DESIGN AND ANALYSIS OF PRESTRESSED FLAT PLATES

by 5408

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SYNOPSIS

In recent years post-tensioned, prestressed flat plate construction has become quite popular for medium- and high-rise buildings. It is estimated that in 1969 flat-plate structures accounted for $5 billion in construction.

The flat-plates are highly indeterminate structures. Many engineers have worked in analysis and have proposed their design analysis and procedure, and many others are still working towards analysis and new findings. This report consists of the work done by different engineers at different times in analysis and design by the load balancing method by T. Y. Lin, with a design example.

INTRODUCTION

A flat slab is a concrete slab so reinforced in two directions as to bring its load directly to supporting columns, generally without the help of any beams or girders. Beams are used where the slab is interrupted, as around stair wells, and at the discontinuous edges of the slab.

Due to advancement of civilization, man tries to move towards bigger cities for the following reasons:

1. For higher education
2. Better employment opportunities
3. Better amenities
4. And the charm of big cities

Due to all these factors, the big cities in the world are overcrowded and the most important problem facing the people in a big city is a problem of suitable accommodation--which is one of the basic needs of every human being. The problem is not as bad in the United States as it is in the other parts of the world and especially in the developing countries where the problem is much worse. Presently every Government in the world is doing its best to meet the growing need of accommodation in the big cities. The main problem is how to solve this problem.

Actually there are two types of development--horizontal and vertical. Due to the high cost of land in the big cities, the horizontal development is not a solution to meet the growing need of suitable accommodation; also you cannot allow the city to grow haphazardly in any direction. As we know, every city has its boundaries, because there always exists a relationship between place of living, place of work and place of shopping. The alternative is vertical development--tall buildings.

There are different types of floor systems, each having its own advantages and disadvantages. The increased use of flat plates as a floor system in high-rise buildings has been largely brought about by obvious economical benefits derived from less expensive form work, elimination of beams, easy installation of electrical and mechanical equipment, smooth ceilings and floors and reduction in story heights. With the advent of climbing cranes and the ability to move larger amounts of materials higher and faster than ever before, it is only natural that flat-plate construction has become more economical for taller buildings.

The architectural trend towards more flexibility and larger column-free space has added impetus to the use of post-tensioned flat plates. The post-tensioning also requires less material which increases its adaptibility to high-rise construction.

LITERATURE REVIEW

Centuries of construction with stone and timber preceded reinforced concrete. Consequently, just as the first motor cars were built to look like horse-drawn carriages, the first reinforced systems were conceived in the image of traditional types. In a timber structure, the planks carried the load to the joists, the joists to the girders and the girders to the columns; so must they in a reinforced concrete structure. Hence, the flat slab had to be invented rather than developed as one of the obvious applications of reinforced concrete.

Because it escaped the imagination of minds trained in the two dimensions of construction with timber and iron, the flat slab was given the treatment of a miracle; while it was endorsed blindly by some engineers, it was resisted savagely by others. Between 1906, when C. A. P. Turner built the first flat slab, and in 1921, when Westergaard and Slater published their comprehensive paper on slabs, the flat slab was a subject of controversy and therefore of intense interest among practicing engineers and college professors. The bone of contention is illustrated dramatically by a comparison made by A. B. McMillan in 1910 which is shown in Fig. 1. The bars indicate the amount of reinforcement required by various design procedures in a 20 x 20 ft. interior panel of an 8 in. thick flat slab intended to carry 200 lbs/sq. ft. Evidently, the material bill for steel could vary by 400 percent depending on the design method chosen. There was no room for argument.

The arguments about the theory did not faze the construction industry. C. A. P. Turner had found entrenched resistance against his invention in 1906. Although the C. A. Bovey building, built at the risk of the inventor, had performed satisfactorily in a load test, Turner was still being asked to put up bond for new work 2 years and a dozen buildings later. But a few years later, the situation changed. By 1913, over 1000 flat slab buildings had been built around the world. During this period Turner's patents were revoked by the Eighth Circuit Court of Appeals in favour of J. L. Drum who had purchased the O. W. Norcross patent for a flat arch which resembled the Turner flat slab and preceded it. This lawsuit evoked a Dickensian hatred of patent lawyers in Turner who sustained a relentless campaign through books, articles, and discussions to seek a recognition of his contribution which he was never to get. In 1914, Edward Godfrey, a disapproving critic of the flat slab wrote, "The flat slab has repeatedly been brought before the engineering profession for consideration and adoption." Turner responded bitterly, "As sponsor for the original successful flat slab construction, the writer may say in answer to Mr. Godfrey, that he has never knowingly submitted it for the adoption of or appropriation by the engineering profession at large."

The appropriation of the flat slab by the engineering profession had its problems created mainly by the comparison shown in Fig. 1. All design methods could not be correct if the variation in results was 400 percent.

The 1912 progress report of the Joint Committee on Concrete and Reinforced Concrete had only one paragraph on the flat slab:

> The continuous flat slab with multiple-way reinforcement is a type of construction used quite extensively, and has recognized advantages for special conditions, as in the case of warehouses with large, open floor space. At present, a considerable difference of opinion exists among engineers as to the form of constants which should be used but experience and tests are accumulating data which it is hoped will in the near future permit the formulation of the principles of design for this form of construction.

The early proof tests had satisfied the building commissioners, but they were not adequate to form a basis for design recommendations. The severe limitations of the ordinary load test were perceived by most engineers. The

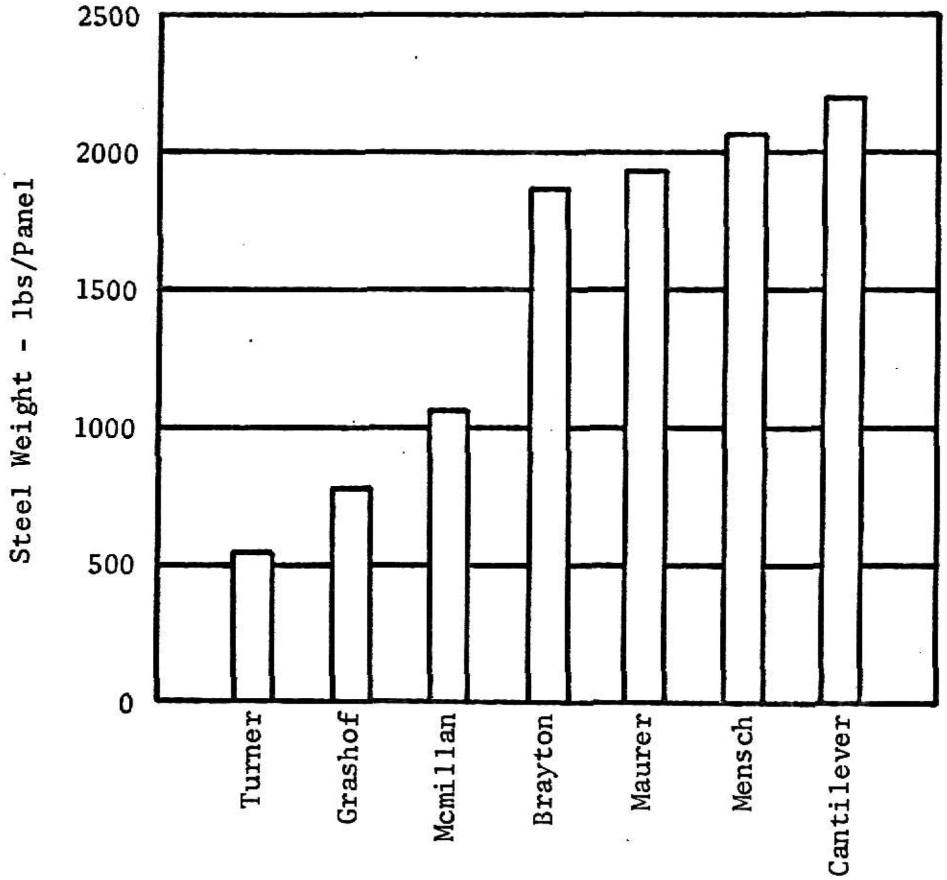


FIG. 1. WEIGHT OF STEEL REQUIRED IN THE INTERIOR PANEL OF A FLAT SLAB BY VARIOUS DESIGN METHODS IN 1910.

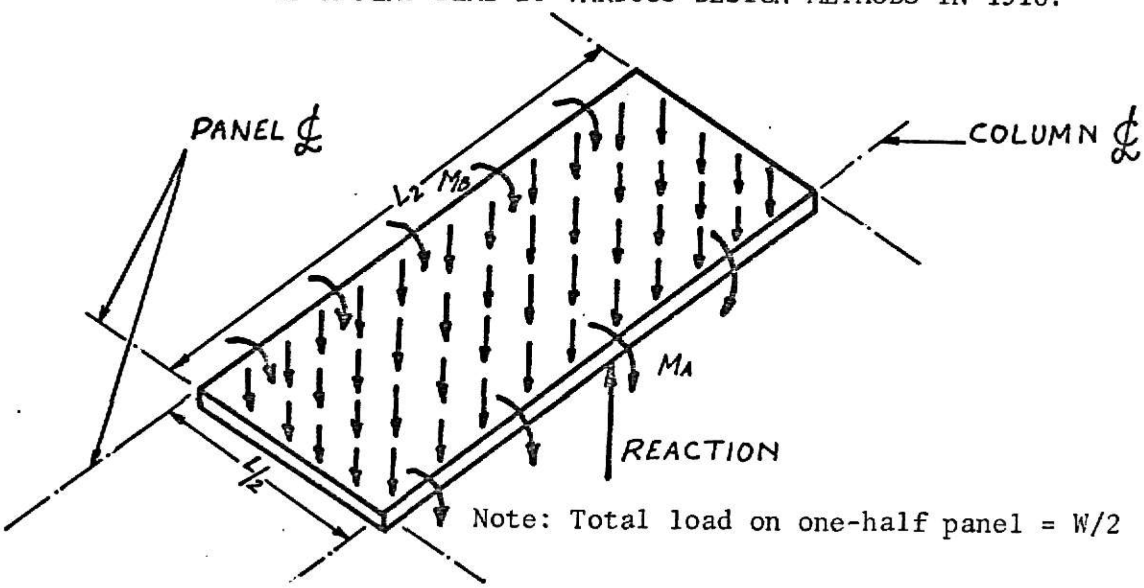


FIG. 2. FREE BODY DIAGRAM OF ONE-HALF OF A TYPICAL INTERIOR PANEL.

fact that a few panels withstood the test load with no apparent sign of distress could attest only to the fact that the same panels might be able to carry the same load if loaded again. The unassailable observation that the flat slab designed by almost any method stood up could not be ignored; nor was it, as evidenced by the sudden boom in flat slab construction. But this fact alone favoured no particular design method.

In 1910, A. R. Lord made testing history when he reported the strain readings he had made during the load test of a flat slab floor of the Deere and Webber building in Minneapolis, Minnesota. With the help of such measurements, an insight could be obtained into the response of flat slabs. Unfortunately, interpretations of the data from this test and a series of others did little toward resolving the mystery of flat slabs.

For the structural engineer, plate action was an entirely new concept. The "crossing beam analogy," thinking of the slab as two perpendicular beams each carrying a certain proportion of the load in relation to their stiffnesses, helped only to foster the still existing illusion that only part of the load need be carried in a given direction. Grashof's work had already been used by mechanical engineers in boiler plate problems. However, this work was represented in American Engineering Literature either simply as formulas without any derivations or as a basis for arriving at questionable conclusions.

Turner was a prolific writer. Nevertheless, it is difficult to make definite statements about his interpretation of the response of a flat slab. In 1909, he wrote, "Such a slab will act at first somewhat like a flat dome and slab combined, but as the deflection gradually increases it will gradually commence to act as a suspension system in which the concrete will merely hold the rods together and distribute the load over them." However, later he resorted to a more conventional explanation, the "mathematical flat plate," which was, "as any mathematician would readily see," a thin plate. His assuming this approach may have been caused by the efforts of the apologists who used poisson's ratio in plate theory as a vehicle to justify Turner's design or by his desire to differentiate his invention clearly from the Norcross patent which was "as a mason would understand it" a slab thick enough to act as an arch.

The admission that the response of the flat slab was similar to that of a medium-thick plate together with the observations indicating very small strains in load tests resulted in conflicting conclusions that are yet to be resolved.

While the majority of the engineering profession classified that flat slab as being beyond the range of pure analysis, J. R. Nichols, a young engineer from Boston, Massachusetts, cut the Gordian knot with a simple straight forward analysis.

Consider one rectangular panel of a flat plate having an infinite number of identical uniformly loaded panels. The plate rests on pin supports. The equilibrium conditions for one-half of the panel are shown in Fig. 2. Because of the symmetry, shears and twisting moments do not exist along the edges of the portion considered. Thus

$$M_A + M_B = M_O = \frac{WL}{8} \qquad (1)$$

Equation (1) does not indicate the relative magnitudes of the positive and negative moments, but it does stipulate that the total moment resistance provided at the two section should add to WL/8. For the interior panel of a flat slab with a round capital and with the assumption that the reaction is distributed uniformly around the capital (ignoring the twisting moments between the slab and the capital) Equation (1) becomes

$$M_O = \frac{WL}{8} \left[1 - \frac{4C}{\pi L} + \frac{1}{3} \left(\frac{C}{L}\right)^3\right] \qquad (2)$$

In the closure to his paper, Nichols suggested that Equation (2) could be approximated by

$$M_O = \frac{WL}{8} \left[1 - \frac{2}{3} \frac{C}{L}\right]^2 \qquad (3)$$

Turner thought the paper "to involve the most unique combination of multifarious absurdities imaginable from either a logical, practical, or theoretical standpoint." H. T. Eddy stated that the fundamental erroneous assumption of the paper appeared in the statement that statics imposed certain lower limits to the applied forces to be resisted by the reinforcement. A. W. Bull was "unable to find a single fact in the paper, nor even an explanation of facts," and considered it to be "a paper of explanations, not only without facts, but contradicted by facts." Newtonian mechanics stood very much condemned.

The observed facts were clearly on Turner's side. He used considerably less steel than would be required by Nichols' analysis, yet his structures performed satisfactorily. Furthermore, the strain readings made by Lord and others indicated very small stresses under both working loads and test loads in flat slabs. When measured steel strains were converted to bending moments through the use of the straight-line formula

$$M = A_s f_s jd \qquad (4)$$

the sum of the moments across the negative and positive moment sections was only a fraction of M_0 as defined by Equation (3). In his discussion of a paper by H. T. Eddy, G. S. Binckley conceded "The crushing weight of practical experience and empirical data under which C. A. P. Turner flattened out Mr. Nichols' purely theoretical paper, tends to induce caution in others."

C. A. P. Turner need never have fought his battle with the conditions of equilibrium. Nichols' analysis was right, but Turner's design was not wrong. The problem arose out of comparing incomparables. Turner realized fully that he could never know exactly what the load on the structure would be. Hence, 'W' in Equ. (3) was only a guess and therefore the whole right-hand side of the Equ. (3) was approximate. To the designer and evidently to the structure, it did not matter whether W or the 1/8 factor was modified. Turner insisted on high-strength steel. Although he might be espousing low steel areas, the force provided was often adequate to satisfy Equ. (3). Furthermore, Turner was well aware of the effects of forces in the plane of the slab.

The first Joint Committee adopted the form but not the intent of Equ. (3). In the final report published in 1916, the design static moment was given as 85% of that based on Nichols' analysis, or

$$M_{od} = 0.107\ WL\ (1 - \frac{2}{3}\frac{C}{L})^2 \tag{5}$$

The 1917 proposed revision and the 1920 approved version of the ACI Building Code contained an even more flagrant violation of statics

$$M_{od} = 0.09\ wL_2\ (L - qc)^2 \tag{6}$$

where qc was twice the distance from center line of the capital to the center of gravity of the periphery of the half capital. The ratio q was taken as 2/3 for round capitals and 3/4 for square capitals. A distinction was made between round and square capitals, although not in exactly the manner implied by Nichols' analysis.

With the publication of the First Joint Committee Report in 1916 and the ACI Building Code in 1920, design procedures had become more or less stabilized. However, the flat slab was still a mystery to a good cross section of the engineering profession. There were too many conflicting reconciliations of the load test data and diverse "theories."

In 1921, Westergaard and Slater published their monumental work on the analysis and design of slabs. This paper included a sound exposition of the theory of plates, ingenious projections of the available theoretical solutions to solve practical problems, and a comprehensive study of the implications of the then available tests on flat slabs and two-way slabs.

One of the accomplishments of the paper was to show conclusively that the moments calculated from measured strains in flat slab tests were not as small as they were interpreted to be. Previously, the moment had been calculated from the measured steel strain using straight line formula

$$M = A_s f_s jd = A_s E_s \varepsilon_s jd \tag{4}$$

which gives a linear relationship between M and ε_s for a given section as shown by line A in Fig. 3. The actual moment-strain curve in a load test is given by Curve B in the same figure. The divergence of the initial portions of the two curves is due to the tensile strength of the concrete. After cracking, Curve B approaches Curve A which is based on the assumption of no tension in the concrete.

The misinterpretations of the test data results from the following series of events: (1) as a result of the applied load a moment M_t had been generated at a section (Fig. 3), (2) this moment caused the steel-strain ε_{st} which was measured, (3) the strain was converted to moment using Curve A [Equ. (4)] and (4) the measured moment M_m, turned out to be a fraction of the actual moment. If this had happened for a beam test, the fallacy would have been obvious, but a lot could be hidden in the unfamiliar concepts of the third dimension.

Objections had been made to the interpretations of test data on this basis almost as soon as Lord's test had indicated extremely small stresses in flat slabs, but no documented effort to reinterpret the data had been made. Slater made a painstaking study of the true moment-strain relationships in reinforced concrete beams and used this study to re-evaluate data from 14 flat slabs, three of which had been tested to failure. He found that

> On the average, the agreement of (his re-evaluated moment co-efficients from test data) the higher computed stresses with (Nichols') analysis is sufficiently close to warrant the belief that if all the sources of error in the measurement of deformations and in the interpretation of test results could be removed, the analysis and the tests would be in substantial agreement.

Nevertheless, Nichols' analysis was not completely endorsed. "There are some indications, however, that there was greater strength in the flat slabs than appears from the conclusion that the test results and the analysis of moments are in fair agreement."

Although the truth of Equ. (3) was conceded, it was not granted that the ratio of the ultimate to the design load would be equal to the ratio of the yield to the design stress. Slater estimated the average factor of safety of the structures he studied to be 3.23 if the design was based on Equ. (5) and 2.72 if based on Equ. (6). Even in the latter case, the factor of safety was "at least as great as that which can be counted upon in the most elementary flexural unit," and therefore satisfactory.

One significant factor was ignored in estimating the factor of safety, the effect of adjacent unloaded panel. This is brought out clearly in the ultimate load for one of the slabs that was tested to failure. The safety factor for slab J tested at Purdue University was based on a capacity of 872 lbs per sq. ft. which was obtained when only one of the four panels was fully loaded.

The Westergaard-Slater treatise on slabs consummated the compromise between analysis and design.

The Second Joint Committee adopted a slightly modified version of Equ. (6) by which the rectangular capital was not recognized,

$$M_{od} = 0.09\ WL\ (1 - \frac{2}{3}\frac{C}{L})^2 \tag{7}$$

with a footnote in fine print. "The sum of the negative and positive moments provided for by this equation is about 72 percent of the moment found by rigid analysis based upon the principles of mechanics. Extensive tests and experience with existing structures have shown that the requirements here stated will give adequate strength."

The 1928 ACI Building Code provided the comfort of Equ. (7) without the discomfort of the footnote. The end had justified the means and the hybrid Equ. (7) graced many a building code in stark defiance of statics.

The 1956 ACI Building Code introduced a new factor F based on the fear that old tests on flat slabs which had C/L ratios on the order of 0.2 might not apply to the modern flat plate which had rather low C/L ratios. Thus Equ. (7) was changed to

$$M_{od} = 0.09\ WLF\ (1 - \frac{2}{3}\frac{C}{L})^2 \tag{8}$$

with

$$F = 1.15 - \frac{C}{L} \text{ but not less than 1} \tag{9}$$

Design by Elastic Analysis, which made its first appearance in the 1941 ACI Building Code, is in essence a two-dimensional approximation to the plate problem. However, it is modified to yield answers comparable to those of the "empirical method."

Westergaard had used the concepts of frame analysis, where applicable, to study critical loadings in flat slabs, and the effect of column stiffness on the distribution of moments had been analyzed with the use of approximate methods in Europe. However, the ACI frame analysis finds its beginnings in a paper by H. D. Dewell and H. B. Hammill published in 1938. The paper was based on a report made in 1929 to the technical committees of the Uniform Building Code, a California edition.

The need for a "rational" method was felt primarily because of the anticipated effects of pattern loadings on the slab and column bending moments. The limitations of the Empirical method as to panel sizes and combinations represented another incentive. The main features of the proposed method were quite similar to those of the current method in ACI 318-56. The bent was one-panel wide. The column-slab joints were assumed to be rigid. Dewell and Hammill assumed in their calculations that the columns were hinged at the middle of the distance from the bottom of the capital to the top of the floor below.

Since the conditions of equilibrium were automatically recognized in the frame analysis, the resulting moments were 100% rather than 72% of the static moments. To eliminate this discrepancy, the negative moments were reduced by 40%. This method was given in the 1933 Uniform Building Code, California

Edition as an alternate method for the design of flat slabs.

The ACI Method was developed under the direction of R. L. Bertin. The desideratum was a frame Analysis method which would give results comparable to those obtained by the Empirical method of design. This was effected by stipulating the design negative moment to be that at a distance xL from the center of the column width, where

$$x = 0.073 + 0.57 \frac{A^1}{L} \tag{10}$$

where A^1 was half the capital width but not greater than L/8.

For a uniformly loaded panel with equal end restraints, the sum of the positive and negative moments can be made equal to the moment given by Equ. (7) by taking the negative moment at a distance xL from the column center line such that

$$\frac{0.09\ WL\ (1 - \frac{2}{3} \times \frac{2A^1}{L})^2}{0.125\ WL} = \frac{(\frac{L}{2} - xL)^2}{L^2/4}$$

For this condition

$$x = 0.076 + 0.565 \frac{A^1}{L} \tag{11}$$

Equation (10) was based on studies covering variations in ratio of adjacent spans, column to slab stiffness, and live to total load.

On the other hand the total design moment was required to be

$$M_{od} \geq \frac{1}{10} W_{av} L \left(1 - \frac{4A_{av}^1}{3L}\right)^2 \tag{12}$$

where W_{av} was the average of the total load on the two adjacent spans and A_{av}^1 was the average of the values of A^1 at the ends of the span considered.

The ACI frame Analysis was modified further in ACI 318-56. Equ. (10) was dropped in favour of a "physical" definition of the critical section for negative moment (distance in the direction of span from center of support to the intersection of the center line of the slab thickness with the extreme 45° diagonal line lying wholly within the concrete section of slab and column or other support, including drop panel, capital or bracket). This definition showed substantial agreement with Equ. (10) for C/L = 0.225 but resulted in more conservative negative moments for small values of C/L. Equ. (11) was also eliminated. Furthermore it was permitted to reduce the design moments, "in such proportion that the numerical sum of the positive and average negative bending moments used in design procedure need not exceed "M_o" as given in Equ. (8), if the structure analyzed was within the range of application of the Empirical method."

However when spans extend beyond 18 ft. two major disadvantages develop in reinforced flat plates:

1. Large thickness slabs are required resulting in heavier dead loads and corresponding increase in column sizes and foundations which makes the structure uneconomical.
2. Slab deflections not only produce cracking in the slab itself but may also crack the room partitions above and below the floors.

Prestressing can be used effectively to overcome these two difficulties. The architectural trend towards more flexibility and larger column free space has added impetus to the use of post-tensioned flat plates. The post-tensioning also requires less material which increases its adaptibility to high-rise construction.

The stresses in prestressed flat plates are highly indeterminate. Possibly Guyon,[1] in the early 1950's was the first to realize that slabs prestressed in two directions behaved analogously to the two-way arch action of thin shell structures. In the late 1950's several prestressed slab research projects were undertaken in the United States. Scordelis[2] studied the ultimate strength of continuous prestressed slabs and proposed several design considerations. In another project Scordelis investigated the load distribution between column and middle strips. Later Rice and Kulka[3] emphasized the need for deflection as an important criterion in the design of prestressed lift slabs. In 1962, Green summarized existing knowledge in the field. He covered the practical details of cable profiles, reversed cable curvature and prestress and frictional losses.

Possibly, the largest stride in the design of prestressed slabs was the publication in 1963 of a paper by T. Y. Lin[4] on the load-balancing method. It was soon made apparent that the tendon profiles could be designed so that the upward cable force neutralized the vertical downward load. This approach bypassed a rigorous analysis of the highly redundant stress system. Furthermore, the method provided for deflection control for the dead load which is generally the major portion of the load.

Koons and Schlegel[6] extended the load balancing approach and presented some practical aids for solving continuity and cable reversal.

In 1963, Saether published a paper in which he applied a structural membrane theory to the solution of prestressed flat slabs. The method, however, was not developed far enough to be used in practical design.

Rozvany and Hampson and Brotchie and Russell developed an elastic approach for the optimum design of prestressed flat plates. Both investigators arrived at the same results, the principal difference being that Rozvany and Hampson used load balancing whereas Brotchie and Russell used moment balancing.

Starting in the early 1960's extensive research on prestressed flat plates was performed at the division of Building Research, Commonwealth Scientific and Industrial Research Organization, Australia. Both draped and straight cables were used. In addition, the thickness of the slab was changed and varying ratios of column to middle strip moments in the different panels were used. The experiments showed that deformation rather than strength was the important criterion in design.

In 1964, Candy[10] developed a procedure for designing flat plates using load balancing method plus ACI 318-63 ultimate strength provisions. Candy also advocated using a column strip of width L/4 to L/3, rather than the customary L/2 width.

Ellen developed a rigorous ultimate load balancing method for designing prestressed flat plates. In 1966, Power devised a practical approach using load balancing in conjunction with yield line theory. Since the percentage of steel in solid slabs is relatively low, this permits the formation of plastic hinges. Power listed four design criteria that must be satisfied, namely:

1. Strength based on an ultimate load basis
2. Camber
3. Deflection
4. Crack resistance

The trend towards high-rise buildings and the commercial availability of high strength, lightweight concrete refocused attention on flat plates in the United States.

In 1967, Grow and Vanderbilt[13] conducted an investigation into the shear strength of 10 post-tensioned light-weight slabs using expanded shale aggregate. From this study a useful formula evolved for checking the shear strength of lightweight prestressed slabs at columns. Subsequently laboratory tests have shown conclusively that structured lightweight concrete has adequate strength and superior fire resistive properties.

In 1968, Wang[14] proposed a method for designing prestressed flat plates using working stresses. However the method was unduly complicated and, furthermore, lacked any mention of ultimate strength check.

Recently, Riley[15] has shown that secondary effects caused by reversed tendon curvature can be eliminated if the tendon can be located to proper profile. However, this technique would involve using a mechanical device to change the natural slope of the tendon at its inflection point. Tendon reversal was assumed to occur at one-tenth the length of the span.

In late 1968, Parme[16] made a rigorous elastic analysis of the distribution of moments and direct forces induced by prestressing flat plates. He also included a set of useful design tables for finding prestressing moments.

Meanwhile in Australia, Rozvany and Woods emphasized the need for giving unbonded tendons a minimum level of average concrete prestress in the event of high live loads or earthquake motions. However in a subsequent discussion Bondy felt that introducing too high a level of average prestress would cause excessive shortening and camber problems. He said that a better solution would be to add bonded unprestressed reinforcement.

ACI-ASCE Committee 423[17] has given a comprehensive report on design recommendations for concrete members prestressed with unbonded tendons. Much of this report is directly applicable to post-tensioned flat plates.

Today, the majority of prestressed flat plate designs are based on some form of load balancing plus service load and ultimate strength checks. In

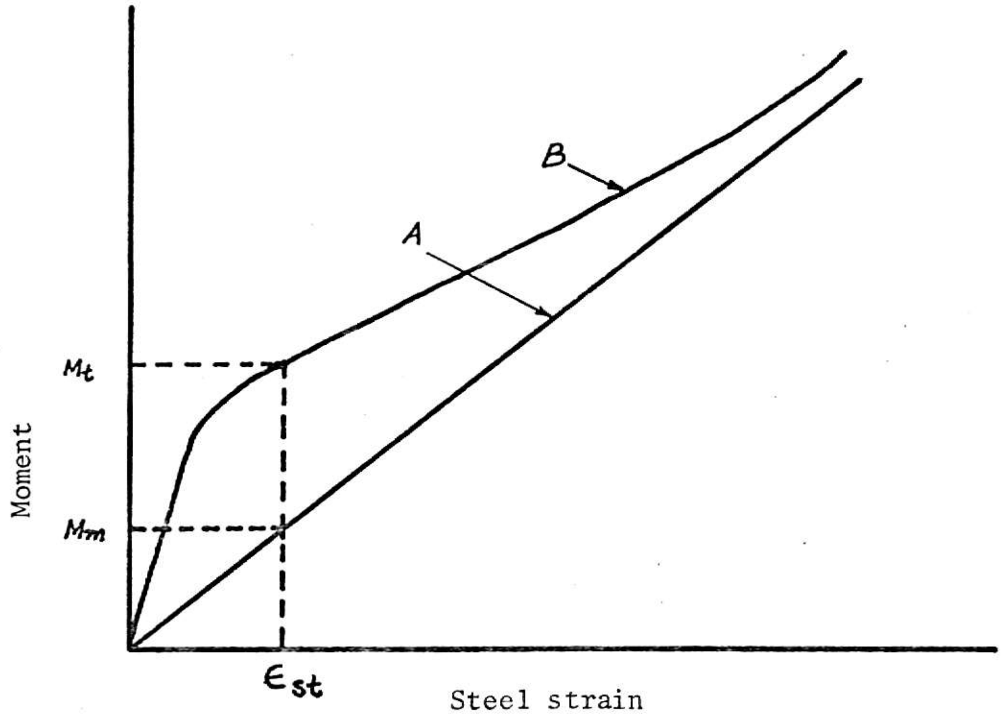


FIG. 3. RELATIONSHIP BETWEEN BENDING MOMENT AND STEEL STRAIN.

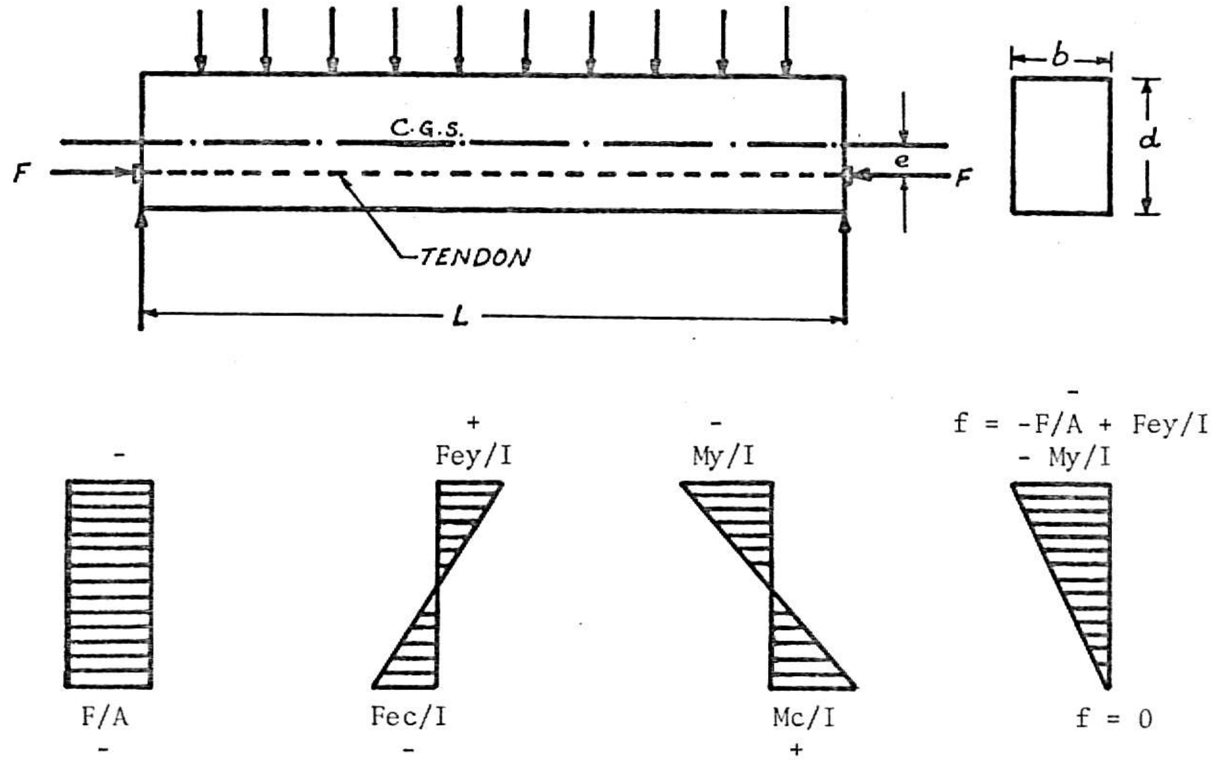


FIG. 4. ELASTIC DESIGN OF PRESTRESSED CONCRETE.

cases where ultimate strength checks are not satisfied, it is general practice to furnish unprestressed bonded reinforcement.

THEORY

Similar to the conventional concrete structures, the design of prestressed concrete has first been based on Elastic theory. In this concept concrete is visualized as being transformed by prestressing into an elastic material, hence the tensile stresses due to the external load are counteracted by the compressive stresses due to the prestress and a set of formulas is derived, taking the shape as shown in Fig. 4.

Then a second concept was developed based on ultimate strength. In this concept prestressed concrete is seen to behave in a manner similar to reinforced concrete. Thus the formula is derived taking the shape

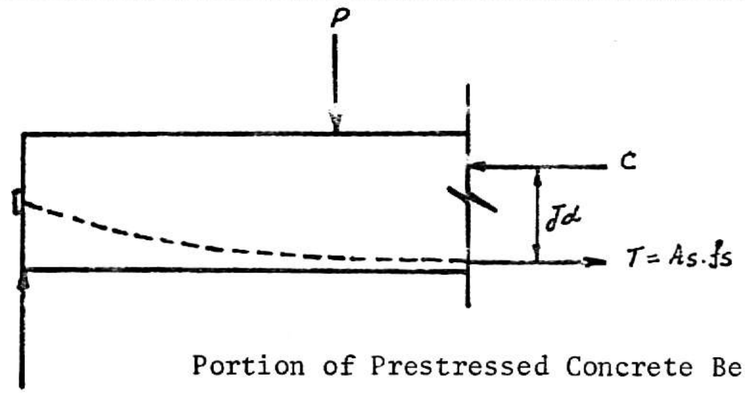
$$M = A_s f_s jd \tag{13}$$

as shown in Fig. 5.

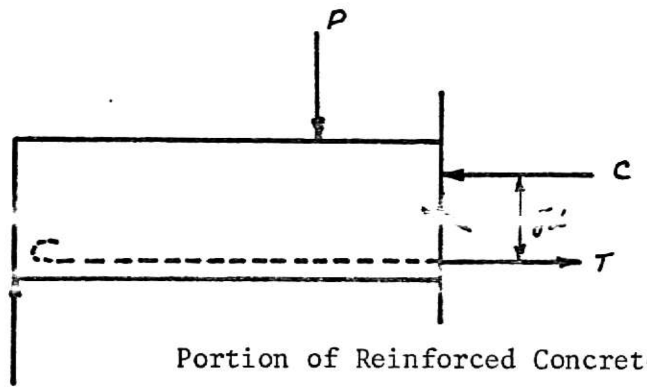
Professor T. Y. Lin[4] introduces a third concept which sees prestressing as primarily an attempt to balance a portion of the load of the structure. This is called load balancing concept.

As far as statically determined structures are concerned the advantage of this method over the two previous methods is not significant, but when dealing with statically indeterminate systems including flat plates and certain thin shells, this load-balancing method offers advantages both in calculating and in visualizing.

According to this Load-Balancing Method, prestressing balances a certain portion of the gravity loads so that flexural members, such as slabs, beams and girders will not be subjected to bending stresses under a given loading condition. This enables the transformation of a flexural member into a member under direct stress and thus greatly simplifies both the design and



Portion of Prestressed Concrete Beam



Portion of Reinforced Concrete Beam

FIG. 5. ULTIMATE DESIGN OF PRESTRESSED CONCRETE.

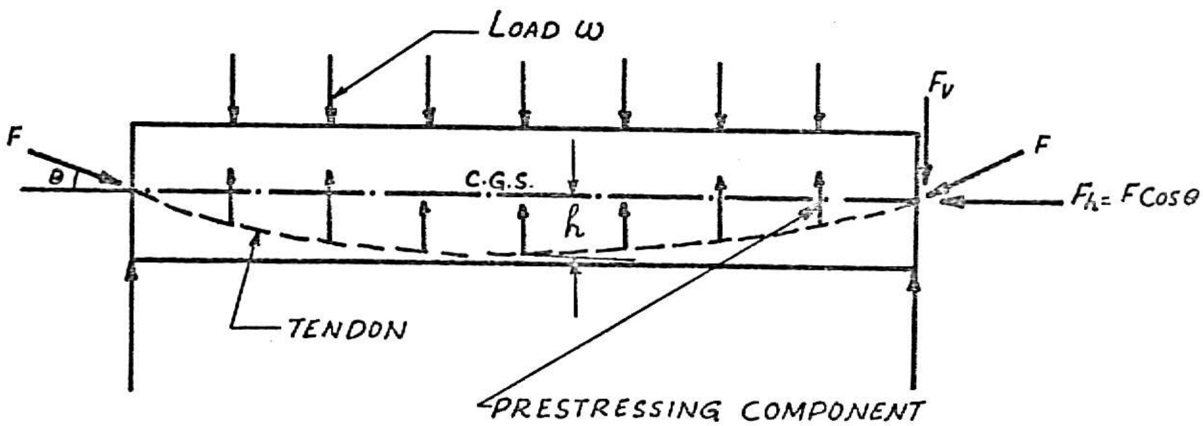


FIG. 6. LOAD BALANCING DESIGN OF PRESTRESSED CONCRETE.

analysis of otherwise complicated structures.

For example, consider a simple beam prestressed with a parabolic tendon as shown in Fig. 6.

F = prestressing force

L = span length

h = sag of parabola

Thus upward uniform force is given by

$$w = \frac{8Fh}{L^2} \tag{14}$$

Thus, for a given downward uniform load 'w,' the transverse load on the beam is balanced, and the beam is subjected only to the axial force F, which produces uniform stress in concrete, $f = \frac{F}{A}$. The change in stresses from this balanced condition can easily be computed by the ordinary formulas in mechanics, $f = \frac{MC}{I}$ (15)

To properly compare the load balancing method with the elastic stress or the ultimate strength method, it will be desirable to briefly examine the life history of a prestressed concrete member under flexure, Fig. 7. While this figure is intended to describe the load-deflection relationship of a member, such as a simple beam, it also applies to the moment-curvature relationship of a section of a member. It will be noted that under static loadings, there are several critical points, as follows:

1. The point of no deflection which usually indicates a rectangular stress block across all sections of a beam.
2. The point of no tension which indicates a triangular stress block with zero stress at either the top or bottom fiber.
3. The point of cracking which generally occurs when the extreme fiber is stressed to the modulus of rupture.

4. The point of yielding at which the steel is stressed beyond its yield point so that complete recovery will not be obtained.

5. The ultimate load which represents the load carried by the member at failure.

On the left of Fig. 7 are shown the various loading conditions to which a beam is subjected, as follows:

1. Girder load = GL

2. Total dead load = DL

3. The working load, made up of dead plus live load = DL + LL

4. A safety factor K_1 applied to obtain the yield point load = K_1(DL + LL)

5. Another safety factor K_2 applied to obtain the ultimate load = K_2(DL + LL)

The conventional elastic design actually consists of matching the (DL + LL) with the point of "no tension" (or some allowable tension) on the beam. Design by ultimate strength consists of matching the K_2(DL + LL) with the ultimate strength of the beam. Design by load balancing consists of matching the DL + K_3LL (where K_3 is zero or much less than one) with the point of no deflection.

Applied Loadings	Stages of Beam Behaviour
DL + K_3LL	No deflection
DL + LL	No tension
K_2(DL + LL)	Ultimate

It is also noted that, regardless of which method is used in the design, it is common practice to check for the behaviour of the beam at the other stages.

Which method is the best to follow depends on the circumstances. Generally it is desirable to choose one which will control the proportioning of the member. If it is not certain that the other requirements will be met automatically, analysis for these other critical stages will be made and modifications of the design may be effected. Since the balance-load point is often indicative of the behaviour during the greater portion of the life span of a prestressed structure, it could deserve more consideration than either the working load or the ultimate load.

A further advantage of the load-balancing method is the convenience in the computation of deflections. Since the loading under which there will be no deflection is already known, the net deflection produced under any load (up to the point of cracking) is simply computed by treating the loading differential acting on an elastic beam. If the effective prestress balances the sustained loading, the beam will remain level regardless of the modulus of elasticity or the flexural creep of concrete.

The two-dimensional load balancing method or the beam method can be applied to the analysis of flat plates.

a. The two-dimensional load balancing method: The two-dimensional load balancing differs from linear load balancing for beams and columns in that the transverse component of the tendons in one direction either adds to or subtracts from that component in the other direction. Thus, the prestress design in the two directions is closely related one to the other. However the basic principle of load balancing still holds and the main aim of the design is to balance a given loading so that the entire structure will possess uniform stress distribution in each direction and will have no deflection or camber under this loading.

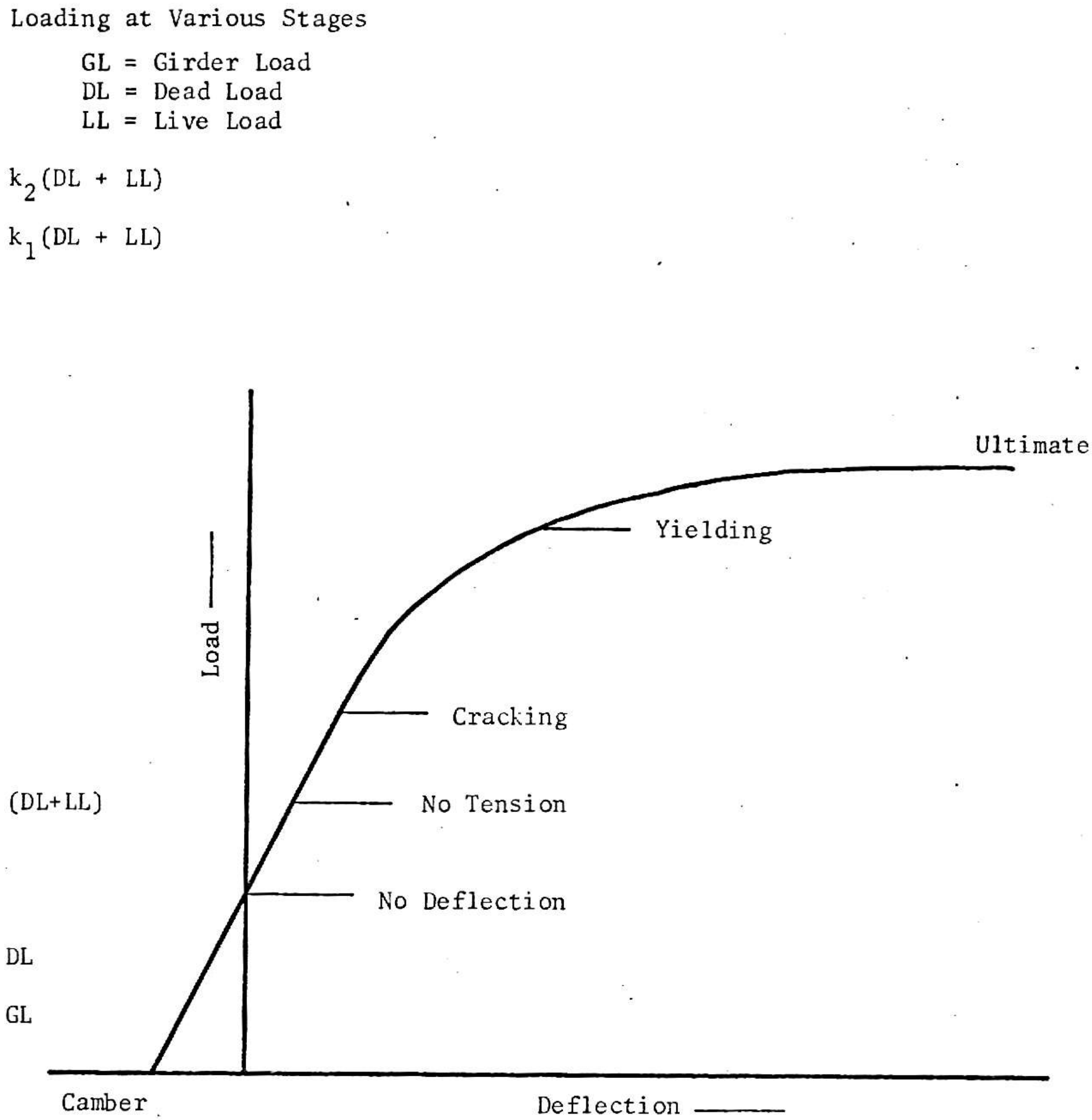


FIG. 7. LIFE HISTORY OF PRESTRESSED MEMBERS UNDER FLEXURE.

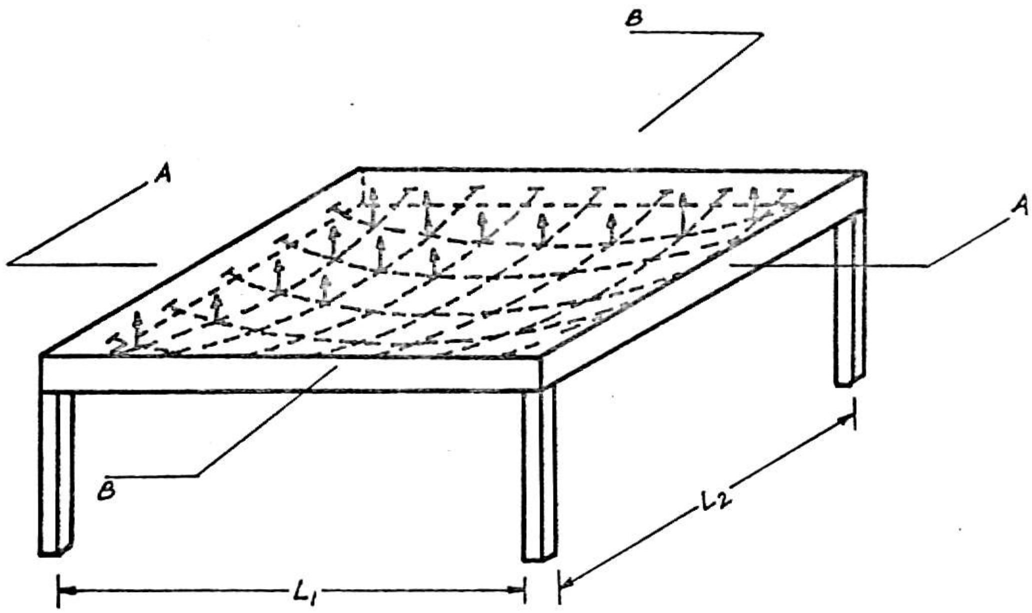
As an example of two-dimensional load balancing, let us consider a two-way slab simply supported on four walls, Fig. 8. The cables in both directions exert an upward force on the slab and if the sum of the upward components balance the downward load w, then we have a balanced design. Thus if F_1 and F_2 are the prestressing forces in two directions per foot width of slab, we have

$$\frac{8F_1h_1}{L_1^2} + \frac{8F_2h_2}{L_2^2} = w \qquad (16)$$

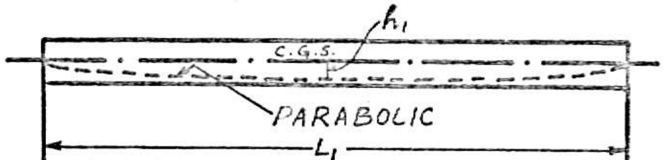
Note that many combinations of F_1 and F_2 will satisfy the above equation. The most economical design is to carry the load only in the short direction (or to carry 0.50 w in each direction in case of a square panel). Practical considerations might suggest different distribution. For example, if both directions are properly prestressed, it is possible to obtain a crack-free slab.

Under the action of F_1, F_2 and w, the entire slab has a uniform stress distribution in each direction equal to F_1/t and F_2/t respectively, where t is the slab thickness. Any change in loading from the balanced amount of w can be analyzed by the elastic theory for slabs.

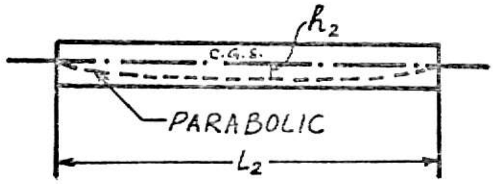
The same principle of load balancing for a slab described above will apply to a grid system if at each intersection the load is balanced by the net upward component of the prestress. Fig. 9 shows a grid system with cables running in two directions x and y producing vertical components V_x and V_y. Near the center of the panel, both V_x and V_y act upward. Near the center of the column line A-A, V_x is acting upward is partly counteracted by V_y acting downward. Over the column both V_x and V_y act downwards.



Isometric View of Slab and Support



Section A-A



Section B-B

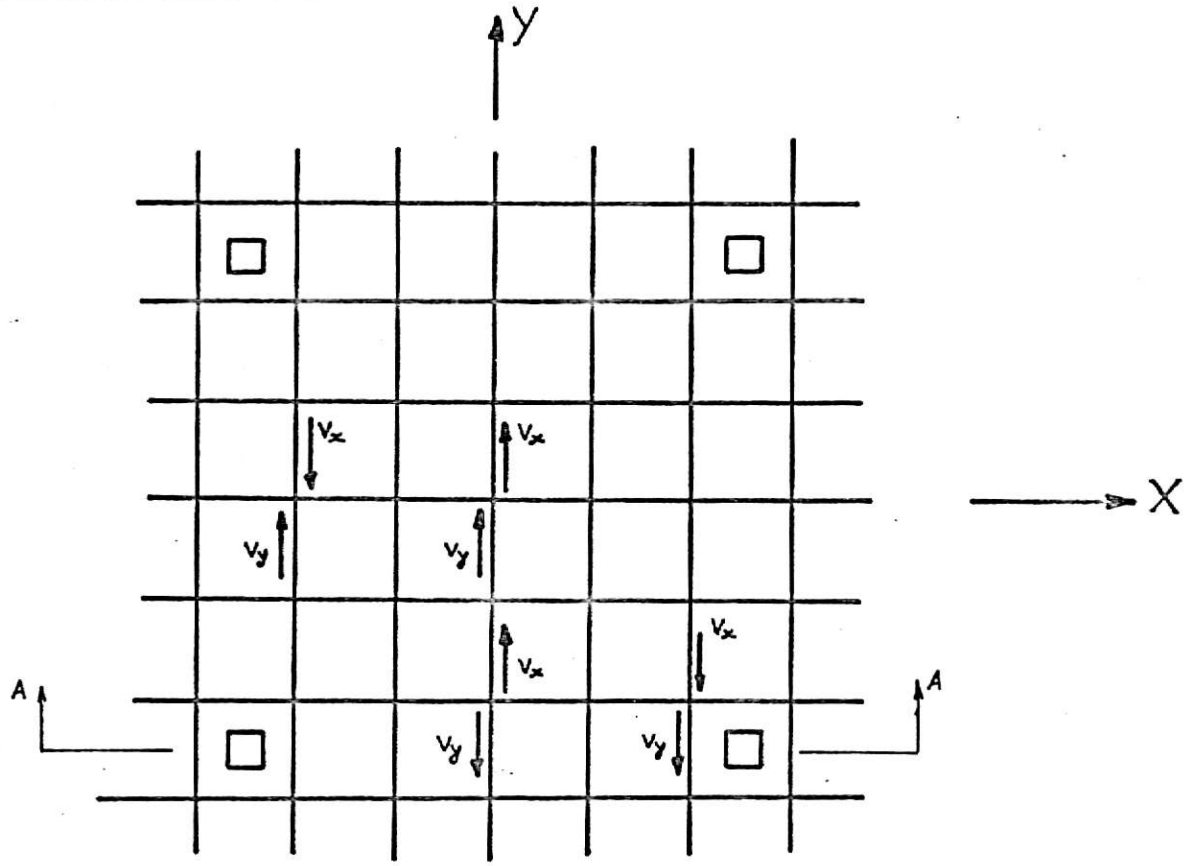
FIG. 8. LOAD BALANCING FOR TWO WAY SLAB.

In the analysis of flat plates, while the method of load balancing for a grid system would be found convenient for certain cases, the beam method probably offers the simplest solution, especially when combined with the concept of load balancing for continuous beams. This beam method assumes continuous knife edge supports along one direction when analysis is being made for moment in the other direction.

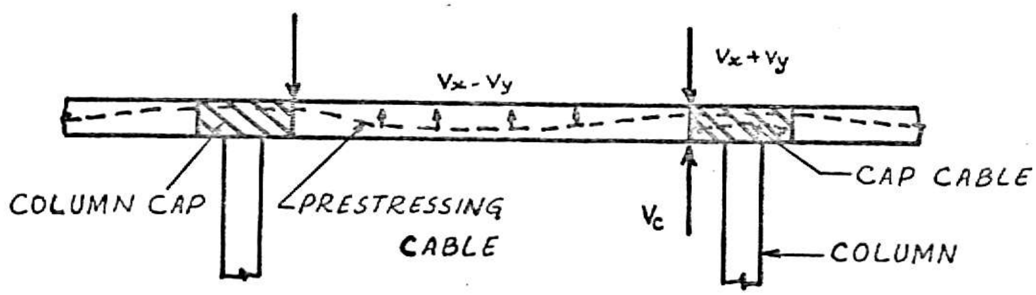
By analyzing a continuous flat plate as a continuous beam, the total moment across any section due to loading and the average position of center line under prestressing can be obtained. But the distribution of the total moment and the variation of the position of the center line along the width of slab still remains to be determined. For flat plates 25% of the total moment is carried by the middle strip and 75% by the column strip--approximately. This can also be partly explained by balanced load concept since at midspan of a middle strip, the cables from both directions act downward, while at midspan of a column strip one set of cables acts upward with the other set acting downward.

Using this concept of load-balancing, an important question arises: what should be the loading to be balanced by the prestressing? The answer to this question may not be simple. As a starting point, it is often assumed that the dead load of the structure or element is completely balanced by the effective prestress. This would mean that a slight amount of camber may exist under initial prestress. In the course of time, when all the losses of prestress have taken place, the structure or element would come back to a level position.

Although it seems logical to balance all the dead load, such balancing may require too much prestress. Since a certain amount of deflection is



Plan of Slab



Section A-A

FIG. 9. LOAD BALANCING IN A GRID SYSTEM.

always permitted for a nonprestressed structure under dead load, it is reasonable to also permit a limited amount of deflection if it does not become objectionable. However, there is a greater tendency in prestressed structures to increase their deflections as a result of creep and shrinkage. Hence the deflection should be limited to a smaller value at the beginning.

When the live load to be carried by the structure is high compared to its dead load, it may be necessary to balance some of the live load as well as the dead load. One interesting approach is to balance the dead load plus 1/2 the live load. If this is done the structure will be subjected to no bending when 1/2 of the live load is acting. Then, it is only necessary to design for 1/2 live load acting up when no live load exists and for 1/2 live load acting down when full live load is on the structure. This idea of balancing the dead load plus 1/2 live load, while theoretically interesting, could result in excessive camber if the live load consists essentially of transient load. Furthermore, it may not result in an economical design.

The load balancing method can be achieved with considerable accuracy because both the gravity load and the prestressing force can often be predicted with precision.

Depending on the accuracy desired in the control of camber and deflection, the amount of loading to be balanced must be chosen. If the limits of error can be estimated and if the significance of deflection or camber control can be assessed, it will not be difficult to design the member so as to possess the desired behaviour. In general the engineer should exercise his judgement when choosing the proper amount of loading to be balanced by prestressing.

In addition to load balancing method, a lot of work has been done by other engineers in the design and analysis of prestressed flat slabs.

In 1963, "Kolbjorn Saether"[7] published a paper in which he applied a "Structural Membrane Theory" to the analysis of prestressed flat slabs.

In its basic form the S.M. theory applies to certain types of thin shell structures. As a system of forces and moments in equilibrium, however, this theory may be transformed from one applying to uniformly thick flat slabs with curved tendons.

The derivations in this paper will show how the column areas stand out from the other areas of a flat slab and how it is subjected to special conditions both as far as the loading as well as the structural behaviour is concerned. In the opinion of the author, any major progress in the design of flat plates will depend on how far and how soon this area will be completely tested, analyzed and understood.

To explain how the two above mentioned systems and methods can be combined in the design of a flat plate, it is necessary to study some of the characteristics of the structural membrane.

The basic S-M shape consists of 2nd degree surface such as hyperboloids and parabolic domes. Each of these areas is subjected to uniform horizontal thrusts in their principal directions. Under this stress pattern the S-M surface is furnished with a uniform upward acting load bearing capacity. The only area that does not express itself as a 2nd degree surface is that next to a column which consists of a circular or elliptical, logarithmical funnel.

One way of loading the column area in a flat slab could be obtained by running the prestressing cables horizontally across the top of the prestressed slab and bending these cables down along the edges of the area. This would render a downward acting line load along the ringbeam of close to uniform intensity.

More consistent with prestressed design, however, is a layout where the tendons are uniformly deflected into concave parabolic curves within the column area. If the tendons are subjected to a uniform tension and spaced uniformly across the column area in both directions they provide a uniformly distributed downward acting load. By giving the area a rectangular instead of an elliptical outline, it is possible to make all cable profiles in each direction identical, Fig. 10.

If the uniform downward acting reactions from each set of tendons are denoted with "P_1" and "P_2" respectively and the extent of area in the two directions by "b_1" and "b_2" we may write the following equations

$$P = (p + p_1 + p_2)b_1b_2 \tag{17}$$

where "p" is the uniform load on the slab to be balanced by the prestressing tendons, and "P" the total balanced load within the Bay. Also

$$P = pL_1L_2 \tag{18}$$

where L_1 and L_2 are the spans in the two directions.

Between the column areas the prestressing tendons must be deflected upward into complex parabolas due to continuity requirements, Fig. 11.

These areas are in the S-M shape referred to as hyperbolic parboloids. In flat plates they may be identified as "Middle areas," Fig. 12. Assuming common tangents at the inflection points and a constant horizontal pull in the tendons it may readily be proved that the total upward reaction in this part of the cables equals that of the downward acting reactions from the same cable within the column area, Fig. 13, with the spacing between cables expressed as

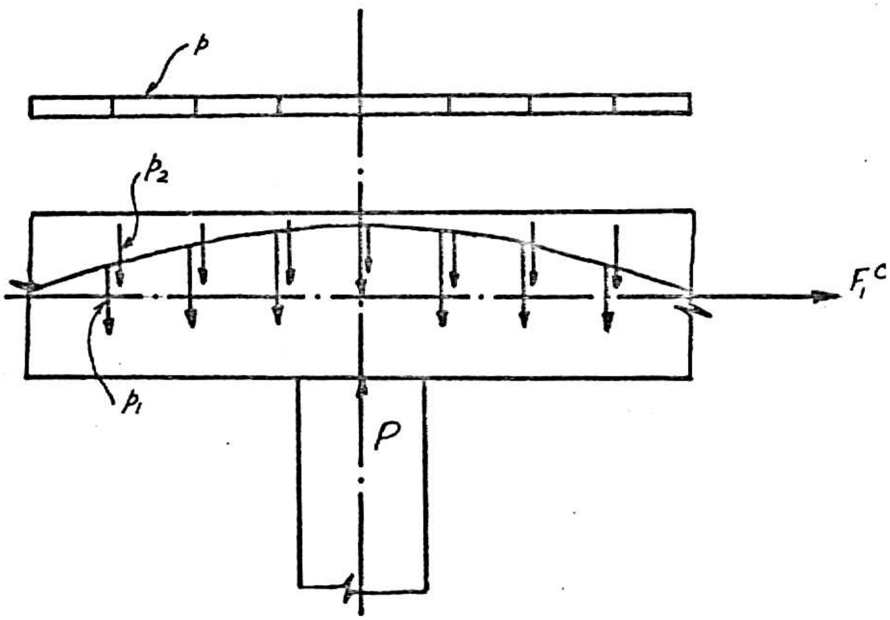


FIG. 10. ARRANGEMENT OF TENDONS IN THE COLUMN AREA.

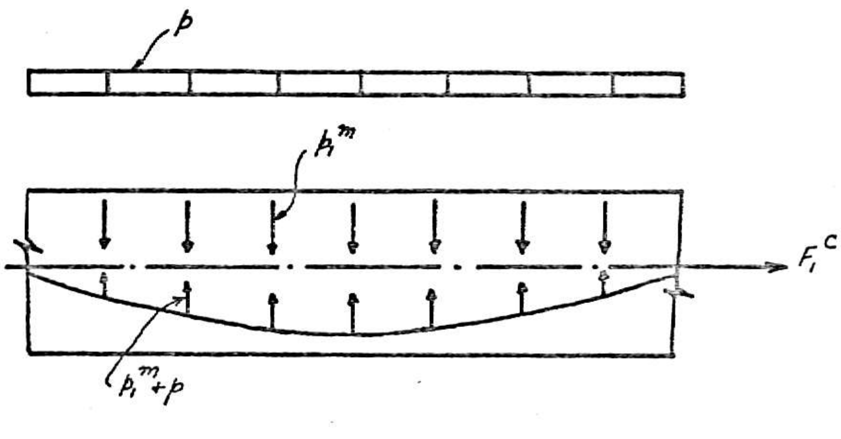


FIG. 11. ARRANGEMENT OF THE TENDONS IN THE MIDDLE AREA.

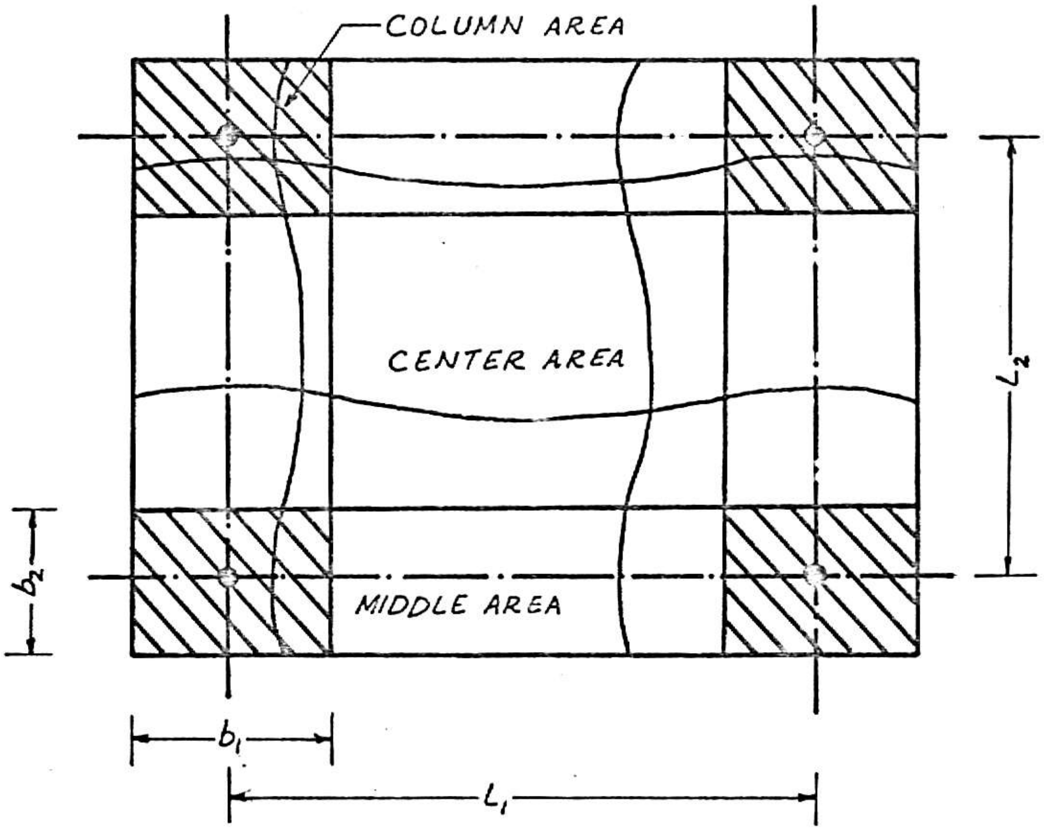


FIG. 12. RELATIVE ARRANGEMENT OF THREE TYPES OF AREA IN A FLAT SLAB.

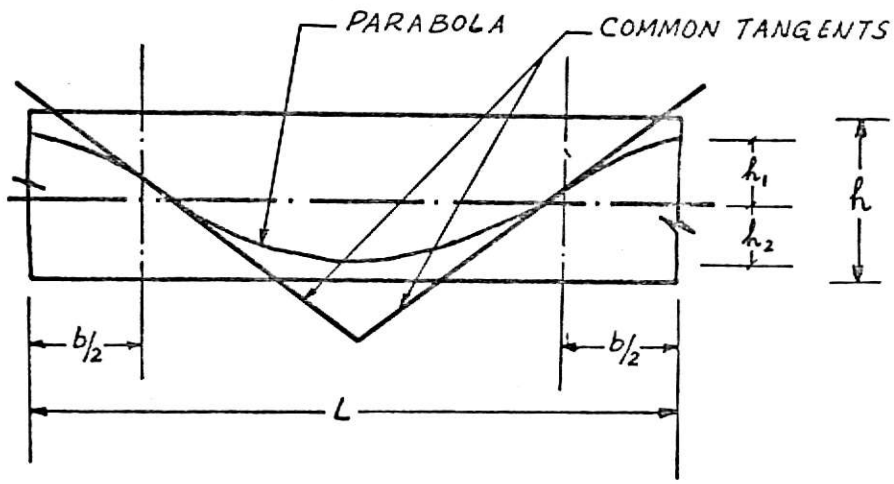


FIG. 13. GEOMETRICAL PROPERTIES OF THE TENDONS.

$$p + p_1^{\,m} = p \frac{b_1}{L_1 - b_1} \tag{19}$$

where $(p + p_1^{\,m})$ is the per square foot equivalent force of the uniform upward acting reactions along the tendons of this load, the part "p" is the necessary force to balance the load on the slab. The part $(p_1^{\,m})$ constitutes an excess capacity within this area. To balance the forces and maintain equilibrium it is therefore required to introduce another set of cables running normal to the first set. By deflecting the latter set downward in a concave parabolic curve, over the set running perpendicularly along the column centerline and by giving the cables a constant horizontal pull, it is possible to create a uniform downward reaction of the magnitude $(p_1^{\,m})$. This balances the force in this area, see Fig. 11.

If the tendons running in this direction are shaped as parabolas with common tangents at the inflection points, and constant spacing and constant horizontal thrust is assumed, it may easily be proved that the per square foot equivalent uniform upward acting reaction is

$$p_1^{\,C} = p_1^{\,m} \frac{b_2}{L_2 - b_2} \tag{20}$$

see Fig. 11.

Similarly from the cables in the other direction

$$p_2^{\,C} = p_2^{\,m} \frac{b_1}{L_1 - b_1} \tag{21}$$

where $p_1^{\,C}$ and $p_2^{\,C}$ are the per square foot tendon reactions in the mid span area. The two sets of tendons in this center area consist of convex parabolas and both furnish upward acting reactions $p_1^{\,C}$ and $p_2^{\,C}$. These must be balanced by the load on the slab "p". This condition may be written as

$$p_1^C + p_2^C = p \tag{22}$$

One important feature of this last equation is the undetermined ratio between p_1^C and p_2^C. This ratio

$$p_1^C/p_2^C = n \tag{23}$$

may be chosen by the designer and may vary from $n = 0$ to $n = \infty$. It is also important to recognise that as soon as this ratio has been selected, the entire load distribution follows automatically. That is, the much discussed ratio between column strip and middle strip is determined from the equations 17 to 22 and not by estimated ratios.

The total load in the direction of L_2 across the bay L_1 is

$$\begin{aligned} W_{L_1} &= p_1^C(L_1 - b_1) + (p_2^m + p)b_1 \\ &= p_2^C(L_1 - b_1) + [p_1^C(\frac{L_1}{b_1} - 1) + p]b_1 \\ &= p_2^C(L_1 - b_1) + p_1^C(L_1 - b_1) + pb_1 \\ &= p(L_1 - b_1) + pb_1 \\ &= pL_1 \end{aligned} \tag{24}$$

Similarly the same is found to be the case in the other direction

$$W_{L_2} = pL_2 \tag{25}$$

This proves that the entire load must be carried in each direction and not as at one time assumed that only part of the load had to be carried in the respective directions.

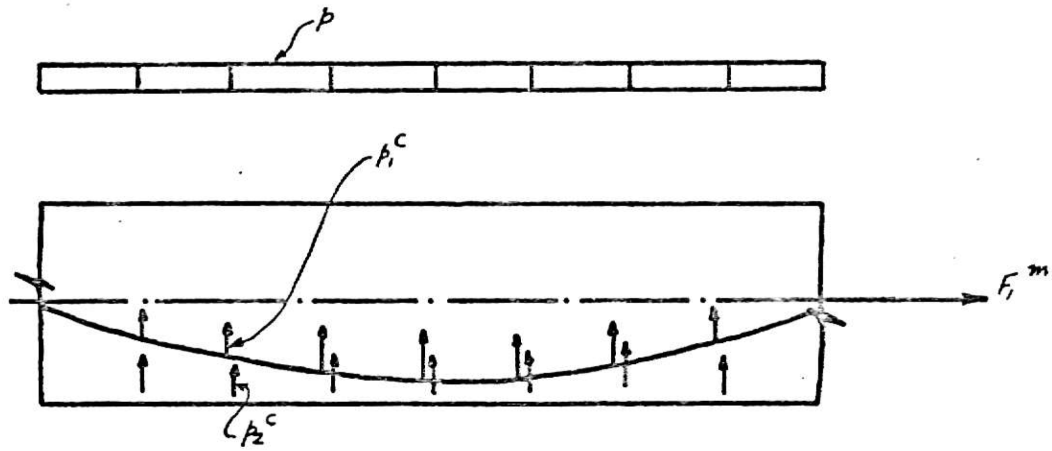


FIG. 14. ARRANGEMENT OF TENDONS IN THE CENTER AREA.

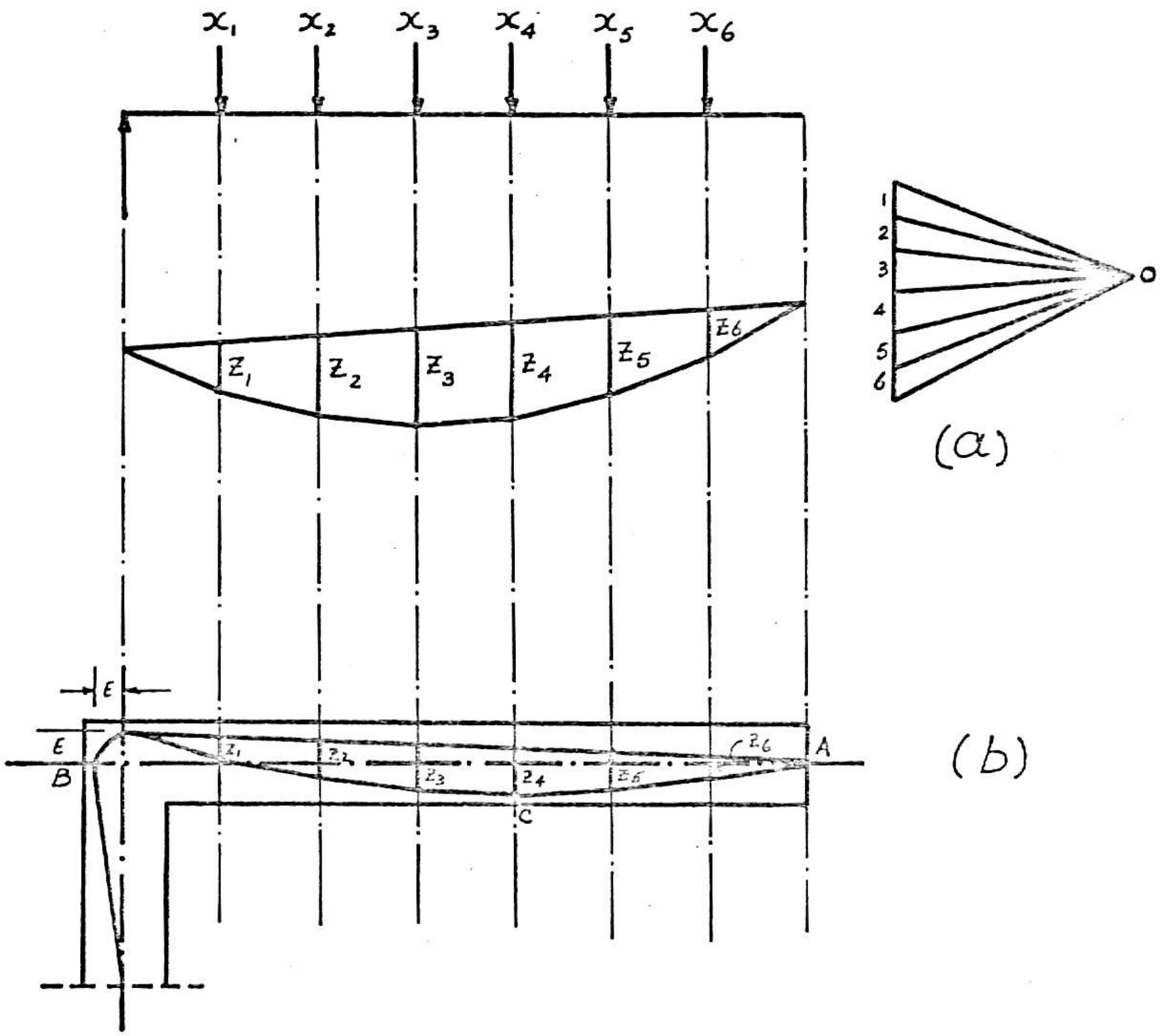


FIG. 15. TENDON CURVE FOR GIVEN LOADING (a) INITIAL (b) FINAL.

From equations 17 to 25 it is readily established that as soon as the ratio $p_1^C/p_2^C = n$ has been chosen and the width 'b_1' and 'b_2' established from the investigations of the column areas, the load carried by the various areas are completely determined and so also the ratio of the load carried by column strip and middle strip.

This theory, however, was not fully developed to be used in practical design, but it does suggest some modifications and further work to be done in the analysis of flat plates, because they are highly indetermined structures.

G. I. N. Rozvany and A. J. K. Hampson[8] together developed an elastic approach for the "optimum design of prestressed flat plates," using load balancing method.

The following assumptions are made:

1. The curvature of the tendons is small and the tendons exert vertical forces only on slab.
2. The friction between the tendons and ducts is negligible for purpose of analysis.
3. The tendons are unbonded until the full design load is developed.

Now if the spacing of the tendons approaches zero, it follows from considerations of equilibrium that

$$\frac{q(x,y)}{s} = \frac{d^2 z_x(x)}{dx^2} + \frac{d^2 z_y(y)}{dy^2} \tag{26}$$

where q(x,y) is the intensity of external loading, 's' is the horizontal component of the cable forces, z_x and z_y are the ordinates of the tendons in the x and y directions.

It will be seen that by replacing the Lagrange equation with Equ. (26)

a. The degree of the differential equation has been reduced from four to two.

b. The differential equation is not partial any more.

c. The tendon ordinates are those of a funicular polygon for the portion of external loading which is carried by the particular tendon.

Assuming a finite number of tendons, the load at any crossing point of two tendons should be Q_{mn} and the load carried by the tendons should be X_{mn} and Y_{mn} respectively. One of these two is arbitrary, the other can be calculated from the equation

$$Q_{mn} = X_{mn} + Y_{mn} \tag{27}$$

<u>Procedure for Design:</u>

The design procedure for flat plates consists of the following operations

1. Assume an arbitrary distribution at each tendon crossing point (X_{mn}, Y_{mn}).
2. Determine the funicular polygon for these forces for each tendon. Assuming unit force in the tendons at first, the funicular polygon will be identical to a bending moment diagram of a simply-supported beam or a cantilever beam as the case may be. The polygon will be called initial tendon curve (Fig. 15a).
3. Place the curve in the plate, moving it vertically and rotating it in such a manner that the anchor points occur at mid-depth of the slab, (Point A in Fig. 15b).
4. Along continuous edges, the eccentricity E is identical for adjoining members (Point B, Fig. 15b). This eccentricity is arbitrary and

it is economical to place the tendons as close to the top of the slab as practicable.

5. The lowest point of the tendons (Point C, Fig. 15b) determines the soffit.

6. Compressive stresses in the concrete due to the tendon anchor reactions would be

$$\frac{S_1}{ta_1} \text{ and } \frac{S_2}{ta_2} \tag{28}$$

where t is the slab thickness, 'a' is the cable spacing, and S_1 and S_2 are the cable forces in the two directions, respectively.

7. Ordinates of the tendons are to be measured from the mid-surface of the slab.

8. Ordinates of the tendons can be multiplied by a constant, provided the tendon force is divided by the same constant (Linear Transformation).

9. If the number of tendon crossing points is too great for convenient solution, divide the slab into equal strips in both x and y directions and substitute one equivalent tendon for each strip.

Criteria for Economy:

For two way prestressed plates, a designer has an infinite choice of distribution of loading between two directions of prestressing. One of the following criteria may be used to find an economical solution.

1. Assuming given depth, the volume of tendon is a minimum.
2. The depth D of the slab is a minimum.

The designer can choose the approach that is appropriate for each particular case.

First Criterion:

Using the first criterion, we find that the distribution of loading is most economical when ΣSL is a minimum. 'S' is the tendon force and 'L' is the tendon length. As we assumed given depth 'D' the final tendon force can be expressed as a function of the maximum tendon ordinate Z_{max} in the initial tendon curve. Using linear transformation, Z_{max} will be reduced to the effective tendon depth 'd', otherwise the tendon will not fit in the plate. Hence all ordinates of the tendon curve are multiplied by d/Z_{max} and from Rule 7, the tendon force is multiplied by the reciprocal value, Z_{max}/d. As we assume unit force in the initial tendon, the final tendon force will be $S = Z_{max}/d$. Multiply all terms by d,

$$d\Sigma SL = d\Sigma LZ_{max}/d = \Sigma Z_{max}L \tag{29}$$

When this expression is a minimum, the quantity of tendon is also minimum for the particular loading and boundary condition.

Second Criterion:

Using the second criterion, a load distribution has to be found where the largest Z_{max} in the slab is a minimum and from Rule 7 this will also mean S_{max} is a minimum.

Load Distribution Patterns:

The following load pattern can be used in two-way prestressing.

1. Load distribution in short direction only.
2. Load distribution in long direction only.
3. Equal distribution in two directions.
4. Transformed membrane method (uniformly stretched membrane).

5. Same as 4, but greater membrane stress in one direction.

6. Minimum tendon method (a) Minimum Volume (b) Minimum Force.

Among all the above load distribution patterns the authors favour the transformed membrane method for most cases and particularly for flat plates.

TRANSFORMED MEMBRANE METHOD (uniformly stretched membrane)

In this method it is assumed that at any tendon crossing, the initial tendon ordinates are identical for the two intersecting tendons

$$Z_x(x,y) = Z_y(x,y)$$

If the tendon spacing approaches zero, the tendon system becomes a uniformly stretched membrane, provided that the horizontal component of the tendon force is the same in each tendon. For this case, it is well known that $\Delta Z = q/S_1$

or

$$\frac{\partial^2 Z(x,y)}{\partial x^2} + \frac{\partial^2 Z(x,y)}{\partial y^2} = \frac{q(x,y)}{S} \qquad (30)$$

It follows from Equ. (30) that for finite cable spacing (Fig. 16)

$$4Aa - \sum_b^e Z = \frac{Qa}{Z} \qquad (31)$$

In Equ. (30) and (31), Za is the initial tendon ordinate at the crossing point a, where

$\sum_b^e Z$ = Sum of the initial tendon ordinates at the adjacent crossing points Z_b, Z_c, Z_d, Z_e.

Qa = Loading at point a.

S = Uniform tendon force, per unit width.

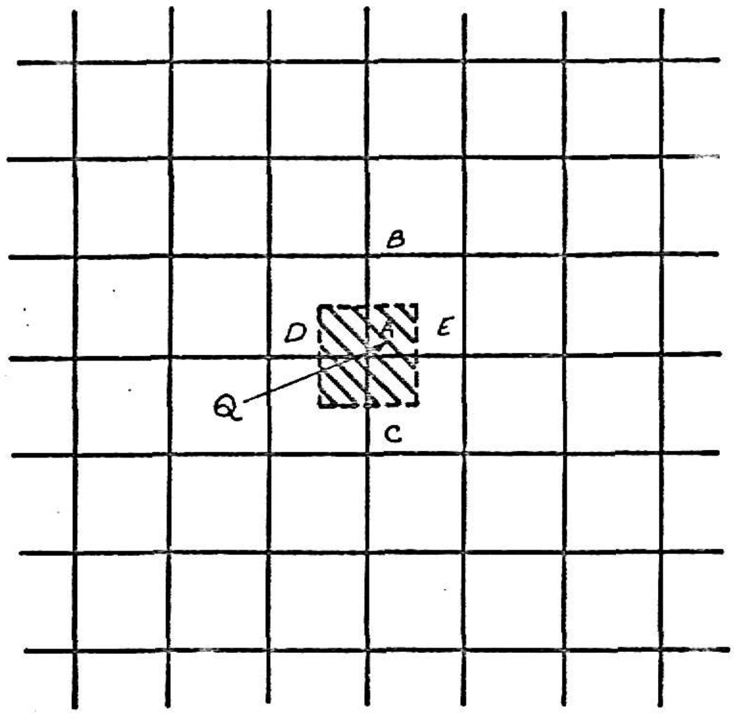


FIG. 16. TENDON GRID IN TRANSFORMED MEMBRANE METHOD.

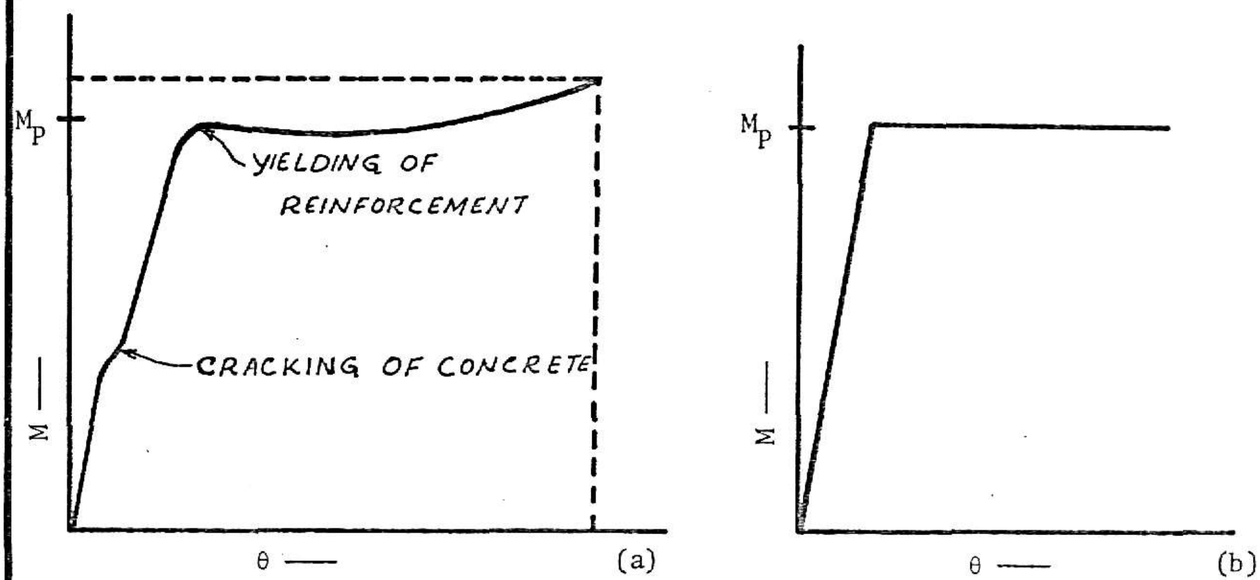


FIG. 17. MOMENT-CURVATURE RELATIONSHIP (a) ACTUAL (b) IDEALIZED.

The design procedure is the same as for balanced load concept, except that instead of operation 1 and 2 ordinates of the initial tendon curves are calculated from Equ. (31).

This method has the following advantages.

a. It is readily applicable to any loading and boundary conditions.

b. It does not require specific mathematical knowledge.

c. The equations are suitable for use in an electronic digital computer.

d. The results nearly satisfy both criteria of economy.

e. The method provides the heaviest tendons (close spacing) at sections where extra live load would cause the greatest elastic bending moments.

J. F. Brotchie and J. J. Russell,[19] together developed an optimum design of prestressed flat plates, using "Moment Balancing Method."

The author says that the method used by Lin was essentially the conventional one of trial and analysis. He proposed a direct design of the flat plate as follows.

A prestressed flat plate within a limited loading range has been found to behave essentially as an elastic plate. This behaviour may therefore be predicted by the equations for elastic behaviour. In the post-elastic range, the moment-curvature relationship is less well known but is assumed to be approximate for analysis, Fig. 17. That is, an elastic-plastic relationship is assumed throughout. With unbonded tendons, the ultimate or plastic moment MP is taken to be the point at which the internal moment reaches a maximum--either through plasticity in the concrete or the steel, by the internal moment arm reaching its maximum value, or from a combination of these factors. This maximum moment is assumed to be given by,

$$Mp = Pfsud^2 \left(1 - 0.59 \frac{Pfsu}{f'c}\right) \tag{32}$$

in which fsu is the stress in the tendons at maximum moments.

To determine the tendons completely, three behaviour criteria are introduced:

1. At ultimate load - maximum moment of resistance reached throughout the critical section, with minimum area of tendons.
2. At service loads and over - minimum deflection and minimum plastic deformation.
3. At sustained service loads - zero bending and deflection at all points.

From 1, the total number or area of tendons is determined, from 2, the spacing and profiles are obtained and from 3, the effective tendon tension is determined.

Design Procedure:

The tendons are merely distributed and tensioned to balance the simplified moments at sustained loads. For square or rectangular panels the procedure is as follows.

1. The plate is divided into panel strips in each direction.
2. The panel strip is considered as a continuous beam supported at the column center lines.
3. The beam is analyzed by conventional moment distribution assuming the column to have zero stiffness. This assumption is available since at zero bending no moment will be transformed to the columns. Negative moments are reduced by Pr_b/π to allow for column size.

4. The distance Z, of the tendon from the middle surface is made proportional to these adjusted moments M with the negative peaks rounded over a width somewhat greater than the columns (0.3L to 0.4L) to reduce friction losses, Fig. 18. The ordinate Z has the maximum value of half plate thickness less the minimum cover required.
5. The number n of tendons per unit width is made proportional to the distribution of critical negative moments across the panel width. The distribution varies with panel shape and is approximated in Fig. 19. For side length ratios greater than 2:1 the distribution for the 2:1 panel might be adopted.
6. The total number N of tendons per panel width is determined from the total ultimate moment at the critical sections or at a single section in the conventional way.
7. The effective tension (after losses) is determined to balance the external moments at the sustained loading, using the equation

$$To = \frac{M}{Nz} \tag{33}$$

8. The plate thickness is determined from the ultimate shear as in conventional design or from the allowable deflection under full live load.

<u>Further Simplification</u>:

As the columns are considered to have zero stiffness, the procedure above is simpler than conventional design. However one more simplification can be introduced. Moment distribution (step 3) may be eliminated and the moment in each panel is calculated as if it were a simply supported beam.

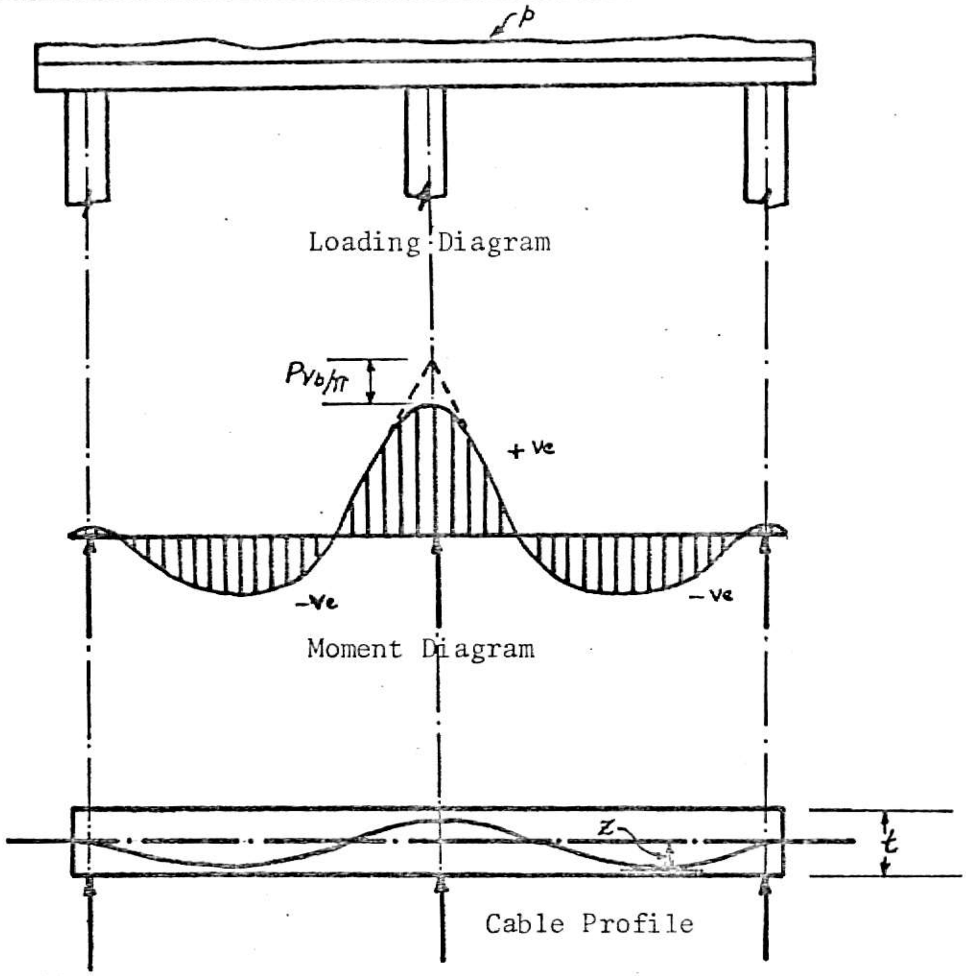


FIG. 18. LOADING, MOMENTS AND RESULTING TENDON PROFILE ALONG PANEL STRIP IN FLAT PLATE STRUCTURE.

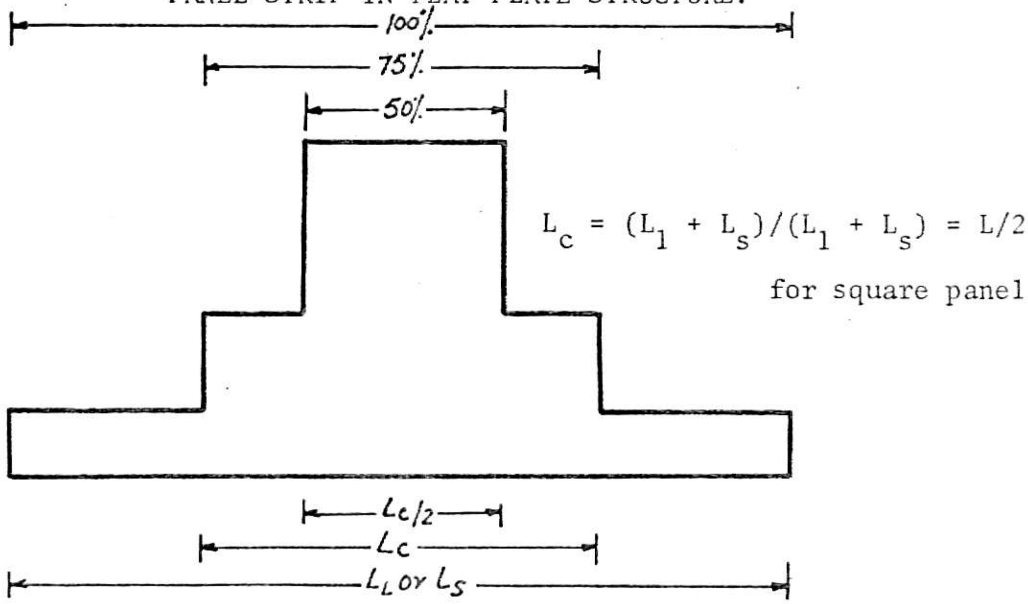


FIG. 19. OPTIMUM TENDON DISTRIBUTIONS ACROSS PANEL STRIP FOR SQUARE OR RECTANGULAR PANELS.

Thus if the tendons are distributed on the assumption that the panel is simply supported at each internal column center line, zero deflection again results. Similarly if any other degree of continuity between the panels is assumed, zero deflection is once more attained. This is because at zero deflection, the degree of continuity between the panels is not utilized, just as continuity with the columns is not required.

General Remarks:

The tendon pattern in an ideal, prestressed flat plate should satisfy the following criteria.

1. The ultimate load should be attained with a minimum number of tendons.
2. Deflection and cracking should be minimized at other loads.
3. Deflection should be zero at sustained loads.
4. In-plane stresses should be a minimum.
5. Tendon profiles should be identical, smooth curves.

The author further says that the condition of zero deflection or zero bending may be considered to be satisfied by one of the two approaches.

A. Load Balancing - in which the upward reaction of the tendon on the concrete directly balances the applied load.

B. Moment Balancing - in which the tendon produces internal moments in the concrete which just balance the external moments due to load.

In a beam these concepts are identical. In a two-way slab they are not. Moment balancing is considered herein, first because it is more general, allowing either horizontal or vertical curvature of the tendons, and second because it gives a unique, simple and optimum solution for any panel when vertical curvature only is considered.

Moment balancing provides optimum performance at overloads and allows this performance to be predicted. At loads above the sustained load the tendon is everywhere in the tensile zone and this, together with the condition that drape is a maximum, means that total tendon elongation is a practical maximum at every load. Hence the tension, and the internal moment in the tendon, are a maximum at each load, so that the net moment in the concrete is a minimum, and bending and deflection are further reduced. Furthermore, the ultimate stress in the tendon is a maximum, and actual ultimate load tends to exceed the design ultimate by a maximum amount.

Hence, optimization is found to occur at various levels of internal behaviour, in addition to those prescribed, and the resultant structure tends to be an optimum in various ways.

These are some of the theories put forward by different engineers in the analysis and design of prestressed flat plates.

DESIGN CONSIDERATIONS

Span Limitations:

For spans ranging up to 35 ft., a prestressed lightweight concrete flat plate system provides a functional and economical solution for low-cost high rise buildings. It should be noted that spans as short as 12 ft. have proved economical depending on the situation. For spans ranging between 35 ft. and 45 ft., a prestressed flat plate system can still be used but drop panels around the columns must be provided to withstand the high bending and shear stresses. A ribbed Waffle system has also worked well. When spans exceed 45 ft., a prestressed beam-Girder system, ribbed Waffle system, or some other system is found practical.

Span-Thickness Ratios:

ACI-ASCE Committee 423 recommends that for

> . . . prestressed slabs continuous over two or more spans in each direction, the span-thickness ratio should generally not exceed 42 for floors and 48 for roofs. These limits may be increased to 48 and 52 respectively, if calculations verify that both short and long term deflection, camber and vibration frequency and amplitude are not objectionable.

For practical purposes a ratio of 45 has been found to be very useful.

Most designers use a thumb-rule to determine the slab thickness. For the usual live loads and normal weight concrete the thickness t (in inches) can be determined from

$$t = \frac{12L}{45}$$

where L = The span length in feet.

Unfortunately, the above formula does not hold in case of very high live loads and for variations in density of concrete.

Average Compressive Prestress:

The average compressive stress, F/A is a good "indicator" of how the design can proceed. The average stress varies inversely with the slab thickness. It would seem, then, that to obtain the minimum thickness of slab, we should use the maximum average prestress. This course, however, does not always produce an economical design because a high average prestress also means a larger prestressing force. In addition, for stresses over 500 psi, there is the danger of excessive elastic shortening, shrinkage, and creep in the slabs. Again we do not want to have too little slab thickness because in long spans there is a sensation of "springiness" when walking on the floors and also a danger of undesirable vibrations. On the other hand, a lower limit of 200 psi is required to minimize cracking and produce a waterproof surface. Thus, an average of about 200 to 350 psi seems ideal for most practical situations.

Elastic Shortening, Shrinkage and Creep:

A precise calculation of shortening effect is difficult. However, if the average compressive stress is kept low, the elastic shortening will be very small with only shrinkage and creep (in that order) becoming important. The following approximate equations can be used to estimate the various shortening effects.

a. Elastic Shortening

$$\varepsilon_e = \frac{F}{AE}$$

where F/A is the average compressive stress and the modulus of elasticity E is calculated from

$$E = 33w^{3/2}\sqrt{f_c'}$$

in which w is the density of the concrete and f_c' is the compressive strength of concrete.

b. Shrinkage

For normal weight concrete,

$$\varepsilon_s = 0.0003$$

For lightweight concrete,

$$\varepsilon_s = 0.0005$$

c. Creep

ε_c may be estimated as twice the elastic shortening.

The total shortening in a slab of length L (in inches), is then given by:

$$\Delta = L(\varepsilon_e + \varepsilon_c + \varepsilon_s)$$

For a typical normal weight concrete slab, with low average compressive stress, the total elastic shortening can be expected to be about 3/4 in. per 100 ft. To minimize shrinkage cracks it is good practice to provide a nominal amount of unprestressed reinforcing steel in the top part of the slab over the columns.

Required Prestressing Force:

In prestressed flat plates the tendon profile is not usually concordant. The least prestressing force will be given when the available tendon drape is a maximum in the controlling span. For an interior span the minimum prestressing force F is given by

$$F = \frac{W_b L^2}{8h}$$

and for a cantilever

$$F = \frac{W_b L^2}{2h}$$

where L is the length of the span, W_b is the balanced superimposed load, and h is the tendon drape.

Since the prestressing force will be the same throughout each slab span, there will be one governing span. The tendon profiles of the remaining spans will then be adjusted accordingly.

Fire Resistance and Cover Requirement:

Adequate data exist that properly designed post-tensioned flat plates satisfy code requirements.

For fire protection purposes and for corrosion protection, building codes require a cover at the top and bottom of the slab. The exact cover requirement should be checked since it will depend on the local building code. Typically, for a 2-hour fire rating a cover requirement might be 1 in. cover at top and 1-1/2 in. at the bottom of the slab. If we use a 1/2 in. diameter tendon, the distance from the surface of the slab to center line would be

Δt support 1 + 1/4 = 1-1/4 in.

Δt midspan 1-1/2 + 1/4 = 1-3/4 in.

Consequently, the total available tendon drape in inches would be

$$t - (1\text{-}1/4 + 1\text{-}3/4) = t - 3$$

where t is the total thickness of the slab.

An economical design should use the largest possible tendon drape in order to minimize the prestressing force.

For structural lightweight concrete, most codes allow some cover reduction.

Allowable Working Stresses:

The stresses caused by the net unbalanced load (W_t - W_b), where W_t is the total superimposed load, must be checked for service load conditions.

For compression . . . . 0.45 f'_c

For tension:

Final . . . . 3 $\sqrt{f'_c}$

Initial . . . . 3 $\sqrt{f'_{ci}}$

where f'_c = design compressive strength of concrete (usually at 28 days).

f'_{ci} = compressive strength of concrete at time of initial prestress.

Two observations warrant mention:

(i) The compressive stress rarely governs; and

(ii) The actual allowable tensile stress according to ACI 318-63 is 6 $\sqrt{f'_c}$. However, the factor 6 is allowed when there is an equal amount of bonded unprestressed reinforcement.

Ultimate Strength:

It is imperative that the design be checked to make sure it satisfies ultimate strength requirements. More specifically, the calculated ultimate moment should satisfy Equ. (24-4) of ACI 318-63.

$$Mu = \phi A_s f_{su}(d - \frac{a}{2})$$

where

Mu = ultimate resisting moment

ϕ = capacity reduction factor (0.9)

A_s = area of prestressing tendons

f_{su} = calculated stress in prestressing steel at ultimate load

d = distance from extreme compression fiber to centroid of prestressing force

$a = A_s f_{su}/0.85\ f'_c b$

When the live load is fairly high, the furnished ultimate strength is often inadequate. Rather than increasing the prestressing force it is general practice to provide unprestressed reinforcement at the critical sections. This reinforcement should be provided over the columns in all cases. A suggested minimum amount is 0.002 times the area of the column strip each way for one-quarter the span.

Shear Strength:

In contrast to unprestressed flat slabs, there is, in prestressed flat plates, a large reserve strength to resist shear failure. However, to prevent any risk of punching shear failure it is advisable to place two or three of the tendons directly over the columns. In addition, most designers strengthen this critical shear area, as well as avoid shrinkage cracks, by providing unprestressed reinforcement over the columns, say 2 #6 bars each way.

There are several methods for determining the shear capacity of prestressed flat slabs--none in complete agreement. The critical section is usually taken one-half the slab thickness away from the face of the column.

For normal weight concrete, according to ACI 318-63, Equ. (26-13), the shear force shall not be taken less than

$$V_{cw} = b^1 d(3.5\ \sqrt{f'_c} + 0.3\ f_{pc}) + V_p$$

where

V_{cw} = shear force at diagonal cracking due to all loads, when such cracking is the result of excessive principal tension stresses in the web

b^1 = minimum width of the web of a flanged member

d = distance from extreme compression to centroid of the pre-stressing force

$f\acute{c}$ = compressive strength of concrete

f_{pc} = compressive stress in the concrete, after all prestress losses have occurred, at the centroid of the cross-section resisting the applied loads, or at the junction of the web and flange when the centroid lies in the flange

V_p = vertical component of the effective prestress force at the section considered.

Note that V_p is usually neglected.

For lightweight concrete, according to ACI 318-63, Equ. (26-13A), the shear force shall not be taken less than

$$V_{cw} = b^1 d[0.5\ F_{sp}\ \sqrt{f\acute{c}} + f_{pc}(0.2 + \frac{F_{sp}}{67})] + V_p$$

where F_{sp} is the ratio of plitting tensile strength to the square root of compressive strength. Again, V_p is usually neglected.

An alternative formula for lightweight concrete is given by Grow and Vanderbilt.

$$V_u = (360 + 0.30\ f_{ce})bd$$

where

V_u = ultimate shear force

f_{ce} = average effective concrete prestress immediately after post-tensioning

b = perimeter of column

d = effective depth of slab

Choice of Column Strip Width:

It is generally desirable to concentrate the load balancing tendons into a narrow column strip. However, it is often difficult to place prestressing tendons close together on column lines. Consequently, there is a practical lower limit to the width of the column strip. Ultimate strength requirements will place an upper limit on the width, because the greater the width, the greater the spacing and hence the smaller the ultimate capacity. Candy suggests using a column strip of width L/4 or L/3. He also assumed that the point of contraflexure (tendon reversal) occurred at these locations. In the U.S.A., the general practice is to use the same column strip width as for ordinary reinforced flat slab design, i.e. L/2. In any case, the choice of column strip width does not appear to be a critical factor.

Tendon Reversal:

The equations given under "Required Prestressing Force" are theoretically true only if the prestressing tendons meet at a point over the supports. In practice, however, the tendons will gradually bend over the supports, so that at some point near the ends, the tendon curvature will be reversed. Experience has shown that continuous prestressing tendons have a natural contraflexure point at about 0.12 of the unsupported span length. Koons and

Schlegel presented a discussion of the tendon reversal and provided charts to take account of this effect.

Riley suggested that if the slopes of the intersecting curves at the contraflexure point are assumed equal, secondary effects will still occur. However, by locating the contraflexure point on a theoretical parabola of length L, and by assuming a difference between the slopes of the intersecting parabolic curves at this point, secondary effects can be shown to be eliminated.

Most designers take cognizance of the fact that tendon reversal occurs, but feel that in practice the effect on design does not warrant the extra work. In any case, the ultimate strength is unaffected.

Tendon Load Distribution:

As in conventional reinforced slab design, it is now common practice to divide the panel into equal column and middle strips. However in contrast to reinforced slab design, in prestressed flat plate design it is necessary to use the same moment or load distribution percentage for both negative and positive moments. The percentage of moment distribution has been thoroughly researched in the United States of America. Probably, a 60 to 40 percent distribution, column strip to middle strip, has been most widely used, although a 65 to 35 percent and a 75 to 25 percent distribution have been used.

Saether suggested that column strips should have three times the tendon concentration of middle strips to satisfy the statical equation according to the membrane theory.

It appears that a precise tendon distribution is not critical. Tests indicate that the ultimate strength is controlled primarily by the total amount of tendons rather than by the tendon distribution.

ACI-ASCE Committee 423 suggests the following:

> For panels with length/width ratios not exceeding 1·33, the following approximate distribution may be used: simple spans, 55 to 60 percent of the tendons are placed in the column strip, with the remainder in the middle strip; continuous spans, 60 to 70 percent of the tendons are placed in the column strip. When length/width ratio exceeds 1·33, a moment analysis should be made to guide the distribution of tendons. For high values of this length/width ratio, only 50 percent of the tendons along the long direction should be placed in the column strip, while 100 percent of the tendons along the short direction may be placed in the column strip. Some tendons should be passed through the columns or at least around their edges.

Tendon Spacing:

There appears to be no rational method for designing the tendon spacing. Some authors suggest a uniform spacing of tendons while some designers varied the spacing parabolically over the bay. If uniform spacing is used a good rule-of-thumb is to make the tendon spacing six times the slab thickness. Again, as for tendon force distribution, it appears that a precise tendon spacing is noncritical so long as there are enough tendons located in the critical column area.

ACI-ASCE Committee 423 suggests that: "The maximum spacing of tendons in column strips should not exceed four times the slab thickness, nor 36 in., whichever is less. Maximum spacing of tendons in the middle strips should not exceed six times the slab thickness, nor 42 in., whichever is less."

High Live Loads and Earthquake Motions:

It is imperative that in the event of high overloads adequate provisions be made to ensure that ultimate flexural capacity governs rather than ultimate shear, as in the latter case collapse would be sudden and without warning. Thus, it would appear essential that yielding take place in the columns rather than the slab.

To avoid sudden collapse in the case of high overloads, Rozvany and Woods suggested that the average concrete prestress be made greater than the modulus of rupture and presented supporting experimental evidence for their theory. However, experience in the United States indicates that it is preferable to keep the average prestress low and to add unprestressed bonded reinforcement, thereby increasing the ultimate strength.

ACI-ASCE Committee 423 recommends that unbonded tendons subject to earthquake loads be able to withstand, without failure, a minimum of 50 cycles of loading corresponding to the following percentages of the minimum specified ultimate strength.

$$60 \pm \frac{2000}{L + 100}$$

where L is the length of the tendon to be used in the structure, in feet.

Bonded vs. Unbonded Tendons:

The overwhelming majority of prestressed flat plates in the United States are constructed using unbonded post-tensioning tendons. Nevertheless, there has been, for many years, dissenting opinion as to the merits of bonded vs. unbonded tendons. Early European practice favored bonded tendons. Even today, some overseas codes (notably in New Zealand) and a few highway agencies in the United States restrict the use of unbonded tendons. The following are apparently the major reasons.

a. Bonded tendons provide a greater ultimate strength than do the same amount of unbonded tendons.

b. Bonded tendons decrease the chance of total collapse in the event of a local failure.

Nevertheless, both the above objections can be overcome by the addition of calculated amounts of unprestressed bonded reinforcement to supplement the unbonded tendon design. The probability of corrosion occurring in either bonded or unbonded tendons appears to be statistically about equal and remote under current practices allowable by codes in the United States. Unbonded tendons are today carefully protected, wrapped, and greased to prevent corrosion.

Recently, a comprehensive investigation was conducted by Mattock, et al., on this precise subject of bonded vs. unbonded tendons, with the following major conclusions:

> Simple span and fully loaded continuous, unbonded, post-tensioned beams, containing additional unprestressed bonded reinforcement and designed according to the provisions of ACI 318-63, will have serviceability characteristics, ductility and strength, equal to or better than those of comparable bonded post-tensioned beams.

DESIGN EXAMPLE

For the purpose of this example we will assume a 10 story flat plate apartment structure 3 bays wide and 5 bays long. The bays are 20' by 20'. The interior columns are 18" x 18" and the exterior ones are 12" x 18", Figs. 20 and 21. The building will be constructed in Kansas.

Loadings:

LIVE LOADS:

Roof = $30\#/F^2$ (SNOW LOAD)

[From--"Minimum design loads in buildings and other structures," American Standards Association, 1955]

Floors = $80\#/F^2$

[From--Table 9.1, Page 404, "Design of concrete structures," by George Winter and others, Mcgraw-Hill Book Company, New York.]

WIND LOADS:

$20\#/F^2$ up to 7th floor

and $30\#/F^2$ from 7th up to roof

The portal method will be used to find the wind moments.

Unbonded tendons will be used in this example and for crack control mild steel will be used.

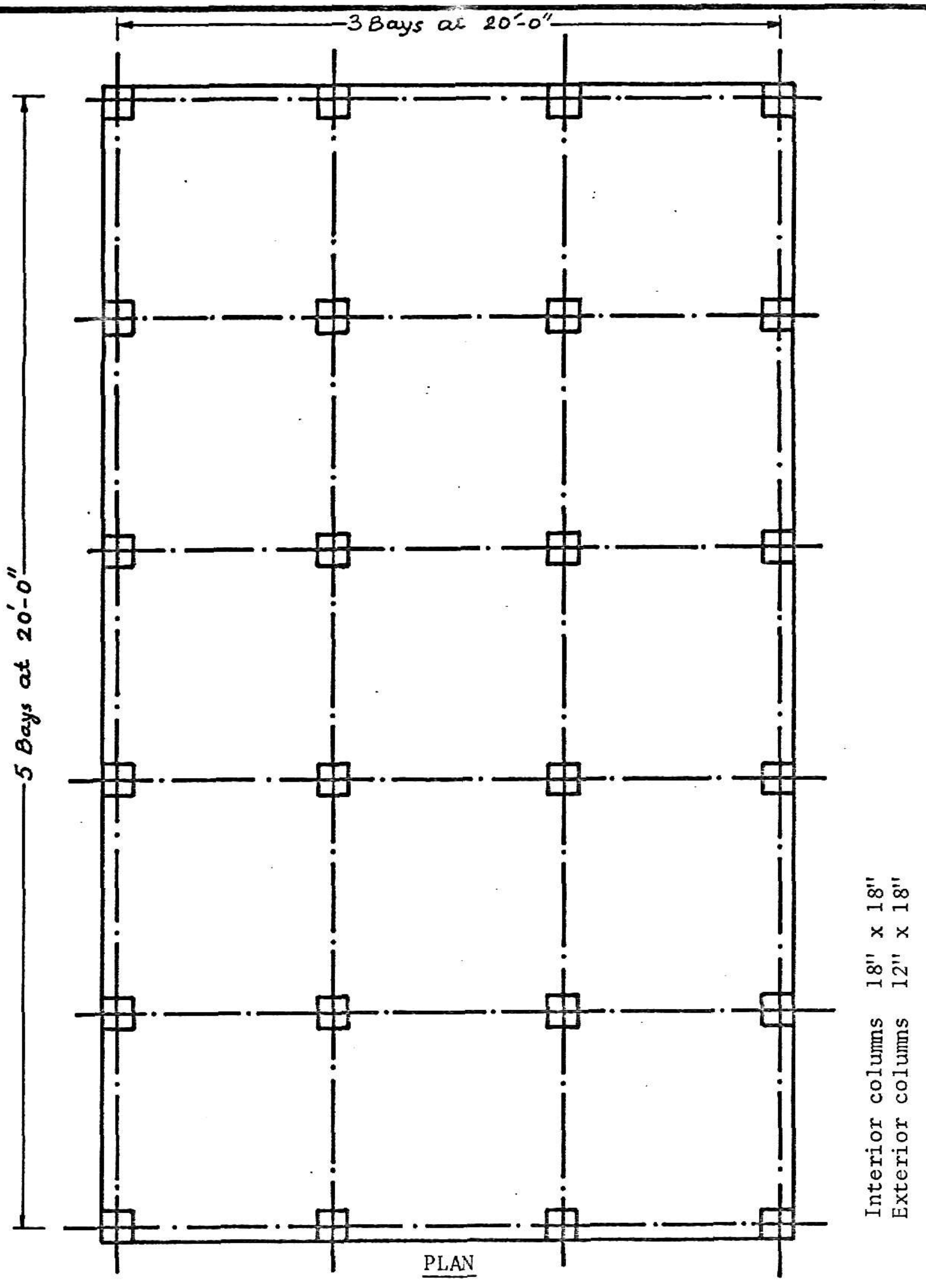


FIG. 20. PLAN OF THE 10-STORY FLAT PLATE APARTMENT BUILDING.

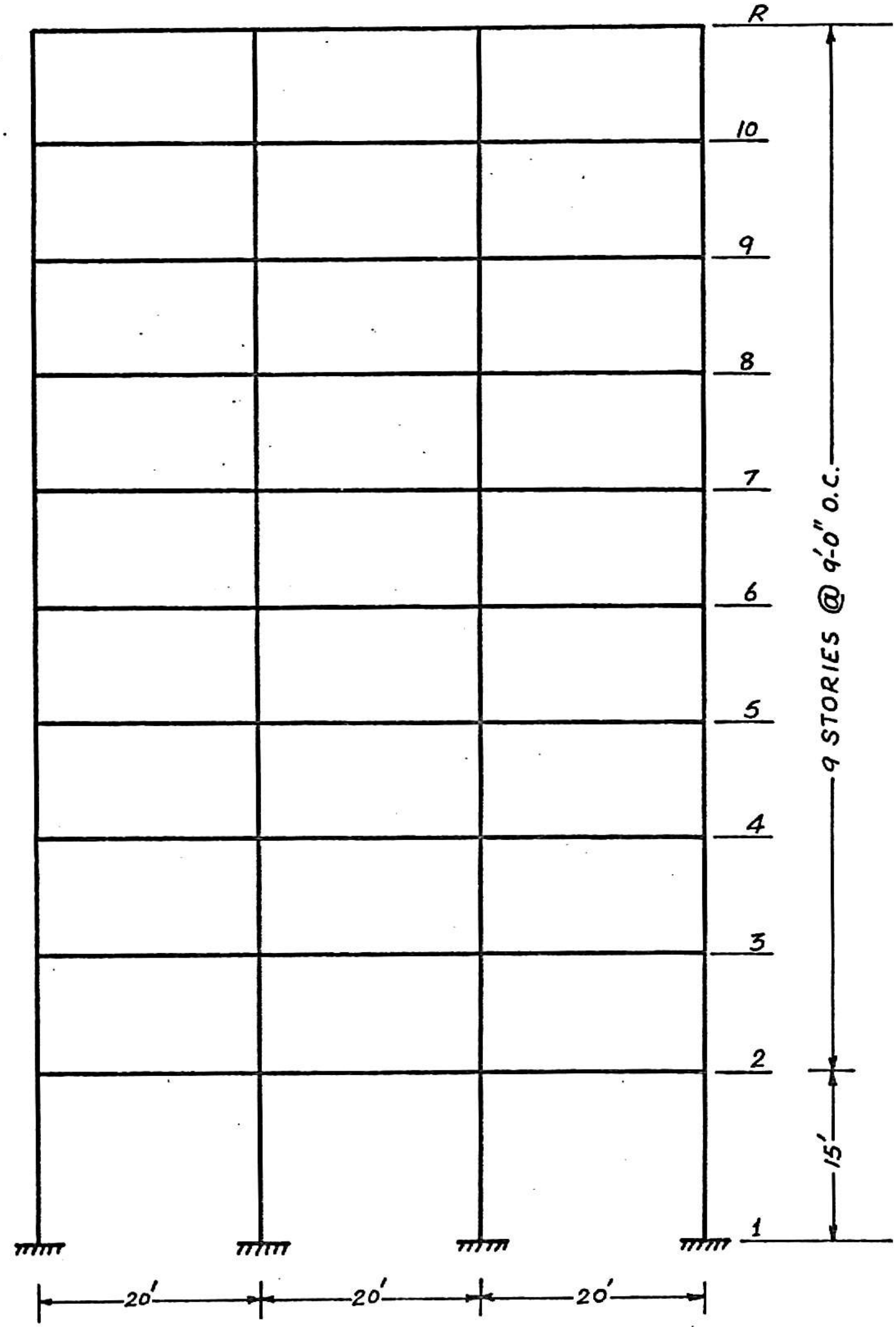


FIG. 21. SECTION OF THE 10-STORY FLAT PLATE APARTMENT BUILDING.

Wind Analysis by Portal Method

Assumptions:

i. The point of contraflexure lies in the middle of the beams and columns.

ii. The interior columns carry twice the shear than the exterior columns.

The wind analysis by portal method is shown in Fig. 22.

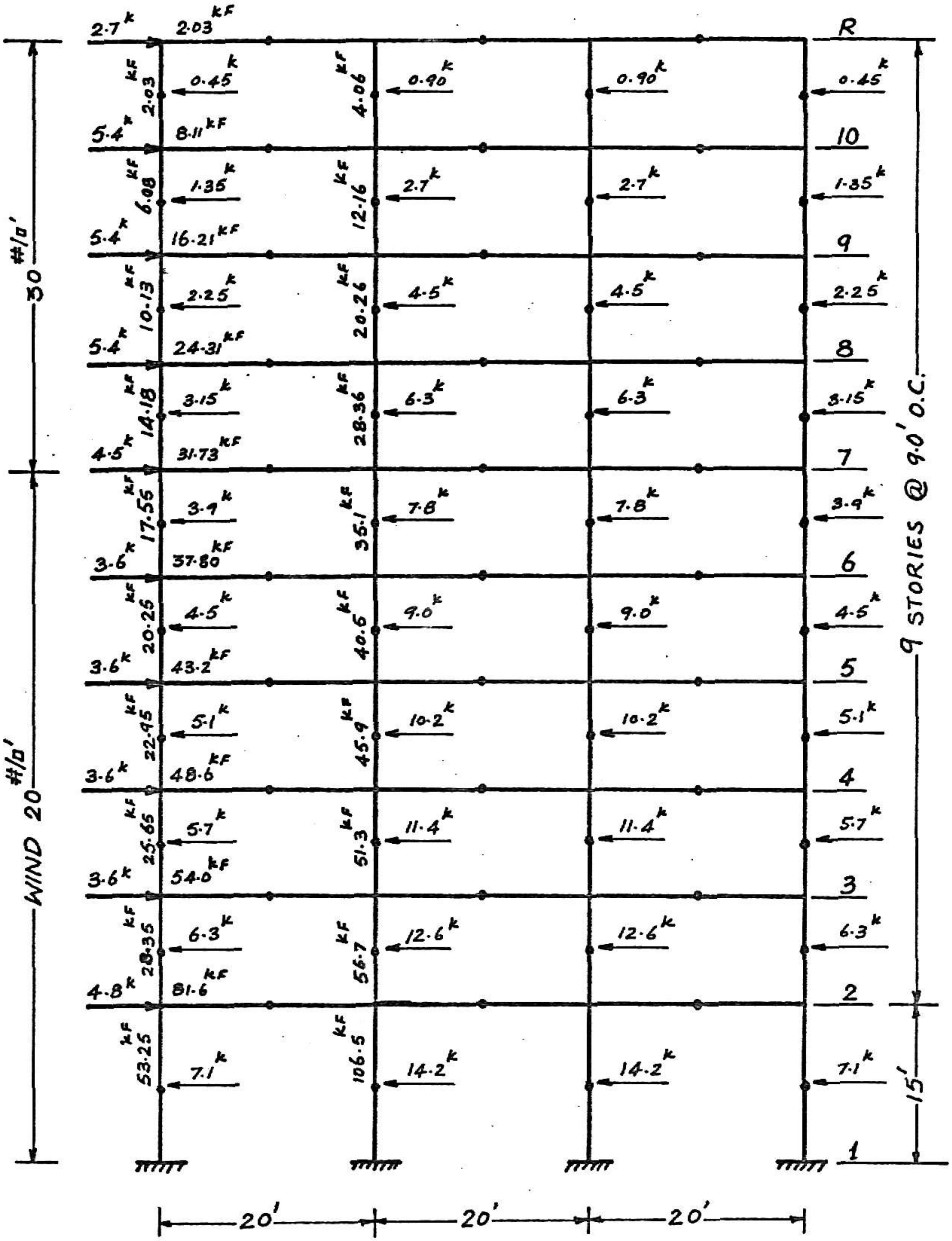


FIG. 22. PORTAL METHOD OF WIND ANALYSIS.

Slab Design

Consider a typical floor.

Let us consider a L/t ratio of 40

$$\frac{L}{t} = \frac{20 \times 12}{t} = 40 \qquad t = 6''$$

Use 6" slab thickness

Loads:

$W_{D.L.} = \frac{6}{12} \times 150 = 75\#/F^2$ (wt. of concrete is taken as 150#/cft)

$W_{Partitions} = \underline{15}\#/F^2$

Total Dead Load $90\#/F^2$

W_{LL}(Floor) $= 80\#/F^2$ (alternate bay loading not required, ACI Standard 318-63, 2103.a.6)

Strip Loads:

D.L. = 20 x 90 = 1800#/ft x 60% C.S. = 1080#/ft = 1.08K/ft

x 40% M.S. = 720#/ft = 0.72K/ft

L.L. = 20 x 80 = 1600#/ft x 60% C.S. = 960#/ft = 0.96K/ft

x 40% M.S. = 640#/ft = 0.64K/ft

Section Properties:

C.S.

$Ac = (10 \times 12) \times 6 = 720 \text{ in}^2$

$Ic = (10 \times 12) \times 6^3/12 = 2160 \text{ in}^4$

$Sc = (10 \times 12) \times 6^2/6 = 720 \text{ in}^3$

M.S.

$Ac = (10 \times 12) \times 6 = 720 \text{ in}^2$

$$Ic = (10 \times 12) \times 6^3/12 = 2160 \text{ in}^4$$

$$Sc = (10 \times 12) \times 6^2/6 = 720 \text{ in}^3$$

For design purposes, use d/2 from face of the column as the critical section--ACI Standard 318-63, 2102.b.

Now the method of moment distribution will be used to find the moments in the framed structure.

The moment distribution of 1K/F of loading on a typical floor, its bending moment and shear force diagrams are as shown in Fig. 23.

$$I_{AD} = 12 \times 18^3/12 = 5832 \text{ in}^4$$

$$I_{BF} = 18 \times 18^3/12 = 8748 \text{ in}^4$$

$$K_{AD} = \frac{3}{4} \times \frac{5832}{4.5} = 972$$

$$K_{BF} = \frac{3}{4} \times \frac{8748}{4.5} = 1458$$

$$K_{AB} = \frac{2160}{20} = 108$$

F.E.M's

$$M^F_{AB} = M^F_{BA} = M^F_{BC} = \frac{we^2}{12} = \frac{1 \times 20^2}{12} = 33.33\text{KF}$$

D.F.'s

$$\delta_{AD} = \delta_{AE} = \frac{972}{972 + 972 + 108} = 0.47$$

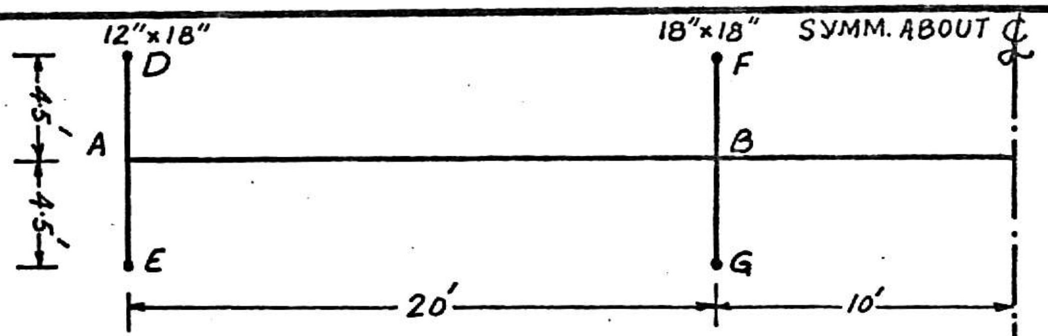
$$\delta_{AB} = \frac{108}{972 + 972 + 108} = 0.06$$

$$\delta_{BF} = \delta_{BG} = \frac{1458}{1458 + 1458 + 108 + 108} = 0.46$$

$$\delta_{BA} = \delta_{BC} = 0.04$$

Max. value of d (slab) = 5" (effective)

The critical section is at a distance of d/2 from face of the column, that is at a distance of

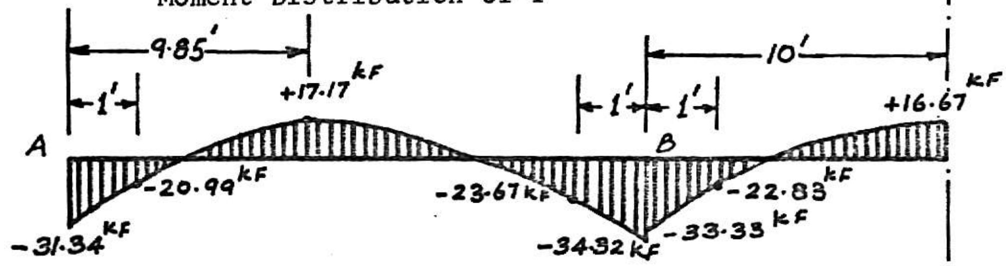


	Col. UP	Col. DN	B.M. RT.
D.F.	0.47	0.47	0.06
F.E.M.			33.33
Distribution	15.67	15.67	1.99
	15.67	15.67	31.34

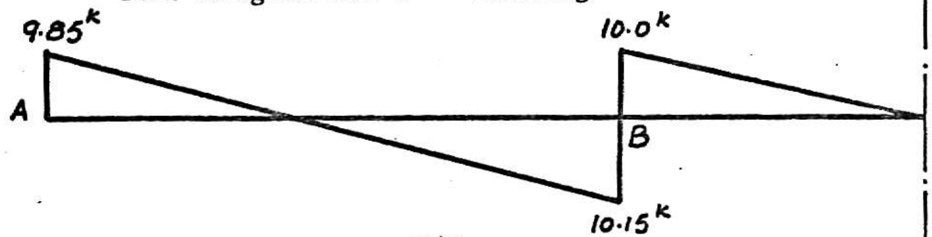
$\xrightarrow[\text{Co.}]{1/2}$

B.M. LT.	Col. UP	Col. DN	B.M. RT.
0.04	0.46	0.46	0.04
33.33			33.33
0.99			
	0.5	0.49	
34.32	0.5	0.49	33.33

Moment Distribution of $1^{K/F}$



B.M. Diagram for $1^{K/F}$ Loading



S.F. Diagram for $1^{K/F}$ Loading

FIG. 23. MOMENT DISTRIBUTION, BENDING MOMENT AND SHEAR FORCE DIAGRAMS FOR $1^{K/F}$ OF LOADING ON A TYPICAL FLOOR.

9" + 5"/2 = 11.5" say 1.0' from center line of the column A

and 9 + 5"/2 = 11.5" say 1.0' from center line of the column B

$$R'_A = R'_B = \frac{wl}{2} = \frac{1 \times 20}{2} = 10K$$

$$R''_A = R''_B = \frac{34.32 - 31.34}{20} = 0.15K$$

$R_A = 9.85K$ $R_B = 10.15$ Fig. 24

The negative moment at a distance of 1.0F from the end A is

$$= 9.85 \times 1 - 1 \times \frac{(1)^2}{2} - 31.34 = \underline{-20.99KF}$$

The negative moment at a distance of 1.0F from end B is

$$10.15 \times 1 - \frac{1 \times 1^2}{2} - 34.32 = \underline{-23.67KF}$$

Let the maximum positive moment occur at a distance x from the end A

$$R_A = wx \qquad x = \frac{R_A}{w} = \frac{9.85}{1} = 9.85 \text{ ft.}$$

Hence the maximum positive moment occurs at a distance of 9.85 ft. from end A

The magnitude of +ve M_{Max} is

$$+\text{ve } M_{Max} = 9.85 \times 9.85 - \frac{1 \times (9.85)^2}{2} - 31.34$$

$$= 97.02 - 48.51 - 31.34 = \underline{17.17KF}$$

$$R'_B = R'_C = \frac{wl}{2} = \frac{1 \times 20}{2} = 10K$$

$$R''_B = R''_C = \frac{33.33 - 33.33}{20} = 0K$$

$R_B = 10K$ $R_C = 10K$ Fig. 25

The negative moment at a distance of 1.0F from the end B

$$= 10 \times 1 - 1 \times \frac{12^2}{2} - 33.33 = \underline{-22.83\text{KF}}$$

Let the maximum positive moment occur at a distance x from the end B

$R_B = wx$

Hence $x = \frac{R_B}{w} = \frac{10}{1} = 10$ ft.

Hence the maximum positive moment occurs at a distance of 10.0 ft. from end B

The magnitude of Max + ve moment is

$$+\text{ve } M_{Max} = 10 \times 10 - 1 \times \frac{10^2}{2} - 33.33$$

$$= 100 - 50 - 33.33 = \underline{16.67\text{KF}}$$

Column Strip

Design Moments at d/2

		A			B	C.L.
D.L.	= 1.08K/F	= -22.57KF	+18.54KF	-25.56KF	-24.66KF	+18.00KF
L.L.	= 0.96K/F	= -20.14KF	+16.48KF	-22.72KF	-21.92KF	+16.00KF
WB	= 1.36KF	= -28.55KF	+23.35KF	-32.19KF	-31.05KF	+22.67KF
WUB	= 0.68KF	= -14.27KF	+11.68KF	-16.10KF	-15.52KF	+11.33KF

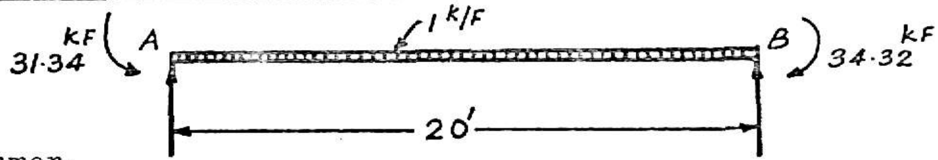
Calculate Post-Tensioning Required for Gravity Loads:

END Bay Controls

use 1" cover on bottom

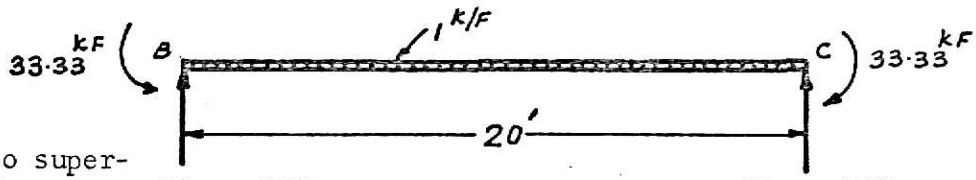
3/4" cover on top

1/2" diameter tendons



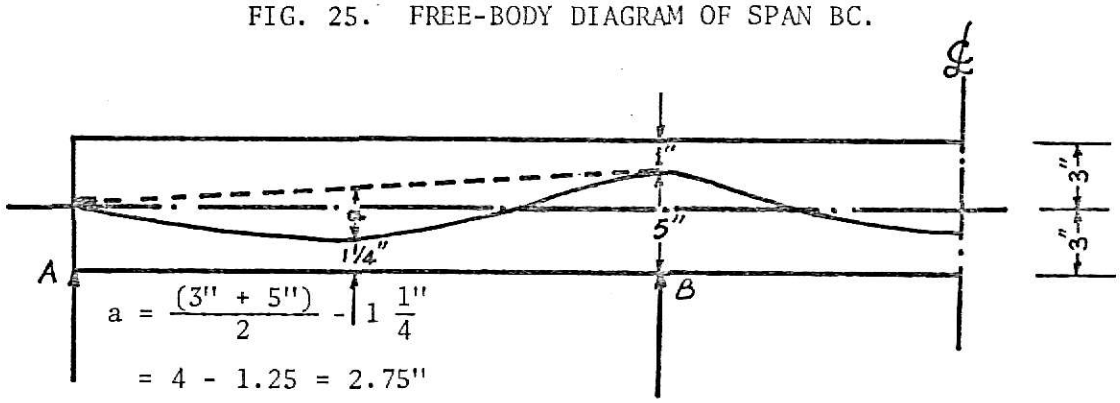
Reaction due to superimposed load	$R'_A = 10K$	$R'_B = 10K$
Reaction due to end moments	$R''_A = 0.15K$	$R''_B = 0.15K$
Total Reaction	$R_A = \overline{9.85}K$	$R_B = \overline{10.15}K$

FIG. 24. FREE-BODY DIAGRAM OF SPAN AB.



Reaction due to superimposed load	$R'_B = 10K$	$R'_C = 10K$
Reaction due to end moments	$R''_B = 0K$	$R''_C = 0K$
Total Reaction	$R_B = \overline{10}K$	$R_C = \overline{10}K$

FIG. 25. FREE-BODY DIAGRAM OF SPAN BC.



$$a = \frac{(3'' + 5'')}{2} - 1\frac{1''}{4}$$

$$= 4 - 1.25 = 2.75''$$

FIG. 26. MAXIMUM VALUE OF "a" FOR END BAY CONTROL.

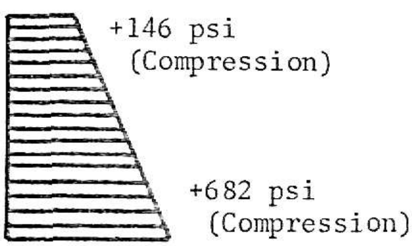


FIG. 27. STRESS DISTRIBUTION UNDER UNBALANCED LOAD.

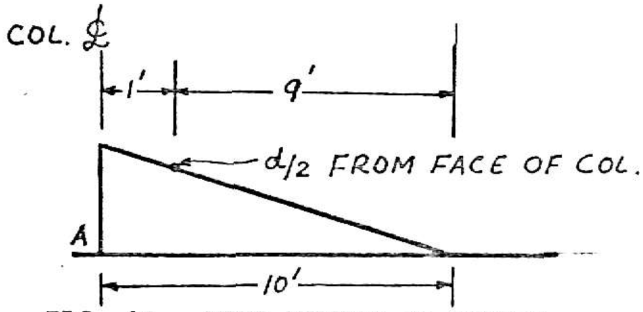


FIG. 28. WIND MOMENT AT COLUMN CENTER LINE.

Design of Column Strip:

The total load $W_T = W_{D.L.} + W_{L.L.}$

$$= 1.08 + 0.96 = 2.04K/F$$

Let us balance 65% of the W_T

$$W_B = 65\% \text{ of } W_T$$

$$= 0.65 \times 2.04 = 1.33K/F$$

$$W_{UB} = W_T - W_B = 2.04 - 1.33 = 0.71K/F$$

Note: Usually 60 to 70% of the total load is balanced, but this should not be less than the $W_{D.L.}$. If this is so then balance the whole $W_{D.L.}$. In case of high live loads compared with the dead loads, some designers suggest that the balanced load should consist of total dead load plus 1/2 the live load. But 60 to 70% is also reasonable. Again the selection of the balanced load depends entirely on the designer.

$$P = \frac{W_B L^2}{8a} = \frac{1.33 \times 20^2 \times 12}{8 \times 2.75} = 290K$$

The force in one 1/2 in. dia. tendon is 24.8K.

Number of tendons $= \frac{290}{24.8} = 11.6$ tendons, say 12 tendons

The exact P = 24.8 x 12 = 298K

$$\frac{P}{A} = \frac{298}{720} = 0.414 \text{ ksi} = \underline{414 \text{ psi}}$$

$$\text{Exact } W_B = \frac{298 \times 8 \times 2.75}{20^2 \times 12} = 1.36KF$$

$$W_{UB} = W_T - W_B = 2.04 - 1.36 = 0.68KF$$

The unbalanced $M_{UB} = -16.10KF$ at Point B (refer to page 72).

The stress due to unbalanced load is

$$\frac{M_{UB}}{S_c} = \frac{16.10 \times 12 \times 1000}{720} = 268 \text{ psi}$$

Hence the concrete fiber stresses under unbalanced load is

$$= \frac{P}{A} \mp \frac{M_{UB}}{S_c} = +414 \mp 268 = +146 \text{ psi Top}$$

$$682 \text{ psi Bottom}$$

At this point, the wind moments should be considered. ACI Standard 318-63, 2605.b allows allowable tension of 6 $\sqrt{f'c}$ if a member contains either bonded tendons or mild steel for crack control in the case of unbonded tendons which are used in this example.

Also, section 1004.a allows a 33 1/3% increase in allowable stresses due to wind moments.

Hence if 5000 psi concrete is used, the allowable tension under unbalanced load and wind would be $1.33 \times 6\sqrt{5000} = 564$ psi.

The available resisting moment for wind

$$= (564 + 146) \times \text{Section Modulus}$$

$$= \frac{710 \times 720}{12 \times 1000} = 42.60\text{KF} \quad \text{at } d/2 \text{ from face of column}$$

Wind moment at Col. ℄ $= \frac{10}{9} \times 42.06 = 44.5$KF <u>Fig. 28</u>

Hence the slab will be able to take wind moment at the 5th floor and above (see Fig. 22, page 67).

"Maximum" and Minimum Moment diagrams at 5th floor level <u>Fig. 29</u>

Wind moment at d/2 from face of Col. A is

$$= \frac{9}{10} \times 43.2 = 38.88\text{KF}$$

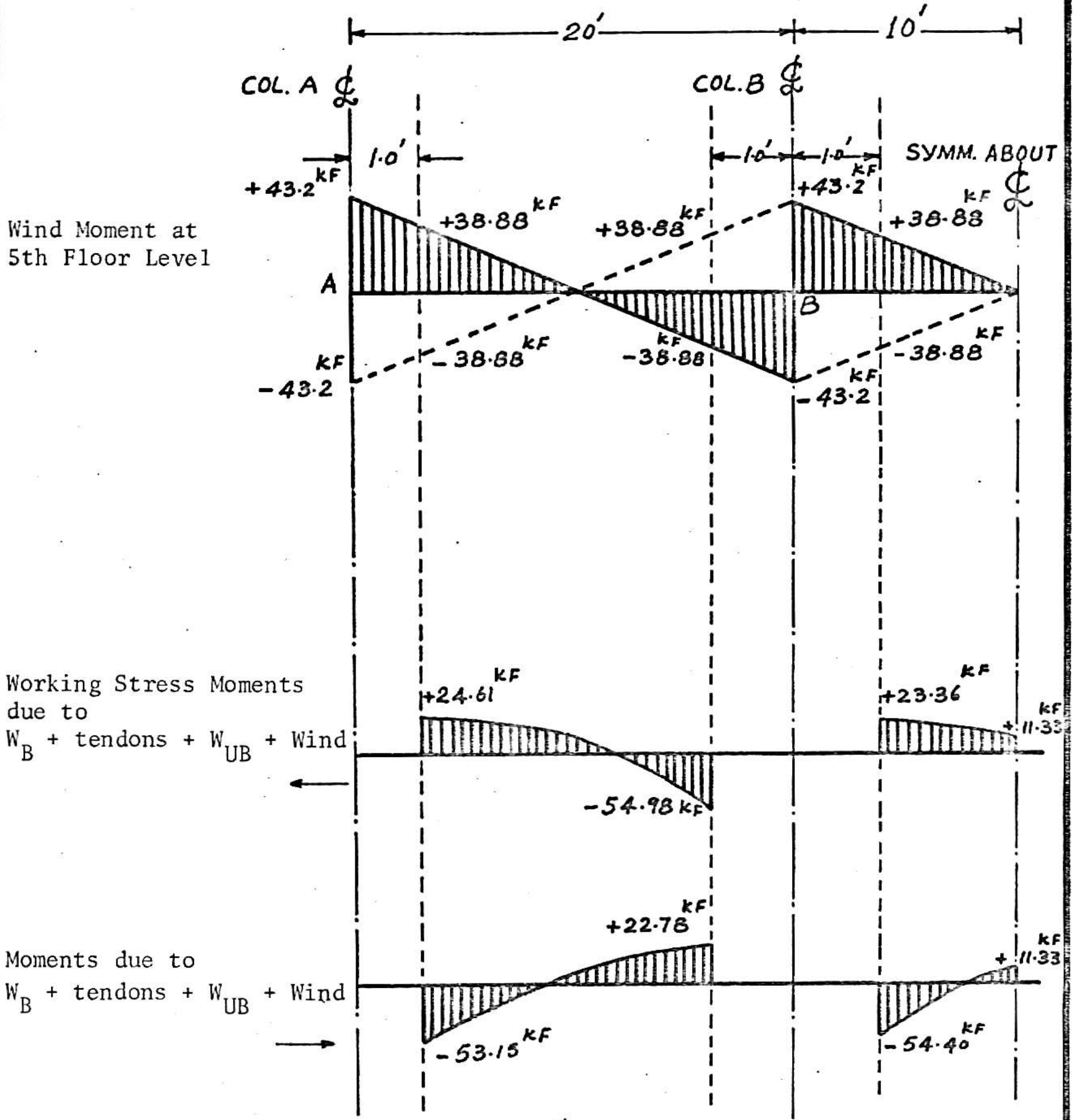


FIG. 29. MAXIMUM AND MINIMUM MOMENT DIAGRAMS AT 5TH FLOOR LEVEL.

Wind moment at d/2 from face of Col. B is

$$= \frac{9}{10} \times 43.2 = 38.88\text{KF}$$

The fiber stresses due to the maximum conditions are as follows:

$$\frac{P}{A} = 414 \text{ psi}$$

at A: Fig. 30

$$= \frac{-53.15 \times 12000}{720} = \mp 884 \text{ psi} + 414 \text{ psi}$$

$$= \frac{+24.61 \times 12000}{720} = \pm 410 \text{ psi} + 414 \text{ psi}$$

at B: Fig. 31

$$= \frac{-54.98 \times 12000}{720} = \mp 913 + 414 \text{ psi}$$

$$= \frac{+23.36 \times 12000}{720} = \pm 391 + 414 \text{ psi}$$

Check for elastic stresses of W_B + tendons + wind - W_{UB} Fig. 32

Max. Wind Moment = ± 38.88

Stress due to this is

$$= \frac{38.88 \times 12000}{720} = \pm 645 \text{ psi} + 414 \text{ psi}$$

The tensile stress is a bit high, but we will make it check in ultimate check.

The ultimate capacity of each section under various loading is as follows:

at A

$$Mu = 1.5\,(-22.57) + 1.8\,(-20.14) = -70\text{KF}$$

$$\text{or } 1.25[(-22.57) + (-20.14) + (-38.88)] = -\underline{102}\text{KF} \quad \text{Max. -ve Mu}$$

$$\text{or } + 38.88 = -4.8\text{KF}$$

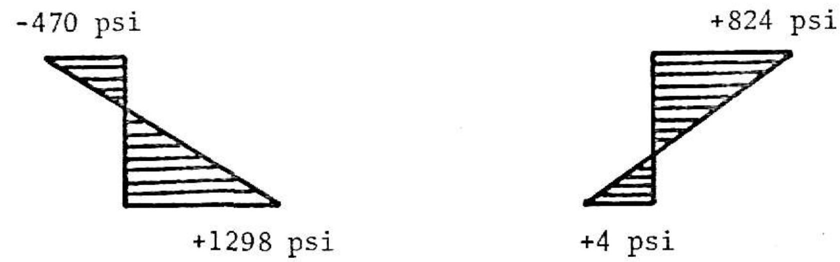


FIG. 30. STRESS DISTRIBUTION UNDER MAXIMUM CONDITIONS AT A.

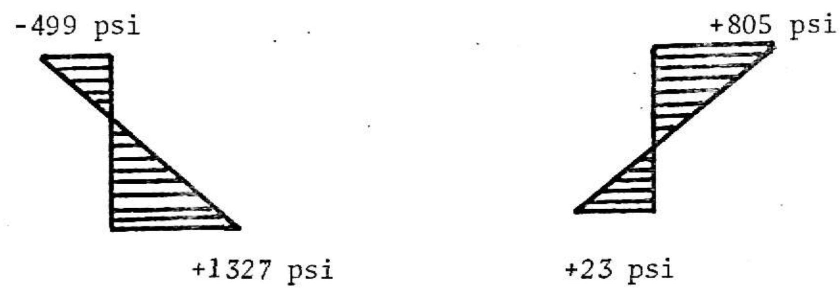


FIG. 31. STRESS DISTRIBUTION UNDER MAXIMUM CONDITIONS AT B.

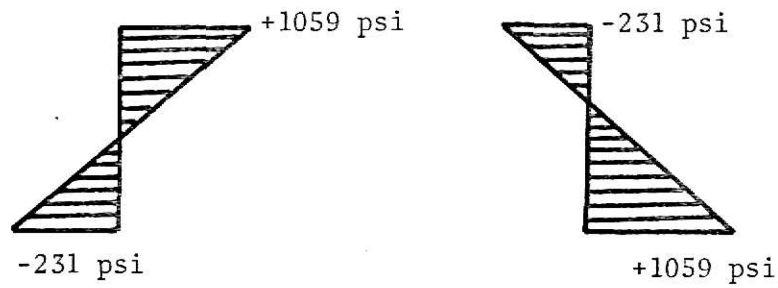


FIG. 32. STRESS DISTRIBUTION UNDER WIND-W_{UB}.

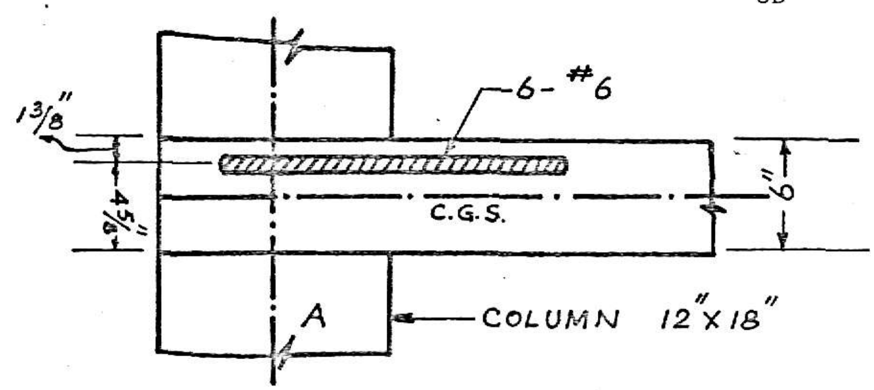


FIG. 33. POSITION OF MILD STEEL BARS FOR -ve MOMENT AT COLUMN A.

or 0.9(-22.57) + 1.1(-38.88) = -63.08KF

or +38.88 = +22.46KF Max. +ve Mu

+ve $M_{u(A-B)}$ = 1.5(18.54) + 1.8(16.48) = 57.41KF

at B LEFT SIDE

Mu = 1.5(-25.56) + 1.8(-22.72) = -79.3KF

or 1.25[(-25.56) + (-22.72) + (-38.88)] = -109KF Max. -ve Mu

or +38.88 = -11.75

or 0.9(-25.56) + 1.1(-38.88) = -65.77KF

or +38.88 = +19.77KF Max. +ve Mu

at B RIGHT SIDE

Mu = 1.5(-24.66) + 1.8(-21.92) = -76.4KF

or

1.25[(-24.66) + (-21.92) + (-38.88)] = -106.5KF Max. -ve Mu

or +38.88 = -9.65

or 0.9(-24.66) + 1.1(-38.88) = -64.96KF

or +38.88 = +20.58KF Max. M_u(+ve)

Ultimate Check

For post-tensioning

$fsu = fse + 15000$ psi — ACI Standard 318-63, 2608.a.3 unbonded tendons

$= 0.6\ fs' + 15000$ — $fse = 0.6\ fs'$ ACI Standard 318-63, 2606.b

$= 0.6 \times 240{,}000 + 15{,}000$

$= 159{,}000$ psi

$As = 0.144 \times 12 = 1.73\ in^2$ — $As \times fsu = 1.73 \times 159000$

$= 275{,}000\# = 275K$

Check at A:

$Mu = \phi As \times fsu[d - \frac{a}{2}]$ — $fc' = 5000$ psi

$d = 3.0$ in

$a = \frac{Asfsu}{0.85\ fc'b}$

$\phi = 0.90$

$$a = \frac{Asfsu}{0.85\ fc'b}$$

$$= \frac{275}{0.85 \times 5 \times (10 \times 12)} = 0.68 \text{ in}$$

$$Mu = \phi Asfsu[d - \frac{a}{2}]$$

$$= 0.9(275)[\frac{3 - 0.68/2}{12}] = 53.86KF$$

Required Mu = 102KF

The balance Mu = 102 - 53.86 = 48.14KF

Provide bonded mild steel bars.

Let us provide 6 #6 bars, Fig. 33. $As = 0.44 \times 6$ $fy = 60{,}000$ psi

$= 2.64 \text{ in}^2$

$$Mu = 0.9\ Asfy[d - \frac{a}{2}] \qquad d = 4.625 \text{ in}$$

$$Asfy = 2.64 \times 60$$

$$= 158KF$$

$$a = \frac{Asfy}{0.85\ fc'b}$$

$$= \frac{158}{0.85 \times 5 \times 10 \times 12} = 0.34$$

$$Mu = 0.9 \times 158[\frac{4.625 - 0.34/2}{12}] = 51.6KF > 48.14KF \quad OK$$

Mu Furnished = 53.86 + 51.6 = 105.46KF

Mu Required = 102KF OK

The maximum +ve moment of 22.33KF would be adequately taken care of since the Mu furnished by the post-tensioning would be the same for +ve or -ve moment. At +ve $M_{u(A-B)}$--again with greater d and only 57.41KF required the post-tensioning is more than adequate.

Check at B:

$$Mu = \phi As\ fs[d - \frac{a}{2}] \qquad a = \frac{Asfsu}{0.85\ fc'b} = 0.68$$

$$= 0.9 \times 275[\frac{5 - 0.68/2}{12}] \qquad d = 5''$$

$$= 95.7KF \qquad Asfsu = 275K$$

Required Mu = 109KF

Balance = 109 - 95.7 = 14.3KF

Let us provide 3-#5 bays, Fig. 34. $As = 3 \times 0.31 = 0.93 \text{ in}^2$

$Asfy = 0.93 \times 60 = 55.8K$

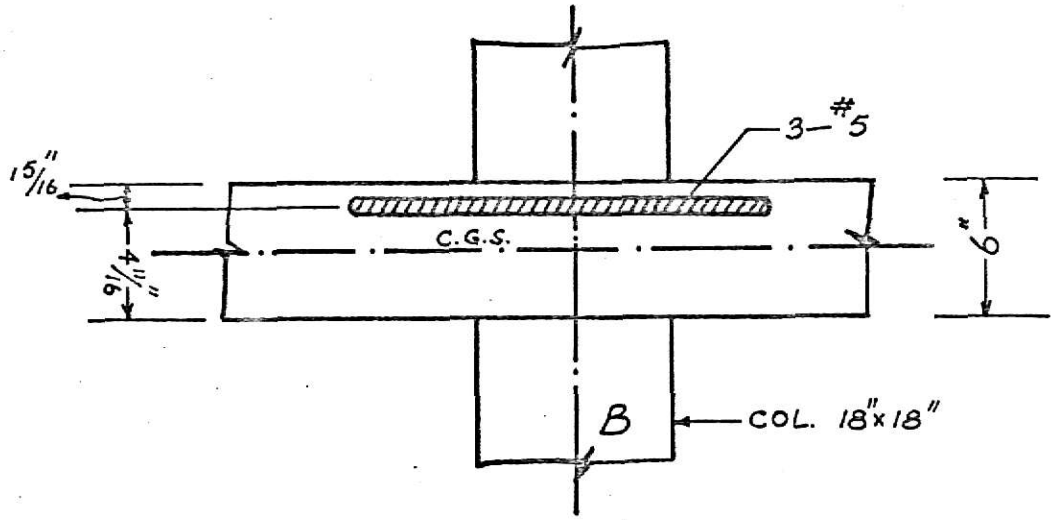


FIG. 34. POSITION OF MILD STEEL BARS FOR -ve MOMENT AT COLUMN B.

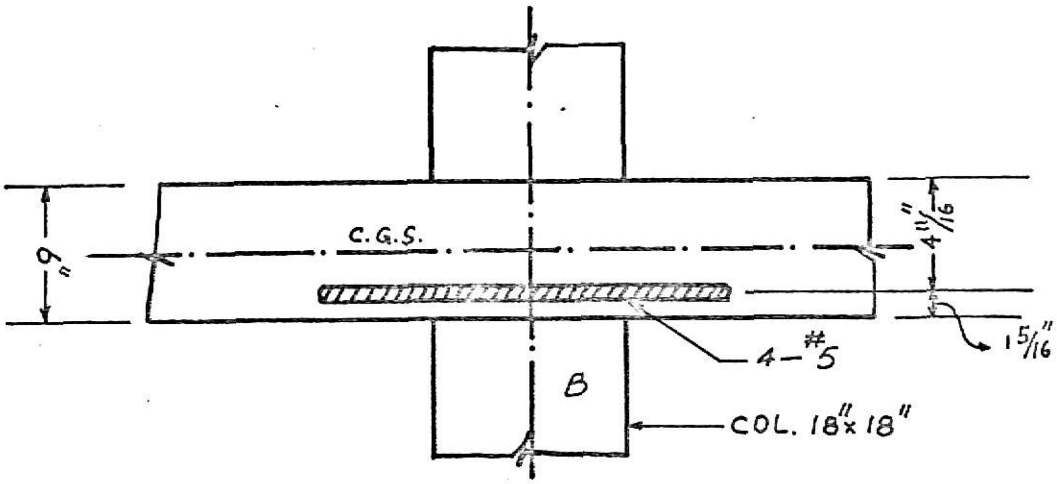


FIG. 35. POSITION OF MILD STEEL BARS FOR +ve MOMENT AT COLUMN B.

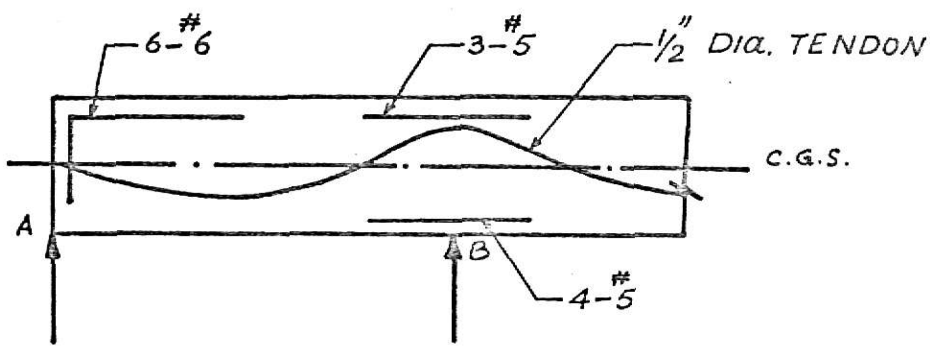


FIG. 36. ELEVATION OF COLUMN STRIP WITH DETAILED REINFORCEMENT.

$Mu = \phi Asfy[d - \frac{a}{2}]$ $a = \frac{Asfy}{0.85\ fc'b} = \frac{55.8}{0.85 \times 5 \times (10 \times 12)}$

$= 0.9 \times 55.8[\frac{4.69 - 0.14/2}{12}]$ $= 0.14$ in

$= 19.64KF$ $d = 6'' - (1'' + 5/16'') = 4.69$ in.

Mu Furnished = 95.7 + 19.64 = 115.34KF

Mu Required = 109KF OK

<u>Check for +ve Mu at B:</u>

Max. +ve Mu = 20.58KF

Let us provide 4-#5 Bars, Fig. 35. $As = 4 \times 0.31 = 1.24\ in^2$

$Asfy = 1.24 \times 60 = 74.4K$

$Mu = \phi Asfy[d - \frac{a}{2}]$ $a = \frac{asfy}{0.85\ fc'b}$

$= 0.9(74.4)[\frac{4.69 - 0.18/2}{12}]$ $= \frac{74.4}{0.85 \times 5 \times (10 \times 12)} = 0.18$ in

$= 25.67KF > 20.58KF$ OK

$d = 6'' - (1'' + 5/16'') = 4.69$ in

Hence the check at all points.

The fully designed column strip of the flat plate of a typical floor with detailed reinforcement is as shown in Fig. 36.

Shear Check

Method I

Check at A:

Refer to Fig. 37. Moment capacity at x - x'

M_u of (6 - #6) = 51.6KF

Assuming 33% of the tendons fall within "a"

33% M_u of tendons = 0.33 x 53.86 = 17.8KF

Refer to Fig. 37:

$c = h + d/2 = 18 + 5/2 = 20.5$ in

$a = b + 3t = 12 + 3(6) = 30$ in

$$\text{Area} = (10 \times 20) - \left(\frac{20.5 \times 30}{144}\right) = 195.7 \text{ in}^2$$

V_D = 195.7 x 0.09 = 17.6K x 1.5 = 26.4K

V_L = 195.7 x 0.08 = 15.6K x 1.8 = 28.1K

54.5K

V_W = 4.3K

V_u = 37.5 x 1.25

= 46.75K

Moment required, M_u = 102KF

M_t = 102 - 51.6 - 17.8

= 32.6KF

$$J_c = \frac{dc^3}{6} + \frac{ct^3}{6} + 2ad(L/2)^2$$

$$= \frac{5 \times (20.5)^3}{6} + \frac{20.5(6)^3}{6} + 2 \times 30 \times 5 \times \left(\frac{20.5}{2}\right)^2$$

$$= 7188 + 738 + 31800 = 39726 \text{ in}^4$$

$$v = \frac{V_u}{A_c} \pm \frac{M_t \ L/2}{J_c}$$

$$= \frac{46750}{426} \pm \frac{32.6 \times 12000 \times 20.5}{2 \times 39726}$$

$= 110 + 100 = 210$ psi $>$ 200 psi a bit high

$A_c = 2cd + ad$

$= 2 \times (20.5)6 + 30(6)$

$= 2460 + 180.0 = 426 \text{ in}^2$

$b_o = 2(20.5) + (12 + 5)$

$= 41 + 17 = 58$ in

Allowable $v_c = 0.6 \ \phi \ F_{sp} \sqrt{fc'}$ ACI Standard 318-63, 1708.a.3

$0.6 \times 0.85 \times 5.5 \times \sqrt{5000} = 200$ psi

Check at B:

Refer to Fig. 38.

$c = h + d = 18 + 5 = 23$ in

$a = b + 3t = 18 + 3(6) = 36$ in

$A_c = 2(a + c)d$

$= 2(23 + 36)5$

$= 590 \text{ in}^2$

$b_o = 4(18 + 5)$

$= 92$ in

M_u of 3-#5 = 19.46KF

33% of M_u tendons = 0.33 x 95.7 = 31.6KF

Total 51.06KF

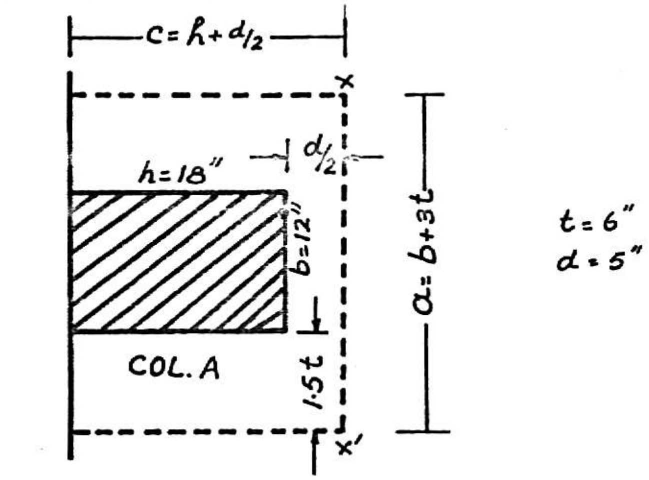


FIG. 37. TOP PLAN AT COLUMN A.

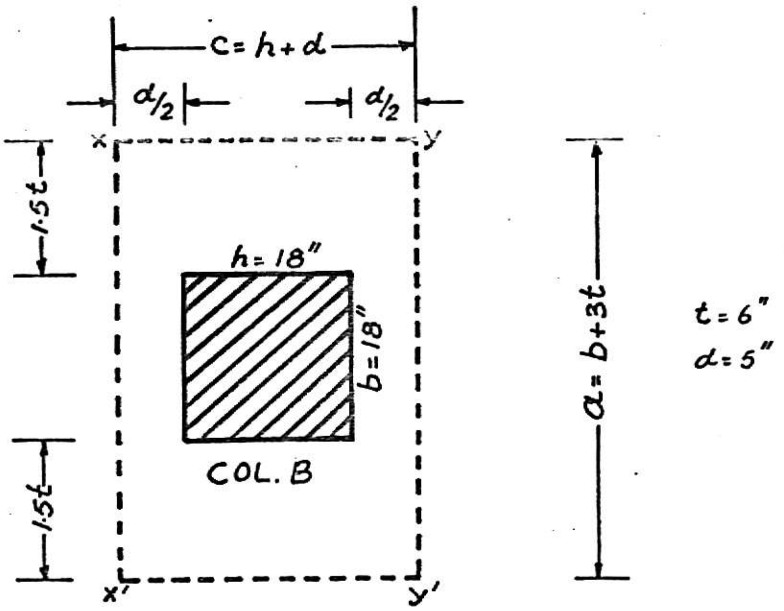


FIG. 38. TOP PLAN AT COLUMN B, METHOD I.

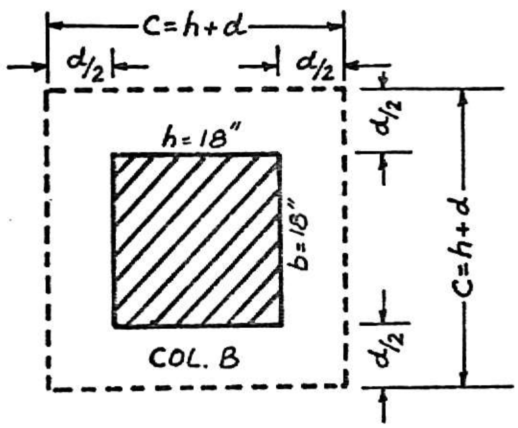


FIG. 39. TOP PLAN AT COLUMN B, METHOD II.

This capacity occurs at both x - x' and y - y' sections

The moment to be transferred from slab to column is = 109KF

$$M_t = 109 - (51.06) - 51.06) = 6.88\text{KF}$$

$$J_c = \frac{dc^3}{6} + \frac{ct^3}{6} + 2ad(L/2)^2$$

$$= \frac{5(23)^3}{6} + \frac{23(6)^3}{6} + 2(36)(5)(23/2)^2$$

$$= 10140 + 828 + 47600$$

$$= 58568 \text{ in}^4$$

$$v = \frac{V_u}{A_c} \pm \frac{M_t(C/2)}{J_c}$$

$$= \frac{88600}{590} \pm \frac{6.88 \times 12000 \times 23}{2 \times 58568}$$

$$= 150 \pm 16.2$$

$$= 166.2 \text{ psi}$$

$$< 200 \text{ psi}$$

OK

$$= 133.8 \text{ psi}$$

$$\text{area} = (20 \times 20) - (\frac{23.0 \times 36}{144}) = 391.7 \text{ in}^2$$

$$V_D = 391.7 \times 0.09 = 35.25\text{K} \times 1.5 = 52.8\text{K}$$

$$V_L = 391.7 \times 0.08 = 31.34\text{K} \times 1.8 = \underline{56.4}\text{K}$$

$$109.2\text{K}$$

$$\underline{V_W} = \qquad = \underline{4.30}$$

$$V_u \qquad 70.9 \times 1.25$$

$$= 88.6\text{K}$$

Check for punching shear

at A

$$V = \frac{V_u}{b_o \times t} = \frac{54500}{58 \times 6} = 157 \text{ psi} < 200 \text{ psi} \quad \text{OK}$$

at B

$$V = \frac{V_u}{b_o \times t} = \frac{109200}{92 \times 6} = 197 \text{ psi} < 200 \text{ psi} \qquad \text{OK}$$

Note: If the shear check fails, then the following are some of the remedies:

a. Increase the size of the column

or b. Increase the fc'

or c. Increase the thickness of the slab

or d. For punching shear provide the drop-panels, but if the architect wants only flat-plate construction, then either increase the depth of slab throughout or increase the size of column for punching shear.

<u>Method II</u>

Although no accurate theoretical method for calculating allowable shear value in prestressing slabs is currently available, a rational procedure will be discussed here. This approach is based on the formulas for beam shear according to ACI Standard 318-63, 2610.b.

This approach is to be thought of as a guide and the designer should use his judgement in evaluating results and determining a subsequent course of action.

The formulas presented here are basically the ACI Standard 318-63, formulas for beam shear modified in the ratio of non-prestressed beam shear to non-prestressed slab shear. Further modifications are aimed at simplifying the use of the equations.

M_{cr} for normal weight concrete can be taken very conservatively as $7.5 \sqrt{fc'}\, s$. Where $7.5 \sqrt{fc'}$ is the modulus of rupture of the concrete and s is the section modulus of the slab. The deviation between M_{cr} as computed by this method vs. the method prescribed by the ACI Code is quite high, therefore M_{cr} is more realistically represented herein as being $1.33(7.5 \sqrt{fc'} s)$.

F_{PC} for a solid slab becomes $\frac{P}{bd}$ (since F_{PC} is taken at centroid), where P is the prestressing force.

The modified formulas are as follows.

For normal weight concrete

$$V_{ci} = 1.2bd\sqrt{fc'}\ (1 + \frac{1.4d}{\frac{M}{V_{ar}} - \frac{d}{2}}) + V_d$$

$$V_{cw} = bd(7.0\sqrt{fc'} + 0.3\ F_{PC}) + V_P$$

$$= bd(7.0\sqrt{fc'} + 0.3\ \frac{P}{bd}) + V_P$$

Neither shall be less than $4bd\sqrt{fc'}$

For slabs where cables are horizontal at supports, $V_P = 0$.

NEGATIVE MOMENTS AT INTERIOR COLUMN "B"

Refer to Fig. 39.

Column Strip

M_1 (Longitudinal) = 4.9 KF/F at d/2 from face of column

M_2 (Transverse) = 4.9 KF/F (assume)

Average M at d/2 from face of column

= (4.9 + 4.9)/2 = 4.9KF/F

Post-Tensioning Forces (Estimated)

P_1 (Longitudinal) = 298/10 = 29.8K/F

P_2 (Transverse) = 29.8K/F (assume)

Average P = (29.8 + 29.8)/2 = 29.8KF

Effective Depth "d" (1/2" dia. tendons)

d_1 (Longitudinal) = 6 - [3/4" cover + 1/2 + 1/4] = 4.5"

d_2 (Transverse) = 6 - [3/4" cover + 1/4] = 5.0"

Average d = (4.5 + 5.0)/2 = 4.75 in

Critical Peripheral Plan

bo = 4(18 + 4.75) = 91.0 in

Ultimate Factor of Safety

$$F.S. = \frac{1.5\ D.L. + 1.8\ L.L.}{D.L. + L.L.} = \frac{1.5(90) + 1.8(80)}{90 + 80} = 1.64$$

D.L. = $90\#/F^2$ L.L. = $80\#/F^2$

Total Load = 90 + 80 = $170\#/F^2$ = $0.170K/F^2$

Peripheral Shears

$$V = [(20 \times 20) - (\frac{18 + 4.75}{12})^2]\ 0.170$$

$$= [400 - 3.6] \times 0.170 = 67.5K$$

$$\text{Average } V_{ar} = \frac{67.5}{\frac{91}{12}} = 8.9K/F \text{ of } b_o$$

$$V_{DL} = 8.9 \times \frac{90}{170} = 4.7K/F \text{ of } b_o$$

V_u Required = 8.9 x 1.64 = 14.6K/F

V_u Supplied

$$\sqrt{fc'} = 70.7 \text{ psi} = 0.071 \text{ ksi}$$

$$V_{ci} = 1.2bd\sqrt{f'c}\ \ (1 + \frac{1.4d}{\frac{M}{V_{ar}} - \frac{d}{2}}) + V_d$$

$$= 1.2(18)(4.75)(0.071)[1 + \frac{1.4(4.75)}{\frac{4.9 \times 12}{8.9} - \frac{4.75}{2}}] + 4.7$$

$$= 7.3[1 + \frac{6.65}{4.24}] + 4.7$$

$$= 7.3 \times 2.57 + 4.7 = 22.4K/F > 14.6K/F$$

$$V_{cw} = bd(7.0\sqrt{fc'} + 0.3 \frac{P}{bd}) + V_P$$

$$= 18(4.75)[7.0 \times 0.071 + 0.3 \times \frac{29.8}{18 \times 4.75}] + 0$$

$$= 85.5[0.497 + 0.105] = 51.0K/F > 14.6K/F$$

$$M_{in} \quad V_{ci} \quad \text{or} \quad V_{cw} = 4bd\sqrt{fc'}$$

$$= 4 \times 18(4.75) \times 0.071 = 23.9K/F$$

$$V_u \text{ supplied} = \phi(V_{ci}) = 0.85(22.4) = 19.0K/F \qquad \text{OK}$$

Slab o.k. in shear

Actually shear will seldom be a critical factor in the design of prestressed slabs. Only punching shear is really critical in the case of flat plates at the column-slab connection, which should be taken care of, and has already been discussed. Moreover to prevent any chance of an abrupt punching shear failure, it is a good practice to pass at least some of the tendons directly over the columns.

The same way the middle strip can be designed, but the only difference is, as no mild steel is provided in the middle strips, keeping the fiber stresses to zero-tension under W_{UB}.

As far as the transverse strips are concerned, the same principle and procedure is applicable.

CONCLUSION

In recent years flat plate construction has become quite popular for medium and high-rise buildings. Following are some of the major factors which have been responsible for this accelerated growth.

a. Simplification of design techniques and this is the single most important reason for the accelerated growth of prestressed flat plate construction.

b. The increasing demand for longer spans, thereby creating more functional interior space without obstructing columns. Up to 35 ft. of span a prestressed lightweight concrete flat plate system provides a functional and economical solution for medium and high-rise buildings.

c. Population growth and high land values have induced a trend towards high-rise buildings.

d. An increasing demand for low-cost housing, hospitals, parking garages, apartment complexes and commercial and institutional buildings.

e. The realization that prestressing can be used effectively to control deflections and thereby minimize the chances of cracking.

f. Ultimate strength is controlled primarily by the total amount of tendons (plus any unprestressed reinforcement) rather than by the tendons distribution.

Currently there is more than $150 billion for building construction. A sizeable amount of this could go to prestressed flat-plate construction.

The most critical portion of the flat-plate construction is the area around the column. More work needs to be done to document the help a slab receives from the post-tensioning in resisting punching shear. There are still some controversial views about the column and middle strip width and the percentage of moments carried by each. A thorough research work is required in these aspects, and engineers are working on these aspects in the United States.

The Joint ACI-ASCE Committee 423 on prestressed concrete is working to eliminate "allowable stresses" as the design criteria in prestressing members. Instead, such criteria as deflections, crack width, natural frequency of vibration and range of stresses under repeated loadings would be used. This would perhaps allow the use of higher tension stresses and more mild steel in such members as shown in our example, resulting in a more economical design.

ACKNOWLEDGEMENT

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APPENDIXES

APPENDIX I - NOTATIONS

M = Bending moment

M_A = Total negative moment acting on one edge of a panel

M_B = Total positive moment acting at panel center line

M_o = Total static moment

M_{od} = Total design static moment

W = Total load on one panel

w_{av} = Average of the total load on two adjacent spans

w = Unit load on one panel

L = Span in direction considered (center-to-center of columns for flat slabs)

L_2 = Span perpendicular to L

b = Short span of a rectangular panel

c = Width of capital in direction considered

A' = C/2

A'_{av} = Average of the values of A' on the two ends of a span

r = Proportion of load assumed to be carried in direction L

A_s = Cross-sectional area of reinforcement

f_s = Steel stress

jd = Internal lever arm for a reinforced concrete section based on the straight line formula

ε_s = Steel strain

E_s = Modulus of elasticity of steel

M_p = Plastic moment

D.L. = Dead load

L.L. = Live load

W_p = Partition load

C.S. = Column strip

M.S. = Middle strip

F = Prestressing force

L = Span length

h = Sag of parabola

I = Moment of inertia about neutral axis

$W_{D.L.}$ = Dead load

$W_{L.L.}$ = Live load

G.L. = Girder load

K_1 & K_2 = Safety factor

t = Slab thickness

d = Effective depth

P = Total balanced load

A_c = Area of the column and middle strip

S_c = Section modulus of column and middle strip

K_{AD} = Stiffness factor

δ or D.F = Distribution factor

F.E.M. or

M^F = Fixed end moments

K/F = Kip per foot

= Center line

B.M. = Bending moment

S.F = Shear force

R = Reaction

W_B = Balanced load

W_{UB} = Unbalanced load

W_T = Total load (D.L. + L.L.)

Mu = Ultimate moment

fc' = Compressive strength of concrete

fsu = Calculated stress in prestressing steel at ultimate load

fse = Effective steel prestress after losses

fs' = Ultimate strength of prestressing steel

fy = Yield strength of steel

ϕ = Capacity reduction factor

h = Depth of column

V_D = Shear due to dead load

V_L = Shear due to live load

V_W = Shear due to wind load

V_u = Shear due to specified ultimate load

b_o = Periphery of critical section of slabs

F_{SP} = Ratio of splitting tensile strength to the square root of compressive strength

V = Shear due to externally applied loads

V_c = Shear carried by concrete

V_{ci} = Shear at diagonal cracking due to all loads, when such cracking is the result of combined shear and moment

V_{cw} = Shear force at diagonal cracking due to all loads, when such cracking is the result of excessive principal tension stresses in the web

V_P = Vertical component of the effective prestress force at the section considered

APPENDIX II - TABLE OF FIGURES

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DESIGN AND ANALYSIS OF PRESTRESSED FLAT PLATES

by

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AN ABSTRACT OF A MASTER'S REPORT

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Due to overcrowding of big cities, the problem facing the city administration is that of providing suitable accommodation. The high-rise buildings are becoming more popular these days for offices and residential buildings. "The post-tensioned flat-plates," construction is used frequently in the high-rise buildings, because of so many advantages which make the structure economical over all.

As we know, flat-plates are highly indeterminate structures, suitable procedures must be found for their analysis and design. Many engineers have done considerable research in this field and still many more are working on it.

In this report I have discussed the necessity, evolution, analysis and design procedure for flat-plates put forward by some of the engineers and the "load-balancing method" in detail which is due to Professor T. Y. Lin.

The moment distribution method developed by the Portland Cement Association is used in the design example. The portal method of analysis has been used for calculating the wind moments and the wind effect has been fully taken into consideration during the design of the flat-plates. The allowable stresses are permitted according to ACI Standard 318-63.

The area around the column is the most critical area in flat-plate construction and proper care has been fully taken into account during the design procedure. The slab is fully checked for shear and also for punching shear.

In the design of flat-plates, the unbonded tendons along with mild steel bars have been used in this report. The design is based on working strength design and is fully checked for ultimate strength design.

Due to increasing popularity and necessity of high-rise buildings, the flat-plate construction is going to play an important role. The "load balancing method," by Professor T. Y. Lin is really a reasonable method for flat-plate design.