ANALYSIS AND OPTIMAL DESIGN OF PRESTRESSED CONCRETE FOLDED PLATES

by

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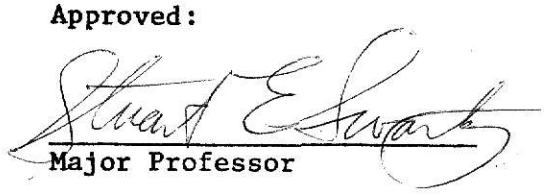
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NOTATIONS

c - torsional moment of inertia of the edge beam

c_d - concrete cover

d_n - thickness of a plate

D_n - longitudinal stress at n^{th} ridge due to prestressing only

DP - direct prestress

E_n - eccentricity of the parabolic prestressing cable, measured at midspan from the line joining the two end points of the cable

EP - eccentric prestress

f_{ci} - concrete strength in compression at transfer

f_{ti} - concrete strength in tension at transfer

f_{co} - concrete strength in compression at final load

f_{to} - concrete strength in tension at final load

h_n - plate width

I_n - moment of inertia of the transverse cross-section of the n^{th} plate about an axis passing through the c.g. of that cross-section and in a direction normal to the plane of the plate

J_n - moment of inertia of a unit longitudinal width of the cross-section of n^{th} plate.

K_n - rotation coefficient of n^{th} plate

ℓ - span of the folded plate

$M_{p_{max}}$ - maximum plate moment

M_{px} - plate moment at a distance x from midspan

$M_{t,p}$ - transverse moment - primary

$M_{t,s}$ - transverse moment - secondary

N_x - total longitudinal shear force

p' - plate load (net) per unit length

P_n - prestressing force in the n^{th} plate

$S_{n,n-1}$ - plate load at joint n, in the direction of joint n-1, per unit length

SU_n - sag constraint of n^{th} plate from upper limit

SL_n - sag constraint of n^{th} plate from lower limit

v_n - in-plane displacement of n^{th} plate

w_t' - component of gravity load in the plane of plate per unit length

x,y - plate coordinate directions

x_n - optimal value of prestressing force in n^{th} plate

y_n - optimal value of sag in n^{th} plate

$\sigma_{x,p}$ - longitudinal stress - primary

$\sigma_{x,s}$ - longitudinal stress - secondary

$\sigma_{x,n}$ - longitudinal stress at n^{th} ridge

$(\sigma_{xd})_n$ - longitudinal stress due to self weight, at n^{th} ridge

$(\sigma_{x\ell})_n$ - longitudinal stress due to live load, at n^{th} ridge

$(\sigma_{xw})_n$ - longitudinal stress due to wind load, at n^{th} ridge

$(\sigma_{xp})_{j,n}$ - longitudinal stress at n^{th} ridge, due to direct prestressing effect on j^{th} plate

$(\sigma_{xe})_{j,n}$ - longitudinal stress at n^{th} ridge, due to eccentric prestressing effect in j^{th} plate

ε_x - strain in x-direction

ε_y - strain in y-direction

τ - shearing stress

θ - angle which the algebraic maximum principal stress makes with x-axis

σ_1 - algebraic maximum principal stress

σ_2 - algebraic minimum principal stress

θ_{x0} - angle which the tangent to the prestressing cable at the end span makes with x-axis

Δ_{n0} - relative displacement between n^{th} and $n+1^{st}$ joints - secondary analysis

Ψ_{n0} - arbitrary rotation of n^{th} plate - secondary analysis

Ψ_n - actual rotation of n^{th} plate

ϕ_n - plate angle w.r.t. horizontal measured clockwise from the direction of the plate pointing towards the next plate

γ_n - plate deflection angle, measured clockwise from the extended direction of the plate to the direction of the next plate

μ - Poisson's Ratio

Chapter 1

INTRODUCTION

A folded plate structure is defined as a system of thin plates spanning longitudinally and monolithically joined to each other along longitudinal joints at some angle other than 180°. The structural behavior of folded plates is characterized by "slab" and "plate" actions. The loads acting normal to each plate cause it to bend transversely between the ridges as a continuous "slab". The plates supported at their ends on end-diaphragms, bend due to the in-plane "plate" loads. This behavior of the folded plate gives rise to longitudinal ridge stresses and transverse bending moments at ridges. With increasing spans, the longitudinal tension assumes very high values. This demands large quantities of steel resulting in an uneconomical design presenting problems in reinforcement congestion, splicing of steel, etc. These problems can be overcome by post-tensioning the folded plates which results in the following advantages:

1) Longitudinal tension is reduced with increasing span, thus saving steel.

2) The deflection of the structure is controlled.

3) A crackless structure which offers greater resistance to weathering agents becomes possible.

It may be stated that the object of prestressing is to place the folded plate into a pure membrane state.

Statement of the Problem

The objective of this thesis is to develop a method for analysis of prestressed folded plates and to determine the optimal selection of prestressing forces and cable locations. First, the tedious analysis of

the folded plate will be carried out by computer and the output will give longitudinal ridge stresses, transverse ridge moments and principal stresses and their directions. The basic method of analysis will be that by Simpson (10). In order to check the computer program, examples already worked out by several authors will be carried out. The computer program will include the analysis for wind load in addition to prestressing and gravity loads. Analysis of folded plate for principal stresses and directions by Guralnick and Swartz (5) will be modified to incorporate the effect of prestressing. For any combination of loadings, a method for the selection of prestressing forces, cable sag and location, leading to an "optimal value" (minimum quantity of cable) will be developed. Folded plate analysis by beam theory will be compared with the Simpson method. The solution of prestressed folded plates using a finite difference method by Wayne Klaiber, Martin J. Gutzwiller and Robert H. Lee (12) will be compared with the method developed in this thesis. Finally, the validity of superposition will be checked.

Previous Work in This Area

Analysis and design of prestressed concrete folded plates has been carried out by John C. Brough and B. H. Stephens (3), treating the folded plate as a beam simply supported on end diaphragms. Analysis of prestressed folded plates by G. S. Ramaswamy (9) assumes the parabolic curve of the cable to be approximated by a sine curve. D. Yitzhaki (14) in his work suggests that shortening effects and bending effects due to prestressing be treated individually. But for different types of prestressing little work has been done to compute the longitudinal stresses and transverse moments, at different points along the longitudinal axis,

treating the direct and eccentric effect of prestressing individually. Also, no one has suggested any systematic method to determine where, and how much prestressing should be applied to take care of various combinations of loadings. The general practice is a "trial and error approach" which may not, in most of the circumstances, lead to an optimum design. Also, no attempt has been made to check the validity of the superposition principle in the case of prestressing and other loads acting simultaneously.

Basic Assumptions

1) The material is elastic, homogeneous and uncracked.

2) The actual deflections are minor relative to plate width and length. Consequently, equilibrium conditions for a given plate may be developed using the configuration of undeflected plate.

3) The principle of superposition is valid. (This assumption will be checked later.)

4) The structure is monolithic and joints are rigid.

5) The length of each plate is more than twice its width.

6) In all plates plane sections remain plane after deformation. (It is, however, to be carefully noted that a plane cross section of the entire structure does not necessarily remain plane after deformation.)

7) Each supporting end diaphragm is infinitely stiff parallel to its own plane, but perfectly flexible, normal to its plane.

8) The strain is assumed to vary linearly across the width of each plate.

Chapter 2 contains a description of the method of analysis used and the development of a computer program to determine longitudinal ridge stresses, transverse ridge moments and principal stresses and their

directions in folded plates. The computer program will analyze prismatic folded plates of any cross-section for gravity load, wind load and prestressing (parabolic draping or straight cable with or without eccentricity). The torsional stiffness of the edge beams is also included.

Chapter 3 contains several numerical examples.

Chapter 4 contains a comparison between the proposed method of analysis, and the beam method of analysis and the method of analysis using the finite difference solution of governing differential equations.

Chapter 5 describes a check on the validity of the principle of superposition.

Chapter 6 deals with the optimal selection of prestressing forces and cable sag. This chapter also includes several numerical examples.

Chapter 2

ANALYSIS OF FOLDED PLATES

2.1 Method of Analysis for Gravity Loads

Analysis of folded plates by H. Simpson (10) has been adopted for the development of the computer program. This is one of the methods recommended by the ASCE Task Committee (8).

Assumptions

1) In addition to the assumptions listed previously, the relative displacement Δ_n, between two adjacent joints, is assumed to vary sinusoidally along the longitudinal axis. If we denote Δ_{no} at midspan to be $(\Delta_{no})_c$,

$$\Delta_{no} = (\Delta_{no})_c \sin \frac{\pi x}{\ell}$$

where ℓ is the span and x is distance measured along the span from the end.

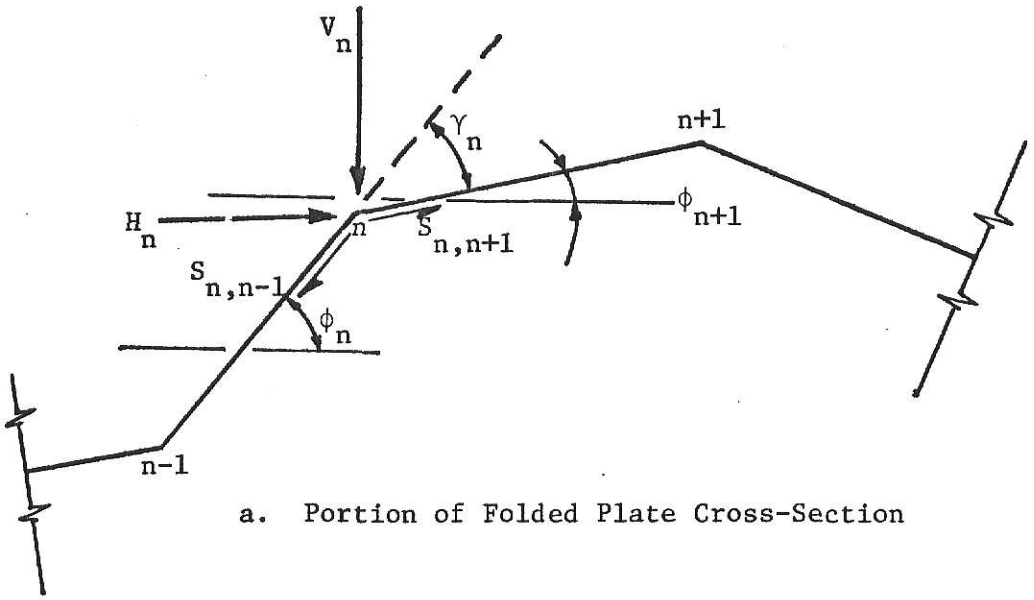
2) End plates are treated as cantilevers.

Outline of Method

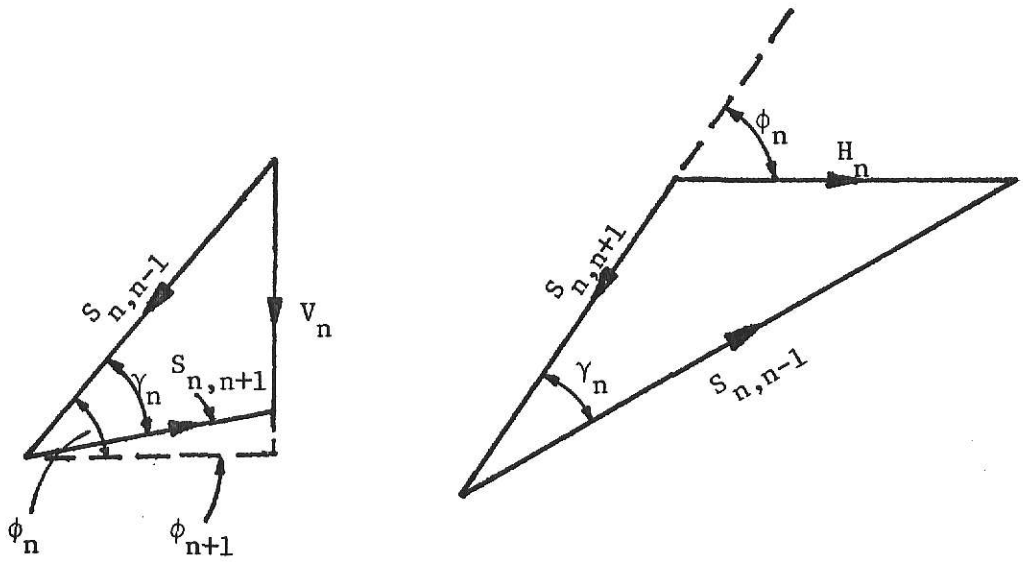
Step 1 - Consider a transverse section of unit length at midspan of the given folded plate. Assuming that the joints do not deflect arrive at joint moments by moment distribution. Compute the reactions at joints and apply equal and opposite forces, V_n and H_n, at these joints. Resolve these applied forces into plate loads. Refer to Fig. 1 for force vector diagram. Compute the longitudinal bending stresses caused by the in-plane loads, $S_{n,n-1}$, etc., assuming the plates to be free to bend independently. These stresses are called free-edge stresses. Next, stress compatibility at the common edges is

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a. Portion of Folded Plate Cross-Section



$$S_{n,n-1} = V_n \text{ Cos } \phi_{n+1}/\text{Sin } \gamma_n \qquad S_{n,n-1} = H_n \text{ Sin } (\phi_n - \gamma_n)/\text{Sin } \gamma_n$$

$$S_{n,n+1} = V_n \text{ Cos } \phi_n/\text{Sin } \gamma_n \qquad S_{n,n+1} = H_n \text{ Sin } \phi_n/\text{Sin } \gamma_n$$

b. Force Diagram at Joint n

Fig. 1 - Force Resolution at a Ridge of a Folded Plate

established by stress-distribution which is analogous to moment distribution. These final stresses are referred to as the primary solution.

Step 2 - The effects of relative joint displacements have now to be taken into account. Having treated the end plates as cantilevers, let joint 3 deflect by an arbitrary amount $\Delta 20$ below the level of joint 2, as shown in Fig. 2. The fixed end moment induced at joints 2 and 3 is $6EJ_2 \Delta_{20}/h_2^2$. Let the rotation of plate 2, Ψ_{20} be such that the magnitude of the moment induced is 6. Since $\Psi_{20} = \Delta_{20}/h_2$, then $\Psi_{20} = h_2/EJ_2$. The arbitrary rotation Ψ_{20} is related to the actual rotation Ψ_2 by, $\Psi_2 = K_2 \Psi_{20}$. These arbitrary moments at joints 2 and 3 are then distributed by moment distribution. Joint moments and reactions are found. Equal and opposite forces are applied at joints, resolved into plate loads, and free-edge stresses and hence, final edge stresses by stress distribution, are computed. This solution corresponds to the secondary solution due to arbitrary rotation of plate 2. Similarly, secondary solutions corresponding to arbitrary rotations of all plates except the first and last are found.

Step 3 - Now, expressions for primary and secondary plate deflections will be derived. (Refer to Fig. 3)

Primary Plate Deflection: Let $\sigma_{x,n}$ and $\sigma_{x,n+1}$ be the fiber stresses of the plate n at midspan. The plate moment at midspan, Mp_{max} is given by

$$Mp_{max} = \frac{1}{2} (\sigma_{x,n} - \sigma_{x,n+1}) \, d_n h_n \cdot h_n/6$$

where d_n is the thickness of the plate and h_n is the width of the plate.

$$Mp_{max} = \frac{1}{12} (\sigma_{x,n} - \sigma_{x,n+1}) \, d_n \cdot h_n^2 \qquad \text{---------- (2.1)}$$

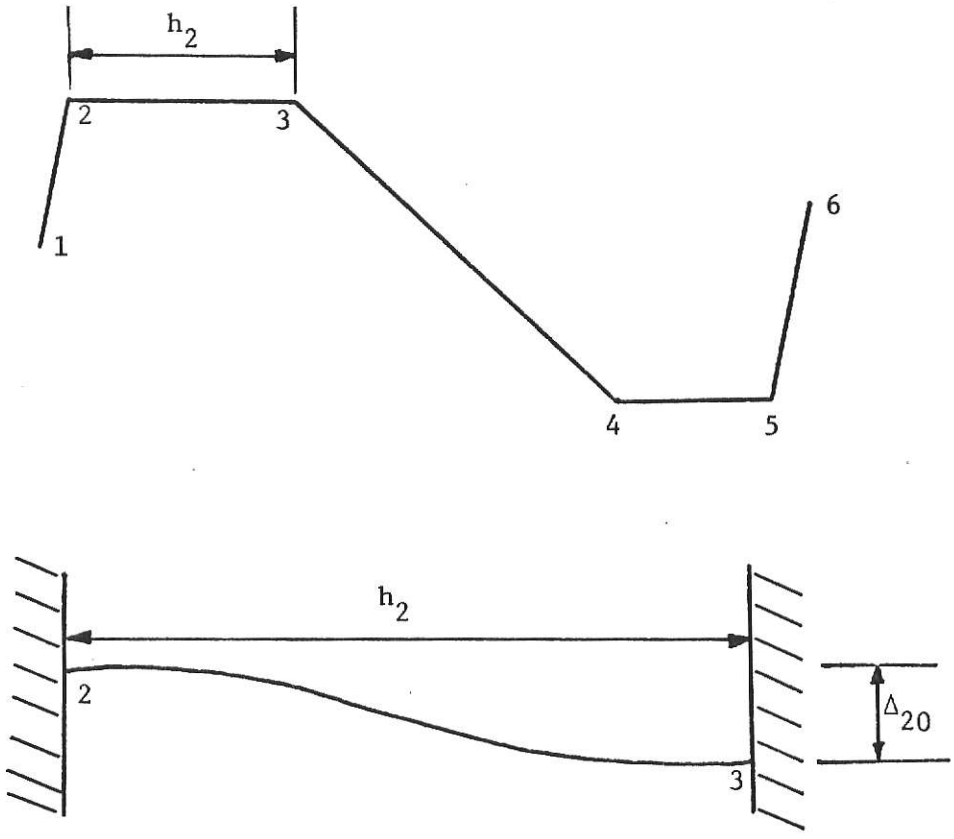


Fig. 2 - Structure Cross-Section Showing
Relative Joint Displacement

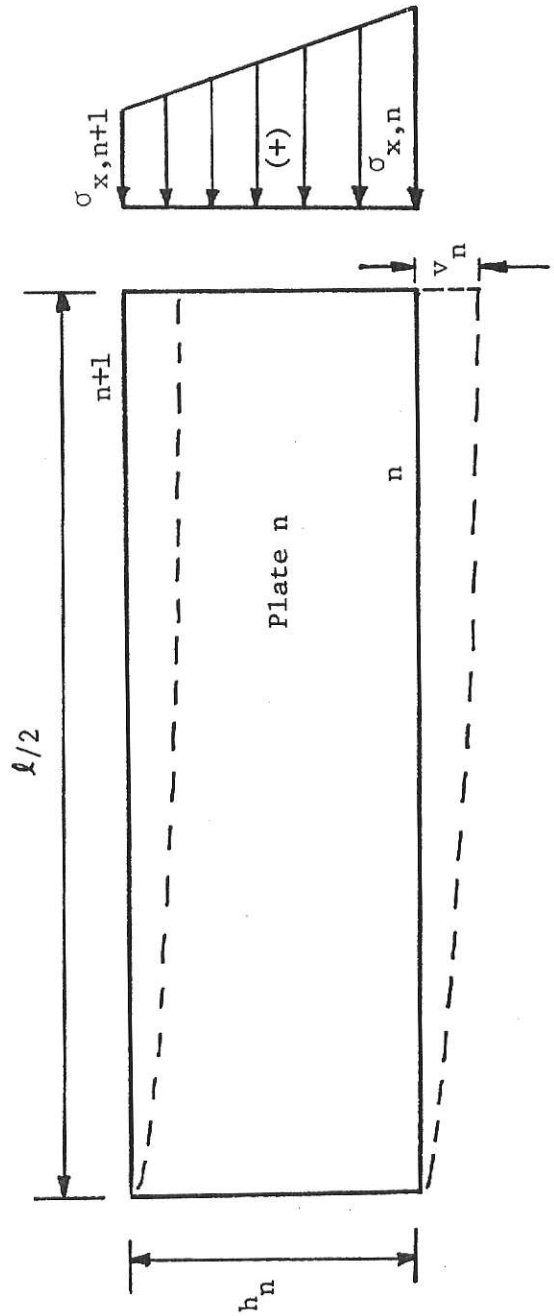


Fig. 3 - Distributed Plate Longitudinal Stresses and Deflections

The maximum plate moment Mp_{max} due to uniform plate load p' is given by

$$Mp_{max} = p'\ell^2/8 \text{ ---------- (2.2)}$$

Equating (2.1) and (2.2) and knowing that the plate deflection at midspan due to uniform load p' is,

$$v_n = \frac{5}{384} (p'\ell^4/EJ_n), \text{ then}$$

$$v_n = \frac{5}{48} \frac{\ell^2}{Eh_n} (\sigma_{x,n} - \sigma_{x,n+1}) \text{ ---------- (2.3)}$$

Secondary Plate Deflection: The loading is proportional to the rotation, Ψ_{no}, of the plate, for which a sine variation along the span has been assumed. Two integrations of this loading will yield the bending moment at the center of the span which will be proportional to $\Psi_{no} (\ell^2/\pi^2)$.

$$Mp_{max} \alpha \Psi_{no} \ell^2/\pi^2$$

Similarly, midspan deflection is proportional to $(\ell^4/\pi^4 EJ_n)\Psi_{no}$. Hence, the deflection at midspan may be obtained by multiplying the bending moment at that section by $\ell^2/\pi^2 EJ_n$. Also,

$$Mp_{max} = \frac{1}{12} (\sigma_{x,n} - \sigma_{x,n+1}) d_n h_n^2$$

Hence, $$v_n = \frac{\ell^2}{\pi^2 Eh_n} (\sigma_{x,n} - \sigma_{x,n+1}) \text{ ---------- (2.4)}$$

Step 4 - From plate deflections v_n, plate rotations, Ψ_n can be computed. (Refer to Fig. 4 and Reference 9)

$$\Psi_n = \frac{1}{h_n} [v_{n+1} (\cot \gamma_n + \cot \gamma_{n-1}) - \frac{v_{n+2}}{\sin \gamma_n} - \frac{v_n}{\sin \gamma_{n-1}}] \text{ -- (2.5)}$$

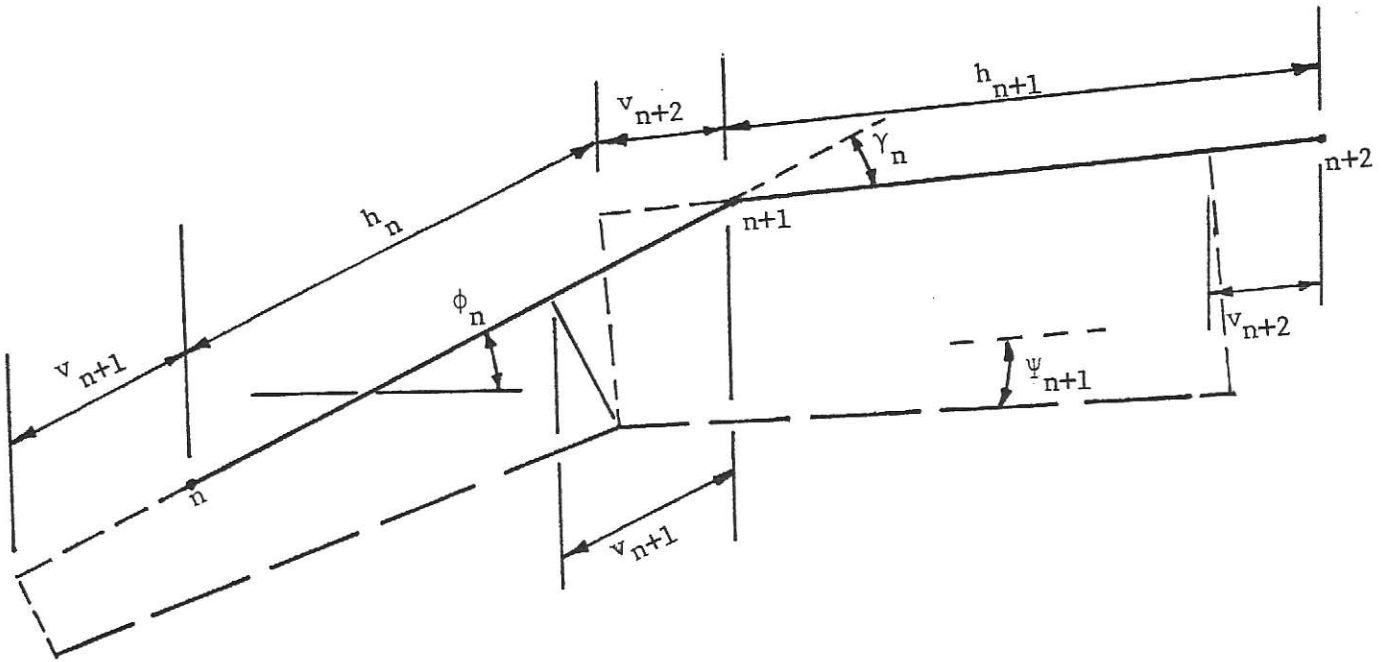


Fig. 4 - Portion of the Cross-Section of a Folded Plate Showing Position of Plates After Deformation

Also we know

$$\Psi_n = K_n \Psi_{no}, \text{ where } \Psi_{no} = h_n/EJ_n \quad \text{---------- (2.6)}$$

From equations (2.5) and (2.6) we can calculate K_n (ie K_2, K_3...) solving (n-2) simultaneous equations.

Step 5 - The final solution of longitudinal stresses and transverse moments, at ridges, is computed by adding K_n times the secondary solution due to rotation of plate n, for all n-2 plates.

Variation of Longitudinal Stresses and Transverse Moments Along a Longitudinal Axis

Primary Solution: $\sigma_{x,p}$ varies parabolically and $M_{t,p}$ remains constant. If $\overline{\sigma}_{x,p}$ is the maximum σ_x at midspan for primary analysis,

$$\sigma_{x,p} = 4 \cdot \overline{\sigma}_{x,p} \cdot x\,(\ell-x)/\ell^2 \quad \text{---------- (2.7)}$$

$$M_{t,p} = \text{constant}$$

where $\sigma_{x,p}$ and $M_{t,p}$ are longitudinal stress and transverse moment from primary analysis.

Secondary Solutions: If $\overline{\sigma}_{x,s}$ and $\overline{M}_{t,s}$ are maximum values at midspan,

$$\sigma_{x,s} = \overline{\sigma}_{x,s} \cdot \sin \frac{\pi x}{\ell}$$

$$M_{t,s} = \overline{M}_{t,s} \cdot \sin \frac{\pi x}{\ell} \quad \text{---------- (2.8)}$$

where $\sigma_{x,s}$ and $M_{t,s}$ are longitudinal stress and transverse moment from secondary analysis.

The computation of principal stresses and their directions as developed by Guralnick and Swartz (9) is the basis for the computer program presented in this thesis.

2.2 Analysis for Wind Loading

Wind loading is essentially load normal to the plane of the plate. It will be a positive pressure on windward plates and negative (suction) pressure on leeward plates. A comprehensive study was made by an ASCE committee on wind forces and a final report was published in the ASCE transactions. It recommends that roofs of buildings be designed for varying pressures depending on their slope. Fig. 5, from Reference (2), gives the value of wind pressure as a function of the slope of the plate and whether it is on the windward or leeward side. These recommendations are based on an assumed wind velocity of about 78 m.p.h. with due allowances for suction and drag effects. The analysis for wind loads is the same as described previously for gravity loads.

2.3 Analysis for Prestressing

The method of analysis is same as that by Billington (1), except that the variation of longitudinal stress along the length of the span will be assumed to be constant as opposed to the sinusoidal variation assumed by Billington. Fig. 6 shows the in-plane load due to eccentric prestressing, and constant plate moment due to eccentric straight cable. The expression for the angle of inclination of the prestressing force with horizontal, at end span, (i.e.) θ_{xo}, can be derived as follows. Referring to Fig. 6.a,

$$e = 4 \cdot e_n \cdot x\ (\ell - x)/\ell^2$$

where e_n is the eccentricity at midspan and e is the eccentricity at a distance x from end.

$$\frac{de}{dx} = \frac{-8 \cdot e_n \cdot x}{\ell^2} + \frac{4 \cdot e_n}{\ell^2}$$

$$= 4 \cdot e_n \cdot (\ell - 2x)/\ell^2$$

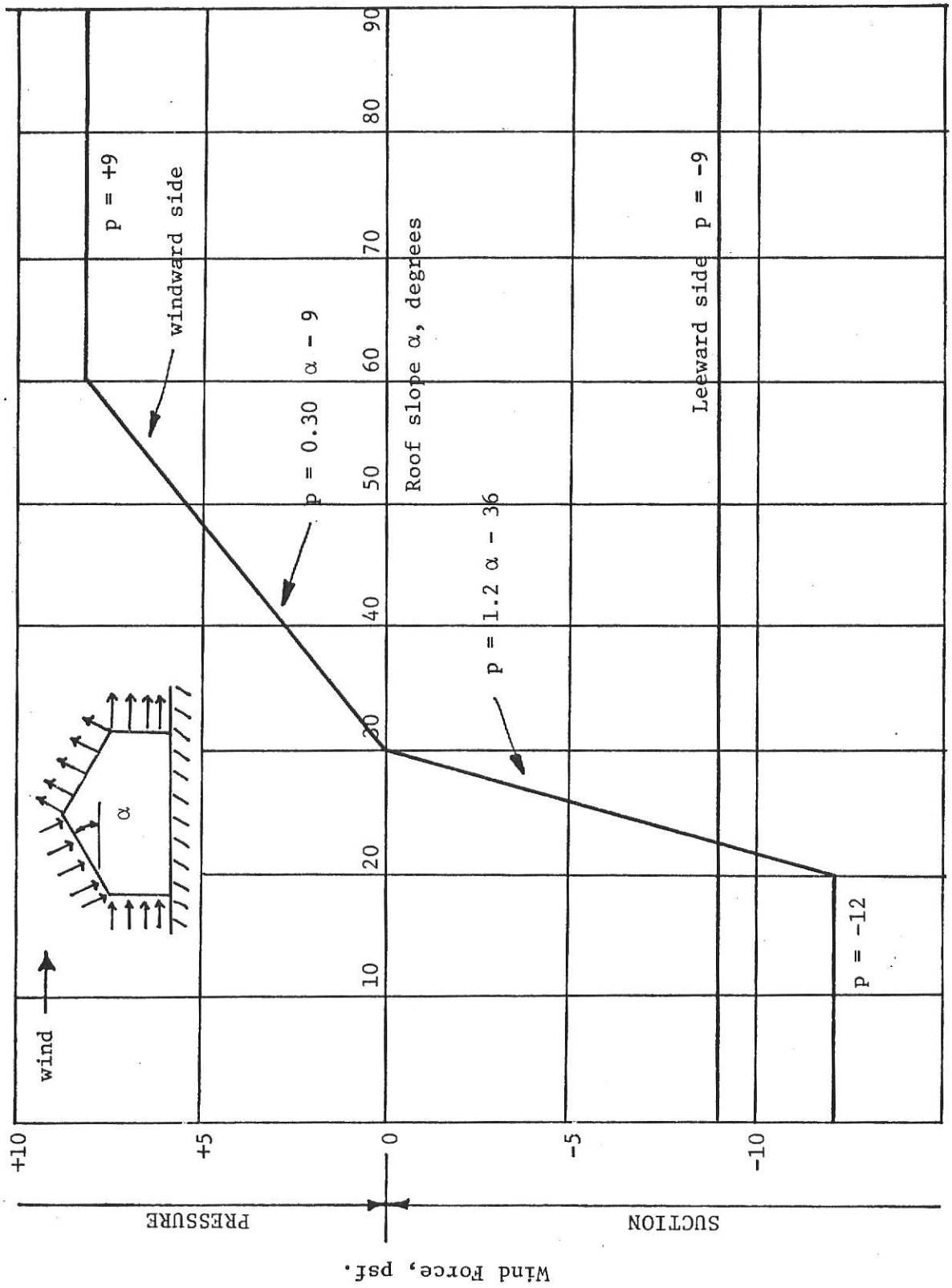
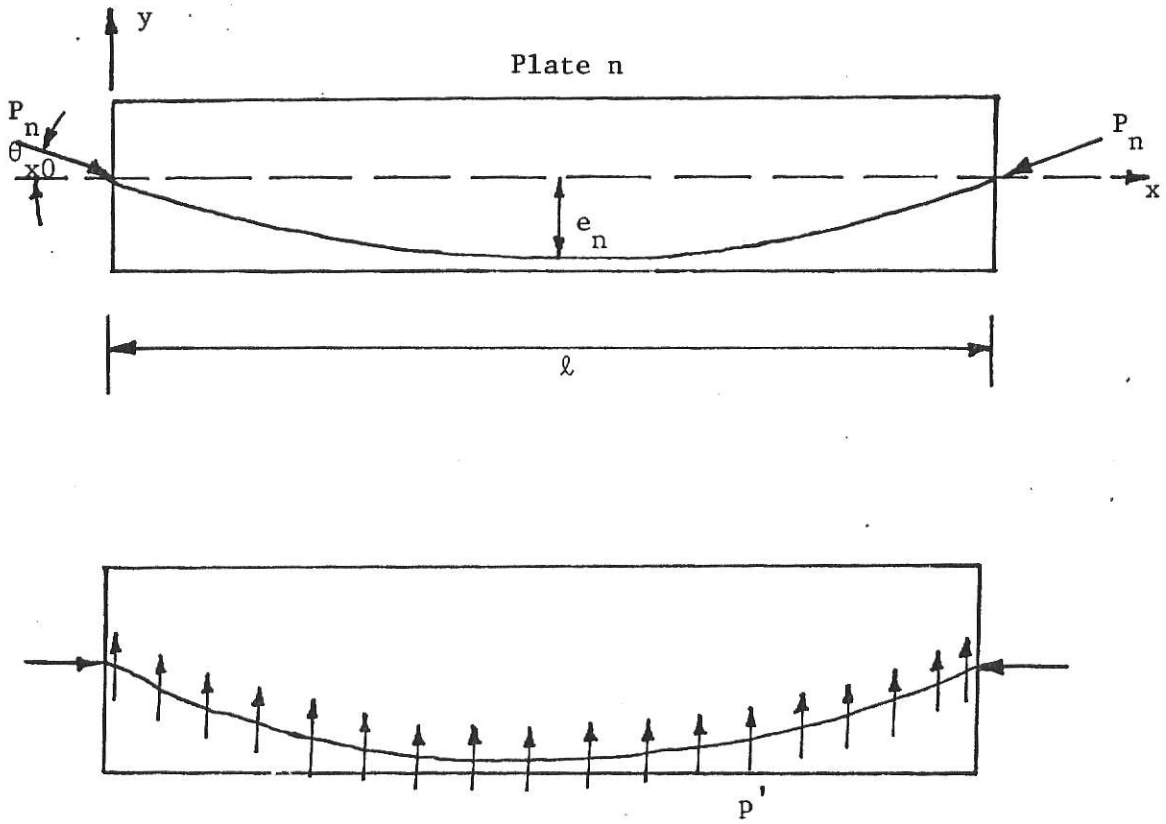
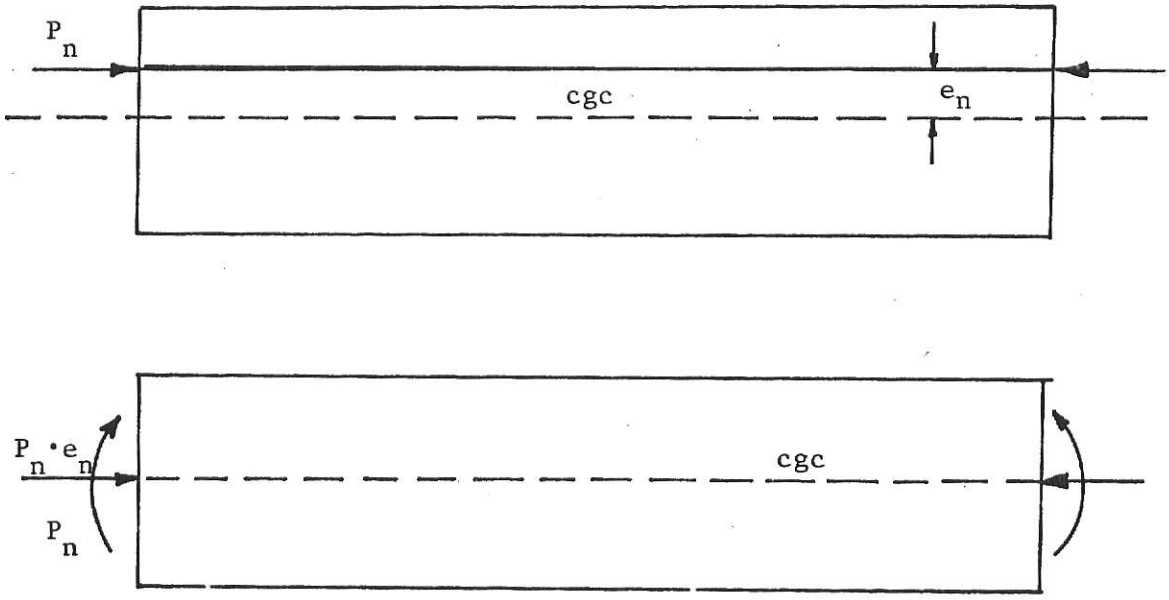


Fig. 5 - Wind Force on Buildings (ASCE Committee on Wind Forces)(2)



a. Parabolic Cable Profile



b. Straight Cable

Fig. 6 - Induced Plate Loads Due to Straight or Draped Prestressed Cables

$$\left.\frac{de}{dx}\right|_{x=0} = \tan \theta_{xo} = 4\, e_n/\ell$$

Hence, $\theta_{xo} = \tan^{-1} (4 \cdot e_n \cdot/\ell)$

and

horizontal component of prestressing force at end span =

$P_n \cdot \cos \theta_{xo}$

where P_n is the prestressing force, in nth plate.

The determination of stresses due to the in-plane loads caused by the draped cables follows the method given in Sec. 2.1. The direct forces due to prestressing create primary and secondary stresses which are also determined using the method described in Sec. 2.1.

2.4 Principal Stresses and Directions

Appendix A contains the method of calculation of principle stresses and directions for gravity loading, as developed by Guralnick and Swartz (5). Here a modification of this procedure to include prestressing effects is discussed.

The secondary solution for principal stresses is unchanged. However, the primary solution has to be modified. In addition to the parallel plate load from gravity loads, there will be a parallel plate load due to parabolic sag of the cable, given by $8\, P_n e_n/\ell^2$.

The method of analysis, once again is split into two, (i.e.):

1) The direct prestress, and

2) All other combinations of uniformly distributed loads including eccentric prestressing.

Since we know the longitudinal edge stresses (primary) corresponding to cases (1) and (2) individually, equilibrium equations can be written individually. In the case of direct prestress alone and considering the

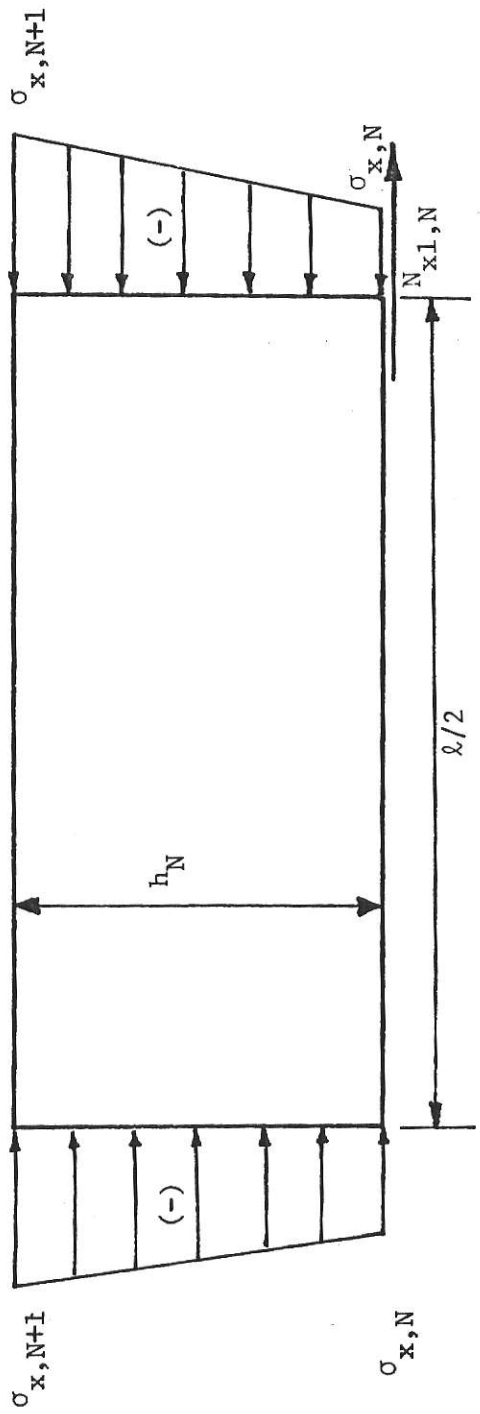


Fig. 7 - Primary Longitudinal Stresses in End Plate
Due to Direct Prestressing

equilibrium diagram, as shown in Fig. 7, the longitudinal stresses are constant along the length of the span and no edge shear force is produced at common ridges or elsewhere. That is,

$$N_{x\ (total)} = N_{x1} + N_{x2}$$

where N_{x1} = shear force due to direct prestressing alone

N_{x2} = shear force due to eccentric prestressing and other uniform loading

But, since $N_{x1} = 0$, then $N_{x\ (total)} = N_{x2}$

This point is to be kept in mind while deriving the normal stress and shearing stress equations; (i.e.) the values of longitudinal stresses used in these equations will not have those due to direct prestressing effect.

In the case of analysis for prestressing, the stress distribution close to the end spans will not be the same as the theoretical one. This is explained by St. Venant's principle. To obtain information about the distribution of stresses in the end-zones, one may refer to stress-distribution in end-block by Guyon (6). The actual stress distribution is quite complicated and beyond the scope of this thesis. But far from end spans the above analysis is found to be very much satisfactory. This will be proved later on when comparison with the finite difference method is carried out.

2.5 Torsional Stiffness of the Edge Beam

The edge beams, as shown in Fig. 8, can be treated as end plates in the analysis. But when the thickness of the beam is much greater than that of the adjacent plate, the torsional stiffness of the edge beam needs to be considered. Previously, the end plates were treated as

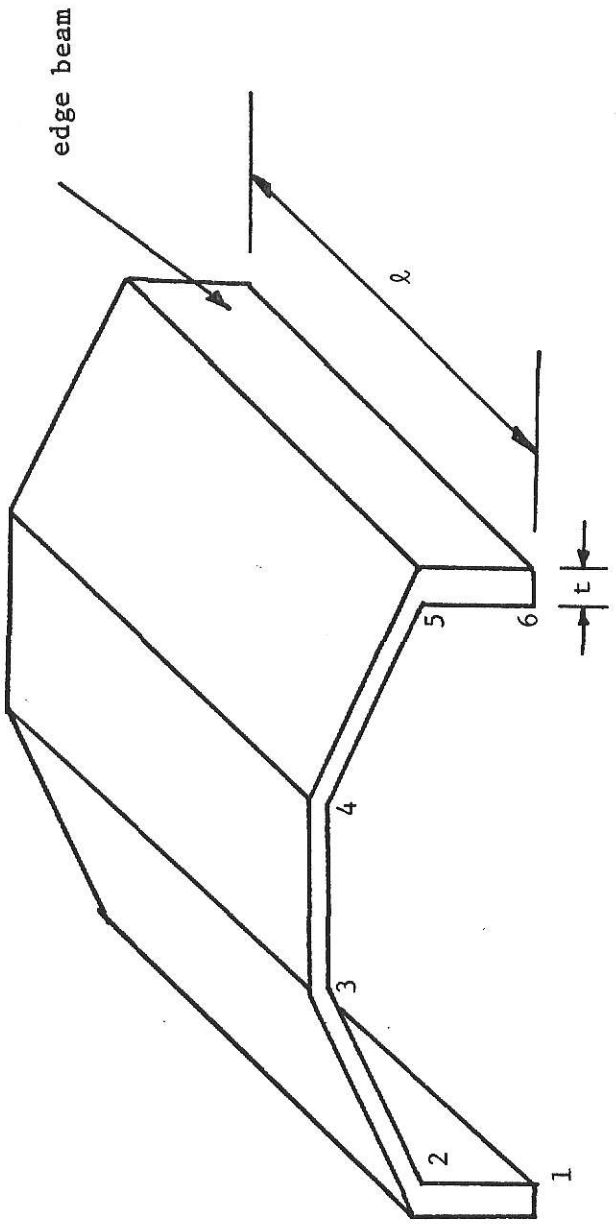


Fig. 8 - Folded Plate With Edge Beams

cantilevers. But in the case of edge beams it is necessary to compute the torsional stiffness of the edge beam in order to compute the distribution factors at joints 2 and N. Yitzhaki (14) has derived an expression for torsional stiffness of the beam which was consistent with his approximation of loading by a Fourier series expression in the case of the primary analysis. To be consistent with this primary analysis, it is necessary to find an expression for torsional stiffness of the edge beam which must be constant along its length, though actually it is not. The ACI Code Specification, as given in Reference 4 , indicates that torsional stiffness of a beam may be calculated by the expression

$$K_t = \frac{9E_{cs}c}{\ell \cdot (1 - c_2/\ell)^3}$$

where E_{cs} - modulus of elasticity of the slab concrete

c_2 - size of the wall, column bracket supporting the edge beam

c - pertains to the torsional rigidity of the edge beam, which for a rectangular cross section is given by Timoshenko, as in Appendix B.

$c_2/\ell = 0$

K_t edge beam $= \frac{9Ec}{\ell}$

K_b adj. plate $= 4EKx\ell = \frac{4E\ d_n^{\ 3}\ell}{12\ h_n}$

Distribution factor at joints 2 and N $= \frac{9Ec}{\ell} : \frac{4E\ d_n^{\ 3}\ell}{12\ h_n}$

$$= \frac{27c}{\ell^2} : \frac{d_n^{\ 3}}{h_n}$$

2.6 Computer Program for the Analysis of Folded Plates

The computer program developed consists of a main program which computes the input to two subroutines. One subroutine carries out moment distribution and stress distribution and the output from it includes longitudinal ridge stresses and transverse ridge moments. The main program after obtaining the results from this subroutine computes necessary coefficients to determine the 'K' values which are then computed from the second subroutine which is a linear equation solver. The final longitudinal ridge stresses, transverse ridge moments and principal stresses and directions are computed in the main program.

The program operates in single precision. If the number of plates is greater than 8, it is recommended to run it in double precision.

The flow chart is given in the following section. The program source listing appears in Appendix C. Input-output details are given in Appendix D.

For any combination of loading, (dead load, live load, wind load and prestress) longitudinal ridge stresses and transverse ridge moments can be found at any section along the length of span. Also for any combination of loading, at any point in the folded plate, principal stresses and their directions can be determined. Flow charts are shown in Figs. 9 and 10.

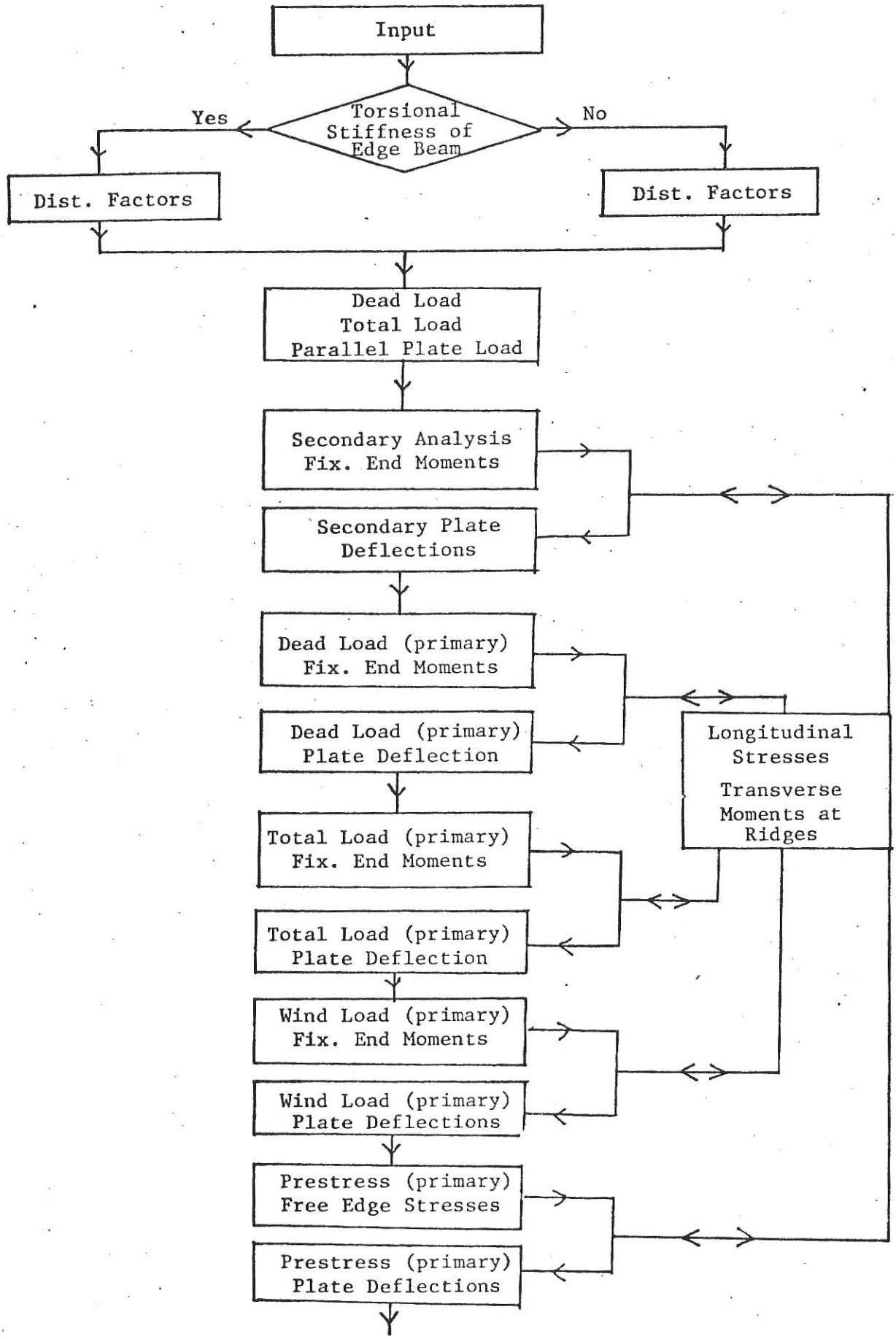


Fig. 9 - Flow Chart for General Folded Plate Analysis - Main Program

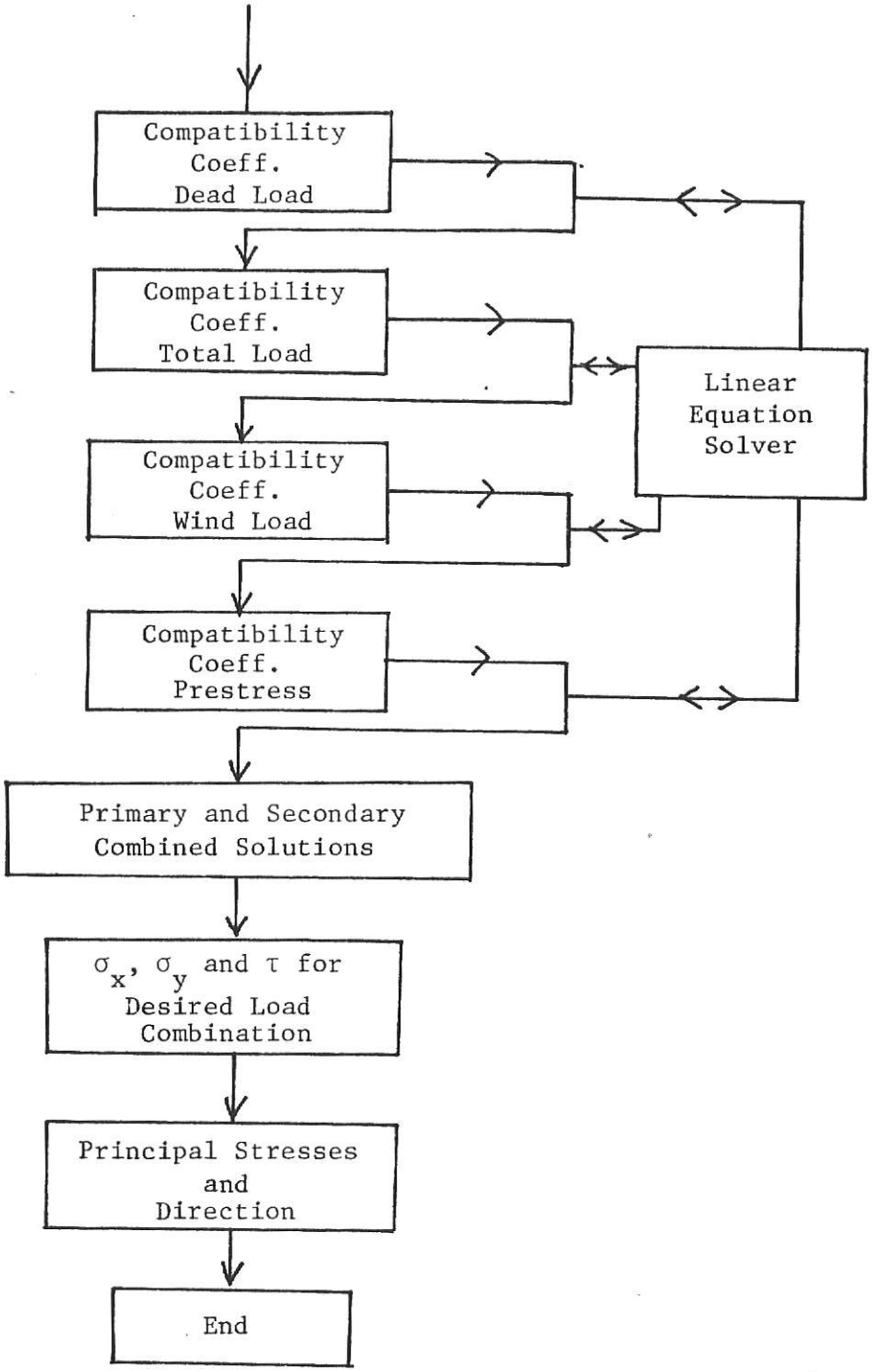


Fig. 9 Continued

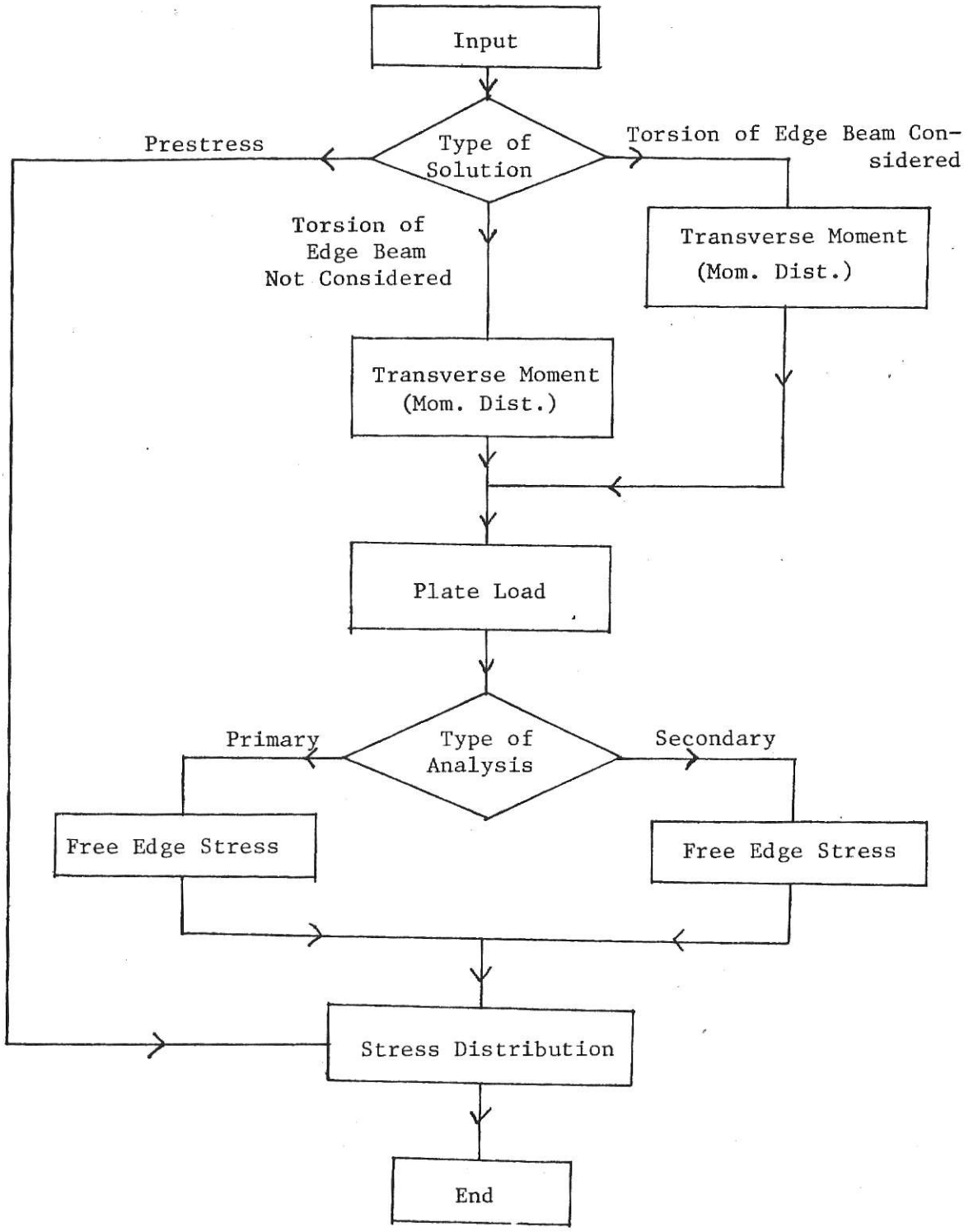


Fig. 10 - Flow Chart for General Folded Plate Analysis, Subroutine for Computation of Internal Tractions

Chapter 3

APPLICATION EXAMPLES

Example 3.1: This is an example worked out in Reference 9. Figure 11 shows the cross-section of the folded plate and gives the geometric properties.

LOADING:

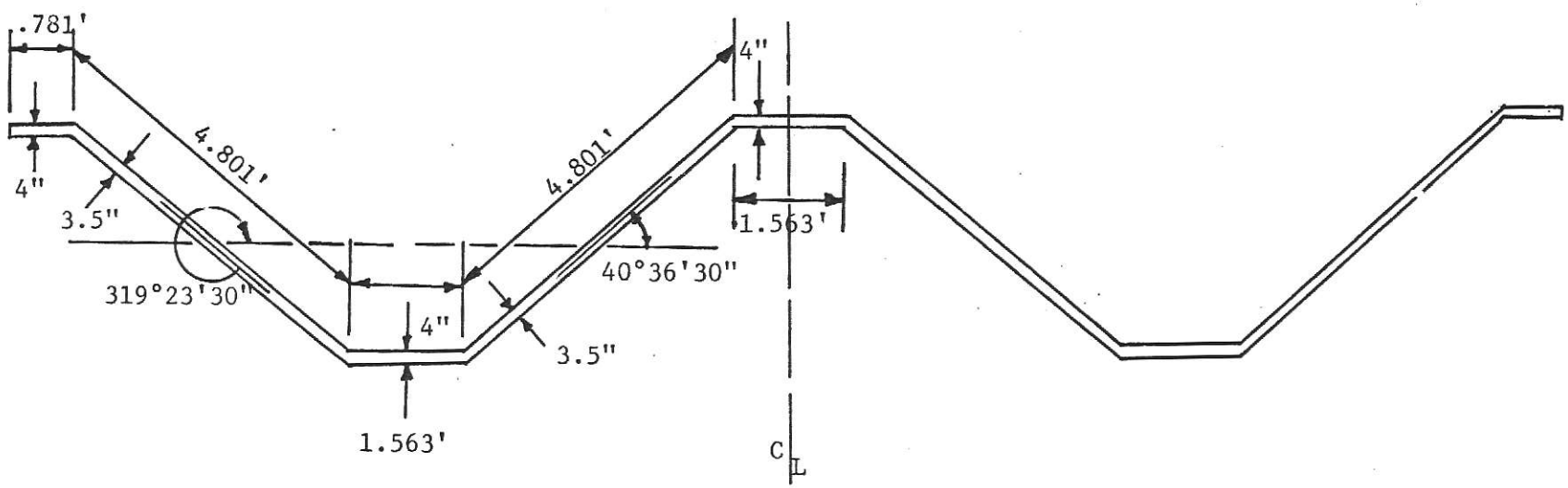
DEAD LOAD - DL = 12.5 psf (598.75 N/m^2) of plate/in. thickness

LIVE LOAD - LL = 15.0 psf (718.5 N/m^2) of surface area

WIND LOAD - Refer to Fig. 5 for wind pressure. According to ACI-Code, whenever wind loading is considered the allowable stress of concrete may be increased 33 1/3% or with same allowable stresses wind load can be reduced by 25%. Taking the latter into consideration,

PLATE	WIND PRESSURE, psf (N/m^2)
1	-9.0 (-431.1)
2	-6.75 (-323.33)
3	-9.0 (431.1)
4	2.25 (107.78)
5	-9.0 (431.1)
6	-6.75 (-323.33)
7	-9.0 (-431.1)
8	2.25 (107.78)
9	-9.0 (431.1)

(1 psf = 47.9 N/m^2)



Span = 60.0'

Dead Load = 12.5 psf. of plate/in. thickness

Live Load = 15.0 psf. of plate surface area

Cross-section symmetrical about C_L

Fig. 11 - Cross-Section of the Folded Plate Analyzed by Ramaswamy (9)

Table 3.1 - Comparison of Computed Results With Those Solutions at Midspan by Ramaswamy (9)

(1) Longitudinal Edge Stresses (psi)

Ridge	Reference 9	Computer Output	
	TL	TL	TL + WL
1	-1077	-1077.00	- 984.09
2	- 729	- 729.22	- 685.71
3	+ 738	738.97	691.34
4	+ 791	792.43	714.04
5	- 757	- 755.19	- 701.62
6	- 757	- 755.19	- 722.74
7	+ 791	792.43	779.09
8	+ 738	738.97	701.39
9	- 729	- 729.22	- 689.02
10	-1077	-1077.00	-1032.94

(2) Transverse Moments (lb-ft)

Ridge	Reference 9	Computer Output	
	TL	TL	TL + WL
1	0	0.00	0.00
2	- 20	- 19.82	- 17.08
3	-986	-985.35	-848.47
4	-953	-953.64	-789.41
5	+270	264.49	269.18
6	+270	264.49	218.33
7	-953	-953.64	-977.24
8	-986	-985.35	-975.34
9	- 20	- 19.82	- 17.08
10	0	0.00	0.00

(1 psi. = 6900 N/m^2; 1 lb-ft. = 1.357 N-m)

The results from the computer output and reference are given in Table 3.1. This example checks the computer program.

Example 3.2: This is an example worked out in Reference 11. This is to check the correctness of the program in the computation of principal stresses and direction. Fig. 12 contains the cross-section of the folded plate and the geometric properties.

LOADING:

DEAD LOAD - Self Weight = 12.5 psf (598.75 N/m^2) of plate/in. thickness

- Roof Load = 10.0 psf (479 N/m^2) on horizontal projection

LIVE LOAD - Snow Load = 15.0 psf (718.5 N/m^2) on horizontal projection

Results are shown in Table 3.2a and Table 3.2b.

Principal Stresses and Directions

The coordinate system for the computations is shown in Fig. 13.

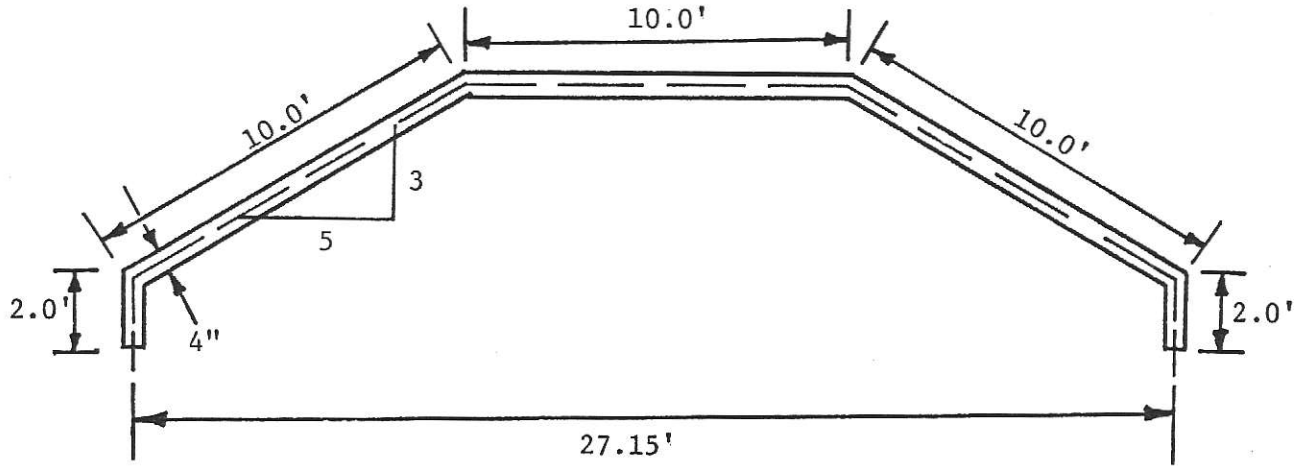
Example 3.3: This is an example of a prestressed folded plate worked out by Yitzhaki (14). This compares the results to his method which represents the loading in terms of Fourier series. The computer results are obtained using the method given in Sec. 2.3, taking into consideration the torsional stiffness of the edge beam. Fig. 14 shows the cross-section of the folded plate and its geometric properties.

LOADING:

DEAD LOAD - DL = 12.0 psf. (598.75 N/m^2)/in. thickness

LIVE LOAD - LL = 14.0 psf. (670.6 N/m^2)

WIND LOAD - From Fig. 5, wind load on each plate is,



Loading: Weight of Concrete = 150 lb/ft.^{3}

Roofing Load = 10 psf. on horizontal projection

Snow Load = 15 psf. on horizontal projection

Span: 50.0'

Fig. 12 - Cross-Section of the Folded Plate Analyzed by Swartz (11)

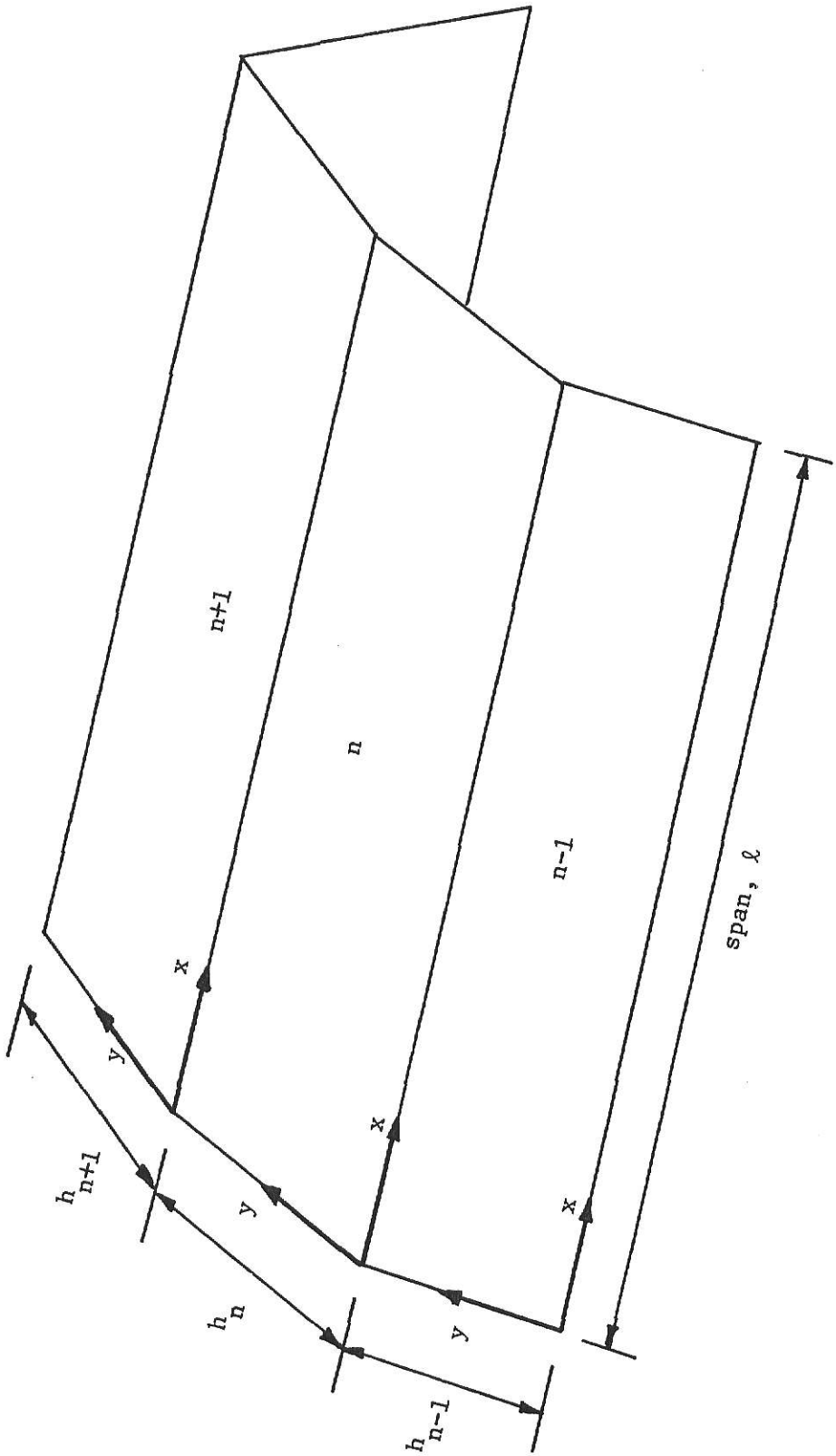


Fig. 13 - Coordinate System for Calculation of Principal Stresses

Table 3.2a - Comparison of Computed Results With Those by Swartz (11)

(1) Longitudinal Ridge Stresses (psi)

x	Reference 11			Comp. Output		
	Ridge 1	Ridge 2	Ridge 3	Ridge 1	Ridge 2	Ridge 3
0	0.0	0.0	0.0	0.0	0.0	0.0
L/4	1056.0	-3.42	-103.67	1038.26	-1.594	-102.87
L/2	1374.13	6.11	-141.25	1351.14	8.35	-140.13

(2) Transverse Moments (ft-lb)

x	Reference 11			Comp. Output		
	Ridge 1	Ridge 2	Ridge 3	Ridge 1	Ridge 2	Ridge 3
0	0.0	0.0	- 687.51	0.0	0.0	- 681.28
L/4	0.0	0.0	-1451.61	0.0	0.0	-1430.55
L/2	0.0	0.0	-1768.11	0.0	0.0	-1740.91

(1 psi. = 6900 N/m^2; 1 lb-ft. = 1.357 N-m)

Table 3.2b - Principal Stresses and Direction

x = 0.0

y	Reference 11			Comp. Output		
	σ_1 (psi)	σ_2 (psi)	θ (deg)	σ_1 (psi)	σ_2 (psi)	θ (deg)
			PL 1			
0	0.0	0.0	45.00	0.0	- 1.04	45.0
h/2	88.63	- 90.15	-44.83	86.16	- 89.73	-44.42
h	114.45	-120.37	-44.33	111.21	-120.13	-43.90
			PL 2			
0	115.86	-118.90	-44.66	115.21	-115.97	-44.90
h/5	104.68	-116.48	-43.50	103.74	-114.66	-43.57
2h/5	91.55	-110.03	-42.33	90.32	-109.39	-42.26
3h/5	76.39	- 99.71	-41.16	74.82	-100.25	-40.83
4h/5	59.04	- 85.60	-39.66	57.17	- 87.38	-39.00
h	39.55	- 67.97	-37.33	37.46	- 71.09	-35.96
			PL 3			
0	39.54	- 68.00	-37.33	39.40	- 67.58	-37.37
h/5	19.29	- 50.01	-31.83	19.28	- 49.72	-31.91
2h/5	3.02	- 34.86	-16.50	3.08	- 34.65	-16.59
h/2	0	- 32.00	0	0.0	- 31.71	0.0

Table 3.2b continued

x = L/4

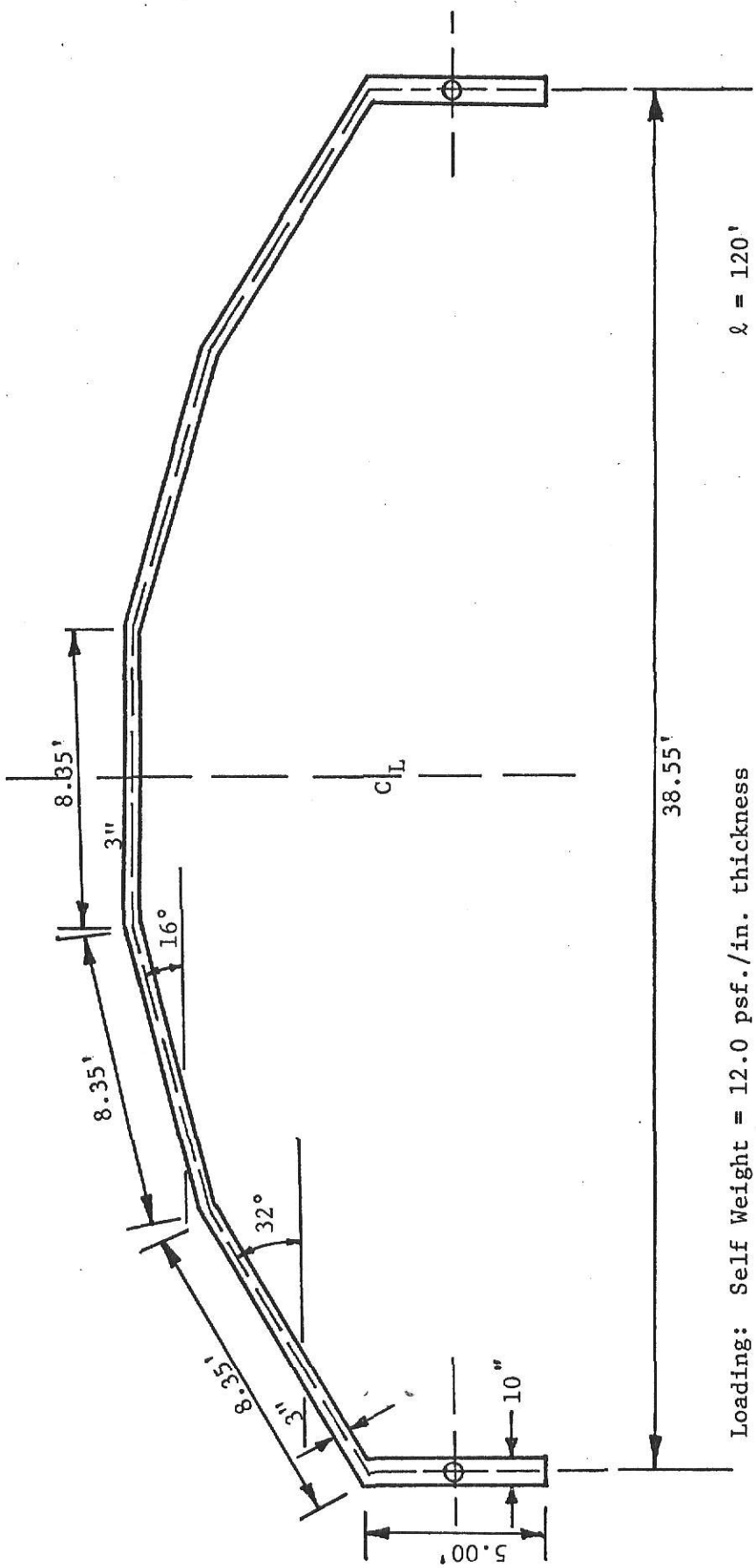
y	Reference 11			Comp. Output		
	σ_1 (psi)	σ_2 (psi)	θ (deg)	σ_1 (psi)	σ_2 (psi)	θ (deg)
			PL 1			
0	1056.00	0.0	0.0	1038.26	- 1.04	0.00
h/2	529.23	- 3.83	- 4.33	521.23	- 5.82	- 4.24
h	49.67	- 57.15	-44.83	48.35	- 57.03	-43.51
			PL 2			
0	50.65	- 56.19	134.66	51.44	- 53.80	134.77
h/5	38.06	- 71.14	131.23	38.12	- 69.61	131.75
2h/5	23.35	- 83.85	127.66	22.94	- 83.07	128.45
3h/5	8.50	- 94.62	123.33	6.41	- 94.38	124.31
4h/5	- 6.95	-104.17	117.50	-10.51	-104.19	118.68
5h/5	-20.71	-114.05	109.50	-26.09	-113.95	110.80
			PL 3			
0	-21.13	-114.11	109.50	-20.84	-113.25	109.58
h/5	-30.75	-107.91	103.50	-30.41	-107.10	103.58
2h/5	-36.18	-104.18	94.83	-35.83	-103.38	94.98
h/2	-36.90	-103.68	90.0	-36.55	-102.87	90.0

Table 3.2b continued

x = L/2

y	Reference 11			Comp. Output		
	σ_1 (psi)	σ_{11} (psi)	θ (deg)	σ_1 (psi)	σ_{11} (psi)	θ (deg)
			PL 1			
0	1374.13	0.0	0.0	1351.14	- 1.04	0.0
h/2	690.12	- 0.62	0.0	679.75	- 2.67	0.0
h	6.11	- 3.29	0.0	8.35	- 6.34	0.0
			PL 2			
0	6.11	- 1.75	0.0	8.35	- 0.76	0.0
h/5	- 8.67	- 23.37	90.0	- 9.11	- 21.34	90.0
2h/5	-15.65	- 52.83	90.0	-17.59	- 51.04	90.0
3h/5	-22.26	- 82.31	90.0	-25.69	- 80.74	90.0
4h/5	-27.94	-111.77	90.0	-32.88	-110.43	90.0
h	-32.17	-141.25	90.0	-38.64	-140.13	90.0
			PL 3			
0	-32.84	-141.25	90.0	-32.47	-140.13	90.0
h/5	-36.75	-141.25	90.0	-36.37	-140.13	90.0
2h/5	-38.70	-141.25	90.0	-38.32	-140.13	90.0
h/2	-38.94	-141.25	90.0	-38.56	-140.13	90.0

(1 psi. = 6900 N/m^2; 1 lb-ft. = 1.357 N-m)



Loading: Self Weight = 12.0 psf./in. thickness

Live Load = 14.0 psf. of plate surface area

Prestress: Force = 480 Kips Eccentricity = 1.25' @ midspan and 0.0' @ end span
parabolic cable profile

Fig. 14 - Cross-Section Analyzed by Yitzhaki (14)

PLATE	WIND PRESSURE, psf (N/m^2)
1	+6.75 (323.33)
2	+0.45 (21.56)
3	-9.00 (-431.1)
4	-9.00 (-431.1)
5	-6.75 (-323.33)
6	-6.75 (-323.33)
7	-6.75 (-323.33)

Note: All these wind loads are modified according to the ACI Code.

Torsional Rigidity of Edge Beam

From Table B.1, in Appendix B $h = 5' = 60"$

for $h/t = 60/10 = 6.0$, $\beta_t = .299$. $t = 10"$

But $\beta_t = \frac{c}{t^3 h} = .299$

$c = .299 \times t^3 \times h = .8652.$

Results are shown in Table 3.3a.

Table 3.3b shows the difference in longitudinal stresses between considering and not considering the effect of torsional stiffness of the edge beam.

Example 3.4: This is an example of a prestressed folded plate given by Billington (1). The example in the reference deals with only gravity loads. The computer output shows the results due to wind load and

Table 3.3a - Comparison of Computed Results With Those by Yitzhaki (14)

Longitudinal Stresses at Midspan (psi)

Ridge	Yitzhaki (14) Total Load	Comp. Output Total Load	Comp. Output TL + Wind Load
1	1429.86	1354.71	1400.61
2	-217.36	-194.79	-285.65
3	-578.47	-531.84	-553.71
4	-520.83	-528.11	-462.56
5	-520.83	-528.11	-462.56
6	-578.47	-531.84	-553.71
7	-217.36	-194.79	-285.65
8	1429.86	1354.71	1400.61

(1 psi. = 6900 N/m^2)

Table 3.3b - Comparison of Results by Considering the Effect of Torsional Stiffness of Edge Beam With Those When This Effect is Neglected

Longitudinal Stresses at Midspan (psi)

Ridge	1	2	3	4	5	6	7	8
Torsion	1354.71	-194.79	-531.84	-528.11	-528.11	-531.84	-194.79	1354.71
No Torsion	1535.99	-361.32	-538.11	-453.37	-453.37	-538.11	-361.32	1535.99

gravity loads. Also, the author of the reference approximates the parabolic deflection curve due to direct prestressing, by a sine curve.

Fig. 15 shows the cross-section of the folded plate and its geometric properties.

LOADING:

DEAD LOAD - Self weight = 12.5 psf (598.75 N/m^2)/in. thickness

Roof load = 10 psf (479 N/m^2)

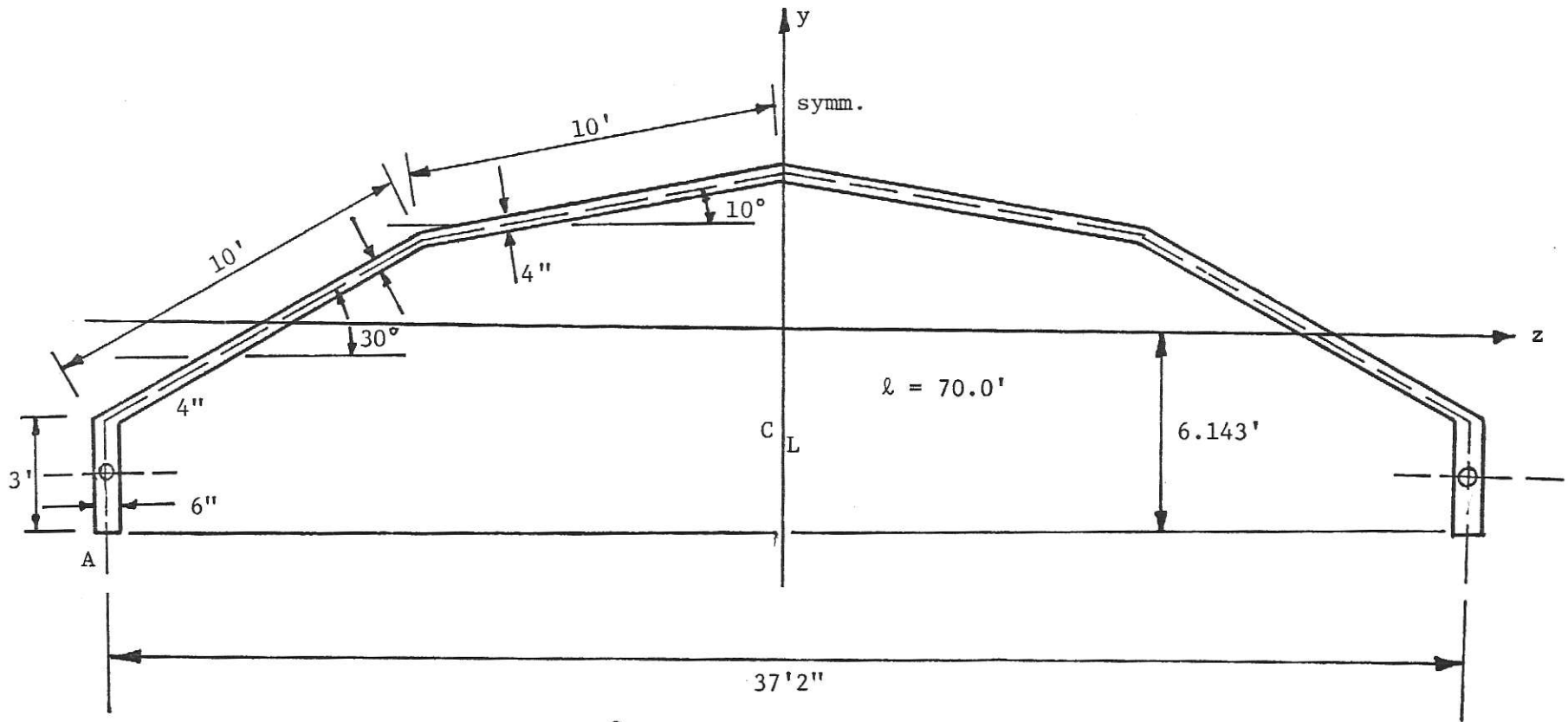
LIVE LOAD - LL = 20 psf (958 N/m^2)

WIND LOAD - From Fig. 5,

PLATE	WIND PRESSURE, psf (N/m^2)
1	+6.75 (323.33)
2	0.0 (0.0)
3	-9.0 (-431.1)
4	-6.75 (-323.33)
5	-6.75 (-323.33)
6	-6.75 (-323.33)

Prestressing Details:

The edge beams are prestressed with parabolic cable with a sag of 12" (.305 m) and prestressing force of 146 Kips (4450 N). Refer to Table 3.4 for the results.



Loading: Self Weight = 150 lb/ft.^{3}
Roof Load = 10 psf. of surface area
Live Load = 20 psf. of surface area

Prestressing: Force = 146 Kips
Eccentricity @ midspan = 1.00'
Eccentricity @ end span = 0.00'
Parabolic cable profile

Fig. 15 - Cross-Section Analyzed by Billington (1)

Table 3.4 - Comparison of the Computed Results With Those by Billington

Longitudinal Ridge Stress at Midspan (psi)

Ridge	Reference TL + PRSTRS.	Comp. Output TL + PRSTRS. (neglecting torsional stiffness)	Comp. Output TL + PRSTRS. + WL	Comp. Output TL + PRSTRS. (including torsional stiffness)
1	0.0	- 2.23	45.54	58.35
2	3.0	8.01	- 35.05	- 20.77
3	-196.0	-192.54	-197.93	-192.63
4	-230.0	-232.89	-260.46	-218.22
5	-196.0	-192.54	-163.13	-192.63
6	3.0	8.01	- 1.135	- 20.77
7	0.0	- 2.23	-111.06	58.35

(1 psi. = 6900 N/m^2)

Transverse Ridge Moments at Midspan (ft-lb.)

Ridge	Reference TL + PRSTRS.	Comp. Output TL + PRSTRS. (neglecting torsional stiffness)	Comp. Output TL + PRSTRS. (including torsional stiffness)	Comp. Output TL + PRSTRS. + WL
1	0.0	0.00	0.0	0.00
2	0.0	0.00	- 235.15	- 30.38
3	- 126.0	- 128.83	- 216.81	- 203.05
4	-1113.0	-1131.40	-1094.02	-1056.93
5	- 126.0	- 128.83	- 216.81	8.93
6	0.0	0.0	- 235.15	30.38
7	0.0	0.0	0.0	0.0

(1 lb-ft. = 1.357 N-m)

Chapter 4

COMPARISON BETWEEN THE PROPOSED METHOD OF ANALYSIS AND THE BEAM METHOD OF ANALYSIS AND METHOD OF ANALYSIS USING FINITE DIFFERENCE SOLUTIONS

In this chapter, a comparison is made between the computer method of analysis including prestressing effects, the approximate beam method of analysis and the rigorous finite difference method. Several examples are worked out and results are discussed. In all these examples the external loading is due to gravity only.

4.1 Beam Method

In this method the folded plate is treated as a simply supported beam of prismatic cross-section. The end-diaphragms are treated as supports. A typical cross-section is shown in Fig 16. If the cross-section is symmetrical, bending is only about zz axis. If the cross-section is asymmetrical, bi-axial bending needs to be considered. Only bending stresses are computed in order to understand the overall behaviour of the folded plate and the degree of approximation involved. The longitudinal stresses can be computed from the following well-known bending formulas.

For uni-axial bending

$$\sigma_x = \frac{M_{zz}}{I_{zz}} \cdot y \qquad \text{---------- (4.1)}$$

For bi-axial bending

$$\sigma_x = \frac{(M_y I_{zz} + M_z I_{yz})z - (M_y I_{yz} + M_z I_{yy})y}{I_{yy} I_{zz} - I_{yz}^2} \qquad \text{---------- (4.2)}$$

The equation of the neutral axis line, passing through the c.g. can be found by equating the equation 4.2 to zero.

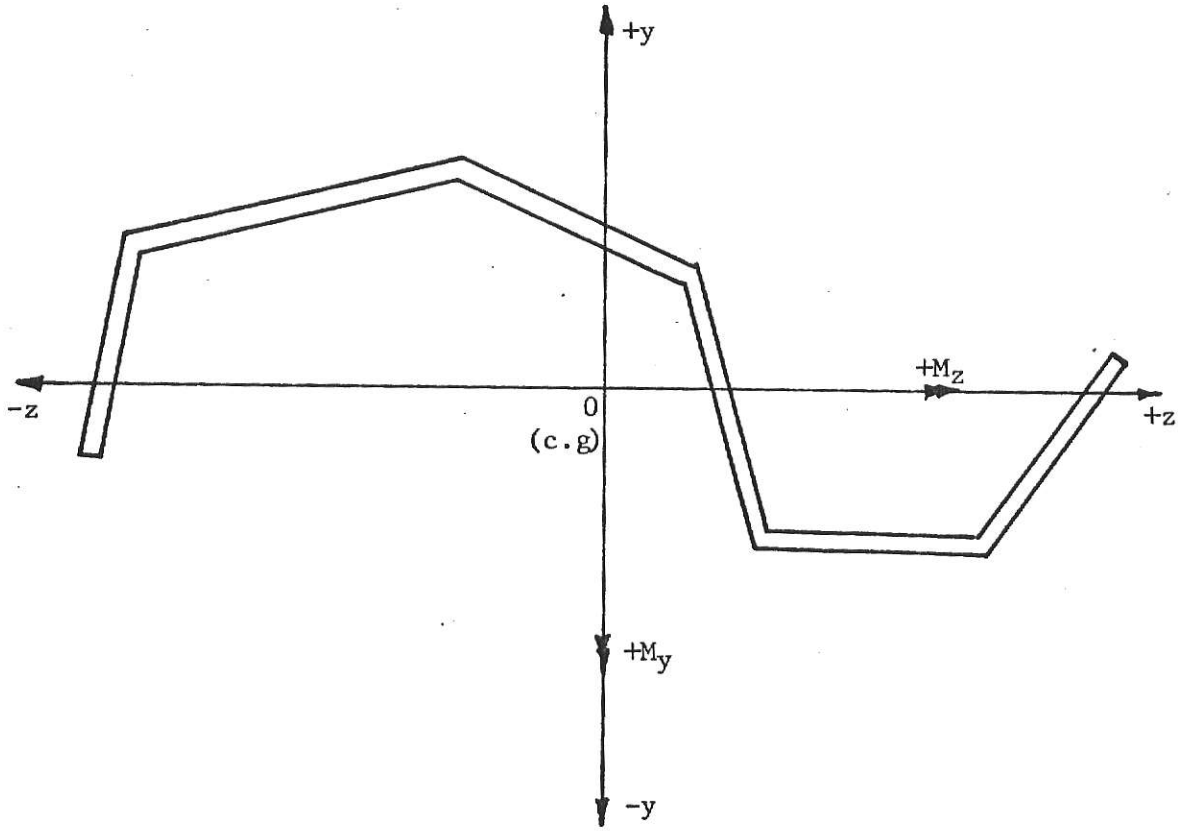


Fig. 16 - Coordinate System and Moment Sign Convention for Beam Method of Analysis

$$(M_y I_{zz} + M_z I_{yz})z - (M_y \cdot I_{yz} + M_z I_{yy})y = 0 \text{ ---------- } (4.3)$$

y,z - co-ordinate system, origin being the centroid of the cross-section. (Refer to Fig. 16)

M_y, M_z - moments about y and z-axes, respectively

I_{zz} - moment of inertia about zz axis

I_{yy} - moment of inertia about yy axis

I_{yz} - product of inertia

(Refer to Appendix E for formulas for moment of inertia)

Equation 4.3 indicates that neutral axis is a straight line.

The procedure to compute σ_x is as follows:

1) Find the centroid of the section and set up the y-z coordinates, the origin being at the centroid, as shown in Fig. 16.

2) Compute I_{zz}, I_{yy} and I_{yz}

   a. For symmetrical sections I_{yz} is equal to zero

   b. For symmetrical sections and symmetrical gravity loading I_{yy} need not be computed

3) Resolve the loadings into y and z components and compute bending moments, M_y, M_z.

$$M_y = W_z \ell^2/8$$

$$M_z = W_y \ell^2/8$$

where W_y, W_z → loading on y and z direction per unit length along x-axis.

4) Find the equation of the line of the neutral axis from equation 4.3.

5) Compute σ_x, at desired points given by y and z, using equation 4.2 or 4.1, whichever is applicable.

The following examples illustrate the application of this method and also indicate the percentage error in this method compared with more exact methods.

Example 4.1: Fig. 15 shows the cross-section of the folded plate and loading details.

1) To locate centroid:

Taking moment about A,

$$\bar{y} = [(1.5 \times 2 \times 1.5) + (3.3333 \times 2 \times 5.5) + (3.333 \times 2 \times 8.875)]/\text{Area} = 6.143'$$

The y, z and origin are as shown in Fig. 17.

2) Computation of I_{zz}

Plate	M.I. (ft.^{4})	
1	$(0.5 \times 3^3/12) + (1.5 \times 4.643^2)$	= 33.461
2	$(3.3333 \times 5^2/12) + (3.3333 \times .643^2)$	= 8.323
3	$(3.3333 \times 1.75^2/12) + (3.3333 \times 2.732^2)$	= 25.730
		67.514

$$I_{zz} = 2 \times (67.514) = 135.028 \text{ ft.}^4$$

Computation of I_{yy} is not necessary.

$$I_{yz} = 0$$

3) Span length = 70.0'

$$W_z = 0; \; W_y = 3.6498 \text{ Kips/ft. } (53321.75 \text{ N/m})$$

$$M_y = 0; \; M_z = -3.6498 \times 70^2/8$$

$$= -2235.5 \text{ K-ft.}$$

(The negative sign is to be consistent with equation 4.2)

4) Neutral axis - because of symmetrical loading and symmetrical cross-section, the line of the neutral axis coincides with the zz centroidal axis.

5) Computation of σ_x:

$$\sigma_x = \frac{-M_z\, y}{I_{zz}} = -\frac{(-2235.5)}{135.028} \times y = 16.5558\ y\ (K/ft.^2)$$

Ridge	y	σ_x (beam)	σ_x(comp.)	Error %
1	6.143'	706.267 psi.	1455.06	51.46
2	3.143'	361.353 psi.	34.54	High
3	-1.851'	-213.500 psi.	-279.24	23.5
4	-3.607'	-414.700 psi.	-146.38	High

(1 psi. = 6900 N/m^2)

The prestressing in the edge beams may be represented by axial thrust and upthrust components, as shown in Fig. 17. Again the axial thrust at an eccentricity of (6.143 - 1.5) = 4.643', from the centroid can be replaced by an axial thrust at centroid and moment which is equal to thrust times eccentricity.

1) Due to axial thrust at centroid:

$$\sigma_x = \frac{2 \times 146}{\text{Area}} = \frac{2 \times 146}{16.3332} = 17.876\ K/ft.^2$$

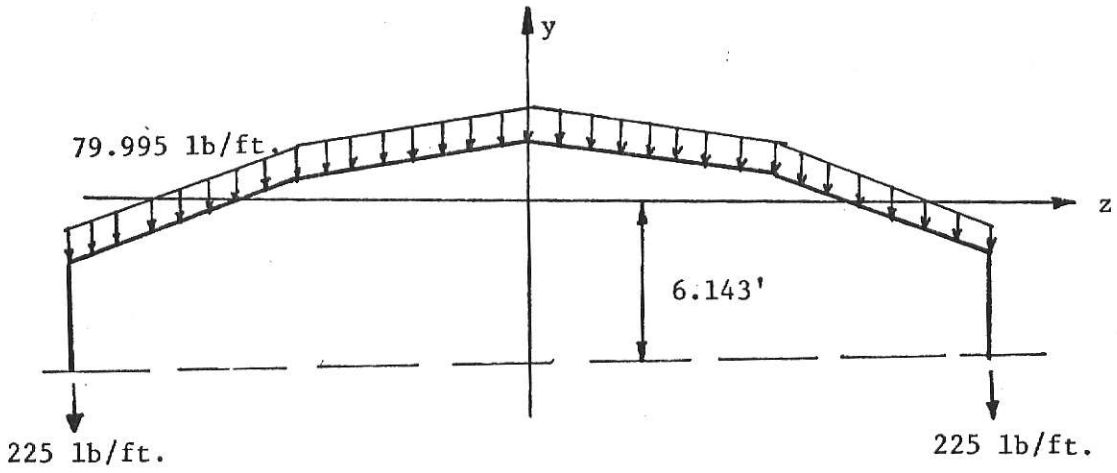
$$= -124.15\ \text{psi.}$$

2) Due to upthrust and moment:

$$\text{Upthrust due to prestress} = \frac{2 \times 146 \times 1 \times 8}{70^2} = .476\ K/ft.$$

$$\text{Moment due to upthrust} = \frac{+\ 476 \times 70^2}{8} = +\ 292\ K\text{-ft.}$$

$$\text{Moment at centroid} = +\ 292 \times 4.643 = +\ 1355.76\ K\text{-ft.}$$



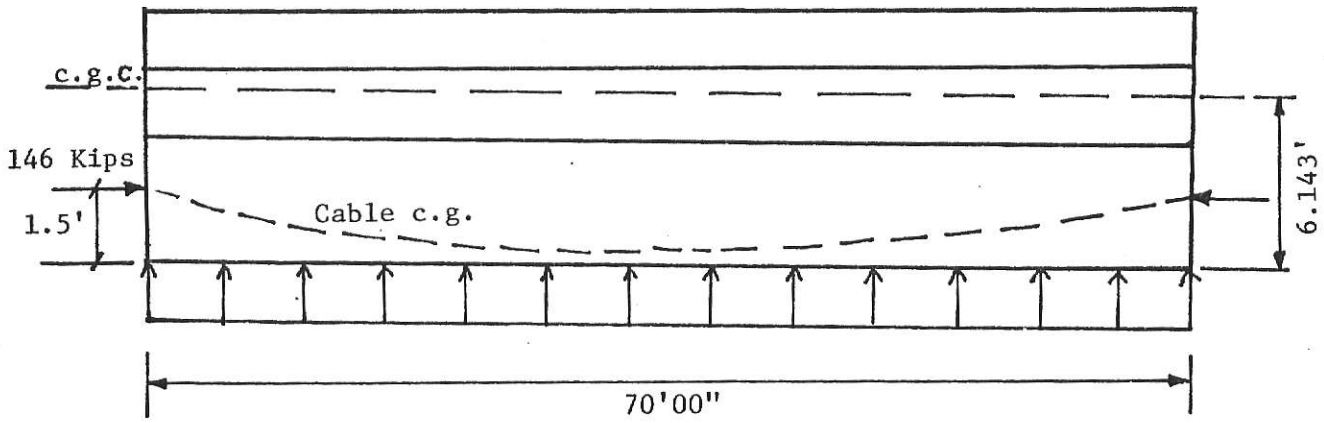


Fig. 17 - Analysis of Billington (1) Folded Plate Using Beam Theory

M_z = Total moment at midspan = + 1647.76 K-ft.

$$\sigma_x = \frac{-M_z \cdot y}{I_{zz}} = -\frac{1647.76}{135.028} \times y = -12.203\ y\ (K/ft.^2)$$

		σ_x (psi)		
Ridge	y	σ_x (beam)	σ_x (comp.)	Error %
1	6.143'	-644.731	-1457.28	55.8
2	3.143'	-390.497	-26.52	High
3	-1.851'	32.709	86.71	62.3
4	-3.607	181.518	-86.51	High

(1 psi. = 6900 N/m^2)

From the above results it is found that except for the nature of the stresses (tension or compression), the error caused in magnitude is considerably significant and especially in the case of prestressing. This implies that for a folded plate of this shape the behavior is not like a beam. One of the reasons for this is the smaller length/depth ratio. In this case, the ratio is

$$r = 70/9.74 = 7.19$$

Also the depth/width ratio is small which influences the beam action.

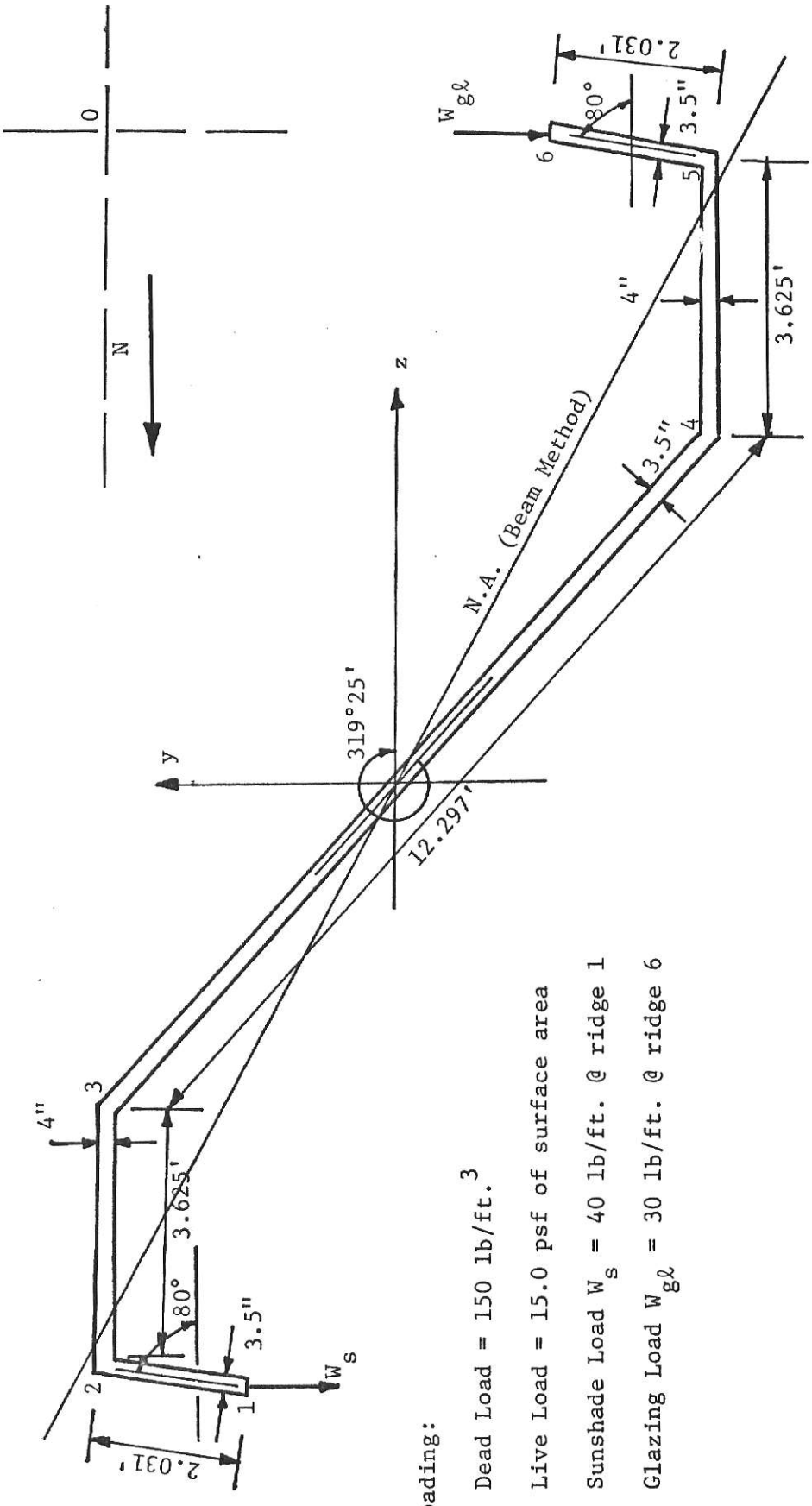
Example 4.2: Fig. 18 shows the cross-section of a north light folded plate roof. This will be analyzed as a beam.

1) To locate centroid:

Taking moment about '0',

$$\bar{z} = [(.593 \times .1765) + (1.207 \times 2.1655) + (3.591 \times 8.648) + (1.207 \times 15.13) + (.593 \times 17.199)]/\text{Area} = 8.647'$$

$$\bar{y} = [(.593 \times 7) + (1.207 \times 8) + (3.591 \times 4) + (.593 \times 1)]/\text{Area} = 4.0'$$



Loading:

Dead Load = 150 lb/ft.^{3}

Live Load = 15.0 psf of surface area

Sunshade Load W_s = 40 lb/ft. @ ridge 1

Glazing Load $W_{g\ell}$ = 30 lb/ft. @ ridge 6

Fig. 18 - Cross-Section of Folded Plate Analyzed by Ramaswamy (9)

2) To compute I_{zz}, I_{yy} and I_{yz}:

I_{zz}

Plate	M.I. ft.^{4}	
1	$(.593 \times 2^2/12) + (.593 \times 3^2)$	= 5.5347
2	$(1.207 \times .333^2/12) + (1.207 \times 4^2)$	= 19.3232
3	$(3.591 \times 8^2/12) + 0$	= 19.152
4	$(1.207 \times .333^2/12) + (1.207 \times 4^2)$	= 19.3232
5	$(.593 \times 2^2/12) + (.593 \times 3^2)$	= 5.5347
		68.8678

$I_{zz} = 68.8678$ ft.^{4}

I_{yy}

Plate	M.I. ft.^{4}	
1	$(.593 \times .353^2/12) + (.593 \times 8.471^2)$	= 42.559
2	$(1.207 \times 3.625^2/12) + (1.207 \times 6.482^2)$	= 52.035
3	$(3.591 \times 9.339^2/12) + 0$	= 26.099
		107.644

$I_{yy} = 2 \times 107.644 = 215.288$ ft.^{4}

I_{xy}

Plate	P.I. ft.^{4}	
1	$I_{\bar{z}\bar{z}} = .0042$, $I_{\bar{y}\bar{y}} = .2039$	$\frac{(-.0042 + .2039)}{2} \sin(160) + (.593 \times 3 \times -8.471)$ = -15.0348
2	$I_{\bar{z}\bar{z}} = 1.3219$, $I_{\bar{y}\bar{y}} = .0111$	$0 + (1.207 \times -4 \times -6.482)$ = -31.2951

Plate		P.I. ft.^{4}
3	$I_{\bar{z}\bar{z}} = .0255$ $I_{\bar{y}\bar{y}} = 45.2480$	$\frac{(45.2480 - .0255)}{2} \sin(278.83) + 0$ $= -22.3430$
4	$I_{\bar{z}\bar{z}} = 1.3219$ $I_{\bar{y}\bar{y}} = .0111$	$0 + (1.207 \times 6.482 \times -4.0)$ $= -31.2951$
5	$I_{\bar{z}\bar{z}} = .0042$ $I_{\bar{y}\bar{y}} = .2039$	$\frac{(.2039 - .0042)}{2} \sin(160) + (.593 \times -3 \times 8.471)$ $= -15.0348$

$I_{xy} = -115.0028$ ft.^{4}

3) Computation of M_y and M_z:

Loading is as shown in the diagram.

$W_y = 1502.34$ lb./ft.

$W_z = 0$

$M_z = +1502.34 \times 60^2/8 = 676.053$ lb. ft. $= 676.053$ K-ft.

(+ sign is consistent with convention)

$M_y = 0$

4) Neutral axis:

$(M_y I_{zz} + M_z I_{yz})z - (M_y I_{yz} + M_z I_{yy})y = 0$

Substituting the values for M_y, M_z, I_{yz}, I_{yy}, and I_{zz},

$[(+676.053) \times (-115.0028)]z - [(676.053) \times (215.2877)]y = 0$

$115.0028z + 215.2877y = 0$

$z + 1.872y = 0$

$y = -.5342z$

$\theta = \tan^{-1}\frac{y}{z} = -28.1103°$

The neutral axis is shown in the figure.

5) Computation of σ_x:

$$\sigma_x = \frac{(M_y I_{zz} + M_z I_{yz})z - (M_y I_{yz} + M_z I_{yy})y}{I_{yy} I_{zz} - I_{yz}^2}$$

$$\sigma_x = (77747.987z + 145545.89y)/1600.746$$

Longitudinal Stresses at Midspan: σ_x (psi)

Ridge	z	y	σ_x (beam)	σ_x (comp.)	Error %
1	-8.647	2.0	1653.72	1636.35	1.06+
2	-8.294	4.0	271.83	304.55	10.7 -
3	-4.6695	4.0	-950.68	-970.88	2.08-
4	4.6695	-4.0	+950.68	964.88	1.47-
5	8.294	-4.0	-271.83	-296.65	8.36-
6	8.647	-2.0	-1653.72	-1611.68	2.6 +

(1 psi. = 6900 N/m^2)

Analysis for Prestressing:

Fig. 19 shows the details in converting the applied prestress into equivalent thrust and plate loads and then replacing them by thrust at the centroid, constant moments and uniform loads in z and y direction.

1) Thrust at centroid = 85.19 + 151.05 = 236.24 Kips

Area of c/s = 7.192 $ft.^2$

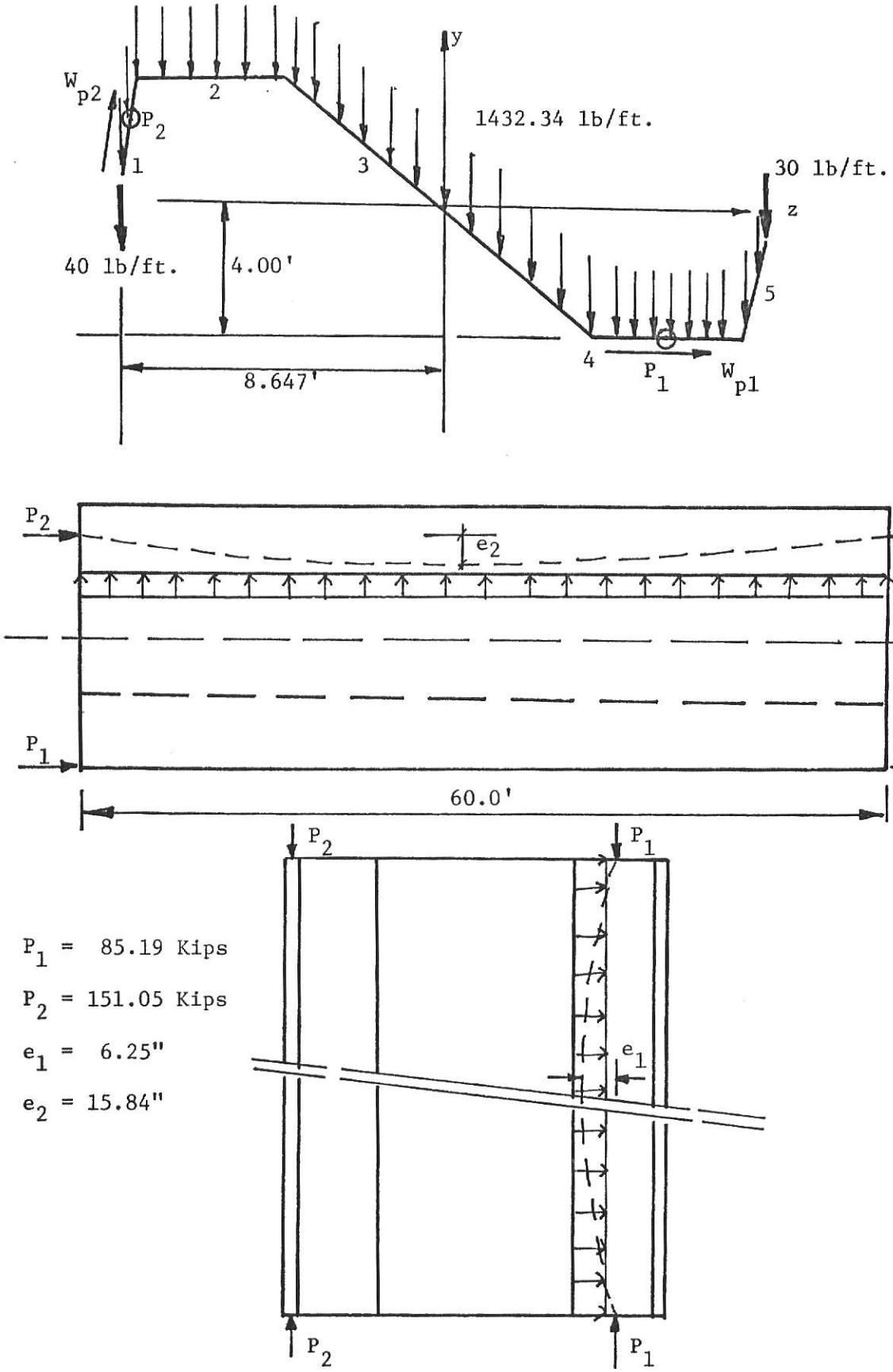
$$\sigma_x = -\frac{236.24}{7.192} = -32.84 \text{ Ksf.} = -228.11 \text{ psi.}$$

2) Bending:

$$M_y = M_{ply} + M_{p4y} + W_z L^2/8$$

$$M_y = +721.62 = 979.11 + .460 \times 60^2/8$$

$$= -50.49 \text{ K-ft.}$$



P_1 = 85.19 Kips

P_2 = 151.05 Kips

e_1 = 6.25"

e_2 = 15.84"

Fig. 19 - Analysis of Ramaswamy (1) Folded Plate Using Beam Theory

$$M_z = M_{plz} + M_{p4z} + W_y L^2/8$$

$$= +255.57 - 604.2 - .097 \times 60^2/8$$

$$= -392.28 \text{ K-ft.}$$

$$\sigma_x = \frac{(M_y I_{zz} + M_z I_{yz})z - (M_y I_{yz} + M_z I_{yy})y}{I_y I_z - I_{yz}^2}$$

$$= [(-50.49 \times 68.8678) + (-392.28 \times -115.0028)]z$$

$$- [(-50.49 \times -115.0028) + (-392.28 \times 215.288)]y/1600.746$$

$$= +41636.16z + 78646.69y/1600.746$$

$$= +26.01z + 49.13y = + (26.01z + 49.13y)$$

			σ_x (psi)		
Ridge	z	y	σ_x (beam)	σ_x (comp.)	Error %
1	-8.647	2.0	-1107.61	-1478.82	25 -
2	-8.294	4.0	-361.49	-491.49	26 -
3	-4.6695	4.0	293.18	401.70	27 -
4	4.6695	-4.0	-749.40	-687.46	9 +
5	8.294	-4.0	-94.73	-126.20	25 -
6	8.647	-2.0	651.39	146.16	345 +

(1 psi. = 6900 N/m^2)

From the above results it is seen that the stresses obtained by beam method agree with that of folded plate analysis in directions (compression or tension) but they are objectionably erroneous in magnitude. Once again the reason may be that due to different amount and direction of prestressing in each plate, relative joint deflections have greater effect on the stresses.

The following example illustrates the close agreement of solutions by beam and folded plate method in the case of v-shaped folded plates subjected to prestressing and gravity loadings.

Example 4.3: Fig. 20 shows the cross-section of the folded plate and its geometric properties. Loading and prestressing details are also given.

1) To locate centroid:

Because of symmetry, $\bar{y} = 2.5'$ from line joining the troughs.

2) To compute I_{zz}, I_{yy} and I_{yz}:

$I_{yz} = 0$ (symmetry)

Only I_{zz} need be considered due to uniaxial bending.

$$I_{zz} = 6 \times (7.07 \times .3333) \times 5^2/12 = 29.455 \text{ ft.}^4$$

3) To compute M_z due to gravity load:

$W_y = 2.758$ K/ft.

$$M_z = W_y L^2/8 = 2.758 \times 60^2/8 = 1241.1 \text{ K-ft.}$$

To compute M_z due to 100 Kips prestress:

$$W_y = \frac{100 \times 2 \times 8}{60^2} \times \sin 45° \times 6 = 1.8856 \text{ K/ft.}$$

$$M_z = 1.8856 \times 60^2/8 = 848.53 \text{ K'}$$

$$\text{Total } M_z = -848.53 + 1241.1 = 392.57 \text{ K'}$$

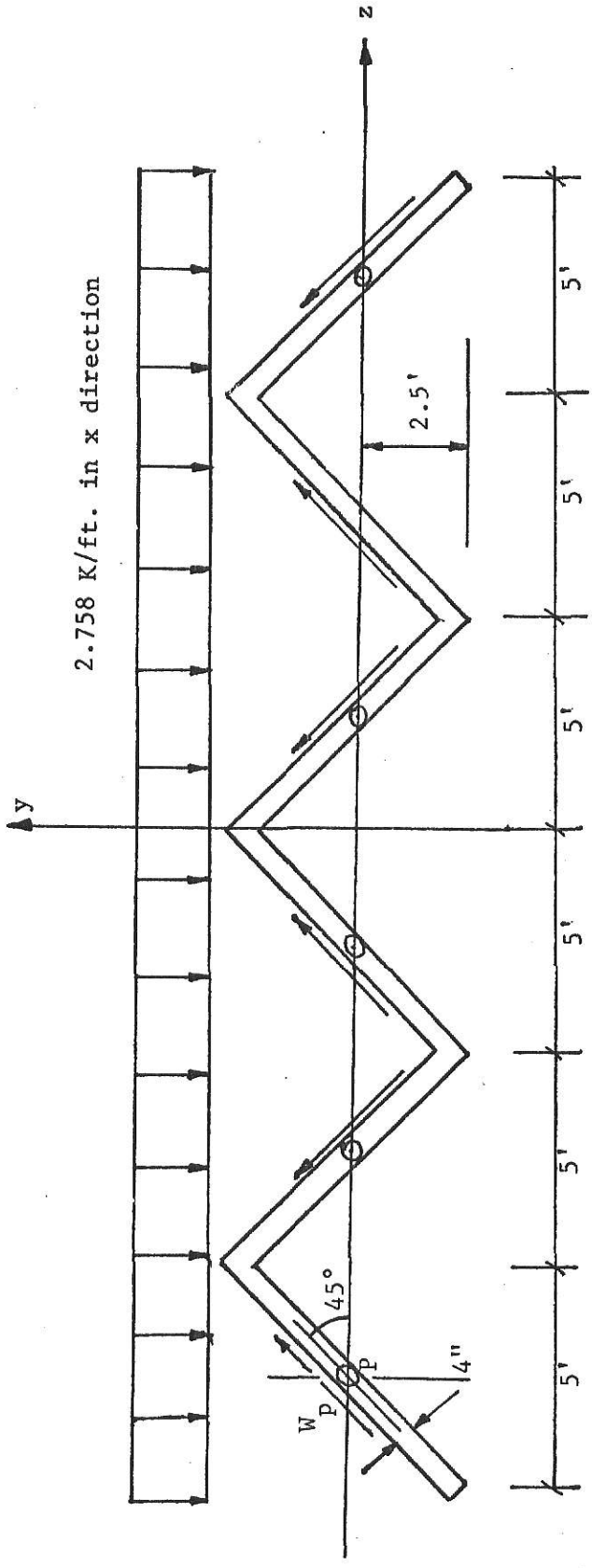
$$\sigma_x = \frac{M_z}{I_{zz}} \cdot y = \frac{392.57}{29.455} \times 2.5 = 33.319 \text{ ksf.}$$

$$= +231.385 \text{ psf. at bottom ridges}$$

4) To compute stress due to direct prestress:

$$\sigma_x = 100 \times 6/\text{Area} = (100 \times 6)/(7.07 \times .3333 \times 6)$$

$$= 42.437 \text{ ksf.} = -294.70 \text{ psi.}$$



P = 100 Kips; e = 2.00'

Fig. 20 - Typical Sawtooth Folded Plate Cross-Section

Ridge	σ_x (psi.) beam	σ_x (psi.) F-P
1	-63.32	-57.07
2	-526.08	-556.68
3	-63.32	-62.76
4	-526.08	-456.51

(1 psi. = 6900 N/m^2)

From the above three examples the validity of the analysis of folded plates by beam method can be summarized as follows, in terms of factors affecting the accuracy and correctness:

1) Span to width ratio
2) Span to depth ratio
3) Shape of the folded plate (plate inclination, width, etc.)
4) Type of loading
5) Prestressing force, and location of prestressing
6) Symmetry

In example 4.1, because of its shape the folded plate tends more or less to behave like a slab and not like a beam. In example 4.2, the beam method gives satisfactory results in the case of gravity loads. However, it fails to do so when the shell is prestressed differently in different plates. In example 4.3, since symmetry is maintained throughout and all the plates are prestressed equally, the relative joint deflections seem to have little effect on the final results. Hence, from these examples, it may be concluded that beam analysis of folded plates will be helpful in understanding the overall behavior of the folded plates in terms of the senses of the stresses. In most of the cases, especially in the case of prestressing, the magnitude of the stresses are found to be significantly erroneous.

4.2 Comparison With More Rigorous Analysis: (Finite Difference Method)

The example for this comparison is that worked out and experimentally checked in Reference 12. This example compares the computer solution of the Simpson Method of analysis with the finite difference solutions. Fig. 21 shows the cross-section and the prestressing details. The authors in their work have computed longitudinal and transverse strains and hence, in order to get the output in terms of strains, the computer program had to be modified. This was done by the biaxial stress-strain relationship

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \mu\sigma_y)$$

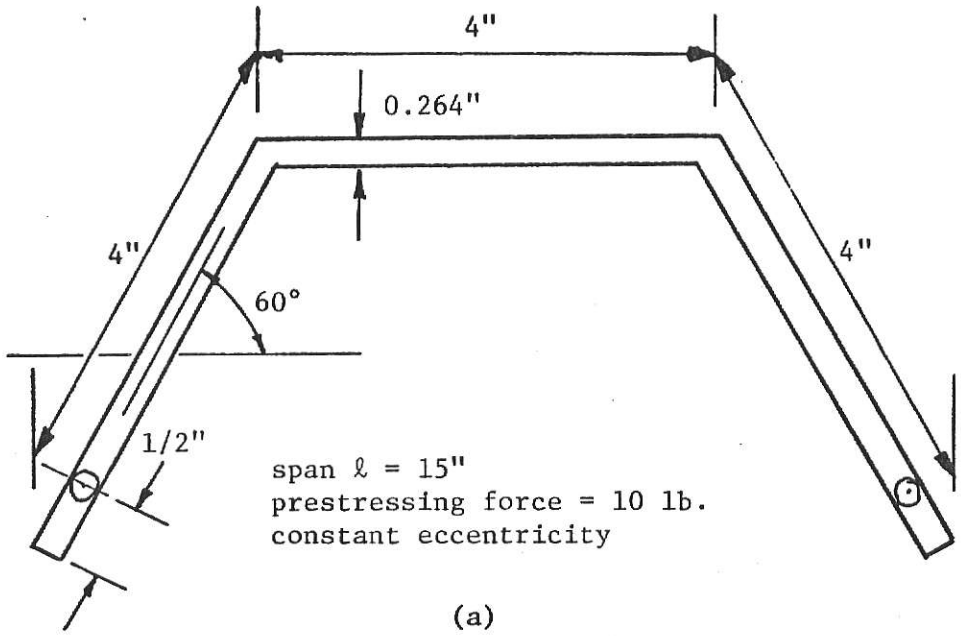
$$\varepsilon_y = \frac{1}{E} (\sigma_y - \mu\sigma_x)$$

where E is modulus of elasticity of the material of the folded plate and μ is the Poisson's ratio.

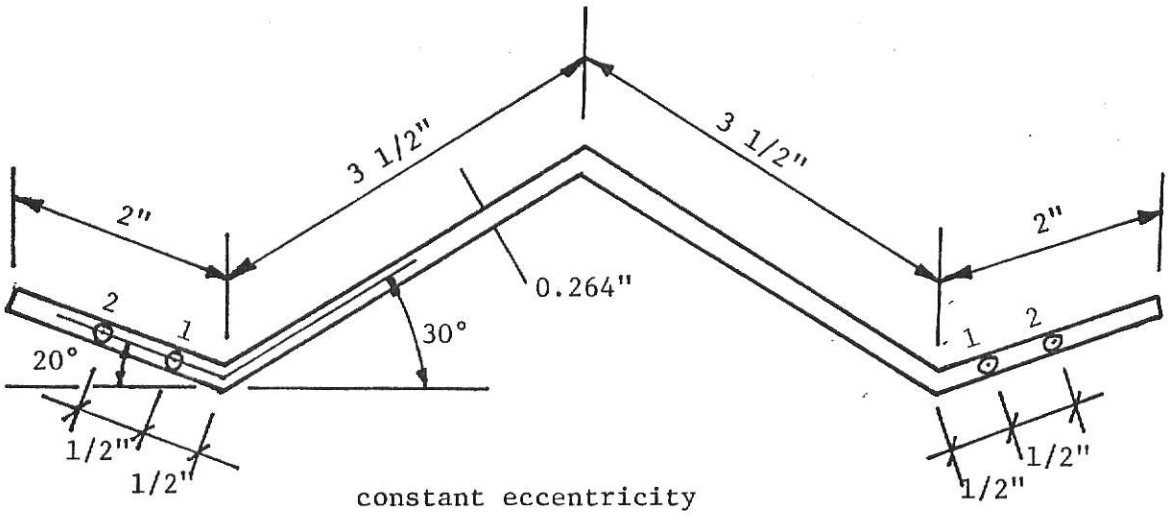
Example 4.4a: The prestressing cable is straight with an eccentricity, as shown in Fig. 21.a. The analysis is for prestressing only. The method of analysis for this type of prestressing has been presented in Chapter 2. Table 4.1 shows the results.

Example 4.4b: Fig. 21.b shows the cross-section of the folded plate and prestressing details. The solutions are compared in Table 4.2 and location of prestressing is 1-1, as shown in the figure.

The pairs of values indicated by the numbers in the parantheses in Table 4.2 in the reference column correspond to strains measured in the upper and lower surface of the model, whereas the corresponding values in the comp. output column correspond to average strain across the section. Keeping this in mind, the average strain for pair #2 from the



(a)



(b)

Fig. 21 - Cross-Sections of Folded Plates Analyzed in Reference 12

Table 4.1 - Comparison of Solutions

Strain, in./in. x 10^{-6}

Plate	Location x	y	Result	Comp. Output	Ref. 12
3	L/4	3h/4	ε_x	-39.87	-38.80
3	3L/8	h/2	ε_y	+ 8.34	+ 8.30
2	L/8	h/2	ε_x	+ 8.22	+ 6.30
2	L/4	h/2	ε_x	+ 8.22	+ 9.50
2	L/4	3h/4	ε_x	+ 8.22	+10.10
2	3L/8	h/2	ε_y	- 2.88	- 3.10
2	L/2	h/2	ε_x	+ 8.22	+10.90
2	L/2	h/4	ε_x	+ 8.22	+11.00
2	L/2	3h/4	ε_x	+ 8.22	+11.00
1	3L/8	h/2	ε_x	-23.84	-22.30
1	L/2	h/4	ε_x	-39.87	-38.80

Table 4.2 - Comparison of Computed Results With Those by the Authors of Reference 12

Strain in./in. x 10^{-6}

Plate	Location x	Location y	Result	Comp. Output	Ref. 12
4	L/4	h/4	ε_x	-30.15	-26.8
3	L/7	6h/7	ε_x	-28.47	-31.0
3	L/2	6h/7	ε_x	-28.35	-25.4 (2)
3	L/4	h/7	ε_x	+10.37	+14.5
3	L/2	h/7	ε_y	- 3.59	- 7.7
3	L/2	3h/7	ε_x	2.63	- 1.4
3	L/4	2h/7	ε_x	+ 2.62	+ 6.9
2	L/4	2h/7	ε_x	-20.64	-16.5 (3)
1	L/2	h/2	ε_x	-24.21	-18.5 (1)
4	L/2	h/2	ε_x	-24.21	-27.7 (1)
4	L/4	h/4	ε_y	+10.57	+13.0
3	L/7	6h/7	ε_y	9.97	+ 8.9
3	L/2	6h/7	ε_x	-28.35	-33.6 (2)
3	L/7	6h/7	ε_x	-28.47	-31.0
3	L/2	h/7	ε_y	- 3.59	- 0.7
2	L/4	h/7	ε_x	-28.39	-33.5
2	L/4	2h/7	ε_x	-20.64	-24.7 (3)
2	L/2	3h/7	ε_x	-12.86	- 9.4
3	L/4	2h/7	ε_x	2.62	- 0.9

Table 4.3 - Comparison of Computed Results With Those by Authors in Reference 12

Strain in./in. x 10^{-6}

Plate	x	y	Result	Comp. Output	Ref. 12	Ref. Avg.	Expt.
4	L/4	h/4	ε_x	-24.09	-24.2	-	-25.9
3	L/7	6h/7	ε_x	-11.9	-11.4	-	-11.6
3	L/2	6h/7	ε_x	-13.64	-14.3	-13.80	-12.3
3	L/4	h/7	ε_x	+ 4.02	+ 4.7	-	+ 4.5
3	L/2	h/7	ε_y	-	+14.7	-	+10.1
3	L/2	3h/7	ε_x	- 3.06	- 2.5	-	- 2.0
3	L/4	2h/7	ε_x	+ 0.63	+ 1.4	-	+ 1.7
2	L/4	2h/7	ε_x	- 9.57	- 9.3	- 9.10	- 8.7
1	L/2	h/2	ε_x	-31.05	-29.6	-28.5	-33.5
4	L/2	h/2	ε_x	-31.05	-27.4	-28.5	-29.6
4	L/4	h/4	ε_y	+ 8.3	+10.4	-	+ 9.3
3	L/7	6h/7	ε_y	+ 4.06	+ 3.4	-	2.8
3	L/2	6h/7	ε_x	-13.64	-13.4	-13.86	-10.4
3	L/7	6h/7	ε_x	-11.90	- 9.4	-	- 8.3
3	L/2	h/7	ε_y	-	-18.4	-	-
2	L/4	h/7	ε_x	-12.98	-13.5	-	- 9.2
2	L/4	2h/7	ε_x	- 9.57	- 8.9	- 9.10	- 6.9
2	L/2	3h/7	ε_x	- 6.58	- 6.3	-	- 3.7
3	L/4	2h/7	ε_x	+ 0.63	- 0.8	-	0

finite difference solution is -29.5 μ in/in. Similarly for pair #1, the average strain from the finite difference solution is -23.5 for which the computer output is -24.21. For pair #3, the f.d. average is -20.6, and the computer result is -20.6. This way, we find the two solutions agree quite closely, considering the two different approaches. The following example, also, is another comparison.

Example 4.4c: Fig. 21.b shows the cross-section of the folded plate and the prestressing details, prestress at 2-2. Table 4.3 contains the comparison of the results.

This example shows the comparison between the results from this thesis, from the finite difference method and from the experiment. It is seen from Table 4.3 that an excellent agreement, from a practical standpoint, exists between these three results. Comparing the results between the Simpson method and the finite difference method in the reference, we may find that a close agreement prevails.

These examples also serve as a check on the working of the computer program in computing principal stresses due to prestressing.

Chapter 5

VALIDITY OF THE PRINCIPLE OF SUPERPOSITION

In this chapter the validity of the superposition principle, in the analysis and design of the prestressing, will be discussed. So far, external loading and prestressing effects have been treated separately. The final results were obtained by superposition. Actually, both the loading and the prestress act simultaneously. The method used and the expressions derived to analyze the folded plate for simultaneous action of load and prestress will be discussed below.

Primary Analysis

This is the same as the one discussed in Chapter 2, to the extent of computing free-edge stresses due to plate load. Now with prestress acting simultaneously, free-edge stresses due to prestress have to be added to the initially computed free-edge stresses. Now stress distribution is carried out and final primary ridge stresses are computed.

Secondary Analysis

There is no modification in the method of the secondary analysis.

In order to compute the primary plate deflections, neither of the expressions (2.3) and (2.4) can be used since they correspond to independent actions of load and prestress. A method to compute primary plate deflection when prestress and load act simultaneously will be discussed below.

Figure 22 shows the final primary stress distribution along the length of the n^{th} plate. The variation is parabolic with a constant component contributed by the direct prestress.

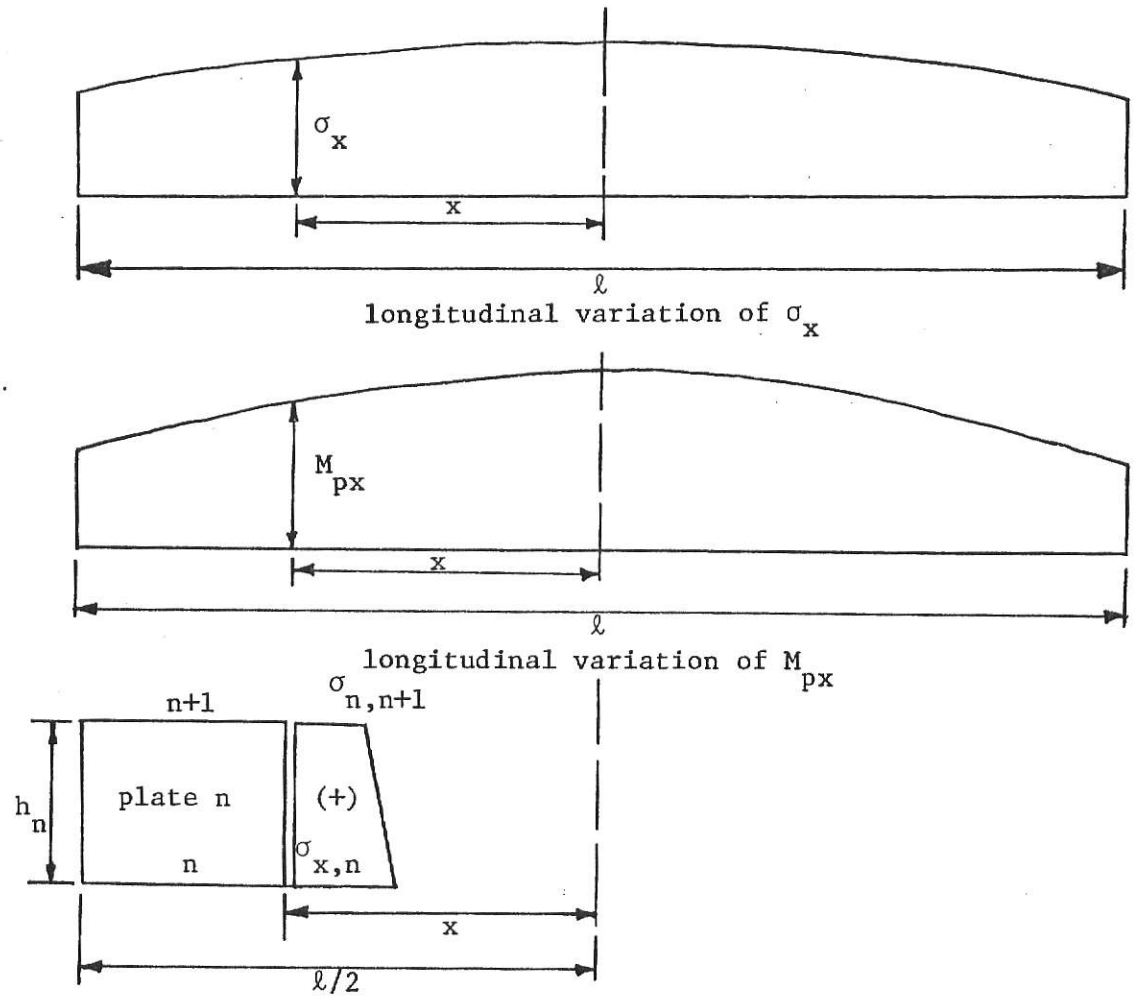


Fig. 22 - Variation of σ_x and M_{px} in a Plate

In order to compute the plate deflection for this type of stress variation, the following expression will be used:

$$\frac{d^2\delta}{dx^2} = -\frac{M_{px}}{EI}$$

where M_{px} = plate moment at a distance x from the center of the plate

δ = plate deflection at distance x

Since the longitudinal stresses vary parabolically along the x-axis, the plate moment M_{px} also varies parabolically. The moment M_{px} at any

point is related to the longitudinal stresses by

$$M_{pc} = (\sigma_{x,n} - \sigma_{x,n+1})\ d_n\ h_n^2/12$$

The moment $M_{p_{max}}$ at midspan is given by

$$M_{p_{max}} = (\bar{\sigma}_{x,n} - \bar{\sigma}_{x,n+1})\ d_n\ h_n^2/12$$

$\bar{\sigma}_{x,n}$ and $\bar{\sigma}_{x,n+1}$ are primary longitudinal stresses at midspan. The plate moment M_{pc} at the end of the span is computed as follows: the variation of the longitudinal stress with x due to direct prestress is zero and that due to uniform load is parabolic. Hence, at the end of the span there will be no primary stresses induced due to the uniform load. Therefore, the primary stresses at the end of the span are only due to direct prestressing and can be found by applying the direct prestress to the primary structure. Hence, the primary plate moment at the end of the span is only due to direct prestressing and is given by

$$M_{pc} = \sigma_{x,n} - \sigma_{x,n+1})\ d_n\ h_n^2/12$$

Knowing M_{pc}, an expression for M_{px} can be derived.

$$M_{px} = M_{p0}\ (1 - \frac{4x^2}{L^2}) + M_{pc} \quad \text{where x is from center}$$

and, $$M_{p0} = M_{p_{max}} - M_{pc}$$

$$\frac{d^2\delta}{dx^2} = -\frac{M_{px}}{EI_n} = -\left[\frac{M_{p0}}{EI_n}\ (1 - \frac{4x^2}{L^2}) + \frac{M_{pc}}{EI_n}\right]$$

$$\frac{d\delta}{dx} = -\frac{M_{p0}}{EI_n}\ (x - \frac{4}{3}\frac{x^3}{L^2}) - \frac{M_{pc}}{EI_n} + c_1$$

But $\frac{d\delta}{dx}$ when $x = 0, = 0. \rightarrow c_1 = 0.$

Therefore, $\frac{d\delta}{dx} = -\frac{M_{p0}}{EI_n}\left(x - \frac{4}{3}\frac{x^3}{L^2}\right) - \frac{M_{pc}}{EI_n}x$

$$\delta = -\frac{M_{p0}}{EI_n}\left(\frac{x^2}{2} - \frac{x^4}{3L^2}\right) - \frac{M_{pc}}{EI_n}\frac{x^2}{2} + c_2 \quad \text{---------- (5.1)}$$

when $x = L/2$, $\delta = 0$.

$$c_2 = \frac{M_{p0}}{EI_n}\left(\frac{L^2}{8} - \frac{L^2}{48}\right) + \frac{M_{pc}}{EI_n}\frac{L^2}{8}$$

Now, deflection at midspan can be obtained by letting $x = 0$ in equation 5.1.

$$\delta = c_2 = \frac{M_{p0}}{EI_n}\left(\frac{5L^2}{48}\right) + \frac{M_{pc}}{EI_n}\frac{L^2}{8} \quad \text{---------- (5.2)}$$

Substituting the values for M_{pc} and M_{p0} in Eqn. 5.2, the primary plate deflection at midspan of the n^{th} plate due to the simultaneous action of prestress can be determined.

Once the primary and secondary plate deflections are computed, deflection compatibility at joints can be established and final stresses can be found out.

Example 5.1: The example 4.4c is considered. First the solutions due to prestress and total load acting separately are computed, using the computer program developed earlier. Then the program is modified for the analysis of prestress and load acting together as discussed above. The solutions due to the combined action of prestress and load are compared with the superimposed results of prestress and load acting

Table 5.1 - Validity of Superposition

Longitudinal Stresses at Midspan (psi)

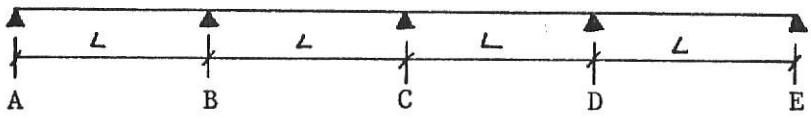
Ridge	Prestress Alone	DL + LL Alone	Prestrs. + TL (Superimposed)	Prestrs. + TL (Acting together)
1	- 5.917	-67.459	-73.376	-73.376
2	-17.322	62.824	45.502	45.502
3	8.708	-60.176	-51.468	-51.468
4	-17.322	62.824	45.502	45.502
5	- 5.917	-67.459	-73.376	-73.376

(1 psi. = 6900 N/m^2)

individually, in Table 5.1. It is seen that the same results are obtained by both the methods. This can be reasoned out as follows:

The analysis of the folded plate consists of primary and secondary parts. The primary analysis deals with moment distribution and stress distribution. Moment distribution of the primary structure (continuous beam on rigid supports) is essentially a linear process. Referring to the figure below, the stiffness equation for the continuous beam can be written as follows:

I = constant



$$\begin{bmatrix} 7EK_b & 2EK_b & 0 \\ 2EK_b & 8EK_b & 2EK_b \\ 0 & 2EK_b & 7EK_b \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} + \begin{bmatrix} \Sigma FEM_B \\ \Sigma FEM_C \\ \Sigma FEM_D \end{bmatrix} = 0 \text{ ---------- (5.3)}$$

where $K_b = I/L$

θ_B, θ_C, θ_D = rotations at B, C and D, respectively.

ΣFEM = summation of fixed end moments at joints.

The equation 5.3 can be solved using the linear error theorem. That is, first a value for θ_B, say, $\theta_B = 1$ will be assumed. Then equation 5.3 can also be represented as follows:

$$(7EK)\ \theta_B + (2EK)\ \theta_C + \Sigma FEM_B = 0 \quad \text{---------- (5.3a)}$$

$$(2EK)\ \theta_B + (8EK)\ \theta_C + (2EK)\ \theta_D + \Sigma FEM_C = 0 \quad \text{---------- (5.3b)}$$

$$(2EK)\ \theta_C + (7EK)\ \theta_D + \Sigma FEM_D = 0 \quad \text{---------- (5.3c)}$$

Corresponding to $\theta_B = 1.0$, θ_C and θ_D can be computed from 5.3a and 5.3b. Also, θ_D can be computed from equation 5.3c and the error involved in assuming $\theta_B = 1.0$ can be found out, (E_1). Now again assuming $\theta_B = 0.0$, the error involved (E_2) can be found out. Hence, the true value of θ_B is given by

$$\theta_{B(true)} = E_1 - E_2/E_1$$

This process is essentially linear. Hence, if given loadings are treated individually the superimposed results will be the same as the one in which the loadings act together.

Since the stress-distribution is analogous to moment distribution, as proved earlier, the same conclusions are obtained. Considering equation 5.2, the first part of the right hand side is due to the stresses induced by loading alone and the second part is due to the stresses induced by prestress alone. The stress compatibility equation discussed in Chapter 2 leads to a tridiagonal matrix. The process of solving a tridiagonal matrix equation is linear and is similar to that discussed above. Hence, the whole of the primary analysis is essentially a linear process in terms of finding stresses, moments and deflections. The secondary analysis is unchanged. This is why the principle of superposition is valid for the analysis of folded plate with prestress and uniform load acting together.

Chapter 6

OPTIMAL SELECTION OF PRESTRESSING FORCE AND CABLE SAG

Most of the work in the selection of prestressing forces and cable layout so far has been essentially a process of trial and error. In the case of prestressing a number of plates, for different combinations of loading conditions such as load at transfer, full live load, combinations of gravity and wind load, this trial and error procedure is tedious and most of the time will not lead to an optimum design. The objective of this chapter is to present a systematic approach to design.

6.1 Principle in Outline

Consider a folded plate subjected to 1) Dead Load 2) Live Load 3) Wind Load and 4) Prestress. The various possible combinations of loading are listed below:

DEAD LOAD + PRESTRESS (Transfer)

DEAD LOAD + LIVE LOAD + PRESTRESS

DEAD LOAD + WIND LOAD + PRESTRESS

DEAD LOAD + WIND LOAD + LIVE LOAD + PRESTRESS

The objective of the prestressing is, under any combination of the loadings as listed above, that the longitudinal stresses (which are most of the time higher than transverse stresses) at the ridges (ridges being points of maximum stresses), should be less than the allowable tensile and compressive stresses. Another objective, from an economy standpoint, is to get an optimum design.

This problem can be solved by "LINEAR PROGRAMMING".

6.2 Outline of Procedure

It is noted that the prestressing force is directly proportional to the cross sectional area of cable. The volume of the cable is proportional to its force if the cable is straight. Otherwise, the volume is approximately proportional to the cable force. The amount of cable and prestressing forces are the major contributions to the cost of prestressing. The function which is to be minimized is called the objective function and will be taken as the total prestressing force. The longitudinal stresses at the ridges are constrained to be less than or equal to the allowable compressive and tensile stresses. This problem can be solved by Linear Programming using an IBM supplied program MPS-360. As a matter of fact, the complexity of the problem lies not in the operational procedures, but in setting up the objective function and constraints.

The requirements in setting up the constraints and uniqueness of the solution will be discussed, in order to have a clear understanding of the prerequisites, before tackling the problem. These requirements are,

1. The objective function should be linear.

2. The constraints should be linear, mutually exclusive and independent.

3. The number of constraints should be greater than or at least equal to the number of variables.

4. The optimum value of the objective function given as a solution by linear programming will be unique. The values assigned to variables may not be unique. For example, if we consider two variables x 1 and x 2, yielding the same amount of profit, say P, then the objective

function, the total profit is given by

$$z = Px_1 + Px_2 \ .$$

Now if the cost of production is also the same for both variables, then under given restraints the solution may be that either n units of x_1, or n units of x_2 or q units of x_1 and (n-q) units of x_2 be produced to achieve the optimum, where $q \leq 1$. But the linear program will merge the variables x_1 and x_2 as one variable and the output will be n units of either x_1 or x_2. Hence, in order to check the uniqueness of the solution after optimization, those variables who are assigned zero will be given certain values and the new optimum value will be found. If this is same as the previous one, the assignment of values to variables is not unique. This situation will be clear when the following examples are worked out.

6.3 General Procedure

Consider the folded plate in Fig. 12. Assuming that plate #1 and plate #5 are each going to contain a cable with parabolic profile, the problem is to find the amount of prestress and sag to take care of live, dead and wind load.

Step 1 - Since the longitudinal stresses will have maximum value at midspan, the folded plate is analyzed for longitudinal stresses, at ridges, at midspan individually for self weight, Live Load and Wind Load (Wind Load modified according to ACI Code). The following notations will be used for longitudinal stress at n^{th} ridge.

Self weight	$(\sigma_{xd})_n$
Live Load	$(\sigma_{x\ell})_n$
Wind Load	$(\sigma_{xw})_n$

Step 2 - Unit prestress and unit sag are applied in plate 1 and longitudinal stresses at midspan at all ridges are determined, due to direct and eccentric prestressing effects individually. A similar procedure is carried out for prestress in plate 5. Following notations will be used for longitudinal stress at n^{th} ridge.

Prestress	Direct	Eccentric
Plate 1	$(\sigma_{xp})_{1,n}$	$(\sigma_{xe})_{1,n}$
Plate 5	$(\sigma_{xp})_{5,n}$	$(\sigma_{xe})_{5,n}$

Step 3 - At optimum value, let the prestressing forces in plates 1 and 5 be x_1 and x_5, respectively. Let the sag in plates 1 and 5 be y_1 and y_5, respectively.

Application of prestress can be conceived as a direct prestress and eccentric prestress action. Direct prestress is the application of a compressive force, equal to prestressing force at the centroid of the plate. Considering prestress in plate 1,

Due to unit direct prestress, stress at midspan for n^{th} ridge = $(\sigma_{xp})_{1,n}$

$\therefore$ Due to x_1 units of direct prestress, stress at midspan for n^{th} ridge = $x_1 \cdot (\sigma_{xp})_{1,n}$

Eccentric prestress is due to the in-plane plate load caused by the parabolic sag of the cable. The plate load p's per unit length

$$p' = \text{prestress} \cdot \text{sag} \cdot 8/L^2$$

Hence, σ_x α plate load α prestress times sag.
This linear proportionality has been proved in Chapter 5. Also, several computer runs to check this relationship were carried out.

Due to unit prestress and unit sag, the stress at midspan for n^{th} ridge by eccentric action of prestress = $(\sigma_{xe})_{1,n}$

$\therefore$ Due to x_1 units of prestress and y_1 units of sag the stress at midspan for n^{th} ridge = $x_1 \cdot y_1 \cdot (\sigma_{xe})_{1,n}$

Hence, the stresses at the ridges due to the above values of the prestressing parameters are given by

$$D_n = x_1 \cdot (\sigma_{xp})_{1,n} + x_1 y_1 (\sigma_{xe})_{1,n} + x_5 (\sigma_{xp})_{5,n} + x_5 y_5 (\sigma_{xe})_{5,n}$$

where D_n is the longitudinal stress at n^{th} ridge at midspan due to prestressing.

Letting $x_1 = E_{11}$

$x_1 y_1 = E_{12}$ $\qquad$ $x_n = E_{n,1}$

$x_5 = E_{51}$ $\qquad$ (i.e.) $x_n y_n = E_{n,2}$ ---------- (6.1)

$x_5 y_5 = E_{52}$

$$D_n = E_{11} (\sigma_{xp})_{1,n} + E_{12} (\sigma_{xe})_{1,n} + E_{51} (\sigma_{xp})_{5,n} + E_{52} (\sigma_{xe})_{5,n} \quad \text{---------- (6.2)}$$

Equations 6.1 and 6.2 are employed to form the expressions for longitudinal stresses due to prestressing at all ridges.

Step 4 - In this step the objective function and the constraints will be set up.

Objective Function: This is an expression for total prestress and hence given by

$$z = E_{11} + E_{51} \text{ (minimize)}$$

Constraints: There are two categories in this. One, the allowable stresses control; two, that the maximum sag a plate can accommodate, controls.

Allowable Stresses Constraint -

Let, f_{ci} = allowable compressive stress at time of transfer

f_{ti} = allowable tensile stress at time of transfer

f_{co} = allowable compressive stress final

f_{to} = allowable tensile stress final

All of the above representations are only for magnitude. Now considering the formation of the constraint at n^{th} ridge:

Longitudinal stress due to prestress alone is given by equation 6.2.

Now, with self weight alone acting along with prestress, the longitudinal stress at n^{th} ridge is given by $D_n + (\sigma_{xd})_n$.

This stress should be less than the allowable stresses. This can be expressed as

$$-f_{ci} \leq D_n + (\sigma_{xd})_n \leq f_{ti}$$

Similar constraints can be formed at the n^{th} ridge, due to various combinations of loading, care being taken to employ proper allowable stresses for the combination of loadings in question.

$$-f_{co} \leq D_n + (\sigma_{xd})_n + (\sigma_{x\ell})_n \leq f_{to}$$

$$-f_{ci} \leq D_n + (\sigma_{xd})_n + (\sigma_{xw})_n \leq f_{ti}$$

$$-f_{co} \leq D_n + (\sigma_{xd})_n + (\sigma_{x\ell})_n + (\sigma_{xw})_n \leq f_{to}$$

It is assumed that combination of wind and self weight act at the time of transfer.

These can also be expressed as follows:

$$-f_{ci} - (\sigma_{xd})_n \leq D_n \leq f_{ti} - (\sigma_{xd})_n$$

$$-f_{co} - (\sigma_{xd})_n - (\sigma_{x\ell})_n \leq D_n \leq f_{to} - (\sigma_{xd})_n - (\sigma_{x\ell})_n \quad \text{---- (6.3)}$$

$$-f_{ci} - (\sigma_{x\ell})_n - (\sigma_{xw})_n \leq D_n \leq f_{ti} - (\sigma_{xd})_n - (\sigma_{xw})_n$$

$$-f_{co} - (\sigma_{xd})_n - (\sigma_{xw})_n - (\sigma_{x\ell})_n \leq D_n \leq f_{to} - (\sigma_{xd})_n - (\sigma_{xw})_n - (\sigma_{x\ell})_n$$

Looking at the above expressions it is seen that the constraint at n^{th} ridge as expressed by D_n, has more than one upper and lower limit. This violates the mutually exclusive property required of the constraint. By observation we can find the controlling lower limit, CL_n, which is algebraically the highest number of the various lower limits, and the controlling upper limit, CU_n, which is algebraically the smallest number of the various upper limits. Hence, a mutually exclusive, independent constraint D_n at n^{th} ridge will have CL_n and CU_n as lower and upper limit. Likewise, constraints at all ridges are found.

$$CL_n \leq D_n \leq CU_n, \quad n = 1 \text{ to total number of ridges.}$$

Sag Constraint - Sag is controlled by the concrete cover requirements and plate width. Assuming for all our problems, the prestressing cable layout is parabolic with maximum sag at midspan and zero eccentricity at the ends, as shown in Fig. 23, the sag can vary between $-(h_n - 2\ c_d)/2$ to $+(h_n - 2\ c_d)_2$. Sag constraint for the n^{th} prestressed plate is given by

$$-(h_n - 2\ c_d)/2 \leq y_n \leq (h_n - 2\ c_d)/2$$

But $y_n = E_{n,2}/E_{n,1}$ from equation 6.1.

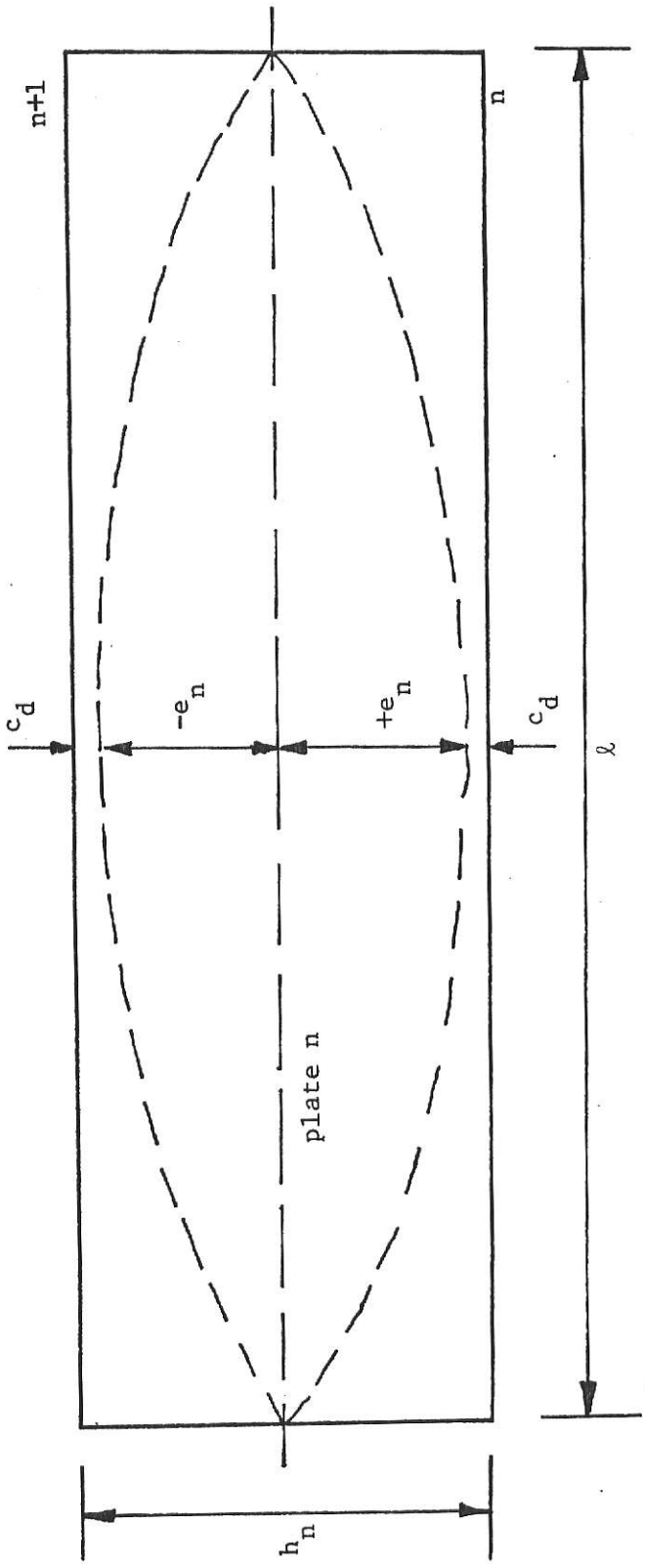


Fig. 23 - Possible Cable Geometry in a Typical Plate

Hence, $-(h_n - 2c_d)/2 \leq E_{n,2}/E_{n,1} \leq (h_n - 2c_d)/2$

$$\rightarrow \quad -\frac{(h_n - 2c_d)}{2} \cdot E_{n,1} \leq E_{n,2} \leq \frac{(h_n - 2c_d)}{2} E_{n,1}$$

$$\rightarrow \quad -E_{n,2} - [(h_n - 2c_d)/2]E_{n,1} = SL_n \leq 0$$

$$E_{n,2} - [(h_n - 2c_d)/2]E_{n,1} = SU_n \leq 0 \qquad \text{---------- (6.4)}$$

The two above constraints as represented by SU_n and SL_n are mutually exclusive and independent. Similar constraints are formed for each prestressed plate.

For the problem shown in Fig. 11, there are six ridges. Hence, six ridge stress constraints using expressions 6.2 and 6.3 are formed. Since two plates are prestressed, four sag constraints using equations 6.4 are formed. Another important constraint is that prestressing forces x_1 and x_5 cannot assume any negative value. This non-negativity constraint need not be formed for the type of IBM-subroutine, used for linear programming. The simplex algorithm can be found in Reference 7.

Step 5 - The objective function and the constraints are used with the IBM-MPS-360 (Linear Programming) and an optimum (minimum) value of total prestressing force and individual values of sag and prestressing force obtained. The formation of simplex tables, in order to give the input, will be clear from the examples. The input-output details are given in the appendix.

Step 6 - Now the uniqueness of the solution will be checked. Suppose for this example, if x_1 and y_1 have some value and x_5 and y_5 have zero, some value very much less than the optimum value but greater than zero is

assigned to x_3 as a lower bound and the new optimum value is tried. If we obtain the same optimum value for the objective function, any combination of x_1 and x_2, but their sum being equal to the optimum objective function value, will satisfy all boundary conditions. If the new solution is different, then the first solution is a unique solution. The examples will illustrate the use of linear programming for the selection of prestressing parameters.

Computer Programs

Step 1 - The analysis of a folded plate for gravity loads and wind load is carried out using the computer program discussed in Chapter 2.

Step 2 - The analysis of a folded plate for prestressing individually in each prestressed plate is carried out by a separate computer program. The flow chart given in Fig. 24 explains the operation of the program. The input includes the geometry number of plates to be prestressed, plate number and whether the structure and the prestressing are symmetrical. The output is longitudinal ridge stresses and transverse ridge moments in all plates at midspan due to prestressing in a given plate. The input, output details are given in Appendix D.

Step 3 - Once the objective function and the constraints are formed, they are fed into an IBM-subroutine MPS-360, to get the optimal values of prestressing forces and sag. The uniqueness of the solution is checked by using the same subroutine.

Example 6.1: For the folded plate shown in Fig. 15 find the optimum prestressing force and sag for edge beams equally prestressed with parabolically draped cables. The loading combinations are given below:

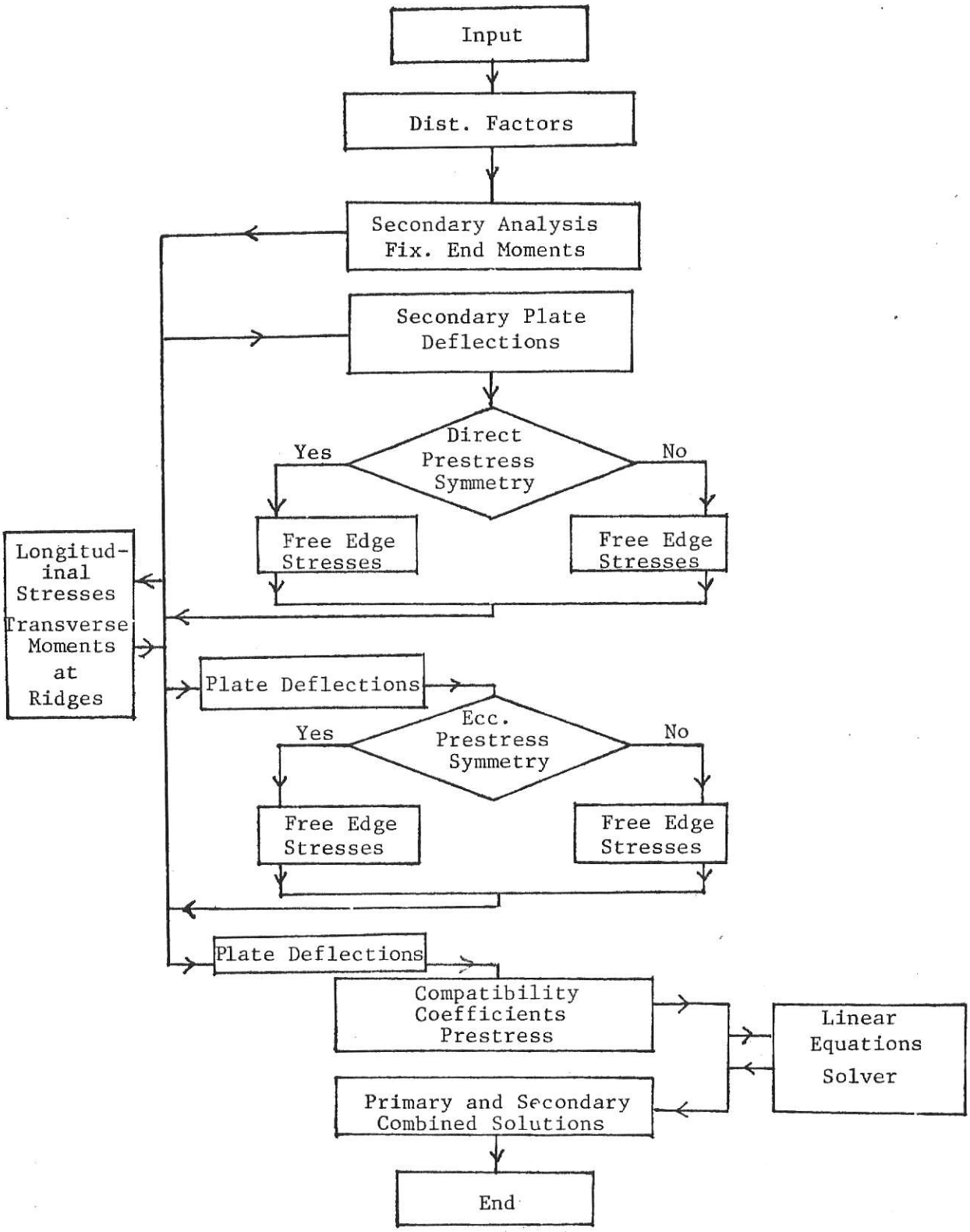


Fig. 24 - Flow Chart for Prestressed Folded Plate Analysis Only, Main Program

DEAD LOAD (Self weight only)

DEAD LOAD + FULL LIVE LOAD

DEAD LOAD + WIND LOAD

DEAD LOAD + FULL LIVE LOAD + WIND LOAD

Solution:

Step 1 - Analyze the folded plate, neglecting torsional stiffness of the edge beam for Dead Load, Wind Load and Live Load. From the computer output the longitudinal stresses for various combinations of loadings are listed in Table 6.1.1.

Step 2 - Since edge beams will be assumed to have equal prestressing, a unit prestressing force of one kip and a unit sag of one foot is applied at each edge beam simultaneously and stresses are computed (Table 6.1.2).

Step 3 - Let x_1 units of prestressing force and y_1 units of sag be the optimal value. Due to these parameters, longitudinal ridge stresses at midspan are, (from equation 6.2)

Ridge	D_n
1	$-5.7587\ E_{11} - 4.2240\ E_{12} = D1$
2	$-1.4610\ E_{11} + 1.2799\ E_{12} = D2$
3	$0.1622\ E_{11} + 0.4318\ E_{12} = D3$
4	$0.2255\ E_{11} - 0.8185\ E_{12} = D4$
5	$0.1622\ E_{11} + 0.4318\ E_{12} = D5$
6	$-1.4610\ E_{11} + 1.2799\ E_{12} = D6$
7	$-5.7587\ E_{11} - 4.2240\ E_{12} = D7$

Because of symmetry,

D1 = D7; D2 = D6; D3 = D5

Table 6.1.1 - Analysis of the Folded Plate for Loading

Longitudinal Stresses at Midspan (psi)

Ridge	DL psi	DL + LL psi	DL + WL psi	DL + WL + LL psi
1	1128.194	1455.829	1175.901	1503.536
2	- 44.733	34.260	- 87.802	- 8.809
3	-196.954	-279.430	-202.296	-284.772
4	- 48.916	-145.941	- 21.362	-118.387
5	-196.954	-279.428	-167.582	-250.056
6	- 44.733	34.258	- 53.842	25.149
7	1128.194	1455.829	1019.352	1346.991

(1 psi. = 6900 N/m^2)

Table 6.1.2 - Analysis of Folded Plate for Prestressing (unit force and sag)

Longitudinal Stresses at Midspan (psi)

Ridge	Direct Prstrs. psi	Ecc. Prstrs. psi
1	-5.7587	-4.2240
2	-1.4610	1.2799
3	0.1622	0.4318
4	0.2255	-0.8185
5	0.1622	0.4318
6	-1.4610	1.2799
7	-5.7587	-4.2240

(1 psi. = 6900 N/m^2)

Step 4 - Selection of prestressing parameters will be based on stresses not exceeding the allowable stresses for the combination of loading given in the problem.

Assume F_c = 5000 psi. and F_{ci} = 4000 psi.

$f_{ci} = 0.6 \times F_{ci} = 2400$ psi. (at transfer)

$f_{co} = .45 \times F_c = 2250$ psi. (at final load)

To be consistent with the assumption used in Reference (1), no tensile stress is allowed. So,

$f_{ti}, f_{co} = 0.$

Constraints Due to Stresses: (From Equation 6.3)

Ridge 1

$-2400 \leq D1 + 1128.19 \leq 0.0$	$-3528.19 \leq D1 \leq -1128.19$	DL
$-2250 \leq D1 + 1455.83 \leq 0.0$	$-3705.83 \leq D1 \leq -1455.83$	DL + LL
$-2250 \leq D1 + 1175.90 \leq 0.0$	$-3425.90 \leq D1 \leq -1175.90$	DL + WL
$-2250 \leq D1 + 1503.54 \leq 0.0$	$-3753.54 \leq D1 \leq \underline{-1503.54}$	DL + LL + WL

Ridge 2

$-2400 \leq D2 - 44.733 \leq 0.0$	$-2355.27 \leq D2 \leq 44.73$	DL
$-2250 \leq D2 + 34.26 \leq 0.0$	$-2284.26 \leq D2 \leq -34.26$	DL + LL
$-2250 \leq D2 - 87.802 \leq 0.0$	$\underline{-2162.20} \leq D2 \leq 87.80$	DL + WL
$-2250 \leq D2 - 8.809 \leq 0.0$	$-2241.19 \leq D2 \leq 8.81$	DL + LL + WL

Ridge 3

$-2400 \leq D3 - 196.54 \leq 0.0$	$-2203.46 \leq D3 \leq 196.54$	DL
$-2250 \leq D3 - 279.43 \leq 0.0$	$-1970.57 \leq D3 \leq 279.43$	DL + LL
$-2250 \leq D3 - 202.30 \leq 0.0$	$-2047.70 \leq D3 \leq 202.30$	DL + WL
$-2250 \leq D3 - 284.77 \leq 0.0$	$\underline{-1965.23} \leq D3 \leq 284.77$	DL + LL + WL

Ridge 4

$-2400 \leq D4 - 48.92 \leq 0.0$	$-2351.08 \leq D4 \leq 48.92$	DL
$-2250 \leq D4 - 145.94 \leq 0.0$	$-2104.06 \leq D4 \leq 145.94$	DL + LL
$-2250 \leq D4 - 21.36 \leq 0.0$	$-2228.64 \leq D4 \leq 21.36$	DL + WL
$-2250 \leq D4 - 118.39 \leq 0.0$	$-2131.61 \leq D4 \leq 118.39$	DL + LL + WL

Ridge 5

$-2400 \leq D5 - 196.95 \leq 0.0$	$-2203.05 \leq D5 \leq 196.95$	DL
$-2250 \leq D5 - 279.43 \leq 0.0$	$-1970.57 \leq D5 \leq 279.43$	DL + LL
$-2250 \leq D5 - 167.58 \leq 0.0$	$-2082.42 \leq D5 \leq 167.58$	DL + WL
$-2250 \leq D5 - 250.07 \leq 0.0$	$-1999.93 \leq D5 \leq 250.07$	DL + LL + WL

Ridge 6

$-2400 \leq D6 - 44.73 \leq 0.0$	$-2353.27 \leq D6 \leq 44.73$	DL
$-2250 \leq D6 + 34.26 \leq 0.0$	$-2284.26 \leq D6 \leq -34.26$	DL + LL
$-2250 \leq D6 - 53.84 \leq 0.0$	$-2196.16 \leq D6 \leq 53.84$	DL + WL
$-2250 \leq D6 + 25.15 \leq 0.0$	$-2275.15 \leq D6 \leq -25.15$	DL + LL + WL

Ridge 7

$-2400 \leq D7 + 1128.19 \leq 0.0$	$-3528.19 \leq D7 \leq -1128.19$	DL
$-2250 \leq D7 + 1455.83 \leq 0.0$	$-3705.83 \leq D7 \leq -1455.83$	DL + LL
$-2250 \leq D7 + 1019.35 \leq 0.0$	$-3269.35 \leq D7 \leq -1019.35$	DL + WL
$-2250 \leq D7 + 1346.99 \leq 0.0$	$-3596.99 \leq D7 \leq -1346.99$	DL + LL + WL

Now the controlling lower and upper limit will be found as discussed earlier for each constraint D. Since D1 = D7, they will be treated as one constraint D1. A similar procedure is applied for D2 and D6, and D3 and D5.

$$-3269.35 \leq D1 \leq -1503.54$$

$$-2162.2 \leq D2 \leq - 34.26$$

$$-1965.23 \leq D3 \leq 167.58$$

$$-2104.06 \leq D4 \leq 21.36$$

Constraints on Sag: Assuming 1 1/2" cover, and that the cable must have a downward sag, the cable will have a sag varying from 0' to (3-2x · 125)/2'.

$$0 \leq y_1 \leq 1.375$$

$$0 \leq \frac{E_{12}}{E_{11}} \leq 1.375$$

$$-1.375\ E_{11} + E_{12} \leq 0.0 = SU_1 \leq 0.0$$

$$- E_{12} \leq 0.0 = SL_1 \leq 0.0$$

Objective Function:

Total prestressing force = prestress in plate 1 + prestress in plate 6

$$= x_1 + x_1 = 2x_1 = 2E_{11}$$

Hence, Objective function $z = E_{11}$ (minimize).

Step 5 - Formation of simplex table to give input to the computer for optimization (Table 6.1.3). Refer to Appendix D for detailed explanation of the input.

Result: E_{11} = 152.793 Kips

$E_{12} = x_1 y_1 = 147.645$

$y_1 = 147.645/152.793 = .9663'$

Prestressing force = 152.793 Kips

Sag = 11.6" (.9663')

Table 6.1.3 - Simplex Table

Structural Variables

	E_{11}	E_{12}		Limits	Range 1
Obj. Function: Force	1.0		N		
Constraints					
D1	-5.7587	-4.2240	≤	-1503.54	1766.35
D2	-1.4610	1.2799	≤	- 34.26	2127.94
D3	0.1622	0.4318	≤	167.58	2132.81
D4	0.2255	-0.8185	≤	21.36	2125.42
SU1	-1.375	1.0	≤	0.0	0.0
SL1	0.0	-1.0	≤	0.0	0.0

Discussion of Results

The resulting prestressing values obtained from the computer output are now used in the computer program which does the analysis of the folded plate. The resulting stresses due to various load combinations are computed and shown in Table 6.1.4. Billington (1) arrived at a prestressing force of 146 Kips (with an initial assumption of sag = 1.0'). If these values of prestress and sag are used to analyze the folded plate along with dead load, live load and wind load, the resulting stresses are those indicated in the far right column on Table 6.1.4. His value of 146 Kips corresponds to only one restraint, namely, no tensile longitudinal stresses at midspan due to dead and full live loading.

It is seen that the optimal solution satisfies all constraints but that the solution using a prestress force of 146 Kips and sag of 1 foot violates no tension constraint at ridge 1 for a given wind load.

Example 6.2: This example deals with a north light folded plate. Fig. 18 shows the cross-section of the folded plate and loading details.

Problem: To find the optimum solution for prestressing parameters when plates are prestressed with parabolic cable profiles. Because of structural assymmetry wind blowing in two directions, north-south (WL1) and south-north (WL2) will be considered. Because of the assymmetrical cross-section, all the plates will be assumed to be prestressed and the optimum solution will be determined. Combinations of loadings are as given below. Assume 15% loss in prestress from initial conditions.

DEAD LOAD (Self weight)

DEAD LOAD + FULL LIVE LOAD

DEAD LOAD + WIND LOAD 1

DEAD LOAD + WIND LOAD 2

Table 6.1.4 - Analysis for Load and Optimum Prestressing

	P = 153 K y = 11.6"				P = 146 K y = 12"
Ridge	DL + PRSTRS. psi	TL + PRSTRS. psi	DL + WL + PRSTRS. psi	TL + WL + PRSTRS. psi	TL + PRSTRS. + WL (ref. 1) psi
1	-375.450	- 47.815	-327.743	- 0.108	46.050
2	- 79.020	- 0.028	-122.090	- 43.097	- 35.252
3	-108.416	-190.892	-113.758	-196.234	-198.049
4	-135.305	-232.330	-107.751	-204.776	-204.966
5	-108.416	-190.890	- 79.043	-161.518	-163.333
6	- 79.021	- 0.030	- 88.130	- 9.139	- 1.295
7	-375.454	- 47.815	-484.292	-156.654	-110.496

DEAD LOAD + FULL LIVE LOAD + WIND LOAD 1

DEAD LOAD + FULL LIVE LOAD + WIND LOAD 2

(reduced wind load because of increased allowable stress)

Step 1 - Analyze the folded plate for Dead Load, Wind Loads and Live Load. From the computer output longitudinal midspan stresses at ridges are listed in Table 6.2.1.

Step 2 - Unit prestressing force of 1 Kip and unit sag of 1 foot is applied to each plate individually and the longitudinal ridge stresses at midspan are computed (Table 6.2.2).

Step 3 - Let the following be the prestressing parameters.

(Initial prestress)

Plate 1 x_1 = Force = E_{11}

y_1 = Sag

$x_1y_1 = E_{12}$

Plate 2 x_2 = Force = E_{21}

y_2 = Sag

$x_2y_2 = E_{22}$

Plate 3 x_3 = Force = E_{31}

y_3 = Sag

$x_3y_3 = E_{32}$

Plate 4 x_4 = Force = E_{41}

y_4 = Sag

$x_4y_4 = E_{42}$

Table 6.2.1 - Analysis for Load

Longitudinal Stresses at Midspan (psi)

Ridge	DL	DL + WL1	DL + WL2	TL	TL + WL1	TL + WL2
1	1155.17	978.24	1124.36	1636.36	1459.42	1605.54
2	222.47	170.53	230.92	304.55	252.61	313.00
3	- 697.70	-591.71	- 686.85	- 970.89	- 864.90	- 960.04
4	697.70	591.71	690.52	964.87	858.88	957.69
5	- 222.47	-170.53	- 235.72	- 296.65	- 244.70	- 309.89
6	-1155.17	-978.243	-1139.50	-1611.68	-1434.75	-1596.02

(1 psi. = 6900 N/m^2)

Table 6.2.2 - Analysis for Prestressing (unit force and sag)

σ_x at Midspan (psi)

Ridge	1	2	3	4	5	6
DP1	-11.966	-5.949	1.360	-0.564	0.169	- 0.305
EP1	- 5.853	-1.161	2.082	-0.713	-0.634	0.226
DP2	- 2.585	-2.004	-2.782	0.847	0.658	- 1.052
EP2	- 2.768	-0.951	0.944	-0.330	0.159	0.203
DP3	1.320	-0.524	-1.411	-1.411	-0.524	1.320
EP3	0.371	-0.091	-0.466	0.466	0.091	- 0.371
DP4	- 1.052	0.658	0.847	-2.782	-2.004	- 2.584
EP4	- 0.203	-0.159	0.330	-0.944	0.951	2.768
DP5	- 0.305	0.169	-0.564	1.360	-5.949	-11.966
EP5	- 0.226	0.634	0.713	-2.082	1.161	5.853

(1 psi. = 6900 N/m^2)

Plate 5 x_5 = Force = E_{51}

y_5 = Sag

$x_5 y_5 = E_{52}$

From equation 6.2, and from Step 2, the following expressions for longitudinal ridge stresses at midspan can be established.

$$D1 = -11.966\ E_{11} - 5.853\ E_{12} - 2.585\ E_{21} - 2.768\ E_{22} + 1.320\ E_{31} + 0.371\ E_{32} - 1.052\ E_{41} - 0.203\ E_{42} - 0.305\ E_{51} - 0.226\ E_{52}$$

$$D2 = -5.949\ E_{11} - 1.161\ E_{12} - 2.004\ E_{21} - 0.951\ E_{22} - 0.524\ E_{31} - 0.091\ E_{32} + 0.658\ E_{41} - 0.159\ E_{42} + 0.169\ E_{51} + 0.634\ E_{52}$$

$$D3 = 1.360\ E_{11} + 2.082\ E_{12} - 2.782\ E_{21} + 0.944\ E_{22} - 1.411\ E_{31} - 0.466\ E_{32} + 0.847\ E_{41} + 0.330\ E_{42} - 0.564\ E_{51} + 0.713\ E_{52}$$

$$D4 = -0.564\ E_{11} - 0.713\ E_{12} + 0.847\ E_{21} - 0.330\ E_{22} - 1.411\ E_{31} + 0.466\ E_{32} - 2.782\ E_{41} - 0.944\ E_{42} + 1.360\ E_{51} - 2.082\ E_{52}$$

$$D5 = 0.169\ E_{11} - 0.634\ E_{12} + 0.658\ E_{21} + 0.159\ E_{22}$$
$$- 0.524\ E_{31} + 0.091\ E_{32} - 2.004\ E_{41}$$
$$+ 0.951\ E_{42} - 5.949\ E_{51} + 1.161\ E_{52}$$

$$D6 = -0.305\ E_{11} + 0.226\ E_{12} - 1.052\ E_{21} + 0.203\ E_{22}$$
$$+ 1.320\ E_{31} - 0.371\ E_{32} - 2.584\ E_{41}$$
$$+ 2.768\ E_{42} - 11.966\ E_{51} + 5.853\ E_{52}$$

These are stresses due to initial prestress. Stresses due to final prestress can be obtained by multiplying D_n by 0.85.

Step 4 -

Objective Function:

Total prestressing force = $E_{11} + E_{21} + E_{31} + E_{41} + E_{51}$

Hence, the objective function to be minimized is

$$z = E_{11} + E_{21} + E_{31} + E_{41} + E_{51}$$

Constraints Due to Stress:

Allowable Stresses $f_c' = 4000$ psi. (at transfer and at final load)

$f_{ti} = 3\sqrt{f_c'} = 190$ psi.

At Transfer

$f_{ci} = .6 \times f_c' = 2400$ psi.

$f_{to} = 6\sqrt{f_c'} = 380$ psi.

$f_{co} = .45\ f_c' = 1800$ psi.

Loss in prestress = 15%

Ridge 1

$-2400 \leq D1 + 1155.17 \leq 190;\ -3555.17 \leq D1 \leq -965.17$ DL

$-2400 \leq D1 + 978.27 \leq 190;\ -3378.24 \leq D1 \leq -788.24$ DL + WL1

$-2400 \leq D1 + 1124.36 \leq 190;\ -3524.36 \leq D1 \leq -934.36$ DL + WL2

$-1800 \leq .85D1 + 1636.36 \leq 380;\ -4042.78 \leq D1 \leq -1478.07$ TL

$-1800 \leq .85D1 + 1459.42 \leq 380;\ -3834.61 \leq D1 \leq -1269.91$ TL + WL1

$-1800 \leq .85D1 + 1605.54 \leq 380;\ -4006.52 \leq D1 \leq -1441.81$ TL + WL2

Ridge 2

$-2400 \leq D2 + 222.47 \leq 190;\ -2622.47 \leq D2 \leq -32.47$ TL

$-2400 \leq D2 + 170.53 \leq 190;\ -2570.43 \leq D2 \leq 19.47$ DL + WL1

$-2400 \leq D2 + 230.92 \leq 190;\ -2630.92 \leq D2 \leq -40.92$ DL + WL2

$-1800 \leq .85D2 + 304.55 \leq 380;\ -2475.84 \leq D2 \leq 88.76$ TL

$-1800 \leq .85D2 + 252.61 \leq 380;\ -2414.84 \leq D2 \leq 149.87$ TL + WL1

$-1800 \leq .85D2 + 313.00 \leq 380;\ -2485.88 \leq D2 \leq 78.82$ TL + WL2

Ridge 3

$-2400 \leq D3 - 697.70 \leq 190;\ -1702.3 \leq D3 \leq 887.7$ DL

$-2400 \leq D3 - 591.71 \leq 190;\ -1808.29 \leq D3 \leq 781.7$ DL + WL1

$-2400 \leq D3 - 686.85 \leq 190;\ -1713.15 \leq D3 \leq 876.85$ DL + WL2

$-1800 \leq .85D3 - 970.89 \leq 380;\ -975.42 \leq D3 \leq 1589.28$ TL

$-1800 \leq .85D3 - 864.90 \leq 380;\ -1100.12 \leq D3 \leq 1464.59$ TL + WL1

$-1800 \leq .85D3 - 960.04 \leq 380;\ -988.19 \leq D3 \leq 1576.52$ TL + WL2

Ridge 4

$-2400 \leq D4 + 697.70 \leq 190;\ -3097.70 \leq D4 \leq -507.70$ DL

$-2400 \leq D4 + 591.70 \leq 190;\ -2991.71 \leq D4 \leq -401.71$ DL + WL1

$-2400 \leq D4 + 690.52 \leq 190;\ -3090.52 \leq D4 \leq -500.72$ DL + WL2

$-1800 \leq .85D4 + 964.87 \leq 380$; $-3252.79 \leq D4 \leq -688.08$ TL

$-1800 \leq .85D4 + 858.88 \leq 380$; $-3128.09 \leq D4 \leq -563.39$ TL + WL1

$-1800 \leq .85D4 + 957.69 \leq 380$; $-3244.34 \leq D4 \leq -679.64$ TL + WL2

Ridge 5

$-2400 \leq D5 - 222.47 \leq 190$; $-2177.53 \leq D5 \leq 412.47$ DL

$-2400 \leq D5 - 170.53 \leq 190$; $-2229.47 \leq D5 \leq 360.53$ DL + WL1

$-2400 \leq D5 - 235.72 \leq 190$; $-2164.28 \leq D5 \leq 425.72$ DL + WL2

$-1800 \leq .85D5 - 296.65 \leq 380$; $-1768.65 \leq D5 \leq 796.06$ TL

$-1800 \leq .85D5 - 244.70 \leq 380$; $-1829.76 \leq D5 \leq 734.94$ TL + WL1

$-1800 \leq .85D5 - 309.89 \leq 380$; $-1753.07 \leq D5 \leq 811.64$ TL + WL2

Ridge 6

$-2400 \leq D6 - 1153.47 \leq 190$; $-1244.87 \leq D6 \leq 1345.17$ DL

$-2400 \leq D6 - 978.24 \leq 190$; $-1421.76 \leq D6 \leq 1168.24$ DL + WL1

$-2400 \leq D6 - 1139.50 \leq 190$; $-1260.50 \leq D6 \leq 1329.5$ DL + WL2

$-1800 \leq .85D6 - 1611.68 \leq 380$; $-221.55 \leq D6 \leq 2343.15$ TL

$-1800 \leq .85D6 - 1434.75 \leq 380$; $-429.71 \leq D6 \leq 2135.0$ TL + WL1

$-1800 \leq .85D6 - 1596.02 \leq 380$; $-239.78 \leq D6 \leq 2324.73$ TL + WL2

Now, it is seen that for each constraint, D_n, there are several upper and lower limits. Controlling ones are selected as discussed in Sec. 6.3, and listed below.

$-3378.24 \leq D1 \leq -1478.07$

$-2414.84 \leq D2 \leq -40.92$

$-975.42 \leq D3 \leq 781.71$

$-2991.71 \leq D4 \leq -688.08$

$-1753.07 \leq D5 \leq 360.53$

$-221.55 \leq D6 \leq 1168.24$

Constraints on Sag: The center of gravity of the cables is assumed to lie at least 6" from the longitudinal edges of each plate in order to take care of the requirements for concrete cover, and distance between two adjacent cables. With this, the maximum sag a plate can accommodate is given by

$$\pm (h_n - 2 \times 0.5)/2.0$$

Plate 1 - sag = y_1 Max. sag = $(2.031 - 2 \times 0.5)/2$

$-.52 \leq y_1 \leq + .52$ $= 0.52'$

$$-.52 \leq \frac{E_{12}}{E_{11}} \leq + .52$$

$$\rightarrow -.52\ E_{11} - E_{12} \leq 0 \text{ ---------- SL1}$$

$$-.52\ E_{11} + E_{12} \leq 0 \text{ ---------- SU1}$$

Plate 2 - sag = y_2 Max. sag = $(3.625 - 2 \times 0.5)/2$

$-1.32 \leq y_2 \leq +1.32$ $= 1.32'$

$$-1.32 \leq \frac{E_{22}}{E_{21}} \leq +1.32$$

$$\rightarrow -1.32\ E_{21} - E_{22} \leq 0 \text{ ---------- SL2}$$

$$-1.32\ E_{21} + E_{22} \leq 0 \text{ ---------- SU2}$$

Plate 3 - sag = y_3 Max. sag = $(12.297 - 2 \times 0.5)/2$

$-5.65 \leq y_3 \leq +5.65$ $= 5.65'$

$$-5.65 \leq \frac{E_{32}}{E_{31}} \leq +5.65$$

$$\rightarrow -5.65\ E_{31} - E_{32} \leq 0 \text{ ---------- SL3}$$

$$-5.65\ E_{31} + E_{32} \leq 0 \text{ ---------- SU3}$$

Plate 4 - sag = y_4 Max. sag = 1.32'

$$-1.32 \leq y_4 \leq +1.32$$

$$-1.32 \leq \frac{E_{42}}{E_{41}} \leq +1.32$$

$$\rightarrow -1.32\ E_{41} - E_{42} \leq 0 \text{ ---------- SL4}$$

$$-1.32\ E_{41} + E_{42} \leq 0 \text{ ---------- SU4}$$

Plate 5 - sag = y_5 Max. sag = .52'

$$-.52 _ y_5 _ +.52$$

$$-.52 \leq \frac{E_{52}}{E_{51}} \leq +.52$$

$$\rightarrow -.52\ E_{51} - E_{52} \leq 0 \text{ ---------- SL5}$$

$$-.52\ E_{51} + E_{52} \leq 0 \text{ ---------- SU5}$$

From simplex table 6.2.3, input is given to the computer program for the optimization process.

The IBM-supplied subroutine, MPS-360 for optimization cannot have negative values for the variables in the problem. But in this example, variables E_{12}, E_{22}, E_{32}, E_{42} and E_{52} can have negative values. Hence, in order to meet the non-negativity condition, each of these variables can be expressed as the difference of two new variables which are individually non-negative variables.

Table 6.2.3 - Simplex Table

Structural Variables

	E_{11}	E_{12}	E_{21}	E_{22}	E_{31}	E_{32}	E_{41}	E_{42}	E_{51}	E_{52}		Limits	Range 1
Obj. Fn: Force	1.0		1.0		1.0		1.0		1.0				
Constraints:													
D1	-11.966	-5.853	-2.585	-2.768	1.320	0.371	-1.052	-0.203	- 0.305	0.226	<	-1478.07	1900.17
D2	- 5.949	-1.161	-2.004	-0.951	-0.524	-0.091	0.658	-0.159	0.169	0.634	<	- 40.92	2373.92
D3	1.360	2.082	-2.782	0.944	-1.411	-0.466	0.847	0.330	- 0.564	0.713	<	781.71	1757.13
D4	- 0.564	-0.713	0.847	-0.330	-1.411	0.466	-2.782	-0.944	1.360	-2.082	<	- 688.08	2303.63
D5	0.169	-0.634	0.658	0.159	-0.524	0.091	-2.004	0.951	- 5.949	1.161	<	360.53	2113.60
D6	- 0.305	0.226	-1.052	0.203	1.320	-0.371	-2.584	2.768	-11.966	5.583	<	1168.24	1389.79
SU1	- 0.52	-1.0									<	0	
SL1	- 0.52	+1.0									<	0	
SU2			-1.32	-1.0							<	0	
SL2			-1.32	1.0							<	0	
SU3					-5.65	-1.0					<	0	
SL3					-5.65	1.0					<	0	
SU4							-1.32	-1.0			<	0	
SL4							-1.32	1.0			<	0	
SU5									- 0.52	-1.0	<	0	
SL5									- 0.52	1.0	<	0	

$$E_{12} = E_{12}' - E_{12}''$$

where $E_{12}' \geq 0$, $E_{12}'' \geq 0$

(i.e.) $E_{n1} = E_{n1}' - E_{n2}''$, n = 1, 2, 3, 4, 5

For example, after this substitution the expression for stresses at Ridge 1, due to prestressing will be

$$D_1 = -11.966\ E_{11} - 5.853\ (E_{12}' - E_{12}'') - 2.585\ E_{21}$$
$$- 2.768\ (E_{22}' - E_{22}'') + 1.320\ E_{31} + 0.371\ (E_{32}' - E_{32}'')$$
$$- 1.052\ E_{41} - 0.203\ (E_{42}' - E_{42}'') - 0.305\ E_{51} - 0.226$$
$$(E_{51}' - E_{52}'')$$

This substitution should be made for all variables in the constraints which can assume negative values. Results from computer output:

Minimum value of total prestressing force = 236.243 Kips.

Optimal Assignment:

1) prestress in plate 1 = 85.196 Kips (E_{11})
   sag x force = 44.299 (E_{12})
   sag = 44.299/85.196 = .52' = 6.25"

2) prestress in plate 4 = 151.051 Kips (E_{41})
   sag x force = 199.388 (E_{42})
   sag = 199.388/151.051 = 1.32' = 15.84"

3) no prestressing in plates 2, 3 and 5.

Step 5 - In order to check the uniqueness of the solution above, 1) first a prestressing force of 10 Kips with sag undetermined is imposed on plate 5 and the optimum is found to be 246.663 Kips (> 236.243).

2) a prestress force of 20 Kips is imposed on plate 2 and the optimum is found out to be 251.627 (> 236.243). 3) a prestressing force of 10 Kips was made minimum in plate 3 and optimum is found to be 243.67 Kips.

In all these trials it is found the new optimum value is greater than the initial solution. Hence, the initial solution appears to be a unique solution. Now with these optimal values of prestress longitudinal stress at midspan at all ridges are tabulated in 6.2.4 and 6.2.5.

Discussion of Results

Looking at the cross-section of the folded plate one may be led to think that plate 3 should be prestressed. If we consider the folded plate as a beam simply supported, we can arrive at the neutral axis, which is inclined to the horizontal by 28°, as shown in Fig. 18. This was derived in section 4, where the beam method of analysis was discussed. The folded plate will bend more or less about this neutral axis. Among the plates which have tensile stresses under applied loading, plates 1 and 4 are inclined steeper to the neutral axis than the other 3 plates. Hence, the maximum upthrust due to prestressing can be achieved only by prestressing these two plates with a parabolic profile. This is what the linear program solution also indicates in its output.

Example 2.3: This example is given in Reference 3. In the reference the authors analyzed and designed a v-shape folded plate, shown in Fig. 25, by beam method.

Problem: Select the prestressing parameters in each plate for the following loading combinations:

Dead Load at Transfer

Dead + Full Live Load

Table 6.2.4 - Stresses at Transfer σ_x psi.

f_{ci} = -2400 psi; f_{ti} = 190 psi.

Ridge	DL	DL + WL1	DL + WL2
1	- 324.362	-501.293	-355.179
2	- 269.033	-320.977	-260.581
3	- 296.000	-190.010	-285.147
4	10.242	- 95.747	3.059
5	- 348.677	-296.731	-361.914
6	-1009.004	-832.075	-993.341

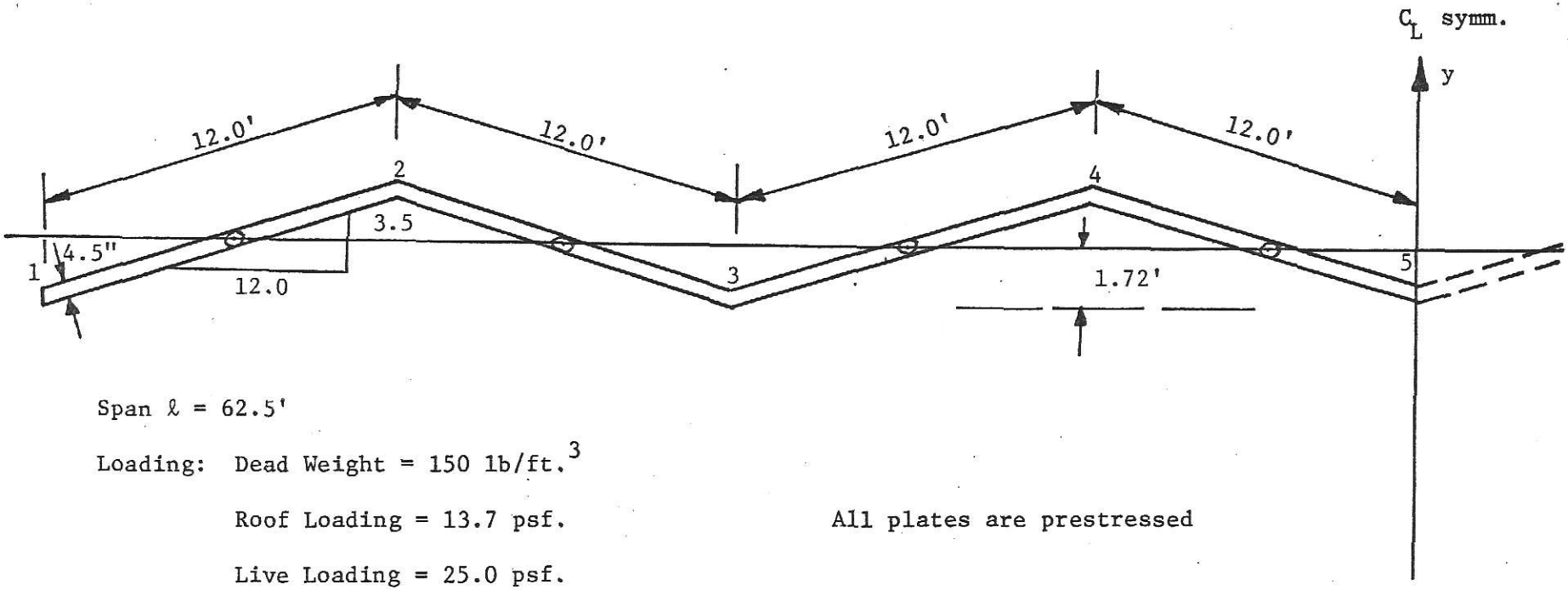
(1 psi. = 6900 N/m^2)

Table 6.2.5 - Stresses at Final Loads σ_x psi.
(15% Loss)

f_{c0} = -1800 psi; f_{t0} = 380 psi.

Ridge	TL	TL + WL1	TL = WL2
1	378.74	347.92	201.81
2	- 113.22	- 104.77	- 165.17
3	- 629.44	- 618.59	- 523.45
4	380.53	373.35	274.54
5	- 403.92	- 417.15	- 351.97
6	-1487.45	-1471.78	-1310.52

(1 psi. = 6900 N/m^2)



Span ℓ = 62.5'

Loading: Dead Weight = 150 lb/ft.^{3}

Roof Loading = 13.7 psf.

Live Loading = 25.0 psf.

All plates are prestressed

Fig. 25 - Cross-Section of Folded Plate Analyzed by J. C. Brough and B. H. Stephens (3)

Step 1 - Analyze the folded plate for dead and live load. The longitudinal stresses, σ_x at midspan are listed in Table 6.4.1.

Step 2 - All the plates will be prestressed with a parabolic cable profile. Unit prestress and unit sag are applied to each plate individually and the stresses are found. Because of the symmetrical shape and loading, prestress in plates 1 and 8, in plates 2 and 7, in plates 3 and 6 and in plates 4 and 5 will be equal. (Table 6.4.2)

Let the following be the prestressing parameters: (Initial prestress)

Plate 1 - force = $x_1 = E_{11}$

sag = y_1

let $x_1 \cdot y_1 = E_{12}$

Plate 2 - force = $x_2 = E_{21}$

sag = y_2

let $x_2 \cdot y_2 = E_{22}$

Plate 3 - force = $x_3 = E_{31}$

sag = y_3

let $x_3 \cdot y_3 = E_{32}$

Plate 4 - force = $x_4 = E_{41}$

sag = y_4

let $x_4 \cdot y_4 = E_{42}$

Now, stresses at ridge D_n due to prestressing alone can be expressed as follows:

$$D1 = -1.927\ E_{11} - 0.561\ E_{12} + 0.498\ E_{21} - .155\ E_{22} - .161\ E_{31}$$

$$- .045\ E_{32} + .049\ E_{41} - .011\ E_{42}$$

Table 6.4.1 - Analysis of the Folded Plate for Load

σ_x (psi) at Midspan

Ridge	Dead Load	Total Load
1	877.359	1490.140
2	-919.192	-1561.190
3	879.015	1492.952
4	-821.996	-1396.108
5	846.987	1438.553
6	-821.996	-1396.108
7	879.015	1492.952
8	-919.192	-1561.190
9	877.359	1490.140

(1 psi. = 6900 N/m^2)

Table 6.4.2 - Analysis of the Folded Plate for Prestressing (unit force and sag)

σ_x (psi) at Midspan

Ridge	DP	EP	DP	EP	DP	EP	DP	EP
1	-1.927	-0.561	0.498	-0.155	- .161	- .045	0.049	-0.011
2	-0.706	0.357	-1.039	0.301	-0.262	0.083	-0.056	+0.030
3	0.165	-0.099	-0.928	-0.283	-0.920	-0.284	0.143	-0.104
4	-0.064	0.029	0.269	0.084	-1.032	0.302	-0.714	0.356
5	0.057	-0.012	-0.183	-0.050	0.461	-0.158	-1.876	-0.552

(1 psi. = 6900 N/m^2)

$$D2 = -0.706\ E_{11} + 0.357\ E_{12} - 1.039\ E_{21} + 0.301\ E_{22} + 0.262\ E_{31} + 0.083\ E_{32} - 0.056\ E_{41} + 0.030\ E_{42}$$

$$D3 = 0.165\ E_{11} - 0.099\ E_{12} - 0.928\ E_{21} - 0.283\ E_{22} - 0.920\ E_{31} - 0.284\ E_{32} + 0.143\ E_{41} - 0.104\ E_{42}$$

$$D4 = -0.064\ E_{11} + 0.029\ E_{12} + 0.269\ E_{21} + 0.084\ E_{22} - 1.032\ E_{31} + 0.302\ E_{32} - 0.714\ E_{41} + 0.356\ E_{42}$$

$$D5 = 0.057\ E_{11} - 0.012\ E_{12} - 0.183\ E_{21} - 0.050\ E_{22} + 0.461\ E_{31} - 0.158\ E_{32} - 1.876\ E_{41} - 0.552\ E_{42}$$

Step 4 -

Objective Function:

$$z = x_1 + x_2 + x_3 + x_4 = E_{11} + E_{21} + E_{31} + E_{41}$$

Constraints Due to Stress: $f_c' = 3750$ psi.

Allowable Stresses: $f_{ci} = .6 \times f_c' = 2250$ psi.

$f_{ti} = 3\sqrt{f_c'} = 184.0$ psi.

$f_{co} = .45 \times f_c' = 1687.5$ psi.

$f_{to} = 6\sqrt{f_c'} = 368.0$ psi.

Loss in prestress = 15%

Ridge 1

$-2250 < D1 + 877.36 < 184.0$; $-3127.36 < D1 < -693.36$ DL

$-1687.5 < D1 + 1490.14 < 368.0$; $-3738.4 < D1 < -1320.16$ TL

Ridge 2

$-2250 \leq D2 - 919.9 \leq 184.0;\ -1330.81 \leq D2 \leq 1103.19$ DL

$-1687.5 \leq .85D2 - 1561.19 \leq 368.0;\ -148.60 \leq D2 \leq 2269.64$ TL

Ridge 3

$-2250 \leq D3 + 879.02 \leq 184.0;\ -3129.02 \leq D3 \leq -695.02$ DL

$-1687.5 \leq .85D3 + 1492.95 \leq 368.0;\ -3741.71 \leq D3 \leq -1323.47$ TL

Ridge 4

$-2250 \leq D4 - 822.0 \leq 184.0;\ -1428 \leq D4 \leq 1006$ DL

$-1687.5 \leq .85D4 - 1396.11 \leq 368.0;\ -342.81 \leq D4 \leq 2675.4$ TL

Ridge 5

$-2250 \leq D5 + 846.98 \leq 184.0;\ -3096.98 \leq D5 \leq -662.98$ DL

$-1687.5 \leq .85D5 + 1438.55 \leq 368.0;\ -3677.71 \leq D5 \leq -1259.47$ TL

Now, controlling upper and lower limit for each constraint will be found.

$-3127.6 \leq D1 \leq -1320.16$

$-148.60 \leq D2 \leq 1103.19$

$-3129.02 \leq D3 \leq -1323.47$

$-342.81 \leq D4 \leq 1006.0$

$-3096.98 \leq D5 \leq -1259.47$

Constraint on Sag: The c.g. of cable is assumed to be at least 10" from longitudinal edge of each plate. With this, for plate 1, the maximum sag will be

$$(h_1 - 2 \times 0.833)/2 = 5.16667.$$

This is the same for each plate.

For Plate 1 - sag = y_1

$$0 \leq y_1 \leq 5.16667$$

$$0 \leq \frac{E_{12}}{E_{11}} \leq 5.16667$$

$$\rightarrow -E_{12} \leq 0 \quad \text{---------- SL1}$$

$$-5.16667\, E_{11} + E_{12} \leq 0 \quad \text{---------- SV1}$$

Similarly, for Plate 2,

$$-E_{22} \leq 0 \quad \text{---------- SL2}$$

$$-5.16667\, E_{21} + E_{22} \leq 0 \quad \text{---------- SV2}$$

For Plate 3,

$$-E_{32} \leq 0 \quad \text{---------- SL3}$$

$$-5.16667\, E_{31} + E_{32} \leq 0 \quad \text{---------- SV3}$$

For Plate 4,

$$-E_{42} \leq 0 \quad \text{---------- SL4}$$

$$-5.16667\, E_{41} + E_{42} \leq 0 \quad \text{---------- SV4}$$

The simplex table for input to the computer is shown in Table 6.4.3.

Results:

Total minimum prestress = 946.119 Kips.

Plate 1 - prestress = 242.97 Kips.

sag = 5.167'

Plate 2 - prestress = 482.01 Kips.

sag = 5.167'

Plate 3 - no prestressing

Table 6.4.3 - Simplex Table

Structural Variables

	E_{11}	E_{12}	E_{21}	E_{22}	E_{31}	E_{32}	E_{41}	E_{42}		Limits	Range 1
Obj. Fn: Force	1.0		1.0		1.0		1.0		N		
D1	-1.927	-0.561	0.498	-0.155	-0.161	-0.045	0.049	-0.011	LT	-1320.16	1807.20
D2	-0.706	0.357	-1.039	0.301	0.262	0.083	-0.056	0.030	LT	1103.19	1251.79
D3	0.165	-0.099	-0.928	-0.283	-0.920	-0.284	0.143	-0.104	LT	-1323.47	1805.55
D4	-0.064	0.029	0.269	0.084	-1.032	0.302	-0.714	0.356	LT	1006.0	1348.81
D5	0.057	-0.012	-0.183	-0.050	0.461	-0.158	-1.876	-0.552	LT	-1259.47	1837.51
SU1	-5.1667	1.0							LT	0.0	
SL1		-1.0							LT	0.0	
SU2			-5.1167	1.0					LT	0.0	
SL2				-1.0					LT	0.0	
SU3					-5.1667	1.0			LT	0.0	
SL3						-1.0			LT	0.0	
SU4							-5.1667	1.0	LT	0.0	
SL4								-1.0	LT	0.0	

Plate 4 - prestress = 221.14 Kips.

sag = 5.167'

Step 6 - In order to check the uniqueness of the solution, a prestressing force of 50 Kips was made minimum in plate 3, and new optimum value of the objective function was found. This was the same as the previous one. But the prestressing force in plate 2 was reduced by 50. Prestressing plates 2 and 3 have equal effects for this loading, and hence, any combination of values for prestressing of plates 2 and 3, the sum of the two being equal to 482 Kips, will be an optimum solution. In order to maintain symmetry equal values will be assigned to plates 2 and 4.

Final Solution:

PF 1 = 242.92 Kips

Sag 1 = 5.167'

PF 3, PF 2 = 482/2 = 241.00 Kips.

Sag 3, Sag 2 = 5.167'

PF 4 = 221 Kips

Sag 4 = 5.167'

The almost equal prestressing in all plates is due to the proximity of the behavior of the folded plate to that of a simply supported beam. The final result for initial prestressing force by the beam method of analysis, as given by authors of Reference 3, was 250 Kips.

CONCLUSIONS

The method of analysis of prestressed folded plates, presented in this thesis, assumes the longitudinal stress due to direct prestressing effect to be constant along the span, as opposed to the sinusoidal approximation by Billington (1). Actually the longitudinal stress at the ends of the span will be zero except at cable locations and beyond a certain distance from the end span it will be a constant along the span. Hence, an improved expression for this variation is desirable. A close agreement between the solutions by Simpson's method and that by the finite difference method was found. The validity of the principle of superposition was proved for the folded plate subjected to uniformly distributed loads and prestressing with straight or parabolic cable drape.

By linear programming, it became possible to determine the optimal cable geometry and prestressing forces in different plates, satisfying allowable stress requirements for the given combinations of loadings. Possible eccentricity of the cable at end span can be included as an additional variable. Also more exact expressions for the volume of the cable can be used; this may require the use of non-linear programming for optimization.

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Appendix A

Principal Stresses and Direction

Shear Stress - Primary Analysis: Refer to Fig. A.1.a

Total shear force at a given level y is,

$$N_x = N_{x,n} - \sigma_{x,n-1}\, d_n y - \frac{d_n y^2}{2h_n} (\sigma_{x,n+1} - \sigma_{x,n})$$

For uniform plate load since σ_x will vary parabolically from zero at supports, to maximum at center of the plate

$$N_x = \frac{4N_{xm}}{\ell} (x - x^2/\ell)$$

$$\tau = \frac{1}{d_n} \frac{\partial N_x}{\partial x} = \frac{4N_{xm}}{\ell d_n} (1 - 2x/\ell)$$

Transverse Normal Stress - Primary Analysis: Refer to Fig. A.1.b

$$\sigma_y = \frac{1}{d_n} (S_{n+1,n}\, d_x + w_t'\, d_x - \frac{T_1 - T_2}{d_x})$$

$$\sigma_y = \frac{1}{d_n} (S_{n+1,n} + w_t' - \frac{8}{\ell^2} \int_y^{h_n} N_{xm}\, d_y)$$

Shear Stress - Secondary Analysis: Refer to Fig. A.1.c

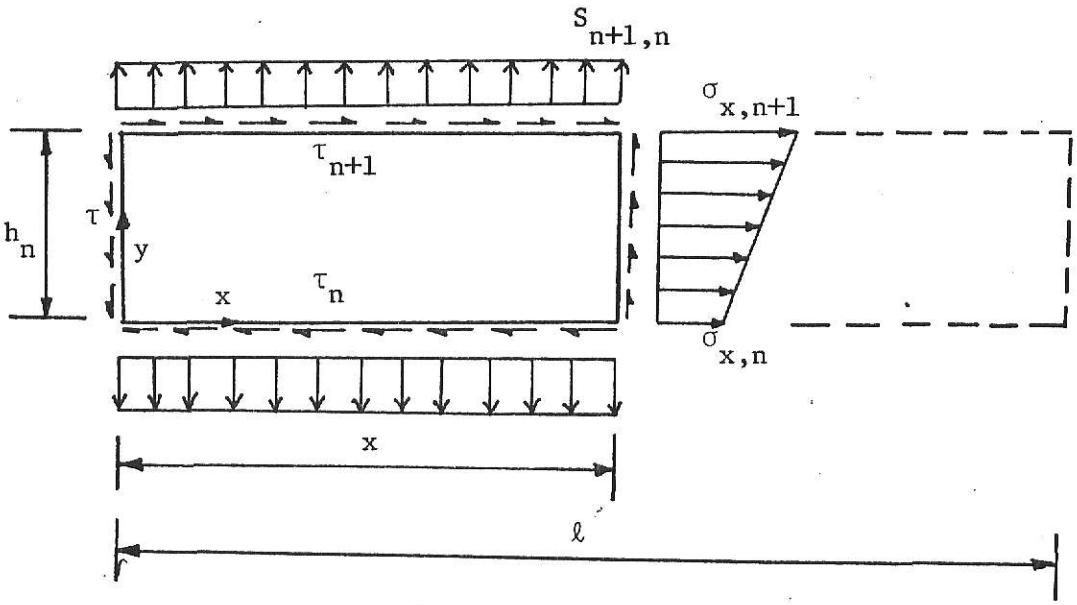
Since plate load varies sinuoidally,

$$N_x = N_{xm} \operatorname{Sin} \frac{\pi x}{\ell}$$

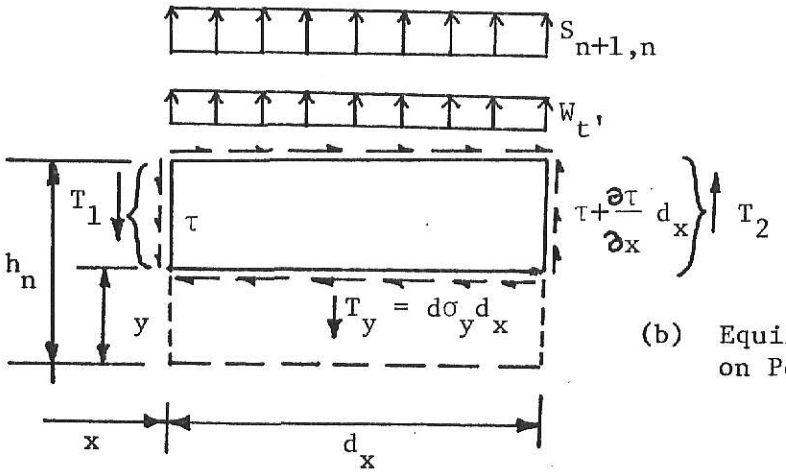
$$\tau = \frac{\pi}{\ell d_n} N_{xm} \operatorname{Cos} \frac{\pi x}{\ell}$$

Transverse Normal Stress - Secondary Analysis:

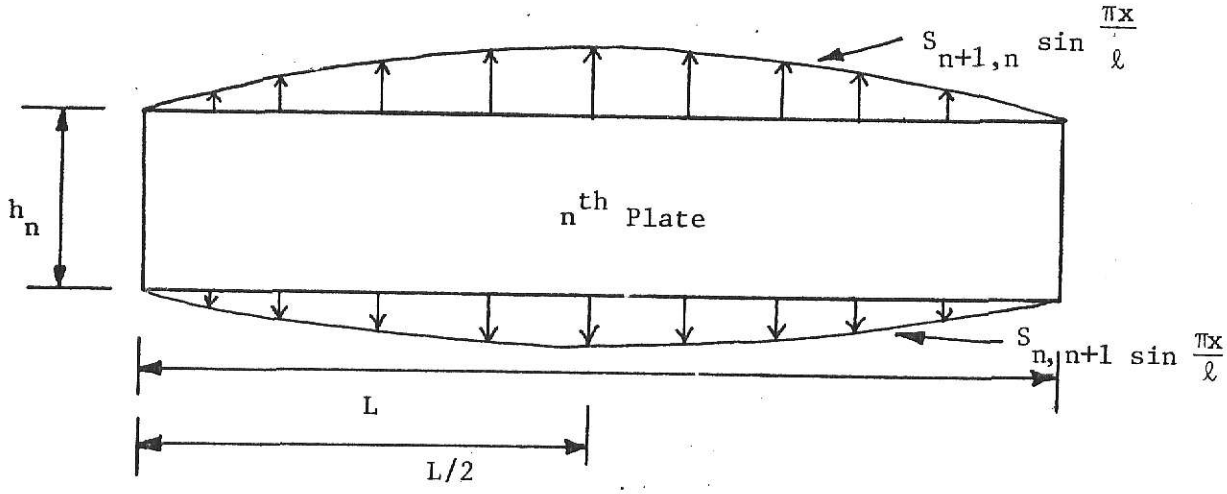
$$\sigma_y = \frac{1}{d_n} (S_{n+1,n} \operatorname{Sin} \frac{\pi x}{\ell} + d_n \int_y^{h_o} \frac{\partial \tau}{\partial x}\, dy)$$



(a) Primary Analysis Loads and Stresses on Portion of n^{th} Plate



(b) Equilibrium of Vertical Forces on Portion of n^{th} Plate



(c) Distribution of Plate Load for Secondary Analysis

Fig. A.1

$$\sigma_y = \frac{1}{d_n} \left(S_{n+1,n} - \frac{\pi^2}{\ell^2} \int_y^{h_n} N_{xm}\, dy\right) \operatorname{Sin} \frac{\pi x}{\ell}$$

Principal Stresses:

$$\sigma_{1,\,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

in which σ_1 is algebraic maximum principal stress and σ_2 is algebraic minimum principal stress.

$$\tan 2\theta = 2\tau/(\sigma_x - \sigma_y)$$

where θ is the angle between x-direction and σ_1.

Appendix B

Torsional Moment of Inertia of a Rectangular Cross-Section

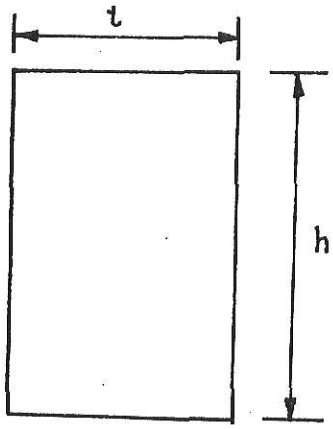


Table B.1

h/t	1.00	1.50	2.00	2.50	3.00	4.00	6.00	8	10	∞
$\beta_t = \frac{c}{t^3 h}$	0.140	0.196	0.229	0.249	0.263	0.281	0.299	0.307	0.313	0.333

Torsional moment of inertia $c = \beta_t t^3 h$ where t and h are as shown in the figure and β_t can be obtained from the above table, as given in Reference 14.

APPENDIX C

Source List of the Computer Programs

```
C*****************************************************************
C*                                                               *
C*     ANALYSIS OF PRISMATIC FOLDED PLATES FOR GRAVITY           *
C*     LOADS, WIND LOAD  AND PRESTRESSING                        *
C*                                                               *
C*                          MAIN  PROGRAM                        *
C*                                                               *
C*****************************************************************

      DIMENSION AL(15),PHI(15),GM(15),H(15),T(15),A(15),TJ(15),
     1AK(15),DAL(15),DAR(15),DL(15),DR(15),WWL(15),WDL(15),WTL(15),
     2FML(15),FMR(15),TMDL(15),ALSDL(15),VDL(15),TMTL(15),ALSTL(15
     3),VTL(15),TMWL(15),ALSWL(15),VWL(15),TMRT(15,15),ALSRT(15,15
     4),VRT(15,15),ALSDP(15),VSDP(15),ALSEP(15),VSEP(15),FSL(15),F
     5SR(15),NP(15),SAG(15),PF(15),Q(15,15),XKDL(15),XKTL(15),XKWL
     6(15),XKDP(15),XKEP(15),WRT(15),DMX(15),TMX(15),WMX(15),SDX(1
     75),STX(15),SWX(15),SDPX(15),SEPX(15),TMDP(15),TMEP(15),DPMX(
     815),EPMX(15),WPL(15),PL(15),RTPL(15,15),DLPL(15),TLPL(15),WL
     9PL(15),PRPL(15),SCPL(15),PPL(15),ALS(15),RTMS(15),ESHP(15),E
     1SHS(15),ESP(15),ESE(15),ERTM(15),ESS(15),PRST1(15),PRST2(15)
     2,THETA(15),CSAG(15)

C  INPUT  TO  THE  PROGRAM

      READ(5,600)N,NPP,STR,GL,VL,SL,E,G,ETJL,ETJR,TL
  600 FORMAT(2I2,8F8.0,F4.0)
      READ(5,601)(PHI(I),GM(I),H(I),T(I),AL(I),SAG(I),PF(I),I=1,N)
  601 FORMAT(7F10.0)
      READ(5,6601) (CSAG(I),I=1,N)
 6601 FORMAT(F10.0)
      READ(5,602)(WWL(I),I=1,N)
  602 FORMAT(6F10.0)
      READ(5,603)(NP(I),I=1,NPP)
  603 FORMAT(6I2)
      READ(5,604)LCD,LCT,LCW,LCPR,NXD,NYD
  604 FORMAT(6I1)

C  COMPUTATION OF DISTRIBUTION FACTORS
      N1=N+1
      N2=N-1
      N3=N-2
      PIVL=3.1415927
      DO 69 I=1,N
      PHI(I)=(PHI(I)*PIVL)/180.0
      GM(I)=(GM(I)*PIVL)/180.0
      A(I)=H(I)*T(I)
      AK(I)=(T(I)**3)/H(I)
   69 CONTINUE
      IF(STR.EQ.2.0) GO TO 80
      AK(2)=0.75*AK(2)
      AK(N2)=0.75*AK(N2)
      AK(1)=0.0
      AK(N)=0.0
      GO TO 81
   80 AK(1)=27.0*ETJL/(SL**2)
      AK(N)=27.0*ETJR/(SL**2)
   81 DO 62 J=2,N
      JO=J-1
      DM=A(JO)+A(J)
      DAL(J)=-A(J)/DM
```

```
      CAR(J)=A(JO)/DM
      DMD=AK(JO)+AK(J)
      DL(J)=AK(JO)/DMD
      DR(J)=AK(J)/DMD
   62 CONTINUE
      DO 63 I=1,N
      WDL(I)=GL*T(I)
      WTL(I)=(GL*T(I))+VL
      IF(TL.EQ.2.0)WTL(I)=(GL*T(I))+(VL*ABS(COS(PHI(I))))
      IF(AL(I).EQ.0.0) WTL(I)=GL*T(I)
      WPL(I)=-((WDL(I)*LCD)+(WTL(I)*LCT))*SIN(PHI(I))
   63 CONTINUE

    C SECONDARY  ANALYSIS

      SOL=0.0
      DO 66 I=1,N
      FML(I)=0.0
      FMR(I)=0.0
      WRT(I)=0.0
   66 CONTINUE
      DO 67 L1=2,N2
      FML(L1)=-6.0
      FMR(L1)=-6.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMDL,ALSDL,TMRT,ALSRT,PL)
      DO 1000 JJ=2,N
 1000 RTPL(JJ,L1)=PL(JJ)
      DO 68 I=1,N
      I1=I+1
      VRT(I,L1)=((ALSRT(I,L1)-ALSRT(I1,L1))*(SL**2))/((PIVL**2)*E*
     1H(I))
   68 CONTINUE
      FML(L1)=0.0
      FMR(L1)=0.0
   67 CONTINUE

    5 PRIMARY  ANALYSIS --- DEAD LOAD (SELF WEIGHT)

      SOL=1.0
      DO 640 I=2,N2
      FML(I)=-((WDL(I)*(H(I)**2))*COS(PHI(I)))/12.0
      FMR(I)=-FML(I)
  640 CONTINUE
      FML(1)=0.0
      FMR(1)=((WDL(1)*(H(1)**2))*COS(PHI(1)))/2.0
      FML(N)=-((WDL(N)*(H(N)**2))*COS(PHI(N)))/2.0
      FMR(N)=0.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WDL,FML,FMR,FSL,FSR,TMDL,ALSDL,TMRT,ALSRT,DLPL)
      DO 641 I=1,N
      I1=I+1
      VDL(I)=(5.0*(ALSDL(I)-ALSDL(I1))*(SL**2))/(48.0*E*(H(I)))
  641 CONTINUE

    C PRIMARY  ANALYSIS -- DEAD LOAD+LIVE LOAD

      SOL=2.0
      DO 642 I=2,N2
      FML(I)=-((WTL(I)*(H(I)**2))*COS(PHI(I)))/12.0
```

```
      FMR(I)=-FML(I)
  642 CONTINUE
      FML(1)=0.0
      FMR(1)=((WTL(1)*(H(1)**2))*COS(PHI(N)))/2.0
      FML(N)=-((WTL(N)*(H(N)**2))*COS(PHI(N)))/2.0
      FMR(N)=0.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WTL,FML,FMR,FSL,FSR,TMTL,ALSTL,TMRT,ALSRT,TLPL)
      DO 643 I=1,N
      I1=I+1
      VTL(I)=(5.0*(ALSTL(I)-ALSTL(I1))*(SL**2))/(48.0*E*(H(I)))
  643 CONTINUE

    C PRIMARY  ANALYSIS -- WIND LOAD

      SOL=3.0
      DO 644 I=2,N2
      FML(I)=-WWL(I)*(H(I)**2)/12.0
      FMR(I)=-FML(I)
  644 CONTINUE
      FML(1)=0.0
      FMR(1)=WWL(1)*(H(1)**2)/2.0
      FML(N)=-WWL(N)*(H(N)**2)/2.0
      FMR(N)=0.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WWL,FML,FMR,FSL,FSR,TMWL,ALSWL,TMRT,ALSRT,WLPL)
      DO 645 I=1,N
      I1=I+1
      VWL(I)=(5.0*(ALSWL(I)-ALSWL(I1))*(SL**2))/(48.0*E*(H(I)))
  645 CONTINUE

    C PRIMARY  ANALYSIS -- DIRECT PRESTRESSING EFFECT

      SOL=4.0
      DO 670 I=1,N
      FSL(I)=0.0
      FSR(I)=0.0
  670 CONTINUE
      DO 671 I=1,NPP
      LN=NP(I)
      ANGLE=ATAN((4.0*(ABS(SAG(LN))))/SL)
      DPR=-(PF(LN)*COS(ANGLE))/A(LN)
      EPRD=PF(LN)*CSAG(I)*COS(ANGLE)
      ZZ=T(LN)*(H(LN)**2)/6.0
      EPR=EPRD/ZZ
      FSL(LN)=DPR-EPR
      FSR(LN)=DPR+EPR
  671 CONTINUE
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMDP,ALSDP,TMRT,ALSRT,PL)
      DO 672 I=1,N
      I1=I+1
      VSDP(I)=((ALSDP(I)-ALSDP(I1))*(SL**2))/(8.0*E*(H(I)))
  672 CONTINUE

    C PRIMARY  ANALYSIS -- ECCENTRIC PRESTRESSING EFFECT

      SOL=5.0
      DO 673 I=1,N
      FSL(I)=0.0
```

```
      FSR(I)=0.0
  673 CONTINUE
      DO 674 I=1,NPP
      LN=NP(I)
      RP=(8.0*SAG(LN)*PF(LN))/(SL**2)
      FSR(LN)=(0.75*RP*(SL**2))/(T(LN)*(H(LN)**2))
      FSL(LN)=-FSR(LN)
  674 CONTINUE
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMEP,ALSEP,TMRT,ALSRT,PL)
      DO 675 I=1,N
      I1=I+1
      VSEP(I)=(5.0*(ALSEP(I)-ALSEP(I1))*(SL**2))/(48.0*E*(H(I)))
  675 CONTINUE
      DO 1001 I=1,N
      PRPL(I)=PF(I)*SAG(I)*8.0/(SL**2)
 1001 CONTINUE

    C DEFLECTION COMPATIBILITY EQUATIONS

      DO 777 II=1,5
      DO 72 I=2,N2
      IO=I-1
      I1=I+1
      AA=((COTAN(GM(IO))+COTAN(GM(I))))/H(I)
      B=1.0/((SIN(GM(IO)))*H(I))
      C=1.0/((SIN(GM(I)))*H(I))
      ROTA=(12.0*H(I))/(E*(T(I)**3))
      DO 70 J=1,N3
      J1=J+1
      Q(IO,J)=(VRT(I,J1)*AA)-(VRT(IO,J1)*B)-(VRT(I1,J1)*C)
      IF(J1.EQ.I) Q(IO,J)=Q(IO,J)-ROTA
   70 CONTINUE
      IF(II.EQ.1)Q(IO,N2)=-((VDL(I)*AA)-(VDL(IO)*B)-(VDL(I1)*C))
      IF(II.EQ.2)Q(IO,N2)=-((VTL(I)*AA)-(VTL(IO)*B)-(VTL(I1)*C))
      IF(II.EQ.3)Q(IO,N2)=-((VWL(I)*AA)-(VWL(IO)*B)-(VWL(I1)*C))
      IF(II.EQ.4)Q(IO,N2)=-((VSOP(I)*AA)-(VSOP(IO)*B)-(VSOP(I1)*C))
      IF(II.EQ.5)Q(IO,N2)=-((VSEP(I)*AA)-(VSEP(IO)*B)-(VSEP(I1)*C))
   72 CONTINUE
      NE=N3
      IF(II.EQ.1)CALL DETSIM(Q,XKDL,NE)
      IF(II.EQ.2)CALL DETSIM(Q,XKTL,NE)
      IF(II.EQ.3)CALL DETSIM(Q,XKWL,NE)
      IF(II.EQ.4)CALL DETSIM(Q,XKOP,NE)
      IF(II.EQ.5)CALL DETSIM(Q,XKEP,NE)
  777 CONTINUE
      DO 1008 J=2,N
      SCPL(J-1)=0.0
      DO 1008 L1=2,N2
      L2=L1-1
      SCPL(J-1)=SCPL(J-1)+(RTPL(J,L1)*((LCD*XKDL(L2))+(LCT*XKTL(L2)
     *)+(LCW*XKWL(L2))+(LCPR*(XKOP(L2)+XKEP(L2)))))
 1008 CONTINUE
      SCPL(N)=0.0
      DO 1009 I=1,N
      I1=I+1
      PPL(I)=((DLPL(I1)*LCD)+(TLPL(I1)*LCT)+(WLPL(I1)*LCW)+(PRPL(I)*
     *LCPR))
 1009 CONTINUE
```

```
C SUMMATION OF PRIMARY AND SECONDARY SOLUTIONS

      PIV=PIVL
      DO 75 I=1,N1
      DMX(I)=TMDL(I)
      TMX(I)=TMTL(I)
      WMX(I)=TMWL(I)
      DPMX(I)=TMDP(I)
      EPMX(I)=TMEP(I)
      SDX(I)=ALSDL(I)
      STX(I)=ALSTL(I)
      SWX(I)=ALSWL(I)
      SDPX(I)=ALSDP(I)
      SEPX(I)=ALSEP(I)
   75 CONTINUE
      DO 1003 I=1,N1
      ALS(I)=(LCD*SDX(I))+(LCT*STX(I))+(LCW*SWX(I))+((SEPX(I))*L
     *CPR)
 1003 CONTINUE
      DO 1004 I=1,N1
      RTMS(I)=0.0
      DO 1004 L1=2,N2
      L2=L1-1
      RTMS(I)=RTMS(I)+((ALSRT(I,L1))*((XKDL(L2)*LCD)+(XKTL(L2)*LCT
     1)+(XKWL(L2)*LCW)+((XKDP(L2)+XKEP(L2))*LCPR)))
 1004 CONTINUE
      ESHS(1)=0.0
      ESHS(N1)=0.0
      ESHP(N1)=0.0
      ESHP(1)=0.0
      DO 1005 I=1,N2
      J=N1-I
      ESHP(J)=ESHP(J+1)+(A(J)*(ALS(J)+ALS(J+1))/2.0)
      ESHS(J)=ESHS(J+1)+(A(J)*(RTMS(J)+RTMS(J+1))/2.0)
 1005 CONTINUE
      DELX=SL/(2.0*(NXD-1))
      X=0.0
      DO 144 I6=1,NXD
      DO 76 I=1,N1
      TMDL(I)=DMX(I)
      TMTL(I)=TMX(I)
      TMWL(I)=WMX(I)
      TMDP(I)=DPMX(I)
      TMEP(I)=EPMX(I)
      ALSDL(I)=((4.0*SDX(I)*X*(SL-X))/(SL**2))
      ALSTL(I)=((4.0*STX(I)*X*(SL-X))/(SL**2))
      ALSWL(I)=((4.0*SWX(I)*X*(SL-X))/(SL**2))
      ALSDP(I)=SDPX(I)
      ALSEP(I)=((4.0*SEPX(I)*X*(SL-X))/(SL**2))
      ESP(I)=(LCD*ALSDL(I))+(LCT*ALSTL(I))+(LCW*ALSWL(I))+(LCPR*(A
     1LSDP(I)+ALSEP(I)))
      DO 761 L1=2,N2
      L2=L1-1
      TMDL(I)=TMDL(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKDL(L2))
      TMTL(I)=TMTL(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKTL(L2))
      TMWL(I)=TMWL(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKWL(L2))
      TMDP(I)=TMDP(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKDP(L2))
      TMEP(I)=TMEP(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKEP(L2))
      ALSDL(I)=ALSDL(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKDL(L2))
      ALSTL(I)=ALSTL(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKTL(L2))
```

```
      ALSWL(I)=ALSWL(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKWL(L2))
      ALSDP(I)=ALSDP(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKDP(L2))
      ALSEP(I)=ALSEP(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKEP(L2))
  761 CONTINUE
      ESE(I)=(LCD*ALSDL(I))+(LCT*ALSTL(I))+(LCW*ALSWL(I))+(LCPR*(A
     1LSDP(I)+ALSEP(I)))
      ERTM(I)=(LCD*TMDL(I))+(LCT*TMTL(I))+(LCW*TMWL(I))+(LCPR*(TMD
     1P(I)+TMEP(I)))
      ESS(I)=ESE(I)-ESP(I)
   76 CONTINUE
      DO 9876 I=1,N1
      ALSDL(I)=(ALSDL(I)*1000.0)/144.0
      ALSTL(I)=(ALSTL(I)*1000.0)/144.0
      ALSWL(I)=(ALSWL(I)*1000.0)/144.0
      ALSDP(I)=(ALSDP(I)*1000.0)/144.0
      ALSEP(I)=(ALSEP(I)*1000.0)/144.0
      ESE(I)=(ESE(I)*1000.0)/144.0
      TMDL(I)=(TMDL(I)*1000.0)
      TMTL(I)=(TMTL(I)*1000.0)
      TMWL(I)=(TMWL(I)*1000.0)
      TMDP(I)=(TMDP(I)*1000.0)
      TMEP(I)=(TMEP(I)*1000.0)
      ERTM(I)=(ERTM(I)*1000.0)
 9876 CONTINUE
      WRITE(6,799) X
      WRITE(6,800)
      WRITE(6,801)
      WRITE(6,8001)
      WRITE(6,802)(I,TMDL(I),ALSDL(I),TMTL(I),ALSTL(I),TMWL(I),ALS
     1WL(I),TMDP(I),ALSDP(I),TMEP(I),ALSEP(I),I=1,N1)
      WRITE(6,803)
      WRITE(6,8003)
      WRITE(6,804)(I,ESE(I),ERTM(I),I=1,N1)
  799 FORMAT( /, 50X,'X=',F6.2,'FT')
  800 FORMAT(/,15X,'DEAD  LOAD',15X,'TOTAL LOAD',15X,'WIND  LOAD',
     114X,'DIR.PRE.STRS',13X,'ECC.PRE.STRS')
  801 FORMAT(/,1X,'RIDGE',7X,'TMT',8X,'LST',11X,'TMT',8X,'LST',11X,
     1'TMT',8X,'LST',11X,'TMT',8X,'LST',11X,'TMT',8X,'LST')
 8001 FORMAT(12X,'(PFT)',6X,'(PSI)',9X,'(PFT)',6X,'(PSI)',9X,'(PFT
     1)',6X,'(PSI)',9X,'(PFT)',6X,'(PSI)',9X,'(PFT)',6X,'(PSI)')
  802 FORMAT(/,3X,I2,4X,2F11.3,3X,2F11.3,3X,2F11.3,3X,2F11.4,3X,2F
     111.4)
  803 FORMAT(//,1X,'RIDGE',9X,'FIN.EDGE.STRS',5X,'FIN.TRNS.MOM')
 8003 FORMAT(19X,'(PSI)',14X,'(PFT)')
  804 FORMAT(/,3X,I2,11X,F10.3,7X,F10.3)
      WRITE(6,8004)
 8004 FORMAT(//,40X,'PRINCIPAL STRESSES AND ORIENTATION')

    C COMPUTATION OF PRINCIPAL STRESSES AND DIRECTION

      DO 1006 I=1,N
      WRITE(6,805) I
  805 FORMAT(/,7X,'PLATE',1X,I2,7X,'Y=0H/4',9X,'Y=1H/4',9X,'Y=2H/4
     1',9X,'Y=3H/4',9X,'Y=4H/4')
      DELY=H(I)/(NYD-1)
      Y=0.0
      DO 1007 J=1,NYD
      SHFP=FSHP(I)-(ALS(I)*T(I)*Y)-(T(I)*(Y**2)*(ALS(I+1)-ALS(I))/
     1(2.0*H(I)))
      SHSP=(4.0*SHFP*(1.0-(2.0*X/SL)))/(SL*T(I))
```

```
      AINTP=(ESHP(I)*(H(I)-Y))-(ALS(I)*T(I)*((H(I)**2)-(Y**2))/2.0
     1)-(T(I)*(ALS(I+1)-ALS(I))*((H(I)**3)-(Y**3))/(6.0*H(I)))
      SIGYP=(PPL(I)+WPL(I)-(8.0*AINTP/(SL**2)))/T(I)
      SHFS=ESHS(I)-(RTMS(I)*T(I)*Y)-(T(I)*(Y**2)*(RTMS(I+1)-RTMS(I
     1))/(2.0*H(I)))
      SHSS=(PIVL*SHFS*COS(PIVL*X/SL))/(SL*T(I))
      AINTS=(ESHS(I)*(H(I)-Y))-(RTMS(I)*T(I)*((H(I)**2)-(Y**2))/2.
     10)-(T(I)*(RTMS(I+1)-RTMS(I))*((H(I)**3)-(Y**3))/(6.0*H(I)))
      SIGYS=((SCPL(I)-((PIVL**2)*AINTS/(SL**2)))*SIN(PIVL*X/SL))/T
     1(I)
      SIGXP=(((ESP(I+1)-ESP(I))*Y)/(H(I)))+ESP(I)
      SIGXS=(((ESS(I+1)-ESS(I))*Y)/(H(I)))+ESS(I)
      SIGX=SIGXP+SIGXS
      SIGY=SIGYP+SIGYS
      SHS=SHSP+SHSS
      SIGXC=SIGX*1000.0/144.0
      SIGYC=SIGY*1000.0/144.0
      PRST1(J)=((SIGX+SIGY)/2.0)+(SQRT((((SIGX-SIGY)/2.0)**2)+(SHS
     1**2)))
      PRST2(J)=((SIGX+SIGY)/2.0)-(SQRT((((SIGX-SIGY)/2.0)**2)+(SHS
     1**2)))
      DIFS=SIGX-SIGY
      IF(DIFS.EQ.0.0) GO TC 1234
      THETA(J)=(ATAN((2.0*SHS)/(SIGX-SIGY)))/2.0
      IF(THETA(J).EQ.0.0) GO TO 324
      GO TO 1333
 1234 THETA(J)=PIVL/4.0
      GO TO 1333
  324 IF(ABS(SIGX-SIGY).LT.2.0) THETA(J)=PIVL/4.0
 1333 OR=ABS((PRST1(J))-SIGY)
      OS=(PRST1(J)-SIGX)
      IF(OR.LT.OS) THETA(J)=THETA(J)+(PIVL/2.0)
      PRST1(J)=(PRST1(J)*1000.0)/144.0
      PRST2(J)=(PRST2(J)*1000.0)/144.0
      THETA(J)=(THETA(J)*180.0)/PIVL
 1007 Y=Y+DELY
      WRITE(6,806)(PRST1(L),L=1,5)
  806 FORMAT(/,1X,'PRIN.STRS1(PSI)',4X,F10.3,5X,F10.3,5X,F10.3,5X,
     1F10.3,5X,F10.3)
      WRITE(6,807)(PRST2(L),L=1,5)
  807 FORMAT(/,1X,'PRIN.STRS2(PSI)',4X,F10.3,5X,F10.3,5X,F10.3,5X,
     1,F10.3,KX,F10.3)
      WRITE(6,808)(THETA(L),L=1,5)
  808 FORMAT(/,6X,'THETA(DEG)', 4X,F10.3,5X,F10.3,5X,F10.3,5X,F10.3,
     *5X,F10.3)
 1006 CONTINUE
  144 X=X+DELX
      STOP
      END
```

```
C*****************************************************************
C*                                                               *
C*                                                               *
C*  ANALYSIS OF THE FOLDED PLATES FOR PRESTRESSING ONLY          *
C*                                                               *
C*                        MAIN  PROGRAM                          *
C*                                                               *
C*                                                               *
C*****************************************************************

      DIMENSION AL(15),PHI(15),GM(15),H(15),T(15),A(15),TJ(15),
     1AK(15),DAL(15),DAR(15),DL(15),DR(15),FML(15),FMR(15),TMDL(15
     2),ALSDL(15),TMRT(15,15),ALSRT(15,15),VRT(15,15),ALSDP(15),
     3VSDP(20),ALSEP(15),VSEP(15),FSL(15),FSR(15),SAG(15),PF(15),Q
     4(15,15),XKDP(15),XKEP(15),VSEP(15),FSL(15),FSR(15),SAG(15),P
     5F(15),Q(15,15),XKDP(15),XKEP(15),WRT(15),SDPX(15),SEPX(15),T
     6 MDP(15),TMEP(15),DPMX(15),EPMX(15),PL(15),CSAG(15),TMEE(15)
     7,ALSEE(15),SEEX(15),EEMX(15),XKEE(15),VSEE(15)

C INPUT TO THE PROGRAM

      READ(5,600) N,STR,SL,E,ETJL,ETJR,SYM
600   FORMAT(I2,6F10.0)
      READ(5,601)(PHI(I),GM(I),H(I),T(I),AL(I),SAG(I),PF(I),I=1,N)
601   FORMAT(7F10.0)
      READ(5,603) (CSAG(I),I=1,N)
603   FORMAT(F10.0)

C COMPUTATION OF DISTRIBUTION FACTORS

      N1=N+1
      N2=N-1
      N3=N-2
      PIVL=3.1415927
      X=SL/2
      DO 69 I=1,N
      PHI(I)=(PHI(I)*PIVL)/180.0
      GM(I)=(GM(I)*PIVL)/180.0
      A(I)=H(I)*T(I)
      AK(I)=(T(I)**3)/H(I)
69    CONTINUE
      IF(STR.EQ.2.0) GO TO 80
      AK(2)=0.75*AK(2)
      AK(N2)=0.75*AK(N2)
      AK(1)=0.0
      AK(N)=0.0
      GO TO 81
80    AK(1)=27.0*ETJL/(SL**2)
      AK(N)=27.0*ETJR/(SL**2)
81    DO 62 J=2,N
      JO=J-1
      DM=A(JO)+A(J)
      DAL(J)=-A(J)/DM
      DAR(J)=A(JO)/DM
      DMD=AK(JO)+AK(J)
      DL(J)=AK(JO)/DMD
      DR(J)=AK(J)/DMD
62    CONTINUE

C SECONDARY ANALYSIS
```

```
      SCL=0.0
      DO 66 I=1,N
      FML(I)=0.0
      FMR(I)=0.0
      WRT(I)=0.0
   66 CONTINUE
      DO 67 L1=2,N2
      FML(L1)=-6.0
      FMR(L1)=-6.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMDL,ALSDL,TMRT,ALSRT,PL)
      DO 68 I=1,N
      I1=I+1
      VRT(I,L1)=((ALSRT(I,L1)-ALSRT(I1,L1))*(SL**2))/((PIVL**2)*E*
     1H(I))
   68 CONTINUE
      FML(L1)=0.0
      FMR(L1)=0.0
   67 CONTINUE

    C PRIMARY ANALYSIS

      NI=N
      IF(SYM.EQ.1.0) NI=N1/2
      DO 669 LN=1,NI
      DO 670 I=1,N
      FSL(I)=0.0
  670 FSR(I)=0.0
      THETA=ATAN((4.0*(ABS(SAG(LN))))/SL)
      FSL(LN)=-(PF(LN)*COS(THETA))/A(LN)
      FSR(LN)=FSL(LN)
      IF(SYM.EQ.1.0) FSR(N1-LN)=FSL(LN)
      IF(SYM.EQ.1.0) FSL(N1-LN)=FSR(LN)
      SOL=4.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMDP,ALSDP,TMRT,ALSRT,PL)
      DO 672 I=1,N
      I1=I+1
      VSDP(I)=((ALSDP(I)-ALSDP(I1))*(SL**2))/(8.0*E*(H(I)))
  672 CONTINUE
      DO 673 I=1,N
      FSL(I)=0.0
  673 FSR(I)=0.0
      FSR(LN)=(6.0*PF(LN)*CSAG(LN)*COS(THETA))/(A(LN)*H(LN))
      FSL(LN)=-FSR(LN)
      IF(SYM.EQ.1.0) FSR(N1-LN)=FSL(LN)
      IF(SYM.EQ.1.0) FSL(N1-LN)=FSR(LN)
      SOL=5.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMEE,ALSEE,TMRT,ALSRT,PL)
      DO 6372 I=1,N
      I1=I+1
      VSEE(I)=((ALSEE(I)-ALSEE(I1))*(SL**2))/(8.0*E*(H(I)))
 6372 CONTINUE
      DO 6733 I=1,N
      FSL(I)=0.0
 6733 FSR(I)=0.0
      PP=(8.0*SAG(LN)*PF(LN))/(SL**2)
      FSR(LN)=(0.75*PP*(SL**2))/(T(LN)*(H(LN)**2))
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```
      FSL(LN)=-FSR(LN)
      IF(SYM.EQ.1.0) FSR(N1-LN)=FSL(LN)
      IF(SYM.EQ.1.0) FSL(N1-LN)=FSR(LN)
      SOL=6.0
      CALL TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,DR,DAL,
     1DAR,WRT,FML,FMR,FSL,FSR,TMEP,ALSEP,TMRT,ALSRT,PL)
      DO 675 I=1,N
      I1=I+1
      VSEP(I)=(5.0*(ALSEP(I)-ALSEP(I1))*(SL**2))/(48.0*E*(H(I)))
  675 CONTINUE

    C DEFLECTION COMPATIBILITY EQUATIONS

      DO 777 II=1,3
      DO 72 I=2,N2
      IO=I-1
      I1=I+1
      AA=((COTAN(GM(IO))+COTAN(GM(I))))/H(I)
      B=1.0/((SIN(GM(IO)))*H(I))
      C=1.0/((SIN(GM(I)))*H(I))
      ROTA=(12.0*H(I))/(E*(T(I)**3))
      DO 70 J=1,N3
      J1=J+1
      Q(IO,J)=(VRT(I,J1)*AA)-(VRT(IO,J1)*B)-(VRT(I1,J1)*C)
      IF(J1.EQ.I) Q(IO,J)=Q(IO,J)-ROTA
   70 CONTINUE
      IF(II.EQ.1)Q(IO,N2)=-((VSDP(I)*AA)-(VSDP(IO)*B)-(VSDP(I1)*C))
      IF(II.EQ.2)Q(IO,N2)=-((VSEE(I)*AA)-(VSEE(IO)*B)-(VSEE(I1)*C))
      IF(II.EQ.3)Q(IO,N2)=-((VSEP(I)*AA)-(VSEP(IO)*B)-(VSEP(I1)*C))
   72 CONTINUE
      NE=N3
      IF(II.EQ.1)CALL DETSIM(Q,XKDP,NE)
      IF(II.EQ.2)CALL DETSIM(Q,XKEE,NE)
      IF(II.EQ.3)CALL DETSIM(Q,XKEP,NE)
  777 CONTINUE

    C COMBINING PRIMARY AND SECONDARY SOLUTIONS

      PIV=PIVL
      DO 75 I=1,N1
      DPMX(I)=TMDP(I)
      EEMX(I)=TMEE(I)
      EPMX(I)=TMEP(I)
      SDPX(I)=ALSDP(I)
      SEEX(I)=ALSEE(I)
      SEPX(I)=ALSEP(I)
   75 CONTINUE
      DO 76 I=1,N1
      TMDP(I)=DPMX(I)
      TMEE(I)=EEMX(I)
      TMEP(I)=EPMX(I)
      ALSDP(I)=SDPX(I)
      ALSEE(I)=SEEX(I)
      ALSEP(I)=((4.0*SEPX(I)*X*(SL-X))/(SL**2))
      DO 76 L1=2,N2
      L2=L1-1
      TMDP(I)=TMDP(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKDP(L2))
      TMEE(I)=TMEE(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKEE(L2))
      TMEP(I)=TMEP(I)+((TMRT(I,L1)*SIN(PIV*X/SL))*XKEP(L2))
      ALSDP(I)=ALSDP(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKDP(L2))
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```
      ALSEE(I)=ALSEE(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKEE(L2))
      ALSEP(I)=ALSEP(I)+ (ALSRT(I,L1)*(SIN(PIV*X/SL))*XKEP(L2))
   76 CONTINUE
      DO 3333 MN=1,N1
      TMDP(MN)=1000.0*TMDP(MN)
      TMEE(MN)=1000.0*TMEE(MN)
      TMEP(MN)=1000.0*TMEP(MN)
      ALSDP(MN)=ALSDP(MN)*1000.0/144.0
      ALSEE(MN)=ALSEE(MN)*1000.0/144.0
      ALSEP(MN)=ALSEP(MN)*1000.0/144.0
 3333 CONTINUE
      WRITE(6,799) LN
      WRITE(6,800)
      WRITE(6,801)
      WRITE(6,8001)
      WRITE(6,802) (I,TMDP(I),ALSDP(I),TMEE(I),ALSEE(I),TMEP(I),AL
     1SEP(I),I=1,N1)
  799 FORMAT(//,10X,'DUE TO PRESTRESS IN PLATE ',I2,' ONLY')
  800 FORMAT(/,15X,'DIR.PRE.STRS',14X,'END.ECNTRCTY',14X,'ECC.PRE.
     1STRS')
  801 FORMAT(/,1X,'RIDGE',5X,'TMT',11X,'LST',11X,'TMT',11X,'LST',1
     11X,'TMT',11X,'LST')
 8001 FORMAT(10X,'(PFT)',9X,'(PSI)',9X,'(PFT)',9X,'(PSI)',9X,'(PFT
     1)',9X,'(PSI)')
  802 FORMAT(/,4X,I2,'*',F13.3,'*',F13.3,'*',F13.3,'*',F13.3,'*',F
     113.3'*',F13.3)
  669 CONTINUE
      STOP
      END
```

```
C*****************************************************************
C*                                                               *
C*   COMPUTATION OF LONGITUDINAL RIDGE STRESSES AND              *
C*                TRANSVERSE RIDGE MOMENTS                       *
C*                                                               *
C*                 SUBROUTINE 'TMALS'                            *
C*                                                               *
C*****************************************************************

      SUBROUTINE TMALS(N,N1,N2,N3,L1,SOL,STR,SL,AL,PHI,GM,H,T,DL,D
     1R,DAL,DAR, W ,FML,FMR,FSL,FSR,TBM ,ALEST,TBMR,ALESTR,PL)
      DIMENSION UBM(15),FML(15),FMR(15),DL(15),DR(15),BLML(15),BLM
     1R(15),BML(15),BMR(15),CML(15),CMR(15),TBM(15),TBMR(15,15),AL
     2(15),RML(15),RMR(15),RLL(15),RLR(15),W(15),H(15),T(15),Z(15)
     3,P(15),PHI(15),GM(15),PL(15),PR(15),R(15),FSL(15),FSR(15),YR
     4ML(15),YRMR(15),YRLL(15),YRLR(15),XRLL(15),XRLR(15),PY(15),P
     5X(15),PYL(15),PXR(15),BSL(15),BSR(15),UBS(15),DAL(15),DAR(15
     6),DSL(15),DSR(15),CSL(15),CSR(15),ALEST(15),ALESTR(15,15),PY
     7R(15),PXL(15)
      PIVL=3.1415927
      IF(SOL.GT.3.0) GO TO 33
       GO TO 39
   33 DO 325 KK=1,N1
      TBM(KK)=0.0
  325 CONTINUE
      GO TO 90
   39 IF(STR.EQ.2.0) GO TO 1

    C MOMENT DISTRIBUTION  (TORSIONAL STIFFNESS OF THE EDGE
      BEAM IS NOT CONSIDERED)

      B=0.0
    5 NM=0
      DO 20 J=2,N
      I=J-1
      K=J+1
      IF(B.GT.0.0) GO TO 2
      UBM(J)=FMR(I)+FML(J)
      BLML(J)=-(UBM(J)*DL(J))
      BLMR(J)=-(UBM(J)*DR(J))
      BML(J)=FMR(I)+BLML(J)
      BMR(J)=FML(J)+BLMR(J)
      IF(J.LT.N) UBM(N)=50.0
      GO TO 3
    2 UBM(N)=50.0
      IF(J.EQ.2) GO TO 3
      IF(J.EQ.N) GO TO 3
      CML(J)=0.5*BLMR(I)
      CMR(J)=0.5*BLML(K)
      IF(B.GT.1.0) CML(3)=0.0
      IF(B.GT.1.0) CMR(N2)=0.0
      UBM(J)=CML(J)+CMR(J)
    3 IF(ABS(UBM(J)).LT.0.000001) NM=NM+1
      IF(J.EQ.2) NM=4
      IF(ABS(UBM(N)).LT.0.000001) NM=NM-1
   20 CONTINUE
      IF(NM.EQ.N1) GO TO 4
      B=B+1.0
      IF(B.EQ.1.0) GO TO 5
    4 IF(B.EQ.0.0) GO TO 6
```

```
      DO 21 M=3,N2
      BLML(M)=-(UBM(M)*DL(M))
      BLMR(M)=-(UBM(M)*DR(M))
      BML(M)=BML(M)+CML(M)+BLML(M)
      BMR(M)=BMR(M)+CMR(M)+BLMR(M)
   21 CONTINUE
      IF(NM.EQ.N1) GO TO 6
      GO TO 5

    C MOMENT  DISTRIBUTION  (TORSIONAL STIFFNESS OF THE EDGE
      BEAM IS CONSIDERED)

    1 B=0.0
   11 NM=2
      BLMR(1)=0.0
      BLML(N1)=0.0
      DO 22 J=2,N
      I=J-1
      K=J+1
      IF(B.GT.0.0) GO TO 7
      UBM(J)=FMR(I)+FML(J)
      BLML(J)=-(UBM(J)*DL(J))
      BLMR(J)=-(UBM(J)*DR(J))
      BML(J)=FMR(I)+BLML(J)
      BMR(J)=FML(J)+BLMR(J)
      GO TO 8
    7 CML(J)=0.5*BLMR(I)
      CMR(J)=0.5*BLML(K)
      UBM(J)=CML(J)+CMR(J)
    8 IF(ABS(UBM(J)).LT.0.000001 ) NM=NM+1
   22 CONTINUE
      IF(NM.EQ.N1) GO TO 10
      B=B+1.0
      IF(B.EQ.1.0) GO TO 11
   10 IF(B.EQ.0.0) GO TO 6
      DO 23 J=2,N
      BLML(J)=-(UBM(J)*DL(J))
      BLMR(J)=-(UBM(J)*DR(J))
      BML(J)=BML(J)+CML(J)+BLML(J)
      BMR(J)=BMR(J)+CMR(J)+BLMR(J)
   23 CONTINUE
      IF(NM.EQ.N1) GO TO 6
      GO TO 11
    6 IF(SCL.EQ.0.0) GO TO 88
      DO 24 J=2,N
      TBM(J)=BMR(J)
   24 CONTINUE
      TBM(1)=0.0
      TBM(N1)=0.0
      GO TO 89
   88 DO 199 J=2,N
      TBMR(J,L1)=BMR(J)
  199 CONTINUE
      TBMR(1,L1)=0.0
      TBMR(N1,L1)=0.0
   89 BMR(1)=0.0
      BML(N1)=0.0
      IF(SCL.EQ.3.0) GO TO 85

    C COMPUTATION OF PLATE LOAD AND FREE EDGE STRESSES
```

```
      DO 25 I=1,N
      I1=I+1
      IF(AL(I).NE.0.0) RML(I)=-(BMR(I)+BML(I1))/AL(I)
      IF(AL(I).NE.0.0) RLL(I)=W(I)*H(I)/2.0
      IF(AL(I).EQ.0.0)RML(I)=0.0
      IF(AL(I).EQ.0.0)RLL(I)=(W(I)*H(I))/2.0
      RMR(I)=-RML(I)
      RLR(I)=RLL(I)
   25 CONTINUE
      RML(1)=0.0
      RMR(1)=0.0
      RML(N)=0.0
      RMR(N)=0.0
      RLL(1)=0.0
      RLR(1)=W(1)*H(1)
      RLL(N)=W(N)*H(N)
      RLR(N)=0.0
      DO 26 J=2,N
      I=J-1
      P(J)=-(RMR(I)+RML(J)+RLR(I)+RLL(J))
      PL(J)=P(J)*(COS(PHI(J)))/(SIN(GM(I)))
      PR(J)=-P(J)*(COS(PHI(I)))/(SIN(GM(I)))
   26 CONTINUE
      PR(1)=0.0
      PL(N1)=0.0
      DO 27 I=1,N
      I1=I+1
      R(I)=PR(I)+PL(I1)
      IF(SCL.EQ.0.0) GO TO 50
      ALBM=R(I)*(SL**2)/8.0
      GO TO 51
   50 ALBM=R(I)*(SL**2)/((PIVL)**2)
   51 FSR(I)=(6.0*ALBM)/(T(I)*(H(I)**2))
      FSL(I)=-FSR(I)
   27 CONTINUE
      GO TO 90
   85 DO 250 I=2,N2
      I1=I+1
      IF(AL(I).EQ.0.0) YRML(I)=0.0
      IF(AL(I).NE.0.0) YRML(I)=-(BMR(I)+BML(I1))/AL(I)
      YRMR(I)=-YRML(I)
      YRLL(I)=W(I)*(COS(PHI(I)))*H(I)/2.0
      YRLR(I)=YRLL(I)
      XRLL(I)=-W(I)*(SIN(PHI(I)))*H(I)/2.0
      XRLR(I)=XRLL(I)
  250 CONTINUE
      YRML(1)=0.0
      YRMR(1)=0.0
      YRML(N)=0.0
      YRMR(N)=0.0
      YRLL(1)=0.0
      YRLR(1)=W(1)*((COS(PHI(1)))*H(1))
      YRLL(N)=W(N)*(COS(PHI(N)))*H(N)
      YRLR(N)=0.0
      XRLL(1)=0.0
      XRLR(1)=-W(1)*(SIN(PHI(1)))*H(1)
      XRLL(N)=-W(N)*(SIN(PHI(N)))*H(N)
      XRLR(N)=0.0
      DO 251 J=2,N
```

```
      I=J-1
      PY(J)=-(YRMR(I)+YRML(J)+YRLR(I)+YRLL(J))
      PX(J)=-(XRLR(I)+XRLL(J))
      PYL(J)=PY(J)*(COS(PHI(J)))/(SIN(GM(I)))
      PYR(J)=-PY(J)*(COS(PHI(I)))/(SIN(GM(I)))
      PXL(J)=-PX(J)*(SIN(PHI(I)-GM(I)))/SIN(GM(I))
      PXR(J)=PX(J)*(SIN(PHI(I)))/SIN(GM(I))
  251 CONTINUE
      PYR(1)=0.0
      PYL(N1)=0.0
      PXR(1)=0.0
      PXL(N1)=0.0
      DO 1002 I=2,N1
 1002 PL(I)=PYL(I)+PXL(I)
      DO 270 I=1,N
      I1=I+1
      R(I)=PYR(I)+PYL(I1)+PXR(I)+PXL(I1)
         ALBM=R(I)*(SL**2)/8.0
      FSR(I)=(6.0*ALBM)/(T(I)*(H(I)**2))
      FSL(I)=-FSR(I)
  270 CONTINUE

    C STRESS  DISTRIBUTION

   90 B=0.0
   13 NM=2
      BSR(1)=0.0
      BSL(N1)=0.0
      DO 29 J=2,N
      I=J-1
      K=J+1
      IF(B.GT.0.0) GO TO14
      UBS(J)=FSR(I)-FSL(J)
      BSL(J)=UBS(J)*DAL(J)
      BSR(J)=UBS(J)*DAR(J)
      CSL(J)=FSR(I)+BSL(J)
      DSR(J)=FSL(J)+BSR(J)
      GO TO 15
   14 CSL(J)=-0.5*BSR(I)
      CSR(J)=-0.5*BSL(K)
      UBS(J)=CSL(J)-CSR(J)
   15 IF(ABS(UBS(J)).LT.0.000001)NM=NM+1
   29 CONTINUE
      IF(B.GT.0.0) GO TO 16
      DSR(1)=FSL(1)-(0.5*BSL(2))
      CSL(N1)=FSR(N)-(0.5*BSR(N))
   16 IF(NM.EQ.N1) GO TO 17
      B=B+1.0
      IF(B.EQ.1.0) GO TO 13
   17 IF(B.EQ.0.0) GO TO 18
      DO 30 J=2,N
      BSL(J)=UBS(J)*DAL(J)
      BSR(J)=UBS(J)*DAR(J)
      DSL(J)=DSL(J)+CSL(J)+BSL(J)
      DSR(J)=DSR(J)+CSR(J)+BSR(J)
   30 CONTINUE
      DSR(1)=DSR(1)-(0.5*(BSL(2)))
      DSL(N1)=DSL(N1)-(0.5*(BSR(N)))
      IF(NM.EQ.N1) GO TO 18
      GO TO 13
```

```
 18 IF(SOL.EQ.0.0) GO TO 850
    DO 31 J=1,N
    ALEST(J)=DSR(J)
 31 CONTINUE
    ALEST(N1)=DSL(N1)
    GO TO 702
850 DO 32 J=1,N
    ALESTR(J,L1)=DSR(J)
 32 CONTINUE
    ALESTR(N1,L1)=DSL(N1)
702 RETURN
    END
```

APPENDIX D

Input-Output Details for the Computer Programs

APPENDIX D

INPUT - OUTPUT DETAILS FOR THE COMPUTER PROGRAMS

PROGRAM NO:1 ANALYSIS OF THE FOLDED PLATES FOR GRAVITY LOADS, WIND LOAD AND PRESTRESSING

INPUT FORMATS: GIVEN IN THE LIST IN APPENDIX C

NOTATIONS FOR THE INPUT PARAMETERS:

N = NUMBER OF PLATES

NPP = NUMBER OF PRESTRESSED PLATES

STR = CONSIDERATION OF TORSIONAL STIFFNESS OF EDGE BEAMS
STR=1.0 IF CONSIDERED
STR=2.0 IF NOT CONSIDERED

GL = UNIT WEIGHT OF THE MATREIAL OF THE FOLDED PLATES (KIPS/CU.FT.)

VL = LIVE LOAD ON THE FOLDED PLATES (KIPS/SQ.FT)

SL = SPAN LENGTH (FT)

ETJL= TORSIONAL MOMENT OF INERTIA OF THE LEFT EDGE BEAM (4TH POWER OF FT)

ETJR= TORSIONAL MOMENT OF INERTIA OF THE RIGHT EDGE BEAM (4TH POWER OF FT)

TL = TYPE OF LIVE LOAD
TL=1.0, IF LOAD IS OVER THE SURFACE AREA OF THE PLATES
TL=2.0, IF LOAD IS OVER THE HORIZONTAL PROJECTION OF THE PLATES

E = A SCALE FACTOR, WHICH CONTROLS THE ACCURACY OF THE PROGRAM IN SINGLE PRECISION. E=75.0 FOR USUAL LOADING VALUES. SAME VALUE CAN BE USED IN DOUBLE PRECISION. E SHOULD NOT BE EQUAL TO ZERO

PHI = ANGLE OF THE PLATE, MEASURED CLOCKWISE FROM THE HORIZONTAL, TO THE DIRECTION THE PLATE, POINTING TOWARDS THE NEXT PLATE (DEGRESS) (REFER FIG.4)

GM = DEFLECTION ANGLE OF THE PLATE, MEASURED CLOCKWISE FROM THE EXTENDED DIRECTION OF THE PLATE TO THE DIRECTION OF THE NEXT PLATE (DEGREES) (REFER FIG.4)

H = WIDTH OF THE PLATE (FT)

T = THICKNESS OF THE PLATE (FT)

AL = HORIZONTAL PROJECTION OF THE WIDTH OF THE PLATE (FT)

SAG = SAG OF THE PARABOLIC CABLE (FT) SIGN CONVENTION SHOWN IN FIG.D.1

PF = PRESTRESSING FORCE (KIPS)

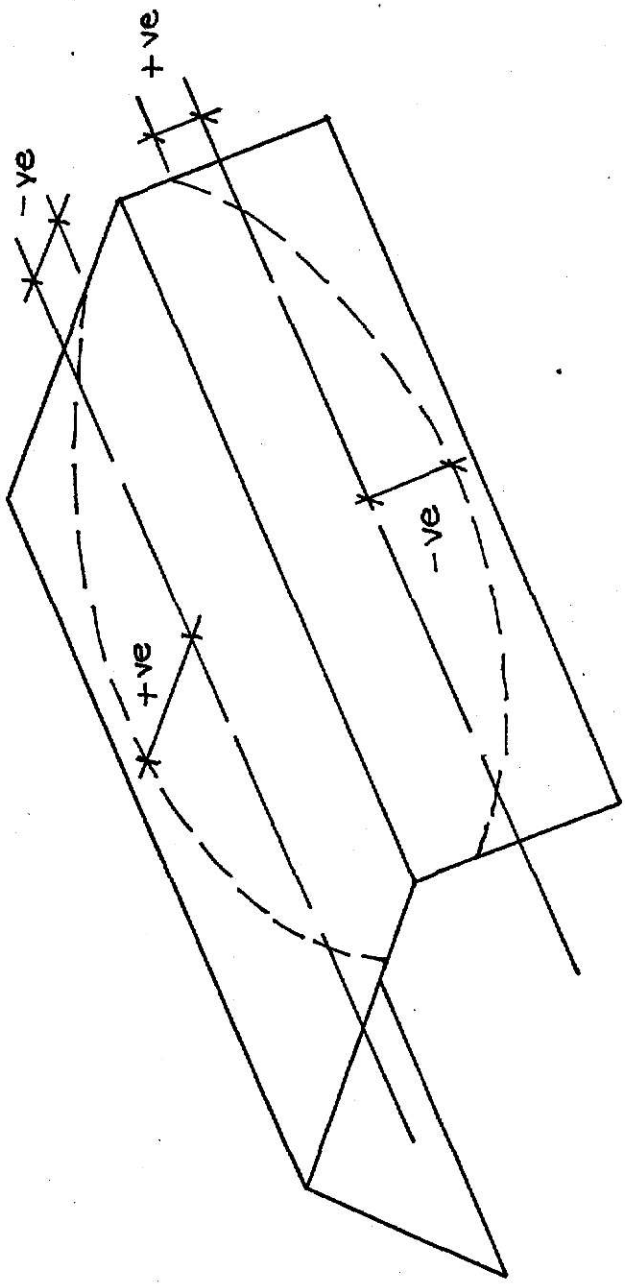


Fig. D.1 - Sign Conventions for Sag and End Eccentricity

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

WWL = WIND LOAD ON THE PLATES (KIPS/SQ.FT)
PRESSURE-- + VE
SUCTION-- - VE
FORMAT STATEMENT SHOULD BE CHANGED TO BE CONSISTENT WITH THE NUMBER OF PLATES

NP = PRESTRESSED PLATE NO.S,IN ASCENDING ORDER

THE FOLLOWING PARAMETERS CONTROL THE DESIRED COMBINATION OF LOADINGS AND POINTS ALONG X AND Y AXIS,WHERE STRESSES ARE TO BE DETERMINED

PARAMETER	USED FOR	INCLUDED	NOT INCLUDED
LCD	DEAD LOAD	1	0
LCT	DEAD+LIVE LOAD	1	0
LCW	WIND LOAD	1	0
LCPR	PRESTRESS	1	0

NX = NO OF POINTS OF INTREST,ALONG X-AXIS.SHOULD BE EQUIDISTANT AND ATLEAST HAVE END AND MIDSPAN POINTS (NX NOT LESS THAN 2)

NY = NO OF POINTS OF INTEREST ALONG Y-AXIS,SHOULD BE EQUIDISTANT AND ATLEAST HAVE THE TWO END POINTS OF THE PLATE WIDTH (NY NOT LESS THAN 2)

CSAG= ECCENTRICITY OF THE PRESTRESSIG CABLE AT THE END SPAN SIGN CONVENTION AS SHOWN IN FIG.D.1 (FT)

PROGRAM NO:2 ANALYSIS OF THE FOLDED PLATES FOR PRESTRESSING ONLY

INPUT FORMATS: GIVEN IN THE LIST IN APPENDIX C

NOTATIONS FOR THE INPUT PARAMETERS:

N,STR,SL,ETJL,ETJR,CSAG,E-- AS GIVEN IN THE PREVIOUS PROGRAM

SYM = SYMMETRY IN PRESTRESSING AND STRUCTURE
SYM=1.0,IF SYMMETRICAL
SYM=0.0,IF NOT SYMMETRICAL

PHI,GM,H,T,AL,SAG,PF -- AS GIVEN IN THE PREVIOUS PROGRAM

OUTPUT PARAMETERS FOR PROGRAMS 1 AND 2:

TMT = TRANSVERSE MOMENT

ALS = LONGITUDINAL STRESS

PRST1= ALGEBRAICALLY MAXIMUM PRINCIPAL STRESS

PRST2= ALGEBRAICALLY MINIMUM PRINCIPAL STRESS

THETA= ANGLE WHICH PRST1 MAKES WITH X-AXIS

PROGRAM NO:3 MPS-360 LINEAR PROGRAMMING FOR OPTIMIZATION

```
//NAME     JOB (STANDARD INFORMATION)
// EXEC KSLP
//CPC.SYSIN DD *

     (JOB CONTROL LANGUAGE REF.FIG.C.2)

/*
//EXEC.SYSIN DD *

     (INPUT DATA DECK REF.FIG.D.3)

/*
```

FORMATION OF SIMPLEX TABLE: REFER TO FIG.6.1.3 AS AN EXAMPLE

STRUCTURAL VARIABLES ARE VARIABLES IN THE PROBLEM,(X1,X2). THE EXTREME LEFT COLUMN WILL HAVE NAME OF THE OBJECTIVE FUNCTION,NAME OF THE CONSTRAINTS AND IF NECESSARY UPPER AND LOWER BOUNDS,IN THAT ORDER. THE COLUMNS CORRESPONDING TO THE VARIABLES(X1,X2) WILL HAVE THE COEFFICIENTS CORRESPONDING TO OBJECTIVE FUNCTION AND CONSTRAINTS.THE INTERSECTION OF THE COLUMN OF VARIABLE WITH THE'UPPER'OR'LOWER'ROW,WILL HAVE THE CORRESPONDING BOUND,IF NECESSARY.THE COLUMN 'LIMITS' WILL HAVE THE UPPER LIMIT FOR THE CONSTRAINTS.FOR OBJECTIVE FUNCTION THERE WILL BE NO LIMIT VALUE.RANGE1 COLUMN WILL HAVE THE RANGE OF OPERATION FOR EACH CONSTRAINTS.

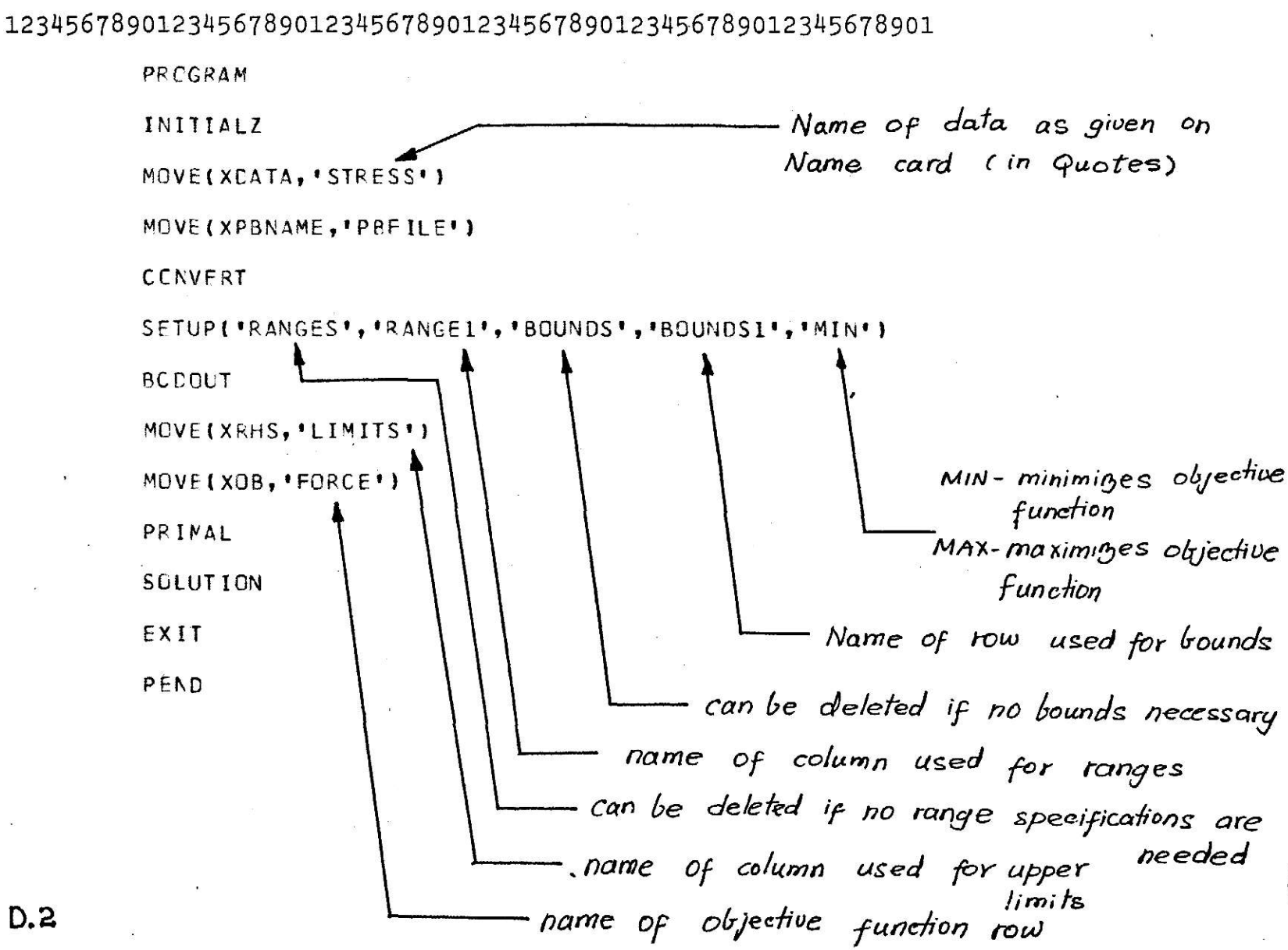

12345678901234567890123456789012345678901234567890123456789012345678901
PROGRAM
INITIALZ
MOVE(XDATA,'STRESS')
MOVE(XPBNAME,'PBFILE')
CONVERT
SETUP('RANGES','RANGE1','BOUNDS','BOUNDS1','MIN')
BCDOUT
MOVE(XRHS,'LIMITS')
MOVE(XOB,'FORCE')
PRIMAL
SOLUTION
EXIT
PEND
Name of data as given on Name card (in Quotes)
MIN- minimizes objective function
MAX- maximizes objective function
Name of row used for bounds
can be deleted if no bounds necessary
name of column used for ranges
can be deleted if no range specifications are needed
name of column used for upper limits
name of objective function row

Fig D.2

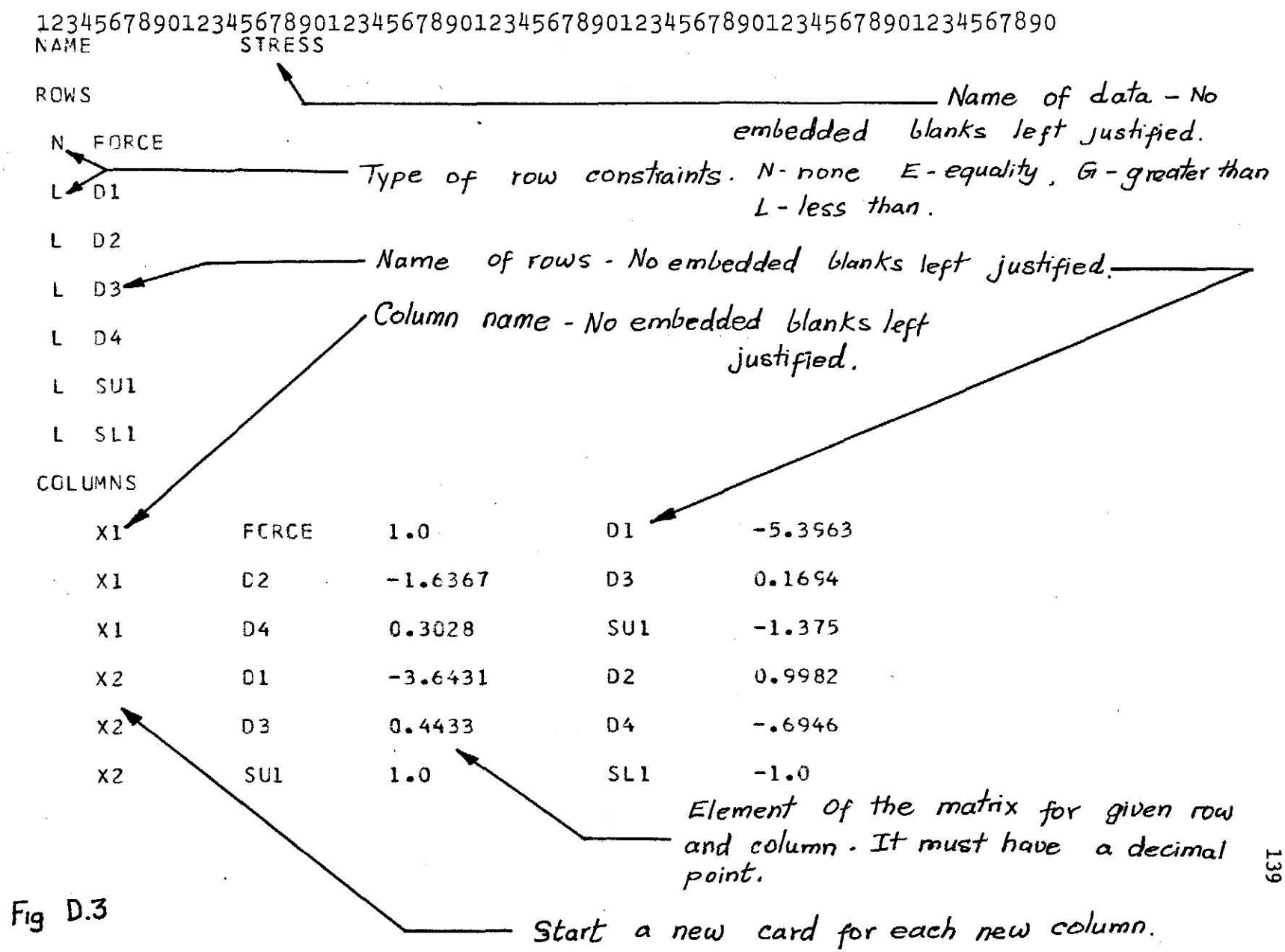
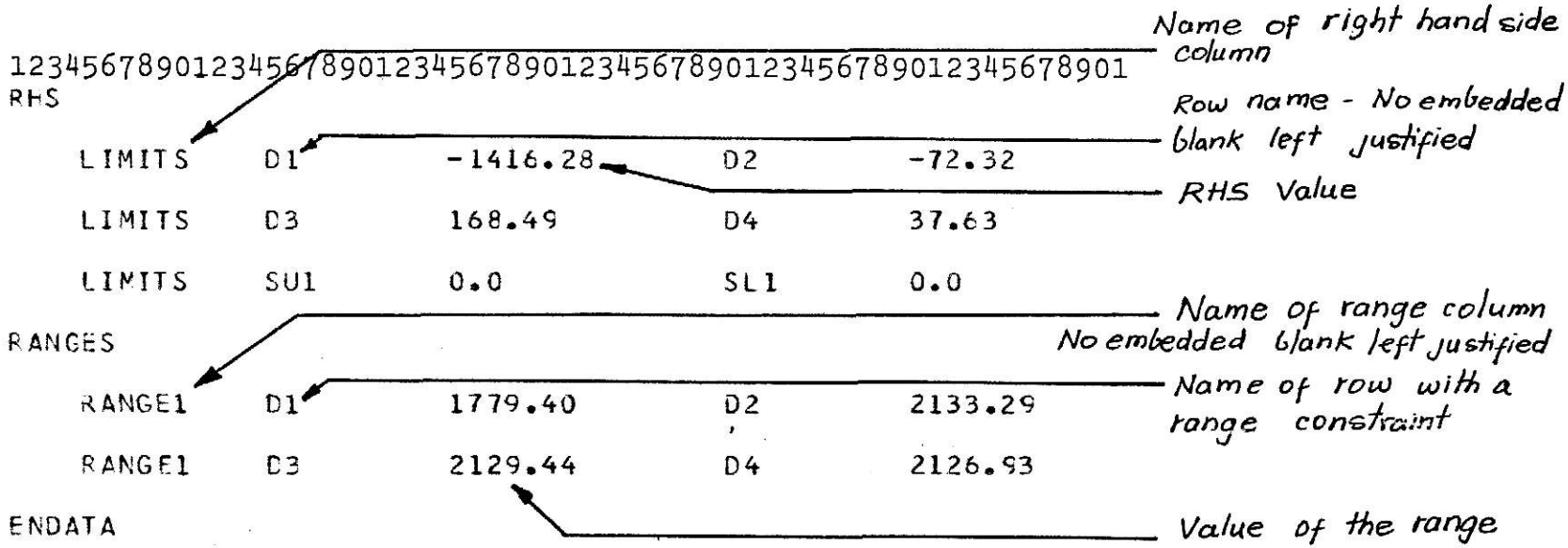

1234567890123456789012345678901234567890123456789012345678901234567890
NAME STRESS
ROWS
N FORCE
L D1
L D2
L D3
L D4
L SU1
L SL1
COLUMNS
X1 FORCE 1.0 D1 -5.3963
X1 D2 -1.6367 D3 0.1694
X1 D4 0.3028 SU1 -1.375
X2 D1 -3.6431 D2 0.9982
X2 D3 0.4433 D4 -.6946
X2 SU1 1.0 SL1 -1.0
Name of data - No embedded blanks left justified.
Type of row constraints. N - none E - equality, G - greater than L - less than.
Name of rows - No embedded blanks left justified.
Column name - No embedded blanks left justified.
Element of the matrix for given row and column. It must have a decimal point.
Start a new card for each new column.

Fig D.3



EACH ELEMENT OF DATA BELONGS TO A CERTAIN FIELD OF PUNCH CARD. THE FIELD TO WHICH AN ELEMENT BELONGS SHOULD BE OBVIOUS FROM THIS EXAMPLE DATA.

FIELD 1	2-5 CARD COLUMNS
FIELD 2	5-12 CARD COLUMNS
FIELD 3	15-22 CARD COLUMNS
FIELD 4	25-36 CARD COLUMNS
FIELD 5	40-47 CARD COLUMNS
FIELD 6	50-61 CARD COLUMNS

Fig D.3 continued

Appendix E

Moment of Inertia and Product of Inertia

Refer to Fig. E.1

$$I_{zz} = A \cdot (h_{Sin\theta})^2/12$$

$$I_{yy} = A \cdot (h_{Cos\theta})^2/12$$

$$I_{yz} = [(I_{y'y'} - I_{z'z'})/2] \text{ Sin } (2\theta)$$

where θ is measured counterclockwise from z-axis to the z'-axis.

With reference to new coordinate system $\bar{y}$, $\bar{z}$,

$$I_{\bar{z}\bar{z}} = I_{zz} + A\ (\bar{y}0)^2$$

$$I_{\bar{y}\bar{y}} = I_{yy} + A\ (\bar{z}0)^2$$

$$I_{\bar{y}\bar{z}} = I_{yz} + A\ (\bar{y}0)(\bar{z}0)$$

where $\bar{y}0$, and $\bar{z}0$ are coordinates of the centroid of the element with respect to $\bar{y}$, $\bar{z}$ axes, as shown in Fig. E.1.

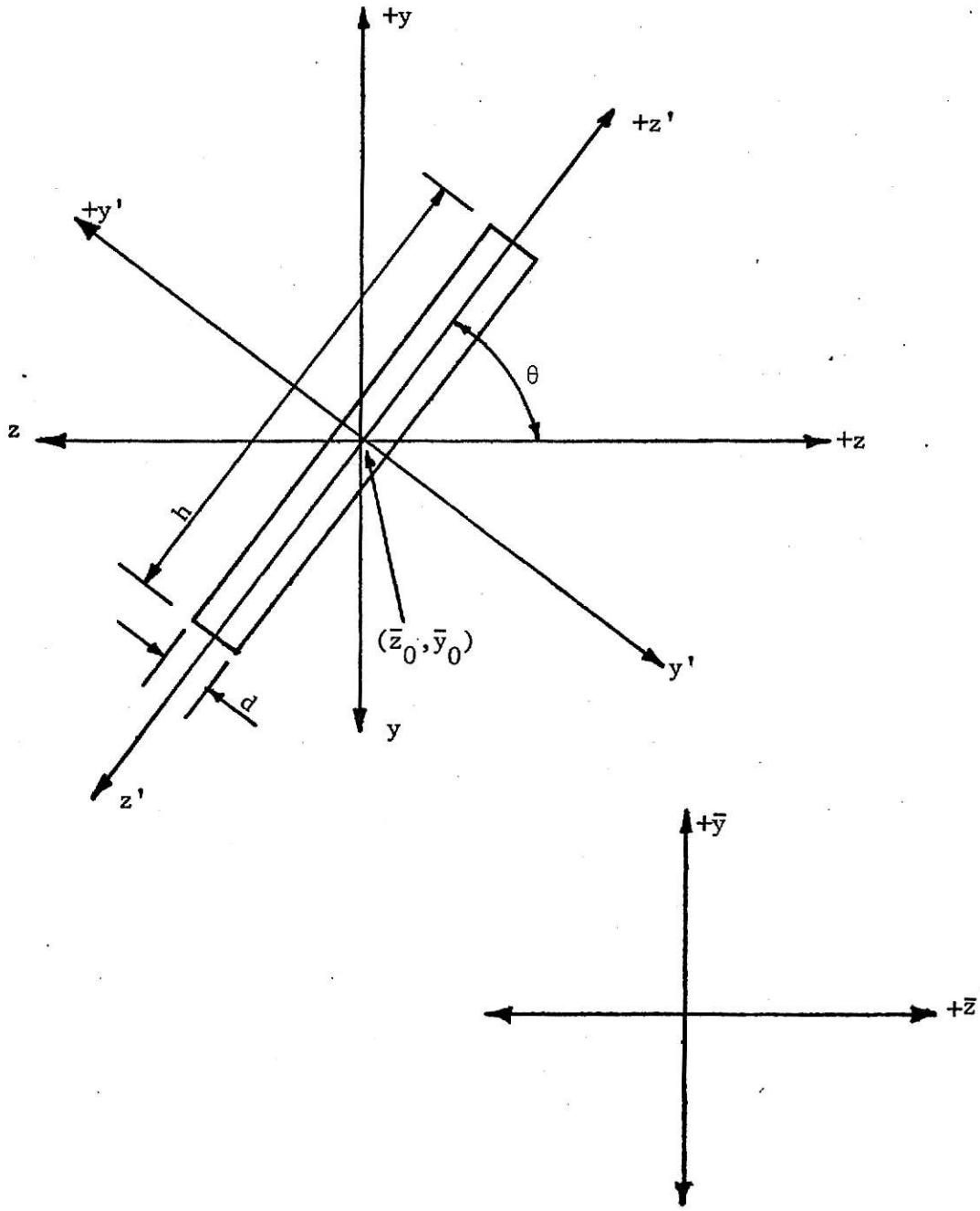


Fig. E.1

ANALYSIS AND OPTIMAL DESIGN OF PRESTRESSED CONCRETE FOLDED PLATES

by

Govindaswamy Rajasekar

B.S., Indian Institute of Technology
Madras, India, 1976

AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements of the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1977

ABSTRACT

The objective of this thesis is to present a method of analysis and design for prestressed concrete prismatic folded plates of any cross-section, the basic approach being that of Simpson. The various layouts of prestressing cable include parabolically draped or straight cable with possible end eccentricities. The method of calculation of principal stresses is modified to include the effect of prestressing. The effect of the torsional stiffness of the edge beam is considered and an analysis for wind load is discussed.

A computer program was written for the above analysis of folded plates and examples worked out by several authors were analyzed and results were compared. From the comparison of the presented method and beam theory, it was found that the beam method of analysis was very approximate. The solutions by the presented method of analysis and that by the rigorous finite difference method were found to have a very close agreement. The validity of superposition was checked for loads and prestressing.

Finally, a method for the selection of prestressing force and cable geometry in various plates, leading to a minimum quantity of cable, under specified combinations of loading, is presented. Several folded plates were analyzed and designed for prestressing using this method of optimal solutions.