TRANSIENT SOLUTIONS OF M/M/s NONSTEADY STATE QUEUEING SYSTEM

by

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B.S. (INDUSTRIAL ENGINEERING)

Chung Yuan Christian College of Science & Engineering

Chung-Li, Taiwan, Rep. of China, 1974

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

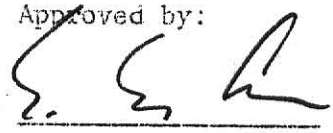
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ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to his major advisor, Dr. E. S. Lee, for his continuous guidance and constant encouragement during the preparation of this thesis.

He also wishes to thank Dr. C. A. Bennett and Dr. Chi-lung Huang for serving on his committee.

He would like to dedicate this work to his parents for their continuous encouragement and assistance.

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CHAPTER 1

INTRODUCTION

Queues, or waiting lines, are very common phenomena, widely encountered in every day life. People wait in lines at supermarket checkout stands, in their cars in busy traffic hours, in banks, at gas stations, at ticket booths and almost everywhere. Wherever the demand for service exceeds the capacity of servers, a queue is formed.

Since waiting is not a pleasant activity, neither customers nor management like to wait. Because long waiting time may cause losses, queueing problems have received the attention of management and operation analysts for a number of years. In fact, queueing models were among the first models of operation research [36]. The problems have been actively researched and written about since 1907 [13].

A basic queueing process can be described as customers leaving the calling population, arriving for service, entering the queueing system, waiting in a queue if no server is available, and leaving the system after being served. This process is depicted in Figure 1.1.

A queueing system can be specified completely in most cases by six basic characteristics [10] which are: arrival pattern (distribution) of customers, service pattern (distribution) of servers, number of service channels, service discipline, system capacity, and size of population. With different basic characteristics of queueing processes, selected queueing models can be developed. Most of these models, which assume that the mean arrival rate and the mean service rate are constant through time and that the system is in a steady state, have been well studied in the past.

Unfortunately this is often not the case in real queueing systems. For instance, service rate has a close relationship with the length of queue (it is quite likely that a server will tend to work faster if the queue is long

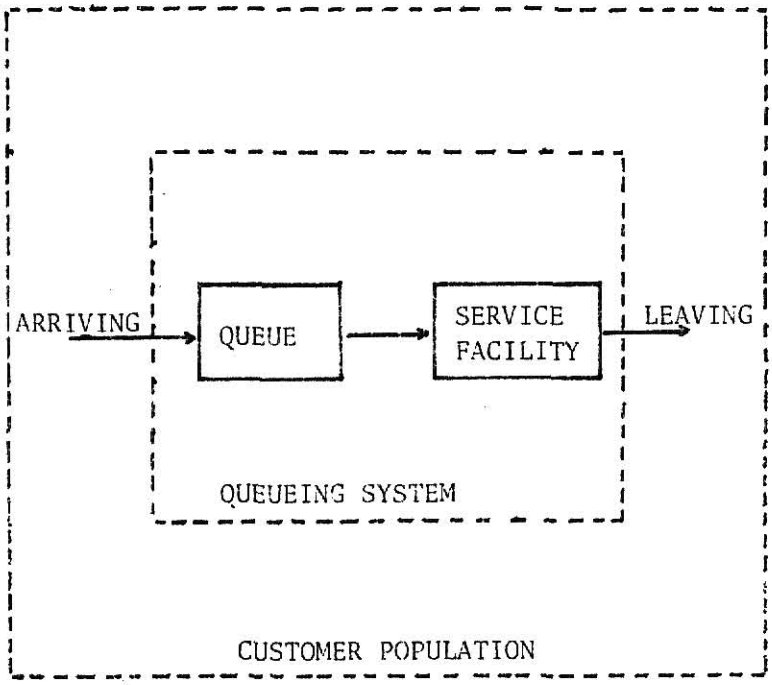


Figure 1.1 Basic queueing processes.

and vice-versa (such as supermarket checkout clerk); customer arrivals change with time of day in supermarkets; and car arrival rates at tool booths peak at the busy periods around 8 A.M. and 5 P.M.

Usually any system is said to be in a transient state when the system's behavior is not typical or not normal. For example, after a car has started, it has to be accelerated to reach the normal speed. During the accelerating period, the system is in a steady state. Transient states occur in queueing systems, especially when the system is still dependent on the initial conditions, or when a steady-state condition responds to sudden changes. When the arrival rate is larger than the capacity of service, the steady state condition never exists no matter how long the system is run. Another important situation is when the arrival rates and service rates are a function of time; in this case, only the transient state has meaning.

As mentioned before, most of the attention in queueing theory analysis has been directed to steady state systems. Although the steady state queueing models have proven very useful in many areas, it is inappropriate to apply these models to situations in which time dependent behavior is very strong. Wagner [41] has listed grocery store checkout stands, bank teller's windows, service station pumps and attendants, telephone trunk lines, and photocopy equipment maintenance men, as cases where the customer's arrival rates have very significant time-dependent behavior. Koopman [16] and Hartman [11] have discussed the time dependent air terminal queueing problem. Figure 1.2 shows typical hourly aircraft arrival rates for J. F. Kennedy and Laguardia airports, as observed by Koopman in 1968 during a month's operations. The average hourly arrival rates are plotted against time of day. Both curves peak around 10 A.M. and 6 P.M. The first peak shows that many incoming aircrafts are scheduled to leave around 10 A.M. for the convenience of the passengers. The second peak shows the arrivals of aircrafts which have been scheduled to arrive in the

Terminal A: J. F. Kennedy data

Terminal B: LaGuardia data

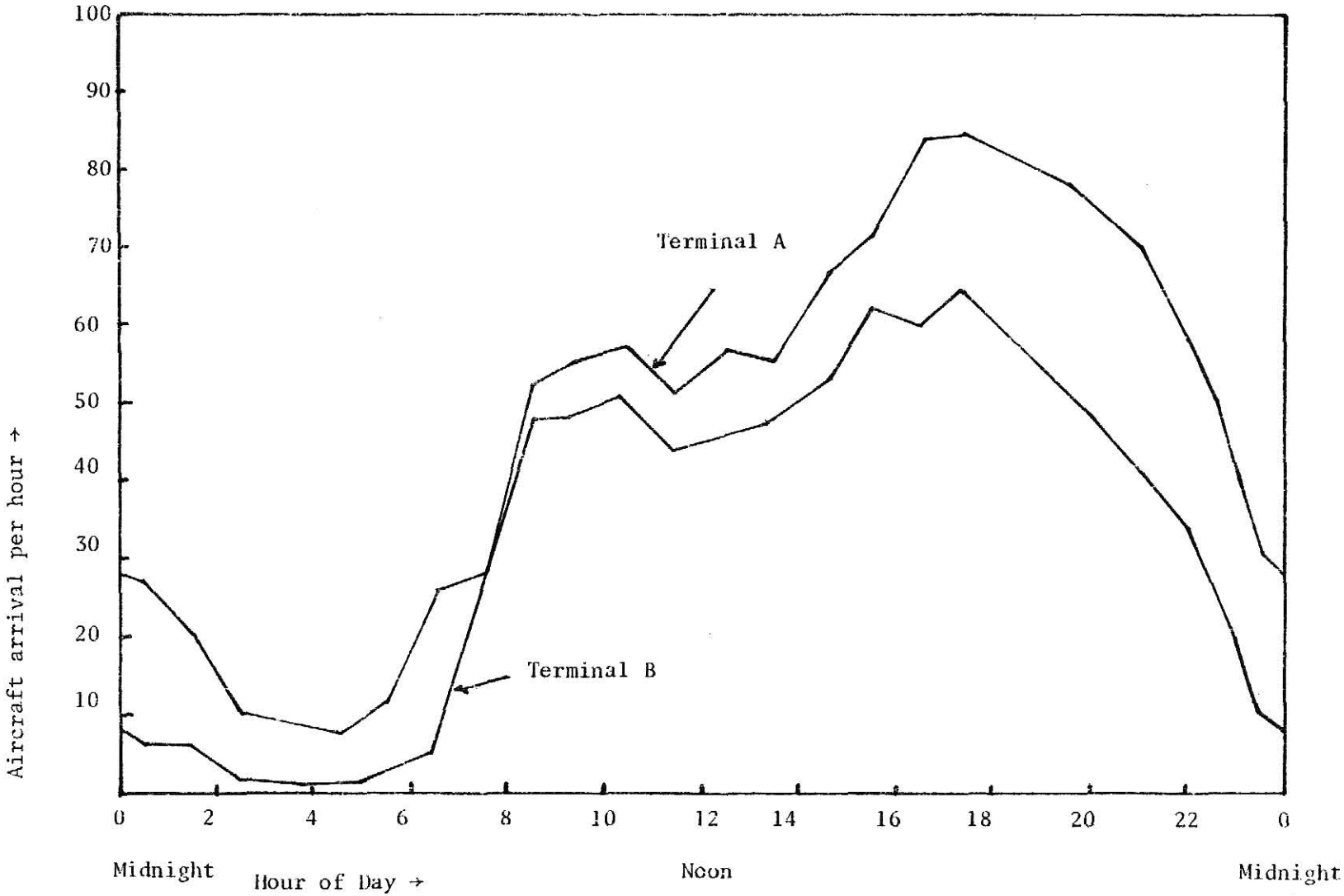


Fig. 1.2 Hourly arrival rates for air terminals A and B.

neighborhood of 6 P.M.

It is very common to use the simulation approach (to interpret queueing problems in transient state) examplified by the well known paper of Galliher and Wheeler [6]. Although the simulation technique possesses versatility and flexibility, it is experimental in nature. This will give rise to such problems as experimental design, run length, number of replications, gathering observations and statistical significance. On the other hand, for most queueing problems the information for making a decision requires only a few simple measurements of effectiveness, such as average waiting time, average queue length, the probability of encountering a queue, etc. Another drawback to simulation is the economical consideration. The effort required for constructing a simulation model and the required computer time can be substantial (roughly stated, the machine time for simulation increases directly proportional to the square of the reliable precision [16]).

In recent years, a good deal of effort has been devoted to the development and application of numerical methods to the transient state queueing problems.

In this study, one of the main objectives is to survey the literature of numerical or approximate approaches. Chapter 2 summarizes the result of a limited study on nonsteady state queues.

Another objective is to demonstrate how to apply one of the simpler numerical techniques in solving real queueing problems. A police patrol car scheduling problem is used in this study. It is first defined in Chapter 3. Chapter 4 presents the solution, with the probability of an urgent call encountering a queue used as a criterion. Chapter 5 presents another solution in which the expected waiting time of an urgent call in the queue is used as a criterion.

Finally, conclusions and recommendations are given in Chapter 6.

CHAPTER 2

NONSTEADY STATE QUEUES

In reviewing the literature, it is convenient to classify the information into the following categories:

1. Explicit expression of transient solutions for some simple stationary queueing systems.
2. Numerical approaches to solve the differential equations of queueing systems directly.
3. Diffusion approximation techniques to approximate the transient solutions of heavy-traffic queueing systems.
4. Discrete time Markov process to approximate the smaller scale service systems.
5. Approximate transform inversion
6. Imbedded Markov chain approach
7. Closure techniques

This is not a exhausive survey; only the references having direct bearing on the current work are included.

Explicit Expression

The procedure for developing the explicit transient solutions for simple stationary queueing systems is very complicated and highly mathematical. Many authors have investigated this subject, including the first known work published by Lederman and Reuter [19]. Additional papers have been presented by *Bailey* [1] and Morse [26] and successive contributions given by Champernowne [2] and Cox and Smith [5]. The explicit formula for the probability $P_n(t)$ that the system is in state n at time t (i.e. the transient solution) of stationary $M/M/1/\infty$ queueing systems is given as:

$$P_n(t) = e^{-(\lambda+\mu)t}[(\mu/\lambda)^{(i-n)/2} \text{ In-i } (2\sqrt{\lambda\mu}\, t)$$
$$+ (\mu/\lambda)^{(i-n+1)/2} I_{n+i+1}(2\sqrt{\mu\lambda}\, t)$$
$$+ (1-\mu/\lambda)(\lambda/\mu)^n \sum_{\ell=n+i+2}^{\infty} (\mu/\lambda)^{\ell/2} I \quad (2\sqrt{\lambda\mu}\, t)]$$

where $I_n(x)$ is the Bessel function of the second kind defined by

$$I_n(x) = \sum_{k=o}^{\infty} [(x/2)^{n+2k}/k!\ (n+k)!], \qquad n > -1$$

where the constant i is the number of units in the system when time is equal to zero. The details of the derivation can be found in the original discussion by Bailey [1] and a clear explanation is given by Gross and Harris [10]. As one can see, even for this simple stationary queueing systems, the formula for the transient solution is very complicated and the result is not very suitable for numerical work. This discourages further exploration of explicit expressions for more complex queueing systems. With the help of modern digital computers, a number of numerical approaches and approximate methods for determining more complicated transient queues have been contributed.

Numerical Approach

The M/M/1 nonsteady state queue has been studied by Clarke [4]. He used a generating function to reduce the difference-differential equation of probabilities to a hyperbolic partial differential equation, which then was solved by applying standard methods and intergration formula for Bessel functions in terms of a Volterra type integral equation. Luchak [22] used Taylor's expansion technique to approximate the solutions of some queueing systems, except that he kept mean service time constant and made arrival rate dependent on time. Leese and Boyd [20] pointed out the difficulties of applying Clark's and Luchak's methods and compared eight methods in terms of the capability of producing satisfactory numerical results with a reasonable amount of computational effort. Among those considered by Leese & Boyd, Wragg's [41] method seemed to be the most powerful. The first method mentioned in Leese and Boyd's paper was the standard numerical method [34], such as the Runge Kutta method, Predictor-Corrector method, etc. to solve the Kolmogorov infinite set of difference-differential equations of state probabilities. Because of its simplicity and ease of use, this direct numerical method has been

applied to integrate a truncated set of probability equations by Koopman [16] and Hartman [11] in the time-varying air-terminal queueing problem, by Ghare et al [9] in job shop problem, and by Kolesar et al [14] in police patrol car scheduling problem. The assumption of truncated systems seems to be reasonable in most applications when there are physical constraints on maximum possible number of arrivals or the probabilities of exceeding a certain amount in the system are very small.

Diffusion Approximation

In the busy period, the arrival rate $\lambda(t)$ gradually increases as a "rush hour" is approaching. The queue builds up during the period when arrival rate exceeds service capacity. After a busy period, the queue finally decays. The studies of this type of situations have been given by Oliver and Samuel [33], Gazis and Potts [8], and May and Keller [23]. The system was analyzed by means of a deterministic model which assumed arrival rate and service time to be constant, instead of describing them in probability distributions.

Newell [30] [31] [32] has presented a method called *diffusion approximation* to improve the deterministic approximation which always tends to underestimate the expected queue length. Newell's method has been applied to solve the heavy traffic queueing problems or the mildly heavy traffic queueing problems.

Discrete Time Markov Process

An approach to the small-scale service system whose short range behavior is important has been described by Neuts [27]. The transient behavior of a substantial class of single server queues may be numerically approximated by such discrete time Markov process as traffic lights, highway merging ramps and many others. Neuts [28-29] has extended the applicability of this process to longer queues and longer time periods. The extension is spohisticated and mathematically involved.

Approximate Transform Inversion

The complicated expression of $P_n(t)$ for M/M/1/∞ queueing systems can be obtained by inversing the complex roots of a certain polynomial equation [4]. Gaver [7] has presented a method which used an approximation technique instead of sophisticated transform inversion. Recently Kotiah [17] has suggested three iterative procedures (method of successive approximations, the series method, and Newton's Method) that give successive rational approximations for complex roots. The rational approximations of complex roots are relatively easy to invert.

Imbedded Markov Chain Approach

With the assumption that arrival and service process parameters are step functions of time, the time horizon is divided into several intervals, during each of which the arrival process is in steady state. Morse [25] has applied the imbedded Markov chain technique to approximate the expected queue length and length variance of nonsteady state single-server queues.

Closure Technique

A different approach for approximating a measurement of effectiveness has been known as closure technique. The infinite set of difference-differential euqations of state probabilities is inverted to a set of limited equations for certain measurements of effectiveness. With a closure assumption, the measurement can be approximated. Rider [35] has used this kind of technique to approximate the time dependent mean length of a nonsteady state M/M/1 queue in terms of the idle probability. Recently Rothkopf and Oren [36] used Clarke's [4] differential equations for the expected length and length variance of a nonsteady state M/M/1 queue. With a closure assumption which employeed the negative binomial distribution to approximate the state probabilities, the mean and variance of queue length were approximated. Adding

a correction term, they extended the applicability to multi-server queues. At the same period of Rothkopf and Oren's work, Chang [3] and his student Wang [40] used different closure assumption to obtain the approximation of expected queue length and length variance of the same systems.

Considering the aspects of simplicity, accuracy and range of applicability, it seems that the standard numerical method is most desirable. Thus it is selected for further study in this research.

CHAPTER 3

PATROL CAR SCHEDULING PROBLEM

An important function of police departments is to provide citizens with rapid and competent emergency service when needed. Indeed, lives and properties are often dependent on the effective performance of this police function.

The patrolling schedule problem is selected as a focus of the application of the numerical technique because the patrol force constitutes the largest portion of police personnel, usually consuming 40 to 50 percent of the total police budget (Larson [18], 1972).

3.1 Brief Description of Patrol Operation

One primary duty of a patrol unit is to prevent crime by removing crime hazards and to deter crime by posing the threat of apprehension, when the patrol unit passes through an assigned geographical response area.

Another primary duty of an unit is to respond to calls for police service. Citizens usually call the police emergency number. After the necessary information is taken down, a police dispatcher assigns or dispatches a patrol unit to the scene. Since rapid response to a call for help is considered to have higher priority than general patrolling, an assignment from the dispatching center interrupts the general preventive patrol.

A dispatcher has a fixed number of patrol units available for dispatching. This number of patrol units constitutes a precinct work force. Most precincts have a centrally located station. Each patrol unit can be assigned to any point within its precinct. If all units in a precinct are simultaneously dispatched, when a call arrives the dispatcher puts the request in queue for later dispatch. There are very few inter-precinct dispatches, and in this study each precinct is assumed to operate independently from the others.

3.2 The Concerns of Administrator

Since it is characteristic of emergency systems that a person's life or well-being may well depend on the immediate dispatch of a unit to a call, a primary objective of any patrol car system is to minimize the possibilities that a call will be placed in the queue, or to minimize the average waiting time once the call is placed in the queue. The administrator faces the problems of how to determine the number of units to be put in each shift and subsequently how to schedule the manpower within each shift, so that the probability of a call encountering a queue and/or the expected waiting time in the queue are insured to be below some specified threshold.

By almost any measurement of effectiveness, patrol performance is improved by increasing the number of patrol cars on duty. However, an administrator would like to optimize the patrol schedule so that he can obtain the most efficient solution within certian constraints.

3.3 Assumptions of the Model

The probabilistic nature of calls for service and service time is such that queueing is the most adaptable model to the patrol dispatching system. This system can best be described as a complicated multiple-server queueing system.

Calls for service, either telephone calls to the radio dispatcher or accidents, crimes, other incidents encountered by the patrol units, are assumed to occur randomly over time as a Poisson process. In other words, the probability that a given number of calls will occur in a given period of time is specified by a Poisson distribution with a mean characteristic of that specific period of time.

It is assumed that each call for service is handled by one single patrol car. In real situations two or more cars sometimes are necessary. A comparison made by Ignall et al [12] shows that the influence of this assumption can be neglected without seriously altering the results.

Service time is defined to be the total "off-air" time associated with a

call for service. This includes travel time to the scene, on-scene service time, and off-scene follow-up time spent on report writing, arrest or booking. The probability of a given amount of service time for a call is assumed to obey an exponential distribution with a constant parameter; mean service time.

A call is placed in queue only when all units are busy servicing prior calls, all calls have the same priority on a first-come first-served basis. During the service period the patrol car is unable for further assignments.

The methodology presented by Kolesar et al [14] is applied to demonstrate how to use the standard numerical method in solving the patrol car scheduling problems defined above.

Chapter 4 describes this problem based on the criterion of a specified probability of an urgent call encountering a queue and Chapter 5 provides the solution using as criterion a specified waiting time of an urgent call in the queue.

CHAPTER 4

THE PROBABILITY OF A CALL ENCOUNTERING A QUEUE AS A CRITERION

Kolesar et al's method [14] is presented in three steps:

Step 1: Calculate the hourly patrol car requirements to meet the specified criterion.

Step 2: Generate schedules for each shift which would satisfy the hourly patrol car requirements calculated in Step 1.

Step 3: Use a more realistic nonsteady state queueing model to evaluate the performance of schedules generated in Step 2 and compare with otner alternatives.

4.1 Step 1: Estimate the Hourly Patrol Car Requirements

The police departmental policy governs the criterion to be specified in the evaluation of patrol car requirements. For instance, the police planner may state his planning objective in the following way, "I wish to assign sufficient units to this time period that at least 90 percent of urgent calls for service are assigned to patrol units without any dispatching delay." The specified threshold is denoted as α (in the above case, α would be 90%).

The relationship between the number of cars assigned in any time period and the threshold α is complicated. The major difficultly stems from the fact that the patrol system is a multi-server nonsteady state queueing system. In this system the call rate, the number of cars on duty and other factors are time dependent. Figure 4.1 shows the typical diurnal variation pattern of the urgent call rate received at a New York City precinct [14]. However, the St. Louis Police Department has overcome the difficulty by using the hourly steady-state assumption of M/M/s queueing model, to obtain an excellent allocation of police patrol cars [24]. In this current study, the same assumptions are made, i.e., the mean of call rate is assumed to be constant for the t^{th} hour and the system is also assumed in a steady state during the same hour.

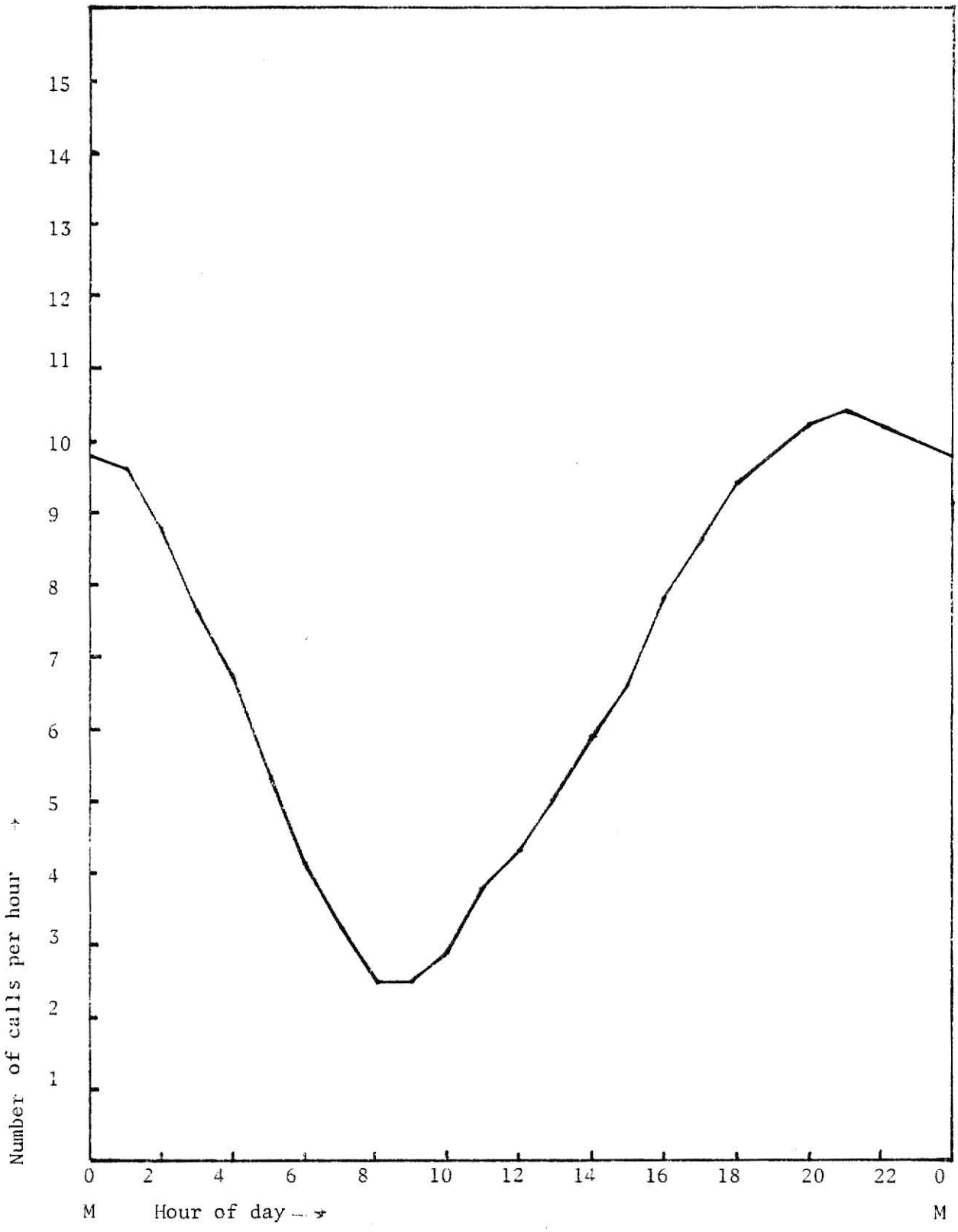


Fig. 4.1 Mean call rates as a function of time.

The state probability, P_{jt}, where j calls are in the system at hour t, can be expressed as:

$$P_{jt} = P_t^j \, P_{ot}/j! \quad \text{when } 1 \leq j \leq N_t \tag{1}$$

$$P_{jt} = P_t^j \, P_{ot}/[N_t^{(j-N_t)} \, N_t!] \quad \text{when } j > N_t, \tag{2}$$

where P_{ot} is the state probability of no call at any time t and can be expressed by:

$$P_{ot} = \{ \sum_{j=0}^{N_t-1} [\rho_t^j / j!] + \rho_t^{N_t} / [N_t! \, (1 - \rho_t/N_t)] \}^{-1} \tag{3}$$

and

$$\rho_t = \lambda_t/\mu$$

where

λ_t = mean call rate at hour t

μ = mean service rate

N_t = number of cars on duty at hour t

t = hour of day (t=0,1,2...23)

With a given threshold α, the car requirement at hour t, denoted by γ_t, can be obtained by searching for the smallest value of N_t, such that

$$\sum_{j=0}^{N_t-1} P_{jt} > \alpha \quad . \tag{4}$$

The physical meaning of the equation (4) is that the probability of at least one car available at any hour is greater than the specified threshold α.

The procedure appears awkward when using hand calculation. It is solved easily by computer. A general solution program has been constructed to generate numerical values for γ_t. The flow diagram of this program is given in Figure 4.2 and the program list is contained in Appendix A.1.

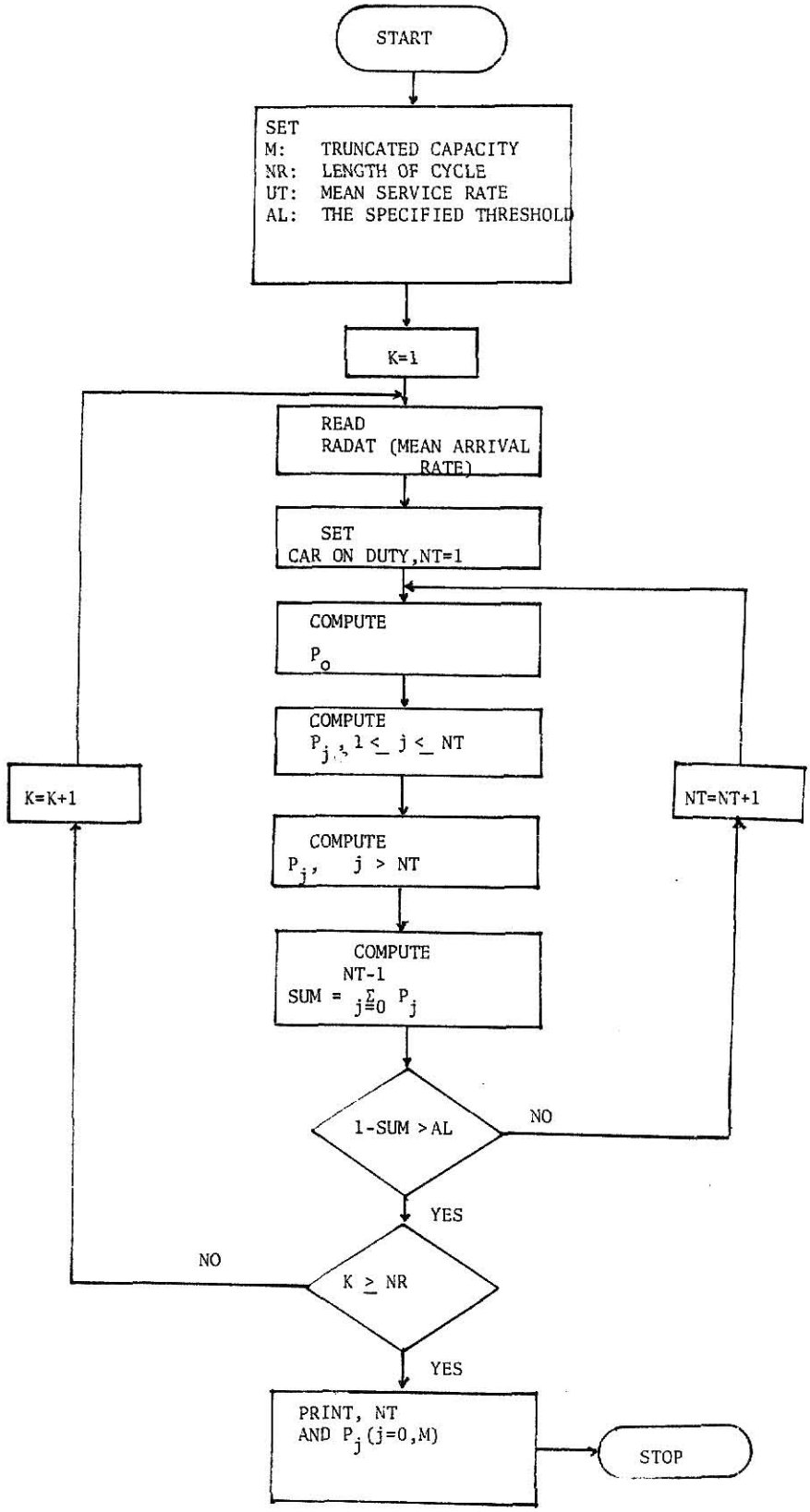


Fig. 4.2. Flow diagram for generating hourly car requirements. Using the probability of a call encountering a queue as a criterion.

The following example illustrates the procedure described above.

Example 4.1

Calculate the number of patrol car requirements based on the following conditions:

1. The threshold α is specified to be at least 90%.
2. The average service time is 30 minutes per service and is assumed to be independent of time.
3. The mean hourly arrival rate is given in Table 4.1.

First one would calculate the constant mean service rate μ:

$\mu = 60/30 = 2$ services per hour.

Then one can start the calculation from midnight, t=0.

From Table 4.1, one finds the mean call arrival rate is 9.8 per hour at t=0. ($\lambda_o = 9.8$ calls per hour) and the traffic density, ρ_o, would be $\lambda_o/\mu = 9.8/2 = 4.9$.

Starting with $N_o=1$, one would then calculate the probability of no call in 0 hour by using equation (3),

$$P_{oo} = 0.24366.$$

This means that the probability of the only patrol car getting an assignment is: 1 - 0.24366 = 0.75634 or approximately 75.6%. After the patrol car is dispatched, the rest of the emergency calls have to be placed in queue. Thus

$$\sum_{j=0}^{N_o-1} P_{jt} = P_{oo} = 0.24366 < 0.90$$

$N_o=1$ is rejected. The search is continued until the criterion is satisfied. At this point, the last tried N_o value would be the required number of cars, μ_t. Following the same procedure, γ_t for every hour of day can be calculated. The computer program listed in Appendix A.1 can be utilized for this calculation.

TABLE 4.1

Recorded mean arrival rate at each hour t.

t	0	1	2	3	4	5	6	7	8	9	10	11
λ_t	9.8	9.6	8.7	7.6	6.7	5.5	4.1	3.2	2.5	2.5	2.9	3.8

t	12	13	14	15	16	17	18	19	20	21	22	23
λ_t	4.3	5.0	5.9	6.6	7.8	8.6	9.4	9.8	10.2	10.4	10.2	10.0

t: Hour of a day (0,1, ...,23).

λ_t: Mean arrival rate at hour t, a constant during each hour.

The solution of this example problem is summarized in Table 4.2. The output is given in Appendix A.2.

4.2. Step 2: Generate Schedule for Each Shift

The set of feasible shift start times S, all of which are assumed to start on the hour, is determined by the police department: Each shift lasts eight hours. It is also assumed that every shift includes a one-hour mealbreak which also begins on the hour. There may be constrained on the earliest, e(i), and latest, $\ell(i)$, hours of a shift that can be used for mealbreak; i+e(i) and i+ℓ(i) denote the earliest and latest possible mealbreak respectively for a shift starting at hour i, $i \in S$.

Let N_i denote the number of cars assigned to work on a shift starting at hour i. $M_{i,t}$ denotes the number of cars working on shift i and assigned to mealbreak at hour t (where $i+e(i) \leq t \leq i+\ell(i)$)

The objective is to find the most efficient shift assignments $\{N_i, i \in S\}$ and $\{M_{i,t}, i \in S \text{ and } i + e(i) \leq t \leq i + \ell(i)\}$ which meet the car requirements γ_t at each hour of a day. This can be accomplished by utilizing an integer linear program;

$$\text{Minimize} \quad \sum_{i \in S} N_i \tag{5}$$

subject to:

$$\sum_{t-i+e(i)}^{i+\ell(i)} M_{i,t} = N_i, \quad i \in S \tag{6}$$

$$\sum_{i \in S,\ t-7 \leq i \leq t} (N_i - M_{i,t}) \geq \gamma_t, \quad t=0,1,2\ldots23 \tag{7}$$

where N_i, $M_{i,t} \geq 0$, and must be integers.

TABLE 4.2

The results of car requirements of Example 4.1

t	0	1	2	3	4	5	6	7	8	9	10	11
γ_t	9	9	8	8	7	6	5	4	4	4	4	5

t	12	13	14	15	16	17	18	19	20	21	22	23
γ_t	5	6	6	7	8	8	9	9	9	9	9	9

t: Hour of day (0,1...,23)

γ_t: Number of cars required in each hour t.

The value of the objective function is the total number of cars used over the day. The first set of constraints insures that every car has a mealbreak, and the second set of contraints insures that cars on duty meet the hourly requirements. This model can easily handle shifts starting on the half hour, quarter hour, etc., as well as shifts and mealbreaks of any length.

Example 4.2

Here the hourly car requirements generated in Example 4.1 are to be satisfied in scheduling. There are three possible shifts starting at 0, 8, and 16 hours, and mealbreaks can only be taken between the second and the fifth hour of each shift. This problem can be formulated as follows:

Minimize: $N_o + N_8 + N_{16}$

Subject to

$$\sum_{t=2}^{5} M_{o,t} - N_0, \text{ at } i=0$$

$$\sum_{t=10}^{13} M_{8,t} = N_8, \quad i=8$$

$$\sum_{t=18}^{21} M_{16,t} = N_{16}, \quad i=16$$

$$N_0 \geq 9$$

$$N_0 - M_{0,2} \geq 8$$

$$N_0 - M_{0,3} \geq 8$$

$$N_0 - M_{0,4} \geq 7$$

$$N_0 - M_{0,5} \geq 6$$

$$N_8 \geq 8$$
$$N_8 - M_{8,10} \geq 4$$
$$N_8 - M_{8,11} \geq 5$$
$$N_8 - M_{8,12} \geq 5$$
$$N_8 - N_{8,13} \geq 6$$

$$N_{16} \geq 9$$
$$N_{16} - M_{16,18} \geq 9$$
$$N_{16} - M_{16,19} \geq 9$$
$$N_{16} - M_{16,20} \geq 9$$
$$N_{16} - M_{16,21} \geq 9$$

N_i, $M_{i,t} \geq 0$ and integer.

There are 15 variables and 18 constraints in this integer linear problem. It is solved by Salklin and Spielberg's subroutine [37] and the results are arranged as follows:

Shift starting at hour 0, cars assigned $N_0 = 10$

Mealbreak assigned: $M_{0,2}=1$ car at hour 2
$M_{0,3}=2$ cars at hour 3
$M_{0,4}=3$ cars at hour 4
$M_{0,5}=4$ cars at hour 5

Shift starting at hour 8, cars assigned $N_8=7$

Mealbreak assigned: $M_{8,10}=2$ cars at hour 10
$M_{8,11}=2$ cars at hour 11
$M_{8,12}=2$ cars at hour 12
$M_{8,13}=1$ car at hour 13

Shift starting at hour 16, cars assigned $N_{16} = 12$

mealbreak assigned: $M_{16,18}=3$ cars at hour 18

$M_{16,19}=3$ cars at hour 19

$M_{16,20}=3$ cars at hour 20

$M_{16,21}=3$ cars at hour 21

4.3 Step 3: Use a More Realistic Nonsteady State Queueing Model to Evaluate the Performance of Schedules

The administrator may be faced with different proposed schedulings such as the current experienced schedules in use, schedules generated under different policies, and schedules calculated by other methods. Before making a choice among the alternatives, he may want to have a more realistic comparison of the results which can be expected from each alternative. Two models can be used for this purpose. One is the simulation model, such as the one described by Kolesar and Walker [15]. As discussed previously in Chapter 1, there are a considerable number of pitfalls one may encounter in using simulation, and its analysis is relatively expensive and cumbersome. The second model involves a M/M/s nonsteady state queue with mean arrival rate as a function of time. The latter one is used for its simplicity. The notations for this model are:

$P_j(t)$: the probability of j calls for service in the system at any time t.

$\lambda(t)$: the mean call rate as a function of time t.

μ : the mean service rate as a constant independent of time

$n(t)$: the number of patrol cars on duty at each hour t(t=0,1,..23); it is a discrete function of hour. During hour t, n(t) is a constant value which is equal to N_i(cars assigned in shift i) - $M_{i,t}$ (cars in shift i and assigned to mealbreak at hour t).

The transition probabilities for this model are expressed by the well-known infinite set of difference-differential equations;

$$P'_0(t) = -\lambda(t) \quad P_o(t) + \mu P_1(t) \tag{8}$$

$$P'_j(t) = \lambda(t)P_{j-1}(t) - [\lambda(t) + j\mu]P_j(t)$$

$$+ (j+1)\mu\, P_{j+1}(t), \qquad 1 \leq j < n(t) \tag{9}$$

$$P'_j(t) = \lambda(t)\, P_{j-1}(t) - [\lambda(t) + n(t)\mu]\, P_j(t)$$

$$+ n(t)\mu\, P_{j+1}(t), \qquad j \geq n(t) \tag{10}$$

In order to integrate this set of equations numerically, it is assumed that m is the maximum number of calls allowed in the system or m is chosen such that the probabilities of m calls or more in system are close to zero, as discussed in Chapter 2. The set of equations can be replaced by:

$$P'_0(t) = -\lambda(t)\, P_o(t) + \mu P_1(t) \tag{11}$$

$$P'_j(t) = \lambda(t)\, P_{j-1}(t) - [\lambda(t) + (j-1)\mu]\, P_j(t)$$

$$+ j\mu P_{j+1}(t), \qquad 1 \leq j < n(t) \tag{12}$$

$$P'_j(t) = \lambda(t)\, P_{j-1}(t) - [\lambda(t) + n(t)\mu]\, P_j(t)$$

$$+ n(t)\mu\, P_{j+1}(t), \qquad n(t) \leq j < m \tag{13}$$

$$P'_m(t) = \lambda(t)\, P_{m-1}(t) - n(t)\mu\, P_m(t) \tag{14}$$

There are a number of standard numerical methods available for solving this type of simultaneous ordinary differential equations. Because n(t) is a discontinuous function of hour, the multi-step predictor-corrector method which uses several steps ahead is not suitable in this case. The fourth order Runge Kutta method which concentrates on single step and does not pass over the discontinuous points is thus selected. The flow diagram of the numerical method developed is shown in Figure 4.3 and the computer program is listed in Appendix B.1.

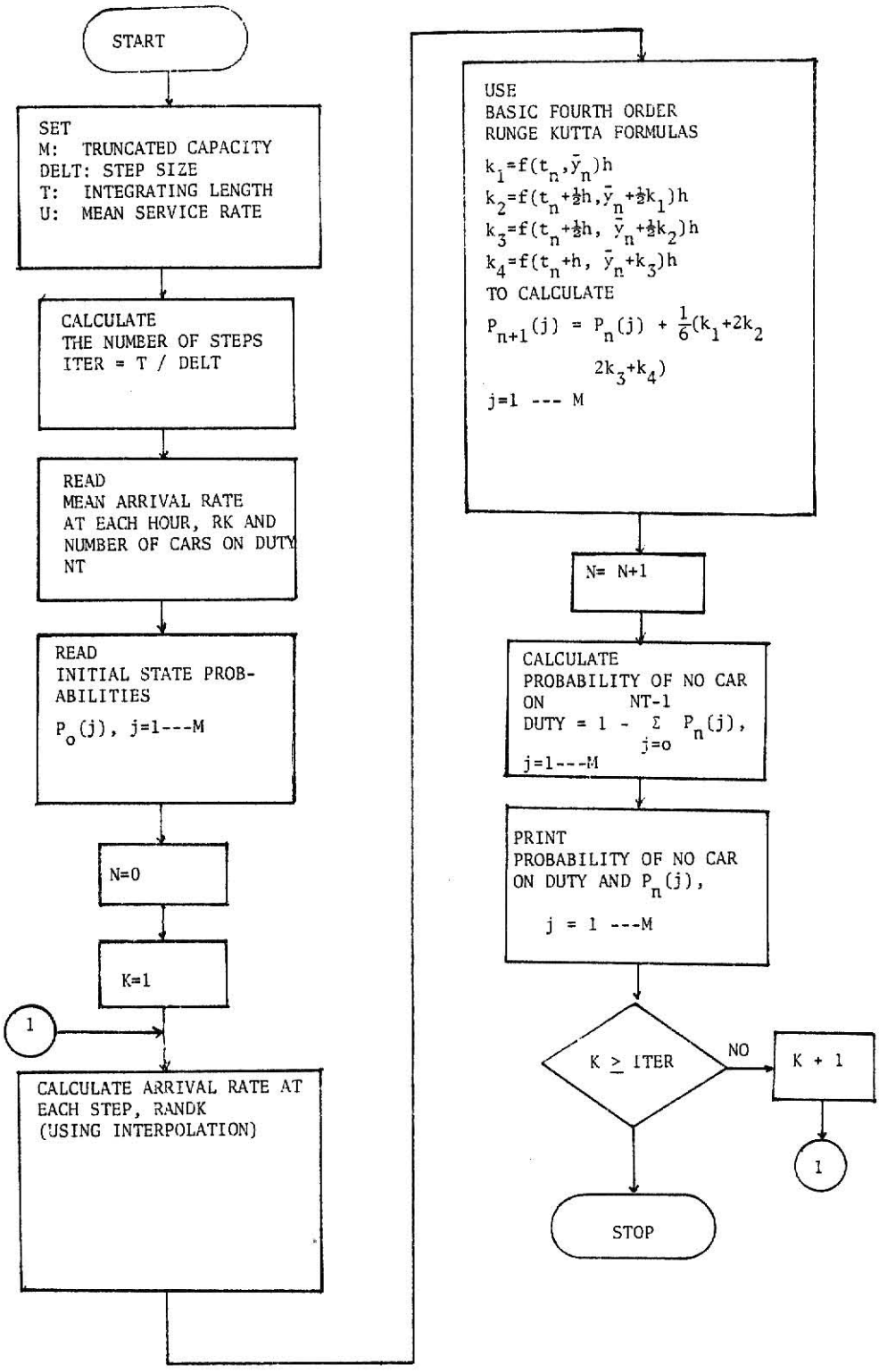


Fig. 4.3 Flow diagram of Runge Kutta method, using the probability of a call encountering a queue as a criterion.

Example 4.3

The same problem which was described in the previous two examples will be further discussed. The results obtained in Example 4.2 are used to calculate the number of cars on duty at each hour t, $n(t) = N_i - M_{i,t}$. The calculated n(t) values are summarized in Table 4.3. The data is also shown graphically in Figure 4.4.

In order to solve this transient state problem, some sets of new information are required beyond those given previously. They are the time varying arrival rate, $\lambda(t)$, the maximum number of calls allowed in the system, m, and the initial values.

The arrival rate function is usually developed by recording the average arrival rate at each hour of a day as listed in Table 4.1. The hourly average arrival rates are connected by straight lines as shown in Figure 4.1. The approximated time-varying arrival rate at any time t, $\lambda(t)$, can be obtained by interpolation.

After several trial runs with assumed values for the maximum allowed calls, m, one could select a value which would assure program convergency with a reasonable amount of computing effort. In this case, m is selected to be 30 calls.

The steady state probabilities of an M/M/s queueing system at hour 0 are used as the initial values.

The problem now can be integrated by the indicated numerical program (See Appendix B.1).

The integration should be run for a longer period of time than 24 hours in order to eliminate the influence of the approximate initial value. Kolesar et al have shown that a two days running period should be sufficient.

TABLE 4.3 The Patrol Cars on Duty at Each Hour t

t	0	1	2	3	4	5	6	7	8	9	10	11
n(t)	10	10	9	8	7	6	10	10	7	7	5	5

t	12	13	14	15	16	17	18	19	20	21	22	23
n(t)	5	6	7	7	12	12	9	9	9	9	12	12

t: Hour of a day (0,1....23).

n(t): Number of patrol cars on duty in each hour t.

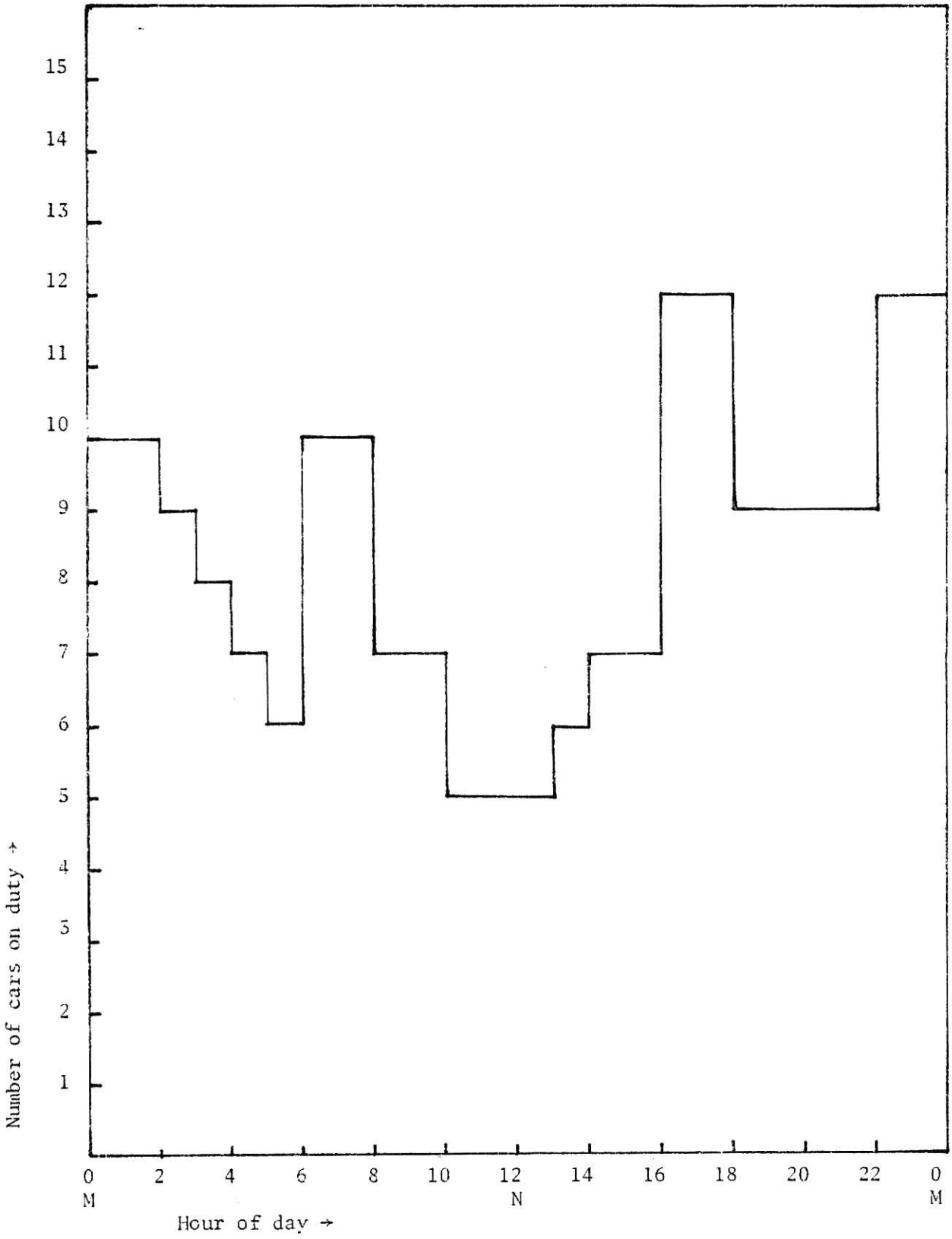


Fig. 4.4 The number of cars on duty during each hour t.

Because of drastic changes in the number of cars on duty and calls for service, the transient responses are very strong at hours 17 and 22. Even step size 0.05 time units does not provide convergence. The sum of state probabilities of m-truncated capacity is used to check convergence. The sum at each time t integrated by step size 0.05 is shown in Figure 4.3. After experimenting with several values, a step size of 0.025 hours is found to provide convergence. The results of integrating is also shown in Figure 4.5 for comparison. There is nowhere that the total probability deviates from unity more than 0.0008. Using step size 0.025 hours, excution time for this problem involving a 48 hour period runs about 2 minutes and 25 seconds. Part of the output is given in Appendix B.2. The probability of no car on duty at each time t is shown in Figure 4.6.

In order to demonstrate the comparison of different alternatives, an experienced schedule used by the New York Police Department [14] is evaluated. The schedule is described below:

Shift starting at hour 0, cars assigned $N_0 = 6$

Mealbreak assigned: $M_{0,3}=1$ car at hour 3

$M_{0,4}=2$ cars at hour 4

$M_{0,5}=3$ cars at hour 5

Shift starting at hour 8, cars assigned $N_8 = 8$

Mealbreak assigned: $M_{8,10}=3$ cars at hour 10

$M_{8,11}=2$ cars at hour 11

$M_{8,12}=2$ cars at hour 12

$M_{8,13}=1$ cars at hour 13

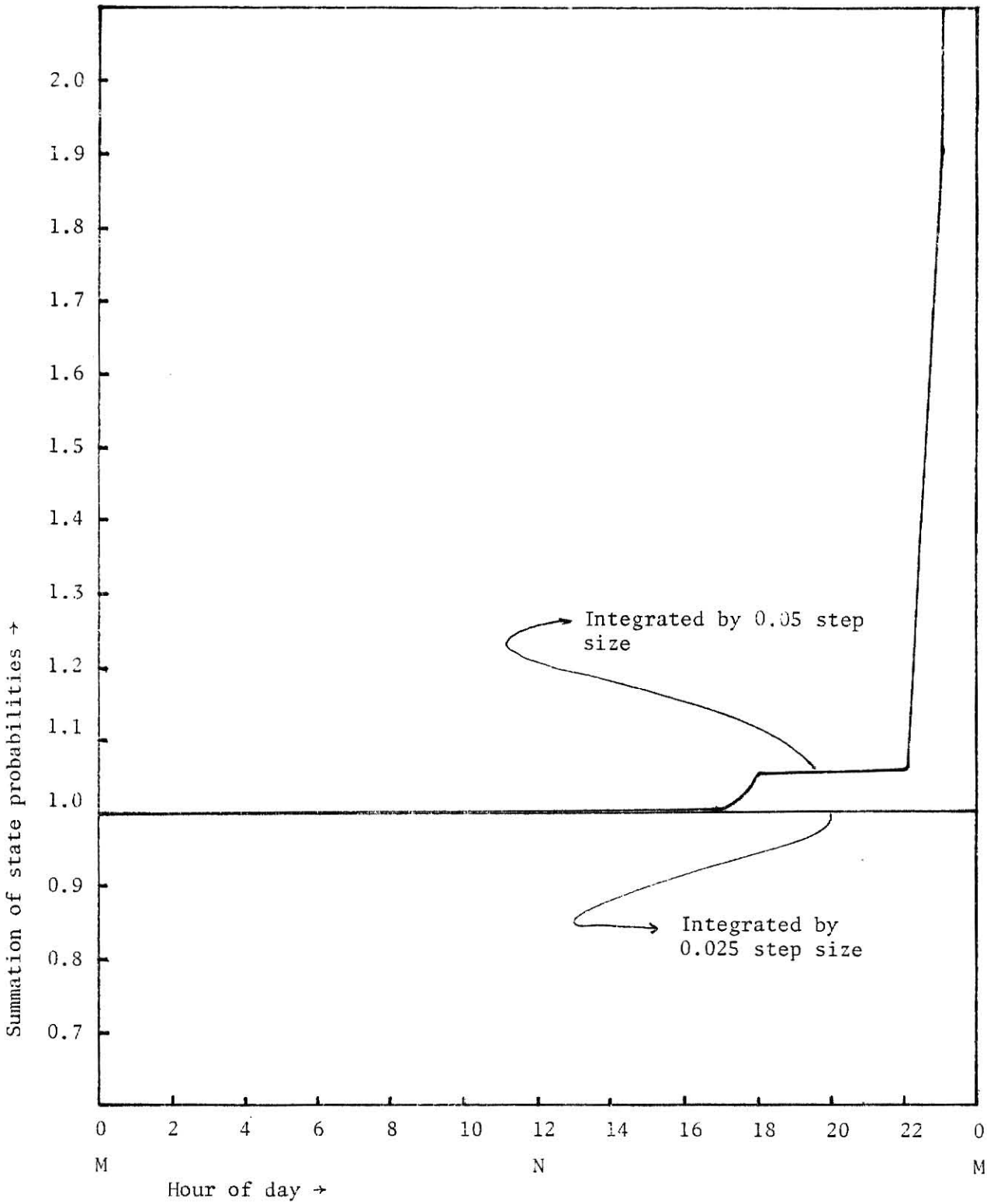


Fig. 4.5 The summation of state probabilities of M-truncated capacity at every time t.

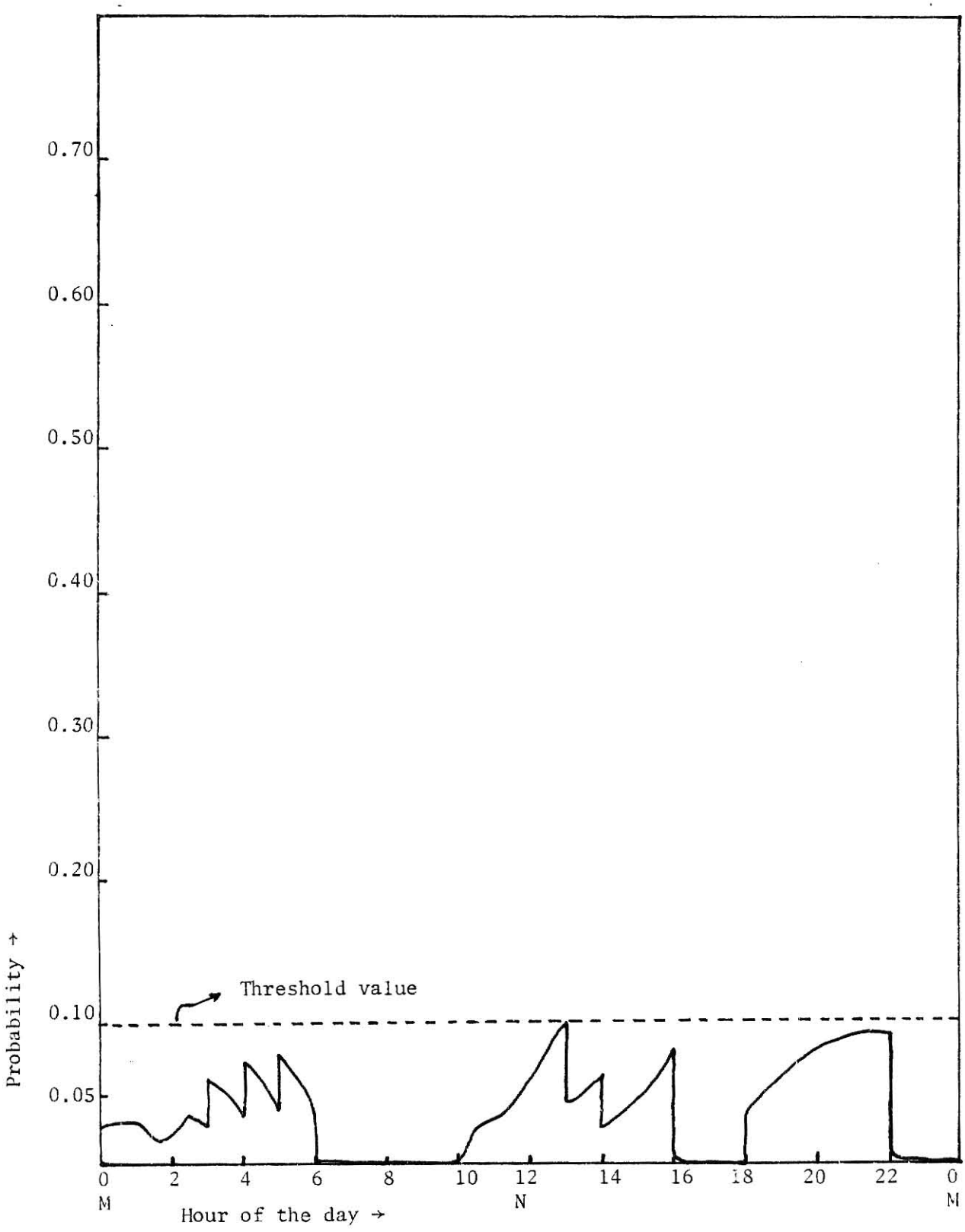


Fig. 4.6 The probability of no car on duty in the schedule with three shifts.

Shift starting at hour 16, cars assigned $N_{16}=10$

Mealbreak assigned: $M_{16,18}=3$ cars at hour 18

$M_{16,19}=3$ cars at hour 19

$M_{16,20}=2$ cars at hour 20

$M_{16,21}=2$ cars at hour 21

the output is partially given in Appendix B.3. The probability of no car on duty at each time t is shown in Figure 4.7.

Comparing the results of these two schedules, it shows that although the experienced schedule uses fewer cars per day, its performance is far inferior. During hour 5 and 6, the probability of a call for service encountering a queue is more than 60 percent.

The same procedure can be used to compare the schedules calculated under different departmental policies or the schedules generated by other methodologies. Considering the resource limitations, administrative concerns, and results of performance, this evaluation technique with a nonsteady state queueing model can help administrators make a reasonable choice among alternatives.

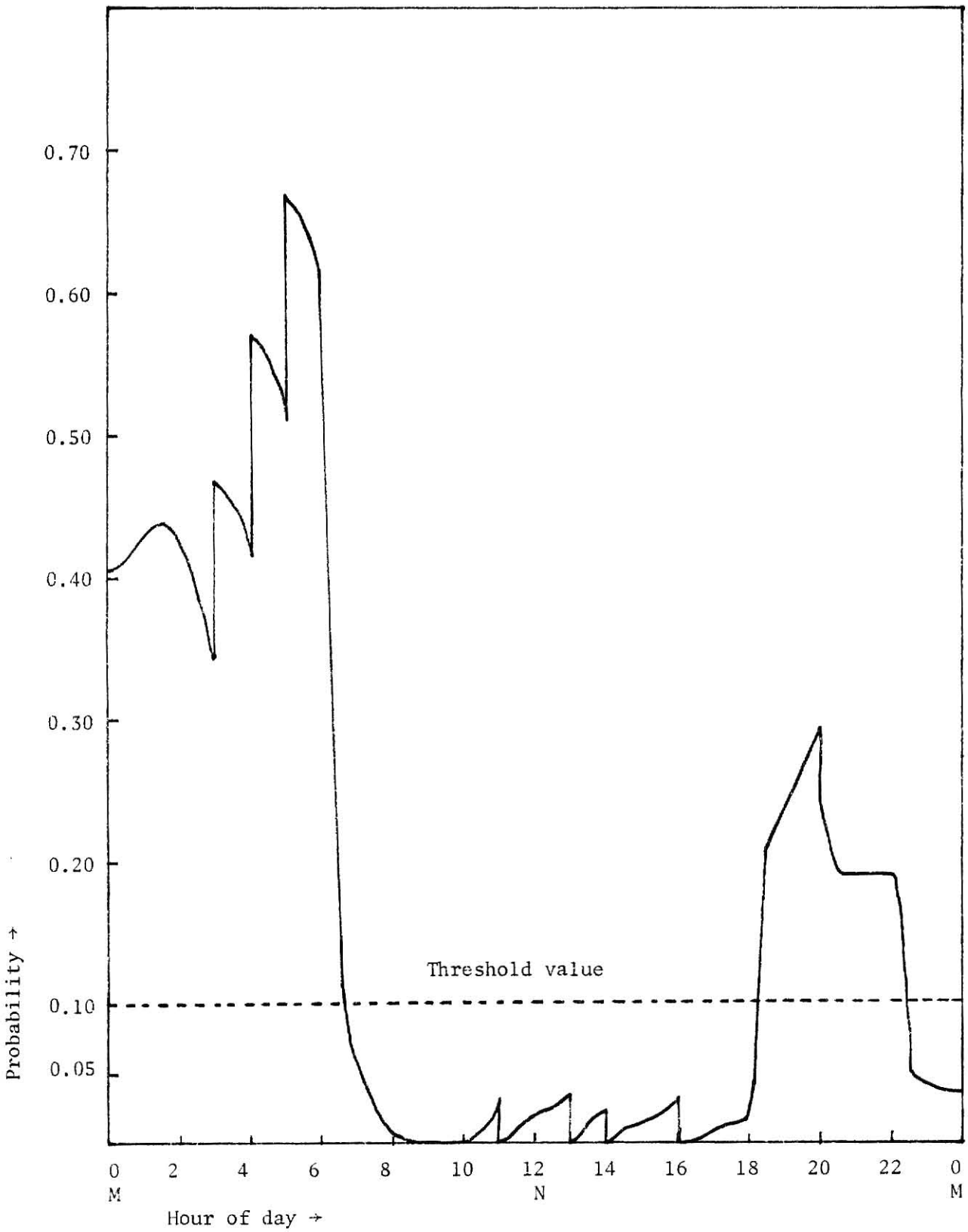


Fig. 4.7. The probability of no car no duty in the experienced schedule.

CHAPTER 5

THE EXPECTED WAITING TIME OF A CALL IN THE QUEUE AS A CRITERION

In Chapter 4, the measurement of patrol service performance was emphasized on the basis of the probability of an urgent call being placed in the queue. This measurement, based on a probability concept, cannot give a clear indication of the effect on an urgent call. However, the measurement of the expected waiting time of urgent calls in the queue seems to be more relevant in gauging the effectiveness of a patrol scheduling system considering the time-critical nature of police services. In this chapter, the level of the time related measurement is used as a specified threshold to calculate the car requirements. The three-step methodology by Kolesar et al has to be modified. The necessary modifications are given in the next several sections.

5.1 Step 1: Estimate the Hourly Patrol Car Requirements

Except for the measurement of patrol performance, the conditions are the same as those in Section 4.1. The objective of the police planner in this case is to assure that the expected waiting time of a call in the queue is below some specific threshold.

The state probability P_{ot} calculated from Equation (3), can be extended to estimate the expected number of calls to be placed in the queue. This number, denoted by L_t, can be calculated by:

$$L_t = [\rho_t^{n_t+1} P_{o,t}]/[n_t ! \ (\frac{1 - \rho_t}{n_t})^2 \ n_t] \qquad (15)$$

The expected waiting time of a call in the queue, W_t, can then be calculated by using Little's formula [21]:

$$W_t = \frac{L_t}{\lambda_t} \qquad (16)$$

With a given threshold α, the car requirement in hour t can be found by finding the smallest value of n_t which also satisfies the condition:

$$W_t < \alpha \tag{17}$$

The mathematical manipulation to solve these equations is quite complicated. A computer program which carries out the task is given in Appendix C.1. and its flow chart is shown in Figure 5.1.

Example 5.1

Here the specified threshold is that the expected waiting time of a call in the queue in any hour should not exceed one minute. The rest of the pertinent information is the same as in Example 4.1.

Again, we start solving the problem at midnight, t=0. The required number of cars would be searched following the procedure below:

Starting with $n_0=1$, P_{00} is calculated by using equation (3):

$$P_{00} = 0.24366$$

The expected number of calls in the queue in hour t=0 is given by Equation (15)

$$L_0 = \left(\frac{9.8}{2}\right)^2 (0.24366)/(1-0.8/z)^z = 0.3846$$

The expected waiting time of a call in the queue in hour t=0 then can be calculated from Equation (16).

$$W_0 = \frac{0.3846}{9.8} = 0.0392 \text{ hours}$$

This value is greater than the specified threshold value of 1 minute (or α = 0.0017 hours). Thus, if only one car is on duty in hour t=0, the performance of the patrol service can not meet the required level. Continuing this trial and error process, one finds 9 is the first value of n_o which meets the requirement.

$$W_o = 0.0088 < \alpha = 0.017$$

Therefore, n_o is estimated to be 9.

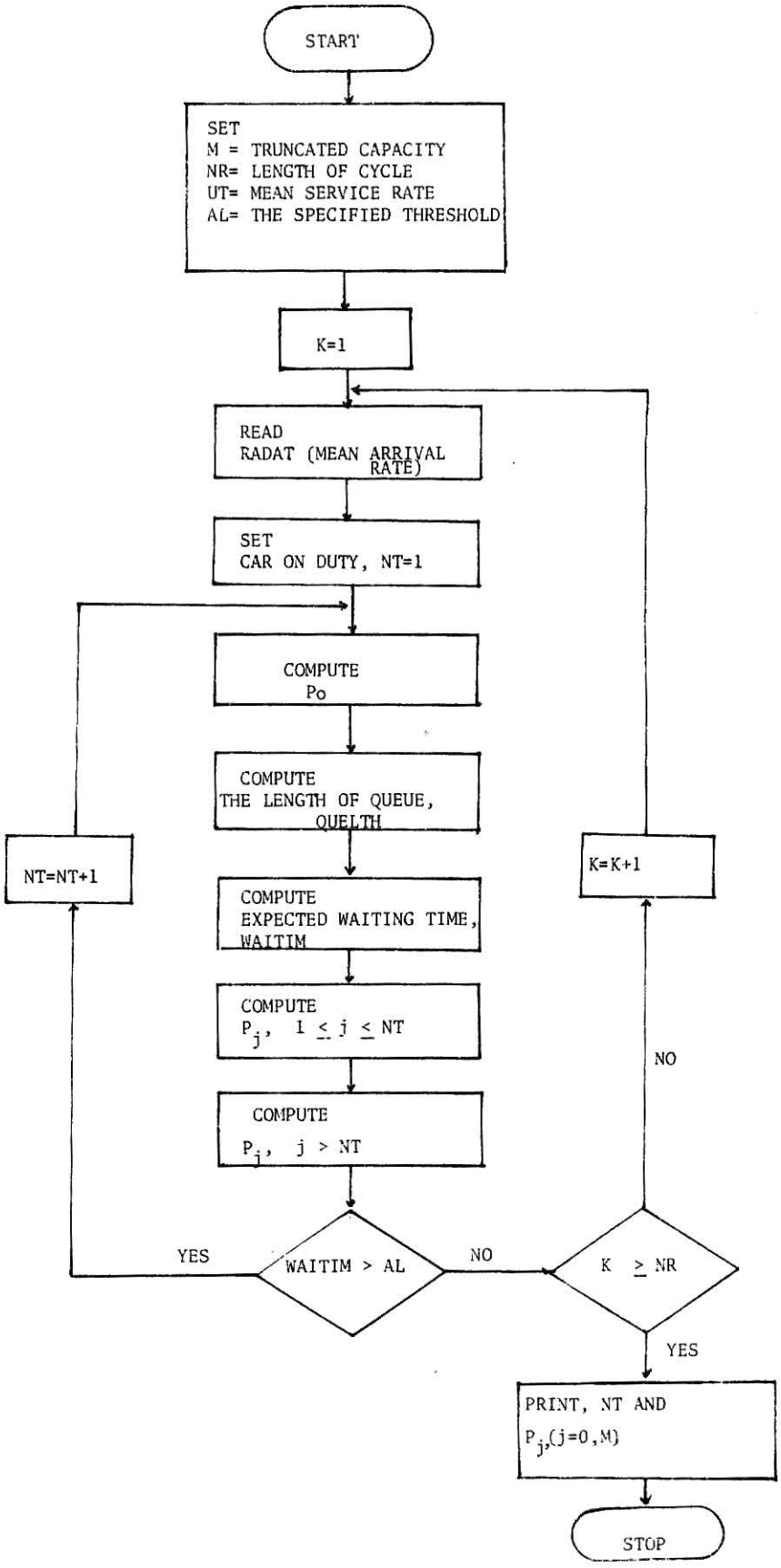


Fig. 5.1. Flow diagram for generating hourly car requirements, using the expected waiting of a call in the queue as a criterion.

Repeating the procedure, the hourly car requirements for the day can be eventually obtained. The final solution is summarized in Table 5.1.

5.2. Step 2: Generate Schedule for Each Shift

The assumptions on the set of feasible shift starting times and the constrains on the earliest and latest mealbreak hours are not changed here. The integer linear program described by Equations (5-7) is used in the same way in arranging shift assignments.

Example 5.2

It is to be pointed out the only difference between the γ_t values generated in Example 4.1 and Example 5.1 lies on γ_7, the car requirement in the 7th hour. One more car is required in the latter case (comparing Tables 4.2 and 5.1). However, both γ_7 values never enter the calculations involved for schedule generating. Therefore, the answers from Example 4.2 can be applied directly in this case.

5.3 Step 3: Use a More Realistic Nonsteady State Queueing Model to Evaluate the Performance of Schedules

The truncated nonstationary queueing model described in Equations (11-14) is again used for the evaluation.

With the new set of criteria, two additional notations are needed: L(t) denotes the expected number of calls in the queue at any time t and W(t) denotes the expected waiting time of a call in the queue at any time t.

When Equations (11-14) are integrated by the Runge-Kutta numerical method, the call arrival and service rates are again assumed to be constant within each integrating step.

Equations (15) and (16) can be extended in solving nonsteady-state cases;

TABLE 5.1

The results of car requirements of Example 4.1

t	0	1	2	3	4	5	6	7	8	9	10	11
γ_t	9	9	8	8	7	6	5	5	4	4	4	5

t	12	13	14	15	16	17	18	19	20	21	22	23
γ_t	5	6	6	7	8	8	9	9	9	9	9	9

t: Hour of day (0,1...,23)

γ_t: Number of cars required in each hour t.

$$L(t) = [\rho(t)^{\acute{n}(t)+1} P_o(t)]/[n(t)!(1-\rho(t)/n(t))^2 \, n(t)] \quad (18)$$

$$(t) = \lambda(t)/\mu \quad (19)$$

and

$$W(t) = L(t)/\lambda(t) \quad (20)$$

The general program which calculates the transient probabilities and the expected waiting time of a call in the queue for the truncated M/M/s non-steady state queue is given in Appendix D.7 and its flow chart is shown in Figure 5.2.

Example 5.3

Since the schedule generated in Example 5.2 is the same as the one calculated in Example 4.2, the cars needed on duty in each hour t are shown in Table 4.2. With the same truncated capacity and initial values of state probabilities, the problem is integrated with a step size of 0.025 hours for a 48 hour period. The output of the solution is given partially in Appendix D.2. The execution time for running this program is approximately 2 minutes and 21 seconds. The character of this schedule is illustrated in Figure 5.3.

The experienced schedule used by a New York police precinct [14] is also evaluated. Part of the output is shown in Appendix D.3. Figure 5.4 shows the character of this experience schedule.

An additional case study is evaluated for further comparison. A more flexible schedule given by Kolesar et al [14] is employed here. This schedule has five start times - 0, 8, 12, 16 and 20. Meal times are allowed at any hour during a shift.

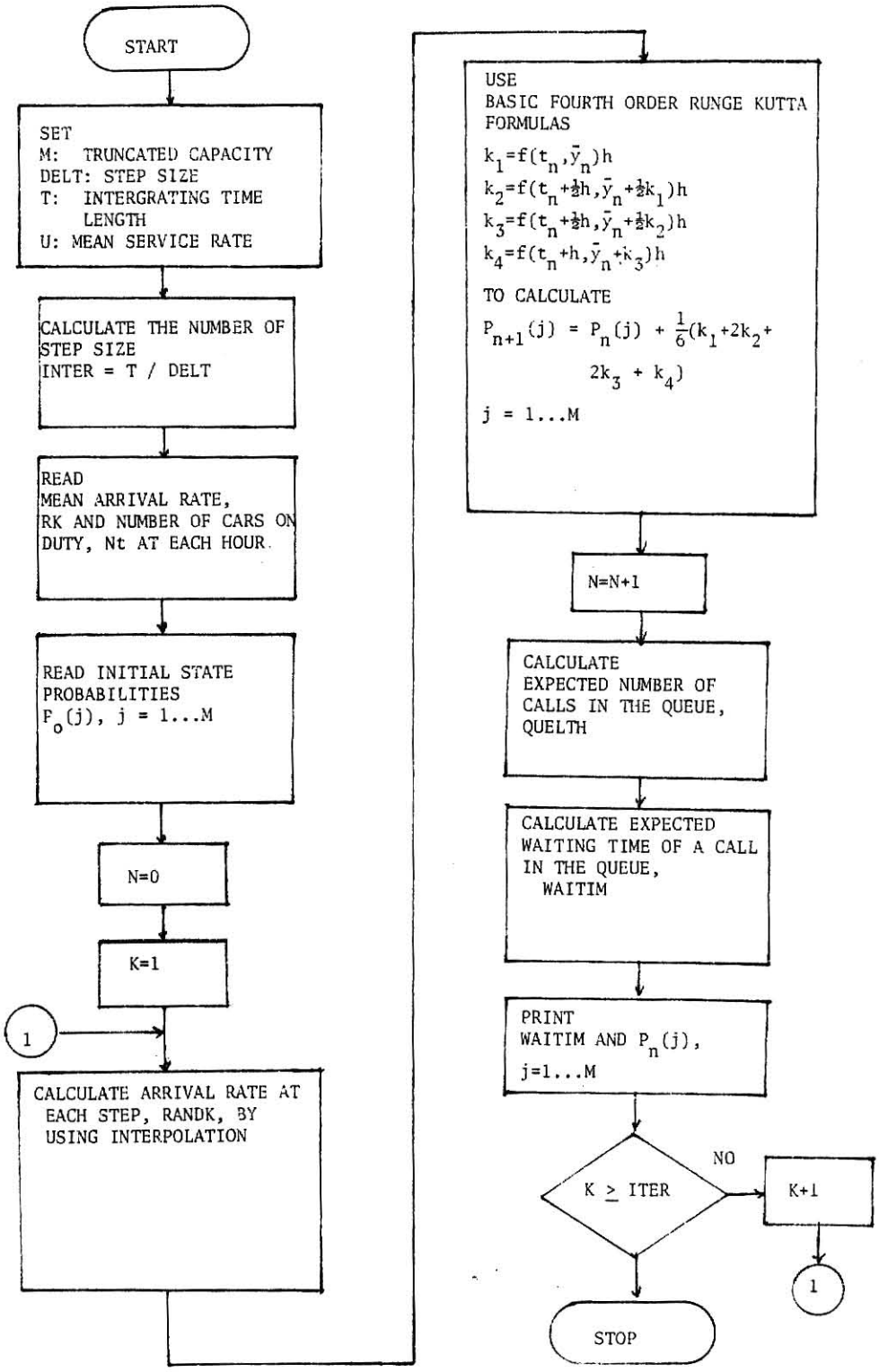


Fig. 5.2. Flow chart of Runge Kutta method, using the expected waiting time of a call in the queue as a criterion.

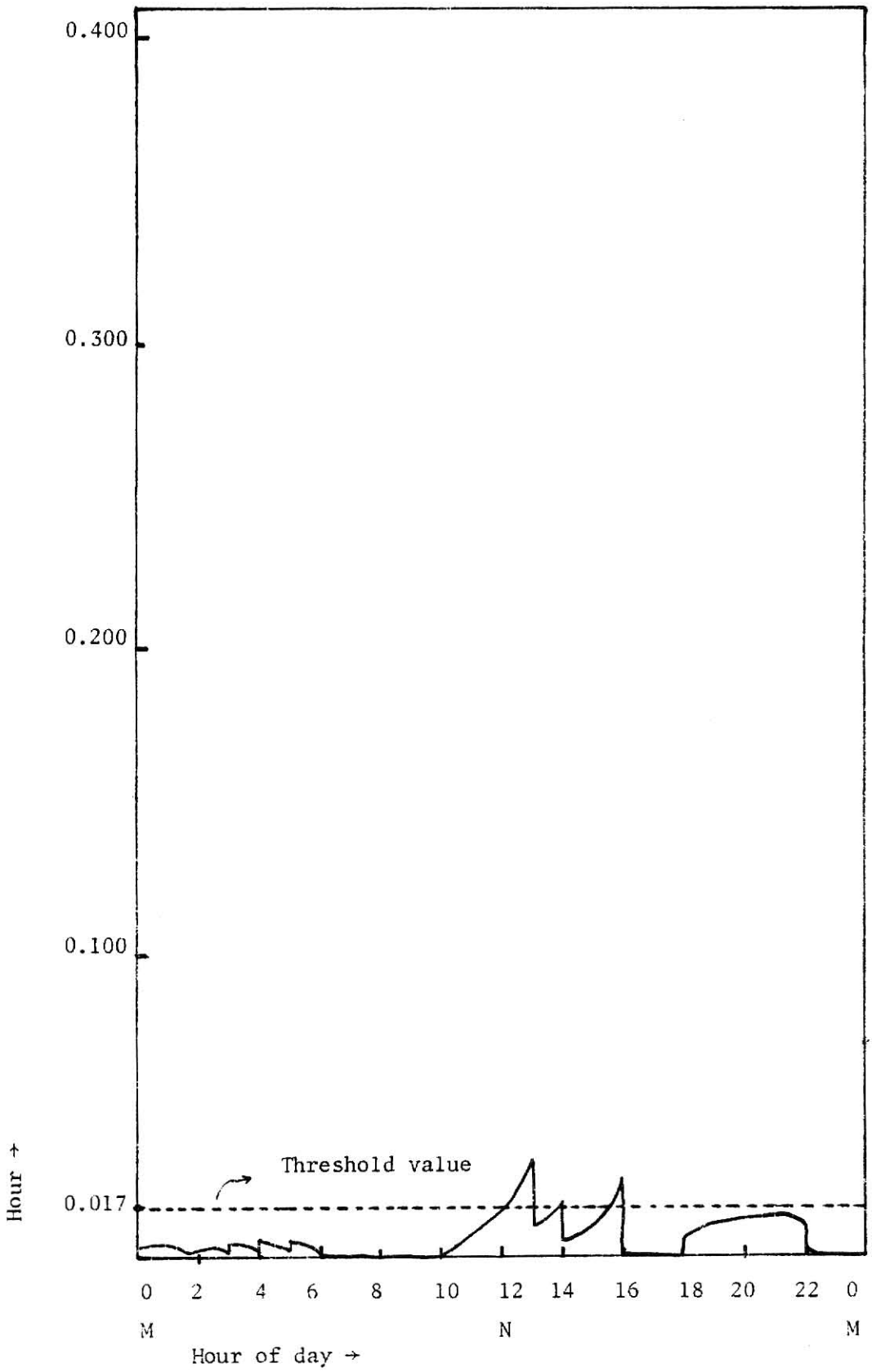


Fig. 5.3. The expected waiting time of a call in the queue in the schedule with three shifts.

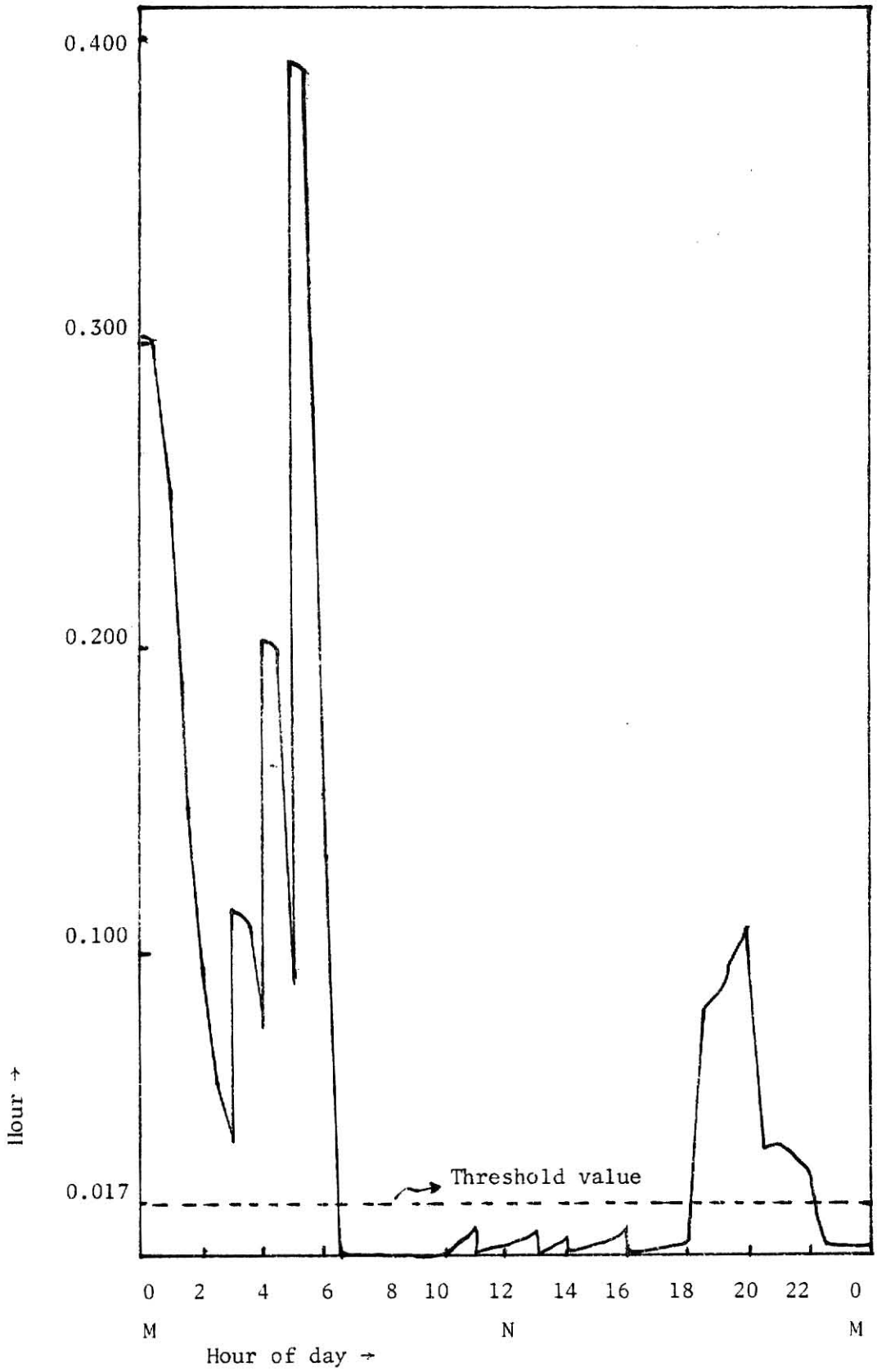


Fig. 5.4. The expected waiting time of a call in the queue in the experienced schedule.

The schedule is duplicated here as follows:

Shift starting at hour 0;	car assigned $N_0 = 7$
mealbreak assigned:	$M_{0,3} = 1$ car
	$M_{0,5} = 1$ car
	$M_{0,6} = 2$ cars
	$M_{0,7} = 3$ cars
Shift starting at hour 8	car assigned $N_8 = 5$
mealbreak assigned:	$M_{8,8} = 1$ car
	$M_{8,9} = 1$ car
	$M_{8,10} = 1$ car
	$M_{8,12} = 1$ car
	$M_{8,14} = 1$ car
Shift starting at hour 12	cars assigned $N_{12} = 2$
mealbreak assigned:	$M_{12,12} = 1$ car
	$M_{12,13} = 1$ car
Shift starting at hour 16	cars assigned $N_{16} = 8$
mealbreak assigned	$M_{16,16} = 2$ cars
	$M_{16,17} = 2$ cars
	$M_{16,18} = 1$ car
	$M_{16,19} = 1$ car
	$M_{16,22} = 1$ car
	$M_{16,23} = 1$ car
Shift starting at hour 20	cars assigned $N_{20} = 2$
mealbreak assigned	$M_{20,20} = 1$ car
	$M_{20,21} = 1$ car

The evaluation of this more flexible schedule is illustrated in Figure 5.5. Part of its output is listed in Appendix D.4. The above discussions illustrate the effectiveness of using the nonstationary queueing model in evaluating alternative proposals for scheduling patrol cars. The schedule generated in Example 5.2 apparently performs better than the experienced schedule, although the criterion $\alpha = 0.017$ hrs is not saisfied entirely. One can see that for the experienced schedule, the average waiting time of a call in the queue is extremely high for most of the night time hours (see Figure 5.4). However, by simply relaxing the constraints on the number of shifts and the starting and ending hours for breaks, one can obtain a schedule with superior performance without increasing the total number of cars required. (See Figure 5.5)

A police administrator can with great ease use this tool, incorporating any departmental policy (constraints) to select the most cost-effective patrol car scheduling.

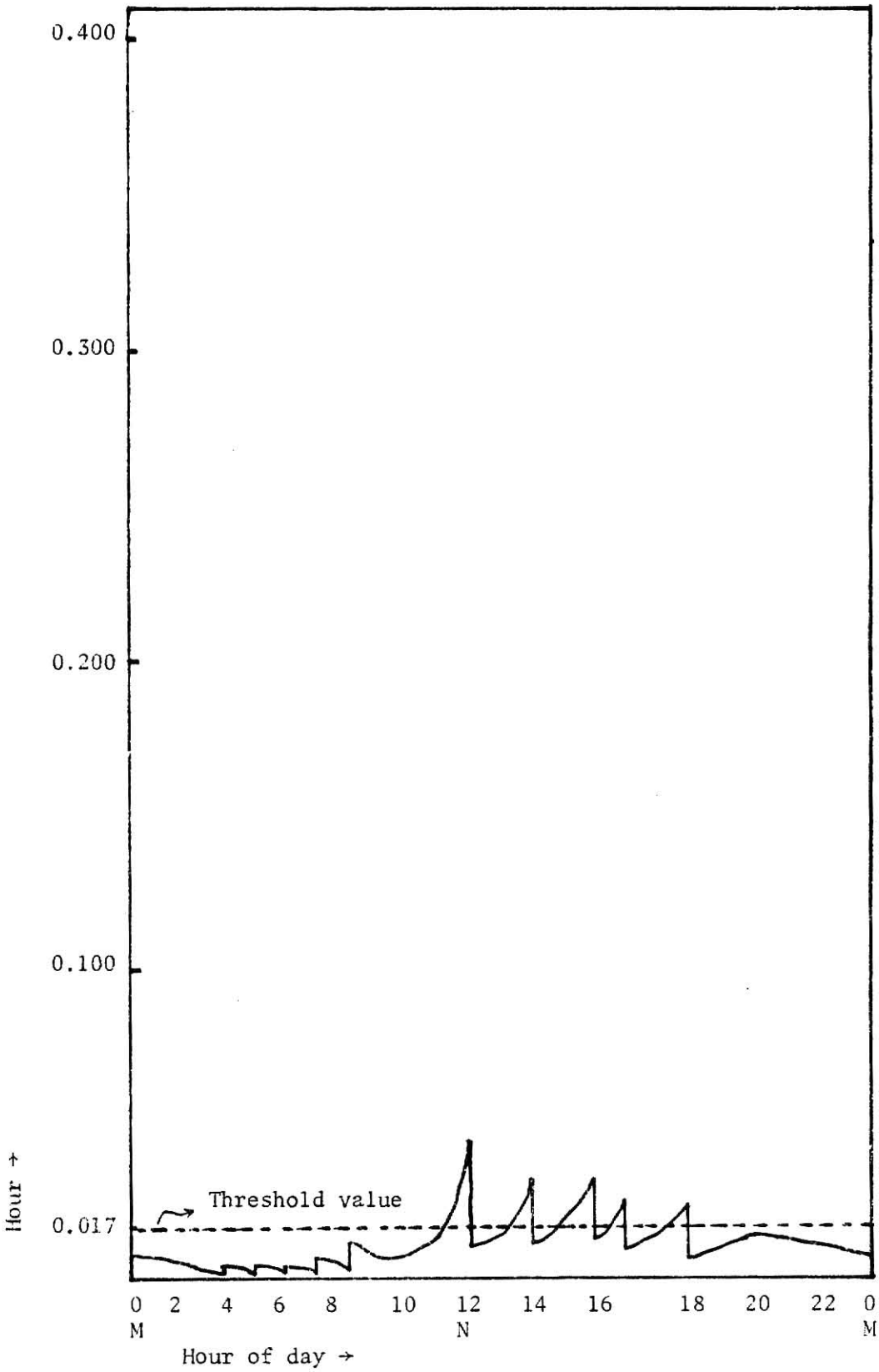


Fig. 5.5. The expected waiting time of a call in the queue in the schedule with five shifts.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

From this study, the following conclusions are made:

1. A police administrator can select a reasonable schedule by applying the M/M/s nonsteady state queueing model to evaluate patrol car scheduling alternatives.
2. The fourth order Runge Kutta method can be used to solve this non-steady state model with reasonable step size.
3. The example discussed in Chapters 4 and 5 showed two strong transient responses at hours 17 and 22, because of drastic changes in the number of available cars and call arrival rates. These strong responses required a much smaller step size in integration. A step size value of 0.025 hours was shown to be able to provide convergence.
4. Considering the information provided by this nonsteady state model, the excution time of approximate 2 minutes and 25 seconds seems to be acceptable.
5. The experienced schedule was evaluated both in Chapters 4 and 5. Two criteria were used in the judgment. The first was based on a threshold probability of an urgent call placed in queue. The second was based on the expected waiting time of a call in the queue. From both points of view, the experienced schedule is extremely inadequate.

Two suggestions are given for future research:

1. Since the strong transient responses occurred at only two hours, it is not very economical that a small step size 0.025 hours is used to cover the entire 48 hour period. Therefore, a variable step size algorithm may be advantageous.
2. The level of patrol performance was measured by the expected waiting time of a call in the queue in Chapter 5. In this case, the Rothkopf

and Oren's closure technique [36], which directly approximate the expected number of calls and variance in the M/M/s nonsteady state queueing system, may be used to replace the fourth order Runge Kutta numerical method to solve the infinite set of difference-differential equations. The expected waiting time of a call in the queue can be calculated from the following equation:

$$W(t) = [(\text{expected waiting time of a call in the system at time } t) - \lambda(t)/\mu]/\lambda(t)$$

Rothkopf and Oren have solved several nonsteady state problems with fixed number of servers. In contrast, the patrol scheduling problem is typefied by varying patrol car requirements on duty at different hours, thus a subroutine for calculating the correction term at each step may be needed. This correction term is used to remove the error caused by negative binomial closure assumption.

REFERENCES

1. Bailey, N. T. J., A Continuous Time Treatment of a Simple Queue Using Generating Functions, Journal Royal Stat. Soc. (B), Vol. 16, 1954, pp. 288-291.

2. Champernowne, D. G., An Elementary Method of Solution of the Queueing Problem with a Single Server and a Constant Parameter, J. Roy. Stat. Soc. (B), Vol. 18, 1956, pp. 125-128.

3. Chang, S. S. L., Simulation of Transient and Time-Varying Conditions in Queueing Networks, Procceding of the Seventh Annual Pittsburg Conference on Modeling and Simulation, U. of Pittsburg, 1977. pp. 1075-1078.

4. Clarke, A. B., A Waiting Line Process of Markov Type, Ann. of Math. Stat., Vol. 27, 1956, pp. 452-459.

5. Cox, D. R., and Simith, W. L., Queues, Metheuen, London, 1961.

6. Galliher, H. P., and Wheeler, R. C., Nonstationary Queueing Probabilities for Landing Congested Aircraft, Operations Research, Vol. 6, 1958, pp.264-275.

7. Gover, D. P., Observing Stochastic Processes and Approximation Transform Inversion, Operations Res., Vol. 14, 1966, pp. 444-459.

8. Gazis, D. C., and Potts, R. B., The Oversaturated Intersection, 2nd International Symposium on the Theory of Traffic Flow, London, OECD Paris, 1965, pp. 221-237.

9. Ghare, P. M., Given, J. M., and Torgersen, P. E., A Machine Release Scheme for the Job Shop Work Center, AIIE Trans., 1(3), September 1969.

10. Gross, G., and Harris, D. M., Fundamental of Queueing Theory, New York: Weily, 1974.

11. Hartman, M. G., The Truncated Time Varying Queue as Model of an Air Traffic Control System, Unpublished M. S. Thesis, The Ohio State University, 1971.

12. Ignall, E., Kolesor, P., and Walker, W., The Use of a Simulation in the Development and Empirical Validation of Analystic Models for Emergency Services, Proceedings of the 1974 Winter Simulation Conference, Washington, D. C., 1974.

13. Johannsen, F. W., Waiting Times and Number of Calls, P. O. Electrical Engineers Jounral, 1907.

14. Kolesar, P. J., Rider, K. L., Crabill, T. B., and Walker, W. E., A Queuing Linear Programming Approach to Scheduling Police Patrol Car, Operations Research, Vol. 23, No. 6, 1975, pp. 1045-1062.

15. Kolesar, P. J., and Walker, W. E., A Simulation Model of Police Patrol Operation: Program Description, R-1625/2-HUD/NYC, The New York City Rand Institute, New York, Feb. 1975.

16. Koopman, B. O., Air-Terminal Queues Under Time Dependent Conditions, Operations Research, Vol. 20, 1972, pp. 1089-1174.

17. Kotiah, T. C., Approximate Transient Analysis of Some Queueing System, Operations Research, Vol. 26, 1978, pp. 333-346.

18. Larson, R. C., Urban Police Patrol Analysis, The MIT Press, Cambridge, Mass., 1972.

19. Ledermann, W., and Reuter, G. E. H., Spectral Theory for the Differential Equations of Simple Birth and Death Processes, Phil. Trans. Royal Soc. London (A), Vol. 246, 1954, pp. 321-369.

20. Leese, E. L., and Boyd, D. W., Numerical Methods for Determining the Transient Behavior of Queues with Variable Arrival Rates, J. Canad. Operational Res. Soc., Vol. 4, 1966, pp. 1-13.

21. Little, J. D. C., A Proof for the Queueing Formula L = λW, Operations Research, Vol. 9, 1961, pp. 383-387.

22. Luchak, G., The Solution of the Single-Channel Queueing Equations Characterized by a Time-Dependent Arrival Rate and a General Class of Holding Times, Operations Research, Vol. 4, 1956, pp. 711-732.

23. May, A. D., and Keller, H. E. M., A Deterministic Queueing Model, Transportation Res. 1, 1967, pp. 117-128.

24. McEwen, T., Project Director, Allocation of Patrol Manpower Resources in the Saint Louis Police Department, Vols. I, II, 1966.

25. Moore, S. C., Approximating the Behavior of Nonstationary Single-Server Queues, Operations Research, Vol. 23, No. 5, 1975, pp. 1011-1031.

26. Morse, P. M., Stochastic Properties of Waiting Lines, Operations Research, Vol. 3, 1955, pp. 255-261.

27. Neuts, M. F., The Single Server Queue in Discrete Time Analysis I., Naval Res. Logist Quarterly, Vol. 20, 1973, pp. 297-304.

28. Neuts, M. F., The M/M/1 Queue with Randomly Varying Arrival and Service Rates, Operations Research, Vol. 15, 1978, pp. 139-157.

29. Neuts, M. F., Further Results on the M/M/1 Queue and Randomly Varying Rates, Opsearch, Vol. 15, 1978, pp. 158-168.

30. Newell, G. F., Approximate Methods for Queues with Application to the Fixed-Cycle Traffic Light, SIAM Rev. 7, 1965, pp. 225-240.

31. Newell, G. F., Queues with Time-Dependent Arrival Rates: I, II, and III, J. Appl. Probability, Vol. 5, 1968, pp. 436-451; 579-590; 591-606.

32. Newell, G. F., Application of Queueing Theory, Chapman and Hall, London, 1971.

33. Oliver, R. M. and Samuel, A. H., Reducing Letter Delays in Post Office, Operations Res., Vol. 10, 1962, pp. 839-892.

34. Ralston, A., A First Course in Numerical Analysis, McGraw-Hill, 1965.

35. Rider, K. L., A Simple Approximation to the Average Queue Size in the Time-Dependent M/M/1 Queue, J. Assoc. Comput. Mach. Vol. 23, 1976, pp. 361-367.

36. Rothkopf, M. H., and Oren, S. S., The Nonstationary M/M/s Queue, Management Science, Vol. 25, No. 6, 1979, pp. 522-554.

37. Salklin, H. and Spielberg, K., Adaptive Binary Programming, IBM Scientific Research Center, 410 East 62nd St., New York, Rep. No. 320-2951, March, 1968.

38. Shannon, Robert E., System Simulation is the Art and Science, Prentice-Hall Inc. 1975.

39. Wagner, H. M., Principle of Operations Research, 2nd Edition, Prentice-Hall, Englewood Cliffs, N.J., 1975.

40. Wang, K. K. G., Continuous Simulation of Queueing Networks with Transient and Time Varying Conditions, Ph.D. Dissertation, State University of New York at Stoneybrook, 1979.

41. Wragg, A., The Solution of an Infinite Set of Differential Difference Equations Occurring in Polymerization and Queueing Problems. Proc. Camb. Phil. Soc., Vol. 59, 1963, pp. 117-124.

APPENDIX A

M/M/s STEADY STATE QUEUE WITH THE PROBABILITY OF A CALL ENCOUNTERING A QUEUE AS A CRITERION

ILLEGIBLE DOCUMENT

THE FOLLOWING DOCUMENT(S) IS OF POOR LEGIBILITY IN THE ORIGINAL

THIS IS THE BEST COPY AVAILABLE

```
$JOB
C*************************************************************************
C
C
C     THIS PROGRAM CALCULATES THE STATE PROBABILITIES OF M/M/S
C     STEADY STATE QUEUEING MODEL AND SEARCH THE NUMBER OF PATROL
C     CARS REQUIRED SUCH THAT THE PROBABILITY OF A CALL ENCOUNTERING
C     A QUEUE IS LESS THAN THE SPECIFIED THRESHOLD.
C
C
C     NP: LENGTH OF CYCLE, IN THIS STUDY IT IS ONE DAY, STARTING
C         FROM 0 HOUR TO 23 HOUR.
C     UT: MEAN SERVICE RATE, A CONSTANT.
C     RADAT: MEAN ARRIVAL RATE, CHANGE WITH HOUR.
C     AL: THE SPECIFIED THRESHOLD, THE PROBABILITY OF A CALL NOT
C         ENCOUNTERING A QUEUE.
C     ROAT: EQUAL TO MEAN ARRIVAL RATE DIVIDED BY MEAN SERVICE RATE.
C     NT: NUMBER OF CARS ON DUTY.
C     FACNT: FACTORIAL OF NT.
C     PTO: PROBABILITY OF ZERO CALL IN THE SYSTEM, PO.
C     PT(J): PROBABILITY OF J CALLS IN THE SYSTEM, OCJ.
C
C
C*************************************************************************
      DIMENSION PT(100)
      READ,M,NP,UT,AL
      PRINT 5
    5 FORMAT('1')
      PRINT 6,M,NP,UT,AL
    6 FORMAT('0','TRUNCATED CAPACITY = ',I3,5X,'PERIOD OF CYCLE = ',I3/
     1' ','MEAN SERVICE RATE = ',F8.4,5X,'SPECIFIED THRESHOLD = ',F8.6)
      DO 80 K=1,NP
      READ,RADAT
      NT=1
      GO TO 20
   10 NT=NT+1
   20 ROAT=RADAT/UT
      FACTJ=1
      SUM=1
      IF(NT.EQ.1) GO TO 30
      NT1=NT-1
      DO 25 J=1,NT1
      FACTJ=FACTJ*J
   25 SUM=SUM+ROAT**J/FACTJ
   30 FACNT=1
      DO 31 I=1,NT
   31 FACNT=FACNT*I
      PLUS=ROAT**NT/(FACNT*(1-ROAT/NT))
      PTO=1./(SUM+PLUS)
      SUMPT=PTO
      IF(NT.EQ.1) GO TO 34
      FACTJ=1
      NT2=NT-1
      DO 33 J=1,NT2
      FACTJ=FACTJ*J
      PT(J)=ROAT**J*PTO/FACTJ
   33 SUMPT=SUMPT+PT(J)
   34 PT(NT)=ROAT**NT*PTO/FACNT
      IF(SUMPT.LE.(1.-AL)) GO TO 10
      SUM=SUMPT+PT(NT)
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      NT3=NT+1
      DO 35 J=NT3,M
      PT(J)=(RCAT**J)*PTO/(FLOAT(NT)**(J-NT)*FACNT)
35    SUMPT=SUMPT+PT(J)
      PNCD=1.-SUMPT
      PRINT 40,K-1,NT,PNCD
40    FORMAT('-',     'AT HOUR',I3,5X,'THE NUMBER OF PATROL CARS REQUIRED
     1 = ',I3,5X,'THE PROBABILITY OF NO CARS ON DUTY = ',F8.6)
      PRINT 45,RADAT
45    FORMAT('0','MEAN ARRIVAL RATE = ',F8.4)
      PRINT 60,PTO,(J,PT(J),J=1,M)
60    FORMAT(' ','P(0) = ',F8.6/' ',6(' ','P(',I2,') = ',F8.6,2X))
      PRINT 70,M,SUM
70    FORMAT(' ','TOTAL PROBABILITY FROM 1 TO ',I2,' CALLS IN THE SYSTEM
     1 = ',F8.6)
80    CONTINUE
      PRINT 85
85    FORMAT('1')
      STOP
      END

$ENTRY
```

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TRUNCATED CAPACITY =  30      PERIOD OF CYCLE =  24
MEAN SERVICE RATE =   2.0000      SPECIFIED THRESHOLD = 0.100000

AT HOUR  0      THE NUMBER OF PATROL CARS REQUIRED =   9      THE PROBABILITY OF NO CARS ON DUTY = 0.072512

MEAN ARRIVAL RATE =   5.5000
P(0) = 0.007361
 P( 1) = 0.036870   P( 2) = 0.088372   P( 3) = 0.144341   P( 4) = 0.176817   P( 5) = 0.173281   P( 6) = 0.141513
P( 7) = 0.099059   P( 8) = 0.060674   P( 9) = 0.033033   P(10) = 0.017985   P(11) = 0.009792   P(12) = 0.005331
P(13) = 0.002902   P(14) = 0.001580   P(15) = 0.000860   P(16) = 0.000468   P(17) = 0.000255   P(18) = 0.000139
P(19) = 0.000076   P(20) = 0.000041   P(21) = 0.000022   P(22) = 0.000012   P(23) = 0.000007   P(24) = 0.000004
P(25) = 0.000002   P(26) = 0.000001   P(27) = 0.000001   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  1      THE NUMBER OF PATROL CARS REQUIRED =   9      THE PROBABILITY OF NO CARS ON DUTY = 0.065088

MEAN ARRIVAL RATE =   9.6000
P(0) = 0.008145
 P( 1) = 0.039115   P( 2) = 0.093876   P( 3) = 0.150201   P( 4) = 0.180241   P( 5) = 0.173032   P( 6) = 0.138425
P( 7) = 0.094920   P( 8) = 0.056952   P( 9) = 0.030374   P(10) = 0.016200   P(11) = 0.008640   P(12) = 0.004608
P(13) = 0.002458   P(14) = 0.001311   P(15) = 0.000699   P(16) = 0.000373   P(17) = 0.000199   P(18) = 0.000106
P(19) = 0.000057   P(20) = 0.000030   P(21) = 0.000016   P(22) = 0.000009   P(23) = 0.000005   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  2      THE NUMBER OF PATROL CARS REQUIRED =   8      THE PROBABILITY OF NO CARS ON DUTY = 0.086605

MEAN ARRIVAL RATE =   8.7000
P(0) = 0.012714
 P( 1) = 0.055304   P( 2) = 0.120286   P( 3) = 0.174414   P( 4) = 0.185676   P( 5) = 0.165018   P( 6) = 0.119638
P( 7) = 0.074346   P( 8) = 0.040426   P( 9) = 0.021982   P(10) = 0.011952   P(11) = 0.006499   P(12) = 0.003534
P(13) = 0.001922   P(14) = 0.001045   P(15) = 0.000568   P(16) = 0.000309   P(17) = 0.000168   P(18) = 0.000091
P(19) = 0.000050   P(20) = 0.000027   P(21) = 0.000015   P(22) = 0.000008   P(23) = 0.000004   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  3      THE NUMBER OF PATROL CARS REQUIRED =   8      THE PROBABILITY OF NO CARS ON DUTY = 0.045681

MEAN ARRIVAL RATE =   7.6000
P(0) = 0.022241
 P( 1) = 0.084515   P( 2) = 0.160579   P( 3) = 0.203400   P( 4) = 0.193230   P( 5) = 0.146855   P( 6) = 0.093008
P( 7) = 0.050490   P( 8) = 0.023883   P( 9) = 0.011392   P(10) = 0.005411   P(11) = 0.002570   P(12) = 0.001221
P(13) = 0.000580   P(14) = 0.000275   P(15) = 0.000131   P(16) = 0.000062   P(17) = 0.000030   P(18) = 0.000014
P(19) = 0.000007   P(20) = 0.000003   P(21) = 0.000002   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR  4      THE NUMBER OF PATROL CARS REQUIRED =   7      THE PROBABILITY OF NO CARS ON DUTY = 0.062656

MEAN ARRIVAL RATE =   6.7000
P(0) = 0.034776
 P( 1) = 0.116499   P( 2) = 0.195136   P( 3) = 0.217902   P( 4) = 0.182493   P( 5) = 0.122270   P( 6) = 0.068268
P( 7) = 0.032671   P( 8) = 0.015635   P( 9) = 0.007483   P(10) = 0.003581   P(11) = 0.001714   P(12) = 0.000820
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P(13) = 0.000393   P(14) = 0.000184   P(15) = 0.000090   P(16) = 0.000043   P(17) = 0.000021   P(18) = 0.000010
P(19) = 0.000005   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR  5     THE NUMBER OF PATROL CARS REQUIRED =   6     THE PROBABILITY OF NO CARS ON DUTY = 0.060376

MEAN ARRIVAL RATE =   5.3000
P(0) = 0.070086
 P( 1) = 0.185723   P( 2) = 0.246095   P( 3) = 0.217376   P( 4) = 0.144013   P( 5) = 0.076327   P( 6) = 0.033711
P( 7) = 0.014895   P( 8) = 0.005576   P( 9) = 0.002904   P(10) = 0.001288   P(11) = 0.000567   P(12) = 0.000250
P(13) = 0.000111   P(14) = 0.000049   P(15) = 0.000022   P(16) = 0.000010   P(17) = 0.000004   P(18) = 0.000002
P(19) = 0.000001   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999599


AT HOUR  6     THE NUMBER OF PATROL CARS REQUIRED =   5     THE PROBABILITY OF NO CARS ON DUTY = 0.065273

MEAN ARRIVAL RATE =   4.1000
P(0) = 0.127643
 P( 1) = 0.261668   P( 2) = 0.268210   P( 3) = 0.183277   P( 4) = 0.093929   P( 5) = 0.038511   P( 6) = 0.015790
P( 7) = 0.006474   P( 8) = 0.002654   P( 9) = 0.001088   P(10) = 0.000446   P(11) = 0.000183   P(12) = 0.000075
P(13) = 0.000031   P(14) = 0.000013   P(15) = 0.000005   P(16) = 0.000002   P(17) = 0.000001   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR  7     THE NUMBER OF PATROL CARS REQUIRED =   4     THE PROBABILITY OF NO CARS ON DUTY = 0.090700

MEAN ARRIVAL RATE =   3.2000
P(0) = 0.199292
 P( 1) = 0.318866   P( 2) = 0.255093   P( 3) = 0.136050   P( 4) = 0.054420   P( 5) = 0.021768   P( 6) = 0.008707
P( 7) = 0.003483   P( 8) = 0.001393   P( 9) = 0.000557   P(10) = 0.000223   P(11) = 0.000089   P(12) = 0.000036
P(13) = 0.000014   P(14) = 0.000006   P(15) = 0.000002   P(16) = 0.000001   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR  8     THE NUMBER OF PATROL CARS REQUIRED =   4     THE PROBABILITY OF NO CARS ON DUTY = 0.042218

MEAN ARRIVAL RATE =   2.5000
P(0) = 0.285328
 P( 1) = 0.356660   P( 2) = 0.222913   P( 3) = 0.092890   P( 4) = 0.029025   P( 5) = 0.009070   P( 6) = 0.002834
P( 7) = 0.000886   P( 8) = 0.000277   P( 9) = 0.000087   P(10) = 0.000027   P(11) = 0.000008   P(12) = 0.000003
P(13) = 0.000001   P(14) = 0.000000   P(15) = 0.000000   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR  9     THE NUMBER OF PATROL CARS REQUIRED =   4     THE PROBABILITY OF NO CARS ON DUTY = 0.042218

MEAN ARRIVAL RATE =   2.5000
P(0) = 0.285329
 P( 1) = 0.356660   P( 2) = 0.222913   P( 3) = 0.092880   P( 4) = 0.029025   P( 5) = 0.009070   P( 6) = 0.002834
P( 7) = 0.000886   P( 8) = 0.000277   P( 9) = 0.000087   P(10) = 0.000027   P(11) = 0.000008   P(12) = 0.000003
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P(13) = 0.000328   P(14) = 0.000161   P(15) = 0.000079   P(16) = 0.000039   P(17) = 0.000019   P(18) = 0.000009
P(19) = 0.000005   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR 15     THE NUMBER OF PATROL CARS REQUIRED =   7     THE PROBABILITY OF NO CARS ON DUTY = 0.058534

MEAN ARRIVAL RATE =   6.6000
P(0) = 0.036585
 P( 1) = 0.120744   P( 2) = 0.199227   P( 3) = 0.219150   P( 4) = 0.180799   P( 5) = 0.119327   P( 6) = 0.065630
P( 7) = 0.030940   P( 8) = 0.014586   P( 9) = 0.006876   P(10) = 0.003242   P(11) = 0.001528   P(12) = 0.000720
P(13) = 0.000340   P(14) = 0.000160   P(15) = 0.000075   P(16) = 0.000036   P(17) = 0.000017   P(18) = 0.000008
P(19) = 0.000004   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR 16     THE NUMBER OF PATROL CARS REQUIRED =   8     THE PROBABILITY OF NO CARS ON DUTY = 0.052061

MEAN ARRIVAL RATE =   7.8000
P(0) = 0.020101
 P( 1) = 0.078393   P( 2) = 0.152866   P( 3) = 0.198726   P( 4) = 0.193758   P( 5) = 0.151131   P( 6) = 0.098235
P( 7) = 0.054731   P( 8) = 0.026681   P( 9) = 0.013007   P(10) = 0.006341   P(11) = 0.003091   P(12) = 0.001507
P(13) = 0.000735   P(14) = 0.000358   P(15) = 0.000175   P(16) = 0.000085   P(17) = 0.000041   P(18) = 0.000020
P(19) = 0.000010   P(20) = 0.000005   P(21) = 0.000002   P(22) = 0.000001   P(23) = 0.000001   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR 17     THE NUMBER OF PATROL CARS REQUIRED =   8     THE PROBABILITY OF NO CARS ON DUTY = 0.083370

MEAN ARRIVAL RATE =   9.6000
P(0) = 0.013381
 P( 1) = 0.057529   P( 2) = 0.123738   P( 3) = 0.177314   P( 4) = 0.190613   P( 5) = 0.163927   P( 6) = 0.117481
P( 7) = 0.072167   P( 8) = 0.038790   P( 9) = 0.020850   P(10) = 0.011207   P(11) = 0.006024   P(12) = 0.003238
P(13) = 0.001740   P(14) = 0.000935   P(15) = 0.000503   P(16) = 0.000270   P(17) = 0.000145   P(18) = 0.000078
P(19) = 0.000042   P(20) = 0.000023   P(21) = 0.000012   P(22) = 0.000007   P(23) = 0.000004   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999995

AT HOUR 18     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.058217

MEAN ARRIVAL RATE =   9.4000
P(0) = 0.009019
 P( 1) = 0.042389   P( 2) = 0.099615   P( 3) = 0.156064   P( 4) = 0.183375   P( 5) = 0.172373   P( 6) = 0.135025
P( 7) = 0.090660   P( 8) = 0.053263   P( 9) = 0.027815   P(10) = 0.014526   P(11) = 0.007586   P(12) = 0.003961
P(13) = 0.002069   P(14) = 0.001080   P(15) = 0.000564   P(16) = 0.000295   P(17) = 0.000154   P(18) = 0.000080
P(19) = 0.000042   P(20) = 0.000022   P(21) = 0.000011   P(22) = 0.000006   P(23) = 0.000003   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR 19     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.072512

MEAN ARRIVAL RATE =   9.8000
P(0) = 0.007361
 P( 1) = 0.036070   P( 2) = 0.088372   P( 3) = 0.144341   P( 4) = 0.176817   P( 5) = 0.173281   P( 6) = 0.141513
P( 7) = 0.099059   P( 8) = 0.060674   P( 9) = 0.033033   P(10) = 0.017985   P(11) = 0.009792   P(12) = 0.005331
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```
P(13) = 0.000001   P(14) = 0.000000   P(15) = 0.000000   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 10      THE NUMBER OF PATROL CARS REQUIRED =   4      THE PROBABILITY OF NO CARS ON DUTY = 0.067218

MEAN ARRIVAL RATE =   2.9000
P(0) = 0.232651
 P( 1) = 0.337345   P( 2) = 0.244575   P( 3) = 0.118211   P( 4) = 0.042852   P( 5) = 0.015534   P( 6) = 0.005631
P( 7) = 0.002041   P( 8) = 0.000740   P( 9) = 0.000268   P(10) = 0.000097   P(11) = 0.000035   P(12) = 0.000013
P(13) = 0.000005   P(14) = 0.000002   P(15) = 0.000001   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 11      THE NUMBER OF PATROL CARS REQUIRED =   5      THE PROBABILITY OF NO CARS ON DUTY = 0.049496

MEAN ARRIVAL RATE =   3.8000
P(0) = 0.148722
 P( 1) = 0.282571   P( 2) = 0.268442   P( 3) = 0.170013   P( 4) = 0.080756   P( 5) = 0.030687   P( 6) = 0.011661
P( 7) = 0.004431   P( 8) = 0.001684   P( 9) = 0.000640   P(10) = 0.000243   P(11) = 0.000092   P(12) = 0.000035
P(13) = 0.000013   P(14) = 0.000005   P(15) = 0.000002   P(16) = 0.000001   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 12      THE NUMBER OF PATROL CARS REQUIRED =   5      THE PROBABILITY OF NO CARS ON DUTY = 0.077382

MEAN ARRIVAL RATE =   4.3000
P(0) = 0.115213
 P( 1) = 0.247707   P( 2) = 0.266285   P( 3) = 0.190838   P( 4) = 0.102575   P( 5) = 0.044107   P( 6) = 0.018966
P( 7) = 0.008155   P( 8) = 0.003507   P( 9) = 0.001508   P(10) = 0.000648   P(11) = 0.000279   P(12) = 0.000120
P(13) = 0.000052   P(14) = 0.000022   P(15) = 0.000010   P(16) = 0.000004   P(17) = 0.000002   P(18) = 0.000001
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 13      THE NUMBER OF PATROL CARS REQUIRED =   6      THE PROBABILITY OF NO CARS ON DUTY = 0.047445

MEAN ARRIVAL RATE =   5.0000
P(0) = 0.081620
 P( 1) = 0.204051   P( 2) = 0.255065   P( 3) = 0.212553   P( 4) = 0.132845   P( 5) = 0.066423   P( 6) = 0.027676
P( 7) = 0.011532   P( 8) = 0.004805   P( 9) = 0.002002   P(10) = 0.000834   P(11) = 0.000346   P(12) = 0.000145
P(13) = 0.000060   P(14) = 0.000025   P(15) = 0.000010   P(16) = 0.000004   P(17) = 0.000002   P(18) = 0.000001
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 14      THE NUMBER OF PATROL CARS REQUIRED =   6      THE PROBABILITY OF NO CARS ON DUTY = 0.052830

MEAN ARRIVAL RATE =   5.9000
P(0) = 0.051552
 P( 1) = 0.152077   P( 2) = 0.224314   P( 3) = 0.220575   P( 4) = 0.162674   P( 5) = 0.095978   P( 6) = 0.047189
P( 7) = 0.023201   P( 8) = 0.011407   P( 9) = 0.005609   P(10) = 0.002758   P(11) = 0.001356   P(12) = 0.000667
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P(13) = 0.002902   P(14) = 0.001550   P(15) = 0.000860   P(16) = 0.000468   P(17) = 0.000254   P(18) = 0.000139
P(19) = 0.000076   P(20) = 0.000041   P(21) = 0.000022   P(22) = 0.000012   P(23) = 0.000007   P(24) = 0.000004
P(25) = 0.000002   P(26) = 0.000001   P(27) = 0.000001   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999995


AT HOUR 20     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.089102

MEAN ARRIVAL RATE =  10.2000
P(0) = 0.006003
 P( 1) = 0.030613   P( 2) = 0.078064   P( 3) = 0.132709   P( 4) = 0.169204   P( 5) = 0.172588   P( 6) = 0.146700
P( 7) = 0.106881   P( 8) = 0.068137   P( 9) = 0.038611   P(10) = 0.021879   P(11) = 0.012398   P(12) = 0.007026
P(13) = 0.003981   P(14) = 0.002256   P(15) = 0.001278   P(16) = 0.000724   P(17) = 0.000411   P(18) = 0.000233
P(19) = 0.000132   P(20) = 0.000075   P(21) = 0.000042   P(22) = 0.000024   P(23) = 0.000014   P(24) = 0.000008
P(25) = 0.000004   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 21     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.098307

MEAN ARRIVAL RATE =  10.4000
P(0) = 0.005418
 P( 1) = 0.028175   P( 2) = 0.073254   P( 3) = 0.126974   P( 4) = 0.165066   P( 5) = 0.171668   P( 6) = 0.148779
P( 7) = 0.110521   P( 8) = 0.071839   P( 9) = 0.041507   P(10) = 0.023582   P(11) = 0.013856   P(12) = 0.008006
P(13) = 0.004626   P(14) = 0.002673   P(15) = 0.001544   P(16) = 0.000892   P(17) = 0.000515   P(18) = 0.000298
P(19) = 0.000172   P(20) = 0.000099   P(21) = 0.000057   P(22) = 0.000033   P(23) = 0.000019   P(24) = 0.000011
P(25) = 0.000006   P(26) = 0.000004   P(27) = 0.000002   P(28) = 0.000001   P(29) = 0.000001   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999998


AT HOUR 22     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.089102

MEAN ARRIVAL RATE =  10.2000
P(0) = 0.006003
 P( 1) = 0.030613   P( 2) = 0.078064   P( 3) = 0.132709   P( 4) = 0.169204   P( 5) = 0.172588   P( 6) = 0.146700
P( 7) = 0.106881   P( 8) = 0.068137   P( 9) = 0.038611   P(10) = 0.021879   P(11) = 0.012398   P(12) = 0.007026
P(13) = 0.003981   P(14) = 0.002256   P(15) = 0.001278   P(16) = 0.000724   P(17) = 0.000411   P(18) = 0.000233
P(19) = 0.000132   P(20) = 0.000075   P(21) = 0.000042   P(22) = 0.000024   P(23) = 0.000014   P(24) = 0.000008
P(25) = 0.000004   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 23     THE NUMBER OF PATROL CARS REQUIRED =   9     THE PROBABILITY OF NO CARS ON DUTY = 0.080515

MEAN ARRIVAL RATE =  10.0001
P(0) = 0.006648
 P( 1) = 0.033240   P( 2) = 0.083100   P( 3) = 0.138501   P( 4) = 0.173128   P( 5) = 0.173129   P( 6) = 0.144276
P( 7) = 0.103055   P( 8) = 0.064410   P( 9) = 0.035784   P(10) = 0.019880   P(11) = 0.011045   P(12) = 0.006136
P(13) = 0.003409   P(14) = 0.001894   P(15) = 0.001052   P(16) = 0.000585   P(17) = 0.000325   P(18) = 0.000180
P(19) = 0.000100   P(20) = 0.000056   P(21) = 0.000031   P(22) = 0.000017   P(23) = 0.000010   P(24) = 0.000005
P(25) = 0.000003   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999
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APPENDIX B

M/M/s NONSTEADY STATE QUEUE WITH THE PROBABILITY OF A CALL ENCOUNTERING A QUEUE AS A CRITERION

B. 1 PROGRAM LIST

```
      $JOB              ,PAGES=60
      C*****************************************************************
      C
      C
      C       THIS PROGRAM CALCULATES TRANSIENT STATE PROBABILITIES AND
      C       THE PROBABILITY OF NO CAR ON DUTY IN THE SYSTEM FOR A NK SERVER
      C       TRUNCATED M MAXIMUM ALLOWED CAPACITY QUEUEING SYSTEM BY THE
      C       FOURTH ORDER RUNGE KUTTA METHOD.
      C
      C
      C       M: MAXIMUM ALLOWED CAPACITY IN THE QUEUEING SYSTEM.
      C       DELT: INTEGRATING STEP SIZE.
      C       T: TOTAL INTEGRATING LENGTH, 48 HOURS SUGGESTED.
      C       U: MEAN SERVICE RATE, A CONSTANT.
      C       RK: MEAN ARRIVAL RATE, CHANGE WITH TIME.
      C       NK: NUMBER OF CARS ON DUTY, CHANG WITH TIME.
      C      PK(J): PROBABILITY OF J-1 CALLS IN SYSTEM.
      C
      C
      C*****************************************************************
 1          DIMENSION D1(100),D2(100),D3(100),D4(100),PK(100)
 2          FO(A,B,U,RANDK)=-RANDK*A+U*B
 3          FLN(A,B,C,U,RANDK,J)=RANDK*A-(RANDK+FLOAT(J)*U)*B+FLOAT(J)*U*C+U*B
 4          FON(A,B,C,U,RANDK,NK)=RANDK*A-(RANDK+FLOAT(NK)*U)*B+FLOAT(NK)*U*C
 5          FM(A,B,U,RANDK,NK)=RANDK*A-FLOAT(NK)*U*B
 6          READ,M,DELT,T,U
 7          M1=M+1
 8          DO 10 J=1,M1
 9     10   READ,PK(J)
10          ITER=T/DELT
11          READ,NK,RK1,RK2
12          ND=1
13          N=1
14          IND=1
15          PRINT 14,M,DELT,T,U
16     14   FORMAT('1',//'0','TRUNCATED CAPACITY = ',I3,5X,'STEP SIZE = ',F8.5
           1/' ','INTEGRATED LENGTH = ',F8.4,5X,'MEAN SERVICE RATE = ',F8.4)
17          PRINT 15,NK,(J-1,PK(J),J=1,M1)
18     15   FORMAT('0',5X,'AT HOUR 0',5X,'PATROL CAR ON DUTY =',I3/' ',
           16(' ','P(',I3,')=',F8.6))
19          DO 70 K=1,ITER
20          RM=(1./DELT)*FLOAT(N)+1.
21          IF(K.LT.RM) GO TO 20
22          READ,NK,RK3
23          RK1=RK2
24          RK2=RK3
25          NH=N
26          IF(NH.GE.48) NH=NH-24
27          IF(NH.GE.24) NH=NH-24
28          PRINT 16,NH,NK
29     16   FORMAT('0','AT HOUR',I3,5X,'PATROL CARS ON DUTY =',I3)
30          N=N+1
31          ND=1
32     20   RANDK=(RK2-RK1)*FLOAT(ND)*DELT+RK1
33          ND=ND+1
34          D1(1)=FO(PK(1),PK(2),U,RANDK)*DELT
35          NK1=NK+1
36          DO 50 J=2,M1
37          IF(J.GE.NK1) GO TO 30
38          D1(J)=FLN(PK(J-1),PK(J),PK(J+1),U,RANDK,J)*DELT
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      GO TO 50
   30 IF (J.EQ.M1)GO TO 40
      D1(J)=FGN(PK(J-1),PK(J),PK(J+1),U,RANDK,NK)*DELT
      GO TO 50
   40 D1(J)=FM(PK(J-1),PK(J),U,RANDK,NK)*DELT
   50 CONTINUE
      D2(1)=FO(PK(1)+1./2.*D1(1),PK(2)+1./2.*D1(2),U,RANDK)*DELT
      DO 51 J=2,M1
      IF(J.GE.NK1)GO TO 31
      D2(J)=FLN(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),
     1PK(J+1)+1./2.*D1(J+1),U,RANDK,J)*DELT
      GO TO 51
   31 IF(J.EQ.M1)GO TO 41
      D2(J)=FGN(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),
     1PK(J+1)+1./2.*D1(J+1),U,RANDK,NK)*DELT
      GO TO 51
   41 D2(J)=FM(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),U,RANDK,NK)*DELT
   51 CONTINUE
      D3(1)=FO(PK(1)+1./2.*D2(1),PK(2)+1./2.*D2(2),U,RANDK)*DELT
      DO 52 J=2,M1
      IF(J.GE.NK1)GO TO 32
      D3(J)=FLN(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),
     1PK(J+1)+1./2.*D2(J+1),U,RANDK,J)*DELT
      GO TO 52
   32 IF(J.EQ.M1)GO TO 42
      D3(J)=FGN(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),
     1PK(J+1)+1./2.*D2(J+1),U,RANDK,NK)*DELT
      GO TO 52
   42 D3(J)=FM(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),U,RANDK,NK)*DELT
   52 CONTINUE
      D4(1)=FO(PK(1)+D3(1),PK(2)+D3(2),U,RANDK)*DELT
      DO 53 J=2,M1
      IF(J.GE.NK1)GO TO 33
      D4(J)=FLN(PK(J-1)+D3(J-1),PK(J)+D3(J),PK(J+1)+D3(J+1),U,RANDK,
     1J)*DELT
      GO TO 53
   33 IF(J.EQ.M1) GO TO 43
      D4(J)=FGN(PK(J-1)+D3(J-1),PK(J)+D3(J),PK(J+1)+D3(J+1),U,RANDK,
     1NK)*DELT
      GO TO 53
   43 D4(J)=FM(PK(J-1)+D3(J-1),PK(J)+D3(J),U,RANDK,NK)*DELT
   53 CONTINUE
      DO 60 J=1,M1
      PK(J)=PK(J)+1./6.*(D1(J)+2.*D2(J)+2.*D3(J)+D4(J))
   60 IF(PK(J).LT.0.) PK(J)=0.
      INDPRI=20*INC
      IF(K.LT.INDPRI) GO TO 70
   61 PRINT 65,K,RANDK,(J-1,PK(J),J=1,M1)
   65 FORMAT('0','STEP',I4,5X,'MEAN ARRIVAL RATE = ',F8.4/' ',6(' ',
     1'P(',I3,')=',F8.6))
      SUM=0
      DO 66 J=1,NK
   66 SUM=SUM+PK(J)
      SUM1=SUM
      DO 68 J=NK1,M1
   68 SUM1=SUM1+PK(J)
      PRINT 69,SUM1
   69 FORMAT(' ','TOTAL PROBABILITY =',F8.6)
      PRINT 63,SUM1-SUM
   63 FORMAT(' ','PROBABILITY NO CAR ON DUTY = ',F8.6)
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92            IND=IND+1
93       70   CONTINUE
94            STOP
95            END

      $ENTRY
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AT HOUR 22      PATROL CARS ON DUTY = 12

STEP 900      MEAN ARRIVAL RATE =  10.1000
 P(  0)=0.096048 P(  1)=0.030816 P(  2)=0.078529 P(  3)=0.133479 P(  4)=0.179301 P(  5)=0.174041
P(  6)=0.148492 P(  7)=0.108880 P(  8)=0.070115 P(  9)=0.040343 P( 10)=0.021045 P( 11)=0.010087
P( 12)=0.004506 P( 13)=0.002069 P( 14)=0.000981 P( 15)=0.000483 P( 16)=0.000246 P( 17)=0.000130
P( 18)=0.000070 P( 19)=0.000038 P( 20)=0.000021 P( 21)=0.000012 P( 22)=0.000007 P( 23)=0.000004
P( 24)=0.000002 P( 25)=0.000001 P( 26)=0.000001 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000739
PROBABILITY NO CAR ON DUTY = 0.008565

STEP 920      MEAN ARRIVAL RATE =  10.0000
 P(  0)=0.006364 P(  1)=0.032157 P(  2)=0.081259 P(  3)=0.136931 P(  4)=0.173116 P(  5)=0.175162
P(  6)=0.147755 P(  7)=0.106911 P(  8)=0.067736 P(  9)=0.038186 P( 10)=0.019404 P( 11)=0.008984
P( 12)=0.003829 P( 13)=0.001644 P( 14)=0.000714 P( 15)=0.000314 P( 16)=0.000141 P( 17)=0.000065
P( 18)=0.000030 P( 19)=0.000015 P( 20)=0.000007 P( 21)=0.000004 P( 22)=0.000002 P( 23)=0.000001
P( 24)=0.000001 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
PROBABILITY NO CAR ON DUTY = 0.006760


AT HOUR 23      PATROL CARS ON DUTY = 12

STEP 940      MEAN ARRIVAL RATE =   9.9000
 P(  0)=0.006716 P(  1)=0.033652 P(  2)=0.084049 P(  3)=0.140162 P(  4)=0.175319 P(  5)=0.175452
P(  6)=0.146338 P(  7)=0.104633 P(  8)=0.065474 P(  9)=0.036427 P( 10)=0.018247 P( 11)=0.008315
P( 12)=0.003478 P( 13)=0.001459 P( 14)=0.000614 P( 15)=0.000260 P( 16)=0.000111 P( 17)=0.000047
P( 18)=0.000021 P( 19)=0.000009 P( 20)=0.000004 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
PROBABILITY NO CAR ON DUTY = 0.006600

STEP 960      MEAN ARRIVAL RATE =   9.8000
 P(  0)=0.007080 P(  1)=0.035050 P(  2)=0.086759 P(  3)=0.143172 P(  4)=0.177203 P(  5)=0.175464
P(  6)=0.144789 P(  7)=0.102413 P(  8)=0.062388 P(  9)=0.034878 P( 10)=0.017275 P( 11)=0.007781
P( 12)=0.003215 P( 13)=0.001331 P( 14)=0.000552 P( 15)=0.000229 P( 16)=0.000096 P( 17)=0.000040
P( 18)=0.000017 P( 19)=0.000007 P( 20)=0.000003 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
PROBABILITY NO CAR ON DUTY = 0.005488


AT HOUR  0      PATROL CARS ON DUTY = 10

STEP 980      MEAN ARRIVAL RATE =   9.7000
 P(  0)=0.007492 P(  1)=0.036507 P(  2)=0.089422 P(  3)=0.146010 P(  4)=0.178775 P(  5)=0.175066
P(  6)=0.142798 P(  7)=0.099770 P(  8)=0.060930 P(  9)=0.033025 P( 10)=0.016071 P( 11)=0.007192
P( 12)=0.003756 P( 13)=0.001795 P( 14)=0.000849 P( 15)=0.000396 P( 16)=0.000182 P( 17)=0.000083
P( 18)=0.000037 P( 19)=0.000016 P( 20)=0.000007 P( 21)=0.000003 P( 22)=0.000001 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000739
PROBABILITY NO CAR ON DUTY = 0.030985
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STEP1000     MEAN ARRIVAL RATE =   9.6000
 P(  0)=0.007831 P(  1)=0.037964 P(  2)=0.092017 P(  3)=0.148678 P(  4)=0.180157 P(  5)=0.174624
P(  6)=0.141035 P(  7)=0.097622 P(  8)=0.059118 P(  9)=0.031819 P( 10)=0.015414 P( 11)=0.007470
P( 12)=0.003619 P( 13)=0.001752 P( 14)=0.000846 P( 15)=0.000408 P( 16)=0.000195 P( 17)=0.000093
P( 18)=0.000044 P( 19)=0.000021 P( 20)=0.000010 P( 21)=0.000004 P( 22)=0.000002 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
PROBABILITY NO CAR ON DUTY = 0.029374


AT HOUR  1     PATROL CARS ON DUTY = 10

STEP1020     MEAN ARRIVAL RATE =   9.1500
 P(  0)=0.008796 P(  1)=0.041615 P(  2)=0.096437 P(  3)=0.155230 P(  4)=0.183590 P(  5)=0.173708
P(  6)=0.136971 P(  7)=0.092585 P(  8)=0.054776 P(  9)=0.028827 P( 10)=0.013678 P( 11)=0.006521
P( 12)=0.003121 P( 13)=0.001498 P( 14)=0.000720 P( 15)=0.000347 P( 16)=0.000167 P( 17)=0.000080
P( 18)=0.000039 P( 19)=0.000019 P( 20)=0.000009 P( 21)=0.000004 P( 22)=0.000002 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
PROBABILITY NO CAR ON DUTY = 0.026201

STEP1040     MEAN ARRIVAL RATE =   8.7000
 P(  0)=0.010582 P(  1)=0.048109 P(  2)=0.109357 P(  3)=0.165725 P(  4)=0.188370 P(  5)=0.171303
P(  6)=0.129841 P(  7)=0.084382 P(  8)=0.048016 P(  9)=0.024321 P( 10)=0.011123 P( 11)=0.005126
P( 12)=0.002380 P( 13)=0.001112 P( 14)=0.000523 P( 15)=0.000247 P( 16)=0.000117 P( 17)=0.000056
P( 18)=0.000027 P( 19)=0.000013 P( 20)=0.000006 P( 21)=0.000003 P( 22)=0.000001 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
PROBABILITY NO CAR ON DUTY = 0.020730


AT HOUR  2     PATROL CARS ON DUTY =  9

STEP1060     MEAN ARRIVAL RATE =   8.1500
 P(  0)=0.013313 P(  1)=0.057464 P(  2)=0.124015 P(  3)=0.178405 P(  4)=0.192449 P(  5)=0.166032
P(  6)=0.119325 P(  7)=0.073481 P(  8)=0.039594 P(  9)=0.018987 P( 10)=0.009152 P( 11)=0.004421
P( 12)=0.002134 P( 13)=0.001028 P( 14)=0.000493 P( 15)=0.000235 P( 16)=0.000112 P( 17)=0.000053
P( 18)=0.000025 P( 19)=0.000012 P( 20)=0.000006 P( 21)=0.000003 P( 22)=0.000001 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000735
PROBABILITY NO CAR ON DUTY = 0.036657

STEP1080     MEAN ARRIVAL RATE =   7.6000
 P(  0)=0.017226 P(  1)=0.069900 P(  2)=0.141819 P(  3)=0.191826 P(  4)=0.194611 P(  5)=0.157976
P(  6)=0.106908 P(  7)=0.062069 P(  8)=0.031598 P(  9)=0.014369 P( 10)=0.006613 P( 11)=0.003077
P( 12)=0.001444 P( 13)=0.000683 P( 14)=0.000325 P( 15)=0.000155 P( 16)=0.000074 P( 17)=0.000035
P( 18)=0.000017 P( 19)=0.000008 P( 20)=0.000004 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000734
PROBABILITY NO CAR ON DUTY = 0.026801
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AT HOUR  3      PATROL CARS ON DUTY =  8

STEP1100      MEAN ARRIVAL RATE =    7.1500
 P(  0)=0.022101 P(  1)=0.084163 P(  2)=0.160231 P(  3)=0.203324 P(  4)=0.193443 P(  5)=0.147173
P(  6)=0.093273 P(  7)=0.050670 P(  8)=0.024125 P(  9)=0.011561 P( 10)=0.005557 P( 11)=0.002671
P( 12)=0.001280 P( 13)=0.000612 P( 14)=0.000291 P( 15)=0.000138 P( 16)=0.000066 P( 17)=0.000031
P( 18)=0.000015 P( 19)=0.000007 P( 20)=0.000003 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000732
PROBABILITY NO CAR ON DUTY = 0.046354

STEP1120      MEAN ARRIVAL RATE =    6.7000
 P(  0)=0.027903 P(  1)=0.059723 P(  2)=0.178179 P(  3)=0.212300 P(  4)=0.189729 P(  5)=0.135704
P(  6)=0.080964 P(  7)=0.041493 P(  8)=0.018711 P(  9)=0.008550 P( 10)=0.003954 P( 11)=0.001847
P( 12)=0.000869 P( 13)=0.000412 P( 14)=0.000196 P( 15)=0.000093 P( 16)=0.000044 P( 17)=0.000021
P( 18)=0.000010 P( 19)=0.000005 P( 20)=0.000002 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000732
PROBABILITY NO CAR ON DUTY = 0.034713


AT HOUR  4      PATROL CARS ON DUTY =  7

STEP1140      MEAN ARRIVAL RATE =    6.0000
 P(  0)=0.036756 P(  1)=0.121204 P(  2)=0.199770 P(  3)=0.219417 P(  4)=0.180643 P(  5)=0.118909
P(  6)=0.065230 P(  7)=0.030750 P(  8)=0.014650 P(  9)=0.007011 P( 10)=0.003354 P( 11)=0.001599
P( 12)=0.000759 P( 13)=0.000359 P( 14)=0.000169 P( 15)=0.000080 P( 16)=0.000038 P( 17)=0.000018
P( 18)=0.000008 P( 19)=0.000004 P( 20)=0.000002 P( 21)=0.000001 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000728
PROBABILITY NO CAR ON DUTY = 0.036796

STEP1160      MEAN ARRIVAL RATE =    5.3000
 P(  0)=0.050697 P(  1)=0.150758 P(  2)=0.224295 P(  3)=0.222470 P(  4)=0.165610 P(  5)=0.098793
P(  6)=0.049310 P(  7)=0.021309 P(  8)=0.009450 P(  9)=0.004283 P( 10)=0.001974 P( 11)=0.000921
P( 12)=0.000433 P( 13)=0.000205 P( 14)=0.000097 P( 15)=0.000046 P( 16)=0.000022 P( 17)=0.000010
P( 18)=0.000005 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000001 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000729
PROBABILITY NO CAR ON DUTY = 0.038756


AT HOUR  5      PATROL CARS ON DUTY =  6

STEP1180      MEAN ARRIVAL RATE =    4.7000
 P(  0)=0.069831 P(  1)=0.185306 P(  2)=0.245785 P(  3)=0.217228 P(  4)=0.143938 P(  5)=0.076358
P(  6)=0.033926 P(  7)=0.012357 P(  8)=0.007024 P(  9)=0.003226 P( 10)=0.001483 P( 11)=0.000683
P( 12)=0.000315 P( 13)=0.000146 P( 14)=0.000068 P( 15)=0.000032 P( 16)=0.000015 P( 17)=0.000007
P( 18)=0.000003 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000000 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000727
PROBABILITY NO CAR ON DUTY = 0.062282
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STEP1000     MEAN ARRIVAL RATE =   9.6000
 P(  0)=0.007235 P(  1)=0.034055 P(  2)=0.083547 P(  3)=0.132728 P(  4)=0.157511 P(  5)=0.148685
P(  6)=0.115954 P(  7)=0.079156 P(  8)=0.047415 P(  9)=0.050018 P( 10)=0.038345 P( 11)=0.025824
P( 12)=0.017921 P( 13)=0.012138 P( 14)=0.008019 P( 15)=0.005168 P( 16)=0.003250 P( 17)=0.001996
P( 18)=0.001198 P( 19)=0.000703 P( 20)=0.000405 P( 21)=0.000229 P( 22)=0.000127 P( 23)=0.000070
P( 24)=0.000038 P( 25)=0.000021 P( 26)=0.000011 P( 27)=0.000006 P( 28)=0.000003 P( 29)=0.000002
P( 30)=0.000001 P(
TOTAL PROBABILITY =1.000735
PROBABILITY NO CAR ON DUTY = 0.436014


AT HOUR  1     PATROL CARS ON DUTY =  8

STEP1020     MEAN ARRIVAL RATE =   9.1500
 P(  0)=0.007786 P(  1)=0.036491 P(  2)=0.085409 P(  3)=0.133058 P(  4)=0.155152 P(  5)=0.144351
P(  6)=0.111530 P(  7)=0.085775 P(  8)=0.050446 P(  9)=0.049405 P( 10)=0.038023 P( 11)=0.027052
P( 12)=0.019565 P( 13)=0.013915 P( 14)=0.009726 P( 15)=0.006676 P( 16)=0.004500 P( 17)=0.002977
P( 18)=0.001934 P( 19)=0.001233 P( 20)=0.000772 P( 21)=0.000475 P( 22)=0.000287 P( 23)=0.000171
P( 24)=0.000100 P( 25)=0.000058 P( 26)=0.000033 P( 27)=0.000019 P( 28)=0.000011 P( 29)=0.000007
P( 30)=0.000004 P(
TOTAL PROBABILITY =1.000735
PROBABILITY NO CAR ON DUTY = 0.438488

STEP1040     MEAN ARRIVAL RATE =   9.7000
 P(  0)=0.008044 P(  1)=0.040581 P(  2)=0.092367 P(  3)=0.139091 P(  4)=0.157070 P(  5)=0.141940
P(  6)=0.107029 P(  7)=0.080958 P(  8)=0.051287 P(  9)=0.046211 P( 10)=0.034629 P( 11)=0.025734
P( 12)=0.018932 P( 13)=0.013769 P( 14)=0.009889 P( 15)=0.007009 P( 16)=0.004899 P( 17)=0.003375
P( 18)=0.002292 P( 19)=0.001533 P( 20)=0.001010 P( 21)=0.000656 P( 22)=0.000420 P( 23)=0.000265
P( 24)=0.000165 P( 25)=0.000101 P( 26)=0.000062 P( 27)=0.000037 P( 28)=0.000023 P( 29)=0.000014
P( 30)=0.000009 P(
TOTAL PROBABILITY =1.000735
PROBABILITY NO CAR ON DUTY = 0.420343


AT HOUR  2     PATROL CARS ON DUTY =  6

STEP1060     MEAN ARRIVAL RATE =   8.1500
 P(  0)=0.011190 P(  1)=0.048216 P(  2)=0.103932 P(  3)=0.149491 P(  4)=0.161531 P(  5)=0.140066
P(  6)=0.101822 P(  7)=0.074920 P(  8)=0.055541 P(  9)=0.041316 P( 10)=0.030732 P( 11)=0.022793
P( 12)=0.016819 P( 13)=0.012322 P( 14)=0.008951 P( 15)=0.006441 P( 16)=0.004586 P( 17)=0.003229
P( 18)=0.002248 P( 19)=0.001546 P( 20)=0.001050 P( 21)=0.000705 P( 22)=0.000467 P( 23)=0.000306
P( 24)=0.000198 P( 25)=0.000127 P( 26)=0.000081 P( 27)=0.000051 P( 28)=0.000032 P( 29)=0.000021
P( 30)=0.000014 P(
TOTAL PROBABILITY =1.000736
PROBABILITY NO CAR ON DUTY = 0.386311

STEP1080     MEAN ARRIVAL RATE =   7.6000
 P(  0)=0.014490 P(  1)=0.054916 P(  2)=0.119927 P(  3)=0.163072 P(  4)=0.166860 P(  5)=0.137385
P(  6)=0.095270 P(  7)=0.067418 P(  8)=0.048452 P(  9)=0.035197 P( 10)=0.025734 P( 11)=0.018866
P( 12)=0.013827 P( 13)=0.010105 P( 14)=0.007350 P( 15)=0.005313 P( 16)=0.003811 P( 17)=0.002711
P( 18)=0.001911 P( 19)=0.001334 P( 20)=0.000922 P( 21)=0.000631 P( 22)=0.000427 P( 23)=0.000287
P( 24)=0.000190 P( 25)=0.000125 P( 26)=0.000082 P( 27)=0.000053 P( 28)=0.000034 P( 29)=0.000022
P( 30)=0.000014 P(
TOTAL PROBABILITY =1.000735
PROBABILITY NO CAR ON DUTY = 0.340084
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AT HOUR  3     PATROL CARS ON DUTY =  5

STEP1100     MEAN ARRIVAL RATE =   7.1500
 P(  0)=0.018610 P(  1)=0.070593 P(  2)=0.133531 P(  3)=0.167835 P(  4)=0.157666 P(  5)=0.118155
P(  6)=0.088524 P(  7)=0.065971 P(  8)=0.048810 P(  9)=0.035974 P( 10)=0.026242 P( 11)=0.019143
P( 12)=0.013943 P( 13)=0.010142 P( 14)=0.007364 P( 15)=0.005331 P( 16)=0.003842 P( 17)=0.002755
P( 18)=0.001962 P( 19)=0.001388 P( 20)=0.000974 P( 21)=0.000678 P( 22)=0.000467 P( 23)=0.000320
P( 24)=0.000217 P( 25)=0.000146 P( 26)=0.000098 P( 27)=0.000065 P( 28)=0.000044 P( 29)=0.000030
P( 30)=0.000021 P(
TOTAL PROBABILITY =1.000765
PROBAPILITY NO CAR ON DUTY = 0.452500

STEP1120     MEAN ARRIVAL RATE =   6.7000
 P(  0)=0.023109 P(  1)=0.082306 P(  2)=0.146619 P(  3)=0.174308 P(  4)=0.155829 P(  5)=0.112112
P(  6)=0.081737 P(  7)=0.060044 P(  8)=0.044245 P(  9)=0.032604 P( 10)=0.023982 P( 11)=0.017593
P( 12)=0.012867 P( 13)=0.009382 P( 14)=0.006820 P( 15)=0.004941 P( 16)=0.003567 P( 17)=0.002564
P( 18)=0.001834 P( 19)=0.001305 P( 20)=0.000923 P( 21)=0.000648 P( 22)=0.000453 P( 23)=0.000314
P( 24)=0.000216 P( 25)=0.000148 P( 26)=0.000101 P( 27)=0.000068 P( 28)=0.000046 P( 29)=0.000032
P( 30)=0.000021 P(
TOTAL PROBABILITY =1.000733
PROBABILITY NO CAR ON DUTY = 0.418563


AT HOUR  4     PATROL CARS ON DUTY =  4

STEP1140     MEAN ARRIVAL RATE =   6.0000
 P(  0)=0.029599 P(  1)=0.096075 P(  2)=0.154989 P(  3)=0.165447 P(  4)=0.131437 P(  5)=0.103805
P(  6)=0.080806 P(  7)=0.061858 P(  8)=0.046653 P(  9)=0.034792 P( 10)=0.025750 P( 11)=0.018962
P( 12)=0.013914 P( 13)=0.010180 P( 14)=0.007426 P( 15)=0.005401 P( 16)=0.003914 P( 17)=0.002827
P( 18)=0.002033 P( 19)=0.001456 P( 20)=0.001037 P( 21)=0.000735 P( 22)=0.000518 P( 23)=0.000363
P( 24)=0.000253 P( 25)=0.000176 P( 26)=0.000122 P( 27)=0.000084 P( 28)=0.000059 P( 29)=0.000042
P( 30)=0.000030 P(
TOTAL PROBABILITY =1.000731
PROBABILITY NO CAP ON DUTY = 0.554630

STEP1160     MEAN ARRIVAL RATE =   5.3000
 P(  0)=0.039046 P(  1)=0.114962 P(  2)=0.169362 P(  3)=0.166836 P(  4)=0.124283 P(  5)=0.094513
P(  6)=0.072420 P(  7)=0.055434 P(  8)=0.042189 P(  9)=0.031860 P( 10)=0.023870 P( 11)=0.017756
P( 12)=0.013125 P( 13)=0.009655 P( 14)=0.007070 P( 15)=0.005158 P( 16)=0.003749 P( 17)=0.002715
P( 18)=0.001959 P( 19)=0.001407 P( 20)=0.001007 P( 21)=0.000717 P( 22)=0.000508 P( 23)=0.000359
P( 24)=0.000252 P( 25)=0.000177 P( 26)=0.000124 P( 27)=0.000086 P( 28)=0.000061 P( 29)=0.000042
P( 30)=0.000029 P(
TOTAL PROBABILITY =1.000729
PROBABILITY NO CAP ON DUTY = 0.510523


AT HOUR  5     PATROL CARS ON DUTY =  3

STEP1180     MEAN ARRIVAL RATE =   4.7000
 P(  0)=0.050696 P(  1)=0.130359 P(  2)=0.165686 P(  3)=0.138702 P(  4)=0.114992 P(  5)=0.093291
P(  6)=0.073977 P(  7)=0.057586 P(  8)=0.044233 P(  9)=0.033654 P( 10)=0.025413 P( 11)=0.019064
P( 12)=0.014213 P( 13)=0.010536 P( 14)=0.007770 P( 15)=0.005702 P( 16)=0.004167 P( 17)=0.003032
P( 18)=0.002198 P( 19)=0.001587 P( 20)=0.001141 P( 21)=0.000817 P( 22)=0.000583 P( 23)=0.000414
P( 24)=0.000293 P( 25)=0.000207 P( 26)=0.000146 P( 27)=0.000103 P( 28)=0.000073 P( 29)=0.000054
P( 30)=0.000040 P(
TOTAL PROBABILITY =1.000725
PROBABILITY NO CAR ON DUTY = 0.653994
```

APPENDIX C

M/M/s STEADY STATE QUEUE WITH THE EXPECTED WAITING TIME OF A CALL IN THE QUEUE AS A CRITERION

C. 1 PROGRAM LIST

```
      $JOB
      C*****************************************************************
      C
      C
      C     THIS PROGRAM CALCULATES THE STATE PROBABILITIES OF M/P/S
      C     STEADY STATE QUEUEING MODEL AND SEARCH THE NUMBER OF PATROL
      C     CARS REQUIRED SUCH THAT THE AVERAGE WAITING TIME OF A CALL IN
      C     THE QUEUE IS LESS THAN THE SPECIFIED THRESHOLD.
      C
      C
      C     M: MAXIMUM NUMBER OF CALLS ALLOWED IN SYSTEM AT SAME TIME.
      C     NR: LENGTH OF CYCLE, IN THIS STUDY IT IS ONE DAY, STARTING
      C         FROM 0 HOUR TO 23 HOUR.
      C     UT: MEAN SERVICE RATE, A CONSTANT.
      C     AL: THE SPECIFIED THRESHOLD, MAXIMUM AVERAGE WAITING TIME
      C         ALLOWED IN THE QUEUE FOR A CALL.
      C     RADAT: MEAN ARRIVAL RATE, CHANGE WITH HOUR.
      C     ROAT: EQUAL TO MEAN ARRIVAL RATE DIVIDED BY MEAN SERVICE RATE.
      C     NT: NUMBER OF CARS ON DUTY.
      C     FACNT: FACTORIAL OF NT.
      C     PTO: PROBABILITY OF ZERO CALL IN THE SYSTEM, PC.
      C     PT(J): PROBABILITY OF J CALLS IN THE SYSTEM, 0<J.
      C     QUELTH: AVERAGE LENGTH OF QUEUE.
      C     WAITTM: AVERAGE WAITING TIME IN THE QUEUE.
      C
      C
      C*****************************************************************
  1         DIMENSION PT(100)
  2         READ,M,NR,UT,AL
  3         PRINT 5
  4       5 FORMAT('1')
  5         PRINT 6,M,NR,UT,AL
  6       6 FORMAT('0','TRUNCATED CAPACITY = ',I3,5X,'PERIOD OF CYCLE = ',I3/
           1' ','MEAN SERVICE RATE = ',F8.4,5X,'SPECIFIED THRESHOLD = ',F8.6)
  7         DO 80 K=1,NR
  8         READ,RADAT
  9         NT=1
 10         GO TO 20
 11      10 NT=NT+1
 12      20 ROAT=RADAT/UT
 13         FACTJ=1
 14         SUM=1
 15         IF(NT.EQ.1) GO TO 30
 16         NT1=NT-1
 17         DO 25 J=1,NT1
 18         FACTJ=FACTJ*J
 19      25 SUM=SUM+ROAT**J/FACTJ
 20      30 FACNT=1
 21         DO 31 I=1,NT
 22      31 FACNT=FACNT*I
 23         PLUS=ROAT**NT/(FACNT*(1-ROAT/NT))
 24         PTO=1./(SUM+PLUS)
 25         QUELTH=(ROAT**(NT+1)*PTO)/(FACNT*(1.-ROAT/NT)**2*NT)
 26         IF(QUELTH.LT.0.) GO TO 10
 27         WAITTM=QUELTH/RADAT
 28         IF(WAITTM.GT.AL) GO TO 10
 29         SUMPT=PTO
 30         IF(NT.EQ.1) GO TO 34
 31         FACTJ=1
 32         NT2=NT-1
```

```
33          DO 33 J=1,NT2
34          FACTJ=FACTJ*J
35          PT(J)=ROAT**J*PTO/FACTJ
36    33    SUMPT=SUMPT+PT(J)
37    34    PT(NT)=ROAT**NT*PTO/FACNT
38          SUM=SUMPT+PT(NT)
39          NT3=NT+1
40          DO 35 J=NT3,M
41          PT(J)=(ROAT**J)*PTO/(FLOAT(NT)**(J-NT)*FACNT)
42    35    SUM=SUM+PT(J)
43          PRINT 40 , K-1,AT,WAITIM
44    40    FORMAT('-','AT HOUR',I3,5X,'THE NUMBER OF PATROL CARS REQUIRED = '
           1,I3,5X,'THE AVERAGE WAITING TIME IN THE QUEUE = ',F8.6)
45          PRINT 45,RADAT
46    45    FORMAT('0','MEAN ARRIVAL RATE = ',F8.4)
47          PRINT 60,PTO,(J,PT(J),J=1,M)
48    60    FORMAT(' ','P(0) = ',F8.6/' ',6(' ','P(',I2,') = ',F8.6,2X))
49          PRINT 70,M,SUM
50    70    FORMAT(' ','TOTAL PROBABILITY FROM 1 TO ',I2,' CALLS IN THE SYSTEM
           1 = ',F8.6)
51    80    CONTINUE
52          PRINT 85
53    85    FORMAT('1')
54          STOP
55          END

    $ENTRY
```

```
TRUNCATED CAPACITY =  30      PERIOD OF CYCLE =  24
MEAN SERVICE RATE =   2.0000      SPECIFIED THRESHOLD = 0.010000

AT HOUR  0      THE NUMBER OF PATROL CARS REQUIRED =   9      THE AVERAGE WAITING TIME IN THE QUEUE = 0.008843

MEAN ARRIVAL RATE =   9.8000
P(0) = 0.007361
 P( 1) = 0.036070   P( 2) = 0.088372   P( 3) = 0.144341   P( 4) = 0.176617   P( 5) = 0.173281   P( 6) = 0.141513
P( 7) = 0.099059   P( 8) = 0.060674   P( 9) = 0.033033   P(10) = 0.017585   P(11) = 0.009792   P(12) = 0.005331
P(13) = 0.002902   P(14) = 0.001580   P(15) = 0.000860   P(16) = 0.000468   P(17) = 0.000255   P(18) = 0.000139
P(19) = 0.000076   P(20) = 0.000041   P(21) = 0.000022   P(22) = 0.000012   P(23) = 0.000007   P(24) = 0.000004
P(25) = 0.000002   P(26) = 0.000001   P(27) = 0.000001   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  1      THE NUMBER OF PATROL CARS REQUIRED =   9      THE AVERAGE WAITING TIME IN THE QUEUE = 0.007749

MEAN ARRIVAL RATE =   9.6000
P(0) = 0.008149
 P( 1) = 0.039115   P( 2) = 0.093876   P( 3) = 0.150201   P( 4) = 0.180241   P( 5) = 0.173032   P( 6) = 0.138425
P( 7) = 0.094920   P( 8) = 0.056952   P( 9) = 0.030374   P(10) = 0.016200   P(11) = 0.008640   P(12) = 0.004608
P(13) = 0.002458   P(14) = 0.001311   P(15) = 0.000699   P(16) = 0.000373   P(17) = 0.000199   P(18) = 0.000106
P(19) = 0.000057   P(20) = 0.000030   P(21) = 0.000016   P(22) = 0.000009   P(23) = 0.000005   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  2      THE NUMBER OF PATROL CARS REQUIRED =   8      THE AVERAGE WAITING TIME IN THE QUEUE = 0.012138

MEAN ARRIVAL RATE =   8.7000
P(0) = 0.012714
 P( 1) = 0.055304   P( 2) = 0.120286   P( 3) = 0.174414   P( 4) = 0.189676   P( 5) = 0.165018   P( 6) = 0.119638
P( 7) = 0.074346   P( 8) = 0.040426   P( 9) = 0.021982   P(10) = 0.011952   P(11) = 0.006499   P(12) = 0.003534
P(13) = 0.001922   P(14) = 0.001045   P(15) = 0.000568   P(16) = 0.000309   P(17) = 0.000168   P(18) = 0.000091
P(19) = 0.000050   P(20) = 0.000027   P(21) = 0.000015   P(22) = 0.000008   P(23) = 0.000004   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999

AT HOUR  3      THE NUMBER OF PATROL CARS REQUIRED =   8      THE AVERAGE WAITING TIME IN THE QUEUE = 0.005438

MEAN ARRIVAL RATE =   7.6000
P(0) = 0.022241
 P( 1) = 0.084515   P( 2) = 0.160579   P( 3) = 0.203400   P( 4) = 0.193230   P( 5) = 0.146855   P( 6) = 0.093008
P( 7) = 0.050490   P( 8) = 0.023983   P( 9) = 0.011392   P(10) = 0.005411   P(11) = 0.002570   P(12) = 0.001221
P(13) = 0.000580   P(14) = 0.000275   P(15) = 0.000131   P(16) = 0.000062   P(17) = 0.000030   P(18) = 0.000014
P(19) = 0.000007   P(20) = 0.000003   P(21) = 0.000002   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000

AT HOUR  4      THE NUMBER OF PATROL CARS REQUIRED =   7      THE AVERAGE WAITING TIME IN THE QUEUE = 0.008583

MEAN ARRIVAL RATE =   6.7000
P(0) = 0.034778
 P( 1) = 0.116499   P( 2) = 0.195136   P( 3) = 0.217902   P( 4) = 0.182493   P( 5) = 0.122270   P( 6) = 0.068268
P( 7) = 0.032671   P( 8) = 0.015635   P( 9) = 0.007483   P(10) = 0.003581   P(11) = 0.001714   P(12) = 0.000820
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P(13) = 0.000393   P(14) = 0.000188   P(15) = 0.000090   P(16) = 0.000043   P(17) = 0.000021   P(18) = 0.000010
P(19) = 0.000005   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR  5     THE NUMBER OF PATROL CARS REQUIRED =   6     THE AVERAGE WAITING TIME IN THE QUEUE = 0.009012

MEAN ARRIVAL RATE =   5.3000
P(0) = 0.070086
 P( 1) = 0.185728   P( 2) = 0.246089   P( 3) = 0.217378   P( 4) = 0.144013   P( 5) = 0.076327   P( 6) = 0.033711
P( 7) = 0.014889   P( 8) = 0.006576   P( 9) = 0.002904   P(10) = 0.001283   P(11) = 0.000567   P(12) = 0.000250
P(13) = 0.000111   P(14) = 0.000049   P(15) = 0.000022   P(16) = 0.000010   P(17) = 0.000004   P(18) = 0.000002
P(19) = 0.000001   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR  6     THE NUMBER OF PATROL CARS REQUIRED =   5     THE AVERAGE WAITING TIME IN THE QUEUE = 0.011063

MEAN ARRIVAL RATE =   4.1000
P(0) = 0.127643
 P( 1) = 0.261668   P( 2) = 0.268210   P( 3) = 0.183277   P( 4) = 0.093929   P( 5) = 0.038511   P( 6) = 0.015790
P( 7) = 0.006474   P( 8) = 0.002654   P( 9) = 0.001088   P(10) = 0.000446   P(11) = 0.000183   P(12) = 0.000075
P(13) = 0.000031   P(14) = 0.000013   P(15) = 0.000005   P(16) = 0.000002   P(17) = 0.000001   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR  7     THE NUMBER OF PATROL CARS REQUIRED =   5     THE AVERAGE WAITING TIME IN THE QUEUE = 0.003807

MEAN ARRIVAL RATE =   3.2000
P(0) = 0.201441
 P( 1) = 0.322306   P( 2) = 0.257844   P( 3) = 0.137517   P( 4) = 0.055007   P( 5) = 0.017602   P( 6) = 0.005633
P( 7) = 0.001802   P( 8) = 0.000577   P( 9) = 0.000185   P(10) = 0.000059   P(11) = 0.000019   P(12) = 0.000006
P(13) = 0.000002   P(14) = 0.000001   P(15) = 0.000000   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR  8     THE NUMBER OF PATROL CARS REQUIRED =   4     THE AVERAGE WAITING TIME IN THE QUEUE = 0.007676

MEAN ARRIVAL RATE =   2.5000
P(0) = 0.285328
 P( 1) = 0.356660   P( 2) = 0.222913   P( 3) = 0.092880   P( 4) = 0.029025   P( 5) = 0.009070   P( 6) = 0.002834
P( 7) = 0.000886   P( 8) = 0.000277   P( 9) = 0.000087   P(10) = 0.000027   P(11) = 0.000008   P(12) = 0.000003
P(13) = 0.000001   P(14) = 0.000000   P(15) = 0.000000   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR  9     THE NUMBER OF PATROL CARS REQUIRED =   4     THE AVERAGE WAITING TIME IN THE QUEUE = 0.007676

MEAN ARRIVAL RATE =   2.5000
P(0) = 0.285328
 P( 1) = 0.356660   P( 2) = 0.222913   P( 3) = 0.092880   P( 4) = 0.029025   P( 5) = 0.009070   P( 6) = 0.002834
P( 7) = 0.000886   P( 8) = 0.000277   P( 9) = 0.000087   P(10) = 0.000027   P(11) = 0.000008   P(12) = 0.000003
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P(13) = 0.000001   P(14) = 0.000000   P(15) = 0.000000   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 10     THE NUMBER OF PATROL CARS REQUIRED =  4     THE AVERAGE WAITING TIME IN THE QUEUE = 0.013180

MEAN ARRIVAL RATE =   2.9000
P(0) = 0.232651
 P( 1) = 0.337345   P( 2) = 0.244575   P( 3) = 0.118211   P( 4) = 0.042852   P( 5) = 0.015534   P( 6) = 0.005631
P( 7) = 0.002041   P( 8) = 0.000740   P( 9) = 0.000268   P(10) = 0.000097   P(11) = 0.000035   P(12) = 0.000013
P(13) = 0.000005   P(14) = 0.000002   P(15) = 0.000001   P(16) = 0.000000   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 11     THE NUMBER OF PATROL CARS REQUIRED =  5     THE AVERAGE WAITING TIME IN THE QUEUE = 0.007983

MEAN ARRIVAL RATE =   3.8000
P(0) = 0.148722
 P( 1) = 0.282571   P( 2) = 0.268442   P( 3) = 0.170013   P( 4) = 0.080756   P( 5) = 0.030687   P( 6) = 0.011661
P( 7) = 0.004431   P( 8) = 0.001684   P( 9) = 0.000640   P(10) = 0.000243   P(11) = 0.000092   P(12) = 0.000035
P(13) = 0.000013   P(14) = 0.000005   P(15) = 0.000002   P(16) = 0.000001   P(17) = 0.000000   P(18) = 0.000000
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 12     THE NUMBER OF PATROL CARS REQUIRED =  5     THE AVERAGE WAITING TIME IN THE QUEUE = 0.013576

MEAN ARRIVAL RATE =   4.3000
P(0) = 0.115213
 P( 1) = 0.247707   P( 2) = 0.266285   P( 3) = 0.190838   P( 4) = 0.102575   P( 5) = 0.044107   P( 6) = 0.018966
P( 7) = 0.008155   P( 8) = 0.003507   P( 9) = 0.001508   P(10) = 0.000648   P(11) = 0.000279   P(12) = 0.000120
P(13) = 0.000052   P(14) = 0.000022   P(15) = 0.000010   P(16) = 0.000004   P(17) = 0.000002   P(18) = 0.000001
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 13     THE NUMBER OF PATROL CARS REQUIRED =  6     THE AVERAGE WAITING TIME IN THE QUEUE = 0.006778

MEAN ARRIVAL RATE =   4.8000
P(0) = 0.081620
 P( 1) = 0.204051   P( 2) = 0.255063   P( 3) = 0.212553   P( 4) = 0.132845   P( 5) = 0.066423   P( 6) = 0.027676
P( 7) = 0.011532   P( 8) = 0.004805   P( 9) = 0.002002   P(10) = 0.000834   P(11) = 0.000348   P(12) = 0.000145
P(13) = 0.000060   P(14) = 0.000025   P(15) = 0.000010   P(16) = 0.000004   P(17) = 0.000002   P(18) = 0.000001
P(19) = 0.000000   P(20) = 0.000000   P(21) = 0.000000   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 14     THE NUMBER OF PATROL CARS REQUIRED =  6     THE AVERAGE WAITING TIME IN THE QUEUE = 0.015218

MEAN ARRIVAL RATE =   5.9000
P(0) = 0.051552
 P( 1) = 0.152077   P( 2) = 0.224314   P( 3) = 0.220575   P( 4) = 0.162674   P( 5) = 0.095978   P( 6) = 0.047189
P( 7) = 0.023201   P( 8) = 0.011407   P( 9) = 0.005609   P(10) = 0.002758   P(11) = 0.001356   P(12) = 0.000667
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P(13) = 0.000328   P(14) = 0.000161   P(15) = 0.000079   P(16) = 0.000039   P(17) = 0.000019   P(18) = 0.000009
P(19) = 0.000005   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000001   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 15      THE NUMBER OF PATROL CARS REQUIRED =   7      THE AVERAGE WAITING TIME IN THE QUEUE = 0.007910

MEAN ARRIVAL RATE =    6.6000
P(0) = 0.036585
 P( 1) = 0.120744   P( 2) = 0.199227   P( 3) = 0.219150   P( 4) = 0.180799   P( 5) = 0.119327   P( 6) = 0.065630
P( 7) = 0.030940   P( 8) = 0.014586   P( 9) = 0.006876   P(10) = 0.003242   P(11) = 0.001528   P(12) = 0.000720
P(13) = 0.000340   P(14) = 0.000160   P(15) = 0.000075   P(16) = 0.000036   P(17) = 0.000017   P(18) = 0.000008
P(19) = 0.000004   P(20) = 0.000002   P(21) = 0.000001   P(22) = 0.000000   P(23) = 0.000000   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 16      THE NUMBER OF PATROL CARS REQUIRED =   8      THE AVERAGE WAITING TIME IN THE QUEUE = 0.006349

MEAN ARRIVAL RATE =    7.8000
P(0) = 0.020101
 P( 1) = 0.078393   P( 2) = 0.152866   P( 3) = 0.198726   P( 4) = 0.193758   P( 5) = 0.151131   P( 6) = 0.098235
P( 7) = 0.054731   P( 8) = 0.026681   P( 9) = 0.013007   P(10) = 0.006341   P(11) = 0.003091   P(12) = 0.001507
P(13) = 0.000735   P(14) = 0.000358   P(15) = 0.000175   P(16) = 0.000085   P(17) = 0.000041   P(18) = 0.000020
P(19) = 0.000010   P(20) = 0.000005   P(21) = 0.000002   P(22) = 0.000001   P(23) = 0.000001   P(24) = 0.000000
P(25) = 0.000000   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 1.000000


AT HOUR 17      THE NUMBER OF PATROL CARS REQUIRED =   8      THE AVERAGE WAITING TIME IN THE QUEUE = 0.011334

MEAN ARRIVAL RATE =    8.6000
P(0) = 0.013381
 P( 1) = 0.057539   P( 2) = 0.123708   P( 3) = 0.177314   P( 4) = 0.190613   P( 5) = 0.163927   P( 6) = 0.117481
P( 7) = 0.072167   P( 8) = 0.038790   P( 9) = 0.020850   P(10) = 0.011207   P(11) = 0.006024   P(12) = 0.003238
P(13) = 0.001740   P(14) = 0.000935   P(15) = 0.000503   P(16) = 0.000270   P(17) = 0.000145   P(18) = 0.000078
P(19) = 0.000042   P(20) = 0.000023   P(21) = 0.000012   P(22) = 0.000007   P(23) = 0.000004   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000001   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 18      THE NUMBER OF PATROL CARS REQUIRED =   9      THE AVERAGE WAITING TIME IN THE QUEUE = 0.006769

MEAN ARRIVAL RATE =    9.4000
P(0) = 0.009019
 P( 1) = 0.042389   P( 2) = 0.099615   P( 3) = 0.156064   P( 4) = 0.183375   P( 5) = 0.172373   P( 6) = 0.135025
P( 7) = 0.090660   P( 8) = 0.053263   P( 9) = 0.027815   P(10) = 0.014528   P(11) = 0.007586   P(12) = 0.003961
P(13) = 0.002069   P(14) = 0.001080   P(15) = 0.000564   P(16) = 0.000295   P(17) = 0.000154   P(18) = 0.000080
P(19) = 0.000042   P(20) = 0.000022   P(21) = 0.000011   P(22) = 0.000006   P(23) = 0.000003   P(24) = 0.000002
P(25) = 0.000001   P(26) = 0.000000   P(27) = 0.000000   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 19      THE NUMBER OF PATROL CARS REQUIRED =   9      THE AVERAGE WAITING TIME IN THE QUEUE = 0.008843

MEAN ARRIVAL RATE =    9.8000
P(0) = 0.007361
 P( 1) = 0.036070   P( 2) = 0.088372   P( 3) = 0.144341   P( 4) = 0.176817   P( 5) = 0.173281   P( 6) = 0.141513
P( 7) = 0.099059   P( 8) = 0.060674   P( 9) = 0.033033   P(10) = 0.017585   P(11) = 0.009792   P(12) = 0.005331
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P(13) = 0.002902   P(14) = 0.001580   P(15) = 0.000860   P(16) = 0.000468   P(17) = 0.000255   P(18) = 0.000139
P(19) = 0.000076   P(20) = 0.000041   P(21) = 0.000022   P(22) = 0.000012   P(23) = 0.000007   P(24) = 0.000004
P(25) = 0.000002   P(26) = 0.000001   P(27) = 0.000001   P(28) = 0.000000   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 20     THE NUMBER OF PATROL CARS REQUIRED =   9     THE AVERAGE WAITING TIME IN THE QUEUE = 0.011423

MEAN ARRIVAL RATE =  10.2000
P(0) = 0.006003
 P( 1) = 0.030613   P( 2) = 0.078064   P( 3) = 0.132709   P( 4) = 0.169204   P( 5) = 0.172588   P( 6) = 0.146700
P( 7) = 0.106881   P( 8) = 0.068137   P( 9) = 0.038611   P(10) = 0.021879   P(11) = 0.012398   P(12) = 0.007026
P(13) = 0.003981   P(14) = 0.002256   P(15) = 0.001278   P(16) = 0.000724   P(17) = 0.000411   P(18) = 0.000233
P(19) = 0.000132   P(20) = 0.000075   P(21) = 0.000042   P(22) = 0.000024   P(23) = 0.000014   P(24) = 0.000008
P(25) = 0.000004   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 21     THE NUMBER OF PATROL CARS REQUIRED =   9     THE AVERAGE WAITING TIME IN THE QUEUE = 0.012935

MEAN ARRIVAL RATE =  10.4000
P(0) = 0.005418
 P( 1) = 0.028175   P( 2) = 0.073254   P( 3) = 0.126974   P( 4) = 0.165066   P( 5) = 0.171668   P( 6) = 0.148779
P( 7) = 0.110521   P( 8) = 0.071839   P( 9) = 0.041507   P(10) = 0.023582   P(11) = 0.013856   P(12) = 0.008006
P(13) = 0.004626   P(14) = 0.002673   P(15) = 0.001544   P(16) = 0.000892   P(17) = 0.000515   P(18) = 0.000298
P(19) = 0.000172   P(20) = 0.000099   P(21) = 0.000057   P(22) = 0.000033   P(23) = 0.000019   P(24) = 0.000011
P(25) = 0.000006   P(26) = 0.000004   P(27) = 0.000002   P(28) = 0.000001   P(29) = 0.000001   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999998


AT HOUR 22     THE NUMBER OF PATROL CARS REQUIRED =   9     THE AVERAGE WAITING TIME IN THE QUEUE = 0.011423

MEAN ARRIVAL RATE =  10.2000
P(0) = 0.006003
 P( 1) = 0.030613   P( 2) = 0.078064   P( 3) = 0.132709   P( 4) = 0.169204   P( 5) = 0.172588   P( 6) = 0.146700
P( 7) = 0.106881   P( 8) = 0.068137   P( 9) = 0.038611   P(10) = 0.021879   P(11) = 0.012398   P(12) = 0.007026
P(13) = 0.003981   P(14) = 0.002256   P(15) = 0.001278   P(16) = 0.000724   P(17) = 0.000411   P(18) = 0.000233
P(19) = 0.000132   P(20) = 0.000075   P(21) = 0.000042   P(22) = 0.000024   P(23) = 0.000014   P(24) = 0.000008
P(25) = 0.000004   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999


AT HOUR 23     THE NUMBER OF PATROL CARS REQUIRED =   9     THE AVERAGE WAITING TIME IN THE QUEUE = 0.010064

MEAN ARRIVAL RATE =  10.0001
P(0) = 0.006648
 P( 1) = 0.033240   P( 2) = 0.083100   P( 3) = 0.138501   P( 4) = 0.173128   P( 5) = 0.173129   P( 6) = 0.144276
P( 7) = 0.103055   P( 8) = 0.064410   P( 9) = 0.035784   P(10) = 0.019880   P(11) = 0.011045   P(12) = 0.006136
P(13) = 0.003409   P(14) = 0.001894   P(15) = 0.001052   P(16) = 0.000585   P(17) = 0.000325   P(18) = 0.000180
P(19) = 0.000100   P(20) = 0.000056   P(21) = 0.000031   P(22) = 0.000017   P(23) = 0.000010   P(24) = 0.000005
P(25) = 0.000003   P(26) = 0.000002   P(27) = 0.000001   P(28) = 0.000001   P(29) = 0.000000   P(30) = 0.000000
TOTAL PROBABILITY FROM 1 TO 30 CALLS IN THE SYSTEM = 0.999999
```

APPENDIX D

M/M/s NONSTEADY STATE QUEUE WITH THE EXPECTED WAITING TIME OF A CALL IN THE QUEUE AS A CRITERION

D. 1 PROGRAM LIST

```
$JOB            ,PAGES=60
C*****************************************************************************
C
C
C      THIS PROGRAM CALCULATES TRANSIENT STATE PROBABILITIES AND
C      THE AVERAGE WAITING TIME IN THE QUEUE FOR NK SERVER
C      TRUNCATED M MAXIMUM ALLOWED CAPACITY QUEUEING SYSTEM BY THE
C      FOUTH ORDER RUNGE KUTTA METHOD.
C
C
C      M: MAXIMUM ALLOWED CAPACITY IN THE QUEUEING SYSTEM.
C      DELT: INTEGRATING STEP SIZE.
C      T: TOTAL INTEGRATING LENGTH, 48 HOURS SUGGESTED.
C      U: MEAN SERVICE RATE, A CONSTANT.
C      RK: MEAN ARRIVAL RATE, CHANGE WITH TIME.
C      NK: NUMBER OF CARS ON DUTY, CHANG WITH TIME.
C      PK(J): PROBABILITY OF J-1 CALLS IN SYSTEM.
C      QUELTH: AVERAGE LENGTH OF QUEUE.
C      WAITIM: AVERAGE WAITING TIME IN THE QUEUE.
C
C
C*****************************************************************************
      DIMENSION D1(100),D2(100),D3(100),D4(100),PK(100)
      FU(A,B,U,RANDK)=-RANDK*A+U*B
      FLN(A,B,C,U,RANDK,J)=RANDK*A-(RANDK+FLOAT(J)*U)*B+FLOAT(J)*U*C+U*B
      FGN(A,B,C,U,RANDK,NK)=RANDK*A-(RANDK+FLOAT(NK)*U)*B+FLOAT(NK)*U*C
      FM(A,B,U,RANDK,AK)=RANDK*A-FLOAT(NK)*U*B
      READ,M,DELT,T,U
      M1=M+1
      DO 10 J=1,M1
   10 READ,PK(J)
      ITER=T/DELT
      READ,NK,RK1,RK2
      ND=1
      K=1
      IND=1
      PRINT 14,M,DELT,T,U
   14 FORMAT('1',//'0','TRUNCATED CAPACITY = ',I3,5X,'STEP SIZE = ',F8.5
     1/' ','INTEGRATED LENGTH = ',F8.4,5X,'MEAN SERVICE RATE = ',F8.4)
      PRINT 15,NK,(J-1,PK(J),J=1,M1)
   15 FORMAT('0','AT HOUR 0',5X,'PATROL CAR ON DUTY =',I3/' ',
     16(' ','P(',I3,')=',F8.6))
      DO 70 K=1,ITER
      RM=(1./DELT)*FLOAT(N)+1.
      IF(K.LT.RM) GO TO 20
      READ,NK,RK3
      RK1=RK2
      RK2=RK3
      NH=K
      IF(NH.GE.48) NH=NH-24
      IF(NH.GE.24) NH=NH-24
      PRINT 16,NH,NK
   16 FORMAT(//'0','AT HOUR',I3,5X,'PATROL CARS ON DUTY =',I3)
      N=N+1
      ND=1
   20 RANDK=(RK2-RK1)*FLOAT(ND)*DELT+RK1
      ND=ND+1
      D1(1)=FU(PK(1),PK(2),U,RANDK)*DELT
      AK1=AK+1
      DO 50 J=2,M1
```

```
37          IF(J.GE.NK1) GO TO 30
38          D1(J)=FLN(PK(J-1),PK(J),PK(J+1),U,RANDK,J)*DELT
39          GO TO 50
40     30   IF (J.EQ.M1)GO TO 40
41          D1(J)=FGN(PK(J-1),PK(J),PK(J+1),U,RANDK,NK)*DELT
42          GO TO 50
43     40   D1(J)=FM(PK(J-1),PK(J),U,RANDK,NK)*DELT
44     50   CONTINUE
45          D2(1)=FO(PK(1)+1./2.*D1(1),PK(2)+1./2.*D1(2),U,RANDK)*DELT
46          DO 51 J=2,M1
47          IF(J.GE.NK1)GO TO 31
48          D2(J)=FLN(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),
           1PK(J+1)+1./2.*D1(J+1),U,RANDK,J)*DELT
49          GO TO 51
50     31   IF(J.EQ.M1)GO TO 41
51          D2(J)=FGN(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),
           1PK(J+1)+1./2.*D1(J+1),U,RANDK,NK)*DELT
52          GO TO 51
53     41   D2(J)=FM(PK(J-1)+1./2.*D1(J-1),PK(J)+1./2.*D1(J),U,RANDK,NK)*DELT
54     51   CONTINUE
55          D3(1)=FO(PK(1)+1./2.*D2(1),PK(2)+1./2.*D2(2),U,RANDK)*DELT
56          DO 52 J=2,M1
57          IF(J.GE.NK1)GO TO 32
58          D3(J)=FLN(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),
           1PK(J+1)+1./2.*D2(J+1),U,RANDK,J)*DELT
59          GO TO 52
60     32   IF(J.EQ.M1)GO TO 42
61          D3(J)=FGN(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),
           1PK(J+1)+1./2.*D2(J+1),U,RANDK,NK)*DELT
62          GO TO 52
63     42   D3(J)=FM(PK(J-1)+1./2.*D2(J-1),PK(J)+1./2.*D2(J),U,RANDK,NK)*DELT
64     52   CONTINUE
65          D4(1)=FO(PK(1)+D3(1),PK(2)+D3(2),U,RANDK)*DELT
66          DO 53 J=2,M1
67          IF(J.GE.NK1)GO TO 33
68          D4(J)=FLN(PK(J-1)+D3(J-1),PK(J)+D3(J),PK(J+1)+D3(J+1),U,RANDK,
           1J)*DELT
69          GO TO 53
70     33   IF(J.EQ.M1) GO TO 43
71          D4(J)=FGN(PK(J-1)+D3(J-1),PK(J)+D3(J),PK(J+1)+D3(J+1),U,RANDK,
           1NK)*DELT
72          GO TO 53
73     43   D4(J)=FM(PK(J-1)+D3(J-1),PK(J)+D3(J),U,RANDK,NK)*DELT
74     53   CONTINUE
75          DO 60 J=1,M1
76          PK(J)=PK(J)+1./6.*(D1(J)+2.*D2(J)+2.*D3(J)+D4(J))
77     60   IF(PK(J).LT.0.) PK(J)=0.
78          INDPRI=20*INC
79          IF(K.LT.INDPRI) GO TO 70
80     61   PRINT 65,K,RANDK,(J-1,PK(J),J=1,M1)
81     65   FORMAT('0','STEP',I4,5X,'MEAN ARRIVAL RATE = ',F8.4/' ',6('  ',
           1'P(',I3,')=',F8.6))
82          SUM=0
83          DO 66 J=1,NK
84     66   SUM=SUM+PK(J)
85          SUM1=SUM
86          DO 68 J=NK1,M1
87     68   SUM1=SUM1+PK(J)
88          PRINT 69,SUM1
89     69   FORMAT(' ','TOTAL PROBABILITY =',F8.6)
```

```
 90          FACNK=1.
 91          DO 71 L=1,NK
 92     71   FACNK=FACNK*L
 93          UP=(RANDK/U)**(NK+1)*PK(1)/NK
 94          DOWN=FACNK*(1.-RANDK/U/NK)**2
 95          QUELTH=UP/DOWN
 96          WAITIM=QUELTH/RANDK
 97          PRINT 67,WAITIM
 98     67   FORMAT(' ','AVERAGE WAITING TIME IN THE QUEUE = ',F8.6,1X,'HR')
 99          IND=IND+1
100     70   CONTINUE
101          STOP
102          END

     $ENTRY
```

```
AT HOUR 22      PATROL CARS ON DUTY = 12

STEP 900      MEAN ARRIVAL RATE =   10.1000
 P( 0)=0.006048 P( 1)=0.030816 P( 2)=0.078528 P( 3)=0.133479 P( 4)=0.170301 P( 5)=0.174041
P( 6)=0.148492 P( 7)=0.108980 P( 8)=0.070115 P( 9)=0.040343 P( 10)=0.021045 P( 11)=0.010087
P( 12)=0.004506 P( 13)=0.002068 P( 14)=0.000981 P( 15)=0.000483 P( 16)=0.000246 P( 17)=0.000130
P( 18)=0.000070 P( 19)=0.000038 P( 20)=0.000021 P( 21)=0.000012 P( 22)=0.000007 P( 23)=0.000004
P( 24)=0.000002 P( 25)=0.000001 P( 26)=0.000001 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000739
AVERAGE WAITING TIME IN THE QUEUE = 0.000431 HR

STEP 920      MEAN ARRIVAL RATE =   10.0000
 P( 0)=0.006364 P( 1)=0.032157 P( 2)=0.081259 P( 3)=0.136931 P( 4)=0.173116 P( 5)=0.175162
P( 6)=0.147765 P( 7)=0.106911 P( 8)=0.067736 P( 9)=0.038186 P( 10)=0.019404 P( 11)=0.008984
P( 12)=0.003829 P( 13)=0.001644 P( 14)=0.000714 P( 15)=0.000314 P( 16)=0.000141 P( 17)=0.000065
P( 18)=0.000030 P( 19)=0.000015 P( 20)=0.000007 P( 21)=0.000004 P( 22)=0.000002 P( 23)=0.000001
P( 24)=0.000001 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.000397 HR


AT HOUR 23      PATROL CARS ON DUTY = 12

STEP 940      MEAN ARRIVAL RATE =    9.9000
 P( 0)=0.006718 P( 1)=0.033602 P( 2)=0.084048 P( 3)=0.140162 P( 4)=0.175319 P( 5)=0.175452
P( 6)=0.146338 P( 7)=0.104623 P( 8)=0.065474 P( 9)=0.036427 P( 10)=0.018247 P( 11)=0.008315
P( 12)=0.003478 P( 13)=0.001459 P( 14)=0.000614 P( 15)=0.000260 P( 16)=0.000111 P( 17)=0.000047
P( 18)=0.000021 P( 19)=0.000009 P( 20)=0.000004 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
AVERAGE WAITING TIME IN THE QUEUE = 0.000366 HR

STEP 960      MEAN ARRIVAL RATE =    9.8000
 P( 0)=0.007080 P( 1)=0.035050 P( 2)=0.086759 P( 3)=0.143172 P( 4)=0.177203 P( 5)=0.175464
P( 6)=0.144789 P( 7)=0.102413 P( 8)=0.063388 P( 9)=0.034878 P( 10)=0.017275 P( 11)=0.007781
P( 12)=0.003215 P( 13)=0.001331 P( 14)=0.000552 P( 15)=0.000229 P( 16)=0.000096 P( 17)=0.000040
P( 18)=0.000017 P( 19)=0.000007 P( 20)=0.000003 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.000337 HR


AT HOUR  0      PATROL CARS ON DUTY = 10

STEP 980      MEAN ARRIVAL RATE =    9.7000
 P( 0)=0.007452 P( 1)=0.036507 P( 2)=0.089422 P( 3)=0.146010 P( 4)=0.178775 P( 5)=0.175066
P( 6)=0.142798 P( 7)=0.099770 P( 8)=0.060930 P( 9)=0.033025 P( 10)=0.016071 P( 11)=0.007192
P( 12)=0.003756 P( 13)=0.001795 P( 14)=0.000849 P( 15)=0.000396 P( 16)=0.000182 P( 17)=0.000083
P( 18)=0.000037 P( 19)=0.000016 P( 20)=0.000007 P( 21)=0.000003 P( 22)=0.000001 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.000288 HR
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AT HOUR  3     PATROL CARS ON DUTY =  8

STEP1100     MEAN ARRIVAL RATE =   7.1500
 P(  0)=0.022101 P(  1)=0.065163 P(  2)=0.160231 P(  3)=0.203224 P(  4)=0.193443 P(  5)=0.147173
P(  6)=0.093273 P(  7)=0.050670 P(  8)=0.024125 P(  9)=0.011561 P( 10)=0.005557 P( 11)=0.002671
P( 12)=0.001200 P( 13)=0.000612 P( 14)=0.000291 P( 15)=0.000138 P( 16)=0.000066 P( 17)=0.000031
P( 18)=0.000015 P( 19)=0.000007 P( 20)=0.000003 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000732
AVERAGE WAITING TIME IN THE QUEUE = 0.002988 HR

STEP1120     MEAN ARRIVAL RATE =   6.7000
 P(  0)=0.027903 P(  1)=0.099723 P(  2)=0.178199 P(  3)=0.212300 P(  4)=0.189729 P(  5)=0.135704
P(  6)=0.080964 P(  7)=0.041498 P(  8)=0.018711 P(  9)=0.008550 P( 10)=0.003954 P( 11)=0.001847
P( 12)=0.000869 P( 13)=0.000412 P( 14)=0.000196 P( 15)=0.000093 P( 16)=0.000044 P( 17)=0.000021
P( 18)=0.000010 P( 19)=0.000005 P( 20)=0.000002 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000732
AVERAGE WAITING TIME IN THE QUEUE = 0.002031 HR


AT HOUR  4     PATROL CARS ON DUTY =  7

STEP1140     MEAN ARRIVAL RATE =   6.0000
 P(  0)=0.036758 P(  1)=0.121204 P(  2)=0.199770 P(  3)=0.219417 P(  4)=0.180643 P(  5)=0.118909
P(  6)=0.065230 P(  7)=0.030750 P(  8)=0.014650 P(  9)=0.007011 P( 10)=0.003354 P( 11)=0.001599
P( 12)=0.000759 P( 13)=0.000359 P( 14)=0.000169 P( 15)=0.000080 P( 16)=0.000038 P( 17)=0.000018
P( 18)=0.000008 P( 19)=0.000004 P( 20)=0.000002 P( 21)=0.000001 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000728
AVERAGE WAITING TIME IN THE QUEUE = 0.003489 HR

STEP1160     MEAN ARRIVAL RATE =   5.3000
 P(  0)=0.050697 P(  1)=0.150798 P(  2)=0.224295 P(  3)=0.222470 P(  4)=0.165610 P(  5)=0.098793
P(  6)=0.049310 P(  7)=0.021309 P(  8)=0.009450 P(  9)=0.004283 P( 10)=0.001974 P( 11)=0.000921
P( 12)=0.000433 P( 13)=0.000205 P( 14)=0.000097 P( 15)=0.000046 P( 16)=0.000022 P( 17)=0.000010
P( 18)=0.000005 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000001 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000729
AVERAGE WAITING TIME IN THE QUEUE = 0.001767 HR


AT HOUR  5     PATROL CARS ON DUTY =  6

STEP1180     MEAN ARRIVAL RATE =   4.7000
 P(  0)=0.069831 P(  1)=0.185306 P(  2)=0.245785 P(  3)=0.217228 P(  4)=0.143938 P(  5)=0.076358
P(  6)=0.033926 P(  7)=0.013557 P(  8)=0.005024 P(  9)=0.003226 P( 10)=0.001483 P( 11)=0.000683
P( 12)=0.000315 P( 13)=0.000146 P( 14)=0.000068 P( 15)=0.000032 P( 16)=0.000015 P( 17)=0.000007
P( 18)=0.000003 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000000 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000727
AVERAGE WAITING TIME IN THE QUEUE = 0.003678 HR
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AT HOUR 22      PATROL CARS ON DUTY = 10

STEP 900      MEAN ARRIVAL RATE =  10.1000
 P( 0)=0.005841 P( 1)=0.029801 P( 2)=0.076094 P( 3)=0.129716 P( 4)=0.166185 P( 5)=0.170828
P( 6)=0.146936 P( 7)=0.108949 P( 8)=0.071250 P( 9)=0.041896 P( 10)=0.022557 P( 11)=0.012484
P( 12)=0.007127 P( 13)=0.004158 P( 14)=0.002543 P( 15)=0.001576 P( 16)=0.000991 P( 17)=0.000633
P( 18)=0.000406 P( 19)=0.000262 P( 20)=0.000169 P( 21)=0.000109 P( 22)=0.000070 P( 23)=0.000045
P( 24)=0.000029 P( 25)=0.000019 P( 26)=0.000012 P( 27)=0.000007 P( 28)=0.000004 P( 29)=0.000003
P( 30)=0.000001 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.003543 HR

STEP 920      MEAN ARRIVAL RATE =  10.0000
 P( 0)=0.006207 P( 1)=0.031426 P( 2)=0.079607 P( 3)=0.134531 P( 4)=0.170660 P( 5)=0.173382
P( 6)=0.146997 P( 7)=0.106030 P( 8)=0.068379 P( 9)=0.038999 P( 10)=0.020164 P( 11)=0.010562
P( 12)=0.005624 P( 13)=0.003056 P( 14)=0.001699 P( 15)=0.000968 P( 16)=0.000566 P( 17)=0.000338
P( 18)=0.000206 P( 19)=0.000128 P( 20)=0.000080 P( 21)=0.000051 P( 22)=0.000032 P( 23)=0.000020
P( 24)=0.000013 P( 25)=0.000008 P( 26)=0.000005 P( 27)=0.000003 P( 28)=0.000002 P( 29)=0.000001
P( 30)=0.000001 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.003341 HR


AT HOUR 23      PATROL CARS ON DUTY = 10

STEP 940      MEAN ARRIVAL RATE =   9.9000
 P( 0)=0.006611 P( 1)=0.033120 P( 2)=0.082977 P( 3)=0.138627 P( 4)=0.173755 P( 5)=0.174300
P( 6)=0.145786 P( 7)=0.104600 P( 8)=0.065747 P( 9)=0.036806 P( 10)=0.018610 P( 11)=0.009473
P( 12)=0.004664 P( 13)=0.002525 P( 14)=0.001328 P( 15)=0.000710 P( 16)=0.000387 P( 17)=0.000215
P( 18)=0.000122 P( 19)=0.000071 P( 20)=0.000042 P( 21)=0.000025 P( 22)=0.000015 P( 23)=0.000009
P( 24)=0.000006 P( 25)=0.000004 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
AVERAGE WAITING TIME IN THE QUEUE = 0.003155 HR

STEP 960      MEAN ARRIVAL RATE =   9.8000
 P( 0)=0.007068 P( 1)=0.034724 P( 2)=0.086034 P( 3)=0.142122 P( 4)=0.176105 P( 5)=0.174603
P( 6)=0.144300 P( 7)=0.102258 P( 8)=0.063446 P( 9)=0.035029 P( 10)=0.017441 P( 11)=0.008119
P( 12)=0.004380 P( 13)=0.002214 P( 14)=0.001128 P( 15)=0.000580 P( 16)=0.000301 P( 17)=0.000159
P( 18)=0.000085 P( 19)=0.000046 P( 20)=0.000026 P( 21)=0.000015 P( 22)=0.000008 P( 23)=0.000005
P( 24)=0.000003 P( 25)=0.000002 P( 26)=0.000001 P( 27)=0.000001 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
AVERAGE WAITING TIME IN THE QUEUE = 0.002962 HR


AT HOUR  0      PATROL CARS ON DUTY =  6

STEP 980      MEAN ARRIVAL RATE =   9.7000
 P( 0)=0.007309 P( 1)=0.035545 P( 2)=0.086096 P( 3)=0.139241 P( 4)=0.169208 P( 5)=0.156229
P( 6)=0.121133 P( 7)=0.091504 P( 8)=0.067003 P( 9)=0.047363 P( 10)=0.032230 P( 11)=0.021085
P( 12)=0.013262 P( 13)=0.008031 P( 14)=0.004696 P( 15)=0.002660 P( 16)=0.001468 P( 17)=0.000792
P( 18)=0.000421 P( 19)=0.000222 P( 20)=0.000116 P( 21)=0.000061 P( 22)=0.000032 P( 23)=0.000017
P( 24)=0.000009 P( 25)=0.000005 P( 26)=0.000003 P( 27)=0.000002 P( 28)=0.000001 P( 29)=0.000001
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000736
AVERAGE WAITING TIME IN THE QUEUE = 0.299708 HR
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STEP1000     MEAN ARRIVAL RATE =   9.6000
 P( 0)=0.007295 P(  1)=0.034955 P(  2)=0.083547 P(  3)=0.132728 P(  4)=0.157511 P(  5)=0.148685
P(  6)=0.115954 P(  7)=0.079156 P(  8)=0.047415 P(  9)=0.050018 P( 10)=0.036345 P( 11)=0.025824
P( 12)=0.017921 P( 13)=0.012138 P( 14)=0.008019 P( 15)=0.005168 P( 16)=0.003250 P( 17)=0.001996
P( 18)=0.001198 P( 19)=0.000703 P( 20)=0.000405 P( 21)=0.000229 P( 22)=0.000127 P( 23)=0.000070
P( 24)=0.000038 P( 25)=0.000021 P( 26)=0.000011 P( 27)=0.000006 P( 28)=0.000003 P( 29)=0.000002
P( 30)=0.000001 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.258182 HR


AT HOUR  1      PATROL CARS ON DUTY =  6

STEP1020     MEAN ARRIVAL RATE =   9.1500
 P(  0)=0.007786 P(  1)=0.036491 P(  2)=0.085409 P(  3)=0.133058 P(  4)=0.155152 P(  5)=0.144351
P(  6)=0.111530 P(  7)=0.085775 P(  8)=0.065446 P(  9)=0.049405 P( 10)=0.036823 P( 11)=0.027052
P( 12)=0.019565 P( 13)=0.013915 P( 14)=0.009726 P( 15)=0.006676 P( 16)=0.004500 P( 17)=0.002977
P( 18)=0.001934 P( 19)=0.001233 P( 20)=0.000772 P( 21)=0.000475 P( 22)=0.000287 P( 23)=0.000171
P( 24)=0.000100 P( 25)=0.000058 P( 26)=0.000033 P( 27)=0.000019 P( 28)=0.000011 P( 29)=0.000007
P( 30)=0.000004 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.146490 HR

STEP1040     MEAN ARRIVAL RATE =   8.7000
 P(  0)=0.009044 P(  1)=0.040801 P(  2)=0.092367 P(  3)=0.139091 P(  4)=0.157070 P(  5)=0.141940
P(  6)=0.107029 P(  7)=0.080998 P(  8)=0.061287 P(  9)=0.046211 P( 10)=0.034629 P( 11)=0.025734
P( 12)=0.018932 P( 13)=0.013769 P( 14)=0.009889 P( 15)=0.007009 P( 16)=0.004899 P( 17)=0.003375
P( 18)=0.002292 P( 19)=0.001533 P( 20)=0.001010 P( 21)=0.000656 P( 22)=0.000420 P( 23)=0.000265
P( 24)=0.000165 P( 25)=0.000101 P( 26)=0.000062 P( 27)=0.000037 P( 28)=0.000023 P( 29)=0.000014
P( 30)=0.000009 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.063779 HR


AT HOUR  2      PATROL CARS ON DUTY =  6

STEP1060     MEAN ARRIVAL RATE =   8.1500
 P(  0)=0.011190 P(  1)=0.048216 P(  2)=0.103932 P(  3)=0.149491 P(  4)=0.161531 P(  5)=0.140066
P(  6)=0.101822 P(  7)=0.074920 P(  8)=0.055541 P(  9)=0.041316 P( 10)=0.030732 P( 11)=0.022793
P( 12)=0.016818 P( 13)=0.012322 P( 14)=0.008951 P( 15)=0.006441 P( 16)=0.004586 P( 17)=0.003229
P( 18)=0.002248 P( 19)=0.001546 P( 20)=0.001050 P( 21)=0.000705 P( 22)=0.000467 P( 23)=0.000306
P( 24)=0.000198 P( 25)=0.000127 P( 26)=0.000081 P( 27)=0.000051 P( 28)=0.000032 P( 29)=0.000021
P( 30)=0.000014 P(
TOTAL PROBABILITY =1.000736
AVERAGE WAITING TIME IN THE QUEUE = 0.057611 HR

STEP1080     MEAN ARRIVAL RATE =   7.6000
 P(  0)=0.014490 P(  1)=0.058916 P(  2)=0.119927 P(  3)=0.163072 P(  4)=0.166860 P(  5)=0.137385
P(  6)=0.095270 P(  7)=0.067418 P(  8)=0.048452 P(  9)=0.035157 P( 10)=0.025734 P( 11)=0.018866
P( 12)=0.013827 P( 13)=0.010105 P( 14)=0.007350 P( 15)=0.005313 P( 16)=0.003811 P( 17)=0.002711
P( 18)=0.001911 P( 19)=0.001334 P( 20)=0.000922 P( 21)=0.000631 P( 22)=0.000427 P( 23)=0.000287
P( 24)=0.000190 P( 25)=0.000125 P( 26)=0.000082 P( 27)=0.000053 P( 28)=0.000034 P( 29)=0.000022
P( 30)=0.000014 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.037559 HR
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AT HOUR  3     PATROL CARS ON DUTY =  5

STEP1100     MEAN ARRIVAL RATE =   7.1500
 P(  0)=0.018610 P(  1)=0.070593 P(  2)=0.133531 P(  3)=0.167835 P(  4)=0.157666 P(  5)=0.118155
P(  6)=0.088524 P(  7)=0.065971 P(  8)=0.048810 P(  9)=0.035874 P( 10)=0.026242 P( 11)=0.019143
P( 12)=0.013943 P( 13)=0.010142 P( 14)=0.007364 P( 15)=0.005331 P( 16)=0.003842 P( 17)=0.002755
P( 18)=0.001962 P( 19)=0.001388 P( 20)=0.000974 P( 21)=0.000678 P( 22)=0.000467 P( 23)=0.000320
P( 24)=0.000217 P( 25)=0.000146 P( 26)=0.000098 P( 27)=0.000065 P( 28)=0.000044 P( 29)=0.000030
P( 30)=0.000021 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.111492 HR

STEP1120     MEAN ARRIVAL RATE =   6.7000
 P(  0)=0.023109 P(  1)=0.082306 P(  2)=0.146619 P(  3)=0.174308 P(  4)=0.155829 P(  5)=0.112112
P(  6)=0.081737 P(  7)=0.060044 P(  8)=0.044745 P(  9)=0.032604 P( 10)=0.023982 P( 11)=0.017593
P( 12)=0.012867 P( 13)=0.009382 P( 14)=0.006820 P( 15)=0.004941 P( 16)=0.003567 P( 17)=0.002564
P( 18)=0.001834 P( 19)=0.001305 P( 20)=0.000923 P( 21)=0.000648 P( 22)=0.000453 P( 23)=0.000314
P( 24)=0.000216 P( 25)=0.000148 P( 26)=0.000101 P( 27)=0.000068 P( 28)=0.000046 P( 29)=0.000032
P( 30)=0.000021 P(
TOTAL PROBABILITY =1.000733
AVERAGE WAITING TIME IN THE QUEUE = 0.074609 HR


AT HOUR  4     PATROL CARS ON DUTY =  4

STEP1140     MEAN ARRIVAL RATE =   6.0000
 P(  0)=0.029588 P(  1)=0.056075 P(  2)=0.154989 P(  3)=0.165447 P(  4)=0.131437 P(  5)=0.103805
P(  6)=0.080806 P(  7)=0.061856 P(  8)=0.046653 P(  9)=0.034792 P( 10)=0.025750 P( 11)=0.018962
P( 12)=0.013914 P( 13)=0.010180 P( 14)=0.007426 P( 15)=0.005401 P( 16)=0.003914 P( 17)=0.002827
P( 18)=0.002033 P( 19)=0.001456 P( 20)=0.001037 P( 21)=0.000735 P( 22)=0.000518 P( 23)=0.000363
P( 24)=0.000253 P( 25)=0.000176 P( 26)=0.000122 P( 27)=0.000084 P( 28)=0.000059 P( 29)=0.000042
P( 30)=0.000030 P(
TOTAL PROBABILITY =1.000731
AVERAGE WAITING TIME IN THE QUEUE = 0.199720 HR

STEP1160     MEAN ARRIVAL RATE =   5.3000
 P(  0)=0.039046 P(  1)=0.114562 P(  2)=0.169362 P(  3)=0.166836 P(  4)=0.124283 P(  5)=0.094513
P(  6)=0.072420 P(  7)=0.055434 P(  8)=0.042189 P(  9)=0.031860 P( 10)=0.023870 P( 11)=0.017756
P( 12)=0.013126 P( 13)=0.009655 P( 14)=0.007070 P( 15)=0.005158 P( 16)=0.003749 P( 17)=0.002715
P( 18)=0.001959 P( 19)=0.001407 P( 20)=0.001007 P( 21)=0.000717 P( 22)=0.000508 P( 23)=0.000359
P( 24)=0.000252 P( 25)=0.000177 P( 26)=0.000124 P( 27)=0.000086 P( 28)=0.000061 P( 29)=0.000042
P( 30)=0.000029 P(
TOTAL PROBABILITY =1.000729
AVERAGE WAITING TIME IN THE QUEUE = 0.088045 HR


AT HOUR  5     PATROL CARS ON DUTY =  3

STEP1180     MEAN ARRIVAL RATE =   4.7000
 P(  0)=0.050624 P(  1)=0.130359 P(  2)=0.165686 P(  3)=0.138702 P(  4)=0.114992 P(  5)=0.093291
P(  6)=0.073977 P(  7)=0.057586 P(  8)=0.044233 P(  9)=0.033654 P( 10)=0.025413 P( 11)=0.019064
P( 12)=0.014213 P( 13)=0.010536 P( 14)=0.007770 P( 15)=0.005702 P( 16)=0.004167 P( 17)=0.003032
P( 18)=0.002198 P( 19)=0.001587 P( 20)=0.001141 P( 21)=0.000817 P( 22)=0.000583 P( 23)=0.000414
P( 24)=0.000293 P( 25)=0.000207 P( 26)=0.000146 P( 27)=0.000103 P( 28)=0.000073 P( 29)=0.000054
P( 30)=0.000040 P(
TOTAL PROBABILITY =1.000725
AVERAGE WAITING TIME IN THE QUEUE = 0.389307 HR
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AT HOUR 22      PATROL CARS ON DUTY =  9

STEP 900      MEAN ARRIVAL RATE =  10.1000
 P( 0)=0.006045 P( 1)=0.030788 P( 2)=0.078400 P( 3)=0.133097 P( 4)=0.169470 P( 5)=0.172633
P( 6)=0.146557 P( 7)=0.106662 P( 8)=0.067946 P( 9)=0.038500 P( 10)=0.021848 P( 11)=0.012414
P( 12)=0.007060 P( 13)=0.004018 P( 14)=0.002288 P( 15)=0.001303 P( 16)=0.000742 P( 17)=0.000422
P( 18)=0.000240 P( 19)=0.000136 P( 20)=0.000077 P( 21)=0.000044 P( 22)=0.000025 P( 23)=0.000014
P( 24)=0.000008 P( 25)=0.000004 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.010263 HR

STEP 920      MEAN ARRIVAL RATE =  10.0000
 P( 0)=0.006332 P( 1)=0.031959 P( 2)=0.080648 P( 3)=0.135668 P( 4)=0.171209 P( 5)=0.172847
P( 6)=0.145439 P( 7)=0.104921 P( 8)=0.066262 P( 9)=0.037234 P( 10)=0.020965 P( 11)=0.011827
P( 12)=0.006683 P( 13)=0.003782 P( 14)=0.002144 P( 15)=0.001216 P( 16)=0.000690 P( 17)=0.000392
P( 18)=0.000223 P( 19)=0.000127 P( 20)=0.000072 P( 21)=0.000041 P( 22)=0.000023 P( 23)=0.000013
P( 24)=0.000007 P( 25)=0.000004 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.009586 HR


AT HOUR 23      PATROL CARS ON DUTY =  9

STEP 940      MEAN ARRIVAL RATE =   9.9000
 P( 0)=0.006651 P( 1)=0.033244 P( 2)=0.083088 P( 3)=0.138450 P( 4)=0.173037 P( 5)=0.173031
P( 6)=0.144213 P( 7)=0.103056 P( 8)=0.064476 P( 9)=0.035857 P( 10)=0.020032 P( 11)=0.011204
P( 12)=0.006280 P( 13)=0.003527 P( 14)=0.001905 P( 15)=0.001119 P( 16)=0.000632 P( 17)=0.000357
P( 18)=0.000202 P( 19)=0.000114 P( 20)=0.000065 P( 21)=0.000037 P( 22)=0.000021 P( 23)=0.000012
P( 24)=0.000007 P( 25)=0.000004 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000738
AVERAGE WAITING TIME IN THE QUEUE = 0.008971 HR

STEP 960      MEAN ARRIVAL RATE =   9.8000
 P( 0)=0.006993 P( 1)=0.034609 P( 2)=0.085648 P( 3)=0.141311 P( 4)=0.174875 P( 5)=0.173149
P( 6)=0.142895 P( 7)=0.101114 P( 8)=0.062644 P( 9)=0.034539 P( 10)=0.019091 P( 11)=0.010578
P( 12)=0.005875 P( 13)=0.003271 P( 14)=0.001825 P( 15)=0.001020 P( 16)=0.000572 P( 17)=0.000321
P( 18)=0.000180 P( 19)=0.000102 P( 20)=0.000057 P( 21)=0.000032 P( 22)=0.000018 P( 23)=0.000010
P( 24)=0.000006 P( 25)=0.000003 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.008400 HR


AT HOUR  0      PATROL CARS ON DUTY =  9

STEP 980      MEAN ARRIVAL RATE =   9.7000
 P( 0)=0.007356 P( 1)=0.036042 P( 2)=0.088297 P( 3)=0.144218 P( 4)=0.176681 P( 5)=0.173182
P( 6)=0.141489 P( 7)=0.099117 P( 8)=0.060792 P( 9)=0.033184 P( 10)=0.018150 P( 11)=0.009963
P( 12)=0.005480 P( 13)=0.003022 P( 14)=0.001670 P( 15)=0.000926 P( 16)=0.000514 P( 17)=0.000286
P( 18)=0.000160 P( 19)=0.000089 P( 20)=0.000050 P( 21)=0.000028 P( 22)=0.000016 P( 23)=0.000009
P( 24)=0.000005 P( 25)=0.000003 P( 26)=0.000002 P( 27)=0.000001 P( 28)=0.000001 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.007865 HR
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STEP1000     MEAN ARRIVAL RATE =   9.6000
 P( 0)=0.007740 P(  1)=0.037535 P(  2)=0.091018 P(  3)=0.147149 P(  4)=0.178436 P(  5)=0.173122
P(  6)=0.139999 P(  7)=0.097C74 P(  8)=0.058933 P(  9)=0.031842 P( 10)=0.015249 P( 11)=0.009368
P( 12)=0.005101 P( 13)=0.002785 P( 14)=0.001524 P( 15)=0.000837 P( 16)=0.000460 P( 17)=0.000254
P( 18)=0.000140 P( 19)=0.000C78 P( 20)=0.000043 P( 21)=0.000024 P( 22)=0.000013 P( 23)=C.000007
P( 24)=0.000004 P( 25)=0.000002 P( 26)=0.000001 P( 27)=0.000001 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.0C0738
AVERAGE WAITING TIME IN THE QUEUE = 0.007359 HR


AT HOUR  1     PATROL CARS ON DUTY =  9

STEP1020     MEAN ARRIVAL RATE =   9.1500
 P(  0)=0.008709 P(  1)=0.041216 P(  2)=0.057533 P(  3)=0.153876 P(  4)=0.182094 P(  5)=C.172419
P(  6)=0.136089 P(  7)=0.092124 P(  8)=0.054633 P(  9)=0.028875 P( 10)=0.015354 P( 11)=0.008208
P( 12)=0.004406 P( 13)=0.002177 P( 14)=0.001287 P( 15)=0.000699 P( 16)=0.000381 P( 17)=0.000208
P( 18)=0.000114 P( 19)=0.000063 P( 20)=0.000034 P( 21)=0.000019 P( 22)=0.000011 P( 23)=C.000006
P( 24)=0.000003 P( 25)=0.000002 P( 26)=0.000001 P( 27)=0.000001 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.004843 HR

STEP1040     MEAN ARRIVAL RATE =   8.7000
 P(  0)=0.010493 P(  1)=0.047715 P(  2)=0.108498 P(  3)=0.164495 P(  4)=0.187078 P(  5)=0.170262
P(  6)=0.129201 P(  7)=0.084121 P(  8)=0.048020 P(  9)=0.024469 P( 10)=0.012586 P( 11)=0.006530
P( 12)=0.003416 P( 13)=0.CC1800 P( 14)=0.000955 P( 15)=0.000505 P( 16)=0.000273 P( 17)=0.000147
P( 18)=0.000080 P( 19)=0.000043 P( 20)=0.000024 P( 21)=0.000013 P( 22)=0.000007 P( 23)=0.000004
P( 24)=0.000002 P( 25)=0.000001 P( 26)=0.000001 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.003356 HR


AT HOUR  2     PATROL CARS ON DUTY =  9

STEP1060     MEAN ARRIVAL RATE =   8.1500
 P(  0)=0.013218 P(  1)=0.057681 P(  2)=0.123262 P(  3)=0.177477 P(  4)=0.191702 P(  5)=0.165723
P(  6)=0.119474 P(  7)=0.073927 P(  8)=0.040133 P(  9)=0.019476 P( 10)=0.009573 P( 11)=0.004763
P( 12)=0.002398 P( 13)=0.001220 P( 14)=0.000627 P( 15)=0.000325 P( 16)=0.000170 P( 17)=C.000090
P( 18)=0.000048 P( 19)=0.000025 P( 20)=0.000014 P( 21)=0.000007 P( 22)=0.000004 P( 23)=0.000002
P( 24)=0.000001 P( 25)=0.000001 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000737
AVERAGE WAITING TIME IN THE QUEUE = 0.002094 HR

STEP1080     MEAN ARRIVAL RATE =   7.6000
 P(  0)=0.017144 P(  1)=0.069613 P(  2)=0.141345 P(  3)=0.191365 P(  4)=0.194372 P(  5)=0.158017
P(  6)=0.107141 P(  7)=0.062365 P(  8)=0.031862 P(  9)=0.014567 P( 10)=0.006762 P( 11)=0.003187
P( 12)=0.001525 P( 13)=0.000740 P( 14)=0.000364 P( 15)=0.000182 P( 16)=0.000092 P( 17)=0.000047
P( 18)=0.000024 P( 19)=0.000013 P( 20)=0.000007 P( 21)=0.000004 P( 22)=0.000002 P( 23)=0.000001
P( 24)=0.000001 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.001299 HR
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AT HOUR  3     PATROL CARS ON DUTY =  0

STEP1100     MEAN ARRIVAL RATE =   7.1500
 P(  0)=0.022045 P(  1)=0.083990 P(  2)=0.159984 P(  3)=0.203130 P(  4)=0.193387 P(  5)=0.147245
P(  6)=0.093406 P(  7)=0.050804 P(  8)=0.024229 P(  9)=0.011641 P( 10)=0.005617 P( 11)=0.002715
P( 12)=0.001312 P( 13)=0.000634 P( 14)=0.000307 P( 15)=0.000149 P( 16)=0.000073 P( 17)=0.000036
P( 18)=0.000018 P( 19)=0.000009 P( 20)=0.000005 P( 21)=0.000002 P( 22)=0.000001 P( 23)=0.000001
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000735
AVERAGE WAITING TIME IN THE QUEUE = 0.002980 HR

STEP1120     MEAN ARRIVAL RATE =   6.7000
 P(  0)=0.027870 P(  1)=0.059628 P(  2)=0.178079 P(  3)=0.212222 P(  4)=0.189725 P(  5)=0.135754
P(  6)=0.081033 P(  7)=0.041555 P(  8)=0.018755 P(  9)=0.008582 P( 10)=0.003977 P( 11)=0.001863
P( 12)=0.000881 P( 13)=0.000420 P( 14)=0.000201 P( 15)=0.000097 P( 16)=0.000047 P( 17)=0.000023
P( 18)=0.000011 P( 19)=0.000005 P( 20)=0.000003 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000731
AVERAGE WAITING TIME IN THE QUEUE = 0.002028 HR


AT HOUR  4     PATROL CARS ON DUTY =  7

STEP1140     MEAN ARRIVAL RATE =   6.0000
 P(  0)=0.036738 P(  1)=0.121153 P(  2)=0.199714 P(  3)=0.219389 P(  4)=0.180650 P(  5)=0.118937
P(  6)=0.065261 P(  7)=0.030775 P(  8)=0.014669 P(  9)=0.007025 P( 10)=0.003365 P( 11)=0.001607
P( 12)=0.000764 P( 13)=0.000363 P( 14)=0.000172 P( 15)=0.000082 P( 16)=0.000039 P( 17)=0.000019
P( 18)=0.000009 P( 19)=0.000004 P( 20)=0.000002 P( 21)=0.000001 P( 22)=0.000001 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000731
AVERAGE WAITING TIME IN THE QUEUE = 0.003487 HR

STEP1160     MEAN ARRIVAL RATE =   5.3000
 P(  0)=0.050684 P(  1)=0.150770 P(  2)=0.224269 P(  3)=0.222462 P(  4)=0.165619 P(  5)=0.098809
P(  6)=0.049325 P(  7)=0.021320 P(  8)=0.009458 P(  9)=0.004289 P( 10)=0.001979 P( 11)=0.000924
P( 12)=0.000435 P( 13)=0.000206 P( 14)=0.000098 P( 15)=0.000047 P( 16)=0.000022 P( 17)=0.000011
P( 18)=0.000005 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000001 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000731
AVERAGE WAITING TIME IN THE QUEUE = 0.001707 HR


AT HOUR  5     PATROL CARS ON DUTY =  6

STEP1180     MEAN ARRIVAL RATE =   4.7000
 P(  0)=0.069822 P(  1)=0.185291 P(  2)=0.245773 P(  3)=0.217227 P(  4)=0.143944 P(  5)=0.076366
P(  6)=0.033932 P(  7)=0.015361 P(  8)=0.007027 P(  9)=0.003229 P( 10)=0.001485 P( 11)=0.000684
P( 12)=0.000316 P( 13)=0.000146 P( 14)=0.000068 P( 15)=0.000032 P( 16)=0.000015 P( 17)=0.000007
P( 18)=0.000003 P( 19)=0.000002 P( 20)=0.000001 P( 21)=0.000000 P( 22)=0.000000 P( 23)=0.000000
P( 24)=0.000000 P( 25)=0.000000 P( 26)=0.000000 P( 27)=0.000000 P( 28)=0.000000 P( 29)=0.000000
P( 30)=0.000000 P(
TOTAL PROBABILITY =1.000727
AVERAGE WAITING TIME IN THE QUEUE = 0.003678 HR
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TRANSIENT SOLUTIONS OF M/M/s NONSTEADY STATE QUEUEING SYSTEM

by

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AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

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1981

ABSTRACT

Queues are very common phenomena, widely encountered in every day life. Most queueing models, which assume that the mean arrival rate and the service rate are constant through time and that the system is in a steady state, have been well studied in the past. Unfortunately this constancy of rate is often not the case in real queueing problems. For instance, customer arrival rates change with the time of day in the supermarket; and car arrival rates at toll booths peak at 8 A.M. and 5 P.M.

It is very common to use the simulation approach to interpret queueing problems in the transient state although there are many pitfalls one may encounter in using it, and its analysis is relatively expensive and cumbersome. With the aid of modern digital computers, a number of numerical approaches and approximate methods for determining the transient solution of nonsteady state queueing problems have been invented.

One of the main objectives of this study is to do a limited survey of the literature of these numerical or approximate approaches.

Because of its simplicity and ease of use, the so-called standard numerical method is applied to police patrol car scheduling problems. Because the number of patrol cars on duty is a discontinuous function of time, the fourth-order Runge Kutta numerical method was selected. The Kolesar et al methodology was applied to solve this patrol scheduling problem.

The results show that the police administrator can select a reasonable schedule by applying a M/M/s nonsteady state queueing model to evaluate patrol scheduling alternatives. The current (experienced) schedule was evaluated, based both on a specified probability of an urgent call being placed in queue as criterion, and on a specified expected waiting time of a call in the queue as criterion. From both points of view, the experienced schedule is extremely inadequate.