A STUDY OF SAMPLING AND SCALE-UP IN SOLIDS MIXING

by

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TABLE OF CONTENTS

IV. CONTACT NUMBER AS AN INDEX OF RADIAL
MIXING IN THE MOTIONLESS MIXER

V. SCALE-UP PROCEDURES FOR BATCH MIXERS--
APPLICATION TO TUMBLING MIXERS

VI. SUMMARY AND RECOMMENDATIONS

CHAPTER I

INTRODUCTION

1.1 GENERAL

Solids mixing may be described as any operation in which energy is applied to a particulate solid system such that the inhomogeneity and concentration gradients tend to diminish. It is a critical operation in many process industries, such as agricultural, pharmaceutical, and ceramic industries. However, it has been much less developed both theoretically and practically compared to other unit operations. Recently there has been a spurt of activity to further develop solids mixing.

Unlike liquid mixing, research on solids mixing has been relatively limited. Probably the statistical nature and discontinuity property of the solid particles hinder the development of this field. For example, the categories of the sampling technique used in a particulate process are far more complex than those used in a liquid process.

The degree of mixedness is a fundamental state of a system and is always evaluated from the sampling results. Fan, et al., (1970) reviewed over thirty different definitions of the degree of mixedness. The difference in the definitions for the criterion reveals the complexity of the mixing process and the uncertainty of various concepts and notions in the field of solids mixing.

Since the mixing action is very complex, it is extremely difficult to formulate an adequate mathematical model describing the action. The practicality and experience still predominate in the design and operation of the

mixing equipment and in the assessment of the quality of a mixture.

1.2 PREVIOUS WORK

The literature on solids mixing has been thoroughly reviewed by Weidenbaum (1953), Gren (1967), Klothen (1969), Fan, et al. (1970), and Fan, et al. (1972a). A brief review of the recent pertinent literature is given below.

Several researchers (Valentin, 1965; Rose, et al., 1965; and Fan, et al., 1970) stressed the following needs in this field.

a. Unification of the mixing index

b. Clarification of the different mixing mechanisms

c. Measurement and control of segregation

d. Systematic study of mixers

e. Modelling and simulation of the mixing process

f. Rules for scale-up and design

g. Synthesis of the mixing process

1.2.1 Statistical Approach to Solids Mixing

Statistical analysis has become the approach most frequently used among investigators because of the random nature of the mixing process. Probably the point at which most analyses begin is that of defining a suitable measure of the degree of mixing. This measure indicates how the composition of the bed being mixed varies from point to point. Most authors (for example, Lacey, 1943; Bourne, 1965; Weidenbaum, 1969) have agreed that the best way to express this degree of mixing is through statistical methods, namely some form of variance which is based upon samples taken from various points in the bed.

Lacey (1943) has shown that for a completely random mixture, the variance in composition among a group of samples drawn from it is given by

$$s_r^2 = \frac{P(1 - P)}{n} \tag{1}$$

where P = overall fraction of a particular type of trace particle

n = number of particles in the sample.

Also, for a completely unmixed system, he has shown that

$$s_0^2 = P(1 - P) \tag{2}$$

This leads to his definition of the degree of mixing any mixture

$$M = \frac{s_0^2 - s^2}{s_0^2 - s_r^2} \tag{3}$$

where

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Fan, et al. (1970) reviewed over thirty different definitions of the degree of mixedness, which differ with the systems used and the experimental procedures followed, especially the sample size. Nevertheless, the relationship between the variance and the sample size is unknown so that mixing indices based upon the variance are dependent upon the sample size, and comparisons among mixing studies in which different sample sizes have been used are therefore of limited value (Williams, 1969). To overcome these difficulties a theoretical description of the relationship between variance and sample size for non-random mixtures must be deduced. Bourne (1965, 1967) gave an interpretation of the results obtained by Poole, et al. (1964), using a statistical theory developed by Landry (1944). Danckwerts (1963) proposed a description of the correlation by correlograms, i.e., the relationship between

the coefficient of correlation of point samples and the distance between the samples. Schofield (1968) showed that the description mentioned can be used to elucidate the mechanism involved in the mixing process. Williams (1969) made a theoretical approach assuming mixing components of uniform particle size. Furthermore, Harnby (1971) has discussed the application of social survey statistical techniques to mixing problems. He mentioned the possibility of describing variance-sample size relationships by correlation theories. Recently, Kristensen (1973) derived a general expression for the variance of the composition of samples drawn from random or non-random mixtures.

Buslik (1973) proposed the negative log of the sample weight required to obtain a standard deviation of 1% as a simple numerical homogeneity index for expressing varying degrees of homogeneity quantitatively. The proposed method is of universal applicability, and a spectrum of index values for homogeneity has been computed for certain mixtures over a very wide range. With a different viewpoint, Akao, et al. (1971) proposed a degree of mixedness for binary mixtures of uniform size particles in regular and random arrangements based on the coordination number. An imaginary or hypothetical particle model was proposed by Akao (1969) in evaluating the distribution of coordination numbers for fine-coarse mixtures.

For convenience of converting each of the degree of mixedness to other forms, conversion formulae are tabulated in TABLE 1. Several converted numerical values for different sizes of samples are presented in Appendix 1.5. It can be seen that M_1, M_4, and M_5 are more dependent upon the size of the sample than others. The form $M_3 = \sigma_0^2 - \sigma^2 / \sigma_0^2 - \sigma_r^2$ approaches unity more rapidly than does the expression $M_8 = \sigma_0 - \sigma / \sigma_0 - \sigma_r$, while the latter form is more convenient in practical application. The comparison of various forms of expression may be the first step in unifying the definition of the degree of

TABLE 1

EQUIVALENT FORMS OF DEGREE OF MIXEDNESS FOR BINARY SYSTEMS

	M_1	M_2	M_3	M_4
M_1	$1 - \dfrac{\sigma}{\sigma_0}$	$2M_1 - M_1^2$	$\dfrac{2M_1 - M_1^2 - \frac{1}{N}}{1 - \frac{1}{N}}$	$\dfrac{1}{\sqrt{N}\,(1 - M_1)}$
M_2	$1 - \sqrt{1 - M_2}$	$1 - \dfrac{\sigma^2}{\sigma_0^2}$	$\dfrac{M_2 - \frac{1}{N}}{1 - \frac{1}{N}}$	$\dfrac{1}{\sqrt{N}\,(1 - M_2)}$
M_3	$1 - \sqrt{1 - M_3\left(1 - \frac{1}{N}\right)}$	$M_3\left(1 - \dfrac{1}{N}\right)$	$\dfrac{\sigma_0^2 - \sigma^2}{\sigma_0^2 - \sigma_r^2}$	$\dfrac{1}{\sqrt{N(1 - M_3)} + M_3}$
M_4	$1 - \dfrac{1}{\sqrt{NM_4}}$	$1 - \dfrac{1}{M_4^2 N}$	$\dfrac{NM_4^2 - 1}{M_4^2(N - 1)}$	$\dfrac{\sigma_r}{\sigma}$
M_5	$\dfrac{M_5(\sqrt{N} - 1)}{M_5(\sqrt{N} - 1) + 1}$	$1 - \dfrac{1}{[1 + M_5(\sqrt{N}-1)]^2}$	$\dfrac{N[M_5^2(\sqrt{N}-1)+2M_5]}{(\sqrt{N}+1)[M_5(\sqrt{N}+1)+1]^2}$	$\dfrac{M_5(\sqrt{N} - 1) + 1}{\sqrt{N}}$

TABLE 1--Continued

	M_1	M_2	M_3	M_4
M_6	$1 - M_6$	$1 - M_6^2$	$\dfrac{1 - M_6^2}{1 - \dfrac{1}{N}}$	$\dfrac{1}{\sqrt{N}\,M_6}$
M_7	$1 - \exp\left[M_7^2 \ln \dfrac{1}{\sqrt{N}}\right]$	$1 - \exp\left[M_7^2 \ln \dfrac{1}{N}\right]$	$\dfrac{1 - \exp\left[M_7^2 \ln \dfrac{1}{N}\right]}{1 - \dfrac{1}{N}}$	$\dfrac{1}{\sqrt{N}\,\exp\left[M_7^2 \ln \dfrac{1}{\sqrt{N}}\right]}$
M_8	$M_8\left(1 - \dfrac{1}{\sqrt{N}}\right)$	$1 - \left[1 - M_8\left(1 - \dfrac{1}{\sqrt{N}}\right)\right]^2$	$\dfrac{1 - \left[1 - M_8\left(1 - \dfrac{1}{\sqrt{N}}\right)\right]^2}{1 - \dfrac{1}{N}}$	$\dfrac{1}{\sqrt{N} - M_8\left(\sqrt{N} - 1\right)}$
M_9	$1 - \sqrt{M_9}$	$1 - M_9$	$\dfrac{1 - M_9}{1 - \dfrac{1}{N}}$	$\dfrac{1}{\sqrt{N}\,M_9}$

TABLE 1--Continued

	M_5	M_6	M_7	M_8	M_9
M_1	$\dfrac{M_1}{(1-M_1)(\sqrt{N}-1)}$	$1-M_1$	$\sqrt{\dfrac{\ln(1-M_1)}{\ln\frac{1}{\sqrt{N}}}}$	$\dfrac{M_1}{1-\frac{1}{\sqrt{N}}}$	$(1-M_1)^2$
M_2	$\dfrac{1-\sqrt{1-M_2}}{\sqrt{1-M_2}\,(\sqrt{N}-1)}$	$\sqrt{1-M_2}$	$\sqrt{\dfrac{\ln(1-M_2)}{\ln\frac{1}{N}}}$	$\dfrac{1-\sqrt{1-M_2}}{1-\frac{1}{\sqrt{N}}}$	$1-M_2$
M_3	$\dfrac{1-\sqrt{1-M_3(1-\frac{1}{N})}}{\sqrt{1-M_3(1-\frac{1}{N})}\,(\sqrt{N}-1)}$	$\sqrt{1-M_3\left(1-\frac{1}{N}\right)}$	$\sqrt{\dfrac{\ln[1-M_3(1-\frac{1}{N})]}{\ln\frac{1}{N}}}$	$\dfrac{1-\sqrt{1-M_3(1-\frac{1}{N})}}{1-\frac{1}{\sqrt{N}}}$	$1-M_3\left(1-\frac{1}{N}\right)$
M_4	$\dfrac{M_4\sqrt{N}-1}{\sqrt{N}-1}$	$\dfrac{1}{\sqrt{N}\,M_4}$	$\sqrt{\dfrac{\ln\sqrt{N}}{\ln\sqrt{N}\cdot M_4}}$	$\dfrac{1-\frac{1}{\sqrt{N}\cdot M_4}}{1-\frac{1}{\sqrt{N}}}$	$\dfrac{1}{N M_4^2}$
M_5	$\dfrac{\frac{\sigma_0}{\sigma}-1}{\frac{\sigma_0}{\sigma_r}-1}$	$\dfrac{1}{M_5(\sqrt{N}-1)+1}$	$\sqrt{\dfrac{\ln\sqrt{N}}{\ln[M_5(\sqrt{N}-1)+1]}}$	$\dfrac{\sqrt{N}\,M_5}{M_5(\sqrt{N}-1)+1}$	$\dfrac{1}{(M_5(\sqrt{N}-1)+1)^2}$

TABLE 1--Continued

	M_5	M_6	M_7	M_8	M_9
M_6	$\dfrac{1 - M_6}{M_6\left(\sqrt{N} - 1\right)}$	$\dfrac{\sigma}{\sigma_0}$	$\sqrt{\dfrac{\ln M_6}{\ln \frac{1}{\sqrt{N}}}}$	$\dfrac{1 - M_6}{1 - \frac{1}{\sqrt{N}}}$	M_6^2
M_7	$\dfrac{\exp[M_7^2 \ln \sqrt{N}] - 1}{\sqrt{N} - 1}$	$\exp\left[M_7^2 \ln \frac{1}{\sqrt{N}}\right]$	$\dfrac{\ln \sigma_0^2 - \ln \sigma^2}{\ln \sigma_0^2 - \ln \sigma_r^2}$	$\dfrac{1 - \exp\left[M_7^2 \ln \frac{1}{\sqrt{N}}\right]}{1 - \frac{1}{\sqrt{N}}}$	$\exp\left[M_7^2 \ln \frac{1}{\sqrt{N}}\right]^2$
M_8	$\dfrac{M_8}{\sqrt{N} - M_8\left(\sqrt{N} - 1\right)}$	$1 - M_8\left(1 - \frac{1}{\sqrt{N}}\right)$	$\sqrt{\dfrac{\ln\left[1 - M_8\left(1 - \frac{1}{\sqrt{N}}\right)\right]}{\ln \frac{1}{\sqrt{N}}}}$	$\dfrac{\sigma_0 - \sigma}{\sigma_0 - \sigma_r}$	$\left[1 - M_8\left(1 - \frac{1}{\sqrt{N}}\right)\right]^2$
M_9	$\dfrac{1 - \sqrt{M_9}}{\sqrt{M_9}\left(\sqrt{N} - 1\right)}$	$\sqrt{M_9}$	$\sqrt{\dfrac{\ln \sqrt{M_9}}{\ln \frac{1}{\sqrt{N}}}}$	$\dfrac{1 - \sqrt{M_9}}{1 - \frac{1}{\sqrt{N}}}$	$\dfrac{\sigma^2}{\sigma_0^2}$

mixedness.

The principle of uncertainty is introduced in a probabilistic or stochastic model. A stochastic process is a random phenomenon that is controlled by statistical laws. This approach seems to be more fundamental and helpful than the deterministic approach in analyzing and understanding the complex mechanisms of solids mixing processes. With this type of model, mathematical intractability can be avoided.

Oyama and Ayaki (1956) proposed a Markov chain model to describe the mixing of particles in a drum mixer but did not conduct experiments to verify the model. Oleniczak (1962) postulated a Poisson process to interchange particles between a volume element and the rest of the mixture. He obtained a stochastic model for the V-type mixer. The distribution of tracer particles was found to be bimodel at a low number of revolutions.

Makarov and Gorbushin (1970) used the Markov process technique to describe the mechanisms of transition of particles in a circular cell model. They proposed this model for the preliminary design of a batch mixer for free flowing materials with closed loop internal circulation. It is assumed that the termination of convective mixing is the determining factor in obtaining the optimum time of mixing because at some time $t_{cov} = t_{opt}$, and the mixing process achieves an equilibrium with the segregation process. The main idea is to divide the internal operating volume of the mixer into a number of zones, each of which has a characteristic particle flow pattern. Assuming that the laws governing the movement of particles through each zone are known, Makarov and Gorbushin (1970) determined the average residence time of particles in each zone and the standard deviation of the residence time distribution in any zone. If the system as a whole is linear, the total average residence time of a particle and the standard deviation for the entire mixer can be

calculated. Experimental verification of the method was presented.

Fan, et al. (1972b) employed a Markov chain model to model the axial mixing of solid particles in a motionless mixer. One-step transition probabilities were determined experimentally for the model. A fairly good agreement with the experimental data was obtained for up to seven steps of the Markov chain, or what was equivalent to seven consecutive passes of the mixture through the mixer.

1.2.2 Scale-up and Design of Solids Mixers

While solids mixing is widely employed and considerable progress has been made in understanding its mechanisms, sufficiently reliable 'design' formulae are not available that permit an engineer to design industrial scale mixers or scale-up small mixers based on the results of laboratory experiments. Comparatively little has been reported on the design and scale-up of solids mixers.

Muller (1967) and Rumpf and Muller (1962) evaluated different mixing elements for a paddle mixer. Muller compared the amount of material lifted by the differently shaped elements across the mixer diameter. He demonstrated that the mixing rate is directly dependent on mixer speed and on the effective surface area of the mixing element. The effective surface is a function of the angle between the shaft and the mixing blade. No generalizations were offered.

Luterek and Cachia (1971) used the Froude number as a criterion for scale-up of V-type mixers. Their method was verified by experiments where two different dry powders were mixed in V-type batch mixers of four different sizes. The scale-up procedure is based on the principle that Froude numbers for the laboratory scale mixer and the full scale mixer must be equal, i.e.:

$$\left(\frac{N_1^2 \, kD_1}{g}\right)_{lab. \, scale} = \left(\frac{N_2^2 \, kD_2}{g}\right)_{full \, scale} \tag{4}$$

Lynch and Ho (1973) presented a standard design procedure for determining the power requirements for double cone and ribbon blenders.

Sawahata (1969) employed the relationship between the circulation and mixing times to estimate the mixing time of a large-scale mixer. The circulation time of the particulate solids in a drum mixer was related to the operating variables as (Sawahata, 1967):

$$T_{HC} = \frac{R^2 (F/V)}{(N/60) h (2R - h)} \, , \, \text{if} \, \left(\frac{N^2 R}{g}\right)_{HC} < 25 \times 10^{-3} \tag{5}$$

where

T_{HC} = average circulation time of solid particles

R   = the radius of the mixer

F/V = filling ratio of particles

h   = thickness of the transportation zone

The thickness of the transportation zone, h, in equation (5) can be related to the filling ratio and Froude number as (Sawahata, 1968, and Sawahata, 1969):

$$\frac{h}{R} = \left(\alpha - \beta \frac{N^2 R}{g}\right) \left(\frac{F}{V}\right) \tag{6}$$

where constants, α and β, were determined by experiments (Sawahata, 1969). For the V-type mixer the following equations hold.

$$T_V = \eta \frac{\sqrt{R}}{(N/60)} \left(\frac{F}{V}\right)^{2/3} , \, \text{if} \, \left(\frac{N^2 R}{g}\right)_V < 7.6 \times 10^{-3} \tag{7}$$

where η is the correction factor.

The lengths of time needed to attain a satisfactory mixed state for the

drum mixer and V-type mixer, respectively (Sawahata, 1968), are

$$\theta_{HC} = 20 \, T_{HC} \tag{8}$$

$$\theta_V = 10 \, T_V \tag{9}$$

If the dynamic similarity exists between two geometrically similar drum mixers, i.e., by holding the Froude number as constant,

$$\frac{N^2 R}{g} = \text{constant} \tag{10}$$

Let

$$\frac{F}{V} = \text{constant} \tag{11}$$

Then, from equation (6), we have

$$\frac{h}{R} = \text{constant} \tag{12}$$

Relating equations (12) and (5), we have

$$N \, T_{HC} = \text{constant} \tag{13}$$

i.e., for two geometrically similar drum mixers,

$$N_I T_I = N_{II} T_{II} \tag{14}$$

Substituting equation (13) into equation (10)

$$T_I \big/ \sqrt{R_I} = T_{II} \big/ \sqrt{R_{II}} \tag{15}$$

If $(\theta_{HC})_I$ represents the mixing time of a small-scale mixer, then the mixing time $(\theta_{HC})_{II}$ of a large-scale mixer loaded with mixtures of the same concen-

tration as the small scale is

$$(\theta_{HC})_{II} = \frac{T_V}{T_I} (\theta_{HC})_I = \frac{\sqrt{R_{II}}}{\sqrt{R_I}} (\theta_{HC})_I \tag{16}$$

A similar result can be obtained by correlating equations (7), (9), and (10) for the V-type mixer as follows:

$$(\theta_V)_{II} = \frac{R_V}{R_I} (\theta_V)_I = \frac{N_I \sqrt{R_{II}}}{N_{II} \sqrt{R_I}} (\theta_V)_I \tag{17}$$

where

$(\theta_V)_{II}$ = the mixing time for a large-scale V-type mixer.

$(\theta_V)_I$ = the mixing time for a small-scale V-type mixer.

Sawahata (1968) presented experimental verification of this method.

1.3 OBJECTIVES

The purposes of the present study are threefold. The first series of studies sought to obtain further information on the statistical nature of the samples in solids mixing by a nonparametric statistical approach. Most of the previous works on the evaluation of the sampling results are parametrically oriented (Harby, 1971; Shinnar and Noar, 1961; and Miles, et al., 1960). They have to assume that the population is distributed with some parameters. The application of nonparametric statistics has its merit in testing hypotheses when we do not assume, or even care about, the normality. Many of the nonparametric tests and other nonparametric procedures are simpler than the usual parametric procedures, and have high power to detect true differences.

The second phase of investigation studied a microscopic and geometric mixing index-contact number. Most of the definitions of the degree of mixed-

ness concern primarily the measurement of the standard deviation or the variance of the spot samples taken from a mixture. Such a viewpoint always neglects the structure inside the spot samples, i.e., it assumes that a completely mixed state exists in any spot sample. This mixing index, first used by Akao, et al. (1973), has the merit of not depending on this assumption. The first part of this second phase was a computer simulation of the distribution of the contact number for a binary system at the completely mixed state. Results were obtained for the two dimensional cubic and hexagonal packings at different concentrations of key components. In the second part of this phase, mixing index was employed to the radial mixing of particles of the same size in a motionless mixer. The results were compared with those made by spot sampling.

The third phase of work investigated the scale-up and design procedures for tumbling mixers. The principle of similarity (Johnstone and Thring, 1957) was exploited to study this category of mixers. If the physical properties of the particles are not far different, it can be reasonably stated that the criteria derived are applicable to the scale-up procedures.

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1.5 APPENDIX

Computer Program and Numerical Comparison of Some Degrees of Mixedness

1-18

```
 1              PN=0.0
 2          4 PN=PN+10.
 3          7 WRITE (6,12) PN
 4         12 FORMAT (1X,//,1X,'NUMBER OF PARTICLES PER SAMPLE = ',F5.0)
 5            WRITE(6,11)
 6         11 FORMAT(3X,'M1',7X,'M2',7X,'M3',7X,'M4',7X,'M5',7X,'M6',7X,'M7',7X,
 7           1'M8',7X,'M9')
 7            DM8=0.0
 8          6 DM3=(1.-(1.-DM8*(1.-1./SQRT(PN)))**2)/(1.-1./PN)
 9            DM1=1.-SQRT(1.-DM3*(1.-1./PN))
10            DM2=DM3*(1.-1./PN)
11            DM4=1./(SQRT(PN*(1.-DM3)+DM3))
12            DM5=(DM1*1.)/((1.-DM1)*(SQRT(PN)-1.))
13            DM6=1.-DM1
14            DM7=SQRT(ALOG(1.0-DM2)/ALOG(1./PN))
15            DM9=1.-DM2
16            WRITE(6,22) DM1,DM2,DM3,DM4,DM5,DM6,DM7,DM8,DM9
17         22 FORMAT(1X,9(1X,F8.5))
18            IF(DM8-1.0) 2,3,3
19          2 DM8=DM8+0.1
20            GO TO 6
21          3 IF(PN-100.) 4,5,5
22          5 IF (PN-1000.) 8,8,9
23          8 PN=PN+100.
24            GO TO 7
25          9 STOP
26            END

        $ENTRY
```

```
NUMBER OF PARTICLES PER SAMPLE =   10.
  M1        M2        M3        M4        M5        M6        M7        M8        M9
 0.00000   0.00000   0.00000   0.31623   0.00000   1.00000   0.00000   0.00000   1.00000
 0.06838   0.13208   0.14675   0.33944   0.03394   0.93162   0.24803   0.10000   0.86792
 0.13475   0.25481   0.28312   0.36632   0.07326   0.86325   0.35742   0.20000   0.74519
 0.20513   0.36818   0.40909   0.39784   0.11935   0.79487   0.44655   0.30000   0.63182
 0.27351   0.47221   0.52468   0.43528   0.17411   0.72649   0.52682   0.40000   0.52779
 0.34189   0.56689   0.62987   0.48051   0.24025   0.65811   0.60282   0.50000   0.43311
 0.41026   0.65221   0.72468   0.53622   0.32173   0.58974   0.67726   0.60000   0.34779
 0.47864   0.72819   0.81909   0.60654   0.42458   0.52136   0.75215   0.70000   0.27182
 0.54702   0.79481   0.88312   0.69810   0.55848   0.45298   0.82936   0.80000   0.20519
 0.61540   0.85208   0.94675   0.82221   0.72999   0.34460   0.91103   0.90000   0.14792
 0.68377   0.90000   1.00000   1.00000   1.00000   0.31623   1.00000   1.00000   0.10000
```

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NUMBER OF PARTICLES PER SAMPLE =   20.
  M1        M2        M3        M4        M5        M6        M7        M8        M9
-0.00000   0.00000   0.00000   0.22361   0.00000   1.00000   0.00000   0.00000   1.00000
 0.07744   0.14925   0.15711   0.24243   0.02424   0.92236   0.23228   0.10000   0.85075
 0.15528   0.28645   0.30152   0.26471   0.05294   0.84472   0.33565   0.20000   0.71355
 0.23292   0.41159   0.43325   0.29150   0.08745   0.76708   0.42074   0.30000   0.58841
 0.31056   0.52467   0.55228   0.32433   0.12973   0.68944   0.49826   0.40000   0.47533
 0.38820   0.62570   0.65863   0.36549   0.18274   0.61180   0.57274   0.50000   0.37430
 0.46584   0.71467   0.75228   0.41861   0.25117   0.53416   0.64702   0.60000   0.28533
 0.54349   0.79159   0.83325   0.48940   0.34286   0.45652   0.72352   0.70000   0.20841
 0.62111   0.85645   0.90152   0.59017   0.47214   0.37889   0.80494   0.80000   0.14355
 0.69875   0.90925   0.95711   0.74227   0.66805   0.30125   0.89600   0.90000   0.09075
 0.77639   0.95000   1.00000   1.00000   1.00000   0.22361   1.00000   1.00000   0.05000
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NUMBER OF PARTICLES PER SAMPLE = 30.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.18257	0.00000	1.00000	0.00000	0.00000	1.00000
0.08174	0.15680	0.16221	0.19883	0.01988	0.91826	0.22393	0.10000	0.84320
0.16349	0.30024	0.31060	0.21826	0.04365	0.83651	0.32399	0.20000	0.69976
0.24523	0.43032	0.44516	0.24189	0.07257	0.75477	0.40674	0.30000	0.56968
0.32697	0.54703	0.56589	0.27127	0.10851	0.67303	0.48253	0.40000	0.45297
0.40871	0.65038	0.67231	0.30877	0.15439	0.59129	0.55586	0.50000	0.34962
0.49046	0.74036	0.76589	0.35831	0.21499	0.50954	0.62966	0.60000	0.25964
0.57220	0.81699	0.84516	0.42677	0.29874	0.42780	0.70661	0.70000	0.18301
0.65394	0.88024	0.91060	0.52758	0.42206	0.34606	0.78993	0.80000	0.11976
0.73568	0.93014	0.96221	0.69074	0.62167	0.26432	0.88455	0.90000	0.06986
0.81743	0.96667	1.00000	1.00000	1.00000	0.18257	1.00000	1.00000	0.03333

NUMBER OF PARTICLES PER SAMPLE = 40.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.15811	0.00000	1.00000	0.00000	0.00000	1.00000
0.08419	0.16129	0.16543	0.17265	0.01726	0.91581	0.21836	0.10000	0.83871
0.16838	0.30840	0.31631	0.19013	0.03803	0.83162	0.31617	0.20000	0.69160
0.25257	0.44134	0.45266	0.21154	0.06346	0.74743	0.39728	0.30000	0.55866
0.33675	0.56011	0.57447	0.23939	0.09536	0.66325	0.47183	0.40000	0.43989
0.42094	0.66469	0.68174	0.27305	0.13653	0.57906	0.54426	0.50000	0.33531
0.50513	0.75511	0.77447	0.31951	0.19170	0.49487	0.61757	0.60000	0.24489
0.58932	0.83134	0.85266	0.38501	0.26950	0.41068	0.69462	0.70000	0.16866
0.67351	0.89340	0.91631	0.48428	0.38743	0.32649	0.77902	0.80000	0.10660
0.75770	0.94129	0.96543	0.65255	0.58729	0.24230	0.87668	0.90000	0.05871
0.84189	0.97500	1.00000	1.00000	1.00000	0.15811	1.00000	1.00000	0.02500

NUMBER OF PARTICLES PER SAMPLE = 50.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.14142	0.00000	1.00000	0.00000	0.00000	1.00000
0.08586	0.16434	0.16770	0.15470	0.01547	0.91414	0.21423	0.10000	0.83566
0.17172	0.31395	0.32035	0.17274	0.03415	0.82828	0.31035	0.20000	0.68605
0.25757	0.44980	0.45796	0.19149	0.05715	0.74243	0.39021	0.30000	0.55120
0.34343	0.56892	0.58053	0.21539	0.08616	0.65657	0.46378	0.40000	0.43108
0.42929	0.67429	0.68805	0.24780	0.12390	0.57071	0.53548	0.50000	0.32571
0.51515	0.76492	0.78053	0.29168	0.17501	0.48485	0.60835	0.60000	0.23508
0.60101	0.84030	0.85796	0.35444	0.24811	0.39899	0.68537	0.70000	0.15920
0.68686	0.90195	0.92035	0.45163	0.36130	0.31314	0.77046	0.80000	0.09805
0.77272	0.94834	0.96770	0.62224	0.56001	0.22728	0.87031	0.90000	0.05166
0.85858	0.98000	1.00000	1.00000	1.00000	0.14142	1.00000	1.00000	0.02000

NUMBER OF PARTICLES PER SAMPLE = 60.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.12910	0.00000	1.00000	0.00000	0.00000	1.00000
0.08709	0.16660	0.16942	0.14142	0.01414	0.91291	0.21097	0.10000	0.83340
0.17418	0.31802	0.32341	0.15633	0.03127	0.82582	0.30575	0.20000	0.68198
0.26127	0.45428	0.46198	0.17476	0.05243	0.73873	0.38461	0.30000	0.54572
0.34836	0.57537	0.58512	0.19811	0.07925	0.65164	0.45738	0.40000	0.42463
0.43545	0.68129	0.69283	0.22868	0.11434	0.56455	0.52947	0.50000	0.31872
0.52254	0.77203	0.78512	0.27039	0.16223	0.47746	0.60093	0.60000	0.22797
0.60963	0.84761	0.86198	0.33071	0.23150	0.39037	0.67786	0.70000	0.15239
0.69672	0.90802	0.92341	0.42568	0.34054	0.30328	0.76342	0.80000	0.09198
0.78381	0.95326	0.96942	0.59716	0.53744	0.21619	0.86496	0.90000	0.04674
0.87090	0.98333	1.00000	1.00000	1.00000	0.12910	1.00000	1.00000	0.01667

NUMBER OF PARTICLES PER SAMPLE =   70.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.11952	0.00000	1.00000	0.00000	0.00000	1.00000
0.08805	0.16934	0.17078	0.13106	0.01311	0.91195	0.20830	0.10000	0.83166
0.17610	0.32118	0.32584	0.14507	0.02901	0.82390	0.30197	0.20000	0.67882
0.26414	0.45851	0.46516	0.16243	0.04873	0.73586	0.37999	0.30000	0.54149
0.35219	0.58034	0.58875	0.18450	0.07380	0.64781	0.45209	0.40000	0.41966
0.44024	0.68667	0.69662	0.21352	0.10676	0.55976	0.52264	0.50000	0.31333
0.52829	0.77749	0.78875	0.25338	0.15203	0.47171	0.59474	0.60000	0.22251
0.61633	0.85230	0.86516	0.31153	0.21807	0.38367	0.67155	0.70000	0.14720
0.70438	0.91261	0.92584	0.40431	0.32345	0.29562	0.75743	0.80000	0.08739
0.79243	0.95691	0.97078	0.57582	0.51824	0.20757	0.86033	0.90000	0.04309
0.88048	0.98571	1.00000	1.00000	1.00000	0.11952	1.00000	1.00000	0.01429

NUMBER OF PARTICLES PER SAMPLE =   80.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.11180	0.00000	1.00000	0.00000	0.00000	1.00000
0.08882	0.16975	0.17190	0.12270	0.01227	0.91118	0.20604	0.10000	0.83025
0.17764	0.32372	0.32732	0.13595	0.02719	0.82236	0.29877	0.20000	0.67628
0.26646	0.46192	0.46776	0.15242	0.04572	0.73354	0.37607	0.30000	0.53808
0.35528	0.58433	0.59173	0.17341	0.06937	0.64472	0.44759	0.40000	0.41567
0.44410	0.69097	0.69972	0.20112	0.10056	0.55590	0.51767	0.50000	0.30903
0.53292	0.73183	0.79173	0.23937	0.14362	0.46708	0.58944	0.60000	0.21817
0.62174	0.85692	0.86776	0.29557	0.20690	0.37826	0.66611	0.70000	0.14308
0.71056	0.91622	0.92782	0.38627	0.30902	0.28944	0.75223	0.80000	0.08378
0.79938	0.95975	0.97190	0.55728	0.50155	0.20062	0.85624	0.90000	0.04025
0.88820	0.98750	1.00000	1.00000	1.00000	0.11180	1.00000	1.00000	0.01250

NUMBER OF PARTICLES PER SAMPLE =   90.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.10541	0.00000	1.00000	0.00000	0.00000	1.00000
0.08946	0.17092	0.17234	0.11577	0.01158	0.91054	0.20409	0.10000	0.82908
0.17892	0.32582	0.32949	0.12838	0.02568	0.82108	0.29600	0.20000	0.67418
0.26838	0.46473	0.46995	0.14408	0.04322	0.73162	0.37268	0.30000	0.53527
0.35784	0.58763	0.59423	0.16415	0.06566	0.64216	0.44369	0.40000	0.41237
0.44730	0.69452	0.70232	0.19072	0.09536	0.55270	0.51336	0.50000	0.30548
0.53675	0.73540	0.79423	0.22755	0.13653	0.46325	0.58482	0.60000	0.21460
0.62621	0.86028	0.86995	0.28200	0.19740	0.37379	0.66135	0.70000	0.13972
0.71567	0.91916	0.92949	0.37173	0.29659	0.28433	0.74764	0.80000	0.08084
0.80513	0.96203	0.97284	0.54093	0.48693	0.19487	0.85258	0.90000	0.03797
0.89459	0.98889	1.00000	1.00000	1.00000	0.10541	1.00000	1.00000	0.01111

NUMBER OF PARTICLES PER SAMPLE =   100.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.10000	0.00000	1.00000	0.00000	0.00000	1.00000
0.09000	0.17190	0.17364	0.10989	0.01099	0.91000	0.20238	0.10000	0.82810
0.18000	0.32760	0.33091	0.12195	0.02439	0.82000	0.29357	0.20000	0.67240
0.27000	0.46710	0.47182	0.13699	0.04110	0.73000	0.36970	0.30000	0.53290
0.36000	0.59040	0.59636	0.15625	0.06250	0.64000	0.44025	0.40000	0.40960
0.45000	0.69750	0.70455	0.18182	0.09091	0.55000	0.50955	0.50000	0.30250
0.54000	0.78840	0.79636	0.21739	0.13043	0.46000	0.58073	0.60000	0.21160
0.63000	0.86310	0.87182	0.27027	0.18919	0.37000	0.65711	0.70000	0.13690
0.72000	0.92160	0.93091	0.35714	0.28571	0.28000	0.74353	0.80000	0.07840
0.81000	0.96390	0.97364	0.52632	0.47368	0.19000	0.84926	0.90000	0.03610
0.90000	0.99000	1.00000	1.00000	1.00000	0.10000	1.00000	1.00000	0.01000

NUMBER OF PARTICLES PER SAMPLE = 200.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.07071	0.00000	1.00000	0.00000	0.00000	1.00000
0.09293	0.17722	0.17811	0.07795	0.00780	0.90707	0.19188	0.10000	0.82278
0.18586	0.33717	0.33887	0.08685	0.01737	0.81414	0.27860	0.20000	0.66283
0.27879	0.47985	0.48226	0.09804	0.02941	0.72121	0.35124	0.30000	0.52015
0.37172	0.60526	0.60830	0.11255	0.04502	0.62828	0.41885	0.40000	0.39474
0.46464	0.71339	0.71698	0.13208	0.06604	0.53536	0.48565	0.50000	0.28661
0.55757	0.80426	0.80830	0.15982	0.09589	0.44243	0.55482	0.60000	0.19574
0.65050	0.87785	0.88226	0.20232	0.14162	0.34950	0.62994	0.70000	0.12215
0.74343	0.93417	0.93887	0.27560	0.22048	0.25657	0.71659	0.80000	0.06583
0.83636	0.97322	0.97811	0.43211	0.38890	0.16364	0.82660	0.90000	0.02678
0.92929	0.99500	1.00000	1.00000	1.00000	0.07071	1.00000	1.00000	0.00500

NUMBER OF PARTICLES PER SAMPLE = 300.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.05774	0.00000	1.00000	0.00000	0.00000	1.00000
0.09423	0.17957	0.18017	0.06374	0.00637	0.90577	0.18628	0.10000	0.82043
0.18845	0.34139	0.34253	0.07114	0.01423	0.81155	0.27059	0.20000	0.65861
0.28268	0.48545	0.48707	0.08049	0.02415	0.71732	0.34131	0.30000	0.51455
0.37691	0.61175	0.61390	0.09266	0.03706	0.62309	0.40728	0.40000	0.38825
0.47113	0.72030	0.72271	0.10917	0.05458	0.52887	0.47262	0.50000	0.27970
0.56536	0.81109	0.81380	0.13283	0.07970	0.43464	0.54053	0.60000	0.18891
0.65959	0.88412	0.88707	0.16960	0.11872	0.34041	0.61470	0.70000	0.11588
0.75381	0.93939	0.94253	0.23452	0.18761	0.24619	0.70106	0.80000	0.06061
0.84804	0.97691	0.98018	0.37993	0.34194	0.15196	0.81281	0.90000	0.02309
0.94227	0.99667	1.00000	1.00000	1.00000	0.05773	1.00000	1.00000	0.00333

NUMBER OF PARTICLES PER SAMPLE = 400.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.05000	0.00000	1.00000	0.00000	0.00000	1.00000
0.09500	0.18097	0.18143	0.05525	0.00552	0.90500	0.18254	0.10000	0.81903
0.19000	0.34390	0.34476	0.06173	0.01235	0.81000	0.26522	0.20000	0.65610
0.28500	0.48878	0.49000	0.06993	0.02098	0.71500	0.33464	0.30000	0.51122
0.38000	0.61560	0.61714	0.08065	0.03226	0.62000	0.39946	0.40000	0.38440
0.47500	0.72433	0.72619	0.09524	0.04762	0.52500	0.46378	0.50000	0.27562
0.57000	0.81510	0.81714	0.11628	0.06977	0.43000	0.53078	0.60000	0.18490
0.66500	0.88778	0.89000	0.14925	0.10448	0.33500	0.60420	0.70000	0.11222
0.76000	0.94240	0.94476	0.20833	0.16667	0.24000	0.69021	0.80000	0.05760
0.85500	0.97897	0.98143	0.34483	0.31034	0.14500	0.80286	0.90000	0.02103
0.95000	0.99750	1.00000	1.00000	1.00000	0.05000	1.00000	1.00000	0.00250

NUMBER OF PARTICLES PER SAMPLE = 500.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.04472	0.00000	1.00000	0.00000	0.00000	1.00000
0.09553	0.18193	0.18229	0.04944	0.00494	0.90447	0.17976	0.10000	0.81807
0.19106	0.34561	0.34637	0.05528	0.01106	0.80894	0.26122	0.20000	0.65439
0.28658	0.49104	0.49202	0.06269	0.01881	0.71342	0.32966	0.30000	0.50896
0.38211	0.61821	0.61945	0.07238	0.02895	0.61789	0.39362	0.40000	0.38179
0.47764	0.72714	0.72860	0.08561	0.04281	0.52236	0.45715	0.50000	0.27286
0.57317	0.81781	0.81945	0.10477	0.06286	0.42683	0.52344	0.60000	0.18219
0.66870	0.89024	0.89202	0.13499	0.09449	0.33130	0.59626	0.70000	0.10976
0.76422	0.94441	0.94630	0.18968	0.15174	0.23578	0.68190	0.80000	0.05559
0.85975	0.98033	0.98229	0.31867	0.28698	0.14025	0.79509	0.90000	0.01967
0.95528	0.99800	1.00000	1.00000	1.00001	0.04472	1.00000	1.00000	0.00200

NUMBER OF PARTICLES PER SAMPLE =   600.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.04082	0.00000	1.00000	0.00000	0.00000	1.0000
0.09592	0.18263	0.18294	0.04516	0.00452	0.90408	0.17756	0.10000	0.8173
0.19184	0.34687	0.34745	0.05052	0.01010	0.80816	0.25905	0.20000	0.6531
0.28775	0.49270	0.49353	0.05732	0.01720	0.71225	0.32572	0.30000	0.5073
0.38367	0.62214	0.62117	0.06624	0.02650	0.61633	0.38899	0.40000	0.3798
0.47959	0.72917	0.73039	0.07845	0.03922	0.52041	0.45189	0.50000	0.2708
0.57551	0.81930	0.82117	0.09617	0.05770	0.42449	0.51759	0.60000	0.1802
0.67142	0.89204	0.89353	0.12425	0.08697	0.32858	0.58989	0.70000	0.1079
0.76734	0.94587	0.94745	0.17547	0.14038	0.23266	0.67520	0.80000	0.0541
0.86326	0.98130	0.98294	0.29855	0.26870	0.13674	0.78871	0.90000	0.0187
0.95918	0.99833	1.00000	1.00000	1.00000	0.04082	1.00000	1.00000	0.0016

NUMBER OF PARTICLES PER SAMPLE =   700.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.03780	0.00000	1.00000	0.00000	0.00000	1.0000
0.09622	0.18318	0.18344	0.04182	0.00418	0.90378	0.17575	0.10000	0.8168
0.19244	0.34785	0.34835	0.04680	0.00936	0.80756	0.25545	0.20000	0.6521
0.28866	0.49400	0.49470	0.05313	0.01594	0.71134	0.32247	0.30000	0.5060
0.38488	0.62163	0.62252	0.06145	0.02458	0.61512	0.38517	0.40000	0.3783
0.48110	0.73074	0.73179	0.07284	0.03642	0.51890	0.44753	0.50000	0.2692
0.57732	0.82134	0.82252	0.08942	0.05365	0.42268	0.51274	0.60000	0.1786
0.67354	0.89343	0.89470	0.11578	0.08104	0.32646	0.58460	0.70000	0.1065
0.76976	0.94699	0.94835	0.16416	0.13133	0.23024	0.66960	0.80000	0.0530
0.86598	0.98204	0.98344	0.28203	0.25383	0.13402	0.78331	0.90000	0.0179
0.96220	0.99857	1.00000	1.00000	1.00001	0.03780	1.00000	1.00000	0.0014

NUMBER OF PARTICLES PER SAMPLE =   800.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.03536	0.00000	1.00000	0.00000	0.00000	1.0000
0.09646	0.18362	0.18385	0.03913	0.00391	0.90354	0.17421	0.10000	0.8163
0.19293	0.34864	0.34907	0.04381	0.00876	0.80707	0.25324	0.20000	0.6513
0.28939	0.49504	0.49566	0.04975	0.01493	0.71061	0.31971	0.30000	0.5049
0.38586	0.62293	0.62361	0.05757	0.02303	0.61414	0.38192	0.40000	0.3771
0.48232	0.73201	0.73293	0.06830	0.03415	0.51768	0.44384	0.50000	0.2679
0.57879	0.82256	0.82361	0.08394	0.05036	0.42121	0.50861	0.60000	0.1774
0.67525	0.89454	0.89566	0.10887	0.07621	0.32475	0.58009	0.70000	0.1054
0.77172	0.94783	0.94907	0.15487	0.12390	0.22829	0.66480	0.80000	0.0521
0.86818	0.98262	0.98385	0.26821	0.24139	0.13182	0.77863	0.90000	0.0173
0.96465	0.99875	1.00000	1.00000	1.00001	0.03535	1.00000	1.00000	0.0012

NUMBER OF PARTICLES PER SAMPLE =   900.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.03333	0.00000	1.00000	0.00000	0.00000	1.0000
0.09667	0.18399	0.18419	0.03690	0.00369	0.90333	0.17289	0.10000	0.8160
0.19333	0.34929	0.34968	0.04132	0.00826	0.80667	0.25133	0.20000	0.6507
0.29000	0.49590	0.49645	0.04695	0.01428	0.71000	0.31733	0.30000	0.5041
0.38667	0.62382	0.62452	0.05435	0.02174	0.61333	0.37911	0.40000	0.3761
0.48333	0.73306	0.73387	0.06452	0.03226	0.51667	0.44063	0.50000	0.2669
0.58000	0.82360	0.82452	0.07937	0.04762	0.42000	0.50503	0.60000	0.1764
0.67667	0.89546	0.89645	0.10309	0.07216	0.32333	0.57616	0.70000	0.1045
0.77333	0.94862	0.94968	0.14706	0.11765	0.22667	0.66060	0.80000	0.0513
0.87000	0.98310	0.98410	0.25641	0.23077	0.13000	0.77450	0.90000	0.0169
0.96667	0.99889	1.00000	1.00000	1.00001	0.03333	1.00000	1.00000	0.0011

NUMBER OF PARTICLES PER SAMPLE = 1000.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.03162	0.00000	1.00000	0.00000	0.00000	1.00000
0.09684	0.18430	0.18448	0.03501	0.00350	0.90316	0.17172	0.10000	0.81570
0.19368	0.34984	0.35019	0.03922	0.00784	0.80632	0.24965	0.20000	0.65016
0.29051	0.49663	0.49713	0.04457	0.01337	0.70949	0.31523	0.30000	0.50337
0.38735	0.62466	0.62529	0.05162	0.02065	0.61265	0.37664	0.40000	0.37534
0.48419	0.73394	0.73467	0.06131	0.03065	0.51581	0.43780	0.50000	0.26606
0.58103	0.82446	0.82529	0.07548	0.04529	0.41897	0.50187	0.60000	0.17554
0.67786	0.89623	0.89713	0.09817	0.06872	0.32214	0.57269	0.70000	0.10377
0.77470	0.94924	0.95019	0.14036	0.11229	0.22530	0.65688	0.80000	0.05076
0.87154	0.98350	0.98448	0.24617	0.22155	0.12846	0.77081	0.90000	0.01650
0.96838	0.99990	1.00000	1.00000	1.00001	0.03162	1.00000	1.00000	0.00100

NUMBER OF PARTICLES PER SAMPLE = 1100.

M1	M2	M3	M4	M5	M6	M7	M8	M9
-0.00000	0.00000	0.00000	0.03015	0.00000	1.00000	0.00000	0.00000	1.00000
0.09698	0.18456	0.18473	0.03339	0.00334	0.90302	0.17069	0.10000	0.81544
0.19397	0.35032	0.35063	0.03741	0.00748	0.80603	0.24816	0.20000	0.64968
0.29095	0.49725	0.49771	0.04252	0.01276	0.70905	0.31336	0.30000	0.50275
0.38794	0.62538	0.62595	0.04926	0.01970	0.61206	0.37444	0.40000	0.37462
0.48492	0.73470	0.73537	0.05854	0.02927	0.51508	0.43528	0.50000	0.26530
0.58191	0.82520	0.82595	0.07212	0.04327	0.41809	0.49905	0.60000	0.17480
0.67889	0.89689	0.89771	0.09390	0.06573	0.32111	0.56958	0.70000	0.10311
0.77588	0.94977	0.95063	0.13453	0.10762	0.22412	0.65354	0.80000	0.05023
0.87286	0.98384	0.98473	0.23716	0.21344	0.12714	0.76748	0.90000	0.01616
0.96985	0.99909	1.00000	1.00000	1.00000	0.03015	1.00000	1.00000	0.00091

CHAPTER II

AN APPLICATION OF NONPARAMETRIC STATISTICS
TO THE SAMPLING IN SOLIDS MIXING

2.1 INTRODUCTION

One of the most important problems in solids mixing is to evaluate the homogeneity of a mixture or the degree of mixedness. For example, in preparing solid animal feed, very small quantities of drugs, vitamins, and minerals are often mixed with large quantities of feeds. These ingredients must be thoroughly distributed, and to test this a proper analysis of the sampling results is required. The results have been analyzed traditionally by using parametric statistical tests (Weidenbaum, 1953; Harnby, 1971). However, the use of nonparametric statistics instead may be advantageous in some cases in the sampling and definitions of the degree of mixedness because of the uncertainty involved in assuming the normality of the distribution of the population. Many of the nonparametric tests are simpler to conduct and have higher power to detect true difference than the usual parametric procedures (Conover, 1971).

One of the purposes of the statistics is to provide measures for the extent of subjectivity that enters into an investigator's conclusions. This is accomplished by setting up a theoretical model for the experiment, for example, the model of tossing a coin. The laws of probability are applied then to this model to determine the probabilities for the various possible outcomes of the experiment under the assumption that chance alone determines the outcome of the experiment. While the description of a theoretical model

may not be easy, the real difficulty lies in finding probabilities associated with the model. To meet the difficulty the model is changed slightly in order to obtain the "exact" solution to the "approximate" problem. Such an approach is called parametric statistics and includes many well-known tests such as t-test, F-test, and chi-square test, which have been employed extensively as statistical methods in investigations of the solids mixing problems (Weidenbaum, 1958).

On the other hand, the problem may be approached without making any change in the model and using rather simple and unsophisticated methods to find the desired probabilities. Thus, an "approximate" solution to the "exact" problem can be found. Such an approach of statistics is termed "nonparametric statistics."

This study was undertaken to show that nonparametric statistics can be applied to analysis of the sampling results in solids mixing. Its uses are demonstrated by several examples.

2.2 NONPARAMETRIC STATISTICS

In nonparametric statistics, the measurement scale need not be numerical. Types of measurement can be classified from the weakest scale to the strongest as follows:

(1) Nominal scale

This merely categorizes the data. For example, the quality of a mixture can be classified by visual inspection as "passed" or "failed." Observations may be classified according to categories.

(2) Ordinal scale

This scale refers to measurements where only the comparisons, namely "greater than," "less than," and "equal to" between measurements

are employed. Observations may be arranged from the smallest to the largest. For example, sample A may be more homogeneous than sample B by visual inspection.

(3) Interval scale

The scale not only provides the relative order of measurement, the size of difference between two measurements is also provided. A zero point and a unit by which the interval between two measurements can be described are defined for this scale. The numerical value of the observation according to this scale is physically meaningful.

(4) Ratio scale

This scale is similar to the interval scale; it has no natural zero but allows measuring ratios. For example, in the definition of the degree of mixedness, we usually associate the degree of mixedness of 0% with the completely segregated state, and 100% with the completely mixed state. The actual numerical value of the degree of mixedness is merely a comparison with an arbitrary reference point at the completely segregated state.

The steps in testing a hypothesis statistically are as follows:

(1) Formulate a null hypothesis H_0, and its alternative H_1, regarding a population parameter, etc.

(2) Select a test statistic which is most powerful.

(3) Describe a rule for accepting or rejecting H_0.

(4) Draw samples and test H_0.

(5) Apply the rule described in Step (3) in order to make a decision as to the acceptance or rejection of H_0.

Rejection of H_0 is equivalent to the statement that H_0 is false. This implies that H_1 is true. On the other hand, acceptance of H_0 does not imply

that H_0 is true. It simply indicates that H_0 has not been shown to be false. Therefore, in determining if a statement is false, make that statement H_0. In determining if a statement is true, make that statement H_1. The procedure for testing a hypothesis is shown in Fig. 1. Type I error (level of significance) is the probability of rejecting a true null hypothesis, and type II error is the probability of accepting a false null hypothesis. The probability of the correct decision in rejecting a false null hypothesis, $1-\beta$, is called power. The critical level is the smallest significance level at which the null hypothesis would be rejected for the given observation.

2.3 APPLICATIONS OF NONPARAMETRIC STATISTICS TO THE SAMPLING IN SOLIDS MIXING

2.3.1 Test of Sampling Techniques

In mixing a certain component with other components, the sample mean will not vary far from its known population mean. The problem is then to determine objectively whether the sample mean and the known population mean are significantly different. The available nonparametric tests, depending on types of sample obtained and types of measurement involved, are given in TABLE 1. If the means from different sets of sample vary significantly, the sampling may have been biased due to location or method (Weidenbaum, 1958). This bias needs to be corrected before further sampling. Notice that the tests listed under the nominal scale may be used in the ordinal scale of measurement, and the tests listed under nominal and ordinal may be used in the interval scale of measurement. Another application of this test of significance for means is to determine which mixer gives a better equilibrium mixture (see part E). For two different mixers, we can determine which is a better mixer based on the comparison of the means in the samples by using the same method of sampling.

THIS BOOK
CONTAINS
NUMEROUS PAGES
WITH DIAGRAMS
THAT ARE CROOKED
COMPARED TO THE
REST OF THE
INFORMATION ON
THE PAGE.

THIS IS AS
RECEIVED FROM
CUSTOMER.

(a) Ho is true

(b) Ho is false
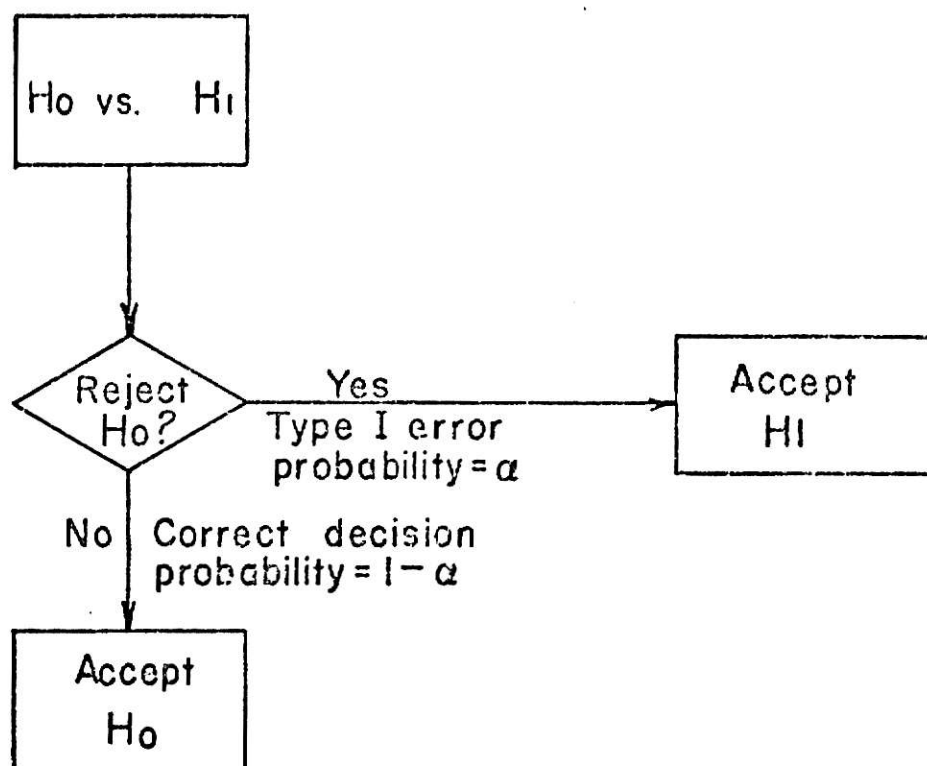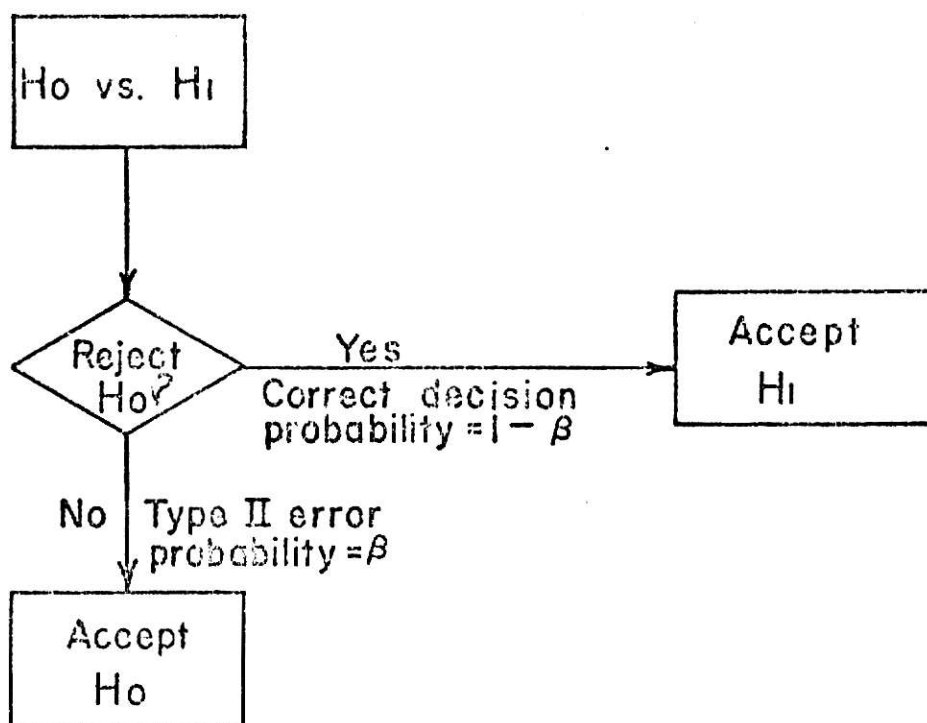
Fig. I.   Two types of error in hypothesis testing.

TABLE 1

NONPARAMETRIC TESTS FOR MEANS

Types of Samples	Measurement Scales		
	Nominal	Ordinal	Interval
Univariate random sample $x_1, \ldots, x_n$	Binomial test	Quantile test	Wilcoxon test
Bivariate sample $(x_1, y_1), \ldots,$ (x_n, y_n)	McNemar test	Sign test	Wilcoxon test Randomization test
Multivariate sample	Cochran test	Friedman test Bell-Doksum test	--
Two random samples $(x_1, \ldots, x_n)$ & $(y_1, \ldots, y_m)$	Chi-square test	Mann-Whitney test Tukey's quick test	Randomization test
Several random samples	Chi-square test	Median test Kruskal-Wallis test Bell-Doksum test Slippage test	--
Other types	Many-way Contingency table	Median test extended Durbin test for BIBD (Balanced Incomplete Block Design)	--

EXAMPLE 1.   Wilcoxon Signed Ranks Test for Means
            to Justify the Sampling Technique

If the population mean is known, the test of means can be a criterion

of the sampling procedure. Harnby (1966) studied the performance of mixing

the materials with segregating tendency (differences in size, density, etc.)

in various industrial mixers. He plotted the "discharge profile" by sampling

at the outlet of a mixer at fixed time intervals until the discharge was com-

pleted. The "discharge profile" plots the percentages of millet in the sam-

ples against sample numbers for each run. In his study the procedure for

sampling in a Rotacube mixer operating with impeller can be tested by using

the Wilcoxon signed ranks test. The data is given in TABLE 2. A nonpara-

metric-statistics package for the IBM 360 NonPAR which includes the Wilcoxon

signed ranks tests (Moe and Kemp, 1971) was used to perform the test. The

results are:

H_0 : The mean $E(X)$ is equal to 0.80

versus

H_1 : $E(X)$ is not equal to 0.80

The sample size is 37

The test statistic is 383.00

At α (level of significance) = 0.100, H_0 is accepted

The critical level $\hat{\alpha}$ = 0.438

Hence, we can conclude that the sampler performed satisfactorily under these

test conditions.

2.3.2 Test of Scale-up Procedures

In scaling up solids mixers, we wish to determine if the larger mixer

produces a "better" mixture than that produced in the pilot mixer. In other

words, we wish to determine if the variation in composition among spot samples

TABLE 2

DATA OF THE DISCHARGE PROFILE IN ROTOCUBE
(WITH IMPELLER OPERATING)
(MEAN = 0.80)   (HARNBY, 1966)

Sample No.	Fraction of millet in sample
1	0.9500
2	0.8625
3	0.8620
4	0.7750
5	0.7188
6	0.6630
7	0.5375
8	0.6500
9	0.7125
10	0.7188
11	0.7190
12	0.7437
13	0.7435
14	0.7313
15	0.7188
16	0.7188
17	0.7150
18	0.7437
19	0.7500
20	0.7563
21	0.7500
22	0.7312
23	0.7750
24	0.8075
25	0.8188
26	0.8438
27	0.8929
28	0.8813
29	0.8895
30	0.9000
31	0.9063
32	0.9288
33	0.9312
34	0.9400
35	0.9563
36	0.9641
37	0.9725

In the larger mixer is smaller than that in the pilot-scale mixer. This can be tested by the Siegel-Tukey test, the Mann-Whitney test, the Mood test, the Freund-Ansari test, or the Klotz inverse normal score test. The usual parametric test for determining the equality of two variances is called the F-test. However, the F-test is fairly sensitive to any departure from normality (Siegel and Tukey, 1960). Therefore, use of a nonparametric test is recommended when the distribution of the population may be nonnormal. Here use of the Mood test for variance is specifically illustrated.

EXAMPLE 2. Mood Test for Scaling-up Procedure

Suppose x_i (i = 1, 2, ..., m) stands for the concentration of a key constituent of the mixture in a pilot mixer, and y_i (i = 1, 2, ..., n) stands for that in a larger mixer. In the Mood test the sample sizes need not be equal (i.e., m not necessarily to equal n). A Mood test was performed on the data given in TABLE 3. The hypothesis and test result are:

H_0 : Var. (X) $\geq$ Var. (Y)

versus

H_1 : Var. (X) < Var. (Y)

The test statistic is 2320.5 at α = 0.050. Since the critical level, $\hat{\alpha}$, is 0.817, H_0 is accepted. Thus, we can conclude that the mixture in the larger mixer is as good as that in the pilot mixer.

2.3.3 Test of the Distribution of a Solids Mixture

It is well-known that the theory of probability is applicable only to events whose frequency of appearance can be either directly or indirectly observed or deduced by logical analysis. Therefore, it may be desirable to determine whether a set of spot samples comes from a certain frequency distribution. In other words, we compare experimental data with those estimated

TABLE 3

CONCENTRATION OF ONE KEY CONSTITUENT
IN MIXTURE OF PILOT AND SCALED-UP MIXERS
AFTER THE SAME NUMBER OF REVOLUTIONS

Sample No.	Concentration in pilot mixer	Concentration in scale-up mixer
i	X_i	Y_i
1	0.227	0.209
2	0.176	0.014
3	0.252	0.165
4	0.149	0.171
5	0.016	0.292
6	0.055	0.271
7	0.234	0.151
8	0.194	0.235
9	0.243	0.147
10	0.099	0.099
11	0.184	0.063
12	0.147	0.184
13	0.088	0.053
14	0.161	0.228
15	0.171	0.271
16	0.174	0.019
17	0.194	0.127
18	0.248	0.151
19	0.206	0.101
20	0.089	0.179

from a theoretical model. In terms of solids mixing, suppose a certain component (say 20% by weight) is to be mixed with another solid mixture. If no segregation in the mixture is assumed, there would be an ideal perfect random mixture concerning the distribution of this particular component in the mixture. The perfect random mixture (which may be theoretically obtained by a perfect mixer) would be a certain distribution of that component with 20% as the mean. The nonparametric tests would enable us to determine whether the distribution of that component in a mixture is normal, Poisson, or Gama, etc. If the variation from sample to sample is too large compared to that obtained from perfect random mixture, some segregation may be expected in the mixture.

A test of goodness of fit usually involves examining a random sample from an unknown distribution $F(X)$ in order to test the null hypothesis H_0 that the unknown distribution is in fact a known, specified function, $F(X)$.

Samples x_1, x_2, ..., x_n are randomly drawn from the population, and are compared to $F(X)$ to see if it is reasonable to say that $F(X)$ is the true distribution function of the random sample. Nonparametric tests of goodness of fit which can be used for this purpose are given in TABLE 4. Here the Lilliefors test is employed to test the normality of the population distribution.

EXAMPLE 3. Lilliefors Test for Normality

Suppose fifty random samples are taken from a solid mixture. The concentration of the key component in the samples is shown in TABLE 5. The test determines if the difference between the normal distribution function and the true distribution function (unknown) is insignificant. If the test results in rejecting the null hypothesis (the distribution of the solid mixture is normal), other types of distribution may be postulated for further testing.

TABLE 4

NONPARAMETRIC TESTS OF GOODNESS OF FIT

Types of samples	Measurement scales		
	nominal	ordinal	interval
Univariate random sample $x_1, \ldots, x_n$	Chi-square test	Komogorov test Cramer-von Mises test for specified population Lilliefors test for unspecified normal population	-
Bivariate random sample $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$	-	Smirnov test Cramer-von Mises test Wald-Wolfowitz test	-
Several random samples	-	Birnbaum-Hall test Smirnov test	-

TABLE 5

CONCENTRATION OF A KEY COMPONENT
IN FIFTY SAMPLES

Sample no.	Concentration, %	Sample no.	Concentration, %
1	23	26	58
2	23	27	58
3	24	28	58
4	27	29	58
5	29	30	59
6	31	31	61
7	32	32	61
8	33	33	62
9	33	34	63
10	35	35	64
11	36	36	65
12	37	37	66
13	40	38	68
14	42	39	68
15	43	40	70
16	43	41	73
17	44	42	73
18	45	43	74
19	48	44	75
20	48	45	77
21	54	46	81
22	54	47	87
23	56	48	89
24	57	49	93
25	57	50	97

H_0 : the random sample has the normal distribution with unspecified mean and variance.

H_1 : the distribution function of x_i's is nonnormal

.

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 55.2$$

$$s = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2 = 10.7$$

The Lilliefors test demands the rejection of H_0 at $\alpha = 0.05$ if the test statistic exceeds its 0.95 quantile, 0.125. In this example, the test statistic is 0.08. Therefore the null hypothesis is accepted.

The acceptance of the null hypothesis does not mean that the parent population is normal, but it does mean that the normal distribution appears to be a reasonable approximation to the true unknown distribution. It is appropriate to assume that the parent population is normally distributed and, therefore, either a nonparametric method or a parametric statistical procedure which assumes a normal parent distribution may be appropriate for further testing with these data.

2.3.4 Test of Significance for Fraction Satisfactory

In quality control we are constantly concerned with the problem of knowing the fraction of the population which meets the quality criterion. The binomial test can be used to solve this problem. For example, in mixing a certain species with others the mixture is considered to be adequate if a certain number, for instance, five or less, of the agglomerates of that particular species appears in any sample. If more than five agglomerates are found, the mixture is considered unsatisfactory.

The hypothesis can be formulated as follows:

H_0 : there are five or less than five agglomerates appearing in a spot sample after mixing, i.e., mixing is adequate.

versus

H_1 : mixing is inadequate.

H_0 is to be tested on the basis of ten spot samples randomly withdrawn from the population (mixture). The assumption is made that each spot sample has the same probability of containing five or less agglomerates, independent of the other samples. If an excessive number of samples are not satisfactory (containing more than five agglomerates per sample), H_0 should be rejected. If we assume that the test statistic T be the total number of unsatisfactory samples, then T has the binomial distribution with parameters p and n. Since n = 10, p = 0.05, from the binomial distribution

$$Pr(T \leq 2) \leq \sum_{i=0}^{T} \binom{n}{i} p^i (1-p)^{n-i}$$

$$= \sum_{i=0}^{2} \binom{10}{i} p^i (1-p)^{10-i}$$

$$= 0.9885$$

and therefore

$$P(T > 2) = 0.0115.$$

The set of points in the sample space which correspond to values of T greater than 2 is called the critical region. Because the probability of locating a point in the critical region when H_0 is true is very small (<0.0115), the decision rule is: reject H_0 if the observed outcome is in the critical region (when T > 2); otherwise, accept H_0.

2.3.5 Test of Significance for Determining when Equilibrium State is Reached

After a certain period of time, further effort spent in the mixing may not improve it significantly. Let $x_1, \ldots, x_n$ represent values of certain forms of the degree of mixedness at times $t_1, \ldots, t_n$, respectively. When equilibrium appears to have been reached the test of trend can be used to determine if equilibrium indeed has been reached. If there is a trend, the equilibrium state obviously has not been reached. The nonparametric tests available for testing trend are the Cox and Stuart test, the Daniel test, runs test, sign test, Spearman's ρ, and Kendall τ.

2.3.6 Test of Significance of Segregation

Numerous definitions for the degree of mixedness to measure the quality of a mixture have been proposed (Fan, et al., 1970). They are generally based on the standard deviation or the variance of the composition of samples taken from different locations in the mixture. They can also be used as measures of the degree of segregation.

To detect the degree of segregation, a mixture can be divided into two regions (not necessarily of equal volume). A number of samples is withdrawn from each region. A test is now applied to determine whether the difference between the means of the two regions could have occurred by chance or whether there is evidence of segregation. Suppose region 1 contains n_1 samples, and let the mean and standard deviation be $\bar{x}_1$ and s_1. Similarly, for region 2, let there be n_2 samples with mean $\bar{x}_2$ and standard deviation s_2.

According to Williams and Birks (1965), the ratio of the difference between two means to the standard error of the difference, R, can be considered as a measure of segregation:

$$R = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{(\frac{1}{n_1} + \frac{1}{n_2})}}$$

where

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Note that the variance of $\bar{x}_1$ from region I is s^2/n_1, and that of $\bar{x}_2$ is s^2/n_2. The variance of the difference between the means is, therefore, $s^2(\frac{1}{n_1} + \frac{1}{n_2})$. Such a measure of the segregation may also be called the segregation index, which can be determined by performing the student's t-test.

A new index of segregation based on the test of means in nonparametric statistics is proposed in the following example.

EXAMPLE 4. Test of Significance of Segregation

Richards (1966) employed William and Birks' (1965) segregation index in his study of segregation according to size of a heap formed by pouring between two vertical plates. The heap is shown in Fig. 2, where the numbers represent the weight percentage of fine particles. The sampling procedure is accomplished by dividing the heap into several small regions, and then withdrawing a sample from the center of each region. Each sample may be analyzed for the percentage composition of fine particles. The sampling results shown in Fig. 2 may be further divided into a central region, region 1, and two outer regions, region 2. Two outer regions may be treated together as one region because of symmetry.

The Mann-Whitney test for significance between the two sample means is

H_0 : $E(X) = E(Y)$, i.e., the mean concentrations in the outer and inner regions are equal.
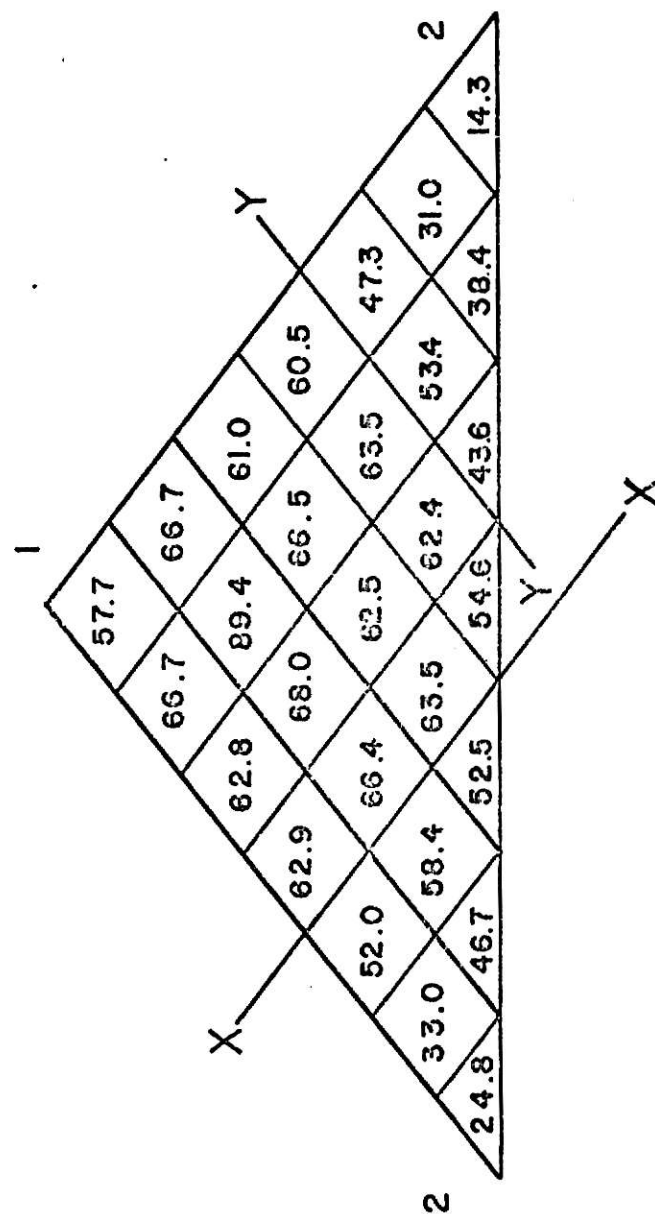
Fig. 2 . Segregation in a heap form by pouring
(Richards , 1966)

versus

H_1 : $E(X) \neq E(Y)$, i.e., they are not equal.

The calculated test statistic is 16.0 at level of significance α = 0.10, H_0 is rejected. The critical level is

$\hat{\alpha}$ = 0.000

The result indicates very marked segregation. This result of the non-parametric test is consistent with the conclusion found by Richards (1966) using the Williams and Birks' (1965) criterion mentioned above. From Fig. 2,

$\bar{x}_1$ = 3.4%, $\bar{x}_2$ = 41.3%

n_1 = 16, n_2 = 12

Hence

$$s^2 = \frac{224 + 1934}{16 + 12 - 2} = 83$$

and

$$R = \frac{63.4 - 41.3}{9.1\left(\frac{1}{16} + \frac{1}{12}\right)} = 6.3$$

Assuming R is distributed identically with student t-distribution, R > 3 indicates evidence of segregation with 99% certainty. Actually, the test statistic of the Mann-Whitney test can be used as an index of segregation without assuming that the population is a normal distribution. Therefore, according to this nonparametric statistical method, this index of segregation is

$$T = S - \frac{n(n+1)}{2}$$

where

$$S = \sum_{i=1}^{n} R(X_i)$$

n = sample size of population 1

The index T may be found by first finding S, the sum of the ranks

assigned to the observation from population one. Thus, in conjunction, the table of the quantiles of the Mann-Whitney test statistic, the degree of segregation can be indicated by this proposed index of segregation. Furthermore, if enough experimental data are available, the quality of the mixture can be graded as poor, fair, good, or excellent, according to the distribution of this statistic.

2.4 CONCLUSIONS

Applicability of the nonparametric statistical tests in the field of solids mixing is demonstrated in this report. The advantages of the nonparametric test are numerous. First, it uses a simple model. Second, it involves less computational effort, and, therefore, is easier and quicker to apply than the parametric statistics tests. Third, much of the theory behind the nonparametric method may be developed rigorously, using simple mathematics.

The procedure and theory behind each test mentioned in this paper are available in standard texts on nonparametric statistics (Conover, 1971; Bradley, 1968). When the sample size is large, a nonparametric statistical computer package, the IBM 360 NonPAR, written in FORTRAN IV (Moe and Kemp, 1971) can be employed instead of a calculator or hand calculation.

The nonparametric tests can be applied to test a variety of hypotheses other than those which are considered in this work. For convenience, the major nonparametric statistical tests are summarized on the following pages.

(1) Tests of goodness of fit:

Chi-square test, Kolmogrov test and Cramer-Von Mises test for specified populations. Lilliefors test for unspecified normal distribution.

(2) Tests for independence between classification of same data:

Three Chi-square tests with technically different H_0's.

(3) Tests for independence between two variates x_i and y_i:

Bell-Doksum test, Olmstead-Tukey test

(4) Tests for linear correlation:

Spearman's ρ and Kendall's τ.

(5) Test for trends:

With univariate sample: Cox and Stuart test

With bivariate sample: Spearman's ρ and Kendall's τ.

(6) One sample tests on paired observations x_i and y_i:

Wilcoxon signed ranks test, Sign test. They can test H_0's true

$(x_i - y_i) = 0, \leq 0, \geq 0$ or $E(X) =, \leq,$ or $\geq E(Y)$.

(7) Tests for specified median $\neq 0$:

Walsh test (This test has the curious feature that the test

statistic changes for each n & α)

Quantile test with $p^* = 0.5$ & $x^* \neq 0$

(8) Two-sample tests for equalities of means of $F(x)$ and $G(x)$.

Wilcoxon-Mann-Whitney U test

Sequential W-M-W test (Alling)

Smirnov two-sample test (general)

Median test (assuming symmetry of distributions)

Cramer-Von Mises two-sample test

Terry-Hoeffding direct normal scores test (uses expected values of

N(0, 1), variates of rank R)

Van der Waerden's inverse normal score test (uses $\phi^{-1}(\frac{Ri}{N+1})$) from

the normal distribution table.

Wald-Wolfowitz runs test

Tukey's quick test

(9) Two-sample tests for equality of dispersion:

Mood's W-test

Klotz inverse normal scores test

Siegel-Tukey test

(10) Tests for means corresponding to a one-way analysis of variance:

Kruskal-Wallis test

Bell-Boksum test

k-sample slippage test

Birbaum-Hall 3-sample test

k-sample Smirnov test

(11) Tests for means corresponding to a two-way analysis of variance:

Friedman test: 1 observation per cell

Friedman test: m observations per cell

Bell-Doksum test

Cochran's test when data are 0's and 1's

Extended Median test

(12) Confidence bands or intervals

Kolmogrov confidence band on distribution function $F(x)$

C. I. (confidence interval) on p-th Quantile test, $x_{p}*$

C. I. on median test using Wilcoxon signed rank statistic

C. I. on median test using Walsh tables

C. I. on difference between two means

C. I. on difference between two means using the Tukey's quick
    test statistic

(13) Tolerance limits:

$$X^{(0)} \leq X^{(1)} \leq \ldots \leq X^{(r)} \leq \ldots X^{(n-m+1)} \leq \ldots \leq X^{(n+1)}$$

$(-\infty)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $(+\infty)$

$(1 - \alpha)$ confidence that fraction q of population lies within

limit determined by sample.

2.5 REFERENCES

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CHAPTER III

DISTRIBUTION OF CONTACT NUMBER—
A MIXING INDEX FOR SOLIDS MIXING

3.1 INTRODUCTION

Most of the available definitions of the degree of mixedness which specifies homogeneity or distribution of the composition in a solid mixture are based on the variance of the concentration of a certain component among constant volume spot samples. This neglects the distribution of particles and packing inside the spot sample, i.e., it assumes that a spot sample is completely mixed. For solid-solid chemical reaction, the rate of reaction is proportional to the, contact points or area among particles. Thus a definition of the degree of mixedness based on the number of contact points appears to be of practical significance.

The contact number is the number of points of contact between two different types of particles for one key particle, a particle species which is selected as a reference. Smith, et al. (1929) determined the relationship between the porosity and the average number of contact points among particles. Selection of the key component simplifies the sampling procedure and broadens its applications in solids mixing, heterogeneous chemical reaction and other operations involving contact between different solid phases.

This chapter is concerned with a computer simulation of the distribution of the contact number for the binary system at the completely mixed state. Results were obtained for the two dimensional cubic and hexagonal packing arrangements at different concentrations of the key component.

3.2 DISTRIBUTION OF CONTACT NUMBER

Let A_i ($i = 0, 1$) denote the i-th component in a binary mixture with regular packing arrangement. When a particle of component A_j is taken randomly from a mixture, the number of all particles which surround and are in contact with that particular sampled particle of component A_j is called the total coordination number, denoted by n_j^*. For a mixture with regularly packed arrangement, n_j^* is constant. Such a sampling is called the coordination number sampling of size n_j^* (Akao, _et al._, 1971). If $c_{i(j)}$ is the coordination number contributed by component A_i given that the sampled particle is component A_j, it can be seen that

$$n_j^* = \sum_{i=0}^{1} c_{i(j)} = c_{1(j)} + c_{0(j)} ; \qquad j = 0, 1$$

For a binary mixture with the regularly packed arrangement, we have

$$n_0^* = n_1^* = n^*$$

If $i \neq j$ ($i, j = 0, 1$), $c_{i(j)}$ is specially called the contact number between particles of components A_i and A_j. If any particle of component A_0 is specially selected as the sampled particle, it is called the key particle and component A_0 is called the key component.

Let the probability of the contact number be $c_{1(0)}$, when the key component is A_0, be $P_{1(0)}$. Then

$$P_{1(0)} = Pr\{M_1 = c_{1(0)} | Y = A_0\}$$

where M_1 is the random variable which represents the contact number and Y is the random variable which can either be A_0 or A_1 depending on the selection of the key particle.

M_1 is distributed binomially at the completely mixed state, i.e.,

$$P_{1(0)} = \binom{n^*}{c_{1(0)}} X^{c_{1(0)}} (1 - X)^{n^* - c_{1(0)}} \tag{1}$$

where X is the concentration of component 1 in the mixture. The theoretical mean and variance of the binomial distribution, respectively, are known to be

$$E(c_{1(0)}) = n^* X \tag{2}$$

$$V(c_{1(0)}) = n^* X (1-X) \tag{3}$$

If we disregard the boundary, we have

$$Pr\{M_1 = 0 | Y = A_o\} = 1 \tag{4}$$

for a completely segregated state. Hence

$$E(c_{1(0)}) = 0 \tag{5}$$

$$V(c_{1(0)}) = 0 \tag{6}$$

3.3 DEGREE OF MIXEDNESS BASED ON CONTACT NUMBER

A mixing index has traditionally been in terms of the variances of spot samples. Since the variance is a macroscopic measure, it provides little information with regard to microscopic characteristics of a mixture. Such information is required in analyzing the compaction characteristics of the mixture or determining the rate of a reaction undergoing inside the mixture.

By comparing the sample mean of the contact number to the scale between the two extremes, i.e., the mean contact number of a completely mixed state and that of a completely segregated state, a measure of degree of mixedness can be defined as:

$$M = \frac{\bar{c}_{1(0)} - [E(c_{1(0)})]_{seg}}{[E(c_{1(0)})]_{mix} - [E(c_{1(0)})]_{seg}} \tag{7}$$

where $\bar{c}_{1(0)}$ is the mean contact number from samples. We have then

> $M = 0$ for the completely segregated state.

> $M = 1$ for the completely mixed state.

3.4 COMPUTER SIMULATION

The distribution of the contact number at the completely mixed state can be simulated on a computer to see the significance of the proposed definition of the degree of mixedness. Let $W(i)$, $i = 1, 2, \ldots, n^*$ denote the i-th position of a particle which is in contact with the sampled particle in the regularly packed arrangement. $W(0)$ is the position for the sampled particle. The cubic and hexagonal packed arrangements of particles are shown, respectively, in Figs. 1 and 2. Random numbers with a uniform distribution are generated to simulate a binary component system. Numbers 0 through 4 represent component A_0 and numbers 5 through 9 component A_1. A sequence of random numbers with (n^*+1) digits corresponds to the particles at positions $W(0)$, $W(1), \ldots, W(n^*)$. Suppose that a number with five digits, 31829, is generated for a two dimensional cubic arrangement which has a coordination number of 4. Then the particles at positions $W(0)$, $W(1)$, and $W(3)$ are of component A_0, and particles at positions $W(2)$ and $W(4)$ are of component A_1. Consequently the contact number for this key particle is $c_{1(0)} = 2$.

The contact number distribution at the completely segregated state is obviously a one-point distribution given by Eq. (4). The numerical experiment was carried out only for the completely mixed state. To attain the numerical stability, two thousand each of five and seven digital random numbers were
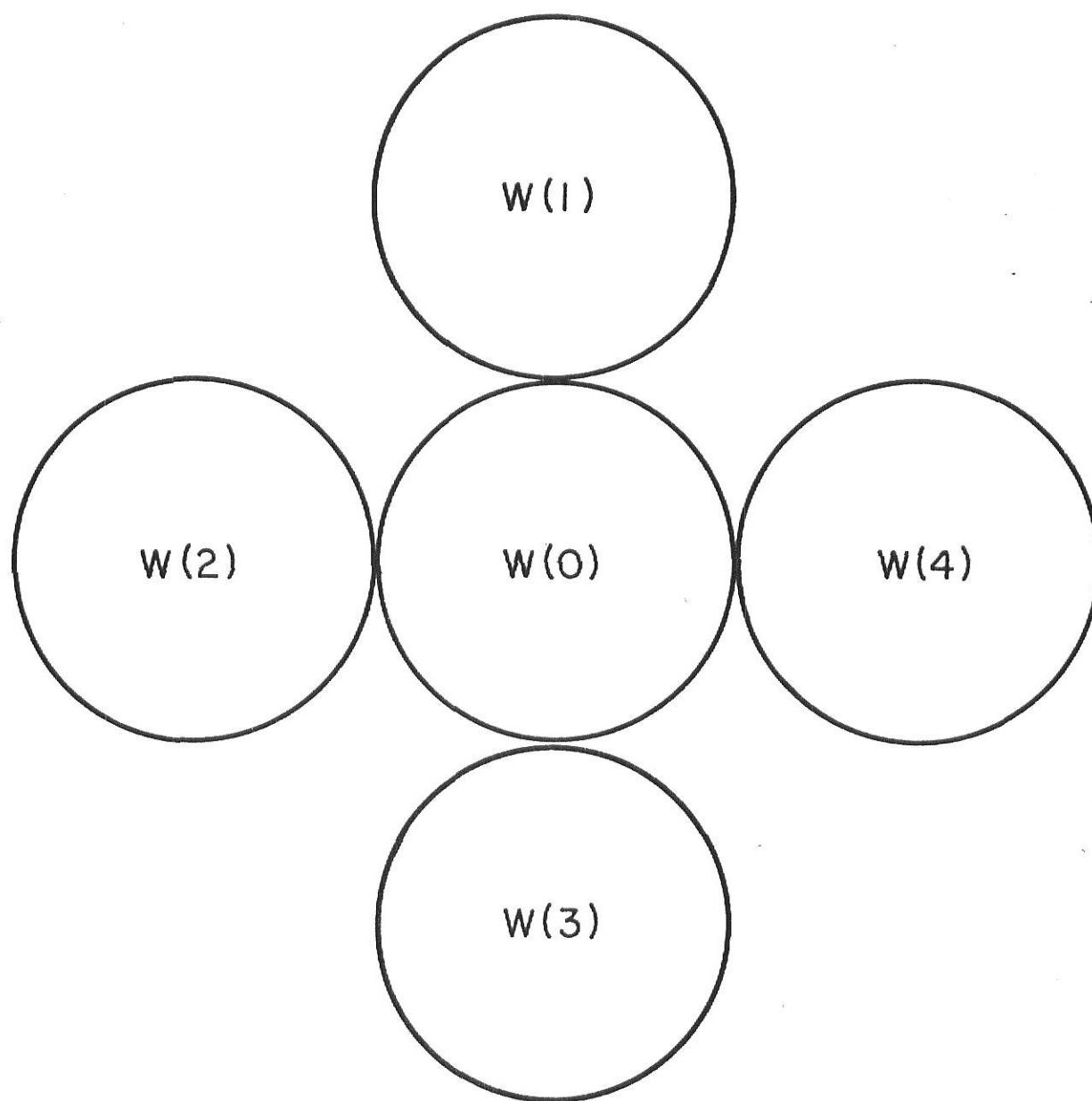
Fig. I.   Two dimensional cubic packing.
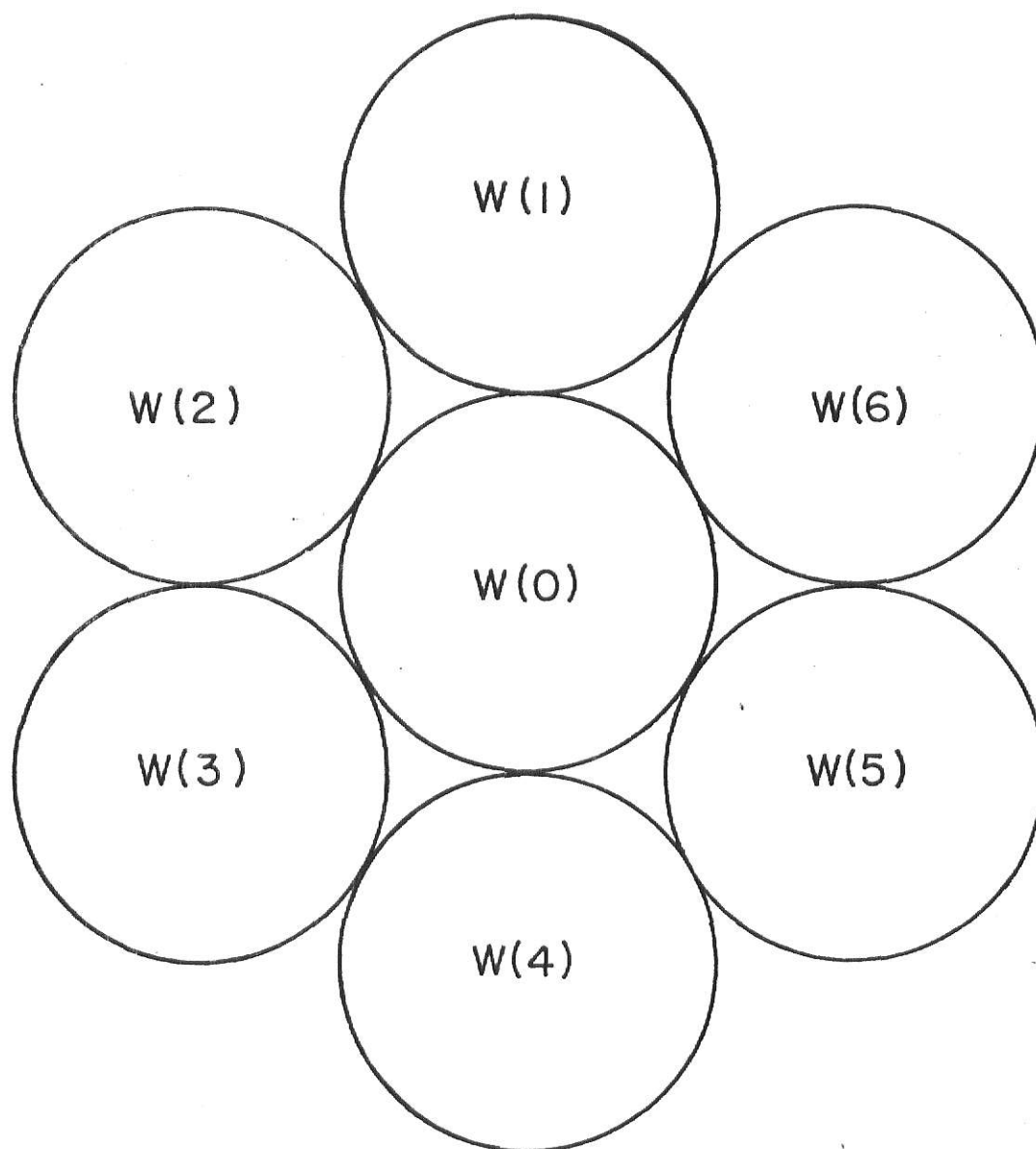
Fig. 2.   Two dimensional hexagonal packing.

generated. The computer program for this simulation is shown in Appendix 3.8. The random number was generated by the Subroutine RANDU in IBM Programmer's Manual (1967) which can produce 2^{29} terms before repeating. The method used in this subroutine is the power-residue method.

3.5 RESULTS AND DISCUSSION

The relative frequencies of the contact number at different concentrations are plotted in Figs. 3 through 5, and tabulated in TABLE 1 for the two dimensional cubic packing arrangement and those for the two dimensional hexagonal packing arrangement in Figs. 6 through 8 and TABLE 2. The theoretical predicted values of those frequencies based on the binomial distribution are also plotted on these figures for comparison. It can be seen that the simulated values are in fairly good agreement with the theoretical values.

The linear correlations of the expected value between the sample mean of the contact number and the concentration and the parabolic correlations between the sample variance and the concentration for the two dimensional cubic packing arrangement are shown, respectively, in Figs. 9 and 10. Similar results for the two dimensional hexagonal packing are shown in Figs. 11 and 12.

Values of the degree of mixedness defined in Eq. (7) are computed from the results of the simulation and are tabulated in TABLE 2 for different concentrations. We can see that the simulated value of the degree of mixedness is reasonably close to the theoretical value of 1.

The distribution of the mean contact number according to the binomial distribution is approximately normal when the sample size is large (Fisz, 1963); therefore, the t-test can be employed to examine if the sample mean is not significantly different from the population mean, which is theoretically the mean of the binomial distribution. For this purpose, let
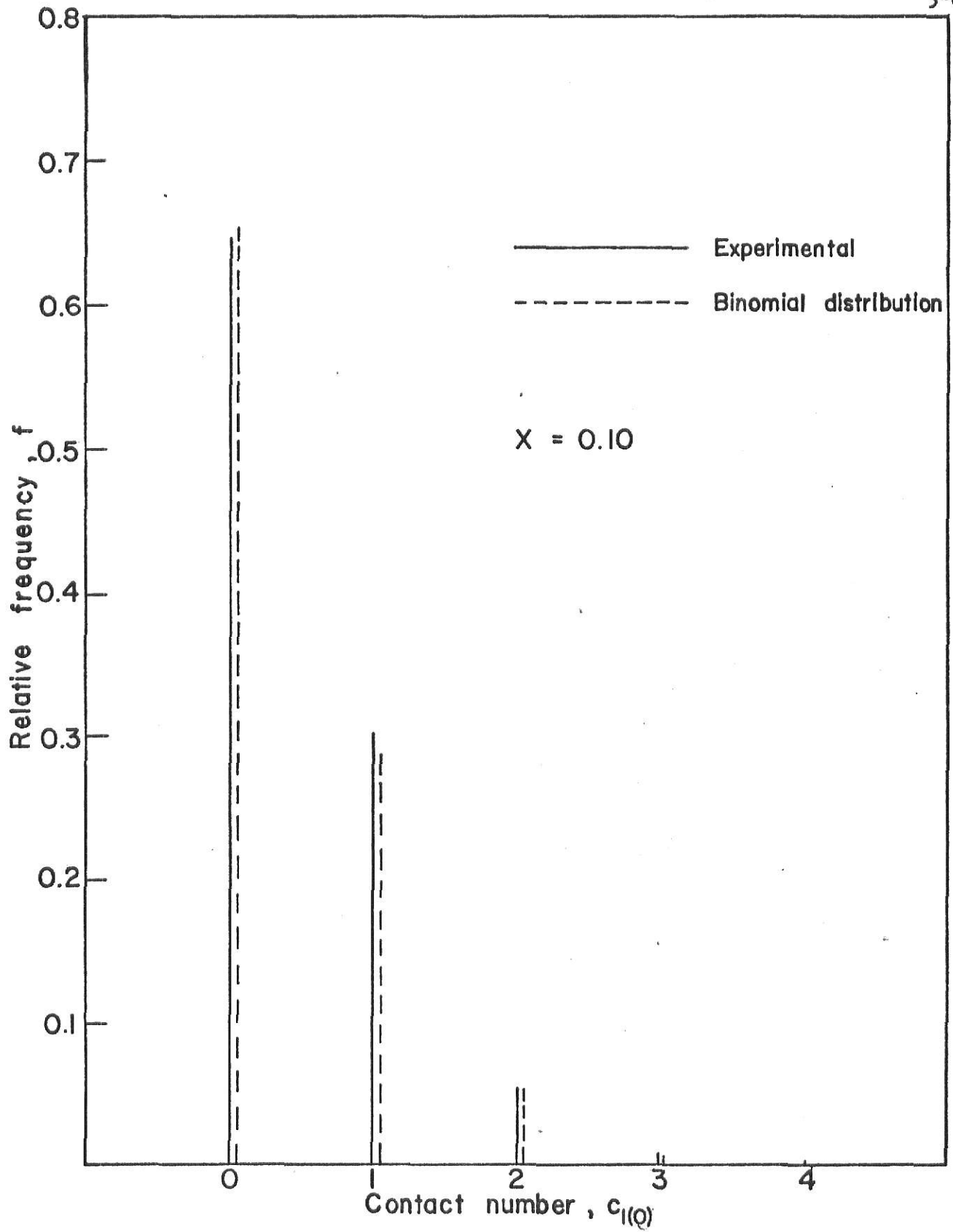
Fig.3. Distribution of contact number in two dimensional cubic packing at completely
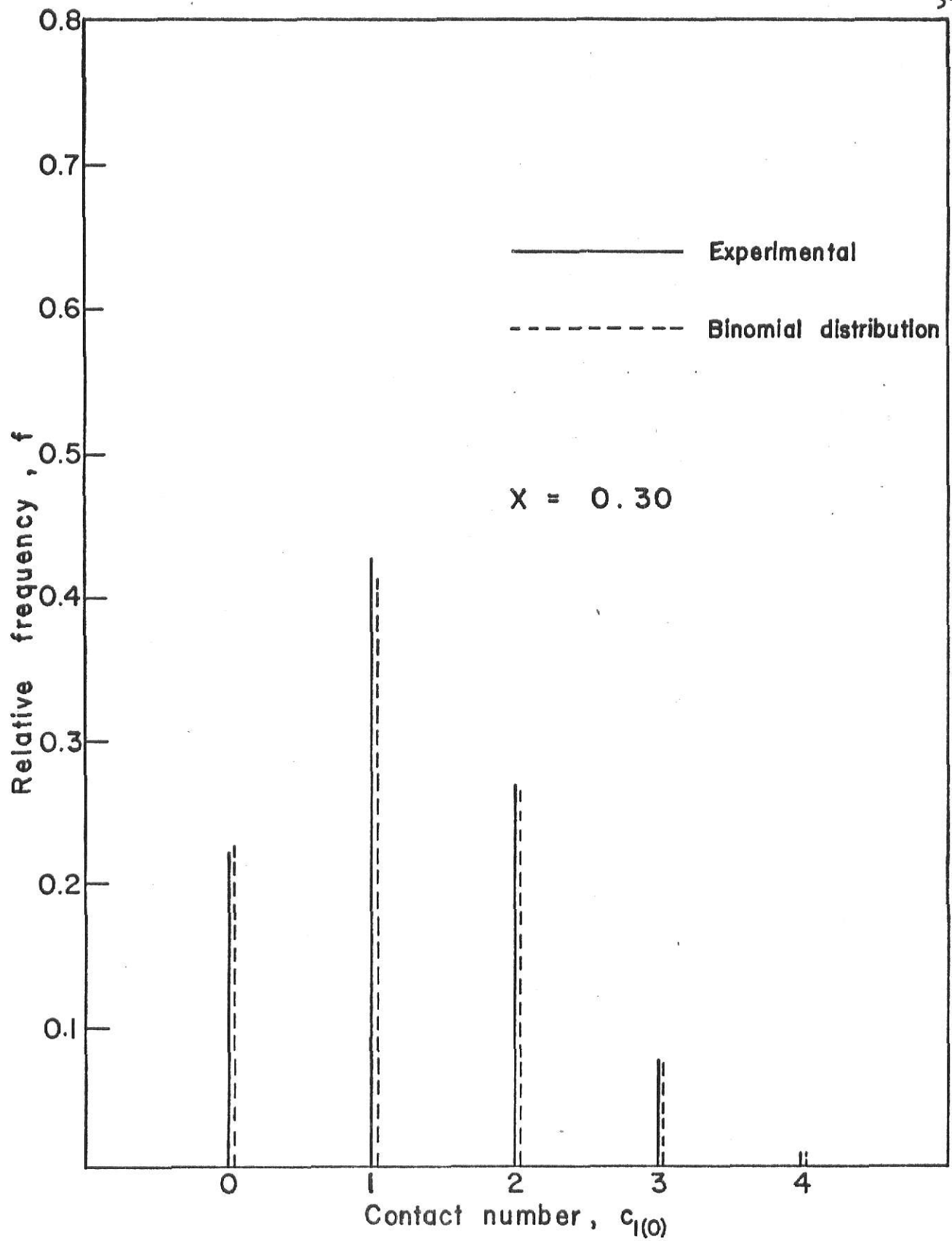
Fig. 4 .   Distribution of contact number in two

dimensional cubic packing at completely
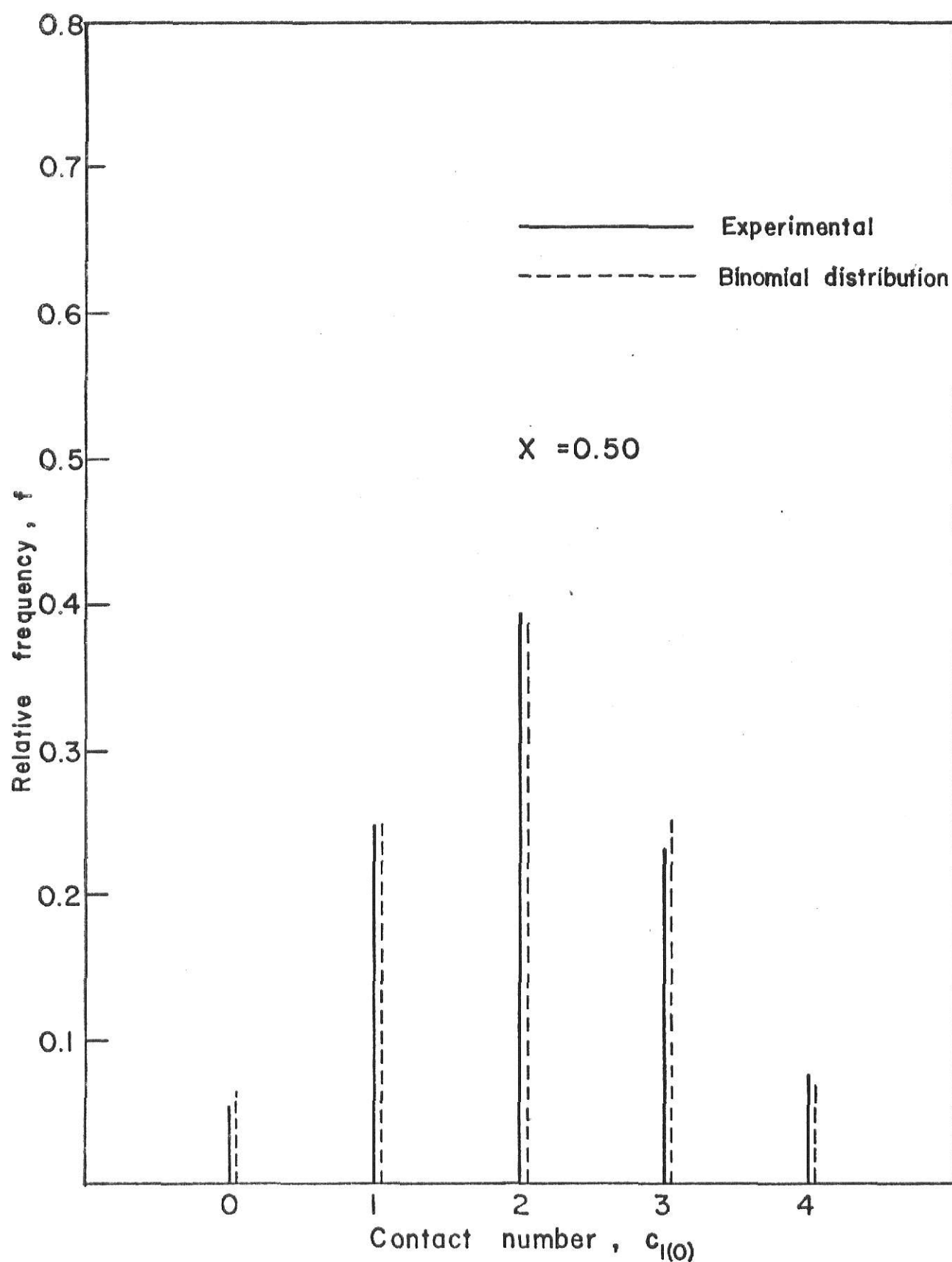
Fig. 5.   Distribution of contact number in two
dimensional cubic packing at completely

TABLE 1

COMPUTER SIMULATION RESULTS OF DISTRIBUTION OF
THE CONTACT NUMBER FOR TWO-DIMENSIONAL
CUBIC PACKING ARRANGEMENT

x_1	$c_1(0)$	Total frequency F	Relative frequency f	Binomial distribution	Sample mean, $\bar{c}_1(0)$	Sample variance
10%	0	1155	0.644531	0.6561	0.412946	0.3652
	1	541	0.301897	0.2916		
	2	89	0.049665	0.0486		
	3	7	0.003960	0.0036		
	4	0	0.0	0.0001		
30%	0	306	0.220939	0.2401	1.232491	0.8340
	1	586	0.423105	0.4116		
	2	371	0.267870	0.2646		
	3	109	0.0787	0.0756		
	4	13	0.009386	0.0081		
50%	0	55	0.056643	0.0625	2.023686	1.0056
	1	238	0.245108	0.25		
	2	381	0.392379	0.375		
	3	223	0.22966	0.25		
	4	74	0.07621	0.0625		
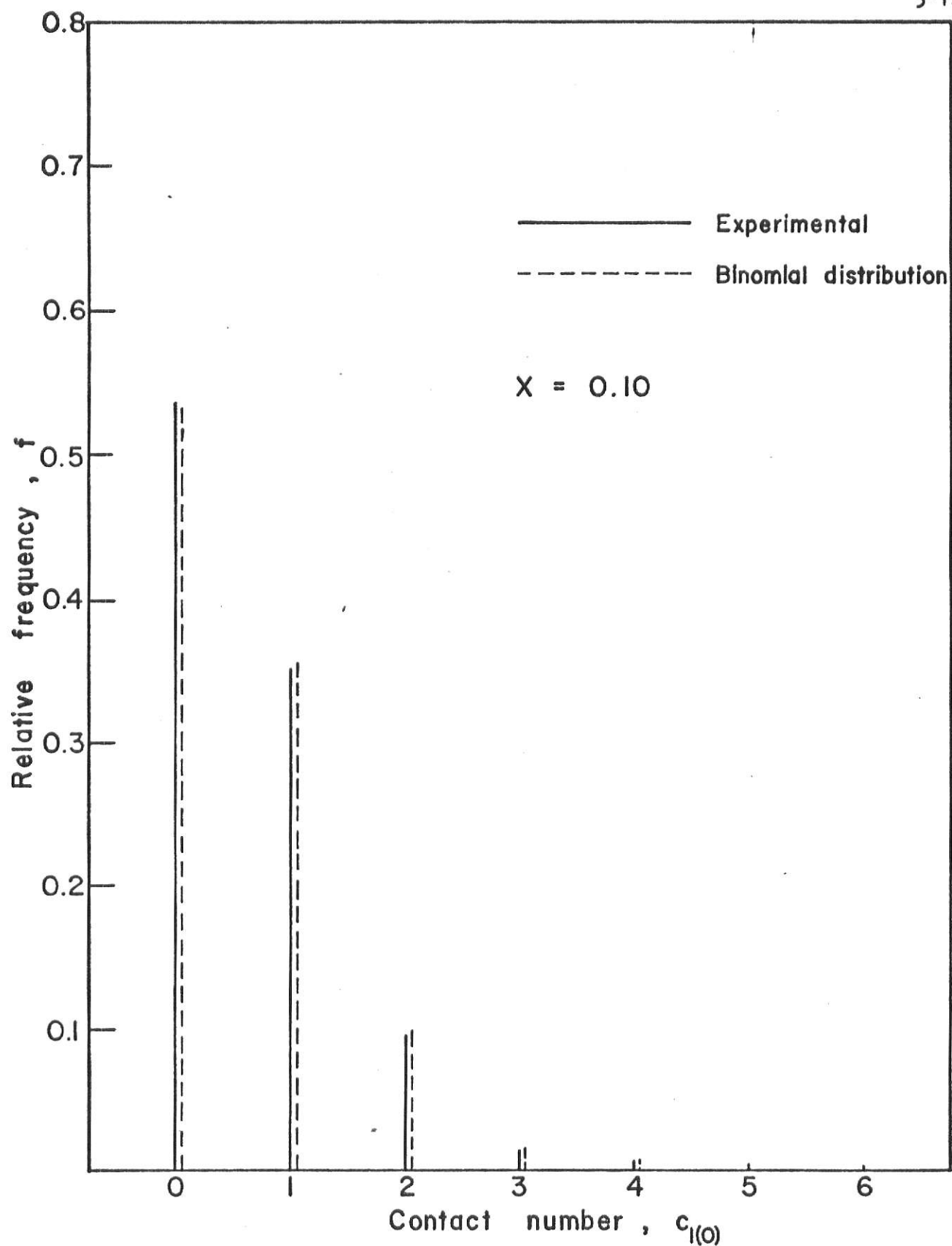
Fig. 6.   Distribution of contact number in two

dimensional hexagonal packing at com-
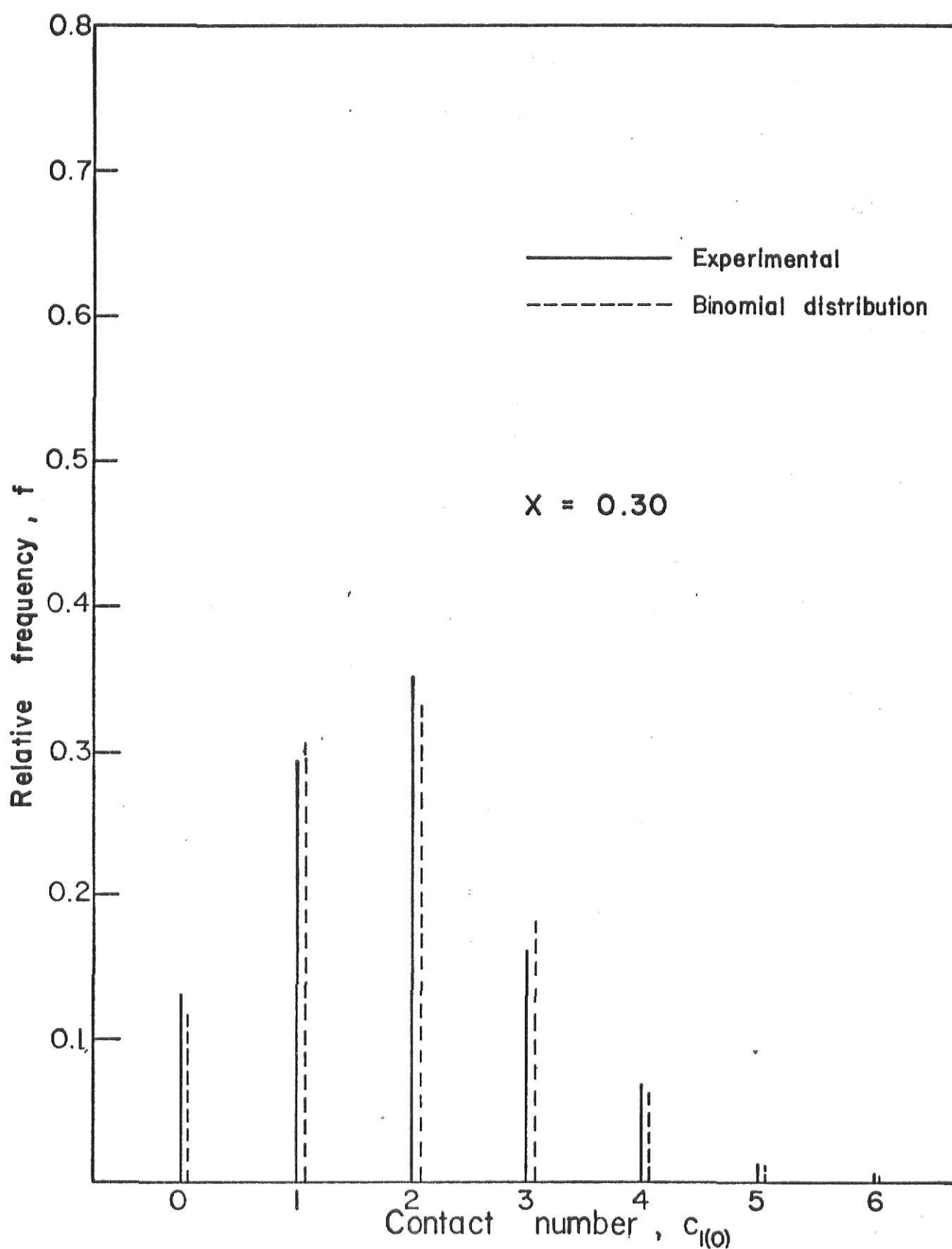
Fig. 7. Distribution of contact number in two dimensional hexagonal packing at completely
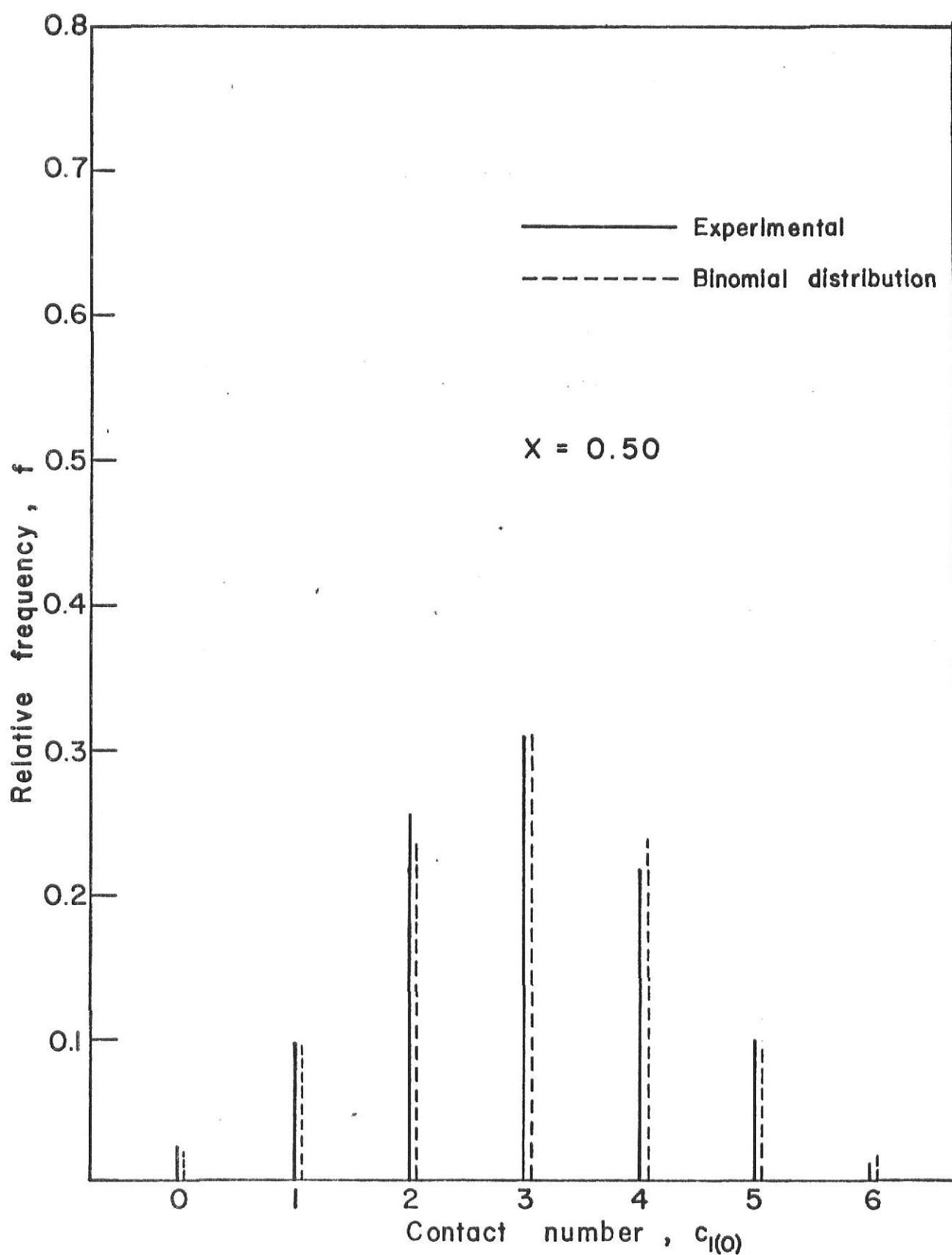
Fig. 8. Distribution of contact number in two dimensional hexagonal packing at com-

TABLE 2

COMPUTER SIMULATION RESULTS OF DISTRIBUTION OF
THE CONTACT NUMBER FOR TWO-DIMENSIONAL
HEXAGONAL PACKING ARRANGEMENT

x_1	$c_1(0)$	Total frequency F	Relative frequency f	Binomial distribution	Sample mean, $\bar{c}_1(0)$	Sample variance
10%	0	970	0.53859	0.531441	0.590783	0.5427
	1	632	0.350916	0.354294		
	2	169	0.093837	0.098415		
	3	26	0.014436	0.01458		
	4	16	0.002221	0.001215		
	5	0	0.0	0.000054		
	6	0	0.0	0.000001		
30%	0	173	0.123571	0.117649	1.786428	1.2837
	1	410	0.292857	0.302526		
	2	487	0.347857	0.324135		
	3	224	0.160000	0.18522		
	4	88	0.062587	0.059535		
	5	15	0.010714	0.010206		
	6	3	0.002143	0.000729		
50%	0	20	0.020661	0.015625	2.933884	1.508
	1	91	0.094008	0.09375		
	2	248	0.256198	0.234375		
	3	296	0.305785	0.3125		
	4	210	0.216942	0.234375		
	5	93	0.096974	0.09375		
	6	10	0.010331	0.015625		
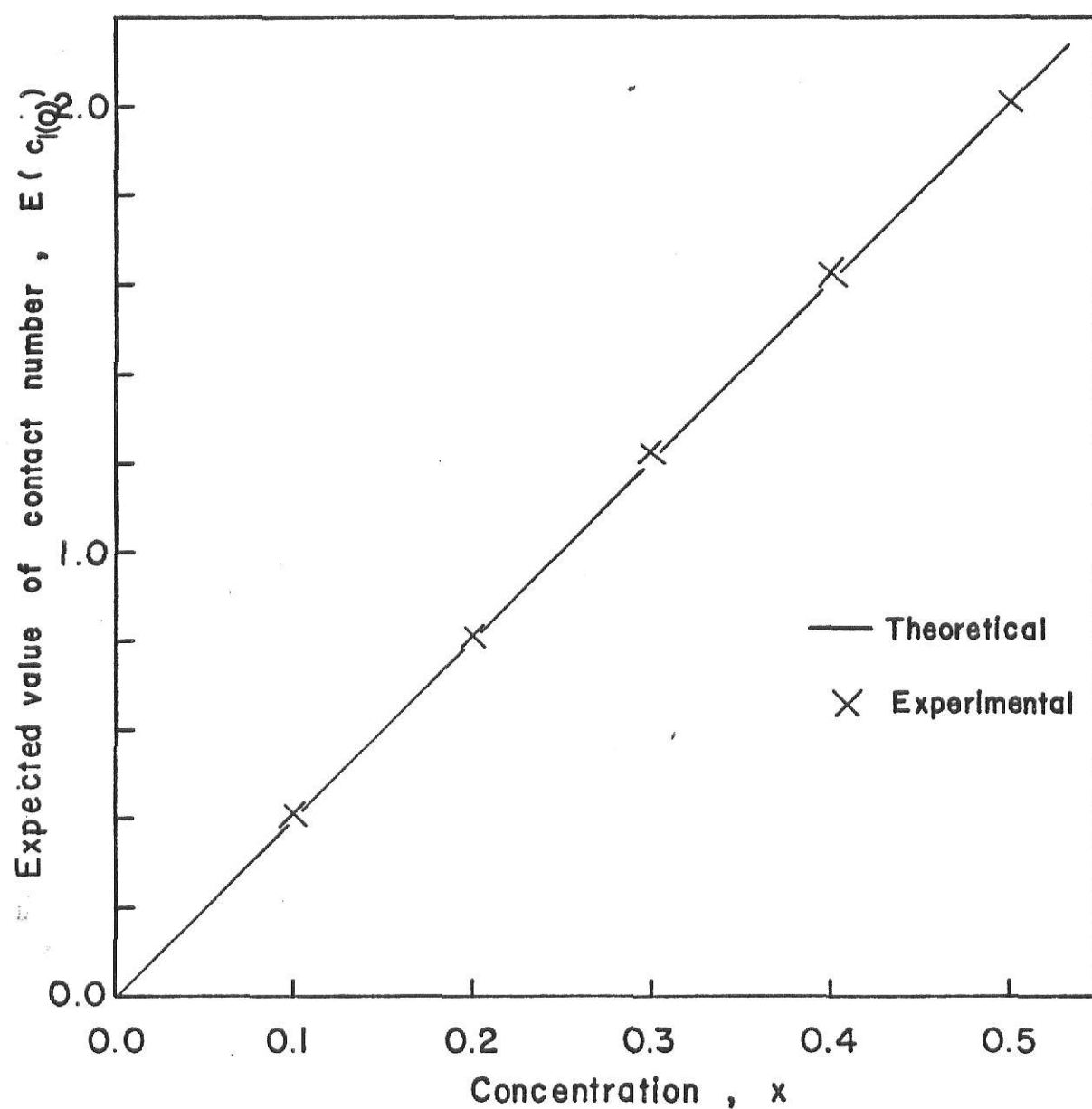
Fig. 9.   Expected value of contact number vs. concentration for two dimensional cubic packing at completely mixed state .
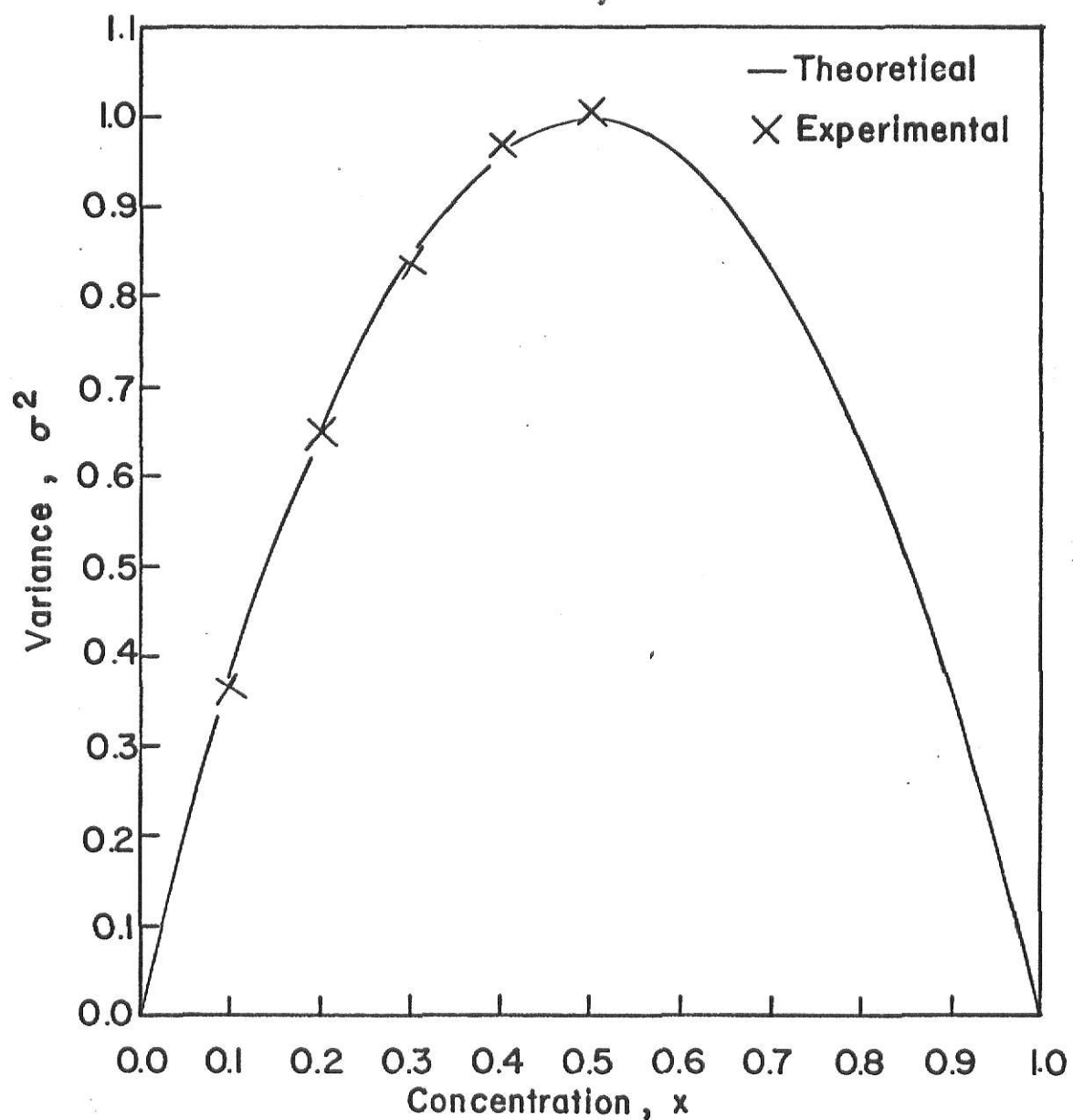
Fig. 10. Variance vs. concentration for two dimensional cubic packing at completely mixed state.
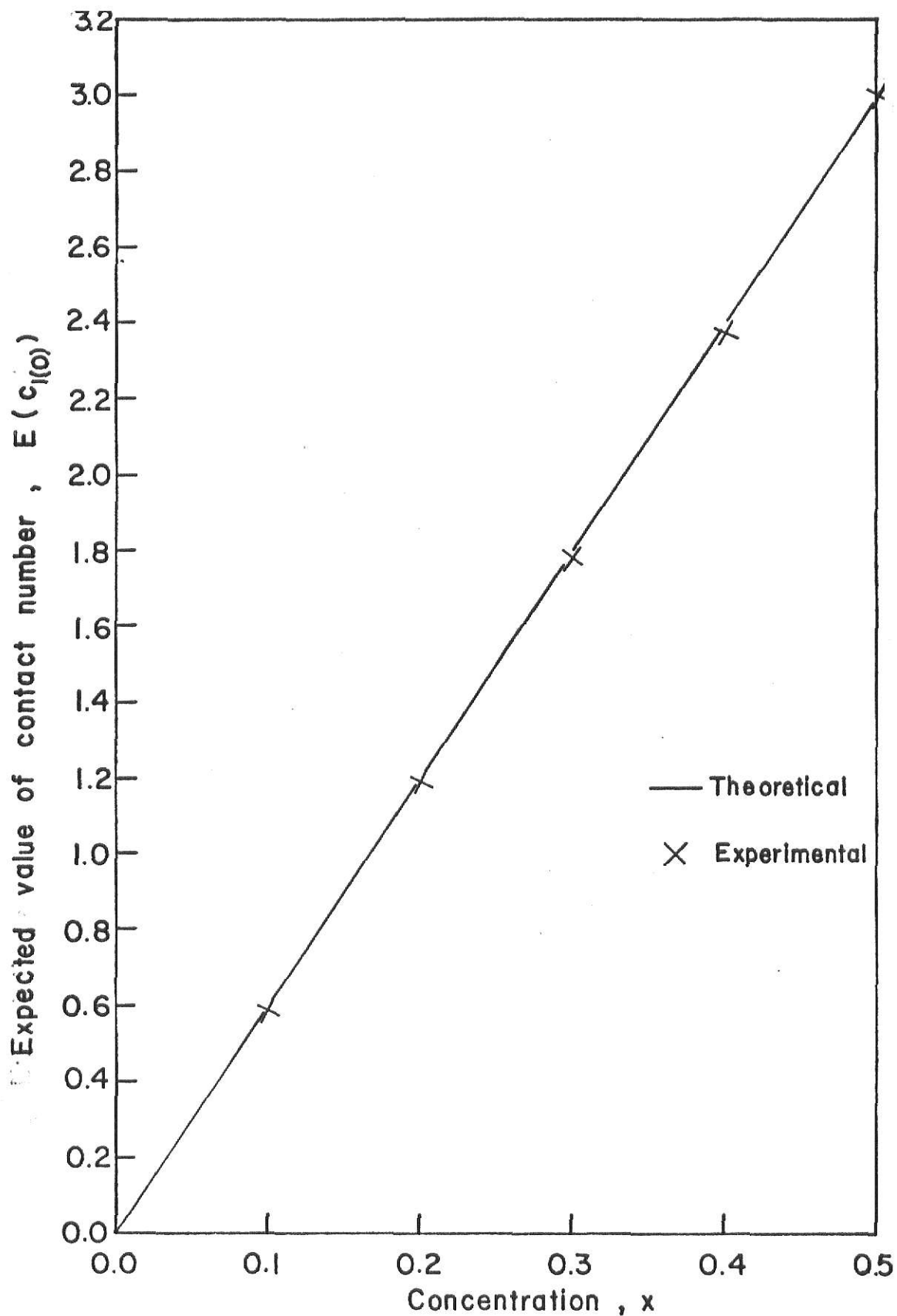
Fig. II.   Expected value of contact
number vs. concentration for
two dimensional hexagonal packing
at completely mixed state.
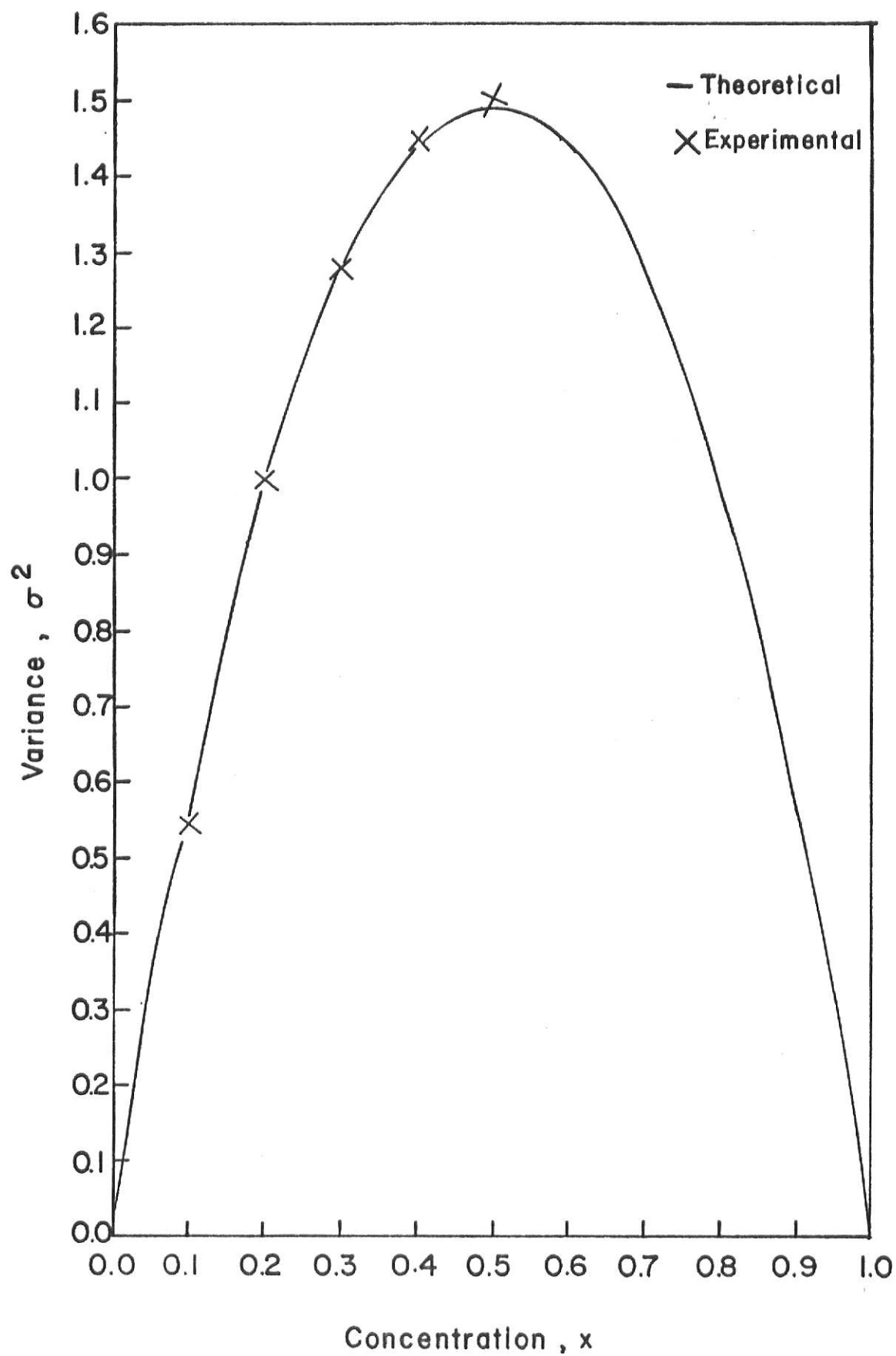
Fig. 12. Variance vs. concentration for two demensional hexagonal packing at completely mixed state.

$$H_0 : E(\bar{c}_{1(0)}) = E(c_{1(0)}), \text{ sample mean} = \text{population mean}$$

$$H_1 : E(\bar{c}_{1(0)}) \neq E(c_{1(0)}), \text{ sample mean} \neq \text{population mean.}$$

Results of the test are summarized in TABLE 3. H_0 is accepted at the 5% significance level.

3.6 CONCLUDING REMARKS

In this chapter concepts of the contact and coordination numbers and a definition of the degree of mixedness based on these concepts are introduced. These concepts and definition are useful in understanding the microscopic and geometric characteristics of solid mixtures, and in analyzing operations and processes involving such mixtures. Results of the computer simulation for the contact number distribution under the completely mixed state agree well with the theoretical prediction, both for the two dimensional cubic and hexagonal packing arrangements at different concentrations of the key component.

TABLE 3

DEGREE OF MIXEDNESS, SAMPLES MEANS AND EXPECTED
VALUE AT THE COMPLETELY MIXED STATE

Packing arrangement	Concentration X_1	Sampled mean, $c_1(0)$	Expected value, $E[c_1(0)]$	Degree of mixedness, M	t-test	
					t-value, $\dfrac{c_1(0) - n^{*}X_1}{\sqrt{n^{*}X_1(1-X_1)/k}}$	Significantly different from the expected value
Cubic	10%	0.4130	0.4	1.0325	0.914	No
	20%	0.8212	0.8	1.0265	1.060	No
	30%	1.2325	1.2	1.0271	1.562	No
	40%	1.6375	1.6	1.0222	1.322	No
	50%	2.037	2.0	1.0118	0.740	No
Hexagonal	10%	0.6070	0.6	1.0116	0.404	No
	20%	1.2335	1.2	1.0280	1.370	No
	30%	1.8188	1.8	1.0103	0.624	No
	40%	2.4083	2.4	1.0035	0.237	No
	50%	3.0081	3.0	1.0027	0.208	No

3.7 REFERENCES

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3.8 APPENDIX

Computer Program for Simulating the Contact Number Distribution

```
 1            DIMENSION IZ(2000),IA1(2000)
 2            READ (5,1) IX
 3          1 FORMAT (I5)
 4            DO 3 I=1,2000
 5            IY=IX*65539
 6            IF(IY)5,6,6
 7          5 IY=IY+2147483647+1
 8          6 IX=IY
 9            YFL=IY
10            YFL=YFL*.4656613E-9
11            IY=YFL*10**5
12            IZ(I)=IY
13          3 CONTINUE
14            WRITE (6,999)
15        999 FORMAT (1X,35H2000 RANDOM NUMBERS ARE AS FOLLOWS:/)
16            WRITE (6,7) (IZ(I),I=1,2000)
17          7 FORMAT (1X,20(1X,I5))
18            DO 99 KAW=1,5
19            N0=0
20            N1=0
21            N2=0
22            N3=0
23            N4=0
24            M0=0
25            M1=0
26            M2=0
27            M3=0
28            M4=0
29            WRITE (6,990) KAW
30        990 FORMAT (1X,76HSIMNLATION OF DISTRIBUTION OF CONTACT NUMBER OF COMI
            1LETELY MIXED STATE AT X=,I1,2H0%)
31            DO 8 I=1,2000
32            IP1=IZ(I)/10000
33            IP2=(IZ(I)-IP1*10000)/1000
34            IP3=(IZ(I)-IP1*10000-IP2*1000)/100
35            IP4=(IZ(I)-IP1*10000-IP2*1000-IP3*100)/10
36            IP5=IZ(I)-IP1*10000-IP2*1000-IP3*100-IP4*10
37            KAW1=10-KAW
38            DO 9 J=1,KAW1
39            IF(IP4-(J-1)) 9,11,9
40          9 CONTINUE
41            MUM1=0
42            MUM2=0
43            MUM3=0
44            MUM4=0
45            MUM5=0
46            MUM6=0
47            KAW2=KAW1+1
48            DO 110 J=KAW2,10
49            IF (IP1-(J-1)) 110,112,110
50        110 CONTINUE
51            MUM1=MUM1+1
52        112 DO 113 J=KAW2,10
53            IF (IP2-(J-1)) 113,114,113
54        113 CONTINUE
55            MUM2=MUM2+1
56        114 DO 117 J=KAW2,10
57            IF (IP3-(J-1)) 117,118,117
58        117 CONTINUE
```

```
59              MUM3=MUM3+1
60       118 DO 120 J=KAW2,10
61              IF (IP5-(J-1)) 120,121,120
62       120 CONTINUE
63              MUM4=MUM4+1
64       121 IA1(I)=MUM1+MUM2+MUM3+MUM4
65              L=IA1(I)+1
66              IA1(I)=IA1(I)+10
67              GO TO (80,85,90,95,96),L
68        80 MO=MO+1
69              GO TO 8
70        85 M1=M1+1
71              GO TO 8
72        90 M2=M2+1
73              GO TO 8
74        95 M3=M3+1
75              GO TO 8
76        96 M4=M4+1
77              GO TO 8
78        11 NUM1=0
79              NUM2=0
80              NUM3=0
81              NUM4=0
82              DO 10 J=1,KAW1
83              IF(IP1-(J-1)) 10,12,10
84        10 CONTINUE
85              NUM1=NUM1+1
86        12 DO 13 J=1,KAW1
87              IF(IP2-(J-1)) 13,14,13
88        13 CONTINUE
89              NUM2=NUM2+1
90        14 DO 17 J=1,KAW1
91              IF(IP4-(J-1)) 17,18,17
92        17 CONTINUE
93              NUM3=NUM3+1
94        18 DO 20 J=1,KAW1
95              IF(IP5-(J-1)) 20,21,20
96        20 CONTINUE
97              NUM4=NUM4+1
98        21 IA1(I)=NUM1+NUM2+NUM3+NUM4
99              K=IA1(I)+1
100             GO TO (30,40,50,60,70),K
101        30 NO=NO+1
102             GO TO 8
103        40 N1=N1+1
104             GO TO 8
105        50 N2=N2+1
106             GO TO 8
107        60 N3=N3+1
108             GO TO 8
109        70 N4=N4+1
110         8 CONTINUE
111             WRITE (6,1001) (IA1(I),I=1,2000)
112      1001 FORMAT (1X,20(4X,I2))
113             NT=NO+N1+N2+N3+N4
114             FNO=NO
115             FN1=N1
116             FN2=N2
117             FN3=N3
118             FN4=N4
```

```
119          FNT=NT
120          FRACNO=FNO/FNT
121          FRACN1=FN1/FNT
122          FRACN2=FN2/FNT
123          FRACN3=FN3/FNT
124          FRACN4=FN4/FNT
125          FRACNT=FRACNO+FRACN1+FRACN2+FRACN3+FRACN4
126          NFO=NO*0
127          NF1=N1*1
128          NF2=N2*2
129          NF3=N3*3
130          NF4=N4*4
131          NFT=NFO+NF1+NF2+NF3+NF4
132          NF20=NO*0**2
133          NF21=N1*1**2
134          NF22=N2*2**2
135          NF23=N3*3**2
136          NF24=N4*4**2
137          NF2T=NF20+NF21+NF22+NF23+NF24
138          WRITE (6,1002)
139     1002 FORMAT (10X,5HA1(0),5X,1HF,5X,7HA1(0)*F,5X,10HA1(0)**2*F,5X,7HF/TC
             2TAL)
140          WRITE (6,1003) NO,NFO,NF20,FRACNO
141     1003 FORMAT (12X,1H0,3X,I4,8X,I4,10X,I5,5X,F8.6)
142          WRITE (6,1004) N1,NF1,NF21,FRACN1
143     1004 FORMAT (12X,1H1,3X,I4,8X,I4,10X,I5,5X,F8.6)
144          WRITE (6,1005) N2,NF2,NF22,FRACN2
145     1005 FORMAT (12X,1H2,3X,I4,8X,I4,10X,I5,5X,F8.6)
146          WRITE (6,1006) N3,NF3,NF23,FRACN3
147     1006 FORMAT (12X,1H3,3X,I4,8X,I4,10X,I5,5X,F8.6)
148          WRITE (6,1007) N4,NF4,NF24,FRACN4
149     1007 FORMAT (12X,1H4,3X,I4,8X,I4,10X,I5,5X,F8.6)
150          WRITE (6,1008) NT,NFT,NF2T,FRACNT
151     1008 FORMAT (8X,5HTOTAL,3X,I4,8X,I4,10X,I5,5X,F8.5)
152          FNFT=NFT
153          FNF2T=NF2T
154          RMEANO=FNFT/FNT
155          VARO=FNF2T-RMEANO**2
156          WRITE (6,800) RMEANO,VARO
157      800 FORMAT (10X,6H MEAN=,F9.6,9HVARIANCE=,F9.6//)
158          MT=MO+M1+M2+M3+M4
159          FMO=MO
160          FM1=M1
161          FM2=M2
162          FM3=M3
163          FM4=M4
164          FMT=MT
165          FRACMO=FMO/FMT
166          FRACM1=FM1/FMT
167          FRACM2=FM2/FMT
168          FRACM3=FM3/FMT
169          FRACM4=FM4/FMT
170          FRACMT=FRACMO+FRACM1+FRACM2+FRACM3+FRACM4
171          MFO=MO*0
172          MF1=M1*1
173          MF2=M2*2
174          MF3=M3*3
175          MF4=M4*4
176          MFT=MFO+MF1+MF2+MF3+MF4
177          MF20=MO*0**2
```

```
178        MF21=M1*1**2
179        MF22=M2*2**2
180        MF23=M3*3**2
181        MF24=M4*4**2
182        MF2T=MF20+MF21+MF22+MF23+MF24
183        WRITE (6,1010)
184   1010 FORMAT (10X,5HA1(1),5X,1HF,5X,7HA1(1)*F,5X,10HA1(1)**2*F,5X,7HF/TC
           3TAL)
185        WRITE (6,1003) M0,MF0,MF20,FRACM0
186        WRITE (6,1004) M1,MF1,MF21,FRACM1
187        WRITE (6,1005) M2,MF2,MF22,FRACM2
188        WRITE (6,1006) M3,MF3,MF23,FRACM3
189        WRITE (6,1007) M4,MF4,MF24,FRACM4
190        WRITE (6,1008) MT,MFT,MF2T,FRACMT
191        FMFT=MFT
192        FMF2T=MF2T
193        RMEAN1=FMFT/FMT
194        VAR1=FMF2T-RMEAN1**2
195        WRITE (6,800) RMEAN1,VAR1
196     99 CONTINUE
197        STOP
198        END
```

CHAPTER IV

CONTACT NUMBER AS AN INDEX OF RADIAL MIXING
IN THE MOTIONLESS MIXER

4.1 INTRODUCTION

Over thirty different mixing indexes in solids mixing have been reviewed by Fan, et al. (1970). Most of these indexes are in terms of the variances of some spot samples and some reference states of a mixture. The diversity of the definitions of these indexes is indicative of the complexity of the mixing process and the uncertainty of various concepts and notions in the field of solids mixing. It appears that these macroscopically and statistically defined mixing indexes cannot provide a sufficiently deep insight into the microscopic and geometric nature of a mixture.

In this chapter a new mixing index proposed by Akao, et al. (1973), which is based on the number of contacts between different particles, was applied to the analysis of radial mixing in a motionless mixer. Although axial mixing in a motionless mixer has been extensively studied (Chen, et al., 1972, 1973a, 1973b), the only work on radial mixing is that by Chen, et al. (1971). In their study a series of cross-sectional photographs were taken every inch along the axis of the total sample in the collector. The cross-sectional view of the collector on each photograph was divided into four quadrants, with each quadrant considered as a sample. In this work, instead of dividing into four parts, each photograph was regarded as a sample. After counting all red and white particles surrounding and in contact with each white particle, which was selected as the key particle, the average contact and coordination numbers for

each sample were calculated, and then the sample mean of the contact number and that of the coordination number were employed to calculate the degree of mixedness for the mixture in this collector. The relationship between the mixing index (so calculated) and the number of helices in the mixer was obtained.

4.2 THEORETICAL

Brothman, et al. (1949) and Coulson and Maitra (1950) used the measure of the interfacial area between two phases of moving media as an index of mixing. Instead of the contact area, the number of contact points among individual particles in a mixture has been proposed by Akao, et al. (1973) as a microscopic and geometric measure of the degree of mixedness.

Let A_i ($i = 0, 1$) denote the i-th component in a binary mixture with regular packing arrangement. When a particle of component A_j is taken randomly from a mixture, the number of all particles which surround and are in contact with that particular sampled particle of component A_j is called the total coordination number, denoted by n_j^*. For a regular packing mixture, n_j^* is constant. Such a sampling is called the coordination number sampling of size n_j^* (Akao, et al., 1971). If $c_{i(j)}$ is the coordination number contributed by component A_i given that the sampled particle is component A_j, it can be seen that

$$n_j^* = \sum_{i=0}^{1} c_{i(j)}$$

$$= c_{1(j)} + c_{0(j)} \quad ; \quad j = 0, 1$$

For a binary mixture with regular packing arrangement, we have $n_o^* = n_1^* = n^*$.

If $i \neq j$ ($i, j = 0, 1$), $c_{i(j)}$ is specifically called the contact number between particles of components A_i and A_j. If any particle of component A_o is

specifically selected as the sampled particle, it is called the key particle and component A_0 is called the key component.

Let the probability of the contact number be $c_{1(0)}$; when the key component is A_0, be $P_{1(0)}$. Then

$$P_{1(0)} = Pr\ (M_1 = c_{1(0)} \mid Y = A_0) \tag{1}$$

where M_1 is the random variable which represents the contact number and Y is the random variable which can either be A_0 or A_1 depending on the selection of the key particle.

M_1 is distributed binomially at the completely mixed state, i.e., (Akao, et al., 1973)

$$Pr\ (M_1 = c_{1(0)} \mid Y = A_0) = (c_{1(0)}^{n^*})\ X^{c_{1(0)}} (1 - X)^{n^* - c_{1(0)}} \tag{2}$$

where X is the concentration of component 1 in the mixture. The theoretical mean and variance of the binomial distribution, respectively, are known to be

$$E(c_{1(0)}) = n^* X \tag{3}$$

$$V(c_{1(0)}) = n^* X\ (1-X) \tag{4}$$

For a mixture at the completely segregated state (Akao, et al., 1973)

$$Pr\ (M_1 = c_{1(0)} \mid Y = A_0) = 1 \tag{5}$$

and hence

$$E(c_{1(0)}) = 0 \tag{6}$$

$$V(c_{1(0)}) = 0 \tag{7}$$

By comparing the mean contact number of any sample to the scale between the two extremes, i.e., the mean contact number of the completely mixed state

and that of the completely segregated state, a measure of the degree of mixedness can be defined as

$$M = \frac{\bar{c}_{1(0)} - [E\,(c_{1(0)})]_{seg}}{[E\,(c_{1(0)})]_{mix} - [E\,(c_{1(0)})]_{seg}} \tag{8}$$

where $\bar{c}_{1(0)}$ is the mean contact number from spot samples. Substituting Eqns. (4) and (6) into Eq. (8), we have

$$M = \frac{\bar{c}_{1(0)}}{n^* X} \tag{9}$$

Note that M takes the values between 0 and 1, but due to sampling fluctuation or measurement error inherently involved in experiments, M may assume a value slightly greater than 1 when a mixture is very close to the completely mixed state.

Most of the conventional mixing indexes are based on the variance of some spot samples. It is difficult to employ such mixing indexes which are macroscopic and statistical in nature for elucidating the relationship between the degree or extent of mixing and the structure of the resulting mixture. Akao, et al. (1973) employed the definition given by Eq. (9) in their studies of the mixing process and structure of mixtures; however, they considered only regularly packed mixtures. The present study extends their work to irregularly packed mixtures.

4.3 EXPERIMENTAL

The experimental setup originally employed by Chen, et al. (1971) is shown in Fig. 1. Motionless mixers with 1, 2, 4, 6, 8 and 12 helices were used by them. Initially, approximately equal quantities of white and red
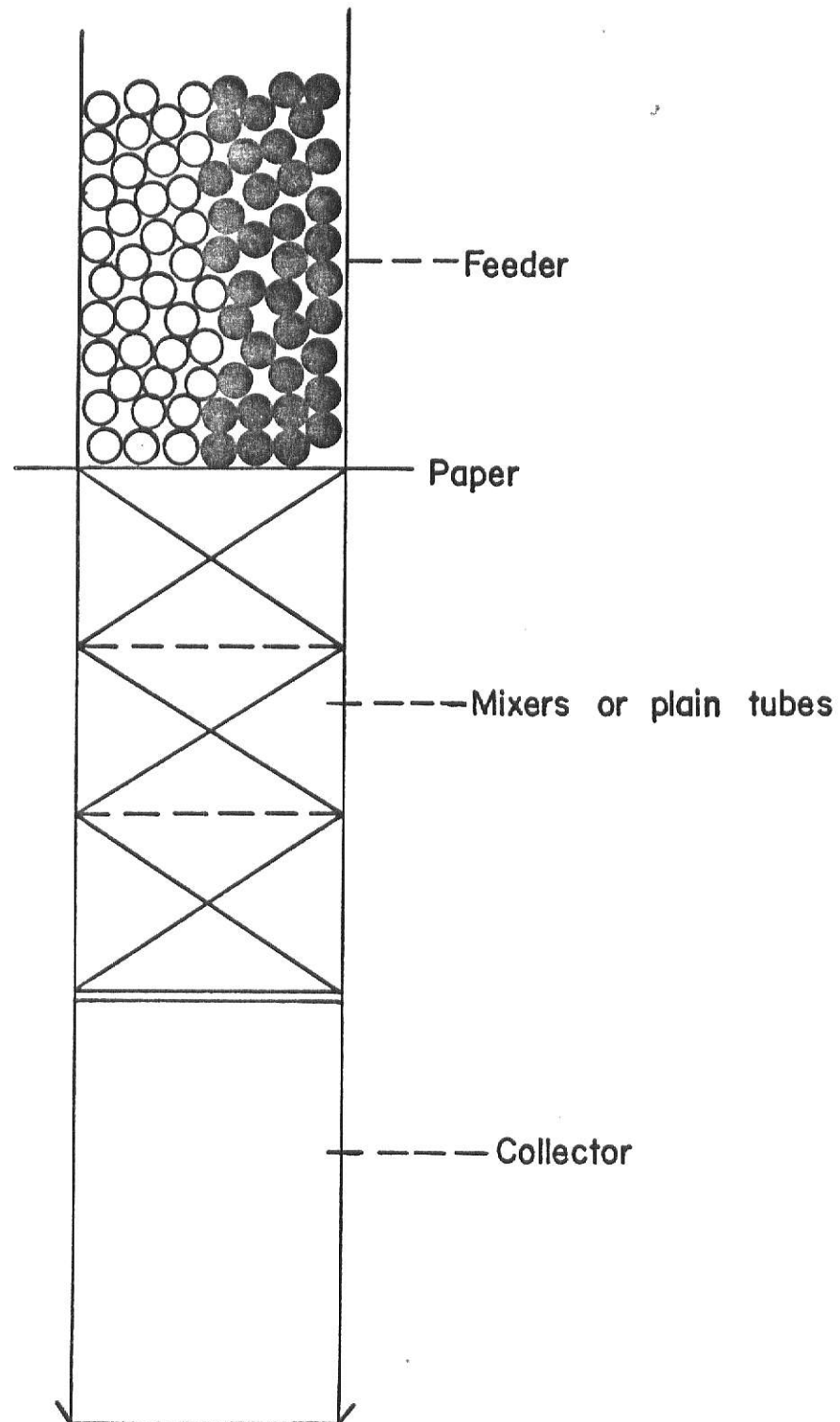
Fig. I . Schematic diagram of experimental setup
(Chen et al., 1971) .

Lucite particles were loaded side by side in the feeder of a mixer. The particles passed through the mixer when the paper at the bottom of the feeder was rapidly pulled out. The particles were accumulated in the collector. A cross-sectional photograph was taken by a Polaroid camera every inch along the collector. Each photograph was divided into four quandrants, with each quadrant considered as a sample. Lucite particles of 1/8-in. and 3/16-in. diameters were used separately in two series of experiments.

In this study each photograph taken by Chen, et al. (1971) was considered as a sample. White particles were selected as key particles, and all red particles surrounding and in contact with each white particle were counted to determine the contact number. The white particles in the outermost layer of each sample were neglected to eliminate the wall effect. Figure 2 shows one of these samples. The number in each white circle designate the contact number for that key particle. Both white and red particles surrounding and in contact with a white one were also counted to obtain the coordination number for each key particle. The mean contact number and the mean coordination number for each sample were calculated, respectively, by dividing the sum of the contact numbers and the sum of the coordination numbers by the total number of key particles.

The degree of mixedness for each sample was calculated by Eq. (9). The average value of the degrees of mixedness for the samples was considered to be the degree of mixedness of the mixture in the collector. TABLES 1 and 2 summarize the results.

4.4 RESULTS AND DISCUSSIONS

Figure 3 shows the values of the new mixing index obtained in this work and the conventional mixing index obtained by Chen, et al. (1971) as functions
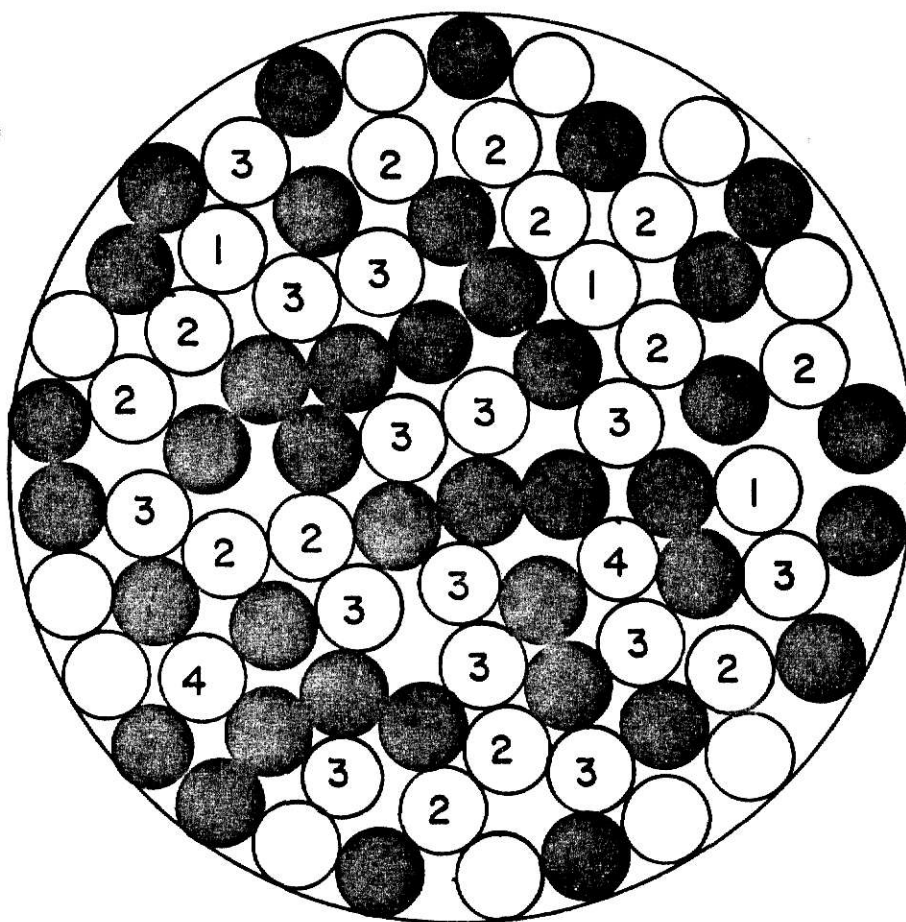
Fig. 2. Schematic diagram of cross-sectional sample.

TABLE 1

DEGREE OF MIXEDNESS VS. NUMBER OF HELICES
FOR MIXING 1/8'' WHITE AND RED
LUCITE PARTICLES

No. of helices	$\bar{c}_1(0)$	n^*	M	m^* (without helices)
0	1.34	4.28	0.602	4.28
0.5	1.59	4.60	0.665	4.26
1	1.62	4.31	0.723	4.15
2	1.83	4.52	0.779	4.28
3	1.95	4.61	0.813	4.31
4	2.31	4.59	0.968	4.32
5	2.32	4.51	0.989	4.27
8	2.39	4.39	1.047	4.23
12	2.50	4.32	1.113	4.25

TABLE 2

DEGREE OF MIXEDNESS VS. NUMBER OF HELICES
FOR MIXING 3/16" WHITE AND RED
LUCITE PARTICLES

No. of helices	$\bar{c}_1(0)$	n^*	M	m^* (without helices)
0	1.33	4.20	0.646	4.20
0.5	1.56	4.57	0.697	4.25
1	1.59	4.35	0.746	4.31
2	1.71	4.43	0.783	4.29
4	2.01	4.12	0.995	4.13
6	2.07	4.14	1.020	4.12
8	2.12	4.10	1.055	4.08
12	2.37	4.45	1.087	4.27
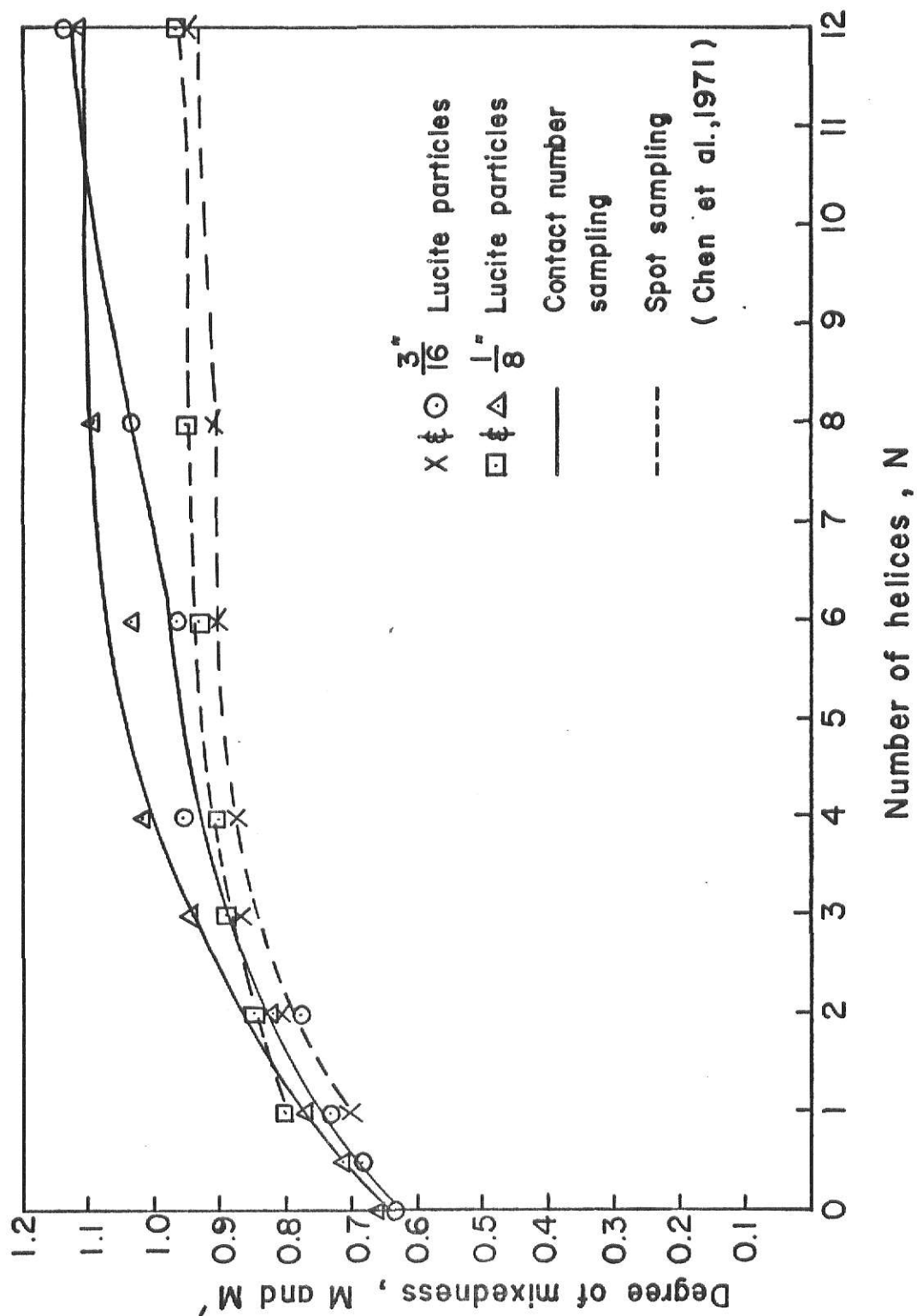
Fig. 3. Degree of mixedness vs. number of helices in mixer.

of the number of helices in the static mixer. The conventional mixing index is defined as (Chen, et al., 1971)

$$M' = 1 - \frac{\sigma^2}{\sigma_o^2} \tag{10}$$

and

$$\sigma^2 = \frac{\sum\limits_{i=1}^{n} (X_i - \bar{X})^2}{n} \tag{11}$$

where

σ^2 = variance after mixing

σ_o^2 = variance before mixing

X_i = composition of red particles in the i-th sample

$\bar{X}$ = overall composition of red particles

n = number of samples

Figure 3 indicates that both mixing indexes increased rapidly as the number of helices increased to four. It also shows that the mixture of the smaller Lucite particles gave rise to higher values of the degree of mixedness than that of the larger Lucite particles. However, the values of M are larger than those of M' for both types of particles. Since the gradient of M with respect to the number of helices is larger than that of M', M appears to be a more sensitive index for the progress of the mixing than the conventional one.

Some of the values of the degree of mixedness in TABLES 1 and 2 are slightly greater than 1. The t-test was employed to see if these values deviate significantly from the theoretically possible maximum of 1. The results, as shown in TABLE 3, indicate that the values do not significantly deviate from 1 with a 95% confidence interval, i.e., their deviations from 1

TABLE 3

RESULTS OF THE T-TEST ON SOME MIXING
INDICES GREATER THAN ONE

Particles	No. of helices	$\bar{c}_1(0)$	M	t-value	Significant value
1/8" white and red lucite	12	2.50	1.113	0.8460	1.96
	8	2.39	1.047	0.3548	1.96
3/16" white and red lucite	12	2.37	1.087	0.6225	1.96
	8	2.12	1.055	0.3799	1.96
	6	2.07	1.020	0.1410	1.96

are due to the experimental error. For example, some of the sample photographs had fuzzy images and some cross-sections of the samples were not truly two dimensional.

TABLE 4 shows the values of the void fraction and mean coordination number of different kinds of packing determined by earlier researchers (Smith, et al., 1929; Graton, et al., 1935; Bernal, et al., 1960). It is obvious that the void fraction decreases when the mean coordination number increases. Although the coordination number in the two dimensional cross-section was counted in this work, intuitively such a relationship still exists.

The mean coordination numbers n^* and m^* for the mixture of 1/8-in. Lucite particles are compared in TABLE 1, and those for the mixture of 3/16-in. Lucite particles are compared in TABLE 2. The mean coordination number of the mixture passed through a mixer is represented by n^*, while m^* represents that passed through an equivalent plain tube with the same length and diameter as the mixer. It can be seen that n^* is consistently greater than m^* for both sizes of particles. This implies that the void fraction of the mixture flowed through a mixer is less than that through plain tubes with the same length as the mixer. In other words, the helices have the significant effect of reducing the void fraction of the mixture. A more random arrangement of particles usually brings about a more compact structure (Rose, et al., 1965; Fuerstenau, et al., 1967).

To see if there is any variation of contact number in the axial direction, the variance for the contact numbers of samples taken along the collector was calculated as

$$\sigma_{axi}^2 = \frac{\sum_{i=1}^{m} (c_{i.} - c^*)^2}{m} \tag{12}$$

TABLE 4

VOID FRACTION VS. MEAN COORDINATION NUMBER

Mode of packing	Size (μ)	Material	Void fraction	Mean coordination number	Source of data
Random packing	3780	Lead shot	0.447	6.9	Smith, et al. (1929)
			0.440	7.3	
			0.426	8.1	
			0.372	9.5	
	6300	Ball-bearing	0.40	7.1	Bernal and Mason (1960)
			0.38	8.5	
Regular packing:		Equal size spheres			Graton and Fraser (1935)
Cubic			0.48	6	
Tetragonal			0.40	8	
Rhombohedral			0.30	10	
Hexagonal			0.26	12	

and

$$c^* = \frac{\sum_{i=1}^{m} c_{i.}}{m}$$

where

σ_{axi}^2 = variance of the contact number in the axial direction

$c_{i.}$ = mean contact number in the i-th 1-in. sample

c^* = mean contact number of the mixture in a collector

m = number of 1-in. samples in a collector

TABLE 5 shows this variance for different numbers of helices and sizes of particles. No systematic correlations with the number of helices can be found for both 3/16-in. and 1/8-in. Lucite particles.

Generally speaking, the mean coordination number indicates that the packing of these mixtures is between the cubic and hexagonal packings. If the sampling procedure is performed three dimensionally, additional information on the structure can be obtained.

4.5 CONCLUDING REMARKS

A microscopic and geometric mixing index which is based on the number of contacts between two kinds of particles was employed to study radial mixing in motionless mixers. It appears that this mixing index is more effective in differentiating the quality of mixtures than the conventional one. Studies of this mixing index involving segregating materials are the subjects of further investigation.

TABLE 5

VARIANCE OF CONTACT NUMBER WITHIN THE SAMPLES
IN THE AXIAL DIRECTION OF THE COLLECTOR

No. of helices	3/16 in. particles	1/8 in. particles
1	0.0858363	0.016183
2	0.0573956	0.0232
4	0.1039	0.02948
6	0.201045	0.01181
8	0.108625	0.01034
12	0.145152	0.0574

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CHAPTER V

SCALE-UP PROCEDURES FOR BATCH MIXERS-- APPLICATION TO TUMBLING MIXERS

5.1 INTRODUCTION

Solids mixing is an operation for changing a non-uniform system of particulate materials into a uniform one. Recent reviews of the literature on this subject (Fan, et al., 1970; Fan, et al., 1971; Fan, et al., 1972) indicate that in spite of its importance in industry, the practice of the art and experience still predominate in the design of mixers. While some works (Sterrett, 1959; Luterek and Cachia, 1971; Lynch and Ho, 1972) have been published on the scale-up and design of solids mixers, generally applicable methodologies and procedures have not yet been developed.

This chapter presents the results of studies on scale-up and design procedures for tumbling mixers. This class of mixers has been widely employed in practice, and the theory and mechanisms of mixing in such mixers have been investigated by several researchers (Lloyd, 1967; Hogg, 1969; Yano, 1957; Carley-Macaley, 1962). Therefore, considerable background information is available for establishing scale-up and design procedures for this class of mixers.

5.2 MECHANISMS OF MIXING IN TUMBLING MIXERS

It is generally considered that mixing of solid particles can proceed through three principal mechanisms (Lacey, 1954): diffusion, convection, and shear. Diffusive mixing refers to the redistribution of particles through the

mixer as a result of random motion of individual particles relative to one another. Convective mixing refers to the transfer of a group of adjacent particles from one location to another in the mixture. Shear mixing is described as the mechanism by which changes in the configuration of the components are effected through the setting up of slipping planes within the mixture. Hogg (1969) suggested that shear mixing should not be regarded as a fundamental mixing mechanism and that actually, shear mixing is always accompanied by convective mixing.

Although all three mechanisms occur to some extent in a tumbling mixer, they vary in importance according to the loading scheme and the mixer type. In drum mixers with end-to-end loading as shown in Fig. 1.a, the diffusion mechanism predominates. However, the convective and shear mechanisms predominate in drum mixers with side-by-side or layer-by-layer loading as shown respectively in Figs. 1.b and 1.c. Since a relatively small number of zones of distinctly different concentrations separated by an appreciable distance exists at the early stage of the mixing operation, convective mixing tends to predominate in this stage (Makarov, 1971). In the later stage of the mixing operation, the number of zones of different concentrations increases. Therefore, the distances of separation of the zones become relatively small, and a sufficiently random mixture is produced through the enhanced operation of the diffusive mechanism. The shift in the predominating mechanism can be observed in the variance versus time plot for a general tumbling mixer shown in Fig. 2. In the region AB, the variance decreases rapidly, exhibiting the characteristics of convective mixing. In the region BC and beyond, the change in the variance is small, exhibiting the characteristics of diffusive mixing. Generally, the time required to complete the diffusive stage is much longer than the time required to complete the convective stage. A tumbling mixer should
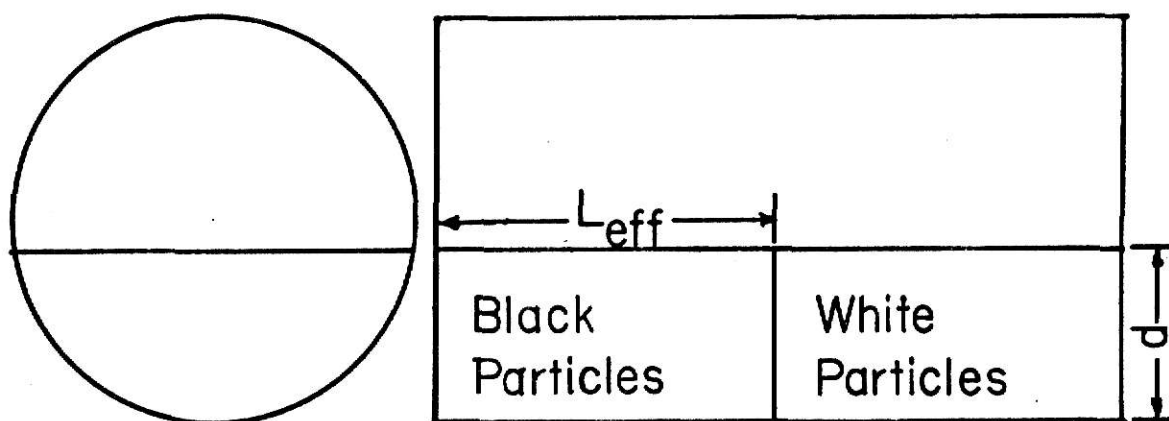
Fig. I.a   Drum mixer with end to end loading.
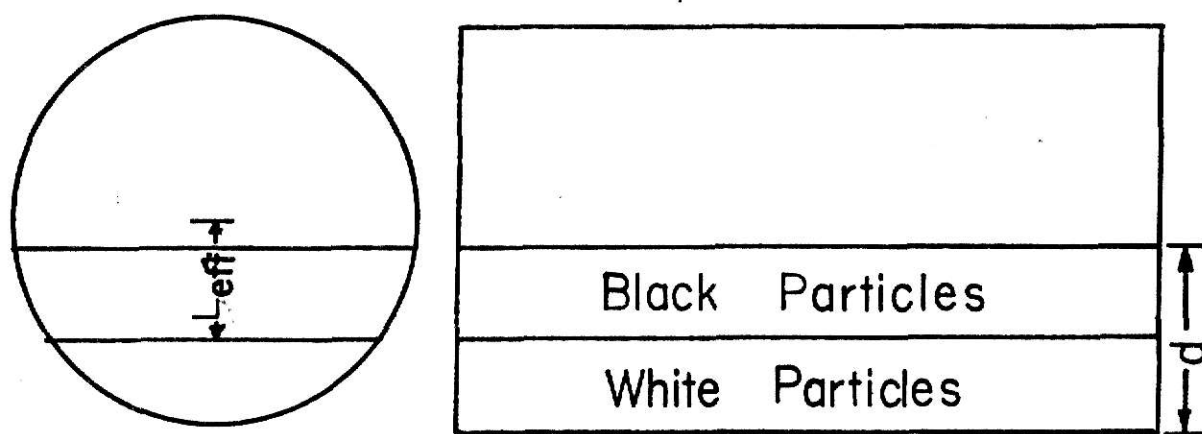
Fig. I. b   Drum mixer with layer by layer loading.
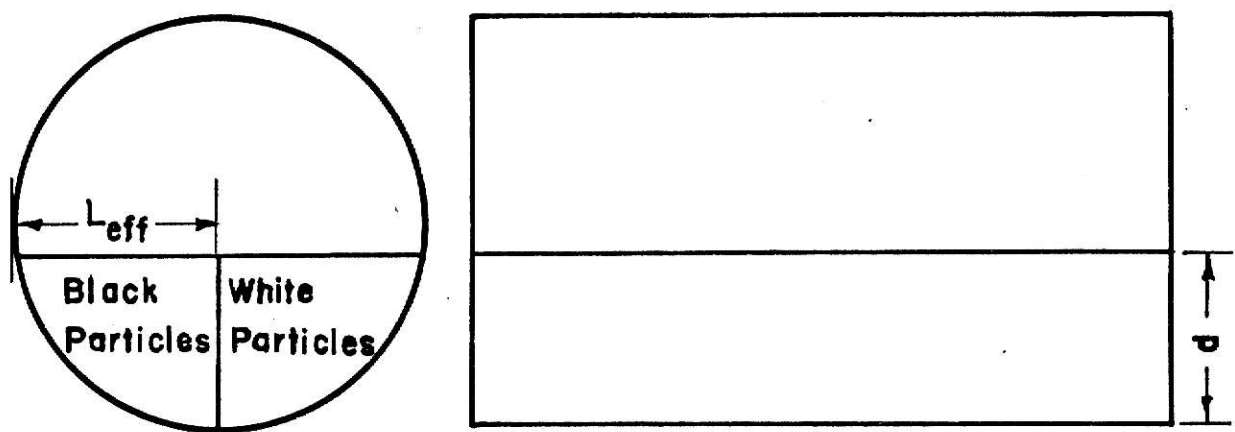
Fig. l.c. Drum mixer with side by side loading.
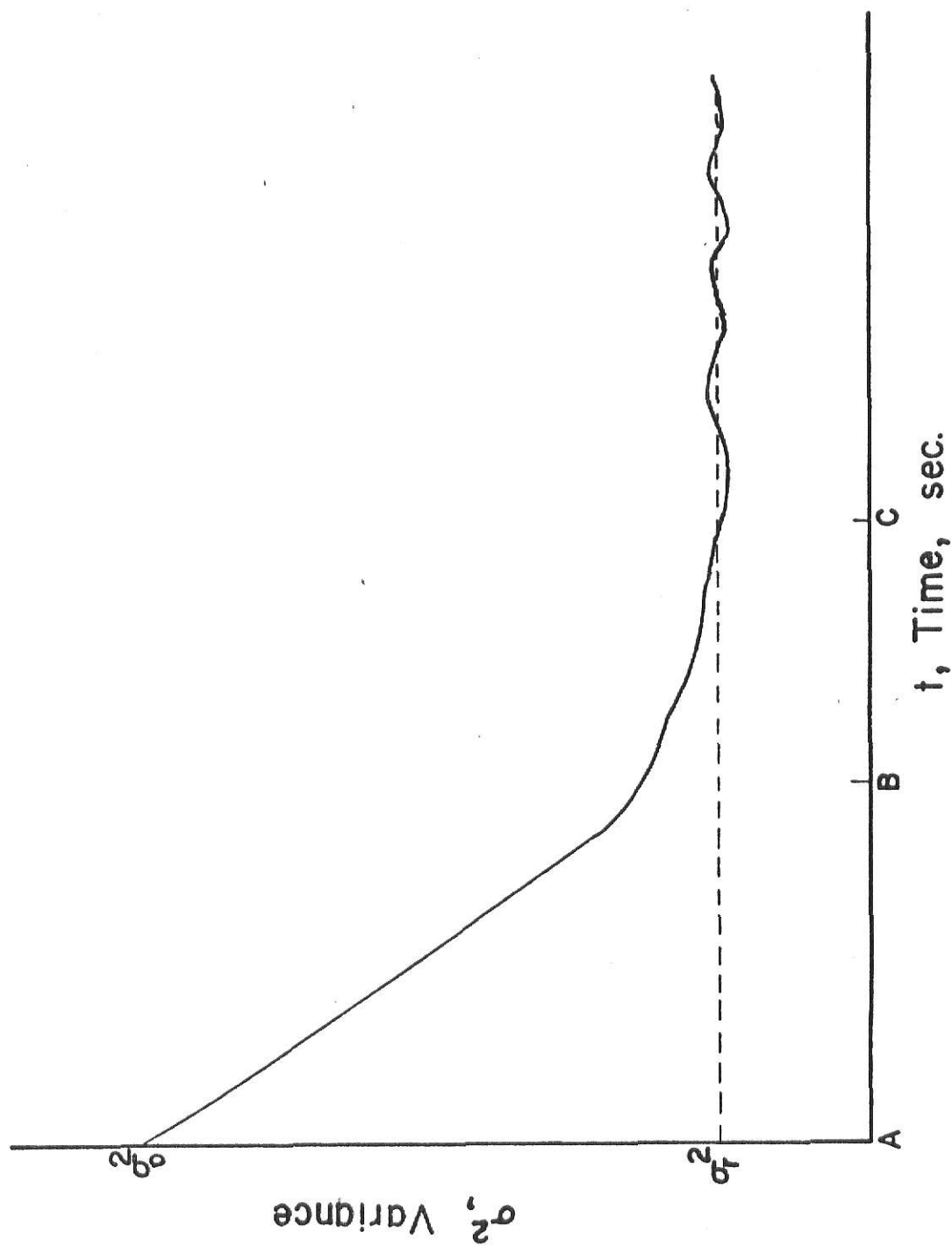
Fig. 2. Typical variance vs. time plot of tumbling mixers.

be designed so that mixing proceeds through an optimal combination of both mechanisms. Reduction of the time over which diffusion takes place will generally decrease the mixing time.

5.3 SCALE-UP PROCEDURES

An important concept for scale-up is the principle of similarity (Johnstone and Thring, 1957). In scaling up a tumbling mixer or, for that matter, any mixer, three types of similarity need be considered: geometric, kinematic and dynamic.

Two systems are said to be geometrically similar when the ratios of the linear dimensions of the prototype and scaled-up vessels are constant. For example, if the ratios of diameters and lengths are equal for two drum mixers, then they are geometrically similar. Kinematic similarity exists between two systems of different sizes when, in addition to being geometrically similar, the ratios of velocities between corresponding points in the two systems are equal. Dynamic similarity exists between two systems of different sizes when, in addition to being geometrically and kinematically similar, the ratios of forces between corresponding points in the two systems are equal.

Since the similarity criteria are ratios of like quantities, they are dimensionless. There are two general methods of arriving at them. Where the differential equations that govern the behavior of the system are unknown, it is possible to derive the similarity criteria by means of "dimensional analysis." Where the differential equations governing a particular process are known, the equations can be transformed into dimensionless form to recover the similarity relations from the parameters in the resulting dimensionless equation. This procedure may be termed "normalization."

The classical principle of similarity can be expressed by an equation of

the form

$$A = f(B, C, D, \ldots) \tag{1}$$

where a dimensionless group A is a function of other dimensionless groups B, C, D, etc. Although the Froude number has been proposed as a criterion of the dynamic similarity in scaling up a tumbling mixer (Luterek and Cachia, 1971; Weidenbaum, 1958), the principle has not been extensively applied to the study of solids mixing. Application of the two procedures to the scale-up of a tumbling mixer is given below.

5.3.1 Derivation of Similarity Criterion with Governing Equation Unknown

Motion of the particulate material being mixed by tumbling in a mixer is extremely complicated and it is difficult, if not impossible, to formulate the equation of motion governing such a system. When the differential equations that govern the behavior of the system are unknown, but all the significant variables which would influence the characteristics of the particle motion are known, it is possible to derive the similarity criteria by means of dimensional analysis, i.e., equation (1) can be derived by applying Buckingham's theorem of dimensional analysis to the system considered. Buckingham's theorem may be stated as follows (Buckingham, 1914; Buckingham, 1915):

1. The solution to every dimensionally homogeneous physical equation has the form

   $$\Phi(\pi_1, \pi_2, \ldots) = 0$$

   where π_1, π_2, $\ldots$ represent a complete set of dimensionless groups of the variables and dimensional constants in the equation.

2. If an equation contains n separate variables and dimensional constants, and these are given dimensional formulas in terms of m primary quantities, then the number of dimensionless groups in a com-

plete set is (n-m).

An application of this theorem to the mixing of nonsegregation material of similar physical properties with the exception of color in tumbling mixers gives (see Appendix 5.10.)

$$f(\frac{N^2 d}{g}, \frac{Kt}{(L_{eff})^2}, \frac{P}{N^3 d^5 \rho}, J, \frac{d}{L_{eff}}) = 0 \qquad (2)$$

The significant variables which appear in this expression are

d = rotating radius of the mixer

g = gravitational acceleration

J = fraction of volume loaded by particles

K = mobility coefficient of the particles

L_{eff} = effective length of the mixer

N = rotational speed of the mixer

P = power needed to drive the mixer

t = mixing time

ρ = true density of the particle

Regardless of the type of tumbling mixers, including drum mixers, V-type mixers, and double cone mixers, d is defined as the rotating radius as shown in Figs. 1.a through 1.g. The effective length, L_{eff}, is different for different loading modes of the material as well as for different mixer geometries. The effective lengths of various mixers are shown in Figs. 1.a to 1.g. Quantitatively, the longer the effective length, the longer the mixing time needed to bring together the components which were originally separated. The reason for defining the effective length as shown in Figs. 1.a to 1.g is to unify the scale-up procedure for the system of tumbling mixers.

An investigation of the relationship between the variance of the mixture
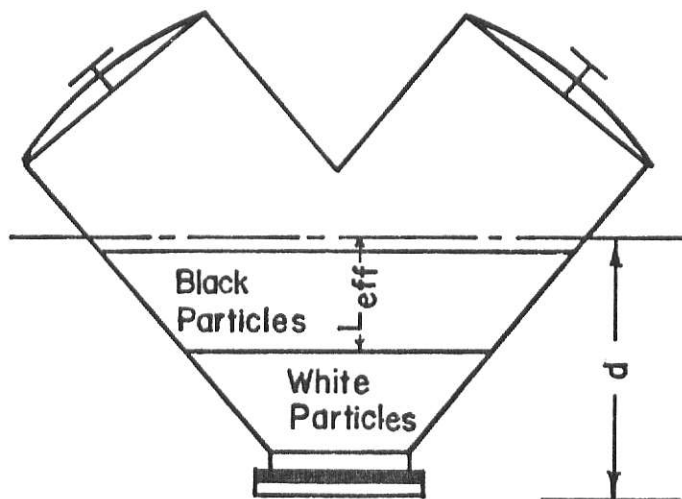
Fig. I.d. V–type mixer with layer by layer loading.
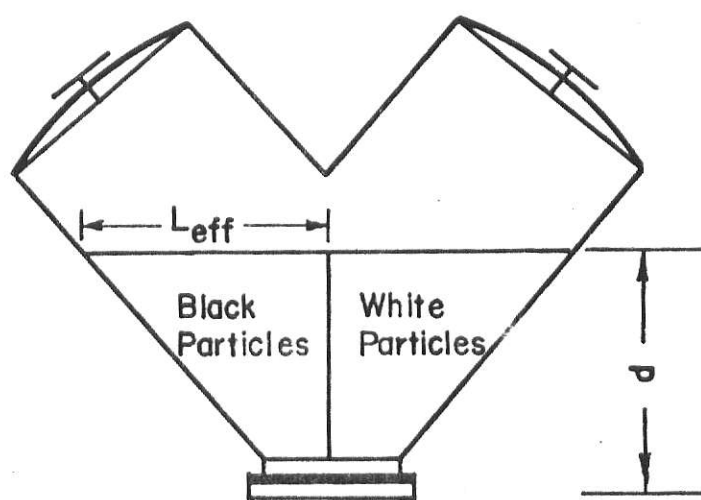
Fig. l.e.   V—type mixer with side by side loading.
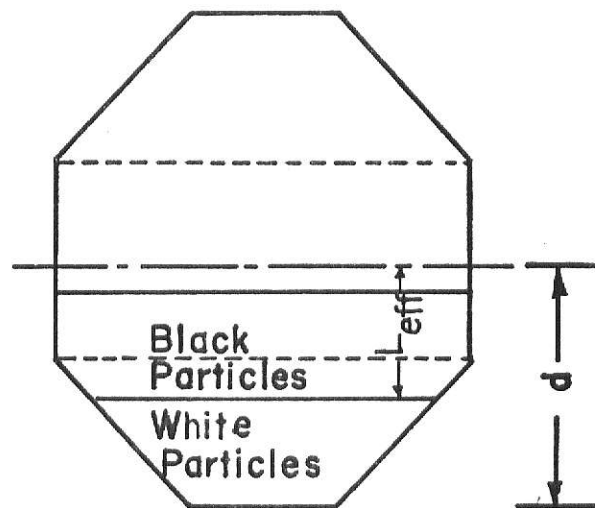
Fig. I.f. Double cone mixer with layer by
layer loading.
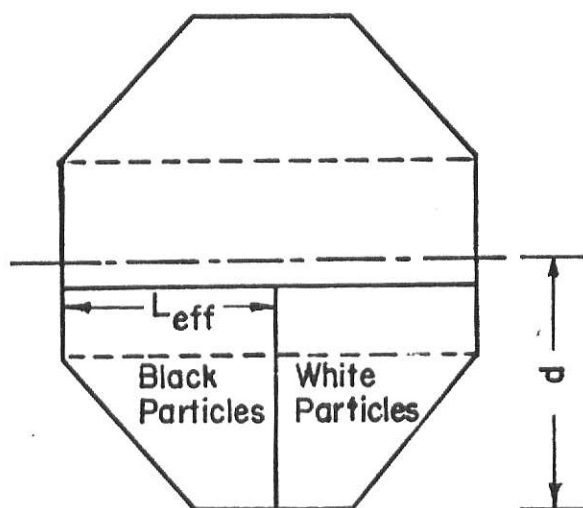
Fig. I.g. Double cone mixer with side by
side loading.

and time of mixing nonsegregating material in tumbling mixers reveals that the variance decreases monotonically at the early stage of mixing. A semi-logarithmic plot of σ^2 as a function of time generally gives rise to a sufficient linear relationship and, therefore, the mobility coefficient can be defined from the slope of such a linear relationship.

The time required for obtaining a desirable degree of mixedness is proportional to the square of the effective length, L_{eff}. Thus, the slope of a semi-logarithmic plot can be taken as $\dfrac{K}{(L_{eff})^2}$ and the mobility coefficient, K, can be recovered from the value of the slope.

For a geometrically similar system, $\dfrac{d}{L_{eff}}$ is constant, and thus eq. (2) can be reduced to

$$f\left(\frac{N^2 d}{g}, \frac{Kt}{(L_{eff})^2}, \frac{P}{N^3 d^5 \rho}, J\right) = 0 \tag{3}$$

Note that the Reynolds number which plays a prominent role in fluid mechanics is absent in this expression because the stress transmitted by granular media is independent of the rate of strain, i.e., they possess no property corresponding to viscosity. However, the Froude number, $\dfrac{N^2 d}{g}$, which arises frequently in dealing with bodies of liquid with free surfaces is present. The Froude number is the ratio of the centrifugal force to the gravity force, and it can be a criterion for the dynamic similarity for a mixer. The dimensionless mixing time, $\dfrac{Kt}{(L_{eff})^2}$, is the criterion for kinematic similarity. This dimensionless number must be used to calculate the mixing time required to attain a desirable state of mixedness.

If we assume that the fraction of volume loaded by particles, J, is constant, then eq. (3) can be simplified as

$$f\left(\frac{N^2 d}{g}, \frac{Kt}{(L_{eff})^2}, \frac{P}{N^3 d^5 \rho}\right) = 0 \tag{4}$$

By letting the Power number, $\dfrac{P}{N^3 d^5 \rho}$, be an independent variable, equation (4) can be written as

$$\frac{P}{N^3 d^5 \rho} = g\left(\frac{N^2 d}{g}, \frac{Kt}{(L_{eff})^2}\right) \tag{5}$$

5.3.2 Derivation of Similarity Criterion with Governing Equations Known

Where the differential equation governing a particular process is known, it is relatively simple to transform the equation into dimensionless analysis. Such an example is a horizontal drum mixer rotating about its own axis with end-to-end loading, as shown in Fig. 1.a. For this mixer, the velocity of particles due to its rotation has no component parallel to the mixer axis. Each particle, as it rolls down the slope, has equal chances of deflecting to either side on each encounter with another particle and therefore, the diffusive mechanism predominates in the mixer. Thus we can write (Lacey, 1954):

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial c^2(x,t)}{\partial x^2} \tag{6}$$

where

c = concentration of one constituent in the mixer

D = diffusion coefficient

t = mixing time

x = coordinate of distance along the mixer axis

Equation (6) can be solved subject to initial and boundary conditions (Hogg, et al., 1966; Cahn, 1966)

$$c(x,0) = 0, \qquad\qquad -\frac{L}{2} \leq x \leq 0$$

$$= 1, \qquad\qquad 0 < x \leq \frac{L}{2}$$

$$\frac{\partial c(x,t)}{\partial x} \Bigg|_{x = -L/2} = 0$$

$$\frac{c(x,t)}{x} \Bigg|_{x = +L/2} = 0$$

The resulting expression is

$$c(x,t) = \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-1)} \exp\left[\frac{-2(k-1)\pi^2 Dt}{L^2}\right] \cdot$$

$$\sin\left[\frac{(2k-1)\pi x}{L}\right] \qquad\qquad (7)$$

The measured variance σ^2 of particles within the bed at any time t is given by

$$\sigma^2(t) = \frac{2}{L} \int_0^{L/2} [c(x,t) - \tfrac{1}{2}]^2 dx \qquad\qquad (8)$$

Substitution of the general expression for c(x,t) given by equation (7) into this equation and integration yield

$$\sigma^2(t) = \frac{2}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \exp\left[-\frac{2(2k-1)\pi^2 Dt}{L^2}\right] \qquad\qquad (9)$$

which simplifies to

$$\sigma^2(t) = \frac{2}{\pi^2} \exp \left[\frac{-2\pi^2 Dt}{L^2}\right] \tag{10}$$

with the exception of a very small t.

It is obvious that for this mixer the mobility coefficient can be directly related to the diffusion coefficient as

$$K = \frac{\pi^2}{2} D \tag{11}$$

and according to the definition of the effective length

$$L = 2L_{eff} \tag{12}$$

equation (10) can be written as

$$\sigma^2(t) = \frac{2}{\pi^2} \exp \left[- \frac{Kt}{(L_{eff})^2}\right] \tag{13}$$

It can be seen that $\ln\sigma^2$ is a linear function of t, except at very small t, and the mobility coefficient, K, can be determined from the slope of this linear relationship. As mentioned previously, K is varied for general tumbling mixers. Recall that the mobility coefficient is generally proportional to the square of the effective length. Time required for mixing particles with similar size and density in tumbling mixers to a desired degree of mixedness can be characterized by the dimensionless number, $\frac{Kt}{(L_{eff})^2}$, as derived previously by dimensional analysis.

The dimension form of equation (6) is

$$[\frac{c}{t}] = [\frac{Kc}{L^2}] \tag{14}$$

Dividing both sides of equation (14) by $[\frac{c}{4t}]$ gives the dimensionless equation

$$\phi\left[\frac{Kt}{(L_{eff})^2}\right] = \text{constant} \tag{15}$$

This is in agreement with what was derived in the previous section.

5.3.3 Mixing Time and Power Requirement

Theoretically speaking, a generalized correlation which includes equation (3) or (5) can be established if sufficient experimental data exist. However, it is not the case at this stage of development in the mixing technology. An alternative approach is to develop a stepwise scale-up procedure in which only one similarity criterion is satisfied at each step.

In scale-up of solids mixers, usually two types of questions are mainly raised. The first is "How long will it take to obtain a mixture with a desired degree of mixedness?" The second is "How much power will be needed to obtain such a mixture?"

Usually the mixing time and power requirement are treated separately (Ho and Lynch, 1972). Since the dimensionless mixing time, $\frac{Kt}{(L_{eff})^2}$, is a criterion for the kinematic similarity, we have

$$\left[\frac{Kt}{(L_{eff})^2}\right]_{lab} = \left[\frac{Kt}{(L_{eff})^2}\right]_{plant} \tag{16}$$

Such a similarity is maintained between the laboratory and plant scale of similar geometries. The first question regarding mixing time can be answered by means of equation (16).

Once the mixing time has been established, we should consider the power requirement. For geometrically and kinematically similar systems, equation (5) can be written as

$$\frac{P}{N^3 d^5 \rho} = \alpha_0 \left(\frac{N^2 d}{g}\right)^{\alpha_1} \tag{18}$$

Equation (18) answers the second question.

5.4 APPLICATION OF SCALE-UP PROCEDURES

This section describes how the general scale-up procedure developed in the preceding section can be applied to each class of tumbling mixers. Since the available information in mixing segregated materials is limited, all applications given below are only concerned with mixing non-segregating materials.

5.4.1 Drum Mixer

Mixing Time. For this class of mixers, and for all other classes of tumbling mixers, equation (16) is applicable provided that the mobility coefficient, K, can be correlated to the operating conditions such as the faction of loading or rotational speed. Such correlations based on the available data (Yano, 1957) are given in Fig. 3 for end-to-end loading. The mobility coefficient and the fraction of the critical rotational speed are correlated by the equation

$$K = a_1 \exp(a_0 f) \tag{19}$$

and

$$f = \frac{N}{\sqrt{g/d}} \tag{20}$$

where f is the fraction of the critical rotational speed and defined by eq. (20). The constants a_0 and a_1 can be estimated by means of a nonlinear least square method (Draper and Smith, 1966) based on the following criterion.
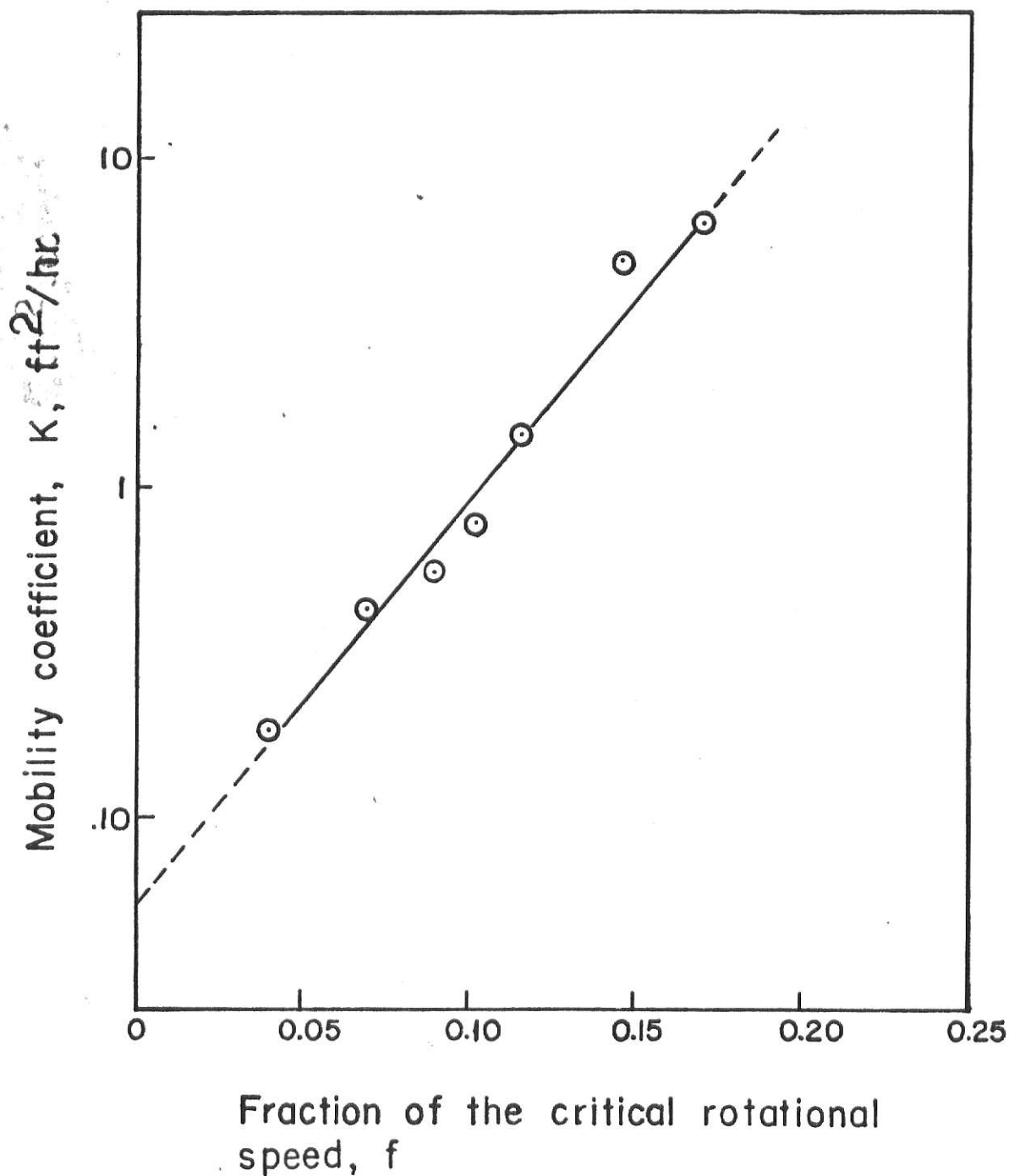
Fig. 3   Correlation between mobility
coefficient and fraction of the
critical rotational speed of
drum mixer with end to end
loading, J=0.135 (Yano, et al.,
1957).

$$T_1 = \sum_{i=1}^{n} [(K)_{i,\,exp} - (K)_{i,\,cal}]^2 \qquad (21)$$

where

$(K)_{i,exp}$ = i-th experimental mobility coefficient

$(K)_{i,cal}$ = i-th calculated mobility coefficient

n = number of data points

T_i = sum of *deviations*

As a computing technique, a combination of the predictor-corrector method (a numerical integration method) and the Gauss-Newton method (an iterative minimum search technique), as proposed by Bard (1967), can be used. The values of the parameters a_0 and a_1 by means of the linearization least square method as well as those determined by the nonlinear least square method are shown in TABLE 1. Apparently no data has been published on side-by-side and layer-by-layer loadings.

Power requirement. For this class of mixers and for all other classes of tumbling mixers, the correlation between the Power and Froude numbers given by equation (18) can be used to estimate the power requirement. A linear relationship obtained by plotting the available data (Rose, 1954) is given in Fig. 6. The constants α_o and α_1 in equation (18) are also estimated by the linearization and nonlinear least square methods. Results of these estimations for all types of drum mixers at different level of loadings are summarized in TABLE 2.

5.4.2 V-Type Mixer

Mixing Time. For this class of mixers the correlation based on the available data (Yano, et al., 1956; Yano, et al., 1957) is given in Fig. 4. Results of the parameter estimation are summarized in TABLE 3.

TABLE 1

ESTIMATION OF PARAMETERS OF THE CORRELATION[*]
BETWEEN THE MOBILITY COEFFICIENT AND THE
FRACTION OF THE CRITICAL ROTATIONAL
SPEED FOR DRUM MIXER WITH END-TO-END
LOADING   (J = 0.135)

nonlinear parameter estimation

$$a_0 = 25.6$$

$$a_1 = 3.27 \times 10^{-2}$$

$$\sigma = \left(\sum_i [Y_{i,\text{ observed}} - Y_{i,\text{ calculated}}]^2 \right)^{1/2}$$

$$= 0.207$$

linear parameter estimation

$$a_0 = 28.6$$

$$a_1 = 2.00 \times 10^{-2}$$

$$\sigma = \left(\sum [\ln Y_{i,\text{ observed}} - \ln Y_{i,\text{ calculated}}]^2 \right)^{1/2}$$

$$= 0.172$$

$$\sigma^1 = \left(\sum [Y_{i,\text{ observed}} - Y_{i,\text{ calculated}}]^2 \right)^{1/2}$$

$$= 1.234$$

[*]$K = a_1 \exp(a_{0f})$

TABLE 2

ESTIMATION OF PARAMETERS OF THE CORRELATION[†]
BETWEEN THE POWER NUMBER AND THE
FROUDE NUMBER FOR DRUM MIXERS

	Nonlinear parameter estimation			Linear parameter estimation			
J	α_0	α_1	$\sigma^{\dagger}$	α_0	α_1	σ	$\sigma^{1\dagger}$
0.125	0.129	-1.2184	0.005095	.131	-1.206	.0234	.0138
0.25	0.152	-0.9658	0.1747	.14093	-0.97642	.253	.0208
0.375	0.1819	-0.9889	0.3391	.1987	-0.9767	.362	.01394
0.50	0.2492	-0.9660	0.2289	.2407	-0.9707	.410	.00348
0.75	0.2039	-0.9458	0.07896	.1435	-0.98902	.092	.12948

[†]
$$\frac{P}{N^3 d^5 \rho} = \alpha_0 \left(\frac{N^2 d}{g}\right)^{\alpha_1}$$

$$\sigma = \left(\sum [Y_{i,\ observed} - Y_{i,\ calculated}]^2\right)^{1/2}$$

$$\sigma^1 = \left(\sum [\ln Y_{i,\ observed} - \ln Y_{i,\ observed}]^2\right)^{1/2}$$
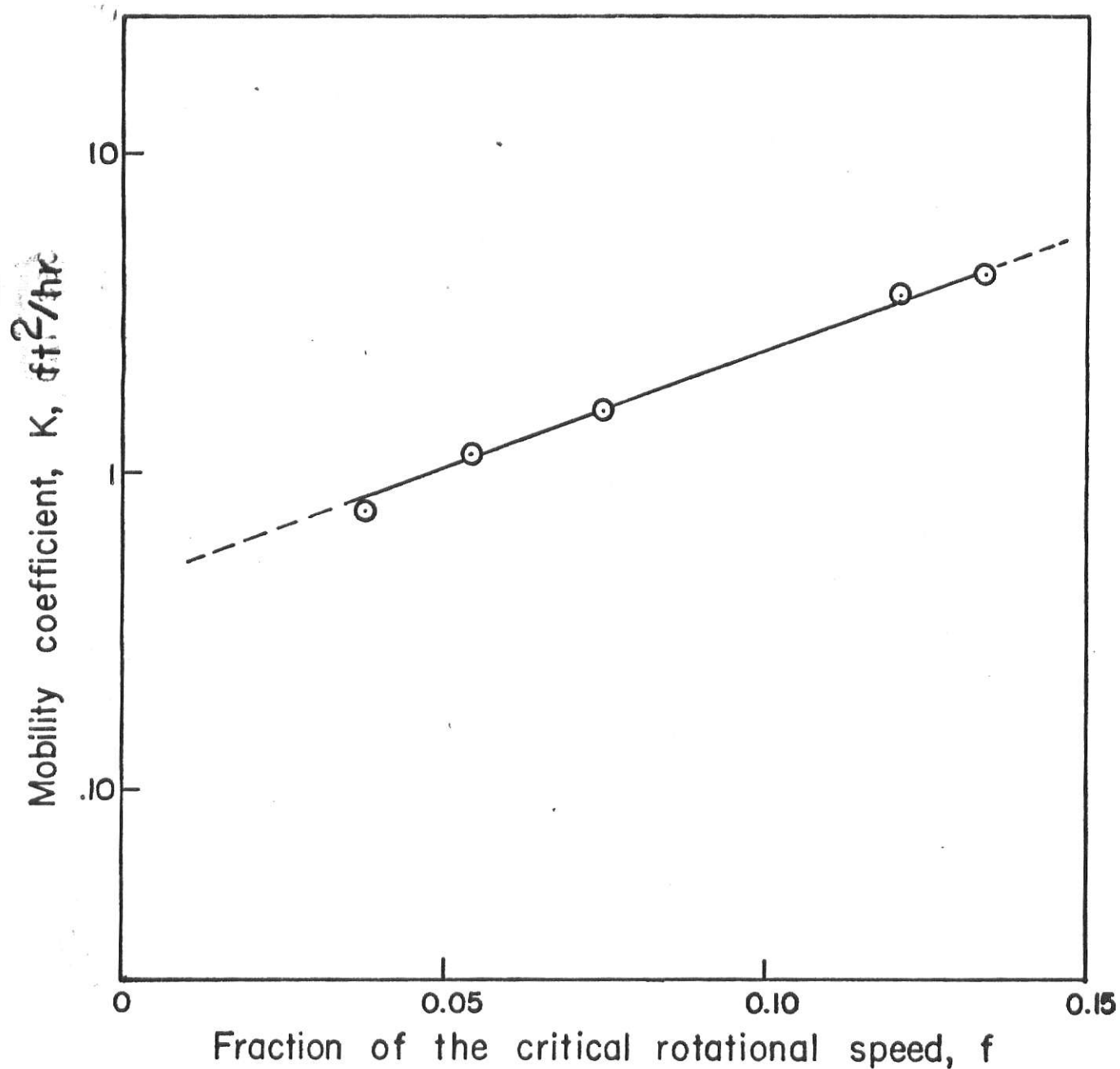
TABLE 3

ESTIMATION OF PARAMETERS OF THE CORRELATION[†]
BETWEEN THE MOBILITY COEFFICIENT AND THE
FRACTION OF THE CRITICAL ROTATIONAL
SPEED FOR V-TYPE MIXER WITH
SIDE-BY-SIDE LOADING
$(J = 0.135)$

nonlinear parameter estimation

$$a_o = 28.3$$

$$a_1 = 1.37 \times 10^{-1}$$

$$\sigma = \left(\sum_i [Y_{i,\ observed} - Y_{i,\ calculated}]^2\right)^{1/2}$$

$$= 0.328$$

linear parameter estimation

$$a_0 = 29.2$$

$$a_1 = 1.24 \times 10^{-1}$$

$$\sigma = \left(\sum [\ln Y_{i,\ observed} - \ln Y_{i,\ calculated}]^2\right)^{1/2}$$

$$= 0.141$$

$$\sigma^1 = \left(\sum [Y_{i,\ observed} - Y_{i,\ calculated}]^2\right)^{1/2}$$

$$= 1.28$$

[†]$K = a_1 \exp(a_{of})$
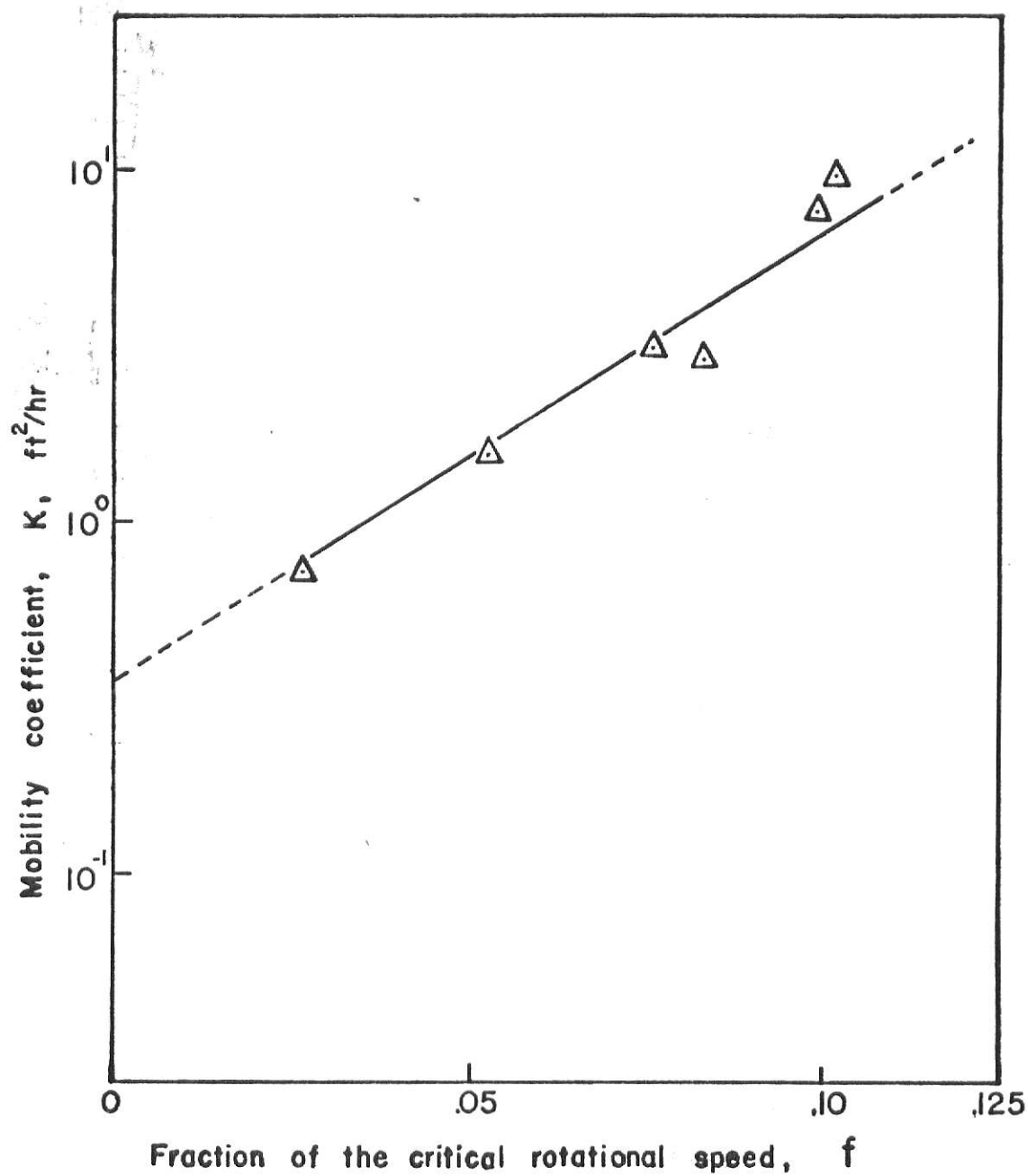
Fig. 4. Correlation between mobility coefficient and fraction of the critical rotational speed of the V-type mixer with side by side loading, J=0.135 (Yano, et al., 1957).

Fig. 5. Correlation between mobility coefficient
and fraction of the critical rotational
speed of double cone mixer with side
by side loading, J=0.135 (Yano, et al., 1957).
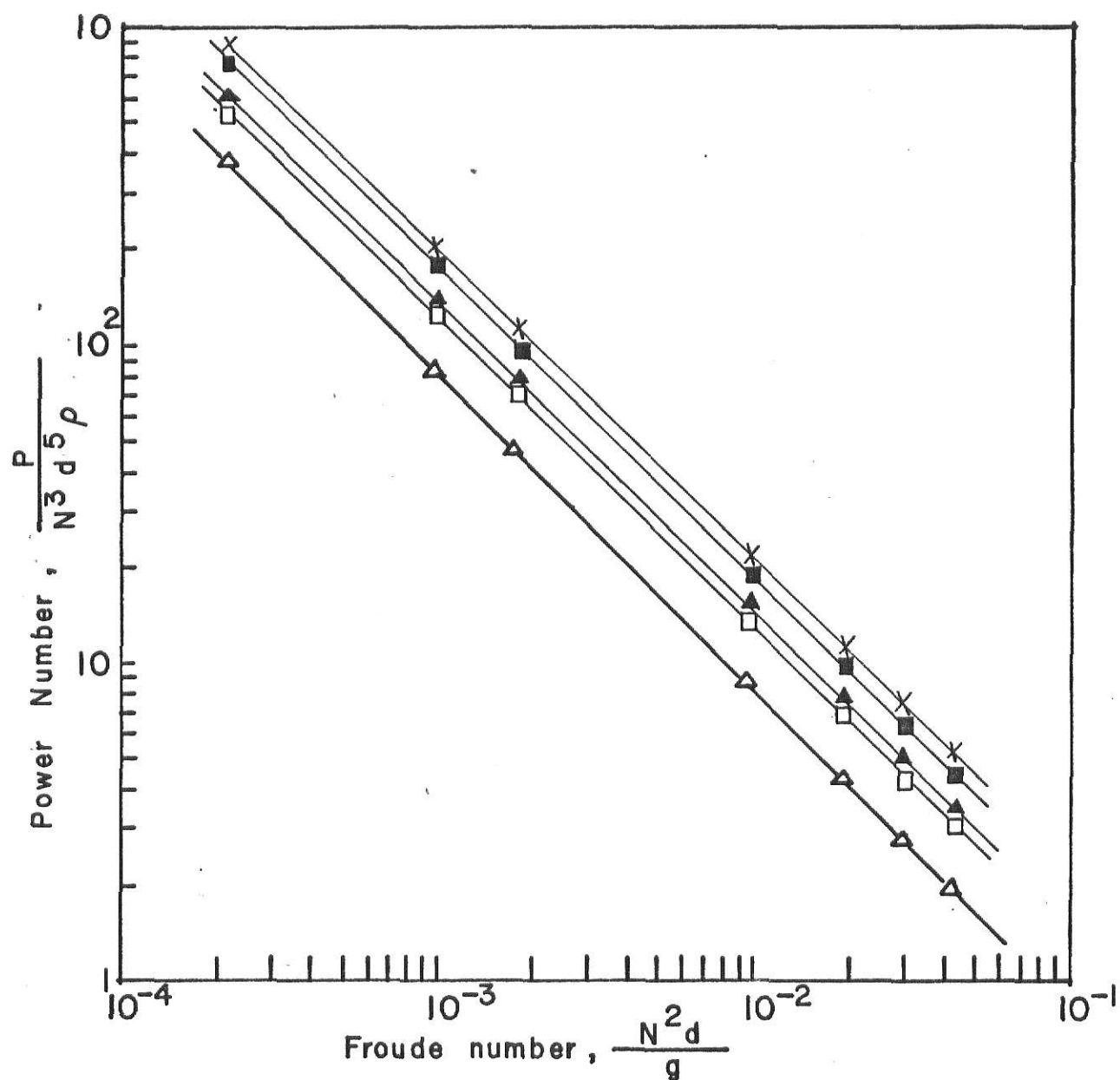
Fig. 6. Power number vs. Froude number
for drum mixers (Rose , 1954)
▲J = 0.75 ; XJ = 0.50 ; ■J = 0.375 ;
□ J = 0.25 ; △J = 0.125 .

Power Requirement. The correlation between the Power and Froude numbers for this kind of mixers is shown in Fig. 7, which is based on the available data summarized in TABLE 7 (Patterson-Kelley Co., 1972). Note that the available data are for the case of 75% loading (J = 0.75) only. The constants α_0 and α_1 of the correlation are summarized in TABLE 4.

5.4.3 Double Cone Mixer

Mixing Time. For this class of mixers the correlation based on the available data (Yano, et al., 1957) is given in Fig. 5. The values of a_0 and a_1 are summarized in TABLE 5.

Power Requirement. Figure 8 gives the correlation between the Power and Froude numbers for double cone mixers based on the available data summarized in TABLES 8 and 9 (Patterson-Kelley Co., 1972; Konline-Sanderson Co., 1965). Note that the available data are for the case of 75% loading (J = 0.75) only. The constants α_0 and α_1 of the correlation given by equation (18) are summarized in TABLE 6.

5.5 OTHER ASPECTS OF SCALE-UP PROCEDURES

Other significant aspects of the scale-up procedures which have not been discussed so far are the optimal rotational speed, the average diameter of particles, and the loading factor.

The optimal rotational speed at which the mixing time is shortest is independent of the types of powders, but is dependent on the diameter of particles and size of mixers. A linear relationship exists between the optimal rotational speed and the rotational radius (Kanise, 1960) as shown below.

$$\frac{(N_{op})^2 d}{g} = \text{constant}$$

TABLE 4

ESTIMATION OF PARAMETERS OF THE CORRELATION[†] BETWEEN THE POWER NUMBER AND THE FROUDE NUMBER FOR V-TYPE MIXERS

nonlinear parameter estimation

$$\alpha_0 = 0.116$$

$$\alpha_0 = 0.793$$

$$\sigma = \left(\sum [Y_{i, \text{ observed}} - Y_{i, \text{ calculated}}]^2 \right)^{1/2}$$

linear parameter estimation

$$\alpha_0 = 0.164$$

$$\alpha_1 = 0.716$$

$$\sigma = \left(\sum [\ln Y_{i, \text{ observed}} - \ln Y_{i, \text{ calculated}}]^2 \right)^{1/2}$$

$$= 0.245$$

$$\sigma' = \left(\sum [Y_{i, \text{ observed}} - Y_{i, \text{ calculated}}]^2 \right)^{1/2}$$

$$= 2.027$$

[†] $\dfrac{P}{N^3 d^5 \rho} = \alpha_0 \left(\dfrac{N^2 d}{g} \right)^{\alpha_1}$
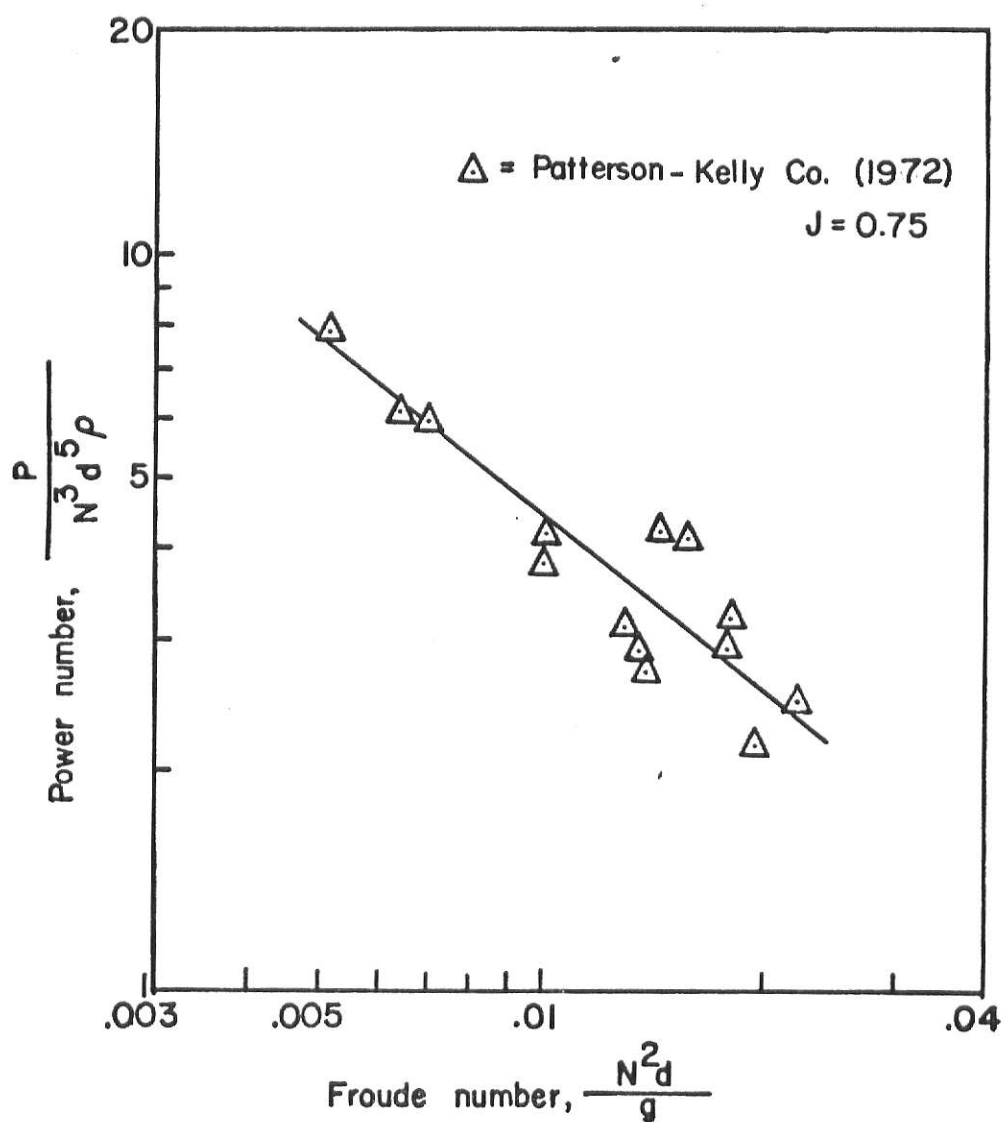
Fig. 7. Power number vs. Froude number for V-type mixers.

TABLE 5

ESTIMATION OF PARAMETERS OF THE CORRELATION[†]
BETWEEN THE MOBILITY COEFFICIENT AND THE
FRACTION OF THE CRITICAL ROTATIONAL
SPEED FOR DOUBLE CONE MIXERS WITH
SIDE-BY-SIDE LOADING
$(J = 0.135)$

nonlinear parameter estimation

$$a_0 = 16.6$$

$$a_1 = 1.7 \times 10^{-1}$$

$$\sigma = \left(\sum_i [Y_{i, \text{observed}} - Y_{i, \text{calculated}}]^2 \right)^{1/2}$$

$$= 0.0453$$

linear parameter estimation

$$a_0 = 16.6$$

$$a_1 = 1.59 \times 10^{-1}$$

$$\sigma = \left(\sum_i [\ln Y_{i, \text{observed}} - \ln Y_{i, \text{calculated}}]^2 \right)^{1/2}$$

$$= 0.0474$$

$$\sigma' = \left(\sum [Y_{i, \text{observed}} - Y_{i, \text{calculated}}]^2 \right)^{1/2}$$

$$= 0.135$$

[†]$K = a_1 \exp (a_0 f)$

TABLE 6

ESTIMATION OF PARAMETER OF THE CORRELATION[†] BETWEEN THE POWER NUMBER AND THE FROUDE NUMBER FOR DOUBLE CONE MIXERS

nonlinear parameter estimation

$$\alpha_0 = 1.25 \times 10^{-2}$$

$$\alpha_1 = 1.45$$

$$\sigma = \left(\sum [Y_{i,\ observed} - Y_{i,\ calculated}]^2 \right)^{1/2}$$

$$= 1.397$$

linear parameter estimation

$$\alpha_0 = 2.53 \times 10^{-2}$$

$$\alpha_1 = 1.291$$

$$\sigma = \left(\sum [\ln Y_{i,\ observed} - \ln Y_{i,\ calculated}]^2 \right)^{1/2}$$

$$= 0.245$$

$$\sigma' = \left(\sum [Y_{i,\ observed} - Y_{i,\ calculated}]^2 \right)^{1/2}$$

$$= 9.433$$

[†] $\dfrac{P}{N^3 d^5 \rho} = \alpha_0 \left(\dfrac{N^2 d}{g} \right)^{\alpha_1}$

TABLE 7

SPECIFICATIONS OF STANDARD TWIN-SHELL MIXERS
(FROM THE CATALOG OF KOMLINE-SANDERSON
ENGINEERING CORPORATION)

Work. Cap. Cu. Ft.	Blender Horsepower	Approx. Speed RPM	Max. Density (lb./ft.^{3})	Maximum Radius of Revolution (inches)
1	1/4	30	165	14-1/8
2	1/3	25	90	17-5/8
3	3/4	25	135	21-3/4
5	3/4	25	65	25
10	2	20	75	31-1/2
20	3	16.5	65	37-1/4
30	3	14.7	55	44-1/2
40	5	13.7	65	49
50	5	13.7	55	51-3/4
60	5	11.2	50	56
75	7-1/2	11	55	59-1/4
100	7-1/2	8.3	50	66-1/4
125	10	8.4	50	70
150	10	7	50	74

TABLE 8

SPECIFICATIONS OF STANDARD DOUBLE CONE MIXERS
(FROM THE CATALOG OF KOMLINE-SANDERSON
ENGINEERING CORPORATION)

Work. Cap. Cu. Ft.	Blender Horsepower	Approx. Speed RPM	Max. Density $(lb./ft.^3)$	Maximum Radius of Revolution (inches)
1	3/4	44	400	14-1/2
2	3/4	40	170	16-5/8
3	3/4	35	110	19-1/8
5	1	32	85	21-5/8
10	2	28.5	110	26
15	2	25	75	29-1/4
20	2	23	55	32-1/2
30	3	21.9	50	35-3/4
40	5	20.8	55	39-1/4
50	7-1/2	18.75	75	42-1/4
60	7-1/2	17.5	55	44-3/4
75	7-1/2	15.75	50	48
100	10	14.8	50	53-3/4
125	15	13.7	55	57
150	15	12.75	50	60-1/2
200	20	11.2	50	67
300	25	8.25	50	76

TABLE 9

SPECIFICATIONS OF STANDARD DOUBLE CONE MIXERS
(FROM THE CATALOG OF THE PATTERSON-
KELLY CO., INC.)

Work. Cap. Cu. Ft.	Blender Horsepower	Approx. Speed RPM	Max. Density (lb./ft.^{3})	Maximum Radius of Revolution (inches)
1	1/4	30	164	14-1/8
2	1/3	37	85	16-3/8
3	3/4	30	125	17-3/4
5	1	30	60	20-1/4
10	2	27	80	23-7/8
20	3	23	55	29-3/4
30	3	21.9	50	33-1/2
40	10	19.5	127	38-3/4
50	7-1/2	20	70	42
60	7-1/2	20	56	44-1/2
75	7-1/2	12	59	46
100	10	14.7	50	52-3/4
125	15	14	50	54-3/4
150	15	7	50	58-3/4
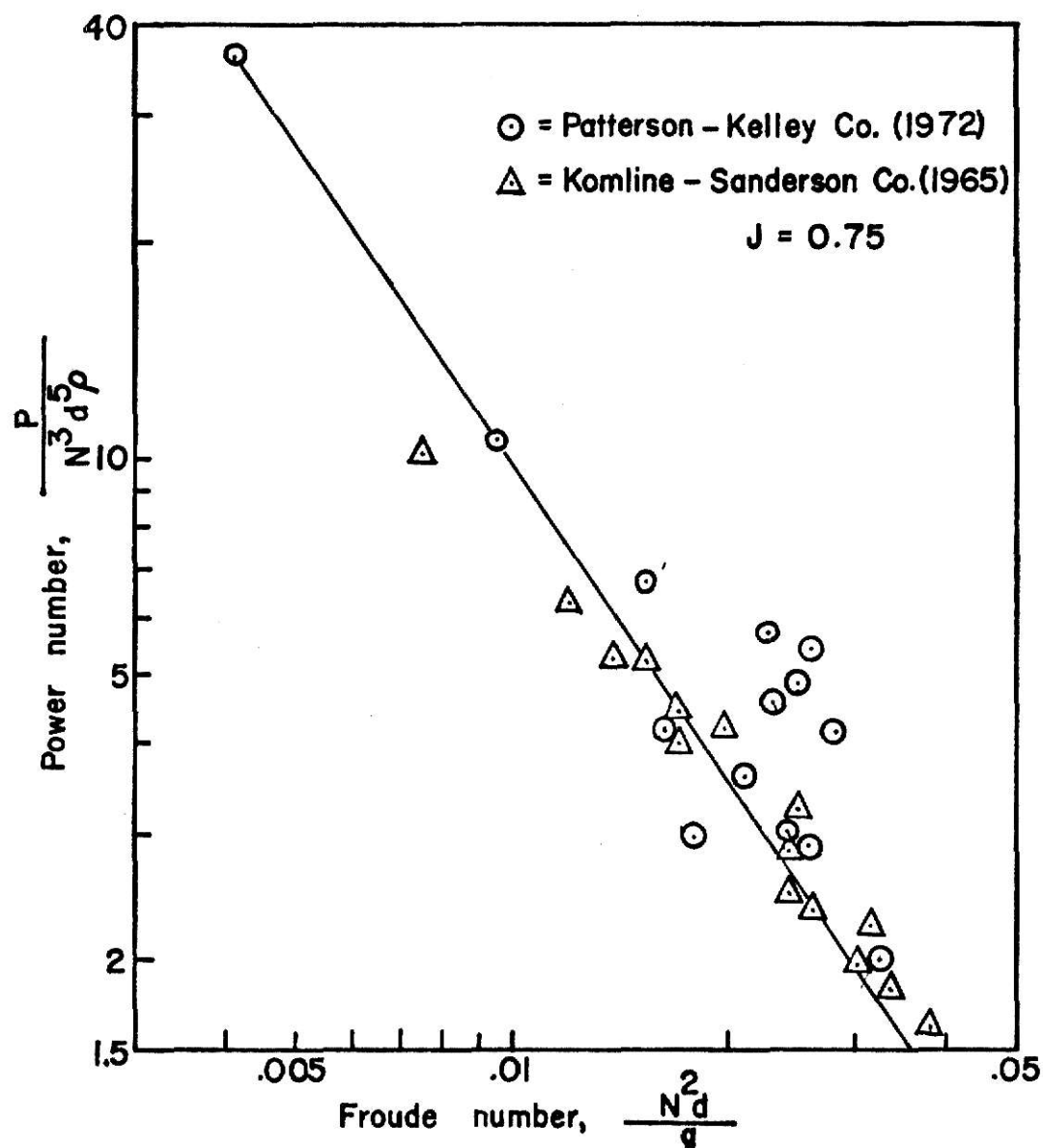
Fig. 8. Power number vs. Froude number for double cone mixers.

where

N_{op} = optimal rotational speed

d = rotational radius of the mixer

g = gravity acceleration

This relationship, together with available data (Kanise, 1960; Patterson-Kelley Co., 1972) are plotted in Fig. 9.

When the rotational speed is optimal, the average diameter of particles can be related to the Froude number for various tumbling mixers as shown in Fig. 10 (Kanise, 1960). This implies that

$$d_{av} = b_0 \left[\frac{(N_{op})^2 d}{g} \right]^{b_1}$$

where

b_0, b_1 = constants

d_{av} = average diameter of particles

If the rational radius is held constant, we have

$$N_{op} \simeq d_{av}^{1/2}$$

Any tumbling mixer attains the maximum degree of mixedness at a certain loading factor which is defined as the ratio of the loading volume to the volume of the mixer. In other words, below or above this loading, the degree of mixedness which can be attained is less than the maximum. The optimum powder loading factor which depends on the physical properties of the particles to be mixed and on many other factors, is usually about 30% volume of the mixer (Yano, et al., 1957; Kanise, 1960).
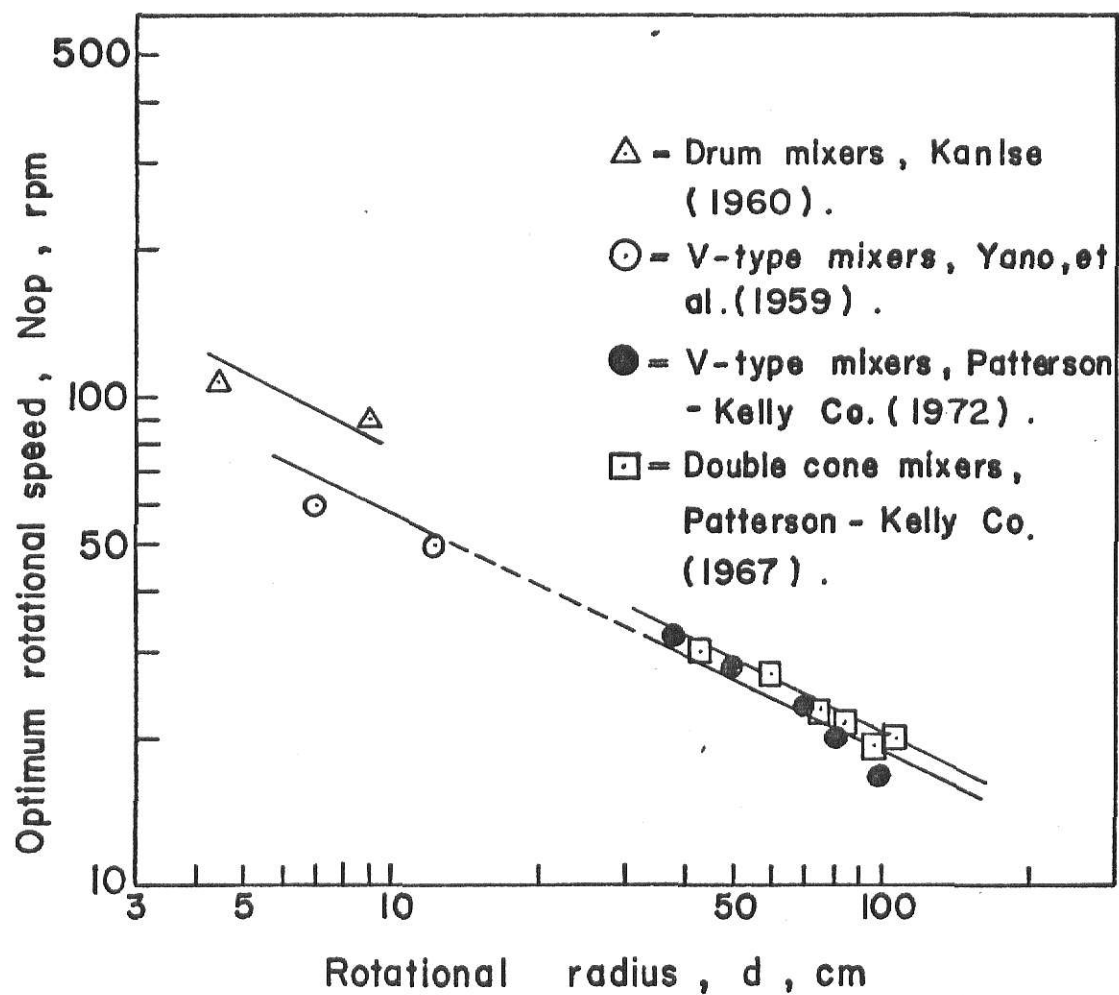
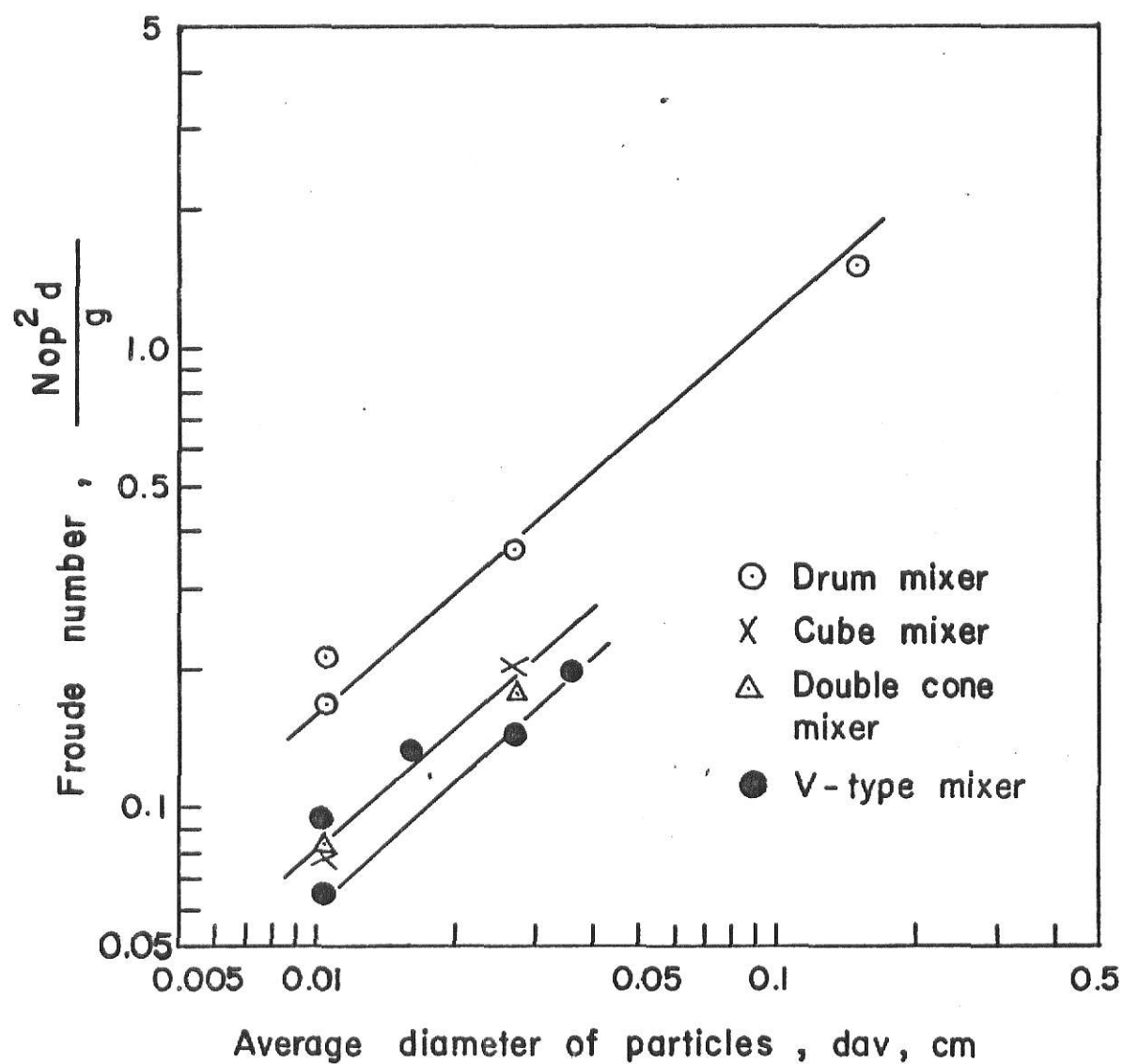Fig. 9. Optimal rotational speed vs. rotational radius for tumbling mixers.

Fig. 10. Froude number vs. average diameter of particles for tumbling mixers (Kanise, 1960).

5.6 COST ESTIMATION

In estimating the cost of solids mixers, the following items must be considered: capital cost of the mixer, capital cost of necessary auxiliary equipment, labor cost, and other operating costs such as power, depreciation, and maintenance.

Approximate capital cost including the auxiliary equipment and power requirements of V-type and double cone mixers are given in Figs. 11 and 12, respectively. This information was obtained from the Patterson-Kelley Company of East Stroudsburg, Pennsylvania. The cost data is for August, 1972, and is subject to change. Depreciation is based on a 20-year working life plus 5% interest on capital. A working time of 2000 hours per year is assumed. The cost of depreciation is calculated by the Sinking-fund method. The annual depreciation cost may be expressed in the equation as follows (Peters and Timmerhaus, 1968):

$$R = (V-V_s) \frac{i}{(1 + i)^{n-1}}$$

where

i = annual interest rate expressed as a fraction

R = uniform annual payments made at the end of each year (this is the annual depreciation cost), dollars

$V-V_s$ = total amount of the annuity accumulated in an estimated service life of n years (original value of property minus salvage value at end of service life), dollars.

The $4/hr. labor charge was assumed for operating both double cone and V-type mixers. Power costs were estimated at 1¢/KWh. The power required can be obtained from Figs. 11 and 12. TABLE 10 was prepared on the basis of costs for mixing 2000 lbs per hour of a 1:1 Justin and Reed wheat mixture. The
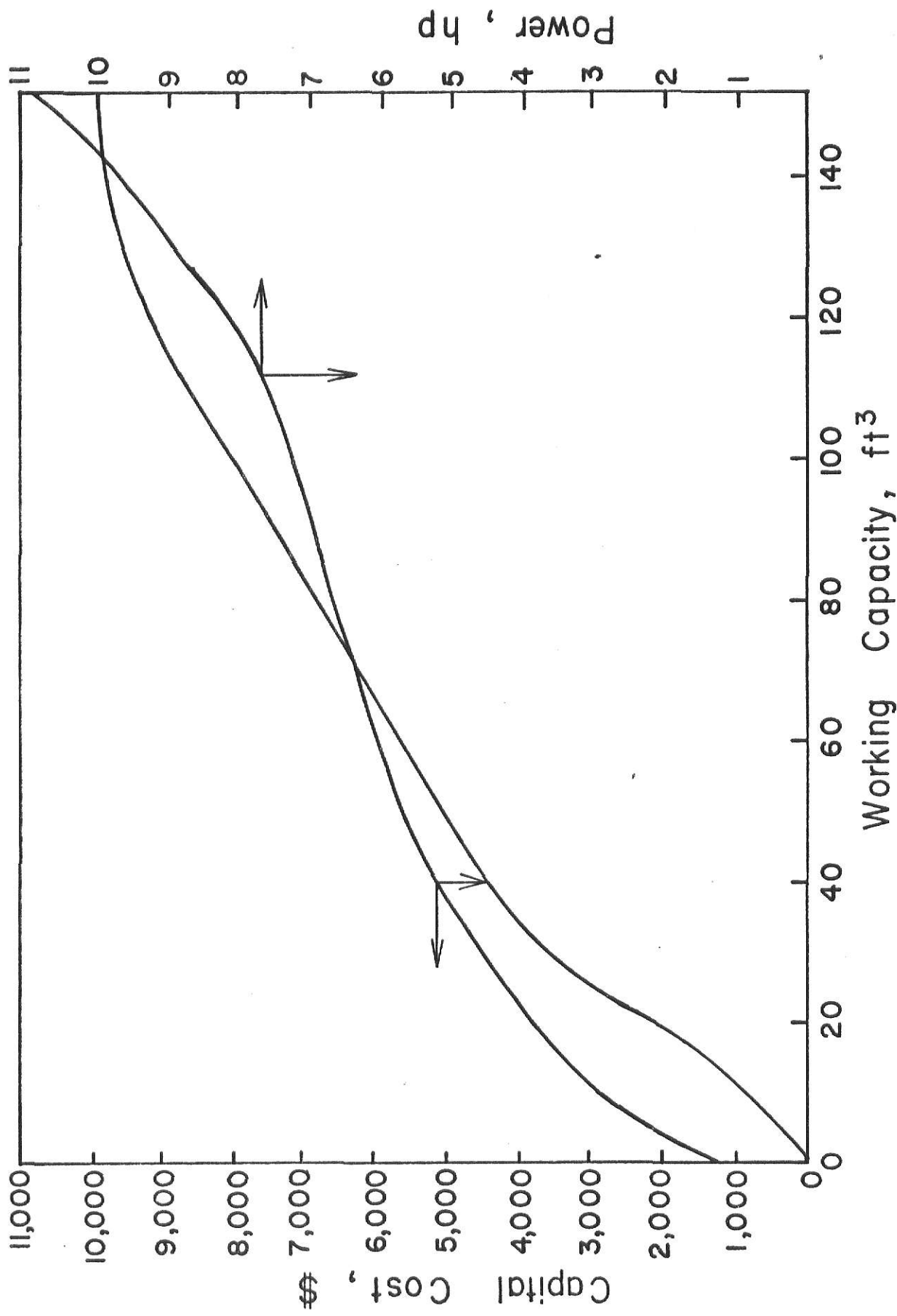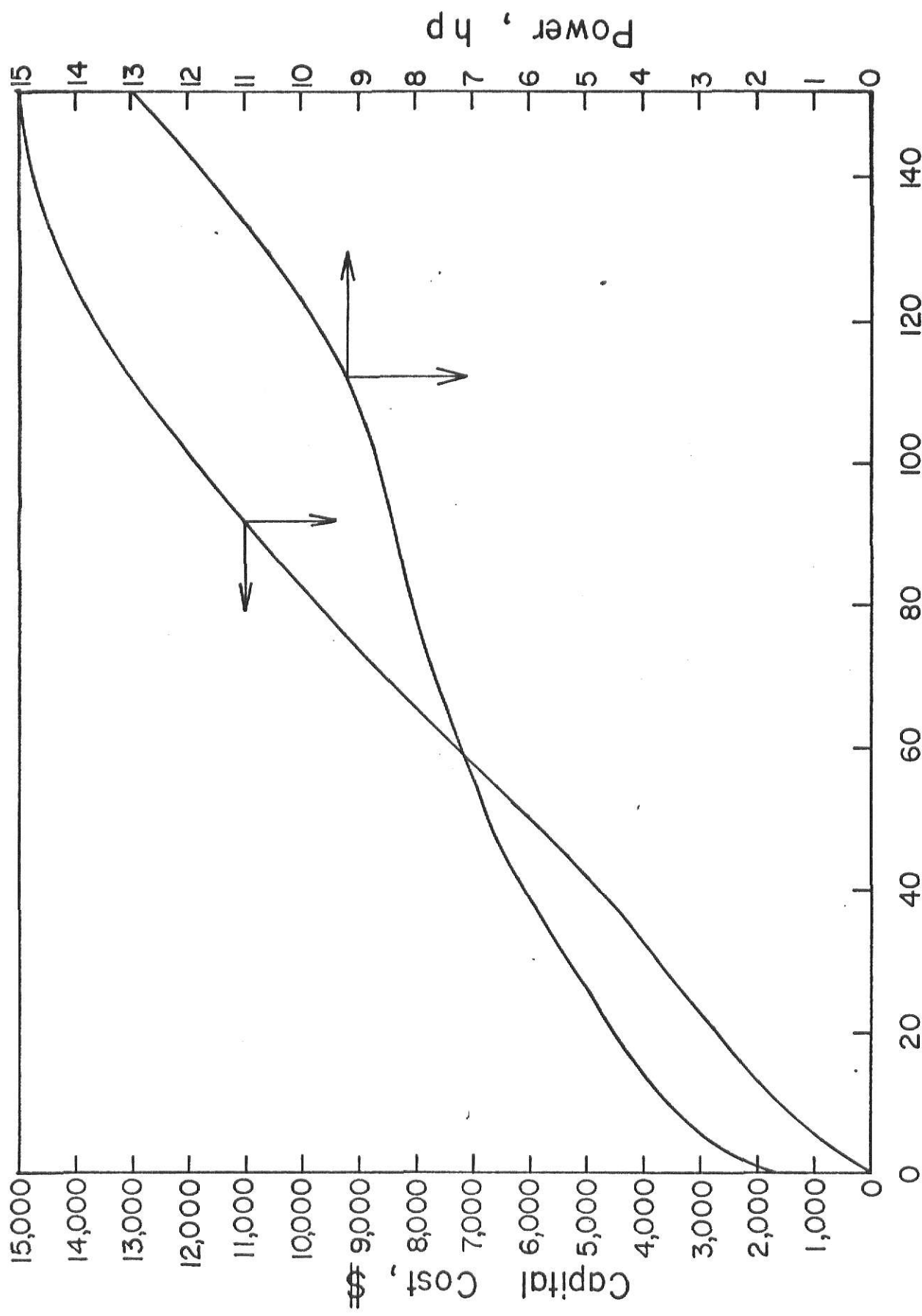
Fig. II. Approximate power requirement and capital cost of V-type mixer (Patterson-Kelly Co., 1972) .

Fig. 12. Approximate power requirement and capital cost of double cone mixers (Patterson - Kelly Co., 1972).

TABLE 10

COSTS OF MIXING 1000 LBS. JUSTIN WHEAT
AND 1000 LBS. REED WHEAT

	V-type	Double cone
Time to fill (min.)	1.5	1.5
Time to mix (min.)	15	10
Time to empty (min.)	1.5	1.5
Number of batches per hour	3.3	4.6
Mixer capacity required {Weight (lb) (Justin and Reed {Volume (ft^3) wheat mixture)	2000 41	2000 41
Mixer, capital cost ($)	5400	6100
Depreciation ($/hr)	0.733	0.828
Power ($/hr)	0.0283	0.0276
Labor ($/hr)	4.0	4.0
Unit cost ($/ton-hr)	1.475	1.057

total cost is obviously dominated by the labor cost. Compared with the labor cost, power and depreciation costs are insignificant. The double cone mixer shows a higher unit cost than the V-type mixer. Harnby (1968) suggested that reducing the number of mixing cycles and having the mixer standing idle for a proportion of its time would reduce labor utilization considerably. Other types of tumbling mixers, such as drum and rotating cube mixers, have not been considered because of the limited information available. Generally speaking, the range of the unit cost of preparing a mixture in a tumbling mixer is between 1 and 2 dollars/ton/hr. The cost information can be updated by taking into account inflation in the recent year. An annual inflation factor of 6.5% has been employed (Friedman, 1973).

5.7 EXAMPLES

The following examples illustrate the procedure to follow when using the general scale-up procedures described previously.

EXAMPLE 1.

A drum mixer is 0.99 ft. in diameter and 1.245 feet in length, and revolves at a speed of 15 r.p.m. Two lots of glass beads with true density of 393 lb/ft^3 and identical physical properties in all respects except for differences in color are end-to-end loaded inside the mixer. The fraction of loading is 0.135. Calculate the power needed to drive the drum mixer, and the mixing time required to obtain a mixture with $\sigma^2 = 0.0001$.

At first we calculate

$$\frac{N^2 d}{g} = \frac{(62.9)^2 \times (0.99/2)}{60^2 \times 32.2} = 1.69 \times 10^{-2}$$

Since J = 0.135, we find from Fig. 6

$$\frac{P}{N^3 d^5 \rho} = 5.10$$

Thus P is calculated as

$$P = \frac{5.10 \times (62.9)^3 \times (0.99/2)^5 \times 393 \times 3.03 \times 10^{-5}}{60 \times 60 \times 60} = .00208 \text{ h.p.}$$

For this mixer we have (see Fig. 1.a)

$$L_{eff} = .6225 \text{ ft.}$$

$$d = 0.99 \text{ ft.}$$

and

$$f = \frac{N}{\sqrt{g/d}} = 0.195$$

From Fig. 3, we find

$$K = 14.7 \text{ ft}^2/\text{hr.}$$

Substitution of these values and

$$\sigma^2 = 0.0001$$

into eq. (13) yields

$$0.0001 = \frac{2}{\pi^2} \exp\left[-\frac{14.7 \ t}{(.6625)^2}\right]$$

Thus the mixing time is

$$t = 12.1 \text{ min.}$$

EXAMPLE 2.

To prepare 50-50 feed mixtures of bean screenings (density, 48 lbs/cu. ft.) and ground sorghum (density, 48 lbs/cu.ft.), two geometrically similar V-type mixers with side-by-side loading are available. The mixers have the following dimensions:

	Experimental Mixer	Full Scale Mixer
d	10"	3.5'
L_{eff}	8"	2.8'

For both mixers, the fraction of the loading, J, under operating condition is 0.135.

The mixing time to attain the sample variance of 0.001 in the experimental mixer which revolves at 5 r.p.m. is two minutes. Calculate the rotational speed required by the full scale mixer to attain the same variance of the composition after 15 minutes of mixing. Also calculate the power requirement and estimate the unit cost ($/ton-hr) of this operation in mixing 250 lbs. each of bean screenings and ground sorghum in the full scale mixer.

For two geometrically similar mixers, we have from equation (16)

$$\left[\frac{Kt}{(L_{eff})^2}\right]_1 = \left[\frac{Kt}{(L_{eff})^2}\right]_2$$

where subscript 1 represents the experimental mixer, and 2 the full scale mixer. The fraction of the critical speed is calculated as

$$f_1 = \frac{N_1}{\sqrt{g/d_1}} = 0.0842$$

From Fig. 4 we obtain

$$K_1 = 5.82 \text{ ft}^2/\text{hr.}$$

Therefore

$$\left[\frac{Kt}{(L_{eff})^2}\right]_2 = \left[\frac{Kt}{(L_{eff})^2}\right]_1 = 0.436$$

and

$$K_2 = 13.7$$

From Fig. 4 we have

$$f_2 = 0.11$$

Thus the rotational speed of the full scale mixer is

$$N_2 = f_2 \sqrt{g/d_2} = 3.19 \text{ r.p.m.}$$

The volume of the material to be mixed is

$$\frac{500}{48} = 10.4 \text{ ft}^3$$

The working capacity (total inner volume) of the mixer is

$$\frac{10.4}{0.135} = 77.1 \text{ ft}^3$$

From Fig. 11 we obtain the power requirement of 6.21 h.p. and the capital cost of $6300. Following the procedure given in the previous section the itemized costs shown in TABLE 11 were obtained. These costs have been converted to the 1973 basis using an annual inflation factor of 6.5%.

CONCLUDING REMARKS

While the scale-up and design procedures given here are applicable only to one class of batch mixers, namely tumbling mixers, there is no doubt that similar procedures can be developed for other classes of batch mixers. However, somewhat different data and additional information is required to develop scale-up and design procedures for continuous mixers.

TABLE 11

SPECIFICATIONS AND COST INFORMATION
FOR EXAMPLE 2

Weight of material (lb)	500
Volume of materials (ft^3)	10.4
Volume of mixer (ft^3)	77.1
Mixer, capital cost ($)	6710
Depreciation ($/hr)	0.781
Power ($/hr)	0.0273
Labor ($/hr)	4.26
Unit cost ($/ton-hr)	1.414

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5.10 APPENDIX

Derivation of Equation (2)

The technique of dimensional analysis is applied to the problem stated as follows:

$$P = f(d, g, J, K, L_{eff}, N, t, \rho) \tag{A-1}$$

Since the fraction of loading is dimensionless itself, it is chosen as the first dimensionless group

$$\pi_1 = J$$

Equation (A-1) is rewritten

$$P = f_1(d, g, K, L_{eff}, N, T, \rho) \tag{A-2}$$

The total number of variables is eight in equation (A-2), and we have chosen to express these in terms of three fundamental dimensions: length (L), mass (M), and time (θ). As is often the case, the maximum number of variables which will not form a dimensionless group is equal to the number of fundamental dimensions. Thus, according to Buckingham's theorem, we would obtain five dimensionless groups. The variables which we choose to be common to all groups are the following: N, L_{eff}, and K. Each of the remaining five variables will in turn be added to the first three, to give the five groups. The dimensionless groups are obtained as follows:

$$\pi_2 = \frac{Kt}{(L_{eff})^2}$$

$$\pi_3 = \frac{N^2 d}{g}$$

$$\pi_4 = \frac{P}{N^3 d^5 \rho}$$

$$\pi_5 = \frac{d}{L_{eff}}$$

$$\pi_6 = Nt$$

The last dimensionless group is the total number of revolutions of the mixer during the period, t, of the mixing operation. It is taken into account by other dimensionless groups. Thus, the final equation obtained is

$$f(\frac{N^2 d}{g} , \frac{Kt}{(L_{eff})^2} , \frac{P}{N^3 d^5 \rho} , J, \frac{d}{L_{eff}}) = 0$$

As in all cases where dimensional analysis is used, it is not certain that the effects are completely described by the variables chosen. For example, the friction force between surfaces of particles and the cylinder has not been included.

CHAPTER VI

SUMMARY AND RECOMMENDATIONS

The major results of this study are summarized below.

1) The nonparametric test was successfully introduced to the field of solids mixing. Wilcoxon signed ranks test for means was employed to justify the sampling technique. Mood's test for variance was used as a criterion for testing the validity of the scale-up procedures. Lilliefor's test was applied in testing the normality of the population. The Mann-Whitney test statistic was used as a measure of segregation for the mixture.

2) The computer simulation on the distribution of the contact number at the completely mixed state indicated that it is a binomial distribution for both the two dimensional cubic and hexagonal packing mixtures.

3) The application of the contact number sampling to radial mixing in a motionless mixer showed that the mixing index based on the contact number adequately represents the state of the mixture. This geometric and microscopic mixing index provides more information about the structure of the mixture than does the conventional mixing index.

4) Dimensional analysis led to the conclusion that dimensionless numbers in the following equation are pertinent to the scale-up of tumbling mixers.

$$f\left(\frac{N^2 d}{g}, \frac{Kt}{(L_{eff})^2}, \frac{p}{N^3 d^5 p}, J\right) = 0$$

The correlation between the mobility coefficient and the fraction of the cri-

tical rotational speed determined the mixing time. The correlation between the Power number and Froude number determined the power requirement.

Further studies are recommended below.

1) Simulate the distribution of the contact number at the completely mixed state of a randomly packed mixture.

2) Conduct studies on the radial mixing of particles of different sizes to gain additional understanding of the characteristics of the motionless mixer.

3) Develop scale-up procedures for batch mixers other than tumbling mixers.

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A STUDY OF SAMPLING AND SCALE-UP IN SOLIDS MIXING

by

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Nonparametric statistical tests are employed to analyze the sampling results of solids mixing. These tests can be performed on the data with different kinds of scale of measurement, such as nominal, ordinal, interval or ratio, without knowing the distribution of the population. In this report the nonparametric statistics are introduced and applied to the study of solids mixing--test of applicability of sampling technique, test of scale-up procedure, test of distribution of solid mixture, test of significance of fraction satisfactory, test of significance of equilibrium state and segregation. An index of segregation is proposed by using the test statistic of the Mann-Whitney test. A nonparametric statistics package written in FORTRAN for the IBM 360 NONPAR was employed to perform the tests.

Concepts of the contact and coordination numbers are introduced. These concepts are useful in understanding the microscopic and geometric characteristics of solid mixtures, and in analyzing operations and processes involving such mixtures. Results of the computer simulation for the contact number distribution under the completely mixed state agree well with the theoretical prediction both for the two dimensional cubic and hexagonal packing arrangements at different concentrations of the key component.

A geometric and microscopic mixing index defined by the contact number was used to study the radial mixing in a motionless mixer. The mixing index, a measure of radial mixing, increased exponentially as the number of helices in the motionless mixer increased. The helices in the mixer have the significant effect of reducing the void fraction of the mixture. The relationship between the coordination number of compaction of the mixture through the mixer was studied. The mean coordination number indicates that the packing of these mixtures are between the cubic and hexagonal packings.

Dimensional analysis has been employed in the study of scale-up procedures for tumbling mixers with nonsegregating materials. A scale-up procedure based on a correlation between the mobility coefficient and the fraction of the critical rotational speed for drum mixers, V-type mixers, and double cone mixers was proposed to determine the mixing time. A correlation of the Power number and Froude number for tumbling mixers was also obtained. The scale-up procedure based on these correlations was obtained on the basis of published data. In addition, a cost estimation procedure was outlined for the mixing of wheat. In general the unit cost was estimated to be about 1-2 \$/ton-hr. for tumbling mixers.