SYNTHESIS OF TRANSFER FUNCTIONS BY MEANS OF INTEGRATED ACTIVE NETWORKS

by 1264

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CHAPTER I

INTRODUCTION

1-1 General Description

The synthesis of a network is based on the exact realization of a specified network function which is a real rational function of the complex frequency variable s, T(s). The network function T(s) is usually obtained by approximating a given transmission characteristic. In order that a satisfactory approximation be achieved, it is usually necessary to have a network function with complex poles, with the total number of poles of the approximating network function required to be reasonably small.

Networks can be divided into two groups, passive networks and active networks. A network containing only passive elements such as resistors, capacitors, and inductors is called a passive network. For such networks, the total energy input $E(t_1)$ is nonnegative for every t_1 and for all possible voltage or current excitations if the initial stored energy is zero.

On the other hand, a network is active if it is not passive. In addition to passive elements, there are active elements in an active network. Active elements generally used at the present time are negative resistance, controlled sources, operational amplifiers, negative impedance converters, and gyrators.

1-2 Restrictions of Passive Networks

Passive networks using only resistors and capacitors are attractive because these elements are cheaper, simpler, and more nearly ideal than are inductors. However, they introduce loss in the pass band. From the characteristics of RC one-ports and two-ports, it is well known that all poles and zeros of the network function are real and negative. Thus using RC networks to meet a given filter specification makes the resultant network more complex than the equivalent RLC network.

The above defects can be eliminated by adding inductors to the network. In fact, by using R, C, and L as network elements, one can realize network functions with complex poles anywhere on the left half s-plane. As a result, inductors are almost always used in passive network design.

An inductor has some inherent shortcomings, i.e., limitations of quality and size. At low frequencies (less than 1 KHz) Q's of inductors are limited to a maximum value of about 100. High Q's are physically impractical because a large, bulky inductor is required. Inductor quality tends to deteriorate rapidly as frequency is lowered. The magnetic coupling between inductive elements in a network presents an additional complication. The problem of how to design a network having satisfactory characteristics without the use of inductors leads to the study of active network synthesis.

1-3 Advantages of Active Networks

When active elements are used in a network, the complete transfer function realization may have gain instead of loss in the pass band. The network function of an active RC network may also have poles anywhere on the s-plane. Therefore inductors can be eliminated without making the total network more complex. Another method of eliminating inductors in a network is to simulate the inductor by active devices such as operational amplifiers or gyrators. In passive network synthesis, transmission zeros are restricted to the left half s-plane. On the other hand, poles of network functions can be extended to the right half s-plane if active devices are used. This means that a specified characteristic can be approximated to a higher degree of accuracy by a reasonably simple network. In a word, RC active networks cannot only realize network functions which are realizable by passive RLC networks but also can be used to realize driving point or transfer characteristics not achievable with passive networks.

Since the poles of the network function of an active network are extended to the right half s-plane, the problem of stability of the complete circuit arises. Fortunately, this problem can be solved by proper design.

Another problem encountered in such network design is the sensitivity of the network function to the change of value of network parameters. The variation of parameters causes the displacement of the position of poles and zeros. As a result, the network will not exhibit the desired performance.

1-4 Active Devices in Integrated Form

Active network design achieved little progress before the appearance of the transistor in 1948 because of such deficiencies of the vacuum tube as large size, high power required, and high cost. Despite the initial parameter variability of the transistor, the modern era of active networks was off and running as transistors overcame the disadvantages of vacuum tubes. The rapid growth of integrated circuit techniques stimulated the recent surge of interest in linear active networks. The main advantages of integrated microcircuits are as follows (Lessor, Maissel, and Thein, 1964):

(1) increased system reliability and reduction of cost obtained by elimination of discrete joints and assembly operations,

(2) reduction of size and weight, and increased equipment density obtained by reducing the packaging levels and electronically inactive structural materials,

(3) increased functional performance obtained by regular and compact distribution of circuit elements,

(4) increased operating speeds due to absence of parasitics and decreased propagation delay, and

(5) reduction in power consumption.

CHAPTER II

ACTIVE NETWORK ELEMENTS

2-1 Negative Resistances

A negative resistance is a two-terminal reciprocal element obeying the relation: $V(t) = -R \times i(t)$. According to this definition, an ideal negative resistance can deliver an infinite amount of power. Depending on the form of their v-i characteristic, these devices are usually divided into two groups, namely, S-type and N-type. The S-type negative resistance is a current-controlled element, i.e., for a given current value, there may be more than one corresponding voltage value. On the other hand, an N-type negative resistance is a voltage-controlled device; there may be more than one current value corresponding to each voltage applied to it.

Negative resistance, at the present time, can be obtained from vacuum tubes, semiconductor devices, or electronic feedback circuits.

A) Vacuum tubes: Transient-time effect of diodes, negative screen-grid resistance of a pentode.

B) Semiconductor devices: Tunnel diode, double-base diode, space-charge diode.

C) Electronic feedback circuits: Series-triode circuit (cascode), simplified Colpitts circuit (or positive feedback circuit), the negative impedance converter, use of operational amplifier.

Theoretically, negative resistances of arbitrary value can be obtained by electronic circuits, but only those of moderate value (several kilohms) have adequate stability.

2-2 Controlled Sources

The controlled source is a unidirectional, nonautonomous active two-port having two pairs of terminals, one controlled and the other controlling. The controlled port has a single valued dependence upon the controlling port. Since the source is unidirectional, the controlling port variables are insensitive of the variables of the controlled port.

There are four kinds of controlled sources. They are voltage-to-voltage transducer, voltage-to-current transducer, current-to-current transducer, and current-to-voltage transducer. For simplicity they are designated as VVT, VCT, CCT, and CVT, respectively.

A) A voltage-to-voltage transducer is shown in Fig. 2.1 and is represented by the voltage-current relation:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

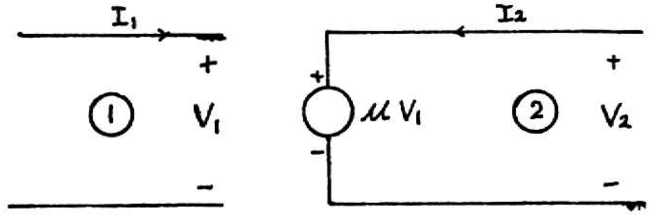


Fig. 2.1.

From this relation, it should be noted that this device has zero-input admittance, zero-output impedance, and an infinite power gain. The factor μ is the forward transfer voltage ratio. Practically, the cathode follower approximates a unity gain VVT.

B) A voltage-to-current transducer is a device in which the output current is dependent upon the input voltage as shown in Fig. 2.2. It is characterized by the following relation:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

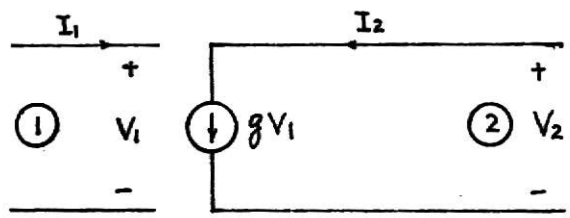


Fig. 2.2.

It is obvious that both input and output impedances are infinite and that the forward transmission is g. This device again provides infinite power gain. An ideal pentode (or field effect transistor) without interelectrode capacitances and with infinite plate (or drain) resistance is a VCT.

C) A current-to-current transducer is a device in which output current depends upon the input current with both input impedance and output admittance zero. Its forward current transfer ratio is α. The CCT is a current amplifier which can supply any amount of output power without consuming any input power. Figure 2.3 is an ideal CCT with current-voltage relation as follows:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

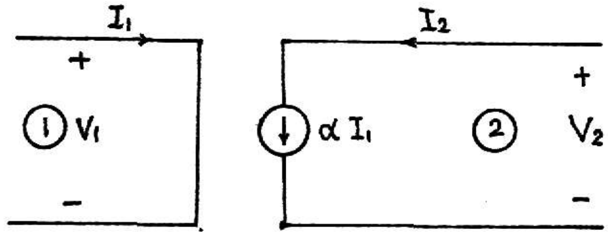


Fig. 2.3.

The grounded base transistor is an example of a nearly ideal CCT.

D) A current-to-voltage transducer is an ideal two-port in which the output voltage is proportional to the input current as shown in Fig. 2.4. Its v-i relation is given as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

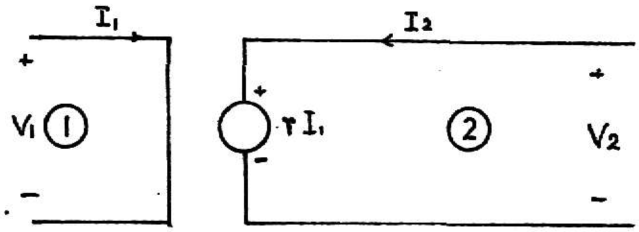


Fig. 2.4.

where r is the forward transfer impedance. Although the input power is zero, the output power is unlimited.

2-3 Operational Amplifiers

The operational amplifier is the most versatile active element at the present time. Any other active elements can be realized by using operational amplifiers. In addition, it is the only active element commercially available in monolithic integrated circuit form at the present.

An ideal operational amplifier is a frequency independent infinite gain VVT with zero input admittance and output impedance. There are two input terminals in an operational amplifier, one marked "+" and the other marked "-". The former is called the noninverting input terminal while the latter is called the inverting input terminal. The output voltage is always of the same polarity as that of the noninverting input terminal and opposite to that of the other input. Its zero input admittance and output impedance characteristic implies that the input excitation will not be affected by the power drawn by the amplifier and that the amplifier can supply any amount of power as required by the load. The symbolic representation of an operational amplifier is shown in Fig. 2.5.

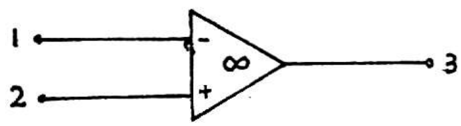


Fig. 2.5.

In practice, the operational amplifier is a nonideal device. Its voltage gain is a function of frequency with a very high value at dc (as high as 100 db) and a monotonic decrease for high frequencies. The voltage gain phase decreases in the same manner. Instead of zero values, the input admittance and output impedance of an operational amplifier have a finite value.

Negative feedback is always used with operational amplifiers for linear applications. This in general can improve the gain stability and linearity of the device and reduce the input admittance and output impedance. Thus a practical (nonideal) operational amplifier can be used satisfactorily as an active network element.

2-4 Negative Impedance Converters

An ideal negative impedance converter is an active two-port in which a negative impedance appears in one port when the other port is terminated with an impedance. There are two kinds of NIC according to the sign of the h parameters. If h_{12} and h_{21} are both negative real constants, the converter is called a voltage inversion type negative-impedance converter (VNIC). It is represented by an h matrix as:

$$[h] = \begin{bmatrix} 0 & -K_1 \\ -K_2 & 0 \end{bmatrix}$$

The symbol of a VNIC is shown in Fig. 2.6,

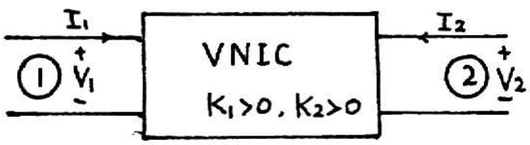


Fig. 2.6.

where K_1 and K_2 are positive real constants. Alternatively, if both h_{12} and h_{21} are positive real numbers, the device is called a current inversion type negative-impedance converter (CNIC). The h matrix of a CNIC is given as:

$$[h] = \begin{bmatrix} 0 & K_1 \\ K_2 & 0 \end{bmatrix}$$

The symbol of a CNIC is shown in Fig. 2.7.

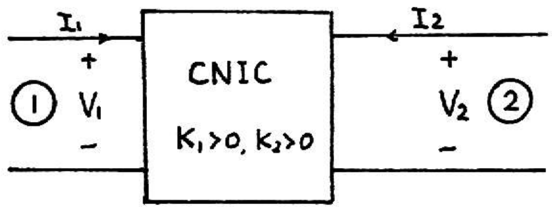


Fig. 2.7.

Practical NIC's are nonideal devices, i.e., h_{11} and h_{22} are not equal to zero. However, by adding appropriate input and output impedances, Z_a and Z_b, respectively, to a nonideal NIC, an ideal NIC can be achieved.

NIC's have several advantages, especially over gyrators. (1) It is possible to design filters with a smaller spread of capacitance value and fewer network elements. (2) There are more design procedures for NIC's. (3) An exact transfer function can be obtained simply by trimming the individual components. (4) An NIC can be built with fewer transistors and with a wider bandwidth than is possible for a gyrator. In general, NIC's are nonideal devices, i.e., h_{11} and h_{22} are not equal to zero. These nonzero values may counteract the negative impedance conversion properties of the device.

2-5 Gyrators

The gyrator is a nonreciprocal two-port whose input impedance is proportional to its load admittance. When a gyrator is loaded with a capacitor its input terminal is characterized as an inductor. The gyrator neither adds nor consumes signal energy in a circuit. Thus it behaves as a lossless active element. Therefore the gyrator by itself can never be unstable.

An ideal gyrator is a positive impedance inverter and is characterized by the following open-circuit impedance matrix:

$$[Z] = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

where r is called the gyration impedance of the gyrator. An ideal gyrator is shown symbolically in Fig. 2.8.

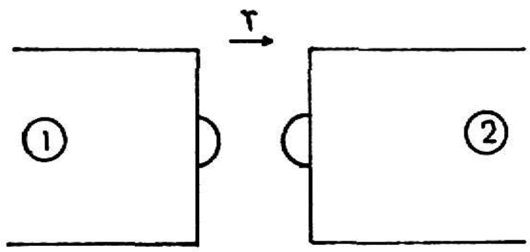


Fig. 2.8.

In practice, a gyrator is a nonideal element, i.e., the zero elements in the Z matrix are not zero, so they act essentially as parasitic impedances. In other words, the gyrator is a lossy device. One way of compensating the effect of these parasitics is by using negative resistances. Another method is to use a resistive network connected in parallel with the nonideal gyrator.

CHAPTER III

CONVENTIONAL ACTIVE-NETWORK SYNTHESIS APPROACHES

In the conventional approach, the synthesis is based on a general network configuration containing passive subnetworks and one or more idealized active elements. Generally the network function of the proposed network is given. By suitable decomposition and partitioning, the parameters characterizing the passive subnetworks for a given network function are obtained. The realization is completed by realizing the passive subnetworks following standard passive-synthesis procedures.

3-1 Linvill's Method

Linvill's method (1964) is used to synthesize transfer impedances using RC passive elements and a negative impedance converter. The basic configuration is shown in Fig. 3.1,

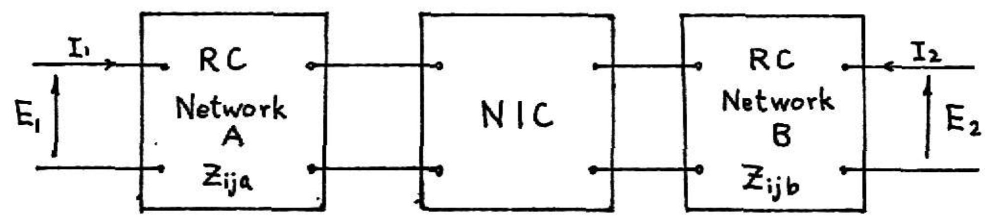


Fig. 3.1.

where Z_{ija} are one Z parameter of RC network A and Z_{ijb} are the Z parameters of RC network B.

The transfer function of Fig. 3.1 is given as

$$\left.\frac{E_2}{I_1}\right|_{I_2=0} = Z_{21} = \frac{Z_{12a}\ Z_{12b}}{Z_{22a} - Z_{11b}} = Z_T(s) = \frac{N(s)}{D(s)} \tag{3.1}$$

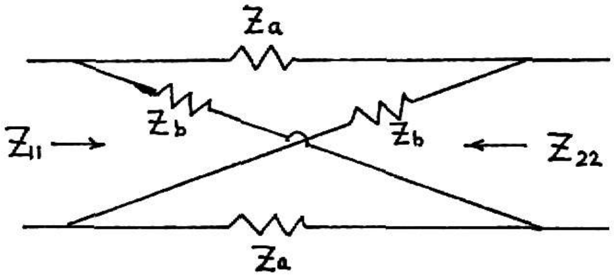
For a given transfer impedance $Z_T(s) = \dfrac{N(s)}{D(s)}$ of order n, Linvill's procedure starts by choosing a polynomial Q(s) of degree equal to, or greater than n. All roots of Q(s) must be simple and lie on the negative real axis. There should be no common roots between Q(s) and D(s). After dividing D(s) and N(s) by Q(s), then it is identified that

$$\frac{D(s)}{Q(s)} = Z_{22a}(s) - Z_{11b}(s) \tag{3.2}$$

$$\frac{N(s)}{Q(s)} = Z_{21a}(s)\ Z_{21b}(s) \tag{3.3}$$

Now D(s)/Q(s) is expanded in partial fractions and positive and negative terms are collected. It is obvious that the first term is $Z_{22a}(s)$ while the second term is $KZ_{11b}(s)$. If N(s) is a constant, i.e., $Z_T(s)$ has infinite zeros, then a Cauer synthesis can be made for $Z_{22a}(s)$ and $KZ_{11b}(s)$ to obtain the passive subnetwork in RC ladder form, and the synthesis is completed.

If N(s) is not a constant, then $Z_T(s)$ has finite transmission zeros. In this case lattices must be used in the passive subnetwork. A lattice of the form



has a driving point impedance of

$$Z_{11} = Z_{22} = \frac{Z_a + Z_b}{2} \tag{3.4}$$

Furthermore, the lattice transfer impedance is

$$Z_{12} = \frac{Z_b - Z_a}{2} \tag{3.5}$$

When Z_{11} and Z_{12} are known, Z_a and Z_b can be obtained from these two relations. Finally, the largest constant multiplier of Z_{12} is selected to permit Z_a and Z_b to still be realized as RC networks.

In equation (3.1), the zeros of N(s) determine the structure of the network, while the zeros of D(s) determine the natural frequencies of the networks. Since an RC ladder network has zeros of transmission only at zero and infinite frequencies, for finite transmission zeros lattice networks or unbalanced equivalent networks must be used. This is the fundamental shortcoming of this method.

3-2 Yanagisawa's Method

The synthesis procedure proposed by Yanagisawa (1957) is used to synthesize an open-circuit transfer voltage ratio using RC passive elements and a negative impedance converter. In this method, a one-port RC network is used. Thus the realization of the RC subnetwork is quite simple even in the case of complex transmission zeros.

For the network of Fig. 3.2, the transfer voltage ratio T_v is

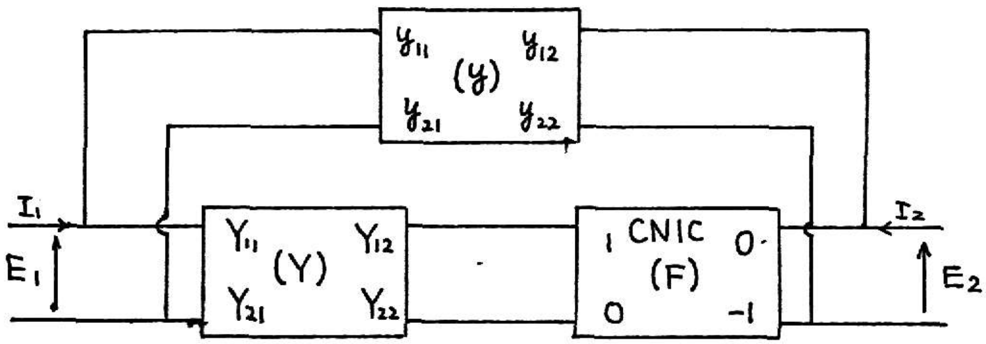


Fig. 3.2.

$$T_v = \left.\frac{E_1}{E_2}\right|_{I_2=0} = -\frac{y_{22} - Y_{22}}{y_{12} - Y_{12}} = \frac{N(s)}{D(s)} \qquad (3.6)$$

It is obvious that the CNIC acts both on the numerator and denominator. Thus the disadvantage of Linvill's method is eliminated.

In order to realize Y_{22} and Y_{12} simultaneously, an RC one-port CNIC configuration is used as in Fig. 3.3. The transfer voltage ratio of this network is

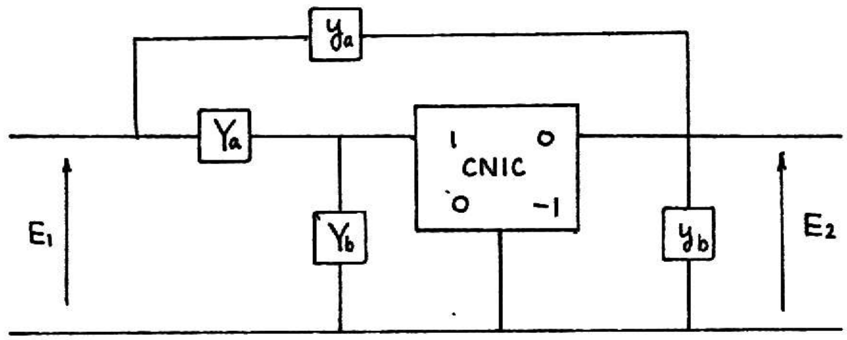


Fig. 3.3.

$$T_v = \frac{y_a - Y_a + y_b - Y_b}{y_a - Y_a} \tag{3.7}$$

Now equation (3.6) is modified to the form

$$T_v = \frac{D(s) + N(s) - D(s)}{D(s)} \tag{3.8}$$

If it is assumed that the order of the transfer function $T_v(s)$ is n, then a polynomial Q(s) of order (n-1) or higher is chosen so that its roots all lie on negative real axis. Both the numerator and denominator of equation (3.8) are divided by Q(s) and compared with equation (3.7) to obtain

$$y_a - Y_a = \frac{D(s)}{Q(s)} = k_\infty s + k_0 + \sum_{i=1}^{n-1} \frac{k_i\ s}{(s - s_i)} \tag{3.9}$$

$$y_b - Y_b = \frac{N(s) - D(s)}{Q(s)} = k_\infty s + k_0 + \sum_{i=1}^{n-1} \frac{k_i\ s}{(s - s_i)}$$

By expanding equation (3.9) into partial fractions, it can be seen that the sum of terms with positive coefficients will be realized by y_a and y_b, and those with negative coefficients by Y_a and Y_b. The degree of Q(s) can be reduced by choosing its factors as common factors of N(s) - D(s), or D(s) if they lie on the negative real axis.

In Fig. 3.3, since y_a and Y_a are in parallel at the input terminal, it is necessary to use a source of low impedance to drive the network. At the output terminal, the parallel impedance y_b can be considered as the output impedance.

It is seen that any real rational function can be realized in the form of Fig. 3.3. This means there is no restriction on the transfer function. But from the sensitivity point of view, it is better to realize second-order stages and cascade them with buffer stages.

3-3 RC: -R Network

By using RC: -R networks, real transmission zeros as well as complex transmission zeros can be realized. To realize real transmission zeros, the well-known zero-producing and zero-shifting techniques used in passive network synthesis can be applied. For zeros on the negative real axis, the procedure is the same as that for the passive RC case.

For zeros on the positive real axis, the zero shifting arm may be realized by a shunt resistance and the transmission zero produced by a parallel combination of -R and C in the series branch. Alternatively, the zero shifting arm may be realized by a positive R in the series branch and the zero generated by a series -R, C in the shunt arm.

The following configuration, Fig. 3.4, is suitable for the realization of complex transmission zeros.

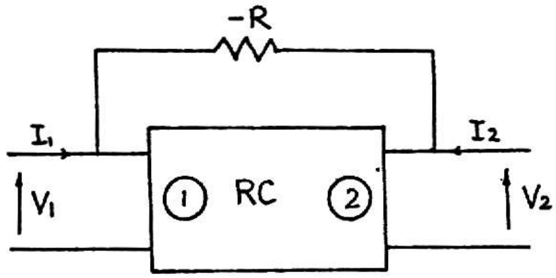


Fig. 3.4.

This two-port has a voltage transfer ratio of

$$T_v = \left.\frac{V_2}{V_1}\right|_{I_2=0} = -\frac{y_{21} - 1}{y_{22} - 1} \tag{3.10}$$

where y_{21} and y_{22} are the y parameters of the RC two-port.

It is supposed that a voltage transfer ratio $T_v = N(s)/D(s)$ is given. Now a polynomial $Q(s)$ is chosen, having negative real roots only, so that $D(s)/Q(s)$ is an RC driving-point impedance function. $N(s)/Q(s)$, on the other hand, can be expressed as

$$\frac{N(s)}{Q(s)} = \frac{N_1(s)}{Q_1(s)} - \frac{N_2(s)}{Q_2(s)} = \frac{N_1(s) + KQ_1(s)}{Q_1(s)} - \frac{N_2(s) + KQ_2(s)}{Q_2(s)}$$

i.e.,

$$N(s) = \left[N_1(s) + KQ_1(s)\right]Q_2(s) - \left[N_2(s) + KQ_2(s)\right]Q_1(s),$$

where K is an arbitrary constant. Thus it follows that

$$\frac{N(s)}{D(s)} = \frac{\dfrac{[N_1(s) + KQ_1(s)]Q_2(s)}{[N_2(s) + KQ_2(s)]Q_1(s)} - 1}{\dfrac{D(s)}{[N_2(s) + KQ_2(s)]Q_1(s)}} \tag{3.11}$$

From equation (3.10) and equation (3.11), the values of y_{21} and y_{22} are

$$-y_{21} = \frac{[N_1(s) + KQ_1(s)]Q_2(s)}{[N_2(s) + KQ_2(s)]Q_1(s)}$$

$$y_{22} = \frac{D(s)}{[N_2(s) + KQ_2(s)]Q_1(s)} + 1 \tag{3.12}$$

In equation (3.12), for a suitable value of K, y_{22} can be made RC realizable and the zeros of y_{21} are always negative real. Thus both $-y_{21}$ and y_{22} can be realized.

For a specified transfer function $T(s) = K \dfrac{N(s)}{D(s)}$, T(s) is modified by substituting s + d for s in T(s) and obtaining

$$T_f(s) = K \frac{N(s + d)}{D(s + d)},$$

so that $T_f(s)$ is passive RC realizable. Finally, the effect of reverse predistortion is removed by replacing each capacitor C by a parallel combination of C and a negative resistor $-R = -1/Cd$. This -RC parallel combination in turn can be replaced by a suitable tunnel diode.

3-4 Kuh's Method

This approach is attractive because a nonideal controlled source, having finite input, output, and feedback impedances, is used as an active device. Its disadvantages are (1) the transfer function is always realized to within a multiplicative constant, and (2) positive real transmission zeros cannot be realized if a common ground is desired.

The structure shown in Fig. 3.5 is proposed by Kuh (1960). Its transfer voltage ratio is

$$\frac{V_2}{V_{in}} = \frac{-y_{21}}{y_{22} + Y_1 - (\mu-1)Y_2} \tag{3.13}$$

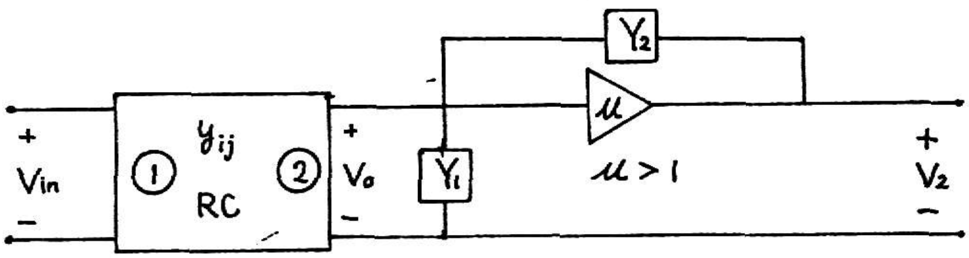


Fig. 3.5.

where y_{12}, y_{22} are short-circuit admittances of the passive RC two-port, Y_1, Y_2 should be RC one-ports, and μ is the voltage transfer ratio. Since Y_1 is in the input terminal of the active element, thus $Y_1(0)$ should not be zero. If $Y_2(0)$ and $Y_2(\infty)$ are finite, the feedback and output impedances can be included in it.

For convenience, it is supposed that Y_1 equals unity. When a specified transfer voltage ratio is given as $N(s)/D(s)$, a polynomial $Q(s)$ with distinct negative real roots, where the degree of Q(s) plus one is greater or equal to the maximum degree of N(s) or D(s), is chosen. If a partial fraction expansion of D(s)/Q(s) is made, the result will be

$$\frac{D(s)}{Q(s)} = k_0 + k_\infty s + \sum_i \frac{sk_i}{s + \sigma_i} - \sum_j \frac{sh_j}{s + \delta_j} \tag{3.14}$$

where k_i and h_j are positive. From equations 3.13 and 3.14 it is obvious that

$$\begin{aligned} y_{22} &= k_0' + k \quad s + \sum_i \frac{k_i s}{s + \sigma_i} + \sum_j \frac{h_j' s}{s + \delta_j} \\ (\mu - 1)Y_2 &= k_0'' + \sum_j \frac{h_j'' s}{s + \delta_j} \end{aligned} \tag{3.15}$$

where $k_0 = k_0' - k_0'' + 1$, $h_j = h_j'' - h_j' > 0$. Because of the suggested decomposition, $Y_2(0)$ and $Y_2(\infty)$ will be finite and positive, and y_{21} will be identified as

$$-y_{21} = \frac{N(s)}{Q(s)} \ .$$

The RC two-port can be realized by using the standard passive RC two-port synthesis technique.

3-5 Horowitz's Method

This method is used to realize an open-circuit transfer impedance. Basically, it consists of a passive RC and a passive RL two-port in cascade. The RL two-port then is converted to an RC network by using active elements. The fundamental network structure is shown in Fig. 3.6 (Horowitz, 1956).

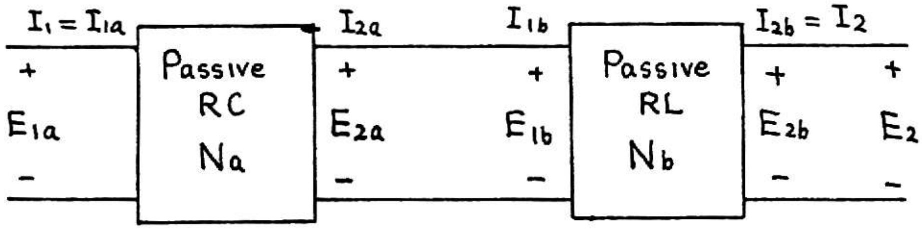


Fig. 3.6.

The transfer impedance function of this network is

$$Z_T(s) = \frac{E_2(s)}{I_1(s)} = \frac{N(s)}{D(s)} = \frac{Z_{21a}(s)Z_{21b}(s)}{Z_{22a}(s) + Z_{11b}(s)}$$

where the degree of N(s) must not be higher than that of D(s). In this expression Z_{21a}, Z_{22a}, and Z_{21b}, Z_{11b} are open-circuit impedance parameters of networks N_a and N_b, respectively. A polynomial Q(s) of degree equal to that of $Z_T(s)$, n, with negative real roots is chosen. Then the following quantities are identified:

$$Z_{21a}(s)Z_{21b}(s) = N(s)/Q(s)$$

and

$$Z_{22a}(s) + Z_{11b}(s) = D(s)/Q(s).$$

The expansion of D(s)/Q(s) in partial fractions yields

$$\frac{D(s)}{Q(s)} = k_0 + \sum_i \frac{k_i}{s + \sigma_i} - \sum_j \frac{h_j}{s + \delta_j} \tag{3.16}$$

where k_i and h_j are positive. The sum of positive terms represents an RC two-port N_a which can be realized by standard passive RC synthesis methods. On the other hand

$$k_0 - \sum_j \frac{h_j}{s + \delta_j} = M$$ represents an RL two-port N_b, if $M > 0$.

The network N_b can be realized with RC elements as suggested by Horowitz in Fig. 3.7.

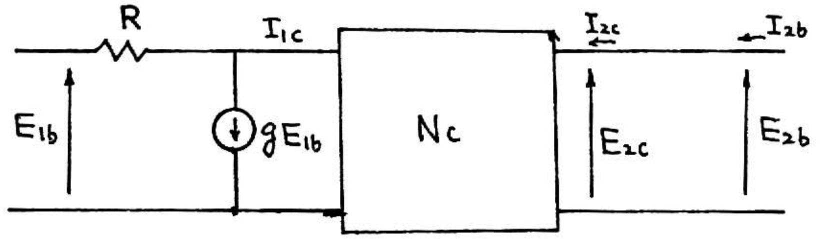


Fig. 3.7.

An analysis of this network gives

$$Z_{11b}(s) = \frac{R + Z_{11c}(s)}{1 + gZ_{11c}(s)}$$

and (3.17)

$$Z_{21b}(s) = \frac{(1 - gR)Z_{21c}(s)}{1 + gZ_{11c}(s)}$$

where Z_{11c} and Z_{21c} are Z parameters of network N_c. Solving (3.17) for Z_{11c} and Z_{21c} gives,

$$Z_{11c}(s) = \frac{Z_{11b}(s) - R}{1 - gZ_{11b}(s)}$$

(3.18)

$$Z_{21c}(s) = \frac{Z_{21b}(s)}{1 - gZ_{11b}(s)} \; .$$

Equation 3.18 assumes that $Z_{21b}(s)$ and $Z_{11b}(s)$ have the same poles. It will be also assumed that $M > 0$ so that $Z_{11b}(s)$ is an RL driving-point impedance. The poles and zeros of $Z_{11c}(s)$ must lie interlaced on the negative real axis with the critical frequency closest to the origin a zero. The zeros of $Z_{11c}(s)$ occur at the roots of

$$Z_{11b}(s) - R = 0,$$

and the poles occur at the roots of

$$Z_{11b}(s) - 1/g = 0.$$

If R and g are chosen so that

$$R \geqslant Z_{11b}(\infty)$$

and

$$1/g \leqslant Z_{11b}(0),$$

then the poles and zeros of $Z_{22c}(s)$ will satisfy the requirements of an RC network. Thus N_c can be realized with zeros of transmission of $Z_{21b}(s)$. It will be necessary to choose a negative g if $Z_{11b}(s)$ has a zero at the origin.

3-6 Synthesis Using a Single-ended Operational Amplifier

This procedure, one of the most frequently used synthesis methods, uses RC passive networks and a single operational amplifier. The configuration shown in Fig. 3.8 has a transfer voltage ratio of

$$T_v = \frac{V_2}{V_1} = \frac{y_{21a}}{y_{12b}} \tag{3.19}$$

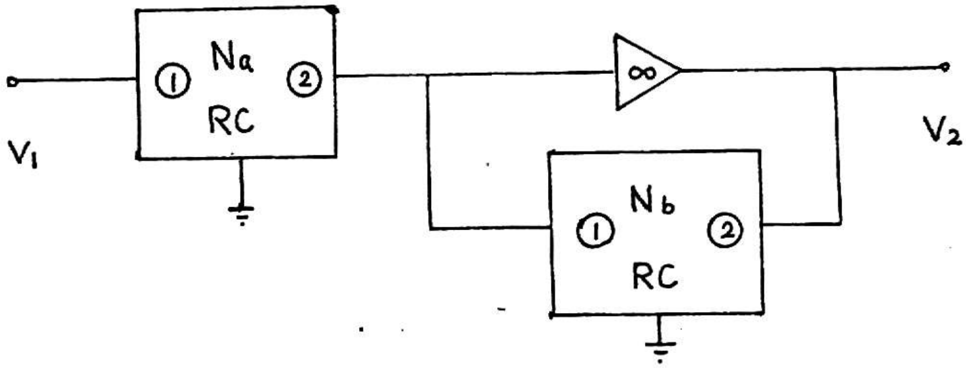


Fig. 3.8.

In equation 3.19, subscripts a and b refer to the networks N_a and N_b, respectively. Since an RC network is reciprocal, $y_{21i} = y_{12i}$.

From equation 3.19, it is obvious that if a transformer is not used, then y_{21a} and y_{12b} can have no positive real transmission zeros. Therefore only voltage transfer ratios with no positive real poles and zeros can be realized using this configuration.

To synthesis a transfer function $T_v = -N(s)/D(s)$, a polynomial $Q(s)$ having distinct negative real roots with degree equal to or higher than that of the maximum degree of $N(s)$ or $D(s)$ is chosen. Then it is found that

$$-y_{21a} = N(s)/Q(s)$$

$$-y_{12b} = D(s)/Q(s).$$

The RC two-ports can be realized by a conventional synthesis procedure. One deficiency of this configuration is that parallel-ladder or other complicated RC two-ports must be used if T_v

has complex poles and zeros.

3-7 Mathews-Seifert Method

The inherent problem of the previous approach, the need for complicated RC two-ports if the transfer zeros are complex, is overcome by using active zero-producing networks. This method was proposed by Mathews-Seifert (1955). Its configuration is shown in Fig. 3.9. The transfer voltage ratio of this

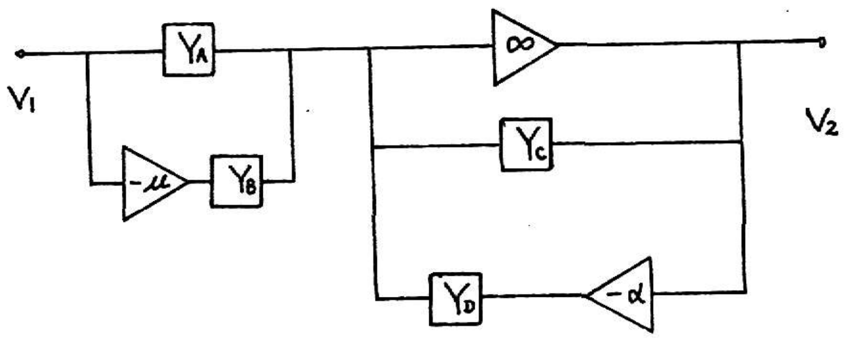


Fig. 3.9.

network is

$$\frac{V_2}{V_1} = - \frac{Y_A - \mu Y_B}{Y_C - \alpha Y_D} \tag{3.20}$$

This transfer function indicates that any real rational function can be realized by this structure.

For a specified transfer function $T_v = -N(s)/D(s)$ of degree n, a polynomial Q(s) having distinct negative real roots and of degree equal to or higher than (n - 1) is chosen. Then it follows that

$$\begin{aligned} N(s)/Q(s) &= Y_{RC}^{(1)} - Y_{RC}^{(2)} \\ D(s)/Q(s) &= Y_{RC}^{(3)} - Y_{RC}^{(4)} \end{aligned} \tag{3.21}$$

where Y_{RC} is a passive RC driving-point admittance. From equations 3.20 and 3.21, the following relations appear:

$$Y_A = Y_{RC}^{(1)}, \quad \mu Y_B = Y_{RC}^{(2)}$$
$$Y_C = Y_{RC}^{(3)}, \quad \alpha Y_D = Y_{RC}^{(4)} \qquad (3.22)$$

From the structure of Fig. 3.9, it is obvious that poles and zeros can be controlled separately by changing the gains of the amplifiers.

3-8 Lovering's Method

The previous method has the advantage of simplicity with no restrictions on the transfer function. However, it uses three active elements. Lovering (1965) advanced a method which uses only two active elements as shown in Fig. 3.10. The transfer

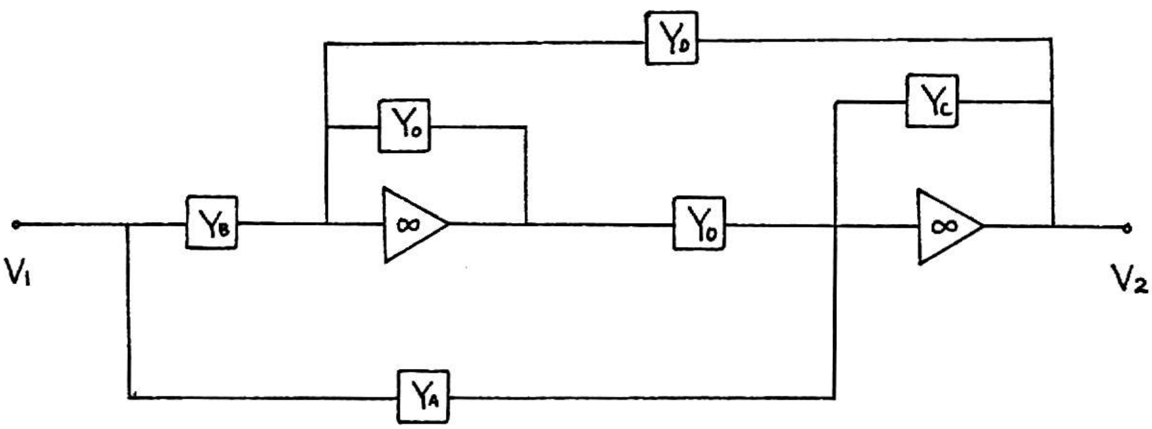


Fig. 3.10.

function for this network is identical to expression 3.20, i.e.,

$$\frac{V_2}{V_1} = - \frac{Y_A - Y_B}{Y_C - Y_D} .$$

Thus the synthesis procedure is the same as before. Since Y_0 does not appear in the transfer function, any suitable preselected nonzero value can be used.

3-9 Synthesis Using a Differential-input Amplifier

The previous three methods using single-ended operational amplifiers have their advantages. Since the integrated operational amplifiers available in the market are usually of the differential input type, it is thus convenient to develop a synthesis method that uses a differential-input operational amplifier. Such a method is proposed by Mitra(1968). The configuration is shown in Fig. 3.11. The transfer function for

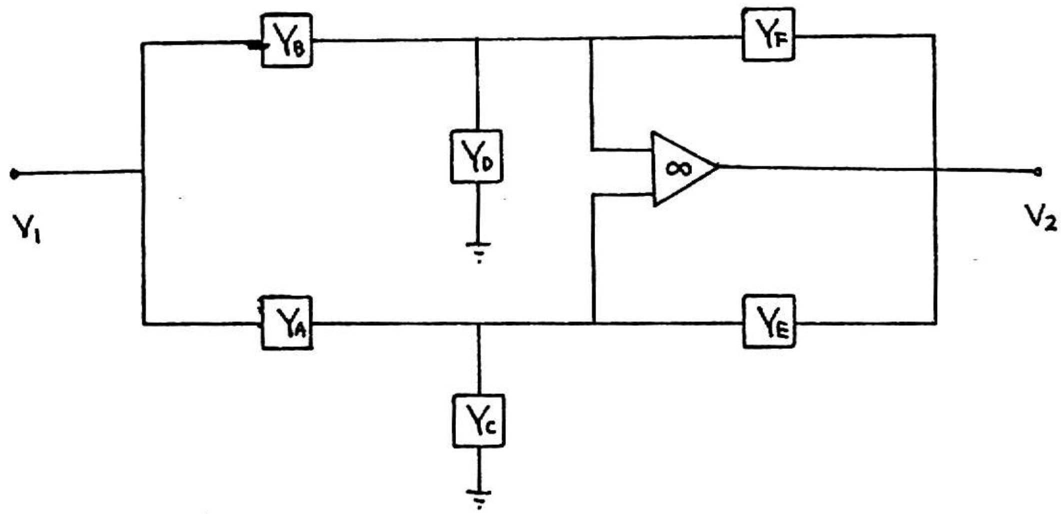


Fig. 3.11.

this network is

$$T_V = \frac{V_2}{V_1} = \frac{Y_A(Y_B + Y_D + Y_F) - Y_B(Y_A + Y_C + Y_E)}{Y_F(Y_A + Y_C + Y_E) - Y_E(Y_B + Y_D + Y_F)} \tag{3.23}$$

If the RC components are chosen so that

$$Y_A + Y_C + Y_E = Y_B + Y_D + Y_F \tag{3.24}$$

then expression 3.23 becomes

$$T_v = \frac{Y_A - Y_B}{Y_F - Y_E}$$

which is identical to equation 3.20. Thus Y_A, Y_B, Y_E, and Y_F can be found by using the same procedure as before, i.e.,

$$Y_A = Y_{RC}^{(1)}, \quad Y_B = Y_{RC}^{(2)}$$

$$Y_F = Y_{RC}^{(3)}, \quad Y_E = Y_{RC}^{(4)} \; .$$

From equation 3.24, it is noted that

$$Y_C - Y_D = (Y_F - Y_E) - (Y_A - Y_B) \doteq \frac{D(s) - N(s)}{Q(s)} \tag{3.25}$$

Expression 3.25 can be expressed then as

$$\frac{D(s) - N(s)}{Q(s)} = Y_{RC}^{(5)} - Y_{RC}^{(6)} \tag{3.26}$$

From equations 3.25 and 3.26, Y_C and Y_D are obtained as

$$Y_C = Y_{RC}^{(5)}, \quad Y_D = Y_{RC}^{(6)} \; .$$

Thus the synthesis is completed by realizing these RC one-ports.

CHAPTER IV

COEFFICIENT-MATCHING APPROACHES

In the coefficient-matching approach, an active-network configuration is selected, and its network function is found by analysis. Then a specified network function is compared with the computed network function. Design equations are obtained by equating like coefficients. For convenience, this approach is generally restricted to the second-order transfer functions. Higher-order transfer functions can be realized by cascading second-order stages, with the use of intervening buffer amplifiers.

4-1 A Second-order Low-pass Filter Section

The active network of Fig. 4.1 was proposed by Sallen and Key (1955) as a low-pass filter section. Its transfer voltage ratio is obtained by analysis as

$$T_v = \frac{V_2}{V_1} = \frac{K\,\dfrac{G_1G_2}{C_1C_2}}{s^2 + \dfrac{G_2}{G_1}(1-K) + \dfrac{G_1}{C_1} + \dfrac{G_2}{C_1}\,s + \dfrac{G_1G_2}{C_1C_2}}$$

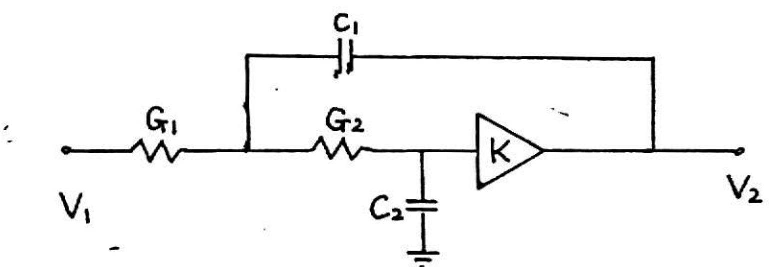


Fig. 4.1.

If a specified transfer voltage ratio is given as

$$T_v = \frac{H}{s^2 + \beta s + \gamma} \tag{4.2}$$

then by equating like coefficients of equations 4.1 and 4.2, H, β, and γ are given as

$$H = K \frac{G_1 G_2}{C_1 C_2}$$

$$\beta = \frac{G_2}{C_2}(1-K) + \frac{G_1}{C_1} + \frac{G_2}{C_1} \tag{4.3}$$

$$\gamma = \frac{G_1 G_2}{C_1 C_2} .$$

In equation 4.3, the number of equations is less than that of unknowns. To solve this set of equations, some element values can be preselected and the remaining unknowns calculated. For example, if

$$C_1 = C_2 = 1$$
$$K = 2,$$

then from 4.3,

$$G_1 = \beta$$
$$G_2 = \gamma/\beta$$
$$H = 2\gamma.$$

4-2 A Second-order High-pass Filter Section

Figure 4.2 shows a second-order high-pass filter proposed by Sallen and Key. This active configuration has a transfer

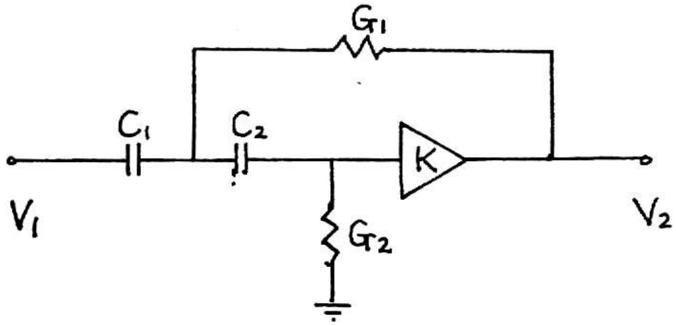


Fig. 4.2.

voltage ratio of

$$T_v = \frac{K\ s^2}{s^2 + \left[\frac{C_1}{G_1}(1-K) + \frac{C_2}{G_2} + \frac{C_2}{G_1}\right]s + \frac{C_1C_2}{G_1G_2}}$$

A specified transfer function can be realized in a manner exactly the same as described before for a low-pass filter.

An alternative approach is the use of a high-pass to low-pass transformation. For a high-pass transfer function $T_v(s)$, a corresponding low-pass characteristic is obtained by replacing s with 1/s, i.e.,

$$T_v'(s) = T_v(1/s) \tag{4.4}$$

Equation 4.4 is a low-pass transfer function. This transfer function can be realized by using the method of section 4.1.

4-3 A Second-order Band-pass Filter Section

An active RC network that can be used to realize a band-pass transfer function has been proposed by Kerwin and Huelsman (1960) as shown in Fig. 4.3. The transfer voltage ratio for

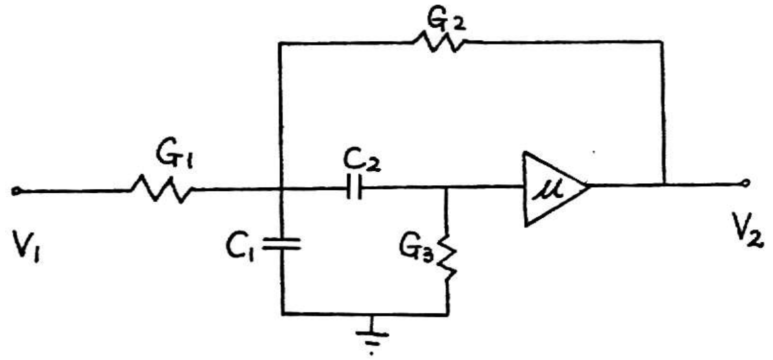


Fig. 4.3.

this network is

$$T_v = \frac{V_2}{V_1} = \frac{\mu \dfrac{G_1}{C_1} s}{s^2 + \left(\dfrac{G_1}{C_1} + \dfrac{G_2}{C_1} + \dfrac{G_2}{C_1} + \dfrac{G_2}{C_2} + \dfrac{G_3}{C_1} - \mu \dfrac{G_2}{C_1}\right)s + \dfrac{G_3(G_1+G_2)}{C_1C_2}} \tag{4.5}$$

The general form of a second-order band-pass transfer voltage ratio is given as

$$T_v = \frac{H_s}{s^2 + \beta s + \gamma} \tag{4.6}$$

Equating like coefficients of equations 4.5 and 4.6 gives H, β, and γ as

$$H = \mu\, G_1/C_1$$

$$\beta = \frac{G_1}{C_1} + \frac{G_2}{C_1} + \frac{G_3}{C_2} + \frac{G_3}{C_1} - \mu \frac{G_2}{C_1} \tag{4.7}$$

$$\gamma = \frac{G_3(G_1 + G_2)}{C_1 C_2}$$

In equation 4.7, since the number of equations is less than the number of unknowns, this set of equations can be solved by preselecting a number of elements. If, for instance,

$$G_1 = G_2 = 1$$
$$C_1 = C_2 = 1$$

then

$$G_3 = \gamma/2$$

$$\mu = 2 + \gamma - \beta$$

$$H = \mu.$$

4-4 An Active RC-Chain Network

This method was suggested by Bach (1960) for the realization of Butterworth filter functions using unity gain voltage amplifiers as the active devices.

A set of low-pass Butterworth filter functions is as follows:

$$\begin{array}{lll}
k = 1 & E_0/E_1 = m/(s + B) & \\
k = 2 & E_0/E_1 = m/(s^2 + 1.414\beta s + B) & (4.8) \\
\vdots & \vdots \quad \vdots \qquad\qquad \vdots & \\
k = n & E_0/E_1 = m/(s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n) &
\end{array}$$

The R-C network chains having transfer functions in the form of equation 4.8 are shown in Fig. 4.4.

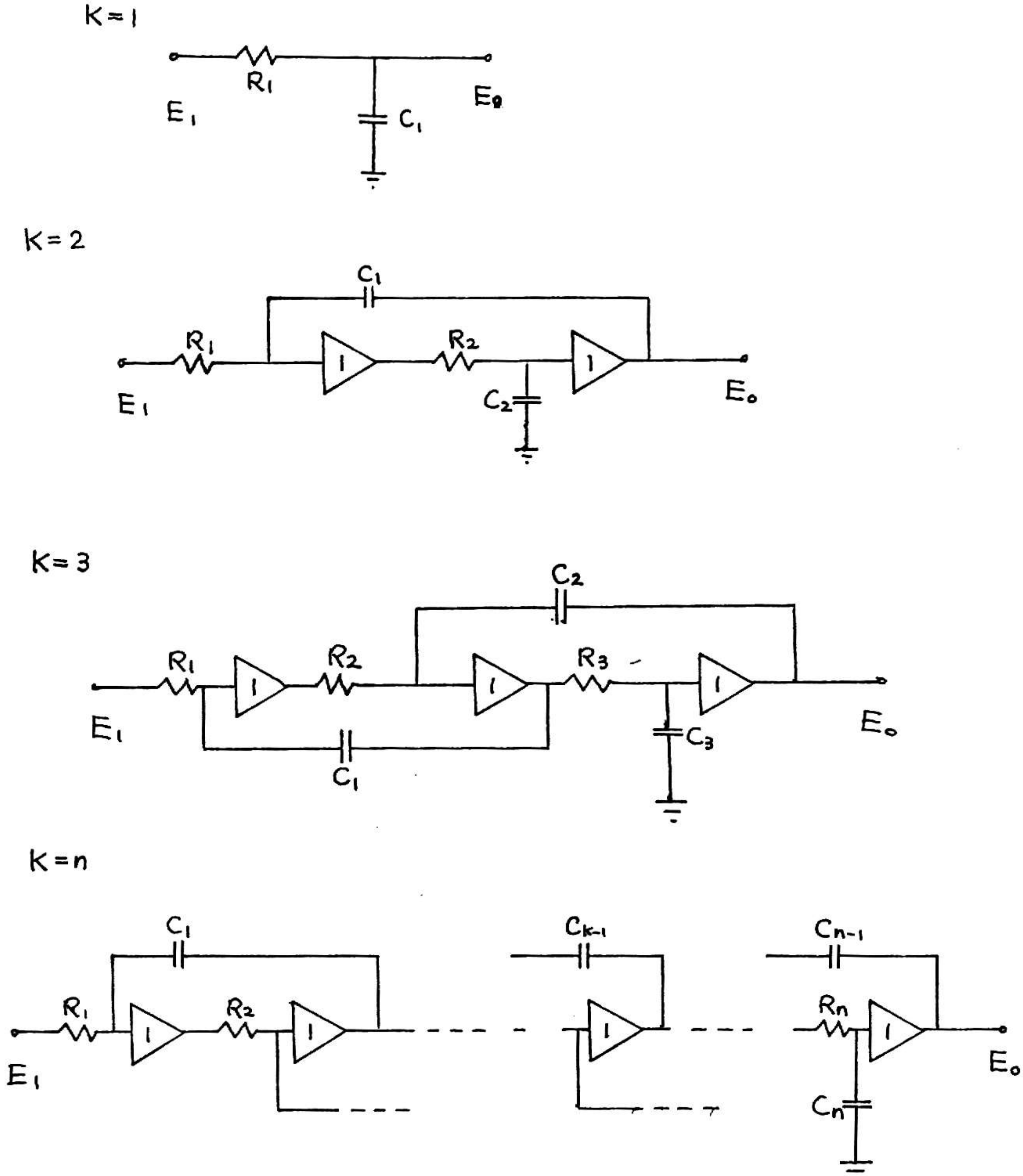


Fig. 4.4.

Thus for the network chains of Fig. 4.4,

$$
\begin{aligned}
&k=1 && E_0/E_1 = \omega_1(s + \omega_1) \\
&k=2 && E_0/E_1 = \omega_1\omega_2/(s^2 + \omega_1 s + \omega_1\omega_2) \qquad (4.9) \\
&k=3 && E_0/E_1 = \omega_1\omega_2\omega_3/(s^3+\omega_1 s^2+\omega_1\omega_2 s+\omega_1\omega_2\omega_3) \\
&\vdots && \vdots \\
&k=n && E_0/E_1 = \prod_{j=1}^{n} \omega_j/(s^n+\omega_1 s^{n-1}+\omega_1\omega_2 s^{n-2}+\ldots+\prod_{j=1}^{k} \omega_j s^{k-1} \\
& && \qquad + \ldots + \prod_{j=1}^{n} \omega_j)
\end{aligned}
$$

where $\omega_K = 1/(C_K R_K)$ is the frequency constant for the K^{th} section. By matching the denominator coefficients of equations 4.8 and 4.9, the values of R's and C's for the desired filter function can be determined. As a result, a particularly simple result is gained:

$$\omega_1 = a_0,\ \omega_2 = a_1/a_0,\ .\ .\ .\ \omega_n = a_n/a_n\text{-}1 = 1/R_n C_n\ .$$

For a high-pass filter, the basic active R-C configuration need be modified only by interchanging the R's and C's. Also, the numerators of the transfer function are multiplied by s raised to a power equal to the order of the filter. A band-pass filter can also be obtained by cascading the desired order of low- and high-pass filter.

If the factored form* of the transfer function is used and equal values chosen for all resistors, the spread of capacitor values is only three to one. But factoring the function in high-order cases is difficult.

4-5 Bohn's Method

The R-C active configuration shown in Fig. 4.5 has been proposed by Bohn (1963). The transfer voltage ratio of this

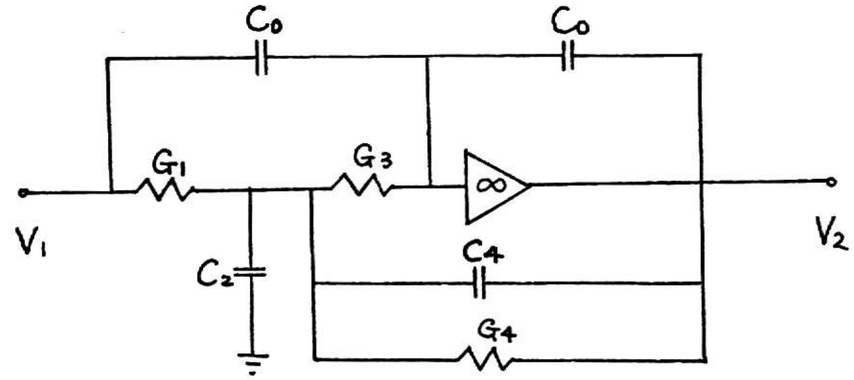


Fig. 4.5.

network is given as

$$\frac{V_2}{V_1} = - \frac{C_0(C_2+C_4)s^2 + C_0(G_1+G_3+G_4)s + G_1G_3}{C_0(C_2+C_4)s^2 + \left[G_3C_4+C_0(G_1+G_3+G_4)\right]s + G_3G_4} \qquad (4.10)$$

The general form of this specified network function is

$$T_V = - \frac{a_2s^2 + a_1s + a_0}{b_2s^2 + b_1s + b_0} \qquad (4.11)$$

---

*For an even order transfer function:

$$E_0/E_1 = 1/\prod_{i=1}^{n/2} (A_is^2 + B_is + C_i)$$

and for an odd order transfer function:

$$E_0/E_1 = 1/(Ds+E) \prod_{i=1}^{(n-1)/2} (A_is^2 + B_is + C_i)$$

By equating like coefficients of equations 4.10 and 4.11, the coefficients of equation 4.11 are given by

$$
\begin{aligned}
a_0 &= C_0(C_2 + C_4) \\
a_1 &= C_0(G_1 + G_3 + G_4) \\
b_1 &= G_3C_4 + C_0(G_1 + G_3 + G_4) = G_3G_4 + a_1 \qquad (4.12) \\
a_0 &= G_1G_3 \\
b_0 &= G_3G_4
\end{aligned}
$$

Equation 4.12 can be solved for the element values. Since the number of unknowns is more than the number of equations, the solution is not unique. Again, some element values must be preselected.

4-6 Brennan and Bridgman's Method

The network shown in Fig. 4.6 was proposed by Brennan and Bridgman (1957). This configuration has a transfer function of

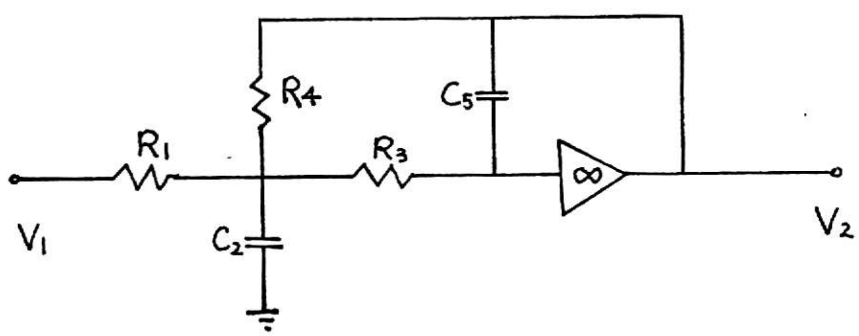


Fig. 4.6.

$$T_v = \frac{V_2}{V_1} = \frac{-1}{(R_1R_3C_2C_5)s^2 + (R_3+R_1+R_3R_1/R_4)C_5s+R_1/R_4} \qquad (4.13)$$

The above transfer function is of the form

$$T_v = -1/a_2s^2 + a_1s + a_0 \qquad (4.14)$$

From equations 4.13 and 4.14 it is easily shown that

$$a_2 = R_1R_3C_2C_5$$

$$a_1 = (R_3 + R_1 + \frac{R_3R_1}{R_4})C_5 \tag{4.15}$$

$$a_0 = R_1/R_4$$

By preselecting some element values, the values of the remaining elements can be determined.

4-7 Kerwin, Huelsman, and Newcomb Method

This method, using three operational amplifiers, proposed by Kerwin, Huelsman, and Newcomb (1967) can be used to realize very high Q pole-pairs with very low Q-sensitivity. The network configuration is shown in Fig. 4.7. If the output is taken

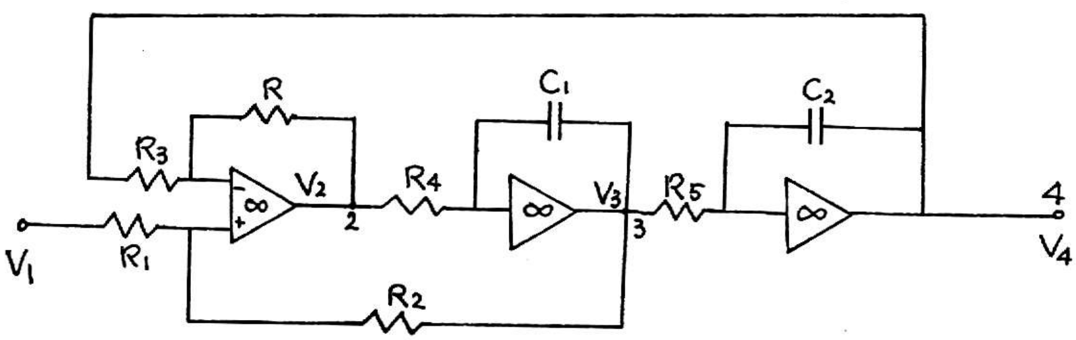


Fig. 4.7.

from terminal 4, then the transfer voltage ratio is given as

$$\frac{V_4}{V_1} = \frac{R_2(R + R_3)}{(R_1+R_2)R_3(s^2C_1C_2R_4R_5 + s\,\dfrac{C_2R_5R_1(R+R_3)}{R_3(R_1 + R_2)} + \dfrac{R}{R_3})} \tag{4.16}$$

This is a low-pass transfer function. If a high-pass characteristic is desired, the output can be taken from terminal 2, and if output is taken from terminal 3, a band-pass response is obtained. If

$$R_1 = R_3 = R_4C_1 = R_5C_2 = 1$$

then equation 4.16 becomes

$$\frac{V_4}{V_1} = \frac{R_2(R + 1)}{(1 + R_2)(s^2 + s\dfrac{R + 1}{R_2 + 1} + R)}$$

When a specified transfer function is of the form

$$T_v = \frac{K}{a_2s^2 + a_1s + a_0} \tag{4.17}$$

the equating of like coefficients gives,

$$\begin{aligned} K &= R_2(1 + R) \\ a_2 &= 1 + R_2 \\ a_1 &= R + 1 \\ a_0 &= R(1 + R_2) \end{aligned} \tag{4.18}$$

Once again in this set of equations, since the number of equations is more than the number of unknowns, the solution is not unique.

CHAPTER V

THE SIMULATED-INDUCTOR APPROACH

Active RC networks give designers the freedom to construct inductorless filters. But filters in this class have a common deficiency, high sensitivity to parameter variation on both active and passive components. Passive RLC networks, on the other hand, are attractive because their sensitivity is extremely low. Orchard (1966) points out in "Inductorless Filters" that a doubly-loaded LC filter has very low sensitivity, too. Thus LC filters are more satisfactory than active RC filters from the point of view of sensitivity.

A simple method of designing a low sensitivity inductorless filter to meet a specified characteristic is to design a conventional RLC filter to meet the specification, then replace each inductor in the filter by capacitors and active devices.

5-1 Simulated Inductors Using Differential Amplifiers

Figure 5.1 (Riordan, 1967) is the basic configuration for the above purpose.

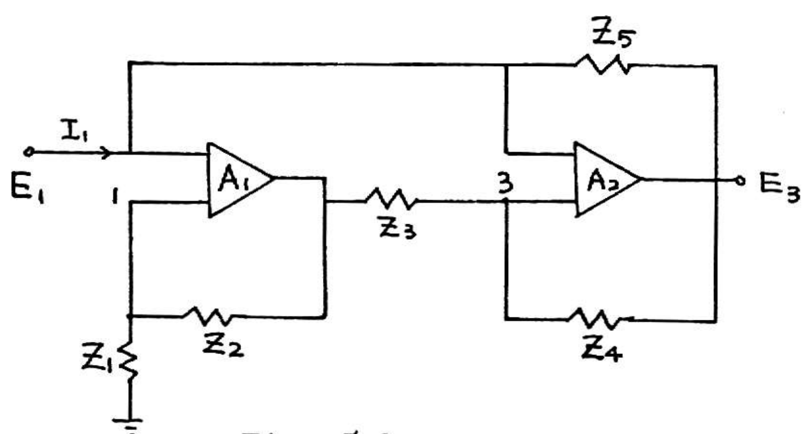


Fig. 5.1.

Here two high-gain differential amplifiers are used. Since the amplifier gain is high,

$$E_2 \doteq E_1\left(1 + \frac{Z_2}{Z_1}\right)$$

Thus it follows that

$$E_3 \doteq E_1\left(1 - \frac{Z_2 Z_4}{Z_1 Z_3}\right)$$

and

$$I_1 = \frac{E_1 - E_3}{Z_5} = \frac{Z_2 Z_4 E_1}{Z_1 Z_3 Z_5} .$$

Therefore the input impedance is

$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} .$$

If either Z_2 or Z_4 is a capacitor C and all other components are resistors R, Z_{in} becomes

$$Z_{in} = j\omega CR^2 ,$$

so the circuit of Fig. 5.1 is characterized as an inductor with inductance of

$$L = CR^2 \text{ henries.}$$

If both Z_2 and Z_4 are capacitors with value C, Z_{in} becomes

$$Z_{in} = - \omega^2 C^2 R^3 .$$

Under this condition, if a variable resistor R_a is connected in series in the input, the circuit becomes a tunable filter with resonant frequency

$$\omega_0{}^2 = \frac{R_a}{C^2R^3} .$$

This filter can be tuned over a wide frequency range but the stability is poor.

5-2 Simulated Inductors Using One Differential Amplifier

Figure 5.2 (Keen and Peters, 1967) shows a simulated inductor circuit using RC networks and one differential amplifier. This circuit has input impedance

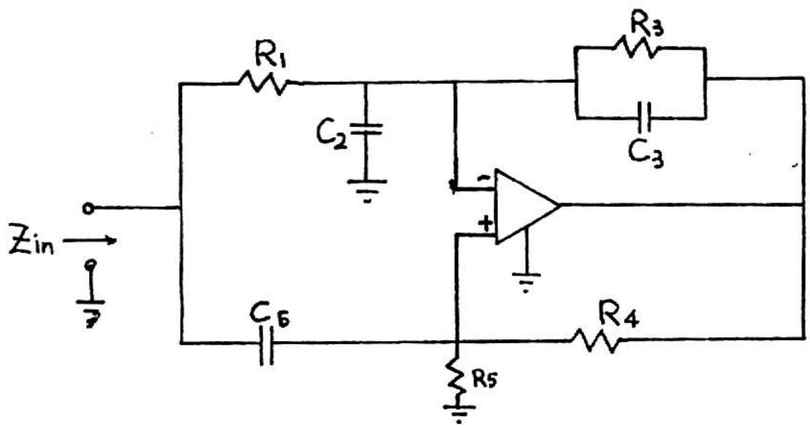


Fig. 5.2.

$$Z_{in}(s) = \frac{a_2s^2 + a_1s + a_0}{b_2s^2 + b_1s + b_0} ,$$

where $a_2 = C_3C_6R_1R_3R_4R_5$

$a_1 = (C_3R_3R_4 + C_6R_4R_5 - C_2R_3R_5)R_1$

$a_0 = R_1R_4 - R_3R_5$

$b_2 = C_6R_1R_3(C_3R_4 - C_2R_5)$

$$b_1 = C_3R_3R_4 + C_6R_1R_4 - C_2R_3R_5$$

$$b_0 = R_4.$$

If
$$a_0 = 0, \text{ i.e., } R_1R_4 = R_3R_5$$

$$b_2 = 0, \text{ i.e., } C_3R_4 = C_2R_5$$

$$C_6R_1 = C_3R_3 ,$$

then the input impedance becomes

$$Z_{in}(s) = C_3R_3R_5s .$$

Thus Fig. 5.2 would act as an inductor with inductance

$$L = C_3R_3R_5 \text{ henries.}$$

5-3 Realization of Ungrounded Inductors Using a Gyrator-type Circuit

This method was proposed by G. J. Deboo (1967). Figure 5.3 is the proposed circuit in which only three operational amplifiers are used. The parameters of this circuit are defined by

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = G \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{5.1}$$

If port 1 is grounded, i.e., $V_1 = 0$, then equation 5.1 becomes

$$\begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = G \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} ;$$

this is the characteristic of a gyrator.

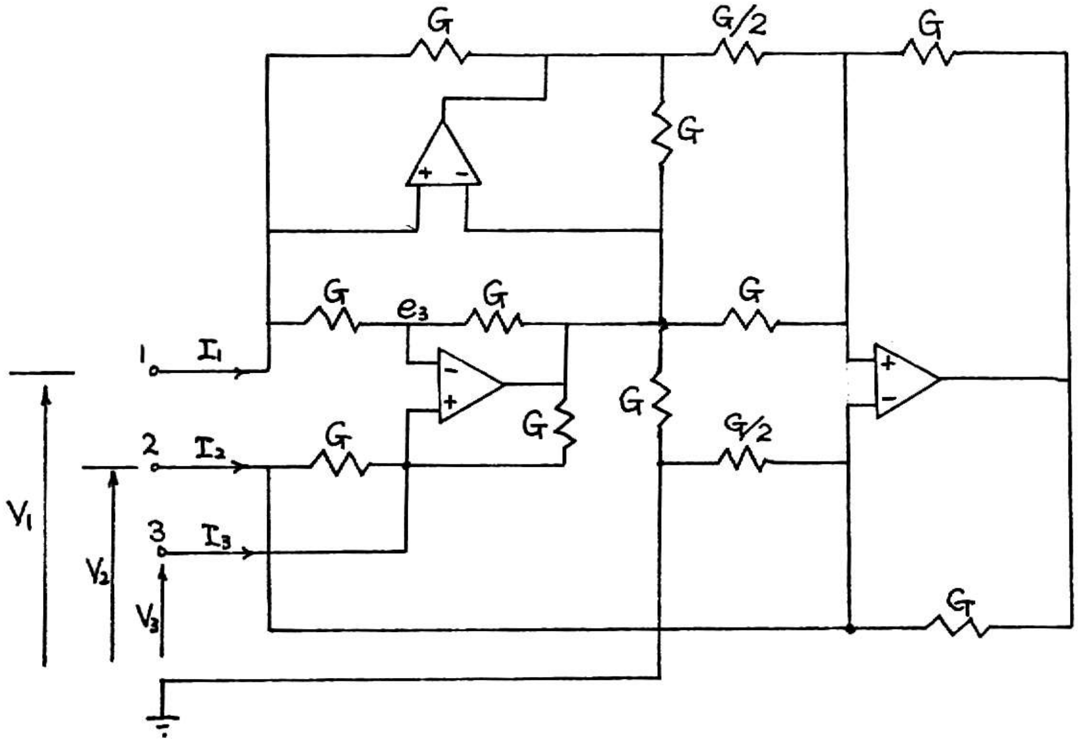


Fig. 5.3.

If port 2 is grounded, i.e., $V_2 = 0$, then the expression 5.1 has the form

$$\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix}.$$

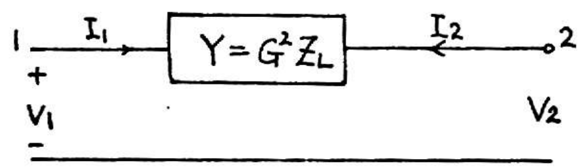
If a load Z_L is connected across port 3, then

$$I_3 = - \frac{V_3}{Z_L} \tag{5.2}$$

From equations 5.1 and 5.2 the following relation is obtained,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = G^2 Z_L \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5.3}$$

The equivalent circuit for equation 5.3 is



It is obvious that Y is an inductor L if Z_L is a capacitor C and that

$$L = C/G^2.$$

5-4 Simulated Floating Inductor by Back-to-Back Gyrators

A gyrator loaded with a capacitor can simulate a grounded inductor. As pointed out by A. G. J. Holt and J. Taylor (1965), such a capacitor loaded gyrator acts as a floating inductor if it is loaded with a similar gyrator. In this method, all four gyration conductances must be perfectly matched, i.e., $g_1 = g_2 = g_3 = g_4$. If this condition is not satisfied, there will be two shunt inductances and a voltage-controlled current source in addition to the required inductances.

5-5 Sheahan's Method to Realize Floating Inductors

The circuit shown in Fig. 5.4 was developed by D. E. Sheahan (1967) and simulates a floating inductance without the serious adjustment and component sensitivity problems of the simulation of section 5.4. In the circuit shown, the output terminal of the capacitor loaded gyrator is connected to a pair of transistors T_1 and T_2, which supply the gyrator with constant-current d-c power. Capacitor C_2 across the power supply terminals provides a low impedance path to a-c signals. The high output

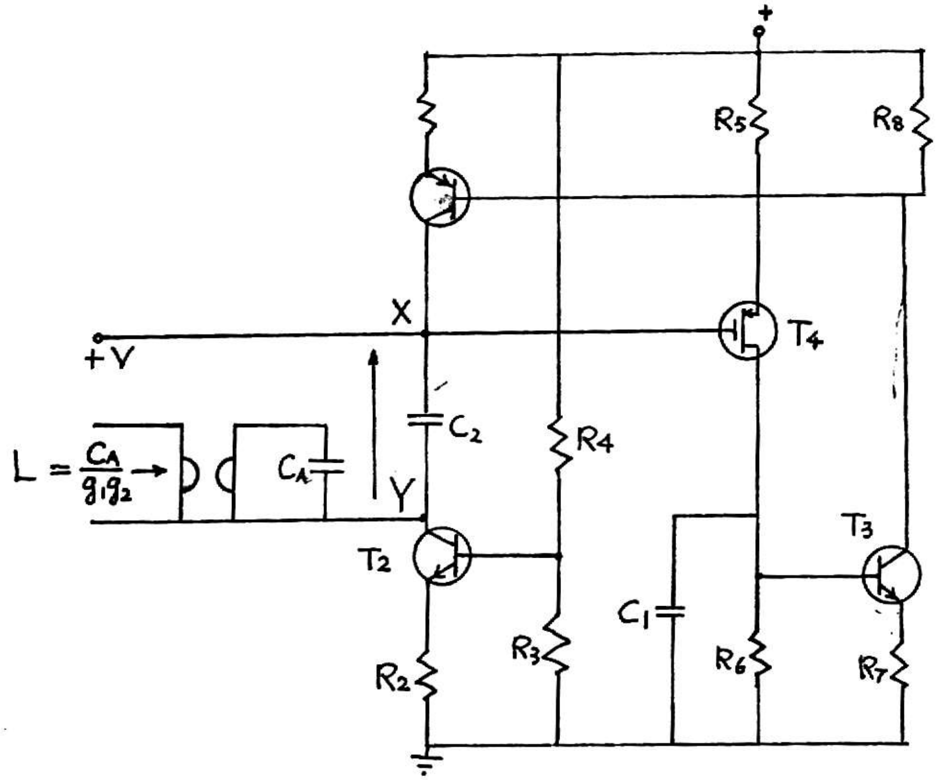


Fig. 5.4.

impedances of T_1 and T_2 insulate the gyrator from ground. The voltage at point X is monitored by a field-effect transistor T_4, and the feedback loop $T_4T_3R_8T_1$ controls the current through T_1 so that it is always equal to that of T_2. Capacitor C_1 decouples the negative feed-back for a-c signal so that it will not lower the maximum insulation resistance. In Fig. 5.4 the insulation resistance between X and Y is 150 K ohm.

CHAPTER VI

PROBLEMS FACING ACTIVE NETWORK SYNTHESIS

Network synthesis using active elements can provide the design of an inductorless filter. In addition, transfer functions with poles and zeros falling on the right half s-plane can also be realized. In contrast with these advantages active networks have certain shortcomings. Among them, sensitivity and stability are most important. Also, in high-frequency applications, the phase shift introduced in the active elements creates another serious problem.

6-1 Stability

In designing an active network, negative feedback is always used. This feedback in turn can lead to instability. An active network may be stable when it is first constructed, yet it will become unstable when element values of both the passive and active components change.

In examining the stability there are some criteria as follows:

(a) Strict Stability: A linear lumped finite (LLF) network is strictly stable if all poles of the corresponding network function are in the left half s-plane.

(b) Marginal Stability: An LLF network is marginally stable if the poles of the corresponding network function are either in the left half plane or single poles on the $j\omega$-axis.

(c) Open-circuit Stability: A one-port is open-circuit stable if poles of the corresponding driving-point impedance Z(s) are in the left half s-plane.

(d) Short-circuit Stability: A one-port is short-circuit stable if poles of the corresponding driving-point admittance Y(s) are in the left half s-plane.

(e) Potential Instability: A one-port with a driving-point impedance $Z_A(s)$ is potentially unstable if there exists a single passive terminating impedance $Z_p(s)$ such that at least one root of equation $Z_A(s) + Z_p(s) = 0$ has a positive real part. A two-port is potentially unstable if there exists at least a pair of passive terminating impedances $Z_1(s)$ and $Z_2(s)$ so that the equation $Z_1(s) + Z_{11}(s) = 0$ has right half s-plane or $j\omega$-axis roots, where $Z_{11}(s)$ is the driving-point impedance of the loaded two-port.

(f) Absolutely Stable: If a one-port is strictly stable for all possible passive terminations, it is absolutely stable. A two-port is absolutely stable if it is strictly stable when terminated with all possible passive input and output impedances.

6-2 Sensitivity

The element values of a network change when temperature, humidity, aging, or bias levels vary. This causes the displacement of the poles and zeros of a network function. As a result, the network may become unstable. Even if instability does not occur, the overall system response may differ from that of the original network. How far is the response changed due to the variation of the element values? This question leads to the problem of sensitivity. There are several methods to measure the sensitivity of an element in the overall system.

A network function $T(s) = \dfrac{N(s)}{D(s)}$ can be written as

$$T(s,K) = \frac{N_1(s) + KN_2(s)}{D_1(s) + KD_2(s)} = \frac{N(s,K)}{D(s,K)} = T_0(s) + \frac{KA(s)B(s)}{1 - KC(s)}$$

where K is the network parameter of interest, and

$$T_0(s) = \frac{N_1(s)}{D_1(s)} = T(s,0), \text{ value of T when } K = 0,$$

$$C(s) = -\frac{D_2(s)}{D_1(s)}$$

$$A(s)B(s) = \frac{N_2(s)D_1(s) - N_1(s)D_2(s)}{D_1^2(s)}$$

It is supposed that $F_K(s) = 1 - K(s) = \dfrac{D(s,K)}{D_1(s)}$. $F_K(s)$ is defined as return difference. And

$$F_K^0(s) = 1 - KC(s) + \frac{KA(s)B(s)}{T^0(s)} = \frac{N(s,K)}{N_1(s)}$$

Here $F_K^0(s)$ is defined as null-return difference. The sensitivity function is defined as

$$S_K^{T(s)} = \frac{d\left[\ln T(s)\right]}{d\left[\ln K\right]} = \frac{dT(s,K)/T(s,K)}{dK/K}$$

$$= \frac{1}{F_K(s)} - \frac{1}{F_K^{0}(s)} .$$

In this sensitivity function, the overall network function T(s) is concerned.

According to the parameter of interest K, there are four kinds of sensitivities.

(a) Phase sensitivity and gain sensitivity

For sinusoidal input, ln T(jω) can be written as

$$\ln T(j\omega) = \ln\left|T(j\omega)\right| + j \arg T(j\omega)$$

$$= \alpha(\omega) + j\beta(\omega)$$

where $\alpha(\omega)$ is the gain function and $\beta(\omega)$ is the phase function, then the sensitivity function can be written as

$$S_K^{T(j\omega)} = A + jB \qquad (6.1)$$

where the real part A is defined as gain sensitivity and the imaginary part B is defined as phase sensitivity. In equation 6.1

$$A = S_K^{\alpha(\omega)} = \frac{d\alpha(\omega)}{dK/K}$$

$$B = S_K^{\beta(\omega)} = \frac{d\beta(\omega)}{dK/K}$$

(b) Polynomial sensitivity

For some applications, a specified band of frequencies such as provided by a band-pass or notch filter is of interest. In this case the denominator polynomial of the transfer function determines the pass-band response. Thus a sensitivity based on this polynomial, called the polynomial sensitivity, is used. It is defined as

$$S_K^{D(s,K)} = \frac{d\left[\ln D(s,K)\right]}{d\left[\ln K\right]} \ .$$

For sinusoidal applications, set $s = j\omega$; then

$$S_K^{D(s,K)} = \frac{d\left[D(j\omega,K)/D(j\omega,K)\right]}{dK/K} \ .$$

(c) Zero sensitivity and pole sensitivity

It is supposed that K takes its nominal value such that $s = s_i$ is a zero (pole) of $T(s,K)$. Then the zero (pole) sensitivity is defined as

$$S_K^{s_i} = \left.\frac{ds_i}{dK/K}\right|_{s=s_i}$$

where $S_K^{s_i}$ is a real number if the root is real; otherwise it is a complex number. In the case of high selectivity band-pass filters, the shift of the resonant frequency is critical. Under this condition, knowledge of this kind of sensitivity becomes quite useful.

(d) Coefficient sensitivity

When the fractional change of each coefficient of a polynomial as a result of the fractional change of the parameter K is needed, the coefficient sensitivity is helpful. For a polynomial

$$Q(s,K) = \sum_{j=0}^{n} Q_j s^j$$

The coefficient sensitivity is defined as

$$S_K^{Q_j} = \frac{dQ_j/Q_j}{dK/K} .$$

Note that the coefficient sensitivity is not a physically measurable quantity.

CHAPTER VII

CONCLUSION

In this report, several active elements and many techniques to realize a given transfer function have been introduced. When a specified transfer function is given, what kind of active element should be chosen for the realization? What method will bring a simplest result? This depends upon what criteria are paramount.

Each active device has its own advantages and deficiencies, and each synthesis method used will give a difference network configuration. No matter what device or method is being used, the final result is the same: an inductorless network which satisfies the need of a specified transfer function.

7-1 Network Classifications

Networks can be divided, according to the components contained, into classes as follows:

(a) Passive RC networks

Only passive resistors and capacitors are used in this class of networks. The poles of network functions of this class are restricted to the negative real axis including origin and infinity and must be simple. The zeros of the driving-point impedance (or admittance) have the same restriction. The zeros of the transfer functions may be located anywhere on the s-plane, but excluding positive real axis if a common ground (three-terminal) network is used.

(b) $\pm R$, C network

In addition to passive resistors and capacitors, this kind of network contains negative resistors. The natural frequencies of this network lie on the entire real axis and must be simple. It is unstable when natural frequencies occur on positive real axis. The zeros of its driving-point impedance (or admittance) lie on the entire real axis, also. The zeros of the transfer function may be located anywhere on the s-plane. For complex zeros parallel networks are required.

(c) RC-gyrator network

Since a capacitor-loaded gyrator acts as an inductor, this kind of network has the same characteristic as an RLC network. For this network the poles may be located anywhere on the left half s-plane including $j\omega$-axis where they must be simple. The location of the zeros of the driving-point impedance (or admittance) are the same as that of the poles. There is no restriction to the location of the zeros of its transfer function. It should be noted that this class of network is non-reciprocal.

(d) $\pm R$, $\pm C$ network

The poles and zeros of this class of network may be located anywhere on the s-plane and may be of any order. Therefore this network is sufficient to realize any rational function of s. When stability is being considered, the natural frequencies of this network must be limited to the left half s-plane and $j\omega$-axis where they must be simple. In addition, this is a reciprocal network. Since RC controlled-sources and RC infinite gain networks may be used to realize NIC's, while RC-NIC networks may be used to realize $\pm R$, $\pm C$ networks, networks containing R, C, controlled-sources, infinite gain, and/or NIC are in this class.

7-2 Properties of Active Devices

(a) Controlled-source

When this device is used in a network, its gain appears in the network function in such a way that different characteristics of the function can be easily obtained without altering the frequency characteristic of the network by changing the gain value. For controlled voltage sources, output impedance is low, so cascade of two stages without a buffer stage is possible. Therefore complicated network functions of higher order can be realized by cascading simple controlled-source networks.

(b) Negative impedance converter

This device together with resistors and capacitors permits the realization of any transfer functions. The realization is easily optimized with respect to sensitivity and number of elements. In determining the value of the network elements, it is not necessary to solve a set of simultaneous nonlinear equations; thus network functions of higher order can be realized easily and quite directly. In addition, input voltages may be summed independently.

(c) Negative resistance

Using a negative resistor as a network element, the poles of the network function can be extended to the positive real axis. Using the tunnel diode is as convenient as a conventional resistor. But the series inductance and parallel capacitor of a tunnel diode makes such a device prone to oscillation. In addition, the tunnel diode is more difficult to fabricate than a transistor. Another difficulty of a $\pm R$, C network is that there are more restrictions to its network function.

(d) Gyrator

When using gyrators as network elements, the realization is based on sum decompositions. Thus sensitivity of this network is lower than that of the controlled source and NIC. Since the Q of an inductor simulated by a capacitor-loaded gyrator depends on the Q of the capacitor used, and it is easy to obtain a high Q capacitor, the Q of the inductor of a capacitor-loaded gyrator is very high. Generally, the gyrator is considered as a passive element; thus it is usually stable.

For the same reason, its gain is very low. Another disadvantage of the gyrator is that the realization of a gyrator needs more active elements than for a controlled source or NIC. In addition, it does not provide low output impedance. Therefore an isolating amplifier must be employed for cascading stages.

(e) Operational amplifier

This active element is the only one which is commercially available in integrated form. The output impedance of the operational amplifier is low; thus a buffer stage is not necessary for cascading stages. Gyrators, NIC's, and other active devices may be realized by using operational amplifiers.

7-3 Comparison of Different Approaches

(a) Conventional active-network approach

In the conventional approach, the synthesis procedure is based on the decomposition of a given network function. Then the characteristics of the passive subnetworks of a given RC active network are obtained. The synthesis is completed by realizing the passive subnetworks using conventional passive network synthesis methods. The configuration of the subnetworks cannot be determined before the final step is completed. In some cases, the realization of the passive subnetworks are difficult. The complete network may differ with different passive synthesis methods.

(b) Coefficient matching approach

In this approach, a network configuration and its network function are given. To realize a specified network function, a set of equations is obtained by equating the like coefficients. This set of equations can be solved by preselecting some element values. This method brings no difficulty in determining the element values. In addition, the network configuration is known at the very beginning. For some realization methods, the spread of element values is as low as 1 to 3.

(c) Simulated inductor approach

The principle of this approach is simple. It begins with a passive network synthesis method to obtain an RLC network. Then the inductors of this network are replaced by active elements. This method is attractive because an RLC network is attractive from the viewpoint of sensitivity. Its shortcoming is that it requires much design work. In addition, this method needs more passive and active elements than any other approach.

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SYNTHESIS OF TRANSFER FUNCTIONS BY MEANS OF INTEGRATED ACTIVE NETWORKS

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With passive RC networks, network functions with poles and zeros anywhere outside of the negative real axis cannot be realized. In addition, these networks introduce loss in the pass band. The above defects can be reduced by adding inductors to the network. However, inductors have some inherent shortcomings, i.e., limitations of quality and size. A better approach is to use active elements in the network. When active elements are used in a network, inductors can be eliminated. Furthermore, the number of network functions that cannot be realized is reduced.

Active devices commonly used at the present time are negative resistances, controlled sources, negative impedance converters, gyrators, and operational amplifiers. Of these devices the operational amplifier is by far the most versatile one for any other active elements can be realized by using these amplifiers. In addition, operational amplifiers are at present the only active element commercially available in monolithic integrated circuit form.

Active network synthesis methods can be classified in different ways. In this report, the classifications are as follows:

A) Conventional active-network synthesis approach
   This approach is based on a general network configuration containing passive subnetworks and one or more idealized active elements. The network function of the proposed network is given. The parameters characterizing the passive subnetworks for a specified

network function are obtained by suitable decomposition and partitioning. The passive subnetworks are then realized following the standard passive synthesis procedures.

B) Coefficient matching approach

In the coefficient matching approach, an active network configuration with known network function is selected. A specified network function is then compared with the known network function. By equating like coefficients, the design equations are obtained. The element values of the required network are determined by preselecting some element values and then solving design equations for the remaining element values.

C) The simulated inductor approach

In this approach an RLC passive network is realized to meet the given specification. Then all inductors are replaced by capacitors and active devices.

There are problems facing active networks, i.e., stability and sensitivity. An originally stable active network can become unstable with change of any of its network parameters. Also, some parameters cause a greater change in the network function than do the others. To determine the sensitivity of any element to the network a number of different criteria have been proposed.

Comparisons of the different active devices and synthesis approach have been given in the last part of this report.