Applications of A Unified Approach to Multiple Attribute Decision Making

by

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TABLE OF CONTENTS

		Page
LIST OF FIGURES		vi
LIST OF TABLES		vii
CHAPTER 1	Introduction	1
CHAPTER 2	Description of the MADM problem	3
	2.1 The basic MADM problem	3
	2.2 Formal MADM researches for cardinal preference of attribute given	4
	2.3 Difficulties of the formal DADM methods	14
CHAPTER 3	A unified approach to MADM	16
	3.1 Screening phase	16
	3.2 Preference ordering phase	18
	3.3 Aggregation phase	18
	3.4 Synthesis phase	21
CHAPTER 4	Problem 1: Industrial land use development problem in the Maasvlakte area	23
	4.1 Introduction	23
	4.2 Formulation of the problem	26
	4.2.1 Specification of the alternatives	26
	4.2.2 Evaluation attributes	28
	4.2.3 Attribute weights	36
	4.3 Solutions by the unified approach	40
	4.4 Discussion	55
CHAPTER 5	Problem 2: Singapore mass transit study	60
	5.1 Introduction	60

		Page
	5.2 formulation of the problem	62
	5.2.1 Specification of the problem	62
	5.2.2 Evaluation attributes	67
	5.2.3 Attribute weights	69
	5.3 Solutions by the unified approach	71
	5.4 Discussion	84
CHAPTER 6	Problem 3: The long-range water resources development study in Hungary	88
	6.1 Introduction	88
	6.2 Formulation of the problem	90
	6.2.1 Specification of the alternatives	90
	6.2.2 Evaluation attributes	92
	6.2.3 Attribute weights	96
	6.3 Solutions by the unified approach	96
	6.4 Discussion	100
CHAPTER 7	Problem 4: Selecting nuclear power plant sites in the Pacific Northwest	101
	7.1 Introduction	101
	7.2 Formulation of the problem	102
	7.2.1 Candidate sites	102
	7.2.2 Evaluation attributes	103
	7.2.3 Attribute weights	108
	7.3 Solutions by the unified approach	110
	7.4 Discussion	122
CHAPTER 8	Concluding remarks	124
	8.1 Discussion for each problem	124

Page

8.2 Evaluation of the four efficient MADM methods 126

8.3 A unified approach to MODM 130

8.4 Future study 133

REFERENCES 135

Appendix 1 Four alternative definitions of discordance index 139

Appendix 2 Detailed calculations of problem 1 142

Appendix 3 Detailed calculations of problem 2 161

Appendix 4 Detailed calculations of problem 3 173

Appendix 5 Detailed calculations of problem 4 182

Appendix 6 Computer programs produce the product-attribute matrix, relative closeness to the ideal solution, and aggregate dominance matrix for LAM, TOPSIS, and ELECTRE, respectively. 199

LIST OF FIGURES

Figure	Title	Page
3.1	A process diagram for a unified approach to MODM	17
4.1	Map of the Maasvlakte area	24
5.1	Map of Singapore	61
5.2	Extended bus service	63
5.3	Basic mass rail transit system	63
5.4	Alternate mass rail transit with Orchard Road	65
5.5	Busway system with bus lanes or with express service	65
5.6	Refined alternate mass rail transit with Orchard Road	68
6.1	Schematic map of the existing water resources system in Hungary	89
6.2	Layout of alternative water resources systems	91
7.1	Utility functions of attribute 2 of problem 4	107
7.2	The poset reflects the synthesized result of nine candidate sites without liquefaction potential	119
7.3	The poset reflects the synthesized result of nine candidate sites with liquefaction potential	121
8.1	A process diagram for a unified approach to MODM	131

LIST OF TABLES

Table	Title	Page
3.1	Example data (Preference orders by four methods)	19
3.2	The results of majority vote based on Table 3.1	19
4.1	Composition of the alternatives of the Maasvlakte intustrial sites	29
4.2	Data of individual activities of prolem 2	31
4.3	Employment shares of industrial activities in the industrialized area of the Rhine-delta area	35
4.4	Decision matrix of the Maasvlakte industrial land use problem	37
4.5	Alternative sets of weights of problem 1	38
4.6	Decision matrix of problem 1 with comparable scale	39
4.7	Weighted average values and preference orders by SAW method	39
4.8	Attributewise preference of problem 1	44
4.9	Preference orders of problem 1 by LAM method	44
4.10	Relative closeness and preference orders of problem 1 by TOPSIS method	48
4.11	Preference orders of problem 1 by ELECTRE mehtod	53
4.12	Preference orders of problem 1 by four MADM method, based on the equal weights.	53
4.13	Majority vote between alternatives based on Table 4.13	54
4.14	Aggregated preference orders of problem 1 by three aggregate techniques, based on the equal weights	54

		Page
4.15	Preference orders of problem 1 by the unified approach	56
4.16	A comparison of results of the Maasvlakte land use problem	56
4.17	Majority vote between alternatives based on Table 4.15	58
4.18	Aggregated preference orders of Table 4.15 and Table 4.17	58
5.1	Summary of preference of Singapore mass transit systems on social and environmental attributes	70
5.2	Weighting schemes of problem 2	70
5.3	Normalized decision matrix of problem 2	73
5.4	Weighted average values and preference orders of problem 2 by SAW method	73
5.5	Attributewise preferences of problem 2	74
5.6	Preference orders of problem 2 by LAM method	74
5.7	Weighted normalized decision matrix of problem 2	77
5.8	Preference orders of problem 2 by TOPSIS method	77
5.9	Preference orders of problem 2 by ELECTRE method	82
5.10	Preference orders of problem 2 by the multidimensional scaling method	82
5.11	Preference orders of problem 2, based on the second set of weights, by the four efficient MADM methods as well as MDS method	83
5.12	Majority vote between alternatives based on Table 5.11	83

		Page
5.13	Preference orders of problem 2 By the unified approach	85
6.1	Decision matrix of problem 3	95
7.1	Probability assessments of attributes by sites	106
7.2	Decision matrix of problem 4	109
7.3	The decision matrix with comparable scale of problem 4	111
7.4	Preference orders of nine candidate sites without liquefaction potential by five MADM methods	116
7.5	Preference orders of nine candidate sites with liquefaction potential by five MADM methods	117
7.6	Aggregate preference orders of nine candidate Sites without liquefaction potential	118
7.7	Aggregate preference orders of nine candidate Sites with liquefaction potential	120
7.8	Preference orders of nine candidate sites obtained by MAUT and the unified approach	123
8.1	IBM 360/50 computational time for four efficient MADM methods	127

Chapter 1
Introduction

In everyday life, it frequently happens that many individuals and groups are party to decision-making involving alternatives. Typically, alternatives are described in terms of various attributes, which are measured in different units and often conflict with one another. Usually a clinical, rather than a scientific, approach is used to deal with multiple attribute decision making (MADM) problems. The methods and applications of MADM have been reviewed thoroughly in Hwang and Yoon (12).

Present-era MADM models have been widely applied in many fields, among them marketing research, economics, management science, and applied regional science, to help in the evaluation of alternatives and the selection of one from a feasible set. Unfortunately, these methods are usually developed in isolation from other methods and the interrelationships among them are not clear. Each MADM method evaluates the alternatives from its own point of view, but no one approach has proven superior under all circumstances. Therefore, a unified approach was first proposed by Hwang and Yoon(12) by means of using several MADM methods simultaneously.

The purpose of this study is to apply the proposed unified approach to four real-world MADM problems and to examine the results of this approach and the MADM methods which are employed in the unified approach.

In chapter two, the basic MADM problem and its common characteristics are described, four efficient MADM methods are reviewed, and the difficulties of applying a single method are discussed.

Chapter three reviews the proposed unified approach to MADM problems. The approach includes four phases: screening phase, preference ordering phase, aggregation phase, and synthesis phase.

Chapters four through seven presents four real-world MADM problems which are selected to illustrate the applications of the proposed unified approach.

Finally, concluding remarks and proposed future study are presented in Chapter 8.

Chapter 2

Description of the MADM problem

2.1 The basic MADM problem

A multiple attribute decision making (MADM) problem is to select an alternative from a set of candidates based upon a set of attributes (criteria). The MADM problem may be expressed concisely in a matrix format. Let m be the number of alternatives, and n the number of attributes according to which each alternative is judged. A decision matrix is a (mxn) matrix whose elements x_{ij} indicate measure of achievement of alternative i with respect to attribute j. Thus, a decision matrix D will have the following form:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccccc} X_1 & X_2 & \cdot & \cdot & \cdot & \cdot & X_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdot & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & \cdot & \cdot & \cdot & \cdot & x_{2n} \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ x_{m1} & x_{m2} & \cdot & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix} \end{array}$$

where the rows $A_i = [\, x_{i1}, x_{i2}, \ldots, x_{in} \,]$ stand for alternatives and the columns $X_j = [x_{1j}, x_{2j}, \ldots, x_{mj}]^T$ for attributes. The problem facing the decision maker is to select an alternative such that the collection of attribute measures is preferable to

that of any other alternative. In general, MADM problems share the following common characteristics:

Conflict among attributes. To a greater or lesser extent, there exists some conflict among attributes. The decision maker would, of course, like to optimize each attribute, but it is rarely possible to develop one alternative which does so. Expansion of the industrial area is usually at the expense of increasing pollution; reducing unemployment may increase inflation; the control of one type of error will often increase another type.

Incommensurable units. The attributes of an MADM problem are measured using different units. The unit of one attribute may measure the additional jobs in man-years, and those of others may be expressed in dollars, acres, or tons of pollutants.

Intangibility. Some attributes might reflect psychological aspects such as aesthetic considerations, annoyance, pain, freedom, security, or other intangible qualities. To ignore such attributes might seriously distort an analysis. When dealing with MADM problems, the quantitative and qualitative effects are often considered simultaneously.

An MADM method is used to help the decision maker evaluate the MADM methods which have the above mentioned common characteristics.

2.2 Formal MADM researches for cardinal preference of attribute given

There has been considerable interest in recent years in the treatment of MADM problems. A variety of procedures have been developed by applied mathematicians, planners, economists, and others, to help in evaluation of alternatives and the selection of one from a feasible set. Each field has introduced methods for it own particular usage, so there exists a large amount of literature on this topic. Overviews of methods for multiple attribute decision making have been given by MacCrimmon (18), and Green and Wind (9). Recently, the various MADM methods were extensively reviewed and well classified, based on different forms of preference information, by Hwang and Yoon (12).

Because of limited space, only the four efficient MADM methods, selected by Hwang and Yoon, are reviewed as follows:

2.2.1 Simple additive weighting method

This method is one of the most simple and widely used methods of the multiple attribute analysis.

This method was first applied to select business investment policy by Churchman, Ackoff, and Arnoff (3) in 1957. The extensions and applications of this method can be found in several other publications like Fishburn (8), Schimpeler Grecco (25), and Khane (13), and Ulvila and Snider (27).

By multiplying the comparable elements with the corresponding preference weights and then summing these products over all attributes, the total score for each

alternative can be calculated as follows:

$$A_i = \sum_{j=1}^{n} w_j \, x_{ij} / \sum_{j=1}^{n} w_j \quad , \qquad \forall i \tag{2.1}$$

where w_j is the weight corresponding to attribute j, and x_{ij} is the outcome of alternative i with respect to attribute j with a comparable scale. If all attributes are treated as benefit criteria, the alternatives can be ranked in descending order according to the total scores.

2.2.2 Linear assignment method.

LAM was developed by Barnardo and Blin (1) for consumer choices among multi-attributed brands. This method was applied to deal with a brand-switching problem by Blin and Dodson (2) and found to compare favorably in predictive value of consumers' preference with other models. From the decision matrix, we can obtain an attribute-wise preference matrix. The attribute-wise preference matrix is expressed as fllows:

Rank	X_1	X_2	X_3
1st	A_1	A_1	A_2
2nd	A_2	A_3	A_1, A_3
3rd	A_3	A	

Then, a product-attribute matrix is defined as a aquare (mxm) nonnegative matrix whose elements π_{ik} represent the frequency (or number) with which alternative i is ranked kth. A

corresponding matrix with the set of weights $\underline{W} = [0.2, 0.3, 0.5]$ becomes

$$\Pi = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} 1st & 2nd & 3rd \end{array} \\ \begin{bmatrix} .50 & .25 & .25 \\ .50 & .20 & .30 \\ .00 & .55 & .45 \end{bmatrix} \end{array}$$

Each element π_{ik} of the product-attribute matrix is simply a measure of the concordance among all attributes on ranking A_i the kth. The larger π_{ik} indicates more concordance in assigning A_i to the kth overall rank. Hence the problem is to find A_i for each k , which maximizes $\sum_{k=1}^{m} \pi_{ik}$. An LP method is applied to solve this problem.

$$\max \sum_{i=1}^{m} \sum_{k=1}^{m} \pi_{ik} p_{ik} \qquad (2.2)$$

subject to

$$\sum_{k=1}^{m} p_{ik} = 1, \qquad i = 1,2,.....,m \qquad (2.3)$$

$$\sum_{i=1}^{m} p_{ik} = 1, \qquad k = 1,2,.....,m \qquad (2.4)$$

where π_{ik} is the $(i,k)^{th}$ element of the product-attribute matrix; p_{ik} is the $(i,k)^{th}$ element of permutation matrix P, i.e., $p_{ik} = 1$ if A_i is assigned rank kth, and $p_{ik} = 0$ otherwise.

The optimal permutation matrix P^* can be found by using linear assignment programming. Applying the P^* to $A = [A_1, A_2, \ldots A_n]$, the optimal ordering may be obtained.

2.2.3 TOPSIS method.

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was developed by Yoon (28). This method is based on the concept which considers the distances to both the ideal and negative-ideal solutions simultaneously by measuring the relative closeness to the ideal solution. The TOPSIS method will be described in the following steps:

Step 1. Construct the normalized decision matrix: The vector normalization technique is employed in this method to transform the different attribute units into a comparable scale, which allows comparison across the attributes. This procedure involves dividing the outcome of each attribute by the norm of the total outcome vector of the attribute at hand. If this normalized outcome is denoted by r_{ij}, then:

$$r_{ij} = x_{ij} \Big/ \sqrt{\sum_{i=1}^{m} x_{ij}^2} \qquad (2.5)$$

Step 2. Construct the weighted normalized decision matrix: This matrix, denoted by V, is obtained by multiplying each element r_{ij} with its corresponding relative weight w_j. The matrix V is equal to

$$V = R \cdot \hat{W} \qquad (2.6)$$

where $\hat{W}$ is a diagonal matrix of order n, with relative weights w_j as successive diagonal elements.

Step 3. Determine ideal and negative-ideal solutions: Let the ideal solution, A^*, and the negative-ideal solution, A^-, be

defined as:

$$A^* = \left\{ \left[\max_i v_{ij} \mid j \in J \right], \left[\min_i v_{ij} \mid j \in J' \right] \mid i=1,2,\cdots,m \right\}$$

$$= \left\{ v_1^*, v_2^*, \cdots, v_n^* \right\} \qquad (2.7)$$

$$A^- = \left\{ \left[\min_i v_{ij} \mid j \in J \right], \left[\max_i v_{ij} \mid j \in J' \right] \mid i=1,2,\cdots,m \right\}$$

$$= \left\{ v_1^-, v_2^-, \cdots, v_n^- \right\} \qquad (2.8)$$

where

J is $\{j = 1, 2, \ldots, n \mid j$ associated with benefit attribute$\}$

J' is $\{j = 1, 2, \ldots, n \mid j$ associated with cost attribute$\}$

Step 4. Calculate the separation measure: The separation of each alternative from the ideal is given by

$$S_{i*} = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^*)^2} \quad , \qquad i=1,2,\cdots,m \qquad (2.9)$$

Similarly, the separation from the negative-ideal is given by

$$S_{i-} = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2} \quad , \qquad i=1,2,\cdots,m \qquad (2.10)$$

Step 5. Calculate the relative closeness to the ideal solution: The relative closeness of A_i with respect to A^* is defined as

$$C_{i*} = S_{i-}/(S_{i*} + S_{i-}) \quad , \quad 0<C_{i*}<1 \quad , \quad i=1,2,\cdots,m \qquad (2.11)$$

It is clear that an alternative A_i is closer to A^* as c_{i*} approach to 1.

Step 6. Rank the preference order: Preference order can be obtained according to the descending order of c_{i*}.

2.2.4 ELECTRE method

The ELECTRE method (Elimination et Choice Translating Reality) was originally developed in France and subsequently applied by Guigou (10), Nijkamp (19), Roy (23) and others in various types of applied research. The ELECTRE method is based on a pairwise comparison of alternatives to examine both the degree of concordance among weights and the discordance differences among weight project outcomes. The concept of an outranking relationship is used in this method. The outranking relationship of $Ap \rightarrow Aq$ says that even though these two alternatives do not dominate each other, the DM accepts the risk of regarding A_p as almost surely better than A_q (24). The first and second steps of ELECTRE are the same as those of TOPSIS. The remaining steps are presented as follows:

Step 3. Determine the concordance and discordance sets: The concordance set c_{pq} of A_p with respect to A_q is composed of all attributes for which A_p is preferred to A_q, or A_p is equal to A_q. This set is defined as:

$$C_{pq} = \{ j | x_{pj} \succeq x_{qj} \} \qquad (2.12)$$

where the symbol $\succ$ denotes: 'preferred to'. The complementary

set is called the discordance set D_{pq} and can be represented as:

$$D_{pq} = \{ j \mid x_{pj} \prec x_{qj} \} \qquad (2.13)$$

where the symbol $\prec$, means 'not preferred to'.

Step 4. Calculate the concordance matrix: It is obvious that C_{pq} will contain more elements as A_p dominates A_q with respect to more attributes. The relative value of the concordance set is measured by means of the concordance index. This index is equal to the sum of weights for those attributes which fall into the concordance set, divided by the sum of all weights. Therefore, the concordance index c_{pq} between A_p and A_q is defined as:

$$c_{pq} = \sum_{j \in C_{pq}} w_j \Big/ \sum_{j=1}^{n} w_j \qquad (2.14)$$

Step 5. Calculate the discordance matrix: The discordance index measures the relative discrepancy between the weighted outcomes of two alternatives. The discordance index may be defined as:

$$d_{pq} = \frac{\sum_{j \in D_{pq}} | v_{pj} - v_{qj} |}{M} \qquad (2.15)$$

where M is defined as:

$$M = \max_{(p,q)} \left(\sum_{j \in J} | v_{pj} - v_{qj} | \right) \qquad (2.16)$$

Thus, M is defined as the maximum of the sum total of the

divergences among two alternatives. Three other alternative discordance index equations exist (see Appendix 1).

Step 6. Determine the concordance matrix: By defining a critical value c for the concordance indicies (in general, the average value of all concordance indices), one may state that A_p will only have a chance of being preferred to A_q if the associated index c_{pq} exceeds the critical value $\bar{c}$; that is to say, if

$$c_{pq} \geq \bar{c} \tag{2.17}$$

In this case, $\bar{c}$ can be defined as:

$$\bar{c} = \sum_{\substack{p=1 \\ p \neq q}}^{m} \sum_{\substack{q=1 \\ q \neq p}}^{m} c_{pq}/m(m-1) \tag{2.18}$$

Next, a Boolean matrix F can be constructed. The elements f_{pq} of matrix F can be defined as

$$f_{pq} = 1, \quad \text{if } c_{pq} \geq \bar{c}$$

$$f_{pq} = 0, \quad \text{if } c_{pq} < \bar{c} \tag{2.19}$$

Matrix F is called the concordance dominance matrix: each element 1 indicates a dominance of one alternative by another in the concordance relationship.

Step 7. Determine the discordance dominance matrix: A_p is more worthwhile with respect to A_q as the discordance index d_{pq} is lower. By defining a critical value $\bar{d}$, A_p can be assumed to receive a higher priority than A_q, if:

$$d_{pq} \leq \bar{d} \tag{2.20}$$

The critical value $\bar{d}$ can be calculated as

$$\bar{d} = \sum_{\substack{p=1 \\ p \neq q}}^{m} \sum_{\substack{l=1 \\ q \neq p}}^{m} d_{pq}/m(m-1) \tag{2.21}$$

The elements of g_{pq} of the discordance matrix G are determined as

$$g_{pq} = 1, \quad \text{if } d_{pq} \leq \bar{d}$$

$$g_{pq} = 0, \quad \text{if } d_{pq} > \bar{d} \tag{2.22}$$

The interpretation of G is analogous to that of F.

Step 8. Determine the aggregate dominance matrix: The aggregate dominance matrix E can be directly calculated as the intersection of the concordance dominance matrix F and the discordance matrix G. The elements e_{pq} of E satisfy the following conditions:

$$e_{pq} = 1, \quad \text{if } f_{pq} = 1 \text{ and } g_{pq} = 1$$

$$e_{pq} = 0, \quad \text{otherwise} \tag{2.23}$$

Step 9. Rank the preference order: If $e_{pq} = 1$, then A_p is preferred to A_q for both the concordance and discordance criteria. This gives the outranking relationship between A_p and A_q : $A_p \rightarrow A_q$. The outranking relationships can be illustrated by graphical representation and give the partial-preference

ordering of the alternatives.

2.3 Difficulty of the formal MADM methods

Different MADM methods involve various types of underlying assumptions, information requirements from DM, and evaluation principles. Two general choice rules were suggested by MacCrimmon (18), and Hwang and Yoon (12). The number of options may be reduced answering some subsequent questions, but one method must still be selected from those that are appropriate. No one method has ever proven superior under all circumstances. Even with the same preference information, each method evaluates problems from different points of view. For instance, the simple weighting method ranks the preference order by the weighted average outcomes; TOPSIS by relative closeness to the ideal solution; linear assignment and ELECTRE methods by the concordance measures. It is somewhat arbitrary to apply one MADM method, which frequently provides only a partial solution. MacCrimmon (18) suggested that when dealing with multiple criterion decisions, a combination of methods is often more effective than a single technique.

Kenney and Nair (16) employed the conjunctive method to identify the candidate nuclear power plant sites in a large area and multi-attribute utility theory to evaluate and rank the candidate sites. Nijkamp (21) used four MADM methods, namely, expected value method, ELECTRE method, entropy method, and discrepancy analysis, to analyze a land use problem in the

northern part of the Netherlands, but the results obtained by these four methods were not aggregated. Hwang and Yoon(12) first proposed a unified approach to MADM by employing some MADM methods simultaneously instead of selecting the most appropriate method for the situation. This unified approach will be introduced in the following chapter.

Chapter 3

A unified approach to MADM

The proposed unified apprach, which was developed by Hwang and Yoon (12), is composed of four phases: screening phase, preference ordering phase, aggregation phase, and synthesis phase. The process diagram for the unified approach to multiple attribute decision making is shown in Figure 3.1 and detailed procedures are given in the following sections.

3.1 Screening phase

This phase screens dominated and unacceptable alternatives by the dominance and conjunctive methods in order to reduce the number of alternatives. Any alternatives remaining after this phase become candiates for further evaluation.

Dominance method: An alternative is dominated if there is another alternative which excels it in one or more attributes and equals it in the remainder. In seeking to select the best of several alternatives, a dominated one can be eliminated, as it is clearly inferior.

Conjunctive method: An alternative is accepted if it passed each of the standard levels. A_i is classified as an acceptable alternative only if

$$x_{ij} \geq x_j^0 , \qquad j = 1, 2, \ldots, n \qquad (3.1)$$

where x_j^0 is the standard level of x_j.

This method is designed to remove unacceptable

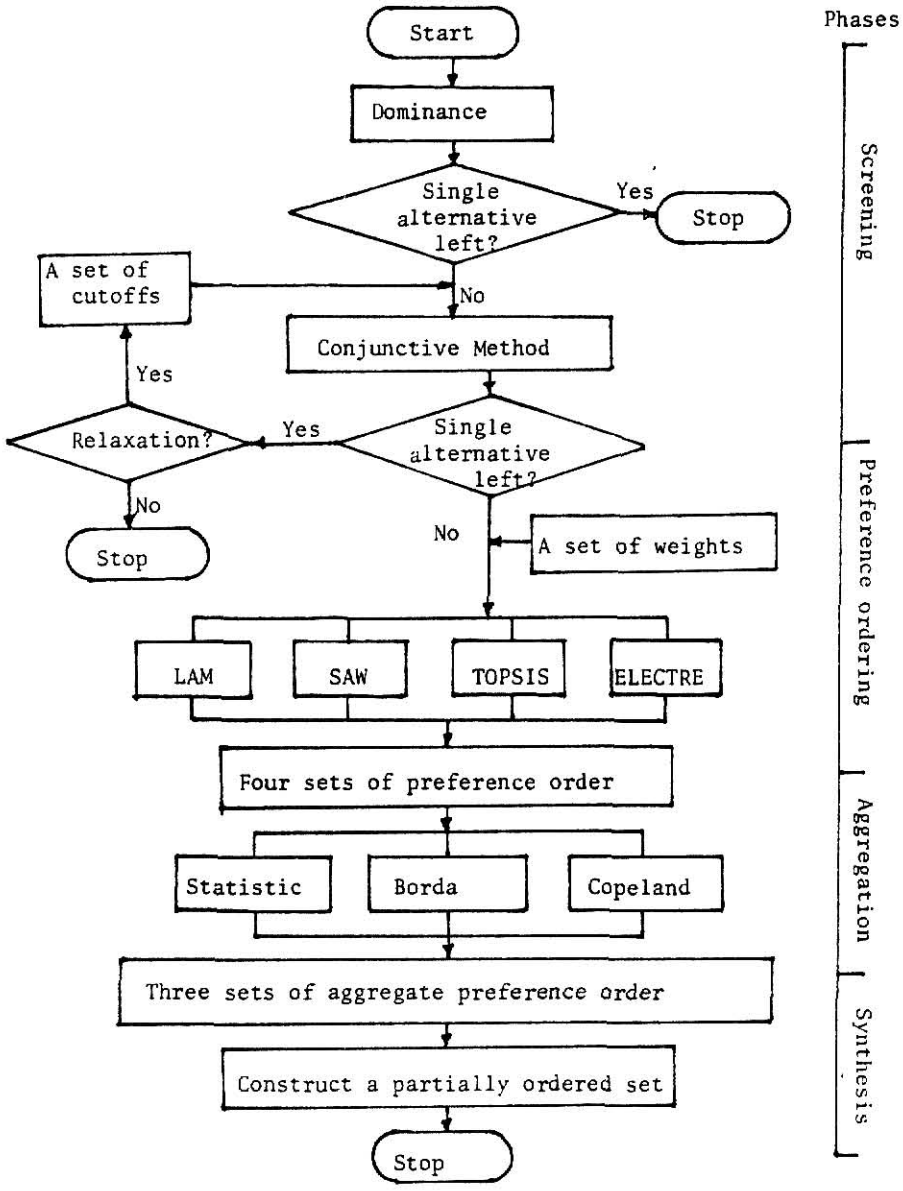


Figure3.1 A process diagram for a unified approach to MADM.

(Adapted from Ref. 12.)

alternatives rather than to evaluate them; hence it is advisable not to set the standard levels high. If none or a single alternartive is left, the relaxation of the standard levels is subject to the decision maker.

3.2 Preference ordering phase

This phase tries to find preference orders among alternatives by employing some MADM methods simultaneously. Four efficient MADM methods -- the linear assignment method, simple additive method, TOPSIS, and ELECTRE, utilizing a weighting scheme -- are selected. These four efficient methods are reviewed in Chapter 2. These methods were developed in different fields and evaluate the problems from different points of view. Therefore, solutions by these four methods may differ even with the same decision matrix and weighting scheme. If these solutions are different, the aggregation process will be needed to aggregate these sets of preference orders. The process is not required if the four methods yield identical preference orders.

3.3 Aggregation phase

The average ranking procedure, Borda method, and Copeland method (7) are applied to aggregate these sets of different preference orders. These aggregation techniques are described via the following example. Their assumed preference orders are shown in Table 3.1.

Table 3.1 Preference orders by four MADM mehtods (example data)

Alternative	MADM method				AVE
	SAW	LAM	TOPSIS	ELECTRE	
1	1	2	1	2	1.5
2	3	3	2	1	2.25
3	2	1	4	3	2.5
4	4	4	3	4	3.75

Table 3.2 The results of majority vote based on Table 3.1.

	A_1	A_2	A_3	A_4	ΣR
A_1	-	M	M	M	3
A_2	X	_	X	M	1
A_3	X	X	_	M	1
A_4	X	X	X	-	0
ΣC	0	1	1	3	
$\Sigma R-\Sigma C$	3	0	0	-3	

* An M in the (j,k) cell implies that A_j M A_k; which also implies that an X appears in the (k,j) cell, implying not (A_k M A_j). The Σ's reflect the number of M's per row or column.

Average ranking procedure: It treats the data in a purely statistical fashion, determining average ranking, standard deviations, etc. The average rank order is shown in the column labeled AVE of Table 3.1. According to the ascending order of the mean rankings, the aggregate rank order by this method is A_1, A_2, A_3, and A_4. If two alternatives have the same average rank, the one with the smaller standard deviation may be ranked ahead.

Borda method: It is based on a well-known majority rule relation which is defined as follows:

$$A_j \; M \; A_k \quad \text{if and only if} \quad \#\{i | A_j \; P_i \; A_k\} > \#\{i | A_k \; P_i \; A_j\} \qquad (3.2)$$

where $A_j \; M \; A_k$ represents that A_j has a majority vote over A_k, $A_j P_i A_k$ means that A_j prefers to A_k, and # denotes the number of elements in a set. According to Table 3.1, A_1 is ranked ahead A_2 by SAM, LAM, and TOPSIS; only ELECTRE prefers A_2 to A_1. Therefore, A_1 has a majority vote over A_2, denoted $A_1 \; M \; A_2$. The results by Borda method are shown in Table 3.2 for the example case. The ordering criterion is based on how many times an alternative wins a majority vote. The last column, lable ΣR, in Table 3.2 represents the number of times an alternative wins in the voting. According to the descending order of ΣR, the aggregate rank order by Borda method can be obtained. If the alternatives have the same ΣR, they are then identified as a group and ranked at the same level. The

aggregate rank of the example is A_1, (A_2, A_3), and A_4.

Copeland method: The Copeland method adds another perspective, starting where the Borda method stops. This method considers not only how many wins an alternative has, but also includes the losses. By subtracting the losses from the wins it provides a measure of the relative strength of voting for an alternative. The row, labeled Σ C, in Table 3.2 represents the number of losses for an alternative. Consider A_1 for example. It has three wins and no losses. Therefore, the score of A_1 is (3-0)=3. The scores for A_2, A_3, and A_4 are 0, 0, -3, respectively. This implies an ordering of A_1, (A_2, A_3), and A_4, ranking the alternatives from highest to lowest score.

These three methods aggregate the original preference orders of the four efficient MADM methods from different perspectives. This represents a rich source of analysis information if it can be meaningfully synthesized. The next presents a method for doing so.

3.4 Synthesis phase

The intent of this phase is to clarify the results of the three previous aggregate methods. The procedure is based on the construction of a partially ordered set (poset). For any pair of alternatives j and k, one alternative is placed above another if and only if it is above it in all the component orderings. For example, if one has the set of orderings

$$K = \{O_1, O_2\}$$

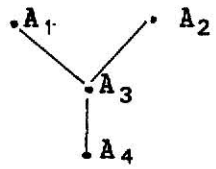
where

O_1 is $A_1 > A_2 > A_3 > A_4$

and

O_2 is $A_2 > A_1 > A_3 > A_4$

and all the elements of O_1 and O_2 come from the same set, i.e., $S = \{A_1, A_2, A_3, A_4\}$, the partial order then can be formed:



where A_4 is always less than any other alternatives, A_3 is always less preferable to A_1 and A_2. The elements A_1 and A_2 are not comparable since in O_1, $A_1 > A_2$ and in O_2, $A_2 > A_1$.

If all the component orderings are identical, the resultant poset is an exact reproduction of the component ordering.

The unified approach including four phases has been described in this chapter. In order to evaluate the effectiveness of applying this unified approach, four real-world MADM problems were selected for illustration in the next four chapters.

Chapter 4

Problem 1: Industrial land use development problem in the Maasvlakte area of the Netherlands

This problem was presented by Nijkamp and Delft (22), in which the method employed for the evaluation of the possible alternatives was the ELECTRE method.

4.1 Introduction

The Maasvlakte is a man-made peninsula in the Rhine-delta area of the Netherlands, near the North Sea. It is located at the southern mouth of the New Waterway, which is the big canal from the North Sea to Rotterdam (see Figure 4.1). The total area of the peninsula is approximately 2460 hectares. Of this about 1300 hectares can be used as an industrial area.

In this area, 520 hectares are already being used for industrial purposes. The use of the remaining area of 780 hectares together with an adjacent area at Europoort of approximately 120 hectares was discussed.

After world War II, the continuous expansion of the Rotterdam harbour has led to a parallel expansion of land use for harbour premises. Rotterdam itself could not afford sufficient locations for concomitant activities and industries. A necessary condition for maintaining Rotterdam's international position was the availability of adequate industrial areas, increasing the attractive features of the Rhine-delta region.

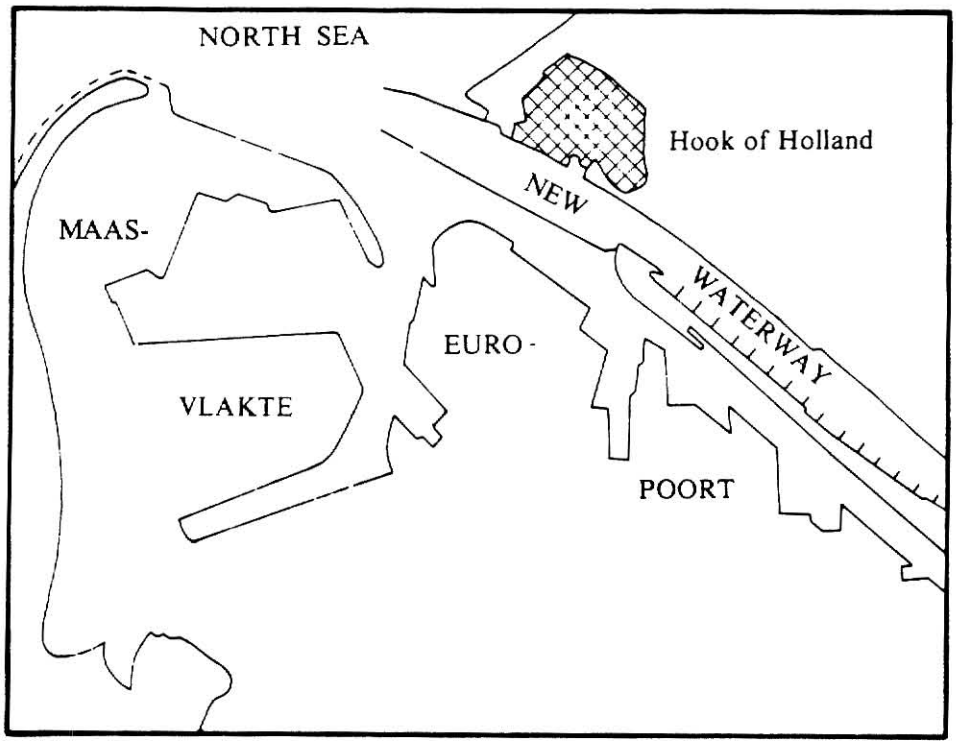


Figure 4.1 Map of the Maasvlakte.(Adapted from Ref. 22.)

This has led to a spatial shift of the harbour activities towards the North Sea (west of Rotterdam).

The area around the New Waterway, which links Rotterdam with the North Sea, was obviously the most ideal site for harbour industries. However, only the southern bank of this canal could be used as a new industrial area; the northern part was mainly occupied as an industrial area.

The southern bank of the New Waterway had been occupied during the post-war period by a great variety of industrial activities. The further expension of creating an additional area was to reclaim land from the North Sea. It is the Maasvlakte peninsula.

The remaining 900 hectares could be utilized by many kinds of industries. The selection of an industrial composition for the area such that the environment conditions would be only marginally affectedand the regional economic structure of the area consolidated was studied by Nijkamp and Delft (6,22). A set of policy objectives and side conditions, which were considered relevant in the evaluation of occupations for the Maasvlakte, are:

Policy objectives

1. A balanced development of the labor market (both in a quantitative and a qualitative sense).
2. Qualitative improvement of the employment structure with regard to both educational and wage levels.
3. Enhancement of the degree of differentiation in the

economic structure of the Rhine-delta region.

4. Efficient exploitation of the port of Rotterdam in its entirety in the long run.

Side conditions

1. Relevant candidates are those geared to location near a deep-water channel.
2. The new industrial activities should not affect a balanced development of population in the Rotterdam agglomeration.
3. A further claim on foreign works in the Rhine-delta region is in general considered to be undesirable.
4. A congestion in physical infrastructure must be avoided in the Rhine-delta area.
5. The extent of environmental pollution in the Rhine-delta region must be reduced.

4.2 Formulation of the problem

4.2.1 Specification of the possible alternatives

First, let us list the candidate activities with a short description of each activity.

1. Integrated steel works. The activity has an annual production capacity of 7.5 million tons of raw steel. Such a large-scale plant is considered an important element in the diversification process of the economic mono-structure of the harbour area, which is largely based on oil refinery and petrochemical industries. The

environmental deterioration caused by the emissions of air pollutants of a large-scale steel plant was the subject of much public discussion.

2. Partial steel works. This candidate entered the discussions as a less polluting alternative for the integrated steel works. This alternative consists of a medium-sized steel plant in which a less polluting production process will be used.
3. Tank storage plant. The eccentric position of the Maasvlakte area with respect to population concentrations makes it a favorable location for this type of activity.
4. Ores and coal transshipment works and integrated processing plants. The processing units of this combined activity will consist of a pelleting plant for iron ores and a coal gasification unit. Particularly the pelleting plant will cause an environmental deterioration by air pollutants. The emission rates of this plant are comparable to those of the integrated steel works.
5. Container terminal. The increasing containerization of loadings necessitates an expansion of the container facilities in the part of Rotterdam which cannot be realized at the present terminals. The Maasvlakte is regard as an alternative location for this space-consuming activity.
6. Ship repair yard and tanker cleaning plant. The facilities offered by this activity for handling very

large ships will considerably add to the attraction of the part of Rotterdam for mammoth tanker sand bulk carriers.

On the basis of this list of candidate activities, five reasonable alternatives which are composed of different combinations of these activities, are selected for the multiple attribute decision analysis. In this previous study (6), a zero-one linear programming model was used to solve this combination problem. The most important side-condition of this model was based on emmission norms for air pollution in the residential areas near the Maasvlakte. By varying these pollution standards (low, medium, and high), three alternatives were selected out of a large number of combinations. They are denoted as alternatives 1, 2, and 3, respectively (see Table 4.1). In addition to the three alternatives determined by means of the linear programming model, two new alternatives were introduced, because of their important role in the public descussions about the industrial structure of the Maasvlakte. These two alternatives, denoted by 4 and 5, include the following respective activities: (1, 3, 4, 6) and (2, 3, 4, 5, 6). Consequently, the total number of alternatives for multiple attribute decision analysis is five.

4.2.2 Evaluation attributes

Next, a set of numerical attributes has to be specified. A short description of these decision attributes is given, including a motivation for their selection.

Table 4.1 Composition of the alternatives of the Maasvlakte industrial site.

Avtivity	Alternative 1	2	3	4	5
1. Integrated steel works				*	
2. Steel works			*	*	
3. Tank works		*	*	*	*
4. Transshipment works	*	*		*	*
5. Container terminal	*	*	*		*
6. Ship and tanker cleaning	*	*	*	*	*

The symbol '*' indicates that an activity is included in the corresponding alternative.

1. Increase in regional welfare from the industrial complex located on the Maasvlakte. This attribute indicates the economic importance of the various alternatives. It is calculated as the sum total of additional value added of that activity and its specific overhead agglomeration effects due to location in the industrial complex on the Maasvlakte.
2. Occupation rate of the total available area. This attribute indicates the extent to which an alternative uses the available industrial area. In view of a less favorable situation on the labor market, a higher occupation rate may prove to be more advantageous. This attribute is defined as the total area occupied by the activities included in that alternative, divided by the total area available for industrial use. The land use of the various candiate activities is shown in Table 4.2.
3. Total demand for labor. The importance of this attribute is based on the situation on the regional labor market. Increasing unemployment during recent years makes it necessary to include regional employment in the selection procedure.
4. Qualitative aspects of labor demand. If a balanced labor market is one of the policy objectives, not only the quantitative aspects but also the qualitative structure of additional labor demand must be considered. This criterion is calculated as a discrepancy index between

Table 4.2 Data of individual activities of Problem 2

Attribute	Activity							
	1	2	3	4	5	6	7	8
X_1	1.838	1.838	0.480	0.450	0.163	0.510	0.957	2.315
X_2	500	200	100	200	250	40	200	100
X_3	24.00	24.00	1.00	2.00	0.48	8.00	6.00	20.00
X_4								
low-skilled	0.40	0.40	0.00	0.49	0.15	0.50	0.48	0.48
medium-	0.48	0.48	0.93	0.42	0.62	0.37	0.47	0.47
high-	0.12	0.12	0.07	0.09	0.23	0.13	0.05	0.05
X_5	82.6	124.0	0.0	72.4	0.0	0.0	510 1	42.9
X_6	8.64	8.64	0.36	4.32	0.00	12.90	26.69	36.89
X_7	10.00	10.00	1.10	1.10	1.10	2.80	4.00	2.80
X_8	15.00	7.50	5.00	30 00	15.00	5.00	20.00	15.00

* Activities 1 - 6 are corresponding to those mentioned in Table 4.1.

* Activities 7 and 8 represent the oil refinery and petrochemical industry, respectively.

* X_1 = value added per hectare (in million Dfl)
  X_2 = land use (in hectares)
  X_3 = demand for labor (per hectare of land use)
  X_4 = share of labor demand (%)
  X_5 = pollution emission (tons per hectare per year)
  X_6 = percentage share of employment in Rhine-delta region
  X_7 = percentage foreign workers
  X_8 = port charge (in million Dfl)

the labor demand of each alternative and the labor supply in the broader Rhine-delta region. The quality of labor demand was divided into three levels of skill(low, medium and high). For each activity, the relative share of these labor levels are shown in Table 4.2. The average percentage share of labor in the mentioned three levels(low, medium, and highly skilled) in the Rhine-delta region is approximately equal to 30, 61, and 9%, respectively. An index of a certain plan i for the qualitative discrepancies on the labor market is constructed as:

$$d_i = \frac{1}{2}\sum_{n=1}^{3}\left|\ell_n^R - \bar{\ell}_{ni}^M\right| \qquad (4.1)$$

where

ℓ_n^R = average percentage share of supply of labor level n (n = 1, 2, 3) in the Rhine-delta area.

$\bar{\ell}_{ni}^M$ = average percentage share of demand for labor level n in the Maasvlakte area associated with plan i.

$\bar{\ell}_{ni}^M$ is calculated on the basis of the activities k (k=1,...,K) constituting plan i. If the set of activities k (k=1,...,K) from plan i is denoted by A_i, then $\bar{\ell}_{ni}^M$ can be calculated as:

$$\bar{\ell}_{ni}^M = \frac{\sum_{k\in A_i} L_k\, \ell_{nk}^M}{\sum_{k\in A_i} L_k} \qquad (4.2)$$

where

ℓ_{nk}^M = the share of labor level n in activity k

L_k = the volume of labor demanded by activity k.

The data on both ℓ^M_{nk} and L_k are in Table 4.2. It is clear that this index can be obtained by using eq. (4.1) and eq. (4.2). This index is a cost attribute.

5. Environmental deterioration as a result of industrial use of the Maasvlakte. In the Rhine-delta area, the additional environmental deterioration is one of the important criteria for evaluating the industrial land use alternatives. This criterion can be measured as the total emission of air pollutants of each alternative.
6. Differentitation in the industrial structure. In the Rhine-delta area, approximately 60% of total industrial employment in this area was generated by the oil refineries and allied petro-chemical industries before the oil crisis. This mono-structured economy increases the drawbacks of the total regional economy in the event of fluctuations in economy development (such as the oil crisis). This criterion is defined by means of the index of specialization for the labor market which was given by Czamanski (4). This index is calculated as the sum total of the absolute differences between the employment shares of the industrial activities of the industrial activities of the alternative and the employment shares of the same activities present in the economic structure of the Rhine-delta region. This specialization index can be written as:

$$S_i = \frac{1}{2} \sum_{k=1}^{K} \left| \frac{E^M_{ki}}{\sum_{k=1}^{K} E^M_{ki}} - \frac{E^R_k}{\sum_{k=1}^{K} E^R_k} \right| \qquad (4.3)$$

where

E^M_{ki} = total employment of activity k in the Maasvlakte area associasted with plan i.

E^R_k = total employment of activity k in the Rhine-delta region.

The employment shares of industrial activities in the Rhine-delta region are shown in Table 4.3 and those in the Maaslvakte area are included in Table 4.2.

7. Demand for foreign labor. The rapid industrialization has resulted in immigrants of unskilled labor from the Mediterranean countries. The drastic change in the demographic structure of these districts after the influx of migrants led to tension between the local population and the migrants. For this reason, location of industries with a large demand for foreign labor will be unfavorable.
8. Efficient harbor exploitation. The favorable situation of the Maasvlakte for attracting large ships and construction of extensive, deep-water channels makes it reasonable to impose the condition that these facilities of the Rotterdam port should be used efficiently. This implies that the location of activities corresponding to

Table 4.3 Employment shares of industrial activities in the industrialized area of the Rhine-delta area.

Industrial activity	Employment of activity i as a percentage of total employment
1. Metallurgical industry	8.64
2. Tank storage	0.36
3. Transshipment works	4.32
4. Container terminals	--
5. Shipyards	10.16
6. Oil refinery	26.69
7. Petrochemical industrial	36.89
8. Others	12.94

these facilities is favored.

Two kinds of criteria can be distinguished, namely, benefit and cost attributes. The first one is formed by the set of attributes for which the statement 'the higher the better' is valid. Attributes 1, 2, 3, 6, and 8 belong to the benefit set. The second set of criteria is based on the opposite statement 'the lower the better'. Attributes 4, 5, and 7 belong to the cost set. The individual data on all separate activities, as well as the resulting data on the plan impacts, are shown in Table 4.2 and Table 4.4, respectively. It is clear that Table 4.4 can be derived from Table 4.2 and Table 4.3 by means of the information on the composition of the alternatives presented above.

4.2.3 Attribute Weights

In this problem, five different sets of weights are specified here. Each weight set is approximately associated with a preference structure of one or more interest groups involved in the decision-making process of the Maasvlakte area. These five different sets of weights are shown in Table 4.5 and described as following:

1. A neutral scenario

   This set of weights assigns equal weights for all criteria. It does not discriminate between the relative importance attached to the regional welfare components. In other words, each attribute is equally important in the future development of the area.

Table 4.4 Decision matrix of the Maasvlakte industrial land use problem.

Alternative	Attribute							
	X_1	X_2	X_3	X_4 (cost)	X_5	X_6	X_7 (cost)	X_8
A_1	109.15	43.30	540	8.10	0.00	86.70	2.10	25.00
A_2	199.15	65.60	940	12.70	14.48	82.40	1.70	55.00
A_3	476.75	65.60	5,340	12.50	24.80	85.00	9.20	33.00
A_4	1077.40	93.30	12,820	13.10	55.78	85.40	9.50	55.00
A_5	566.75	87.80	5,740	13.00	39.28	81.10	8.60	63.00

* Attribute 1, 2, 3, 5,6, and 8 are benefit attributes; the others are cost benefit attributes.

X_1 = Total value added(in million Dfl)

X_2 = Occupation rate

X_3 = Labor demand(in man-year)

X_4 = Discrepancy index for labor levels

X_5 = Pollution (1,000 tons per year)

X_6 = Specialization index

X_7 = Percentage foreign workers

X_8 = Port charge(in million Dfl)

Table 4.5 Alternative sets of weights of problem 1.

Attribute	Weighting scheme				
	1	2	3	4	5
1	.125	.100	.167	.095	.097
2	.125	.067	.167	.095	.064
3	.125	.233	.125	.095	.161
4	.125	.200	.083	.095	.161
5	.125	.033	.042	.381	.226
6	.125	.133	.125	.095	.097
7	.125	.133	.042	.095	.129
8	.125	.100	.250	.048	.064

2. An employment scenario

The employment scentario places more emphasis on both the quantitative and the qualitative aspects of the labor market (attributes 3 and 4). In this scenario some other criteria indirectly related to the future situation on the labor market also receive relatively high priority, namely, the degree of differentiation in the regional economic structure (attribute 6) and the number of foreign works to be employed in the area concerned (criterion 7). Thus, this scentario aims at a balanced social and labor development for the Rhine-delta area.

3. A labor scenario

This set of weights assigns a rather high priority to the expected returns for the Rotterdam Port Council (attribute 8). In addition to the financial revenues of the port, some other related criteria are also considered to be rather important, namely, the increase in regional value added (attribute 1) and the occupation rate of the Maasvlakte area (attribute 2). This scenario can be seen as a future growth scenario for the port of Rotterdam.

4. An environmental scenario

The environmental scentario focuses particularly on environmental protection (attribute 5). This scenario can be considered as future stabilization variant for the Rhine-delta area.

5. An integrated labor-environmental scenario

This scenario reflects in particular strong preferences with respect to the employment aspects and the environmental aspects (attributes 3, 4 and 5). Moreover, the position of foreign labors receives some attention (criterion 7).

By means of the information reflected in Table 4.4 and 4.5, the unified approach to the multiple Attribute analysis discussed in the proceeding chapter will be applied and illustrated.

4.3 Application of the unified approach

There are five different sets of weights used in the analysis, but this problem will be demonstrated with the first set of weights (equal weights). The detail calculation is shown in Appendix 2.

4.3.1 Screening phase

The total number of possible alternative combination to be investigated separately would be equal to 2^6-1, but this combination problem was screened by the zero-one linear programming model and five alternatives were left for future multiple attribute decision analysis. Examining these five alternatives by the dominance method, these five alternatives are not dominated.

4.3.2 Preference ordering phase

The first set of weights is:

[0.125 0.125 0.125 0.125 0.125 0.125 0.125 0.125]

where the weights are normalized.

Simple additive weighting method

The simple additive weighting method requires a comparable scale for all elements in the decision matrix. Criteria 1, 2, 3, 6 and 8 are benefit criteria and should be divided by the absolute maximum value of this criterion. These were assumed to equal 107.440; 100; 12,820; 100; 63 by Nijkamp and Delft(22). Regarding the remaining cost criteria 4, 5 and 7, the maximum permissible values are also assumed to equal 100; 55.780 and 10, respectively. The comparable scale is obtained by using eq. (4.4) for benefit criteria and by eq. (4.5) for the cost criteria, they are,

$$r_{ij} = x_{ij} \; / \; x_j^* \qquad j=1,2,3,6,8 \qquad (4.4)$$

and

$$r_{ij} = 1 - x_{ij} \; / \; x_j^* \qquad j=4,5,7 \qquad (4.5)$$

where

x_{ij} = element of the alternative i about the attribute j

$x_j^* = \max_i x_{ij}$

According to these two transformation equations, the comparable scale decision matrix can be calculated (see Table 4.6).

Based on the first set of weights, the weighted average values for the alternatives are

Table 4.6 Decision matrix of Problem 1 with comparable scale.

Alternative	attribute							
	1	2	3	4	5	6	7	8
1	.101	.433	.042	.919	1.000	.867	.790	.397
2	.185	.656	.073	.873	.740	.824	.830	.873
3	.443	.656	.417	.875	.555	.850	.080	.524
4	1.000	.933	1.000	.869	.000	.854	.050	.140
5	.933	.878	.448	.870	.296	.811	.873	1.000

Table 4.7 Weighted average values and preference orders of problem 1 by SAW method.

Alternative	Weight 1		Weight 2		Weight 3		Weight 4		Weight 5	
	value	rank	value	rank	value	rank	value	rank	value	rank
1	.569	4	.526	5	.454	5	.700	1	.630	1
2	.632	2	.586	3	.609	3	.651	2	.623	2
3	.550	5	.555	4	.572	4	.552	3	.545	5
4	.697	1	.777	1	.870	1	.489	5	.603	3
5	.621	3	.626	2	.732	2	.507	4	.547	4

$A_1 = 0.5686$

$A_2 = 0.6318$

$A_3 = 0.5499$

$A_4 = 0.6974$

$A_5 = 0.6211$

Therefore, the preference orders are $[A_4, A_2, A_5, A_1, A_3]$

In like manner, the preference orders for the other sets of weights are shown in Table 4.7.

Linear assignment method

From the decision matrix (Table 4.4), we can easily obtain the following attribute-wise preference (Table 4.8).

According to the attribute-wise preference matrix and the first set of weights, the product-attribute matrix can be calculated as:

$$\Pi = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1st & 2nd & 3rd & 4th & 5th \end{array} \\ \begin{bmatrix} .375 & .125 & .000 & .000 & .500 \\ .125 & .188 & .250 & .438 & .000 \\ .000 & .125 & .563 & .313 & .000 \\ .375 & .188 & .063 & .000 & .375 \\ .125 & .375 & .125 & .250 & .125 \end{bmatrix} \end{array}$$

The optimal pemutation matrix P^* is

$$P^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Table 4.8 Attributewise preference of Problem 1.

Rank	Attribute							
	1	2	3	4	5	6	7	8
1st	A_4	A_4	A_4	A_1	A_1	A_1	A_2	A_5
2nd	A_5	A_5	A_5	A_3	A_2	A_4	A_1	A_2, A_4
3rd	A_3	A_2, A_3	A_3	A_2	A_3	A_3	A_5	
4th	A_2		A_2	A_5	A_5	A_2	A_3	A_3
5th	A_1	A_1	A_1	A_4	A_4	A_5	A_4	A_1

Table 4.9 Preference orders of Problem 1 by LAM method.

Alternative	Weighting scheme				
	1	2	3	4	5
1	5	5	5	1	1
2	4	4	4	2	4
3	3	3	3	3	3
4	1	1	1	5	5
5	2	2	2	4	2

Applying the permutation matrix P^* to A, we find the following optimal order

$$A \cdot P^* = [A_4, A_5, A_3, A_2, A_1]$$

Similarly, the results for other sets of weights by the LAM can be obtained and are shown in Table 4.9. The product-attribute matricesbased on the other sets of weights are shown in Appendix 2.1.

TOPSIS method

Step 1. Calculate the normalized decision matrix:

The outcomes of each criterion are divided by the norm of the total outcome vector of the criterion at hand.

$$R = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccccccc} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 \end{array} \\ \begin{bmatrix} .082 & .264 & .036 & .301 & .000 & .461 & .131 & .231 \\ .150 & .400 & .062 & .472 & .196 & .438 & .106 & .508 \\ .359 & .400 & .354 & .465 & .335 & .452 & .575 & .305 \\ .812 & .569 & .851 & .487 & .754 & .454 & .594 & .508 \\ .427 & .535 & .381 & .483 & .531 & .431 & .537 & .582 \end{bmatrix} \end{array}$$

Step 2. Calculate the weighted nomalized decision matrix:

Based on the first set of weights, the weighted normalized decision matrix is

$$V = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{bmatrix} .010 & .033 & .004 & .038 & .000 & .068 & .016 & .029 \\ .019 & .050 & .008 & .059 & .024 & .055 & .013 & .063 \\ .045 & .050 & .044 & .058 & .042 & .056 & .072 & .038 \\ .101 & .071 & .106 & .061 & .094 & .057 & .074 & .063 \\ .053 & .067 & .048 & .060 & .066 & .054 & .067 & .073 \end{bmatrix}$$

Step 3. Determine the ideal and negative-ideal solutions:

$A^* = \{.101, .071, .106, .038, .000, .058, .013, .073\}$

$A^- = \{.010, .033, .004, .061, .094, .054, .074, .029\}$

Step 4. Calculate the separation measures:

The separation of each alternative from the ideal solution can be obtained by

$$S_{i*} = \sqrt{\sum_{j=1}^{8}(v_{ij} - v_j^*)^2}, \quad i=1,2,3,4,5 \qquad (4.6)$$

$S_{1*} = .1486$ $S_{2*} = .1347$

$S_{3*} = .1196$ $S_{4*} = .1149$

$S_{5*} = .1167$

Similarly, the separation of each alternative from the negative-ideal one can be obtained by

$$S_{i-} = \sqrt{\sum_{j=}^{8}(v_{ij} - v_j^-)^2} \quad i=1,2,3,4,5 \qquad (4.7)$$

$S_{1-} = .1130$ $S_{2-} = .1007$

$S_{3-} = .0769$ $S_{4-} = .1461$

$S_{5-} = .0873$

Step 5. Calculate the relative closeness to the ideal solution:

$$C_{i*} = S_{i-}/(S_{i*}+S_{i-}), \quad i=1,2,3,4,5 \qquad (4.8)$$

$C_{1*} = .4319$ $C_{2*} = .4278$

$C_{3*} = .3914$ $C_{4*} = .5598$

$C_{5*} = .4279$

Step 6. Rank the preference order:

According to the descending order of C_{i*}, the preference order based on the first set of weights is

$[A_4, A_1, A_5, A_2, A_3]$

The results solved by TOPSIS are shown in Table 4.10. The detail calculation is in Appendix 2.2.

ELECTRE method

Step 1. Calculate the normalized decision matrix:

This step is the same as Step 1 of the TOPSIS method.

Step 2. Calculate the weighted normalized decision matrix:

This step is the same as Step 2 of the TOPSIS method.

Step 3. Determine the concordance and discordance sets:

concordance sets	discordance sets
$C_{12} = \{4, 5, 6\}$	$D_{12} = \{1, 2, 3, 7, 8\}$
$C_{13} = \{4, 5, 6, 7\}$	$D_{13} = \{1, 2, 3, 8\}$
$C_{14} = \{4, 5, 6, 7\}$	$D_{14} = \{1, 2, 3, 8\}$
$C_{15} = \{4, 5, 6, 7\}$	$D_{15} = \{1, 2, 3, 8\}$
$C_{21} = \{1, 2, 3, 7, 8\}$	$D_{21} = \{4, 5, 6\}$
$C_{23} = \{2, 5, 7, 8\}$	$D_{23} = \{1, 3, 4, 6\}$
$C_{24} = \{4, 5, 7, 8\}$	$D_{24} = \{1, 2, 3, 6\}$
$C_{25} = \{4, 5, 6, 7\}$	$D_{25} = \{1, 2, 3, 8\}$
$C_{31} = \{1, 2, 3, 8\}$	$D_{31} = \{4, 5, 6, 7\}$
$C_{32} = \{1, 2, 3, 4, 6\}$	$D_{32} = \{5, 7, 8\}$
$C_{34} = \{4, 5, 7\}$	$D_{34} = \{1, 2, 3, 6, 8\}$
$C_{35} = \{4, 5, 6\}$	$D_{35} = \{1, 2, 3, 7, 8\}$

Table 4.10 Preference orders of Problem 1 by TOPSIS method.

Alternative	Wetght 1		Weight 2		Weight 3		Weight 4		Weight 5	
	RC	Rank	RC	Rank	RC	Rank	RC	Rank	RC	Rank
1	.4319	2	.2685	5	.1765	5	.7271	1	.5461	1
2	.4278	4	.2714	4	.3440	4	.6358	2	.4839	2
3	.3914	5	.3577	3	.3568	3	.5209	3	.4482	4
4	.5598	1	.7229	1	.8015	1	.2717	5	.4514	3
5	.4279	3	.4185	2	.5681	2	.3211	4	.3533	5

* RC represents the relative closeness to the ideal solution.

$C_{41} = \{1, 2, 3, 8\}$ $\quad D_{41} = \{4, 5, 6, 7\}$

$C_{42} = \{1, 2, 3, 6, 8\}$ $\quad D_{42} = \{4, 5, 7\}$

$C_{43} = \{1, 2, 3, 6, 8\}$ $\quad D_{43} = \{4, 5, 7\}$

$C_{45} = \{1, 2, 3, 6\}$ $\quad D_{45} = \{4, 5, 7\}$

$C_{51} = \{1, 2, 3, 8\}$ $\quad D_{51} = \{4, 5, 6, 7\}$

$C_{52} = \{1, 2, 3, 8\}$ $\quad D_{52} = \{4, 5, 6, 7\}$

$C_{53} = \{1, 2, 3, 7, 8\}$ $\quad D_{53} = \{4, 5, 6\}$

$C_{54} = \{4, 5, 7, 8\}$ $\quad D_{54} = \{1, 2, 3, 6\}$

Step 4. Calculate the concordance matrix:

An element c_{pq} of the concordance matrix C is calculated as

$$c_{pq} = \sum_{j \in C_{pq}} w_j / \sum_{j=1}^{8} w_j \qquad (4.9)$$

Then the concordance matrix is

$$C = \begin{bmatrix} -- & .375 & .500 & .500 & .500 \\ .625 & -- & .438 & .438 & .500 \\ .500 & .563 & -- & .375 & .375 \\ .500 & .563 & .625 & -- & .500 \\ .500 & .500 & .625 & .500 & -- \end{bmatrix}$$

Step 5. Calculate the discordance matrix:

An element d_{pq} of the discordance matrix D is obtained as

$$d_{pq} = \frac{\sum_{j \in D_{pq}} |v_{pj} - v_{qj}|}{M} \qquad (4.10)$$

In this case, M is defined as the maximum of the sum

total of the divergences between two alternatives.

$$M = \max_{(p,q)} \left(\sum_{j \in J} |v_{pj} - v_{qj}| \right) \qquad (4.11)$$

where the expression (p,q) stands for all values of p and q.

The discordance matrix is then obtained as:

$$D = \begin{bmatrix} -- & .1506 & .2278 & .6015 & .3712 \\ .1102 & -- & .1478 & .4625 & .2277 \\ .2691 & .2294 & -- & .3743 & .1539 \\ .3985 & .2999 & .1300 & -- & .1009 \\ .3249 & .2218 & .0665 & .2577 & -- \end{bmatrix}$$

Step 6. Determine the concordance dominance matrix:

The average value of all concordance indices is taken as the threshold value, then

$$\bar{c} = .5125$$

The concordance dominance matrix is

$$F = \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 1 & - & 0 & 0 & 0 \\ 0 & 1 & - & 0 & 0 \\ 0 & 1 & 1 & - & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Step 7. Determine the discordance dominance matrix:

The average value of all discordance indices is taken as the threshold value, then

$$\bar{d} = .2563$$

The discordance dominance matrix is

$$G = \begin{bmatrix} - & 1 & 1 & 0 & 0 \\ 1 & - & 1 & 0 & 1 \\ 0 & 1 & - & 0 & 1 \\ 0 & 0 & 1 & - & 1 \\ 0 & 1 & 1 & 1 & - \end{bmatrix}$$

Step 8. Determine the aggregate dominance matrix:

The aggregate dominance matrix E is defined by means of its typical elements e_{pq} as

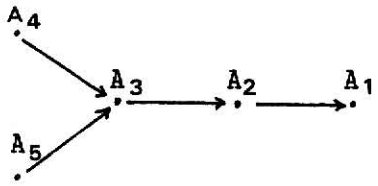
$$e_{pq} = f_{pq} \cdot g_{pq} \qquad (4.12)$$

The aggregate dominance matrix is

$$E = \begin{bmatrix} - & 0 & 0 & 0 & 0 \\ 1 & - & 0 & 0 & 0 \\ 0 & 1 & - & 0 & 0 \\ 0 & 0 & 1 & - & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Step 9. Rank the preference order:

The E matrix gives the following outranking relationships: $A_2 \longrightarrow A_1$, $A_3 \longrightarrow A_2$, $A_4 \longrightarrow A_3$, $A_5 \longrightarrow A_3$. These relationships can be illustrated by graphical representation,



The final preference order by ELECTRE method is

$[(A_4, A_5), A_3, A_2, A_1]$

In similar manner, the preference orders for the other sets of weights are obtained and shown in Table 4.11. The detail calculation is Appendix 2.3.

4.3.3 Aggregation phase

Table 4.12 shows the preference ranking of four effective MADM methods based on the equal weights.

By means of statistical methods, the mean rankings of A_1, A_2, A_3, A_4, and A_5 are 4.0, 3.5, 4.0, 1.13, and 2.37, respectively.

The results of majority vote are given in the Table 4.13. Aggregate ranking by both the Borda and Copeland methods can be obtained by reading the descending order of $\sum R$ and $(\sum R - \sum C)$ of Table 4.13. The three aggregate rankings from Table 4.12 and Table 4.13 are given in Table 4.14.

4.3.4 Synthesis phase

According to Table 4.13, the partially ordered (poset) is constructed as:

Table 4.11 Preference orders of Problem 1 by ELECTRE method.

Alternative	Weighting scheme				
	1	2	3	4	5
1	5	5	5	1	1
2	4	4	4	2	3
3	3	3	3	3	2
4	1.5	1	1	5	5
5	1.5	2	2	4	4

Table 4.12 Preference orders of Problom 1 by four MADM methods, based on the equal weights.

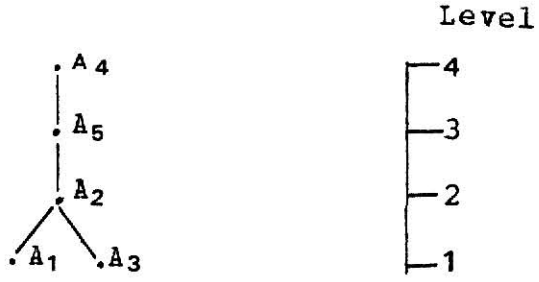
Alternative	MADM method				Ave.
	SAW	LAM	TOPSIS	ELECTRE	
1	4	5	2	5	4
2	2	4	4	4	3.5
3	5	3	5	3	4
4	1	1	1	1.5	1.13
5	3	2	3	1.5	2.37

Table 4.13 Majority vote between alternatives based on Table 4.13.

	A_1	A_2	A_3	A_4	A_5	ΣR
A_1	-	X	X	X	X	0
A_2	M	-	X	X	X	1
A_3	X	X	-	X	X	0
A_4	M	M	M	-	M	4
A_5	M	M	M	X	-	3
ΣC	3	2	2	0	1	
$\Sigma R-\Sigma C$	-3	-1	-2	4	2	

Table 4.14 Aggregated preference orders of Problem 1 by three aggregate techniques, based on the equal weights.

Rank	Average	Borda	Copeland
1st	A_4	A_4	A_4
2nd	A_5	A_5	A_5
3rd	A_2	A_2	A_2
4th	A_1, A_3	A_1, A_3	A_3
5th			A_1



The poset reflects the synthesized preference order, e.g., A_1 and A_3 tied in level 1, A_2 in level 2, A_5 in level 3, and A_4 in level 4. Thus, based on the equal weights, the final preference ranking by this unified approach is $[A_4, A_5, A_2, (A_1, A_3)]$.

Similarly, by means of the procedures employed in the aggregate phase and the synthesis phase, the synthesized preference orders for the other sets of weights can be obtained. These final results by the unified approach are concisely presented in Table 4.15.

4.4 Discussion

The results in Table 4.15 demonstrate that alternative 4 is the best with the neutral scenarial, the employment scenario and the harbor scenario, whereas the environmental scenario and the mixed employment-environmental scenario places much emphasis on alternative 1. The results obtained by Nijkamp and Delft only identified the best associated with the corresponding sets of weights. The comparison of the results by the unified approach with those of Nijkamp and Delft is given in Table 4.16.

It is clear that different priorities may lead to

Table 4.15 Preference orders of Problem 2 by the unified approach.

Alternative	Weighting scheme				
	1	2	3	4	6
1	4.5	5	5	1	1
2	3	4	4	2	2
3	4.5	3	3	3	4
4	1	1	1	5	4
5	2	2	2	4	4

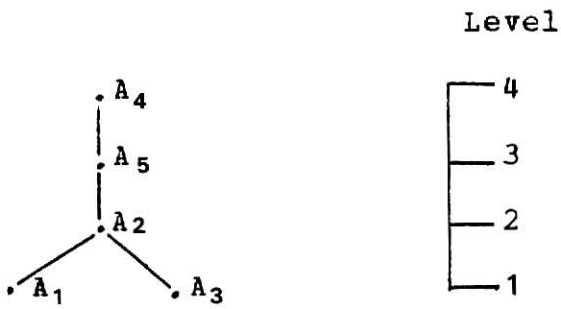
Table 4.16 A comparison of results of the Maasvlakte land use problem.

Rank	Nijkamp and Delft					Unified approach				
	W_1	W_2	W_3	W_4	W_5	W_1	W_2	W_3	W_4	W_5
1	A_4	A_4	A_4	A_1	A_1	A_4	A_4	A_4	A_1	A_1
2						A_5	A_5	A_5	A_2	A_2
3						A_2	A_3	A_3	A_3	A_3, A_4, A_5
4						A_1, A_3	A_2	A_2	A_5	
5							A_1	A_1	A_1	

different outcomes. So far, one unambiguous solution was not obtained in this case. We may try to get an ultimate solution by considering this to be a group decision making problem. Therefore, five committee members are assumed to be involved in the decision-making process. Each set of weights represents the corresponding decision maker's priority. Then the aggregate phase and the synthesis phase can be repeated to derive the unambiguous solution.

According to Table 4.15, the mean ranking of A_1, A_2, A_3, A_4, and A_5 are 3.3, 3, 3.5, 2.4, and 2.8, respectively. The majority votes are given in the Table 4.17. Three aggregate rankings of Table 4.15 are given in Table 4.18.

The poset, composed of 4 levels with A_1 and A_2 tied for the last rank, is formed as:



The ultimate solution of considering the final selection process to be a group decision making problem is $[A_4, A_5, A_2, (A_1, A_3)]$.

Finally, this ultimate solution as well as the results of the unified approach based on five different sets of weights (Table 4.15) are presented to the decision

Table 4.17 Majority vote between alternatives based on Table 4.15.

	A_1	A_2	A_3	A_4	A_5	$\sum C$
A_1	-	X	X	X	X	0
A_2	M	-	M	X	X	2
A_3	X	X	-	X	X	0
A_4	M	M	M	-	M	4
A_5	M	M	M	X	-	3
$\sum C$	3	2	3	0	1	
$\sum R-\sum C$	-3	0	-3	4	2	

Table 4.18 Aggregate preference orders of Table 4.15 and Table 4.17.

Rank	Average	Borda	Copeland
1st	A_4	A_4	A_4
2nd	A_5	A_5	A_5
3rd	A_2	A_2	A_2
4th	A_1	A_1, A_3	A_1, A_3
5th	A_3		

makers and the selection in favor of A_4 is suggested.

Chapter 5
Singapore Mass Transit Study

This problem was presented by Massan (19). The method employed for the analysis was the ELECTRE method.

5.1 Introduction

The area of Singapore (Figure 5.1), about the size of Chicago, is approximately 238 square miles, but this nation has a population of 2.4 million.

The urban area has historically been centered near the mouth of the Singapore River on the southern coast of the island. About 70 per cent of Singapore's inhabitants lives within a five-mile radius that encompasses the older Town area. This includes major residential developments in the Queenstown corridor, Tea Payoh and Kallang New Town. Further to the west lies the Turong Industrial Estate, with a potential for more than 600 modern factories and a population of 400,000, while to the east the Bedok New Town is beginning to take shape, ultimately to house 120,000 to 150,000 persons.

The locations and intensities of Singapore's development give rise to highly concentrated travel in the primary urban area along the southern shore of the island. Rising personal incomes are expected to produce two main effects: more people will be able to afford cars, and non-car owners will travel more. These changes will cause the quality of public transport service to deteriorate as rising volumes of both private

Figure 5.1 Map of Singapore. (Adapted from Ref. 27.)

vehicles and buses result in greater congestion.

The aim of the Singapore Mass Transit Study is to assist Government in formulating a transportation program and policy (26).

5.2. Formulation of the problem

5.2.1 Specification of the alternatives

(1) Extended bus service (Figure 5.2). This alternative is the extension of present mass transit service (buses on shared rights of way). Although it relies on continued bus operation in mixed traffic, this alternative provides several improvements over present services. These include utilization of proposed expressways for limited stop express services connecting major activity centers, and more widespread reservation of bus lanes on congested Central Area Streets. A total of 147 local and 14 express routes make up the alternative.

(2) Basic mass rail transit system (Figure 5.3). The rail rapid transit facility in this system has a length of almost 23 miles. It follows the southern belt of development, connecting the Jurong Town center and Bedok via the Queenstown corridor and the Center Area, and a branch from this route near the Kallang River passes to the north serving Tea

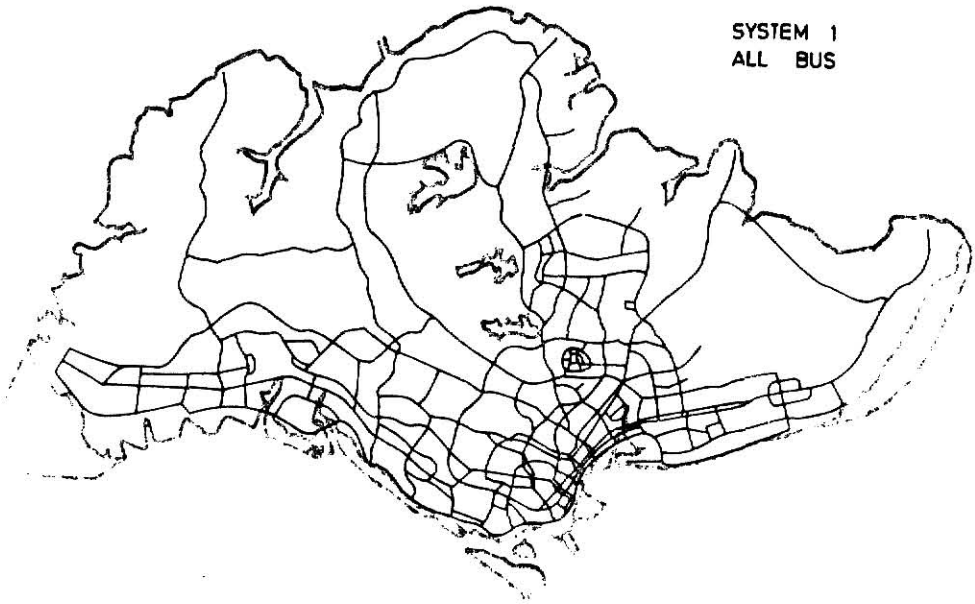


Figure 5.2 Extended bus service.

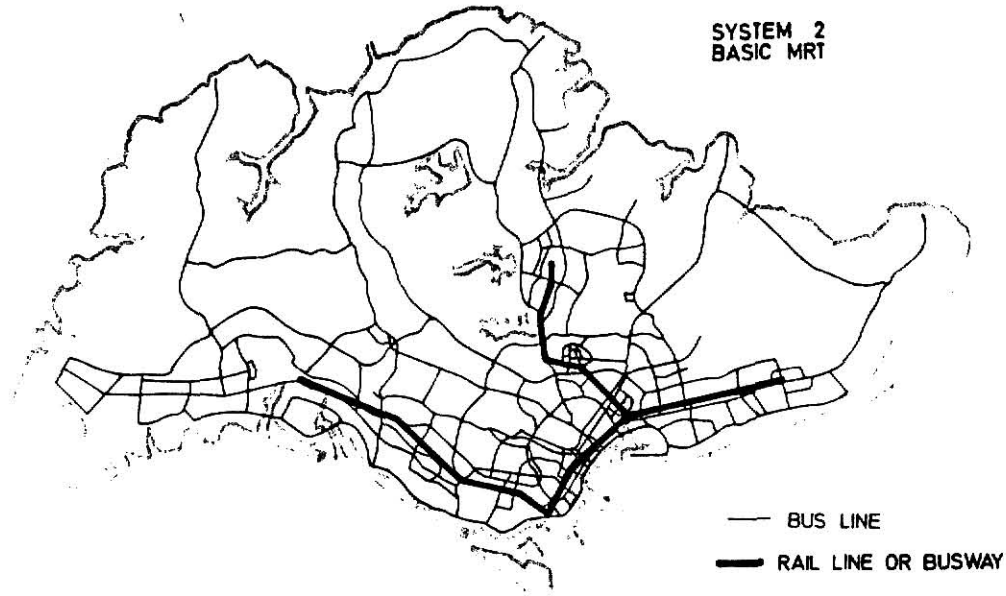


Figure 5.3 Basic mass rail transit system.

(Adapted from Ref. 27.)

Payoh and Ang Mo Ki. The central section between Henderson Road and Crawford Street is underground, as is the portion through Tea Payoh. Exepect for these six miles, construction is either at grade or on aerial structure. In addition to rail service, many of the bus routes of Alternative 1 and some additional ones complement the rail facility.

(3) Alternate mass rail transit with Orchard Road route (Figure 5.4). A computer-based review of travel demand patterns was undertaken to define this alternative. The resulting abstract plan was modified to yield a rail system that is similar in many respects to Alternative 2. The south coastal line extends two miles further to the west in Jurong and lies about a half mile south of the basic MRT in Queenstown corridor. The Tea Payoh line branches off at Stamford Road and follows Orchard Road rather than traversing the Kallang Basin. This line also extends a half mile further north in Ang Mo Kio. The section from Outram Road to Crawford Street and to Tea Payoh (nine miles of its 27-mile length) is underground. The concept of complementary bus services is also similar to that of Alternative 2.

(4) Busway system with bus lanes (Figure 5.5). This alternative incorporates grade-separated roadways

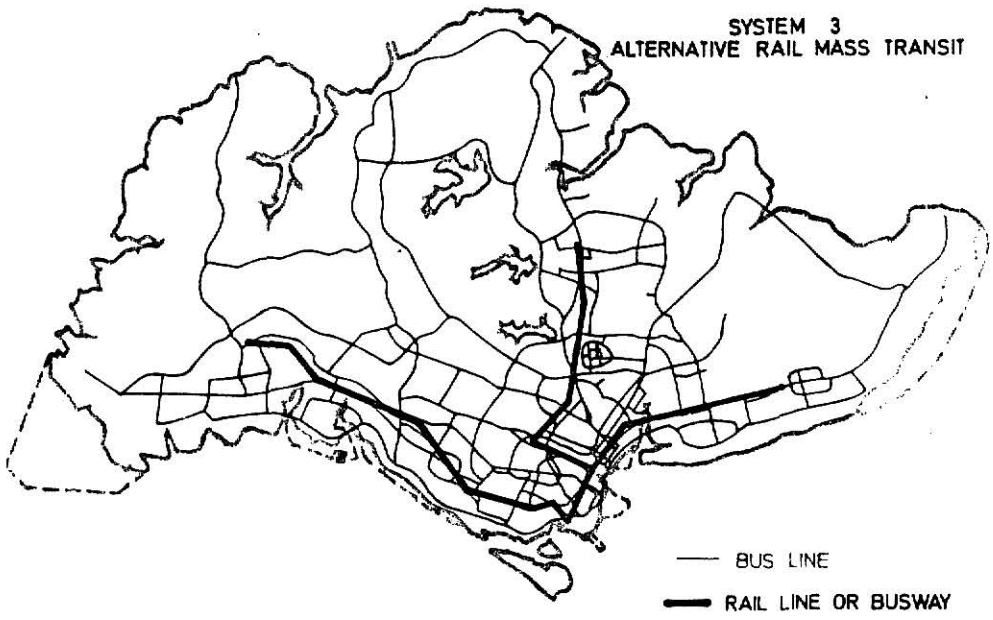


Figure 5.4 Alternate mass rail transit with Orchard Road.

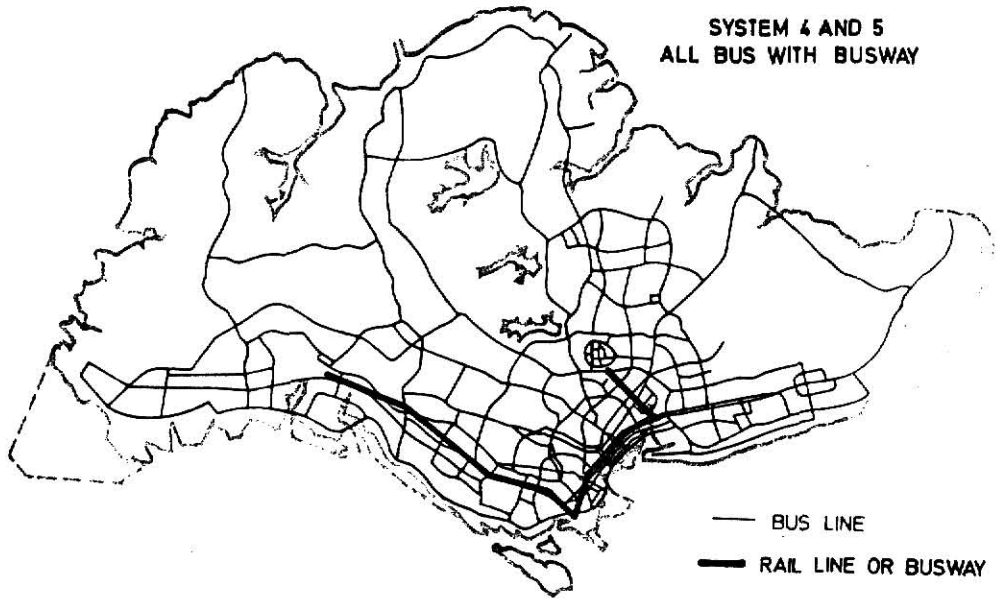


Figure 5.5 Busway system with bus lanes or with express services.
(Adapted from Ref. 27.)

for exclusive use by buses to penetrate and traverse highly congested areas. The operating concept of this alternative utilizes the flexibility of buses to circulate over the road network and collect loads of passengers before joining the busway for the line-haul portion of the journey. The busway in this alternative generally follows the route of the Basic MRT between Jurong and Tea Payoh. The ramps permit buses to enter and leave the busway at Tea Payoh, Jurong, North Buona Vista and Geylang. Elsewhere passengers will board and alight at strategically located stations on the busways. About 4.5 miles of its 14-mile length are underground. There is widespread reservation of traffic lanes for bus use -- on several of the proposed expressways, on major arterial roads, and on most of the Central Area streets that serve as bus routes.

(5) Busway system with express service (Figure 5.5). This alternative is physically similar to alternative 4, but its operating concept is quite different. Services using the busway were defined so as to proceed between major origin and destination areas with minmum numbers of stops at intermediate stations. Some routes leave the busway in the Central Area to serve Orchard Road

and Chinatown directly rather than by transfers to other routes as in Alternative 4. These changes produce faster, more direct service than in Alternative 4.

(6) Refined alternative mass rail transit with Orchard Road (Figure 5.6). This alternative, subject to refinement during preliminary evaluation of Alternativer 2 and 3, was defined as having the same terminal stations and Queenstown alignment as Alternative 2, whereas the Tea Payoh line serves the Orchard and Scotts Roads the same ways as Alternative 3. The overall length of Alternative 6 is 24 miles with 9.7 miles underground.

5.2.2 Evaluation attributes

There are eight attributes used to measure how well the alternatives serve various travel needs and development potentials. These attributes are described as follows:

(1) Service to major development areas: Location of the alternative and its stations with respect to areas of new development or redevelopment.

(2) Access to jobs: Service afforded by the alternative between principle residential developments and employment centers.

(3) Access to Central Area: Extent to which the system supports the viability of highly concentrated

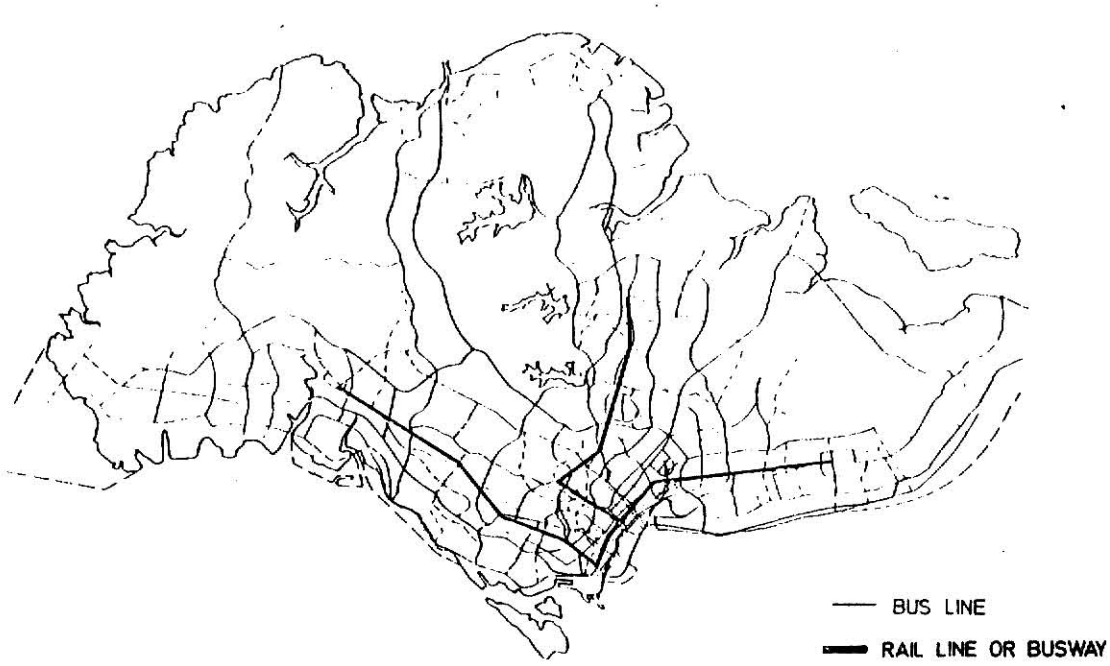


Figure 5.6 Refined alternate mass rail transit with Orchard Road.
(Adapted from Ref. 27.)

Central Area development by making it accessible to the work force and population at large.

(4) Relocation requirements: Dislocation of jobs, residences, and community facilities necessary to construct the system.

(5) Environmental impact: Effects on the urban environment caused by noise.

(6) Environmental impact: Effects on the urban environment caused by intrusion.

(7) Environmental impact: Effects on the urban environment caused by atmospheric pollution.

(8) Pedestrain amenity: Effects of the system on the safety, freedom and comfort of pedestrian movement.

These eight attributes are measured from 1 (poor performance) to 5 (good performance). The decision matrix is given in Table 5.1.

5.2.3 Attribute weights

Three sets of weights were applied in the multidimensional scaling analysis (19), reflecting priorities which might be assigned to the corresponding attributes by interested parties. The first set of weights gives equal weighting to all attributes; the second emphasizes environmental factors; and the third, access to jobs. These sets of weights are given in Table 5.2.

Table 5.1 Summary of performance of Singapore Mass Transit Alternatives on social and enviormental attributes.

Alternative	Attribute							
	1	2	3	4	5	6	7	8
1	2	2	2	5	1	5	1	2
2	4	3	3	3	3	3	4	3
3	5	4	4	4	2	2	4	4
4	4	3	3	2	3	2	2	3
5	4	5	5	2	3	2	3	3
6	5	4	4	3	3	3	4	4

Table 5.2 Weighting schemes of Problem 2.

Attribute	Weighting scheme		
	1	2	3
1	1	1	3
2	1	1	3
3	1	1	2
4	1	2	1
5	1	3	1
6	1	3	1
7	1	3	1
8	1	2	1

5.3 Application of the unified approach

5.3.1 Screening phase

In this phase, the dominated and acceptable/infeasible alternatives can be eliminated by dominance and conjunctive methods. An alternative is dominated if there is another alternative which excels it in one or more attributes and equals it in the remainder. Examining these six mass transit alternatives by the dominance method, alternative 2 is dominated by alternative 6 and alternative 4 is dominated by alternatives 5 and 6. Thus alternatives 2 and 4 are removed. The number of alternatives for further evaluation is reduced to four.

5.3.2 Preference ordering phase:

This problem will be demonstrated only with the set of weights which emphasized access to jobs.

This set of weights is

$$\underline{W} = \{\ 2 \quad 3 \quad 2 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\ \}$$

The weights are usually normalized to sum to 1. The normalized weights are given as follows:

$$\underline{\hat{W}} = \{\ .167 \quad .250 \quad .167 \quad .083 \quad .083 \quad .083 \quad .083 \quad .083\ \}$$

Simple additive weighting method.

The attributes of this problem are measured from 1 for poor performance to 5 for good performance. This means that the attributes are comparable. These attribute

values can be normalized by the equation of

$$r_{ij} = x_{ij} / x_j^* \qquad (5.1)$$

where x_j^* is equal to 5 (good performance).

The normalized decision matrix is given in Table 5.3. Based on the set of weights which emphasizes access to jobs, the weighted average values for the alternatives are

$$A_1 = 0.4660, \qquad A_3 = 0.7662,$$

$$A_5 = 0.7664, \qquad A_6 = 0.7828.$$

Then the overall preference orders are $[A_6, A_5, A_3, A_1]$. In this manner, the preference orders based on the other sets of weights are shown in Table 5.4.

Linear assignment method

According to the decision matrix (Table 5.1), the attributewise preferences are easily obtained and shown in Table 5.5.

From the attributewise rankings, the product-attribute matrix, based on the second weight scheme, is given by

$$\Pi = \begin{array}{c} \\ A_1 \\ A_3 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{cccc} \text{1st} & \text{2nd} & \text{3rd} & \text{4th} \end{array} \\ \begin{bmatrix} 0.1660 & 0.0000 & 0.0000 & 0.8330 \\ 0.1665 & 0.4580 & 0.3330 & 0.0415 \\ 0.4585 & 0.0415 & 0.3745 & 0.1245 \\ 0.2080 & 0.4995 & 0.2915 & 0.0000 \end{bmatrix} \end{array}$$

The optimal permutation matrix is obtained as

Table 5.3 The normalized decision matrix of Problem 2.

Alternative	Attribute							
	1	2	3	4	5	6	7	8
1	0.4	0.4	0.4	1.0	0.2	1.0	0.2	0.4
3	1.0	0.8	0.8	0.8	0.4	0.4	0.8	0.8
5	0.8	1.0	1.0	0.4	0.6	0.4	0.6	0.6
6	1.0	0.8	0.8	0.6	0.6	0.6	0.8	0.8

Table 5.4 Weighted average values and preference of Problem 2 by SAW method.

Alternative	Weight 1		Weight 2		Weight 3	
	Value	Rank	Value	Rank	Value	Rank
1	.5125	4	.4660	4	.5000	4
3	.6625	3	.7662	3	.7250	2
5	.6000	2	.7664	2	.6750	3
6	.7125	1	.7828	1	.7500	1

Table 5.5 Attribute-wise preferences of Problem 2.

Rank	Attribute							
	1	2	3	4	5	6	7	8
1st	A_3, A_6	A_5	A_5	A_1	A_5, A_6	A_1	A_3, A_6	A_3, A_6
2nd		A_3, A_6	A_3, A_6	A_3		A_6		
3rd	A_5			A_5	A_3	A_3, A_5	A_5	A_5
4th	A_1	A_1	A_1	A_5	A_1		A_1	A_1

Table 5.6 Preference orders of Problem 2 by LAM method.

Alternative	Weighting Scheme		
	1	2	3
1	4	4	4
3	2	3	3
5	3	1	1
6	1	2	2

$$P^* = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Applying the permutation matrix P^* to A, the optimal preference ranking is $[A_5, A_6, A_3, A_1]$.

Similarly, the results by LAM can be obtained and are shown in Table 5.6. The product-attribute matrices based on the other sets of weights are given in Appendix 3.1.

Note:

Applying this method, only the ordinal data are needed in the decision matrix. This information requirement is attractive. But this method may produce unsatisfying results or several optimal rankings by linear programming. The product-attribute matrix based on the first set of weights is

$$\Pi = \begin{array}{c} \\ A_1 \\ A_3 \\ A_5 \\ A_6 \end{array} \begin{array}{c} \begin{array}{cccc} 1st & 2nd & 3rd & 4th \end{array} \\ \begin{bmatrix} 0.3125 & 0.0000 & 0.0000 & 0.6875 \\ 0.1875 & 0.3750 & 0.3438 & 0.0938 \\ 0.2188 & 0.0938 & 0.4688 & 0.2188 \\ 0.2813 & 0.5313 & 0.1875 & 0.0000 \end{bmatrix} \end{array}$$

The optimal permutation matrix is obtained by LP as

$$P^* = \begin{matrix} A_1 \\ A_3 \\ A_5 \\ A_6 \end{matrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

According to the optimal permutation matrix P^*, the preference ranking is $[A_3, A_6, A_5, A_1]$. But it is clear that Alternative 6 is preferred over Alternative 3 by comparing the fourth row of the product-attribute matrix, corresponding to Alternative 6, with the second row, corresponding to Alternative 3. Alternative 6 is more frequently ranked 1st and 2nd and less frequently ranked 3rd and 4th than Alternative 3.

TOPSIS method

The weight normalized decision matrix is given in Table 5.7. Ideal and negative-ideal solution are determined as

$$A^* = \{ .167 \quad .250 \quad .167 \quad .083 \quad .050 \quad .083 \quad .066 \quad .066 \}$$
$$A^- = \{ .067 \quad .100 \quad .067 \quad .033 \quad .017 \quad .033 \quad .017 \quad .033 \}$$

The calculated separation measures are

$S_{1*} = .2174$ $\qquad$ $S_{1-} = .0704$

$S_{3*} = .0815$ $\qquad$ $S_{3-} = .1746$

$S_{5*} = .0814$ $\qquad$ $S_{5-} = .1987$

$S_{6*} = .0763$ $\qquad$ $S_{6-} = .1724$

Table 5.7 Weighted normalized decision matrix of Problem 2.

Alternative	Attribute							
	1	2	3	4	5	6	7	8
1	.025	.025	.025	.125	.037	.188	.037	.050
3	.063	.050	.050	.100	.075	.075	.150	.100
5	.050	.063	.063	.050	.112	.075	.112	.075
6	.063	.050	.050	.075	.112	.112	.150	.100

Table 5.8 preference orders of Problem 2 by TOPSIS method.

Alternative	Weighting scheme		
	1	2	3
1	4	4	4
3	2	3	2
5	3	1	3
6	1	2	1

The relative closeness to the ideal solution is calculated as

$C_{1*} = .2447$

$C_{3*} = .6780$

$C_{5*} = .7094$

$C_{6*} = .6933$

According to the descending order of C_{i*}, the preference order based on the second set of weights is $[A_6, A_3, A_5, A_1]$.

The solutions of TOPSIS are shown in Table 5.8. The deatiled calculation is in Appendix 3.2.

ELECTRE method

The concordance and discordance sets are determined as follows:

Concordance sets	Discordance sets
$C_{13} = \{4, 6\}$	$D_{13} = \{1, 2, 3, 5, 7, 8\}$
$C_{15} = \{4, 6\}$	$D_{15} = \{1, 2, 3, 5, 7, 8\}$
$C_{16} = \{4, 6\}$	$D_{16} = \{1, 2, 3, 5, 7, 8\}$
$C_{31} = \{1, 2, 3, 5, 7, 8\}$	$D_{31} = \{4, 6\}$
$C_{35} = \{1, 4, 6, 7, 8\}$	$D_{35} = \{2, 3, 5\}$
$C_{36} = \{1, 2, 3, 4, 7, 8\}$	$D_{36} = \{5, 6\}$
$C_{61} = \{1, 2, 3, 4, 7, 8\}$	$D_{61} = \{5, 6\}$
$C_{53} = \{2, 3, 5, 6\}$	$D_{53} = \{1, 4, 7, 8\}$

$C_{56} = \{2, 3, 5\}$ $D_{56} = \{1, 4, 6, 7, 8\}$

$C_{61} = \{1, 2, 3, 5, 7, 8\}$ $D_{61} = \{4, 6\}$

$C_{63} = \{1, 2, 3, 5, 6, 7, 8\}$ $D_{63} = \{4\}$

$C_{65} = \{1, 4, 5, 6, 7, 8\}$ $D_{65} = \{2, 3\}$

Based on the concordance sets, and the second set weights, the concordance matrix is calcualted as

$$C = \begin{bmatrix} -- & .166 & .166 & .166 \\ .833 & -- & .499 & .833 \\ .833 & .583 & -- & .500 \\ .833 & .916 & .582 & -- \end{bmatrix}$$

The average value of all concordance indicies is taken as the threshold value, then

$\bar{c} = .5758$

According to the concordance matrix and the threshold value indices, the concordance dominance matrix is determined as

$$F = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 1 & 1 & - & 0 \\ 1 & 1 & 1 & - \end{bmatrix}$$

The discordance matrix is obtained from the discordance sets and the weighted normalized decision matrix of Table 5.7:

$$D = \begin{bmatrix} -- & .7388 & .8006 & .7670 \\ .1329 & -- & .2002 & .0665 \\ .1998 & .1998 & -- & .1998 \\ .1328 & .0332 & .1669 & -- \end{bmatrix}$$

The average value of all concordance indicies is taken as threshold value, then

$\bar{d} = .3221$

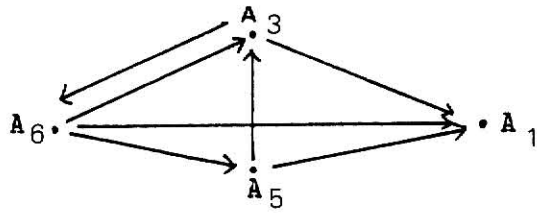
The discordance dominance matrix is obtained from the threshold value of discordance indices and the discordance indices:

$$G = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 1 & 1 \\ 1 & 1 & - & 1 \\ 1 & 1 & 1 & - \end{bmatrix}$$

Based on the concordance dominance matrix and the discordance dominance matrix, the aggregate dominance matrix is obtained as

$$E = \begin{bmatrix} - & 0 & 0 & 0 \\ 1 & - & 0 & 1 \\ 1 & 1 & - & 0 \\ 1 & 1 & 1 & - \end{bmatrix}$$

The overall ranking relationships of the aggregate dominance matrix can be illustriated by graphical representation,



From the above graphical representation, A_1 is clearly dominated by the others. A_1 is therefore ranked last. Yet, the relations among A_3, A_5, and A_6 are not clear at this stage. Next, by increasing the threshold value of the concordance index and decreasing the threshold value of the discordance indix to 0.90 and 0.10, respectively, the outranking relationship between A_6 and A_3 is $A_6 \longrightarrow A_3$. The final preference order by ELECTRE method is then obtained as

$[A_6, A_5, A_3, A_1]$

Similarly, the preference orders based on the other sets of weights are obtained and shown in Table 5.9. The detail calculation is shown in Appendix 3.3.

This problem was solved by Massam (19), using the multidimensional scaling method. The solutions of this method are shown in Table 5.10.

5.3.3 Aggregation phase

Based on the second set of weights, the solutions of the four effective MADM methods as well as the multidimensional scaling method as shown in Table 5.11. The results of majority vote are given in Table 5.12. Three aggregate rankings by mean ranking, Borda method, and Copeland method from Table 5.11 and 5.12 are given as

Table 5.9 Preference orders of Problem 2 by ELECTRE method.

Alternative	Weighting scheme		
	1	2	3
1	3.5	4	4
3	2	3	2
5	3.5	2	3
6	1	1	1

Table 5.10 preference orders of Problem 2 by the multidimensional scaling method.

Alternative	Weighting scheme		
	1	2	3
1	6	4	4
2	4	2	5
3	2	3	2
4	5	6	6
5	3	5	3
6	1	1	1

Table 5.11 Preference orders of Problem 2 , based on the second set of weights, by the four efficient mthods as well as MDS method.

Alternative	MADM method					Ave.
	SAW	LAM	TOPSIS	ELECTRE	MDS	
1	4	4	4	4	3	3.8
3	3	3	3	3	2	2.8
5	2	1	1	2	4	2
6	1	2	2	1	1	1.4

Table 5.12 Majority vote between alternatives based on Table 5.11.

	A_1	A_3	A_5	A_6	$\sum R$
A_1	-	X	X	X	0
A_3	M	-	X	X	1
A_5	M	M	-	X	2
A_6	M	M	M	-	3
$\sum C$	3	2	1	0	
$\sum R-\sum C$	-3	-1	1	3	

follows:

Rank	Ave.	Borda	Copeland
1st	A_6	A_6	A_6
2nd	A_3	A_3	A_3
3rd	A_5	A_5	A_5
4th	A_1	A_1	A_1

5.3.4 Synthesis phase

The rankings by these three techniques are identical, the evaluation process, based on the second set of weights, can be determined here. The result after the aggregation phase is $[A_6, A_3, A_5, A_1]$.

In the same manner, the results based on three different sets of weights by the unified approach are concisely presented in Table 5.13.

5.4 Discussion

In this problem. Massam tried to determine the best alternative by the multidimensional scaling method. It is clear that since the dominated alternatives have no chance of being the best, they may therefore be removed to reduce the number of alternatives. In other words, the problem can be simplied.

Hwang and Yoon (12) mentioned that 'each pair of alternatives given in the MDS judgment must be ranked in

Table 5.13 Preference orders of Problem 2 by the unified approach.

Alternative	Weighting scheme		
	1	2	3
1	4	4	4
3	2	3	2.5
5	3	2	2.5
6	1	1	1

terms of similarity. This requires quite fine discrimination and may be the most difficult for the DM among the MADM methods.' Massam defined the measure of dissimilarity (or similarity) by the following equation:

$$MOD_{ii'} = | 0.5 - c_{ii'}| \qquad (5.2)$$

where $MOD_{ii'}$ is the measure of dissimilarity between A_i and $A_{i'}$, and $c_{ii'}$ is the concordance index which was defined as

$$c_{ii'} = \frac{\sum_{j \in C_{ii'}} w_j + \frac{1}{2}\sum_{j \in T_{ii'}} w_j}{\sum_{j=1}^{n} w_j} \qquad (5.3)$$

where

$C_{ii'} = \{j \mid$ with respect to attribute j, plan i is preferred plan $i'\}$

$T_{ii'} = \{j \mid$ with respect to attribute j, plan i equals to plan $i'\}$

w_j = given weight cooresponding to attribute j.

A symmetrical matrix to measure dissimilarity can be obtained by equation(5.2), and the value in any cell MOD is close to zero when A_i and $A_{i'}$ are not clearly dissimilar and is close to 0.5 when one alternative clearly dominates the other. It is clear that $c_{ii'}$ and $c_{i'i}$ have a complementary relationship. But the measure of dissimilarity in equation (5.2) took no account of the discordance indices, because $d_{ii'}$ has no inherent complementary relationship with $d_{i'i}$. This lack of

complementarity means that the discordance matrix can not easily be converted to a symmetrical matrix. The information contained in the discordance matrix differs significantly from that contained in the concordance matrix to make the information content of C and D complementary. Therefore, the obtaining of a measure of dissimilarity from the discordance matrix needs further study.

Again, an ultimate solution can be obtained by considering this problem having three sets of weights to be a group decision-making problem. Three committe members may set their own priorities. Then the aggregation phase and synthesis phase can be repreated to derive an unambiguous solution. According Table 5.13, this solution is obtained as A_6, (A_3, A_5), A_1 . Selection in favor of A_6 is suggested.

Chapter 6

Problem 3: Selecting a reasonable alternative for long-range water resources development in Hungary

This problem was presented by David and Duckstein (5), and Keeney and Nair (16).

6.1 Introduction

The Tisza River basin, having a total area of 130,000 sq km, is shared by five countries. The region considered in Hungary is surrounded by mountains, and covers about 30,000 sq km. The main river following through this study area is the Tisza River. This area is shown in Figure 6.1.

The development of the water resources system was started in the middle of the last century by flood control and river regulation works. Presently needed is the further development of the water supply for agricultural purposes, industrial and domestic supplies, navigation, and riparian recreation. Careful management of both the quantity and quality of natural supplies has been introduced in the recent years.

The long range goals (55 years) of the water resources systems development are:

(1) The water supply should be regulated to satisfy requirements in quality and quantity.

(2) Flood protection should be provided along the Tisza River and its tributaries.

(3) Drainage and disposal of excess water produced by the system should be provided.

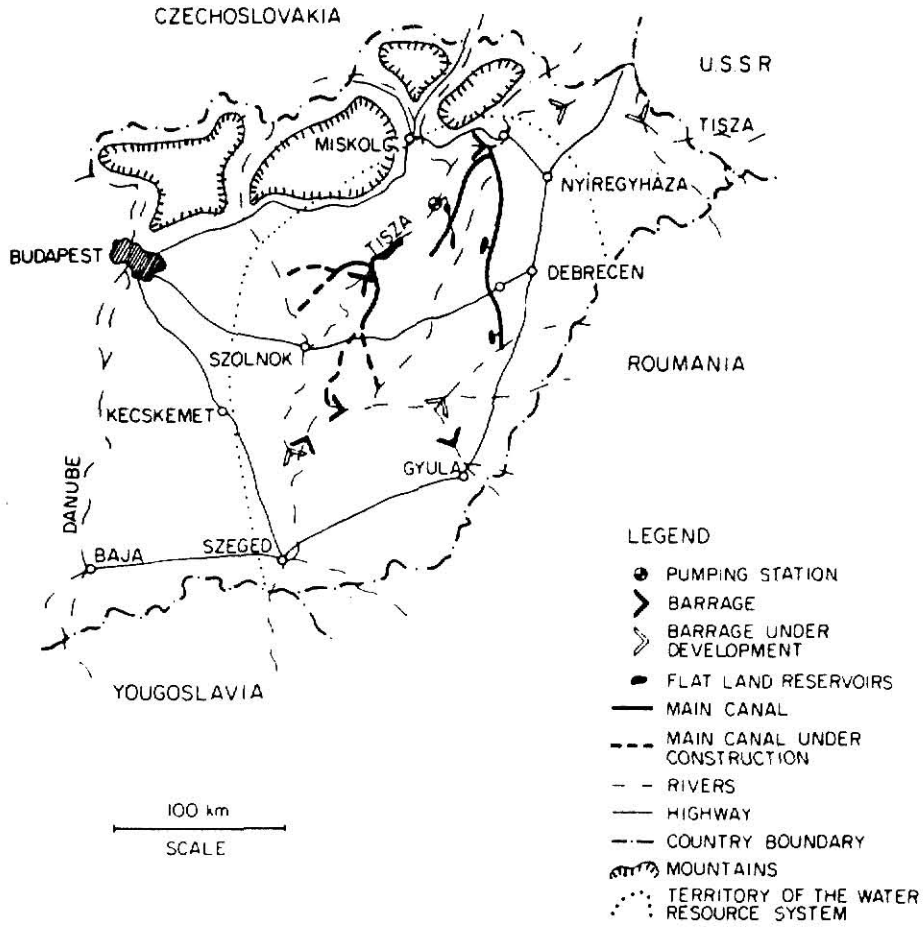


Figure 6.1 Schematic map of the existing water resources system in Hungary. (Adapted from Ref. 5.)

(4) The natural, social and economic resources needed to implement and operate the system should be kept to a minimum.

(5) Environment, including ecology and aesthetic value, should be preserved.

(6) The proposed system should be flexible enough to meet future requirements.

6.2 Formulation of the problem

6.2.1 Specification of the alternatives

The basic layouts of the alternatives are shown in Figure 6.2 and a short description for each alternative is given as follows:

(1) Danube-Tisza interbasin transfer using a multi-purpose canal reservoir system. The system utilizes the water resources of both the Tisza and Danube rivers. The water is transferred all year arround from the Danube by a gravity canal in the flat-land area and by a pumped canal reservoir system in the Borzsong-Cserhat Mountains.

(2) Pumped reservoir system in the northeastern part of the region. This pumped reservoir system is supplied only from the Tisza River and developed in the hilly region of the Sators and Bukk mountains. In this system the natural supply of water is available only four to five months per year. It would use all of the natural supply of this part of the Tisza River basin but not all of the

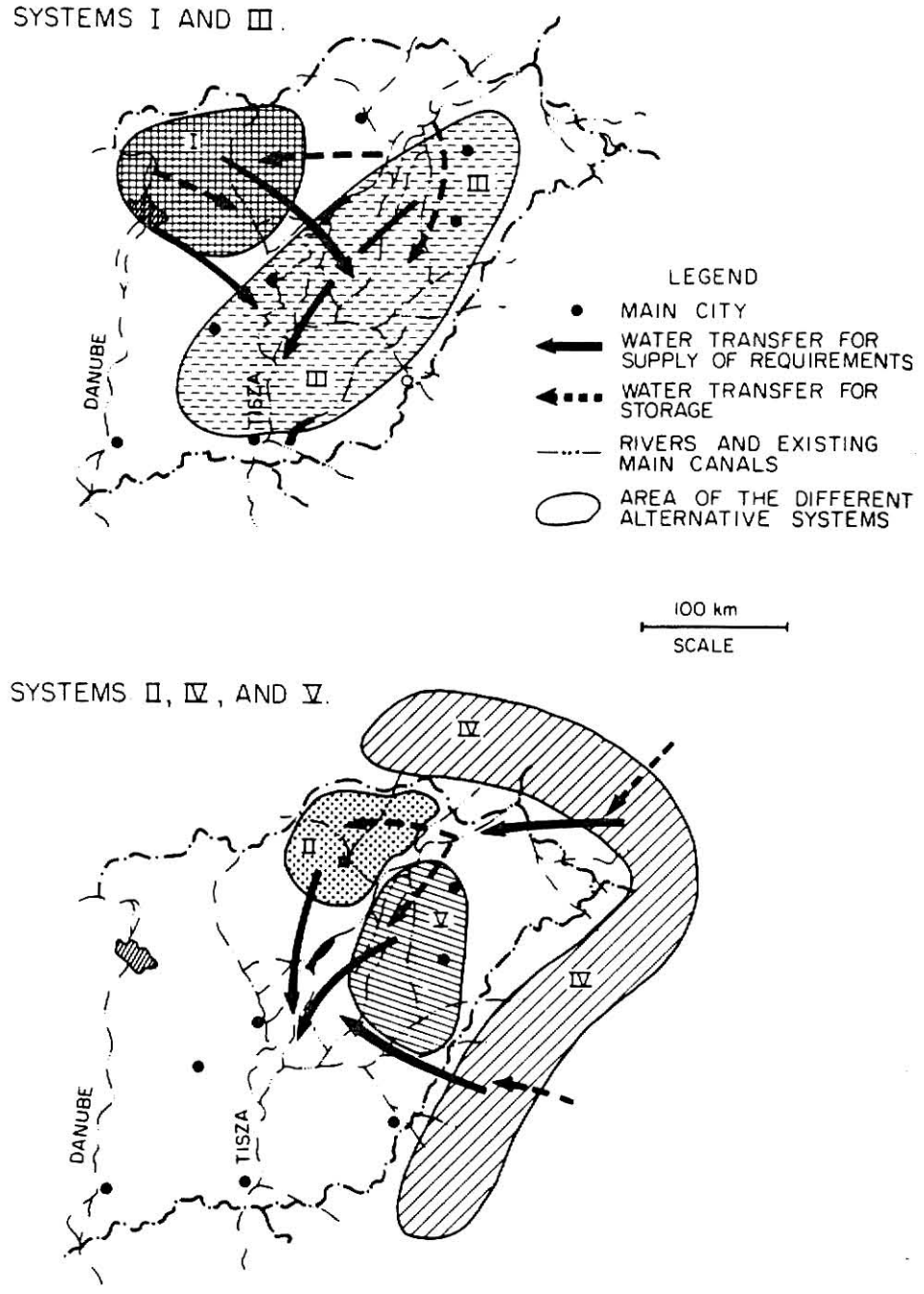


Figure 6.2 Layout of alternative water resources systems.
(Adapted from Ref. 5.)

available storage capacity.

(3) Flat land reservoir system. The third system could be developed on the flatland part of the region and would be composed of 2- to 4-m-deep reservoirs, using water from the Tisza River. Only a limited area of 5.5 sq km could be used for such reservoirs.

(4) Mountains reservoir system in upper Tisza River basin. This system would be located in the upper Tisza River basin outside of Hungary. It uses and regulates the water resources of the Tisza River by gravity. Extensive international cooperation must be initiated and sustained.

(5) Groundwater storage system. This system would be developed mainly on the flatland part of the region, especiaaly in the eastern part. The system would use the Tisza water and stored grounwater resources would be composed of underground storage spaces. Such places are limited.

6.2.2 Evaluation attributes

(1) Yearly costs. These costs include the investment and operation costs.

(2) Probability of water shortage. Since the primary purpose of the development is to provide water, the probability of water storage is considered as a separate attribute.

(3) Water quality. The environmental and social impact of water quality is rated very good, good, fair, or bad.

(4) Energy. The energy index, β, is the ratio between peak energy produced and total energy consumed.

$$\beta = E_p/E \qquad (6.1)$$

where E_p represents the peak energy produced and E the total energy consumed. E_p can be calculated as:

$$E_p = E - E_L \qquad (6.2)$$

where E_L is the energy losses.

It is clear that the energy index should be as high as possible.

(5) Recreation. This attribute may become one of the most important uses by 2030. It is rated very good, good, fair, and bad.

(6) Flood protection. By 2030, 3.5 million peple are expected to live in areas under under flood protection. The performance index of flood protection is measured by probability of flood.

(7) Land and forest use. This index is expressed by the area occupied.

(8) Manpower impact. The effects of the project should include an upgrading of available jobs to provide better income distribution. The performance index for manpower is described qualitatively as very good, good, fair, or bad.

(9) Environmental impact. The water resources system can help to shape a better environment in the vicinity of the project; this process is called 'environmental

architecture'. The performance index for the quality of the environmental architecture is rated very good, good, fair, or bad.

(10) Interational cooperation. It should be possible to link the system with another system implemented at a later date in a neighboring region. This specification has the further advantage of opening interational cooperation possibilities. The index for this attribute may be very easy, easy, fairly difficult, or difficult.

(11) Development possibility. The system should be flexible enough to withstand substantial revisions of storage design and conveyance capacities. The performance index for this attribute may be very good, good, fair, or bad.

(12) Sensitivity. The system should be able to cope with serval types of uncertainties, such as the natural uncertainty inherent in forecasting, the strategic uncertainty due to the future policy, the economic uncertainty, and technological uncertainty. The sensitivity of the system to various uncertainties is rated as not sensitive, sensitive, or very sensitive.

The subjective indices for attributes 3, 5, 8, 9, 10, 11, and 12 were first given by David and Duckstein(5). Keeney(14) assigned '80' for very good, very easy, and not sensitive; '60' for good and easy; '40' for fair, fairly difficult, and sensitive; '20' for bad, difficult, and very sensitive. The decision matrix is given in Table 6.1.

Table 6.1 Decision matrix of Problem 3.

Alternative	Attribute											
	1	2	3	4	5	6	7	8	9	10	11	12
1	99.6	4	80	.7	80	1.0	90	80	80	80	80	80
2	85.7	19	60	.5	60	.5	80	80	60	60	60	80
3	101.1	50	20	.01	40	1.5	80	60	20	40	40	20
4	95.1	50	80	.1	20	.5	60	40	60	20	20	40
5	101.8	50	40	.01	20	2.0	70	40	40	40	40	20

Attribute

1	cost (10^9 ft/yr)
2	probablity of water shortge (%)
3	water quality (subjective)
4	energy factor (= energy produced/energy consumed)
5	recreation (subjective)
6	flood protection (probability of flood %)
7	land and forest use (1,000 ha)
8	manpower impact (subjective)
9	environment impact (subjective)
10	international cooperation (subjective)
11	development possibility (subjective)
12	sensitivity (subjective)

Attributes 1, 2, 6, and 7 are cost attributes; the others are benefit attributes.

6.2.3 Attribute weights

This problem was solved by ELECTRE AND MAUT methods. The weights given by David and Duckstein for concordance analysis are:

$$[2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \quad 1 \quad 2 \quad 2 \quad 1 \quad 1]$$

The order of scaling constants corresponding to their associated attributes given by keeney is:

$$k_2 > k_6 > k_3 = k_{10} > k_8 > k_1 > k_5 = k_9 > k_4 = k_7 > k_{11} = k_{12}$$

The scaling constants were then assessed as follows:

$k_1 = .150$, $k_2 = .243$, $k_3 = .189$ $k_4 = .090$,

$k_5 = .132$, $k_6 = .200$, $k_7 = .090$, $k_8 = .165$,

$k_9 = .132$, $k_{10} = .189$, $k_{11} = .034$, $k_{12} = .034$.

A normalization procedure is employed to normalize these two sets of weights and to yield two sets of normalized weights

$$[.1 \quad .1 \quad .1 \quad .1 \quad .1 \quad .1 \quad .05 \quad .05 \quad .1 \quad .1 \quad .05 \quad .05]$$

and

$$[.091 \quad .147 \quad .115 \quad .080 \quad .121 \quad .055 \quad .100 \quad .080 \quad .115 \quad .021 \quad .021]$$

6.3 Application of the unified approach

6.3.1 Screening phase

According to Table 6.1, attribute 7 of alternative 2 is equal to that of alternative 3, but the other attributes of alternative 2 are preferred to those of alternative 3.

Therefore, A_3 is eliminated by dominance method; A_1, A_2, A_4, and A_5 become the candiates for further evaluation.

6.3.2 Preference ordering phase

Simple additive method:

In this case, the number of benefit attributes is greater than that of cost attributes. Therefore eq. (6.1) for benefit attributes and eq. (6.2) for cost attributes are employed to transform the decision matrix (Table 6.1) into the comparable decision matrix. These two equations are

$$r_{ij} = x_{ij} / x_j^* , \qquad j \in J \tag{6.3}$$

and

$$r_{ij} = x_{ij} / x_j^{min} , \qquad j \in J' \tag{6.4}$$

The comparable decision matrix D is

$$D = \begin{matrix} A_1 \\ A_2 \\ A_4 \\ A_5 \end{matrix} \begin{pmatrix} .86 & 1.0 & 1.0 & 1.0 & 1.0 & .50 & .44 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & ,21 & .75 & .71 & .75 & 1.0 & .50 & 1.0 & .75 & .75 & .75 & 1.0 \\ .90 & .08 & 1.0 & .14 & .25 & 1.0 & 1.0 & .50 & .75 & .25 & .25 & .50 \\ .84 & .08 & .50 & .01 & .25 & .25 & .57 & .50 & .50 & .50 & .50 & .25 \end{pmatrix}$$

Based on the first set of weight, the weighted average values for A_1, A_2, A_4, and A_5 are 0.908, 0.730, 0.550, and 0.385, respectively. The preference order is then

$[A_1, A_2, A_4, A_5]$

Based on the second set of weights, the weighted average values for A_1, A_2, A_4, and A_5 are 0.925, 0.786, 0.567, and 0.392, respectively. The preference order is also

$[A_1, A_2, A_4, A_5]$

Linear assignment method:

The following attribute-wise preference matrix is obtained from the decisin matrix (Table 6.1).

Rank	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}
1st	A_2	A_1	A_1, A_4	A_1	A_1	A_2, A_4	A_4	A_1, A_2	A_1	A_1	A_1	A_1, A_2
2nd	A_4	A_2		A_2	A_2, A_4		A_5		A_2, A_4	A_2	A_2	
3rd	A_1	A_4, A_5	A_2	A_4		A_1	A_2	A_4, A_5		A_5	A_5	A_4
4th	A_5		A_5	A_5	A_5	A_5	A_1		A_5	A_4	A_4	A_5

According to the attribute-wise preference matrix and the two sets of weights, the product-attribute matrices, shown in Appendix 6.1, can be calculated. The optimal permutation matrices for both weighting schemes are identical.

$$P^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The preference ranking for both weighting schemes is

$[A_1, A_2, A_4, A_5]$

TOPSIS method:

The detailed calculation by TOPSIS is shown in Appendix 6.2. Based on the first weighting scheme, the closeness to the ideal solution for A_1, A_2, A_4, and A_5 are .839, .704, .370, and .126, respectively. Based on the second weighting scheme, they are .820, .714, .400, and .129, respectively. Again, the

preference orders based on both weighting schemes are identical. That is

$$[A_1, A_2, A_4, A_5]$$

ELECTRE method:

Appendix 6.3 shows the detailed calculations by the ELECTRE method. The aggregate dominance matrices for both weighting schemes are identical. The aggregate dominance matrix is

$$E = \begin{bmatrix} - & 1 & 1 & 1 \\ 0 & - & 1 & 1 \\ 0 & 0 & - & 1 \\ 0 & 0 & 0 & - \end{bmatrix}$$

Also, the preference order for both weighting schemes are

$$[A_1, A_2, A_4, A_5]$$

The solution by MAUT, including A_3, is given as follows:

Alternative	Utility value
1	.832
2	.831
3	.503
4	.648
5	.521

This results imply that the preference order is

$$[A_1, A_2, A_4, A_5]$$

Because the preference orders for both weighting schemes obtained by these five MADM methods are identical, the process may be terminated here.

6.4 Discussion

Because this problem is to evaluate five water resource alternatives with the aim of selecting one alternative for long-range development, any dominated alternative, which has no possibility of being the best , can be removed. In this cases, one advantage of applying the unified approach is demonstrated by the screening phase, which reduced the number of alternatives by the dominance method to simplify the given decision matrix.

This problem shows that an identical solution is obtained by five different MADM methods in the preference ordering phase, allowing the unified approach to be terminated at this stage.

Chapter 7

Problem 4: Selecting Nuclear Power Plant Sites in the Pacific Northwest

This problem was presented by Keeney and Nair (16), in which the evaluation of the candidate sites was carried out by multiattribute utility theory.

7.1 Introduction

The Washington Public Power Supply System in the State of Washington authorized a study to identify, select, and rank potential new sites in the Pacific Northwest suitable for 3000 MWe that might be required after 1984. This study was conducted by Woodward-Clyde Consulants in 1975.

The study area of this siting problem was composed of approximatedly 170,000 square miles, including the entire State of Washington, the major river basins in Oregon and Idaho, and major river basins of the Oregon coast.

The objective of this study was to recommend sites that would have a high likelihood for successful licensing and that would therefore be most suitable for the detailed site-specific studies.

The approach used by Keeney and Nair to conduct this study consisted of two steps: 1) to identify the candidate sites, 2) to evaluate and rank the candidate sites.

The problem demonstrated here focuses on evaluation of the candidate sites.

7.2 Formulation of the problem

7.2.1 Candidate sites

A screening process was applied to identify the candidate sites. After the screening procedure, there were nine potential candidate sites identified which were involved in the ranking analysis of multiattribute utility theory. These nine sites will be evaluated by the unified approach.

The assessments for each site were based on existing information, site visits, and data developed during the study by Woodward-Clyde Consultants. Each attribute for each site was assessed by specialists in each of the relevant disciplines. Thus, the assessments represent the professional judgement of individuals based on their expertise and on all information currently available concerning the candidate sites.

Site visits indicated that a potential for liquefaction of existing foundation materials under earthquake loading existed at sites 2, 3, and 4. Because the likelihood of liquefaction at these sites could not be ascertained without site-specific studies, two cost estimates were made for each: one if the elimination of liquefaction potential is not necessary, and two, if it is found to be necessary. The additional costs to remove the liquefiable foundation materials and replace them with suitable compacted fill were incorporated in the capital costs associated with site grading and are reflected in the annual differential site costs.

7.2.2 Evaluation attributes

(1) The site population factor. This attribute is an index developed by the U.S. Atomic Energy Commission to indicate the relative human radiational hazard associated with a nuclear facility. The site population factor for site S, denoted SPF(S), is defined by

$$SPF(S) = \sum_{r=1}^{50} P(r)\, r^{-2} \Big/ \sum_{r=1}^{50} Q(r)\, r^{-2} \qquad (7.1)$$

where r is distance in miles from site S, P(r) is the population living between r-1 and r miles of S, and Q(r) is the population that would live between r-1 and r miles of site S. The r^{-2} is meant to account for the decrease in radiation exposure hazard with distance.

(2) Loss of salmonids. Two separate indices were required to measure the salmonid impact adequately. These are the number of fish in a stream, and the percentage of fish loss. The reason for using these two indices, rather than simply the number of fish lost, is that the distribution of the population of salmonids in each stream is distinct. Therefore, the loss of 2,000 fish in a stream of 2,000 is a more important loss than 2000 fish in a stream of 50,000. For the Columbia River (over 300,000 salmonids), only the number of salmonids lost is important, since it is virtually impossible that a large percentage of these fish would lost due to a specific

nuclear power plant.

(3) Biological impacts at site. A subjective scale was developed by two experienced ecologists to measure the biological impact. This is a qualitative scale of potential short- and long-term impacts which could result from the construction and operation of a power plant on a site. The impact range from '0' for no impact to '8' for maximum impact. Three main impacts captured by this scale are those on native timber or sagebrush communities, habitats of rare or endangered species, and productive wetlands.

(4) Socioeconomic impact. The subjective scale for socioeconomic impact was created by a sociologist/planner associated with the siting team. The scale, from 0 (for no impact) to 7 (maximum impact), considers the effects of rapid population growth, overloading of municipal service systems, impacts on cultural institions, alternation of the social order, increased demand for capital improvements, changes in the tax base, impacts on municipal administrative services, alteration of land use patterns, and revenue lags in public financing of capital projects. The purpose of such a scale is to identify a number of impact levels.

(5) Length of transmission intertie through environmentally sensitive areas (i.e., land that was not cleared out,

cultivated, or urbanized). This attribute is measured in miles. The length was assessed from field visits to each of the sites.

(6) Annual differential cost between sites. The differential is calculated relative to the lowest-cost site for which the differential cost is set at zero. Costs such as the major plant components are not included in this attribute since these would be the same for all sites.

Attribute 1, site population factor, and attribute 5, length of power transmission intertie passing through environmentally sensitive areas, were treated deterministically. For attribute 3, biological impact at site, and attribute 4, socioeconomic impact, probability estimates for intervals were used. Normal probability distributions were used for attribute 2 and attribute 6. The resulting data are shown in Table 7.1.

Applying these data in the unified approach, the expected scales are calculated for attributes 3 and 4; the mean values are chosen as the impact index of attribute 6; the utility values represent the index of attribute 2, loss of salmonids, because two separate measures -- the number of salmonids in a stream and percent loss of salmonids -- were required to describe the possible impact on salmonids. The utility functions for attribute 2, given by Keeney and Robilliard (17), are shown in Figure 7.1. Y is defined as the number of salmonids in a stream (in thousands) and Z as the percentage

Table 7.1 Probability assessments of attributes by sites.*

Attribute	Site																	
	S1		S2[b]		S3[b]		S4[b]		S5		S6		S7		S8		S9	
X_1 (SPF)	0.057		0.04		0.025		0.048		0.044		0.023		0.052		0.011		0.018	
X_2 (loss of salmonids)																		
$\bar{x}_2$	8		8		8		15		15		15		15		1		1	
$\sigma^2_{x_2}$	16		16		16		56.25		56.25		56.25		56.25		4.62		3.33	
y	75		75		75		5.5		17		5		3		430		365	
	Range	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*
X_3 (biological impact at site)	1–2	0.9	1–2	0.9	1–2	0.8	2–3	0.2	3–4	0.2	3–4	0.2	1–2	0.3	0–1	0.1	0–1	0.7
	2–3	0.1	2–3	0.1	2–3	0.2	3–4	0.8	4–5	0.5	4–5	0.5	2–3	0.6	1–2	0.5	1–2	0.3
									5–6	0.3	5–6	0.3	3–4	0.1	2–3	0.4		
	Range	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*	*Range*	*Prob.*
X_4 (socioeconomic impact)	1–2	0.2	1–2	0.25	1–2	0.3	2–3	0.2	1–2	0.2	2–3	0.1	2–3	0.2	2–3	0.1	1–2	0.05
	2–3	0.65	2–3	0.55	2–3	0.45	3–4	0.5	2–3	0.45	3–4	0.55	3–4	0.5	3–4	0.4	2–3	0.6
	3–4	0.15	3–4	0.1	3–4	0.15	4–5	0.3	3–4	0.2	4–5	0.3	4–5	0.2	4–5	0.4	3–4	0.2
			4–5	0.1	4–5	0.1			4–5	0.15	5–6	0.05	5–6	0.1	5–6	0.1	4–5	0.15
X_5 (length of inertic through sensitive environments, mi)	1		1		7		6		12		1		0		0		0	
X_6 (differential cost 10^6/y, 1955 dollars)																		
$\bar{x}_6$	2.035		0 (4.929)		1.535 (15.343)		1.933 (7.613)		12.347		17.713		4.834		10.936		11.423	
$\sigma^2_{x_6}$	0.259		0 (1.518)		0.147 (14.713)		0.234 (3.622)		9.528		19.609		1.461		7.475		8.155	

[a] X_1 and X_5 were treated deterministically; probability estimates for intervals were used for X_3 and X_4, and normal distributions were used for X_2 and X_6.
[b] Figures in parentheses include additional costs associated with elimination of possible liquefaction potential.

*Adapted from Ref. 16.

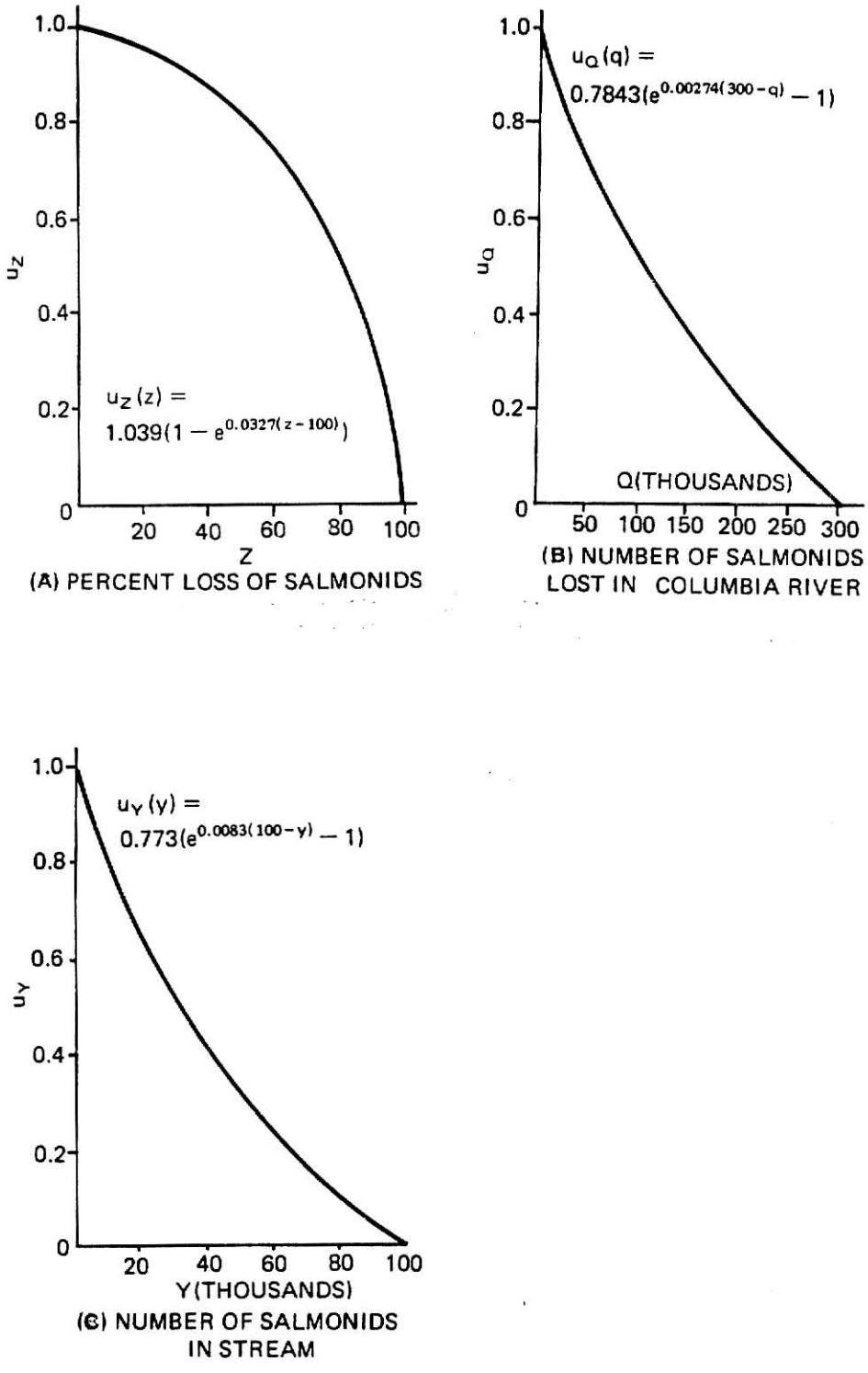


Figure 7.1 utility functions of attribute 2 of problem 4.

(Adapted from Ref. 16.)

lost. Because attribute 2 is a composite of Y and Z, x (y,z) is defined. If a stream has less than 100,000 salmonids, a utility function u_2 was found to be

$$u_2(x) \equiv u_2(y,z) = u_z(z) + u_y(y) - u_z(z)\,u_y(y), \quad y \leqslant 100 \qquad (7.2)$$

u_y and u_z are illustrated in Figure 7.1.a and 7.1.b. For streams with more than 300,000 salmonids, an appropriate utility function was

$$u_2(x) \equiv u_2(y,z) = 0.568 + 0.432u_Q(q), \quad y > 300 \qquad (7.3)$$

where Q, defined as the number of salmonids lost, is Y times Z. The utility function is shown in Figure 7.1.c. The decision matrix (Table 7.2), involved in the unified approach, can then be obtained from Table 7.1.

7.2.3 Attribute weights

The ordering of the scaling constants (k_i) corresponding to their associated attributes (X_i) were determined by five members of the project team as follows:

$$k_6 > k_1 > k_2 > k_4 > k_5 > k_3$$

The scaling constants were then assessed

$$k_1 = 0.358,\ k_2 = 0.218,\ k_3 = 0.013,\ k_4 = 0.104,\ k_5 = 0.059,\ k_6 = 0.400$$

The sum of these scaling constants is not equal to 1 so a normalization procedure is employed to normalize these constants. The set of normalized weights applied in the unified approach is obtained as

$$\underline{W} = (\ .311 \quad .189 \quad .011 \quad .090 \quad .051 \quad .347\ )$$

7.3 Solutions by the unified approach

Table 7.2 Decision matrix of Problem 4 for the unified approach.

Site	Attribute					
	1	2	3	4	5	6
1	.057	.9895	1.60	2.45	1	2.035
2	.040	.9895	1.60	2.55	1	0.000(4.929)
3	.025	.9895	1.60	3.60	7	1.535(15.343)
4	.048	.9978	3.30	2.80	6	1.933(7.613)
5	.044	.9936	4.60	3.80	12	12.347
6	.023	.9980	4.60	3.80	1	17.713
7	.052	.9988	2.30	3.70	0	4.834
8	.011	.9913	1.80	4.00	0	10.936
9	.018	.9926	0.50	2.95	0	11.423

Attribute

1 Site population factor

2 Loss of salmonids

3 Biological impact

4 Socioeconomic impact

5 Length of intertie through sensitive environmental areas.

6 Differential cost

Both the liquefaction and no-liquefaction cases are examined by the unified approach.

7.3.1 Screening phase

A screening phase by means of conjunctive method was used to identify the the candidate sites. Examining these nine candidate sites by dominance method, none of the nine sites is dominated. Hence all of them enter the preference ordering phase.

7.3.2 Preference ordering phase

Simple additive weighting method:

The decision matrix with the comparable scale, derived from Table 7.2 by means of the linear scale transformation, is shown in Table 7.3. The weighted average values for the candidate sites without liquefaction potential are:

A_1=0.5289, A_2=0.7133, A_3=0.7393, A_4=0.5844, A_5=0.3908

A_6=0.4254, A_7=0.5303, A_8=0.6288, A_9=0.6073

The preference order by SAW is obtained as

(A_3, A_2, A_8, A_9, A_4, A_1, A_7, A_6, A_5)

The weighted average values for the candidate sites with liquefaction potential are:

A_1=0.5829, A_2=0.6167, A_3=0.4639, A_4=0.4732, A_5=0.3908,

A_6=0.4254, A_7=0.5303, A_8=0.6288, A_9=0.6073

The preference order is then obtained as

(A_8, A_2, A_9, A_1, A_7, A_4, A_3, A_6, A_5)

Table 7.3 The comparable decision matrix for the problem of selecting nuclear power plant sites

Site	Attribute					
	1	2	3	4	5	6
1	.0000	.9895	.6522	.3875	.9167	.8851
2	.2982	.9895	.6522	.3625	.9167	1.0000 (.7217)
3	.5614	.9895	.6304	.3625	.4167	.9133 (.1388)
4	.1579	.9978	.2826	.1000	.5000	.8909 (.5702)
5	.2281	.9936	.0000	.3000	.0000	.3029
6	.5965	.9980	.0000	.0500	.9167	.0000
7	.0877	.9988	.5000	.0600	1.0000	.7271
8	.8070	.9913	.6087	.0000	1.0000	.3826
9	.6842	.9926	.8261	.2625	1.0000	.3551

* Figures in parentheses are the comparable scale of the differential costs which are associated with liquefaction potential.

Linear assignment method:

The product-attribute matrix of nine candidate sites without liquefaction potential is given in Appendix 5.1.a. The optimal permutation matrix is then obtained as

$$P^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The preference order by LAM is obtained as

$$\{A_2, A_3, A_4, A_1, A_7, A_8, A_9, A_5, A_6\}$$

The product-attribute matrix of nine candidate sites with liquefaction potential is shown in Appendix 5.1.b. The preference order by LAM for this case is

$$\{A_1, A_7, A_2, A_4, A_8, A_9, A_5, A_3, A_6\}$$

TOPSIS method:

The detail calculation by this method is given in Appendix 7.2.a. and 7.2.b. The closeness to the ideal solution and rank orders by TOPSIS method are given as follows:.

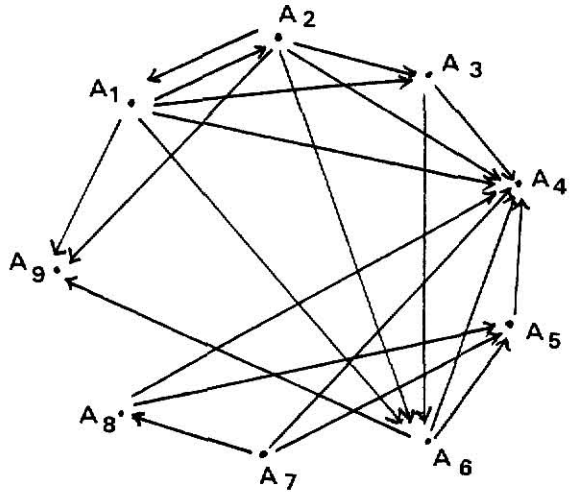
Site	No-liquefaction potential		Liquefaction potential	
	Closeness	Order	Closeness	Order
A1	0.6159	3	0.5812	3
A2	0.7487	2	0.6397	1
A3	0.8218	1	0.3831	7
A4	0.6585	4	0.4870	6
A5	0.2952	9	0.3161	9
A6	0.3025	8	0.3661	8
A7	0.5711	5	0.5558	5
A8	0.5279	6	0.6079	2
A9	0.4860	7	0.5629	4

ELECTRE method:

The detailed calculations by ELETRE method are shown in Appendix 7.3.a and 7.3.b. The aggregation dominance matrix corresponding to the case without liquefaction potential is given as

$$E = \begin{bmatrix} - & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & - & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & - & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & - & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & - & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & - & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & - \end{bmatrix}$$

The outranking relationships of the aggregate matrix can be illustrated by the graphical representation,



From the above graph , A_8, not dominated by any other alternative, is ranked first. A_2, A_3, and A_9 are each dominated only once. A_9 is, however, prefered to A_2 and A_3 according to the concordance indices c_{92} and c_{93}, whose values are .5620. But $d_{93} = .4835$, and $d_{93} = .4202$, are much larger than $\bar{d} = .2760$.

Therefore, it may not be said that A_9 is preferred to A_2 and A_3. In a similar manner, A_1 and A_6 are at the same ranking level. The preference order by ELECTRE method for the case without liquefaction potential is

$$[A_8, (A_2, A_3, A_9), A_4, A_7, (A_1, A_6), A_5]$$

Similarly, the preference order for the case with liquefaction potential is

$$[(A_7, A_8), A_9, A_1, A_2, A_6, A_3, A_4, A_5]$$

This problem was analyzed by multiattribute utility theory. The preference orders by the four efficient MADM methods as well as MAUT for both the no-liquefaction and liquefaction cases are shown in Table 7.4 and Table 7.5, respectively.

7.3.3 Aggregation phase

The results by three aggregation techniques for both the no-liquefaction and liquefaction cases are given in Table 7.6 and Table 7.7, respectively.

7.3.4 Synthesis phase

According to Table 7.6, the partially ordered set, shown on Figure 7.2, is constructed. The poset reflects the synthesized preference order for the no-liquefaction case $[A_3, A_2, A_4, A_1, (A_7, A_8), A_9, (A_5, A_6)]$. According to Figure 7.3, the synthesized result for the liquefaction case is

$$[(A_1, A_2), (A_7, A_8), A_9, A_4, A_3, (A_5, A_6)]$$

Table 7.4 Preference orders of nine candidate sites without liquefaction potential by five MADM methods.

Site	MADM method					Ave.
	SAW	LAM	TOPSIS	ELECTRE	MAUT	
1	6	4	3	7.5	3	4.7
2	2	1	2	3	2	2
3	1	2	1	3	1	1.6
4	5	3	4	5	4	4.2
5	9	8	9	9	6	8.2
6	8	9	8	7.5	9	8.3
7	7	5	5	6	5	5.3
8	3	6	6	1	8	4.8
9	4	7	7	3	7	5.6

Table 7.5 Preference orders of nine candidate sites with liquefaction potential by five MADM methods.

Site	DADM method					Ave.
	SAW	LAM	TOPSIS	ELECTRE	MAUT	
1	4	1	3	4	1	2.6
2	2	3	1	5	2	2.6
3	7	8	7	7	6	7.0
4	6	4	6	8	5	5.8
5	9	7	9	9	4	7.6
6	8	9	8	6	9	8.0
7	5	2	5	1.5	3	3.3
8	1	5	2	1.5	8	3.5
9	3	6	4	3	7	4.6

Table 7.6 Aggregate preference orders of nine candidate sites without liquefaction potential.

Rank	Average	Borda	Copeland
1	A_3	A_3	A_3
2	A_2	A_2	A_2
3	A_4	A_4	A_4
4	A_1	A_1	A_1
5	A_8	A_7	A_7
6	A_7	A_8	A_8
7	A_9	A_9	A_9
8	A_5	A_6	A_6
9	A_6	A_5	A_5

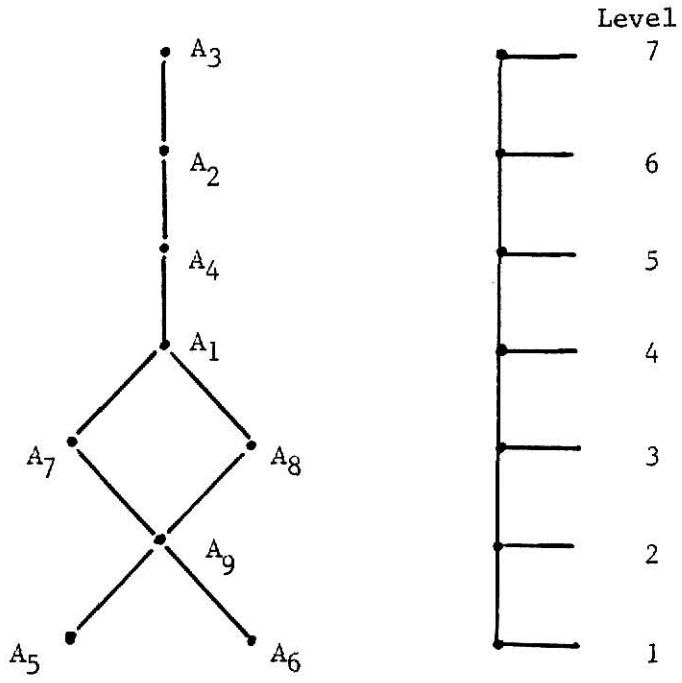


Figure 7.2 The poset reflects the synthesized result of Table 7.6.

Table 7.7 Aggregate preference orders of nine candidate sites with liquefaction potential.

Rank	Average	Borda	Copeland
1	A_1, A_2	A_1, A_2	A_1, A_2
2			
3	A_7	A_8	A_8
4	A_8	A_7	A_7
5	A_9	A_9	A_9
6	A_4	A_4	A_4
7	A_3	A_3	A_3
8	A_5	A_6	A_6
9	A_6	A_5	A_5

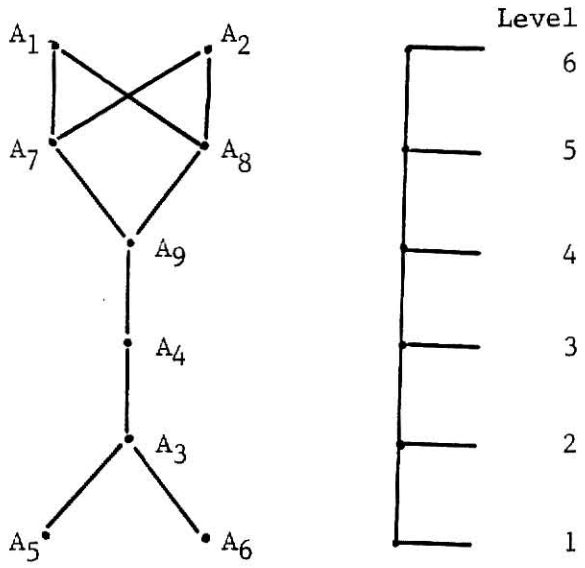


Figure 7.3 The poset reflects the synthesized result of Table 7.7.

7.4 Discussion

In response to both financial and time constraints, a screening process incorporating the conjunctive method was used to identify the candidate sites from a large study area. This process shows the usefulness of the conjunctive method.

These candidate sites were then evaluated by multiattribute utility theory. But assessment of the utility functions is a complex process. The assessment process may be carried out by continually asking questions of the decision maker (15). The problems associated with such an approach are mandatory involvement of the DM in much work and the need of an analyst who is experienced in questioning the DM in order to obtain a satisfactory reflection of real preferences.

The results of the unified approach and MAUT are shown in Table 7.8. In the no-liquefaction potential case, the overall ranking by MAUT and the unified approach are not identical, but solutions by these two approaches both rank A_3 first and A_2 second. In the liquefaction case, A_1 is ranked first and A_2 second by MAUT. This solution is similar to that by the unified approach, in that A_1 and A_2 are ranked equally. However, the information requirement for the unified approach, calling only for the decision matrix and weighting scheme, is much simpler than that for MAUT.

According to Table 7.8, A_2 is shown to be a stable solution even in the liquefaction case. If a detailed site-specific study is necessary, A_1, A_2, and A_3 are recommended.

Table 7.8 The preference orders of nine candidate sites obtained by MAUT and the unified approach.

Site	No-liquefaction potential		Liquefaction potential	
	MAUT	Unified approach	MAUT	Unified approach
1	3	4	1	1.5
2	2	2	2	1.5
3	1	1	6	7
4	4	3	5	6
5	6	8.5	4	8.5
6	9	8.5	9	8.5
7	5	5.5	3	3.5
8	8	5.5	8	3.5
9	7	7	7	5

Chapter 8
Concluding remarks

8.1 Discussion for each problem

This study deals with applications of the unified approach to four real-world MADM problems. The results obtained by four effective MADM methods as well as the unified approach for each problem have been demonstrated in chapters 4, 5, 6, and 7. A short discussion for each problem is given here.

Problem 1. Land use development problem in the Maasvlakte area

The problem has 5 alternatives, 8 alternatives, and 5 sets of weights.

Based on the first, third, and fifth weighting schemes, the preference orders by the four efficient MADM methods are different. Therefore, the aggregation and synthesis phase must be applied in order to derive the unified preference order. In this case, the different weighting schemes yield different results. This analysis is done as if five decision makers had independently analyzed the situation based on their own individual priorities. An eclectic solution among these priorities is achieved by repeating the aggregation and synthesis phases.

Problem 2. Signapore Mass Transit Study

The problem has 6 alternatives, 8 attributes, and 5 sets of weights.

This problem shows how the dominated alternatives can be removed by the dominance method to simplify the decision matrix. The preference orders by the four efficient MADM methods as well as the Multidimensional scaling method are not identical. The aggregation and synthesis phase are used to synthesize these results. A drawback of the linear assignment method, which may produce an unsatisfactory solution, has been found.

Problem 3. Selecting a water resources system in Hungary

The problem has 5 alternatives, 12 attributes, and 2 sets of weights.

One of the five proposed alternatives is eliminated by the dominance method to reduce the number of alternatives from five to four for further analysis by the unified approach. In this case, preference orders by the four MADM methods and multiattribute utility theory are identical. The unified approach is then terminated at the end of the preference ordering phase.

Problem 4. Selecting nuclear power plant sites in the Pacific Northwest

The problem has 9 alternatives, 6 attributes, and 1 set of weights.

The solutions of this problem, obtained by different MADM methods, are also non-identical.

Multiattribute utility theory requires transforming the impact measurements of an attribute into utilities. The

decision maker has to be involved in the process of assessing the utility functions. The information requirement of MAUT is more complex and involved than that of the unified approach.

8.2 Evaluation of the four efficient MADM methods and the unified approach:

These four MADM methods have some common good points: 1) the procedures are well understood by a wide range of people. 2) the computational expense for each method, shown in Table 8.1, is low. The individual advantages and disadvantages are discussed as follows:

Simple weighting method. The weak points of this method are: 1) the comparable form is converted in a rather crude way, 2) the total score for each alternative, obtained by straightforward summation, may ignore interactions among the attributes. Due to these two weaknesses, this method may conceal the significant differences among the alternative outcomes. However, this method is very widely used, as it is quite simple and leads to a unique preference order.

Linear assignment method.

Applying this method, the only information required is attribute-wise rankings and attribute weights. This simple requirement eliminates difficulties encountered in constructing appropriate cardinal indices on attributes. Yet this method may produce an unsatisfactory ranking. This weak point was seen in application of this method to the Singapore

Table 8.1 IBM 360/50 computational time for the four efficient MADM methods.

		MADM problem			
		1	2	3	4
No. of alternatives		5	4	4	9
No. of attributes		8	8	12	6
No. of weighting schemes		5	3	2	1
Computational Time (sec)	SAW	.14	.12	.16	.14
	LAM	.79	.64	.58	.72
	TOPSIS	.52	.59	.60	.52
	ELECTRE	2.01	1.15	1.17	1.40

* The computer program for LAM produces the product-attribute matrix; the others yield the weighted average outcomes, relative closeness to the ideal solution, and aggregate dominance matrix for SAW, TOPSIS, and ELECTRE, respectively.

Mass Transit Study.

TOPSIS method.

This method judges alternatives from both optimistic and pessimistic viewpoints simultaneously, although it considers distances from both ideal and negative-ideal solutions, the procedure of this method is simple and gives an undisputable preference order.

ELECTRE method. This method is capable of providing a relative evaluation of alternatives with respect to each other. Another advantage is that the information contained in the concordance matrix differs from that contained in the discordance matrix, making the information content of concordance indices and discordance indices complementary. The basic problem with this method is that it does not always yield a unique solution. We may increase the threshold value of the concordance index and decrease the threshold value of the discordance index, so that stronger preference relationships are imposed. However, the determination of these threshold values is rather arbitrary.

Unified approach. The unified approach displays the following strong points:

(1) The burden of selecting an appropriate MADM method is lifted.

(2) The number of alternatives may be reduced by dominance and conjuctive methods in the screening phase.

(3) Though including four phases and employing several

methods simultaneously, the logic and process of this approach are quite easily understood and explained.

(4) This approach, providing an eclectric solution as well as the results by four efficient MADM methods, gives a better understanding of ranking relationships among alternatives.

(5) Any other efficient method can be added or replace the less efficient method easily. These changes will not affect the logic of the unified approach.

(6) The only information required is a decision matrix and attribute weights.

A disadvantage of this approach is found in the Borda and Copeland methods, employed in the aggregation phase. These two methods, based on the majority rule, do not consider the strength of preference orders and may produce a weak ordering due to cyclic results.

According to the advantages and disadvantages mentioned above, the four efficient MADM methods and the unified approach are very easily applied by the analyst or the decision makers; information requirements for these methods are not extensive; the computational expense for each method is quite low. Yet different MADM methods usually yield different preference orders, because they judge the same problem from different points of view. For these reasons, the unified approach is worth carrying out in that it may provide better understanding of relationships among alternatives to enhance the likelihood

of good decisions.

8.3 A unified approach to MODM:

There also exists a variety of multiple objective decision making (MODM) methods. A review of various MADM methods and their applications is contained in Hwang and Masud (12). The MADM models try to design the 'best' alternative based on their own assumptions and points of view, such that results obtained by different MODM methods are often not identical. Thus, the concept of a unified approach may also be applied to MADM problem. A unified approach, composed of screening phase, preference solving phase, and aggregation phase, is therefore proposed.

Mathematically, the MODM methods can be formed as:

$$\max \; [\, f_1(\underline{x}),\; f_2(\underline{x}),\; \ldots\ldots,\; f_k(\underline{x}) \,]$$
$$\text{s.t.} \quad x \in \underline{X} \qquad (8.1)$$

A process diagram for unified approach to MODM is shown in Figure 8.1 and described as follows:

Screening phase: In this phase, non-conflicting objective functions will be removed to reduce the number of objective functions. A payoff table is constructed by solving the ideal solution for each objective function.

Preference solving phase: An MODM problem will be solved by appropriate methods based on the preference information given. If the DM does not give any preference information, the

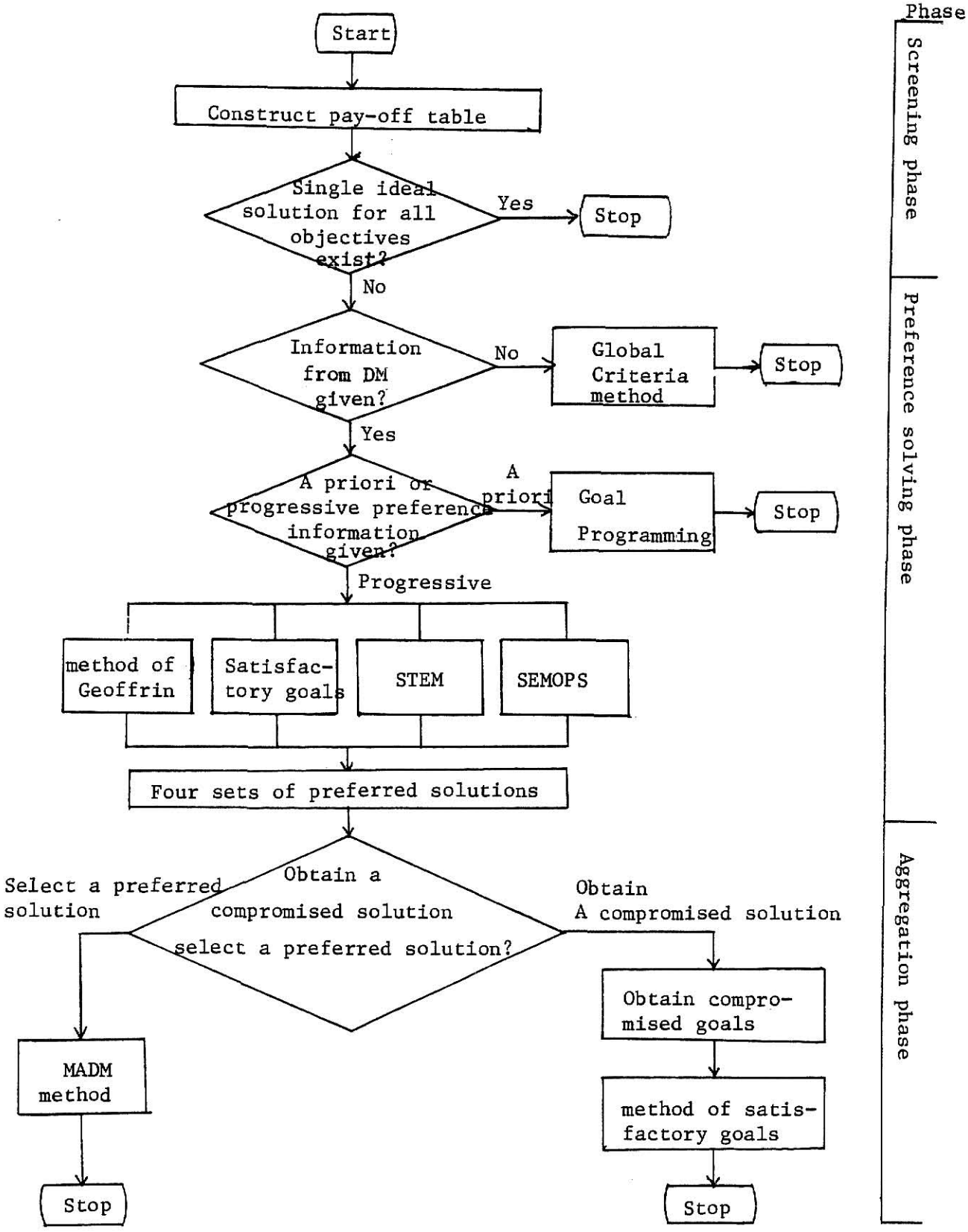


Figure 8.1 A process diagram for a unified approach to MODM.

global criterion method is employed in this phase. If the DM gives priorities for the objective functions, the MODM problem may be solved by goal programming. Four MODM methods - method of satisfactory goals, method of Goeffrin, STEM method, and SEMOPS method - are selected to obtain the preferred solution, if the DM can be involved in the decision process progressively for each iteration. If the preferred solution, obtained by the four MODM methods, do differ, a aggregation procedure will be included in the next step.

Aggregation phase: This phase attempts to obtain a compromised solution , or select a preferred solution by the MADM methods among the results yielded in the previous phase.

(1) Obtain a Compromised solution. This compromised solution may be obtained by the following steps.

Step 1. Obtain a set of compromised goals:

A solution, which has the shortest total distance to the preferred solutions, can be obtained by solving the following problem:

$$\min d = \sum_{i=1}^{n} \| f^0 - f^n \|$$
$$\text{s.t.} \quad f_k^{min} \leqslant f_k^0 \leqslant f_k^{max} \qquad (8.2)$$

where f_k^{min} = minimum value of the kth objective among the preferred solutions.

f_k^{max} = maximum value of the kth objective among the preferred solutions.

$\underline{f}^n$ = Preferred solution by the nth method.

$\underline{f}^0$ = compromised goals.

The solution, which may not be a preferred solution, can be considered as a set of compromised goal levels. In the next step, the objective which receives the most attention may be imposed.

Step 2. Optimize the most important objective:

The most important objective f_M is maximized subject to the original constraints and additional constraints formed by the rest of the objective functions:

$$\begin{aligned} &\max\ f_M(\underline{x}) \\ &\text{s.t.}\quad x \in \underline{X} \qquad (8.3) \\ &\qquad f_j \geq f_j^0,\ j = 1,2,\ldots,\ k,\ \text{and}\ j \neq M \end{aligned}$$

The final compromised solution $(f_1^0, f_2^0, \ldots, f_M, \ldots, f_k^0)$ is then obtained.

(2) Select a preferred solution:

The aggregation procedure may be carried out another way. Let each solution constitute an alternative and the values of the objective constitute the attributes of each alternative. Then a preferred solution can be selected from the four solutions by employing a MADM method or the unified approach to MADM.

8.4 Future study:

A computer program including the whole procedure of the unified approach to MADM by modifying the LAM and ELECTRE

method could be constructed to obtain the synthesis solution by running the program only once.

The Borda and Copeland methods, employed in the aggregation phase, are very simple. However, these methods do not always give strong preference orders due to cyclic results. This weakness could be countered by utilizing group decision methods, which are efficient and produce unique solutions.

Since only four real-life problems have been tested by the unified approach, employing four MADM methods, we cannot compare in depth the efficiency of the unified approach with that of those which were not applied in this study. A more comprehensive evaluation of MADM methods will be reached by dealing with additional real-life problems.

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Appendix 1

Four alternative definitions of discordance index

Appendix 1. Four alternative definitions of discordance index

A discordance index is designed to measure the degree to which a certain alternative p is less desirable than a competing alternative q. Four equations of a discordance index have been introduced.

(1) The definition suggested by Hwang and Yoon (12) is:

$$d = \frac{\max_{j \in D_{pq}} \left| v_{pj} - v_{pj} \right|}{\max_{j \in J} \left| v_{pj} - v_{qj} \right|}$$

This discordance index (between alternatives p and q) represents the maximum differences between the pairwise weighted outcomes of all attributes belonging to the discordance set (D_{pq}), divided by the maximum dicordance difference between alternative p and q.

(2) The second one, defined by Guigou (10), is:

$$d_{pq} = \frac{\max_{j \in D_{pq}} \left| v_{pj} - v_{qj} \right|}{\max_{\substack{j \in D_{pq} \\ (p,q)}} \left| v_{pj} - v_{qj} \right|}$$

Where the expression (p,q) stands for all values of p and q. This discordance index between alternatives p and q is equal to the maximum difference the pairwise weighted outcomes with respect to all attributes belonging to the discordance set D_{pq}, divided by the

maximum difference between all arbitrary pairs of alternatives.

(3) The third denifition, suggested by Delft and Nijkamp (6), is:

$$d_{pq} = \frac{\left(\sum_{j \in D_{pq}} \left| v_{pj} - v_{qj} \right| \right) \Big/ t_{pq}}{\max_{\substack{j \in D_{pq} \\ (p,q)}} \left| v_{pj} - v_{qj} \right|}$$

where t_{pq} represents the number of elements of D .This discordance index measures the ratio of the average discordance difference between alternatives p and q with repect to the overall maximum discordance difference.

(4) The fourth one, given by Nijkamp (20), is:

$$d_{pq} = \frac{\sum_{j \in D_{pq}} \left| v_{pj} - v_{qj} \right|}{\max_{(p,q)} \left(\sum_{j \in J} \left| v_{pj} - v_{qj} \right| \right)}$$

in this case, the denominator is defined as the maximum of the sum total of the divergence among two alternatives. This definition takes into account the weighted discrepancy between alternatives p and q with respectto all those attributes for which alternative p does not dominate alternative q.

Appendix 2 Detailed calculations of problem 1

==== THE LINEAR ASSINGMENT METHOD ====

*** INDUSTRIAL LAND USE PROBLEM IN THE MAASVLAKTE ***

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	109.15	43.30	540.	-8.10	0.00	86.70	-2.10	25.00
A2	199.15	65.60	940.	-12.70	-14.48	82.40	-1.70	55.00
A3	476.75	65.60	5340.	-12.50	-24.80	85.00	-9.20	33.00
A4	1077.40	93.30	12820.	-13.10	-55.78	85.40	-9.50	55.00
A5	566.75	87.80	5740.	-13.00	-39.28	81.10	-8.60	63.00

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.375	0.125	0.000	0.000	0.500
A2	0.125	0.188	0.250	0.438	0.000
A3	0.000	0.125	0.563	0.313	0.000
A4	0.375	0.188	0.063	0.000	0.375
A5	0.125	0.375	0.125	0.250	0.125

- -

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.100	0.067	0.233	0.200	0.033	0.133	0.133	0.100

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.366	0.133	0.000	0.000	0.500
A2	0.133	0.083	0.283	0.499	0.000

A3	0.000	0.200	0.532	0.266	0.000
A4	0.400	0.183	0.050	0.000	0.366
A5	0.100	0.400	0.133	0.233	0.133

- - - -

0.097 0.064 0.161 0.161 0.226 0.097 0.129 0.06

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.484	0.129	0.000	0.000	0.386
A2	0.129	0.258	0.225	0.387	0.000
A3	0.000	0.161	0.613	0.225	0.000
A4	0.322	0.129	0.032	0.000	0.516
A5	0.064	0.322	0.129	0.387	0.097

- - - -

**** TOPSIS METHOD ****

*** INDUSTRIAL LAND USE PROBLEM IN THE MAASVLAKTE ***

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	109.15	43.30	540.	-8.10	0.00	86.70	-2.10	25.00
A2	199.15	65.60	940.	-12.70	-14.48	82.40	-1.70	55.00
A3	476.75	65.60	5340.	-12.50	-24.80	85.00	-9.20	33.00
A4	1077.40	93.30	12820.	-13.10	-55.78	85.40	-9.50	55.00
A5	566.75	87.80	5740.	-13.00	-39.28	81.10	-8.60	63.00

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.082	0.264	0.036	-0.301	0.000	0.461	-0.131	0.231
A2	0.150	0.400	0.062	-0.472	-0.196	0.438	-0.106	0.508
A3	0.359	0.400	0.354	-0.465	-0.335	0.452	-0.575	0.305
A4	0.812	0.569	0.851	-0.487	-0.754	0.454	-0.594	0.508
A5	0.427	0.535	0.381	-0.483	-0.531	0.431	-0.537	0.582

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.010	0.033	0.004	-0.038	0.000	0.058	-0.016	0.029
A2	0.019	0.050	0.008	-0.059	-0.024	0.055	-0.013	0.063
A3	0.045	0.050	0.044	-0.058	-0.042	0.056	-0.072	0.038
A4	0.101	0.071	0.106	-0.061	-0.094	0.057	-0.074	0.063
A5	0.053	0.067	0.048	-0.060	-0.066	0.054	-0.067	0.073

	IDEAL SOL.	N-IDEAL SOL.
X1	0.1015	0.0103
X2	0.0711	0.0330
X3	0.1064	0.0045
X4	-0.0376	-0.0609
X5	0.0000	-0.0942
X6	0.0576	0.0539
X7	-0.0133	-0.0742
X8	0.0727	0.0288

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1486	0.1130	0.4319
A2	0.1347	0.1007	0.4278
A3	0.1196	0.0769	0.3914
A4	0.1149	0.1461	0.5598
A5	0.1167	0.0873	0.4279

- -

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.100	0.067	0.233	0.200	0.033	0.133	0.133	0.100

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.008	0.018	0.008	-0.060	0.000	0.061	-0.017	0.023
A2	0.015	0.027	0.015	-0.094	-0.006	0.058	-0.014	0.051
A3	0.036	0.027	0.083	-0.093	-0.011	0.060	-0.076	0.030
A4	0.081	0.038	0.198	-0.097	-0.025	0.060	-0.079	0.051
A5	0.043	0.036	0.089	-0.097	-0.018	0.057	-0.071	0.058

	IDEAL SOL.	N-IDEAL SOL.
X1	0.0812	0.0082
X2	0.0381	0.0177
X3	0.1983	0.0084
X4	-0.0602	-0.0974
X5	0.0000	-0.0249
X6	0.0613	0.0573

X7	-0.0141	-0.0789
X8	0.0582	0.0231

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.2075	0.0761	0.2685
A2	0.1988	0.0741	0.2714
A3	0.1463	0.0815	0.3577
A4	0.0791	0.2064	0.7229
A5	0.1357	0.0966	0.4158

- -

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.167	0.167	0.125	0.083	0.042	0.125	0.042	0.250

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.014	0.044	0.004	-0.025	0.000	0.058	-0.006	0.058
A2	0.025	0.067	0.008	-0.039	-0.008	0.055	-0.004	0.127
A3	0.060	0.067	0.044	-0.039	-0.014	0.056	-0.024	0.076
A4	0.136	0.095	0.106	-0.040	-0.032	0.057	-0.025	0.127
A5	0.071	0.089	0.048	-0.040	-0.022	0.054	-0.023	0.145

	IDEAL SOL.	N-IDEAL SOL.
X1	0.1356	0.0137
X2	0.0950	0.0441
X3	0.1064	0.0045
X4	-0.0250	-0.0404
X5	0.0000	-0.0317
X6	0.0576	0.0539
X7	-0.0045	-0.0249
X8	0.1454	0.0577

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1885	0.0404	0.1765
A2	0.1528	0.0801	0.3440

A3	0.1262	0.0700	0.3568
A4	0.0447	0.1806	0.8015
A5	0.0932	0.1225	0.5681

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.097	0.064	0.161	0.161	0.226	0.097	0.129	0.064

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.008	0.017	0.006	-0.048	0.000	0.045	-0.017	0.015
A2	0.015	0.026	0.010	-0.076	-0.044	0.042	-0.014	0.032
A3	0.035	0.026	0.057	-0.075	-0.076	0.044	-0.074	0.019
A4	0.079	0.036	0.137	-0.078	-0.170	0.044	-0.077	0.032
A5	0.041	0.034	0.061	-0.078	-0.120	0.042	-0.069	0.037

	IDEAL SOL.	N-IDEAL SOL.
X1	0.0788	0.0080
X2	0.0364	0.0169
X3	0.1370	0.0058
X4	-0.0485	-0.0784
X5	0.0000	-0.1703
X6	0.0447	0.0418
X7	-0.0137	-0.0766
X8	0.0372	0.0148

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1521	0.1829	0.5461
A2	0.1520	0.1425	0.4839
A3	0.1372	0.1114	0.4482
A4	0.1841	0.1514	0.4514
A5	0.1596	0.0872	0.3533

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===== ELECTRE METHOD =====

*** INDUSTRIAL LAND USE PROBLEM IN THE MAASVLAKTE ***

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	109.15	43.30	540.	-8.10	0.00	86.70	-2.10	25.00
A2	199.15	65.60	940.	-12.70	-14.48	82.40	-1.70	55.00
A3	476.75	65.60	5340.	-12.50	-24.80	85.00	-9.20	33.00
A4	1077.40	93.30	12820.	-13.10	-55.78	85.40	-9.50	55.00
A5	566.75	87.80	5740.	-13.00	-39.28	81.10	-8.60	63.00

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.082	0.264	0.036	-0.301	0.000	0.461	-0.131	0.231
A2	0.150	0.400	0.062	-0.472	-0.196	0.438	-0.106	0.508
A3	0.359	0.400	0.354	-0.465	-0.335	0.452	-0.575	0.305
A4	0.812	0.569	0.851	-0.487	-0.754	0.454	-0.594	0.508
A5	0.427	0.535	0.381	-0.483	-0.531	0.431	-0.537	0.582

CONCORDANCE SETS

	X1	X2	X3	X4	X5	X6	X7	X8
C12	0	0	0	1	1	1	0	0
C13	0	0	0	1	1	1	1	0
C14	0	0	0	1	1	1	1	0
C15	0	0	0	1	1	1	1	0
C21	1	1	1	0	0	0	1	1
C23	0	1	0	0	1	0	1	1
C24	0	0	0	1	1	0	1	1
C25	0	0	0	1	1	1	1	0
C31	1	1	1	0	0	0	0	1
C32	1	1	1	1	0	1	0	0
C34	0	0	0	1	1	0	1	0
C35	0	0	0	1	1	1	0	0
C41	1	1	1	0	0	0	0	1
C42	1	1	1	0	0	1	0	1
C43	1	1	1	0	0	1	0	1
C45	1	1	1	0	0	1	0	0
C51	1	1	1	0	0	0	0	1
C52	1	1	1	0	0	0	0	1
C53	1	1	1	0	0	0	1	1
C54	0	0	0	1	1	0	1	1

DISCORDANCE SETS

	X1	X2	X3	X4	X5	X6	X7	X8
D12	1	1	1	0	0	0	1	1
D13	1	1	1	0	0	0	0	1
D14	1	1	1	0	0	0	0	1
D15	1	1	1	0	0	0	0	1
D21	0	0	0	1	1	1	0	0
D23	1	0	1	1	0	1	0	0
D24	1	1	1	0	0	1	0	0
D25	1	1	1	0	0	0	0	1
D31	0	0	0	1	1	1	1	0
D32	0	0	0	0	1	0	1	1
D34	1	1	1	0	0	1	0	1
D35	1	1	1	0	0	0	1	1
D41	0	0	0	1	1	1	1	0
D42	0	0	0	1	1	0	1	0
D43	0	0	0	1	1	0	1	0
D45	0	0	0	1	1	0	1	1
D51	0	0	0	1	1	1	1	0
D52	0	0	0	1	1	1	1	0
D53	0	0	0	1	1	1	0	0
D54	1	1	1	0	0	1	0	0

* SYMBOL 1 INDICATES THAT AN ATTRIBUTE IS INCLUDED IN THE CORRESPONDING CONCORDANCE/DISCONCORDANCE SETS

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.010	0.033	0.004	-0.038	0.000	0.058	-0.016	0.029
A2	0.019	0.050	0.008	-0.059	-0.024	0.055	-0.013	0.063
A3	0.045	0.050	0.044	-0.058	-0.042	0.056	-0.072	0.038
A4	0.101	0.071	0.106	-0.061	-0.094	0.057	-0.074	0.063
A5	0.053	0.067	0.048	-0.060	-0.066	0.054	-0.067	0.073

** CONCORDANCE INDEX MATRIX **

0.0000	0.3750	0.5000	0.5000	0.5000
0.6250	0.0000	0.5000	0.5000	0.5000
0.5000	0.6250	0.0000	0.3750	0.3750
0.5000	0.6250	0.6250	0.0000	0.5000
0.5000	0.5000	0.6250	0.5000	0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5125

** CONCORDANCE DOMINANCE MATRIX **

0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	1	1	0	0
0	0	1	0	0

** DISCORDANCE INDEX MATRIX **

0.0000	0.1506	0.2278	0.6015	0.3712
0.1102	0.0000	0.1478	0.4625	0.2277

0.2691	0.2294	0.0000	0.3743	0.1539
0.3985	0.2999	0.1300	0.0000	0.1009
0.3249	0.2218	0.0665	0.2577	0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.4419

AVERAGE OF DISCORDANCE INDICES = 0.2563

** DISCORDANCE DOMINANCE MATRIX **

0	1	1	0	0
1	0	1	0	1
0	1	0	0	1
0	0	1	0	1
0	1	1	0	0

** AGGREGATE DOMINANCE MATRIX **

0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	1	0	0

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.100	0.067	0.233	0.200	0.033	0.133	0.133	0.100

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.008	0.019	0.008	-0.060	0.000	0.061	-0.017	0.023
A2	0.015	0.027	0.015	-0.094	-0.006	0.058	-0.014	0.051
A3	0.036	0.027	0.083	-0.093	-0.011	0.060	-0.076	0.030
A4	0.081	0.038	0.198	-0.097	-0.025	0.060	-0.079	0.051

A5 0.043 0.036 0.089 -0.097 -0.018 0.057 -0.071 0.058

** CONCORDANCE INDEX MATRIX **

0.0000	0.3660	0.4990	0.4990	0.4990
0.6330	0.0000	0.3330	0.4660	0.4990
0.5000	0.7330	0.0000	0.3660	0.3660
0.5000	0.6330	0.6330	0.0000	0.5330
0.5000	0.5000	0.6330	0.4660	0.0000

AVERAGE OF CONCORDANCE INDEICES = 0.5078

** CONCORDANCE DOMINANCE MATRIX **

0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	1	1	0	1
0	0	1	0	0

** DISCORDANCE INDEX MATRIX **

0.0000	0.1219	0.2720	0.7142	0.3862
0.1003	0.0000	0.2119	0.6048	0.2719
0.2388	0.2003	0.0000	0.4429	0.1257
0.2858	0.1980	0.0477	0.0000	0.0527
0.2570	0.1643	0.0297	0.3520	0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.4355

AVERAGE OF DISCORDANCE INDICES = 0.2539

** DISCORDANCE DOMINANCE MATRIX **

0	1	0	0	0
1	0	1	0	0
1	1	0	0	1
0	1	1	0	1

0 1 1 0 0

** AGGREGATE DOMINANCE MATRIX **

0 0 0 0 0

1 0 0 0 0

0 1 0 0 0

0 1 1 0 1

0 0 1 0 0

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.167	0.167	0.125	0.083	0.042	0.125	0.042	0.250

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.014	0.044	0.004	-0.025	0.000	0.058	-0.006	0.058
A2	0.025	0.067	0.008	-0.039	-0.008	0.055	-0.004	0.127
A3	0.060	0.067	0.044	-0.039	-0.014	0.056	-0.024	0.076
A4	0.136	0.095	0.106	-0.040	-0.032	0.057	-0.025	0.127
A5	0.071	0.089	0.048	-0.040	-0.022	0.054	-0.023	0.145

** CONCORDANCE INDEX MATRIX **

0.0000 0.2500 0.2920 0.2920 0.2920

0.7510 0.0000 0.5010 0.4170 0.2920

0.7090 0.6670 0.0000 0.1670 0.2500

0.7090 0.8340 0.8340 0.0000 0.5840

0.7090 0.7090 0.7510 0.4170 0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5213

** CONCORDANCE DOMINANCE MATRIX **

0 0 0 0 0

```
  1   0   0   0   0
  1   1   0   0   0
  1   1   1   0   1
  1   1   1   0   0

  **  DISCORDANCE INDEX MATRIX  **
 0.0000  0.2617  0.3094  0.8362  0.5684
 0.0614  0.0000  0.1794  0.5819  0.3092
 0.1153  0.1856  0.0000  0.5274  0.2628
 0.1638  0.1098  0.0492  0.0000  0.0742
 0.1415  0.0826  0.0300  0.3197  0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =  0.4113
AVERAGE OF DISCORDANCE INDICES =  0.2585

  **  DISCORDANCE DOMINANCE MATRIX  **
  0   0   0   0   0
  1   0   1   0   0
  1   1   0   0   0
  1   1   1   0   1
  1   1   1   0   0

  **  AGGREGATE DOMINANCE MATRIX  **
  0   0   0   0   0
  1   0   0   0   0
  1   1   0   0   0
  1   1   1   0   1
  1   1   1   0   0

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  **  WEIGHTS  **
```

	X1	X2	X3	X4	X5	X6	X7	X8
	0.095	0.095	0.095	0.095	0.381	0.095	0.095	0.048

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.008	0.025	0.003	-0.029	0.000	0.044	-0.012	0.011
A2	0.014	0.038	0.006	-0.045	-0.075	0.042	-0.010	0.024
A3	0.034	0.038	0.034	-0.044	-0.128	0.043	-0.055	0.015
A4	0.077	0.054	0.081	-0.046	-0.287	0.043	-0.056	0.024
A5	0.041	0.051	0.036	-0.046	-0.202	0.041	-0.051	0.028

** CONCORDANCE INDEX MATRIX **

0.0000	0.5710	0.6660	0.6660	0.6660
0.4280	0.0000	0.6190	0.6190	0.6660
0.3330	0.4750	0.0000	0.5710	0.5710
0.3330	0.4280	0.4280	0.0000	0.3800
0.3330	0.3330	0.4280	0.6190	0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5066

** CONCORDANCE DOMINANCE MATRIX **

0	1	1	1	1
0	0	1	1	1
0	0	0	1	1
0	0	0	0	0
0	0	0	1	0

** DISCORDANCE INDEX MATRIX **

0.0000	0.0698	0.1357	0.3511	0.2009
0.1726	0.0000	0.0922	0.2886	0.1356
0.3458	0.1995	0.0000	0.2158	0.0718
0.6489	0.4835	0.3034	0.0000	0.1749
0.4846	0.3164	0.1454	0.1608	0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.5384

AVERAGE OF DISCORDANCE INDICES = 0.2499

** DISCORDANCE DOMINANCE MATRIX **

0	1	1	0	1
1	0	1	0	1
0	1	0	1	1
0	0	0	0	1
0	0	1	1	0

** AGGREGATE DOMINANCE MATRIX **

0	1	1	0	1
0	0	1	0	1
0	0	0	1	1
0	0	0	0	0
0	0	0	1	0

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.097	0.064	0.161	0.161	0.226	0.097	0.129	0.064

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.008	0.017	0.006	-0.048	0.000	0.045	-0.017	0.015
A2	0.015	0.026	0.010	-0.076	-0.044	0.042	-0.014	0.032
A3	0.035	0.026	0.057	-0.075	-0.076	0.044	-0.074	0.019
A4	0.079	0.036	0.137	-0.078	-0.170	0.044	-0.077	0.032
A5	0.041	0.034	0.061	-0.078	-0.120	0.042	-0.069	0.037

** CONCORDANCE INDEX MATRIX **

0.0000	0.4840	0.6130	0.6130	0.6130

```
  0.5150  0.0000  0.4830  0.5800  0.6130
  0.3860  0.5800  0.0000  0.5160  0.4840
  0.3860  0.4830  0.4830  0.0000  0.4190
  0.3860  0.3860  0.5150  0.5800  0.0000

AVERAGE OF CONCORDANCE INDICES  =  0.5059

  **  CONCORDANCE DOMINANCE MATRIX  **
  0  0  1  1  1
  1  0  0  1  1
  0  1  0  1  0
  0  0  0  0  0
  0  0  1  1  0

  **  DISCORDANCE INDEX MATRIX  **
  0.0000  0.0810  0.1833  0.4787  0.2578
  0.1480  0.0000  0.1398  0.4072  0.1832
  0.3204  0.2100  0.0000  0.2958  0.0842
  0.5213  0.3829  0.2013  0.0000  0.1260
  0.4092  0.2677  0.0985  0.2348  0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =  0.4998
AVERAGE OF DISCORDANCE INDICES =  0.2516

  **  DISCORDANCE DOMINANCE MATRIX  **
  0  1  1  0  0
  1  0  1  0  1
  0  1  0  0  1
  0  0  1  0  1
  0  0  1  1  0

  **  AGGREGATE DOMINANCE MATRIX  **
  0  0  1  0  0
```

1 0 0 0 1

0 1 0 0 0

0 0 0 0 0

0 0 1 1 0

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Appendix 3 Detailed calculations of problem 2

==== THE LINEAR ASSINGMENT METHOD ====

**** SINGAPORE MASS TRANSIT SYSTEMS ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	2.000	2.000	2.000	5.000	1.000	5.000	1.000	2.000
A2	5.000	4.000	4.000	4.000	2.000	2.000	4.000	4.000
A3	4.000	5.000	5.000	2.000	3.000	2.000	3.000	3.000
A4	5.000	4.000	4.000	3.000	3.000	3.000	4.000	4.000

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.063	0.063	0.063	0.125	0.188	0.188	0.188	0.125

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.313	0.000	0.000	0.688
A2	0.188	0.375	0.344	0.094
A3	0.219	0.094	0.469	0.219
A4	0.281	0.531	0.188	0.000

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.250	0.000	0.000	0.750
A2	0.188	0.438	0.313	0.063
A3	0.313	0.063	0.438	0.188
A4	0.250	0.500	0.250	0.000

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**** TOPSIS METHCD ****

**** SINGAPORE MASS TRANSIT SYSTEMS ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	2.000	2.000	2.000	5.000	1.000	5.000	1.000	2.000
A2	5.000	4.000	4.000	4.000	2.000	2.000	4.000	4.000
A3	4.000	5.000	5.000	2.000	3.000	2.000	3.000	3.000
A4	5.000	4.000	4.000	3.000	3.000	3.000	4.000	4.000

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.400	0.400	0.400	1.000	0.200	1.000	0.200	0.400
A2	1.000	0.800	0.800	0.800	0.400	0.400	0.800	0.800
A3	0.800	1.000	1.000	0.400	0.600	0.400	0.600	0.600
A4	1.000	0.800	0.800	0.600	0.600	0.600	0.800	0.800

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.063	0.063	0.063	0.125	0.188	0.188	0.188	0.125

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.025	0.025	0.025	0.125	0.037	0.188	0.037	0.050
A2	0.063	0.050	0.050	0.100	0.075	0.075	0.150	0.100
A3	0.050	0.063	0.063	0.050	0.112	0.075	0.112	0.075
A4	0.063	0.050	0.050	0.075	0.112	0.112	0.150	0.100

	IDEAL SOL.	N-IDEAL SOL.
X1	0.0625	0.0250
X2	0.0625	0.0250

X3	0.0625	0.0250
X4	0.1250	0.0500
X5	0.1125	0.0375
X6	0.1875	0.0750
X7	0.1500	0.0375
X8	0.1000	0.0500

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1581	0.1352	0.4610
A2	0.1225	0.1474	0.5461
A3	0.1431	0.1237	0.4638
A4	0.0919	0.1596	0.6347

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.167	0.250	0.167	0.083	0.083	0.083	0.083	0.083

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.067	0.100	0.067	0.083	0.017	0.083	0.017	0.033
A2	0.167	0.200	0.134	0.066	0.033	0.033	0.066	0.066
A3	0.134	0.250	0.167	0.033	0.050	0.033	0.050	0.050
A4	0.167	0.200	0.134	0.050	0.050	0.050	0.066	0.066

	IDEAL SOL.	N-IDEAL SOL.
X1	0.1670	0.0668
X2	0.2500	0.1000
X3	0.1670	0.0668
X4	0.0830	0.0332
X5	0.0498	0.0166
X6	0.0830	0.0332
X7	0.0664	0.0166
X8	0.0664	0.0332

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.2174	0.0704	0.2447
A2	0.0815	0.1716	0.6780

A3	0.0814	0.1987	0.7094
A4	0.0763	0.1724	0.6933

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** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.050	0.050	0.050	0.125	0.025	0.125	0.025	0.050
A2	0.125	0.100	0.100	0.100	0.050	0.050	0.100	0.100
A3	0.100	0.125	0.125	0.050	0.075	0.050	0.075	0.075
A4	0.125	0.100	0.100	0.075	0.075	0.075	0.100	0.100

	IDEAL SOL.	N-IDEAL SOL.
X1	0.1250	0.0500
X2	0.1250	0.0500
X3	0.1250	0.0500
X4	0.1250	0.0500
X5	0.0750	0.0250
X6	0.1250	0.0500
X7	0.1000	0.0250
X8	0.1000	0.0500

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1658	0.1061	0.3901
A2	0.0901	0.1479	0.6213
A3	0.1146	0.1392	0.5485
A4	0.0791	0.1500	0.6549

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===== ELECTRE METHOD =====

**** SINGAPORE MASS TRANSIT SYSTEMS ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	2.000	2.000	2.000	5.000	1.000	5.000	1.000	2.000
A2	5.000	4.000	4.000	4.000	2.000	2.000	4.000	4.000
A3	4.000	5.000	5.000	2.000	3.000	2.000	3.000	3.000
A4	5.000	4.000	4.000	3.000	3.000	3.000	4.000	4.000

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.400	0.400	0.400	1.000	0.200	1.000	0.200	0.400
A2	1.000	0.800	0.800	0.800	0.400	0.400	0.800	0.800
A3	0.800	1.000	1.000	0.400	0.600	0.400	0.600	0.600
A4	1.000	0.800	0.800	0.600	0.600	0.600	0.800	0.800

CONCORDANCE SETS

	X1	X2	X3	X4	X5	X6	X7	X8
C12	0	0	0	1	0	1	0	0
C13	0	0	0	1	0	1	0	0
C14	0	0	0	1	0	1	0	0
C21	1	1	1	0	1	0	1	1
C23	1	0	0	1	0	1	1	1
C24	1	1	1	1	0	0	1	1
C31	1	1	1	0	1	0	1	1
C32	0	1	1	0	1	1	0	0
C34	0	1	1	0	1	0	0	0
C41	1	1	1	0	1	0	1	1
C42	1	1	1	0	1	1	1	1
C43	1	0	0	1	1	1	1	1

DISCORDANCE SETS

	X1	X2	X3	X4	X5	X6	X7	X8
D12	1	1	1	0	1	0	1	1
D13	1	1	1	0	1	0	1	1
D14	1	1	1	0	1	0	1	1
D21	0	0	0	1	0	1	0	0
D23	0	1	1	0	1	0	0	0
D24	0	0	0	0	1	1	0	0
D31	0	0	0	1	0	1	0	0
D32	1	0	0	1	0	0	1	1
D34	1	0	0	1	0	1	1	1
D41	0	0	0	1	0	1	0	0
D42	0	0	0	1	0	0	0	0
D43	0	1	1	0	0	0	0	0

* SYMBOL 1 INDICATES THAT AN ATTRIBUTE IS INCLUDED IN THE CORRESPONDING CONCORDANCE/DISCONCORDANCE SETS

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.063	0.063	0.063	0.125	0.188	0.188	0.188	0.125

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.025	0.025	0.025	0.125	0.037	0.188	0.037	0.050
A2	0.063	0.050	0.050	0.100	0.075	0.075	0.150	0.100
A3	0.050	0.063	0.063	0.050	0.112	0.075	0.112	0.075
A4	0.063	0.050	0.050	0.075	0.112	0.112	0.150	0.100

** CONCORDANCE INDEX MATRIX **

0.0000	0.3125	0.3125	0.3125
0.6875	0.0000	0.6875	0.6250
0.6875	0.5000	0.0000	0.3125
0.6875	0.8750	0.8750	0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5729

** CONCORDANCE DOMINANCE MATRIX **

0	0	0	0
1	0	1	1
1	0	0	0
1	1	1	0

** DISCORDANCE INDEX MATRIX **

0.0000	0.6216	0.5946	0.7027
0.2973	0.0000	0.1351	0.1622
0.4054	0.2703	0.0000	0.2973
0.2703	0.0541	0.0541	0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.4625

AVERAGE OF DISCORDANCE INDICES = 0.3221

** DISCORDANCE DOMINANCE MATRIX **

0	0	0	0
1	0	1	1
0	1	0	1

1 1 1 0

** AGGREGATE DOMINANCE MATRIX **

0 0 0 0

1 0 1 1

0 0 0 0

1 1 1 0

- -

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8
0.167	0.250	0.167	0.083	0.083	0.083	0.083	0.083

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8
A1	0.067	0.100	0.067	0.083	0.017	0.083	0.017	0.033
A2	0.167	0.200	0.134	0.066	0.033	0.033	0.066	0.066
A3	0.134	0.250	0.167	0.033	0.050	0.033	0.050	0.050
A4	0.167	0.200	0.134	0.050	0.050	0.050	0.066	0.066

** CONCORDANCE INDEX MATRIX **

0.0000 0.1660 0.1660 0.1660

0.8330 0.0000 0.4990 0.8330

0.8330 0.5830 0.0000 0.5000

0.8330 0.9160 0.5820 0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5758

** CONCORDANCE DOMINANCE MATRIX **

0 0 0 0

1 0 0 1

1 1 0 0

```
  1   1   1   0

  **  DISCORDANCE INDEX MATRIX  **

  0.0000  0.7338  0.8006  0.7670

  0.1329  0.0000  0.2002  0.0665

  0.1994  0.1998  0.0000  0.1998

  0.1329  0.0332  0.1669  0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =  0.4996

AVERAGE OF DISCORDANCE INDICES =  0.3027

  **  DISCORDANCE DOMINANCE MATRIX  **

  0   0   0   0

  1   0   1   1

  1   1   0   1

  1   1   1   0

  **  AGGREGATE DOMINANCE MATRIX  **

  0   0   0   0

  1   0   0   1

  1   1   0   0

  1   1   1   0

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -


  **  WEIGHTS  **

      X1      X2      X3      X4      X5      X6      X7      X8

      0.125   0.125   0.125   0.125   0.125   0.125   0.125   0.125

  **  WEIGHTED NORMALIZED DECISION MATRIX  **

      X1      X2      X3      X4      X5      X6      X7      X8

A1    0.050   0.050   0.050   0.125   0.025   0.125   0.025   0.050

A2    0.125   0.100   0.100   0.100   0.050   0.050   0.100   0.100

A3    0.100   0.125   0.125   0.050   0.075   0.050   0.075   0.075
```

A4 0.125 0.100 0.100 0.075 0.075 0.075 0.100 0.100

** CONCORDANCE INDEX MATRIX **

0.0000 0.2500 0.2500 0.2500

0.7500 0.0000 0.6250 0.7500

0.7500 0.5000 0.0000 0.3750

0.7500 0.8750 0.7500 0.0000

AVERAGE OF CONCORDANCE INDICES = 0.5729

** CONCORDANCE DOMINANCE MATRIX **

0 0 0 0

1 0 1 1

1 0 0 0

1 1 1 0

** DISCORDANCE INDEX MATRIX **

0.0000 0.6842 0.6842 0.7368

0.2105 0.0000 0.1579 0.1053

0.3158 0.2632 0.0000 0.2632

0.2105 0.0526 0.1053 0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.4750

AVERAGE OF DISCORDANCE INDICES = 0.3158

** DISCORDANCE DOMINANCE MATRIX **

0 0 0 0

1 0 1 1

0 1 0 1

1 1 1 0

** AGGREGATE DOMINANCE MATRIX **

0 0 0 0

1 0 1 1

0 0 0 0 172

1 1 1 0

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Appendix 4 Detailed calculations of problem 3

Appendix 4.1

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****   THE LINGO ASSIGNMENT METHOD   ****

****   SELECTING A LONG-RANGE WATER RESOURCE SYSTEMS   ****

  **  DECISION MATRIX  **
      X1      X2      X3      X4      X5      X6      X7      X8      X9      X10     X11     X12
A1  -99.8  -4.00   80.00    0.70   80.00   -1.00  -60.00   80.00   80.00   80.00   80.00   80.00
A2  -85.7  -19.00   60.00    0.50   60.00   -0.50  -80.00   40.00   60.00   60.00   60.00   60.00
A3  -95.1  -50.00   80.00    0.10   20.00   -0.50  -40.00   40.00   60.00   20.00   20.00   40.00
A4 -101.8  -50.00   40.00    0.01   20.00   -2.00  -70.00   40.00   40.00   40.00   40.00   20.00

  **  WEIGHTS  **
      X1      X2      X3      X4      X5      X6      X7      X8      X9      X10     X11     X12
     0.100   0.100   0.100   0.100   0.100   0.100   0.050   0.050   0.100   0.100   0.050   0.050

  **  PRODUCT-ATTRIBUTE MATRIX  **
A1    0.6600  0.1000  0.2000  0.0500
A2    0.2000  0.6000  0.2000  0.3000
A3    0.1500  0.2500  0.3250  0.2750
A4    0.0000  0.0500  0.2750  0.6750

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

  **  WEIGHTS  **
      X1      X2      X3      X4      X5      X6      X7      X8      X9      X10     X11     X12
     0.151   0.147   0.115   0.055   0.080   0.121   0.065   0.100   0.000   0.115   0.021   0.021

  **  PRODUCT-ATTRIBUTE MATRIX  **
A1    0.6160  0.1150  0.2120  0.0550
A2    0.2120  0.5790  0.2100  0.0000
A3    0.1730  0.2490  0.2795  0.2995
```

A4 0.0000 0.0558 0.2995 0.6465

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Appendix 4.2

**** TOPSIS METHOD ****

**** SELECTING A LONG-RANGE WATER RESOURCE SYSTEMS ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
A1	-99.6	-4.00	80.00	0.70	60.00	-1.00	-90.00	80.00	80.00	80.00	80.00	80.00
A2	-85.7	-19.00	60.00	0.50	60.00	-0.50	-80.00	80.00	60.00	60.00	60.00	80.00
A3	-95.1	-50.00	80.00	0.10	20.00	-0.50	-40.00	40.00	60.00	20.00	20.00	40.00
A4	-101.8	-50.00	40.00	0.01	20.00	-2.00	-70.00	40.00	40.00	40.00	40.00	20.00

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
A1	-0.52	-0.05	0.60	0.81	0.77	-0.43	-0.62	0.63	0.65	0.73	0.73	0.66
A2	-0.45	-0.26	0.45	0.58	0.58	-0.21	-0.55	0.63	0.49	0.55	0.55	0.66
A3	-0.50	-0.68	0.60	0.12	0.19	-0.21	-0.28	0.32	0.49	0.18	0.18	0.33
A4	-0.53	-0.68	0.30	0.01	0.19	-0.85	-0.48	0.32	0.32	0.37	0.37	0.16

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
0.100	0.100	0.100	0.100	0.100	0.100	0.050	0.050	0.100	0.100	0.050	0.050

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
A1	-0.05	-0.01	0.06	0.08	0.08	-0.04	-0.03	0.03	0.06	0.07	0.04	0.03
A2	-0.04	-0.03	0.04	0.06	0.06	-0.02	-0.03	0.03	0.05	0.05	0.03	0.03
A3	-0.05	-0.07	0.06	0.01	0.02	-0.02	-0.01	0.02	0.05	0.02	0.01	0.02
A4	-0.05	-0.07	0.03	0.00	0.02	-0.09	-0.02	0.02	0.03	0.04	0.02	0.01

	IDEAL SOL.	N-IDEAL SOL.
X1	-0.0448	-0.0532
X2	-0.0055	-0.0682
X3	0.0596	0.0299
X4	0.0808	0.0012
X5	0.0773	0.0192
X6	-0.0213	-0.0853
X7	-0.0138	-0.0311
X8	0.0316	0.0158
X9	0.0649	0.0324
X10	0.0730	0.0183

X11	0.0365	0.0091
X12	0.0329	0.0082

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.0254	0.1483	0.8394
A2	0.0491	0.1168	0.7040
A3	0.1289	0.0757	0.3699
A4	0.1495	0.0215	0.1260

- -

** WEIGHTS **

X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
0.091	0.147	0.115	0.055	0.080	0.121	0.055	0.130	0.080	0.115	0.021	0.021

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
A1	-0.05	-0.01	0.07	0.04	0.06	-0.05	-0.03	0.06	0.05	0.08	0.02	0.01
A2	-0.04	-0.04	0.05	0.03	0.05	-0.03	-0.03	0.06	0.04	0.06	0.01	0.01
A3	-0.05	-0.10	0.07	0.01	0.02	-0.08	-0.02	0.03	0.04	0.02	0.00	0.01
A4	-0.05	-0.10	0.03	0.00	0.02	-0.10	-0.03	0.05	0.03	0.04	0.01	0.00

	IDEAL SOL.	N-IDEAL SOL.
X1	-0.0407	-0.0484
X2	-0.0080	-0.1002
X3	0.0686	0.0343
X4	0.0445	0.0006
X5	0.0616	0.0154
X6	-0.0256	-0.1032
X7	-0.0152	-0.0342
X8	0.0632	0.0316
X9	0.0519	0.0260
X10	0.0840	0.0210
X11	0.0153	0.0038
X12	0.0138	0.0035

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.0327	0.1493	0.8203
A2	0.0495	0.1234	0.7137
A3	0.1320	0.0880	0.4001
A4	0.1533	0.0227	0.1287

Appendix 4.3

```
=====     ELECTRE METHOD     =====

****   SELECTING A LONG-RANGE WATER RESOURCE SYSTEMS   ****

  **  DECISION MATRIX  **

A1  -99.6 -4.00 80.00  0.70 60.00 -1.00-90.00 80.00 80.00 80.00 90.00 80.00

A2  -85.7-19.00 60.00  0.50 60.00 -0.50-60.00 80.00 60.00 60.00 60.00 60.00

A3  -65.1-50.00 60.00  0.10 20.00 -0.50-40.00 40.00 60.00 20.00 20.00 40.00

A4 -101.8-50.00 40.00  0.01 20.00 -2.00-70.00 40.00 40.00 40.00 40.00 20.00

  **  NORMALIZED DECISION MATRIX  **

      X1     X2     X3     X4     X5     X6     X7     X8     X9     X10    X11    X12

A1   -0.52 -0.05  0.60   0.81   0.77 -0.43 -0.62  0.63   0.85   0.73   0.73   0.66
A2   -0.45 -0.26  0.45   0.58   0.58 -0.21 -0.55  0.63   0.49   0.55   0.55   0.66
A3   -0.50 -0.68  0.60   0.12   0.19 -0.21 -0.28  0.32   0.49   0.18   0.18   0.33
A4   -0.53 -0.64  0.30   0.01   0.19 -0.85 -0.48  0.32   0.32   0.37   0.37   0.16

  **  WEIGHTS  **

      X1     X2     X3     X4     X5     X6     X7     X8     X9     X10    X11    X12

     0.100  0.100  0.100  0.100  0.100  0.100  0.050  0.050  0.100  0.100  0.050  0.050

  **  WEIGHTED NORMALIZED DECISION MATRIX  **

      X1     X2     X3     X4     X5     X6     X7     X8     X9     X10    X11    X12

A1   -0.05 -0.01  0.06   0.08   0.08 -0.04 -0.03  0.03   0.08   0.07   0.04   0.03
A2   -0.04 -0.03  0.04   0.06   0.06 -0.02 -0.03  0.03   0.05   0.05   0.03   0.03
A3   -0.05 -0.07  0.06   0.01   0.02 -0.02 -0.01  0.02   0.05   0.02   0.01   0.02
A4   -0.05 -0.07  0.03   0.00   0.02 -0.09 -0.02  0.02   0.03   0.04   0.02   0.01

  **  CONCORDANCE INDEX MATRIX  **

  0.0000  0.7500  0.7500  0.9500

  0.3500  0.0000  0.6500  0.9500

  0.3500  0.3500  0.0000  0.8500

  0.0500  0.0500  0.4000  0.0000

AVERAGE OF CONCORDANCE INDICES =  0.5542

  **  CONCORDANCE DOMINANCE MATRIX  **

  0  1  1  1
```

```
  0  0  1  1
  0  0  0  1
  0  0  0  0

  **  DISCORDANCE INDEX MATRIX  **
 0.0000  0.0764  0.1002  0.0169
 0.2971  0.0000  0.0703  0.0084
 0.7846  0.5360  0.0000  0.0671
 0.9831  0.7560  0.3489  0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =  0.4063
AVERAGE OF DISCORDANCE INDICES =  0.3373

  **  DISCORDANCE DOMINANCE MATRIX  **
  0  1  1  1
  1  0  1  1
  0  0  0  1
  0  0  0  0

  **  AGGREGATE DOMINANCE MATRIX  **
  0  1  1  1
  0  0  1  1
  0  0  0  1
  0  0  0  0
```

- -

```
  **  WEIGHTS  **
     X1     X2     X3     X4     X5     X6     X7     X8     X9     X10    X11    X12
    0.091  0.147  0.115  0.055  0.080  0.121  0.055  0.100  0.080  0.115  0.021  0.021

  **  WEIGHTED NORMALIZED DECISION MATRIX  **
     X1     X2     X3     X4     X5     X6     X7     X8     X9     X10    X11    X12
```

```
A1    -0.05 -0.01  0.07  0.04  0.06 -0.05 -0.03  0.06  0.05  0.08  0.12  0.01
A2    -0.04 -0.04  0.05  0.03  0.05 -0.03 -0.03  0.06  0.04  0.06  0.01  0.01
A3    -0.05 -0.10  0.07  0.01  0.02 -0.03 -0.02  0.02  0.04  0.02  0.00  0.01
A4    -0.05 -0.10  0.03  0.00  0.02 -0.10 -0.03  0.03  0.03  0.04  0.01  0.00

  **  CONCORDANCE INDEX MATRIX  **

  0.0000  0.7240  0.7340  0.9460

  0.3880  0.0000  0.3310  0.9460

  0.3820  0.3710  0.0000  0.8650

  0.0550  0.0550  0.4630  0.0000

AVERAGE OF CONCORDANCE INDICES  =  0.5642

  **  CONCORDANCE DOMINANCE MATRIX  **

  0  1  1  1

  0  0  1  1

  0  0  0  1

  0  0  0  0

  **  DISCORDANCE INDEX MATRIX  **

  0.0000  0.0918  0.1190  0.0192

  0.2809  0.0000  0.0820  0.0096

  0.7671  0.5351  0.0000  0.0630

  0.9808  0.7761  0.3763  0.0000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =  0.3943

AVERAGE OF DISCORDANCE INDICES =  0.3422

  **  DISCORDANCE DOMINANCE MATRIX  **

  0  1  1  1

  1  0  1  1

  0  0  0  1

  0  0  0  0

  **  AGGREGATE DOMINANCE MATRIX  **
```

0 1 1 1

0 0 1 1

0 0 0 1

0 0 0 0

Appendix 5 Detailed calculations of problem 4

==== THE LINEAR ASSINGMENT METHOD ====

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	0.0000
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-1.5350
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-1.9330
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** WEIGHTS **

X1	X2	X3	X4	X5	X6
0.311	0.189	0.011	0.090	0.051	0.347

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.090	0.005	0.005	0.364	0.017	0.017	0.063	0.063	0.374
A2	0.347	0.050	0.050	0.017	0.328	0.017	0.063	0.063	0.063
A3	0.000	0.392	0.045	0.322	0.000	0.000	0.063	0.114	0.063
A4	0.000	0.000	0.536	0.000	0.000	0.090	0.373	0.000	0.000
A5	0.000	0.000	0.000	0.279	0.000	0.311	0.000	0.352	0.056
A6	0.000	0.189	0.311	0.017	0.017	0.017	0.000	0.095	0.352
A7	0.206	0.017	0.017	0.000	0.347	0.011	0.090	0.311	0.000
A8	0.328	0.017	0.017	0.000	0.011	0.536	0.000	0.000	0.090
A9	0.028	0.328	0.017	0.000	0.279	0.000	0.347	0.000	0.000

==== THE LINEAR ASSINGMENT METHOD ====

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	-4.9290
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-15.3430
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-7.6130
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** WEIGHTS **

X1	X2	X3	X4	X5	X6
0.311	0.189	0.011	0.090	0.051	0.347

** PRODUCT-ATTRIBUTE MATRIX **

A1	0.437	0.005	0.005	0.017	0.017	0.017	0.063	0.063	0.374
A2	0.000	0.050	0.397	0.017	0.328	0.017	0.063	0.063	0.063
A3	0.000	0.045	0.045	0.322	0.000	0.000	0.063	0.461	0.063
A4	0.000	0.000	0.189	0.347	0.000	0.090	0.373	0.000	0.000
A5	0.000	0.000	0.000	0.279	0.000	0.311	0.347	0.005	0.056
A6	0.000	0.189	0.311	0.017	0.017	0.017	0.000	0.095	0.352
A7	0.206	0.364	0.017	0.000	0.000	0.011	0.090	0.311	0.000
A8	0.328	0.017	0.017	0.000	0.358	0.189	0.000	0.000	0.090
A9	0.028	0.328	0.017	0.000	0.279	0.347	0.000	0.000	0.000

**** TOPSIS METHCD ****

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	0.0000
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-1.5350
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-1.9330
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.493	0.332	-0.191	-0.255	-0.066	-0.074
A2	-0.346	0.332	-0.191	-0.265	-0.066	0.000
A3	-0.216	0.332	-0.203	-0.265	-0.460	-0.056
A4	-0.415	0.335	-0.393	-0.374	-0.394	-0.071
A5	-0.380	0.333	-0.548	-0.291	-0.788	-0.451
A6	-0.199	0.335	-0.548	-0.395	-0.066	-0.647
A7	-0.449	0.335	-0.274	-0.384	0.000	-0.177
A8	-0.095	0.333	-0.215	-0.416	0.000	-0.399
A9	-0.156	0.333	-0.095	-0.306	0.000	-0.417

** WEIGHTS **

X1	X2	X3	X4	X5	X6

0.311 0.189 0.011 0.090 0.051 0.347

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.153	0.063	-0.002	-0.023	-0.003	-0.026
A2	-0.107	0.063	-0.002	-0.024	-0.003	0.000
A3	-0.067	0.063	-0.002	-0.024	-0.023	-0.019
A4	-0.129	0.063	-0.004	-0.034	-0.020	-0.024
A5	-0.118	0.063	-0.006	-0.026	-0.040	-0.156
A6	-0.062	0.063	-0.006	-0.036	-0.003	-0.224
A7	-0.140	0.063	-0.003	-0.035	0.000	-0.061
A8	-0.030	0.063	-0.002	-0.037	0.000	-0.139
A9	-0.048	0.063	-0.001	-0.028	0.000	-0.145

	IDEAL SOL.	N-IDEAL SOL.
X1	-0.0296	-0.1532
X2	0.0633	0.0628
X3	-0.0010	-0.0060
X4	-0.0229	-0.0374
X5	0.0000	-0.0402
X6	0.0000	-0.2244

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1263	0.2026	0.6159
A2	0.0780	0.2324	0.7487
A3	0.0484	0.2234	0.8218
A4	0.1050	0.2024	0.6585
A5	0.1844	0.0772	0.2952
A6	0.2272	0.0985	0.3025
A7	0.1266	0.1686	0.5711
A8	0.1393	0.1558	0.5279
A9	0.1461	0.1381	0.4860

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**** TOPSIS METHOD ****

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	-4.9290
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-15.3430
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-7.6130
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.493	0.332	-0.191	-0.255	-0.066	-0.062
A2	-0.346	0.332	-0.191	-0.265	-0.066	-0.151
A3	-0.216	0.332	-0.203	-0.265	-0.460	-0.471
A4	-0.415	0.335	-0.393	-0.374	-0.394	-0.234
A5	-0.380	0.333	-0.548	-0.291	-0.788	-0.379
A6	-0.199	0.335	-0.548	-0.395	-0.066	-0.544
A7	-0.449	0.335	-0.274	-0.384	0.000	-0.148
A8	-0.095	0.333	-0.215	-0.416	0.000	-0.336
A9	-0.156	0.333	-0.095	-0.306	0.000	-0.351

** WEIGHTS **

X1	X2	X3	X4	X5	X6

0.311 0.189 0.011 0.090 0.051 0.347

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.153	0.063	-0.002	-0.023	-0.003	-0.022
A2	-0.107	0.063	-0.002	-0.024	-0.003	-0.053
A3	-0.067	0.063	-0.002	-0.024	-0.023	-0.163
A4	-0.129	0.063	-0.004	-0.034	-0.020	-0.081
A5	-0.118	0.063	-0.006	-0.026	-0.040	-0.132
A6	-0.062	0.063	-0.006	-0.036	-0.003	-0.189
A7	-0.140	0.063	-0.003	-0.035	0.000	-0.051
A8	-0.030	0.063	-0.002	-0.037	0.000	-0.116
A9	-0.048	0.063	-0.001	-0.028	0.000	-0.122

	IDEAL SOL.	N-IDEAL SOL.
X1	-0.0296	-0.1532
X2	0.0633	0.0628
X3	-0.0010	-0.0060
X4	-0.0229	-0.0374
X5	0.0000	-0.0402
X6	-0.0217	-0.1886

	IDEAL SEPARATION	N-IDEAL SEPARATION	RELATIVE CLOSENESS
A1	0.1237	0.1716	0.5812
A2	0.0839	0.1489	0.6397
A3	0.1485	0.0922	0.3831
A4	0.1181	0.1121	0.4870
A5	0.1469	0.0679	0.3161
A6	0.1706	0.0985	0.3661
A7	0.1148	0.1436	0.5558
A8	0.0959	0.1487	0.6079
A9	0.1019	0.1312	0.5629

- -

===== ELECTRE METHOD =====

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	0.0000
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-1.5350
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-1.9330
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.493	0.332	-0.191	-0.255	-0.066	-0.074
A2	-0.346	0.332	-0.191	-0.265	-0.066	0.000
A3	-0.216	0.332	-0.203	-0.265	-0.460	-0.056
A4	-0.415	0.335	-0.393	-0.374	-0.394	-0.071
A5	-0.380	0.333	-0.548	-0.291	-0.788	-0.451
A6	-0.199	0.335	-0.548	-0.395	-0.066	-0.647
A7	-0.449	0.335	-0.274	-0.384	0.000	-0.177
A8	-0.095	0.333	-0.215	-0.416	0.000	-0.399
A9	-0.156	0.333	-0.095	-0.306	0.000	-0.417

CONCORDANCE SETS

	X1	X2	X3	X4	X5	X6
C12	0	1	1	1	1	0
C13	0	1	1	1	1	0
C14	0	0	1	1	1	0

DISCORDANCE SETS

	X1	X2	X3	X4	X5	X6
D12	1	0	0	0	0	1
D13	1	0	0	0	0	1
D14	1	1	0	0	0	1

C15	0	0	1	1	1	1
C16	0	0	1	1	1	1
C17	0	0	1	1	0	1
C18	0	0	1	1	0	1
C19	0	0	0	1	0	1
C21	1	1	1	0	1	1
C23	0	1	1	1	1	1
C24	1	0	1	1	1	1
C25	1	0	1	1	1	1
C26	0	0	1	1	1	1
C27	1	0	1	1	0	1
C28	0	0	1	1	0	1
C29	0	0	0	1	0	1
C31	1	1	0	0	0	1
C32	1	1	0	1	0	0
C34	1	0	1	1	0	1
C35	1	0	1	1	1	1
C36	0	0	1	1	0	1
C37	1	0	1	1	0	1
C38	0	0	1	1	0	1
C39	0	0	0	1	0	1
C41	1	1	0	0	0	1
C42	0	1	0	0	0	0
C43	0	1	0	0	1	0
C45	0	1	1	0	1	1
C46	0	0	1	1	0	1
C47	1	0	0	1	0	1
C48	0	1	0	1	0	1
C49	0	1	0	0	0	1
C51	1	1	0	0	0	0
C52	0	1	0	0	0	0
C53	0	1	0	0	0	0
C54	1	0	0	1	0	0
C56	0	0	1	1	0	1
C57	1	0	0	1	0	0
C58	0	1	0	1	0	0
C59	0	1	0	1	0	0
C61	1	1	0	0	1	0
C62	1	1	0	0	1	0
C63	1	1	0	0	1	0
C64	1	1	0	0	1	0
C65	1	1	1	0	1	0
C67	1	0	0	0	0	0
C68	0	1	0	1	0	0
C69	0	1	0	0	0	0
C71	1	1	0	0	1	0
C72	0	1	0	0	1	0
C73	0	1	0	0	1	0
C74	0	1	1	0	1	0
C75	0	1	1	0	1	1
C76	0	1	1	1	1	1
C78	0	1	0	1	1	1
C79	0	1	0	0	1	1
C81	1	1	0	0	1	0
C82	1	1	0	0	1	0
C83	1	1	0	0	1	0
C84	1	0	1	0	1	0
C85	1	0	1	0	1	1
C86	1	0	1	0	1	1
C87	1	0	1	0	1	0

D15	1	1	0	0	0	0
D16	1	1	0	0	0	0
D17	1	1	0	0	1	0
D18	1	1	0	0	1	0
D19	1	1	1	0	1	0
D21	0	0	0	1	0	0
D23	1	0	0	0	0	0
D24	0	1	0	0	0	0
D25	0	1	0	0	0	0
D26	1	1	0	0	0	0
D27	0	1	0	0	1	0
D28	1	1	0	0	1	0
D29	1	1	1	0	1	0
D31	0	0	1	1	1	0
D32	0	0	1	0	1	1
D34	0	1	0	0	1	0
D35	0	1	0	0	0	0
D36	1	1	0	0	1	0
D37	0	1	0	0	1	0
D38	1	1	0	0	1	0
D39	1	1	1	0	1	0
D41	0	0	1	1	1	0
D42	1	0	1	1	1	1
D43	1	0	1	1	0	1
D45	1	0	0	1	0	0
D46	1	1	0	0	1	0
D47	0	1	1	0	1	0
D48	1	0	1	0	1	0
D49	1	0	1	1	1	0
D51	0	0	1	1	1	1
D52	1	0	1	1	1	1
D53	1	0	1	1	1	1
D54	0	1	1	0	1	1
D56	1	1	0	0	1	0
D57	0	1	1	0	1	1
D58	1	0	1	0	1	1
D59	1	0	1	0	1	1
D61	0	0	1	1	0	1
D62	0	0	1	1	0	1
D63	0	0	1	1	0	1
D64	0	0	1	1	0	1
D65	0	0	0	1	0	1
D67	0	1	1	1	1	1
D68	1	0	1	0	1	1
D69	1	0	1	1	1	1
D71	0	0	1	1	0	1
D72	1	0	1	1	0	1
D73	1	0	1	1	0	1
D74	1	0	0	1	0	1
D75	1	0	0	1	0	0
D76	1	0	0	0	0	0
D78	1	0	1	0	0	0
D79	1	0	1	1	0	0
D81	0	0	1	1	0	1
D82	0	0	1	1	0	1
D83	0	0	1	1	0	1
D84	0	1	0	1	0	1
D85	0	1	0	1	0	0
D86	0	1	0	1	0	0
D87	0	1	0	1	0	1

C89	1	0	0	0	1	1	D89	0	1	1	1	0	0
C91	1	1	1	0	1	0	D91	0	0	0	1	0	1
C92	1	1	1	0	1	0	D92	0	0	0	1	0	1
C93	1	1	1	0	1	0	D93	0	0	0	1	0	1
C94	1	0	1	1	1	0	D94	0	1	0	0	0	1
C95	1	0	1	0	1	1	D95	0	1	0	1	0	0
C96	1	0	1	1	1	1	D96	0	1	0	0	0	0
C97	1	0	1	1	1	0	D97	0	1	0	0	0	1
C98	0	1	1	1	1	0	D98	1	0	0	0	0	1

* SYMBOL 1 INDICATES THAT AN ATTRIBUTE IS INCLUDED IN THE CORRESPONDING CONCORDANCE/DISCONCORDANCE SETS

** WEIGHTS **

X1	X2	X3	X4	X5	X6
0.311	0.189	0.011	0.090	0.051	0.347

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.153	0.063	-0.002	-0.023	-0.003	-0.026
A2	-0.107	0.063	-0.002	-0.024	-0.003	0.000
A3	-0.067	0.063	-0.002	-0.024	-0.023	-0.019
A4	-0.129	0.063	-0.004	-0.034	-0.020	-0.024
A5	-0.118	0.063	-0.006	-0.026	-0.040	-0.156
A6	-0.062	0.063	-0.006	-0.036	-0.003	-0.224
A7	-0.140	0.063	-0.003	-0.035	0.000	-0.061
A8	-0.030	0.063	-0.002	-0.037	0.000	-0.139
A9	-0.048	0.063	-0.001	-0.028	0.000	-0.145

** CONCORDANCE INDEX MATRIX **

0.000	0.341	0.341	0.152	0.499	0.499	0.448	0.448	0.437
0.909	0.000	0.688	0.810	0.810	0.499	0.759	0.448	0.437
0.847	0.590	0.000	0.759	0.810	0.448	0.759	0.448	0.437
0.847	0.189	0.240	0.000	0.598	0.448	0.748	0.626	0.536
0.500	0.189	0.189	0.401	0.000	0.448	0.401	0.279	0.279
0.551	0.551	0.551	0.551	0.562	0.000	0.311	0.279	0.189
0.551	0.240	0.240	0.251	0.598	0.688	0.000	0.677	0.587
0.551	0.551	0.551	0.373	0.720	0.720	0.373	0.000	0.709

0.562 0.562 0.562 0.463 0.720 0.810 0.463 0.341 0.000

AVERAGE OF CONCORDANCE INDICES = 0.5132

** CONCORDANCE DOMINANCE MATRIX **

0 0 0 0 0 0 0 0 0

1 0 1 1 1 0 1 0 0

1 1 0 1 1 0 1 0 0

1 0 0 0 1 0 1 1 1

0 0 0 0 0 0 0 0 0

1 1 1 1 1 0 0 0 0

1 0 0 0 1 1 0 1 1

1 1 1 0 1 1 0 0 1

1 1 1 0 1 1 0 0 0

** DISCORDANCE INDEX MATRIX **

0.000 0.233 0.301 0.085 0.115 0.299 0.057 0.414 0.356

0.003 0.000 0.131 0.002 0.001 0.151 0.013 0.265 0.207

0.069 0.129 0.000 0.013 0.001 0.085 0.078 0.199 0.142

0.097 0.244 0.256 0.000 0.059 0.273 0.070 0.396 0.358

0.569 0.685 0.687 0.502 0.000 0.305 0.452 0.490 0.413

0.701 0.782 0.718 0.663 0.252 0.000 0.555 0.407 0.356

0.157 0.342 0.410 0.158 0.097 0.254 0.000 0.361 0.327

0.415 0.496 0.432 0.385 0.037 0.007 0.262 0.000 0.037

0.403 0.484 0.420 0.393 0.005 0.001 0.273 0.081 0.000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.3071

AVERAGE OF DISCORDANCE INDICES = 0.2760

** DISCORDANCE DOMINANCE MATRIX **

0 1 0 1 1 0 1 0 0

1 0 1 1 1 1 1 1 1

1 1 0 1 1 1 1 1 1

1 1 1 0 1 1 1 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 1 0 0 0 0

1 0 0 1 1 1 0 0 0

0 0 0 0 1 1 1 0 1

0 0 0 0 1 1 1 1 0

** AGGREGATE DOMINANCE MATRIX **

0 0 0 0 0 0 0 0 0

1 0 1 1 1 0 1 0 0

1 1 0 1 1 0 1 0 0

1 0 0 0 1 0 1 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 1 0 0 0 0

1 0 0 0 1 1 0 0 0

0 0 0 0 1 1 0 0 1

0 0 0 0 1 1 0 0 0

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===== ELECTRE METHOD =====

**** SELECTING NUCLEAR POWER PLANT SITES ****

** DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.0570	0.9895	-1.6000	-2.4500	-1.0000	-2.0350
A2	-0.0400	0.9895	-1.6000	-2.5500	-1.0000	-4.9290
A3	-0.0250	0.9895	-1.7000	-2.5500	-7.0000	-15.3430
A4	-0.0480	0.9978	-3.3000	-3.6000	-6.0000	-7.6130
A5	-0.0440	0.9936	-4.6000	-2.8000	-12.0000	-12.3470
A6	-0.0230	0.9980	-4.6000	-3.8000	-1.0000	-17.7100
A7	-0.0520	0.9988	-2.3000	-3.7000	0.0000	-4.8340
A8	-0.0110	0.9913	-1.8000	-4.0000	0.0000	-10.9360
A9	-0.0180	0.9926	-0.8000	-2.9500	0.0000	-11.4230

** NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.493	0.332	-0.191	-0.255	-0.066	-0.062
A2	-0.346	0.332	-0.191	-0.265	-0.066	-0.151
A3	-0.216	0.332	-0.203	-0.265	-0.460	-0.471
A4	-0.415	0.335	-0.393	-0.374	-0.394	-0.234
A5	-0.380	0.333	-0.548	-0.291	-0.788	-0.379
A6	-0.199	0.335	-0.548	-0.395	-0.066	-0.544
A7	-0.449	0.335	-0.274	-0.384	0.000	-0.148
A8	-0.095	0.333	-0.215	-0.416	0.000	-0.336
A9	-0.156	0.333	-0.095	-0.306	0.000	-0.351

CONCORDANCE SETS

	X1	X2	X3	X4	X5	X6
C12	0	1	1	1	1	1
C13	0	1	1	1	1	1
C14	0	0	1	1	1	1

DISCORDANCE SETS

	X1	X2	X3	X4	X5	X6
D12	1	0	0	0	0	0
D13	1	0	0	0	0	0
D14	1	1	0	0	0	0

C15	0	0	1	1	1	1
C16	0	0	1	1	1	1
C17	0	0	1	1	0	1
C18	0	0	1	1	0	1
C19	0	0	0	1	0	1
C21	1	1	1	0	1	0
C23	0	1	1	1	1	1
C24	1	0	1	1	1	1
C25	1	0	1	1	1	1
C26	0	0	1	1	1	1
C27	1	0	1	1	0	0
C28	0	0	1	1	0	1
C29	0	0	0	1	0	1
C31	1	1	0	0	0	0
C32	1	1	0	1	0	0
C34	1	0	1	1	0	0
C35	1	0	1	1	1	0
C36	0	0	1	1	0	1
C37	1	0	1	1	0	0
C38	0	0	1	1	0	0
C39	0	0	0	1	0	0
C41	1	1	0	0	0	0
C42	0	1	0	0	0	0
C43	0	1	0	0	1	1
C45	0	1	1	0	1	1
C46	0	0	1	1	0	1
C47	1	0	0	1	0	0
C48	0	1	0	1	0	1
C49	0	1	0	0	0	1
C51	1	1	0	0	0	0
C52	0	1	0	0	0	0
C53	0	1	0	0	0	1
C54	1	0	0	1	0	0
C56	0	0	1	1	0	1
C57	1	0	0	1	0	0
C58	0	1	0	1	0	0
C59	0	1	0	1	0	0
C61	1	1	0	0	1	0
C62	1	1	0	0	1	0
C63	1	1	0	0	1	0
C64	1	1	0	0	1	0
C65	1	1	1	0	1	0
C67	1	0	0	0	0	0
C68	0	1	0	1	0	0
C69	0	1	0	0	0	0
C71	1	1	0	0	1	0
C72	0	1	0	0	1	1
C73	0	1	0	0	1	1
C74	0	1	1	0	1	1
C75	0	1	1	0	1	1
C76	0	1	1	1	1	1
C78	0	1	0	1	1	1
C79	0	1	0	0	1	1
C81	1	1	0	0	1	0
C82	1	1	0	0	1	0
C83	1	1	0	0	1	1
C84	1	0	1	0	1	0
C85	1	0	1	0	1	1
C86	1	0	1	0	1	1
C87	1	0	1	0	1	0

D15	1	1	0	0	0	0
D16	1	1	0	0	0	0
D17	1	1	0	0	1	0
D18	1	1	0	0	1	0
D19	1	1	1	0	1	0
D21	0	0	0	1	0	1
D23	1	0	0	0	0	0
D24	0	1	0	0	0	0
D25	0	1	0	0	0	0
D26	1	1	0	0	0	0
D27	0	1	0	0	1	1
D28	1	1	0	0	1	0
D29	1	1	1	0	1	0
D31	0	0	1	1	1	1
D32	0	0	1	0	1	1
D34	0	1	0	0	1	1
D35	0	1	0	0	0	1
D36	1	1	0	0	1	0
D37	0	1	0	0	1	1
D38	1	1	0	0	1	1
D39	1	1	1	0	1	1
D41	0	0	1	1	1	1
D42	1	0	1	1	1	1
D43	1	0	1	1	0	0
D45	1	0	0	1	0	0
D46	1	1	0	0	1	0
D47	0	1	1	0	1	1
D48	1	0	1	0	1	0
D49	1	0	1	1	1	0
D51	0	0	1	1	1	1
D52	1	0	1	1	1	1
D53	1	0	1	1	1	0
D54	0	1	1	0	1	1
D56	1	1	0	0	1	0
D57	0	1	1	0	1	1
D58	1	0	1	0	1	1
D59	1	0	1	0	1	1
D61	0	0	1	1	0	1
D62	0	0	1	1	0	1
D63	0	0	1	1	0	1
D64	0	0	1	1	0	1
D65	0	0	0	1	0	1
D67	0	1	1	1	1	1
D68	1	0	1	0	1	1
D69	1	0	1	1	1	1
D71	0	0	1	1	0	1
D72	1	0	1	1	0	0
D73	1	0	1	1	0	0
D74	1	0	0	1	0	0
D75	1	0	0	1	0	0
D76	1	0	0	0	0	0
D78	1	0	1	0	0	0
D79	1	0	1	1	0	0
D81	0	0	1	1	0	1
D82	0	0	1	1	0	1
D83	0	0	1	1	0	0
D84	0	1	0	1	0	1
D85	0	1	0	1	0	0
D86	0	1	0	1	0	0
D87	0	1	0	1	0	1

C89	1	0	0	0	1	1		D89	0	1	1	1	0	0
C91	1	1	1	0	1	0		D91	0	0	0	1	0	1
C92	1	1	1	0	1	0		D92	0	0	0	1	0	1
C93	1	1	1	0	1	1		D93	0	0	0	1	0	0
C94	1	0	1	1	1	0		D94	0	1	0	0	0	1
C95	1	0	1	0	1	1		D95	0	1	0	1	0	0
C96	1	0	1	1	1	1		D96	0	1	0	0	0	0
C97	1	0	1	1	1	0		D97	0	1	0	0	0	1
C98	0	1	1	1	1	0		D98	1	0	0	0	0	1

* SYMBOL 1 INDICATES THAT AN ATTRIBUTE IS INCLUDED IN THE CORRESPONDING CONCORDANCE/DISCONCORDANCE SETS

** WEIGHTS **

X1	X2	X3	X4	X5	X6
0.311	0.189	0.011	0.090	0.051	0.347

** WEIGHTED NORMALIZED DECISION MATRIX **

	X1	X2	X3	X4	X5	X6
A1	-0.153	0.063	-0.002	-0.023	-0.003	-0.022
A2	-0.107	0.063	-0.002	-0.024	-0.003	-0.053
A3	-0.067	0.063	-0.002	-0.024	-0.023	-0.163
A4	-0.129	0.063	-0.004	-0.034	-0.020	-0.081
A5	-0.118	0.063	-0.006	-0.026	-0.040	-0.132
A6	-0.062	0.063	-0.006	-0.036	-0.003	-0.189
A7	-0.140	0.063	-0.003	-0.035	0.000	-0.051
A8	-0.030	0.063	-0.002	-0.037	0.000	-0.116
A9	-0.048	0.063	-0.001	-0.028	0.000	-0.122

** CONCORDANCE INDEX MATRIX **

0.000	0.688	0.688	0.499	0.499	0.499	0.448	0.448	0.437
0.562	0.000	0.688	0.810	0.810	0.499	0.412	0.448	0.437
0.500	0.590	0.000	0.412	0.463	0.448	0.412	0.101	0.090
0.500	0.189	0.587	0.000	0.598	0.448	0.401	0.626	0.536
0.500	0.189	0.536	0.401	0.000	0.448	0.401	0.279	0.279
0.551	0.551	0.551	0.551	0.562	0.000	0.311	0.279	0.189
0.551	0.587	0.587	0.598	0.598	0.688	0.000	0.677	0.587
0.551	0.551	0.898	0.373	0.720	0.720	0.373	0.000	0.709

0.562 0.562 0.909 0.463 0.720 0.810 0.463 0.341 0.000

AVERAGE OF CONCORDANCE INDICES = 0.5132

** CONCORDANCE DOMINANCE MATRIX **

0	1	1	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1	1
0	0	1	0	0	0	0	0	0
1	1	1	1	1	0	0	0	0
1	1	1	1	1	1	0	1	1
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	0	0	0

** DISCORDANCE INDEX MATRIX **

0.000	0.166	0.312	0.090	0.128	0.334	0.063	0.461	0.397
0.115	0.000	0.146	0.002	0.001	0.168	0.018	0.296	0.231
0.591	0.476	0.000	0.313	0.117	0.094	0.494	0.393	0.310
0.324	0.286	0.268	0.000	0.066	0.305	0.185	0.441	0.400
0.559	0.482	0.268	0.263	0.000	0.340	0.449	0.536	0.453
0.666	0.551	0.148	0.403	0.241	0.000	0.525	0.405	0.351
0.154	0.159	0.305	0.042	0.109	0.283	0.000	0.402	0.364
0.398	0.282	0.050	0.144	0.041	0.008	0.248	0.000	0.041
0.380	0.265	0.014	0.149	0.005	0.001	0.256	0.087	0.000

MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS = 0.2754

AVERAGE OF DISCORDANCE INDICES = 0.2614

** DISCORDANCE DOMINANCE MATRIX **

0	1	0	1	1	0	1	0	0
1	0	1	1	1	1	1	0	1
0	0	0	0	1	1	0	0	0

0 0 0 0 1 0 1 0 0

0 0 0 0 0 0 0 0 0

0 0 1 0 1 0 0 0 0

1 1 0 1 1 0 0 0 0

0 0 1 1 1 1 1 0 1

0 0 1 1 1 1 1 1 0

** AGGREGATE DOMINANCE MATRIX **

0 1 0 0 0 0 0 0 0

1 0 1 1 1 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 0 0 1 0 0 0 0

0 0 0 0 0 0 0 0 0

0 0 1 0 1 0 0 0 0

1 1 0 1 1 0 0 0 0

0 0 1 0 1 1 0 0 1

0 0 1 0 1 1 0 0 0

- - - -

Appendix 6 The computer programs produce the product-attribute matrix, relative closeness to the ideal solution, and aggregate dominance matrix for LAM, TOPSIS, and ELECTRE, respectively.

Appendix 6.1 The computer program for LAM produces the produce-attribute matrix.

```
      $JOB            JSS,TIME=45,PAGES=200,RUN=CHECK
      C$OPTIONS  PAGES=30
    1       DIMENSION AL(10),AT(12)
    2       DIMENSION SR(10,12)
    3       DIMENSION A(12,12,12),R(12,12),SUM(12,12),W(12),AA(20),B(20)
    4       REAL MAX
    5       INTEGER I,J,K,L,M,N,P,PP,NT,IT,POS,I1,Q
      C   THIS PROGRAM DEMONSTRATES THE PRODUCT-ATTRIBUTE MATRIX OF LAM METHOD
    6       READ(5,91) (AA(K),K=1,20)
    7       READ(5,91) (B(K),K=1,20)
    8       WRITE(6,90)
    9       WRITE(6,92) (AA(K),K=1,20)
   10       WRITE(6,92) (B(K),K=1,20)
      C   READ ALTERNATIVE #, ATTRIBUTE #, AND WEIGHT #
   11       READ(5,10) M,N,L
   12       READ(5,94)(AT(I),I=1,N)
   13       READ(5,94)(AL(I),I=1,M)
   14       READ(5,20) ((R(I,J),J=1,N),I=1,M)
   15    90 FORMAT('1')
   16    91 FORMAT(20A4)
   17    92 FORMAT(//14X,20A4/)
   18    10 FORMAT(3I5)
   19    20 FORMAT(8F10.3)
   20    94 FORMAT(8A3)
   21    95 FORMAT(17X,8(3X,A3,1X)/)
   22    96 FORMAT(14X,A3,2F7.2,F7.0,5F7.2/)
   23    97 FORMAT(14X,A3,5F7.3/)
   24    98 FORMAT(18X,12F7.3/)
   25       DO 1110 I=1,M
   26       DO 1110 J=1,N
   27       DO 1110 Q=1,M
   28       A(I,J,Q)=0
   29  1110 CONTINUE
   30       DO 1115 I=1,M
   31       DO 1115 J=1,N
   32       SR(I,J)=R(I,J)
   33  1115 CONTINUE
   34       WRITE(6,70)
   35    70 FORMAT(/14X,'  **  DECISION MATRIX  **'/)
   36       WRITE(6,95)(AT(J),J=1,N)
   37       WRITE(6,96) (AL(I),(R(I,J),J=1,N),I=1,M)
      C   DETERMINE ATTRIBUTEWISE PREFERENCE
   38       DO 1120 J=1,N
   39       DO 1120 I=1,M
   40       I1=I+1
   41       IF(I1.GT.M) GO TO 1120
   42       MAX=R(I,J)
   43       POS=I
   44       DO 1138 P=I1,M
   45       IF(R(P,J)-MAX) 1138,1138,1131
   46  1131 MAX=R(P,J)
   47       POS=P
   48  1138 CONTINUE
   49       TEM=R(I,J)
   50       R(I,J)=R(POS,J)
   51       R(POS,J)=TEM
   52  1120 CONTINUE
   53       DO 1160 J=1,N
   54       DO 1160 Q=1,M
   55       NT=0
```

```
56          DO 1155 I=1,M
57          IF(SR(I,J)-R(Q,J)) 1155,1154,1155
58     1154 A(I,J,Q)=1
59          NT=NT+1
60     1155 CONTINUE
61          DO 1158 I=1,M
62          A(I,J,Q)=A(I,J,Q)/NT
63     1158 CONTINUE
64     1160 CONTINUE
65          DO 1200 K=1,L
66          WRITE(6,78)
67       78 FORMAT(/14X,'  **  WEIGHTS  **'/)
68          READ(5,20) (W(J),J=1,N)
69          WRITE(6,95)(AT(J),J=1,N)
70          WRITE(6,98) (W(J),J=1,N)
      C    CALCULATE THE PRODUCT-ATTRIBUTE MATRIX
71          DO 1150 I=1,M
72          DO 1150 Q=1,M
73          SUM(I,Q)=0
74          DO 1150 J=1,N
75          SUM(I,Q)=SUM(I,Q)+W(J)*A(I,J,Q)
76     1150 CONTINUE
77          WRITE(6,83)
78       83 FORMAT(/14X,'  **  PRODUCT-ATTRIBUTE MATRIX  **  '/)
79          WRITE(6,97) (AL(I),(SUM(I,Q),Q=1,M),I=1,M)
80          WRITE(6,89)
81       89 FORMAT(/14X,34(2H- )//)
82     1200 CONTINUE
83          WRITE(6,90)
84          STOP
85          END

      $ENTRY
```

Appendix 6.2 The computer program for TOPSIS produces the relative closeness to the ideal solution.

```
      $JOB          JSS,TIME=45,PAGES=200,RUN=CHECK
      C$OPTIONS  PAGES=30
    1       DIMENSION AA(20),BB(20),AL(10),AT(12)
    2       DIMENSION R(15,15),RNW(15,15),W(15)
    3       DIMENSION SFI(10),SFN(10),RC(10),SI(10),SN(10)
    4       REAL MAX(15),MIN(15),NORM
      C    THIS PROGRAM DEMONSTRATES THE TOPSIS METHOD
    5       READ(5,91) (AA(K),K=1,20)
    6       WRITE(6,90)
    7       WRITE(6,92) (AA(K),K=1,20)
    8       READ(5,91) (BB(K),K=1,20)
    9       WRITE(6,92) (BB(K),K=1,20)
      C    READ ALTERNATIVE #, ATTRIBUTE #, AND WEIGHT #
   10       READ(5,10) M,N,L
   11       READ(5,94)(AT(J),J=1,N)
   12       READ(5,94)(AL(I),I=1,M)
   13     20 FORMAT(8F10.3)
   14     90 FORMAT('1')
   15     91 FORMAT(20A4)
   16     92 FORMAT(//14X,20A4/)
   17     94 FORMAT(8A3)
   18     95 FORMAT(17X,8(3X,A3,1X)/)
   19     96 FORMAT(14X,A3,1X,8F7.3/)
   20     97 FORMAT(14X,A3,8F7.3/)
   21     98 FORMAT(18X,8F7.3/)
      C    READ DECISION MATRIX
   22       READ(5,20) ((R(I,J),J=1,N),I=1,M)
   23     10 FORMAT(3I5)
   24       WRITE(6,70)
   25     70 FORMAT(/14X,'  **  DECISION MATRIX  **'/)
   26       WRITE(6,95)(AT(J),J=1,N)
   27       WRITE(6,96) (AL(I),(R(I,J),J=1,N),I=1,M)
      C    CALCULATE NORMALIZED DECISION MATRIX
   28       DO 101 I=1,M
   29       DO 101 J=1,N
   30       R(I,J)=R(I,J)/5
   31    101 CONTINUE
   32       WRITE(6,71)
   33     71 FORMAT(/14X,'  **  NORMALIZED DECISION MATRIX  **'/)
   34       WRITE(6,95)(AT(J),J=1,N)
   35       WRITE(6,97) (AL(I),(R(I,J),J=1,N),I=1,M)
   36       DO 1000 K=1,L
      C    READ A SET OF WEIGHTS
   37       READ(5,20) (W(J),J=1,N)
   38       WRITE(6,78)
   39     78 FORMAT(/14X,'  **  WEIGHTS  **'/)
   40       WRITE(6,95)(AT(J),J=1,N)
   41       WRITE(6,98) (W(J),J=1,N)
   42       DO 105 J=1,N
   43       DO 104 I=1,M
      C    CALCULATE WEIGHTED NORMALIZED DECISION MATRIX
   44    104 RNW(I,J)=R(I,J)*W(J)
   45    105 CONTINUE
   46       WRITE(6,72)
   47     72 FORMAT(/14X,'  **  WEIGHTED NORMALIZED DECISION MATRIX  **'/)
   48       WRITE(6,95)(AT(J),J=1,N)
   49       WRITE(6,97) (AL(I),(RNW(I,J),J=1,N),I=1,M)
   50       DO 900 J=1,N
   51       MAX(J)=RNW(1,J)
   52       MIN(J)=RNW(1,J)
```

```
53          DO 900 I=1,M
54          IF(RNW(I,J)-MAX(J)) 910,910,915
55        915 MAX(J)=RNW(I,J)
56          GO TO 900
      C   DETERMINE IDEAL AND NEGATIVE-IDEAL SOLUTIONS
57        910 IF(RNW(I,J)-MIN(J)) 920,900,900
58        920 MIN(J)=RNW(I,J)
59        900 CONTINUE
60          WRITE (6,80)
61         80 FORMAT(/18X,'IDEAL SOL.  N-IDEAL SOL.'/)
62          DO 940 J=1,N
63          WRITE(6,81) AT(J),MAX(J),MIN(J)
64         81 FORMAT(14X,A3,F10.4,5X,F10.4)
65        940 CONTINUE
66          WRITE (6,84)
67         84 FORMAT(/14X,'  IDEAL SEPARATION    N-IDEAL SEPARATION  RELATIVE CL
           *OSENESS'/)
68          DO 950 I=1,M
69          SI(I)=0
70          SN(I)=0
71          DO 960 J=1,N
72          SI(I)=SI(I)+(RNW(I,J)-MAX(J))**2
73          SN(I)=SN(I)+(RNW(I,J)-MIN(J))**2
74        960 CONTINUE
      C   CALCULATE THE SEPARATION MEASURES
75          SFI(I)=SQRT(SI(I))
76          SFN(I)=SQRT(SN(I))
      C   CALCULATE RELATIVE CLOSENESS TO THE IDEAL SOLUTION
77          RC(I)=SFN(I)/(SFI(I)+SFN(I))
78          WRITE(6,85) AL(I),SFI(I),SFN(I),RC(I)
79         85 FORMAT(/14X,A3,1X,3(F10.4,10X))
80        950 CONTINUE
81          WRITE(6,89)
82         89 FORMAT(/14X,34(2H- )//)
83       1000 CONTINUE
84          WRITE(6,90)
85          STOP
86          END

      $ENTRY
```

Appendix 6.3 The computer program for ELECTRE produces aggregate dominance matrix.

```
      $JOB            JSS,TIME=45,PAGES=200,RUN=CHECK
      C$OPTIONS  PAGES=30
    1       DIMENSION AA(20),BB(20),AL(10),AT(12)
    2       DIMENSION R(10,12),RNW(10,12),W(12),CO(10,10),DCON(10,10)
    3       REAL STDD(10,10),MSTDD,RY,RX,NORM,DCD(10,10)
    4       INTEGER C(10,10,12),D(10,10,12),COD(10,10),DCOD(10,10),ADOM(10,10)
      C
      C   THIS PROGRAM DEMONSTRATES THE ELECTRE METHOD
      C
    5       READ(5,91) (AA(K),K=1,20)
    6       WRITE(6,90)
    7    90 FORMAT('1')
    8       WRITE(6,92) (AA(K),K=1,20)
    9       READ(5,91) (BB(K),K=1,20)
   10       WRITE(6,91) (BB(K),K=1,20)
      C    READ ALTERNATIVE #, ATTRIBUTE #, AND WEIGHT #
   11       READ(5,10) M,N,L
   12       READ(5,94)(AT(J),J=1,N)
   13    91 FORMAT(20A4)
   14    92 FORMAT(14X,20A4//)
   15    20 FORMAT(8F10.3)
   16    52 FORMAT(14X,'C',2I1,1X,8I3,T49,'D',2I1,2X,8I3)
   17    61 FORMAT(14X,5F8.4/)
   18    65 FORMAT(14X,5I3/)
   19    93 FORMAT(17X,8(3X,A3,1X)/)
   20    94 FORMAT(8A3)
   21    95 FORMAT(17X,8(3X,A3,1X)/)
   22    96 FORMAT(14X,A3,2F7.2,F7.0,5F7.2/)
   23    97 FORMAT(14X,A3,8F7.3/)
   24    98 FORMAT(18X,12F7.3/)
   25       READ(5,94)(AL(I),I=1,M)
      C    READ DECISION MATRIX
   26       READ(5,20) ((R(I,J),J=1,N),I=1,M)
   27    10 FORMAT(3I5)
   28       WRITE(6,70)
   29    70 FORMAT(/14X,'  **  DECISION MATRIX  **'/)
   30       WRITE(6,93)(AT(J),J=1,N)
   31       WRITE(6,96) (AL(I),(R(I,J),J=1,N),I=1,M)
      C    CALCULATE NORMALIZED DECISION MATRIX
   32       DO 100 J=1,N
   33       RS=0
   34       DO 102 I=1,M
   35   102 RS=RS+R(I,J)**2
   36       NORM=RS**0.5
   37       DO 103 I=1,M
   38   103 R(I,J)=R(I,J)/NORM
   39   100 CONTINUE
   40       WRITE(6,71)
   41    71 FORMAT(/14X,'  **  NORMALIZED DECISION MATRIX  **'/)
   42       WRITE(6,95)(AT(J),J=1,N)
   43       WRITE(6,97) (AL(I),(R(I,J),J=1,N),I=1,M)
   44       DO 125 I=1,M
   45       DO 125 IN=1,M
   46       DO 125 J=1,N
   47       IF(I-IN) 121,124,121
   48   121 IF(R(I,J)-R(IN,J)) 123,122,122
   49   122 C(I,IN,J)=1
   50       D(I,IN,J)=0
   51       GO TO 125
   52   123 C(I,IN,J)=0
```

```
      D(I,IN,J)=1
      GO TO 125
  124 C(I,IN,J)=0
      D(I,IN,J)=0
  125 CONTINUE
      WRITE(6,85)
   85 FORMAT(/14X,'CONCORDANCE SETS',T53,'DISCORDANCE SETS'/)
      WRITE(6,55) (AT(J),J=1,N),(AT(J),J=1,N)
   55 FORMAT(T21,8A3,T55,8A3/)
      DO 53 I=1,M
      DO 53 IN=1,M
      IF(I.EQ.IN) GO TO 53
      WRITE(6,52) I,IN,(C(I,IN,J),J=1,N),I,IN,(D(I,IN,J),J=1,N)
   53 CONTINUE
      WRITE(6,54)
   54 FORMAT(14X,'* SYMBOL 1 INDICATES THAT AN ATTRIBUTE IS INCLUDED IN
     *THE'/15X,' CORRESPONDING CONCORDANCE/DISCONCORDANCE SETS',/)
      DO 1000 K=1,L
C     READ A SET OF WEIGHTS
      WRITE(6,78)
   78 FORMAT(/14X,'  **  WEIGHTS  **'/)
      WRITE(6,95)(AT(J),J=1,N)
      READ(5,20) (W(J),J=1,N)
      WRITE(6,98) (W(J),J=1,N)
      DO 155 J=1,N
      DO 154 I=1,M
C     CALCULATE WEIGHTED NORMALIZED DECISION MATRIX
  154 RNW(I,J)=R(I,J)*W(J)
  155 CONTINUE
      WRITE(6,72)
   72 FORMAT(/14X,'  **  WEIGHTED NORMALIZED DECISION MATRIX  **'/)
      WRITE(6,95)(AT(J),J=1,N)
      WRITE(6,97) (AL(I),(RNW(I,J),J=1,N),I=1,M)
      DO 210 I=1,M
      DO 210 IN=1,M
      RS=0
      RX=0
      RY=0
      DO 205 J=1,N
C     CALCULATE THE CONCORDANCE INDEX MATRIX
      RS=RS+C(I,IN,J)*W(J)
      RX=RX+D(I,IN,J)*ABS(RNW(I,J)-RNW(IN,J))
      RY=RY+ABS(RNW(I,J)-RNW(IN,J))
  205 CONTINUE
      CO(I,IN)=RS
      DCO(I,IN)=RX
      STDD(I,IN)=RY
  210 CONTINUE
      MSTDD=-99999
      DO 254 I=1,M
      DO 254 IN=1,M
      IF(STDD(I,IN)-MSTDD) 254,254,252
  252 MSTDD=STDD(I,IN)
  254 CONTINUE
      SUM=0
      SDCON=0
C     CALCULATE THE DISCORDANCE INDEX MATRIX
      DO 256 I=1,M
      DO 256 IN=1,M
      DCON(I,IN)=DCO(I,IN)/MSTDD
```

```
108          SUM=SUM+CO(I,IN)
109          SDCON=SDCON+DCON(I,IN)
110        256 CONTINUE
      C     CALCULATE THE CONCORDANCE THRESHOLD VALUE
111          A=SUM/(M*(M-1))
112          WRITE(6,73)
113         73 FORMAT(/14X,'  **  CONCORDANCE INDEX MATRIX  **'/)
114          WRITE(6,61) ((CO(I,IN),IN=1,M),I=1,M)
115          WRITE(6,79) A
116         79 FORMAT(/14X,'AVERAGE OF CONCORDANCE INDEICES =',F8.4/)
      C     CALCULATE THE DISCORDANCE THRESHOLD VALUE
117          B=SDCON/(M*(M-1))
      C     DETERMINE THE CONCORDANCE DOMINANCE MATRIX
118          DO 410 I=1,M
119          DO 415 IN=1,M
120          IF(I-IN) 411,414,411
121        411 IF(CO(I,IN)-A) 412,413,413
122        412 COD(I,IN)=0
123            GO TO 415
124        413 COD(I,IN)=1
125            GO TO 415
126        414 COD(I,IN)=0
127        415 CONTINUE
128        410 CONTINUE
129            WRITE(6,74)
130         74 FORMAT(/14X,'  **  CONCORDANCE DOMINANCE MATRIX  **'/)
131            WRITE(6,65) ((COD(I,IN),IN=1,M),I=1,M)
132            WRITE(6,75)
133         75 FORMAT(/14X,'  **  DISCORDANCE INDEX MATRIX  **'/)
134            WRITE(6,61) ((DCON(I,IN),IN=1,M),I=1,M)
135            WRITE(6,99) MSTDD
136         99 FORMAT(/14X,'MAX SUM OF DIVERGENCE BETWEEN ANY TWO PLANS =',F8.4)
137            WRITE(6,80) B
138         80 FORMAT(/14X,'AVERAGE OF DISCORDANCE INDICES =',F8.4/)
      C     DETERMINE THE DISCORDANCE DOMINANCE MATRIX
139          DO 660 I=1,M
140          DO 665 IN=1,M
141          IF(I-IN) 661,664,661
142        661 IF(DCON(I,IN)-B) 662,662,663
143        662 DCOD(I,IN)=1
144            GO TO 665
145        663 DCOD(I,IN)=0
146            GO TO 665
147        664 DCOD(I,IN)=0
148        665 CONTINUE
149        660 CONTINUE
150            WRITE(6,76)
151         76 FORMAT(/14X,'  **  DISCORDANCE DOMINANCE MATRIX  **'/)
152            WRITE(6,65) ((DCOD(I,IN),IN=1,M),I=1,M)
      C     DETERMINE THE AGGREGATE DOMINANCE MATRIX
153          DO 800 I=1,M
154          DO 800 IN=1,M
155          ADOM(I,IN)=COD(I,IN)*DCOD(I,IN)
156        800 CONTINUE
157            WRITE(6,77)
158         77 FORMAT(/14X,'  **  AGGREGATE DOMINANCE MATRIX  **'/)
159            WRITE(6,65) ((ADOM(I,IN),IN=1,M),I=1,M)
160            WRITE(6,89)
161         89 FORMAT(/14X,34(2H- )//)
162       1000 CONTINUE
```

```
163           WRITE(6,90)
164           STOP
165           END

      $ENTRY
```

Applications of A Unified Approach to Multiple Attribute Decision Making

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Abstract

This study deals with applications of a unified approach to MADM, developed by Hwang and Yoon, to four real-world problems.

The unified approach is composed of four phases. They are: screening phase, preference ordering phase, aggregation phase, and synthesis phase.

Screening phase: The dominated and unacceptable alternatives may be removed by dominance and conjective methods in order to reduce the number of alternatives.

Preference ordering phase: Four efficient Multiple Attribute Decision Making (MADM) methods -- linear assignment, simple additive weighting, TOPSIS, and ELECTRE -- evaluate the decision problems independently.

Aggregation phase: The average ranking procedure, Borda method, and Copeland method are applied to aggregate the preference orders obtained by different MADM methods.

Synthesis phase: The intent of this phase is to clasify the results of the three previous aggregate techniques. The procedure is based on the construction of a partially ordered set (poset).

The first real-world problem, 'Industrial land use development problem in the Maasvlakte area of the Netherlands', has 5 alternatives, 8 attributes, and 5 sets of weights. The second one, 'The Singapore mass transit study', has 6 alternatives, 8 attributes, and 3 sets of weights. The third

one, 'Selecting a long-range water resources system in Hungary', has 5 alternatives, 12 attributes, and 2 sets of weights. The last problem, 'Selecting nuclear power plant sites in the Pacific Northwest', has 9 alternatives, 6 attributes, and 1 weighting scheme.

The results show that preference orders obtained by the four efficient MADM methods are usually non-identical. The unified approach is, therefore, very useful and worth while in that it will provide a better understanding of ranking relationships among alternatives and enhance the likelihood of good decisions.

A unified approach to multiple objective decision making (MODM), similar to this study, is proposed.