Also, thickness selection had to be coordinated carefully to get meaningful results about the effect of thickness on the *CTOA* fracture criterion. The details of the specific material and specimen type that were selected for this project will be discussed in more detail in Chapter 4.

After selecting the material and specimen type, the experimental setup and experimental procedure were planned to accomplish the *CTOA* fracture tests. The experimental setup was designed to provide the ability of fracturing the specimen and collecting all of the important data parameters. The fracture procedure consisted of fatigue precracking at low load level to provide a sharp crack in the fracture specimen. After fatigue precracking, the fracture specimen was fractured slowly by applying displacement-controlled load exertion to induce stable crack growth. During the fracture tests, it was necessary to measure applied load, load line displacement, and crack extension. Also, the specimen surface was monitored continuously during the fracture test to allow measuring *CTOA* on the surface of the specimen using the optical microscopy (*OM*) and digital image correlation (*DIC*) techniques. Details and results of the fracture testing are given in chapters 5 and 7.

The third aspect of the research plan included the computational simulation of the stable tearing process. Three-dimensional finite element analysis of the specimen was used in conjunction with the experimentally determined load and load line displacement data to find the value of the *CTOA* that enables the simulation to predict the maximum applied load correctly. Also, the computational aspect of the test procedure included two-dimensional finite element analysis that was used to find the plane strain core height. The *CTOA* value found from the three-dimensional elastic plastic finite element analysis was used with two-dimensional elastic plastic finite element analysis to predict the amount of constraint (plane strain core height) that the material thickness exerts. By correlating these two analyses, it may therefore be possible to use the simpler (and therefore less expensive) two-dimensional analysis in place of the more complex three-

dimensional finite element analysis in maximum load prediction. Details and results of the computational simulations of the stable tearing process are given in chapters 6 and 7.

Chapter 4

Test Material and Test Specimen

As mentioned in Chapter 3, the first step of the current research project was to select an appropriate test material and test specimen with which to evaluate the *CTOA* thickness effects. It was also necessary to determine preliminary stress-strain properties for the material as well as designing and fabricating an appropriate set of grips. This chapter will provide a discussion of the motivation and details involved with the material and specimens selection processes.

4.1 Material Selection

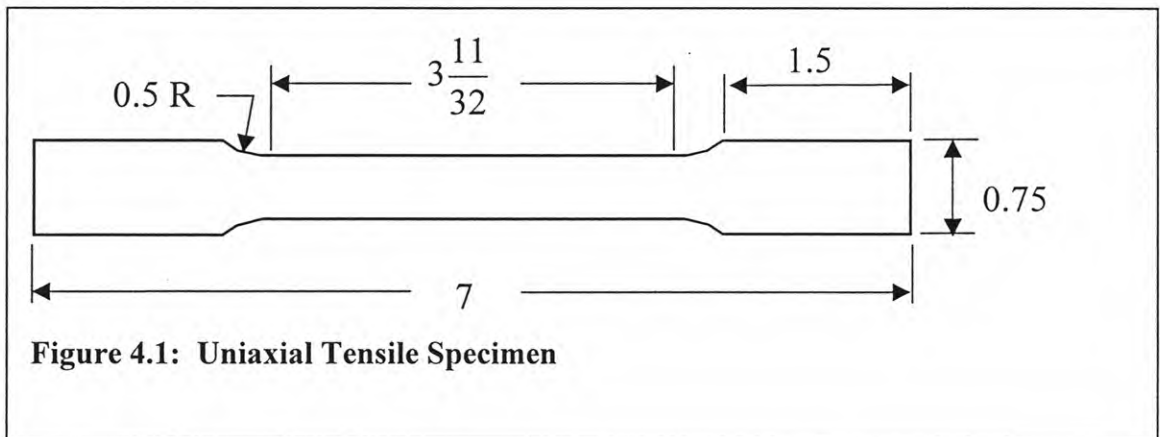
The material selection decision was based on availability of literature on the material and feasibility of using the crack tip opening angle (*CTOA*) characterization for that material. Literature availability means *CTOA* characterization of the material has been performed and reported for at least one thickness. This was desired to provide a baseline comparison for the data to be generated in this project. Feasibility of the *CTOA* characterization refers to the usefulness of the critical *CTOA* as a fracture design parameter for the particular material. Thus, it was decided to use a material that has applications in which stable-tearing fracture is likely to occur in typical applications. Therefore, a material used as aircraft structural and/or skin elements such as 2024-T3, 2024-T351 and 7075-T651 Aluminum alloys were considered [62]. Also 304 Stainless Steel, which is used for nuclear reactor pressure vessels [89], was considered as well. Some literature was identified about *CTOA* characterization of thin sheets of 304 Stainless Steel and plates of 7075-T651 Aluminum alloy [62]. However, researchers at

NASA Langley research center [8,58,64] conducted the most comprehensive *CTOA* characterization on thin sheets of 2024-T3 Aluminum alloy. Since the aim of the current research project was to characterize thickness effect on the critical *CTOA*, it was desired to run the same caliber research as that of NASA Langley research center. Thus, the most appropriate material choice would have been 2024-T3 Aluminum so the work by NASA Langley research center can be used as a base case for the current research project. Based on discussions with material vendors, 2024-T3 Aluminum is not available in thick plates and the closest material was 2024-T351 Aluminum. The “51” in 2024-T351 designates that the material has been stress relieved by stretching [3]. With regards to feasibility of *CTOA* characterization on the selected material, 2024-T351 Aluminum is an aircraft grade material, an application where overloading of the material occurs repeatedly and the material in service is susceptible to stable growing cracks. Thus, *CTOA* may be an appropriate fracture design criterion for 2024-T351 Aluminum.

4.2 Determination of Material Properties

After acquiring the material, the next step was to determine its uniaxial stress-strain properties that would be used with the finite element based elastic-plastic fracture analysis phase of the *CTOA* characterization procedure. Uniaxial tensile tests were conducted to determine the monotonic stress-strain properties for the acquired 2024-T351 plate using ASTM standard E8-91 as a guide for designing specimens and running tests [90]. A total of six uniaxial tensile specimens were tested, three specimens were cut along the rolling direction of the plate and three were cut along the transverse direction of the plate. As Figure 4.1 illustrates, the tensile specimens were 0.25 in thick flat dog-bone shaped with a 7 in overall length. The gripping sections were 1.5 in long and 0.75 in

wide which blend, using 0.5 in radius fillets, into 0.5 in wide and $3 \frac{11}{32}$ in long reduced gage section. The uniaxial tests were run using a 22 KIP digitally controlled servo-hydraulic material testing machine with hydraulic wedge grips. The testing machine load cell was used for measuring the load and an axial strain extensometer was used for measuring axial strain. The MTS Basic Test Ware software was used to run the tests (at 0.01 in/min displacement rate) and collect the load and the strain data during tests. For each specimen, the stress was calculated by dividing the load by the original cross sectional area of the particular specimen.



After collecting the stress-strain curves, the elastic modulus, the elastic stress limit, the yield stress, and the ultimate stress were determined for the material in both the rolling and the transverse directions. The elastic modulus for each specimen was determined by finding the slope of the initial linear region of the stress-strain curve. The elastic modulus for the material in each direction was then considered as the average of the three specimens tested for each of the directions. The elastic material limits were then identified visually by looking at the stress-strain curve and determine the point at which the stress-strain curve starts to go non-linear. Also, 0.2% offset yield stresses were

determined. For comparison purposes, published tensile data for 2024-T3 and 2024-T351 were obtained from MIL-Hand Book-5G [91]. Table 4-1 includes both the measured material properties and the data obtained from the MIL-Hand Book. Figure 4.2 shows the measured uniaxial stress strain curves for both of the longitudinal and the transverse uniaxial tensile specimens as well as the curves digitized from the MIL Hand Book-5G [91] for the 2024-T3 and –T351 along the longitudinal direction. It is obvious that the elastic modulus for the longitudinal directions, obtained from the MIL Hand Book-5G and the determined from experimental measurements, agreed well with each other. However, the Elastic Modulus measured for the transverse direction was slightly lower than that of the longitudinal direction. The yield stress reported for the 2024-T351 (longitudinal direction) was slightly lower than the corresponding yield stress measured in the current project. MIL Hand BOOK-5G also reports that the 2024-T3 alloy would have higher yield stress but lower ultimate stress than the -T351. Based on the measurements conducted in the current research for the 2024-T351, it was found that the longitudinal direction had higher yield stress but slightly lower ultimate stress than the transverse direction. The Possion’s ratio was not measured in this project but it was reported to be 0.3 in MIL Hand BOOK-5G [91] for both the –T3 and –T351 alloys.

Table 4-1: Uniaxial Material Properties

Property (ksi)	2024-T351 Measured Data		Data from MIL Hand Book-5G [91]	
	Longitudinal	Transverse	2024-T351 Longitudinal	2024-T3 Longitudinal
Elastic Modulus	10740	7560	10700	10500
Elastic Limit	52.7	37	45	47
0.2% Yield Stress	54	48.4	48.15	50
Ultimate Stress	69	70.4	64.17	63

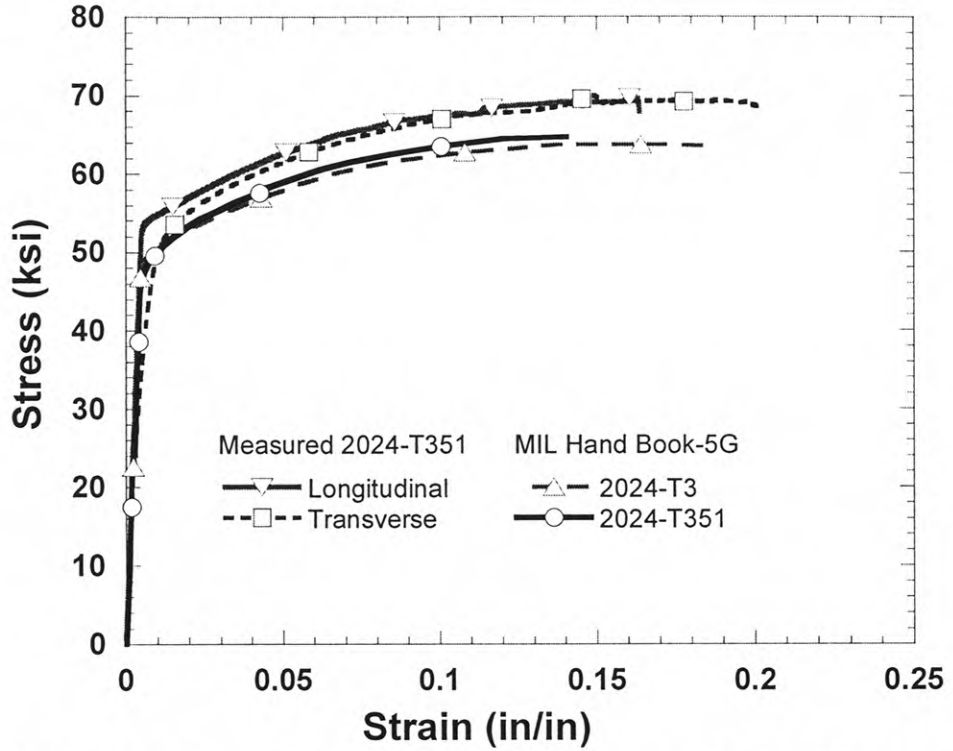


Figure 4.2: Uniaxial Stress Strain Curves Obtained From Measurements and MIL Hand Book [91]

4.3 Fracture Specimen Design and Thicknesses

After selecting the material, a decision had to be made regarding the fracture specimen geometry and the thicknesses to be evaluated in this study. A variety of standard specimen types may be used to test the critical value of the *CTOA* such as compact tension C(T), middle crack tension M(T), single edge crack SEC, and bending specimen. Based on conversations with NASA researchers [7], it was concluded that the C(T) specimen, shown in Figure 4.4: , would be the best choice for measuring the *CTOA*. M(T) specimens are likely to undergo plastic collapse during large deformation stable tearing tests [58] and bending type specimens are primarily suitable for testing relatively

thick specimen. Based on the work by Dawicke [8,58,64], 0.090 in thick 6 in wide 2024-T3 Aluminum specimens exhibited about 0.5 in of stable crack growth, which was found to be suitable for characterizing the critical *CTOA*. Since a thicker specimen was expected to exhibit less stable crack growth, the larger 8 in C(T) specimen was selected for the current work to make sure that a sufficient amount of stable crack growth is observed during the tests. Also, NASA researchers [7], who conducted comprehensive *CTOA* characterization on thin sheets of 2024-T3 Aluminum, were consulted about the specimen thicknesses. Based on these consultations, it was determined that there is interest in characterizing larger thicknesses materials that are more typical of structural elements. For the purposes of this research project, it was decided to evaluate four thickness of the 2024-T351 alloy. Coordinating with Dawicke [7], the chosen thicknesses were 0.090, 0.25, 0.50, and 1.0 in.

All 2024-T351 specimens were machined out of the same plate to avoid lot-to-lot variation. The plate was 1 in. thick, 48 in. long along the rolling direction and 48 in. wide in the transverse direction. To carve the specimens out of the plate, the plate was cut into sixteen square 12x12 in. plates. Thus, for the 1 in. thick specimen, only final surface machining was required in the thickness direction. To produce specimens with thickness less than 1 in., one of the resulting 12x12 in. plates was sliced to produce a sheet/plate whose thickness was close to the desired thickness, then the surfaces of the resulted sheet/plate was milled to produce the desired thickness

The design of the 8 in C(T) fracture specimen was based on ASTM standards E399-83, E813-89, E1152-87, and E1290-93 [16,28,29,50]. The specimens were fabricated in the L-T orientation which means that the crack would grow in the transverse direction. The L-T orientation was chosen for comparison purpose with NASA work [8, 42 below, 43 below, 58]. Refer to Figure 4.3 for an illustration of the specimen orientation. Figure 4.4 shows a drawing of the 8 in C(T) specimen which was modified from the standards recommended design by changing the loading hole diameter from 2 in to 1 in. Changing the specimen loading hole diameter was mandated by changes made to the clevis grips loading hole diameter, which will be explained in more detail later. Analyses were performed using two-dimensional finite element analysis (FRANC2D/L [92]) to verify that changing the loading hole diameter did not have significant effect on the stress concentration in the vicinity of the crack tip. The difference between the maximum stresses in the specimen for both cases (1 in. and 2 in. loading hole diameters)

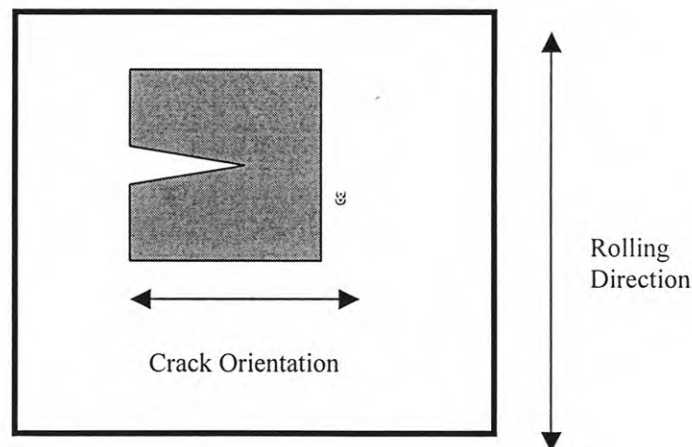


Figure 4.3: Illustration of the L-T Orientation

was less than 1%. Also, the difference in the vertical displacement at 0.04 in behind the crack tip for both cases was less than 1%. This distance corresponds to one finite element length behind the crack tip, and will be described in a later chapter as the reference point for all computational and experimental *CTOA* measurements. Therefore, it was concluded that changing the loading hole diameter from 2 in to 1 in did not have any significant effect on the fracture specimen. A 3 in long chevron crack starter (measured from the center of the loading hole to the crack tip) was machined into the specimen to facilitate the initiation of fatigue cracking.

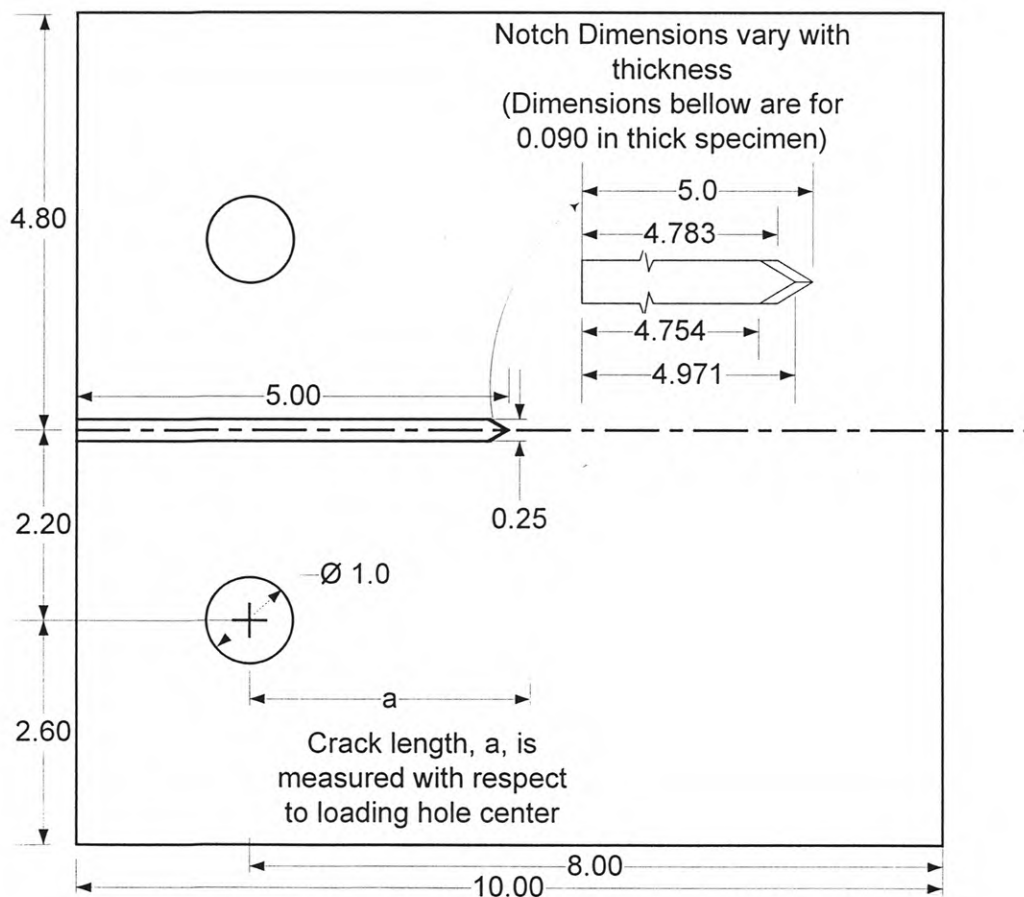


Figure 4.4: 8in C(T) Specimen.

4.4 Design of Grips and Anti Buckling Guide

To grip the specimen in the material testing machine, it was necessary to design and machine appropriate clevis-type grips suitable for the 8 in. C(T) specimen. Also, since the thinner specimens would exhibit in plane buckling during testing, it was necessary to design and fabricate anti-buckling guiding plates for the grip/specimen arrangement.

The clevis grips were designed using ASTM standards E399-83, E813-89, E1152-87, and E1290-93 as a guide [16,28,29,50]. Since the clevis grips provided in the ASTM standards E399-83, E813-89, E1152-87, and E1290-93 [16,28,29,50] were very massive, modification to the clevis grips were made. The clevis grips in the standards were designed to test specimen whose thickness equals to half of its width. Thus, for 8 in. C(T) specimen the clevis grips design provided in the standard is sufficient to support 4 in. thick specimen. Because the thickest specimen tested in the current work was 1 in., the dimensions of the clevis grips were changed such that the cross sectional area of the main grip section was reduced from 8x8 in. to 3x3 in. This, in turn, lead to changing the loading pin diameter from 2 in. to 1 in. to ensure that the clevis cross section had enough material to support the applied load during the tests. Changing the grip loading hole diameter mandated changing the C(T) specimen loading hole diameter, as mentioned earlier. Figure 4.5 provides detailed drawings of the clevis grips. Stress analysis using the finite element method was conducted to make sure that the clevis grips were capable of withstanding the maximum loads anticipated during the fracture tests.

During preliminary testing, it was noticed that thinner specimens would buckle. To remedy the buckling problem, spacers were used to constrain the specimen in the grip

section and an anti-buckling guide device was used to prevent buckling of the specimen away from the grips. The spacers consisted of 3.25 x 4.5 in. L-shaped brackets with 1 in clearance holes. Two pair of the L-spacers were used in the upper and lower clevis grips to keep the specimen centered with respect to the grips. The edges of the spacers were necessary to prevent rotation of the spacers. The brackets were made to constrain the 0.25 in. thick specimen. For the 0.090 in. thick specimen another pair of 0.080 in. thick plates (with 1 in. diameter clearance hole) were placed between the L-shaped spacers and the grips. The anti-buckle-guiding device consisted of two 8 in. wide, 11.75 in. long, and 0.375 in. thick Aluminum plates with a 1x3 in. window cut into the plates to allow for monitoring the crack. The plates had five 0.25 in. diameter holes drilled 0.375 in. away from the top and bottom edges. The buckle-guiding plates sandwich the specimen with a pair of 0.035 in. thick Teflon sheets separating the inner surfaces of the plates from both surfaces of the specimen. Two finger-tight rows of bolts were used to fasten the two plates together. Figure 4.6 illustrates the assembly of the anti-buckling constraints to the specimen.

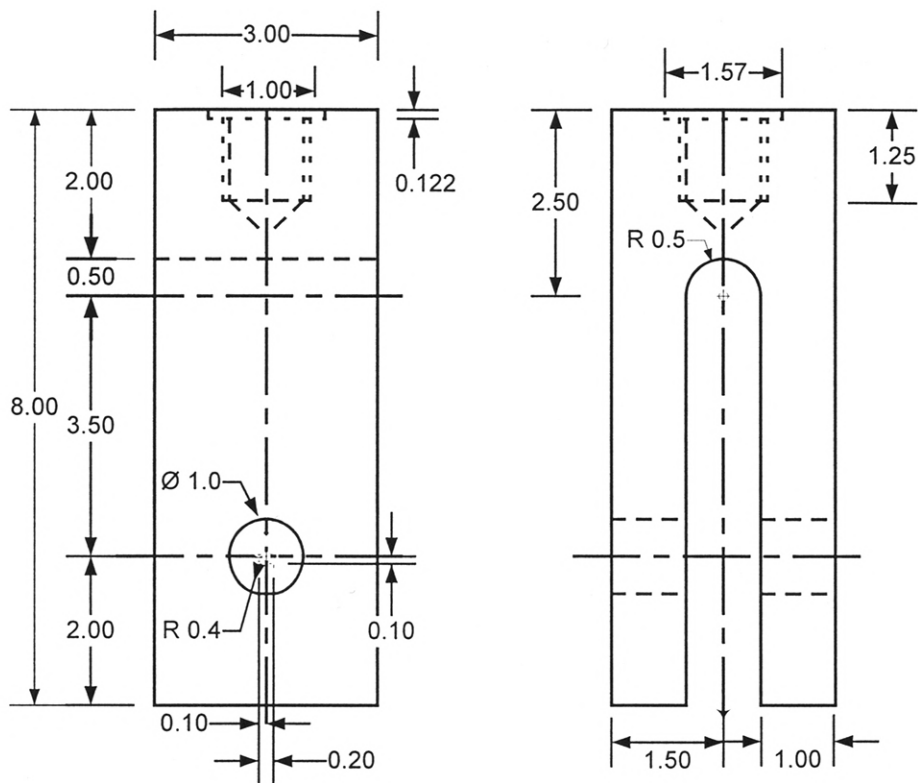


Figure 4.5: Clevis Grip Design

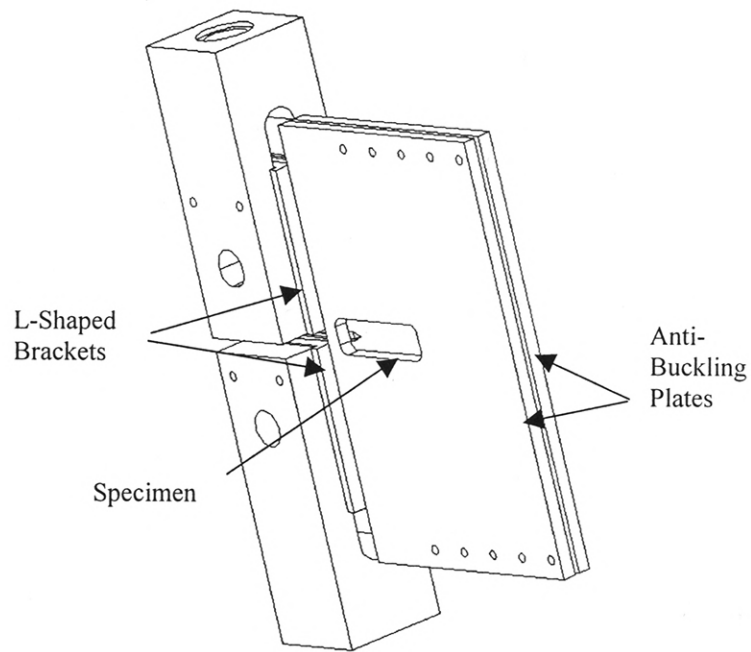


Figure 4.6: Anti-Buckling Constraints Attachment

Chapter 5

Experimental Procedures

The next step in this research project involved preparing for and performing the fracture tests using the 8 in. C(T) specimens fabricated from the 2024-T351 alloy (0.090, 0.25, 0.5, and 1.0 in. thicknesses). The focus of the experimental procedure was to measure *CTOA* experimentally during the entire stable tearing process, determine the fracture loads, and capture the load vs. load line displacement and load vs. crack extension behaviors of the specimens as they fracture.

5.1 Test Setup

As Figure 5.1 depicts, the test set up used for the fracture testing in this research project consisted of a material testing machine, clevis grips, an external LVDT, a microscope, a CCD camera, a VCR, computer hardware/software for controlling the machine, and computer hardware/software for facilitating the image acquisition.

Two servo hydraulic digitally controlled material testing machines were used in the investigation. The smaller of the two machines had a 22-kip load frame and was controlled with an MTS Test Star II digital controller. This machine was used to test all specimens except the 1 in. thick specimens, which were tested on a larger machine due to load frame limitations. The larger material testing machine had a 55-kip load frame, and was controlled with an Instron 8500 PLUS digital controller. The larger machine was used to test the 1 in. thick specimens. Both machines were equipped with load cells and LVDT units that provided load measurements and accurate sensing of actuator position.

The clevis grips, described in the previous chapter, were attached to the actuator and the load cell using fatigue rated studs.

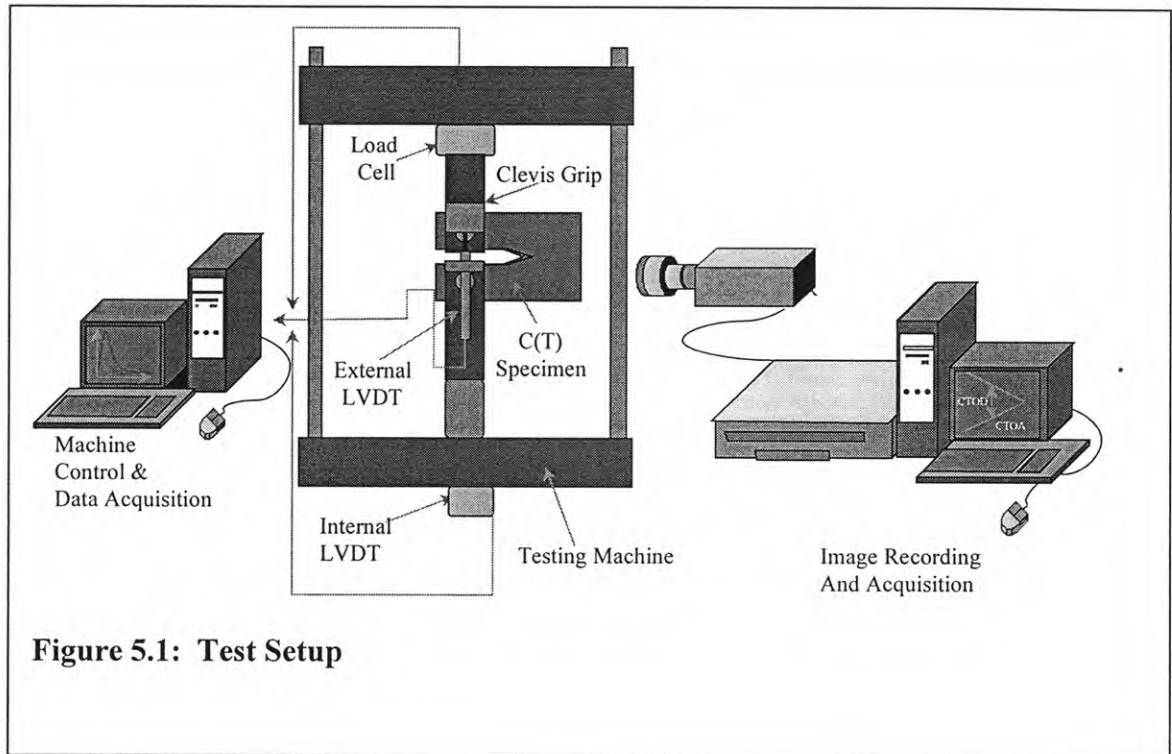


Figure 5.1: Test Setup

The microscope used for all *CTOA* measurements was an Olympus zoom-field stereoscopic microscope with 1X objective lens and a zoom capability ranging from 0.7-7. The CCD camera had a 1/3 in.chip with a 768 (H) x 494 (V) CCD array and a C mount lens attachment. The camera output could be sent to the VCR where it can be taped or it can be sent to the digital image acquisition board where images could be acquired and stored on the computer. The combination of the CCD camera and the digital image acquisition board produced images that were 480 (V) x 640 (H) pixels. As described in a later section, two different optical techniques were used to measure the *CTOA* during fracture testing. The first technique employed direct optical microscopy

measurement using the digital image analysis software tools and the second technique employed the digital image analysis technique. Both of these measurement methods used the microscope/CCD camera set-up with the digital image analysis workstation. The Image Pro Plus (IPP) Software was used on the image analysis workstation for image acquisition and analysis.

In an attempt to obtain more accurate load line displacement measurements during the fracture testing, an external LVDT was mounted directly to faces of the clevis grips, as shown in Figure 5.2. Load-line displacement measurements could also be obtained directly from the on-board LVDT located in the actuator housing of the testing frame. The external LVDT used in this research was spring loaded with a 2 in. travel. The external LVDT output was fed into an analog input channel on the testing machine

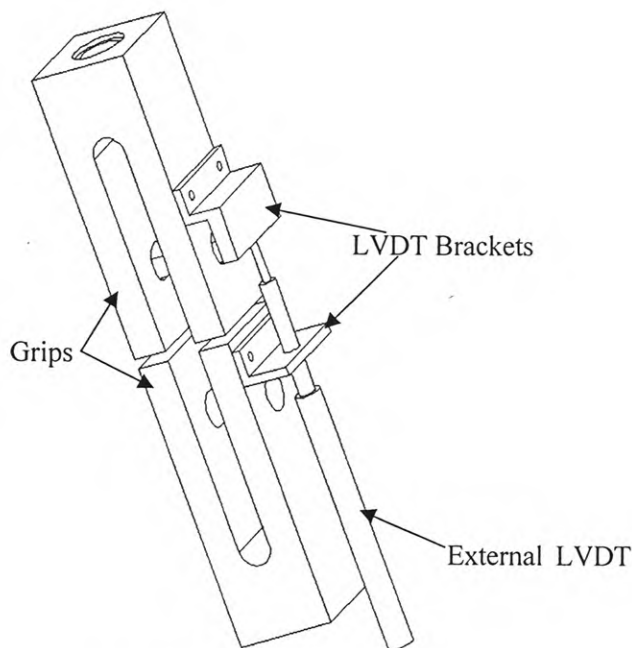


Figure 5.2: External LVDT Attachment

controller and its signal was logged during the tests. Due to controller limitations, the external LVDT was used only with the smaller testing machine (i.e., the external LVDT signal was not available for the 1 in. thick 2024-T351 tests).

5.2 Fracture Testing Procedure

The experimental *CTOA* characterization process employed in this research project included the fatigue pre-cracking and the fracture testing of the 8 in. C(T) specimens. As required in ASTM Standard E1152-87 and E1290-93 [29,50], all specimens were fatigue pre-cracked prior to the actual fracture testing to provide a sharp crack emanating from the chevron region. Based on these guidelines, the pre-crack was grown to a length corresponding to a crack length to specimen width ratio of 0.4. Since the slot and chevron notch machined into the specimen was 3 in. long, measured from the center of the loading pin, the fatigue pre-crack length had to be 0.2 in. long from the tip of the chevron. For the fatigue pre-cracking, the specimen was loaded into the clevis grips and the testing machine was programmed to apply a sinusoidal load signal to the specimen. The means and amplitudes of the pre-cracking load wave for each specimen thickness were selected in accordance with standards ASTM E1152-87 and E1290-93 [29,50]. To ensure the formation of a sharp fatigue crack, standards ASTM E1152-87 and E1290-93 [29,50] gave guidelines to keep the fatigue load levels such that the ratio of the stress intensity factor range to the material modulus of elasticity is less than $0.001\sqrt{\text{in}} \left(\frac{\Delta K}{E} \leq 0.001 \right)$ with the minimum to maximum load ratio (R) not to exceed 0.1. Also, Dawicke [7] advised that the maximum stress intensity factor during specimen pre-cracking should remain below $12 \text{ ksi}\sqrt{\text{in}}$ to keep the plastic zone at the tip of the pre-

crack relatively small. Taking both of these guidelines into consideration, the appropriate means, amplitudes, minimum, and maximum values of the pre-cracking load waves for the entire specimen thicknesses were calculated as shown in Table 5.1. During pre-cracking the microscope and the CCD camera were used to monitor the crack length. Images of the specimen surfaces were obtained periodically and the crack length was measured using the Image Pro Plus (IPP) digital image analysis software. When the pre-crack length reached the desired value (0.2 in, as measured from the chevron tip), the pre-cracking was terminated. Accurate measurement of the pre-crack length for each specimen was quite important since this value would need to be used during the FEM-based fracture modeling of each test.

Table 5.1: Pre-cracking Loads (lb)

Thickness (in)	Amplitude (lb.)	Mean (lb.)	Min (lb.)	Max (lb.)	Kmax @ a=0.2 in*
0.09	160	195.6	35.6	355.6	10.17
0.25	500	611.1	111.1	1111.1	11.44
0.5	1000	1222.2	222.2	2222.2	11.44
1.0	2000	2444.4	444.4	4444.4	11.44

* a = 0.2 inches was the final length of the pre-crack

The actual stable tearing fracture tests performed on the pre-cracked C(T) specimens consisted of an incremental displacement controlled ramp waveform (rate = 7.884×10^{-5} in/sec) that would pull the specimen apart until stable tearing was initiated and would then be paused at that position. The specified displacement rate was similar to the displacement rate used by Dawicke *et al.* during the NASA Langley CTOA tests [8,58,64] and it complies with ASTM standards E1152-87 and E1290-93 [29,50]. During the fracture tests, the load and load line displacement from both the external and internal

LVDT were recorded using the testing machine data acquisition system[†]. The data was logged every 10 seconds using the MTS Test Star II system on the smaller frame machine and using the Instron Max software on the larger load frame machine. Before starting the tests, the image resolution was calibrated to determine the image scale in terms of pixels per inch. A nominal 7500 pixel/in. image resolution was used with the optical microscopy technique but the actual resolution would change slightly between different tests. As the fracture test was initiated, the VCR was set to start recording. The image of the crack was displayed on the computer screen and an auxiliary monitor. When a noticeable growth in the crack was observed (approximately 0.02 in.) the testing machine was put on hold. Then a record was obtained of the load, the VCR counter, the internal LVDT reading, and the external LVDT reading when it was used. After that, the ramp displacement wave was resumed until another crack growth increment was observed. This process was repeated until complete specimen fracture occurred. Using the supporting tripod adjustments, the microscope was moved periodically in the horizontal and vertical directions to track the crack tip and keep it in the field of view. Measuring the *CTOA* and the crack length was achieved later by reviewing the VCR tape.

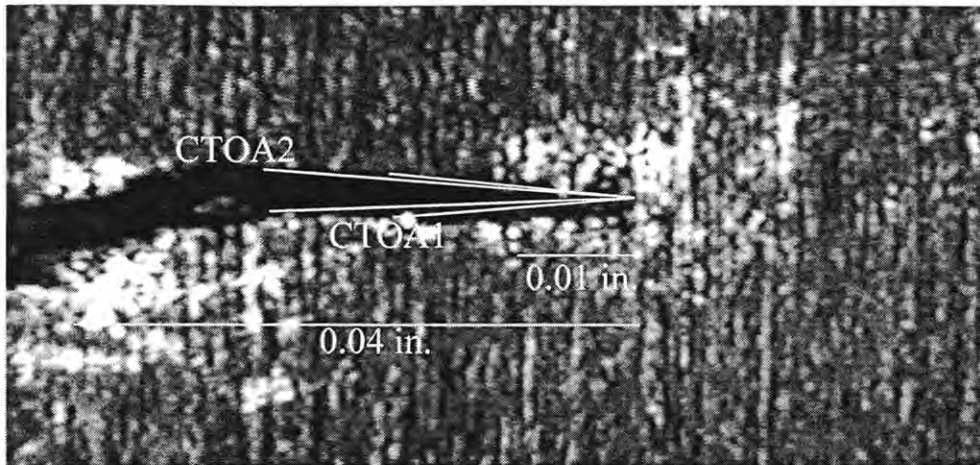
5.3 CTOA Measurement Procedure

The *CTOA* was measured using direct optical microscopy (OM) and digital image correlation (DIC) techniques. The next two subsections describe in detail both of the OM and DIC techniques.

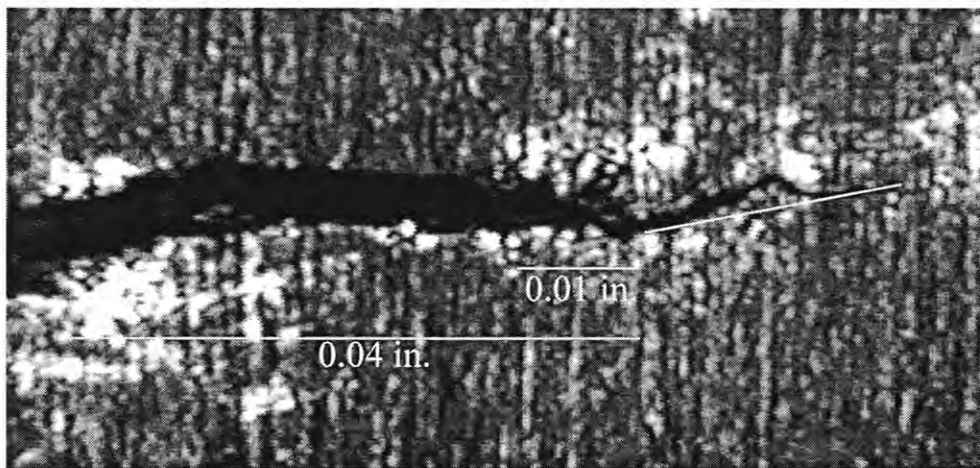
[†] External LVDT was not used on the 2024-T351 1 in. thick specimens

[73,25] used the more general incremental theory of plasticity that is valid during the material loading and unloading as well. The most commonly used yield criterion in elastic-plastic material modeling is the Von Mises yield criterion, it was also employed in the software used in this project. Modeling the deformation process would involve an assumption about the magnitude of the inflected strains in the material. When the strains are considered small then the strains inside an element within the finite elements mesh are related linearly to the nodal displacements [100]. When large strains are present inside the element then the strains are not related linearly to the nodal displacements [100]. The deformation field in the vicinity of the crack tip usually contains large strains that may locally invalidate the small strain assumption, however, several researchers such as Newman [73] showed that the small strain assumption produced good results for stable crack growth in 2024-T3 Aluminum specimens. Furthermore, Newman *et al.* [42] proved that FEM analysis based on small strain assumption provides good simulation of the large deformation field in a notched specimen. Finite element analysis with large strain have also been developed to investigate fracture mechanics such as the work of McMeeking and Hom [25], however, such analyses were mainly conducted to model crack blunting and initiation and have not been fully extended to model the stable crack growth.

Experimental investigation of the near tip crack stress and strain fields [74,75,76] revealed the existence of a three dimensional zone near the tip of the crack. Therefore to obtain an accurate simulation of the fracture behavior will require a three-dimensional finite element analysis. Although three-dimensional finite element analysis of the stable tearing is possible, such as the work of Dawicke *et al.* [64], it has proven to be more time consuming and thus more expensive than the two-dimensional analyses [77]. Shan and



Before Crack Extension



After Crack Extension

Figure 5.3: Illustration of the Optical Microscopy Measurements

5.3.2 Digital Image Correlation

The DIC technique is an experimental tool that provides the ability to measure full field displacement through the manipulation of digital images by the computer. The principle of the digital image correlation is based on locating a small region of a random speckle pattern on the surface of the specimen after it deforms. Although the DIC technique was not necessary for measuring the *CTOA*, it served for checking the *CTOA* values measured with the OM method. Sutton *et al.* [65] developed the DIC technique and applied it successfully to fracture mechanics research where he was able to measure various fracture parameters such as *K*, *J*, and *CTOA* [67,2,4]. A DIC setup was put together with help from Sutton who provided the author with training and suggestions on applying the DIC technique [66]. The DIC technique requires speckle coating of the specimen surface, image acquisition, and image correlation (locating small regions after deformation). Post processing of the DIC displacement field is often necessary to extract other significant parameters.

Various methods exist for applying the speckle pattern to the surface of the specimen. For the current research, Sutton [66] found the most appropriate method is to paint the specimen surface with white acrylic paint, dust the painted surface with Xerox toner powder, and then place the specimen in an oven until the Xerox toner cures (approximately 20 minutes at 200 °F). The same image acquisition setup used to acquire images for the OM method was also used to obtain images for the DIC method. The system available for the current work has an image area of 640(H) x 480(V) pixel and 8 bit intensity resolution with zero representing black, 255 representing white, and the numbers in between are linearly related to the gray shades. A digital image of the

speckled surface is acquired before and after the specimen deformation. The images used for the Digital Image Correlation technique had 4550 pixel/in. resolution. Comparing the two images by the computer allows the determination of the local displacements in the speckle pattern, which corresponds to the deformation of the specimen surface. The deformed and undeformed image comparison was accomplished by the Sub-Pixel Digital Image Correlation (SPDIC) analysis software [68] that was acquired through NASA COSMIC.

The SPDIC software divides the undeformed image into smaller subregions, referred to as subsets or submatrices, the center location in each subregion indicates a point on the specimen surface representing this subregion. A typical subset size varies between 30x30 pixels to 60x60 pixels. The task of the SPDIC software is to find where those selected subsets, from the undeformed image, are located in the deformed image. Figure 5.4 shows how the SPIDC software defines a subset in the deformed image and locates the corresponding pattern in the undeformed image. The degree of similarity between two subsets from the undeformed and deformed images may be judged based on computing a correlation coefficient. A number of correlation coefficients may be defined such as the cross correlation coefficient and the least square correlation coefficient. For two submatrices of size nxn, Sutton and coworkers [69,70] define the cross correlation coefficient (C_f) by:

$$C_f = 1 - \frac{\sum F(x,y) * G(x^*, y^*)}{\sqrt{\sum (F(x,y))^2 * (G(x^*, y^*))^2}} \quad (5.1)$$

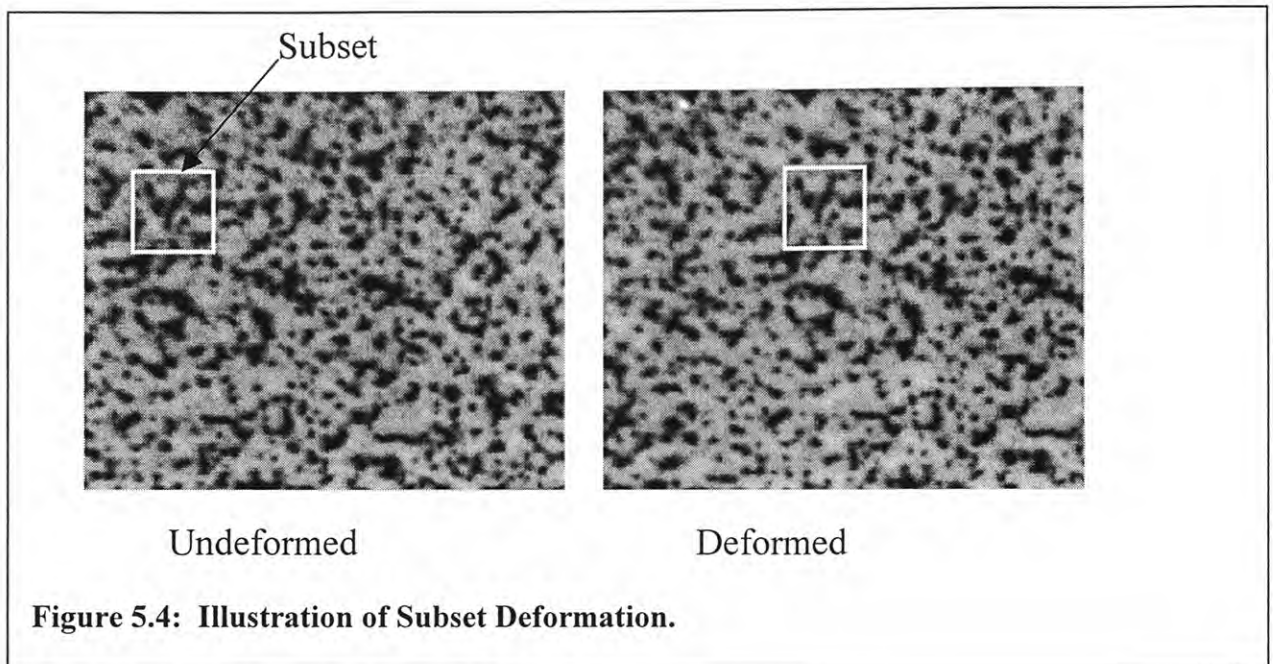
where $F(x,y)$ is the gray level value at the coordinate (x,y) for the undeformed image and $G(x^*,y^*)$ is the gray level value at the coordinate (x^*,y^*) for the deformed image. The coordinates (x,y) and (x^*,y^*) are related by the deformation between the two images. For the same submatrices and quantities, Peters *et al.* [71] define the least square coefficient (C) as:

$$C = \sum (G(x^*,y^*) - F(x,y))^2 \quad (5.2)$$

The least square correlation coefficient is absolute while the cross correlation coefficient is relative with respect to the intensity in the original (undeformed) subset. Thus, the cross correlation coefficient has an advantage over the least square correlation coefficient since it provides a relative indication of measurement goodness. (Cross correlation coefficient very close to zero indicates a good measurement while cross correlation coefficient on the order of 0.2 indicates a poor measurement)

The SPDIC software selects a subset from the undeformed image then computes the correlation coefficient between that subset and a number of candidate subsets from the deformed images. The deformed image subset producing the minimum correlation coefficient corresponds to the undeformed image subset. Thus, the DIC technique is based on searching for possible subsets until the matching subset is found. Once the location of the corresponding subset in the deformed image is determined, the displacement is found by subtracting the coordinates of the subset in the undeformed image from the coordinates of the

subset in the deformed image. Strain fields and other important quantities can be calculated from the displacement field.



For verification purpose, the DIC setup was applied in rigid body translation tests that involved positioning a speckled specimen on a translation stage and acquiring images of the specimen before and after translation. The specimen was translated in a plane normal to the optical axis of the camera in both the horizontal and vertical directions. Figure 5.5 presents the results of a typical rigid body translation test where the vertical axis shows the results of the DIC technique and the horizontal axis shows the values of the translation table movement. Based on these results, the DIC setup that was used in the current research project produced accuracy within 0.0004 in. when used to predict rigid body translation.

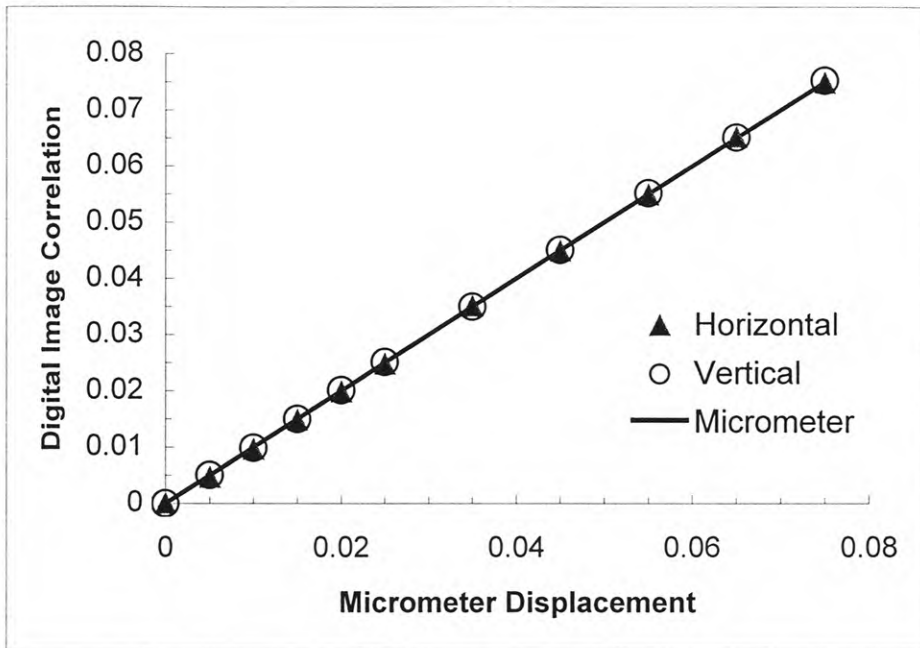


Figure 5.5: Measurement of Rigid Body Displacement Using Digital Image Correlation

As pointed out earlier, through surface displacement measurement, the DIC technique has the potential of measuring useful fracture mechanics parameters such as the stress intensity factor (K_I), the *J-integral*, and the *CTOA*. For measuring the stress intensity factor (K), McNeill [67] proposed that the DIC measured displacement has rigid body component (constant) and K -dominant displacement component. By performing a least square fit to minimize the error between the experimental and theoretical displacement, McNeill [67] was able to determine the stress intensity factor (K) and the rigid body displacement. The DIC setup used in the current research was tried for measuring K_I for one of the 0.090 in. 2024-T351 8 in. C(T) specimen tests. Two load levels (250 and 350 lb.) at three different crack lengths (3.06, 3.1, and 3.2 in) were

used. Table 5.2 shows the resulted K_I values and compares it with the K values calculated using the ASTM closed form solution for the C(T) specimen geometry. In general, the results of the DIC measurements of K_I were in good agreement with the calculated K_I based on load and crack length. The highest difference between the measured and calculated values of K was within 20% and that was less than the highest error reported by McNeill [67]. There were difficulties associated with measuring K using the DIC technique. Some of those difficulties are associated with determining the exact location of the crack tip. Also, it is critical to look at the measured displacement data points and eliminate possible bad data points. Looking at the cross correlation coefficient and eliminate data points with high cross correlation coefficients is one method. Also, it is necessary to select a measurement region located outside the plastic zone associated with K . Thus, it is necessary to avoid overloading of the specimen (stay in linear elastic range). If overloading occurs, it is necessary to keep track of the plastic zone size based on the maximum load and crack length.

Table 5.2: Listing of the K measurements using the DIC technique

Crack Length (in)	Load (lb.)	DIC Measured K (Ksi $\sqrt{\text{in}}$)	Closed Form Solution [16] for K (Ksi $\sqrt{\text{in}}$)	Difference (%)
3.06	250	5.43	6.83	-20.5%
3.1	250	6.81	6.92	-1.6%
3.2	250	6.37	7.15	-10.9%
3.06	350	9.11	9.56	-4.7%
3.1	350	9.38	9.68	-3.1%
3.2	350	9.52	10.01	-4.9%

The results of the conducted rigid body translation test and the K measurements proved that the DIC technique can be applied in the current research project to measure

the *CTOA* successfully. Figure 5.6 illustrates how the DIC technique is used for measuring the *CTOA* value. Measuring the *CTOA* value using the DIC technique uses an approximation stating that the deformation encountered by a small region close to the crack line is the same as the deformation encountered by the crack boundary itself. Thus in Figure 5.6 the deformation of the dotted squares and the dotted circle halves is approximated to be the same. As Figure 5.6 shows, after deformation, the dotted squares move to the solid black squares and the dotted circle halves move to the solid black half circles. Using the digital image correlation technique, the displacement of the upper and lower dotted squares can be determined. The approximation mentioned above dictates that ($\delta_u = d_u$ and $\delta_l = d_l$). The normal and tangential directions to the crack line also need to be estimated from the images. Once the upper and lower displacements, and the normal and tangential directions to the crack line are determined, the *CTOA* can be determined from the geometrical consideration of Figure 5.6 by the following equation:

$$CTOA = \arctan\left(\frac{(d_u - d_l).n}{2r}\right) \quad (5.3)$$

For measuring the critical *CTOA* with the DIC technique, Dawicke *et al.* [8] provide the following procedures:

- Choose a particular crack tip location and deformed image, D, for analysis, and choose a "reference" image, R.
- Estimate the crack tip location in the deformed image, D.

- In the region within 0.25-1.25 mm behind the crack tip in image, D, select several pairs of subsets (each pair has a subset above the crack line and one subset below the crack line).
- Compute the crack opening displacements for upper and lower subset.
- Subtract the displacement vector for each pair of subsets.
- Estimate the normal vector to the crack line, n .
- Compute the $CTOA$ for each pair of subset using Equation (5-5).

The measurement of $CTOA$ is the average of all the measured $CTOA$ values for each pair of subsets. Using the DIC setup in the Mechanical Testing and Evaluation Laboratory (MTEL), these procedures were followed in this research project to evaluate the $CTOA$ for one specimen (0.090 2024-T351), as a check, for comparison to the OM measured critical $CTOA$ values.

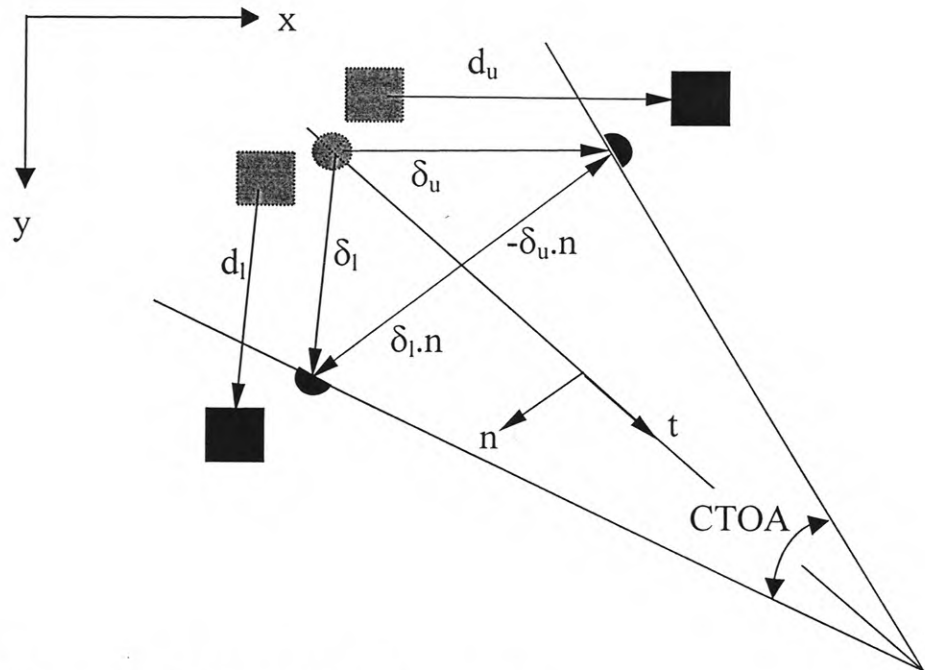


Figure 5.6: Illustration of DIC CTOA measurements

Chapter 6

Computational Procedures

As outlined earlier, the *CTOA* fracture characterization procedure followed in this research project included both experimental fracture testing and numerical simulation of the stable tearing process. The numerical simulation of the stable tearing process was conducted after completing the fracture tests and collecting the maximum applied loads, as described in the previous chapter. It was necessary to perform the numerical analysis of the stable tearing process in order to complete the *CTOA* fracture characterization process. Experimentally determined critical *CTOA* values were measured on the surface of the specimen and may not infer the *CTOA* value at the interior of the specimen. Thus, the analytical aspect of the *CTOA* characterization procedures provides a means for investigating the *CTOA* variation across the thickness of the specimen. In addition, the analytical aspect of the characterization procedure allows for determining the critical *CTOA* value that makes the results of the analyses, which are based on the finite element analysis method, match the experimentally measured average maximum applied load for each thickness group. This method of arriving at the *CTOA* through analysis is referred to as the generation phase [40].

The generation phase provides a mean of arriving at the *CTOA* by using the average of the measured maximum applied load determined from the fracture testing for each thickness group. As Figure 6.1 shows, the generation phase involves iterative use of the finite element method with critical *CTOA* fracture criteria to predict the maximum

applied load. An initial guess of the critical *CTOA* was selected based on the measured critical *CTOA* on the specimen surface. Then, the finite element simulation was conducted and the maximum load predicted by the finite element analysis was compared to the measured maximum applied load. This procedure was repeated while systematically changing the critical *CTOA* until a satisfactory match between the measured and computed maximum applied loads was achieved. Systematic alteration of the *CTOA* value involved updating the *CTOA* value (new input to the finite element model) by multiplying the old value (previous guess) of the *CTOA* with the ratio of the experimentally determined load to the predicted load based on the old *CTOA* input (previous guess).

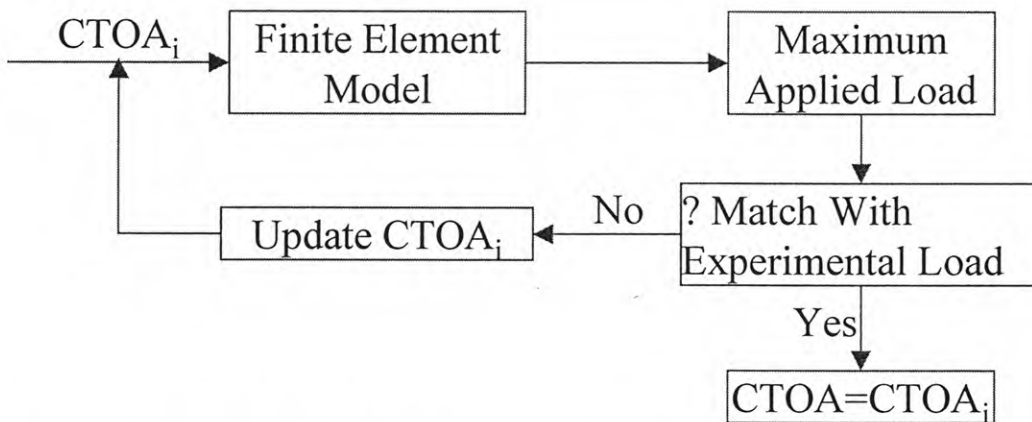


Figure 6.1: Schematic of the Computational Characterization Procedure

The numerical procedure involved meshing of the 8 in C(T) specimen geometry and conducting two and three-dimensional finite element based fracture mechanics with *CTOA* fracture criterion. The remainder of this chapter provides an overview of finite element based fracture mechanics, the particular meshing of the specimen geometry used

in this project, the three-dimensional finite element simulation used to predict critical *CTOA*, and the two-dimensional finite element simulation used to predict plane strain core height.

6.1 Finite Element Based Fracture Analysis

Modeling the fracture behavior of an elastic-plastic material using the finite element analysis involves geometry meshing, elastic-plastic deformation analysis, fracture criterion, and crack propagation. The finite element mesh must meet the constraint of being sufficiently dense in the high stress concentration areas (cracks) to provide accurate solution. At the same time, the mesh should have larger elements in areas experiencing lower stress gradient such that minimal computation time and computer memory are required but good approximations are maintained. The elastic-plastic deformation analysis requires (1) a material model, (2) a plasticity theory, (3) a yield criteria, and (4) a strain assumption (small/large). Researchers have commonly used either a Ramberg-Osgood or multilinear representation of the stress-strain curve to incorporate the constitutive stress strain behavior of the material into the model [62, 83]. Due to the software that was employed, the current research made use of the multilinear representation of the stress strain curve. Multilinear representation of the stress-strain curve utilizes a set number of points to represent the experimentally determined stress-strain curve. Between each two points, the curve is then represented with straight lines (i.e., multilinear). Figure 6.2 shows how closely the multilinear representation models the experimentally determined stress-strain curve for the 2024-T351 Aluminum. Table 6.1 lists the discrete stress-strain points that were used in Figure 6.2 to produce the

multilinear simulation of the true stress-strain material behavior. The data in Table 6.1 was used in both of the two and three-dimensional finite element analyses.

Table 6.1: Multilinear Representation Data Points

Point number	Strain (%)	Stress (Ksi)
1	0	0
2	0.004907	52.7
3	0.0275	58.5
4	0.06	63.6
5	0.1	67.3
6	0.13	69.0
7	0.161	69.0

The plasticity theory is a very important aspect of the elastic-plastic deformation analysis since it simulates the physics involved in the material deformation process. Although some researchers such as Goan *et al.* [72] based their analysis on the deformation theory of plasticity that is valid only for monotonic loading, several others

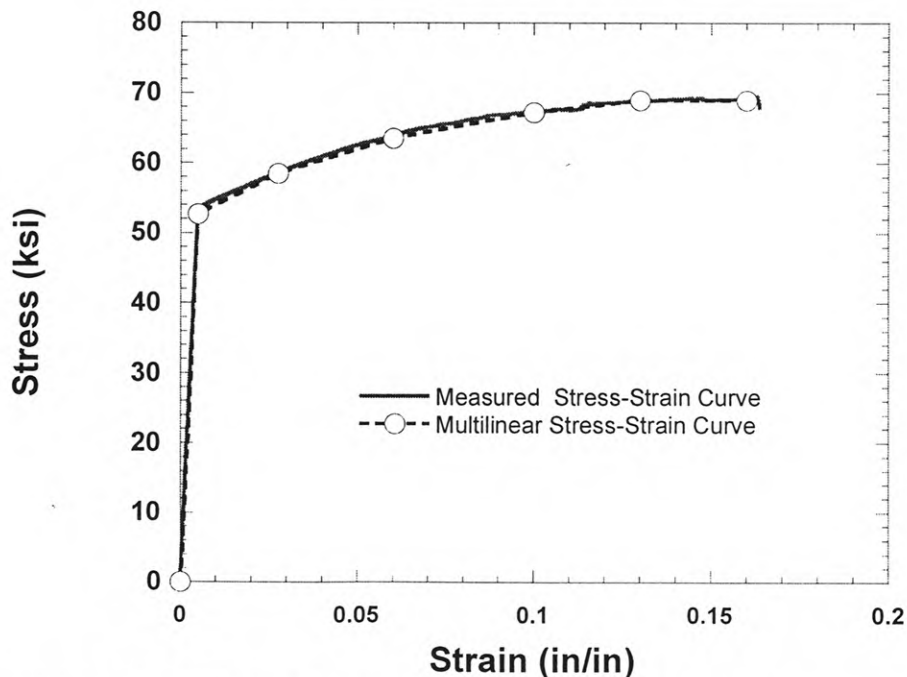


Figure 6.2: Multilinear Representation of Stress Strain Curve.

[73,25] used the more general incremental theory of plasticity that is valid during the material loading and unloading as well. The most commonly used yield criterion in elastic-plastic material modeling is the Von Mises yield criterion, it was also employed in the software used in this project. Modeling the deformation process would involve an assumption about the magnitude of the inflected strains in the material. When the strains are considered small then the strains inside an element within the finite elements mesh are related linearly to the nodal displacements [100]. When large strains are present inside the element then the strains are not related linearly to the nodal displacements [100]. The deformation field in the vicinity of the crack tip usually contains large strains that may locally invalidate the small strain assumption, however, several researchers such as Newman [73] showed that the small strain assumption produced good results for stable crack growth in 2024-T3 Aluminum specimens. Furthermore, Newman *et al.* [42] proved that FEM analysis based on small strain assumption provides good simulation of the large deformation field in a notched specimen. Finite element analysis with large strain have also been developed to investigate fracture mechanics such as the work of McMeeking and Hom [25], however, such analyses were mainly conducted to model crack blunting and initiation and have not been fully extended to model the stable crack growth.

Experimental investigation of the near tip crack stress and strain fields [74,75,76] revealed the existence of a three dimensional zone near the tip of the crack. Therefore to obtain an accurate simulation of the fracture behavior will require a three-dimensional finite element analysis. Although three-dimensional finite element analysis of the stable tearing is possible, such as the work of Dawicke *et al.* [64], it has proven to be more time consuming and thus more expensive than the two-dimensional analyses [77]. Shan and

coworkers proposed a method based on two-dimensional finite element analysis as a substitute for the full three-dimensional finite element analysis to predict failure loads in fracture specimen [77,78,79]. They suggested dividing the specimen into two surface layers where a plane stress condition is assumed and a central layer where a plane strain condition is assumed. However, the thickness of each layer must be determined. Usually, three-dimensional analysis is needed to determine the thickness of each layer. Newman *et al.* [73] proposed an alternative two-dimensional substitution for the three-dimensional finite elements in predicting failure load in thin specimen by proposing “a mixed state of stress”. They assumed a plane stress condition to exist throughout the specimen except for a small region around the crack tip, referred to as plane strain core, where plane strain state of stress is assumed to exist as illustrated in Figure 6.3. Usually, three-dimensional analysis is needed to determine the height of the plane strain core. Newman and coworkers [73, 83] applied two-dimensional finite element analysis with a plane-strain core to predict failure load within 10% in fracture specimens. Dawicke, *et al.* [58] analyzed the fracture behavior of thin sheets using a plane stress finite element model, and showed a 15% error in estimating the measured maximum applied load and as much as 40% in the measured displacement data. Incorporating the plane strain core to their analysis improved their ability to predict the failure loads to within 2% for the M(T) specimens and within 4% for the C(T) specimens. They also reported that the computed displacement agreed well with the measured values.

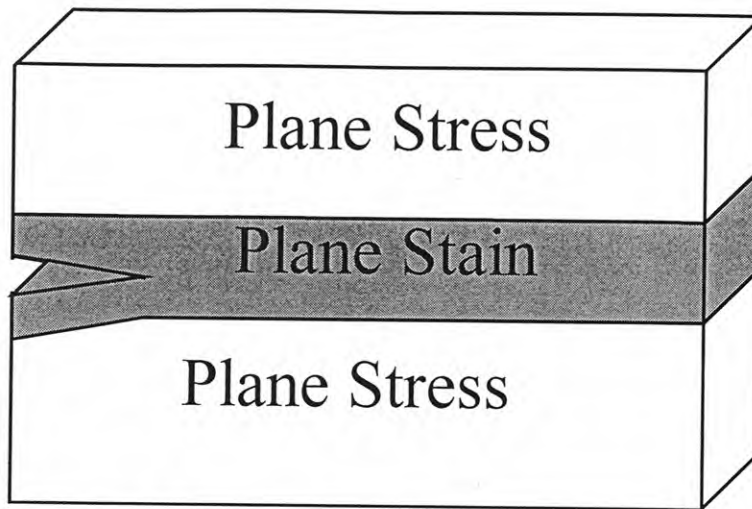


Figure 6.3: Schematics of the Plane Strain Core

Since the tearing process is three dimensional, investigating the thickness effect on the critical *CTOA* requires three-dimensional finite element analysis to accurately model the fracture behavior of the specimen as the thickness increases. Thus, the analytical aspect of the test procedures in the current research project involved three-dimensional finite element simulation of the fracture specimens using the ZIP3D [83] code (obtained from NASA Langley) in order to simulate the tearing process observed during the fracture tests. The three-dimensional finite element analysis produced the appropriate critical *CTOA* value and this completed the fracture characterization process. Since the two-dimensional finite element analysis with plane strain core showed a good promise of predicting the fracture loads accurately, the second step of the analytical characterization procedures was to estimate the size of the plane strain core for each specimen thickness. Estimating the plane strain core height was achieved using the FRANC2D/L program that was modified by James [80] to incorporate critical *CTOA* fracture criteria. Details of the plane strain core height estimation involved using the

critical $CTOA$ value produced by the ZIP3D [83] analysis along with an iterative procedure to determine which value of the plane strain core height produced the most accurate estimation of the fracture load. The rest of this chapter describes the meshing of the 8 in C(T) specimen (used in both 2D and 3D analyses), the ZIP3D three-dimensional finite element program, and the FRANC2D/L two-dimensional finite element program.

6.2 Meshing of the 8 in C(T) Specimen

This section describes the details of the two and three-dimensional finite element mesh of the 8 in C(T) specimen, described in detail in Chapter 4. The two-dimensional finite element mesh was developed using the CASCA geometry meshing software [81]. Only one half of the C(T) specimen was meshed, as illustrated by Figure 6.4, due to symmetry about the crack plane. All elements were quadrilateral with the size of all the

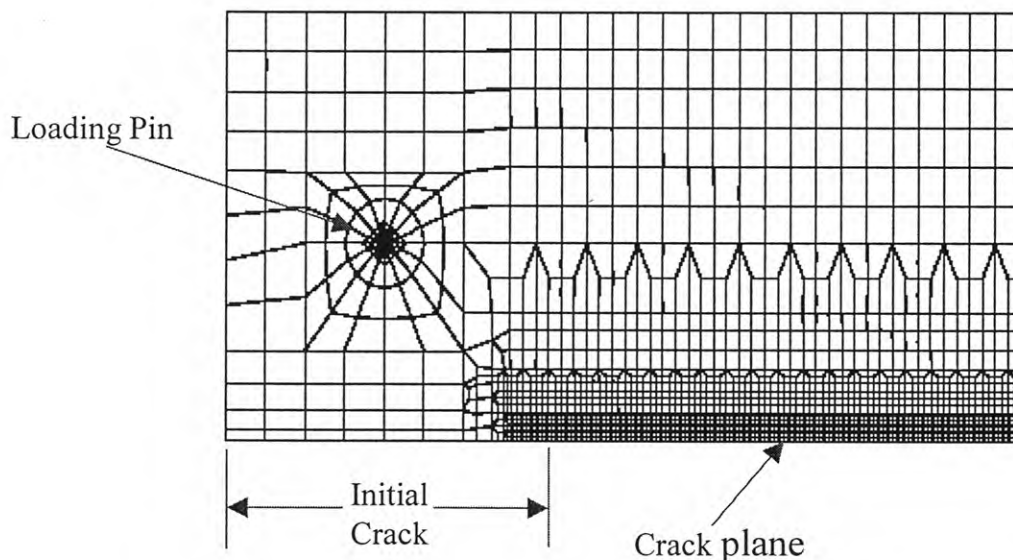


Figure 6.4: Two-dimensional Mesh of the C(T) Specimen.

crack face elements set at 0.04 in. The 0.04 in. element size along the crack plane was based on the convergence analysis conducted by Newman [42]. Newman [42] found that increasing the smallest element size over 0.04 in will give inaccurate solution and decreasing the smallest element size under 0.04 will not increase the accuracy of the results. Also, the chevron notch was not modeled because the specimens were pre-cracked at low load levels to produce sharp fatigue cracks. Thus, leaving the chevron notch out of the mesh would not affect the analysis. A total of 6711 nodes and 2158 elements were used for generating the two-dimensional mesh. To simulate the loading condition, the elements inside the loading hole were assigned material properties with high stiffness (similar to the loading pin steel material). A unit load in the vertical (y) direction was imposed at the center of the loading pin while the horizontal (x) displacement at the same node was set to zero. The material properties for the specimen geometry were assigned using the multilinear stress-strain behavior described earlier.

To create the three-dimensional mesh, the two-dimensional mesh was extruded in the thickness direction. As Figure 6.5 illustrates, only one fourth of the 8 in. C(T) specimen was modeled due to symmetry about the crack plane and the mid thickness of the specimen. Before extruding the two-dimensional mesh into the three-dimensional mesh, the mesh was optimized. The mesh optimization was accomplished using the **mopt2d.f** Fortran code available from NASA Langley research center. The mesh was extruded using the **mcvt8to3d.f** Fortran code also available from NASA Langley research center. The resulting elements were eight-noded isoperimetric. Five layers with a total of 13662 nodes and 10790 elements were used to generate the three-dimensional mesh, illustrated in Figure 6.5. As in the case for the two-dimensional mesh, the elements

inside the loading pin were assigned high stiffness steel properties and a unit vertical (y) displacement was applied at each of the 6 nodes coinciding with the center of the loading pin. Also, zero horizontal (x) displacements were imposed at these nodes. The plane coinciding with the center of the specimen was used as symmetry plane. Similar to the two-dimensional case, multilinear stress-strain representation was used to simulate the material properties of the specimen.

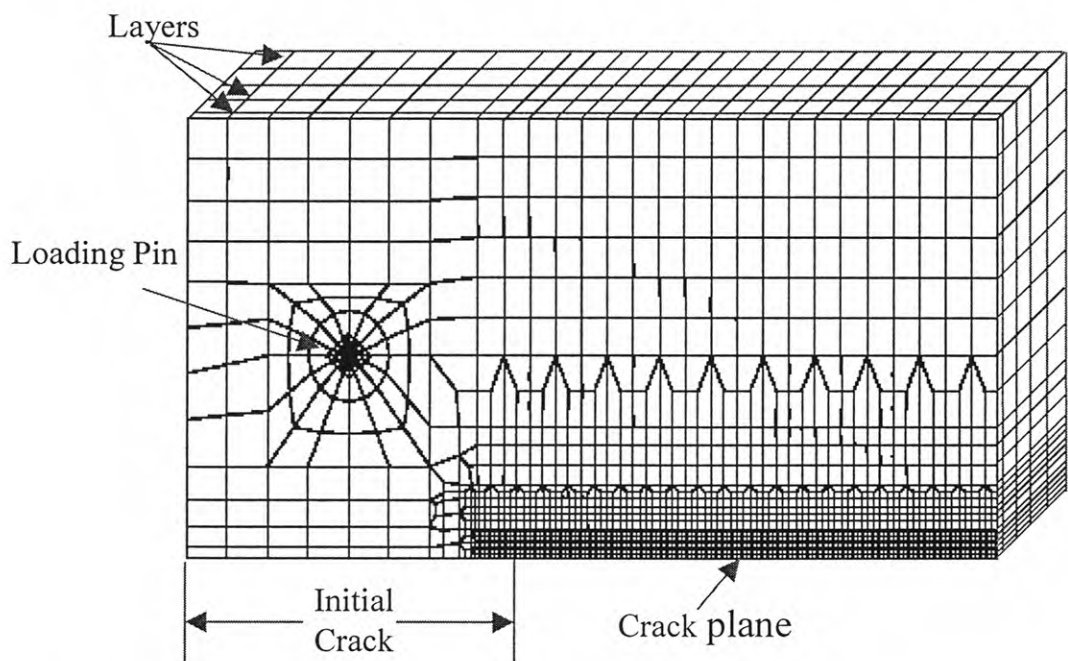


Figure 6.5: Three-dimensional mesh of the C(T) Specimen

6.3 Three-Dimensional Elastic-Plastic Fracture Mechanics with *CTOA* Fracture Criteria

The three-dimensional elastic-plastic finite element-based fracture analysis was achieved using the ZIP3D program developed by Shivakumar and Newman [83]. ZIP3D is capable of running stress and deformation analysis on elastic and elastic-plastic bodies. The ZIP3D code uses eight-noded isoparametric elements and it utilizes small-strain assumption. The elastic-plastic analyses in the ZIP3D code are based on the incremental theory of plasticity, Von-Mises yield criterion, isotropic hardening, and Drucker's flow rule. The constitutive stress-strain relation supported in ZIP3D includes Ramberg-Osgood, multilinear representation and elastic-perfectly plastic. However, the constitutive stress-strain relation used in this work was the multilinear representation specified earlier in Table 6.1.

To implement the critical *CTOA* fracture criterion, ZIP3D calculates the *CTOD/CTOA* value at the first free node behind the current crack tip (0.04 in. behind the current crack tip) that is located on the center of the specimen[†], as illustrated in Figure 6.1. When the *CTOA* value at this node reaches the specified critical value, ZIP3D releases the next set of tied nodes as illustrated in top portion of Figure 6.6. It should be noted that the input *CTOA* value to ZIP3D specifies the critical *CTOA* at the center of the specimen, thus, it would have a different value from the *CTOA* measured experimentally on the specimen surface. ZIP3D also calculates the *CTOA* value for all the crack front nodes across the specimen thickness including the *CTOA* value at the surface of the

[†] Center of the specimen is at $z=0$ where z is measured along specimen thickness.

specimen. Thus, comparison of measured and calculated surface *CTOA* values would be possible.

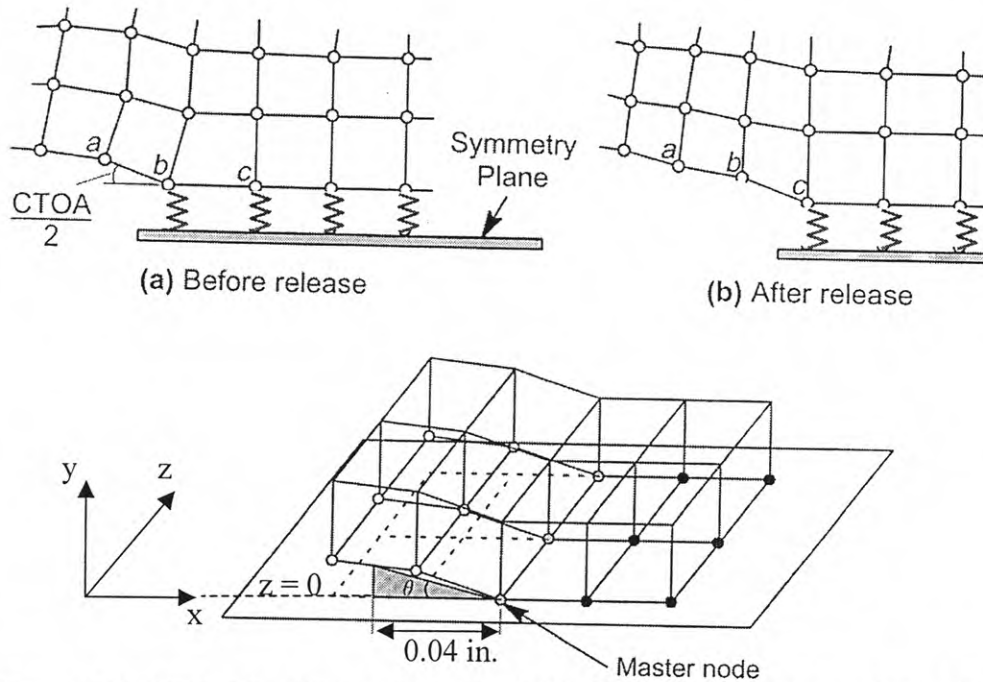


Figure 6.6: CTOA Implementation in Finite Element Analyses. (from [101]).

To use the ZIP3D code an ASCII input file must be prepared to specify the boundary conditions and the input parameters such as the *CTOD* and the loading parameter. The analyses were run in displacement control (i.e. one unit displacement was applied to the nodes corresponding to the middle of the loading pin). Also, the input file instructs the program what type of analysis to run and what type of output to print out. Runs were made for each specimen thickness, iterating on the *CTOA* while matching the average of the measured maximum applied load for each thickness group. The results of these runs will be discussed in detail in Chapter 7.

6.4 Two-Dimensional Elastic-Plastic Fracture Mechanics with CTOA Fracture Criteria and Plane Strain Core

As mentioned previously, the two-dimensional finite element analyses were conducted to estimate the plane strain core height that was defined earlier in Figure 6.3. After the critical *CTOA* values at the center of the specimen were determined for each thickness using the three-dimensional finite element analyses, the resulted critical *CTOA* values were used with the two-dimensional finite element analyses to determine the appropriate plane strain core heights for each thickness.

The two-dimensional analyses were conducted using the FRANC2D/L program that was modified by James [80] to handle elastic-plastic materials and layered structures. Also, James [80] incorporated the critical *CTOA* fracture criterion to FRANC2D/L. Although the FRANC2D/L program can handle many element types, the two-dimensional mesh used to generate the three-dimensional mesh (Figure 6.4Figure 6.5) was used in the two-dimensional analysis. Thus, four sided bilinear elements were used in the two-dimensional analyses. FRANC2D/L uses the Von-Mises yield criterion and the incremental theory of plasticity to model the behavior of elastic-plastic materials. To specify the plane strain core height in FRANC2D/L, all elements are assigned plane stress analysis type initially and then the elements near the crack plane are switched to plane strain analysis type. Determining the plane strain core height is an iterative process where an initial plane strain core height is used in the analysis. The maximum applied loads predicted by the two-dimensional finite element analyses were compared to the maximum applied load obtained from the experimental measurements. The strain core height was changed systematically until a good agreement between the two-dimensional

and experimental average maximum applied load for each thickness group was reached.

The results of the two-dimensional finite elements analysis will be discussed in Chapter

7.

Chapter 7

Results and Discussion

This chapter presents the results and associated discussions pertaining to both the experimental and computational phases of this research. The next chapter provides conclusions and recommendations supported by these results.

7.1 Results of the Experimental Investigation

As mentioned earlier, the experimental portion of the current investigation consisted of running fracture tests on 2024-T351 8 in C(T) specimens of four different thicknesses (0.090, 0.25, 0.5, 1.0 in). Data collected from these tests include applied loads, load line displacement, and crack extension. In addition, the *CTOA* at the specimen surface was measured during the entire stable tearing process using the optical microscope. The remainder of this section discusses the experimental findings of this research and ties these results to the goals of the research that involved determination of thickness effect on the critical *CTOA* fracture criteria.

7.1.1 Effect of Specimen Thickness on Maximum Applied Load

As mentioned earlier, the load vs. load line displacement behavior was captured for all fracture tests performed in this research. Figure 7.1 illustrates schematics of the test setup and the collected load vs. load line displacement curve. As described in Chapter 5, the load cell was used to measure the applied loads and the internal LVDT sensors of both the 22- and 55-kip machines were used to measure the load line

displacement. In addition, an external LVDT sensor was used to provide a second set of load line displacement measurements for all of the specimens tested on the 22-kip machine. The first portion of the load vs. load line displacement curve (rising load with increasing crack length) represents the stable tearing region while the peak and the tail of the curve represent unstable crack extension. While only the maximum applied load was required for the finite element simulation (as discussed earlier), the entire load vs. load line displacement curve was used for overall comparison. The next two subsections discuss how the specimen thickness affects the measured maximum applied load and load vs. load line displacement curves.

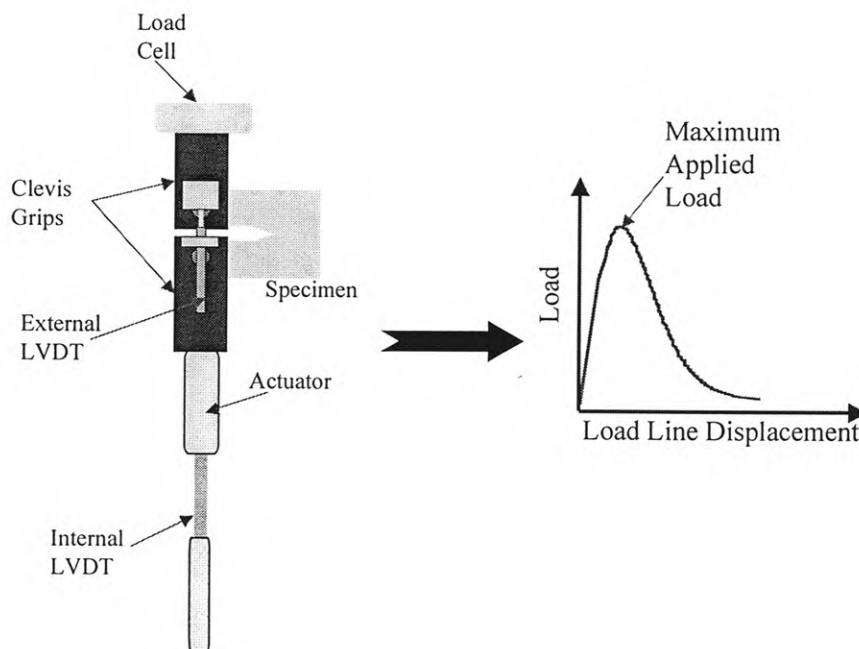


Figure 7.1: Illustration of Load and Load Line Displacement Measurements

7.1.1.1 Thickness Effect on the Maximum Applied Load

Table 7.1 presents the experimentally determined maximum applied loads for all of the tested specimens as well as the average, standard deviations, and previously published values (if available) for each thickness group. It should be noted that “NASA” data [93,99] for the 0.090 and 0.25 in. thicknesses were obtained using 2, 4, and 6 in wide C(T) specimen with $\frac{a_i}{W} = 0.4$ while the “Newman” data [62] for the 0.5 in. thick specimen were obtained using 8 in. wide C(T) specimen with $\frac{a_i}{W} = 0.3, 0.4, \text{ and } 0.7$. To make the comparison presented in Table 7.1, the published data was interpolated and extrapolated to obtain approximate values for the maximum applied load corresponding to the 8 in. C(T) with $\frac{a_i}{W} = 0.4$ used in this research. As expected and as is illustrated in Figure 7.1, the maximum applied load is minimum for the 0.090 in thick specimen and maximum for the 1 in. thick specimen. In general, all multiple tests performed for each specimen thickness had good agreement. The most deviation in the maximum applied load measurements for a given thickness was observed for the 0.25 in. thick specimens where the standard deviation was approximately 4% of the average value of the measured maximum applied load. For the remainder of the thicknesses, the standard deviations of the maximum applied loads were within 2% of the computed average for each thickness group.

Table 7.1: Summary of Fracture Loads

Specimen		Maximum Applied Load (kip)			Published	Difference
Thickness (in.)	Test #	Measured	Average	Standard Deviation	Maximum Loads (kip)	%
0.090	1	3.224	3.20	0.06	2.966*	7%
	2	3.132				
	3	3.237				
0.25	1	8.574	8.43	0.35	9.5**	13%
	2	8.698				
	3	8.032				
0.5	1	17.478	17.10	0.41	16.28***	5%
	2	16.661				
	3	17.149				
1.0	1	28.506	28.74	0.33	N.A.	N.A.
	2	28.977				

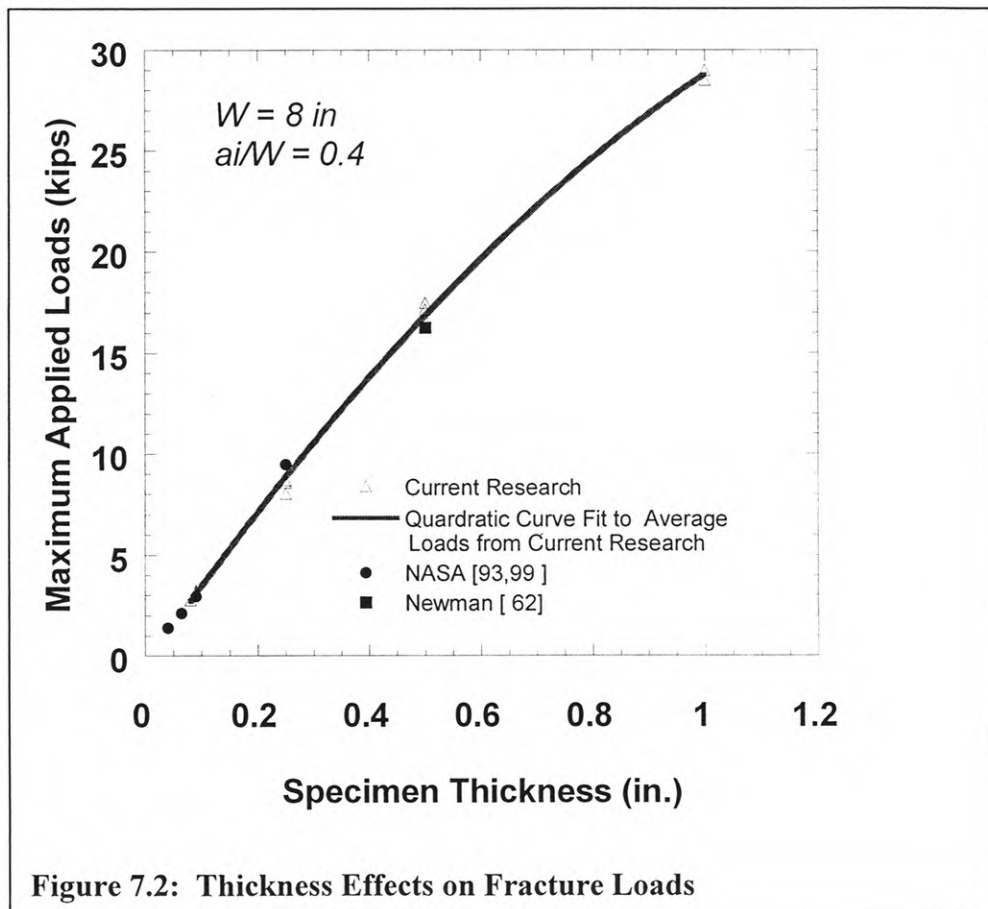
* NASA Data [99]

** NASA Data [93]

*** Newman Data [62]

Figure 7.2 shows a plot of the maximum applied loads for each specimen tested in this project, as a function of specimen thickness. For comparison purposes, maximum applied loads obtained from published data [62, 93,99] were also included in this plot. As can be seen in Figure 7.2, the interpolated maximum applied loads from Newman's data [62] compare well with the data obtained in the current work. The difference between maximum applied loads based on the interpolation of Newman's data and the average of the measured maximum applied loads for the 0.5 in. thick specimen in the current work was within 5%. For the 0.25 in. thick specimen, the difference between the average maximum applied load measured in the current research and the extrapolated maximum applied load from NASA's work [93] was approximately 13%. Part of the discrepancy was due to the existence of a slightly longer machined notch in one of the 0.25 in. thick specimens tested in the current work. Taking into considerations the longer

initial crack length of that particular specimen suggests that the average maximum applied load would be slightly higher than what is reported in Table 7.1, however, the difference between the corrected average maximum applied load value from the current work and the predicted maximum applied load from NASA's work would still be approximately 10%. Based on the variability in the maximum applied load and the deviation from published data for the 0.25 in. thick specimens tested in this research, it would appear that variation/irregularity in the tested material or slight differences in the test procedure may exist between the two researches (current and NASA [99]).



In conversations with Newman [94] concerning the discrepancies in the 0.25 in. specimens, he suggested that residual stresses in the machined specimens may have had an effect. As described in Chapter 4, all specimens used in this research were machined from a single 1 in. plate of material while specimens from NASA [99] and Newman [62] were machined from as-received plates of the appropriate thickness. Newman [94] suggested that the residual stresses would affect the local condition at the crack tip without having a noticeable effect on the maximum applied load. On the other hand, Hill *et al.* [95] reported that residual stress affects both the driving force for fracture and constraint conditions at the crack tip and thus alter the fracture loads in brittle behaving materials. Also, Sharples *et al.* [102] conducted ductile fracture testing on 2024 Aluminum to investigate the effects of residual stress on load carrying capacity of M(T) specimen. They also concluded that residual stress had large influence the maximum applied load. However, the specimens tested in the current research project were not tested for residual stress and no further investigation was conducted to verify whether the 0.25 in. thick specimen contain residual stresses.

For the 0.090 in. thick specimens, it is obvious from Figure 7.2 that both of the maximum applied load values measured in the current work and extrapolated from NASA's work [99] agreed well with each other. The difference between the maximum applied load values was within 7%. This provides a good agreement, considering that Dawicke tested 2024-T3 Aluminum while 2024-T351 Aluminum was tested in the current work. For the 1 in. thick specimen no data was identified in the literature for 2024-T351 8 in. C(T).

7.1.1.2 Measured Load vs. Load Line Displacement Behavior

Figure 7.3 shows the load vs. load line displacement behavior for all 8 in. C(T) specimens tested in this project. Generally, the load vs. load line displacement behavior shows good repeatability between specimens of the same thickness. The 0.25 in. thick specimens show the most scattering in the data. As justified earlier, this scattering in the data of the 0.25 in. thick specimens is due in part to the fact that one of the 0.25 in. thick specimens had a slightly longer machined notch than the other two. The longer initial crack length results in lower maximum applied load, which was the case in the measured loads for the 0.25 in thick specimen with the longer initial crack. Another factor that may have attributed to the variability in the load vs. load line displacement behavior of the 0.25 in. thick specimens is that in one of the specimens, the crack rolled away from the initial crack orientation and grew in the perpendicular direction (along the grain direction of the material) almost immediately after crack initiation while in the other specimens the crack maintained its original orientation. This is believed to be the reason for the higher loads in that particular specimen after the maximum load was reached. However the maximum applied load agreed well with the other specimens.

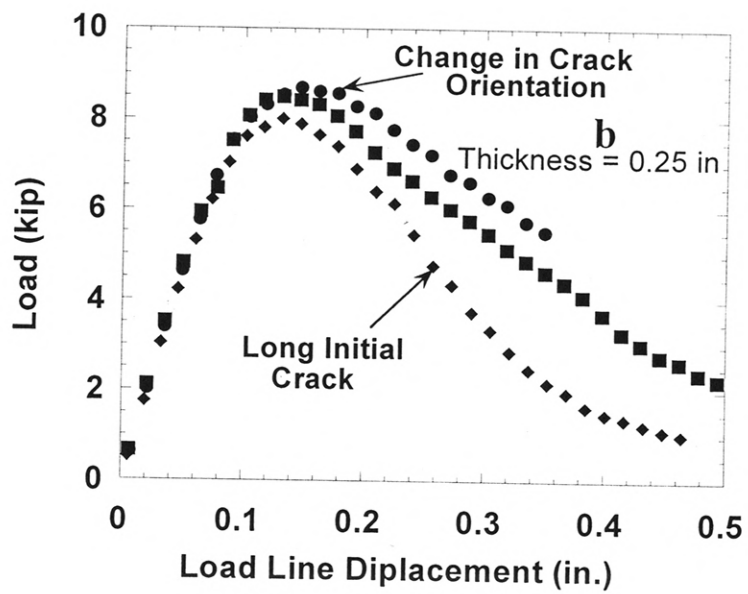
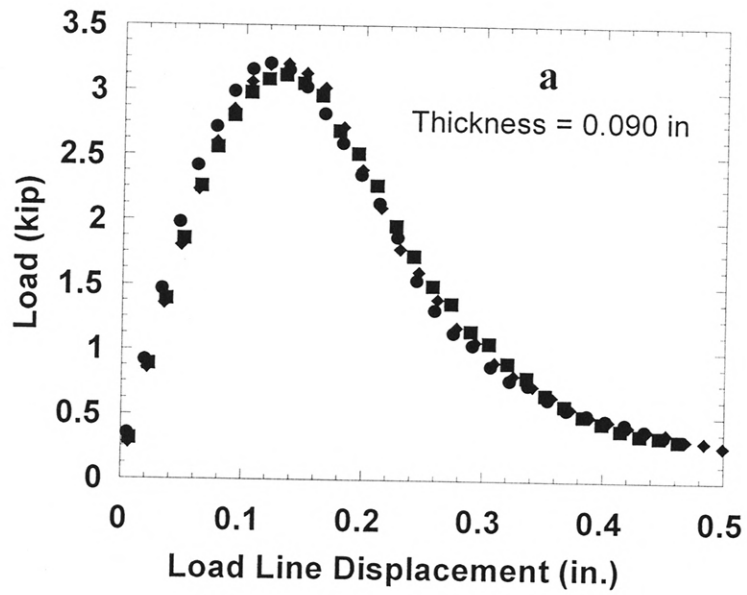
For the 0.5 in. thickness, excellent agreement was observed for the measured load vs. load line displacement for all three specimens. It should be noted however, that two of the 0.5 in. thick specimens were tested using the 22-kip machine while the other was tested using the 55-kip machine. The 0.50 in. thick specimen tested on the 55-kip machine shows slightly higher load line displacement (based on its internal LVDT measurements) values than the specimens tested on the 22-kip machine in the initial

region prior to maximum load and near the tail of the curve. Comparisons of the load vs. load line displacement behavior of the specimens tested on the 22-kip machine using measurements from both of the internal LVDT and the external LVDT suggest that the internal LVDT has produced higher load line displacement than the external LVDT. The higher displacements in the internal LVDT readings may be attributed to the deformation in the piston of the testing machine and the load frame or may be due to slight differences in the calibration of each sensor. This lead to conclude that the internal LVDT measurements of load line displacements might be inaccurate. Figure 7.4 shows the shift in the load line displacement recorded by both the external and internal LVDT for typical 0.090 and 0.5 in. thick specimens. As can be seen in this figure, it appears that the difference between the external and internal LVDT displacement measurements increased as the specimen thickness (and therefore applied load) was increased.

Since only the internal LVDT displacement measurements were available on the 55-kip machine, it was therefore desired to estimate a mapping function that would predict what the external LVDT would record had it been used on the 55-kip machine setup. This was only necessary to provide better comparison for the load vs. load line displacement curves determined from both measurements and finite element simulation. The mapping function would attempt to correct for the uncertainty in the internal LVDT measurements by reflecting what the external LVDT would measure. Details of the mapping procedure are given in Appendix A. Since the external LVDT was attached to the clevis grips (as described in Chapter 4), deformation in the grips would also introduce uncertainty in the external LVDT measurements. To account for the uncertainty in the

internal LVDT displacement measurements, it is possible to estimate the deformation in the grips and subtract it from the measurements. The uncertainty in the external LVDT reading becomes more significant at higher loads, as the grips will experience higher deformation. However, finite element stress analysis using the Pro Mechanica™ software [104] suggests that the deformation in the grips was small in comparison with the measured load line displacement values. Thus, no correction was attempted for the external LVDT displacement measurements.

As mentioned previously, the external LVDT was used to measure displacements for the 0.090, 0.25, and two of the 0.5 in. thick specimens, which were tested on the 22-kip machine. As illustrated in Figure 7.3 c, using the mapping procedure, described in Appendix A, had little effect on the load line displacement measurement for the 0.5 in. thick specimen fractured using the 55-kip machine. In the region prior to maximum load and at maximum load, the mapping function brought the displacement values measured with the internal LVDT of the 55-kip machine closer to the load line displacement values measured with the external LVDT for the 0.5 in thick specimens tested on 22-kip machine. After the maximum applied load was reached, however, the displacement values from both the 55-kip LVDT internal machine and the external LVDT on the 22-kip machine were in good agreement and the mapping function did not alter the internal LVDT measured displacement noticeably. For the 1 in. thick specimens the mapping function produced significantly different values from the load line displacement measurements made by the internal LVDT of the 55-kip machine.



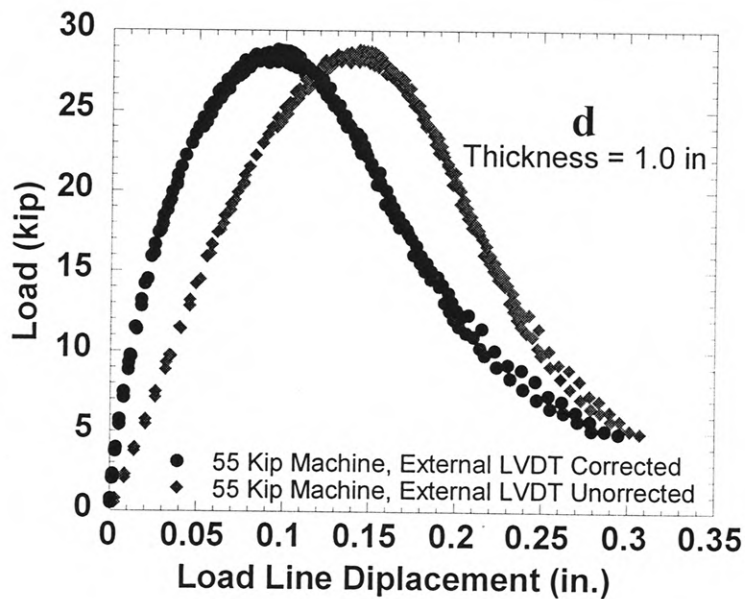
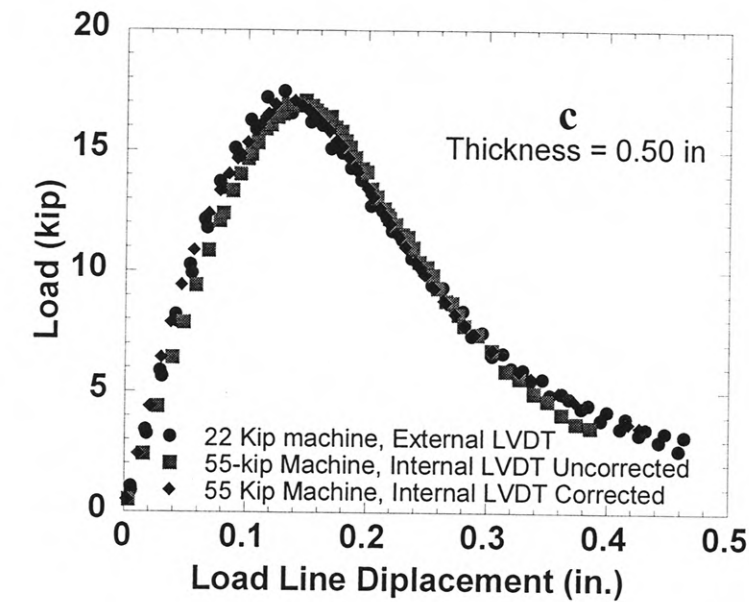


Figure 7.3: Load vs. Load Line Displacement Behavior for the (a) 0.09, (b) 0.25, (c) 0.5, and (d) 1 in. Thick Specimens

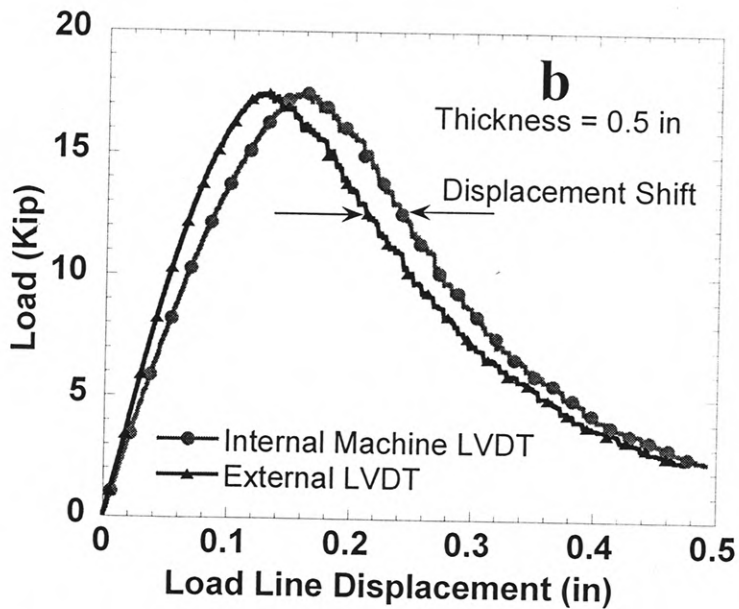
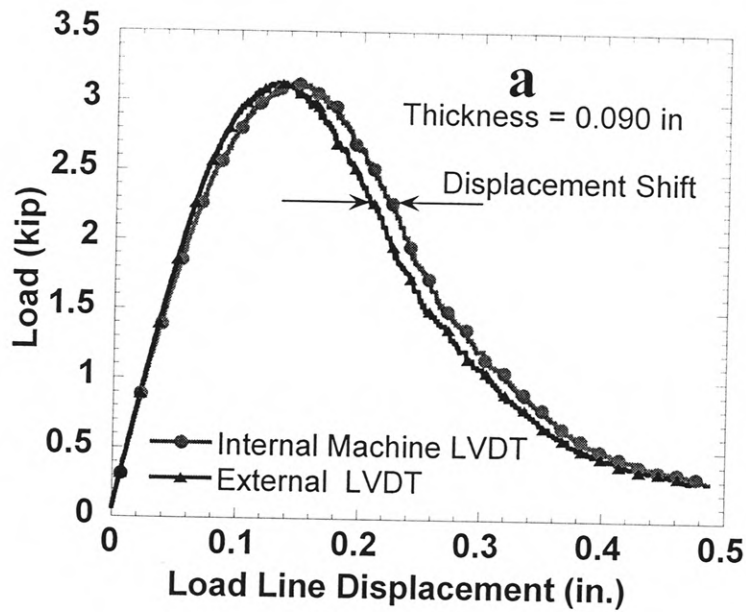
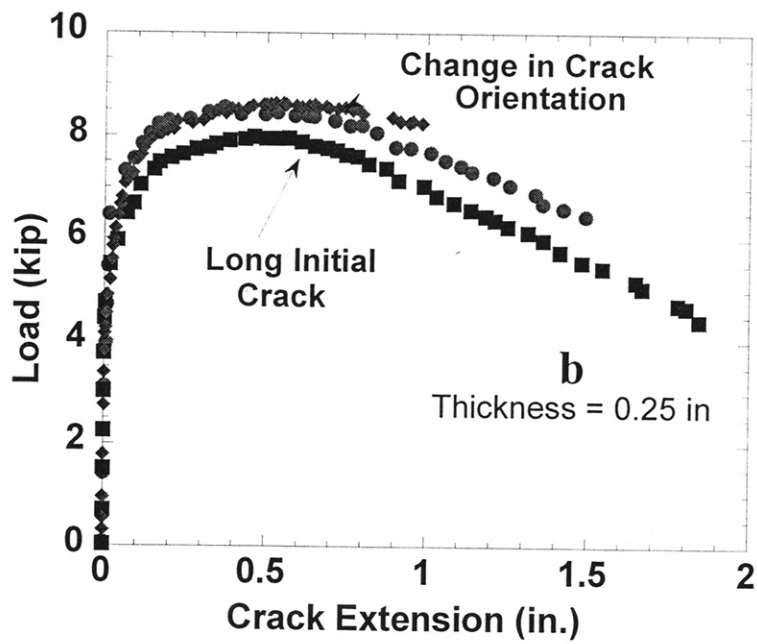
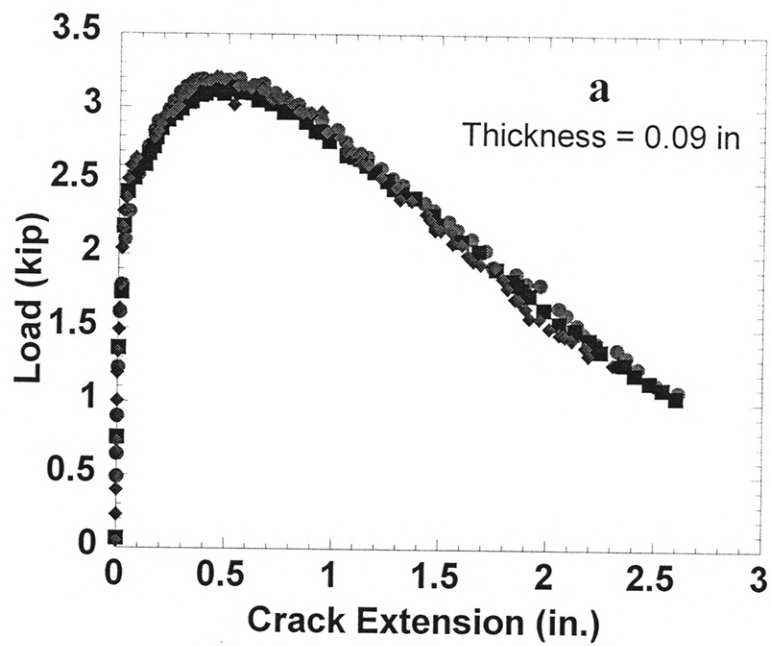


Figure 7.4: Difference Between the Internal and External LVDT Load Line Displacement Measurements for Typical (a) 0.09 and (b) 0.5 in. Thick Specimens

7.1.2 Measured Load vs. Crack Extension Behavior

Figure 7.5 presents the measured load vs. crack extension behavior for all fracture tests performed in this research. The loads vs. crack extension profiles show good repeatability for specimens having the same thickness. As in the load vs. load line displacement behavior, the most scattering of the load vs. crack extension occurred for the 0.25 in. thick specimen. As justified in the previous section, the scattering in the data was due in part to the longer initial crack for one specimen and change of crack orientation from across to along the rolling/grain direction mode of another. Dawicke [98] ran fracture testing on 0.28 in. thick 2324-T39 Aluminum alloy in which he observed scattering in the load vs. load crack extension data comparable to the scattering observed for the 0.25 in. thick specimen tested in the current work.



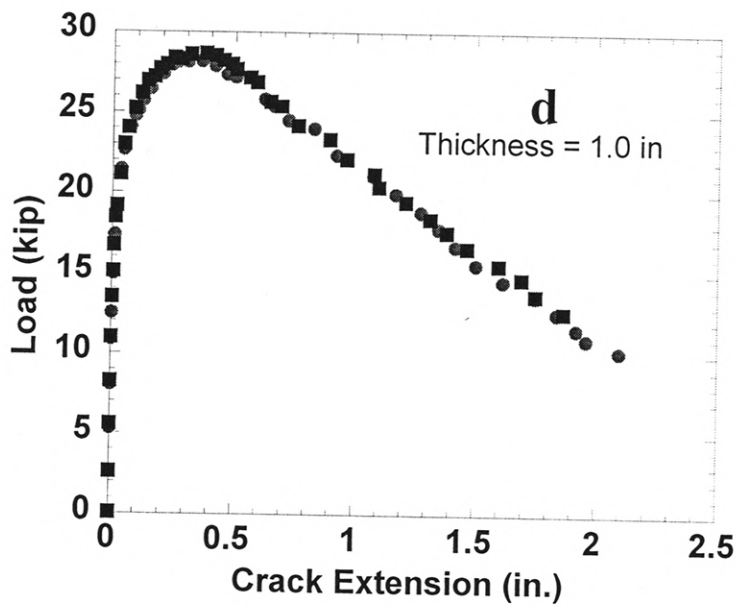
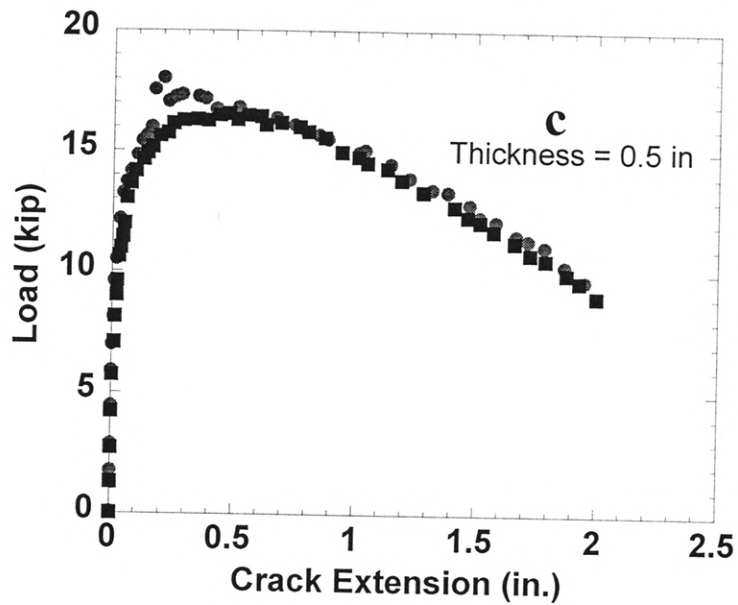


Figure 7.5: Measured Load vs. Crack Extension Behavior for the (a) 0.09, (b) 0.25, (c) 0.5, and (d) 1 in. Thick Specimens

It should be noted that the occurrence of maximum applied load in Figure 7.5 marks the onset of instability. Thus, it was possible to measure the amount of stable crack extension that was exhibited by each specimen before the fracture process became unstable. Figure 7.6 presents the length of the stable crack extension for all specimen thickness tested in this project. It is obvious in Figure 7.6 that the amount of stable crack growth decreases with increasing specimen thickness. The average stable crack

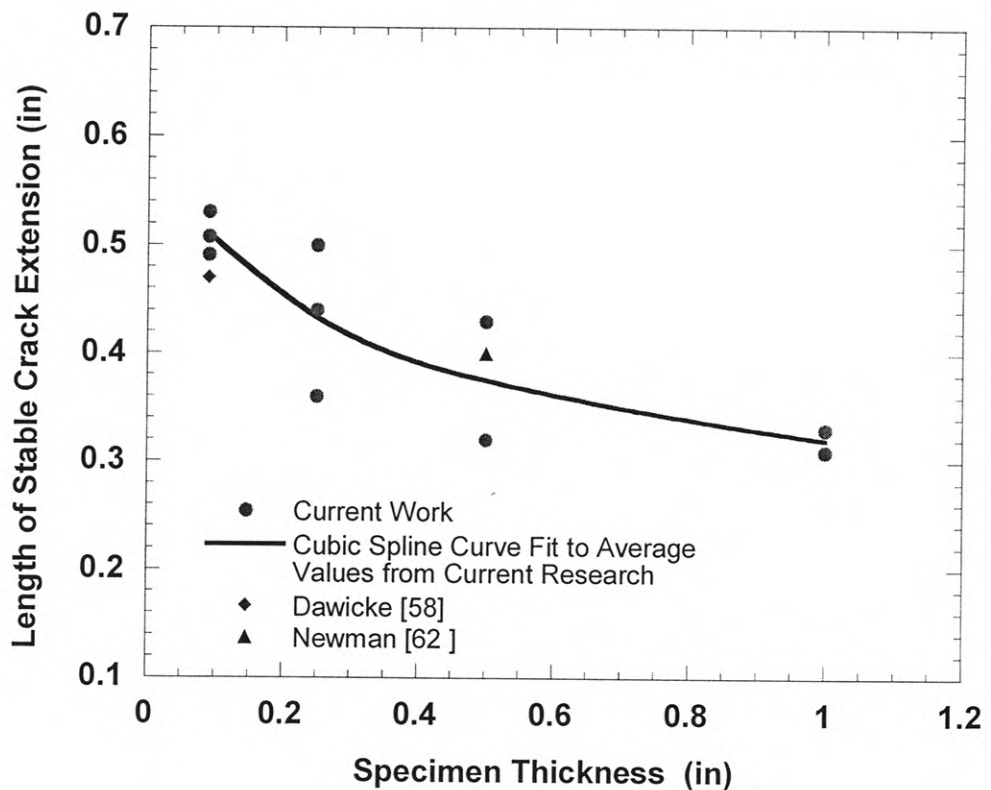


Figure 7.6: Effect of Thickness on Stable Crack Extension

extension value decreased from 0.51 in. for the 0.090 in thick specimens to 0.32 in. for the 1 in. thick specimens. Reasonable scatter was observed for the thinnest and thickest specimens (0.09 and 1.0 in.) while the specimens with intermediate thicknesses (0.25 and 0.5 in.) exhibited high scatter. For comparison purposes, the stable crack extension observed by Newman [62] for the 0.5 in. thick specimen with $\frac{a_i}{W}=0.5$ and Dawicke *et al.* [58] for the 0.09 in. thick specimen with $\frac{a_i}{W}=0.4$ were also plotted on Figure 7.6. The stable crack extension observed for the 0.090 in. thick specimen is slightly higher than the amount of stable crack extension observed by Dawicke [8] for the 6 in. wide 0.09 in. thick C(T) specimen. This is expected since a C(T) specimen with a larger width (W) should produce more stable crack extension than a narrower specimen of the same thickness. The data reported by Newman [62] for the 0.5 in. thick specimen with $\frac{a_i}{W}=0.5$ fell within the scattering band of the stable crack extension measured in this project for the 0.5 in. thick specimen. Although a specimen with a lower $\frac{a_i}{W}$ ratio should exhibit more stable crack extension than a similar specimen with higher $\frac{a_i}{W}$ ratio, the comparison in Figure 7.6 between the data from Newman [62] and current research did not demonstrate that due to the high variability observed in the crack extension length measured for the 0.5 in. thick specimens tested in this project.

In an attempt to collapse the load vs. crack extension data in Figure 7.5 for all thicknesses, the nominal applied stress at the crack tip (σ) (loading parameter) was calculated using the following formula for the C(T) specimen:

$$\sigma = \frac{P}{A} + \frac{My}{I} = \frac{P}{B(w-a)} \left(1 + 3 \frac{w+a}{w-a} \right) \quad (7.1)$$

where P is the measured load, A and I are the cross sectional area and moment of inertia for the remaining ligament of the specimen respectively, M is the moment induced by the applied load (P) at the midline of the remaining ligament, B is the specimen thickness, W is the specimen width, and a is the crack length measured from the applied load (or center of the specimen loading pin). Figure 7.7 shows the resulting nominal applied stress-crack extension behaviors for all specimens using this equation. For all but the 1.0 in. thick specimens the nominal applied stress collapsed crack extension curves on top of each other with a scatter band of 6 ksi. For the 1.0 in. thick specimen, the nominal stress-crack

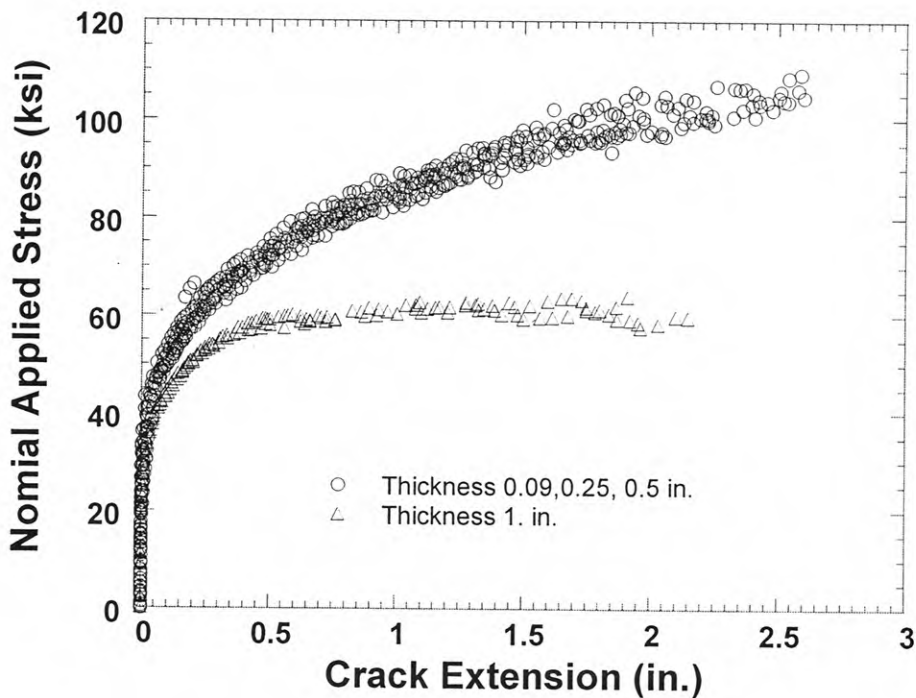
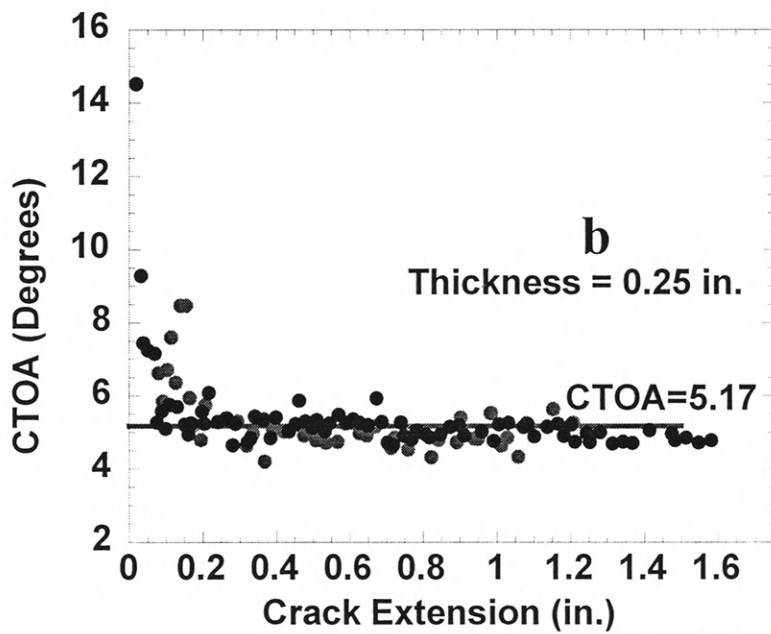
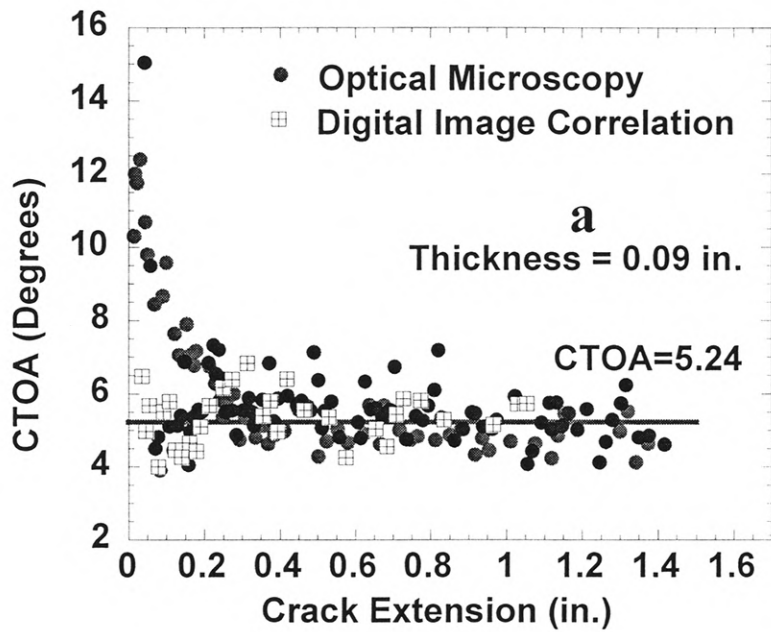


Figure 7.7: Nominal Stress vs. Crack Extension

extension curve agreed with the stress-crack extension curve for the thinner specimens initially then it started to deviate at a crack extension of about 0.15 in. After about 0.55 in of crack extension, the stress-crack extension curve became flat and remained flat for the rest of the fracture process.

7.1.3 Measurements of *CTOA* at Specimen Surface

As described in Chapter 5, the *CTOA* during the entire stable tearing process was measured on the surface of each specimen using optical microscopy and digital image analysis. The digital image correlation techniques was also used to measure the *CTOA* value on the surface of one of the 0.090 in thick specimen. Figure 7.9a shows the experimentally measured *CTOA* value versus crack extension for the 0.090 in. thick specimen in which both measurement techniques were used. Both of the *CTOA* measurements from the optical microscopy and the digital image correlation agree well with each other. The *CTOA* vs. crack extension behavior of the four specimen thicknesses exhibits a similar behavior where the *CTOA* starts at a high value then rapidly decreases to a constant value. As Figure 7.8 exhibits, the measured *CTOA* values had a fair amount of scattering but was generally within ± 1.5 degree. Many researchers observed the rapid transition in the measured *CTOA* vs. crack extension behavior from an initially high value to a constant value after a short amount of stable crack extension [8, 47, 52, 57, 58]. Dawicke *et al.* [8,58] noted that the length of the transition region



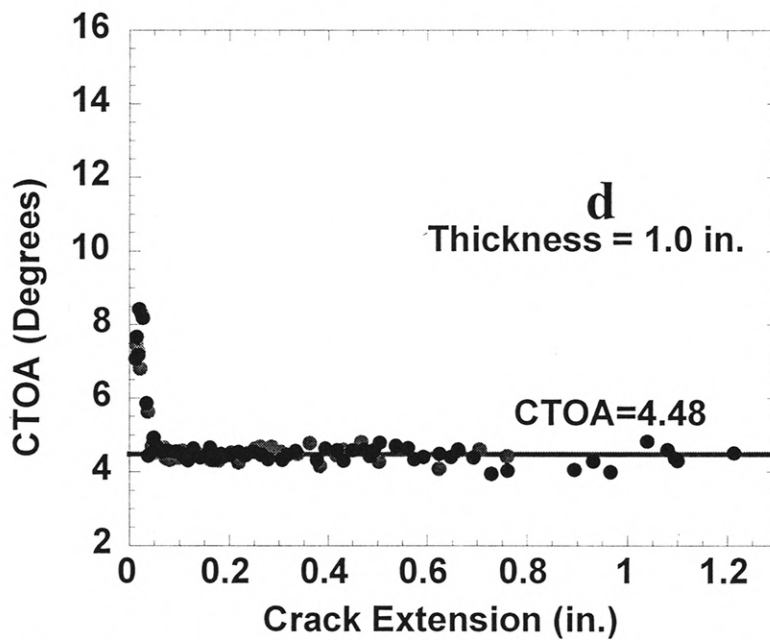
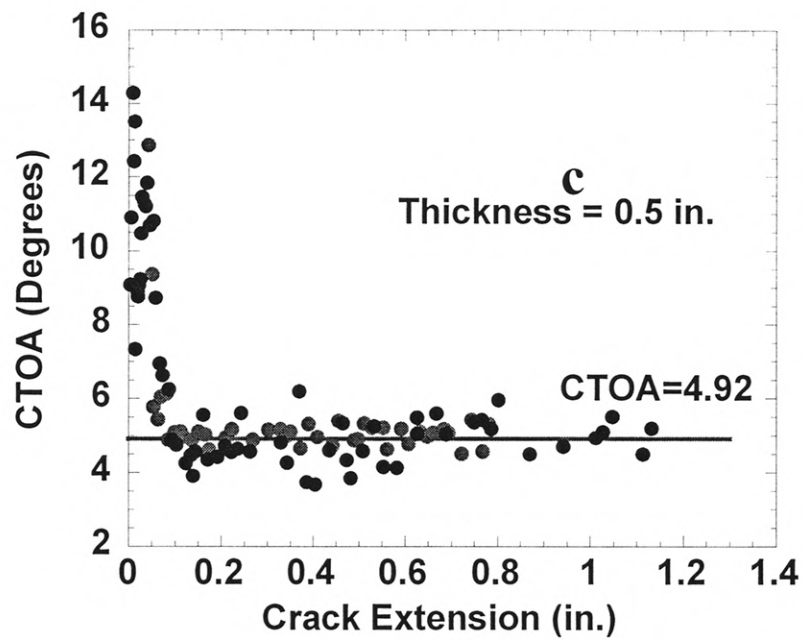


Figure 7.8: CTOA vs. Crack Extension for the (a) 0.09, (b) 0.25, (c) 0.5, and (d) 1 in. Thick Specimens

(crack extension at which $CTOA$ starts becoming constant) was on the order of the specimen thickness for the 0.090 in thick 2024-T3 specimen. For the current work, Figure 7.8 shows that for the 0.090 in thick specimens, the $CTOA$ came to a constant value after 0.2 in of crack extension. The observed transition length was about 0.25 in. for the 0.25 in. thick specimen, and it was about 0.1 in. for each of the 0.5 and 1.0 in thick specimens. Guan *et al.* [96] performed analysis and measurements of the initial deformation field in the vicinity of stably growing cracks in thin Aluminum specimens and they also observed the rapid initial decrease in the $CTOA$. They explained the initial transition in the $CTOA$ Δa curve by the blunting process that accompanies crack growth initiation. Guan *et al.* [96] showed that the $CTOA$ is related to the effective plastic strain in vicinity of the crack tip. Thus crack blunting, which was reported to precede crack growth initiation by many researchers [5,30,38,40,47,57] and was also observed in the current work, induces large plastic strains near the crack tip and thus increases the $CTOA$ initially.

Figure 7.8 also shows the average $CTOA$ value (in the constant plateau region) measured for each specimen thickness. These values are denoted with a solid black line in the plot and represent the numerical average of the data points in the “leveled off” or “constant” region of the $CTOA$ vs. Δa plots, for each thickness tested. Although experimental $CTOA$ values for the .25, .5 and 1 in. thicknesses are not available in the published literature. Dawicke *et al.* [8,58] has reported experimental $CTOA$ for the 0.09 in. thick 2024-T3 Aluminum material to be 6.0 degrees. This is about 0.76 degrees higher than the experimentally determined value for the 0.90 in. thick specimen tested in

the current research. Part of the difference between the two values (current research and Dawicke *et al* [8,58]) is due to the material difference (-T3 and -T351). Another reason may be associated with the test procedure and the instruments used in both researches. A more detailed discussion of the experimental *CTOA* values will be provided in a later section when they are compared to the computational *CTOA* values and such factors as *CTOA* variation through the thickness, crack tunneling, and material constraint effects are addressed. As can be clearly seen from Figure 7.8 the *CTOA* vs. Δa data for all thicknesses contain a large amount of scatter. This scatter was a maximum for the 0.09 in. thick specimens at approximately ± 1.5 degrees and was less (approximately ± 1.0 degree) for the other thicknesses. The high degree of variability in the *CTOA* measurements was observed in the work of Dawicke *et al.* [8, 58,98] who reported ± 1 degree scatter in the *CTOA* measurements. On the other hand, Liu *et al.* [97] reported high scattering in the *CTOA* measurement (for five different alloys of Aluminum and Steel CT3) such that the distribution of the data obtained seemed random, although they reported that the *COA*- Δa had similar characteristics (initially high then constant) to the *CTOA* - Δa curve observed in the current work. They also observed approximately 1.5 degree scatter band and good repeatability on the *COA*- Δa resistance curve, where *COA* is a measure of crack opening angle based on crack opening displacement measured away from the crack tip (crack mouth), as discussed earlier in Chapter 2. Liu *et al.* [97] explained the scattering in the *CTOA* measurements by the local inhomogeneity and impurity in the material which result in voids and coalescence. The *COA*- Δa curve exhibits less scattering than *CTOA*- Δa because *COA* is measured at a large enough

distance behind the crack tip such that the local inhomogeneity in the material would be averaged out [52].

7.1.4 Fracture Surfaces Examination

After testing was complete, the fracture surfaces of each test specimen in the current work were visually examined to investigate all aspects of the crack growth process. Figure 7.9 shows typical fracture surfaces for the 2024-T351 specimens of thickness (1, 0.5, 0.25, and 0.09 in) tested in this research. For each of the specimens, the fracture surface shows the chevron notch and the fatigue pre-cracking surface as well as the stable tearing region. The fatigue pre-cracking region is the bright region near the chevron notch in Figure 7.9 while the stable tearing region is the darker region. The purpose of the chevron notch and the pre-cracking process was to grow a symmetric/straight front sharp fatigue crack with which to start the stable tearing process. However, the fatigue pre-cracking crack fronts were not always symmetrical as can be seen in Figure 7.9. Upon detailed examination of all the fatigue pre-cracks for all of the specimens tested, three different behaviors were exhibited; flat across the specimen (observed in one of the 1.0 in. thick specimens and one of the 0.25 in. thick specimens); curved front with more fatigue crack growth near the specimen center than the surface (observed for all 0.5 in. thick specimens and one of the 1.0 in. thick specimen); and asymmetrical fatigue crack front where the crack grew more near one surface than the other surface (observed for all other specimens). In general the variation in fatigue crack growth across specimen thickness was less than 5% of the final fatigue pre-crack length and was as high as 7 % for two of the 0.5 in. thick specimens. Since the variation in the

fatigue pre-crack length was not very high, the shape of the fatigue crack front was not expected to have influenced the maximum applied loads.

As can be seen in Figure 7.9, the 1.0 in. thick specimen had flat fracture throughout the fracture process with shear lips covering only a small portion of the final fracture surface and contained in a small area in the vicinity of the specimen surface. For the 0.09, 0.25, and 0.5 in. thick specimens, the final fracture region showed a transition from flat to slanted fracture. In the flat and the transition regions, the fracture surfaces had a slightly brighter shade near the middle of the specimen in comparison with the edges. The flat fracture and transition regions had a triangular shape with its base starting

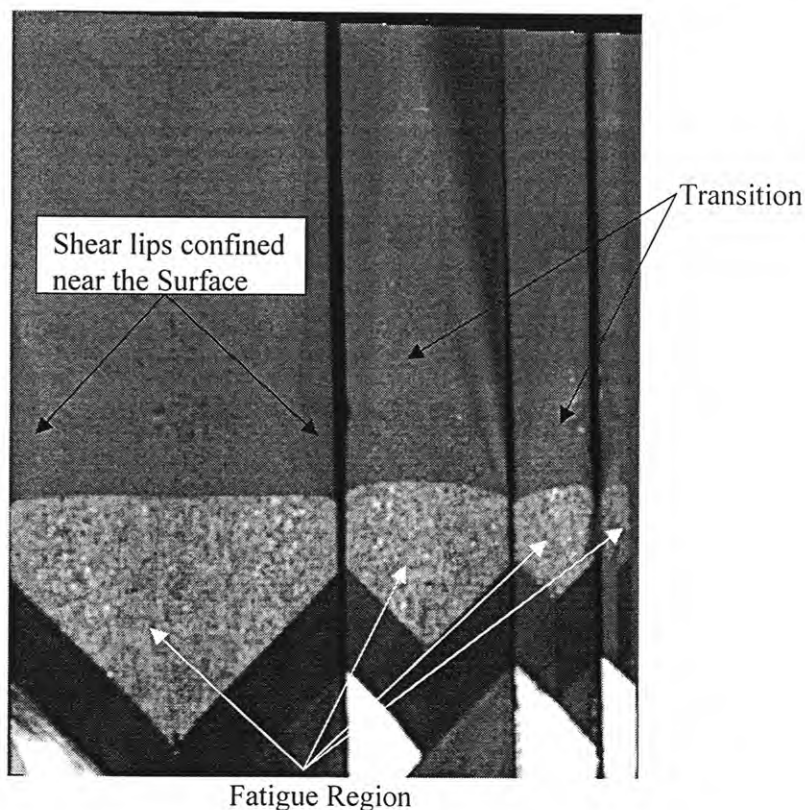


Figure 7.9: Fracture Surface Examination

at the end of the fatigue pre-crack front. The crack grew normal to the material rolling/grain direction for all but a single 0.25 in. thick specimen where the crack rolled away from the initial orientation and grew along the grain/rolling direction, with final fracture actually occurring perpendicular to the initial crack plane. Newman *et al.* [42] conducted conclusive examination of fracture surfaces for the 0.090 in. thick 6 in. wide 2024-T3 Aluminum C(T) specimens in which they made quantitative measurements of the tunneled crack fronts. They concluded that a straight flat crack front produced by fatigue crack growth would transition to a tunneled crack front (higher crack growth on the interior than the surface of the specimen) after a short amount of crack extension. They also reported that the initially flat fracture regime would transition to a fully slant-

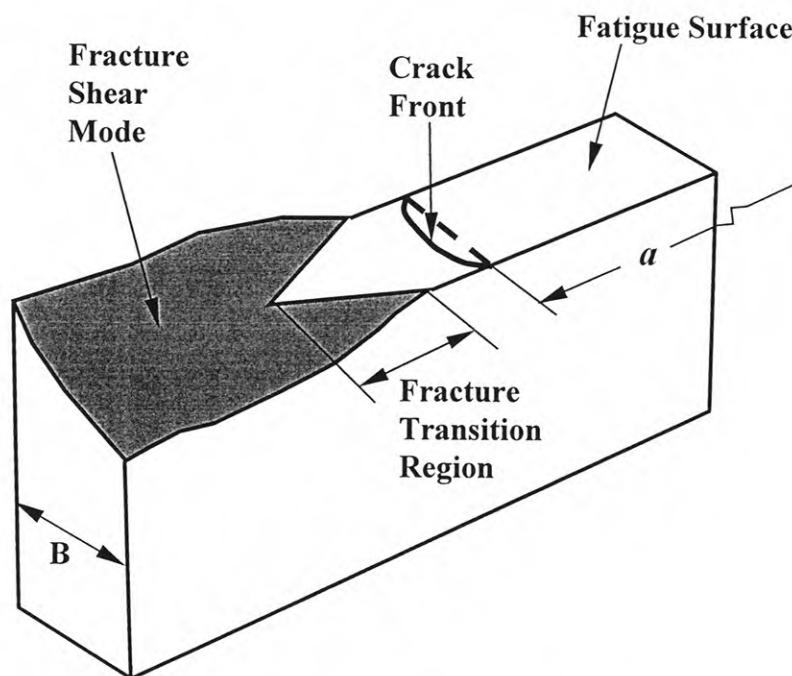


Figure 7.10: Schematic of Flat to Slant Fracture Transition

fracture after a transitional length of crack extension as illustrated by the schematics of Figure 7.10. The initial flat fracture region starts out the full width of the specimen and then decreases in width toward the center of the specimen (forming a flat triangular region), with slant fracture occurring on both sides of the reducing or diminishing flat fracture region. Finally, the final fracture region transitioned completely to single or double slant fracture. Crack tunneling is often accompanied by the formation of shear lips near the specimen surfaces where the crack curvature increases as the depth of the shear lips building near the surfaces increases. [103].

While the length of the flat fracture region could not be quantified based on the fracture surface images provided in Figure 7.9, the extent of the transition region (flat fracture length + transition region length) was seen to have increased with increasing specimen thickness. This supports the fact mentioned by Sih [103] where he pointed out that more flat fracture would be observed as the specimen thickness increases. From the images presented in Figure 7.9, it was possible to measure the total flat and flat-to-slant transition lengths for the 0.09, 0.25, and 0.5 in. thick specimens, which were 0.1, 0.7, and 2.90 in. respectively. Using three dimensional SEM microscopy, Newman *et al.* [42] reported that the total flat and transition (from flat to slant fracture) length for the 6 in. wide 0.09 in. thick 2024-T3 Aluminum C(T) specimen was 0.13 in. This is similar to the transition length observed for the 0.090 in. thick specimens tested in the current project considering the resolution of the measurements.

Lloyd *et al.* [63] reported that crack tunneling would induce variation in the *CTOA* measured across the specimen thickness for thin 2024-T3 Aluminum specimens

causing higher *CTOA* values near the specimen surface than the interior. However, they reported that both of the surface and center *CTOA* values approached a similar value after a short amount of crack extension. Thus a comprehensive characterization of the thickness effects on the *CTOA* would require thorough characterization of crack tunneling effects on the *CTOA*. One of the techniques used to evaluate crack tunneling effects is to run fracture tests as described in the current work, but stop at various stages of stable crack growth and mark the crack front at that crack location using fatigue crack growth then fracture the specimen into two halves. While such an investigation is possible, it is difficult, time consuming, and costly since many specimen have to be broken to capture the crack profiles at various increments of crack growth. For these reasons the crack tunneling effects were not investigated in the current research. Dawicke [7] broke over 20 specimens to capture the crack front profiles for thin 2024-T3 Aluminum specimens and he concluded that crack tunneling did not have significant effect on the maximum applied load.

7.2 Results of the Computational Simulations

Once all the fracture testing was complete, two-dimensional and three dimensional, elastic-plastic, FEM-based fracture analyses were performed to simulate these tests. The three-dimensional analyses were used to complete the *CTOA* characterization process (generation phase) while the two dimensional analyses provided estimates for the plane strain core height necessary to account for the constraint effects.

Before proceeding, it should be noted that, as described in the previous section, the actual fracture surfaces of the tested C(T) specimens consisted of three distinct

regions, including an initially flat region, a transition region, and a slant (single or double) region. However, all modeling (simulations) performed in this project assumed flat fracture with a straight crack front. (no more changes after this point)

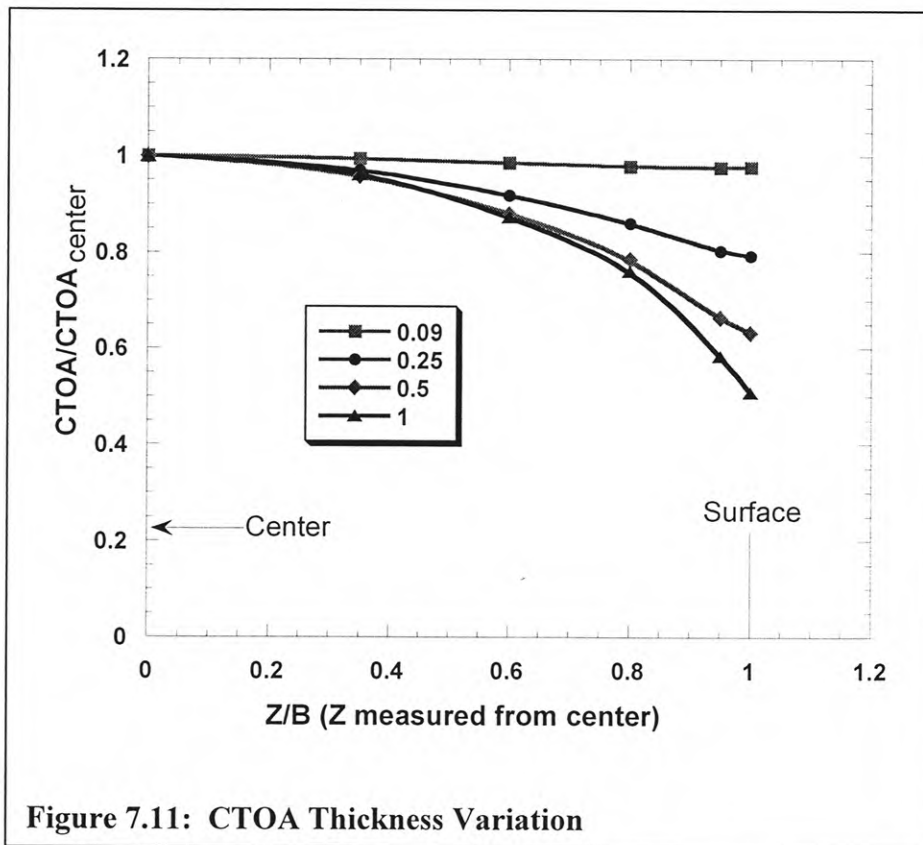
The following sections will provide discussion of these research findings on thickness effect on the *CTOA* and plane strain core height, as well as comparison of the load vs. load line displacement and load vs. crack extension curves obtained from both the experiments and the FEM-based computational analyses.

7.2.1 Effects of Specimen Thickness on *CTOA*

As described in Chapter 6, the critical *CTOA* input to the ZIP3D fracture simulations was varied until the maximum applied load produced by ZIP3D matched the average experimental value for each thickness group. The ZIP3Dcritical *CTOA* input that produced the closest match to the experimental applied maximum load was then recorded for each thickness. It should be noted that the experimental results represent the critical values measured on the surface of the specimens during the stable tearing process. As described earlier, this is in contrast to ZIP3D, where the output is based on the *CTOA* value specified at the center of the specimen. Due to the effects of the free surface and the higher constraint on the interior of the specimens, it would be expected that the *CTOA* at the center of the specimen would have a higher value than the *CTOA* on the surface of the specimen. In effect, the crack flanks on the outer surfaces of the test specimens would “sag” with respect to the crack flanks on the interior of the specimen. In order to make valid comparisons between the critical *CTOA* values from both the measurements

and the ZIP3D output, it was necessary to consider the *CTOA* value produced by ZIP3D at the surface of the specimen.

Examining the output of ZIP3D indicated that the *CTOA* value maintains a maximum value at the center of the specimen then it decreases across the thickness and reaches a minimum value at the surface. Furthermore, the change between the *CTOA* value at the center of the specimen and its surface increases with increasing specimen thicknesses. Figure 7.11 illustrates the variation of the *CTOA* across the thickness of the



specimen, for the four thicknesses tested in this research, based on the ZIP3D output. Comparison of thickness effects on the *CTOA* variation across the specimen thicknesses was accomplished in Figure 7.11 by scaling the *CTOA* value at any given distance along

the thickness with respect to the *CTOA* value at the center of the specimen. As this figure shows, the drop in the *CTOA* values on the surface of the specimen has a minimum value (2%) for the 0.090 in. thick specimen and a maximum value (50%) for the 1 in. thick specimen. As explained earlier, the *CTOA* variation across the specimen thickness is caused primarily by the material constraint, with less constraint (more deformation) at the surface and more constraint (less deformation) at the center. Figure 7.12 compares the *CTOA* values determined from ZIP3D (at specimen center and surface) for all of the tested specimen thicknesses, as well as the average experimentally determined critical *CTOA* values, shown earlier in Figure 7.8. For comparison purposes, previously published critical *CTOA* data obtained from ZIP3D (at specimen center) are also included in this figure [62, 93,99]. In addition, Table 7.2 provides a listing of all the numerical values for the ZIP3D critical *CTOA* values on the specimen surface and center as well as the experimentally measured *CTOA* values on the surface of the specimen. Moreover, Table 7.2 provides the difference between the *CTOA* values determined from ZIP3D on the specimen surface and center and the difference between the experimentally determined *CTOA* and the surface *CTOA* from ZIP3D.

As shown earlier in Figure 7.11, Figure 7.12 also demonstrates that the difference between the ZIP3D-determined *CTOA* values on both the center and the surface of the specimen increases with increasing specimen thickness. While there is negligible difference (2%) between the surface and center values of the *CTOA* based on the ZIP3D analysis for the thinnest specimen (0.090 in.), more significant variation was observed for the remaining thicknesses. Figure 7.12 also shows that the *CTOA* values measured on the specimen surface fall between the ZIP3D center and surface *CTOA* values for all but the thinnest tested specimen (0.090 in.). For the 0.090 in thick specimen, where plane stress behavior is approached, the surface *CTOA* value based on the three dimensional finite-element results over predicts the *CTOA* measured on the surface by 5%. In the thicker specimens (0.25-1.0 in.) more significant difference was observed between the measured

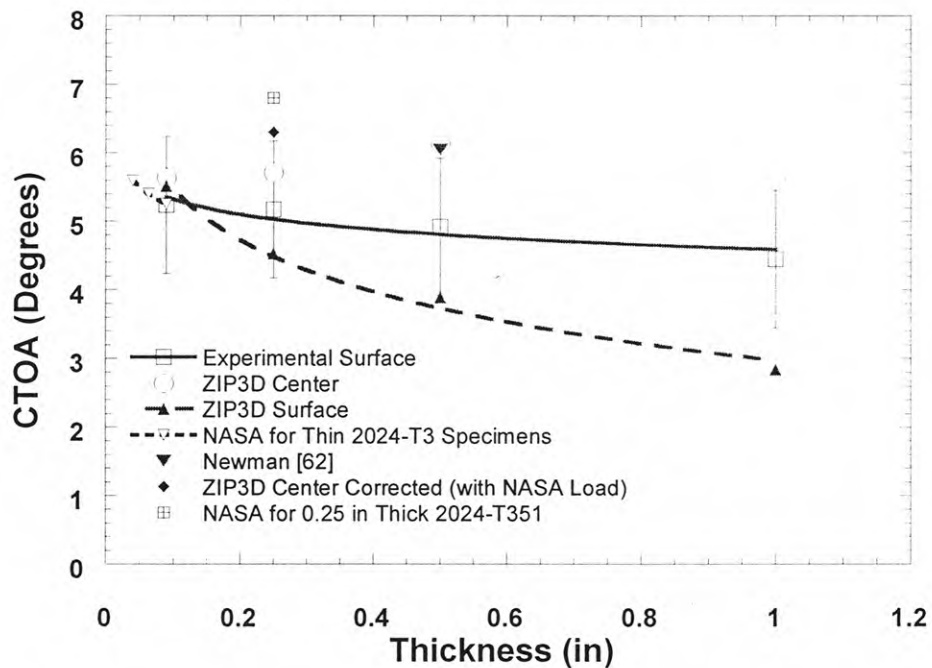


Figure 7.12: Thickness Effects on the Crack Tip Opening Angle

CTOA values and the surface *CTOA* values determined from the ZIP3D analysis.

The ZIP3D values at the specimen center imply that no definite trend was established between the center *CTOA* value and the material thickness. As can be seen, the *CTOA* value increases significantly from the 0.09 to the 0.25 in. thickness, and then decreases for the 0.5 in. and the 1.0 in. thicknesses. Trying to validate the results of the current research project with data obtained from published [62, 93,99] imply an anomaly in the ZIP3D determined center *CTOA* value for the 0.25 in. thick specimen. Also this comparison (current results with published data) indicates the existence of two ranges in which center *CTOA* value decreases with increasing specimen thickness. Considering the small difference between the *CTOA* values from measurement and ZIP3D analysis (5%) for the 0.090 in. thick, it is appropriate to conclude that the *CTOA* values determined for the 0.090 in. thick. specimen in this project follow the trend established by NASA [99] for thin materials. Also, for the 0.5 in. thick specimen, the *CTOA* value determined from ZIP3D at the center of the specimen agrees well with the *CTOA* value (at center of specimen) generated using the fracture load reported by Newman [62]. Considering the soundness of the ZIP3D determined *CTOA* values at the center of the specimen for the 0.090 and 0.50 in. thick specimens would indicate that the *CTOA* value for the 0.5 in. thick specimen does not follow the trend established by NASA [99] for thin specimens. This leads to the proposition of the existence of a critical thickness where the *CTOA* vs. thickness behavior will deviate from thin specimen trend established by NASA [99]. Comparing the *CTOA* value determined at the center of the specimen from the ZIP3D analysis using the maximum applied loads measured in this project with the *CTOA* value

reported by NASA [93] for the 0.25 in. thick specimen reveals a 16% difference. However, using the maximum applied load determined by NASA [93] with the ZIP3D computational procedures employed in this project yield a 9% difference from the *CTOA* value generated using the applied maximum load measured in the current research project. This supports the reasoning behind the anomaly suspected in the *CTOA* value at the center of the specimen using ZIP3D and the maximum applied load measured in the current research. This is in agreement with the scattering in the load vs. load line displacement and the load vs. load crack extension curves, reported earlier. While a numerical value for critical thickness at which the ZIP3D center *CTOA* value deviates from the behavior established for thin specimens by NASA [93] is not available in the published literature, it is suspected that this critical value falls between 0.09 and 0.25 in.

Considering the surface *CTOA* from both the measurements and the ZIP3D analysis provided that the surface *CTOA* value decreases with increasing specimen thickness. Comparing the surface *CTOA* value from both the measurements and ZIP3D analysis shows that the ZIP3D determined *CTOA* value slightly overestimates the measured *CTOA* for the 0.090 in. thick specimen. For the remainder of the tested specimen thicknesses, the surface determined *CTOA* values considerably underestimate the measured *CTOA* values. Table 7-2 provides the magnitudes of the difference between the surface *CTOA* from the measurements and ZIP3D analyses. It is also obvious in Figure 7.12 that the ZIP3D generated surface *CTOA* values fall within the ± 1.5 degree scatter band (reported earlier in Figure 7.8 for all except the 1.0 in thick specimen. Another explanation of the difference between the measured and computed surface

CTOA values is due to the possible inaccuracy in ZIP3D due to the lack of considering crack tunneling in the ZIP3D analysis.

Overall, the data presented in this section helps in the understating of thickness effect on the *CTOA*. It is clear that material induced constraint would cause thick material to exhibit different *CTOA*-thickness behavior from thin material. Also, differences between measured and computed *CTOA* values on the surface of the specimen would call for including crack tunneling in future investigations. However, at this point, incorporating crack tunneling into the three dimensional analyses are not supported in the ZIP3D version used in the current research.

Table 7.2: Measured and Computational (ZIP3D) CTOA Values

Thickness in.	ZIP3D		Measured on Surface	Difference	
	Center	Surface		Center-Surface	Measured-ZIP3D Surface
0.090	5.64	5.52	5.24	2%	5%
0.25	5.71	4.53	5.17	21%	12%
0.5	6.14	3.89	4.92	37%	21%
1.0	5.59	2.84	4.45	49%	36%

7.2.2 Thickness Effects on the Plane Strain Core Height

As chapter 6 outlined, the final stage of the computational characterization process entailed using the elastic plastic finite element fracture mechanics capability of FRANC2D\L for determining the plane strain core (PSC) height necessary to account for the 3-dimensional constraint in each of the specimen thicknesses. This was accomplished by using the CTOA values generated from the ZIP3D analysis at the center of the specimens with the FRANC2D\L analyses. The input PSC height value to FRANC2D\L was varied until the maximum applied load predicted by the FRANC2D\L analyses produced the closest match to the measured maximum applied load. NASA researchers [99] reported testing thin specimens from which they concluded that the plane strain core height would increase with increasing specimen thickness. Furthermore, they generalized that the height of the PSC would be approximately equivalent to the thickness of the specimen. Figure 7.13 presents a plot containing the PSC height values for the various specimen thicknesses tested in the current work and compares them to the PSC values reported by NASA [99] for the thinner specimens. It is worth noting that the PSC height value computed by NASA researches [99] for the 0.063 in. thick specimen was higher

than the thickness of the specimen. However, they reported X-ray diffraction measurements revealed significant residual stresses present in the 0.063 in. thick specimens. When they accounted for the effect of these residual stresses, they found that the PSC value would indeed follow the trend of the other specimen and would have a height of approximately 0.06. Figure 7.13 shows both of the values reported by NASA [99] for the 0.063 in thick specimen (computed based on measurements and corrected for residual stress).

It is seen from Figure 7.13 that for the 0.090 in. thick specimen the PSC height value determined from the current work matches the PSC value reported by NASA [99]. Also, this figure demonstrates how the PSC height increases with increasing specimen thickness. However, based on the results of the current work, the slope of the curve is not unity as reported by Dawicke *et al.* [8,58]. The slope of the PSC height vs. thickness linear regression shown in Figure 7.13 was approximately 0.62 in. It is also obvious that the PSC height value for the 0.25 in. thick specimen deviates from the linear regression curve fit established using data from the other thicknesses.

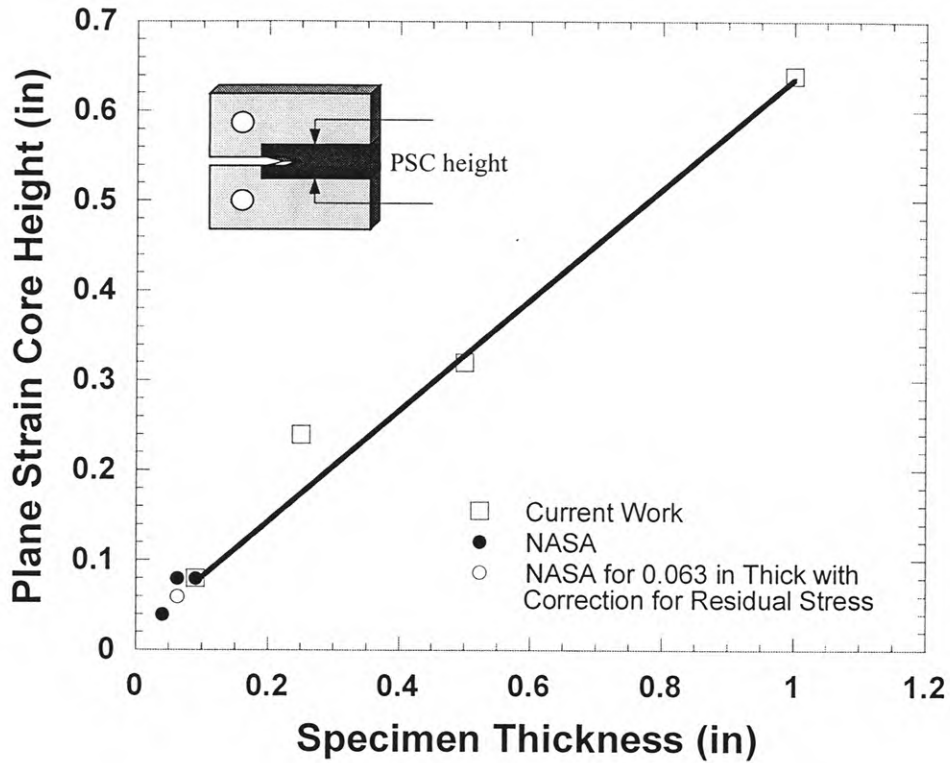


Figure 7.13: Effect of Specimen Thickness on PSC Height

Figure shows 7.14 compares the maximum applied load predicted from the two-dimensional FEM-based analyses (using plane stress only and plane stress with PSC) to the experimental measurements of the maximum applied load for all the tested thicknesses. The FRANC2D\L analysis using plane stress produced good prediction of the maximum applied the 0.090 in. thick specimen (within 3%) while overestimating the maximum applied load by more than 10% for the remaining thicknesses. Dawicke *et al.* [58] reported that the plane stress two-dimensional analyses over predicted the maximum applied load for the 6 in. wide, 0.090 in. thick 2024-T3 Aluminum specimen by 15%.

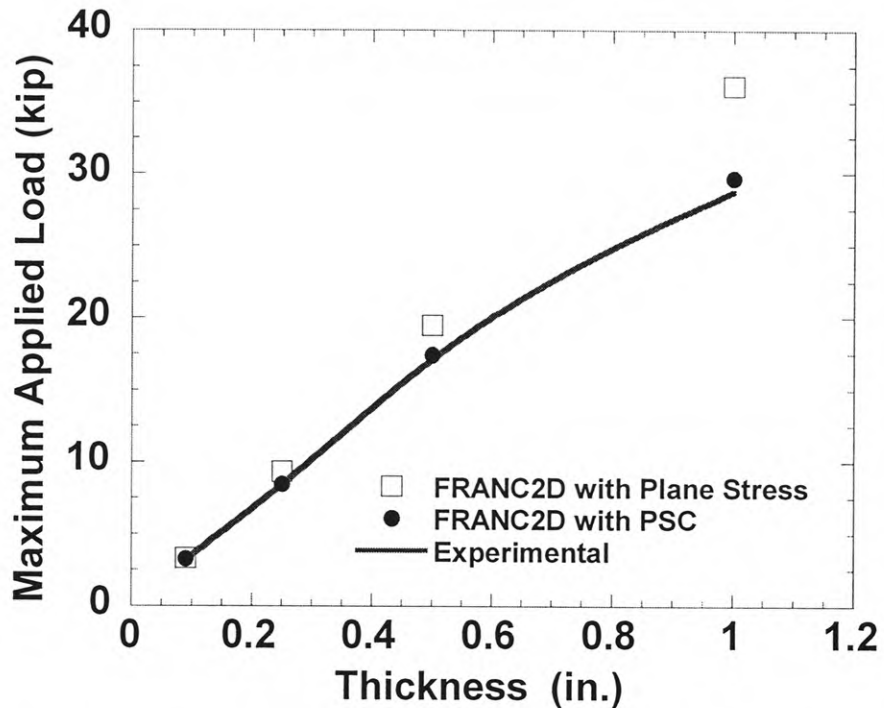


Figure 7.14: Comparison of Maximum Load Prediction of FRANC2DL with and without PSC

The difference between the plane stress-based analysis and the experimentally determined maximum applied load increases with increasing specimen thickness. Incorporating the PSC into the FRANC2DL analysis brought the maximum applied load prediction closer to the experimentally determined loads. The maximum difference between the measured maximum load and FRANC2DL with PSC prediction was observed as 3% for the 1.0 in. thick specimen.

Figure 7.15 also shows comparison between the load line displacement at maximum applied load for the values obtained from measurements and FRANC2DL analyses (with plane stress only and plane stress with PSC). Figure 7.15 shows that the

load line displacement at maximum applied load decreases with increasing specimen thickness. The FRANC2D\L analysis with plane stress overestimates the load line displacement at maximum applied load by approximately 10% for the thinner specimens (0.090 and 0.25 in.) and up to 56% for the thickest specimen (1 in. thick). As illustrated in Figure 7.15, incorporating the PSC into the FRANC2D\L analysis slightly improved predicting the load line displacement value corresponding to the maximum applied load for the 0.090 in thick specimen. For the remainder of the thicknesses, incorporating the PSC height had caused the FRANC2D\L to underestimate the measured load line displacement values at maximum applied load. While no effect was observed for the accuracy of the prediction for the 0.25 in. thick specimen, the accuracy of the prediction had slight improvement for the 0.5 in. thick specimen and substantial improvement was observed in the prediction accuracy for the 1.0 in. thick specimen. Dawicke *et al.* [58] reported that the plane stress two-dimensional analysis over predicted the load line displacement at maximum applied load for the 6 in. wide, 0.090 in. thick 2024-T3 Aluminum specimen by 40%.

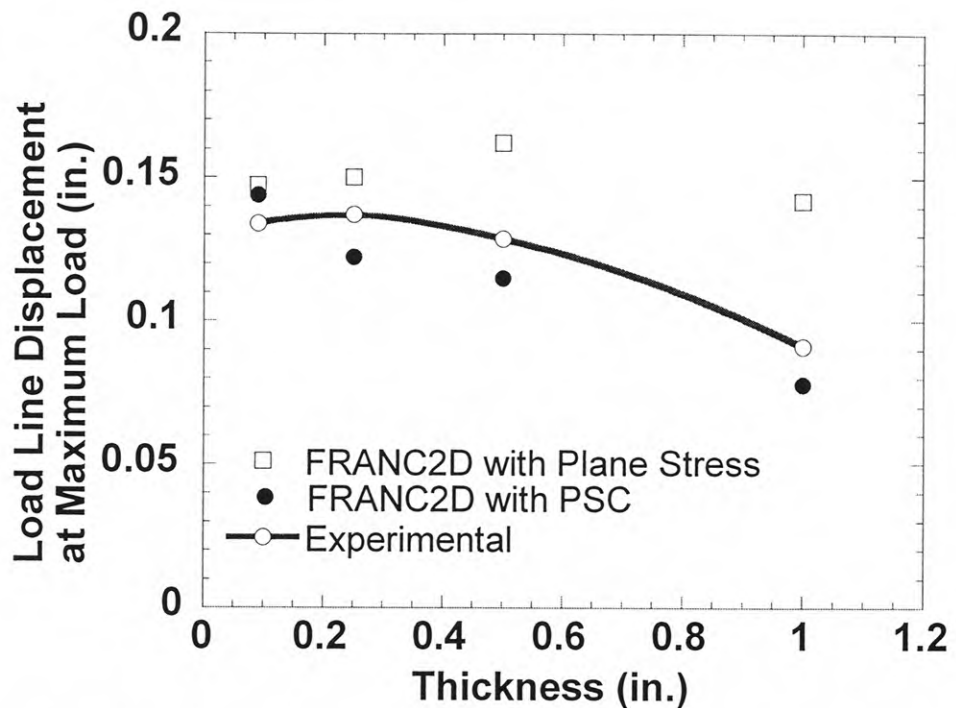


Figure 7.15: Comparison of Load Line Displacement Prediction of RANC2DL with and without PSC

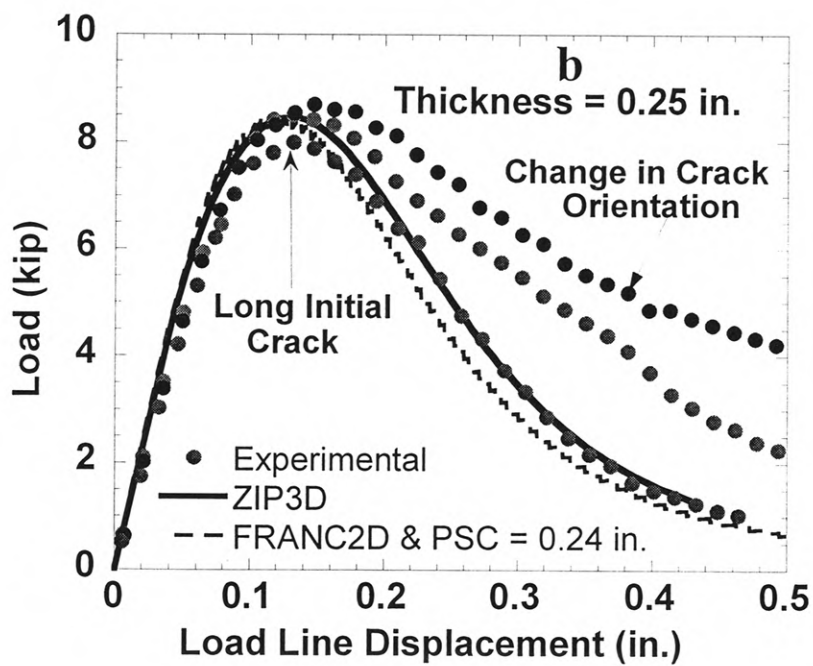
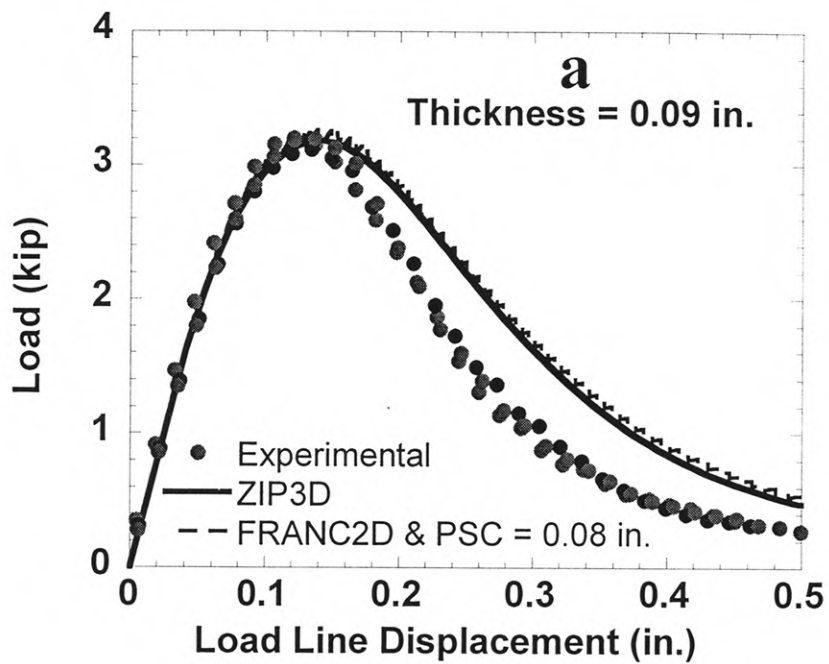
7.2.3 Comparison of Simulation with Experimental Load vs. Load line displacement Behavior

Figure 7.16 contains a plot showing a comparison between the measured load vs. load line displacement curves and the computed load vs. load line displacement curves based on the three and two-dimensional finite element analyses for all tested specimen thicknesses. As shown in Figure 7.16 a, the computed load vs. load line displacement curves from the two finite element based computational methods (three-dimensional and two-dimensional with PSC) agreed very well with each other. Initially, both of the computationally determined curves agreed closely with the experimentally determined load vs. load line displacement until the maximum values was reached. After the

maximum load was reached, the computationally determined load vs. load line displacement behavior deviated from the experimentally measured load vs. load line behavior for the 0.090 in. thick specimens. Both computational procedures have over predicted the experimentally measured loads in the region beyond the maximum applied load.

For the 0.25 in thick specimen, Figure 7.16 b shows that both of the two and three-dimensional computationally determined load vs. load line displacement curves agree with each other initially until the maximum load was reached. In the range beyond the maximum load the two-dimensional FEM based analyses under predicted the curve generated by the three-dimensional FEM-based analyses. Considering the scattering in the load vs. load line displacement for the 0.25 in thick specimen (discussed earlier), the computational procedure produced a reasonable simulation of the experimental data.

For the 0.5 in. thick specimen, the two-dimensional FEM-based analysis matched the three dimensional FEM-based analysis initially. At approximately 0.06 in. of load line displacement and beyond, the two-dimensional finite element analysis seems to have underestimated the load line displacement produced by the three dimensional finite element analyses. However, the maximum loads produced by both analyses agreed well. With regard to experimental data, both computational procedures (two-dimensional with PSC) and three-dimensional) produced good simulation of the experimental load vs. load line displacement. The three-dimensional FEM-generated curve lays on the edge of the scatter band in the range prior to maximum load. After maximum load, excellent agreement was shown between the three-dimensional computed and experimentally



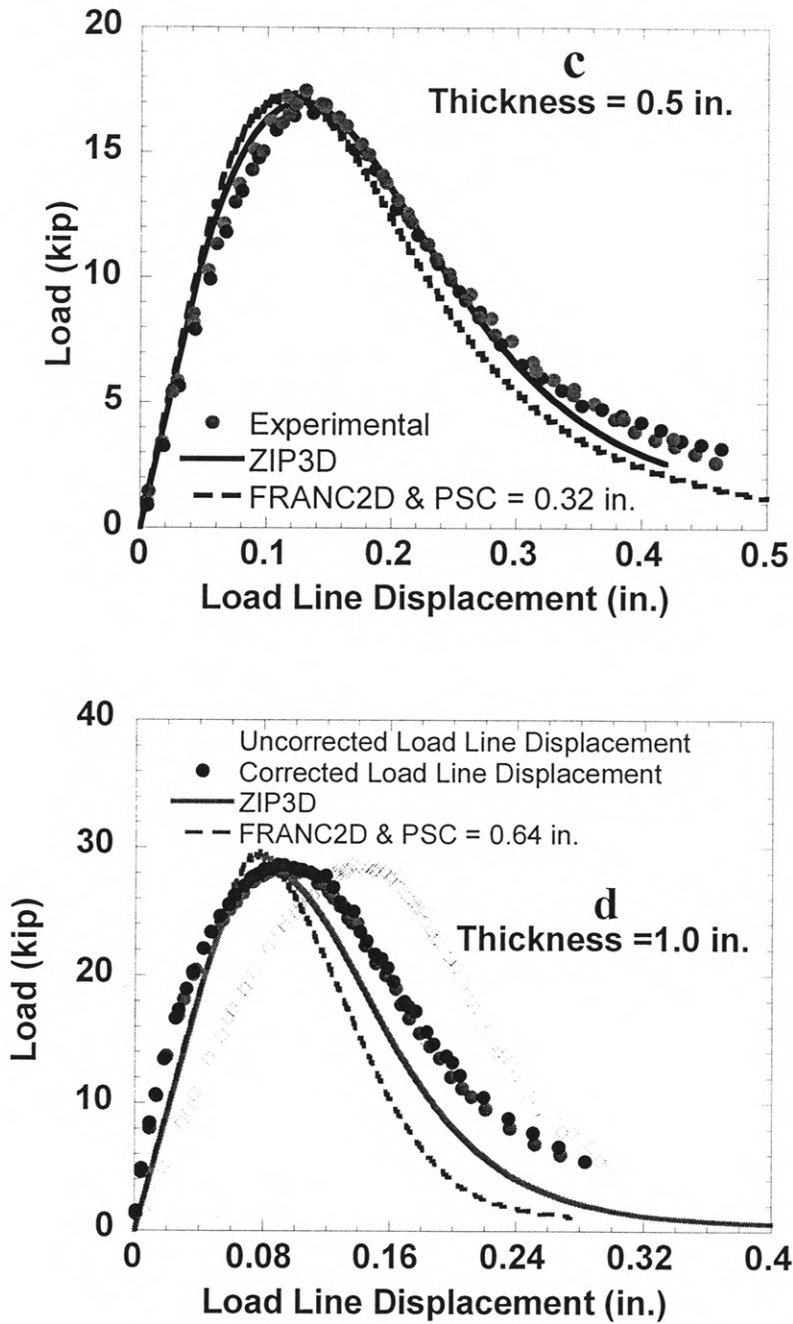


Figure 7.16: Comparison of Measured and Computational Load vs. Load Line Displacement Behavior for the (a) 0.09, (b) 0.25, (c) 0.5, and (d) 1 in. Thick Specimens

determined load vs. load line displacement curves until 0.3 in. of load line displacement. After 0.3 in of load line displacement the three-dimensional FEM-determined load line displacement underestimated the experimentally measured load line displacement data.

For the 1.0 in. thick specimen, Figure 7.16 shows the load vs. load line displacement curves. The measured load vs. load line displacement curves based on the corrected and uncorrected load line displacements were included in Figure 7.16 to show how the correction of the measured load line displacement brings the curve closer to the simulation. Both of the computationally determined curves agreed well with each other initially, however, the FRANC2D analysis with plane strain core overestimated the maximum applied load. The ZIP3D analysis on the other hand predicted the correct load. After the point of maximum applied load, the FRANC2D\L analysis underestimated the displacement predicted by the ZIP3D analyses. It is seen from Figure 7.16 that both the computationally determined curves have underestimated the measured displacement throughout the measured load line displacement range. However, the experimental load line displacement values were based on the correction procedure described in Appendix A. Thus, there was no mean of determining the accuracy of the load vs. load line displacement measurement.

7.2.4 Comparison of Simulation with Experimental Load-Crack Extension Behavior

Figure 7.17 contains a plot showing a comparison between the measured load vs. crack extension curves and the computed load vs. crack extension curves based on the three and two dimensional FEM-based analyses for all tested specimen thicknesses. As shown in Figure 7.17 a, the computed load vs. crack extension curves from the two FEM

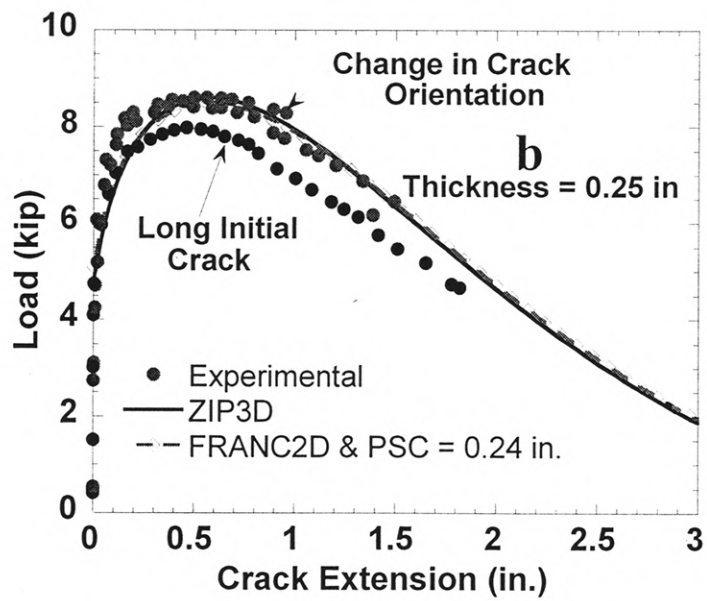
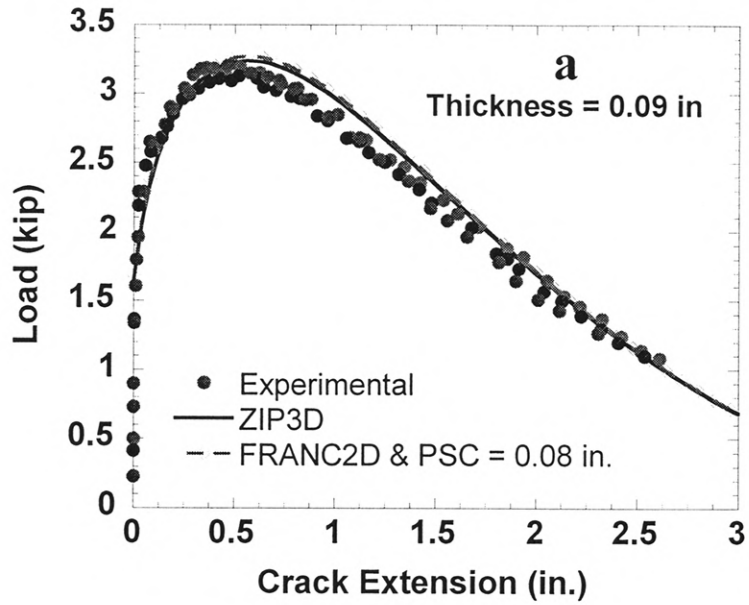
based computational methods (three-dimensional and two-dimensional with PSC) agreed very well with each other. Initially, both of the computational load vs. crack extension curves lay at the lower edge of the scattered band of the experimentally measured data until the maximum load value was reached. Beyond maximum load, both of the computationally determined load vs. crack extension curves lay on the upper edge of the scatter band.

For the 0.25 in. thick specimen, Figure 7.17 b shows that both the two and three-dimensional computationally determined load vs. crack extension curves agreed very well. Considering the scattering in the load vs. crack extension curves for the 0.25 in. thick specimen, the computational procedure produced a reasonable simulation of the experimental data. The computationally determined load vs. crack extension curves lay consistently on the upper edge of the scattering band of the data.

For the 0.5 in. thick specimen, the two-dimensional FEM-based analysis with PSC matched the three dimensional FEM-based analyses closely in the initial part of the load vs. crack extension curve. The two-dimensional FEM-based analyses have slightly overestimated the loads produced by the three-dimensional finite element analysis beginning at approximately 1.26 in. of crack extension until the tail of the curve. Both of the computationally determined load vs. crack extension curves matched the experimental load vs. crack extension curve initially. Deviation between computational and experimental curves is observed in the crack extension range extending from 0.31-1.36 in. where both computational procedures have overestimated the measured loads.

The experimental and computational curves then agree well again beginning at 1.36 in. of crack extension until the end of the collected load vs. crack extension data points.

Figure 7.17 d shows the computationally and experimentally determined load vs. crack extension curves for the 1 in. thick specimen. The load vs. crack extension curve obtained using FRANC2D\L with PSC overestimated the ZIP3D generated curve initially. After approximately 0.8 in. of stable crack extension both of the computationally determined curves (ZIP3D and FRANC2D\L) matched each other closely. With regards to the experimental data, ZIP3D simulated the experimental data closely in the initial part until 0.55 in. of crack extension. Beyond 0.55 in. of crack growth, the ZIP3D results started to deviate from the experimentally determined curve. The FRANC2D\L with PSC analysis simulated curve deviated from the experimentally measured load vs. crack extension at an early stage (approximately 0.1 in. of crack extension) and overestimated the loads for the rest of the measured stable crack extension behavior.



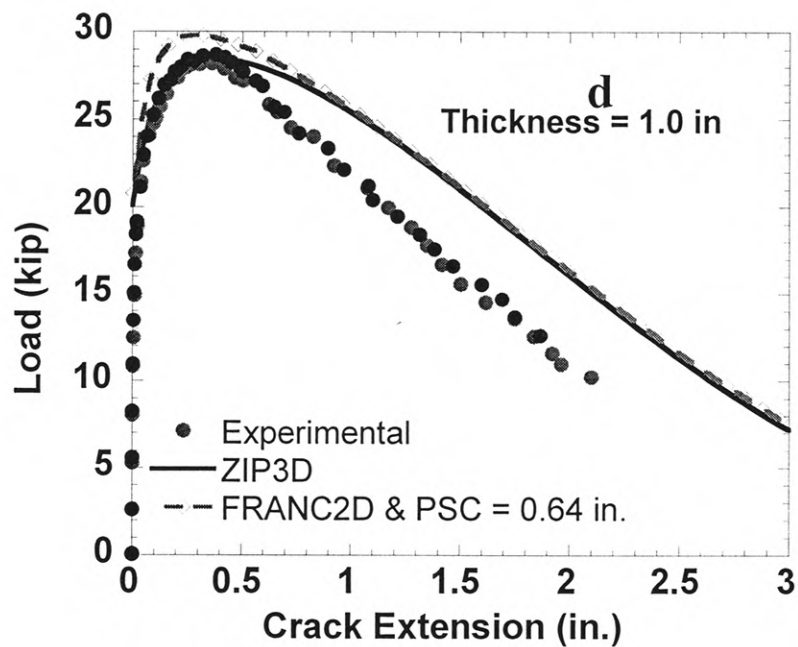
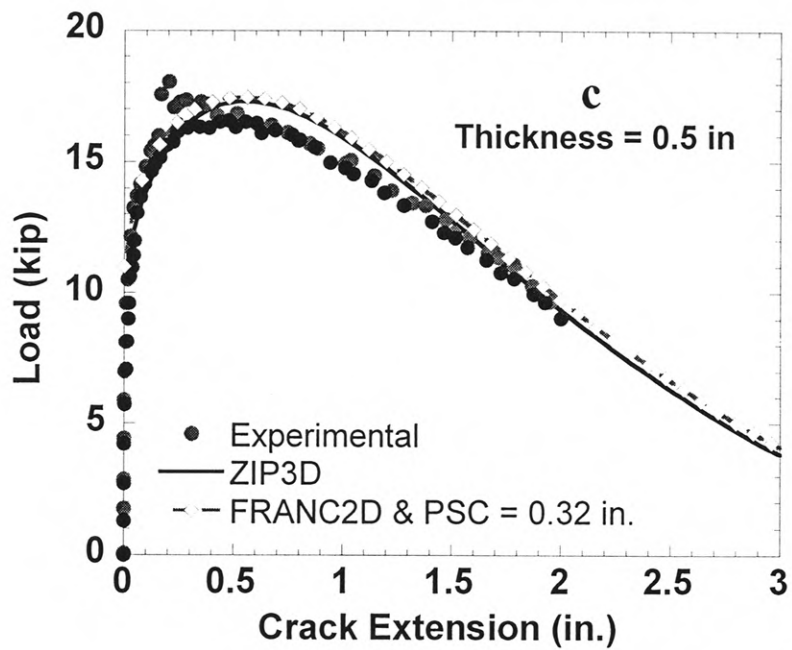


Figure 7.17: Comparison of Measured and Computational Load vs. Crack Extension Behavior for the (a) 0.09, (b) 0.25, (c) 0.5, and (d) 1 in. Thick Specimens

Chapter 8

Conclusions and Recommendations for Future Work

8.1 Summary and Conclusions

The objectives of this research project were to experimentally and computationally investigate the thickness effect on the *CTOA* fracture parameter and the plastic plane strain core height that is necessary to account for the material induced constraint in the two-dimensional FEM-based fracture analyses. Based on the work performed in this research to meet these objectives, the following conclusions are supported.

- 1) Stable tearing tests were conducted successfully on 8 in C(T) specimen with four thicknesses (0.090, 0.25, 0.50, 1.0 in.) for which the load, load line displacement, crack extension, and surface *CTOA* were measured. Measurements were used to obtain load vs. load line displacement, load vs. load crack extension, and *CTOA* vs. crack extension behaviors for all specimens. Reasonable repeatability was obtained among the specimens of each thickness group.
- 2) Digital image correlation was successfully used to measure rigid body displacement, linear elastic stress intensity factor, and *CTOA* at the surface of the specimen. It was then used to verify the *CTOA* measurements made by the optical microscope on one of the 0.090 in. thick specimens. The difference between the two measurement methods for the average *CTOA* in the constant *CTOA*- Δa region is approximately 4%.

- 3) Experimentally measured *CTOA* at the surface of the specimens (using OM) exhibited an initially high value followed by a leveling-off to a “constant” value after a short amount of crack extension. This behavior is in agreement with published *CTOA* characterization data from Aluminum alloys in thin sheet form.
- 4) The surface measurements of *CTOA* obtained in this research exhibited a ± 1.5 degree scatter band which is similar to the scatter band reported in the literature. Also, the average *CTOA* value (computed from the constant or “leveled-off” *CTOA* range) from the experimentally determined surface *CTOA* decreased with increasing specimen thickness.
- 5) Three-dimensional elastic-plastic FEM-based fracture analyses (ZIP3D) were performed to determine *CTOA* at the center of the specimen and study the variation through the specimen thickness. These analyses showed that the *CTOA* has a maximum value at the center of the specimen thickness and decreases toward the exterior of the specimen and attains a minimal value at the surface of the specimen. The difference between the computationally determined surface and center *CTOA* values increased with increasing specimen thickness.
- 6) The experimentally measured surface *CTOA* values were found to lie between the computationally determined center and surface values.
- 7) The three-dimensional finite element analyses produced reasonable simulations of the measured load vs. load line displacement behavior and load vs. crack extension behavior of the thicknesses evaluated. It should be noted that the maximum applied loads were always predicted accurately since they were used to generate the *CTOA* at the center of the specimen. The deviation then appeared in

the ability of the three-dimensional FEM-based analyses in predicting crack extension and load line displacement. Overall, the FEM-based analyses produced excellent approximation in predicting the load line displacement and crack extension.

- 8) Two-dimensional elastic-plastic FEM-based fracture analyses were employed to determine the plane strain core height necessary to account for the three-dimensional constraint present in the various thicknesses of test specimens. The plane strain core height was found to increase with increasing specimen thickness. The plane strain core height values are on the order of specimen thickness for the thinnest specimens (0.090 and 0.25 in.), but were found to be less than specimen thickness for the other two thicknesses (0.50 and 1.0 in.). Incorporating the plane strain core into the analyses improved the accuracy of the two-dimensional FEM analyses in predicting maximum applied loads and displacements.
- 9) The two-dimensional FEM analyses using plane stress with PSC were able to produce reasonable simulation of experimentally determined load vs. load line displacement and load vs. load crack extension behaviors of the entire tested specimen thicknesses except for the 1.0 in. thick specimen. Again, since the maximum applied loads were used to generate the PSC height, the maximum load predictions were generally accurate with most inaccuracy occurring for the 1.0 in thick specimen. Overall the two-dimensional finite element analysis using plane stress with PSC produced reasonable approximation of load line displacements and crack extensions.

- 10) The fracture surfaces for the 1.0 in. thick specimens exhibit flat fracture throughout the stable tearing process while a transition from flat to slant fracture was observed for all specimens of the other thicknesses. The length of the flat + transition region increases with increasing specimen thickness.
- 11) Overall, based on the results of this research, it has been shown that the application of the critical *CTOA* characterization process becomes increasingly more involved as specimen thickness increases. This is due primarily to the effects of tunneling and *CTOA* variation through the thickness that must be addressed to correctly characterize the critical *CTOA*. However, as standard practices are developed and verified, it is believed that the critical *CTOA* can be effectively characterized for thicker specimens.

8.2 Recommendations for Future Work

Upon completion of this project, a number of areas of potential future work have been identified that could provide additional insight into the effect of thickness on the applicability of the *CTOA* fracture criterion. A summary of these suggested areas is provided below.

- 1) Investigate crack tunneling both experimentally and computationally to determine how this phenomena affects the *CTOA* variation across specimen thickness and how increasing the thickness of the specimen influences that variation.
- 2) Evaluate additional thicknesses in the range of 0.090-0.250 in. to determine whether there is a transitional thickness at which material constraint would force the *CTOA*-thickness dependence to change.

- 3) Perform residual stress analyses on the test specimens and incorporate any residual stress into the analysis.
- 4) Use the *CTOA* values determined in this project (in conjunction with 2D and 3D FEM-based fracture analyses) to simulate the residual strength of more complicated pre-cracked test panels to evaluate the feasibility of using the *CTOA* fracture criterion in structural applications.
- 5) Conduct more thorough fracture surface examinations (using both optical and SEM) and determine the effects of transition from flat to slant fracture.

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Appendix A

This appendix illustrates the computational procedure employed to correct for the load line displacement measurements recorded by the internal LVDT of the 55 kip machine. The procedure involved two-stage mapping that utilizes the load line displacement from the internal LVDT of the 55 kip machine to estimate what the external LVDT would read if it was used. The first part of the mapping procedure entailed using the 0.5 in thick specimens data to correlate the internal LVDTs displacements for both machines. The next step was to develop a correlation between the external LVDT and the internal LVDT of the 22 kip machine. Establishing correlation of the internal LVDT involved two steps. First, a function that would predict the LVDT displacement of the 22 kip machine based on the load was determined. To establish such a function, the measured loads were normalized with respect to the maximum load then a third order polynomial function was fitted to the rising portion of the normalized load and a sixth order polynomial was fitted to the descending portion of the normalized load. The following equation illustrates the proposed two-ranged function:

$$V_{2kip} = \begin{cases} Ax^3 + Bx^2 + Cx & V \leq V@P_{max} \\ Dx^6 + Ex^5 + Fx^4 + Gx^3 + Hx^2 + Ix + K & V > V@P_{max} \end{cases} \quad (A.1)$$

$$x = \frac{P}{P_{max}}$$

where P is the load, $P_{\max}$ is the maximum load, V is measured load line displacement, and $V_{22\text{kip}}$ is the fitted function that predicts load line displacement. The function in the above equation was fitted to the data obtained for the 0.5 in thick specimen tested on the 22 kip machine. Table A.1 shows the obtained values of the coefficients based on the data for both 0.5 in thick specimens tested on the 22 kip machines. Also, Table A.1 shows the average of the coefficients. The average values of the coefficients were considered the appropriate values.

The next step of correlating the internal LVDTs of both machines involved using the function obtained above to evaluate the load line displacement for the 0.5 in thick specimen tested on the 55 kip machine based on the measured loads for that specimen. Next, a power law curve fit was determined to map the measurements of the 55 kip machine LVDT displacement ($V_{55\text{kip}}$) to the 22 kip machine LVDT displacement ($V_{22\text{kip}}$). Equation shows how the mapping this mapping function was find:

$$V_{22\text{kip}} = a(V_{55\text{kip}})^b \quad (\text{A.2})$$

where a and b are determined from the curve fit. The value for a was 1.125315 and the value for b was 1.017334.

After the characteristics of the two internal LVDT were mapped, it was necessary to map the characteristics of the external LVDT to the internal LVDT of the 22 kip machine. A power law curve similar to that used in the above equation was used to relate the external LVDT measurements to the internal LVDT of the 22 kip machine. This power law function is defined in the following equation as:

$$V_{external} = c(V_{22Kip})^d \quad (A.3)$$

where c and d are determined from the curve fitting procedure. The c and d coefficients were determined for each specimen thickness. Next the average coefficient values for each specimen thickness were found then extrapolated using second order polynomial to determine the c and d value for the 1 in thick specimen. Table A.2 presents the c and d values obtained for all specimens where the external LVDT was used.

Table A.1: Coefficients of Equation A.1

Coefficient	First Specimen	Second Specimen	Average
A	0.12222	0.137666	0.129943
B	-0.10055	-0.13425	-0.1174
C	0.130744	0.143298	0.137021
D	1.789774	-14.7398	-6.47501
E	-8.53224	50.07785	20.77281
F	15.37351	-66.1148	-25.3707
G	-14.1456	41.56701	13.71068
H	7.432386	-11.5815	-2.07456
I	-2.47932	0.333311	-1.07301
J	0.723343	0.616717	0.67003

Table A.2: Coefficients of Equation A.3

	c	d
0.09 in. Thick Specimen 2	1.003403	1.045699
0.09 in. Thick Specimen 3	1.019428	1.047949
0.25 in. Thick Specimen 1	1.000631	1.049767
0.25 in. Thick Specimen 2	1.005762	1.054656
0.25 in. Thick Specimen 3	1.026181	1.066369
0.5 in. Thick Specimen 1	1.077048	1.139574
0.5 in. Thick Specimen 2	1.067722	1.11105
1 in. Thick Specimen Extrapolated	1.4237	1.4545