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Mr. Friedmann

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Kirmser Undergraduate Research Award Essay

The idea of choosing a topic for my ENGL 415 paper with seemingly limitless options made me nervous. However, with the help of library resources and engineering librarian Jason Coleman, I discovered a topic I became passionate about: evaluating optimization models to reduce evacuation times for flash flooding in developing countries. Without the help of Jason Coleman and the many library research tools he provided me with, I would have been unable to identify this. By deciding on a topic, identifying research strategies, using library research tools, and finding and evaluating research, this project turned from a stressful task to one of the most enjoyable assignments I have completed at K-State.

In a broad sense, what I most want to do with my life is to help others. Specifically, I would love to help people in developing countries. I went to China with my dad in middle school for a month and this past summer I visited impoverished villages in the country of Georgia. During both trips, I realized how privileged I am to live in America. Because of this, I aspire to help those who are less privileged. As an industrial engineer, I know my optimization skills are valuable in places where resources are scarce. Additionally, I really enjoy operations research, or the study of mathematical optimization tools and how they can be applied in real life. Combining these, I realized opportunities existed to construct mathematical models which optimized evacuation procedures in developing countries. My goal was to pursue my passions and explore what using my degree to help others would look like from a global perspective.

While I chose my topic by focusing on my passions, those passions were originally too broad for a research paper. I had spent hours searching for research articles on Scopus but was unable to find what I was looking for. After discussing my hopes for the paper with my professor, Mr. Friedmann, I was referred to engineering librarian Jason Coleman. Mr. Coleman did an excellent job of helping me narrow my topic to specifically focus on flash flooding in developing countries. After this, he showed me how to use library research tools, such as the Web of Science, to identify useful articles. He introduced me to the ability to add multiple keywords and topics, as well as combine past searches to produce different mixes of results. Once we found the set of results I was looking for, the search was narrowed within a specific timeframe and exclusively to the field of engineering and operations research. This produced a list of journal articles which were highly useful in producing my paper.

My overall research strategy was to identify reliable, useful information on optimization models that would positively impact the organization my research was tailored to: the United Nations Office for Disaster Risk Reduction (UNDRR). I organized my research with Zotero, a reference management software taught by Mr. Coleman. This stored all my research, allowed me to create folders for organization and highlight text within each source. Zotero also proved highly valuable by providing a bibliography and in-text citations for my paper.

Two sources significantly informed my project by supplying different mathematical optimization strategies. While other research in this field was available, I selected the two models with the broadest application and greatest adaptability. This would be most beneficial to the UNDRR to account for differences between countries in which the models could be applied. Both sources gave me an in-depth understanding of their models and provided case studies in which they were applied in real life. I was also able to learn how their mathematical models worked to identify key benefits and flaws in their formulations.

As a result of writing my paper, I have learned the K-State library research tools are way more helpful than I previously thought. In my meeting with Mr. Coleman, he showed me different databases to find the articles I needed. The most beneficial database I used was the Web of Science, which offers resources from all the databases the K-State library has access to. Additionally, I used ProQuest Central and the Transportation Research Information Services (TRIS) database. After meeting with Mr. Coleman and learning more about how to utilize K-State library research tools, I realized the importance of having a clear topic, using multiple keywords, and limiting the search criteria to find reliable articles in short periods of time.

When deciding if information I found in research articles was trustworthy, I checked the number of citations it had to other publications, how current it was, and how professionally it was written. I also had to evaluate whether an article was worth reading in full. First, I evaluated whether the source would be beneficial to the UNDRR. If so, I planned to read it in full. However, my methods for evaluating sources changed throughout my research. While I first planned on reading every source that seemed relevant, I quickly realized this would require too much time. Therefore, I identified sources most critical to my topic and read those carefully. The remaining sources, however, were skimmed for additional information I deemed important.

Using the K-State library resources effectively turned a mundane class assignment into an avenue to pursue my passions. They gave me tools to identify my passions and a topic I wanted to consider as a lifelong career. It was a joy to use a class assignment to explore using my degree to help the less fortunate in developing countries. When this assignment became less about getting an “A” in my class and more about pursuing my passions, I realized how important library resources are. K-State library resources do more than offer information to students — they serve as the doorway to turn one’s degree into a lifelong pursuit of learning.

OPTIMIZATION MODELS FOR FLASH FLOODING IN DEVELOPING COUNTRIES

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UNDRR

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List of Abbreviations

UNDRR……………………………………………United Nations Office for Disaster Risk Reduction

ISDR…………………………………………….International Strategy for Disaster Risk Reduction

UN……………………………………………………………………………………….United Nations

MCR2030……………………………………………………………..Making Cities Resilient by 2030

MIP…………………………………………………………………...Mixed Integer Programming

RP…………………………………………………………………………………...Relief Point

EP…………………………………………………………………………...Evacuation Point

1. ABSTRACT

This paper details two optimization models which are designed to increase the number of lives saved from flash flooding in developing countries. The evacuation planning model is prescriptive and built to minimize road link failure probability as civilians evacuate the area. The operational planning model is formulated and applied during a flash flood and focuses on minimizing the number of trips taken by public transportation services to evacuate civilians. Both models are discussed in detail and evaluated according to applicability, ability to predict crowd behavior, road link maneuverability with rising water depths, ability to identify risk-averse relief points, and assumption requirements. After evaluating both models, this paper recommends the evacuation planning model as best suited for countries with poor infrastructure or rural environments with limited technological resources. The operational planning model is found to be best suited for countries with large cities and public transportation networks. The paper recommends that the UNDRR hire an industrial engineer familiar with operations research and CPLEX optimization software. This engineer would coordinate with local officials to develop flash flood evacuation plans by tailoring one of these models to fit the specific needs of that country.

2. INTRODUCTION

Established in 1999, the United Nations Office for Disaster Risk Reduction (UNDRR) was created to oversee the International Strategy for Disaster Reduction (ISDR). The ISDR was created by the United Nations (UN) as its primary system for the “coordination of disaster reduction and to ensure synergies among the disaster-reduction activities of the United Nations system and regional organizations and activities in socio-economic and humanitarian fields” [1, pp. 2-3]. With this goal, the UNDRR works to offer leadership to accelerate global disaster risk reduction efforts through its five regional offices that maintain strong relationships with national and local governments, intergovernmental organizations, and the private sector [2].

In 2015, following the third UN World Conference on Disaster Risk Reduction, the UNDRR was tasked with overseeing the implementation, follow-up, and review of the Sendai Framework — a development agenda enacted by the UN to provide member states with concrete actions to reduce disaster risk by 2030 [3]. As stated within the Sendai Framework, "the pursuance of this goal requires the enhancement of the implementation capacity and capability of developing countries ... facing specific challenges, including the mobilization of support through international cooperation ..." [3, p. 12]. This goal requires the UNDRR to utilize its regional offices and existing relationships with different UN member states to determine how to best mobilize resources to enhance risk reduction efforts in developing countries.

In conjunction with the Sendai Framework, the UNDRR has constructed a list of ten essentials [4] for a country to improve disaster risk reduction efforts. While this operational framework — Making Cities Resilient by 2030 (MCR2030) — does propose useful objectives, it does little to expand on them. Essential 8: "Increase Infrastructure Resilience" suggests assessing the capacity of critical infrastructure and improving it when needed but fails to propose any in-depth solutions. The lack of clarity fails to provide developing countries with risk reduction strategies for specific natural disasters and instead offers general principles to aim for. While differences in geographical locations prevent a uniform strategy, providing an in-depth list of risk reduction measures would allow countries to best determine how to address their distinct situations.

While the need to improve in risk reduction exists across all natural disasters, flooding proves particularly deadly. From 2000 to 2019, flooding was one of the deadliest natural disasters and accounted for 104,614 deaths — approximately 9% of all deaths related to natural hazards [5]. With annual average losses from flooding reaching over USD 40 billion in recent years [6, p. 12], a dire need exists for enhanced risk minimization strategies. This is exaggerated by the forecasted increase in coming years in the occurrence and frequency of flooding resulting from climate change and global warming [5]. The combined loss of human life and predicted increase in flooding worldwide exposes the urgency of addressing this natural disaster. Figure 1, below, shows civilians evacuating New Delhi, India, in 2023, where almost 100 deaths resulted from flash flooding.

Figure 1. Flash Flooding Leads to Evacuation in New Delhi, India (Reproduced from [7])

Identifying potential mathematical optimization models to supply developing countries with specific tools to aid in risk minimization would provide effective, tangible methods of reaching the goals outlined by MCR2030 instead of leaving nations to identify such methods themselves. Most of these optimization models involve minimizing or maximizing an objective function, or goal, that is subject to a list of constraints. These constraints narrow a broad objective function to only work under the parameters assigned based on real-life constrictions.

One of the most common methods uses a Mixed Integer Programming (MIP) model, meaning that some decision variables are restricted to integer values at the optimal solution. Utilizing this model would greatly assist in the construction of constraints. For example, integer values could describe the number of relief points, or “locations that can be used for the accommodation of evacuated people” [8, p. 2]. Since there cannot be partial numbers of relief points, integer values are needed. However, when determining the least-cost path of evacuation, planners should use non-integer values to calculate distance. A relief supply network could be developed to visualize and solve this problem. Labeling nodes for evacuation zones, distribution centers for humanitarian aid, and flooding relief points [9] allow the assessment of transportation costs. With extensive research performed in this area due to consumers’ demand for quicker transportation of goods, an MIP model can be modified to determine the best evacuation routes during floods.

The UNDRR's primary objective is to help decision makers across the globe better understand and act on risk. With the number of people in need of humanitarian assistance because of natural disasters projected to double to 200 million annually by 2050 [10], the need for additional support in disaster risk management continues to rise. With the UNDRR's access and reach as the lead agency for the coordination of disaster risk reduction within the United Nations, it would have the technical information and data needed to utilize the latest mathematical optimization models to reduce evacuation times. Therefore, the purpose of this report is to evaluate mathematical tools and optimization strategies that identify a least-cost path of evacuation for refugees during a flood and how the UNDRR can utilize these tools to equip developing countries with the proper amount of preparation and planning.

3. EVACUATION MODELS

In sections 3.1 through 3.2, I discuss two optimization models that have been developed to aid in flash flood evacuation efforts. Two methods have been proposed to construct an objective function for a model: minimizing risk and minimizing the number of evacuation trips. The evacuation planning model, discussed in section 3.1, aims to determine the most risk-averse paths for evacuation. The operational planning model, discussed in section 3.2, focuses on minimizing the number of evacuation trips that different modes of public transportation take to evacuate civilians.

3.1 Evacuation planning

The first optimization method I discuss is the evacuation planning model. The objective of this model is to minimize risk by directing proportions of population centroids through various road links to relief points (RPs) in accordance with their stated capacities. The model was developed by Nitheesh et al. [8] in three iterations. The three iterations of the model and its mathematical formulations are found in sections 3.1.1 through 3.1.3.

3.1.1 Iteration 1

For the first iteration of this model, Nitheesh et al. constructed a base model representing previous research performed in risk-averse evacuation trip distribution models. In this model, emphasis is given on a population centroid's (k_n) proximal location to any RP (s_n). The population centroids are dispersed amongst RPs in order of 1) closest proximity and 2) capacity. The size of every s_n shown in the first iteration represents its capacity. Figure 2, shown below, depicts the visual representation of the mathematical function of the model. The lines connecting every k_n and s_n represent road links, which are viable routes through which any k_n can travel. The arrows between each k_n and s_n represent the allocated trip distributions from any k_n to s_n after the iteration has been performed. This iteration neglects failure probabilities for any road link and assumes each RP has a fixed capacity.

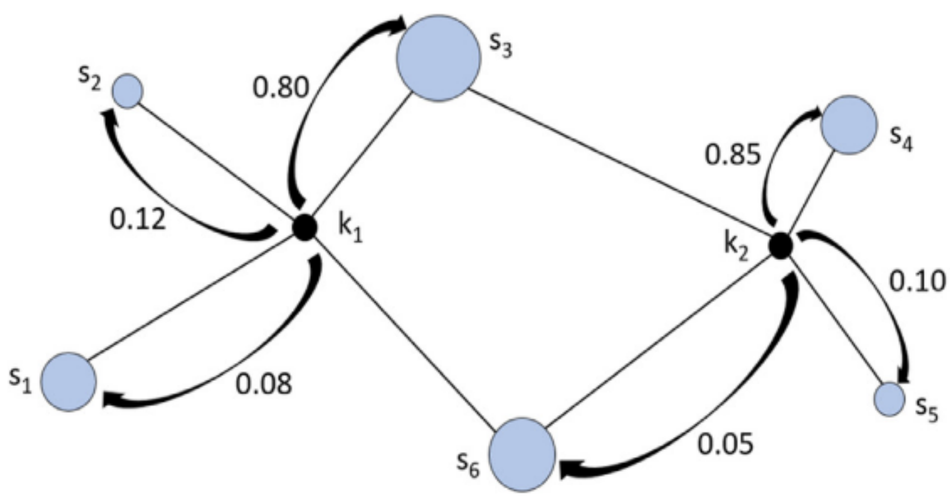

Figure 2. Hypothetical Network for Iteration 1 (Reproduced from [8])

As shown in Figure 2, the first iteration allocates populations to several RPs:

1. Centroid k_1 distributes 80% of its population to s_3, the closest RP with the largest capacity, followed by 12% to s_2 (smallest capacity) and the remaining 8% to s_1 (medium capacity). Centroid k_1 is connected to RP s_6 but does not allocate any resources to it.

2. Centroid k_2 distributes 85% of its population to s_4, the closest RP with medium capacity, followed by 10% to s_5 (smallest capacity) and the remaining 5% to s_6. Centroid k_2 is connected to RP s_3 but does not allocate any resources to it.

The purpose of this model is to minimize trip distribution distance by dispersing populations to their nearest RPs. This is proven in the hypothetical model, as centroid k_2 disperses most of its population to RP s_4, which has less capacity than s_3, which is not allocated to any of k_2's population. Table 1 defines each variable used in the mathematical functions for iterations 1 through 3.

Table 1. Variable Definitions for Iterations 1-3 (Reproduced from [8])

Notations.

Symbol	Definition
i	Population centroid
j	Relief point
N	Number of population centroids
M	Number of relief points
a	Link (or) edge in a path, where $a \in S_{ij}$, the collection of links on a path chosen by the analyst between i and j
R	Number of links (or) edges in a path
d_i	Demand at a population centroid
c_j	Capacity at a relief point
f_j	Facility establishment cost at a relief point (i.e. per capita capacity enhancement cost)
x_{ij}	Decision variable indicating the proportion of population to be evacuated from population centroid i to relief point j
l_{ij}	Distance between population centroid i and relief point j
ξ_{ij}^{a}	Failure probability associated with the road link a between i and j
B	Budget available for overall capacity enhancement in the network

As shown above, Table 1 describes the meaning for each variable used in the mathematical function. Figure 3, below, shows iteration 1's mathematical representation.

$$\min z = \sum_{i=1}^{M} \sum_{j=1}^{N} l_{ij} x_{ij}$$

$$\sum_{i=1}^{N} d_i x_{ij} \leq c_j \quad (\text{for } j = 1 \text{ to } M)$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad (\text{for } i = 1 \text{ to } N)$$

$$0 \leq x_{ij} \leq 1$$

Figure 3. Mathematical Representation for Iteration 1 (Reproduced from [8]).

The objective of this function is to minimize z, where z equals the product of the distance between population centroid i and RP j and the decision variable that determines what percentage of the population can be evacuated from i to j. This product is summed from i=1 to M and j=1 to N. The first constraint equation sums the product of population centroid demand and the proportion of the population allocated to a given RP from i=1 to N and requires that this product is less than or equal to the capacity at any RP. This prevents the total number of evacuees leaving k_n from ever exceeding the capacity at s_n. The second constraint requires the sum of the proportions of population being evacuated from k_n to equal 1. This ensures that all evacuees leave k_n and prevents an infeasible scenario of more civilians evacuating than are currently at k_n. The final constraint requires the proportion of evacuees be a value between 0 and 1. There cannot be a negative proportion of evacuees departing nor can there be more civilians evacuating than are present.

3.1.2 Iteration 2

While most previous research focuses on any k_n's proximity to s_n, Nitheesh et al. propose an alternative solution that accounts for the failure probability of any given road link. This is implemented by evaluating landslide susceptibility maps in the specified region to determine proclivity for natural disasters. These maps describe the potential for future landsliding in the area based on its natural geographical features [11]. All notations remain the same as in iteration 1, except the objective of the model is changed to prioritize road links with the lowest failure probability. On the hypothetical network, this is seen through color-coding each road link according to its failure probability. The color-coding key is shown in Table 2 below.

Table 2. Road Link Colors Corresponding to Failure Probabilities (Adapted from [8])

Road Link Color	**Failure Probability**
Green	Very Low
Yellow	Low
Blue	Medium
Gray	High
Red	Very High

The population centroids are dispersed among RPs in order of 1) road link failure probability and 2) capacity. This iteration assumes each RP is given a fixed resource capacity. Figure 4, below, shows the second iteration of the evacuation trip distribution on a hypothetical network.

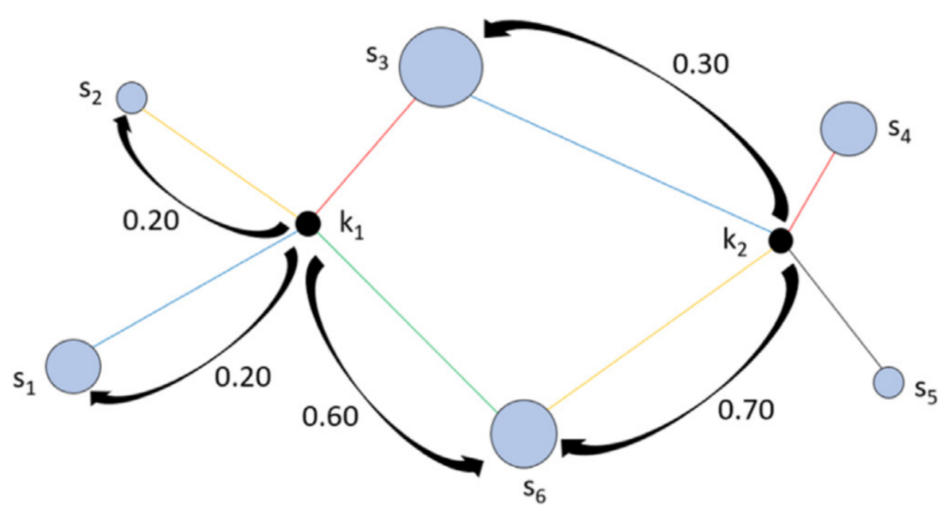

Figure 4. Hypothetical Network for Iteration 2 (Reproduced from [8])

As shown in Figure 4, iteration 2 allocates populations to several RPs:

1. Centroid k_1 distributes 60% of its population to s_6, the connected road link with the lowest failure probability. This is followed by 20% distributed to both s_2 and s_3, which have the following lowest failure probabilities. Centroid k_1 is connected to RP s_3 but does not allocate any resources to it.
2. Centroid k_2 distributes 70% of its population to s_6, the connected road link with the lowest failure probability. The remaining 30% of its population is distributed to s_3, the road link with the next lowest failure probability. Centroid k_2 is connected to RPs s_4 and s_5 but does not allocate any resources to them.

This model minimizes the overall risk of the network by dispersing populations to RPs with the lowest failure probabilities. This is proven in the model, as centroid k_1 disperses most of its population to RP s_6, which is farther away and has less capacity than s_3. Further, centroid k2 disperses all its population to RPs s3 and s6, which are farther away from s4 and s5. Figure 5, below, shows iteration 1's mathematical representation.

$$\min z = \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} (\Pi_{a=1}^{R} \ \xi_{ij}^{a})$$

$$\sum_{i=1}^{N} d_i x_{ij} \leq c_j \quad (\text{for } j = 1 \text{ to } M)$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad (\text{for } i = 1 \text{ to } N)$$

$$0 \leq x_{ij} \leq 1$$

Figure 5. Mathematical Representation for Iteration 2 (Reproduced from [8])

The objective of this function is to minimize *z*, where *z* equals the product of the proportion of evacuees departing population centroid *i* to RP *j* times the constant π from a=1 to *R* and the failure probability assigned to road link *a* between *i* and *j*. The final two constraints are identical to the previous iteration.

3.1.3 Iteration 3

The third and final iteration proposed by Nitheesh et al. is almost identical to the second, with the exception that the capacity at s_n can be increased. After solving the model, the capacity increase required for s_n can be identified. Figure 6, below, shows the third iteration of evacuation trip distribution on a hypothetical network.

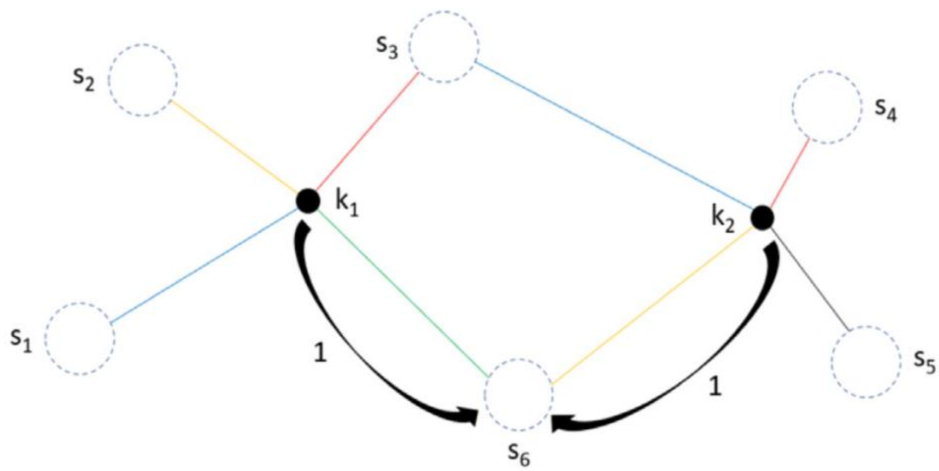

Figure 6. Hypothetical Network for Iteration 3 (Reproduced from [8])

As shown in Figure 6, iteration 3 allocates both population centroid's k_1 and k_2 to s_6. Since the capacity at s_n can be increased to any amount, all k_n are allocated to the s_n with the lowest failure probability road link.

Susceptibility maps are used to predetermine whether enhancing capacity at any RP will minimize risk. The highest proportion of people from either population centroid is allocated to the RP with the lowest risk path. Without a specified budget, all of a given centroid is routed to the RP with the lowest failure probability path. This RP is expected to enhance its capacity to support the resulting increase in resource supply. Figure 7, below, shows iteration 3's mathematical representation.

$$\min z = \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij}(\Pi_{a=1}^{R}\ \xi_{ij}^{a}) + \sum_{j=1}^{M} \left[\sum_{i=1}^{N} d_i x_{ij} - c_j \right] f_j$$

$$\sum_{j=1}^{M} x_{ij} = 1 \quad (\text{for } i = 1 \text{ to } N)$$

$$\sum_{j=1}^{M} \left[\sum_{i=1}^{N} d_i x_{ij} - c_j \right] f_j \leq B$$

$$0 \leq x_{ij} \leq 1$$

Figure 7. Mathematical Representation for Iteration 3 (Reproduced from [8])

The objective of this function is to minimize z. First, the sum of the product of population centroid *i*'s demand times the proportion of evacuees minus the capacity at a RP *j* is taken from *i=1* to *N*. This is then multiplied by the facility establishment cost at RP *j* from *j*=1 to M and the total is added to the objective function of iteration 2, which accounted for the sum of failure probabilities for all road links traveled. The constraints that require the total proportions of evacuees to equal 1 and any proportion to be between 0 and 1 are kept from iterations 1 and 2. Additionally, a constraint is added to ensure that capacity enhancement costs do not exceed the available budget *B*. This is accounted for by constraining the new, capacity enhancement section of the objective function in this iteration to be less than or equal to the available budget.

3.2 Operational Planning

The operational planning model uses modes of public transportation to evacuate an uncertain number of people in a highly populated area in real time. Its objective is to find the minimum number of vehicles and their trips required to save all evacuees before water reaches its predicted areas. A three-hour short notice evacuation time is given to the model [12, p. 4], which allows one hour for decision making and two for evacuating civilians from the designated evacuation points (EPs) to RPs. Different service times are provided at each EP and RP (shelter) to account for flood propagation. Public transportation is used to retrieve resources (civilians) from their respective EPs and take them to their designated shelters. To account for random population distribution, Poisson distribution is employed to give the most accurate prediction of EP locations. Civilians are expected to know where the nearest EP is and travel to it immediately following an evacuation warning. Figure 8, below, shows specific routing assignments to each node with a specified closing time window.

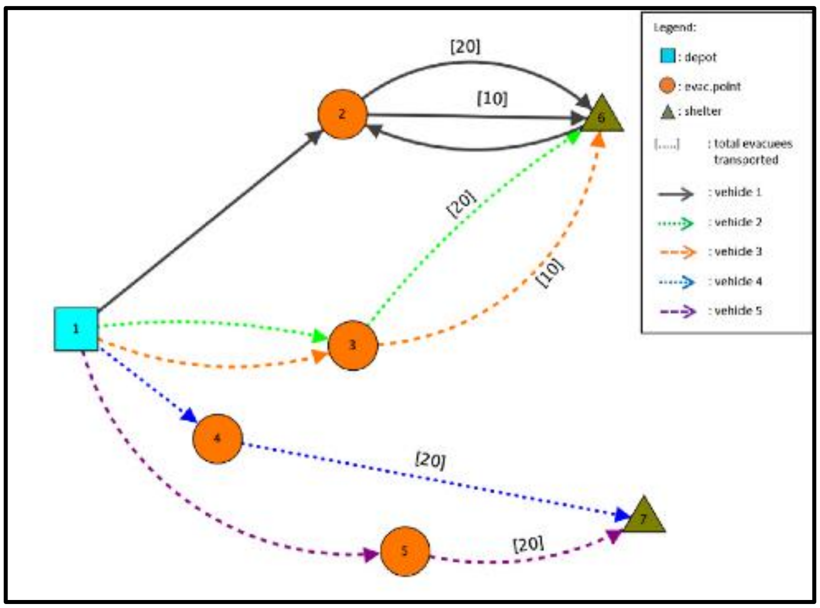

Figure 8. Optimized Vehicle Routing Assignments (Reproduced from [12]).

Table 3 displays the results obtained from this model.

Table 3. Results Obtained from the Operations Planning Model (Adapted from [12])

Vehicle	# Trips	# Evacuees	Evacuation Time (min.)
1	2	30	100
2	1	20	90
3	1	10	80
4	1	20	70
5	1	20	60

As shown in Figure 8 and Table 3, routing assignments are given as follows:

1. Vehicle 1 travels from the Depot to Node 2, where it takes two trips to evacuate 30 civilians in 100 minutes.
2. Vehicles 2 and 3 travel from the Depot to Node 3. Vehicle 2 takes one trip to evacuate 20 civilians in 90 minutes, while Vehicle 3 takes one trip to evacuate 10 civilians in 80 minutes.
3. Vehicle 4 is routed from the Depot to Node 4, where it takes one trip to evacuate 20 civilians in 70 minutes.
4. Vehicle 5 departs from the Depot to Node 5, where it takes one trip to evacuate 20 civilians in 60 minutes.

Tables 4 through 6, shown below, define the sets, parameters, and decision variables for the Operation Planning model's mathematical representation.

Table 4. Sets for Operational Planning's Mathematical Representation (Adapted from [12])

Set	**Description**
0	The depot
C	$\{1, 2, 3, ...N\}$ — a set of EPs
S	$\{N+1, N+2, ..., N+M\}$ — *a set of temporary shelters*
V	$0 \cup C \cup S$ — a set of all nodes
K	Set of homogenous vehicles
R	Set of vehicle trips

Table 5. Parameters for Operational Planning's Mathematical Representation (Adapted from [12])

Parameter	Description
d_i	Demand (number of evacuees) at an EP $i \in C$
t_{ij}	Traveling time associated with arc (i, j) where $i, j \in V$, $i \neq j$
Q	Capacity of vehicles
L_i	Capacity of shelter $\in S$
st	Service time at any node in $C \cup S$
T_{max}	Maximum evacuation completion time (clearance time)
M	A big number

Table 6. Decision Variables for Operational Planning's Mathematical Representation (Adapted from [12])

Decision Variable	Description
x_{ijkr}	1 if vehicle $k \in K$ in trip $r \in R$ travels directly from node $i \in V$ to node $j \in V$; 0 otherwise
z_k	1 if vehicle $k \in K$ is dispatched; 0 otherwise
y_{ikr}	1 if vehicle $k \in K$ in trip $r \in R$ visits node $i \in C \cup S$; 0 otherwise
f_{kr}	1 if vehicle $k \in K$ is assigned to trip $r \in R$; 0 otherwise
p_{ikr}	Number of demand (evacuees) from EP $i \in V$ picked up by vehicle $k \in K$ in trip $r \in R$
q_{ikr}	Number of evacuees dropped off to shelter $i \in S$ by vehicle $k \in K$ in trip $r \in R$
a_{ikr}	Arrival time of vehicle $k \in K$ in trip $r \in R$ at node $i \in V$

The given sets, parameters, and decision variables are then utilized to form the objective function, given below in Figure 9.

$$\min \sum_{j \in C} \sum_{k \in K} x_{0jk1}$$

Figure 9. The Operational Planning Model's Objective Function (Reproduced from [12])

As shown in Figure 9, the objective of the function is to minimize the number of vehicles dispatched to move all evacuees from EPs to shelters. Figure 10, below, shows the first 10 constraints for the Operational Planning's mathematical representation.

$$\sum_{j \in C} x_{0jk1} = z_k \quad \forall k \in K$$

$$f_{k1} = z_k \quad \forall k \in K$$

$$\sum_{r \in R} f_{kr} \geq 1 \quad \forall k \in K$$

$$\sum_{i \in C} \sum_{j \in S} x_{ijkr} \leq f_{kr} \quad \forall r \in R, \forall k \in K$$

$$\sum_{i \in V} \sum_{j \in V} \sum_{r \in R} x_{ijkr} \leq M \cdot z_k \quad \forall k \in K$$

$$\sum_{i \in V, i \neq h} x_{ihkr} \leq \sum_{j \in C \cup S, h \neq j} x_{hjkr} \quad \forall h \in C, \forall k \in K, \forall r \in R$$

$$\sum_{j \in C} x_{hjk(r+1)} \leq \sum_{i \in C} x_{ihkr} \quad \forall h \in S, \forall k \in K, \forall r \in 1, 2, \ldots |R| - 1$$

$$\sum_{k \in K} \sum_{r \in R} p_{ikr} = d_i \quad \forall i \in C$$

$$\sum_{i \in V, i \neq j} \sum_{k \in K} \sum_{r \in R} x_{ijkr} \geq 1 \quad \forall j \in C$$

$$\sum_{i \in C} p_{ikr} \leq Q \cdot z_k \quad \forall k \in K, \forall r \in R$$

Figure 10. Constraints 1-10 of the Operations Planning Model (Reproduced from [12])

Constraint 1 requires each dispatched vehicle to start initially from the depot [12]. For all vehicles used, constraints 2 and 3 assign them to a trip to pick up civilians. Constraint 4 makes sure "a vehicle will end the trip from a shelter on the second, third, and subsequent trips" [12]. If a vehicle is not deployed, constraint 5 prevents it from traveling on a path. The sixth constraint forces the vehicle to leave the EP immediately after evacuees are onboard. Constraint 7 ensures the vehicle returns to the same EP as in the previous trip unless all evacuees have been cleared. Constraint 8 guarantees the transportation of all evacuees. The greater-than or equal-to symbol in constraint 9 allows for any EP to be serviced by more than one vehicle if needed. Lastly, constraint 10 prevents any vehicle from acquiring more demand than its available capacity. Figure 11, below, shows constraints 11-16 in the model.

$$p_{ikr} \leq Q \cdot y_{ikr} \quad \forall i \in C, \forall k \in K, \forall r \in R$$

$$q_{ikr} \leq Q \cdot y_{ikr} \quad \forall i \in S, \forall k \in K, \forall r \in R$$

$$\sum_{i \in V, i \neq j} x_{ijkr} = y_{jkr} \quad \forall j \in C, \forall k \in K, \forall r \in R$$

$$\sum_{i \in C} x_{ijkr} = y_{jkr} \quad \forall j \in S, \forall k \in K, \forall r \in R$$

$$\sum_{i \in C} p_{ikr} = \sum_{i \in S} q_{ikr} \quad \forall k \in K, \forall r \in R$$

$$\sum_{k \in K} \sum_{r \in R} q_{ikr} \leq L_i \quad \forall i \in S$$

Figure 11. Constraints 11-16 of the Operations Planning Model (Reproduced from [12])

Constraints 11 and 12 describe the relationships between each vehicle on its respective trip and each vehicle and its number of evacuees dropped off to a shelter [12]. The respective number of trips and number of evacuees must be less than or equal to the vehicle capacity times the binary variable y_{ikr}, which indicates whether a vehicle is dispatched on a path. Constraint 13 allows only one path from any point entered by a vehicle because it can only enter one way to reach an EP. Similarly, constraint 14 allows only one entrance point to the shelter. The number of civilians retrieved from an EP must be the same as the amount dropped off at a shelter, according to constraint 15. Lastly, constraint 16 states that the number of evacuees dropped off to a shelter must be less than or equal to the capacity of the given shelter. Figure 12, below, shows constraints 17-19 of the model.

$$a_{ikr} + (lt \cdot p_{ikr}) + t_{ij} \leq a_{jkr} + M \cdot (1 - x_{ijkr}) \quad \forall i \in C, \forall j \in C \cup S, \forall k \in K, \forall r \in R$$

$$a_{ikr} + (ult \cdot q_{ikr}) + t_{ij} \leq a_{jk(r+1)} + M \cdot (1 - x_{ijk(r+1)}) \quad \forall i \in S, \forall j \in C, \forall k \in K, \forall r \in 1, 2, \ldots |R| - 1$$

$$a_{ikr} \leq b_i \quad \forall i \in V, \forall k \in K, \forall r \in R$$

Figure 12. Constraints 17-19 of the Operations Planning Model (Reproduced from [12])

Constraint 17 accommodates different closing time windows at each EP by protecting trip continuity [12]. The same continuity protections are added for shelters in constraint 18. Next, constraint 19 makes sure each vehicle has serviced its designated EP before the flood reaches it. Figure 13, below, shows constraints 20-26 of the model.

$$x_{ijkr} \in \{0,1\} \quad \forall i,j \in V, \forall k \in K, \forall r \in R$$

$$z_k \in \{0,1\} \quad \forall k \in K$$

$$y_{ikr} \in \{0,1\} \quad \forall i,j \in C \cup S, \forall k \in K, \forall r \in R$$

$$f_{kr} \in \{0,1\} \quad \forall k \in K, \forall r \in R$$

$$p_{ikr} \geq 0 \quad \forall i \in C, \forall k \in K, \forall r \in R$$

$$q_{ikr} \geq 0 \quad \forall i \in C, \forall k \in K, \forall r \in R$$

$$a_{ikr} \geq 0 \quad \forall i \in V, \forall k \in K, \forall r \in R$$

Figure 13. Constraints 20-26 of the Operations Planning Model (Reproduced from [12])

Constraint 20-23 constrain binary variables x_{ijkr}, z_k, y_{ikr}, and f_{kr} to values of either 0 or 1 [12]. The final three constraints require non-negative values for the number of evacuees picked up from the EP, the number of evacuees dropped off to the shelter, and the arrival time of any vehicle.

4. EVALUATION OF THE MODELS

The feasibility of these optimization models can be evaluated based on several criteria. In sections 4.1 through 4.5, I evaluated both optimization models on these five criteria:

applicability, ability to predict crowd behavior, road link maneuverability with rising water depths, ability to identify risk averse relief points, and assumption requirements.

4.1 Applicability

Applicability is the first model trait that I discuss. Sudden flooding makes it impossible for people to move and increases disaster risk, so people need to be evacuated quickly [13, p. 18]. As a result, the point of a flash flood in which an optimization model is designed to be applied is critical. Sections 4.1.1 and 4.1.2 evaluate both the evacuation and operational planning models regarding their respective timeframes of application.

4.1.1 Evacuation Planning

The planning model is prescriptive [8], meaning that it is constructed and applied by a planner before a flash flood strikes. This person also uses susceptibility maps to determine where to place EPs and RPs. This preparation could prove vital to saving more lives because it allows the city to take immediate action. However, this model fails to account for sudden changes in road and traffic conditions. For example, a flash flood warning announced by the city may result in masses of civilians attempting to flee through road links that are incapable of sustaining that much traffic. As a result, there may be delays in travel time that force civilians to travel in unsafe conditions.

4.1.2 Operational Planning

The operation model is designed to be applied in real time. This allows the planner a better assessment of the situation, allowing for more precise EP and RP locations. However, it also requires foreknowledge and quick action from civilians. Residents of the city would need to memorize where the nearest EP is relative to their current position and move there quickly and in an orderly manner. Further, it assumes the proper public transportation officers are available to operate vehicles and public authorities know exactly how many vehicles will be needed. Another assumption of this model is that traffic conditions will be negligible, which is not reasonable to assume. Were this model implemented, extended research should be performed to account for potential variability in traffic flow due to many civilians evacuating on foot or in personal vehicles.

4.2 Ability to predict crowd behavior

In sections 4.2.1 through 4.2.2, I evaluate the evacuation and operational planning models based on each one's ability to predict crowd behavior. Moussaid et al. [14, p. 1] found that up to 70% of pedestrians walk in groups at any given time. Additionally, studies conducted by Bode et al. [15, p. 1] have revealed the presence of social groups increases evacuation times. With the large number of social groups in public areas and research proving the number of social groups directly impact evacuation times, it's important to find a model that accounts for crowd behavior. Without attention to this criterion, one cannot expect a realistic output from an optimization model focusing on evacuating civilians.

4.2.1 Evacuation planning

The evacuation planning model is designed to be performed before a flash flood occurs. Due to the predictive nature of this model, crowd fluctuations are unaccounted for and population centers are assumed to be precisely where the planner has predicted them to be. This could prove disastrous if flash flooding were to occur during an event in which population centers were abnormally condensed (i.e. a holiday). In this case, EPs closest to these population centers would be packed with evacuating civilians while EPs further away would be underutilized. The model attempts to resolve this by assigning road links with longer travel times for evacuees, but the increase in population size may still prove too extreme. In this case, either RP capacities nearby would need to be expanded or the planner would need to determine additional RPs in real time and convey the information to the civilians.

4.2.2 Operational planning

In the evacuation operation model, the Poisson distribution is used to evaluate the best EP locations within a given city. According to Kissell and Posering, the Poisson distribution is "...a discrete distribution that measures the probability of a given number of events happening in a specified time period" [16, p. 126]. In this case, Poisson distribution accounts for the different responses of each civilian to an evacuation order [12, p. 2]. The event in this scenario represents the location of civilians in any given city sector. This has a mean rate of occurrence, as civilians have predictable flow patterns to different areas of the city (e.g., place of work, in home, and place of other activities). Thus, applying Poisson distribution is the most accurate way to determine EP locations and account for a population's stochastic nature at different time intervals. While this method is unable to account for atypical events (e.g., a cultural holiday) that lead to skewed proportions of the population in certain city sectors, its assessment of EPs based on average population distributions is most effective for a model being applied in real time.

4.3 Road link maneuverability with rising water depths

In sections 4.3.1 through 4.3.2, I discuss the maneuverability of any road link for each optimization model with rising water depths. With masses of civilians needing to evacuate the given area, the model must account for how the road links will support the influx of traffic. As little as six inches of moving water can knock someone over and a foot of water can move a vehicle [17], so keeping road links open to traffic is critical to reducing civilian injuries and fatalities during flash flooding. Figure 14, below, shows citizens of Lahore, Pakistan, evacuating flash flooding with a severely limited range of motion.

Figure 14. Flash Flooding Slows Evacuations in Lahore, Pakistan (Reproduced from [18])

4.3.1 Evacuation planning

In the evacuation planning model, traffic times are not accounted for. This model is specifically designed to evaluate the most risk-averse evacuation route possibilities before a flash flood hits. Due to its prescriptive nature, Nitheesh et al. have chosen to disregard traffic patterns. This is an obvious weakness of the model, as traffic flow can greatly influence civilians' abilities to move through the risk-averse routes to their relief points. Additionally, no adjustments are made for how travel times are affected by rising water levels. As Dias et al. illustrated, an increase in water level directly relates to civilian travel time, with the most significant impact occurring when depths surpass one's knees [5]. A path with greater distance may prove quicker and more risk-averse than one that requires civilians to wade through floodwaters.

4.3.2 Operational planning

In the first iteration of the operational planning model, a closing window and service times for each EP and RP are used to maximize the number of civilians transported from an EP before water levels become unsafe. The customized service window predicts how long it will take civilians to load and unload at any EP and RP. While these features allow the model to account for water depth levels, they fail to account for potential traffic times. For example, one EP may have a city bus traveling to its location with only minutes before its water levels become unsafe. If the bus encounters standing traffic, it would be unable to reach the civilians before the safe time window closes. Further analysis must be conducted to account for this additional constraint.

4.4 Ability to identify risk averse relief points

Musolino et al. conducted a study in 2022 that aimed to improve civilian safety during evacuation in flash flooding events [19, p. 1943]. While previous research in this field has often based evacuation plans on the shortest distance paths to an evacuation point, it has become clear that the shortest path is not always the safest [19, p. 1960]. This idea was further supported in research by Nitheesh et al., who found the lowest risk path — the path least likely to reach unsafe flood levels — does not have to be the shortest path [8, pp. 15-16]. Thus, the model's method for determining risk averse relief points must be evaluated. In sections 4.4.1 through 4.4.3, I evaluate the evacuation and operational planning models based on each one's ability to identify risk averse relief points.

4.4.1 Evacuation planning

Iteration 1 of the evacuation planning model sends population centroids to their nearest RPs first. Once these are at capacity, civilians are sent to other RPs in order of decreasing proximity to the centroid. This successfully minimizes road link distance and is most often used in similar research. In iteration 2, the population centroids are disbursed in accordance with the lowest failure probabilities of any given road link. Most civilians travel down the road link with the lowest failure probability, but once that road link's capacity has been reached future civilians will follow in order of increasing failure probabilities. Iteration 3 follows the same order as the previous iteration but assumes capacities at any RP can be enhanced in accordance with a designated budget for such enhancements. Without a specified budget in the model presented by Nitheesh et al., all population centroids travel to their RP with the lowest failure probability. This iteration minimizes risk but needs additional information about the RP capacities and resources (i.e., time, money, and labor) needed to enhance them.

4.4.2 Operational planning

In the evacuation operation model, EPs are determined beforehand by calculating the average walking time from the Indonesian Disaster Management Board [12, p. 13]. This source was primarily chosen by Insani et al. due to their case study's focus on Indonesia. EPs are mapped out in the city according to population centers and civilian travel time. Additionally, several shelters are selected from public spaces to send civilians away from the flooding. This model provides common spaces where evacuees can congregate but also assumes they will be available.

4.5 Assumption requirements

Optimization models such as those discussed in this paper require assumptions about circumstances outside the scope of the objective function to achieve a solution. These assumptions, while not precisely correct, guide the model toward an effective solution. In sections 4.5.1 through 4.5.2, I discuss the assumptions required and whether these assumptions compromise the reliability of each model.

4.5.1 Evacuation planning

The evacuation planning model's main weakness is its failure to account for traffic conditions during flash flooding. While Nitheesh et al. include failure probabilities in any given path in the second and third iterations of their model [8], they also admit the need for future studies to evaluate the risk as it relates to traffic assignments. The failure probabilities are determined based on an area's susceptibility to natural disasters, but this has no correlation to traffic flow.

Another assumption made is that this model can handle significantly higher quantities of EPs and RPs without failure. The simplified model proposed by Nitheesh et al. allows one to easily grasp the model's concept but assumes no issues will arise if it were significantly upscaled [8, p. 16]. The model also does not account for the possibility of one road link being a member of multiple paths. Nitheesh et al. assume that any one EP will travel through exactly one road link to reach one RP. However, the possibility exists in a larger network that efficiency may be increased if civilians from one EP travel through a network of road links, dropping off various proportions of its population at different RPs along the way to its destination. Further research should be performed to determine how to account for this possibility.

In the second and third iteration, Nitheesh et al. predict failure probabilities of each path using region-specific susceptibility maps [8]. However, such maps only predict an area's vulnerability to natural disasters and are not precise indicators. The disaster will likely not occur in exact alignment with the susceptibility map's predictions.

The third iteration assumes that any RP can increase capacity, so long as it does not violate the budget constraint. For the capacities of RPs which are determined to be fixed, enhancement costs can be set to be significantly larger values [8, p. 15]. This iteration assumes the availability of a financial budget for capacity enhancement but does not account for the resulting man-hours and labor availability required to enhance such capacities. A planner would have to determine these unknowns and incorporate them into the model to assess whether capacity enhancement is feasible for any RP.

4.5.2 Operational planning

The evacuation operation's greatest failure is that traffic conditions are negligible [12, p. 16]. This is unreasonable because any one area may experience traffic congestion due to mass evacuation. In this scenario, public transportation used to evacuate civilians will be unable to move in a timely fashion to their designated RPs and will therefore be unable to evacuate all civilians within the assigned time. This would occur in areas where flooding is rapidly approaching, as panic from civilians may incite traffic congestion which prohibits the amount of flow through a road link.

The model also assumes 60% of the total population will choose to use public transportation to evacuate [12, p. 4], with the remaining 40% presumed to remain in place during the flooding. This does not, however, account for road space consumed by civilians' personal modes of transportation. Personal civilian transportation will consume greater road space and increase the likelihood of traffic buildup, therefore delaying evacuation vehicles sent by the city.

The capacity of any EP is based on the Poisson distribution of population sizes and the average civilian walking speed. This walking speed was derived by Insani et al. from the standard times set by the Indonesian Disaster Management Board [12, p. 13]. While the model's formulation is completely designed by Insani et al., these standard walking times maximize the strategic placement of EPs within a given city. However, these walking speeds fail to account for demographic differences within a city. For example, a sector with a larger elderly population will likely have a slower average walking speed than one with a younger population. A more precise measurement of walking speeds that considers this would increase the model's accuracy. Figure 15, below, shows residents of Melbourne, Australia, rescuing someone without the ability to evacuate independently.

Figure 15. Australians Assisting Dependent Evacuee (Reproduced from [20])

As shown in Figure 15, civilians without the capabilities to evacuate on their own must rely on others, which increases the evacuation time for all involved parties. Additionally, Insani et al. fail to consider variations in geographical features in different city sectors [12, p. 16]. This could result in some evacuation vehicles requiring an unreasonable amount of time to evacuate civilians from an EP if the geographical conditions prove challenging (e.g., one-lane roads with tight turns for a large bus, large incline/declines, road construction).

This model also assumes that authorities know the exact number of vehicles that are required to be ready once the evacuation warning is issued [12, p. 16]. The number of vehicles available in the evacuation's preliminary stages is historically insufficient [12, p. 16]. Insani et al. address this by minimizing the number of vehicles required and implementing a time horizon in which drivers can evacuate civilians. The three-hour time window given to plan and initiate evacuation efforts accounts for potential lag time in the initial stages of the evacuation.

5. CONCLUSION

Flash flooding will turn serene environments into chaos as panicking civilians flee in any direction. As an office specializing in reducing risk, identifying ways to improve civilian evacuation strategies should be at the forefront of the UNDRR's efforts against flash flooding. Successfully implementing mathematical optimization strategies to aid evacuation efforts, particularly in developing countries with poor infrastructure, could prove essential in saving lives. It is important to identify an appropriate model for such instances and develop constraints which simulate real-life conditions.

The evacuation planning model determines its RPs, EPs, and road links in advance of a flash flood. This is valuable because a plan can be well developed ahead of any potential flood. However, this also raises concerns regarding the reliability and feasibility of the model. Since this model is prescriptive, the UNDRR could assess any region's susceptibility maps, geographical features, and population distributions to develop an evacuation plan. This would be most beneficial to countries with lower levels of education and limited technological resources, as local officials could receive an evacuation plan without needing to understand the mathematical process behind its formulation.

The operational planning model is planned in real time, minimizing the number of public transportation vehicles needed to evacuate civilians from RPs in a city. This model's instant applicability allows for the most accurate results, but also faces short time constraints and issues relaying information to all parties. The use of this model requires it to be applied in real time. Were the UNDRR to recommend this model, it would require someone (either a UNDRR representative or local official) with an in-depth understanding of how to apply the model to be residing in the area when a flash flood warning is issued. This model is most beneficial to countries with large cities that rely heavily on public transportation and contain residents with the mathematical foreknowledge to understand how to operate such models.

6. RECOMMENDATIONS

Based on my conclusions, I recommend that the UNDRR takes the following actions:

1. Hire an industrial engineer with foreknowledge of operations research and its mathematical optimization strategies
2. Coordinate the industrial engineer and country officials to develop a specific flash flooding evacuation plan for a given area
3. Recommend the evacuation planning model to developing countries with poor infrastructure or rural environments with limited technological resources
4. Recommend the operational planning model to large cities in developing countries which have public transportation infrastructure

An industrial engineer with a detailed knowledge of operations research can accurately assess and apply these mathematical optimization strategies to fit any given place effectively. While both the evacuation and operational planning models offer a solid framework to begin an evacuation plan, many unknowns exist. Each country has distinct geographical and cultural features that must be inputted into the model. Further, an industrial engineer would also possess the ability to use CPLEX or another optimization software to successfully build and run one of these models. Without an understanding of such software, the model is unable to be evaluated in an appropriate timeframe.

The industrial engineer must coordinate with local officials to design and implement a model that will respond well to the needs of any specific country. This must be done in person and on-site, so the engineer can observe the geographical landscape and identify critical constraints that would have potentially been unaccounted for otherwise. This coordination allows the engineer to tailor a model to suit any given country's needs. It also creates an opportunity for the engineer to teach some of these processes to city officials or qualified personnel. Over time, nationals may acquire enough knowledge on this subject to implement and adjust models for their countries independently.

The evacuation planning model should be recommended to developing countries which have poor infrastructure or for rural environments with limited technological resources. This model was designed to run in advance of a flash flood. For countries without infrastructure to implement evacuation procedures quickly, this type of model would work best. Similarly, rural areas of developing countries are often severely lacking in technological resources and would be unable to provide enough public transportation to evacuate civilians. Further, a lack of high-speed internet access or nationals with the degree of education required to use an optimization software like CPLEX would prove adjusting a model in real time impossible.

The operational planning model should be implemented in developing countries which contain large cities with large public transportation networks. This model is easiest to apply in this scenario, as many civilians already depend on public transportation for traveling. The industrial engineer could develop an initial optimization model that utilizes current public transportation routes and perform adjustments as needed when a flood occurs. Larger cities may also have nationals possessing higher levels of education. In this case, the engineer could teach these people how to run optimization software while on-site and eventually train them to run the model on their own.

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