

Neutron Flux Characterization of the Kansas State University TRIGA Mark
II's Northeast Beam Port

by

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B.S., Kansas State University, 2016

A THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

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2019

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Abstract

A high-resolution, multi-dimensional flux characterization was performed for the Kansas State University TRIGA Mark II's Northeast Beam Port for the purpose of informing future experimental work, such as detector characterization, done using the beam. First, the Beam Port geometry was added to the existing reactor model. Then the in-core fission rates were tallied using MCNP to provide a source term for the beam transport. The program ADVANTG was used for automated generation of weight windows to accelerate the convergence of tallies in the beam port model. A tally was then collected at the end of the beam port and results are presented.

Results from this tally were used to produce simulated responses and response functions for two detectors: a gold foil-based passive spectrometer and a standard set of Bonner Spheres. An experiment was conducted with both measurement devices to obtain responses. These measurements showed decent shape agreement and a small magnitude bias relative to the simulated results. Finally, the measurements, simulated flux, and response functions were used to unfold a final set of spectra using three different unfolding techniques. The Doroshenko directed divergence and Gravel (modified Sand-II) methods produced physically-realistic spectra which successfully fit the measured data, while MAXED did not.

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Acknowledgments

I would like to begin by thanking my father, John, who taught me about toughness, humility, and responsibility - a set of values that proved more useful in grad school than the most complex of equations. I thank my mother, Donna, who continually pushed me to challenge my intellect as a child and showed me the value of critical thinking. I thank my sister, Brooke, whose constant encouragement and friendship inspired me to strive to do my best while remaining at K-State. I would also like to thank my roommates and brothers at the Catholic Household of Chi Rho for providing an invaluable support system through my time in grad school. I greatly appreciate all of the help, teamwork, and friendship from my fellow graduate students at KSU - Dan, Richard, Rabab, Kevin, Ye, and Mike. I would like to thank two undergraduate researchers, Keith and Tyler, for their company and contributions to the work presented in this thesis. I thank Dr. Amir Bahadori and Dr. William Dunn for being members of my committee. And finally, I would like to thank Dr. Jeremy Roberts for taking me on in his research group and offering continued guidance and encouragement. I could have never guessed five years ago that his fascinating presentation of nuclear physics would have inspired this academic adventure of mine.

Dedication

To my family

Chapter 1

Background and Introduction

1.1 Neutron Spectrometry

In the last century, nuclear technology has proven itself to be a field of tremendous power and utility. However, these great capabilities are often limited by our true understanding of the neutronic systems that underlie them. Having proper methods to accurately obtain knowledge of how these systems operate and evolve raises this fundamental limit on our capabilities. Therefore, from the beginning of nuclear science, measurement techniques have had an indispensable niche in supporting the development of these technologies, and today, that is no different.

In the field of neutron detection, a huge body of scientific methods and technologies has developed that allow an experimenter or analyst to reliably and accurately obtain a most fundamental property of these systems, the neutron flux spectrum. Active scintillation spectrometers and passive foil or wire activation detectors represent a portion of the different ways information on a neutron spectrum can be obtained. Though these detectors allow us to find information on a neutronic system, a large field of mathematics has allowed even greater understanding by allowing even more information to be extracted. Spectral deconvolution techniques have allowed a small series of measurements to be translated into very large group neutron fluxes, a feat once considered impossible.

This field, neutron spectrometry, represents the field of research within which the topics discussed in this thesis lie. The author will show how simulation, measurement, and deconvolution techniques have been fit together in a specific application to ultimately widen the current field of scientific knowledge.

1.2 Current State of the KSU TRIGA Mark II's Northeast Beam Port

In our case, the neutronic system in question is Kansas State University's TRIGA Mark II research reactor. Four beam ports penetrate the reactor structure and allow neutrons to stream from the reactor core to the outside for ex-core irradiations. Upon building the reactor, an initial flux characterization of these ports was conducted, reporting the magnitude of the flux, as well as the fast-to-total ratio of these ports¹. Separate works have been published that further characterized the flux in one of the beam ports, the northeast beam port,^{2,3}; however, since then, a collimator has replaced the wooden plugs that previously occupied the northeast beam port, significantly changing its geometry. The reactor core has also undergone neutronic significant change, including burnup and the shuffling/replacement of fuel elements.

These changes, and the relatively low resolution of the initial measurements, leave a notable gap in our understanding of the reactor system. The changes within this northeast beam port are likely to have a significant effect on both the spectral, angular, and radial characteristics of the flux, as well as its absolute magnitude. Minimal simulation and experimental work has been performed to characterize of this updated system. Experimentalists currently are either required to do a form of flux characterization every time this beam port is used, or rely on a questionable dataset to complete calculations and simulations related to their devices. Also, the potential radiological hazard associated with any partially unknown source of radiation is worthy of noting.

1.3 Justification and Outline of Conducted Research

It is important, then, to do a full characterization of the northeast beam port to fill this gap. A better understanding of this port would increase the experimental capability for the purpose of detector characterization, inform experimental modeling, and improve the safety of anyone using the northeast beam port.

Simply stated, this work seeks to understand the spectral properties of the neutrons departing from the northeast beam port. Is it possible to build a model to simulate the port's flux, including the radial, spectral, and angular distributions of this flux, and then conduct an experiment to validate that model?

This research aims to compute this flux, with all mentioned dimensions with a very high fidelity and resolution. Then, the goal is to verify these flux values using experimental spectrometric techniques. It has been hypothesized that this beam will contain a significant contribution from fast neutrons relative to the other beam ports because it, unlike other ports, penetrates the graphite reflector surrounding the core. The neutrons departing from this beam should also be predominantly monodirectional.

To achieve these goals and test the hypotheses, first, a review of useful neutron spectrometers and neutron spectrum deconvolution methods will be described in Chapter 2. Then, in Chapter 3, the whole modeling effort conducted will be described, including any preliminary simulated results. Following that, a description of the experimental campaign used to validate these models will be presented in Chapter 4 along with a series of unfolding results that show a set of solutions consistent with the experimental data. Chapter 5 will include conclusions, contributions to the field, and proposed future work.

Chapter 2

Review of Available Methods and Data

2.1 Beam Port Characterization Experiments

A useful property when measuring charged particles is that it is possible for all of the particle's energy to be deposited into a detector, meaning the response of the detector will be directly proportional to the particles energy. Neutrons, unfortunately, do not share this property with charged particles, making the process of obtaining spectral information about any neutronic environment less straightforward. Instead, neutrons have to be measured indirectly - a neutron will react and produce some measurable secondary effect within a detection medium. This effect is often not equal to (or even proportional to) the energy of the incident neutron. That does not mean, however, that spectral information cannot still be obtained about a neutron flux. In practice, with a set of different detectors, each having a response uniquely dependent on the speed of neutrons, a collection of measurements can be compiled to produce information on the entire spectrum. Here, we will explore two common ways neutron spectra are investigated experimentally in practice.

2.1.1 Bonner Sphere Spectrometer Set

A major breakthrough came for the neutron dosimetry community in the early 1960s when Bramlett et. al. first introduced their “new type of neutron spectrometer”⁴. The “Bonner Sphere Spectrometer” (BSS), named after one of the original Rice University authors, was a first-of-its-kind, active neutron spectrometer. The device allows measurements to be taken across the entire range of energies found within or around a nuclear reactor or other common neutron source. Fundamental to the design is the thermal-sensitive LiI crystal that serves as the detection medium. First, a measurement is taken using the bare sensor. Then, a small high density polyethylene sphere is placed around the crystal. The purpose of this configuration is twofold: the sphere will remove a portion of thermal neutrons from the system and will thermalize faster neutrons so they can be more readily absorbed by the crystal. Following this measurement, multiple, increasingly larger sphere sizes are used to isolate faster regions of the neutron spectrum by increasing this removal and thermalization process. After taking this set of measurements, information will have been obtained representing regions across the entire neutron spectrum. Using specialized mathematical techniques (which will be detailed in the next section) these measurement can be used to finally obtain a full, high resolution neutron spectrum.

2.1.2 Gold Foil Activation

Foil activation, specifically through the use of gold foils, is a standard way of measuring a neutron flux. The foils are placed into the neutron flux. Neutrons activate the foils causing them to become radioactive and emit gamma rays. These gamma rays can be measured, and the equations that govern the activation process can be used to calculate the incident flux. With a gold foil alone, a single response can be obtained which yields the absolute total flux of a system. Gold, in particular, is useful for this process for a number of reasons. First, its (n, γ) reaction has a very high thermal cross section. It also has a 2.7 day half life, which is ideal, as it will not quickly decay away, yet it also does not take an unreasonable amount of time to reach a substantial fraction of saturation. The gamma ray produced by gold has an

energy of approximately 412 keV, ideal for measuring with a high-purity germanium (HPGe) detector. It is also malleable, meaning that it can be easily worked into different forms depending on the desired measurement. Though a single gold foil cannot give information on the spectrum, there are a number of ways in which gold can be used to help measure the entire spectrum. Adding a cadmium filter around the gold will produce a two-group flux, since cadmium will absorb a large portion of the thermal neutrons and isolate gold's major resonance. Gold can also be used as part of a multi-foil experiment. Different kinds of foils with different responses can be used to cover the whole spectrum. Also, gold has been used with a set of Bonner spheres, acting as the primary detection medium within the center of the spheres⁵⁶. This has been done in-beam at many different research and medical isotope production reactors.

2.2 Spectral Unfolding Methods

2.2.1 Formulation

In practice, active and passive detectors and measurement techniques can appear very different. However, for the purpose of mathematical formulation, it is possible, and indeed useful, to abstract their similarities. The above mentioned detection methods resemble each other in that they provide a set of unique, discrete responses in the presence of an unknown neutron flux. These discrete responses can vary greatly among different neutron environments. For example, the LiI detection crystal of the Bonner Sphere Spectrometer (BSS) responds strongly to thermal neutrons and weakly to fast neutrons, whereas certain reactions utilized in activation foil analysis, such as $^{27}\text{Al}(n, \alpha)^{24}\text{Na}$, will not occur below specific, fast, threshold neutron energies; thus the detector is considered to have a fast response. These largely energy-dependent responses, often related to material properties and reaction cross sections, can be considered functions. A detector *response function* is the measurable effect exhibited by a detector in a particular geometric configuration as a function of energy to a particular neutron source. A response function is generally a function of the system geometry and the

source emission profile (as a function of space, angle, and energy), but for fixed source-detector configurations, the response can be considered unidimensional in energy.

With this understanding of the response and response function, the relationship between the flux, response function, and response for detector configuration k can be represented as a Fredholm integral equation of the first kind, i.e.,

$$N_k + \epsilon_k = \int_E R_k(E)\phi(E)dE. \quad (2.1)$$

Here, N_k represents the measured response of the detector, ϵ_k is the unknown error associated with N_k , $R_k(E)$ is the response function, and $\phi(E)$ is the flux. By placing a detector k with associated response function $R_k(E)$ in a neutron environment $\phi(E)$, a response N_k is exhibited that deviates by ϵ_k from the expected response due to noise or other sources.

As posed, this formulation bears a certain number of issues. It is impossible to derive an arbitrary, continuous function for the flux from a finite set of equations. With any discrete number of measurements, the solution remains non-unique; the equation is thus satisfied by an infinite number of solutions. The fact that the ϵ_k term is unknown implies that, just as there is a limit to the confidence one can have in a real measurement, there, too, is a limit on the confidence in the final spectrum.

A number of approximations serve to remedy these dilemmas. An energy bin structure can be introduced, with energy bins $\Delta E_i (i = 1, \dots, n)$, which discretize the problem. Then, for a problem with m detector configurations, equation Eq. (2.1) is rewritten as

$$N_k + \epsilon_k = \sum_i R_{ki}\phi_i, \quad k = 1, \dots, m. \quad (2.2)$$

Response functions are generally computed, using either deterministic or Monte Carlo methods, and are therefore more naturally accommodated by this equation. The unknown ϵ_k term can be handled by removing it entirely, although more sophisticated approaches to solving will utilize a known variance, σ^2 , related to the measurement environment, into the analysis and propagate this error into the final ϕ .

2.2.2 Proposed Methods of Solution

The process of obtaining the solution for the unknown neutron spectrum from the set of possible solutions is known as spectral deconvolution (or unfolding), and given the infinite number of valid solutions to the ill-posed problem, the volume of existing deconvolution methods is, perhaps unsurprisingly, commensurate to the number of solutions themselves. One can feasibly produce a valid solution using a method derived entirely from the realm of mathematics. In fact, an expansive literature exists on pure mathematical methods to solving ill-posed, linear, inverse problems such as the one posed. However, though these may produce solutions that satisfy the equation, what can be said of the accuracy of these solutions?

Unfortunately, solution spectra derived from these methods can exhibit features that lack a physical basis or lack features present in the actual neutron spectrum. Though the neutron spectrum under inspection is technically unknown, a strong theoretical foundation allows us to realize the issues with these methods. Nearly a century's worth of effort has gone into understanding neutronics and reactor physics after James Chadwick's 1932 discovery of the neutron. Since then, this effort has ultimately led to the development of a robust and comprehensive theory of nuclear reactor physics. It is this knowledge that is crucial for assisting in the discrimination of derived neutron spectra. For a method to bear utility, it must weave this theoretical information into its analysis.

A first step in applying this information is through qualitative inspection of the solution spectrum. It is common practice to study an unfolded flux spectrum and determine whether or not it matches our *a priori* understanding of a neutron flux. Although this inspection, however useful, can allow the acceptance of a solution, it does not increase the likelihood that a physically reasonable solution will actually be arrived upon.

Another path to include this information is by imposing constraints on the deconvolution. In this way, the problem is considered an optimization, specifically a minimization of the difference in the measured and calculated responses, subject to certain physical constraints. Examples of these constraints include jump limitation and regional enforcing of increase or decrease, and will be discussed in more detail later.

Undoubtedly, the most common solution to the dilemma of theory inclusion is the utilization of a ‘default spectrum’. This default spectrum, also often referred to as a guess spectrum or trial spectrum, is a theoretically-based, *a priori* solution hypothesis - the best guess at the final answer. If we have access to the true solution, cast as a perfect guess, the reliance on the uninformed mathematics behind deconvolution is completely alleviated. If the default spectrum is nearly correct (general shape and magnitude), but exhibits a handful of minor inaccuracies, the task of the unfolding method is *tuning*, not building, the true solution. This refashions the entire problem of unfolding to one of inference, where substantial weight is placed on the construction of the best default spectrum, and not necessarily on the math-mechanics of the method.

Before continuing, it may be helpful to provide a discussion of the fidelity of our current reactor models to demonstrate the viability of such a refashioning. Let’s take for example a typical light water reactor (LWR). In the general case, the neutron spectrum present in such a reactor can be justifiably simplified to the superposition of three features⁷. The fast region is comprised of fission neutrons, commonly modeled via Watt’s distribution; the thermal end resembles a gas dependent on reactor temperature and can be modeled as a Maxwell-Boltzmann distribution; and downscattering, a slowing down process leading to an energy shape inversely proportional to the neutron energy, affixes the two peaks. The constants associated with any of these models are all physically meaningful and can be selected heuristically, aided by other known reactor parameters, measurements, or standards, which minimizes guess-work. Unfolding, viewed as a tuning process and initialized with a model-based default spectrum, will, with high likelihood, produce a physically valid solution spectrum because it is just a slight alteration to some spectrum that is, *by construction*, physically valid.

This LWR default spectrum model provides a reasonable starting point for unfolding; however, it is deficient in its capacity to capture finer resolution, spatially or temporally dependent, in-core intricacies. The interest in neutron spectrometry is certainly not limited to only in-core applications, so other, more general approaches for constructing default spectra are necessary. To achieve this level of detail in a default spectrum model, we must turn

our attention to numerical approaches, a methodology that allows a departure from, or maybe, remarkable enhancement to, the usefulness of analytical models. The computing power available to modern reactor theorists have allowed the generation of incredibly complex representations of neutronic environments with high fidelity and often reasonable computational times. Subdivided into two major classes, stochastic and deterministic methods, with both bearing unique advantages, these numerical approaches are the contemporary standard for obtaining a default spectrum. Programs such as MCNP and Serpent have paved a road from the domain of typical or simplified problems into that of any well-understood or semi-understood neutronic problem and grant deconvolution methods value in almost any conceivable experimental configuration.

Sand II

The Sand II algorithm was first introduced in a 1967 paper from the Air Force Weapons Lab by McElroy et. al. describing ‘A Computer-Automated Iterative Method for Neutron Flux Spectra determination by Foil Activation’⁸. Sand II was built specifically with foil activation unfolding in mind, using a specially-designed, iterative, spectral adjustment procedure. The procedure employed a user-provided default spectrum and built-in cross-section libraries to compose the response matrix.

The algorithm is a relatively simple procedure meant to minimize the differences in the measured and calculated responses. First, the default spectrum and response functions are folded together to obtain a set of calculated responses. Then, these are compared with each measured response to obtain correction factors for each foil. A weighting function, which is simply the differential flux times the differential cross sections normalized to unity, is then obtained for each foil. These weighting functions map a difference in a single activity onto energy space. A correction function is produced, then, as a weighted average of the log of the correction factors. This function is exponentiated transform these correction functions back into linear changes (as opposed to logarithmic changes). The correction functions are applied to the current step’s flux, and a new flux is obtained.

The many steps from above can be written in a concise format,

$$\phi_j^{k+1} = \phi_j^k \exp\left(\frac{\sum_i W_{ji}^k \log\left(\frac{N_i}{\sum_{j'} R_{ij'} \phi_{j'}^k}\right)}{\sum_i W_{ji}^k}\right), \quad (2.3)$$

where the function W_{ji}^k is provided by

$$W_{ji}^k = \frac{R_{ij} \phi_j^k}{\sum_{j'} R_{ij'} \phi_{j'}^k}. \quad (2.4)$$

Because Sand II is iterative, there needs to be a criterion defined for solution acceptability. A user provides a value which represents the degree to which each differential flux value for two successive iterations must agree for a solution to be considered achieved.

Because the purpose of Sand-II is foil activation in a typical fission reactor environment, a bin structure is imposed with 621 points between 10^{-10} and 18 MeV, representing the range covered by neutrons in that environment. A library is distributed with the code that contains 29 foil activation cross sections, plus built-in cover data (Cd, etc.) that could be applied to the foil cross sections.

Accommodating the somewhat unpredictable or error-prone nature of experimentation, there is also a feature built into the code that will, given a user-defined threshold σ , discard individual foil measurements whose values differ by more than σ the solution spectrum. This threshold allows the user to mathematically (and automatically) classify and remove outlying data. The solution is then recomputed without this discarded value.

The Sand II algorithm has been applied widely since its introduction in the 1960s. Many other codes have built upon the foundation of the algorithm since then, such as Sand IV and MSANDB⁹. The popular unfolding code Gravel uses the same algorithm but with one small change¹⁰. In particular, the weighting function is updated via

$$W_{ji}^k = \frac{R_{ij} \phi_j^k}{\sum_{j'} R_{ij'} \phi_{j'}^k} \frac{N_i^2}{\sigma_i^2}, \quad (2.5)$$

to include a term that incorporates the measurement errors into the solution-finding process. Another feature of Gravel in its current distribution is the removal of the required input

format of Sand II, meaning the application is no longer locked to the domain of foil activation, but can be applied to any unfolding situation, given the proper, user-provided response functions.

Doroshenko et. al.

In 1975, an algorithm was introduced by Doroshenko et. al. to tackle the problem of spectral unfolding¹¹. The basis of the algorithm was to provide a solution that satisfies Eq. (2.2) while also maximizing a quantity called ‘directed divergence’. Directed divergence, also referred to as entropy, is a measure related to the likelihood that a neutron spectrum will take a particular shape. A more complete treatment of entropy will be handled later. In the case of Doroshenko’s algorithm, the entropy is defined as

$$J = - \sum_i N_i \ln\left(\frac{N_i}{\sum_j \phi_j R_{ij}}\right). \quad (2.6)$$

An iterative scheme can then be produced that will, after sufficient iteration, converge on a flux spectrum that will maximize the entropy J of the spectrum. This relationship is shown in the following equation,

$$\phi_j^{k+1} = \phi_j^k \left(\frac{\sum_i \frac{R_{ji}}{\sum_j R_{ij} \phi_j^k}}{\sum_i \frac{R_{ji}}{N_i}} \right). \quad (2.7)$$

Over iterations, this algorithm will compute the ratio between calculated and measured responses, similar to the correction factors described in the Sand II algorithm. These are used to produce a weighting function that will correct the flux more in the regions that produce the largest deviations in calculated-to-measured responses. A measure of error can be computed to compare responses,

$$\chi^2 = \frac{1}{N} \sum_{i=1}^N \left[\frac{\sum_j R_{ij} \phi_j^k - N_i}{N_i} \right]^2, \quad (2.8)$$

which will tend to rapidly converge on a finite value, so the change in χ is used as a convergence criteria,

$$\delta\chi^k = \chi^k - \chi^{k-1}. \quad (2.9)$$

Similar to Sand II, the solution found in this way is guaranteed to be positive, given R is positive. One advantage exhibited by this algorithm is its relative simplicity and easiness to implement. Its basis in information theory means it has a grounding in mathematics that is related to neutronic behavior, an obvious advantage for the application.

The unfolding code SPUNIT¹² was one of the earlier codes to apply the Doroshenko method, and since then it has been used in several other unfolding codes, including BUNKI¹³, AFITBUNKI¹⁴, and NSDUAZ¹⁵.

MAXED

Other maximum-entropy unfolding methods have been proposed since Doroshenko, such as Itoh and Tsunoda (1989), but in 1998, a Fortran-based, maximum-entropy, unfolding code from Reginatto and Goldhagen was introduced that has since seen wide-spread use across the field of neutron spectral unfolding¹⁶.

This algorithm, MAXED, begins with Eq. (2.2) but adds the additional constraint

$$\Omega = \sum_i \frac{\epsilon_i^2}{\sigma_i^2}, \quad (2.10)$$

where Ω is a user-selected parameter that relates the measurement error of the detectors to the allowable deviation in the measured and calculated responses.

The unique feature of MAXED is that entropy, while normally considered an information measure of the solution spectrum itself, is here instead characterized as a value associated with the deviation between the default and solution spectra. The code will then search for the maximum entropy solution that is most similar to the default spectrum using the following definition for entropy:

$$S = - \sum_j [\phi_j \ln(\frac{\phi_j}{\phi_j^{DEF}}) + \phi_j^{DEF} - \phi_j]. \quad (2.11)$$

In this way, the in-built hypotheses in the starting spectrum are more likely conserved, as they are considered parts of the *a priori* information inclusion process described earlier.

To solve this equation, it first can be shown that the maximization of the entropy as defined above is associated with the Lagrangian

$$L(\phi_j, \epsilon_i, \lambda_i, \mu) = - \sum [\phi_j \ln(\frac{\phi_j}{\phi_j^{DEF}}) + \phi_j^{DEF} - \phi_j] - \sum \lambda_i [\sum R_{ij} \phi_j - N_i - \epsilon_i] - \mu [\sum (\frac{\epsilon_i}{\sigma_k})^2 - \Omega], \quad (2.12)$$

and that λ_i and μ are Lagrange multipliers. Reginatto and Goldhagen show that variation of L is equivalent to the maximization of the following potential function, Z ,

$$Z = \sum \phi_j^{DEF} \exp(- \sum \lambda_i R_{ij}) - [\Omega \sum (\lambda_i \sigma_i)^2]^{1/2} - \sum N_i \lambda_i. \quad (2.13)$$

Although many methods can be applied to maximize the function, the chosen method for MAXED was simulated annealing for its ability to discriminate between local and global optima. This process will give the λ_i that maximize Z which then can be substituted into

$$\phi_j = \phi_j^{DEF} \exp(- \sum_i \lambda_k R_{ij}) \quad (2.14)$$

and

$$\epsilon_i = \frac{\lambda_i \sigma_i^2}{2} (\frac{4\Omega}{\sum_j (\lambda_j \sigma_j)^2})^{1/2} \quad (2.15)$$

to provide a solution flux and errors associated each response.

The method is shown to work in a variety of cases and been included in many comparative studies¹⁷. Much of the emphasis in Reginatto and Goldhagen's work is on the importance of inference in obtaining the solution. Without proper *apriori* information, the solution should not be considered useful.

A Note on Entropy

Although a formal or complete treatment of entropy extends beyond the scope of this work, a note will be included here, because two of the above methods are derived from this information theory.

Entropy is a quantifiable measure of the potential information contained within some signal¹⁸. The higher the entropy, the less likely it is for information to be encoded within the signal. In other words, the signal (or data, or vector), is considered to be more random and more likely in processes that produce random behavior.

As a measure, entropy is very important in the theory of kinetic gases and heat transfer, since both involve numerous, stochastic interactions that tend to degrade inhomogeneities in data over time. Neutrons, often large in quantity, behave similar to gases. This is one of the reasons that allow diffusion theory to be applied successfully to model neutronic behavior. Since the neutron environments are often well understood, it is highly unlikely that sharp spectral features will form in solution spectra outside of those expected. Therefore, neutron spectra that develop features in the unfolding process may be, rightfully, treated skeptically, since there's no known reason for this 'information' to have been created. It is for this reason that the maximization of entropy has seen such widespread use in the neutron spectral unfolding.

Other Methods

Although the three above methods are among the most popular, they do not begin to represent the whole of unfolding techniques.

Several codes have been developed that utilize parametric models. Since neutron spectra are often governed by known, analytic models, unfolding can be considered a process perturbing model coefficients instead of energy bin values. This aids in the default spectrum selection, since it becomes a matter of selecting an analytic model and initial coefficients instead of providing a numerical default spectrum vector. These models can be simply selected based on the neutron environment in which the experiment was conducted. Codes like FRUIT,

MITOM, BONMA, BON94, and MINUIT all use parametric models for unfolding.

A simple mathematical least squares inversion can be done to obtain a solution. The codes LOUHI and STAY'NL are among those that use this inversion technique. STAY'NL is particularly useful for its ability to incorporate error from the measurements, response functions, *and* default spectrum into its final solution.

A variety of other unfolding techniques have been successfully applied to the problem of spectral unfolding. These include genetic algorithms, like in the code BONDI-97; artificial neural networks, seen in BUMS; and many other iterative and non-iterative methods such as the Backus-Gilbert methods, the Bryan-Caudill Model, and the code BON.

Chapter 3

Northeast Beam Port Modeling

3.1 Introduction

This section contains a complete description of the neutronic modeling for the NEBP. It is often necessary, as in this case, to model a neutronic environment before attempting to measure it, as the modeling results can inform certain measurement methods and parameters. The measurement systems used in later analyses required calculated response functions that incorporated the flux's angular dependence obtained from this modeling effort. The final result from this section also provides the default spectrum to be used in the unfolding of the neutron spectrum after measurement. Many problems were encountered during this analysis, often related to computational difficulty/time, that lengthened the process of obtaining the final results. These problems, along with the applied solutions, are presented. We begin with the NEBP model's KCODE implementation in MCNP, followed by a conversion to a steady-state problem, the application of variance reduction, and finally, the beam-aperture tally results. The steps here ultimately led to success in solving a difficult transport problem and these steps taken can likely be applied to similar situations, such as other reactor beam transport problems or, more broadly speaking, any problem with severe collimation of an isotropic source.

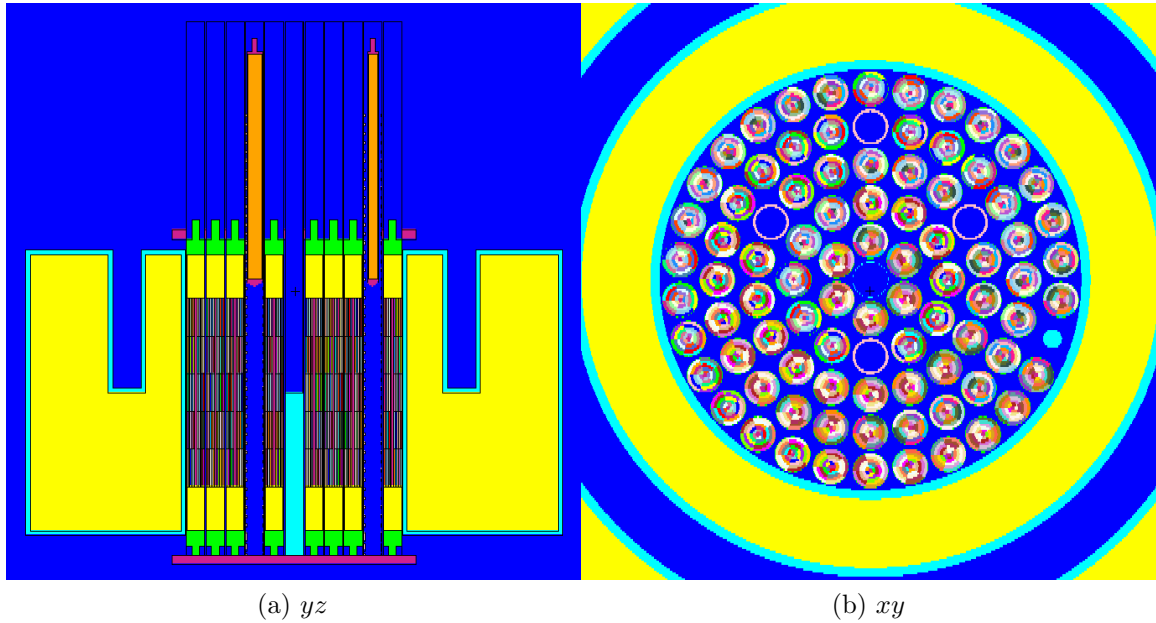


Figure 3.1: *A yz and xy slice of the existing reactor model before the addition of the beam port.*

3.2 Addition of Northeast Beam Port

3.2.1 Existing Model

In the summer of 2017, a reactor modeling effort was initiated at KSU. The goal of the effort was to produce, from the ground up, a Python-based suite that would interpret geometric and material data related to KSU's TRIGA Mark II research reactor and produce input files for various transport codes. The model, though still currently under development, contains a majority of the features from the actual reactor, including any of those which would have a major neutronic impact. There are 85 fuel elements, containing fresh fuel, that are able to be automatically discretized axially, radially, and azimuthally into up to 10,000 cells. The four control rods are also included, and a user may specify their exact position to match that of in-core experiments. The central thimble, grid plates, and source location are all also included in the core model, which is surrounded by a graphite reflector. This model can be seen in Fig. 3.1 and can be used to create input files for different transport codes, such as MCNP¹⁹.

3.2.2 The Beam Port Geometry

Using this model as a starting point, the northeast beam port (NEBP) was then added. The NEBP, also known as the penetrating beam port or the fast beam port, is in essence a large, collimating cavity in the reactor’s structure that allows neutrons to stream from the core to the outside of the reactor. The primary purpose of this beam is to provide a monodirectional neutron source, the strength of which can be scaled easily via reactor control. This beam can be used for detector calibrations and testing, neutron imaging, or sample irradiation for neutron activation analysis. The NEBP is unique in the fact that it penetrates the graphite reflector providing, in theory, the hardest neutron spectrum of the four beam ports at KSU due to the absence of significant moderating material between the fuel and the beam port, as all other beam ports are located outside of the graphite reflector.

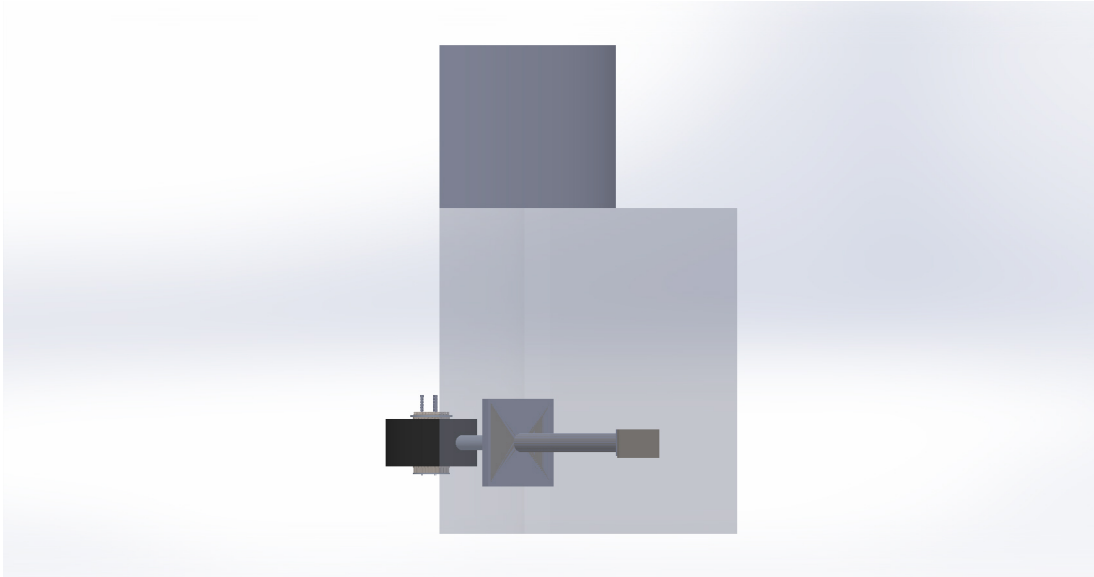


Figure 3.2: *An angled view of the NEBP with the reactor with a quarter section of the reactor structure.*

The NEBP geometry is relatively simple. It is comprised of two, six-foot, cylindrical segments; the inner and outer segments being 6 in and 8 in in diameter, respectively. A lead “shadow shield” sits at the junction of these two sections to help reduce the gamma component of the outgoing particle stream. The shadow shield is 4 ft x 4 ft and 4 in thick and contains a hole through the center, through which the outer portion of the NEBP’s 6 in section extends.

The walls of each section are made of 11/32" thick aluminum. Originally, these sections contained removable wooden plugs that would block neutron streaming; however, water leaks in the port caused damage to these plugs, and, at present, they have been removed.

A collimator is situated within the 8" section. This was designed by a KSU senior design team attempting to further constrain the neutron beam. The collimator is an aluminum tube of 0.75" ID and 1" OD. An aluminum plate is affixed to this tube on the reactor side and contains a throughhole to maintain line of sight with the reactor core. Borated polyethylene disks have been stacked around the tube to act as the primary absorption medium for collimating neutrons. The borated polyethylene extends the entire distance of the collimator.

3.2.3 MCNP Implementation

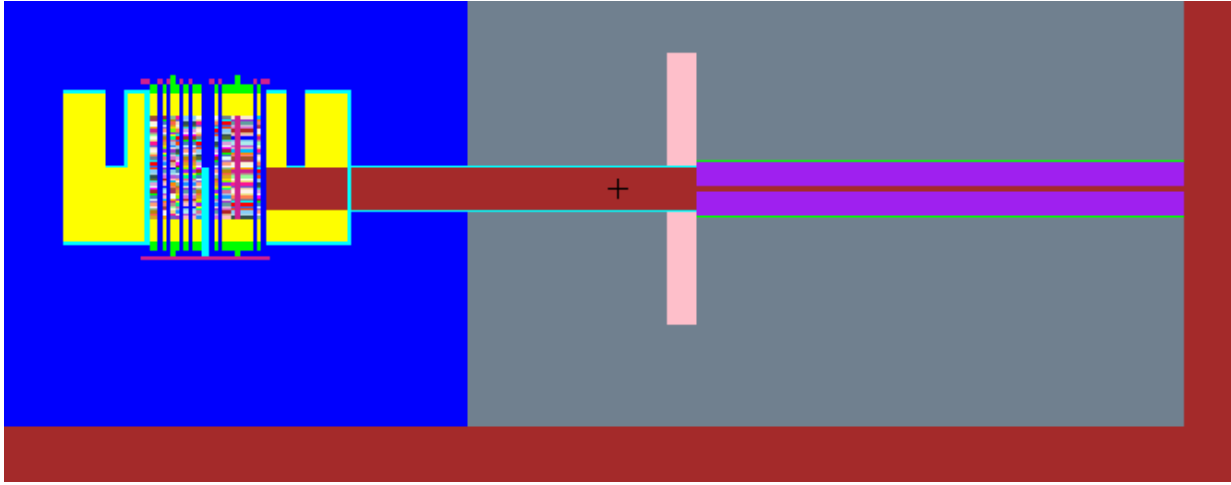


Figure 3.3: *An xz view of the NEBP in mcnp.*

The NEBP itself consists primarily of cylinders and planes, so its implementation in the model was relatively straightforward. The surrounding concrete structural region was modeled as a large cylindrical cell, with the assumption that the geometric intricacies a great distance from the NEBP would have a minimal effect on the flux in the NEBP. As described above, the NEBP, including both sections and the shadow shield, was added within this concrete region. A penetration was added to the existing graphite reflector. The reactor

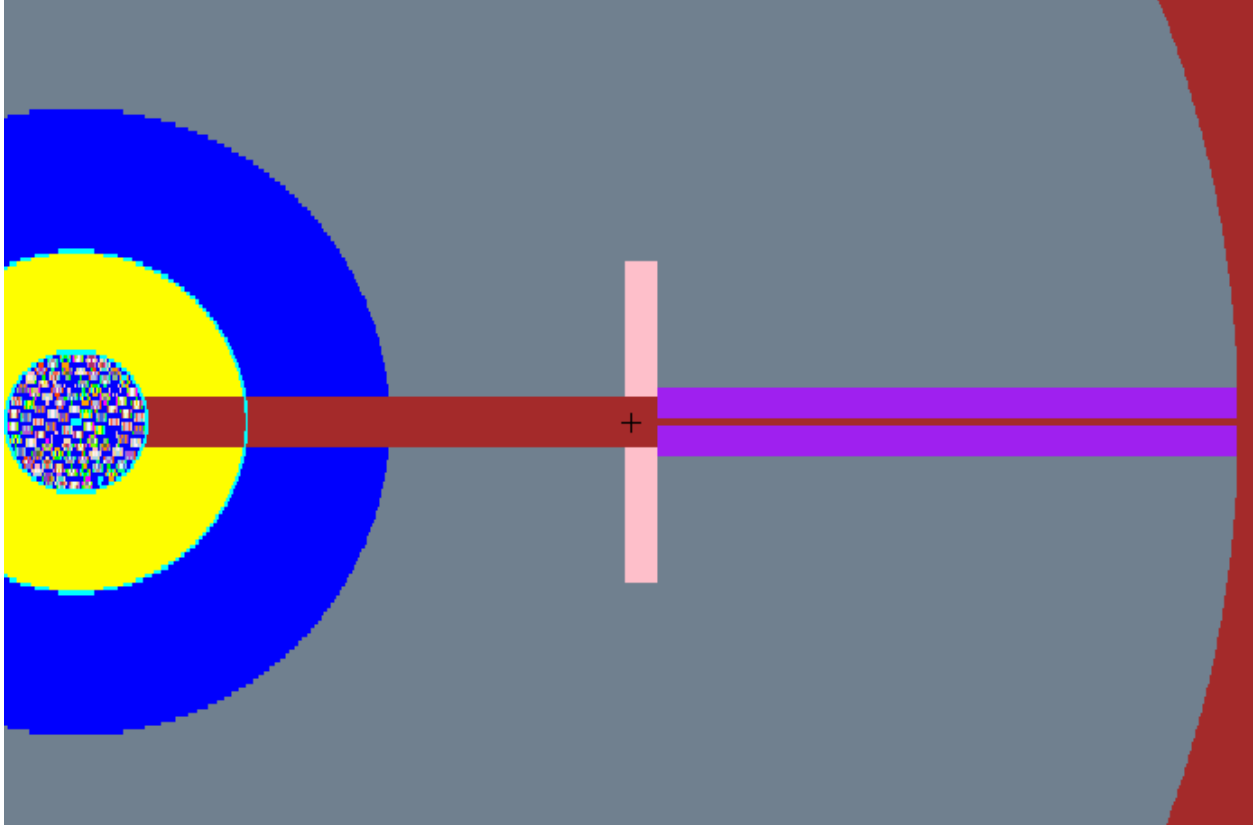


Figure 3.4: *An xy view of the NEBP in mcnp.*

documentation was unclear about some specifics regarding this penetration, so it was assumed that it was not radially encased by any aluminum, but the aluminum surrounding the reflector was left intact to not let water in. The materials used for the NEBP can be seen in Table 3.2.3. Material data in the model came from `pyne`²⁰.

Table 3.1: *Materials used in the NEBP model.*

Material	Density g/cm^3	Composition (Wgt. Fraction)
Aluminum	2.7	^{27}Al (1.000)
Lead	11.34	^{204}Pb (0.0138) ^{206}Pb (0.2396) ^{207}Pb (0.2207) ^{208}Pb (0.5259)
Air	0.001239	^{12}C (0.0001) ^{14}N (0.7147) ^{16}O (0.205) ^{40}Ar (0.0346)
Borated Polyethylene	0.95	^1H (0.1253) ^2H (0.000028) ^{10}B (0.0184) ^{11}B (0.0816) ^{12}C (0.7657) ^{13}C (0.0090)
Concrete	2.4	^1H (0.0221) ^2H (0.00000509) ^{12}C (0.002458) ^{13}C (0.0000288) ^{16}O (0.5741) ^{17}O (0.000232) ^{23}Na (0.0152) ^{24}Mg (0.000988) ^{25}Mg (0.00013) ^{26}Mg (0.000149) ^{27}Al (0.01998) ^{28}Si (0.2802) ^{29}Si (0.0147) ^{30}Si (0.010066) ^{39}K (0.0093) ^{40}K (0.0000012) ^{41}K (0.000709) ^{40}Ca (0.04157) ^{42}Ca (0.00029) ^{43}Ca (0.00006223) ^{44}Ca (0.00098) ^{46}Ca (0.00000197) ^{48}Ca (0.00009622) ^{54}Fe (0.00036) ^{56}Fe (0.00592) ^{57}Fe (0.000139) ^{58}Fe (0.0000189)
Steel	8.03	^{12}C (0.00039536233) ^{13}C (0.0000046336713) ^{28}Si (0.0045932784) ^{29}Si (0.00024167903) ^{30}Si (0.00016499257) ^{31}P (0.0002299977) ^{32}S (0.00014207158) ^{33}S (0.0000011567992) ^{34}S (0.0000067532956) ^{36}S (0.000000016825336) ^{50}Cr (0.007929925) ^{52}Cr (0.1590272) ^{53}Cr (0.018379631) ^{54}Cr (0.0046613454) ^{55}Mn (0.0099999) ^{54}Fe (0.03961618) ^{56}Fe (0.64489415) ^{57}Fe (0.015159803) ^{58}Fe (0.0020528512) ^{58}Ni (0.062157351) ^{60}Ni (0.024767425) ^{61}Ni (0.0010945963) ^{62}Ni (0.003547273) ^{64}Ni (0.00093242912)

3.3 Fission Rate Tallying and Steady-State Source Generation

3.3.1 SDEF Generation

The ultimate goal here is to obtain, via simulation, the flux departing from the beam port. However, the existing model is setup only for KCODE problems. The difficult nature of this transport problem, due to the low probability of a simulated particle streaming down the collimator and exiting the reactor, requires some form of variance reduction. Because of an incompatibility between KCODE problems and the code ADVANTG (selected for variance reduction and used to generate weight windows), this problem must first be converted into an SDEF-type problem. This conversion required the collection of spatially-dependent fission site data. This “fission map” could provide the set of probability vectors to inform the source sampling in the steady-state problem. This was done in the following way. First, the fuel cells were discretized axially and radially. It was decided to use 40 axial sections and 5 radial sections for each of the 85 fuel elements to create a total of 17,000 individual cells. This resolution ensured that the details in the fission rate profiles were adequately captured. Then a fission rate tally was created for these cells. This was done using an F4 tally (flux averaged over a cell) with an FM (tally multiplier) card which would multiply the flux by the total fission cross section for the fresh fuel material. With these tallies in place, the KCODE problem was then run for a total of 4,400 cycles where the first 10 were discarded and each cycle consisted of 100,000 particles. This guaranteed the statistics for the tallies had properly converged.

After running this problem, a python script was used to parse the output and generate an SDEF source to be used in the next step; this card is:

```
SDEF ERG=D1 RAD=D2 AXS=0 0 1 POS=D3 EXT=FPOS=D4
```

The ERG (energy) distribution used for the source was MCNP’s built-in Watt spectrum, indicated by a -3 on the source probability card. The source’s spatial distribution is a series

of cylinders co-located with the respective fuel elements within the core. The **POS** (position) variable contained the center points of each of the fuel elements, whose probability of sampling is proportional to their integrated fission rate. The **EXT** (extent) variable means length of the cylinder, in this case the length of the fuel meat, and refers to a series of distributions dependent on the **POS** of the fuel. Because base MCNP does not easily allow for nested dependent distributions, the **RAD** (radial) distribution was considered an independent variable, and the distribution used was the normalized, averaged radial fission rate across each fuel element.

3.3.2 Fission Tally Results

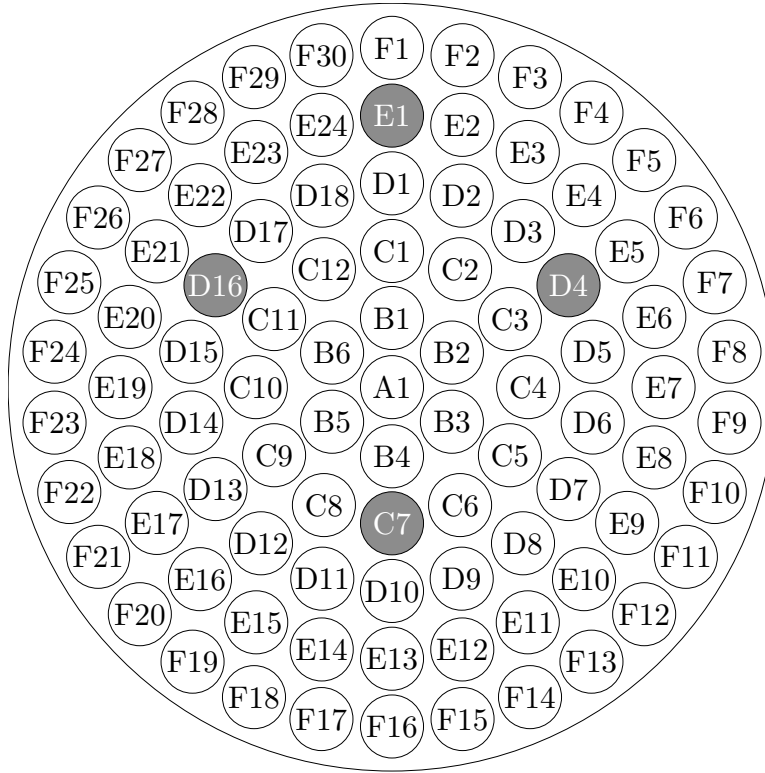


Figure 3.5: *A map of the named elements in the core.*

For reference, a map of the core is provided in Fig. 3.5. Along with the source, several plots were made of this fission rate data.

The heatmap in Fig. 3.6 demonstrates how the fissions are distributed within a typical



Figure 3.6: *The fission rate distribution within the element B1.*

fuel element. Because there was no azimuthal discretization in the tallies, the values here are rotationally symmetric. Although it is hard to draw any quantitative conclusions from this image, it is useful for visualizing fission rates within an element. The fission rate appears to be a maximum at the center of the rod, both axially and radially.

In Fig. 3.7, one can see the radial behavior of the fission rates within ring B. Although the curves here are different, they differ primarily in magnitude and not shape. The other rings not pictured here all show similar behavior. The fission rate density is highest towards the center of the rod and lowest in the outermost region.

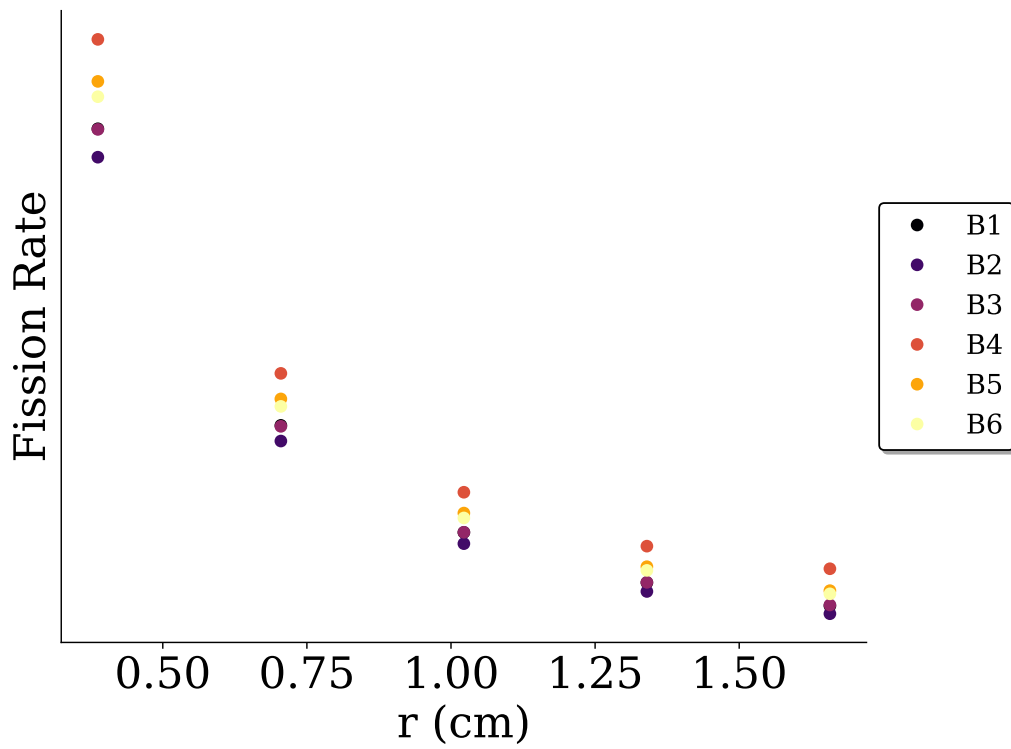


Figure 3.7: *The radial fission rate within the B ring.*

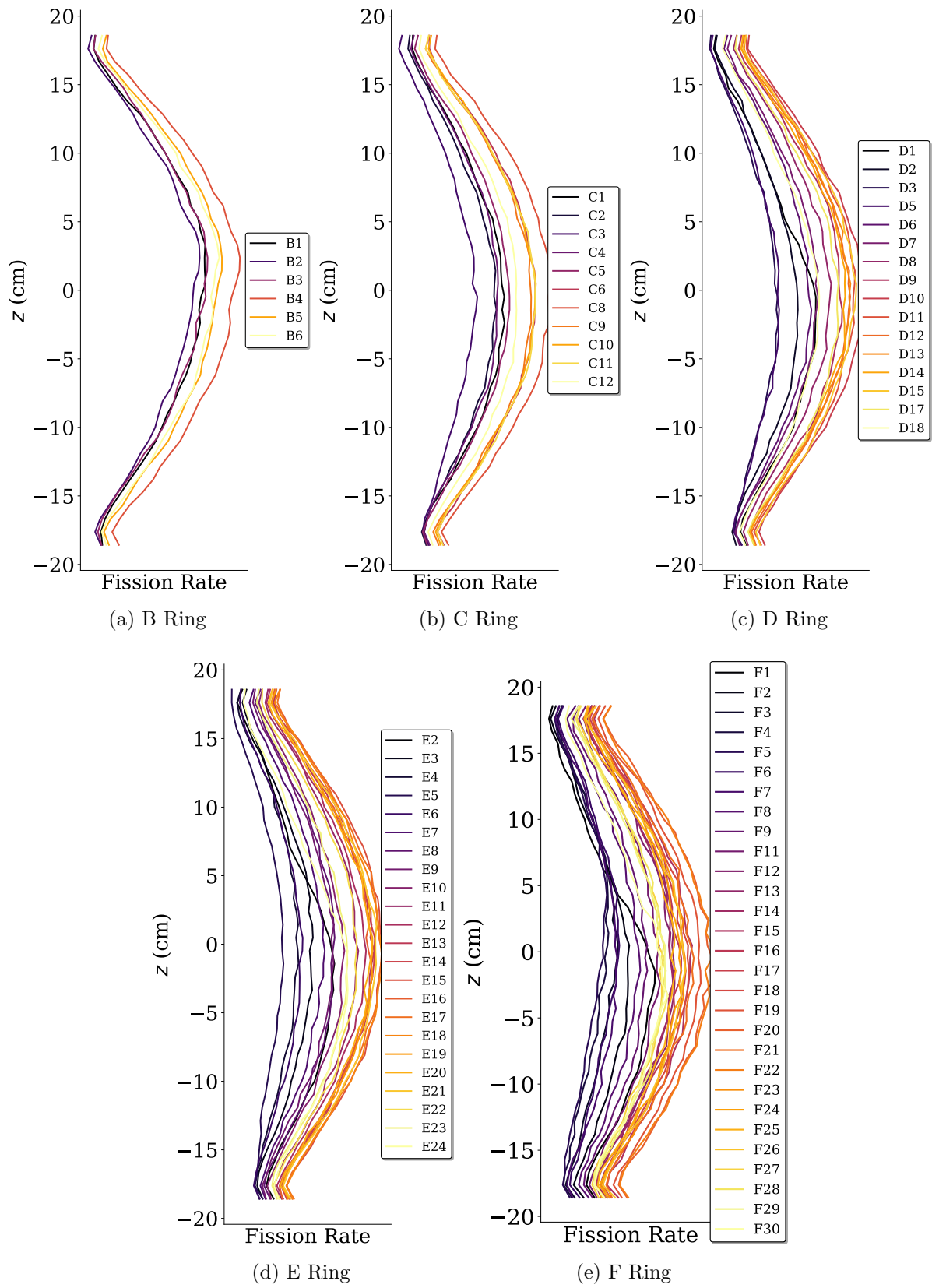


Figure 3.8: *The axial fission rates within each ring of the core.*

Figure 3.8 shows the axial fission rate densities within each ring of the core. Each of these rings shows a cosine-like distribution with a slight upturn at each extrema. Neutron flux within a *roughly* cylindrical geometry, like a reactor core, will *roughly* exhibit a cosine shape, which causes a similar shape for the fission distribution. The upturn is caused by the graphite on each end of the fuel elements which reflects neutrons back towards the rod. There are two rings in particular that show additional behavior outside of these two phenomena. The first is the depression within the B ring below 0 cm. This is caused by the central thimble, which currently occupies the A1 location. Moderating water which would normally occupy that location is replaced by this thimble which is currently modeled as an aluminum rod that extends halfway up the core. This lack of neutron moderation means lower thermal flux in that region and results in the depression.

The second is the depressed fission rate within the lower region of F1-F9. This is caused by the addition of the NEBP. Recall, the beam port includes a penetration within the graphite reflector. This penetration is situated 8.3 cm below the z axis. This lack of reflection means more neutrons will stream out of the core within that region, thus lowering the local flux and fission rate. This effect can be seen to a degree within the E ring, but it is much less pronounced.

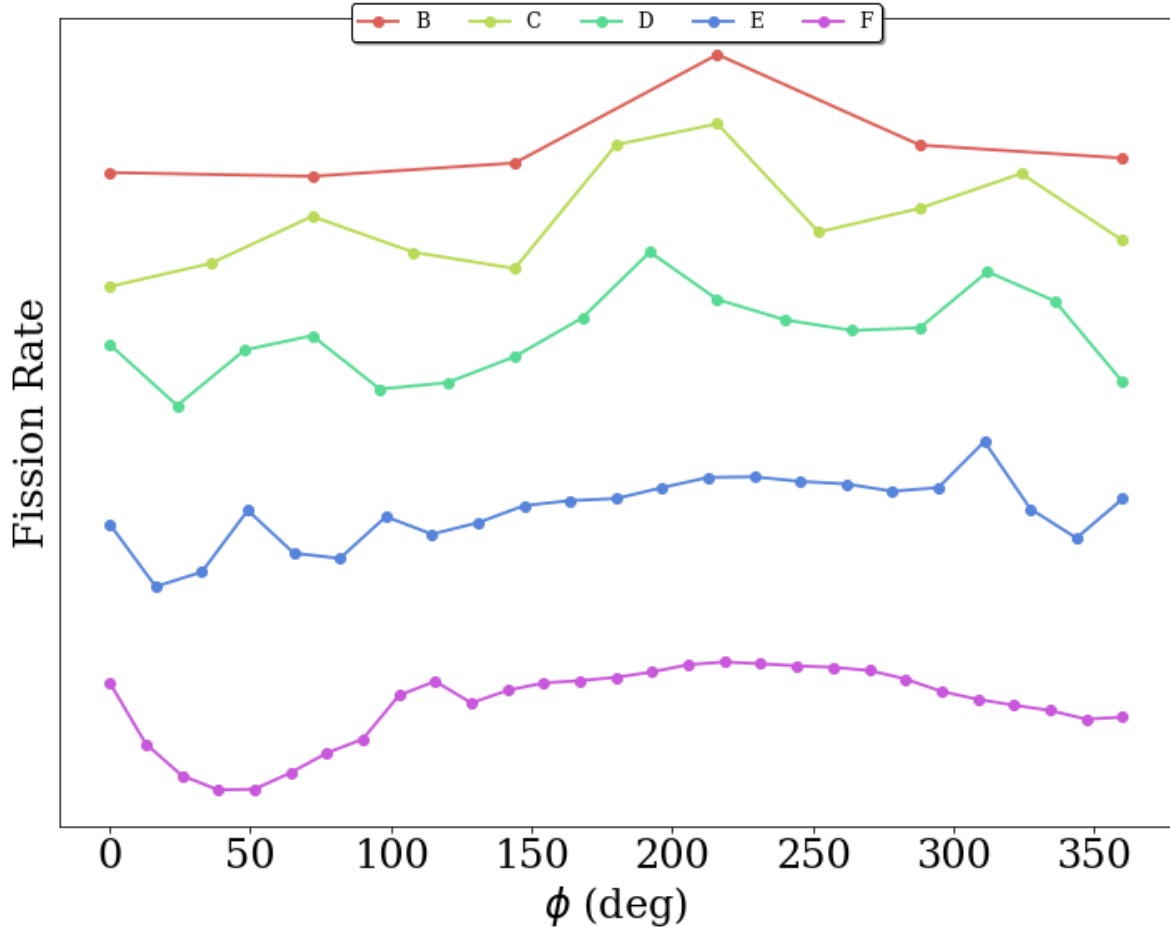


Figure 3.9: *The fission rate density for each element in the core as a function of core position.*

By studying Fig. 3.9, one can again see the effect the NEBP has on the F ring's fission rates. This depression is seen when sweeping past the NEBP from around 12° - 75° . Other peaks and valleys seen in the figure are likely the cause of control rod, central thimble, and source placement.

3.4 Application of ADVANTG

After obtaining the fission rate tallies and creating an appropriate SDEF card, the MCNP problem formulation is now only missing a final tally. The tallies are setup here to capture the maximum detail and understanding of what is happening at the outer aperture of the

NEBP. First, the outer face of the NEBP is used in a surface tally (F2) to find the fluence departing. This tally was divided spatially using different concentric cylinders, providing information on seven regions of the NEBP as shown in Fig. 3.10. The first three are all equivalent-area tallies within the collimator itself. This will provide an understanding of the radial distribution with the main beam. The fourth segment was the exterior face of the aluminum collimator. Then, the last three were equivalent area tallies within the borated polyethylene collimator. This would give an idea for the quantity of neutrons that managed to escape the main beam but still contribute to detectors, dose, etc., outside of the beam.

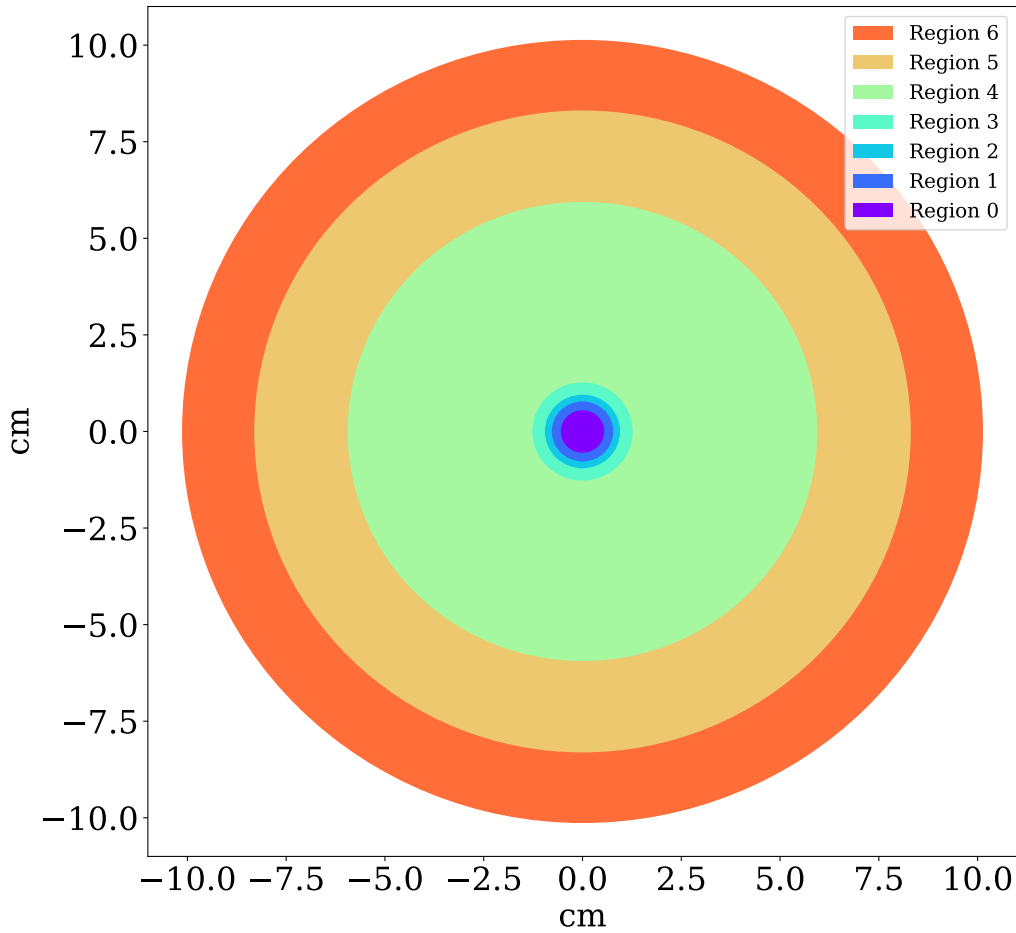


Figure 3.10: *The radial regions used for tallying.*

Then, both angular and spectral groupings were used to bin each of these regions. For the angle, a custom, fine, 109-group structure was used. The first 20 bins were tenth degree increments between 0° and 2° , where 0° refers to a partition edge on the beam's main axis.

Following that, a resolution of one degree increments were used between 2° and 90° . All of the backward fluence was captured by one bin between 90° and 180° .

For the energy structure, the scale252 group structure was used. This medium-resolution structure was selected because of the trade off between wanting to capture finer spectral details and the run-time to resolve statistics.

With the tally now in place, MCNP was run, first without any variance reduction (VR). Unsurprisingly, only a handful of the several million simulated particles survived to the end of the NEBP and were captured by the tally. It became quickly apparent that this problem needed of some form of VR to be able to achieve decent tally results in any reasonable amount of time.

For this application, the code ADVANTG was select to provide the VR needed to do the transport. ADVANTG, the Automated VArIaNce reducTion Generator, generates weight window parameters for a fixed-source, continuous-energy MCNP problem²¹. This is accomplished by solving the 3-D discrete ordinates neutron (or photon or coupled neutron-photon) transport problem using the Denovo package. The adjoint solution obtained in this way is used to generate the weight window mesh to be used in the MCNP problem. ADVANTG also creates biasing parameters for the source. Both the weight windows and biasing parameters work together to provide a significant reduction in computational time for obtaining accurate tally results on difficult transport problems. In the case of the NEBP, the large source-to-tally distance, small tally solid angle, and large amount of present moderating and absorbing materials present a challenging transport problem, but one that ADVANTG can prove extremely useful to when applied.

However, there were first a few issues that needed to be addressed with the formulation of the transport problem before ADVANTG could be used effectively. Recall that the NEBP aligns 50° azimuthally from the main x -axis. Denovo²² solves problems on a 3-D fixed mesh, meaning that capturing the details of the NEBP will be counterintuitive, as the axis of the beam does not directly lie along one of the axes. In addition to this, the full transport problem requires a large amount of memory to solve, which became a limiting factor when selecting ADVANTG parameters. Because of these issues, the model needed to be rotated

so that way the beam could lie on axis and the mesh resolution could be altered in a way that fully captured the effects of the beam and collimator while not wasting memory solving for fluxes in the structural material outside of the beam. Alterations were then made to the input-generating python scripts to allow the user to rotate the entire model by altering a single parameter. The model was then rotated 50° clockwise so the NEBP lied on the positive x -axis. The KCODE problem was rerun to verify the rotated model still produced the same fission rates and that no biases were introduced. The results from this verification did match the original fission-rate tallies.

Table 3.2: *Input parameters used for ADVANTG.*

model	mcnp
method	cadis
outputs	mcnp
mcnp_input	ksun.inp
mcnp_tallies	11
mesh_refinement	mcnp
mesh_x	-55 -22 22 184 368
mesh_y	-55 -22 -12 -2 2 12 22 55
mesh_z	-35 -16 -11 -5 0 35
mesh_x_ints	17 25 25 25
mesh_y_ints	75 14 17 22 17 14 17
mesh_z_ints	17 14 35 14 17
anisn_library	bplus
denovo_pn_order	4
denovo_quad_num_azi	15
denovo_quad_num_polar	12
denovo_x_blocks	8
denovo_y_blocks	8
denovo_z_blocks	1

ADVANTG was then run with the NEBP problem. The parameters selected are shown in Table 3.4. Several of the parameters are self-explanatory, but the non-intuitive ones will be discussed here. First, `cadis` refers to the method used to solve the transport problem. `mesh_x`, etc. refer to the major planes where the resolution of the mesh changes and `mesh_x_ints`, etc. refers to the number of elements between two planes in the mesh. This mesh was selected to focus on the core and NEBP, so the region surrounding the beam is

high resolution and the regions outside of the core and beam are low resolution. The `bplus` library was selected because it contained all of the materials used in the model and had a relatively large number of groups (47). The degree of anisotropy modeled is adjusted by the `denovo_pn_order`, which was selected to be 4. A higher order in this case would increase the runtime, but not necessarily produce a more accurate solution for this particular problem. The parameters, `denovo_quad_num_azi` and `denovo_quad_num_polar` refer to the number of angles used by the problem. Because the problem has a high angular dependence (a beam of neutrons streaming down a long collimator), these were selected to be higher than the defaults. Increasing this parameter also helped to reduce ray effects present within the solution. Finally, `denovo_x_blocks`, etc. refer to the parallelization scheme. The computer used for this calculation had 64 cores available, so these values 8, 8, and 1 all multiply to that value. Using these parameters, ADVANTG was run and biasing parameters and a weight window input file was produced.

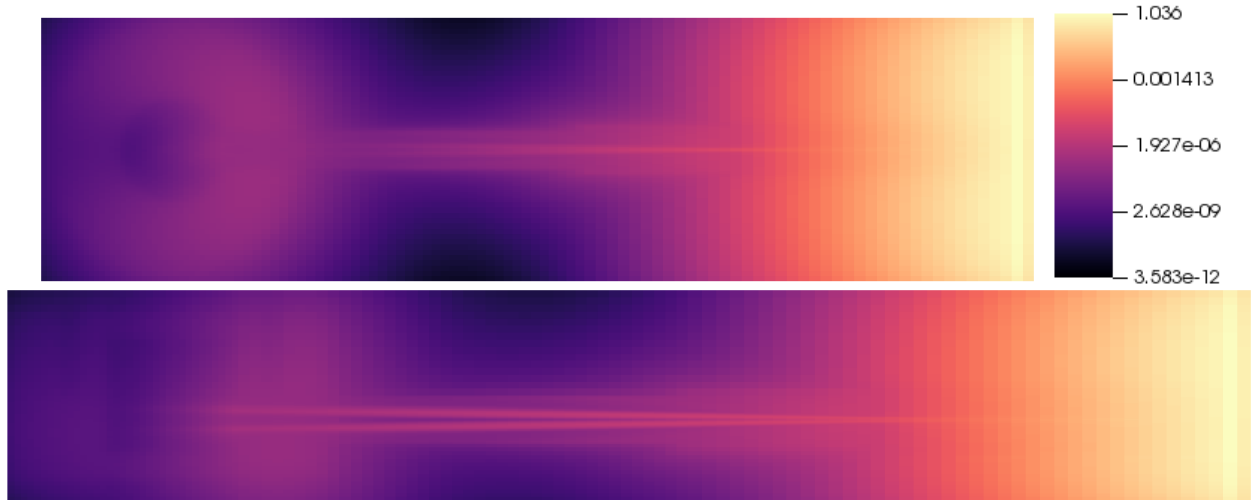


Figure 3.11: *XY (top) and XZ (bottom) slices of the adjoint solution for group 6 (4.9659 - 6.0653 MeV) produced from ADVANTG. Units here are considered arbitrary as the weight window values produced from this solution will be normalized in the MCNP problem.*

As an example, the adjoint flux solution for group 6 (4.9659 - 6.0653 MeV) can be seen in Fig. 3.11. This solution allows us to see how the weight windows will be defined. Regions with a high value have high "importance," i.e., neutrons in these regions are more likely to contribute to the response of interest. As a particle moves from a region of low importance

to one of higher importance, it is split into two or more identical particles of reduced weight such that the total weight is preserved. This splitting process leads to a greater number of particles reaching the detector and, hence, lower uncertainties. The geometry of the core and beam are roughly apparent in the figure. The side of the core closest to the beam is weighted higher than the side furthest, since neutrons born there are most likely to make it to the tally. The highest values occur closest to the tally to ensure much splitting, and therefore, more particles tracked in the region of the tally. In the reverse direction, some particles are killed ("rouletted"), while others are kept alive with increased weight, again, so that the total weight, on the average, is preserved. With this game of splitting and rouletting, the overall computational effort of moving particles is focused on regions of phase space of the highest importance to the response of interest.

3.5 Tally Results

This section contains several plots and qualitative discussion on the three dimensions (Region, Angle, and Energy) of tally data that were generated from the MCNP model of the NEBP.

Shown in Fig. 3.12 is the differential flux as a function of energy. This has been integrated over both angle as well as the seven tally regions. The spectrum resembles that which one would find within a reactor core. Centered around 1 MeV is the Watt fission distribution, the intermediate region is comprised of $1/E$ slowing down neutrons, and peaking around 3×10^{-8} MeV is the Maxwellian distribution. This is likely because the line-of-sight neutrons streaming from the core are the primary component to this flux tally. The borated polyethylene within the collimator is a very strong absorber and also very long, so it is unlikely that moderated neutrons escape and manage to contribute to the tally.

There are also two regions seen in this plot that are heavily affected by statistical noise, the slower half of the $1/E$ region and the slow tail of the Maxwellian distribution. This is because there is such a low probability that a neutron at that energy will be tallied. Specifically, within the scale252 bin structure, there are several narrow bins within the $1/E$ region, and tally probability is proportional to the bin width times the true flux, therefore, it

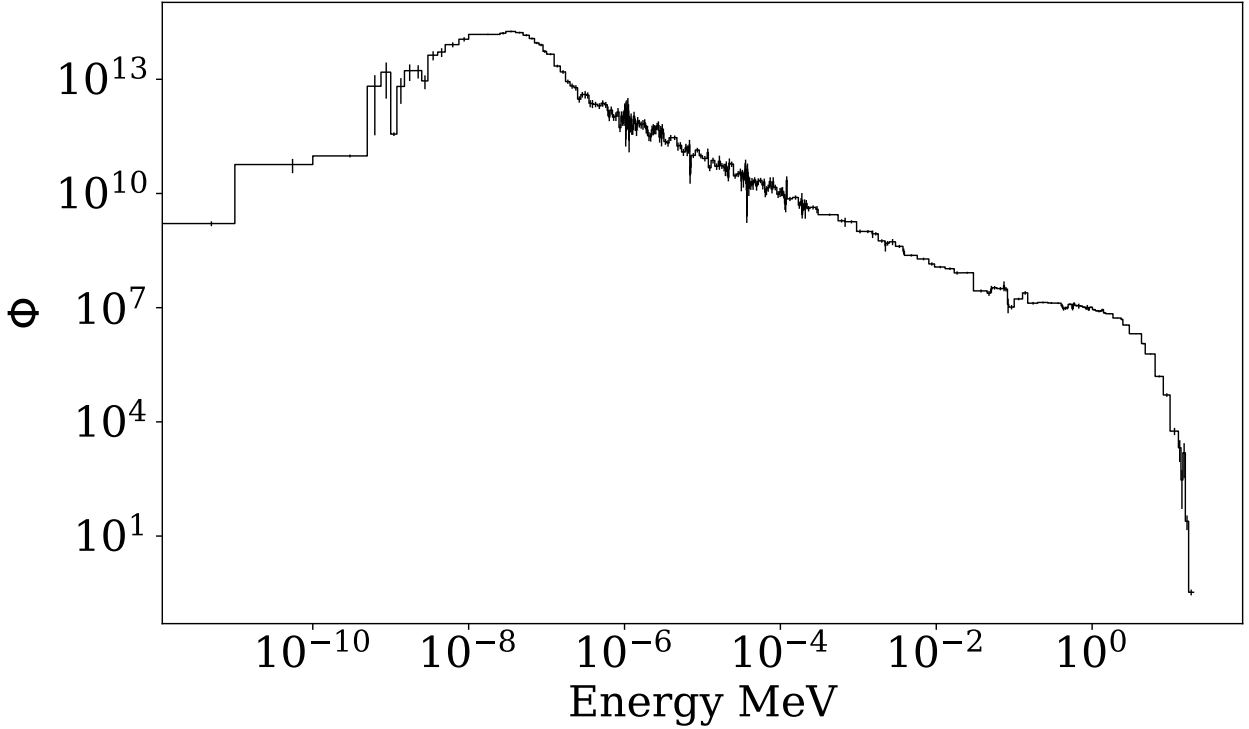


Figure 3.12: *The differential flux as a function of energy.*

is reasonable to see high errors within that region. The flux in the tail end of the Maxwellian distribution is also very low, causing the poor resolution within that region. These could ultimately be resolved via simulation of more particles.

The energy-dependent flux can be seen broken down into its separate regional tallies within Fig. 3.13. Recall, regions 0-2 are equal-area tallies within the beam, region 3 refers to the aluminum collimator, and regions 5-7 are equal-area tallies within the borated polyethylene. This plot gives us a first glance at comparing the magnitudes within each region. Over three orders of magnitude lie between the highest and lowest regional fluxes. The noise problem Fig. 3.12 is also amplified here, as separating the tallies by region further slows individual tally bin convergence. However, it is still possible to draw meaningful conclusions from this plot. First, apart from statistical noise, there is no obvious difference between the spectra within the in-beam regions. It is in the aluminum where the flux begins to deviate, being separated both by magnitude and also exhibiting a flatness within the fast region. Differences seen in the borated polyethylene regions are much more obvious, as the spectra are clearly

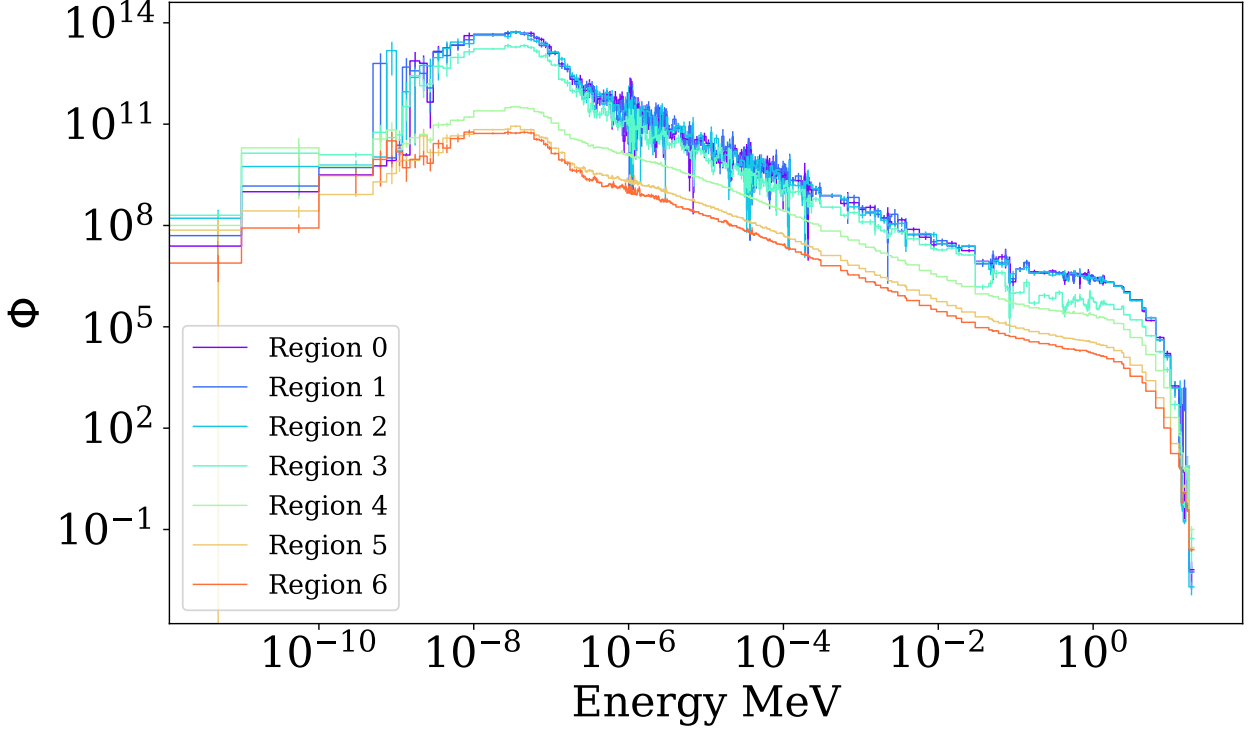


Figure 3.13: *The differential flux plotted against energy and broken into each tally region.*

moderated variants of those in-beam. The farther from the center of the beam, the more flat the fast region becomes. The $1/E$ region begins to demonstrate some curvature, which is likely in relation to the absorption and scattering cross sections of the borated polyethylene. The Maxwellian distributions within these sections are also flattened and exhibit much less drastic tapering in the slowest regions.

The flux as a function of angle can be seen in Fig. 3.14. Right away, it is apparent that the collimator is effectively constricting the neutron beam. The fluxes in the plot span over eight orders of magnitude. This incredibly sharp upturn at the right is due to the line-of-sight neutrons again being the largest contributor to the flux. Outside of that, the rest of this flux is governed by a smooth exponential tapering caused by scattering within the collimator.

Seen in Fig. 3.15 is a closer look at the forward peak in the angular flux. Within cosine values of 0.9999 and 1.0000 a jump of over five orders of magnitude occurs. This allows us to comment on the relationship between scattering and absorption within the collimator. The bounds on the sharpness of the peak match the maximum line-of-sight angle within the

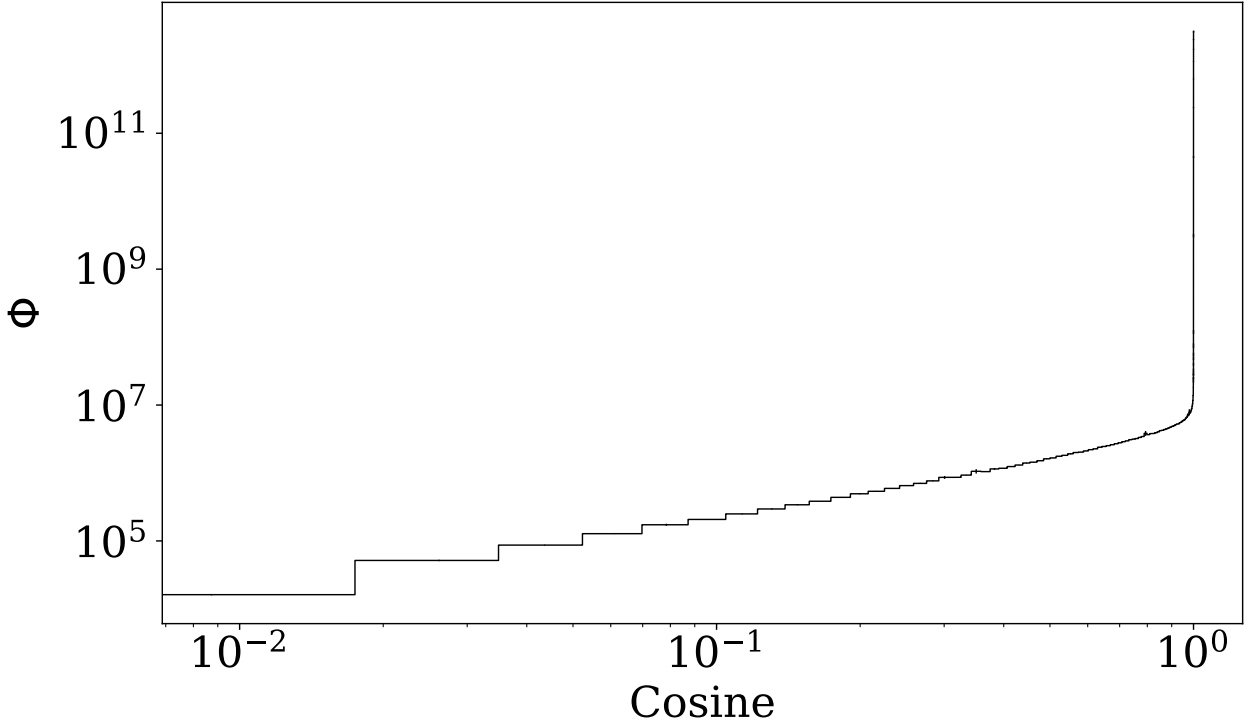


Figure 3.14: *The energy-integrated flux as a function of angle.*

aluminum tube in the beam. The neutrons within this peak are managing to stream down the collimator from the core without interacting on the way. This suggests that absorption dominates interactions within the collimator. If scattering was more likely, this peak would be much smoother, but, instead, the neutrons entering the collimator are being removed.

Here, in Fig. 3.16 is the angular flux broken into tally regions. Similar to the energy dependent flux plotted above, there is little deviation that can be seen between regions 0-2, although there is slight differences in the magnitudes. The aluminum section, section 3, exhibits much more isotropy, as the flux seen on this plot appears much higher than the others, it is the first non-void material encountered by the neutrons. This anisotropy is still much higher in the borated polyethylene sections. Each region has a peak located close to 1.000; however, it is not always the highest at 1.000. This is likely due to the single or double scatterings that have to happen near the end of the beam to move a neutron to those regions.

In Fig. 3.17 is the detailed view of the data from Fig. 3.16. This shows greater detail on the peak specifics of the angular fluxes. The in-beam regions are all forward peaked,

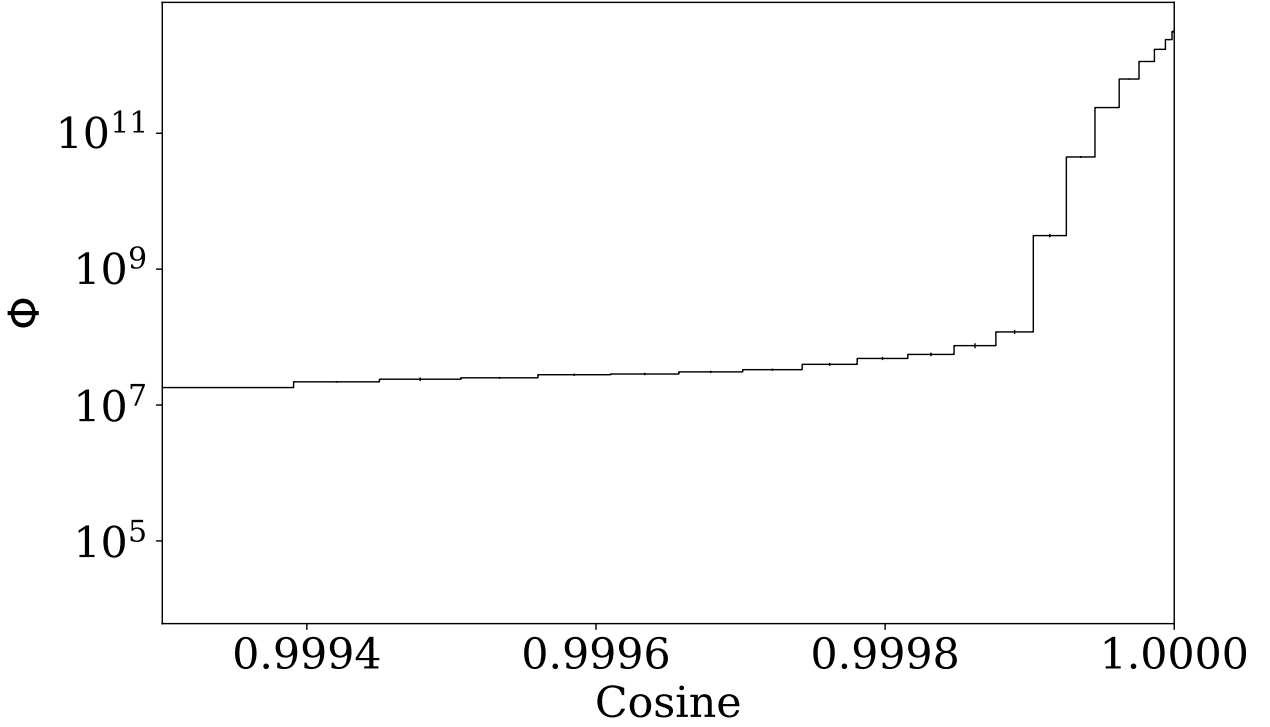


Figure 3.15: *A detailed view of the NEBP angular flux.*

with the highest bin being at $\cos = 1.000$; however, the distribution tends to broaden when moving outward. This trend continues in the aluminum section. In region 4 in the borated polyethylene, this peak appears slightly off axis, which is related to the scattering or “edge-clipping” effects that would transport a neutron to that region. In this view, regions 5 and 6 appear isotropic, likely because any neutron to encounter either of those tally planes would have scattered several times, effectively diffusing the spectrum.

Here, finally, we can see a direct comparison of the fluxes within each tally region in Fig. 3.18. There is very little difference between the magnitude of the tallies in the beam. Then, moving outward through each region in the collimator, we see a tapering that leads to a difference of two orders of magnitude in the flux from the innermost to the outermost region.

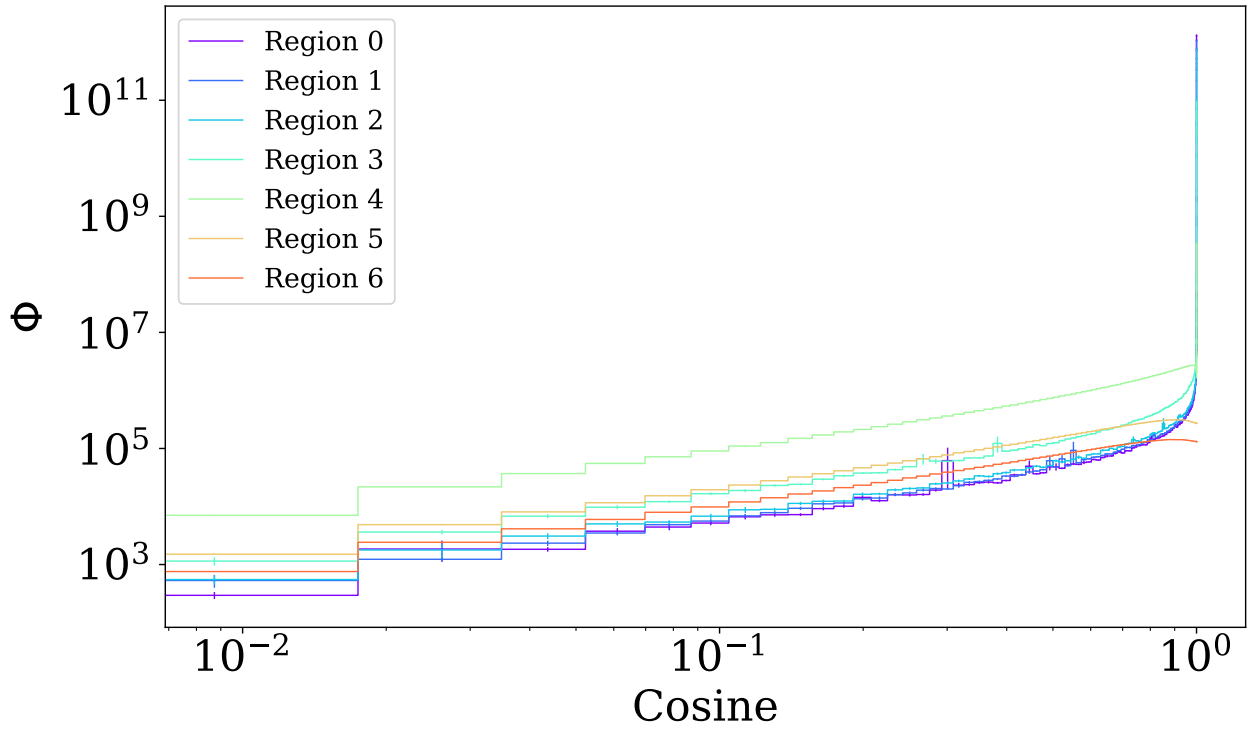


Figure 3.16: *The NEBP radial flux broken into tally regions.*

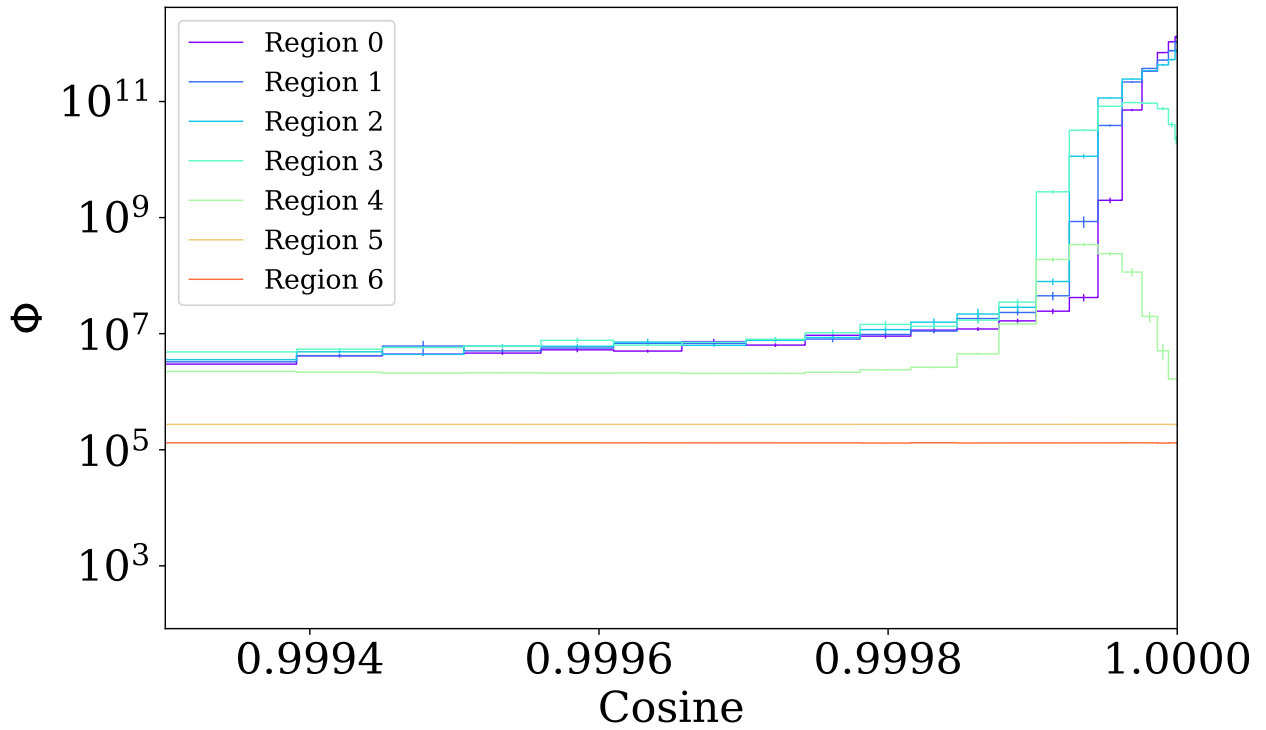


Figure 3.17: *A detailed view of the angular flux broken into tally regions.*

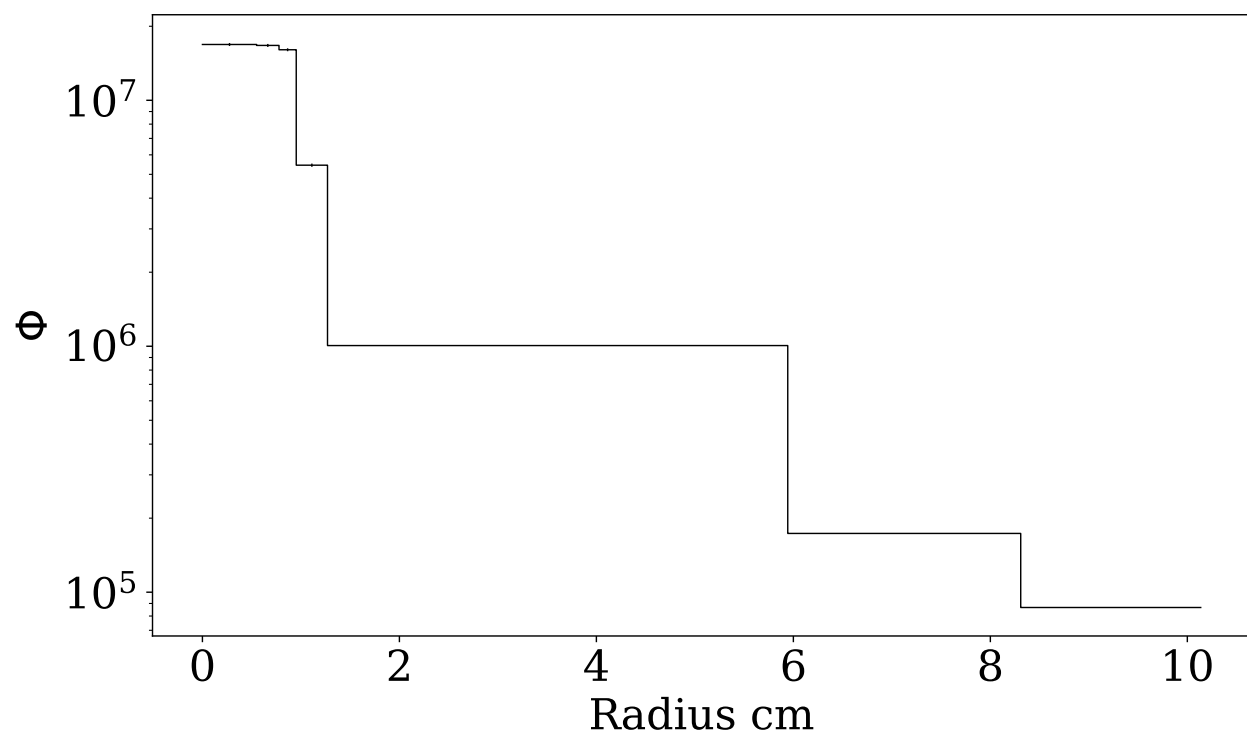


Figure 3.18: *The NEBP flux as a function of tally region.*

Chapter 4

Experimental Validation

4.1 Modeling

4.1.1 NEBP Decoupling

Before conducting any flux measurement of the beam port, the experimental setups were first modeled in MCNP¹⁹. The purpose of this is twofold: (1) the model provides detector responses to aid in the selection of experimental parameters and a metric for the accuracy of initially measured results, and (2) response functions are generated for the devices that are ultimately used in the unfolding of the final spectrum. However, due to the (computational) size of the full reactor model, it would be very expensive to generate these experimental responses and response functions. Also, the beam port flux and not the in-core spectrum is what we desire to measure, so the response functions would not relate back to the proper flux using the full model. Necessarily, a decoupled model of the beam and experimental configurations must be used for this portion of the simulation. This will simplify the geometry and transport significantly, allowing for reduced computation times and also for the ultimate unfolding of the correct neutron spectrum. As to the validity of this decoupling, as long as the full surface current leaving the outer boundary of the full model is captured in the decoupled model, then this approach is neutronically equivalent to a non-decoupled approach. In our case, as indicated by the flux results from the previous chapter, the vast majority of departing

neutrons leave the beam through the collimator void. The tallies include a large area around this beam, too, to account for any neutrons that escape the beam. This flux value decreases rapidly further from the center of the beam, and so it is believed that any neutrons that escape the collimator, leave through the structural material around the collimator, and still manage to interact in any detector will contribute in a statistically insignificant way.

The decoupled model includes the 8" outer portion of the NEBP, which would account for any backscattering from either detector. The tally from the full model was converted into a steady-state source placed at the outer face of the NEBP. The collimator void and space outside of the NEBP contain universes that allow different detectors to be swapped in and out of the model. The entire decoupled model is wrapped in python to allow for variation in input parameters and for the easy extraction and processing of output data.

4.1.2 Gold Foil Tube

The gold foil tube is a kind of neutron spectrometer. In design, it is principally based on the bonner sphere spectrometer. Gold foils, which preferentially absorb thermal neutrons are spaced with sections of high density polyethylene (HDPE). HDPE is a strong neutron moderator, meaning that the response of each foil will be somewhat independent of the others, since neutrons are slowing down and being absorbed between foils. This design also is advantageous in that it solves issues with beam alignment and only requires a single irradiation to generate multiple responses.

The gold foil device can be seen in Fig. [4.1](#). The tube is comprised of three separate materials, the gold foils, the HDPE separators, and the aluminum containment tube. Each of the gold foils are 5 mm in diameter and 0.1 mm thick. The HDPE sections were 1 in thick cylinders. A 1 mm depth, 5 mm OD hole was drilled into the front of each section where the gold foil was situated. All of these sections were sandwiched together and encased in 0.75 in OD, 1/24 in thick aluminum tubing. Aluminum was chosen because of its relative insensitivity to neutrons. The whole apparatus was comprised of 12 HDPE pieces and foils, with the first, exposed foil being situated 1 in from the start of the tubing.

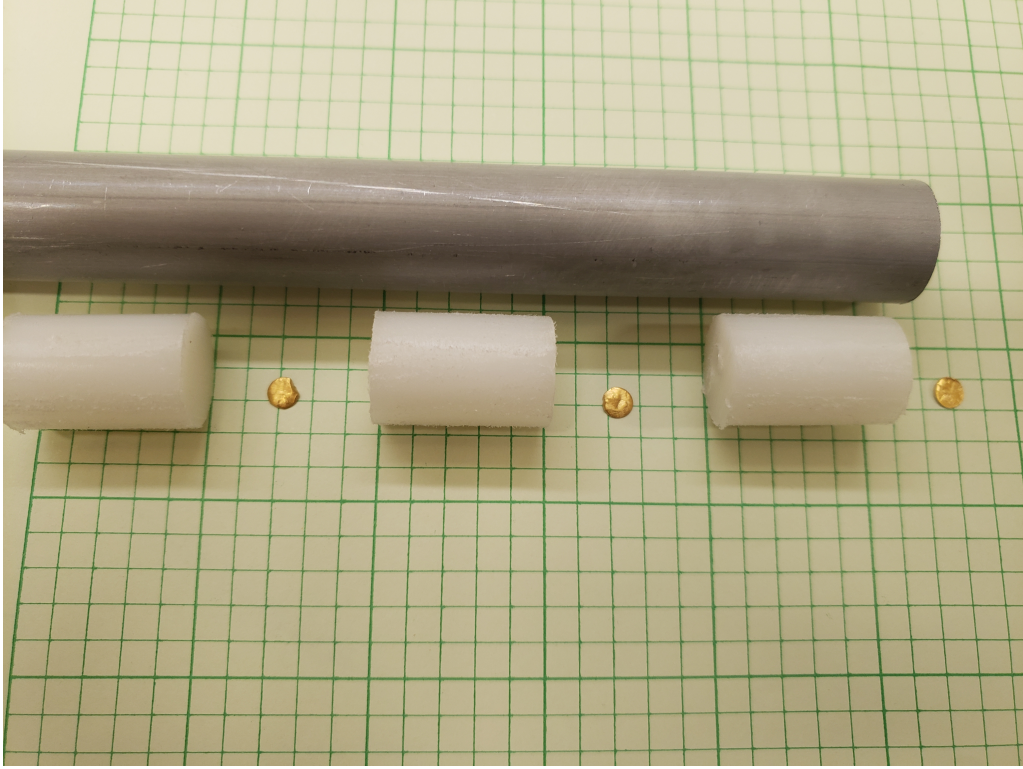


Figure 4.1: *The gold foil tube (disassembled).*

This device was modeled in MCNP, where **F4** flux tallies, folded with the $^{198}\text{Au}(n,\gamma)$ cross sections were used with an SCX card to produce the response functions.

4.1.3 Bonner Sphere Spectrometer

A second, active detector was also used in the spectrometric section of the NEBP analysis, too, the Bonner Sphere Spectrometer (BSS). The BSS, while explained more thoroughly in Chapter 2, use a series of plastic spheres surrounding a LiI detector to produce independent neutronic responses.

The main feature of the device was the $4 \times 4 \text{ mm}^2$ detection crystal located at the center of the sphere, although smaller details were also captured with the model, including the photomultiplier tube and some of the detector casing material. The device was positioned a distance of 28 in from the NEBP aperture. Response functions were generated for the bare response, as well as the 2, 3, 5, 8, 10, and 12 in spheres. The **F4** flux tally over the detection

crystal was folded with the (n,t) reaction cross section and an SCX card was again used to produce response functions.

4.1.4 Response Functions

Post processing for the gold foil tube and BSS response functions is relatively straightforward. The response function value for energy j , R_j , is multiplied by the number of energy groups, n_{groups} and then converted from b to cm^2 . This is because the SCX card, which allows a user to tally based on what source distribution bin a particle was born in, will naturally be weighted by the probability of a particle being born in that bin. To generate the response functions, all bins were considered equiprobable, which means that the true result requires the simple multiplying by the number of bins. The cm^2 conversion is because flux is generally understood in $\text{cm}^{-2} \text{ s}^{-1}$, so using response functions in these units allows one to avoid that conversion later.

$$R_j = R_{j,mcnp} \times n_{groups} \times 10^{-24} \quad (4.1)$$

The response functions for the gold foil tube are shown in Fig. 4.2. As the distance from the tube front increases, many interesting features form and decay. The first response very closely resembles the $^{198}\text{Au}(\text{n},\gamma)$ reaction because it is exposed to the source and backscattering contributes relatively little to this response. However, the addition of HDPE tapers the thermal tail of this response very quickly and shifts peak behavior towards the right. The four sections that follow the first actually have an even higher response than the exposed foil. Throughout the epithermal region, the relatively slow curvature doesn't seem much affected by the increasing foil distance, but in the fast region, a peak forms after the 2 in response, and then grows (relatively) and is pushed to the right as foil distance increases. Overall, the goal of producing response functions that are independent was met to at least some degree with this device design.

The trends in the BSS are similar to that of the gold foil tubes. The bare response is thermally dominated, but as HDPE spheres are added, a peak begins to form and then is

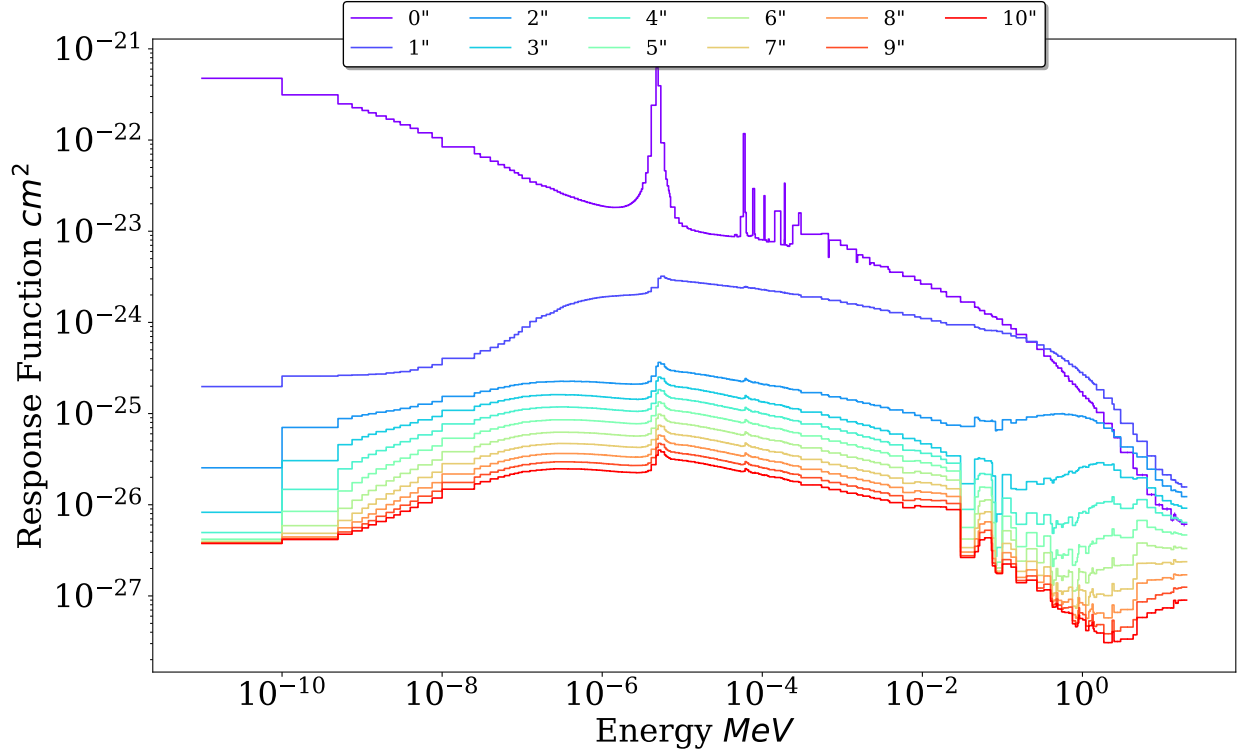


Figure 4.2: *The response functions for the gold foil tube.*

pushed towards the right. The main difference between the two devices, besides number of detectors, is their respective responses to thermal neutrons. Even though the thermal response is depressed in the gold foil tube, the BSS is almost completely insensitive to thermal neutrons, and there is a much larger difference between the fast and thermal response. This is likely due to the spherical geometry used. The BSS have lots of room around the detector for neutrons to thermalize, meaning that the fast peaks will grow to be very sharp. In the foil tube's case, a scattered neutron is much more likely to escape the device and not be absorbed by the foil, meaning that the contribution to the fast portion of the spectrum will be much lower, relatively.

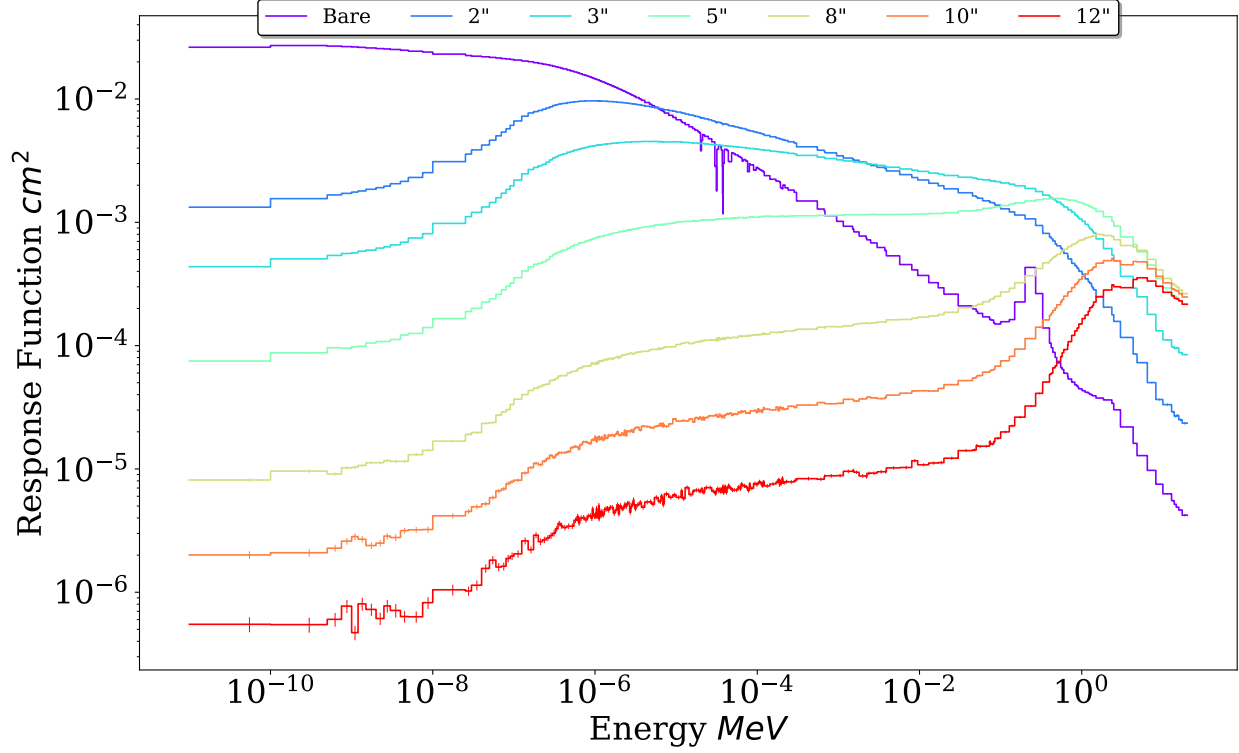


Figure 4.3: *The response functions for the Bonner Sphere Spectrometer.*

4.2 Experimental Procedures

4.2.1 Gold Foil Tube

First, the foils were individually weighed and those values are shown in Table 4.2.1. The device was then assembled as described in the modeling section. Some of the outer shielding around the beam port was removed to access the collimator. Initially, an attempt was made to insert the device into the collimator, but deformation in the aluminum portion of the collimator prevented insertion. The first few inches of the device were then ground down with a belt sander to reduce the diameter of the device slightly. The device was then inserted into the reactor. Reactor power was brought to 100kW(th). The reactor remained at power for approximately 2.5 hours. Following the irradiation, the reactor was shut down. A cooling period was necessary for the short-lived isotopes produced in the collimator and device to decay. Then, the device was removed, and from the device, the individual foils were removed and bagged. The foils were moved to an individual counting station, where they

were sequentially counted using an HPGE.



Figure 4.4: *A view of the gold foil tube inserted into the NEBP collimator.*

Table 4.1: *The masses of the gold foils.*

Foil ID	Position (in.)	Foil Mass (g) $\pm$ 0.1 mg
2	0	0.0320
13	1	0.0312
4	2	0.0320
5	3	0.0318
6	4	0.0319
7	5	0.0325
8	6	0.0326
9	7	0.0320
10	8	0.0318
11	9	0.0326
12	10	0.0317

4.2.2 Bonner Sphere Spectrometer

For the BSS irradiation, the device was positioned vertically in a manner that aligned the detection crystal with the center of the beam at a distance of 28". The detector was set to acquire for 300 s live time and then the reactor was brought to 1kW(th). The lower level discriminator setting was electronically adjusted to reduce the detector dead time, without

removing the ${}^6\text{Li}(n,t)$ peak. Spectra were acquired for the bare, 2", 3", 5", 8", 10", and 12" configurations. For the 12" sphere, a live time of 600 seconds was used to help achieve better counting statistics. Following the experiment, the reactor was shut down.

4.3 Postprocessing and Results

4.3.1 Gold Foil Tube

The postprocessing necessary for a foil activation experiment involves taking the measured activities, which are reported from the gamma spectrometry software, and backing out the saturation activities for each foil. Specifically, the saturation activity is defined as

$$A_{sat} = A_{meas} \frac{R_{meas}}{R_{sat} n_a K I_{rel}}, \quad (4.2)$$

where R_{meas} is the ratio that corrects for decay during measurement, R_{sat} is the saturation ratio which will be explained below, n_a is the number of sample atoms (equal to $\rho N_A / M$), K is the isotopic abundance, and I_{rel} is the relative intensity of the counted gamma ray.

The term R_{meas} is the ratio of the activity at the beginning of the gamma counting to the activity assuming the decay is constant during counting. It is defined by

$$R_{meas} = \frac{\lambda t_{meas}}{1 - e^{-\lambda t_{meas}}}, \quad (4.3)$$

in which λ is the decay constant and t_{meas} is the measurement time.

The saturation ratio, R_{sat} , is the ratio between the activity at the time of measurement and the saturation activity. Although this value generally assumes constant production during irradiation followed by a period of decay with no production, a method has been developed that captures transient behavior in reactor fluxes which will be detailed here.

The production and decay of radioisotopes is expressed mathematically through the bateman equation,

$$\frac{dN(t)}{dt} = C(t) - \lambda N(t) \quad (4.4)$$

where $N(t)$ is the number of radioactive nuclei in the sample, $C(t)$ is the production term in nuclei per second, and λ is the decay constant for the isotope in consideration. The change in radionuclides is simply the production minus the loss. The relationship between activity and radionuclides, $A(t) = \lambda N(t)$ allows us to convert Eq. (4.4) into the following,

$$\frac{dA(t)}{dt} = \lambda(C(t) - A(t)). \quad (4.5)$$

We can then divide each of the terms by C_{sat} which is the isotopic production at nominal power. This value is equal to the saturation activity, since the activity at $t = \infty$ for nominal power is equal to the production at that power.

$$\frac{dA(t)/C_{sat}}{dt} = \lambda\left(\frac{C(t)}{C_{sat}} - \frac{A(t)}{C_{sat}}\right) \quad (4.6)$$

Two facts allow us to simplify Eq. (4.6) into a useful form. First, because $A_{sat} = C_{sat}$, any $A(t)/C_{sat} = A(t)/A_{sat}$, so we can replace these terms with a defined term, $R_{sat}(t)$ which is the ratio of the activity at time t to the saturation activity. Second, although we do not have access to the $C(t)$ or C_{sat} terms since they require the (currently unknown) flux, we do have access to power data. Because isotopic production is approximately proportional to reactor power, i.e., $C(t) \propto P(t)$, we therefore have $C(t)/C_{sat} \approx P(t)/P_{nominal}$. We will replace this power ratio with the term $P_f(t)$, which is simply the time dependent ratio of the power to the nominal power, or

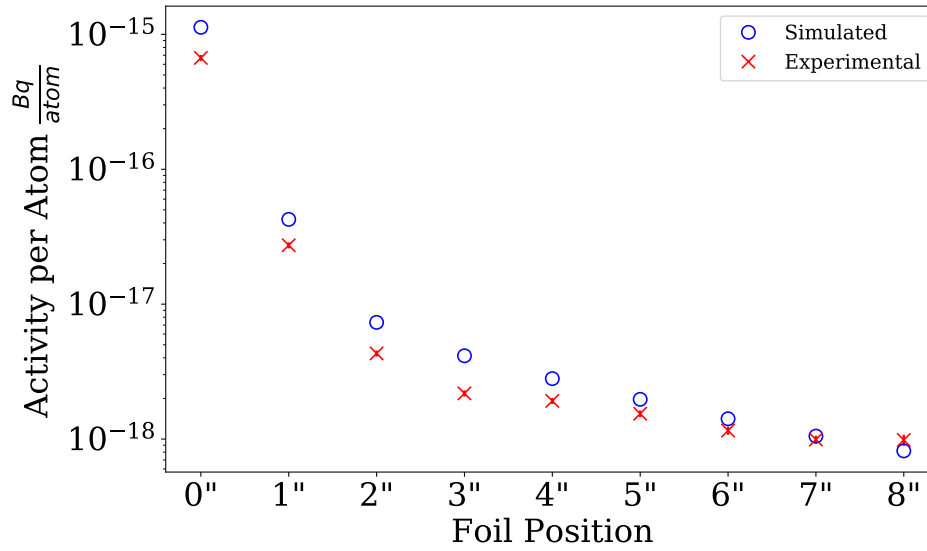
$$\frac{dR_{sat}(t)}{dt} = \lambda(P_f(t) - R_{sat}(t)). \quad (4.7)$$

This equation can be solved to find R_{sat} at the time of measurement using a numerical solver. Here, `scipy's odeint`²³ was used. The data for $P_f(t)$ is simply normalized strip chart data collected from an ex-core fission chamber.

A comparison of the simulated and experimentally determined activities is shown in Fig. 4.5.

Table 4.2: *The correction factors and foil activities.*

Position (in.)	A_{meas} (Bq)	R_{meas}	R_{sat}	A_{sat} (Bq)
0	$1483.73 \pm 5.03\%$	1.00089	0.02268	65468.28
1	$58.60 \pm 5.11\%$	1.00590	0.02262	2605.89
2	$8.92 \pm 5.18\%$	1.03805	0.02196	422.02
3	$4.06 \pm 5.32\%$	1.06581	0.0204	212.01
4	$2.58 \pm 5.17\%$	1.14023	0.01574	186.96
5	$1.60 \pm 5.63\%$	1.27976	0.01340	153.12
6	$0.72 \pm 7.03\%$	1.26894	0.00794	115.22
7	$0.37 \pm 6.78\%$	1.27340	0.00483	97.03
8	$0.17 \pm 9.35\%$	1.41587	0.00256	95.65

**Figure 4.5:** *A comparison of the simulated and experimentally obtained foil activities.*

The shape seems to agree well, but there is an apparent bias causing an overprediction in the simulated results. The ratio between the simulated and experimental results is an average of 1.41. This overprediction is much higher in the foils with less HDPe attenuating the beam, corresponding to more error in the prediction of the thermal regions of the flux.

4.3.2 Bonner Sphere Spectrometer

The BSS data was processed by taking the area under the Li reaction peaks and dividing that value by the live time of the detector. These values are presented in Table 4.3.2.

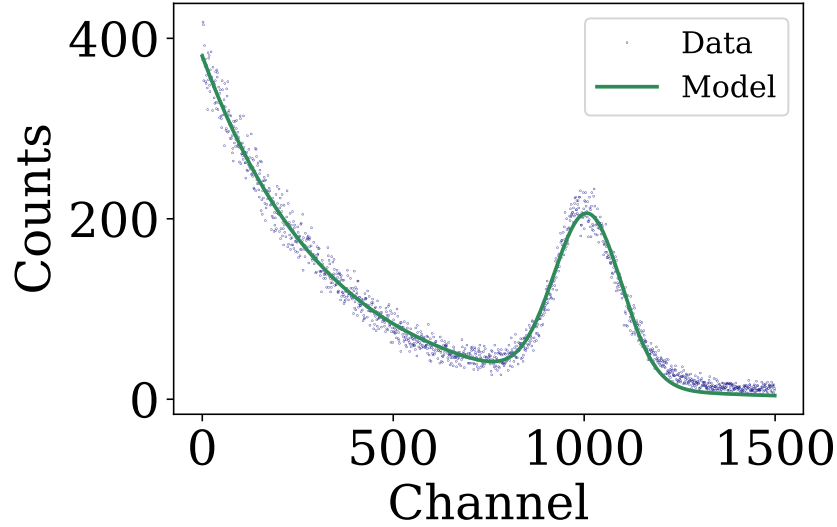


Figure 4.6: *The 8" sphere's gamma-ray spectrum and the model used to find the integral response.*

A typical spectrum obtained from a BSS is shown in Fig. 4.6. The area under the Li reaction peaks was found by fitting a model to the data. The model is shown in Eq. (4.8). It is the superposition of a decaying exponential function to fit the noise, and a normal distribution to fit the reaction. The C term in the normal represents the area under the curve, since the area of an unscaled normal distribution is unity.

$$c(E) = Ae^{-BE} + C \frac{1}{\sqrt{2\pi D^2}} e^{-(E-F)^2/2D^2} \quad (4.8)$$

Shown in Fig. 4.7 is a comparison of the simulated and experimentally obtained Bonner sphere responses. Similar to the foil tube results, the simulated responses overpredict the experimental results. Here, the average ratio of simulated-to-experimental results is 1.50. The smaller spheres and bare configuration show a greater overprediction, which was another trend seen in the foil tube results.

Table 4.3: *The BSS count rates from the experiment.*

Sphere (in.)	Count Rate s^{-1}
Bare	1626.76 ± 2.33
2	1062.40 ± 1.88
3	3891.13 ± 1.72
5	417.24 ± 1.18
8	139.72 ± 0.68
10	70.75 ± 0.49
12	37.43 ± 0.25

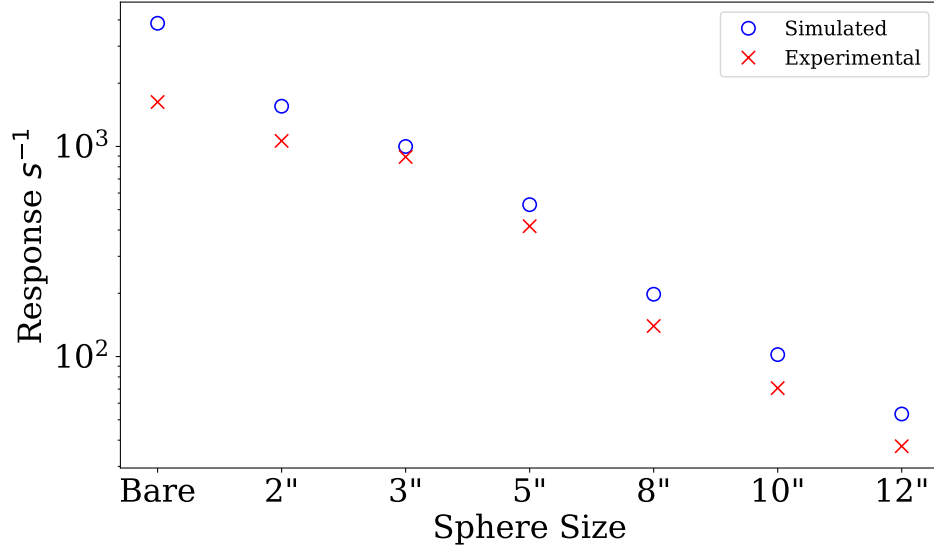


Figure 4.7: *A comparison of the simulated and experimentally obtained BSS responses.*

4.4 Spectral Unfolding

4.4.1 Methods and Parameters

The detector responses and response functions were unfolded with the methods described in chapter two to obtain a set of solution spectra. The responses from the foil tube and the BSS were first considered separately, then combined into a single vector to produce a third spectrum for each unfolding method. The Gravel, Doroshenko, and MAXED unfolding algorithms were used. For both Gravel and Doroshenko, the termination criteria was set at 50 iterations. For MAXED, the parameter Omega was set using the number of detectors for each dataset, 9 for the foil tube, 7 for the BSS, and 16 for the combined case. Then, the

MAXED unfolding was repeated, but with a scaled default spectrum. In this case, scaled means that the default spectrum was normalized so the average of the computed responses was equal to the average of the experimentally obtained responses before unfolding.

4.4.2 Results and Discussion

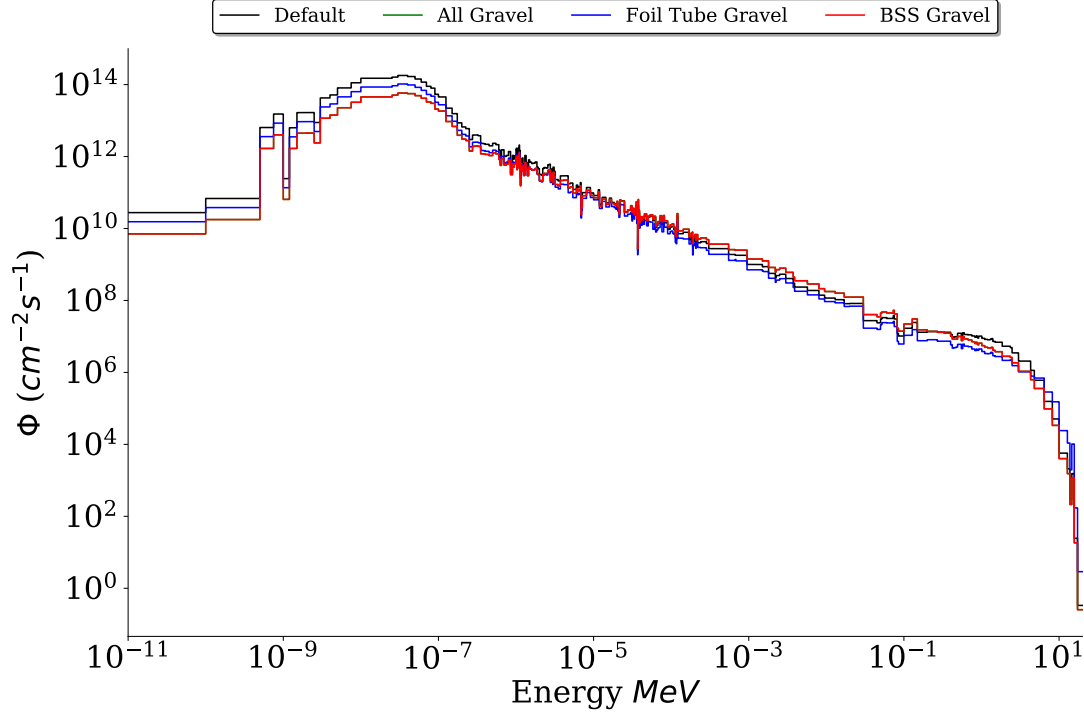


Figure 4.8: *The unfolded NEBP spectra obtained from using the Gravel method.*

Here, the Gravel results are presented in Fig. 4.8. As seen in the figure, the foil tube and BSS results show good agreement with one another. All of the major features from the default spectrum are preserved. Some differences appear between the two datasets, where the foil tube shows a slightly higher thermal flux and the BSS is larger through a portion of the fast region. The combined results cannot be seen as they are covered by the foil tube data. This is due to the fact that the gravel algorithm will base its manipulation on relative error, meaning that smaller error (more well-known) responses will affect the final results more, and the errors from the foil tube are much smaller than the BSS results.

Seen in Fig. 4.9 are the results from the MAXED unfolding. In the BSS and combined

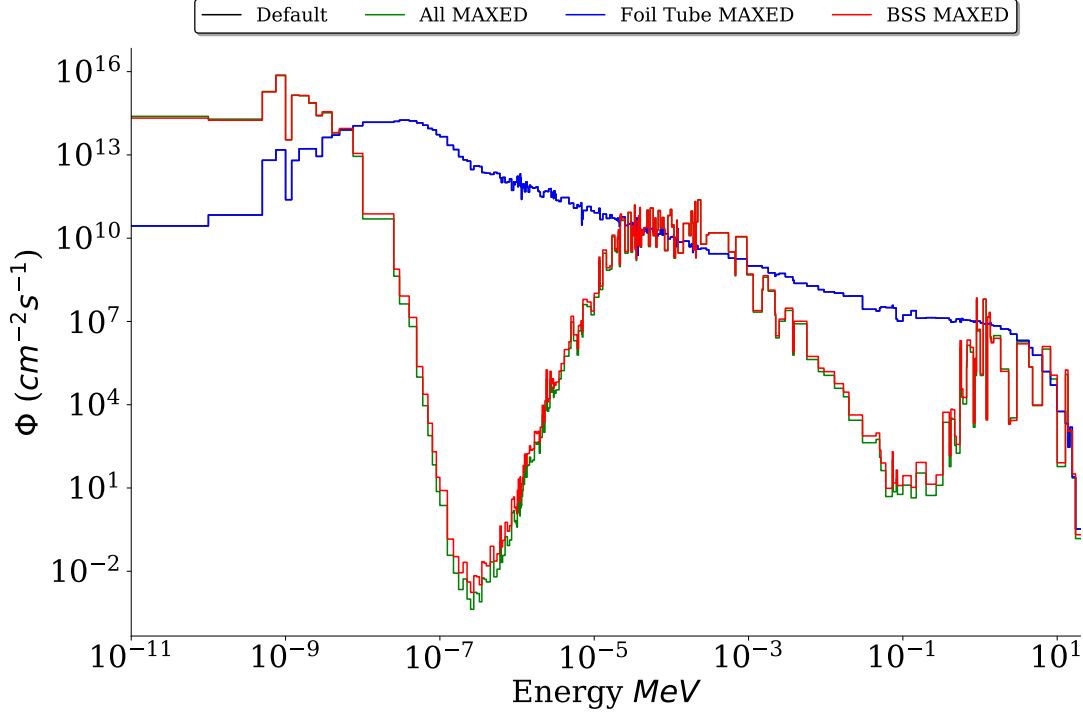


Figure 4.9: *The unfolded NEBP spectra obtained using the MAXED method.*

cases, the results produced appear very unphysical. This is likely due to the fact that the algorithm is trying to fit the data while retaining some values close to the defaults spectrum to maximize the cross entropy between the two spectra. The default spectrum remains unchanged for the foil tube case, which indicates that the default spectrum actually represents a decent fit of the data, although it is still not as close as in the Gravel case.

It was hypothesized that scaling the defaults spectrum would remove some of the unphysicalities of the solution spectra; however, the results in Fig. 4.10 still appear erratic and are very similar in characteristics to the unscaled versions. The BSS and combined cases are still rather unphysical, and the foil tube solution remains unperturbed.

The results in Fig. 4.11 are from the Doroshenko unfolding. Unlike MAXED, these results appear stable, like the Gravel data. Here, though, the differences between the datasets are more exemplified. This is most obvious in the foil tube dataset. The fast end of the spectrum appears flattened as it tries to fit some of the foil data whose responses lie in that end. Those responses have high error and likely this feature is due to a bias in the measurement, and

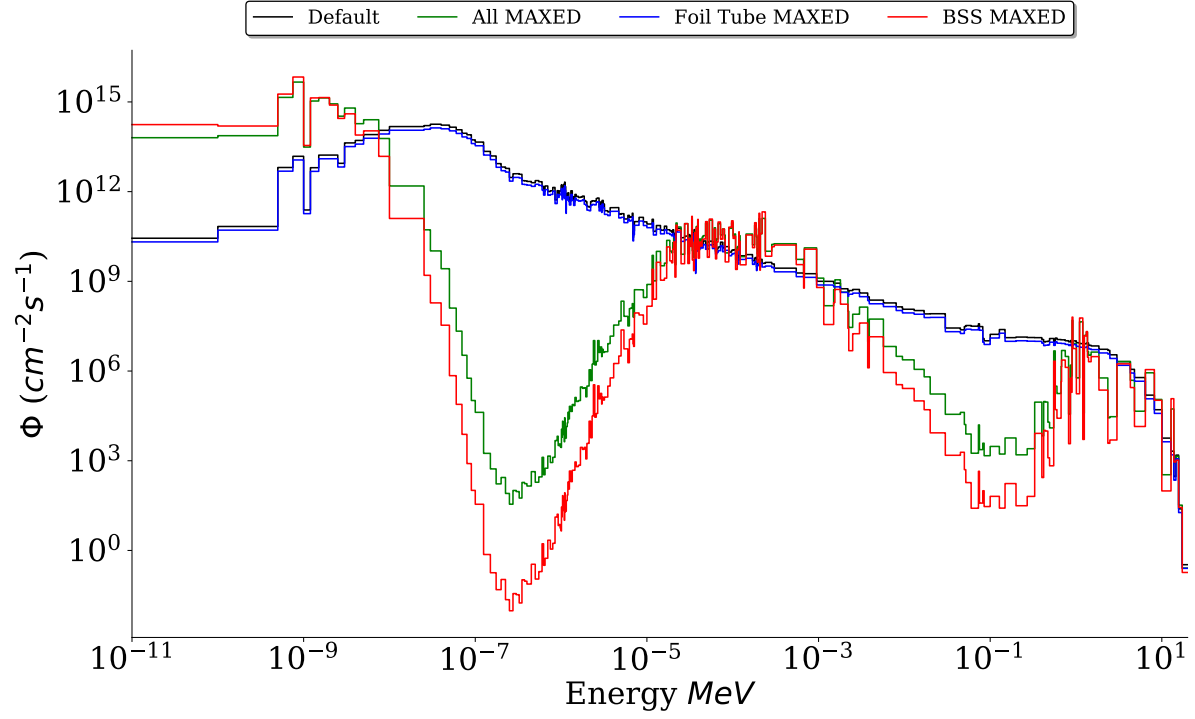


Figure 4.10: *The unfolded NEBP spectra obtained from scaling the default spectrum then unfolding with the MAXED method.*

not an actual spectral feature. The other two datasets unfolded into reasonable-appearing spectra, with some slight variation in both the center of the maxwellian distribution, as well as in the flatness of the fast region.

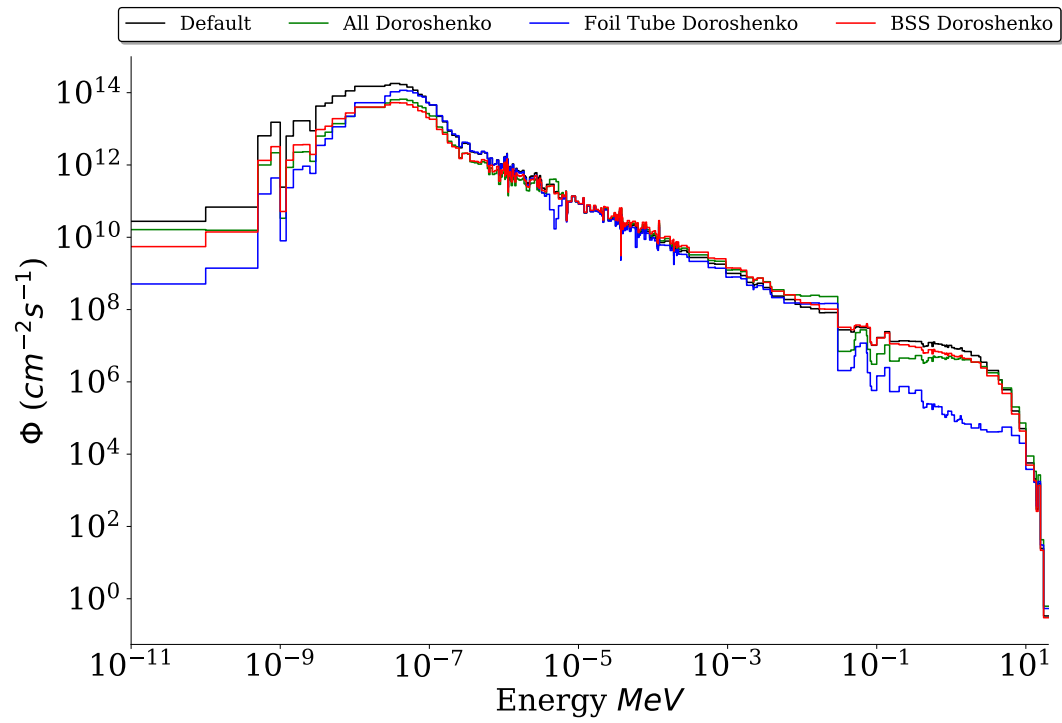


Figure 4.11: *The unfolded NEBP spectra obtained from unfolding with the Doroshenko method.*

Chapter 5

Conclusions and Future Work

5.1 Conclusions

First, through the assistance of the variance reduction methods described, it is possible to find the neutron spectrum for a collimated reactor beam port. A high resolution flux spectrum was obtained via Monte Carlo simulation for the KSU TRIGA Mark II's Northeast Beam Port featuring dimensional data in energy, angle, and the radial direction. Two separate experiments were conducted which collected data that demonstrated decent shape-agreeance with the simulated spectrum and differed in magnitude from the simulated work by similar values. Of the two experiments, one involved the creation of a novel type of neutron spectrometer which solved issues like beam alignment and allowed several different responses to be obtained using only one irradiation and one type of foil (gold). Successful deconvolution through Doroshenko Directed Divergence and Gravel using the experimental data led to a set of solution spectra that is consistent with a typical reactor spectrum shape, the original simulated spectrum shape, and the experimental data. The final shape of the spectrum does not match the original hypothesis of being predominately fast, but rather resembles an in-core neutron flux. The final flux is approximately monodirectional. Also, a significant majority of the flux departing from the beam port lives with the void of the collimator.

5.2 Contributions

Through this analysis, a much more complete picture of the NEBP has been obtained. This replaces the sparse existing data, which allows experimentalists to better understand the environment that their devices will be tested in. The set of experiments conducted provides a set of benchmarks for future simulations of the NEBP. This effort led to several additions to the KSU reactor model, making it more complete. Many of the steps taken in this analysis could be easily extended to other reactor beam problems, and the success found here allows other analysts the confidence to invest time attempting to simulate their beam in this way. The foil tube design used here is an easy to manufacture device that can be adapted and used as a new way to characterize a neutron beam.

5.3 Future Research

Following this work, the other three beam ports at KSU can be characterized in a similar way. The framework provided here will easily extend to those other beam ports and allow the analyses to be done much quicker. This includes the experimental methods utilized. Work can also be done to study the difference made on the spectrum by operating the reactor at different powers/temperatures, as well as the effect that different control rod positionings could have on the spectrum. The experiments conducted in this research should be redone regularly to provide a continually evolving understanding of the NEBP spectrum.

The foil tube design alone also provides a framework for more research to be conducted. There are many different parameters the device could be optimized for. This includes the spacer widths and materials, different or multiple foil types, the inclusion of cadmium or some other covering material in the design, and different overall geometric concepts.

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