Simulation based model for component replenishment in multi-product ATO systems with shared resources

by

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Abstract

With increasing product complexity and customization, Assemble-to-Order (ATO) systems have gained a lot of popularity in recent years. ATO systems have the advantage of delivering customer orders at shorter leadtimes by manufacturing components to stock. However, for an on-time delivery of the final assembled product, the corresponding components must be replenished and be available when needed for assembly in a timely yet cost-effective manner.

This research investigates the production and subcontracting decisions in the multi-product ATO systems. We also provide insights on the following main research questions: (1) how to allocate shred in-house resource among various components? and (2) how does randomness in the service times impact these decisions?

We consider a manufacturing system where the components can either be shared manufactured in-house or can be procured by dedicated subcontractors, with each having finite manufacturing capacity. In addition, the components have stochastic lead times, and component availability is critical to satisfying the demand of the final product. Using, Monte Carlo simulation approach, we encompass a wide range of possible scenarios and provide insights on when to use shared resource for producing one component versus another, when it is optimal to source components from outside vendors. Using numerical experiments, we analyze different practical scenarios: (i) In-house manufacturer is cheaper, (ii) External subcontractor is cheaper, (iii) Using shifted exponential distribution (adding a constant delay in exponential distribution). Further, we observe that if the service times are shifted exponential distribution then the optimal policy tends to subcontract more often compared to when the service times are exponential distribution.

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Finally and most importantly, I would like to thank and dedicate this thesis to my mother, my father and my siblings. Their unconditional love, prayers, care, sacrifice and support encourages me to remain motivated and keeps me going. This thesis would not have been possible without them.

Dedication

To my parents

Chapter 1

Introduction

In the context of todays fast-paced and competitive manufacturing industry, the customer order must be completed and delivered by a certain due date, if not before. The product complexity is increasing and to counter these complexities and to deliver the products with shorter lead times, a number of different manufacturing strategies have been implemented. In literature the common manufacturing strategies are engineered-to-order (ETO), make-to-order (MTO), make-to-stock (MTS) and assemble-to-order (ATO).

These different strategies are designed according to the nature of the product and the nature of the demand. In ETO systems the customer demand is highly complicated and the product is only being used for a specific environment. A manufacturer spends thousands of hours on engineering and research, and produce a highly customized product. Examples of these products are commercial HVAC (Heating, Ventilation, and Air Conditioning), defense systems, oil drilling rigs and industrial cranes. These systems are common in engineering industries.

While in MTO system, the customer places an order that may not require a lot of engineering hours. The manufacturer produces it based on customer specification. Typically MTO systems have shorter lead time compared to ETO system. Examples of these systems include aerospace industries, steel industries, and automotive industries.

In MTS system, the manufacturer produces products based on demand forecast and retains them in inventory stocks. When the customer places an order, the manufacturer simply satisfies the demand through inventory stock. The demands are either backordered or lost if the stock is empty at the demand arrival. This type of manufacturing system adds up holding costs, although they are countered with minimal opportunity losses. Examples of products being manufactured are groceries, clothes, and other merchandise.

Taking into perspective the aforementioned strategies and products, ATO systems combine the benefits of both MTO and MTS systems to provide shorter lead times for custom products. In ATO systems, components or sub-assemblies are stored in inventories. Upon demand arrival for the final assembled product, for quick delivery process, these components are assembled together to produce the final product. The manufacturing of these components take significant time and resources, so the inventory is stocked according to projected demand. It is critical to have all components available at the time of arrival of customer order to ensure shorter lead time and high service levels. If any of the component is not available then the customer order is lost/back-ordered.

Interest in ATO systems has increased alongside the growing need to produce customized products while maintaining shorter lead times. In this paper, we will analyze an ATO system where the manufacturer has in-house manufacturing capacity for the components. Alternatively, these components can also be subcontracted to an external supplier. Understanding the complexity in developing methods to find the optimal solution, we use a simulation-based approach to determine near optimal manufacturing policy and inventory levels to achieve high service levels and cost minimization on a case-by-case basis.

1.1 Assemble-to-Order Systems

As companies need to make its mark on a fiercely competitive industry, manufacturers upgrade their facilities to gain a technical edge over one another. Furthermore, as such a personal item used by consumers on a continual basis, customization and demographic tailoring becomes necessary due to customer diversity. The most advanced, high-quality cell phone models are the prime example of personalizable technology. This industry is both rapidly changing and involves numerous components. For example, a single smartphone consists of display screen, microphone, speakers, sensors, camera, Bluetooth module, battery, and many other parts. Based on the customer requirements, the cell phone is assembled from these components.

The business model governing this type of manufacturing is known as an assemble-to-order system. To reiterate this manufacturing system, products are assembled from constituent parts only after an order has been placed. The ATO system from the products perspective could have single product system and multiple products system, while from the resource perspective, could have dedicated or shared resources.

For single product, the assembly of pre-fabricated parts takes very little time, but the manufacturing of those individual components may be expensive or time consuming. The main concern in this system is that insufficient inventory of a component may cause delays in order fulfillment, while too much inventory sitting unused could increase costs thereby reducing a companys profits and inhibit its relevance in a competitive market. This clearly makes inventory level policies an important consideration for the manufacturing industry.

In a multi-product system, an assembly line is capable of producing multiple different products from their respective different components, some of which may apply to multiple final products. This system faces the same concerns and decisions that a single product system does, but with added complexity and more variables involved.

A shared resource is comprised of a single-server production server that is shared among different components. This adds complexity while deciding when to produce one component versus another components using the shared resource.

1.1.1 Single Product Systems

As mentioned above, the world of personal electronics is a prime example of assemble-to-order systems for customized devices. From the earliest Apple iPods with a range of colors and storage capacities, to current PCs tailored to the consumer's work and play needs, this industry has demonstrated a need for fast-paced development, attention to detail, and a balance between construction or sourcing of parts at the macro and micro levels.

As a large-scale example in the transportation industry, we may consider aviation. The United States' largest aerospace company, Boeing Co., has over 153,000 employees and generated over a hundred billion dollars of revenue in 2018. Zazulia (2019) elaborates how Boeing has historically been quick to outsource when there may be cost savings, recognizing that with some 367,000 parts comprising even its smallest 737 jet, not every piece of the plane can be built within its own plants. Instead, Boeing must utilize a complex web of hundreds of suppliers to produce elements like engines and fuselages all the way down to details like airline-specific colors, branding, and exit signs. Fuselages are built in Wichita by the Boeing spin-off Spirit Aerosystems, and sub-assembly work may even be outsourced beyond Boeing. However, eventually, all parts make their way to Renton, Washington Allison (2010). There, the components are assembled into a 737 jet, flight-tested, and delivered to the customer Anonymous (2018). Fig 1.1 displays a single product system.

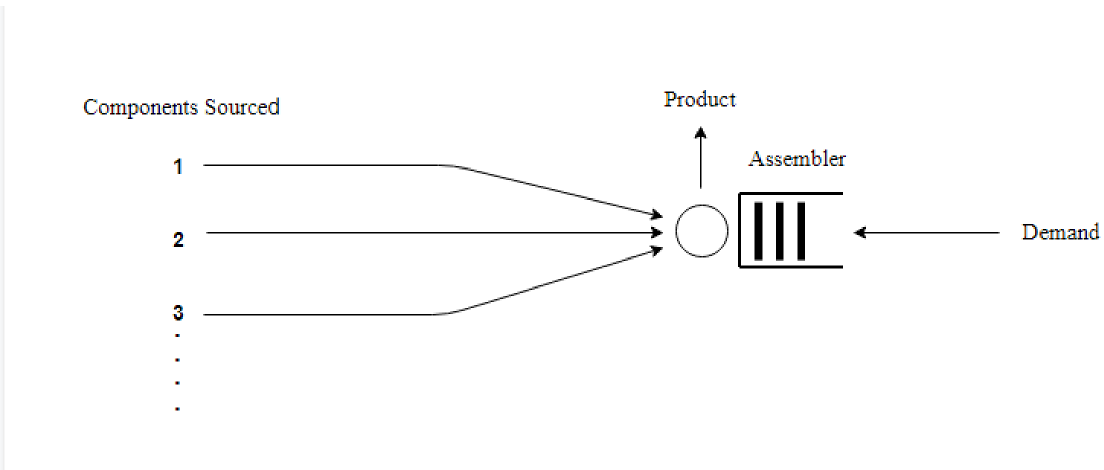

Figure 1.1: *A Typical Single Product System*

Boeing also provides an example of the delicate balance between in-house manufacturing and sourcing from external subcontractors and how it can go terribly wrong. When work began on the 787 Dreamliner, a first order plane was scheduled for delivery in 2008. However, Boeing gave external suppliers too much responsibility in an effort to cheapen the massive project. Delays quickly piled up due to parts shortages, software issues, problems with foreign and domestic supply chains, and even incorrect fastener installation. After sorting out miscommunications, performance issues, and even electrical fire problems, the first 787 was finally delivered in 2011, three years behind schedule (Anonymous (2013)).

An assembly line for Boeing's mainstream airplane is a fine example of a single product system, where the components are stocked up and assembled together to manufacture the product.

1.1.2 Multi-Product System

Next, we discuss some examples of multi-product ATO system. Fig 1.2 displays a multi-product system with multiple components. Automotive manufacturers today have a vast line of products to choose from. Currently Ford offers a "Build and Price" capability online where consumers can create their own customized cars or truck from the roof down. The customers chooses the bed configurations, paint color, various technology packages, additional style elements like chrome or sport appearance, trailer hitches, wheels, interior designs, power-train options and a dozen more options. Pricing is shown as the truck is designed, and subsequently an order can be placed. At the manufacturing line, the parts dictated by the order must be assembled to build the truck quickly and efficiently Anonymous (2019).

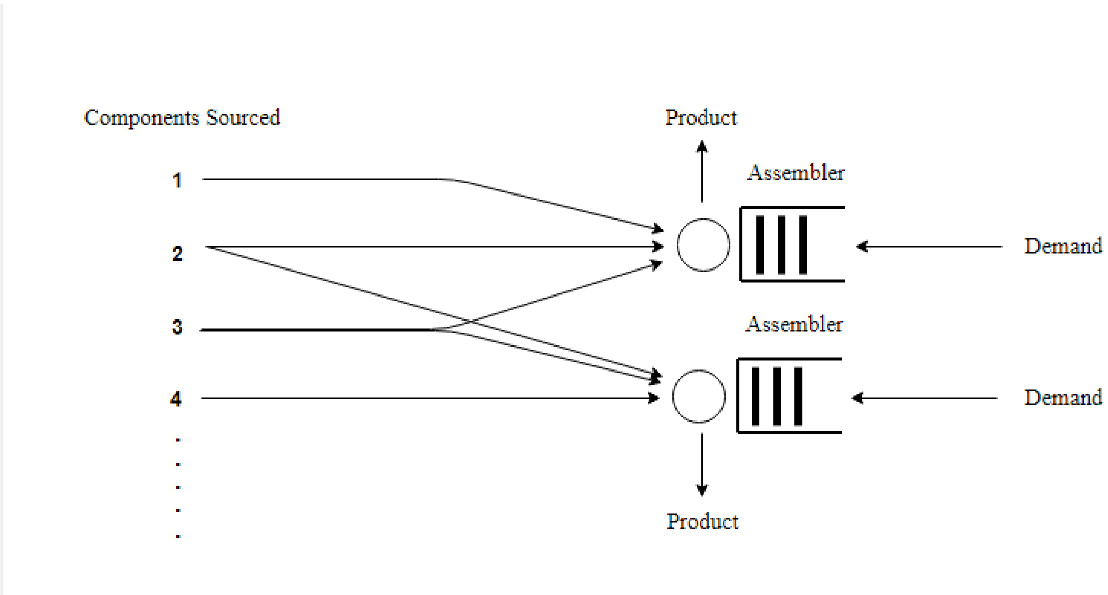

Figure 1.2: *Multi-Product System*

Multiple products are manufactured at an assembly line. It is very common to see many components being cross-shared among different products. For example, different models have the same seats, stereos, speakers, engines, switches, fuses, etc. All these components need to be stocked for faster operation.

1.1.3 ATO System with Shared Resource

Many components undergoing similar manufacturing operations that do not require dedicated assembly lines.

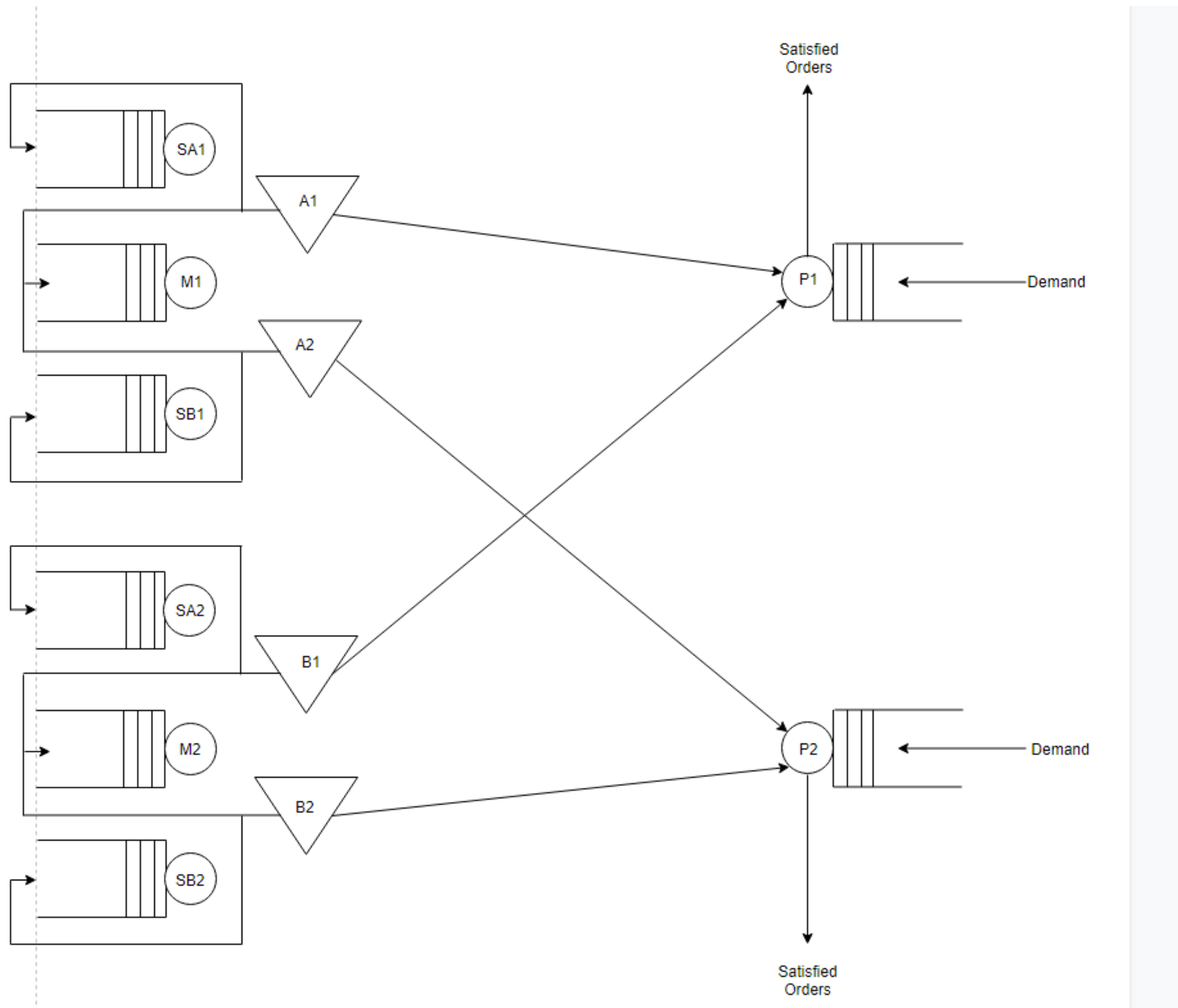

Figure 1.3: *Multi-Product with Shared Resource*

Instead the same resource undergoes slight modification according to the design specifications of the product. Not only does shared resources for different components result in effective resource management, but it also eliminates huge capital costs incurred at the installation of new dedicated machinery. Shared resources or sometimes also referred to as flexible resources are highly common among the manufacturing industries. A multi-components system with shared resource can be seen in Fig 1.3. Examples include packaging industries, where the machines are calibrated for various sized products.

1.2 Subcontracting

Subcontracting is the practice of assigning some portion of the scope of work to an outside vendor by the original contractor either due to lack of technology, costs, or service level requirements. Subcontracting is predominantly present in complex operations such as manufacturing, construction and information technology.

Sourcing to subcontractors can reduce costs by capitalizing on the suppliers' specialized knowledge or resources. A manufacturer may have the capability to produce a component, but a subcontractor has the facilities and resources to mass-produce the same component, yielding more cost-efficiency and higher production rate. For example, an automotive manufacturer such as Chevrolet sources out parts like rear and front bumpers from a specialized subcontractor like Flex-N-Gate in order to meet customer demand and reduce cost. Another advantage could be in terms of meeting service levels requirements. Often times, the manufacturing facilities have long queues of products due to resource limitations. Subcontracting in this scenario could alleviate the burden from the in-house manufacturer and reduce the overall lead time of the products.

1.3 Challenges with Production and Subcontracting Decisions

These examples demonstrate the pervasiveness of assemble-to-order systems across multiple industries. The challenges faced in manufacturing of ATO system are

- **Limited in-house capacity:** The availability of the components at the assembly line is critical to satisfy the demand. Typically, the manufacturer has limited capacity to cope up with the demand leading to stockout of components. In this case, the demand is either backordered or lost at an higher cost. It is also common that the components share the same resource. In this case, it is critical to decide when to produce a certain component using the capacity at the shared resource.

- **Lead time tradeoffs:** It is a well known fact that the lead time is directly related to the spare capacity. With the randomness in the demand, the in-house manufacturer could be burdened resulting in longer queues for components and thus resulting in higher leadtimes. The main challenge is to find the balance between leadtimes and costs.

- **High capital costs vs subcontracting:** A manufacturer deciding to install another manufacturing assembly line for the product incurs heavy capital costs. To mitigate these high costs, the components could be subcontracted to vendors at a lower costs. However, the vendors could have higher lead time so the decision maker need to find the tradeoffs between cost and lead time.

- **Multi-product ATO system:** The literature on production and subcontracting strategies for ATO system is limited. The multi-product ATO system increases the complexity of the problem.

1.4 Research Tasks and Main Contribution

The main objective of the research is to investigate policies for the component replenishment in a general ATO system with shared resource. Within the question of inventory management for our ATO system, the manufacturer has following options available to satisfy the demand of the component:

1. Use capacity of the shared resource to produce one of the components.
2. Subcontract components to external supplier.

The choice between these options can depend on processing time for orders, customer demand for the product, and costs and lead times for in-house production or subcontractors. Typically, in MTS or MTO systems, dual index policies could be used for such decision where we have thresholds on the inventory to decide the replenishment decisions for components.To the extent of our knowledge, there is no know optimal policy for our proposed system. We plan to analyze such policy in our proposed multi-product ATO system. In this paper, we will use a simulation based approach to investigate the following research questions.

RQ1: What are the optimal inventory levels and threshold levels for the components? How do the threshold levels impact the total cost?

RQ2: How does our decision to make in-house vs subcontract change with fluctuating the demand?

RQ3: How does distribution of service times impact our policy?

We answers these research questions in our thesis by designing a complex simulation of an assemble to order system. Our system consists of multi-product where each product is composed of multi-components. Multiple scenarios are modeled with respect to cost and service times at different servers. These scenarios are analyzed to find the direction towards optimality.

There are two main contributions of this thesis:

Firstly, we develop a simulation based model to analyze production and subcontracting decisions in multi-product ATO system with shared resource. This type of manufacturing system is very common in practice. However, there is limited research on subcontracting in ATO system due to complexity in solving such a system. For exact analysis we present a Markov decision process model but faced the curse of dimentionality in terms of state and action space. To overcome this, we use Monte-Carlo simulation and answer when to subcontract a component for a product and when to use capacity at the shared resource to manufacturer a component.

Secondly, we analyze the impact of system parameters such as service costs/ service rates of the manufacturer and the subcontractor on the optimal decision. The simulation also enables us to use different distribution assumptions in the service time such as shifted exponential distribution which is more practical to fit the proposed manufacturing system.

1.5 Thesis Outline

The thesis is organized as follows:

In chapter 2 of the thesis we provide a detailed literature review of different types of assemble-to-order systems. It is followed by the subcontracting strategies implemented and analyzed in different manufacturing settings. The last literature review is about simulation techniques and the how the data is analyzed to determine inventory control policies.

In chapter 3, we develop our model as a Markov decision process. We discuss the state space and action space of the system. With the model being extremely complex, we propose how simulation of the system can reduce the complexity to reach near optimal solution. We provide insights for research question **RQ1** in this chapter.

In chapter 4, we discuss the impact of system parameters on the production and subcontracting decisions. Using different practical scenarios such as: manufacturer is cheaper, subcontractor is cheaper, and other distribution assumptions, we show how the results changes and what a supply chain manager should do under these scenarios. We provide insights on research questions **RQ2** and **RQ3** in this chapter.

In chapter 5, we conclude the thesis and discuss future work possibilities and corroborate our work.

Chapter 2

Literature Review

In this chapter we review the relevant literature. This chapter is further organized into three sections. In Section 2.1 we will discuss Assemble-to-Order (ATO) systems. Section 2.2 will focus around different subcontracting strategies. Section 2.3 will elaborate on simulation based approaches.

2.1 Assemble-to-Order Systems

There have been a number of studies conducted involving ATO systems. To further elaborate on these studies, we have classified them into single product system and multi-product system categories.

Single Product Systems: ‘Yano (1987) studies a single product system where the assembly part shortages arise from late arrivals. The research develops an algorithm to find optimal solutions for the assembly line where the lead time for these components is stochastic, to minimize the inventory holding and tardiness cost. Kumar (1989) performs a generalized study for the inventory cost for a factory stockroom which supplies the sub-assemblies. The components can only be subcontracted. The main concern of his study is the effect of lead time variability by the subcontractor, the number of sub-assemblies used in the product, and the service levels on the inventory cost. Chu et al. (1993), present an iterative algorithm for optimality for ordering the components. Here again the components are provided by a subcontractor for the assembly line. Furthermore, their proposed algorithm is for an n-product component system. The aforementioned studies all focus on optimal decisions for ATO systems with a known demand and random component reorder lead times.

Later, Song and Yao (2002) use greedy algorithms to solve a single product system where the demand arrives via Poisson process and the refill rates for the components are independent and identically distributed. The components are made to stock before being assembled together. The paper also discusses how lead time variability undermines the performance of the ATO unlike the standard M/G/∞. Gallien and Wein (2001) analyze a single product ATO system where the refill rates for the components are independent but not identically distributed.

Benjaafar and ElHafsi (2006) model a single product ATO system with multiple customer classes.

Multi-Product Systems:

Song (1998) was the first to analyze order fill rate as a performance measure on base-stock systems, using a multivariate compound Poisson demand process and constant lead times. In this model, customers order different items in different quantities, and the demand for each component is superimposed to find a compound Poisson process. Certain assumptions are made, such as that unfulfilled demand is backlogged at positive cost, and in-stock components are shipped to the customer without waiting for out-of-stock parts to arrive. Versatile estimation bounds for the optimal order fill rate are developed, as well as a procedure for exact calculation. Song concludes that the fill rate of an individual component is not an ideal indicator of order fill rate correlated across all items. Later, Song (2000) analyzes an ATO system where components are held in stock and assembled only when customer orders are realized, again assuming a multivariate compound Poisson demand process and constant replenishment lead times. The paper develops a model for estimating the order fill rate of the system. Lu et al. (2005) analyze a multi-product ATO system to evaluate expected backorders using bounds and approximations with appropriate parameters. ElHafsi et al. (2008) studies the optimal production and inventory allocation policies of multi-product

ATO systems with a modular nested design. The optimal production policy is shown to be of base-stock type with levels dependent on the inventory of other components. The optimal inventory allocation policy is shown to be a multi-level rationing policy, and a simple heuristic is proposed compared to optimal policy. Gao et al. (2010) analyze a multi-product ATO with multiple classes of demand, where replenishment depends on independent unreliable machines. Again, they assume that demand arrives according to a Poisson process and allow two kinds of stockout. The paper applies a matrix-geometric solution approach to compute order-based and item-based fill rate. Zhou and Chao (2012) develop a simple Stein-Chen approximation relating to order-based fill rate and component-based fill rates for a multi-product ATO system.

Manufacturing Systems with Shared Resources:

Resource sharing in manufacturing enables performance and service levels improvement while keeping low capacity costs (Jordan and Graves (1995), Sheikhzadeh et al. (1998)). Later, Amirteimoori (2013) studies decision-making in shared resource efficiency by evaluating the model as a two-stage data envelopment analysis. His study includes real life situations where some production outcomes have failure capability and may undergo repair operations.

Garcia-Santiago et al. (2015) exhibit a meta-heuristics approach for complex shared resource energy production planning. They focus on comparing simulation data with genetic algorithms to minimize the energy consumption. Another stream of literature by Simeonova et al. (2005) involves a simulation based proposal to improve the plant efficiency, where the manufacturing setup comprises of two parallel manufacturing servers and shared resources. The simulation outcome reveals that a feedback strategy for rescheduling is efficient but further is required for real life scenarios.

2.2 Subcontracting Strategies

Timely outsourcing and subcontracting in manufacturing industries proves to minimize cost and increase service levels Yao et al. (2010). Taking in consideration the real life scenarios encircling lead times, customer variability and cost implications the decision making process becomes complex. Kumar and Vannelli (1986) propose cost minimization for production facility where multiple products and components are manufactured. Subcontracting is introduced for disaggreagation of the facility, their study focuses on both financial and non-financial parameters.

Manufacturing versus subcontracting decision is analyzed by Lee and Zipkin (1989) through Dynamic programming algorithm. The evaluated scenario permits the manufacturer to satisfy demand through in-house manufacturing, subcontracting or a combination of both. Zero replenishment lead time policy is considered for all cases. Furthermore, their proposed model is extended to entertain backlogging and bounded inventory within the problem.

Rivera-Gómez et al. (2016) build a stochastic optimal control model where the production and subcontracting are evaluated for a manufacturing system. The key highlight of their work involves taking deterioration of quality as increasing functions. The model is approached as a stochastic dynamic programming problem. Rivera-Gómez et al. (2018) further elaborate their findings and then formulate a simulation based approach to minimize cost and obtain control polices for subcontracting and other manufacturing factors.

Sinha and Krishnamurthy (2014) analyze a manufacturing system where a multiple products are assembled from multiple components. They propose an approximate method to determine the production and subcontracting decisions. However, their work could only solve for small examples and does not provide bounds on the solution obtained from the approximate method. Our work extends their analysis to a simulation model and also discusses the impact of relaxing distribution assumption of service times on the optimal solution.

2.3 Simulation Approach

With unprecedented outcome possibilities in varying real life scenarios, simulation offers a unique concept of mimicking the real structural flow. Studying and analyzing these simulations enables us to predict and decide optimal solutions prior to implementation and reduce adverse consequences through predictive analysis Ören (2011). Simulation modeling can be applied in many if not all environments to visualize dynamics, save resources and handle uncertainty. Different simulation techniques are implemented to predict various behaviors in numerous areas for example 3D modeling, health-care, aerospace, fluid dynamics, operations research etc.

Several studies have been conducted for simulation based analysis in production systems. Huang et al. (1983) study simulation analysis for just-in-time techniques with kanban for inventory policies for multistage production systems. Simulating production operations and analyzing data for inventory and production systems have been studied by (Rezg et al. (2004), Köchel and Nieländer (2005), Kämpf and Köchel (2006), Olhager and Persson (2006), Dias et al. (2018)).

Rezg et al. (2004) simulate a production system with n machines ($n \geq 1$), The machines in their model are also prone to failure and have time based preventive maintenance policy scheduled. Inventory control and maintenance strategies are formed by analyzing simulated data and evaluating genetic algorithms. Köchel and Nieländer (2005) suggest a simulated approach for a multi-facility/multi-echelon inventory problem. However, computational limitation restricts their study for an elaborate optimal control policies.

Kämpf and Köchel (2006) model a capacitated stochastic lot-sizing system where inventory and production control policies are optimized using simulations. The manufacturing times are random and unsatisfied demand is backlogged. Their model also incorporates sequencing and lot size rule to maximize profits. Olhager and Persson (2006) report and review the behavior and the design of policy structure for production and inventory control systems using simulation. They also highlight the benefits of simulation as a risk free environment for predictive analysis. Dias et al. (2018) propose a framework for scheduling and predictive control through simulation based optimization in air separation units. The scheduling problem addresses the dynamic behaviour of the system by state space model.

Chapter 3

Stochastic Model for ATO System

3.1 Introduction

In this chapter, we develop stochastic models to production and subcontracting decisions for our assemble to order system with multiple products. With limited literature on subcontracting decisions for ATO systems, there is a significant need of decision making in the subcontracting of components for multi-product systems for relevant manufacturing facilities. Our model for this problem involves an elaborate layout where the finished products are assembled together with their respective components.

The components in our system are made to stock and are held at the inventory at the assembly line. They can be produced by either; using in-house manufacturing capacity or by subcontracting through external vendors. Both the subcontractor and the manufacturing facility have finite production capacity, which entails stochastic lead times.

In the proposed ATO system, multiple products are assembled from respective components. Components that have similarity in terms of their design or manufacturing processes could be manufactured using the shared capacity at the in-house manufacturer. This idea can be conceptualized where multiple components require to undergo similar operations, the manufacturer instead of investing on more capacity to create parallel dedicated resources, can use capacity on the shared resource for different components. This allows the manufacturer to produce multiple products without dedicating an assembly line for a single product. Furthermore, this completely nullifies the huge capital costs incurred in constructing a new assembly line. However, the same decision could have impact on the lead time.

In contrast, high demands and unforeseen bottlenecks in the supply chain require the manufacturer to sometimes subcontract these components instead of manufacturing in-house at the shared resources. This raises the following research questions: (1) For general ATO systems, what are some good production and subcontracting policies? (2) How do we schedule shared in-house resource among different components?

3.2 Multi-Product ATO System

Sinha and Krishnamurthy (2014) proposed an Markov decision model for multi-product ATO system. We leverage the same problem but extends the analysis to provide insights on optimal production and subcontracting decisions for large systems and the impact of relaxing distribution assumptions. The model consists of two products i = 1,2. Each product is manufactured from its two respective components C_{ij}, where $j = 1, 2$. The products are assembled at a facility where each assembly line requires both corresponding components. For example, the final products are large and small wheels, large wheel would be assembled from a large tire and a large rim (components) and respectively for the small wheels. The components are stored and held at respective inventory locations. W_{ij} represents the locations for components C_{ij} $j = 1, 2$. Products i are assembled at location S_{ij}. The assembly time for S_{ij} is assumed to be zero. Product i's demand follow a Poisson process $N_i(t)$, $t >= 0$ with rate λ_i. At the demand arrival if the both corresponding components C_{ij} for product i are available at W_{ij}, the demand is satisfied. If one of either components C_{ij} is unavailable then the demand is unsatisfied and lost sale cost incurs. Fig 3.1 displays our supply chain model. Our model can be extended to system with n products and m components.

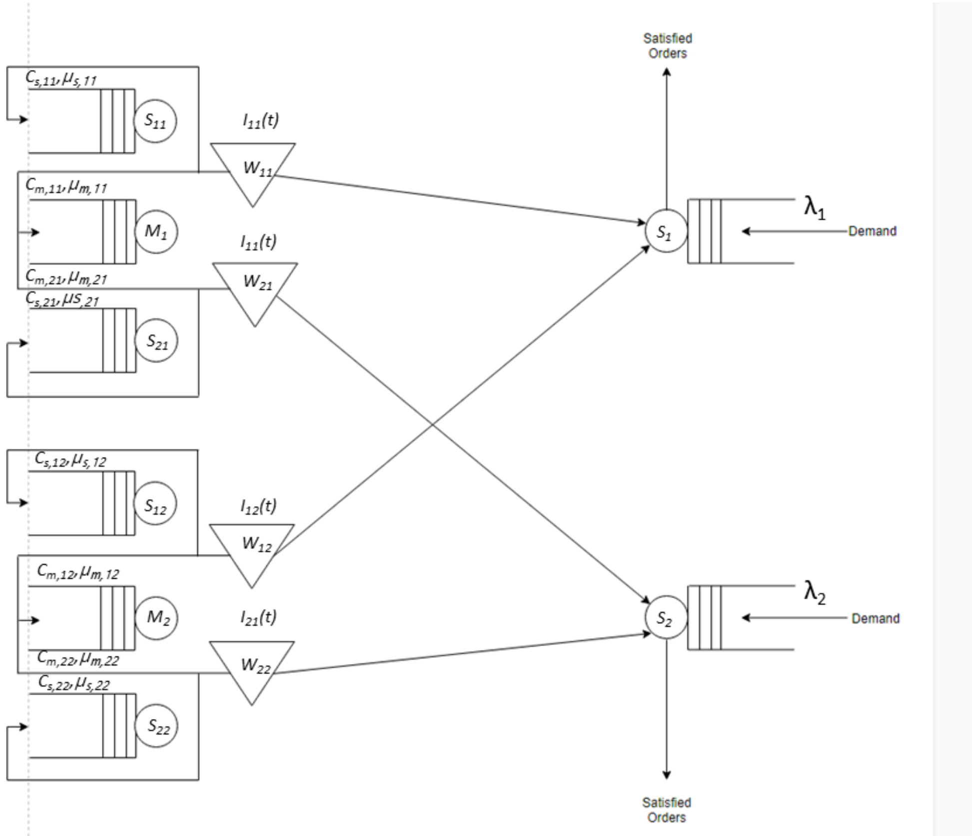

Figure 3.1: *Supply Chain Model*

$I_{ij}(t)$ is the net inventory at a given time t for the components C_{ij} at location W_{ij}. The inventory of the components can be replenished by either subcontracting through an external source S_{ij} or use the in-house manufacturing capacity M_j. Both component inventory replenishment sources can manufacture either C_{1j} or C_{2j}.

The subcontracting source S_i and in-house manufacturing unit M_j are modeled as single server queues. The service times for both facilities; subcontractor and in-house are exponentially distributed with mean $\mu_{s,ij}^{-1}$ and $\mu_{m,ij}^{-1}$ respectively. The cost rate for each server to manufacturer component C_{ij} is denoted by $c_{s,ij}$ and $c_{m,ij}$. Holding cost of the inventory is denoted by h_{ij} and we let l_i be the lost sale cost for product i whose demand was unsatisfied because of either component unavailability. In the next section we analyze the model as a Markov Decision process.

3.3 Markov Decision Process Formulation

To analyze the dynamics of the system we can model it as a continuous-time Markov chain problem. Let χ_j , $j = 1, 2$ denote a subsystem j which represents the manufacturing process of components C_{1j} and C_{2j}. The subcontracting units are S_1j and S_2j, while the in-house manufacturing facility is M_j.

The key elements for the Markov chain process are as follows:

Decision epoch: A state change triggers the actions, i.e either on demand arrival or product manufactured.

State Space Σ: The state of the system is represented with four inventory values i.e $\sigma = (I_{11}, I_{21}, I_{12}, I_{22})$, $\sigma \ \ \epsilon \ \Sigma$ where I_{ij} is the net inventory position of the component C_{ij}.

Action Space A : The action space is expressed as, $A = A_1 \ * A_2$ where A_j, $j = 1, 2$; where the action set of the subsystem χ_j with $\alpha_{j,k_j} = (m_j, s_{1j}, s_{2j})$, $\alpha_{j,k_j} \ \epsilon \ A_j$,k$_j = 1, 2...12$. If the component C_{ij} is to be manufactured at the in-house facility M_j, m_j is assigned the value i and has the value 0 when the component is being outsourced. Correspondingly if the component C_{ij} is being being outsourced by the external subcontractor S_{ij}, s_{ij} takes the value i and is assigned value 0 if it is manufactured in-house. Tables 3.1 and Table 3.2 represent the action space for each of subsystems χ_j.

Transition Probabilities: Let $p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})$ be denoted as the transition probability for any of the states $\sigma = (I_{11}, I_{21}, I_{12}, I_{22})$ to state $\sigma^{'} = (I^{'}_{11}, I^{'}_{21}, I^{'}_{12}, I^{'}_{22})$ using actions α_{1,k_1} $\epsilon \ A_1$, $\alpha_{2,k_2} \ \epsilon \ A_2$.

We define $v = \Sigma^2_{i=1}\lambda_i + \Sigma^2_{i=1}\Sigma^2_{j=1}(\mu_{m,ij} + \mu_{s,ij}) + C$, where C is the normalizing factor. The transition probability are defined as follows:

Product demand arrival i : $I^{'}_{ij} = I_{ij} - 1$, $j = 1, 2$; then the transition probability will be as $p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})$ is :

$$p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2}) = \lambda_i/v, \forall i = 1, 2$$

Table 3.1: *Action Space for* χ_1

A_1	M_1	S_{11}	S_{21}
$\alpha_{1,1}$	1	1	2
$\alpha_{1,2}$	1	0	2
$\alpha_{1,3}$	1	1	0
$\alpha_{1,4}$	1	0	0
$\alpha_{1,5}$	2	1	2
$\alpha_{1,6}$	2	0	2
$\alpha_{1,7}$	2	1	0
$\alpha_{1,8}$	2	0	0
$\alpha_{1,9}$	0	1	2
$\alpha_{1,10}$	0	0	2
$\alpha_{1,11}$	0	1	0
$\alpha_{1,12}$	0	0	0

Table 3.2: *Action Space for* χ_2

A_2	M_2	S_{12}	S_{22}
$\alpha_{2,1}$	1	1	2
$\alpha_{2,2}$	1	0	2
$\alpha_{2,3}$	1	1	0
$\alpha_{2,4}$	1	0	0
$\alpha_{2,5}$	2	1	2
$\alpha_{2,6}$	2	0	2
$\alpha_{2,7}$	2	1	0
$\alpha_{2,8}$	2	0	0
$\alpha_{2,9}$	0	1	2
$\alpha_{2,10}$	0	0	2
$\alpha_{2,11}$	0	1	0
$\alpha_{2,12}$	0	0	0

Component production completion C_{1j}: $I_{1j}^{'} = I_{1j} + 1$ and the transition probability $p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})$ are as follows:

$$p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2}) = \Sigma_j(l_{m,1,kj}\mu_{m,1j} + l_{s,1,kj}\mu_{s,1j})/v$$

where $l_{m,1,kj}\mu_{m,1j}$ and $l_{s,1,kj}\mu_{s,1j}$, $i = 1, 2$ are the indicator function that will take value 1 if the M_j and S_{ij} respectively are producing component C_{ij} under action $A_{j,kj}$ and 0 else.

Component production completion C_{2j} $I_{2j}^{'} = I_{2j} + 1$ and the transition probability

$p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})$ as follows:

$$p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2}) = \Sigma_j(l_{m,2,kj}\mu_{m,2j} + l_{s,2,kj}\mu_{s,2j})/v$$

where $l_{m,1,kj}\mu_{m,1j}$ and $l_{s,1,kj}\mu_{s,1j}$, $i = 1, 2$ are the indicator function that will take value 1 if the M_j and S_{ij} respectively are producing component C_{ij} under action $A_{j,kj}$ and 0 otherwise.

Conclusively $I^{'}_{ij}$ $=I_{ij}$,$\forall$ i, $j = 1, 2$ and the transition probability $p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})$ is given by;

$$p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2}) = (v - \Sigma_{ij}(\lambda_i + (l_{m,i,kj}\mu_{m,ij} + l_{s,i,kj}\mu_{s,ij})/v))$$

Cost Equation: Let $h(\sigma) = \Sigma_i\Sigma_j h_{ijmax}(I_{ij}, 0)$ as the total holding cost of the inventory and $ls(\sigma) = \Sigma_i ls_i max(-I_{ij}, 0)$ as the total lost sale cost. Let $c(\alpha_{1,k_1}, \alpha_{2,k_2}) = \Sigma_{i,j}(c_{m,ij}l_{m,i,kj}+ c_{s,ij}l_{s,i,kj}$ defined as the total production cost for $\alpha_{1,k1}$ and $\alpha_{2,k2}$ where $l_{m,i,kj}$ or $(l_{s,i,kj})$ are binary variables which take value; 1 for in-house manufacturing unit in process and 0 for subcontractor in process for components C_{ij}. This results in production cost at facility is initiated only if the action is set to produce.

Now we construct a Bellman cost equation (see Equation (3.1)) with the value function where $V_t()$ is the value function at any given state σ and decision time t.

$$V_t(\sigma) = h(\sigma) + ls(\sigma) + \min_{(\alpha_{1,k_1}, \alpha_{2,k_2})\epsilon} \left(c(\alpha_1, \beta_2) + \eta\Sigma_{\sigma'} p(\sigma^{'}|\sigma, \alpha_{1,k_1}, \alpha_{2,k_2})V_{t+1}(\sigma^{'})\right) \tag{3.1}$$

The objective minimizes the value function, $V_t(\sigma)$ at each state σ and determines the optimal action $(\alpha^*_{1,k1}, \alpha^*_{2,k2})$. The model described above faces a huge set of problems relating to the dynamic analysis of the optimal policy. The state space of the system Σ and action space A is huge. For example in our model as $I_{ij}, i, j = 1, 2$ varying from -100 to 100 we have over a billion states and and 144 actions. In addition the optimal value function $V^*_t(\sigma)$

may not be convex in $I_{1j}, j = 1,2$ for the state space Σ and action space α. Subsystem χ_j can be visualized in Figure 3.2.

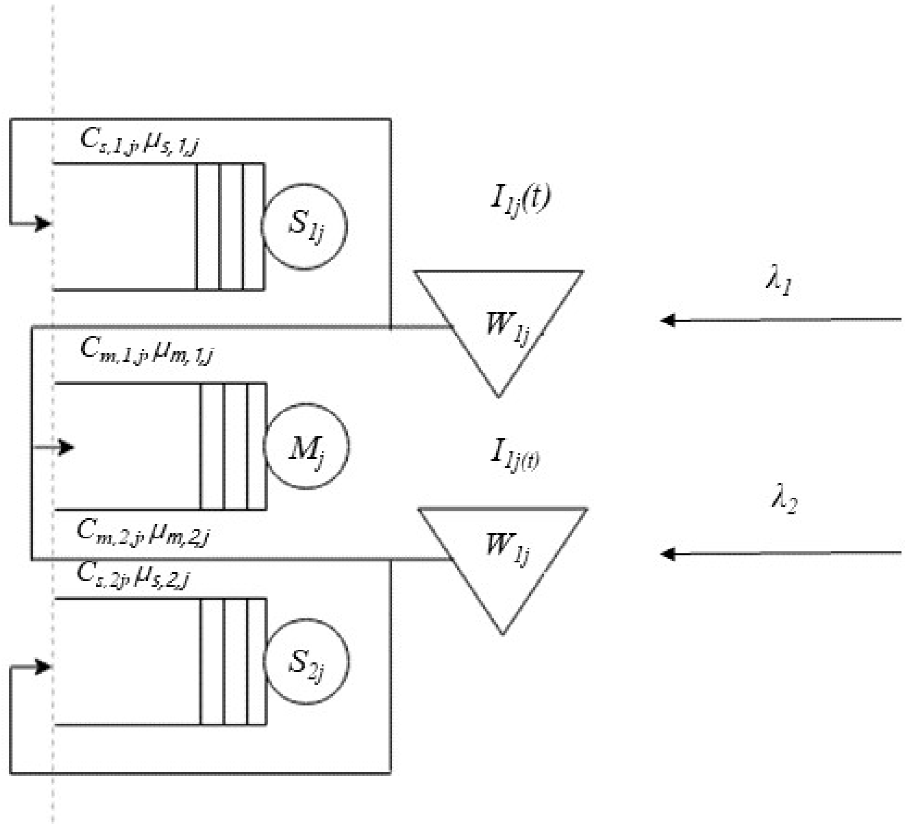

Figure 3.2: *Subsystem* $\chi_j, j = 1,2$

Note that for a two product and their respective two component MDP model has 144 actions and $(|I| + 1)^4$ states when the inventory of each component varies from 0 to $|I|$. This high level of complexity makes the problem formulation by MDP extremely difficult. Additionally, the MDP model is limited to Markovian assumption with service rates and demand arrival rate. Many manufacturing systems often follow other distributions such as shifted exponential. Given the complexity in determining the optimal solution, we use Monte-Carlo Simulation approach to sample potential actions of a given state and identify a near optimal solution.

3.4 Monte Carlo Simulation

We model our proposed ATO system using a discrete-event simulation engine inside MATLAB R2014 (namely Simulink). The simulation model is designed to capture various dynamics of the system such as switching between production and subcontracting,non-zero assembly time, lost sales, randomness in service and demand process, etc.

The simulation model is designed to handle two types of policy: (1) randomized action for each state, and (2) threshold based policy.

Randomized action for each state:

At the beginning of the simulation, we specify a particular action pair $(A_1(\sigma), A_2(\sigma))$, $A_i(\sigma) \in A_i, i = 1, 2$ to a state $\sigma = (I_{11}, I_{21}, I_{12}, I_{22}), \sigma \in \Sigma$. This data is stored in a file which will later be used to decide if a components needs to be manufactured using in-house manufacturer or the subcontractor. For example, for a state $\sigma = (1, 1, 1, 1)$ with actions $A_i(\sigma) = (0, 1, 2), i = 1, 2$, the simulation model should send all four components to their respective subcontractor when the inventory for all components are 1. We run the simulation model with the given action list. After the simulation is complete, we record the results and update the action list with new set of actions chosen. An elaborate way of choosing the the action space is to assign completely random actions for all the state space. A randomly assigned action space table (see Table 3.3) is as follows:

Table 3.3: *States and Corresponding Random Action*

I_{11}	I_{12}	I_{21}	I_{22}	m_{11}	m_{12}	m_{21}	m_{22}	s_{11}	s_{12}	s_{21}	s_{22}
0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	1	1	0	1	1	0	1	0	0
0	0	0	2	1	1	1	0	0	0	0	1
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
2	3	4	5	1	1	0	1	0	0	1	0
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
9	9	9	8	1	0	1	0	0	1	0	1
9	9	9	9	1	0	1	1	0	1	0	0

Note that randomly assigning actions to each state could result in millions of possible combinations even for a small-sized problem with inventory varying from 0 to 10. This requires lot of simulation runs and could also make the sampling difficult.

Threshold based policy:

It is common to see in manufacturing industries where a threshold based policy is implemented by the supply chain manager. Whenever the current inventory of a component is lower than the threshold value, then that component is manufactured using capacity at the in-house manufacturer to make the components at a faster rate. On the other hand, if the current inventory of a component is at least the threshold value, then it is subcontracted to an external vendor.In our simulation model, we use threshold base switching policy for our experiments which helps to reduce the number of simulations while giving practical policies to the supply chain manager.

Before the start of the simulation, the inventory I_{ij} of each component C_{ij} is set to the maximum value of B_{max}. The simulation begins with the demand arrival of product i, $i = 1, 2$. We assume that the demand arrival process of each product i follows Poisson process. Next, for a product i, an event is triggered which initiates the manufacturing of its two respective components $C_{ij}, j = 1, 2$ (see Figure 3.3). We ensure that the held inventory of each component is in the range $(0, B_{max})$, i.e., $0 \leq I_{ij} \leq B_{max}$.

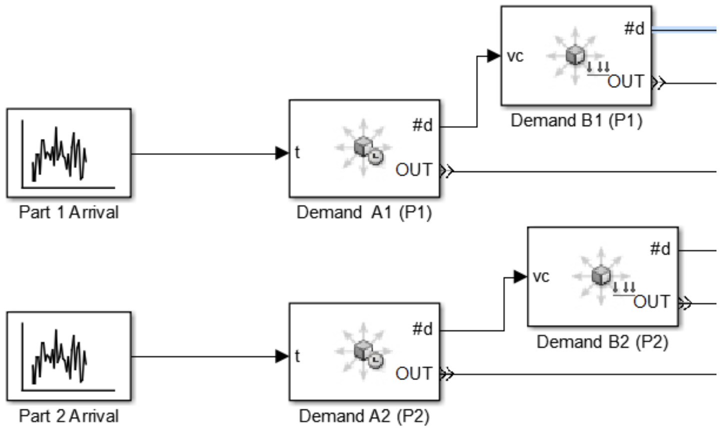

Figure 3.3: *Demand Arrival for Product i*

At the demand epoch, the state space of the system i.e. $(I_{11}, I_{21}, I_{12}, I_{22})$ the current inventory of the respective components C_{ij}, is compared to their respective threshold values to determine whether the component needs to undergo service at the in-house facility or subcontractor.

When the demand of product i arrives and there is available inventory stock for components $C_{ij}, j = 1, 2$ then we reduce the inventory of the components, i.e. $I'_{ij} = I_{ij} - 1$. This means we are consuming from the available inventory of the components. This also sends a signal to produce those consumed components through the policy mentioned before. If in any case, the current held inventory of the component is unavailable, the replenishment event of that particular component is ignored. This demand lost incurs a loss of sales cost which is constant for every unsatisfied component. This routes the component entity to lost sales cost accumulator before the entity is destroyed. The state space of the system remains constant i.e $I_{ij} = I_{ij}$.

The demand for each component triggers an attribute allocation process. Figure 3.4 shows the "Set Attribute" block in Simulink to specify attributes to an entity. For our multi-product system, every component event is associated with both types of products and components. The attributes are as follows:

- The "ServiceTimeMan" attribute stores the exponentially distributed random service times for components for in-house manufacturer. We also test the model with other distributions.

- The "Inhouse" attribute specifies the simulation if the component needs to be manufactured using in-house capacity or using subcontractor. We use the thresholds to decide the value of this attribute.

- The "Create" attribute checks for stockout situation and destroys the entity suggesting that the demand is lost at a cost.

- The "ServiceTimeSub" attribute stores the exponentially distributed random service times for components for subcontractor. We also test the model with other distributions.

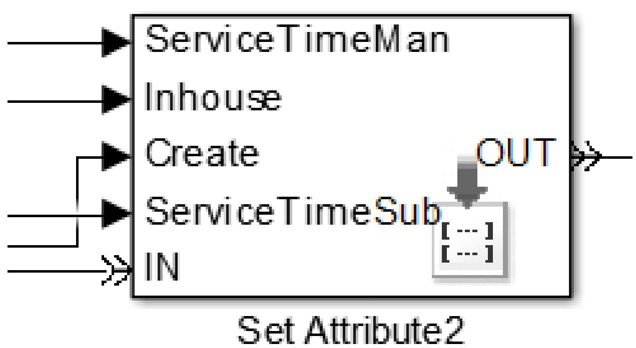

Figure 3.4: *Attribute Allocation*

After the allocation of the attributes, the components undergo manufacturing process using the pre-specified threshold policy. For products i, the components C_{ij} will either be sent to the in-house shared resource or will be manufactured by a dedicated subcontractor. For example, if current inventory I_{ij} of component C_{ij} is 4 and threshold limit is set for 5 then the "Inhouse" is set to direct the component to the shared in-house manufacturer M_j. On the other hand, if current inventory I_{ij} of component C_{ij} is 6 and threshold limit is set for 5 then the "Inhouse" is set to direct the component to the corresponding subcontractor S_{ij}. We assume that all servers in our system follow a first-in first-out (FIFO) queuing model. The service time for each facility is present inside the attributes of the components. The facilities will operate for the associated service time until the component production is completed. In our simulation model, we also capture the costs at the shared resources M_j and along with the dedicated subcontractors S_{ij} (see Figure 3.5).

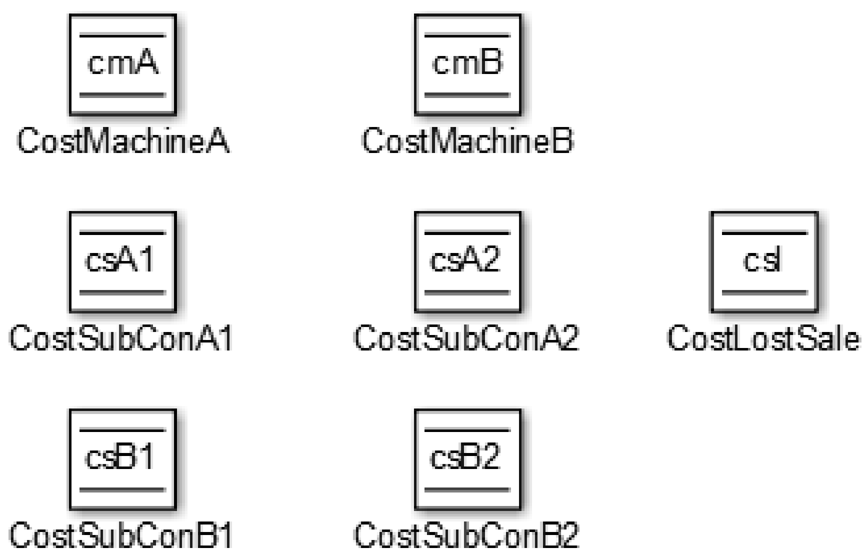

Figure 3.5: *Service and Lost Sale Costs Being Stored Inside Simulation*

As each component is attributed with their random service times, the utilization of the servers for service completion of the component is corresponding to the service time. The cost of in-house manufacturing and subcontracting facility is measured according to the rate of operation of the servers to produce a component to a respective cost factor.

Upon the service completion of the components C_{ij}, the components replenish the inventory and next state space of system becomes; $I'_{ij} = I_{ij} + 1$. The manufactured component is added to the dynamic inventory I_{ij}. For the next component the system will read the action space respective to the real-time state space.

Following the production of the components, each component is again switched and routed according to get assembled to produce the product i. Regardless of the service line, all components are routed towards the assembler to produce the final product. The assembler also follows FIFO queuing method and the time taken to assemble the component is assumed to be very small. To produce the final product i both components need to be available.The total number of products i manufactured are measured and stored. Figure 3.6 and Figure 3.7 display the complete simulation model before and after switching takes place respectively.

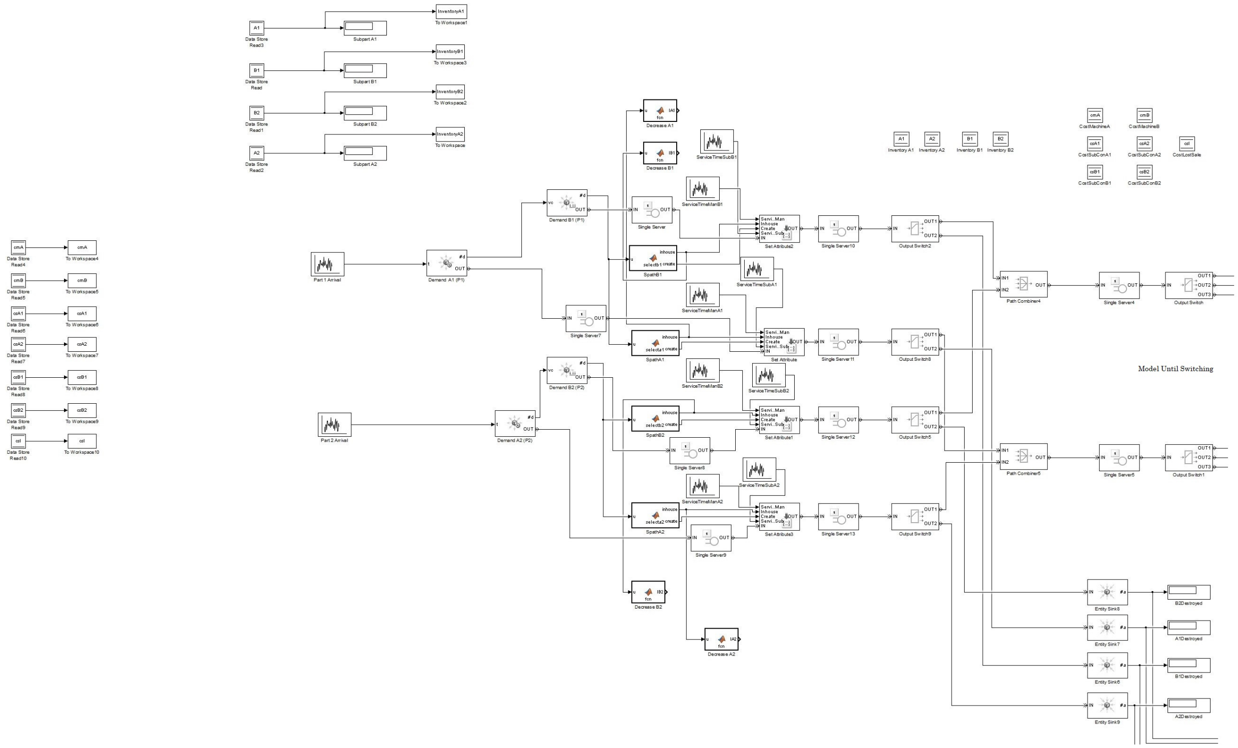

Figure 3.6: *Model Before Switching*

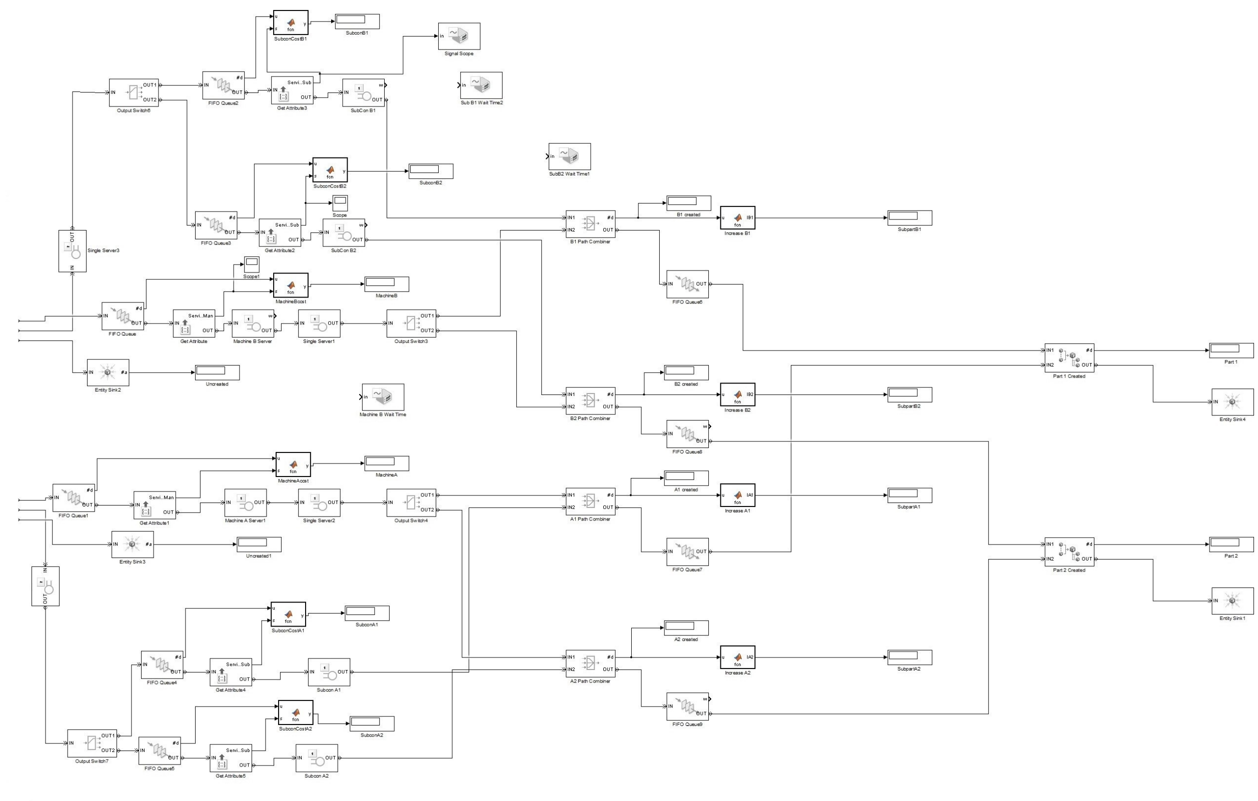

Figure 3.7: *Model After Switching*

We present the steps followed through the simulation:

- Set the maximum number of iterations for the simulation and simulation time till the system reaches steady state.
- Run each simulation with given inventory thresholds.
- Use inventory thresholds to decide whether the components need to be produced by in-house manufacturer or subcontractor.
- Calculate the total cost that comprises of costs for all servers, inventory costs, and lost sales costs.
- Update the inventory thresholds and run the simulation again.
- analyze results from multiple simulation runs to further narrow down the selection of inventory thresholds.

We run each simulation for sufficient time by which we can observe steady state. In each simulation run, we update the thresholds value and calculate the total cost. The total cost includes the rate of mean inventory costs, lost of sale costs and cost of service of the components throughout the simulation.

Challenges with Simulation

We faced the following challenges with the simulation model.

- Given the amount of time one simulation run takes, it is impossible to run all combination of inventory thresholds. We deal with this limitation, by using a greedy heuristics where we analyze few hundreds runs of simulations and analyze the pattern in costs and thresholds. This helps to set the range of random threshold values in the next runs.
- Capturing custom results from the simulation is difficult.

- Randomized policy does not yield any good results and it requires lot of computation power to even analyze 1% of the samples.

Chapter 4

Impact of System Parameters on Production and Subcontracting Decisions

In this chapter, we analyze the impact of system parameters such as: manufacturing and subcontracting costs, service rates, distribution assumption on the production and subcontracting decisions in a multi-product ATO system. For our system we analyze three case studies: (1) Manufacturer is cheaper, (2) Subcontractor is cheaper, (3) Exponentially shifted service time.

Case Study 1: Manufacturer is cheaper

With high demands and the internal capacity undergoing heavy utilization, the supply chain has an option to outsource the components to the external subcontractor. This allows the supply chain manager to share the load externally and supply the products to the customers with shorter lead times. However, outsourcing these components to the subcontractor incurs additional costs. For example at a car manufacturing site, the in-house manufacturing facility has the capability to manufacture specific bumpers. When the demand is high, the supply chain manager faces capacity issues, subcontracting of these bumpers are considered.

With the product having more specifications, the subcontractor could charge higher and the operation time may be extended.

Case Study 2: Subcontractor is cheaper

We model the system in this case when manufacturing a product at the internal facility may cost more than normal. This is common in manufacturing lines where production of a specific component may require more resources and cost. In this case the subcontractor can provide the item at a lower cost but it may incur additional lead times. If the subcontractor's cost of product is cheaper but due to high lead times, back-ordering and loss of sales cost can occur. To counter these problems and establish a steady flow at the assemble system, the supply chain manager needs to establish balance between the subcontracting and manufacturing in-house capacity to satisfy customers.

Case Study 3: Shifted exponential service time

In many cases when an order is placed for manufacturing there is a delay before the actual manufacturing process starts. For example machine setup time, specifications of the product, sending in documentation, work order confirmation and payments. In this scenario, the supply chain manager accounts for the constant delays and utilizes the output to streamline the supply chain. To develop this problem we modify our system by adding a constant delay to our service time, at both subcontractor and in-house facility. This system is modelled and analyzed for both of the above systems where the manufacturer is cheaper and also where the subcontractor is cheaper.

Note: The best results

4.1 Experiments 1: Manufacturer is Cheaper

For Experiment 1, we assume that the manufacturer is cheaper than the external subcontractor. We also consider that the component $C_{1j}, j = 1, 2$ are expensive to manufacture than

Table 4.1: *System Parameters for Experiment 1*

Subcontractor's Parameter		**Manufacturer's Parameters**	
$c_{s,1j}, j = 1,2$	35	$c_{m,1j}, j = 1,2$	15
$c_{s,1j}, j = 1,2$	25	$c_{m,2j}, j = 1,2$	10
$\mu_{s,i1}, i = 1,2$	1	$\mu_{m,i1}, i = 1,2$	2
$\mu_{s,i2}, i = 1,2$	1.5	$\mu_{m,i2}, i = 1,2$	3
System Parameters		**Additional Costs**	
B_{max}	9	$ls_i, i = 1,2$	80
$\lambda_i, i = 1,2$	1.5	$h_{ij}, i = 1,2$	2

the component $C_{2j}, j = 1,2$ at both the subcontractor and in-house facility. The in-house facility has two production lines and each is shared across two components $C_{i1}, i = 1,2$ and $C_{i2}, i = 1,2$. The manufacturer is twice as faster than the subcontractor ($\mu_{m,1j} = 2\mu_{s,1j}$). The system parameter are as follows in Table 4.1:

Using the above mentioned system parameters, we run our simulation model for 300 scenarios. Each scenario gives us the result for the total cost associated with the randomly assigned threshold values. After running the simulation for 650 simulation hours and 10 replications, these values for each iteration are sorted according to the minimum cost with a 95% CI. The top ten percent of the data which yields the minimum cost is analyzed to direct us towards optimality. Our result for the first experiment is as follows in Table 4.2:

Table 4.2: *Threshold Values for Experiment 1*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
97.59 ± 8.28	9	9	2	9
107.61 ± 6.8	9	9	9	2
111.36 ± 7.23	9	8	9	8
117.32 ± 7.46	9	7	7	8
123.84 ± 6.94	9	2	5	8

Once, we observe the threshold values for our best solutions, we perform statistical analysis and further limit the threshold values to minimize computation efforts. For instance, if the top results show that the thresholds between 0 and 5 yields lowest costs then we limit the random threshold generation for subsequent simulation runs to range $0-5$. The distribution of the threshold values for the lowest cost is displayed in Fig 4.1

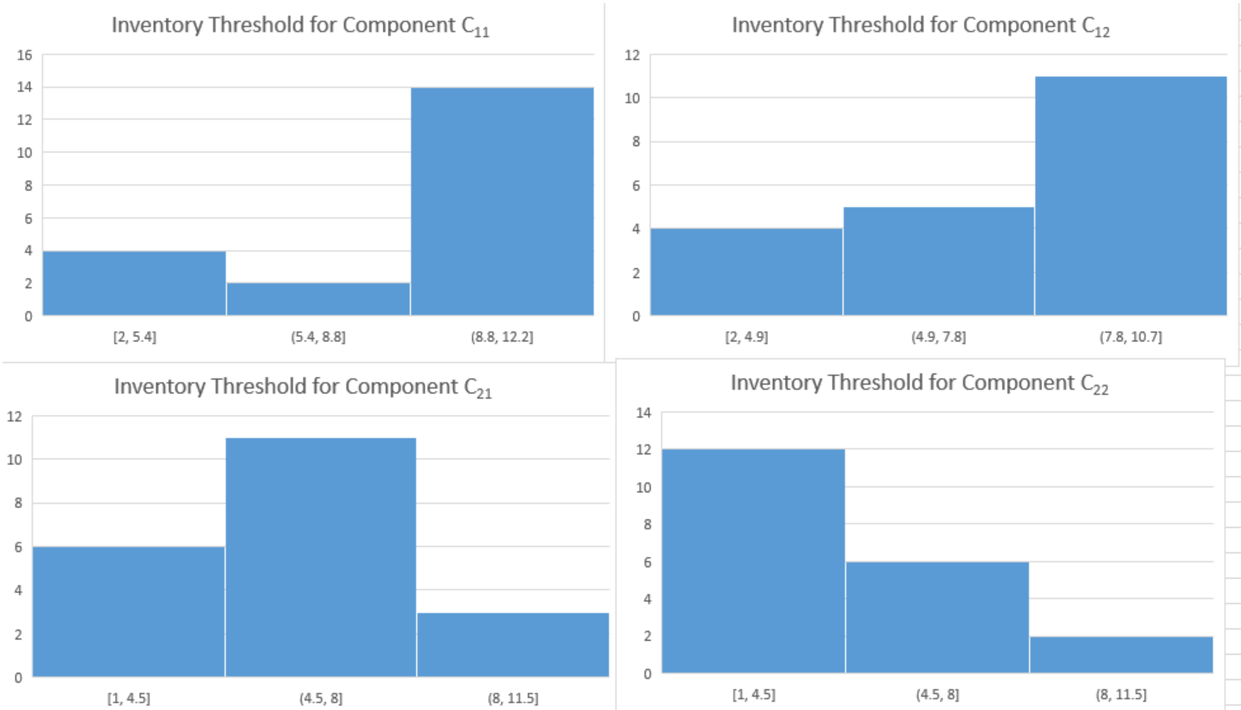

Figure 4.1: *Distribution of Threshold Values for Experiment 1*

These graphs indicate that the general trend towards optimality is having the threshold values for C_{11} and C_{12} be higher. By constructing the general trend we now limit the threshold values to be higher and run the simulation for multiple iterations again. We limit the threshold values for C_{11} and C_{12} to be random between 5-9 and run the simulation again for 650 simulation hours with 300 scenarios. After 10 replications of the best results with a CI of 95% the data is as given in Table 4.3 below:

Table 4.3: *Limited Threshold Values for Experiment 1*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
108.62 ± 10.21	9	5	9	8
111.72 ± 6.76	9	7	9	9
111.98 ± 8.67	9	6	9	9
116.32 ± 8.27	9	9	7	1
118.43 ± 13.7	9	9	6	2

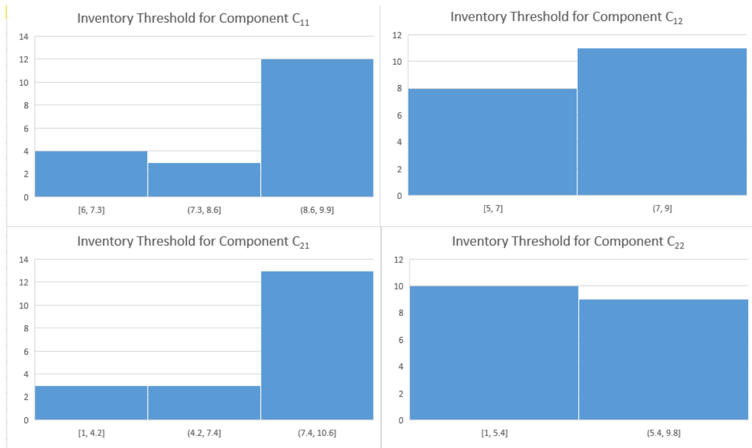

Figure 4.2: *Distribution of Limiting Threshold Values for Experiment 1*

The resulted data in Fig 4.2 suggests that trend towards the optimality may require the cut-off values to be higher. We perform the simulation run again with the same simulation hours and scenarios but now limiting C_{ij} i=1,2 and$j$ = 1 to be between 5 and 9. The accumulated best data is replaced and with a CI of 95% the data is as follows:

Table 4.4: *Extending Limiting Threshold Values for Experiment 1*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
94.51 ± 6.27	9	9	8	9
101.65 ± 7.98	9	9	6	8
108.99 ± 6.61	9	9	9	3
108.74 ± 8.43	9	8	9	8

The above given Table 4.4 corroborates our direction towards optimality; as our threshold value increases the total cost of the system is reduced. After acquiring this data we again perform distribution analysis of the threshold values (see Fig 4.3)

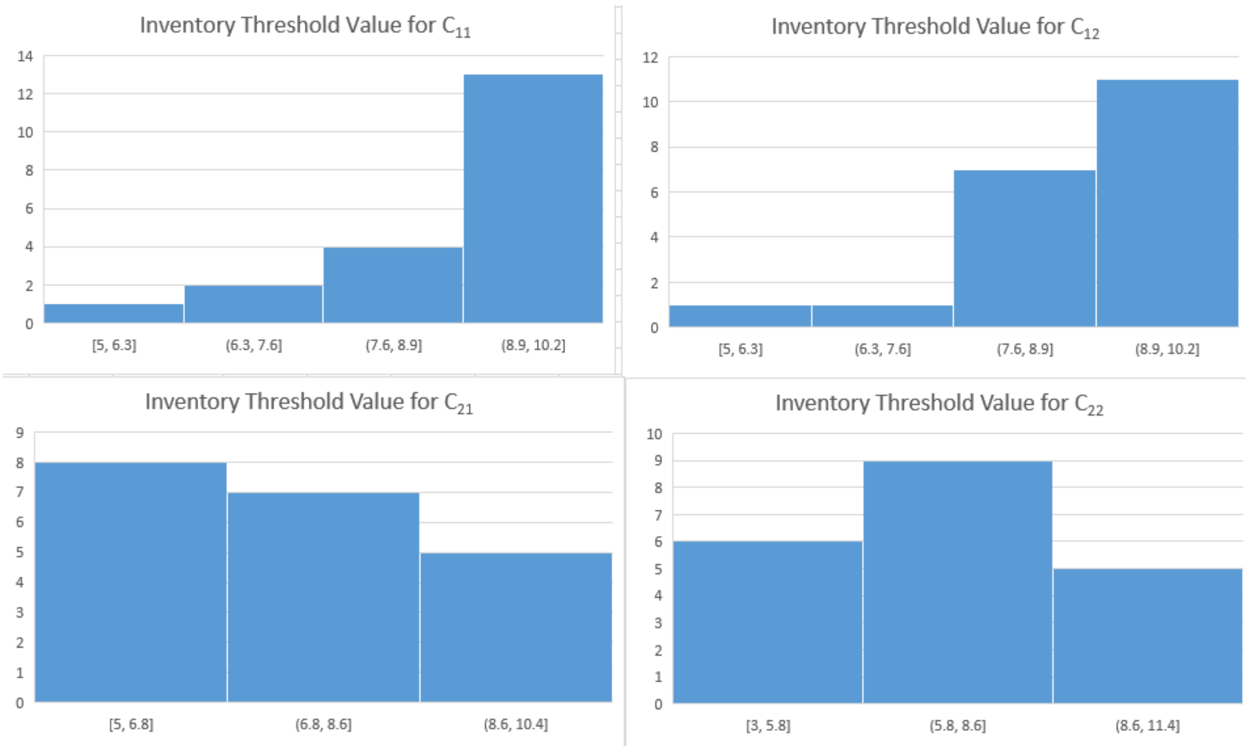

Figure 4.3: *Distribution of Extended Limiting Threshold Values for Experiment 1*

This approximate analysis by simulating multiple scenarios and by applying Monte Carlo simulation techniques exhibits that restricting the threshold values of C_{ij} towards a higher number enables us to direct towards the optimal solution.

4.2 Experiments 2: Subcontractor is Cheaper

For Experiment 2, we assume that the subcontractor is cheaper than the internal manufacturer. We also consider that the component $C_{1j}, j = 1, 2$ is expensive to manufacturer than the component $C_{2j}, j = 1, 2$ at both the subcontractor and in-house facility. The in-house facility has two production lines and each is shared across two components $C_{i1}, i = 1, 2$ and $C_{i2}, i = 1, 2$. The manufacturer is twice as faster than the subcontractor ($\mu_{m,1j} = 2\mu_{s,1j}$). The system parameter are as follows in Table 4.5

Table 4.5: *System Parameters for Experiment 2*

Subcontractor's Parameter		Manufacturer's Parameters	
$c_{s,1j}, j = 1, 2$	15	$c_{m,1j}, j = 1, 2$	35
$c_{s,1j}, j = 1, 2$	10	$c_{m,2j}, j = 1, 2$	25
$\mu_{s,i1}, i = 1, 2$	1	$\mu_{m,i1}, i = 1, 2$	2
$\mu_{s,i2}, i = 1, 2$	1.5	$\mu_{m,i2}, i = 1, 2$	3
System Parameters		**Additional Costs**	
B_{max}	9	$ls_i, i = 1, 2$	80
$\lambda_i, i = 1, 2$	1.5	$h_{ij}, i = 1, 2$	2

This setting indicates that although the subcontractor requires twice as much time to produce the components, the cost is lower at the subcontractor. All the other parameters are the same as experiment 1. The simulation hours for this experiment at 300 and we conduct 300 different scenarios.

After 10 replications of the best data, the result with a 95% CI is as follows (see Table 4.6):

Table 4.6: *Threshold Values for Experiment 2*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
98.70 ± 2.12	2	2	1	2
100.83 ± 4.38	3	1	1	1
100.62 ± 3.77	4	2	3	4
101.12 ± 3.43	4	2	1	5
101.51 ± 2.33	1	2	2	2

The data suggests that the trend towards optimality is by keeping lower threshold values for C_{ij}. We construct a histogram (Fig 4.4) for the threshold value distribution to enable us to better predict trend direction of optimality.

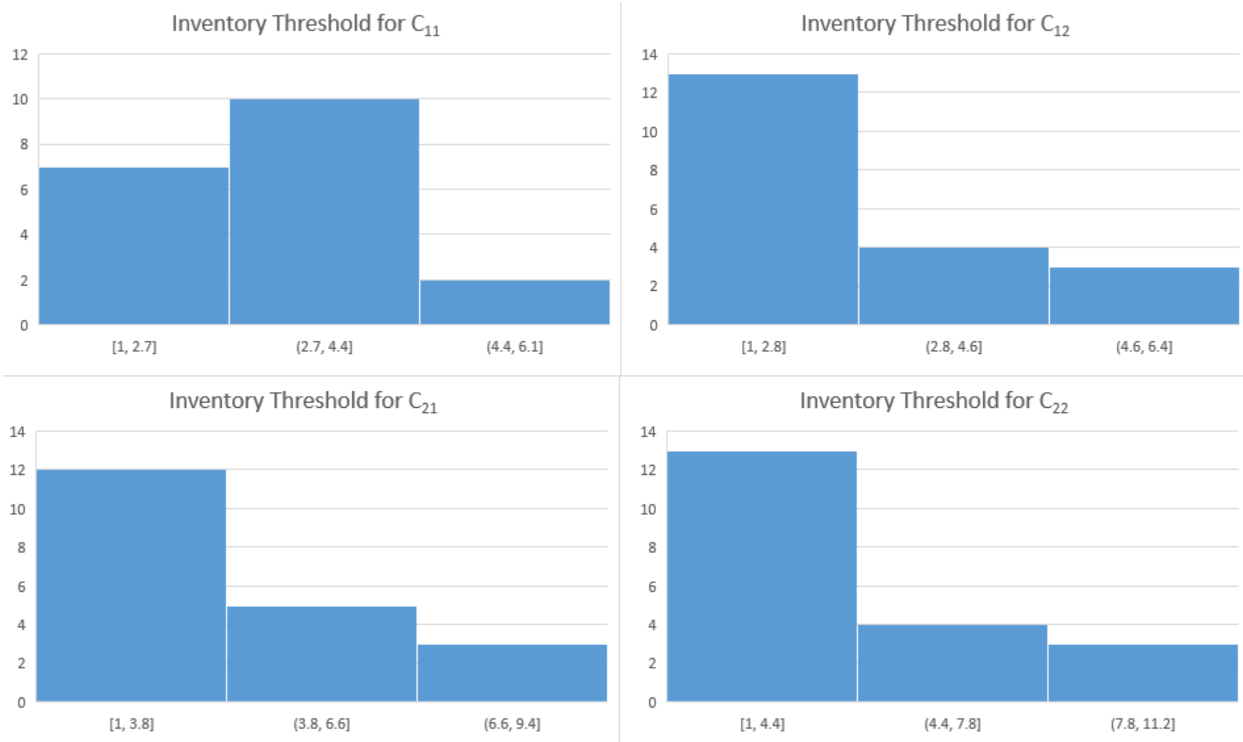

Figure 4.4: *Distribution of Threshold Values for Experiment 2*

This approximate analysis by simulating multiple scenarios and by applying Monte Carlo simulation techniques exhibits that restricting the threshold values of C_{ij} towards a lower number enables us to direct towards the optimal solution.

4.3 Experiments 3: Shifted Exponential Service Times

In our last experiment we model two cases. The system parameter settings for each of these cases is identical to both the above mentioned experiments, however the service time for each of the servers i.e in-house and subcontractor, $\mu_{s,ij}$ *and* $\mu_{m,ij}$ where $i, j = 1, 2$ have a constant delay denoted by d is added.

We split our experiment 3 in to two parts; 3a and 3b, where 3a represents the system where the manufacturer is cheaper and all severs have a constant d, 3b represents the system where the subcontractor is cheaper and all servers have a constant delay d.

4.3.1 Experiment 3a: Manufacturer is Cheaper

This experiments parameter settings are identical to the experiment 1, however a constant delay is added to all the service time $\mu_{k,i,j}$ where $k = s, m$ and $i, j = 1, 2$.

The system parameter for this experiment are given as follows Table 4.7:

Table 4.7: *System Parameters for Experiment 3a*

Subcontractor's Parameter		**Manufacturer's Parameters**	
$c_{s,1j}, j = 1, 2$	35	$c_{m,1j}, j = 1, 2$	15
$c_{s,1j}, j = 1, 2$	25	$c_{m,2j}, j = 1, 2$	10
$\mu_{s,i1} + d, i = 1, 2$	1+10%	$\mu_{m,i1} + d, i = 1, 2$	2+10%
$\mu_{s,i2} + d, i = 1, 2$	1.5 +10%	$\mu_{m,i2} + d, i = 1, 2$	3 +10%
System Parameters		**Additional Costs**	
B_{max}	9	$ls_i, i = 1, 2$	80
$\lambda_i, i = 1, 2$	1.5	$h_{ij}, i = 1, 2$	2

After running the simulation for 300 simulation hours and 100 scenarios the best data is replicated 10 times, the results with 95% CI are as follows as follows:

Table 4.8: *Threshold Values for Experiment 3a*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
105.32 ± 13.3	2	2	8	9
96.76 ± 10.67	8	4	8	8
103.19 ± 12.74	3	9	8	8
106.77 ± 7.28	5	9	7	8
107.84 ± 10.45	1	1	6	8

To better predict the general trend of optimality we construct a distribution histogram.

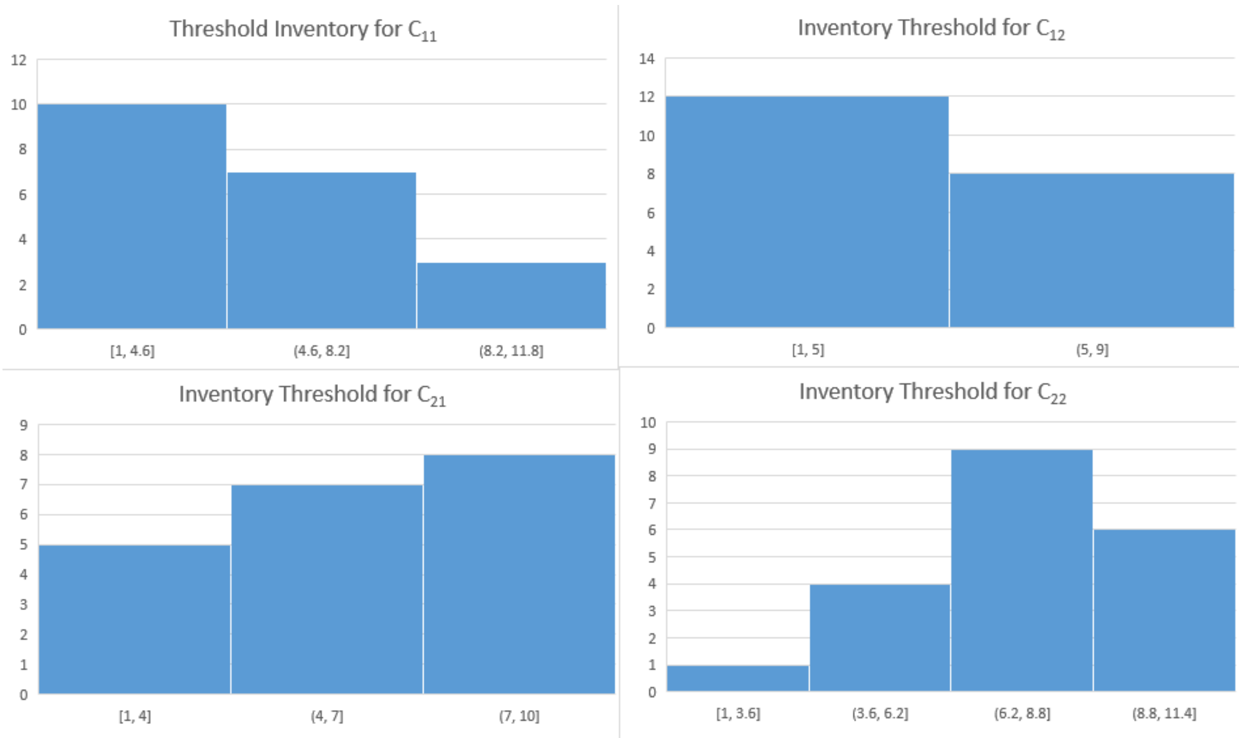

Figure 4.5: *Distribution of Threshold Values for Experiment 3a*

Reducing the randomness by adding a constant delay our experimental data differs from the data in experiment 1. The coefficient of variation in in this experiment is ¡1. The approximate analysis by simulating multiple iterations and by applying Monte Carlo simulation techniques the impact of the delay exhibits that restricting the threshold values of C_{11} and C_{12} towards a lower cut-off number and C_{21} and C_{22} towards a higher cut-off number rather than how we had a higher threshold value for routing between subcontractor and manufacturer in experiment 1.

4.3.2 Experiment 3b: Subcontractor is Cheaper

This experiments parameter settings are identical to the experiment 2, however a constant delay is added to all the service time $\mu_{k,i,j}$ where $k = s, m$ and $i, j = 1, 2$. The system parameters are presented in Table 4.9.

Table 4.9: *System Parameters for Experiment 3b*

Subcontractor's Parameter		**Manufacturer's Parameters**	
$c_{s,1j}, j = 1, 2$	15	$c_{m,1j}, j = 1, 2$	35
$c_{s,1j}, j = 1, 2$	10	$c_{m,2j}, j = 1, 2$	25
$\mu_{s,i1} + d, i = 1, 2$	1+10%	$\mu_{m,i1} + d, i = 1, 2$	2+10%
$\mu_{s,i2} + d, i = 1, 2$	1.5+10%	$\mu_{m,i2} + d, i = 1, 2$	3+10%
System Parameters		**Additional Costs**	
B_{max}	9	$ls_i, i = 1, 2$	80
$\lambda_i, i = 1, 2$	1.5	$h_{ij}, i = 1, 2$	2

Using similar simulation run time and scenarios, the best results with 10 replications are presented with 95% CI in Table 4.10.

Table 4.10: *Threshold Values for Experiment 3b*

Total Cost	C_{11}	C_{12}	C_{21}	C_{22}
99.12 ± 6.33	2	2	8	9
98.53 ± 5.76	1	1	6	8
99.87 ± 6.16	2	2	1	8
100.93 ± 4.87	2	2	3	8
100.12 ± 7.87	4	1	8	1

The general trend towards optimality is again difficult to predict with the resulted data so we construct a distribution to see how does the system behave to the threshold values (see Figure 4.6).

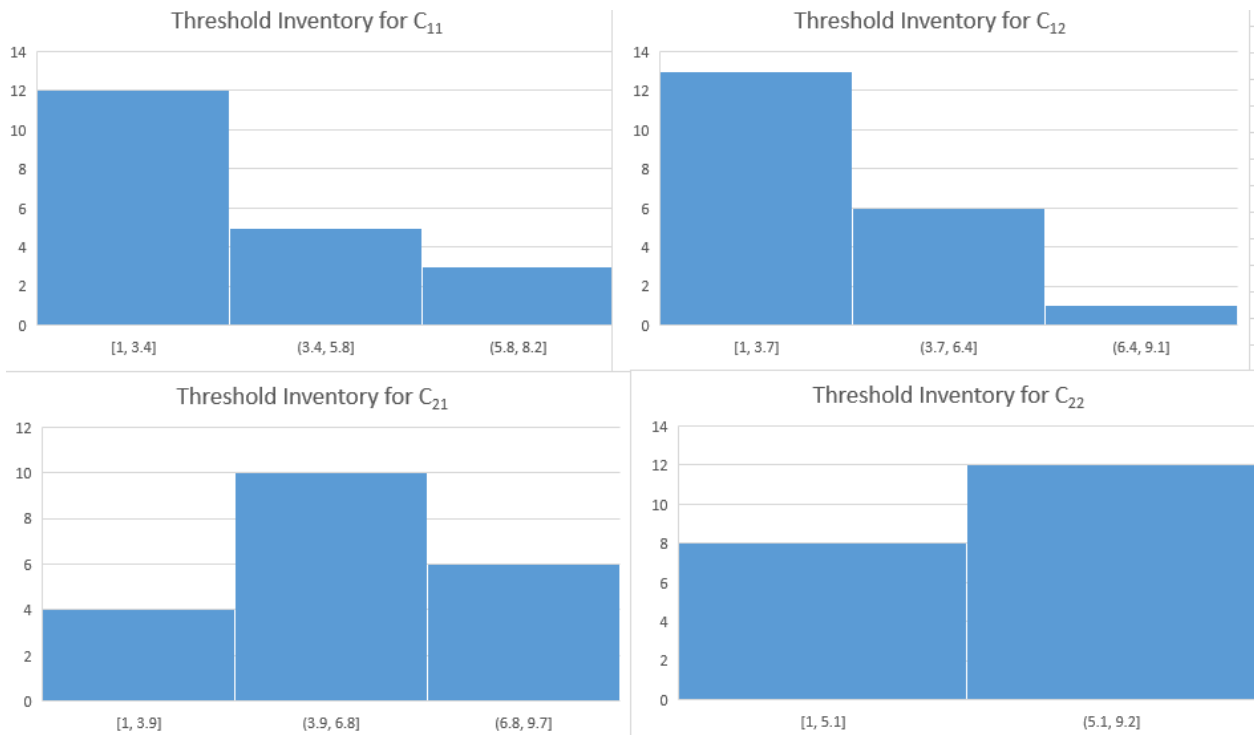

Figure 4.6: *Distribution of Threshold Values for Experiment 3b*

This approximate analysis by simulating multiple iterations and by applying Monte Carlo simulation techniques exhibit the trend to differ from experiment 2. In experiment 2 it was evident for better performance to have the threshold values lower, adding a delay decreases the randomness and increases randomness in the new threshold values as the coefficient of variation is 1. The data reveals by restricting the threshold values of C_{11} and C_{12} towards a low range, C_{21} in the mid range and C_{22} towards a higher cut-off number enables us to direct towards the optimal solution.

Chapter 5

Conclusion and Future Direction

We present an inventory control problem for this thesis. The motivation of this problem comes from the demand of complex and customized products increasing in toady's fast-paced environment. Within the highly competitive market era, manufacturers need to shorten lead times and costs to outperform their counterparts and grow in the environment. We highlight some common manufacturing strategies i.e. engineered-to-order (ETO), make-to-order (MTO), make-to-stock (MTS) and assemble-to-order (ATO) systems. As products become more complex, ATO manufacturing systems have become a commonly opted strategy for many manufacturers.

We consider a multi-product and multi-component system, where the components can either be manufactured at the in-house facility or by the subcontractor. The in-house facility is shared across multiple components, while the subcontractors are dedicated to each component. The shared resource is common in manufacturing systems where the manufacturer instead of setting up new manufacturing lines shares the resources to manufacturer different items.

To model our proposed problem, we first use Markov Decision Process (MDP) model and but faced issues with the curse of dimensionality. For our proposed system, the state space is directly proportional to the number of components and the action space is directly proportional to the number of components are the servers. For such complex problems, simulation based approaches have been found useful. We develop a simulation model using Matlab 2014 (Simulink) for our proposed system. The simulation model comes with several computational challenges and we show how by assigning random actions to each possible state space requires exceptional computational effort. Later, we propose a threshold based policy which is easy to use and more common in practice for routing the components to either in-house or subcontractor and determine near-optimal thresholds.

We analyze three scenarios i.e. 1. Manufacturer is cheaper, 2. Subcontractor is cheaper and 3. Shifted exponential service times. We use multiple iterations and Monte Carlo techniques to narrow down optimal decision path by limiting the threshold values. The results for the first experiment indicate that the direction towards optimality is by having the threshold values kept close to the base stock level for all components. However maintaining high inventory levels increases the holding cost but reduces lost sales costs and production costs. The next scenario, i.e. 2. Subcontractor is cheaper. The total cost of the system for identical simulation time as experiment 1, results in cheaper production costs. The threshold values are towards the lower end suggesting that components should be subcontracted earlier to reduce the total production costs.

For experiments 3a and 3b, we add a constant delay by having a shifted exponential time for the service times. This is more common in practice. We see that by restricting the threshold values of components C_{11} and C_{12} tend towards a lower cut-off inventory value while components C_{21} and C_{22} tend towards a higher cut-off inventory value. This result differs from out initial experiment without the delay. While for experiment 3b the trend towards optimality is executed by limiting the threshold values of C_{11} and C_{12} towards a low, C_{21} in the mid range and C_{22} towards a higher cut-off number enables us to direct towards the optimal solution. The variation of the threshold values in the last experiments can be explained by the decrease in randomness and coefficient of variation being < 1.

The comparison of the best threshold values for all the experiments is given below (see Figure 5.1:

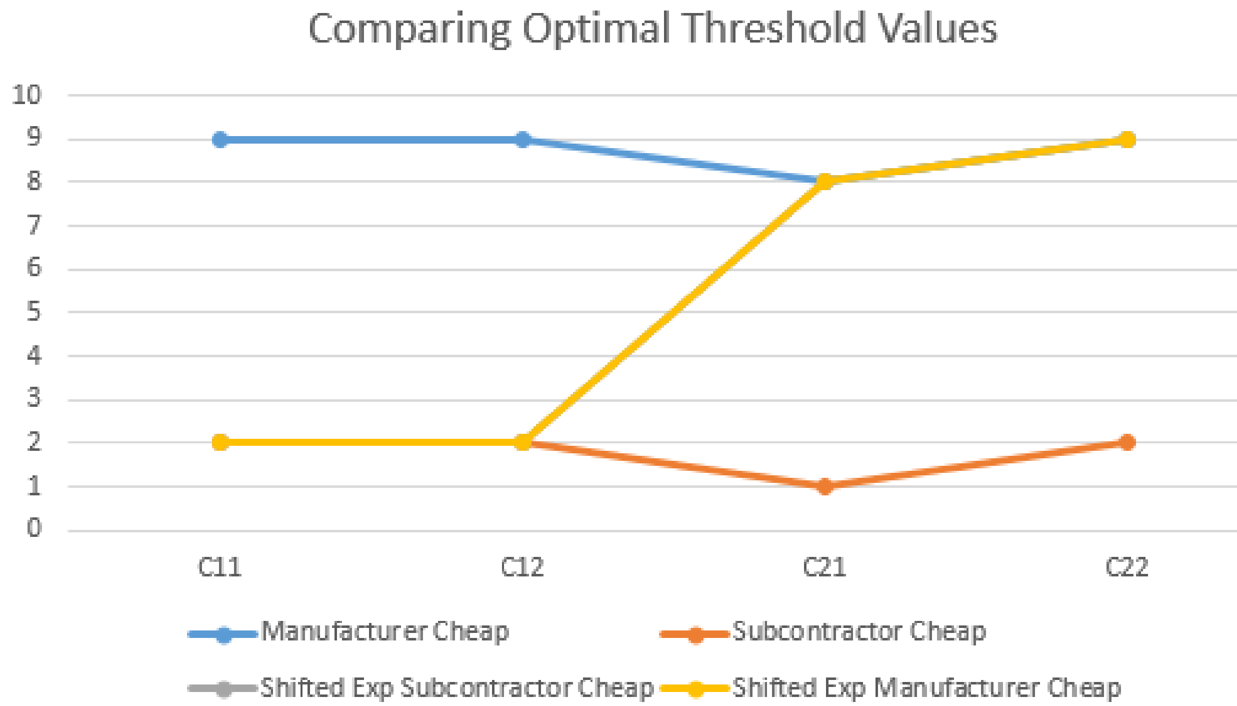

Figure 5.1: *Comparison of Optimal Threshold Values*

Our research gives us insights on how to determine optimal inventory and control policies for supply chain of complex manufacturing systems by simulation. Most manufacturing systems are highly complicated and MDP model formulation becomes extensively exhausting. Simulation of these complex systems for random scenarios and then by restricting the model towards optimal trends, progresses us towards optimality. Note that we are not making any claims that the proposed simulation model gives optimal solution but the insights from our model gives useful insights to supply chain managers in ATO systems.

Future work may include analyzing more products and components. We would also like to analyze other replenishment policies such as lattice based policy and compare with the threshold based policy.

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