STANDBY REDUNDANCY IN RELIABILITY
A REVIEW

by

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THIS BOOK CONTAINS NUMEROUS PAGES WITH DIAGRAMS THAT ARE CROOKED COMPARED TO THE REST OF THE INFORMATION ON THE PAGE.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Reliability is a relatively new field whose conception is primarily due to the complexity, sophistication, and automation inherent in modern technology. The problems of maintenance repair and field failures became severe for the military equipment used in World War II. In the late 1940's and early 1950's reliability engineering appeared on the scene. The fields of communication and transportation were perhaps the first to witness rapid growth in complexity as advances in electronics and control systems spurred equipment manufacturers.

In the field of electronics, there is a tendency towards increasing complexity and correspondingly high orders of reliability are becoming more difficult to achieve.

In the military sphere, the reliability of electronic equipment has become a major problem, and is recieving a high order of priority, particularly in connection with guided missiles and similar devices.

In the commercial field, high reliability is equally important. Computers, for instance, now play a major role in industrial affairs, being complex as well as expensive. A computer inoperative for a day can cause not only inconvenience but also financial loss.

Full automation of flight control of aircraft in takeoff, flight and landing may be expected in the not too distant future. This is a sphere where extreme reliability is, for obvious reasons, vital. For instance, even allowing for manual overriding of the automatic systems, a pilot might not have time to take over manual control in the event of equipment failure or inaccuracy in inspection.

A further example of the requirement for high reliability is in the instrumentation associated with the generation of energy and materials by the use of nuclear reaction. Here the instrumentation response must be immediate, so that remedial action can be taken, otherwise the results can be catastropic or mav necessiate complete shutting down and isolation of the plant concerned for a considerable period. All this points to the need for increasing reliability at all levels of design and development to keep in step with greater complexity and the increased demands for greater accuracy and performance.

TERMINOLOGY

Reliability is the probability that the system will function satisfactorily at least for a given period of time when used under stated conditions.

The Reliability function is the same probability expressed as a function of time.

Let T be a non-negative continuous random variable which represents the useful life of a component. The failure law of the component can be described in any of the several ways. Perhaps the most fundamental formulation is in terms of the cumulative distribution function (CDF) $F(t)$, defined as the probability that the unit fails before time t, and which we write as

$$
\begin{aligned}
F(t) &= P(T<t) \\
&= \int_0^t f(x)\,dx
\end{aligned}
$$

where $f(x)dx$ is the failure density and describes how the failure probability is spread over time.

The CDF is also referred to as the unreliability function. The reliability function $R(t)$, the probability that the component lives for atleast time t, is given by

$$
\begin{aligned}
R(t) &= P(T>t) \\
&= 1-P(T\leq t) \\
&= 1-F(t)
\end{aligned}
$$

<u>Hazard</u> <u>function</u> (also called instantaneous failure rate) is defined as the failure probability per unit time t, given that failure has not yet occured at time t. Thus the hazard rate $Z(t)$ is

$$Z(t)=f(t)/R(t)$$

where $f(t)$ is the failure density

and $R(t)$ is the reliability function.

The constant failure rate (CFR) model is one of the most important concepts in reliability. The corresponding failure law is called the negative exponential or sometimes simply as exponential and is widely used in the reliability theory because of its mathematical tractability.

<u>'Cold' standby unit</u> is a standby unit which is under no stress and hence subject to no failure while on standby.

<u>'Warm' or 'Hot' standby unit</u> is a standby unit which is under stress and hence subject to failure while on standby.
(A unit is referred to as a warm standby unit if the failure rate of the standby unit is less than that of the operating unit. A unit is referred to as a hot standby unit if the failure rate of the standby unit is equal to that of the operating unit.)

<u>Mean Time To Failure (MTTF)</u> is the mathematical expectation or mean of the random operating time until the first failure.

$$\begin{aligned} \text{MTTF} &= \int_0^{\infty} t\, f(t)\, dt \\ &= \int_0^{\infty} R(t)\, dt \end{aligned}$$

<u>Mean Time Between Failures (MTBF)</u> is the mathematical expectation or mean of the random operating time between failures for systems or units with repairs.

<u>Mean Repair Time (MRT)</u> is the mathematical expectation of the time in the operation of a repairable structure spent in elimi-

nating failure.

Instantaneous Availability is defined as the probability that the system is operational at any given time.

Average Uptime Availability, $A(t)$ is the proportion of time in the specified interval $(0, T)$ that the system is available for use.

Steady State Availability : Availability is termed as steady state, $A(\infty)$, when the time interval considered is very large and is given by

$$A(\infty) = \lim_{T \to \infty} A(T)$$

k-out-of-n:G The system is good only if at least k of its n elements are good.

k-out-of-n:F The system is failed if and only if atleast k of its n elements are failed.

Methods of improving reliability

There are several methods by which we can improve the reliability of a system. They are

(1) Using large safety factors,

(2) Reducing the complexity of the system,

(3) Increasing the reliability of the components,

(4) Practising a planned maintenance and repair schedule,

(5) Using redundancy.

The idea of operating components below their basic ratings is fundamental to various fields of engineering design. The rated parameters can be voltages, currents, powers, force or torque loads, velocities, temperatures, humidity etc. In the structural engineering field the idea of the safety factor is a basic one.

All too frequently equipment is often poorly designed or overly complex. In many cases, by proper design it is possible to simplify the system, thus improving the reliability.

Another approach to bettering system reliability is to improve the reliability of all the constituent components. Although it is often possible to trade time, money, size and weight to buy improved reliability, obviously such an approach soon reaches limits.

Reliability may also be improved by practising a planned maintenance and repair schedule.

Redundancy is used to mean, in a broad sense, the creation of alternate paths in a system structure to improve reliability.

Redundancy can be of two types:

(1) Parallal redundancy

(2) Standby redundancy

In parallel redundancy all the elements of the system are active and are functioning simultaneously.

In standby redundancy, one or more units will be operating while the other units are standing by. When the operating unit fails the standby unit takes over in place of the failed unit.

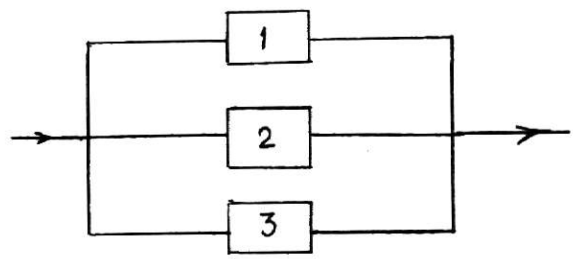

Fig.1 Parallel redundant system

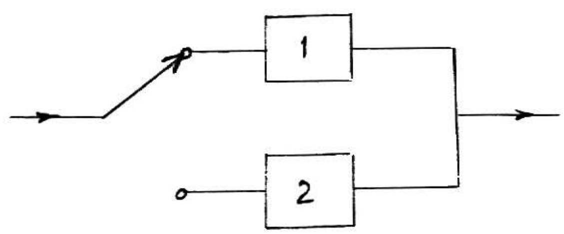

Fig.2 Standby redundant system

The four engines of a bomber are an example of parallel redundancy, in that failure of one or more engines will not necessarily result in a plane crash. A spare tire in the trunk of a car is a standby redundant element. The decision and switching device in this system is, of course, the driver.

Standby redundant systems can be classified into two distinct categories, viz. ,

(1) Non-repairable systems

(2) Repairable systems

A nonrepairable system is a system whose operation after failure is considered impossible.

A repairable system is a system whose operation can be restored after failure by making the necessary repairs.

Table 1 shows the classification of references on standby redundant systems. The following attributes aid in reading the table.

A : Nonrepairable standby redundant systems

B : Repairable standby redundant systems

C : Preventive maintenance standby redundant systems

D : Priority standby redundant systems

E : Optimization of standby redundant systems

F : Miscellaneous systems

G : Restricted repair facility

H : Identical units

I : Two unit systems

J : Cold standby

K : Perfect and instantaneous switching

* : Text book

Table 2(a) and Table 2(b) show the classification of references on standby redundancy with regard to failure and repair time distributions respectively.

Table 1. Classification of references on standby redundant systems

Reference	A	B	C	D	E	F	G	H	I	J	K
1		x		x			x	x	x		x
2		x						x	x		x
3				x			x		x	x	x
4				x			x			x	x
5 *											
6 *											
7				x							
8				x			x		x	x	x
9 *											
10		x							x		
11						x					
12			x					x			x
13		x						x	x	x	x
14		x						x	x	x	x
15		x						x	x	x	x
16											
17 *											
18			x				x	x	x	x	x
19			x				x		x	x	x
20		x					x	x			x
21		x					x	x	x	x	
22			x				x	x		x	x
23		x					x	x		x	x
24		x					x	x			x
25			x				x	x	x	x	x
26			x				x	x	x	x	x
27		x					x		x	x	
28		x					x	x		x	
29			x				x	x		x	
30						x	x	x	x	x	x
31		x					x			x	x
32		x					x	x	x	x	x
33		x					x	x	x		x
34		x					x	x	x	x	x
35		x					x	x	x		x
36		x					x	x		x	x
37		x					x	x	x	x	x
38		x					x	x	x	x	x
39			x								
40						x					
41		x					x	x	x	x	x
42					x						
43					x						
44					x						
45		x					x	x	x		
46			x				x	x	x	x	x
47			x				x	x	x	x	x
48 *											
49		x						x	x		x
50			x						x	x	x

Table 1. (cont'd) Classification of references on standby redundant systems

Reference	A	B	C	D	E	F	G	H	I	J	K
51			x					x	x	x	x
52		x					x	x	x	x	x
53		x					x	x		x	x
54		x						x	x	x	
55		x					x	x		x	x
56		x						x	x		x
57		x		x					x		x
58 *											
59						x					
60		x						x	x		x
61						x	x			x	
62		x						x	x		x
63					x						
64		x					x	x		x	x
65		x					x	x	x		x
66			x				x	x	x		x
67					x						
68			x					x		x	x
69					x						
70		x					x	x	x	x	x
71					x						
72					x						
73		x					x		x	x	x
74						x	x	x		x	x
75			x		x				x	x	
76			x		x			x	x	x	x
77				x			x		x	x	x
78		x						x	x	x	
79		x					x	x	x		
80		x					x	x		x	x
81		x						x	x		
82			x	x	x				x	x	x
83			x					x	x	x	x
84				x					x		x
85		x		x					x	x	
86		x						x	x		x
87		x						x	x	x	
88			x				x	x	x	x	
89		x							x		x
90		x						x	x	x	
91		x					x	x	x	x	x
92		x					x	x	x	x	x
93						x				x	
94						x					
95		x							x	x	
96					x						
97		x							x	x	x
98		x						x		x	x
99		x						x			x

Table 1.(cont'd) Classification of references on standby redundant systems

Reference	A	B	C	D	E	F	G	H	I	J	K
100		x					x	x	x	x	x
101	x										
102						x	x	x		x	x
103 *											
104					x						
105					x						
106						x					
107 *											
108						x					
109		x					x	x	x	x	
110		x						x	x	x	
111						x	x	x	x	x	
112		x					x	x		x	x
113		x					x	x	x	x	x
114		x					x	x	x		x
115		x							x	x	x
116		x							x		
117			x					x	x	x	
118			x				x	x	x	x	x
119		x					x	x			x
120		x					x	x	x	x	
121			x						x	x	
122			x				x		x	x	x
123		x						x	x	x	x
124		x					x	x	x		x
125				x					x		x
126		x					x				x
127							x				x
128				x			x			x	x
129		x					x	x	x		x
130			x				x	x			x
131					x						
132					x						
133		x					x	x		x	x
134			x				x	x	x		x
135						x					
136 *											
137					x						
138					x						
139		x					x	x	x		x
140	x							x	x	x	x
141		x						x	x	x	x
142		x						x		x	x

Table 2(a). Classification of references on standby redundancy with regard to failure time distributions

Failure distribution	References
Exponential	1,2,14,15,20,21,23,24,28,29,37,38,59, 60,63,65,70,77,79,87,92,95,101,109,111, 114,116,117,124,126,128,129,141,142
Erlang	56,57,98,99,133,139
Marshall-Orlkin	100
General	3,4,8,12,18,19,22,23,25,26,27,28,29,34, 35,36,45,47,49,50,51,52,53,54,55,66,73, 75,77,78,85,86,88,90,91,99,110,113,118, 119,120,121,122,123,125,127,130,133, 134,140

Table 2(b). Classification of references on standby redundancy with regard to repair time distributions

Repair distribution	References
Exponential	12,14,18,23,28,29,51,53,55,77,117,119,127
Erlang	99,100
Gamma	70
Marshall-Orlkin	100
General	1,2,3,4,8,15,19,20,21,22,23,24,25,26,28,29, 34,35,36,37,38,45,47,49,50,52,54,56,57,60, 60,63,65,66,70,73,75,77,78,79,85,86,87,88, 90,91,92,95,98,99,109,110,111,113,114,116, 118,120,121,122,123,124,125,126,128,129,130 133

NONREPAIRABLE SYSTEMS

By a nonrepairable system we mean a system whose operation after failure is considered completely impossible or infeasible. However, this does not necessarily mean that a structure of the given type cannot be repaired, as in the case for example, with electro-vacuum devices, meteorological rocket equipment, or ballistic missiles. The very concept of nonrepairable structure is characterized principally by the specific use of the equipment and not by its form.

Basically a nonrepairable structure should be understood in practice as a structure whose failure during operation leads to irrepairable consequences. In this sense, for example, an electronic computer that is being used to control a complicated chemical process, where any interruption in the normal technological process leads to an irreversible disruption of the process, can be thought of as a nonrepairable system. At the same time it is clear that when failure occurs, a computer can be repaired and made fit for other use. However within the limits of a concrete technological operation a computer is essentially an unrepairable structure.

First, consider systems in which the sensing and switching device is assumed to be perfect and instantaneous.

<u>Two unit cold standby nonrepairable system</u>

Let

$f(t)$=pdf for system life

$f_1(t)$=pdf for primary unit life

$f_2(t)$=pdf for secondary unit life

The probability element (p.e.) $f(t)\,dt$ is the probability that the system fails in the incremental interval dt. This event corresponds to the union of two simpler disjoint events.

E1: The primary unit fails sometime before t,

E2: The secondary unit functions for the balance of the interval (0, t) and fails in the interval dt.

Let z stand for the length of life of the primary unit, and (t-z) for the length of life of the standby. Now we can restate E1 and E2 as

E1: The primary lives for period z and fails in dz

E2: The standby lives for period (t-z) and fails in dt

It can be seen that

$$P(E1)=f_1(z)\,dz \tag{1}$$

and

$$P(E2)=f_2(t-z)\,dt \tag{2}$$

and the system failure in dt is related to their product

$$f_1(z)\,dz\; f_2(t-z)\,dt \tag{3}$$

Averaging out z by integration

$$f_S(t)\,dt=\int_0^t f_1(z)\,dz\; f_2(t-z)\,dt \tag{4}$$

so that the system failure pdf

$$f_S(t)=\int_0^t f_1(z)\; f_2(t-z)\,dz \tag{5}$$

Only in the special case of constant failure rate (CFR) units is equation (5) easy to solve. In that case equation (5) becomes

$$f_S(t) = \int_0^t \lambda_1 e^{-\lambda_1 z}\, \lambda_2\, e^{-\lambda_2 (t-z)}\, dz$$

$$= \frac{\lambda_1 \lambda_2}{\lambda_1-\lambda_2}\, e^{-\lambda_2 t} + \frac{\lambda_1 \lambda_2}{\lambda_2-\lambda_1}\, e^{-\lambda_1 t} \tag{6}$$

Integrating over (t,∞) we obtain the reliability function

$$R_S(t) = \frac{\lambda_1\, e^{-\lambda_2 t}}{\lambda_1 - \lambda_2} + \frac{\lambda_2\, e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} \tag{7}$$

$$= e^{-\lambda_1 t} + AR(2) \tag{8}$$

The term $e^{-\lambda_1 t}$ is the reliability of the first component and AR(2) is the additional reliability due to second component.

The form of equation (7) is symmetric in the subscripts 1 and 2 and thus it is clear that it does not matter which of the two components is chosen as primary and which one as secondary (or standby).

The Mean Time To Failure (MTTF) obtained by integrating equation (7), over $(0,\infty)$ is

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \tag{9}$$

In the case of an n-unit cold standby system (one unit operating and (n-1) standby units), proceeding as above yields

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \ldots\ldots + \frac{1}{\lambda_n} \tag{10}$$

<u>Identical elements in series</u> [101] :

First consider an application using only one operating element with one standby element available. The operating and standby elements have identical failure rates of λ. The reliability of the combination is

$$R(t) = e^{-\lambda t} + \lambda t\, e^{-\lambda t}. \tag{11}$$

and

$$MTTF = 2/\lambda \tag{12}$$

For (n+1) identical elements, with one operating and n sequential standby elements, the equations for reliability and MTTF are

$$R(t) = e^{-\lambda t}[1 + \lambda t + (\lambda t)^2/2! + \cdots + (\lambda t)^n/n!] \tag{13}$$

$$MTTF = n/\lambda \tag{14}$$

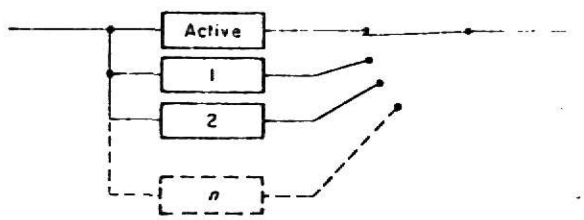

Fig.3 One active element and n sequential standby's

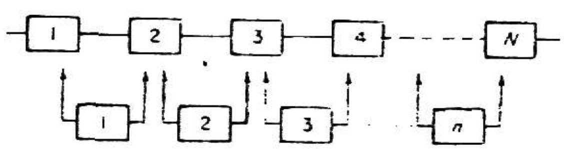

Fig.4 Active elements(N) in series and standby elements(n)

Fig.4 shows a configuration with N identical operating elements in series and n standby elements. The standby elements are not necessarily sequenced and all elements have a failure rate of λ. Total failure of the system will occur only on the (n+1) th failure. The reliability of the configuration is given by

$$R(t) = e^{-N\lambda t}[1 + N\lambda t + (N\lambda t)^2/2! + \cdots + (N\lambda t)^n/n!] \qquad (15)$$

and

$$MTTF = \frac{n+1}{N\lambda} \tag{16}$$

Two unit 'warm' standby nonrepairable system:

We consider now the case where the standby component has failure law

$f_2^*(t)$ while on 'warm' standby

$f_2(t)$ while in active service

The system fails in the incremental interval dt if Ea or Eb occurs, where

Ea: The primary lives for time z and fails in dz, the secondary has not failed while on standby, takes over and lives for time (t-z), finally failing in dt.

Eb: The primary fails in dt but the secondary cannot take over because it has failed while on standby.

The system failure probability element is thus

$$f_S(t)dt = P(Ea) + P(Eb)$$

$$= \int_0^t f_1(z)\, dz\, R_2^*(z) f_2(t-z) dt \tag{17}$$

$$+ f_1(t)\, dt\, [1 - R_2^*(t)]$$

For CFR components this becomes

$$f_s(t) = \int_0^t \lambda_1 e^{-\lambda_1 z} dz\, e^{-\lambda_2^* z} \lambda_2 e^{-\lambda_2(t-z)} + \lambda_1 e^{-\lambda_1 t}(1 - e^{-\lambda_2^* t}) \tag{18}$$

The system reliability is found by integrating equation (18) over (t,∞) and rearranging terms to obtain

$$R_s(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1+\lambda_2^*-\lambda_2}\left[e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2^*)t}\right] \tag{19}$$

The mean life of the system is

$$MTTF = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}\left[\frac{\lambda_1}{\lambda_1+\lambda_2^*}\right]$$

The added mean life for the standby element is thus reduced by the factor $\lambda_1/(\lambda_1 + \lambda_2^*)$ for warm as compared with cold standby.

<u>Two unit 'shared load' standby system</u> [6]

The assumption is that two components start functioning together at time $t=0$ with constant failure rates λ_1 and λ_2. If one component fails, then the other carries on alone, but with higher failure rates, say λ_1^* and λ_2^*. The system fails in the incremental interval dt if any of the following three events occur.

Ex: Component 1 fails in interval dz and component 2 carries on alone for (t-z) at increased failure rate, finally failing in dt

Ey: Component 2 fails in interval dz and component 1 carries on alone for time (t-z) at increased failure rate finally failing in dt.

Ez: Both components fail in dt.

The system probability element, p.e., of failure in dt is thus

$$f_S(t)\,dt = P(Ex) + P(Ey) + P(Ez)$$

$$= \int_0^t f_1(z)dz\, R_2(z)\, f_2{*}\,(t-z)\, dt$$

$$+ \int_0^t f_2(z)dz\, R_1(z) f_1{*}(t-z)dt + f_1(t)\, f_2(t)\, (dt)^2 \qquad (21)$$

The last term is neglected because it is a differential of higher order in dt.

For CFR components we have

$$f_S(t) = \lambda_1\lambda_2{*} \int_0^t e^{-\lambda_1 z}\, dz\, e^{-\lambda_2 z}\, e^{-\lambda_2{*}(t-z)}$$

$$+ \lambda_1{*}\lambda_2 \int_0^t e^{-\lambda_2 z}\, dz\, e^{-\lambda_1 z}\, e^{-\lambda_1{*}(t-z)} \qquad (22)$$

Integrating over (t,∞) we find

$$R_s(t) = \frac{\lambda_1\lambda_2{*}}{\lambda_1+\lambda_2-\lambda_2{*}}\left[\frac{e^{-\lambda_2{*}t}}{\lambda_2{*}} - \frac{e^{-(\lambda_1+\lambda_2)t}}{\lambda_1+\lambda_2}\right]$$

$$+ \frac{\lambda_1{*}\lambda_2}{\lambda_1+\lambda_2-\lambda_1{*}}\left[\frac{e^{-\lambda_1{*}t}}{\lambda_1{*}} - \frac{e^{-(\lambda_1+\lambda_2)t}}{\lambda_1+\lambda_2}\right]. \qquad (23)$$

As one of the redundancy techniques for an unrepairable system, Yamada and Osaki [140] supposed an unrepairable two unit standby redundant system in which each new component for replacement is supplied by an order. The stochastic behavior of such a model was analyzed by applying a unique modification of Markov renewal process.

The model consists of an unrepairable two unit standby redundant system in which a new unit is supplied by an order with lead time. This system is composed of unit 1 and unit 2. At time 0 unit one begins to operate and unit 2 is in standby. If unit 1 fails, unit 2 takes over its operation, and a new unit is ordered to replace unit 1. At the end of the lead time the new unit is delivered and replacement of unit 1 is made. During the lead time and replacement time, failure of unit 2 might take place. Then a new unit for replacement of unit 2 is ordered after replacement of unit 1. The replacement begins to operate when unit 2 fails. On the other hand if unit 2 is operating after replacement of unit 1 the new replacement unit is put into standby. The system behaves in the same fashion repeatedly. Fig.5 shows a configuration of the systems behavior.

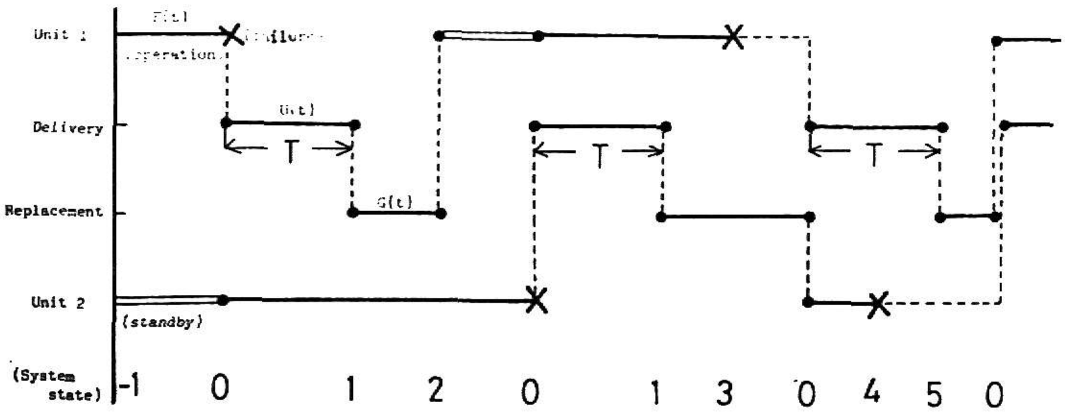

Fig. 5 Unrepairable two unit standby redundant system

The following assumptions are made for the above model:

(1) The system consists of two identical units, i.e., unit 1 and unit 2 are statistically identical.

(2) The failure time of a unit and replacement time of the new unit have arbitrary distributions F(t) and G(t) with finite means $1/\lambda$ and $1/\mu$ respectively. The lead time for an ordered unit has a degenerate distribution

$$U(t) = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$$

i.e. the lead time is constant T.

(3) A unit in standby never deteriorates or fails in the standby interval.

(4) We can obtain an identical unit for each replacement by ordering it.

(5) Each switchover is perfect and instantaneous.

Under the assumptions above, the stochastic behavior of the model is discussed by defining the following time instants:

time instant #-1: A unit begins to operate.

time instant #0 : One unit fails and the other begins to operate. A replacement unit is ordered.

time instant #1 : A new unit for replacement is delivered and replacement is initiated. The other unit is operating.

time instant #2 : Replacement of a new unit is finished and a new unit begins to be in standby. The other unit is operating.

time instant #3 : During replacement of a new unit the other unit fails.

time instant #4 : During delivery of a new unit the other fails.

time instant #5 : A new unit for replacement is delivered and begins to be replaced. The other unit is in the failed condition.

Let l be the mean recurrence time for state 0. Then the expected number of orders (Mo) and replacements (Mr) per unit of time in the steady state are

$$Mo = Mr = 1/\,l \tag{24}$$

where

$$l = \frac{1}{\lambda} + \int_{T}^{\infty} F(t)\left[1 - G(t-T)\right] dt \tag{25}$$

Imperfect switching:

There are several ways that switch failure can occur in standby redundant systems. The possible modes of switch failure depend on the particular switching mechanism and system. Two posibilities are considered here.

Let us first look at a situation where the switch simply fails to operate when called upon. If the ith sensing and switching device has reliability of only r_{Si}, then equation (8) requires insertion of the factor r_{Si} with the second term. The general case would be

$$R_S(t) = e^{-\lambda_1 t} + r_{s1}\,[AR_{(2)} + r_{S2}[AR_{(3)} \dots \dots]] \dots]. \tag{26}$$

It will be assumed that the switch is a complex piece of equipment and has a constant failure rate of λ_S. Thus the reliability function for the switch is

$$R_S(t) = e^{-\lambda_s t} \tag{27}$$

and the switch can fail before it is needed.

For a two unit standby system, if we consider the special case where all subsystems have a constant failure rate, then the expression for reliability is

$$R_S(t) = e^{-\lambda t}\left[1+\frac{\lambda}{\lambda_s}(1-e^{-\lambda_s t})\right] \tag{28}$$

and for a three unit standby system

$$R_S(t) = e^{-\lambda t}\left[1+\frac{\lambda}{\lambda_s}(1-e^{-\lambda t})\right]+e^{-\lambda t}(\lambda/\lambda_s)^2\left[1-e^{-\lambda_s t}-\lambda_s t e^{-\lambda_s t}\right] \tag{29}$$

REPAIRABLE SYSTEMS

It has long been known that one way to improve reliabiity is to use redundant units which are repairable. By a repairable system we mean a system whose operation after failure can be restored by carrying out the needed repair work. Basically, a repairable system should be understood in practice as a system that can continue to fulfill its function after the elimination of the failure which caused the curtailment of its operation.

Pepair maintenance of the systems can be of two types, viz,

1) Corrective maintenance,
2) Preventive maintenance.

If repair is carried out only upon failure of the unit then the unit is said to have undergone corrective maintenance. On the other hand if repair is carried out before a unit fails then the unit is said to have undergone preventive maintenance.

Repair of failed units (both operating and standby) can be done for either unrestricted or restricted service as explained below.

Unrestricted repair is a repair scheme in which a failed unit is immediately put in for repair, that is, we assume that the number of repair facilities is sufficient for simultaneous repair of all failed units of the system.

Restricted repair is a repair scheme in which not more than 'n' failed units can undergo repair at any moment of time, that is, we assume there are precisely 'n' repair facilities.

First consider the case of perfect and instantaneous switch-over with unrestricted repair facilities.

Perfect and instantaneous switchover (unrestricted repair facility)

Epstein and Hosford [14] derived the expression for the reliability of a two unit redundant system with constant failure rate and constant repair time. The system is composed of two identical units with constant failure rate λ and it takes exactly τ time units to repair a failed unit.

The study was extended to a two identical unit system with constant failure and repair rates, λ and μ respectively.
From [14, equation (14)], the expression for reliability is

$$R(t) = \frac{s_1 e^{s_2 t} - s_2 e^{s_1 t}}{s_1 - s_2} \tag{30}$$

where

$$S1 = \frac{-(2\lambda + \mu) + \sqrt{4\lambda\mu + \mu^2}}{2}$$

and

$$S2 = \frac{-(2\lambda + \mu) - \sqrt{4\lambda\mu + \mu^2}}{2}$$

and the mean time between failures is

$$\mathrm{MTBF}= \frac{2\lambda+\mu}{\lambda^2} \tag{31}$$

If $\mu=0$ (which means that repairs are impossible) the MTBF is $2/\lambda$. The ratio of the two mean times is $1+\mu/2\lambda$, which is a measure of improvement due to repair.

Gaver [15] discussed reliability properties of a standby redundant system which consisted of two identical units with constant failure rates λ and independent but otherwise arbitrarily distributed repair times. It is of some interest that several of the important operational measures deduced depend in detail upon the form of the distribution of repair times and not simply upon certain simple averages or moments of repair time.

The Laplace-Stieltjes transform (LS transform) of the distribution of time to system failure is

$$\psi(s) = \frac{\lambda^2}{s+\lambda[1-\hat{g}(s+\lambda)]} \frac{[1-\hat{g}(s+\lambda)]}{s+\lambda} \tag{32}$$

and

$$\mathrm{MTTF}= \frac{2}{\lambda}\left[\frac{1-\frac{1}{2}\hat{g}(\lambda)}{1-\hat{g}(\lambda)}\right] \tag{33}$$

where $\hat{g}(.)$ is the LS transform of the distribution of repair time distribution.

Srinivasan [115] considered a two unit redundant repairable system having arbitrarily distributed failure and repair rates.

Needed notation is

i	subscript referring to unit number $i=1, 2$
t	time $t>0$
$F_i(t)$	failure time distribution of unit i
$G_i(t)$	repair time distribution of unit i

From [115, equation (15)], the mean time to system failure is

$$\mathrm{MTTF}= \frac{1}{\lambda_1}+\frac{\frac{1}{\lambda_2}+\frac{1}{\lambda_1}\int_0^\infty G_1(t)\,dF_2(t)}{1-\int_0^\infty G_1(t)\,dF_2(t)\int_0^\infty G_2(t)\,dF_1(t)} \tag{34}$$

where

$$\frac{1}{\lambda_i} = \int_0^\infty t\,dF_i(t)$$

As a special case let

$$F_i(t)=1-e^{-\lambda_i t}$$

then

$$\mathrm{MTTF}= \frac{1}{\lambda_1}+\frac{(1/\lambda_2)+(1/\lambda_1)g_1(\lambda_2)}{1-g_1(\lambda_2)g_2(\lambda_1)}. \tag{35}$$

Osaki [85] has also derived similar results using renewal theory.

Kodama and Deguchi [56] provide an analysis for a two unit redundant system with identical units having Erlang failure and general repair time distributions. The Laplace transforms of the distribution of time to system failure and mean time to failure are derived.

Kodama [57] extended this result to a case of two unit redundant system with dissimilar units and with Erlang failure and general repair time distributions.

Ramanarayanan [98] studied the n-unit cold standby system when the operating unit has Erlang failure time.

Ramanarayanan and Usha [99] treated the n-unit (1-out-of-n:G) warm standby system with Erlang failure and general repair time distributions and its dual. In model 1 the operating unit has Erlang failure time and general repair time. Model 2 treats the case when the operating unit has general failure time and Erlang repair time. The integral equations and Laplace transformations of the availability and reliability are derived using renewal theory.

Usha and Ramanarayanan [133] considered two models in which one distribution is general and the other is general Erlang.

Arora [2] considered a 2-unit warm standby redundant system with repair under the assumption that the repair of the failed unit must be completed within a specified time known as Maximum Repair Time (MRT). The failure time of the operating and standby units are exponential distributions, whereas the repair time is general.

Perfect but noninstantaneous switchover (unrestricted repair)

The preceeding studies on reliability characteristics of standby redundant systems are made assuming that the subsystem in standby is instantaneously put into the active state upon failure of the active subsystem. This assumption may not be realistic in certain complex systems because bringing a system in the standby to the active state may require a series of operations involving some time element. The time required for bringing the standby subsystem to the operating standby state is called as switchover time.

Srinivasan [116] considered a standby redundant system where the switchover is not instantaneous. The two identical units have a constant failure rate and the repair and switchover times are distributed arbitrarily. A policy specifying the instant at which action is initiated on the standby subsystem in order to bring it to the operating standby state is proposed, and under this proposed policy the Laplace transforms of the probability density function of time to system failure and the expected time to system failure are derived.

Sinha and Kapil [109] employed Markov renewal process to derive the MTTF, availability and expected number of visits to a certain state of a two identical unit cold standby system with two types of repairs. The units have exponential distribution of failure times and arbitrary distribution for other times.

In practice, when a unit fails it takes time to locate a fault, isolate the unit from the main system, and to decide what type of repair is needed. Time taken in these later activities is called 'analysis time'. Agarwal and Kumar [1] employed a semi-Markov process technique to obtain the mean time to failure and steady state availability for a two unit standby redundant system with failure analysis time and two types of repairs.

Restricted repair capabilities

Hitherto it has been assumed that the repair times of the failed units are independently distributed. This implies that there exists a fairly large number of independent repair facilities which take up each unit as, and when, it fails. However in many practical situations, it is not feasible to have more than a single repair facility, in which case the units that fail must queue for repair.

Perfect and instantaneous repair (restricted repair)

Gnedenko [17] analyzed a two unit warm standby system with one repair facility and subject to exponential failure and general repair time distributions.

The reliability of a system having two identical and repairable units, one of which is operating while the other is in cold standby, was treated by Muth [70] It was assumed that each unit has independent exponentially distributed failure times and independent gamma distributed repair times. The gamma distribution has an advantage of covering a wide variety of shapes; the exponential distribution and the fixed time are special cases. The failure rate is and the mean time to repair is . Formulas are derived in [70] which express the system reliability and expected time to system failure, specifically as a function of the capability for one repair, two repairs, etc. Important conclusions are:

a) The benefit of repair capability is negligible when $\lambda\tau > 0.5$

b) The choice of repair distribution does not influence the results when $\lambda\tau < 0.2$, and

c) The effect of repair capability is comparable to that of additional standby units when $\lambda\tau < 0.01$

Kumugai [64] considered the case of an n-identical unit system having two repair facilities and subject to general failure and exponential repair time distributions.

Srinivasan and Gopalan [113] discussed the reliability and availability chracteristics of a two unit cold standby system when the units are identical, there exists a single repair facility, and the failure and repair times are both generally distributed. The Laplace transform of the probability density function of the downtime of the system is explicitly obtained when repair time is exponentially distributed. A general method of calculating the moments of the total downtime, when the repair time is arbitrarily distributed is indicated.

Srinivasan and Gopalan [114] extended this study to the case of a two unit warm standby system with a single repair facility. The failure times of the operating and the standby units are assumed to be exponentially distributed and the repair times are generally distributed.

Srinivasan and Gopalan [112] analyzed the availability and reliability of a single server, 1-out of-n:G system under the assumption that either the failure or the repair rate of a unit is constant and there is a single repair facility.

Gopalan [20] extended this study to an n-unit system with (n-1) independent and identically distributed warm standbys, with a single repair facility.

Gopalan and Saxena [23] further extended the study to a situation where the repair facility becomes temporarily unavailable after repairing all waiting units. Explicit expressions for the Laplace transform of the mean downtime of the system and for the mean time to failure have been obtained.

Gopalan and Venkatachalam [22] analyzed a 1-out of-n+1 system with one operating unit and n warm standbys and a single repair facility. Integral equations are set up for various transition probabilities in the general system by using the idea of regenerative points. The Laplace transform for the mean downtime of the system, the reliability function and the mean time to system failure have been explicitly obtained when n=2. The steady state availability has also been discussed.

The number of repairs up to time t for a 2-unit cold standby system with general repair and failure distributions was studied by Parthasarathy and Ramanarayanan [91]

Weins [139] analyzed the reliability characteristics of a hot standby sytem with two identical units. The distribution of unit times to failure is a form of the Erlang distribution. There is a single repair facility and the unit repair times are arbitrarily distributed.

Ramanarayanan and Subramanian [100] considered a 2-unit cold standby system with Marshall-Orlkin bivariate exponential life and repair times. The steady state probabilities and the steady state availability are obtained.

Subramanian and Natarajan [124] have obtained the availability measures of the repairable two unit warm standby system with the assumption that the failure time distribution of the online unit is exponential while the failure time distribution of the standby and the repair time distributions are arbitrary. Other characteristics like the expected number of repairs in (0, t) and the expected frequency of failures in (0, t) are derived by Gopalan and Natesan [33]

Perfect but noninstantaneous switchover (restricted service)

Kalpakam and Hameed [45] analyzed a two unit warm standby redundant system with a single repair service facility and perfect but noninstantaneous switchover. The availability and reliability analysis is done and many known results are derived as special cases.

Gopalan and Marathe [27] discussed the availability of a two dissimilar unit system with noninstantaneous switchover ("slow switch") when there is a single repair facility.

Gopalan and Marathe [28] extended this to the case of a 1 server 1-out of-n:G system with noninstantaneous switchover. Two models were considered here. In model 1, the failure time and the switchover time are distributed arbitrarily and the repair time is distributed exponentially. In model 2, the failure time and switchover time are distributed exponentially while the repair time is distributed arbitrarily. An explicit expression for the Laplace transform of the availability of the system was obtained.

Miscellaneous systems

Subramanian and Sharma [129] considered a two unit warm standby redundant system with one repair facility. After each repair completion, the repair facility is not available for a random 'dead time'. This is the preparation time needed before another repair can be initiated. Laplace transforms of availability and reliability are obtained.

Khalil [55] analyzed a 1-out of-(n+1):G system where one unit is active and the other n units are in cold standby. The system has one repair facility which begins repair only when a queue of length K units exists. The failure time of the active unit is arbitrary and the repair time distribution is exponential. The Laplace transform of the system time to failure distribution is derived.

Kapur and Kapoor [49] assumed a 'delay time' (preparation or waiting time) for repair facilities. If the delay time finishes before the failure of a unit, repair can be made immediately. Otherwise, the repair must wait until the delay time is over, causing a delayed repair. The system has one operating unit with a constant failure rate and a warm standby unit with an arbitrarily distributed failure time. Only one repair facility is available and the delay time is measured from the instant at which the system is put into operation, that is, a unit just becomes active. The Laplace transforms of

a) The first passage time to system's failure,

b) The expected number of failures during (0, t), and

c) The probability that the system fails at time t

are all derived by a unique modification of Markov renewal process.

Repairable systems with imperfect switchover :

The perfect switching models are easiest to develop, but unfortunately in practice we generally encounter imperfect switching.

Imperfect switchover (unrestricted service)

Prakash [95] evaluated the reliability characteristics of a standby redundant equipment consisting of two components with an imperfect sensing and switching mechanism. The model consists of two components A and B with sensing and switching mechanism S. This is studied based on the following assumptions:

1) Unit A operates and at its failure, switch S senses and switches to B. The probability of this type of switching is β and the failure time of A and B follow the exponential distribution.

2) Unit A operates and while A is still alive, S fails, and in failing, the switch remains on A. Then A operates until its failure. The probability of this type of failure of S is p1.

3) Unit A is operating and while A is still alive S fails, and in failing, switches to B. The probability of this type of failure is p2.

4) Unit A is operating and while A is still alive S fails. The signal path through S becomes open or shorted and the equipment dies at the time S fails, the probability for which is p3.

5) On failure the equipment goes to repair. The repair follows a general distribution.

The behavior of a standby redundant complex system under imperfect switchover devices with general waiting and repair time distributions for both types of components has been studied by Das [11] The probability of successful operation of a failure sensing and switchover device is a constant. The repair time of this sensing and switching device follows an exponential distribution. The Laplace transforms of various state probabilities have been derived.

Osaki [87] analyzed a standby redundant repairable system having two identical units, the failure rate of which is assumed constant and the switchover imperfect. The repair time of each unit has an arbitrary distribution. The failure time and the repair time of the switch is distributed exponentially. For this system the switch is used only instantaneously to change a unit from the standby state into the operating state. The Laplace transform of the distribution of the time to failure of the system and the mean time to failure of the system are derived using the relationship between Markov renewal processes and signal flow theory.

Nakagawa and Osaki [78] extended the model in [87] to an arbitrary failure time distribution of the main unit. In another model it was assumed that the failure of the switch could be detected only when it is used.

Nakagawa [79] considered a two unit warm standby system where the operating and standby units have constant failure rates and general repair times, and the switchover is imperfect. The switch has two different failure modes. It switches the operating unit out of operation when it should not, or it does not put the standby in operation when it should. The stochastic behavior of system failure is derived.

Imperfect switchover (restricted service)

Khalil [54] derived the mean time to failure of a two unit cold standby system with random switchover time and two types of repairs. The failure and repair distributions are general and there is a single repair service facility. If the repair is made quickly as compared to the time to failure of the working unit then it is proved that the limiting distribution of time to failure is exponential.

The system discussed in [54] has to be considered operating even if one unit has failed and the other is in the process of switchover. Thus there is a difficulty in defining the reliability of the system and it is more pertinent to ask for pointwise availability of the system rather than the reliability. The Laplace transform of the pointwise availability of such a system was obtained by Subramanian and Ravichandran [120]

Table 3 shows the classification of references on repairable standby redundant systems.

Table 3. Classification of references on repairable (corrective) standby redundant systems

System configuration	References
Repairable systems	1,2,10,13-15,20,21,23,24,27,28,31-38,41,45, 49,52-57,60,62,64,65,70,73,78-81,85-87,89-92,95,97-100,109,110,112-116,119,120,123, 124,126,127,129,133,139,141,142
Switching :	
Perfect and instantaneous	1,2,13-15,20,23,24,31-38,41,49,52,53,55-57, 60,62,64,65,70,73,80,86,89,91,92,97-100, 112-115,119,123,124,126,127,129,133,139,141 142
Imperfect	10,21,27,28,45,54,78,79,81,85,87,90,95,109, 110,116,120
Cold standby	13-15,21,23,27,28,31,32,34,36-38,41,52-55, 64,70,73,78,80,85,87,90-92,95,97,98,100,109 110,112-115,120,123,133,141,142
Warm standby	1,2,10,20,24,33,35,45,49,56,57,60,62,65,79, 81,86,89,99,114,116,119,124,126,127,129,139
Two units	1,2,10,13-15,21,27,31,32,34,36-38,41,52-55, 62,65,70,73,78,79,81,85-87,89-92,95,97,100, 109,110,113-116,120,123,124,129,139,141
n units	20,23,24,28,33,35,45,49,56,57,60,64,80,98, 99,112,119,126,133,142
Identical	1,2,13-15,20,21,23,24,28,32-38,41,45,49,52-56,60,62,64,65,70,78-81,86,87,90-92,98-100, 109,110,112-114,119,120,123,124,129,133,139 141,142
Dissimilar	10,27,31,57,73,85,89,95,97,115,116,126,127
Repair facility :	
Restricted	1,20,21,23,24,27,28,31-38,41,45,52,53,55,64 65,70,73,79,80,91,92,100,109,112-114,119, 120,124,126,127,129,133,139
Unrestricted	2,10,13-15,49,54,56,57,60,62,78,81,85-87,89 90,95,97-99,115,116,123,141,142

PREVENTIVE MAINTENANCE OF STANDBY REDUNDANT SYSTEMS

System improvement through preventive maintenance (PM) is as old as the sayings, "A stitch in time saves nine", or, "Prevention is better than cure". If a unit is characterized by a failure rate that increases with age, it may be wise to replace it before it has aged too greatly.

A commonly considered replacement policy [5] is the policy based on age (age replacement). Such a policy is in force if a unit is always replaced at the time of failure or T hours after its installation, whichever occurs first, where T is a constant. If T is a random variable then the replacement policy is referred to as a random age replacement policy.

Preventive maintenance and repair can be done with either unrestricted or restricted service.

Preventive maintenance (unrestricted service)

Mine and Asakura [68] presented a 1-out of-n+1:G standby redundant system with repair and preventive maintenance. The units have a general failure distribution and negative exponential repair and inspection times. Also the replacement interval is a random variable with an arbitrary distribution.

Osaki and Asakura [83] analyzed a two unit system assuming that the failure, repair inspection and preventive maintenance times are all arbitrary. The Laplace transform of time to first system down is derived and the MTTF is obtained from it. It was further shown that under suitable conditions the preventive maintenance policy for the system is effective in the sense that it increases the MTTF of the system.

Nakagawa and Osaki [75] discussed a two identical unit cold standby system with repair and preventive maintenance. The Laplace transform of the pointwise availability and steady state availability are derived by applying a unique modification of the regeneration point technique under the assumption that all distributions are arbitrary. A theorem gives the optimum preventive maintenance time (which is unique) and a finite solution of the equation under certain conditions.

Subramanian and Ravichandran [121] argued that the expression for the pointwise availability as given in [75] is incorrect and the correct expression was derived in [121]

Mahmoud [66] considered a warm standby system under the assumption that the standby unit has a constant failure rate.

Dirickx and Kistner [12] analyzed a 1-out of-n system with warm standby units with a constant failure rate. The repair times of the failed units are distributed exponentially.

Kapur and Kapoor [49] considered a two dissimilar unit system and derived the Laplace transform of the first passage time distribution to system failure, the expected number of system failures in a certain time interval and the probability that the system fails at time t, using unique modifications of Markov renewal process.

Srinivasan [117] derived the expected time to failure of a two unit cold standby redundant system with delayed switchover and with preventive maintenance.

Preventive maintenance (restricted service)

Osaki [88] considered a standby redundant model with two identical units; the failure time of each obeys an arbitrary distribution and the repair time of each failed unit an arbitrary distribution. The time to begin preventive maintenance policy and the preventive maintenance time are also arbitrary but different. Under these conditions the Laplace transform of the time distribution to failure and the MTTF are derived by applying the relationship between Markov renewal process and the signal flow graphs.

Kapoor and Kapur [47] extended the model in [88] to a case where the standby unit is warm and its failure time is distributed exponentially. A two unit cold standby system subject to intermittent availability was also discussed.

Venkatachalam [134] analyzed the availability and reliability characteristics of a two unit standby redundant system under the assumption that all the distributions are arbitrary.

Gopalan and D'Souza [18] analyzed the availability and reliability of a two identical unit system with a cold standby and a single repair service facility. The repair and the PM time of the unit are exponentially distributed. The times at which the operating unit is sent for PM or repair are assumed to be governed by two (distinct) general distributions.

Gopalan and D'souza [19] extended the analysis in [18] to two dissimilar units and where the pdf's of the time to failure, the time to send to PM, repair time, and PM time are all different and arbitrarily distributed. Similar results were obtained by Subramanian [122] using the concept of imbedded regenerative stochastic processes.

Gopalan and Venkatachalam [22] considered a 1-out of-n:G standby redundant system with (n-1) cold standbys and subject to repair and PM. The pdf's of times to failure and PM are arbitrary while the repair and PM rates are constant.

Gopalan and Saxena [25] analyzed a two unit cold standby system with 1-server subject to PM under the assumption that the service facility is taken up for PM immediately after completion of the rth repair ($r>1$) and the pdf's of the time to failure of the unit, the repair time and the time for PM are all arbitrary.

Gopalan and Saxena [26] extended the analysis in [25] to a situation where the service facility waits for PM after the completion of the rth repair and waiting follows an arbitrary distribution.

Kapoor and Kapur [47] analyzed a 2-unit warm standby redundant system with delay.

Subramanian and Natarajan [130] discussed an n-identical unit warm standby redundant system with r repair facilities and PM under the assumption that the failure time of the unit on line is arbitrary and all other times are distributed negative exponentially.

Table 4 shows the classification of references on preventive maintenance standby redundant systems.

Table 4. Classification of references based on PM of systems

System configuration	References
Preventive maintenance	4,12,18,19,22,25,26,29,39,46,47,50,51,66,68,75,76,82,83,88,117,118,121,122,130,134
Repair facilities :	
Unrestricted	4,12,50,51,68,75,76,82,83,88,117,118,121
Restricted	18,19,22,25,26,29,46,47,66,122,130,134
Cold standby	4,18,19,22,26,29,46,47,50,51,68,75,76,82,83,117,121,122
Warm standby	12,47,66,118,130,134
Component type :	
Identical	12,18,22,25,26,29,46,47,51,66,68,75,76,83,88,117,118,121,130,134
Dissimilar	4,19,50,82,122
Two unit	4,18,19,25,26,46,47,50,51,66,75,76,82,83,88,117,118,121,122,134
n-unit	12,22,29,130
Distribution of time to PM :	
Uniform distribution	12,22,29,130
Arbitrary	4,18,19,22,25,26,29,46,47,50,66,75,88,118,121,122,134
Distribution of PM time :	
Exponential	18,22,50,51,75,117,122,130
Arbitrary	4,19,25,26,29,46,47,88,118,134

PRIORITY STANDBY REDUNDANT SYSTEMS

In many industrial processes the provision of spare machines is necessary for very high reliability. The reliability of a system with standby units is further increased if failed units are repaired. Most analyses assume that each unit can perform the desired system function with the same reliability. However when system cost and complexity are large, the system might well have different units. Each unit is individually capable of performing the same nominal system function, but with different degrees of reliability and desirability. An example of this situation is a system consisting of three devices, viz, an electrical device, a battery operated device and a purely mechanical device. The reliability of such a system can be greatly improved by assigning priorities to the units for their operating and repair schedules.

Two repair disciplines are often used in practice, viz,

1) Head of line : This is 'first come, first served', i.e., the units are repaired serially in the order of arrival for repair.

2) Preemptive resume : The unit with higher priority is taken in repair immediately by suspending the repair of the lower priority unit. When the lower priority unit is taken up in the repair facility, the repair starts from the point it left earlier.

Osaki [84] discussed a 2-unit standby priority redundant system under the assumption that the failure times and the repair times of the priority unit (p-unit) follow an arbitrary distribution. The ordinary unit (o-unit) when it operates has a constant failure rate and no repair facility. Preemptive resume repair is followed.

Osaki [85] extended the analysis in [80] to a situation where the failure times of both p-unit and o-unit follow arbitrary distributions.

Subramanian and Ravichandran [128] considered a two unit priority redundant system under the assumption that the failure rate of the o-unit is constant but the failure and repair time distributions of p-unit are arbitrary.

Arora [4] discussed a two unit cold standby system with priority when the repair time for the o-unit is distributed exponentially but the failure time and repair time of the p-unit and the failure time of the o-unit are distributed arbitrarily.

Buzacott [8] analyzed a two unit priority standby system when both priority and ordinary units are considered to have general failure time and repair time distributions. Both preemptive resume and head of line repair disciplines are considered.

Subramanian and Ravichandran [125] considered the system discussed in [8] with the additional assumption that the failure rate of the p-unit and the o-unit while in standby or on line are constants.

Arora [3] analyzed a two unit priority standby redundant system when the priority unit has a finite repair capability.

Table 5 shows classification of references on priority standby redundant systems.

Table 5 . Classification of references on priority standby redundant systems.

Classification	References
Priority standby redundant systems	3,4,8,57,77,85,125,128
Repair discipline type	
Head of line	4,8,85
Preemptive resume	3,4,8,77,125,128

OPTIMIZATION IN STANDBY REDUNDANCY

In studying the use of redundancy to increase the reliability of various systems, we encounter the problem of not only guaranteeing certain minimum reliability standards, but also of designing the systems so as to effect this reliability criterion as economically as possible, with the least total expenditure on standby units for the system as a whole.

In practice such expenditures may be numerical measures of various characteristics of the system such as its cost, weight or size. The selection of the characteristics is determined by the concrete form of the system and its intent. It is usually possible to distinguish one most important characteristic, which in the case of space systems will most probably be weight.

In the literature, techniques such as integer programming, Lagrange multipliers, pseudo-Boolean programming and steepest descent are applied to determine the optimal number of standby units.

Table 6 shows the classification of references based the formulation of optimization problem. Table 7 shows the classification of references based on the optimization techniques employed to solve the problem.

Table 6. Formulation of optimization problems	
Formulation of problem	References
Optimum redundancy allocation, maximization of system reliability subject to cost constraints.	7,42,104,105,131, 137,138
"cost" minimization problems subject to system reliability constraint	43,44

Table 7. Optimization techniques employed	
Optimization technique	References
Integer programming	43,69,131
Dynamic programming	96
Lagrange multipliers and the Kuhn-Tucker conditions	7,67
Pseudo-Boolean programming	44
Method of steepest descent	58
Miscellaneous	42,71,72,104,105, 137,138

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STANDBY REDUNDANCY IN RELIABILITY
A REVIEW

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ABSTRACT

The technique of rédundancy is often used in practice when it becomes necessary to improve the reliability of a system beyond that obtained with simpler techniques such as simplifying, selecting better parts etc. In parallel redundancy all the elements of the system are active and functioning simultaneously. In standby redundancy, one or more units will be operating while the other units are standing by. The provision of standby spares is necessary for systems which are required to operate continuously over long periods of time. Systems used on space vehicles, such as power plants and life support systems, are prominent examples. The reliability of system with standby redundancy is increased further if units which have failed can be repaired.

This report is a review of the literature related to standby redundancy. The literature from early 1950's to 1982 is categorized and reviewed.