OPTIMAL TRAFFIC ASSIGNMENTS AND ECONOMIC ANALYSES OF TRANSPORTATION SYSTEMS BY THE DISCRETE MAXIMUM PRINCIPLE

by

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1. INTRODUCTION

The urban transportation planners and the highway designers have developed two important tools to evaluate various transportation improvement alternatives. They are (1) traffic assignment and (2) economic analysis of the transportation system.

Traffic assignment is the process of allocating personal or vehicular trips in an existing or proposed system of travel facilities [19], and economic analysis deals with the minimization of the sum of travel time cost, operating cost and the investment cost of the transportation system. Both of these processes are invaluable from a transportation planning viewpoint, in that they allow proposed facilities to be tested for traffic carrying ability before they are built.

Since 1950, many methods for traffic assignment have been developed and refined until all methods may be classified under three groups—judgement, two path analysis and network analysis [19].

In the judgement method, senior members of the highway department proportion the traffic between old and new facilities on the basis of their own evaluations.

The two path analysis considers assignment to one freeway route and one arterial route on a proportional basis, and diversion curves are formulated from empirical studies. A diversion curve shows the percentage of traffic split between a freeway path and an arterial street path based on such parameters as time ratio, distance ratio, or a combination of the two.

Early traffic assignment usage was concerned with the above mentioned techniques, but because of the obvious limitations of these techniques a "network" approach has been adopted by most agencies responsible for transportation studies. The network analysis considers the traffic assignment within the whole transportation system.

Campbell [2] presented a procedure to assign traffic to expressways in 1956. In 1957, Moore [12] and Dantzig [6], developed algorithms for selecting the shortest route through a network. Other techniques have also been developed since 1957. These techniques are linear programming [4], Shimbel's algorithm [16], and the Road Research Laboratory algorithm [22]. Moore's algorithm is a widely adopted method used with most computer traffic assignment programs.

Today, most traffic assignment methods are primarily of the "all or nothing" type, that is, the traffic between two zones is assigned to a single route regardless of the traffic volume on that route. The route selected is the minimum time path between the zones. The "all or nothing" assignment technique is not realistic in that it does not allow for increased travel time as link traffic volumes approach or exceed link capacities.

It can be concluded that all the above techniques use a constant travel time-volume relationship. The constant travel time-volume relationship poorly approximates the functional relationship between the link travel time and link traffic volume. Therefore, a non-linear travel time-volume relationship has been introduced which satisfies three conditions. First, there exists a proper travel time under free flow or near zero traffic volume conditions. Second, at low volumes travel times must increase slightly with increased traffic volume. Third, as link capacity is reached, travel time must increase rapidly to reflect the congestion conditions. A travel time-volume relationship which satisfies these three conditions represents the 'real world' situation.

In order to take into consideration a non-linear travel time-volume relationship new methods are needed. Attempts to provide such a realistic relationship have resulted in some revised computational procedures such as those developed during the course of the Chicago Area Transportation Study [3]. Wallace [19] has used a systems approach in order to solve traffic assignment problems. Dynamic programming wherein non-linear time functions are employed has been successfully used by Tillman, Pai, Funk and Snell [18]. A continuous research has been carried on by Snell, Funk and their associates on the traffic assignment and economic analysis of transportation systems by using a discrete version of the maximum principle at Kansas State University. Yang and Snell [23] have used the maximum principle [7,14], to solve the traffic assignment problems. They have considered a constant travel time-volume relationship. Snell, Funk and Blackburn [8], have again employed the discrete maximum principle to assign the traffic optimally in any transportation system by taking into account a non-linear travel time-volume relationship.

Numerous methods for economic analysis of any urhan transportation system have been devised in the past few decades. Four principle methods are: (1) the annual cost method, (2) the present worth method, (3) the benefit-cost ratio method, and (4) the rate of return method [1,10,13,15]. No matter which method was used, the analyses made in the past have restricted themselves to comparing alternatives for a single link or a single route of a transportation network. The overall system effect of improvements was completely ignored.

Realizing this deficiency, some recent studies have compared alternatives through complete network analysis. In the Chicago Area Transportation study [22], five alternative freeway systems were developed. In 1958, Garrison and Marble [9] presented a linear programming formulation for the economic analysis of the transportation network. Wallace [19] has employed a systems approach to solve cost minimization problems. Wang, Funk, and Snell [20,21] have used the discrete maximum principle to solve the cost minimization problems. They have considered three different cases of the investment cost.

This report attempts a systematic, elementary and exhaustive presentation of the use of the discrete maximum principle to solve traffic assignment and cest minimization problems in transportation systems. In section 2 the optimal traffic assignment pattern is obtained, considering the constant travel time-volume relationship based on the study made by Yang and Snell [23.24]. In section 3 the behavior of the non-linear travel time-volume relationship is thoroughly explained in a very simplified form. Also, the development of the mathematical model which represents the 'real world' situation is discussed in detail, and an optimal traffic assignment pattern, using the non-linear travel time-volume relationship which is based on the paper of Funk, Snell and Blackburn [8] is presented. A single copy (multi origin single destination) street network is considered. Section 4 considers a multicopy traffic flow network with a non-linear travel time-volume relationship [17]. Two different formulations are studied. In the first formulation turn penalties are considered and in the second formulation turn penalties are assumed to be zero. A comparison is made of the numerical results of the two formulations. In section 5, which is based on Wang, Snell, and Funk [20,21], the economic analysis of the transportation network is studied. Two cases of investment cost have been investigated in detail. The development of the non-linear travel time model, which expresses the travel time as a function of the investment cost and of traffic volume, is described comprehensively.

2. TRAFFIC ASSIGNMENT USING CONSTANT TRAVEL TIME FUNCTION

Traffic assignment is the process of allocating a given set of trip interchanges to a specific transportation system. The problem involving a rectangular system with linear time functions will be considered first. In this case of linear time function the link travel times remain constant as link volumes increase, in other words the link travel time is independent of the corresponding link volume. In Fig. 1, the link travel times are plotted as a function of link volumes. This can mathematically be explained as follows:

let

t = time required by one vehicle to travel a unit distance along a link (unit travel time) in hours per mile per vehicle,

k = free flow travel-time, which is constant, in hours per mile per vehicle,

then for a linear time function

$$t = k.$$

This is the simplest functional relationship that is available to approximate the true travel time curve. This constant travel time function does not provide for greatly increased travel time as traffic volume increases, even though traffic volume may approach link capacity, which is maximum number of vehicles a link can accomodate in unit time. A non-linear travel time-volume relation is discussed in Sec. 3 of this report.

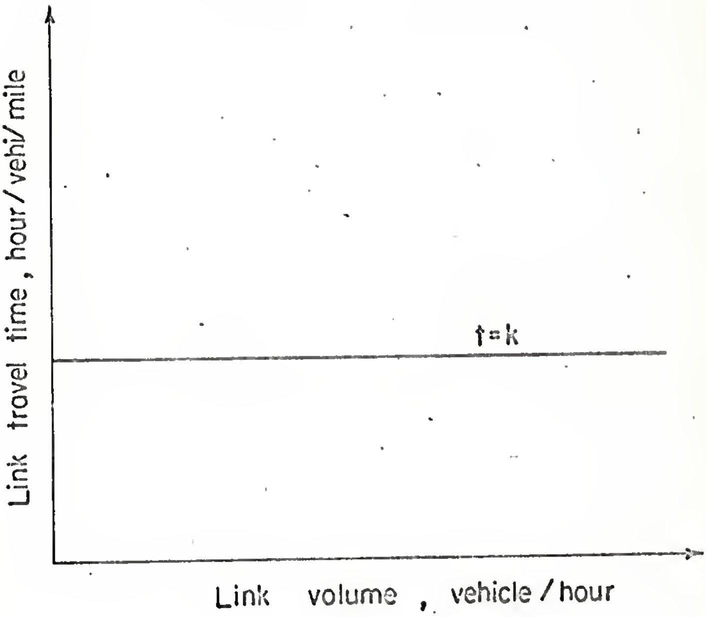


Fig. 1. Constant travel time - volume curve .

FORMULATION OF THE PROBLEM

Suppose that there is a network of traffic-flow as shown in Fig. 2. A network is a combination of all links and nodes. Node is a point where segments of the street network connect and link is a connection between two nodes, representing a segment of the street network.

To simplify the problem of notations a rectangular network is shown, however, the network need not be rectangular in order to solve it by the discrete version of the maximum principle.

Let

$V^{n,m}$ = the total number of vehicles entering the network just before node (n,m),

$\theta_1^{n,m}$ = fraction of the vehicles entering node (n,m) on the horizontal link which leaves on the horizontal link,

$\theta_2^{n,m}$ = fraction of the vehicles entering node (n,m) on the vertical link which leaves on the vertical link,

$x_1^{n,m}$ = the horizontal flow (or the trip volumes assigned to the horizontal link) from node (n,m),

$x_2^{n,m}$ = the vertical flow (or the trip volume assigned to the vertical link) from node (n,m),

$x_3^{n,m}$ = the total cumulative travel time up to node (n,m+1) on the horizontal link (or the cumulative travel time on horizontal links from node (n,1) including the horizontal link immediately beyond node (n,m)),

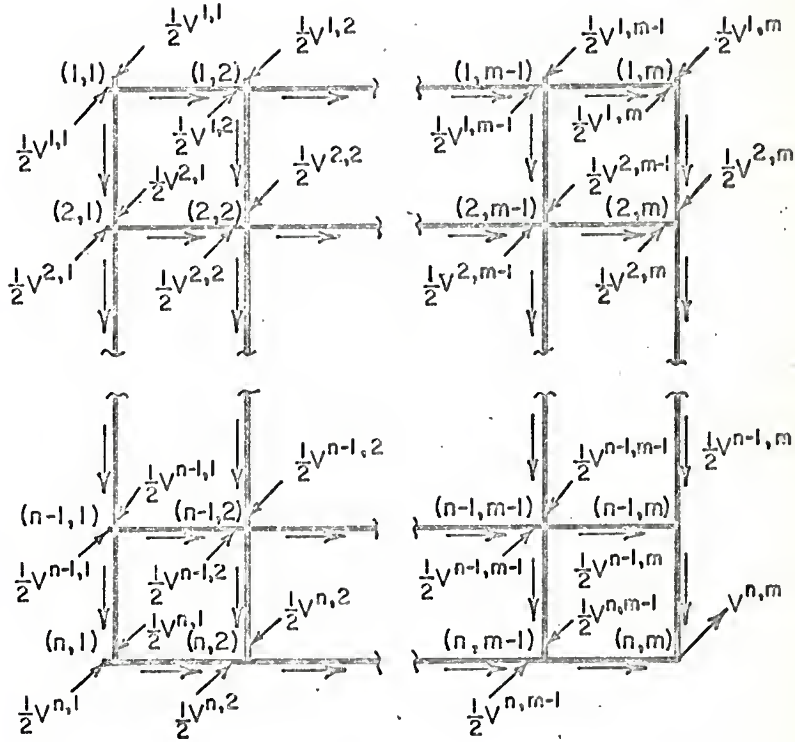


Fig. 2. nxm network .

$x_4^{n,m}$ = the total cumulative travel time up to node (n+1,m) on the vertical links (or the cumulative travel time on vertical links from node (1,m) including the vertical link immediately beyond node (n,m)),

$n = 1, 2, \ldots, N, \qquad m = 1, 2, \ldots, M.$

The problem is to determine a sequence of $\theta_1^{n,m}$ and $\theta_2^{n,m}$ in order to minimize the total cumulative travel time, which is the time required for all the vehicles in the network of (NxM) nodes, starting from different origins, to reach the destinations.

$$S = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{N,m} \quad .$$

Assume that (1) the link travel time does not vary with link volume, and that (2) $V^{n,m}$ can be split up so that $V^{n,m}/2$ enters the vertical link and $V^{n,m}/2$ enters the horizontal link, respectively, just ahead of the node as shown in Fig. 3. The second assumption allows the calculation of number of turns made at a node, thus allowing inclusion of turn delay penalties in the system.

Considering each node as a stage the performance equations for a typical interior node (n,m) in a network are as follows:

$$x_1^{n,m} = \theta_1^{n,m} \left(x_1^{n,m-1} + \frac{V^{n,m}}{2}\right)$$

$$+ (1 - \theta_2^{n,m})\left(x_2^{n-1,m} + \frac{V^{n,m}}{2}\right), \; x_1^{n,0} = 0, \tag{1}$$

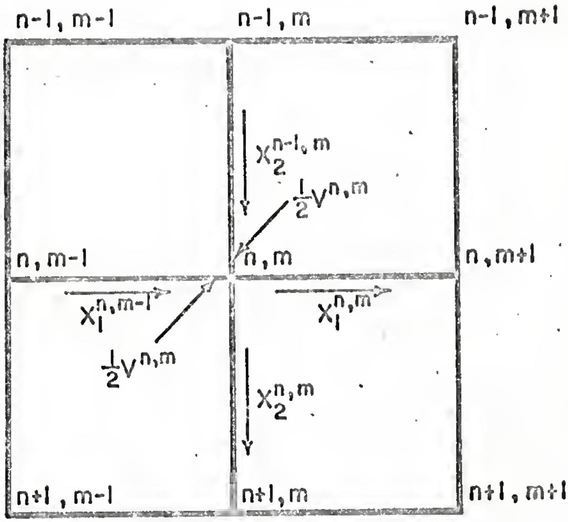


Fig. 3 . Typical interior node of a rectangular network .

$$x_2^{n,m} = (1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2})$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + \frac{V^{n,m}}{2}), \quad x_2^{0,m} = 0, \tag{2}$$

$$x_3^{n,m} = x_3^{n,m-1} + K_1^{n,m} x_1^{n,m} + K_L^{n,m} (1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2})$$

$$= x_3^{n,m-1} + K_1^{n,m}\big(\theta_1^{n,m}(x_1^{n,m-1} + \frac{V^{n,m}}{2}) +$$

$$+ (1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2})\big)$$

$$+ K_L^{n,m}(1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2}), \quad x_3^{n,0} = 0, \tag{3}$$

and

$$x_4^{n,m} = x_4^{n-1,m} + K_2^{n,m}\, x_2^{n,m} + K_R^{n,m} (1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2})$$

$$= x_4^{n-1,m} + K_2^{n,m}\big((1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2})$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + \frac{V^{n,m}}{2})\big)$$

$$+ K_R^{n,m}(1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2}), \quad x_4^{0,m} = 0, \tag{4}$$

$n = 1, 2, \ldots, N, \qquad m = 1, 2, \ldots, M,$

where $K_j^{n,m}$, $j = 1,2$, represent the travel time coefficients for the horizontal and vertical links, immediately beyond node (n,m), respectively, and $K_L^{n,m}$ and $K_R^{n,m}$ represent the left-turn and right-turn penalties respectively at node (n,m).

The Hamiltonian function and the adjoint variables can be written as

$$H^{n,m} = \sum_{i=1}^{4} z_i^{n,m} x_i^{n,m}$$

$$= z_1^{n,m}\left(\theta_1^{n,m}(x_1^{n,m-1} + \frac{V^{n,m}}{2}) + (1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2})\right)$$

$$+ z_2^{n,m}\left((1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2}) + \theta_2^{n,m}(x_2^{n-1,m} + \frac{V^{n,m}}{2})\right)$$

$$+ z_3^{n,m}\{x_3^{n,m-1} + K_1^{n,m}\left(\theta_1^{n,m}(x_1^{n,m-1} + \frac{V^{n,m}}{2})\right.$$

$$\left.+ (1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2})\right)$$

$$+ K_L^{n,m}(1 - \theta_2^{n,m})(x_2^{n-1,m} + \frac{V^{n,m}}{2})\} + z_4^{n,m}\{x_4^{n-1,m}$$

$$+ K_2^{n,m}\left((1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2}) + \theta_2^{n,m}(x_2^{n-1,m} + \frac{V^{n,m}}{2})\right)$$

$$+ K_R^{n,m}(1 - \theta_1^{n,m})(x_1^{n,m-1} + \frac{V^{n,m}}{2})\}, \tag{5}$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m}\theta_1^{n,m} + z_2^{n,m}(1 - \theta_1^{n,m})$$

$$+ z_3^{n,m}K_1^{n,m}\theta_1^{n,m} + z_4^{n,m}[K_2^{n,m}(1 - \theta_1^{n,m}) + K_R^{n,m}(1 - \theta_1^{n,m})], \tag{6}$$

$$z_2^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_2^{n-1,m}}$$

$$= z_1^{n,m}(1 - \theta_2^{n,m}) + z_2^{n,m}\theta_2^{n,m} + z_3^{n,m}[K_1^{n,m}(1 - \theta_2^{n,m})$$

$$+ K_L^{n,m}(1 - \theta_2^{n,m})] + z_4^{n,m} K_2^{n,m} \theta_2^{n,m} , \tag{7}$$

$$z_3^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_3^{n,m-1}} = z_3^{n,m} , \tag{8}$$

$$z_4^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_4^{n-1,m}} = z_4^{n,m} . \tag{9}$$

The objective function to be minimized is as follows

$$S = \sum_{n=1}^{N} c_3 x_3^{n,M} + \sum_{m=1}^{M} c_4 x_4^{N,m} = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{N,m} . \tag{10}$$

Therefore,

$$c_1 = 0, \qquad c_2 = 0, \tag{10a}$$

$$c_3 = 1, \qquad c_4 = 1, \tag{10b}$$

and

$$z_1^{n,M} = c_1 = 0, \qquad n = 1, 2, \ldots, N, \tag{11a}$$

$$z_2^{N,m} = c_2 = 0, \qquad m = 1, 2, \ldots, M, \tag{11b}$$

$$z_3^{n,M} = c_3 = 1, \qquad n = 1, 2, \ldots, N, \tag{11c}$$

$$z_4^{N,m} = c_4 = 1, \qquad m = 1, 2, \ldots, M. \tag{11d}$$

From equations (8), (9), (11c) and (11d) we have

$$z_3^{n,m} = z_4^{n,m} = 1, \quad n = 1, 2, \ldots, N; \quad m = 1, 2, \ldots, M. \tag{12}$$

It is worth noting that the Hamiltonian function is linear in the decision variables, therefore, the optimal decisions, $\theta_j^{n,m}$, $j = 1, 2$, which are determined to minimize $H^{n,m}$ are either the upper bound ($\theta_j^{n,m} = 1.0$) or the lower bound ($\theta_j^{n,m} = 0.$) of the decision variables.

COMPUTATIONAL PROCEDURE

There are several computational procedures which can be used to solve this type of problem. One of them is as follows:

Step 1. Assume θ's for all nodes; $\theta_1^{n,m}$ and $\theta_2^{n,m}$ at any node should either be zero or one.

Step 2. Start at node (1,1) and work forward through the network, calculate all the values of $x_i^{n,m}$ from equations (1) through (4).

Step 3. Start from the destination node and work backward to calculate the values of the adjoint variables, $z_i^{n,m}$, $i = 1, 2$, at each node, from equations (6), (7), (11), and (12).

Step 4. Minimize the Hamiltonian function at each stage, in turn, thus determining the desired value of the decision variables at each node.

Step 5. Return to step 2 and repeat the process until two successive sets of decision-variable are identical.

NUMERICAL EXAMPLE

The technique is illustrated in the following simple numerical example. A 2x3 traffic-flow network is shown in Fig. 4. The link travel time coefficients are as shown. The direction of flow in each link is preassigned. For convenience, assume $K_L^{n,m} = 3$ and $K_R^{n,m} = 1$, for n = 1, 2; m = 1, 2, 3. Assume $V^{1,1} = 5$, $V^{1,2} = 5$, $V^{2,3} = -10$, and all other $V^{n,m} = 0$. The problem is to find the minimum path from the origins (1,1) and (1,2) to the destination (2,3).

Step 1. θ's are assumed as follows:

$$\theta_1^{1,1} = 1, \quad \theta_2^{1,1} = 0, \quad \theta_1^{1,2} = 1, \quad \theta_2^{1,2} = 0.$$

The values of θ's which are determined from the configuration are:

$$\theta_1^{1,3} = 0, \quad \theta_2^{1,3} = 1, \quad \theta_1^{2,1} = 1, \quad \theta_2^{2,1} = 0,$$

$$\theta_1^{2,2} = 1, \quad \theta_2^{2,2} = 0.$$

Step 2. Calculating forward starting from node (1,1) for x's by applying equations (1) through (4), we obtain

$$x_1^{1,1} = 5, \quad x_2^{1,1} = 0, \quad x_3^{1,1} = 132.5, \quad x_4^{1,1} = 0,$$

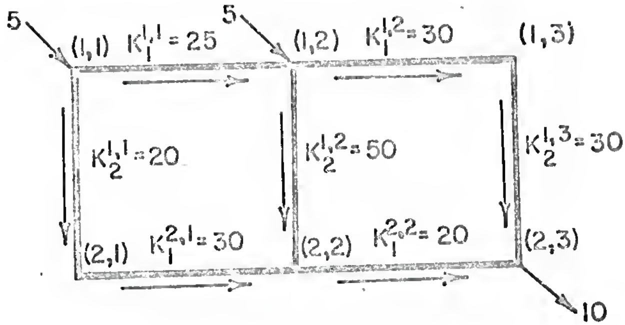


Fig. 4. Network for numerical problem .

$$x_1^{1,2} = 10, \quad x_2^{1,2} = 0, \quad x_3^{1,2} = 440, \quad x_4^{1,2} = 0,$$

$$x_1^{1,3} = 0, \quad x_2^{1,3} = 10, \quad x_3^{1,3} = 440, \quad x_4^{1,3} = 310,$$

$$x_1^{2,1} = 0, \quad x_2^{2,1} = 0, \quad x_3^{2,1} = 0, \quad x_4^{2.1} = 0,$$

$$x_1^{2,2} = 0, \quad x_2^{2,2} = 0, \quad x_3^{2,2} = 0, \quad x_4^{2,2} = 0.$$

The total time for the assumption is

$$S = \sum_{n=1}^{2} x_3^{n,3} + \sum_{m=1}^{3} x_4^{2,m} = 750.$$

Step 3. From equations (11a) and (11b) we have

$$z_1^{1,3} = z_1^{2,3} = 0, \qquad z_2^{2,1} = z_2^{2,2} = z_2^{2,3} = 0.$$

By applying the recurrence equations, given by equations (6), (7), (11) and (12) for adjoint variables, we now work backward starting from node (2,3).

$$z_1^{2,2} = 0, \quad z_1^{2,1} = 0, \quad z_2^{1,3} = 0, \quad z_2^{1,2} = 23,$$

$$z_2^{1,1} = 33, \quad z_1^{1,2} = 31, \quad z_1^{1,1} = 61.$$

Step 4. Because of the boundary conditions the decision variables at nodes (1,3), (2,1), and (2,2) are fixed. Hence, we need to minimize the Hamiltonian functions at nodes (1,2) and (1,1) to minimize the total travel time.

There are four possible combinations of choosing $\theta_1^{1,2}$ and $\theta_2^{1,2}$ at node (1,2). The corresponding Hamiltonian can be obtained as follows:

(1) The combination of $\theta_1^{1,2} = 1$, and $\theta_2^{1,2} = 0$, gives

$$x_1^{1,2} = 10, \quad x_2^{1,2} = 0, \quad x_3^{1,2} = 440, \quad x_4^{1,2} = 0,$$

and

$$H^{1,2} = 750.$$

(2) The combination of $\theta_1^{1,2} = 0$, and $\theta_2^{1,2} = 1$ gives

$$x_1^{1,2} = 0, \quad x_2^{1,2} = 10, \quad x_3^{1,2} = 132.5, \quad x_4^{1,2} = 507.5,$$

and

$$H^{1,2} = 870.$$

(3) The combination of $\theta_1^{1,2} = 1$, and $\theta_2^{1,2} = 1$ gives

$$x_1^{1,2} = 7.5, \quad x_2^{1,2} = 2.5, \quad x_3^{1,2} = 365, \quad x_4^{1,2} = 125,$$

and

$$H^{1,2} = 780.$$

(4) The combination of $\theta_1^{1,2} = 0$, and $\theta_2^{1,2} = 0$ gives

$$x_1^{1,2} = 2.5, \quad x_2^{1,2} = 7.5, \quad x_3^{1,2} = 215, \quad x_4^{1,2} = 382.5,$$

and

$$H^{1,2} = 947.5.$$

The minimum of the Hamiltonian is $H^{1,2} = 750$, and the decisions are $\theta_1^{1,2} = 1$ and $\theta_2^{1,2} = 0$. These values are used for the next iteration, and we obtain

$$z_1^{1,1} = 61, \quad z_2^{1,1} = 33, \quad z_3^{1,1} = 1, \quad z_4^{1,1} = 1.$$

Similarly for node (1,1), we obtain

(1) For $\theta_1^{1,1} = 1$, and $\theta_2^{1,1} = 0$, we obtain

$$x_1^{1,1} = 5, \quad x_2^{1,1} = 0, \quad x_3^{1,1} = 132.5, \quad x_4^{1,1} = 0,$$

and

$$H^{1,1} = 437.5.$$

(2) For $\theta_1^{1,1} = 1$, and $\theta_2^{1,1} = 1$, we obtain

$$x_1^{1,1} = 2.5, \quad x_2^{1,1} = 2.5, \quad x_3^{1,1} = 62.5, \quad x_4^{1,1} = 50,$$

and

$$H^{1,1} = 347.5.$$

(3) For $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 1$, we obtain

$$x_1^{1,1} = 0, \quad x_2^{1,1} = 5, \quad x_3^{1,1} = 0, \quad x_4^{1,1} = 102.5,$$

and

$$H^{1,1} = 264.5.$$

(4) For $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 0$, we have

$$x_1^{1,1} = 2.5, \quad x_2^{1,1} = 2.5, \quad x_3^{1,1} = 70, \quad x_4^{1,1} = 52.5,$$

and

$$H^{1,1} = 357.5.$$

The minimum of the Hamiltonian is $H^{1,1} = 264.5$ with the decisions of $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 1$.

According to the procedure illustrated above, the results of the second iteration are as follows:

Step 1. Assume

$\theta_1^{1,1} = 0, \quad \theta_2^{1,1} = 1, \quad \theta_1^{1,2} = 1, \quad \theta_2^{1,2} = 0.$

$\theta^{n,m}$'s fixed by the boundary conditions are:

$\theta_1^{1,3} = 0, \quad \theta_2^{1,3} = 1, \quad \theta_1^{2,1} = 1, \quad \theta_2^{2,1} = 0, \quad \theta_1^{2,2} = 0,$

$\theta_2^{2,2} = 0.$

Step 2.

$x_1^{1,1} = 0, \quad x_2^{1,1} = 5, \quad x_3^{1,1} = 0, \quad x_4^{1,1} = 102.5,$

$x_1^{1,2} = 5, \quad x_2^{1,2} = 0, \quad x_3^{1,2} = 157.5, \quad x_4^{1,2} = 0,$

$x_1^{1,3} = 0, \quad x_2^{1,3} = 5, \quad x_3^{1,3} = 157.5, \quad x_4^{1,3} = 155,$

$x_1^{2,1} = 5, \quad x_2^{2,1} = 0, \quad x_3^{2,1} = 165, \quad x_4^{2,1} = 102.5,$

$x_1^{2,2} = 5, \quad x_2^{2,2} = 0, \quad x_3^{2,2} = 265, \quad x_4^{2,2} = 0,$

and

$$S = \sum_{n=1}^{2} x_3^{n,3} + \sum_{m=1}^{3} x_4^{2,m} = 680.$$

Step 3.

$z_1^{2,2} = 0, \quad z_1^{2,1} = 20, \quad z_2^{1,3} = 0, \quad z_2^{1,2} = 31, \quad z_2^{1,1} = 61,$

$z_1^{1,2} = 23, \quad z_1^{1,1} = 33.$

Step 4.

At node (1,2):

For $\theta_1^{1,2} = 1$, and $\theta_2^{1,2} = 0$, we obtain

$x_1^{1,2} = 5$, $x_2^{1,2} = 0$, $x_3^{1,2} = 157.5$, $x_4^{1,2} = 0$,

and

$H^{1,2} = 312.5$.

For $\theta_1^{1,2} = 0$, and $\theta_2^{1,2} = 1$, we have

$x_1^{1,2} = 0$, $x_2^{1,2} = 5$, $x_3^{1,2} = 0$, $x_4^{1,2} = 252.5$,

and

$H^{1,2} = 367.5$.

For $\theta_1^{1,2} = 1$, and $\theta_2^{1,2} = 1$, we have

$x_1^{1,2} = 2.5$, $x_2^{1,2} = 2.5$, $x_3^{1,2} = 75$, $x_4^{1,2} = 125$,

and

$H^{1,2} = 335$.

For $\theta_1^{1,2} = 0$, and $\theta_2^{1,2} = 0$, we have

$x_1^{1,2} = 2.5$, $x_2^{1,2} = 2.5$, $x_3^{1,2} = 82.5$, $x_4^{1,2} = 127.5$,

and

$H^{1,2} = 345$.

The minimum of the Hamiltonian gives the optimal decision of $\theta_1^{1,2} = 1$ and $\theta_2^{1,2} = 0$.

At node (1,1):

For $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 1$, we have

$x_1^{1,1} = 0, \quad x_2^{1,1} = 5, \quad x_3^{1,1} = 0, \quad x_4^{1,1} = 102.5,$

and

$H^{1,1} = 267.5.$

For $\theta_1^{1,1} = 1$, and $\theta_2^{1,1} = 0$, we have

$x_1^{1,1} = 5, \quad x_2^{1,1} = 0, \quad x_3^{1,1} = 132.5, \quad x_4^{1,1} = 0,$

and

$H^{1,1} = 437.5.$

For $\theta_1^{1,1} = 1$, and $\theta_2^{1,1} = 1$, we have

$x_1^{1,1} = 2.5, \quad x_2^{1,1} = 2.5, \quad x_3^{1,1} = 62.5, \quad x_4^{1,1} = 50,$

and

$H^{1,1} = 347.5.$

For $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 0$, we have

$x_1^{1,1} = 2.5, \quad x_2^{1,1} = 2.5, \quad x_3^{1,1} = 70, \quad x_4^{1,1} = 52.5,$

and

$H^{1,1} = 357.5.$

The minimum of the Hamiltonian is $H^{1,1} = 267.5$ with the optimal decision of $\theta_1^{1,1} = 0$, and $\theta_2^{1,1} = 1$.

Thus, we find that when the iterative process is repeated, the last two consecutive sets of decision variables are identical and gives

$$S = 680.$$

Therefore, it is determined that for $V^{1,1} = 5$, $V^{1,2} = 5$, and $V^{2,3} = -10$, the least-time path for $V^{1,1}$ is $(1,1) \to (2,1) \to (2,2) \to (2,3)$; and the least-time path for $V^{1,2}$ is $(1,2) \to (1,3) \to (2,3)$.

The number of iterations may increase with increasing dimension of the network. Figure 5 shows the number of iterations versus the total travel time for the numerical problem solved above.

CONCLUSION

For linear travel times the maximum principle method is identical with Moore's "Algorithm A.". Therefore, it may be concluded that the method provides an additional variational proof of Moore's algorithm.

Interested readers are referred to references [23] and [24] for more complex numerical examples than the one presented here.

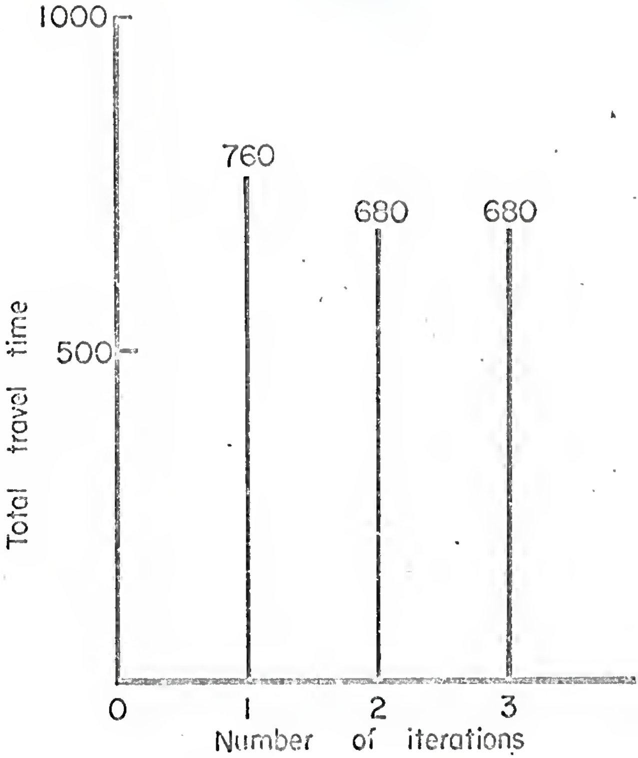


Fig. 5. Number of iterations versus total travel time.

3. TRAFFIC ASSIGNMENT USING NONLINEAR TRAVEL TIME FUNCTION

The constant travel time function to approximate the functional relationship of the travel time-volume in Sec. 2 is a poor approximation. The travel time-volume relationship is introduced and a nonlinear travel time function which makes it possible to simulate congestion on urban streets in a more realistic manner is presented. This nonlinear travel time function is applied to a single copy (single destination) urban network and the optimal traffic assignment pattern is obtained.

TRAVEL TIME-VOLUME RELATIONSHIPS

Numerous studies [11,19] have shown that a predictable relationship exists between speed and volume on urban streets and freeways. Figure 1 shows the typical relationship between the link traffic volume and link operation speed. Since the travel time is the reciprocal of speed, the curve shown in Fig. 1 can be converted to a travel time-volume curve as shown in Fig. 2. It can be explained from Figs. 1 and 2 that, when there are no vehicles on the street, the speed of any vehicle traveling on the street will be maximum, or it will require minimum travel time. This is shown by Point A in Fig. 1, and Point A' in Fig. 2. As the number of vehicles per hour increases on the link, obviously the speed of a vehicle on the link, will decrease. This decrease of the speed of vehicles, is linear as far as three fourths of the link capacity on a freeway is reached. The curve in Fig. 1 is almost linear up to Point B, the corresponding point in Fig.

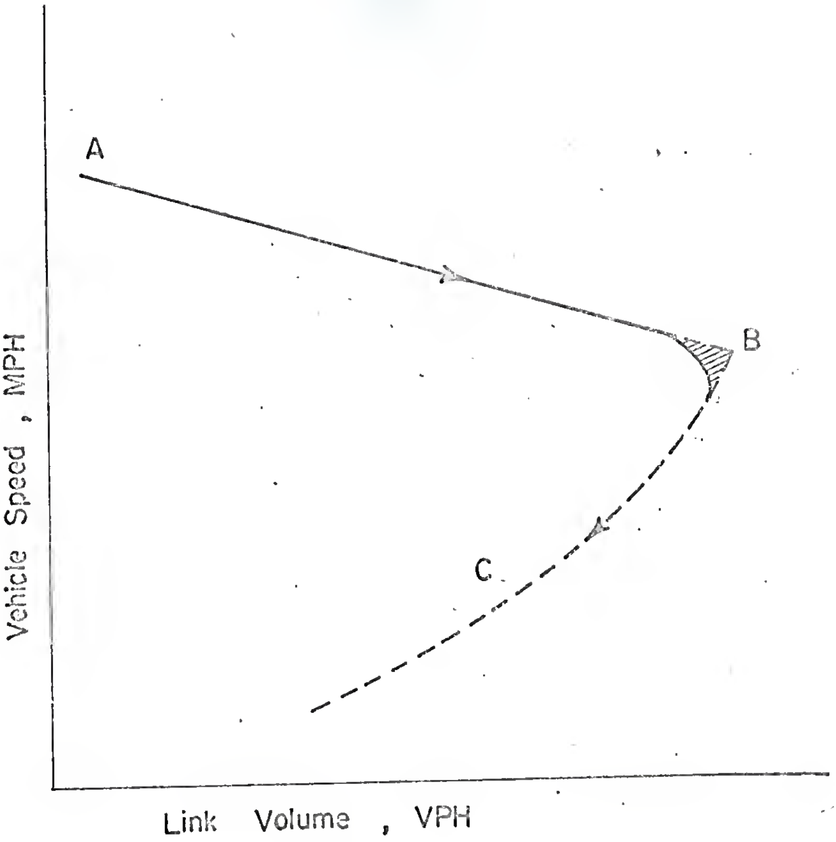


Fig. 1. Typical Speed Volume Curve .

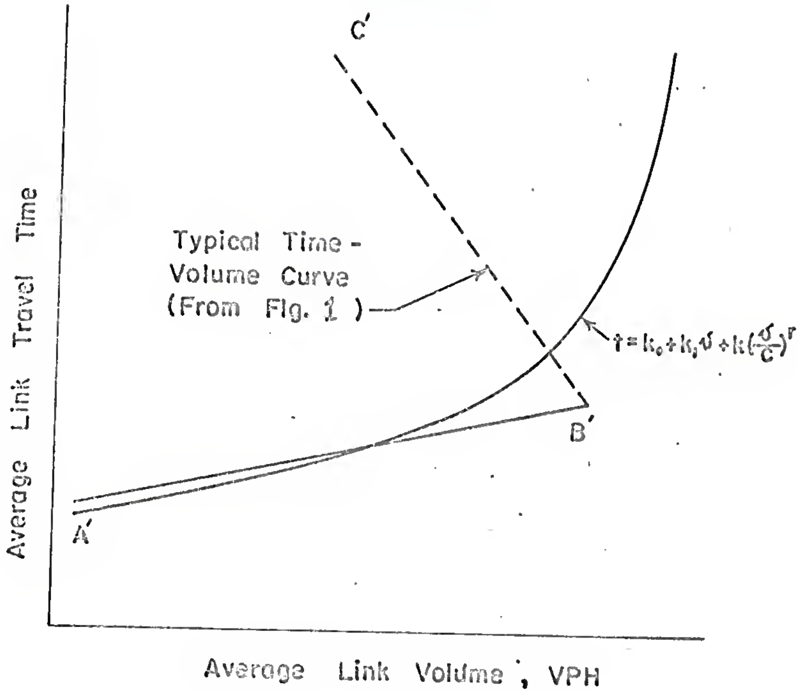


Fig. 2. Typical Travel - Time Volume Relationship.

2 of the travel-time volume is B'. Beyond this volume additional vehicles cause an increasingly rapid reduction in the average speed of vehicles on the freeway. Point B in Fig. 1 indicates the link capacity. If the volume at any time exceeds this limit, there will be congestion. At the link capacity, the flow on the link becomes very unstable and a slight incident can cause a reduction in average speed. This can be indicated by dotted line on both the curves. Beyond capacity the travel time increases considerably.

To describe adequately the relationship between the link travel time and the link volume, there are three conditions to be satisfied. First, there exists a proper travel time under free flow or near zero traffic volume conditions. Second, at low volumes travel times must increase slightly with increased traffic volume. Third, as link capacity is reached, travel time must increase rapidly to reflect the congestion condition.

A constant travel time function

$$t = k$$

which is used in Sect. 2 is the simplest functional relationship to approximate the travel time curve shown in Fig. 2. It is apparent from Fig. 2 that a constant travel time function is a poor approximation of the true travel time-volume relationship. The constant travel time function can meet only the first condition, that is, it can provide only for free flow travel times at near zero link traffic volumes. The constant function does not provide for greatly increased travel times as traffic volume increases even though traffic volume may approach link capacity.

The typical travel time-volume relationship over the range represented by the points A'B'C' on the curve can be approximated by the following non-linear functional relationship between the link travel time and the link volume [1]

$$t = k_0 + k_1 v + k_2 \ (v/C)^r \tag{2}$$

where

t = link travel time in hours per vehicle,

k_0 = constant representing travel time at free flow conditions,

k_1, k_2 = empirically derived constants,

v = link volume in vehicles per hour,

C = link capacity in vehicles per hour,

r = empirically derived exponent.

This equation, equation (2), contains three terms which are required to approximate the important characteristics of the typical time-volume curve in Fig. 2. The first term represents the travel time at free flow or near-zero volume conditions. The second term serves to increase travel times as link volume increases. The increase in travel time due to a unit increase in volume depends on the magnitude of the constant k_1. Thus, the first two terms of equation (2) represent the linear portion of the time-volume curves between the points A' and B' as shown in Fig. 2.

The third term, $k_2(v/C)^r$, represents the effect of congestion on the travel time for the facility under consideration.

The magnitude of this effect will depend on the value of the exponent r and the constant k_2 and if the link volume remains small compared to the capacity of the link this term should contribute little to the link travel time t. As the link volume nears capacity (v>C) the travel time becomes so great that in effect the link has been closed to additional traffic.

In Fig. 2 the dashed segment of the curve A'B'C' represents conditions of congestion and thus represents an undesirable operating region. In using equation (2) the operation of the system beyond the point B' is difficult because the third term acts as a constraint which prevents the system from operating in the B' and C' region. Operation is, however, possible in this range but as the expense of greatly increased travel time.

STATEMENT OF THE PROBLEM

An example of traffic assignment problem which is the minimization of total accumulative travel time for an urban street and freeway network is studied. The example network is shown in Fig. 3 together with the trip origin and destination. The network is composed of two classes of streets, arterial streets and collector streets. Each class of street is characterized by a travel time function which is as follows:

$$t_a^{n,m} = k_{a0}^{n,m} + k_{a1}^{n,m} v^{n,m} + k_{a2}^{n,m} \left(\frac{v^{n,m}}{C^{n,m}}\right)^r \tag{3a}$$

$$t_c^{n,m} = k_{c0}^{n,m} + k_{c1}^{n,m} v^{n,m} + k_{c2}^{n,m} \left(\frac{v^{n,m}}{C^{n,m}}\right)^r \tag{3b}$$

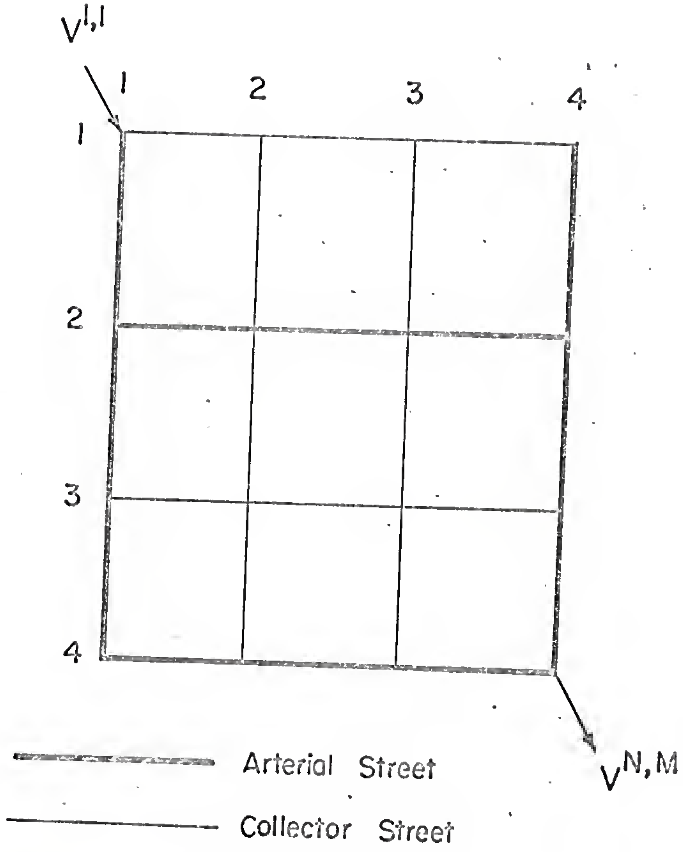


Fig. 3. SIMPLE 4x4 NETWORK

where

t_a = link travel time on arterial streets,

t_c = link travel time on collector streets.

FORMULATION OF THE PROBLEM

In general, for a typical interior node of a rectangular network, as shown in Fig. 4, the performance equations are as follows:

$$x_1^{n,m} = \theta^{n,m} (x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}), \qquad x_1^{n,0} = 0, \tag{4}$$

$$x_2^{n,m} = (1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}), \qquad x_2^{0,m} = 0, \tag{5}$$

$$x_3^{n,m} = x_3^{n,m-1} + T_3^{n,m}(\theta^{n,m}, x_1^{n,m-1}, x_2^{n-1,m}, v^{n,m}) , \qquad x_3^{n,0} = 0, \tag{6}$$

$$x_4^{n,m} = x_4^{n-1,m} + T_4^{n,m}(\theta^{n,m}, x_1^{n,m-1}, x_2^{n-1,m}, v^{n,m}), \qquad x_4^{0,m} = 0, \tag{7}$$

$$n = 1, 2, \ldots, N; \qquad m = 1, 2, 3, \ldots, M,$$

where

$x_j^{n,m}$ = a state variable representing the number of vehicles on link j immediately beyond node (n,m), j = 1, 2, in which j = 1 denotes the horizontal link and j = 2 denotes the vertical link,

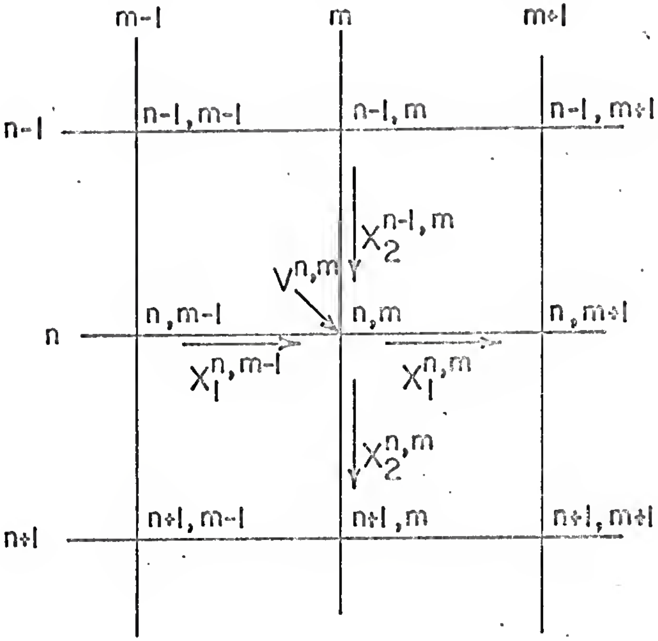


Fig. 4. Typical interior node showing assumed network flow directions.

$x_3^{n,m}$ = a state variable representing the accumulated travel time on horizontal links from node (n,1) including the horizontal link immediately beyond node (n,m),

$x_4^{n,m}$ = a state variable representing the accumulated travel time on vertical links from node (1,m) including the vertical link immediately beyond node (n,m),

$T_j^{n,m}$ = the relationship between total vehicle hours on the horizontal link (j=3) or on the vertical link (j=4) immediately beyond node (n,m) and the number of vehicles on that link,

$v^{n,m}$ = the number of vehicles entering or leaving the network at node (n,m),

$\theta^{n,m}$ = the decision variable that represents the fraction of the vehicles which enter the node and leave on the horizontal link, at the node (n,m).

The objective function to be minimized, which is the total cumulative travel time of all trips in the system, is given by

$$S = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{N,m} \quad . \tag{8}$$

In this formulation of the problem, the travel time-volume relationship of equation (2) with r = 10 is used, therefore, $T_3^{n,m}$ and $T_4^{n,m}$ are given by

$$T_3^{n,m} = [k_{10}^{n,m} + k_{11}^{n,m} x_1^{n,m} + k_{12}^{n,m} \{\frac{x_1^{n,m}}{C_1^{n,m}}\}^{10}]\, x_1^{n,m} , \tag{9}$$

$$T_4^{n,m} = [k_{20}^{n,m} + k_{21}^{n,m} x_2^{n,m} + k_{22}^{n,m} \{\frac{x_2^{n,m}}{C_2^{n,m}}\}^{10}]\, x_2^{n,m} \tag{10}$$

where $C_1^{n,m}$ and $C_2^{n,m}$ are the link capacities of horizontal link and vertical link, respectively, immediately beyond node (n,m).

The Hamiltonian function and the adjoint variables can be written as

$$H^{n,m} = \sum_{i=1}^{4} z_i^{n,m} x_i^{n,m}$$

$$= z_1^{n,m}\, \theta^{n,m}\, (x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_2^{n,m}\, (1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_3^{n,m}\, \{x_3^{n,m-1} + T_3^{n,m}\, (\theta^{n,m},\ x_1^{n,m-1},\ x_2^{n-1,m},\ v^{n,m})\}$$

$$+ z_4^{n,m}\, \{x_4^{n-1,m} + T_4^{n,m}\, (\theta^{n,m},\ x_1^{n,m-1},\ x_2^{n-1,m},\ v^{n,m})\}$$

$$= z_1^{n,m}\, \theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + z_2^{n,m}(1 - \theta^{n,m})$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + z_3^{n,m}\{x_3^{n,m-1} + k_{10}^{n,m}\theta^{n,m}$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + k_{11}^{n,m}[\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m}$$

$$+ v^{n,m})]^2 + k_{12}^{n,m}\ C_1^{n,m}[\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})/C_1^{n,m}]^{11}\}$$

$$+ z_4^{n,m}\{x_4^{n-1,m} + k_{20}^{n,m}(1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ k_{21}^{n,m}\ [(1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})]^2$$

$$+ k_{22}^{n,m}C_2^{n,m}[(1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})/C_2^{n,m}]^{11}\} \qquad (11)$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m}\theta^{n,m} + z_2^{n,m}(1-\theta^{n,m}) + z_3^{n,m}\{k_{10}^{n,m}\theta^{n,m}$$

$$+ 2k_{11}^{n,m}(\theta^{n,m})^2(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ 11k_{12}^{n,m}(\theta^{n,m})^{11}[(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})/C_1^{n,m}]^{10}\}$$

$$+ z_4^{n,m}\{k_{20}^{n,m}(1-\theta^{n,m}) + 2k_{21}^{n,m}(1-\theta^{n,m})^2(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ 11k_{22}^{n,m}(1-\theta^{n,m})^{11}[(x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m})/C_2^{n,m}]^{10}\} , \qquad (12)$$

$$z_2^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_2^{n-1,m}}$$

$$= z_1^{n,m-1}, \tag{13}$$

$$z_3^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_3^{n,m-1}} = z_3^{n,m}, \tag{14}$$

$$z_4^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_4^{n-1,m}} = z_4^{n,m}, \tag{15}$$

$$z_1^{n,M} = 0, \qquad n = 1, 2, \ldots, N, \tag{16}$$

$$z_2^{N,m} = 0, \qquad m = 1, 2, \ldots, M, \tag{17}$$

$$z_3^{n,M} = 1, \qquad n = 1, 2, \ldots, N, \tag{18}$$

$$z_4^{N,m} = 1, \qquad m = 1, 2, \ldots, M. \tag{19}$$

From equations (14), (15), (18) and (19), we obtain

$$z_3^{n,m} = z_4^{n,m} = 1, \qquad \begin{array}{l} n = 1, 2, \ldots, N, \\ m = 1, 2, \ldots, M. \end{array} \tag{20}$$

The optimal sequence of the decision variable, $\theta^{n,m}$ to minimize the total cumulative travel time is obtained from

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}} = 0 \tag{21a}$$

or

$$H^{n,m} = \text{minimum.} \tag{21b}$$

Equation (21a) gives

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}} = 0$$

$$= (z_1^{n,m} - z_2^{n,m} + k_{10}^{n,m} - k_{20}^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ [2k_{11}^{n,m}\theta^{n,m} - 2k_{21}^{n,m}(1-\theta^{n,m})](x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})^2$$

$$+ 11k_{12}^{n,m}C_1^{n,m}(\theta^{n,m})^{10}[(x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m})/C_1^{n,m}]^{11}$$

$$- 11k_{22}^{n,m}C_2^{n,m}(1-\theta^{n,m})^{10}[(x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m})/C_2^{n,m}]^{11}. \tag{22}$$

The second partial derivative of the Hamiltonian with respect to the decision variable, $\theta^{n,m}$, which is used at the computational procedure, is

$$\frac{\partial^2 H^{n,m}}{\partial(\theta^{n,m})^2} = 2(k_{11}^{n,m} + k_{21}^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})^2$$

$$+ 110k_{12}^{n,m}C_1^{n,m}(\theta^{n,m})^9[(x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m})/C_1^{n,m}]^{11}$$

$$+ 110k_{22}^{n,m}C_2^{n,m}(1-\theta^{n,m})^9[(x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m})/C_2^{n,m}]^{11}. \tag{23}$$

This formulation of the problem assumes essentially that all flows along the links, in the network under consideration are in the direction of the increasing super script. This implies that the network destination node is the point (N,M); i.e., the node farthest to the right and down. If this is not the case, the network has to be renumbered in such a fashion that it fits the above assumption. For instance, in a network such as that shown in Fig. 5, the network needs to be subdivided into four quadrants (networks) as shown in Fig. 6. Now, each quadrant would be treated as an independent problem except for the boundary perturbations caused by flows entering from adjacent quadrants.

In a case such as shown in Figs. 5 and 6, we have following equations:

$$M^{I} + M^{II} = M^{III} + M^{IV} = M+1 = 9,$$

$$N^{I} + N^{IV} = N^{II} + N^{III} = N+1 = 8,$$

where

M^{I} = number of columns on quadrant I,

M^{II} = number of columns on quadrant II,

M^{III} = number of columns on quadrant III,

M^{IV} = number of columns on quadrant IV,

N^{I} = number of rows on quadrant I,

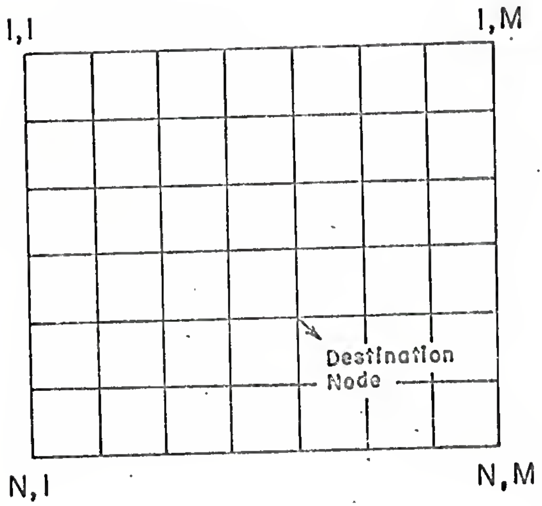


Fig.5 Typical NxM Network .

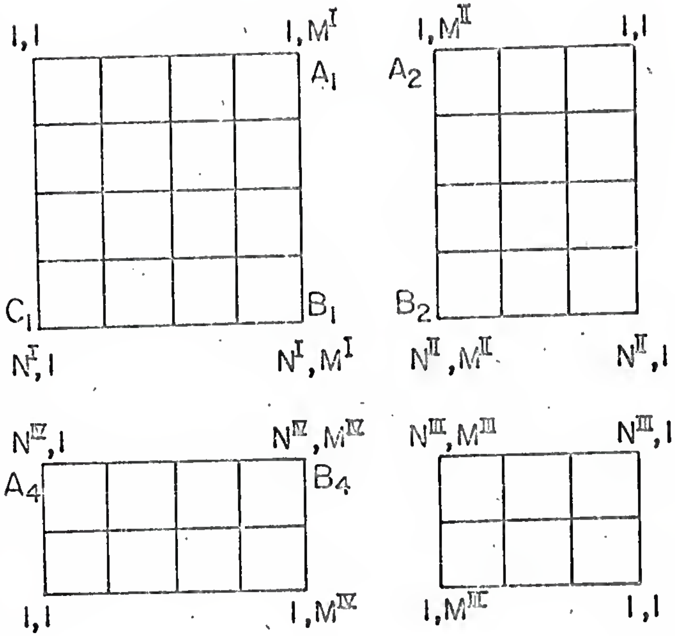


Fig. 6. Subdivided network

N^{II} = number of rows on quadrant II,

N^{III} = number of rows on quadrant III,

N^{IV} = number of rows on quadrant IV.

The links along the interior boundaries of the various quadrants are common to each of the two adjacent networks and must reflect the flows from each quadrant. For example, the links along the interior boundary $A_1\ B_1$ of quadrant I is common to the links of boundary A_2B_2 of the adjacent quadrant II, and C_1B_1 of quadrant I is common to A_4B_4 of quadrant IV.

COMPUTATIONAL PROCEDURE

By using equations (4) through (22) the optimal sequence of decision variables, $\theta^{n,m}$, can be found. The particular algorithm used to accomplish this is as follows:

Step 1. Assign the proper values to the system parameters. These parameters include the empirically found constants, $k^{n,m}_{j,\ell}$ and the exponent 'r' for the travel time equations and the input or output ($v^{n,m}$) for each node (n,m).

Step 2. Assume a set of decision variables, $\theta^{n,m}$, at each node in the network. It is worth mentioning again that $0 \leq \theta^{n,m} \leq 1$.

Step 3. Use equations (4) through (7) to obtain the state variables, $x_1^{n,m}$, $x_2^{n,m}$, $x_3^{n,m}$, and $x_4^{n,m}$ at each node of the network. Start at n = m = 1 and proceed to n = N, m = M.

Step 4. Calculate the values of the adjoint variables $z_1^{n,m}$ and $z_2^{n,m}$. Work backward, starting at n = N, m = M and proceeding to n = m = 1.

Step 5. Calculate $\frac{\partial H^{n,m}}{\partial \theta^{n,m}}$ and $\frac{\partial^2 H^{n,m}}{\partial (\theta^{n,m})^2}$ by equations (22) and (23), using the values of $x_i^{n,m}$ and $z_i^{n,m}$ obtained above.

Step 6. Compute a new sequence of decision variables $\theta^{n,m}$ from the following equation.

$$(\theta^{n,m}) \text{ revised} = (\theta^{n,m})_{old} + \Delta\theta^{n,m} \tag{24}$$

where $\Delta\theta^{n,m}$ is given by

$$\Delta\theta^{n,m} = - \frac{\partial H^{n,m}}{\partial \theta^{n,m}} \bigg/ \frac{\partial^2 H^{n,m}}{\partial (\theta^{n,m})^2} \tag{24a}$$

Step 7. Return to step 3 and repeat the procedure until the new set of decision variables is sufficiently close to the previous set to indicate adequate convergence.

It is worth noting that when the optimal point is not reached, a revised set of decision variables given by equation (24) are assumed and the computations are repeated. For minimization of the Hamiltonian, $H^{n,m}$, the second derivative of the Hamiltonian with respect to the decision variable,

$$\frac{\partial^2 H^{n,m}}{\partial (\theta^{n,m})^2}$$

is positive. When the first derivative of the Hamiltonian with respect to the decision variable,

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}}$$

is negative, then the increment of the decision variable, $\Delta\theta^{n,m}$, should be positive, and if

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}}$$

is positive, $\Delta\theta^{n,m}$ should be negative in order that the decision variable approaches to the optimal point. The magnitude and the sign of the increment $\Delta\theta^{n,m}$ is given by equation (24a).

In the case of a multi-quadrant problem such as that shown in Figs. 5 and 6 the above procedure is carried through on cycle for the 1st quadrant (I) as if it were a total problem. Then one cycle is carried out for the 2nd quadrant (II), using the volume, previously obtained on the horizontal links of quadrant one (I) adjacent to the common boundary of quadrants one and two as inputs to the second quadrant at the common boundary nodes. For this cycle it is assumed that quadrant three does not exist at all. One cycle is carried out on quadrant three, using the volumes on vertical links adjacent to the common boundary between quadrants two and three as inputs at the boundary nodes common to quadrants two and three and ignoring quadrant four. Quadrant four is also handled in a similar fashion. On the second and subsequent cycles the boundary inputs are taken to be the values obtained on the previous cycles for the adjacent quadrants. In this way an assignment can be made for an arbitrarily located destination node.

NUMERICAL EXAMPLES

The technique described above is illustrated in the following two simple numerical examples.

Example 1

Figure 7 shows a 4 x 4 traffic flow network. The link travel time coefficients are given by the following equations:

$t_a = 10 + .06v + 10(v/180)^{10}$ (arterial streets and frontage road)

$t_f = 5 + .02v + 10(v/360)^{10}$ (freeway)

where

$k_{j0}^{n,m} = 10$, $j = 1, 2$ for arterial streets,

$k_{j0}^{n,m} = 5$, $j = 1, 2$ for freeways,

$k_{j1}^{n,m} = .06$, $j = 1, 2$ for arterial streets,

$k_{j1}^{n,m} = .02$, $j = 1, 2$ for freeways,

$k_{j2}^{n,m} = 10$, $j = 1, 2$ both for arterial street and freeways,

$C_j^{n,m} = 180$, $j = 1, 2$ for arterial streets,

$C_j^{n,m} = 360$, $j = 1, 2$ for freeways.

The central business district which is the destination, is assumed to be at node (4,4). The direction of flow in each link is preassigned. The input volumes are also shown in Fig. 7. The problem is to find an optimal traffic assignment along the links for minimum path from origins to the destination.

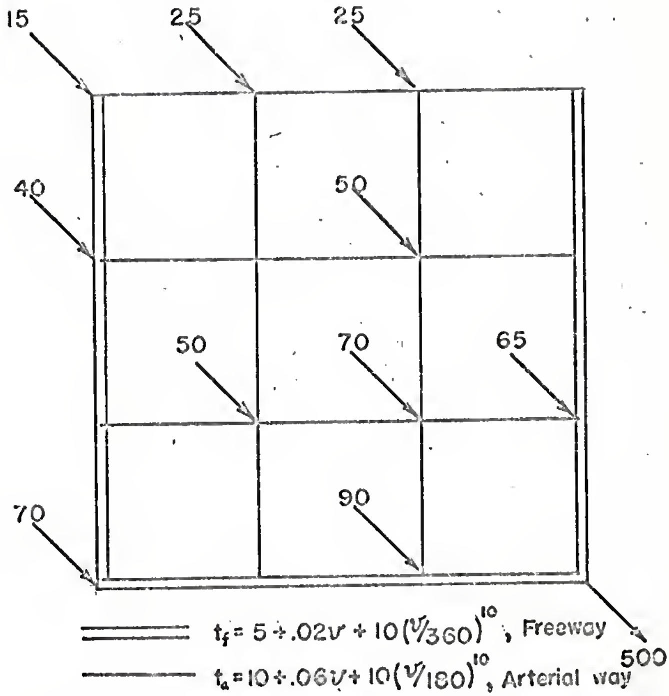


Figure 7. NETWORK FOR NUMERICAL EXAMPLE I.

The optimal sequence of the decision variables was obtained by an IBM 360/50 computer and the final traffic assignment is presented in Fig. 8. The total accumulated travel time is 14,076 time units. It has taken 32 iterations.

Since the destination node is at the last node (4,4) itself the problem is a single quadrant problem.

Example 2

A 5x5 traffic flow network is shown in Fig. 9. The link travel time coefficients are given by the following equations:

$$t_c = 12 + .08v + 10(v/150)^{10} \text{ (collector streets)}$$

$$t_a = 10 + .06v + 10(v/180)^{10} \text{ (arterial streets and frontage road)}$$

$$t_f = 5 + .02v + 10(v/360)^{10} \text{ (freeway)}$$

where

$k_{j0}^{n,m} = 12$, $j = 1, 2$ for collector streets,

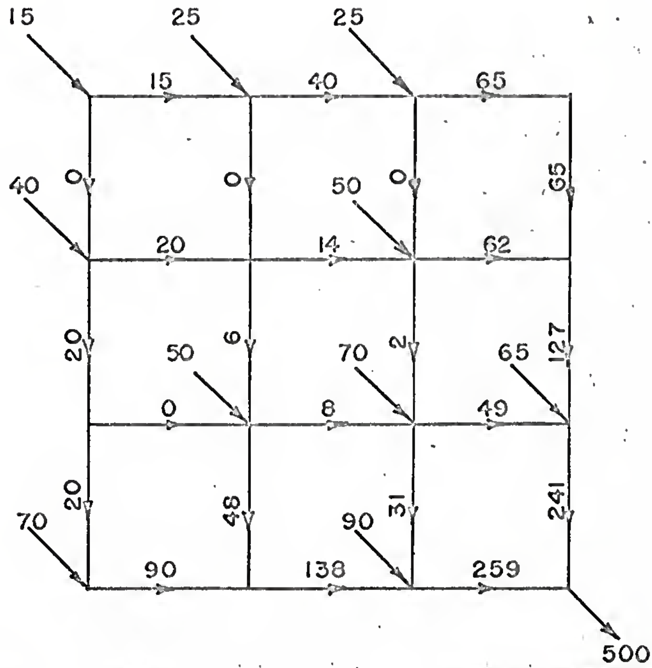
$k_{j0}^{n,m} = 10$, $j = 1, 2$ for arterial streets,

$k_{j0}^{n,m} = 5$, $j = 1, 2$ for freeways,

$k_{j1}^{n,m} = .08$, $j = 1, 2$ for collector streets,

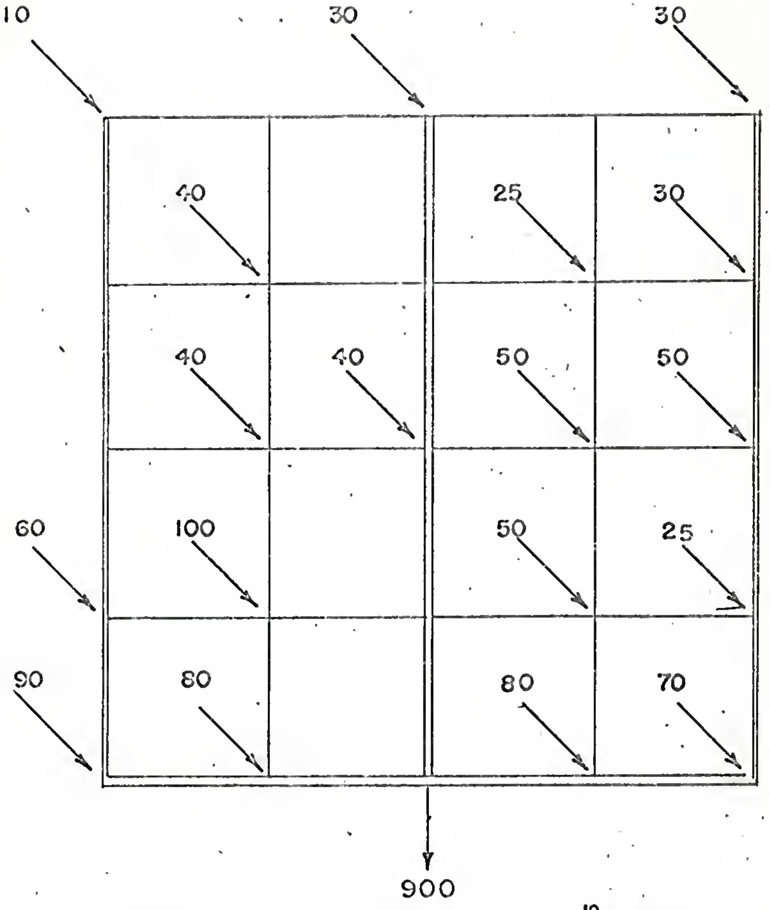
$k_{j1}^{n,m} = .06$, $j = 1, 2$ for arterial streets,

$k_{j1}^{n,m} = .02$, $j = 1, 2$ for freeways,



TOTAL ACCUMULATED TRAVEL TIME = 14076 (Time Units)

Figure 8. OPTIMAL TRAFFIC ASSIGNMENT FOR NUMERICAL EXAMPLE I.



$t_f = 5 + .02v + 10(v/360)^{10}$, Freeway

$t_a = 10 + .06v + 10(v/360)^{10}$, Arterial Street

$t_c = 12 + .03v + 10(v/150)^{10}$, Collector Street

Figure 9. NETWORK FOR NUMERICAL EXAMPLE 2.

$k_{j2}^{n,m} = 10$, $j = 1, 2$ for all three types,

$C_j^{n,m} = 150$, $j = 1, 2$ for collector streets,

$C_j^{n,m} = 180$, $j = 1, 2$ for arterial streets,

$C_j^{n,m} = 360$, $j = 1, 2$ for freeways.

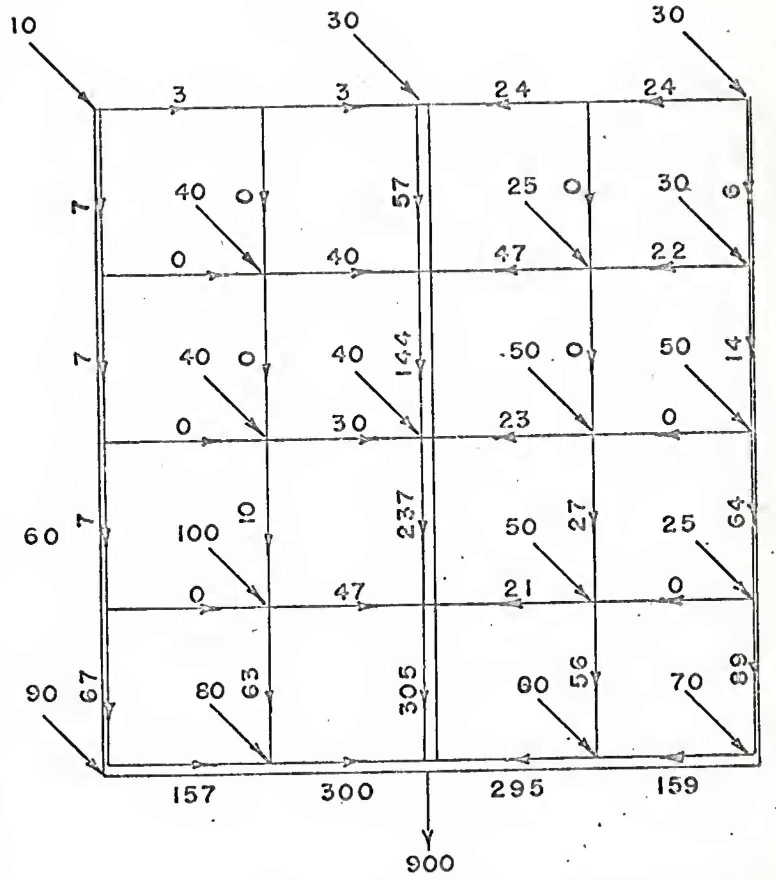
The destination is assumed to be at node (5,3). The direction of flow in each link is preassigned. The input volumes are also shown in Fig. 9. The problem is to find an optimal traffic assignment along the links so that all the vehicles reach the destination node (5,3) in minimum time.

As the destination node is not the last node (5,5), the problem becomes a two-quadrants problem.

The results were obtained by an IBM 360/50 computer and they are presented in Fig. 10. The total travel time is 27,777 time units. It takes 85 iterations.

COMPUTATIONAL CHARACTERISTICS

Assignment by the maximum principle is achieved through a series of iterations until desired convergence has occurred. Each iteration is a feasible solution to the problem although not necessarily the optimal one. To begin this iteration process in this study, it is first assumed that the vehicles entering a node would be divided equally between the horizontal and the vertical links when they leave the node, i.e.,



TOTAL ACCUMULATED TRAVEL TIME = 27,776 TIME UNITS

Figure 10. OPTIMAL TRAFFIC ASSIGNMENT FOR NUMERICAL EXAMPLE 2.

$$\theta_1^{n,m} = .5, \qquad \begin{array}{l} n = 1, 2, \ldots, N, \\ m = 1, 2, \ldots, M. \end{array}$$

This provides the first feasible solution from which subsequent iterations are made. The numerical results for both problems will now be discussed in detail.

Example 1: Using an IBM 360/50 computer the optimal total system travel time obtained is 14,076 time units and the convergence is obtained after 32 iterations. The computer took approximately 155 seconds to execute this program. Compilation time is approximately 129.6 seconds thus leaving 25.4 seconds for 32 iterations, or approximately .793 second per iteration.

It can be seen from Fig. 11 and Table 1 that the total system travel time calculated for each of the first few iterations is considerably greater than the final total travel time. Table 1 shows the total travel time at each iteration. We can see from Table 1 that the total travel time for first iteration is considerably high, namely 782,542,080 time units, and at the end of iteration 2 it is 1,215,121 time units. But it drops down to 17,894 time units at the end of 3rd iteration. From iteration 4 to iteration 19, the total travel time fluctuates, then from iteration 20 onwards it fluctuates very slowly until it converges to 14,076 time units in 32nd iteration.

The iteration process is stopped when

$$\left| \frac{S_{new} - S_{old}}{S_{new}} \right| \leq .00001$$

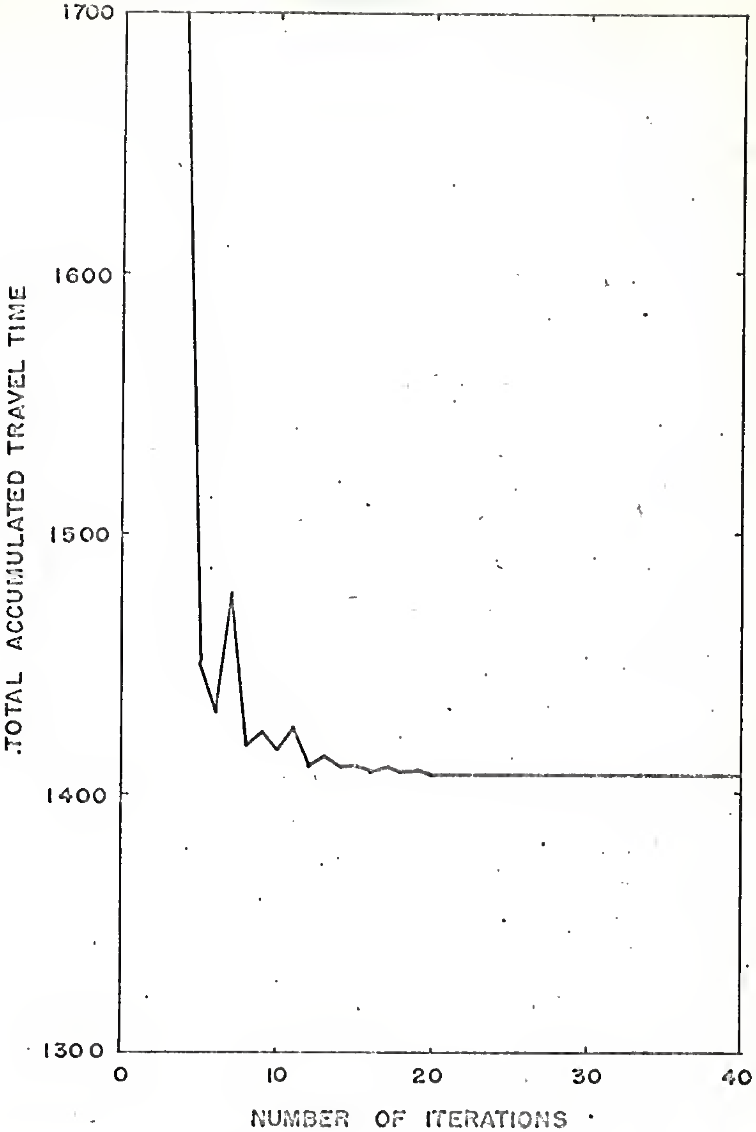


Figure II. TOTAL ACCUMULATED TRAVEL TIME VS. NUMBER OF ITERATIONS.

Table 1. Total travel time at each iteration.

Iteration No.	Total Travel Time
1	782,542,080
2	1,215,121
3	17,894
4	67,980
5	14,449
7	14,764
8	14,176
11	14,253
16	14,090
20	14,082
30	14,077
31	14,076
32	14,076

where S_{new} is total travel time in the present iteration, and S_{old} total travel time in the previous iteration. At this point the volumes on all the links of the network do not change appreciably in subsequent iterations.

Example 2: Using an IBM 360/50 computer the optimal total system travel time is 27,777 time units and the convergence occurred after 85 iterations. The computer took approximately 208.8 seconds to execute the program. The compilation time is approximately 129.6 seconds thus leaving 79.2 seconds for 85 iterations, or approximately .932 seconds per iteration.

As shown in Fig. 12 and Table 2 the total system travel time calculated for each of the first few iterations is again considerably greater than the final total travel time. Table 2 shows the total travel time at each iteration. Referring to Table 2 we note that the total travel time for first iteration is considerably high, namely 1,814,837,800 time units, and it drops to 904,504 time units for 2^{nd} iteration. It again drops to 37,143 time units in the 3^{rd} iteration. The quick convergence at these iterations can be attributed to the computational procedures employed here based on the maximum principle algorithm. It fluctuated from iteration 4 to iteration 18, and then started converging slowly to 27,777 time unit in 85 iterations.

In this problem also the iteration process is stopped when

$$\left| \frac{S_{new} - S_{old}}{S_{new}} \right| \leq .00001 \quad .$$

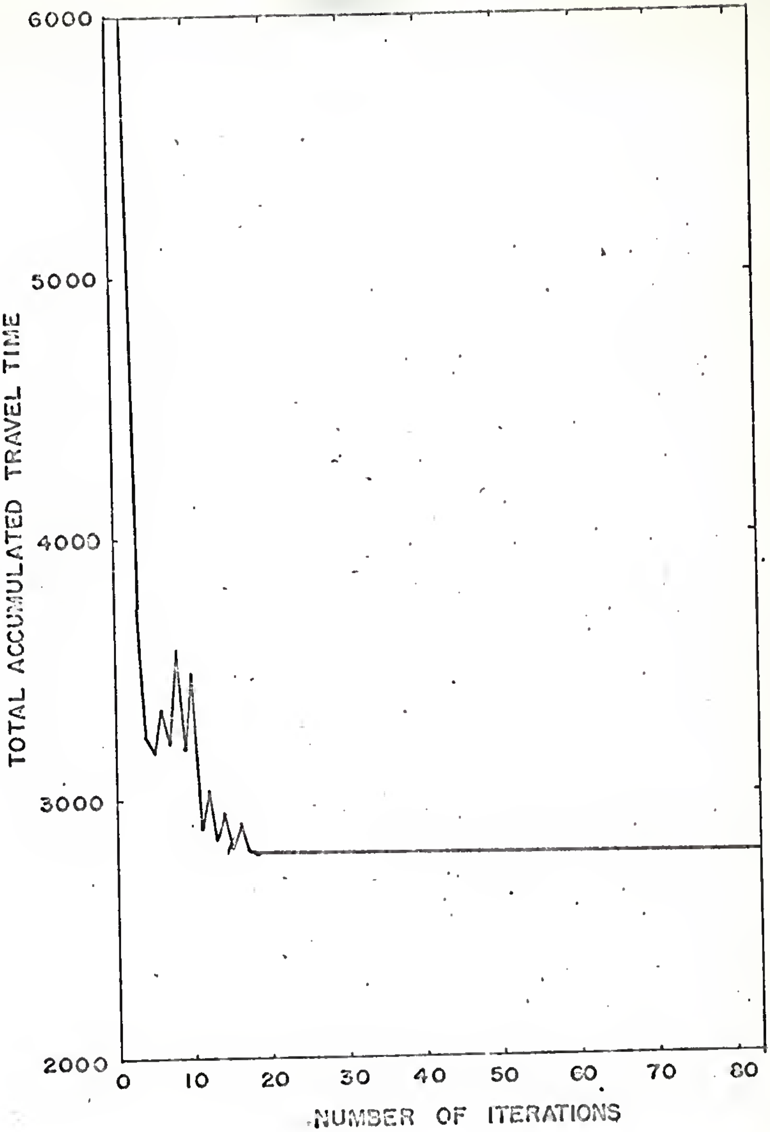


Figure 12. TOTAL ACCUMULATED TRAVEL TIME VS. NUMBER OF ITERATIONS.

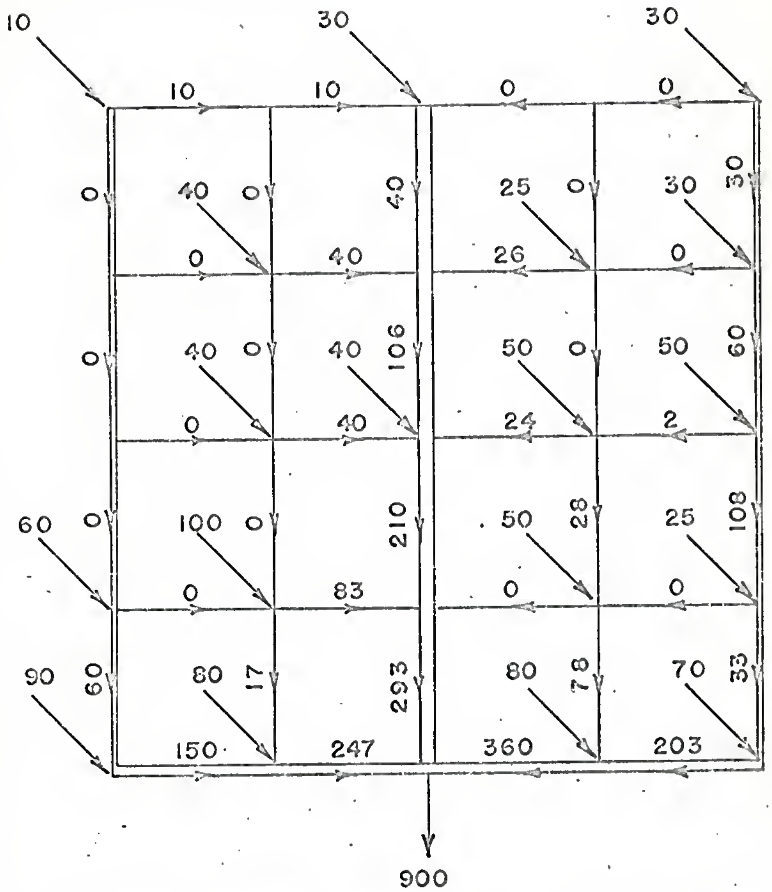
Table 2. Total Travel Time at each Iteration.

Iteration No.	Total Travel Time
1	1,814,837,800
2	904,504
3	37,143
4	32,378
5	31,775
8	35,831
10	35,069
11	28,728
12	30,389
15	28,086
20	27,820
40	27,800
70	27,781
84	27,777
85	27,777

At this point the volumes on all the links of the network do not change appreciably in subsequent iterations.

As we have noted, the computational time per iteration increased as the number of nodes increased. It can be said that the computational time per iteration increases approximately linearly with increase in the number of nodes (or in other words the size of the network).

Figures 13, 14 and 15 illustrate the traffic assignment at the end of 5th, 32nd and 70th iteration respectively for numerical example 2. Comparison of these assignments with the optimal traffic assignment as shown in Figure 10 can be made. We note first that as the number of iterations increase the total travel time decreases as it should be. The total travel time at the end of 5th iterations is 31,774 time units. At the end of 32nd and 60th iteration the values are 27,806 and 27,781 time units respectively and finally it converges to 27,777 at 85th iterations. We also note that traffic assignment at different nodes does not remain the same as the number of iterations increases, but it gradually tends towards the optimal. For example consider node (1,1). The input volume is 10 vehicles. In the 5th iteration all the 10 vehicles are assigned to the horizontal link while no vehicle is on the vertical link. At the end of 32nd iteration 9 vehicles are assigned to horizontal link while one vehicle is assigned to the vertical link; and at the end of 70th iteration 4 vehicles have been assigned to the horizontal link and 6 vehicles to the vertical link and finally 3 vehicles are assigned



TOTAL TRAVEL TIME = 31,774 (Time Units)

Figure 13. TRAFFIC ASSIGNMENT ITERATION 5 FOR NUMERICAL EXAMPLE 2.

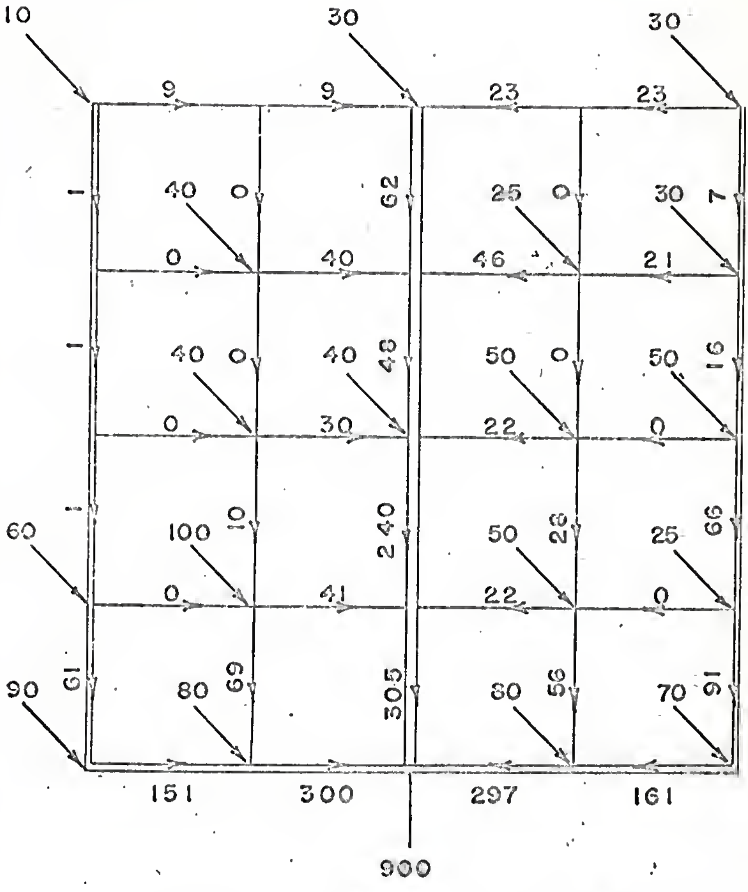
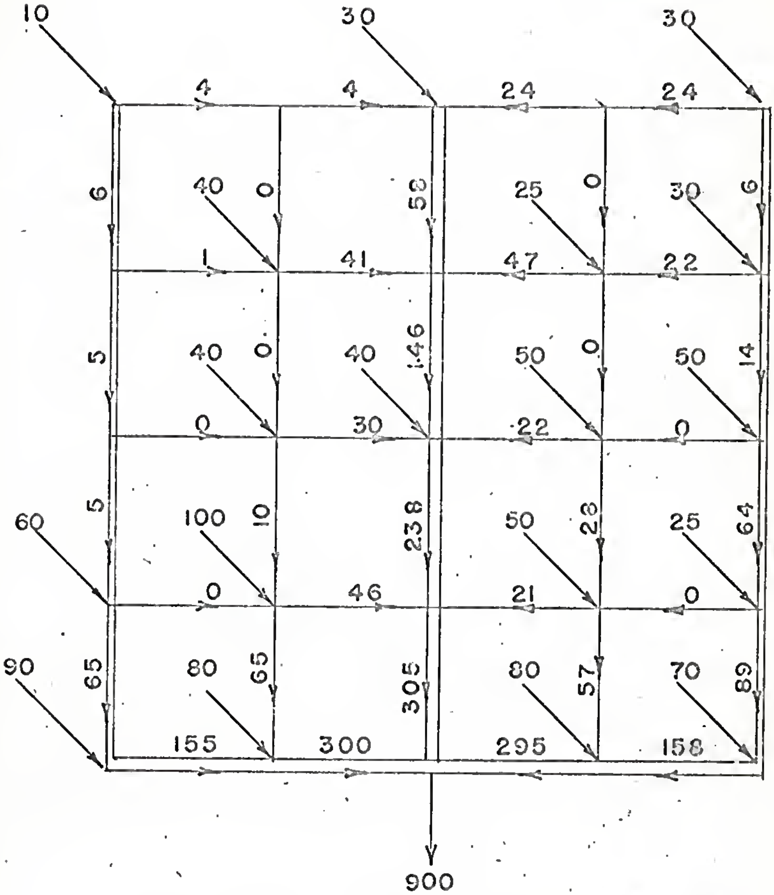


Figure 14. TRAFFIC ASSIGNMENT AT ITERATION 32 FOR NUMERICAL EXAMPLE 2,



TOTAL TRAVEL TIME = 27,781 (Time Units)

Figure 15. TRAFFIC ASSIGNMENT AT ITERATION 70 FOR NUMERICAL EXAMPLE 2.

on the horizontal link while 7 vehicles on the vertical link, which is the optimal traffic assignment. In a similar fashion we can explain the gradual change in the traffic assignment at different nodes. At node (1,3) the assignment on vertical link increases from 40 to 58 and to 62 as iterations increase from 5th to 32nd and to 70th. At the same time the assignment on the horizontal link is zero at the end of 5th iteration but it increased to 23 at the end of 32nd iteration and then to 24 at the end of 70th and 85th iterations.

4. TRAFFIC ASSIGNMENT USING NONLINEAR TIME FUNCTION WITH MULTI-COPY NETWORK

Single copy (multi-origin single destination) assignments, based on constant and nonlinear travel time-volume relationships, have been studied in sections 2 and 3. In this section the traffic assignment for a multi-copy network is considered. In a multi-copy network we have a multi-origin and multi-destination network. This represents an actual situation, since we usually do not have a single destination in actual practice. At the computational procedure to obtain an optimal solution, the multi-copy problem is reduced to a series of constrained single copy problems. Two numerical examples, one considering the turn penalty and the other without penalty, are presented.

4-1. Multi-Copy Network With Turn Penalty

STATEMENT OF THE PROBLEM

The problem is to obtain an optimum traffic assignment to a network which minimizes the total accumulative travel time. The prototype urban network shown in Fig. 1 is composed of three classes of streets; freeways, arterial streets and collector streets. The trip distribution pattern is also given and is composed of three copies; that is three zones of destination and numerous zones of origin. In the problem shown in Fig. 1 all trips are destined to zone A (copy 1), zone B (copy 2), and zone C (copy 3). It is assumed that each trip will be made by a separate vehicle.

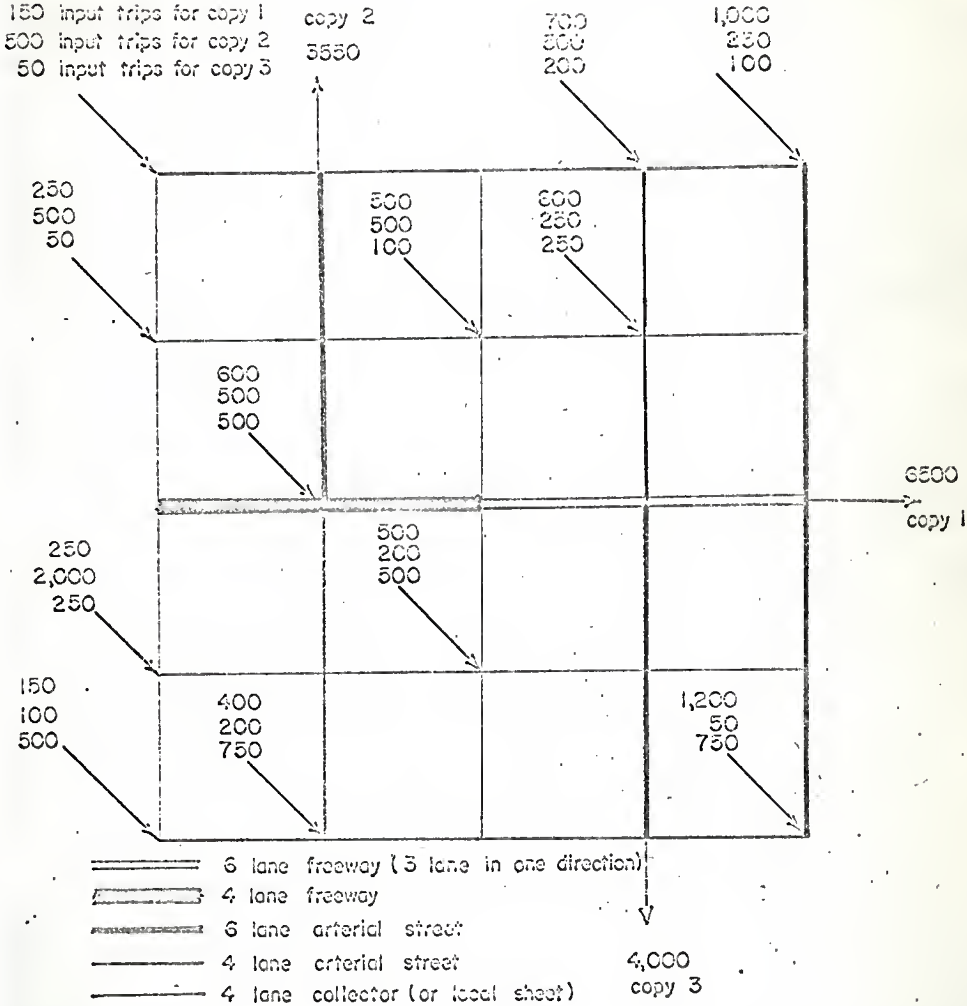


Figure 1. Multicopy example.

Each class of street is characterized by a travel time function as follows:

$$t_f = 1 + 9 \times 10^{-5}(\frac{v}{\ell}) + 2x(\frac{v/\ell}{1800})^{10} \quad (1)$$

$$t_a = 2 + 10 \times 10^{-5}(\frac{v}{\ell}) + 2x(\frac{v/\ell}{900})^{10} \quad (2)$$

$$t_c = 2.5 + 10 \times 10^{-5}(\frac{v}{\ell}) + 2x(\frac{v/\ell}{750})^{10} \quad (3)$$

where

t_f = link travel time in minutes on freeways

t_a = link travel time in minutes on arterial streets

t_c = link travel time in minutes on collector streets

In general, the following nonlinear functional relationship represents the link travel time and the link volume.

$$t = k_0 + k_1 \ (\frac{v}{\ell}) + k_2 \ (\frac{v}{C\ell})^r \quad (4)$$

where

t = link travel time for vehicle,

k_0 = constant representing time at free flow conditions,

k_1, k_2 = empirically derived constants,

v = link volume in vehicles per hour,

C = lane capacity in vehicles per lane per hour,

ℓ = number of lanes making up the link in one direction,

r = empirically derived exponent.

Equation (4) is similar to equation (2) of section 3. Since k_1, k_2, v and ℓ are given constants for each link, equation (4) can be written as

$$t = k_0 + k_1^{'}v + k_2^{'}(v)^r \qquad (5)$$

where

$$k_1^{'} = k_1/\ell ,$$

$$k_2^{'} = \left(\frac{k_2}{C\ell}\right)^r .$$

Also included in this problem is a penalty accessed to any vehicle which makes a right or left hand turn. The turn penalties are volume independent and in this case are assumed as follows:

k_L = 0.3 minutes (left turn penalty),

k_R = 0.1 minutes (right turn penalty).

FORMULATION OF THE PROBLEM WITH TURN PENALTY

To facilitate the formulation of the problem consider a typical interior network node, at (n,m) as shown in Fig. 4 of section 3. The performance equations associated with that node are as follows:

$$x_1^{n,m} = \theta_1^{n,m}\ (x_1^{n,m-1} + v^{n,m}/2) + (1 - \theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2), \qquad (6)$$

$$x_1^{n,0} = 0, \tag{6a}$$

$$x_2^{n,m} = (1 - \theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2) + \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) \ , \tag{7}$$

$$x_2^{0,m} = 0 \ , \tag{7a}$$

$$x_3^{n,m} = x_3^{n,m-1} + T_3^{n,m}(\theta_1^{n,m},\ \theta_2^{n,m},\ x_1^{n,m-1},\ x_2^{n-1,m},\ vv^{n,m},\ k_L), \tag{8}$$

$$x_3^{n,0} = 0 \ , \tag{8a}$$

$$x_4^{n,m} = x_4^{n-1,m} + T_4^{n,m}(\ {}_1^{n,m},\ \ {}_2^{n,m},\ x_1^{n,m-1},\ x_2^{n-1,m},\ v^{n,m},\ vh^{n,m},\ k_R) \tag{9}$$

$$x_4^{0,m} = 0 \ , \tag{9a}$$

$$n = 1,\ 2,\ \ldots,\ N; \qquad m = 1,\ 2,\ \ldots,\ M.$$

where

$x_j^{n,m}$ = a state variable representing the number of vehicles on link j immediately beyond node (n,m), j = 1, 2, in which j = 1 denotes the horizontal link and j = 2 denotes the vertical link,

$x_3^{n,m}$ = a state variable representing the accumulated travel time on horizontal links from node (n,1) including the horizontal link immediately beyond node (n,m),

$x_4^{n,m}$ = a state variable representing the accumulated travel time on vertical links from node (1,m) including the vertical link immediately beyond node (n,m),

$T_3^{n,m}$ = the relationship between total vehicle minutes on the horizontal link immediately beyond node (n,m) and the number of vehicles on that link,

$T_4^{n,m}$ = the relationship between total vehicle minutes on the vertical link immediately beyond node (n,m) and the number of vehicles on that link,

$v^{n,m}$ = the number of vehicles entering or leaving the network at node (n,m). It is assumed that $v^{n,m}$ can be split so that $v^{n,m}/2$ enters the vertical link, and the horizontal link, respectively, just ahead of node (n,m),

$vh^{n,m}$ = the number of vehicles on the same links in the same direction as $x_1^{n,m}$, obtained from previous copies,

$vv^{n,m}$ = the number of vehicles on the same links in the same direction as $x_2^{n,m}$, obtained from previous copies,

$\theta_1^{n,m}$ = the decision variable that represents the fraction of the vehicles which enter the node on a horizontal link and leave on the horizontal link, at node (n,m),

$\theta_2^{n,m}$ = the decision variable that represents the fraction of the vehicles which enter the node on a vertical link and leave on the vertical link, at the node (n,m).

The object function to be minimized is given by

$$S = \sum_{n=1}^{N} c_3 x_3^{n,M} + \sum_{m=1}^{M} c_4 x_4^{N,m} = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{N,m} \tag{10}$$

where

$$c_1 = 0, \quad c_2 = 0, \quad c_3 = 1, \quad c_4 = 1 . \tag{10a}$$

The Hamiltonian and the adjoint variables can be written as

$$H^{n,m} = \sum_{i=1}^{4} z_i^{n,m} x_i^{n,m}$$

$$= z_1^{n,m}\left(\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2)\right)$$

$$+ z_2^{n,m}\left((1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2) + \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2)\right)$$

$$+ z_3^{n,m}\{x_3^{n,m-1} + k_{10}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}\}$$

$$+ k_{11}'^{n,m}\{\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}\}^2$$

$$+ k_{12}'^{n,m}\{\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}\}^{11}$$

$$+ k_L(1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2)\}$$

$$+ z_4^{n,m}\{x_4^{n,m-1} + k_{20}'^{n,m} \ (1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m})$$

$$+ k_{21}'^{n,m}((1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m})^2$$

$$+ k_{22}'^{n,m}((1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m})^{11}$$

$$+ k_R(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)\} \qquad (11)$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m} \ \theta_1^{n,m} + z_2^{n,m} \ (1-\theta_1^{n,m}) + z_3^{n,m} k_{10}'^{n,m} \ \theta_1^{n,m}$$

$$+ z_4^{n,m} \ k_{20}'^{n,m} \ (1-\theta_1^{n,m}) + z_3^{n,m} \ 2 \ k_{11}'^{n,m} \ \theta_1^{n,m} \ (\theta_1^{n,m}$$

$$(x_1^{n,m-1} + \frac{v^{n,m}}{2}) + (1-\theta_2^{n,m})(x_2^{n-1,m} + \frac{v^{n,m}}{2}) + vh^{n,m})$$

$$+ 11\ k'^{n,m}_{12}\ \theta^{n,m}_1 \Big(\theta^{n,m}_1\ (x^{n,m-1}_1 + \frac{v^{n,m}}{2})$$

$$+ (1-\theta^{n,m}_2)(x^{n-1,m}_2 + \frac{v^{n,m}}{2}) + vh^{n,m}\Big)^{10}$$

$$+ z^{n,m}_4\ 2\ k'^{n,m}_{21}(1-\theta^{n,m}_1)\Big((1-\theta^{n,m}_1)(x^{n,m-1}_1 + \frac{v^{n,m}}{2})$$

$$+ \theta^{n,m}_2(x^{n-1,m}_2 + \frac{v^{n,m}}{2}) + vv^{n,m}\Big)$$

$$+ 11\ k'^{n,m}_{22}(1-\theta^{n,m}_1)\Big((1-\theta^{n,m}_1)(x^{n,m-1}_1 + \frac{v^{n,m}}{2})$$

$$+ \theta^{n,m}_2(x^{n-1,m}_2 + \frac{v^{n,m}}{2}) + vv^{n,m}\Big)^{10}$$

$$+ z^{n,m}_4 k_R(1-\theta^{n,m}_1)\ , \qquad (12)$$

$$z^{n-1,m}_2 = \frac{\partial H^{n,m}}{\partial x^{n-1,m}_2}$$

$$= z^{n,m}_1(1-\theta^{n,m}_2) + z^{n,m}_2\ \theta^{n,m}_2 = z^{n,m}_3\ k'^{n,m}_{10}(1-\theta^{n,m}_2)$$

$$+ z^{n,m}_4\ k'^{n,m}_{20}\ \theta^{n,m}_2 + z^{n,m}_3\ 2\ k'^{n,m}_{11}(1-\theta^{n,m}_2)$$

$$\Big(\theta^{n,m}_1(x^{n,m-1}_1 + \frac{v^{n,m}}{2}) + (1-\theta^{n,m}_2)(x^{n-1,m}_2 + \frac{v^{n,m}}{2}) + vh^{n,m}\Big)$$

$$+ 11\ k'^{n,m}_{12}(1-\theta^{n,m}_2)\Big(\theta^{n,m}_1(x^{n,m-1}_1 + \frac{v^{n,m}}{2})$$

$$+ (1-\theta^{n,m}_2)(x^{n-1,m}_2 + \frac{v^{n,m}}{2}) + vh^{n,m}\Big)^{10}$$

$$+ z_4^{n,m}\ 2\ k_{21}^{\prime n,m}\ \theta_2^{n,m}\Big((1-\theta_1^{n,m})(x_1^{n,m-1} + \frac{v^{n,m}}{2})$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + \frac{v^{n,m}}{2}) + vv^{n,m}\Big)$$

$$+ 11\ k'^{n,m}_{22}\ \theta_2^{n,m}\Big((1-\theta_1^{n,m})(x_1^{n,m-1} + \frac{v^{n,m}}{2})$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + \frac{v^{n,m}}{2}) + vh^{n,m}\Big)^{10}$$

$$+ z_3^{n,m} k_L (1-\theta_2^{n,m}), \tag{13}$$

$$z_3^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_3^{n,m-1}}$$

$$= z_3^{n,m}, \tag{14}$$

$$z_4^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_4^{n-1,m}}$$

$$= z_4^{n,m}, \tag{15}$$

$$z_1^{n,M} = 0, \qquad n = 1, 2, \ldots, N, \tag{16}$$

$$z_2^{N,m} = 0, \qquad m = 1, 2, \ldots, M, \tag{17}$$

$$z_3^{n,M} = 1, \qquad n = 1, 2, \ldots, N, \tag{18}$$

$$z_4^{N,m} = 1, \qquad m = 1, 2, \ldots, M. \tag{19}$$

where

$k'^{n,m}_{j,q}$ = the parameters in the travel time-volume relationship in which j = 1, 2 denotes horizontal and vertical links respectively and q = 0, 1, 2 denotes coefficient numbers.

From equations (14), (15), (18), and (19), we obtain

$$z_3^{n,m} = z_4^{n,m} = 1 \qquad \begin{array}{l} n = 1, 2, \ldots, N, \\ m = 1, 2, \ldots, M. \end{array} \tag{20}$$

The optimal sequence of the decision variables, $\theta_1^{n,m}$ and $\theta_2^{n,m}$ which minimize the total cumulative travel time are obtained from

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = 0 \tag{21a}$$

and

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = 0 \tag{21b}$$

or

$$H^{n,m} = \text{minimum} . \tag{22}$$

Equations (21a) and (21b) give

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = 0$$

$$= z_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) - z_2^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ k_{10}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + 2k_{11}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2)$$

$$+ vh^{n,m}] + 11k_{12}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2)$$

$$+ vh^{n,m}]^{10} - k_{20}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$- 2k_{21}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$[(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]$$

$$- 11k_{22}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)[(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]^{10}$$

$$- k_R(x_1^{n,m-1} + v^{n,m}/2), \tag{22a}$$

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = 0$$

$$= -z_1^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + z_2^{n,m}(x_2^{n-1} + v^{n,m}/2)$$

$$- k'^{n,m}_{10}(x_2^{n-1,m} + v^{n,m}/2) - 2k'^{n,m}_{11}(x_2^{n-1,m} + v^{n,m}/2)$$

$$[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}]$$

$$- 11k'^{n,m}_{12}(x_2^{n-1,m} + v^{n,m}/2)[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}]^{10}$$

$$- k_L(x^{n-1,m} + v^{n,m}/2) + k'^{n,m}_{20}(x_2^{n-1,m} + v^{m}/2)$$

$$+ 2k'^{n,m}_{21}(x_2^{n-1,m} + v^{n,m}/2)[(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]$$

$$+ 11k'^{n,m}_{22}(x_2^{n-1,m} + v^{n,m}/2)[(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]^{10} . \qquad (22b)$$

The second partial derivatives of the Hamiltonian with respect to the decision variables, $\theta_1^{n,m}$ and $\theta_2^{n,m}$, which are used at the computational procedure are

$$\frac{\partial^2 H^{n,m}}{\partial(\theta_1^{n,m})^2} = 2k_{11}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)^2$$

$$+ 110k_{12}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)^2$$

$$[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2) + (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2)$$

$$+ vh^{n,m}]^9 + 2k_{21}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)^2$$

$$+ 110k_{22}'^{n,m}(x_1^{n,m-1} + v^{n,m}/2)^2 \; [(1-\theta_1^{n,m})$$

$$(x_1^{n,m-1} + v^{n,m}/2) + \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]^9$$

(23a)

$$\frac{\partial^2 H^{n,m}}{\partial(\theta_2^{n,m})^2} = 2k_{11}'^{n,m}(x_2^{n-1,m} + v^{n,m}/2)^2$$

$$+ 110k_{12}'^{n,m}(x_2^{n-1,m} + v^{n,m}/2)^2[\theta_1^{n,m}(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ (1-\theta_2^{n,m})(x_2^{n-1,m} + v^{n,m}/2) + vh^{n,m}]^9$$

$$+ 2k_{21}'^{n,m}(x_2^{n-1,m} + v^{n,m}/2)^2$$

$$+ 110k_{22}'^{n,m}(x_2^{n-1,m} + v^{n,m}/2)^2[(1-\theta_1^{n,m})(x_1^{n,m-1} + v^{n,m}/2)$$

$$+ \theta_2^{n,m}(x_2^{n-1,m} + v^{n,m}/2) + vv^{n,m}]^9 \quad . \qquad (23b)$$

COMPUTATIONAL PROCEDURE FOR MULTI-COPY NETWORK

Through the use of equations (6) through (23) the optimum sequence of decision variables, $\theta_1^{n,m}$, $\theta_2^{n,m}$, $n = 1, 2, \ldots, N$; $m = 1, 2, \ldots, M$ can be found. The computational procedure used for each copy is essentially identical to that presented in section 3 for single copy problem except that we have two decision variables $\theta_1^{n,m}$ and $\theta_2^{n,m}$ at each node (n,m) instead of just one decision $\theta^{n,m}$ as in section 3.

To solve a multi-copy problem, the following procedure is employed.

Step 1. Choose a single copy at random.

Step 2. Obtain the optimal traffic assignment for this single copy network using the computational procedure presented in section 3.

Step 3. Choose another single copy left at random.

Step 4. Consider the volumes obtained from previous assignments (or previous copy or copies) as fixed, which are given as $vv^{n,m}$ and $vh^{n,m}$, obtain the optimal traffic assignment for the single copy from the copy selected in step 3.

Step 5. Return to step 3 and continue until all copies have been assigned.

NUMERICAL RESULTS

The total accumulated travel time for copy 1 is 33,027 minutes and the convergence is obtained in 58 iterations. Fig.

2 shows the input volumes $v^{n,m}$ and the optimal traffic assignment.

For copy 2, according to the procedure explained in step 4, the link volumes $vv^{n,m}$ and $vh^{n,m}$ are obtained from the optimal traffic assignment of copy 1, which have the same directions of traffic flow as for copy 2. These volumes $vv^{n,m}$ and $vh^{n,m}$ at each link and the input volumes, $v^{n,m}$ for copy 2 are shown in Fig. 3.

The total travel time for copy 2 is 51,206 minutes, which includes the total time obtained on copy 1. The converge occurred after 57 iterations. Fig. 4 shows the input volumes and the optimal traffic assignment for copy 2.

For copy 3 again, according to the procedure presented in step 4, the link volumes, $vv^{n,m}$ and $vh^{n,m}$ are obtained from the optimal traffic assignments of copy 1 and copy 2, which have the same traffic flow directions as for copy 3. These traffic flow volumes, $vv^{n,m}$ and $vh^{n,m}$ at each link and the input volumes $v^{n,m}$ for copy 3 are presented in Fig. 5. The total travel time for copy 3 is 74,752 minutes, which includes the total times obtained on copy 1 and copy 2. The convergence occured in 53 iterations. Fig. 6 shows the input volumes and the optimal traffic assignment on copy 3.

Fig. 7 shows the final traffic assignment for all the three copies.

Now the multi-copy problem without turn penalty will be considered.

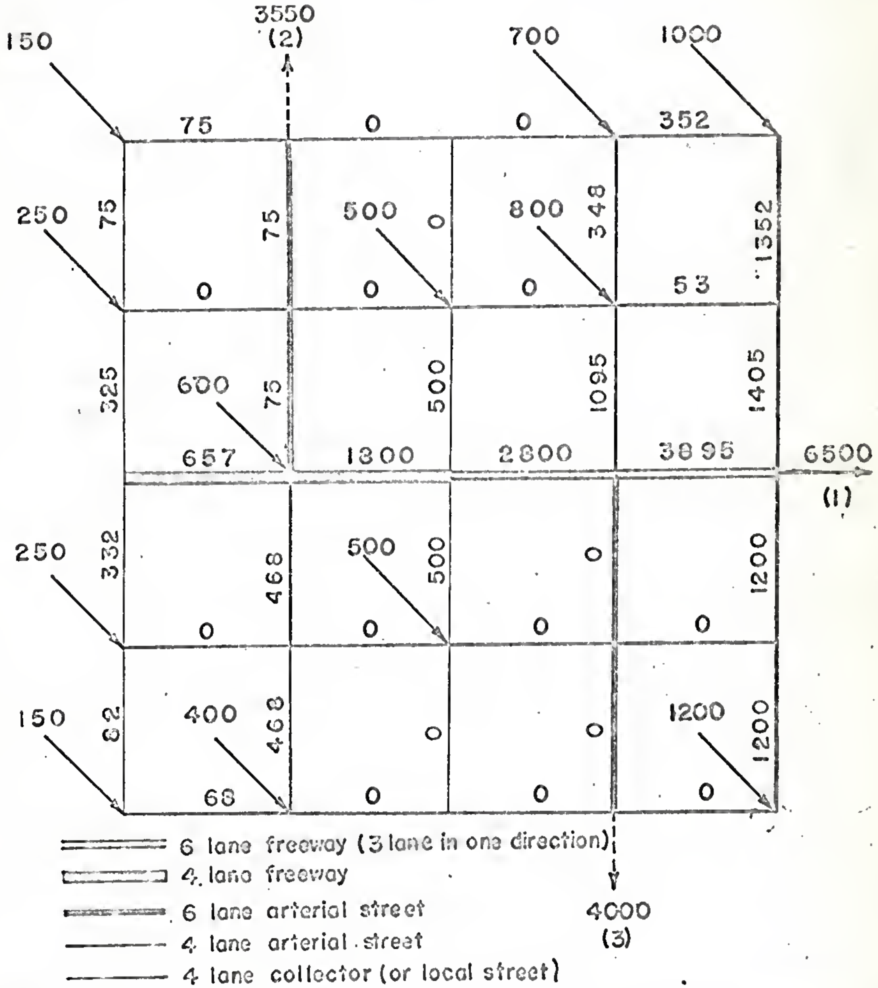


Figure 2. INPUT VOLUMES v^{rs} AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY I.

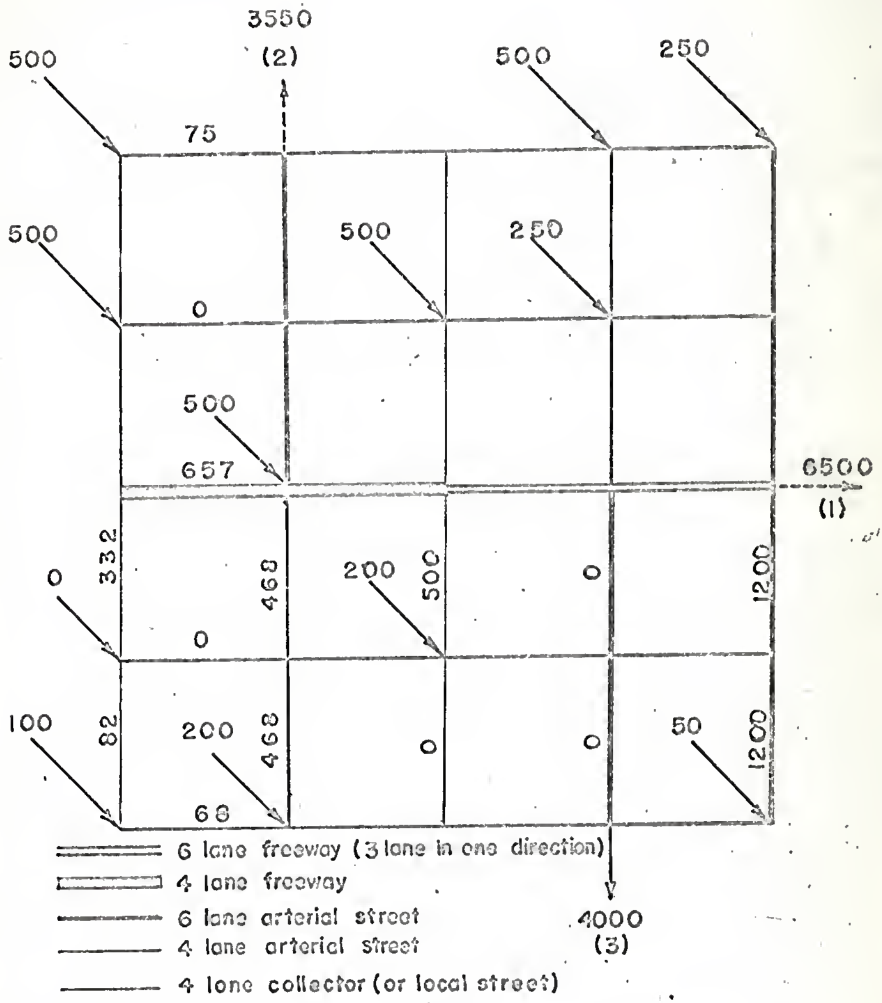


Figure 3 INPUT VOLUMES $v^{n,m}$, $vv^{n,m}$ AND $vh^{n,m}$ FOR COPY 2/1.

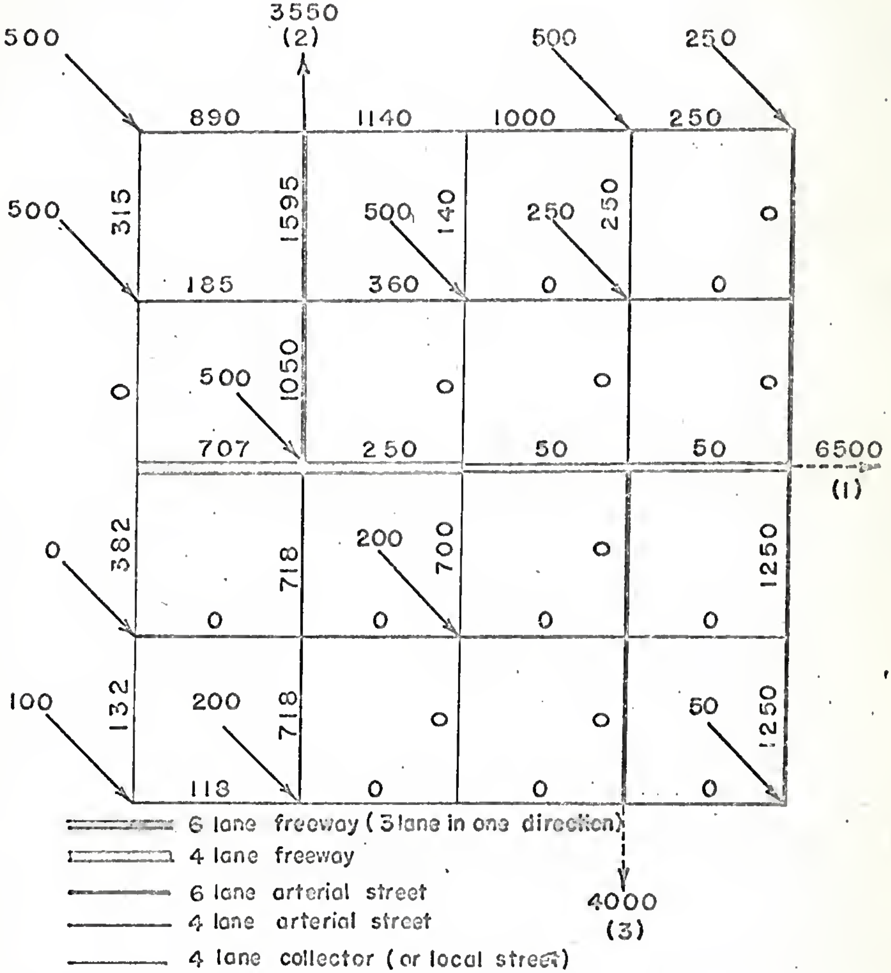


Figure 4. INPUT VOLUMES $v^{n,m}$ AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY 2/1.

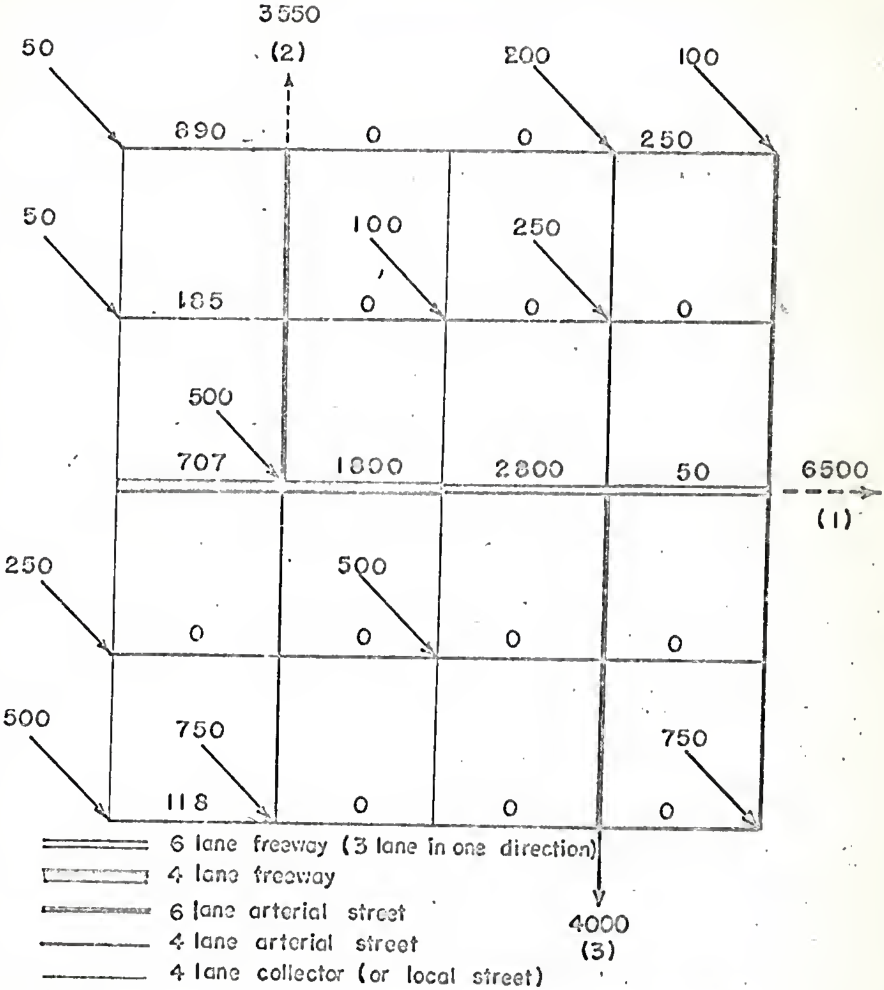


Figure 5. INPUT VOLUMES $v^{n,m}$; $vv^{n,m}$ AND $vh^{n,m}$ FOR COPY 3/2/1.

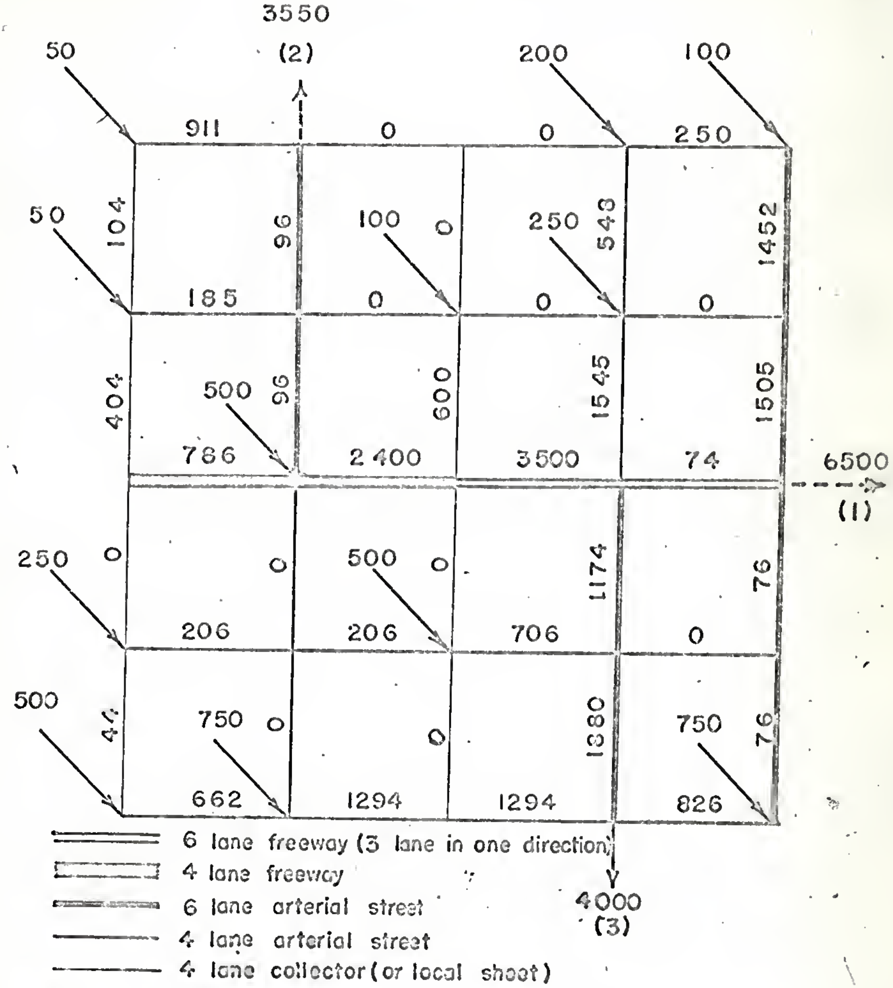
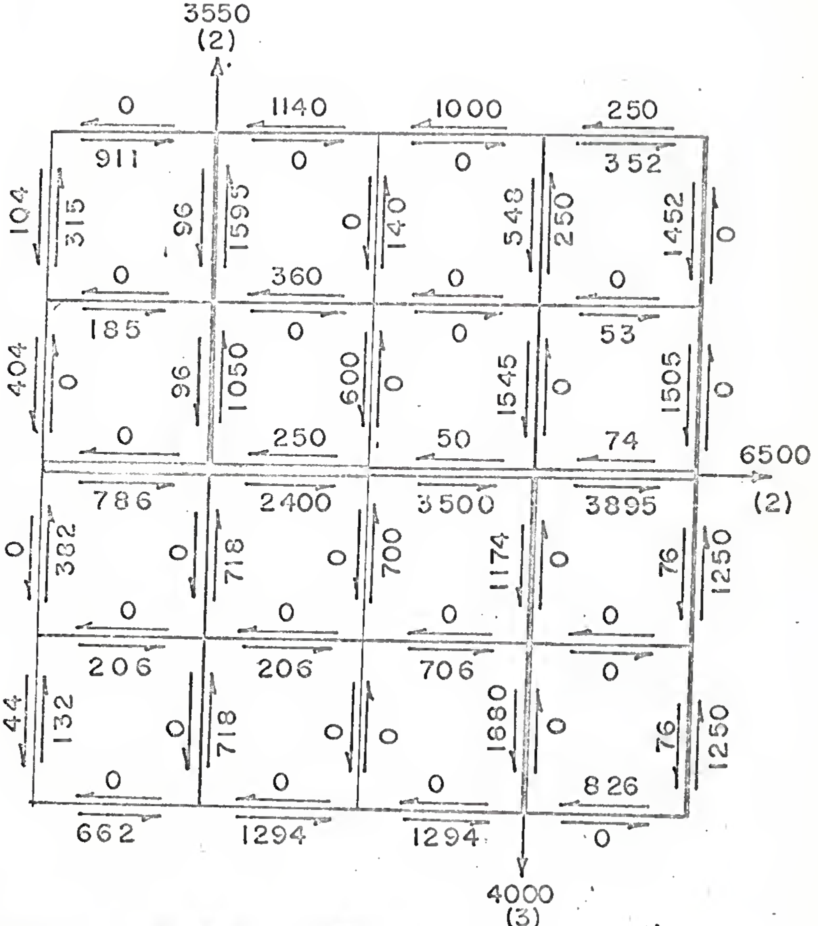


Figure 6. INPUT VOLUMES v^{nm} AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY 3/2/1.



6 lane freeway (3 lane in one direction)
4 lane freeway
6 lane arterial street
4 lane arterial street
4 lane collector (or local street)

Figure 7. FINAL TRAFFIC ASSIGNMENT FOR 3 COPIES

4-2. Multicopy Network Without Turn Penalty

STATEMENT OF THE PROBLEM

The statement of the problem is essentially the same as for the problem with turn penalty, with the only difference that there is no penalty accessed to any vehicle which makes a right or left turn.

FORMULATION OF THE PROBLEM WITHOUT TURN PENALTY

Considering each node as a stage, the performance equations for a typical interior node (n,m) of a rectangular network can be written as follows:

$$x_1^{n,m} = \theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}), \quad x_1^{n,0} = 0, \tag{1}$$

$$x_2^{n,m} = (1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}), \quad x_2^{0,m} = 0, \tag{2}$$

$$x_3^{n,m} = x_3^{n,m-1} + T_3^{n,m}(\theta^{n,m}, x_1^{n,m-1}, x_2^{n-1,m}, v^{n,m}, vh^{n,m}),$$

$$x_3^{n,0} = 0, \tag{3}$$

$$x_4^{n,m} = x_4^{n-1,m} + T_4^{n,m}(\theta^{n,m}, x_1^{n,m-1}, x_2^{n-1,m}, v^{n,m}, vv^{n,m}),$$

$$x_4^{0,m} = 0, \tag{4}$$

$n = 1, 2, \ldots, N \qquad m = 1, 2, \ldots, M.$

The objective function, which is the total accumulated travel time of all the trips in the system of the network, to be minimized can be given by the following equation;

$$S = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{N,m} \quad . \tag{5}$$

The Hamiltonian function and the adjoint variables can be written as follows:

$$H^{n,m} = \sum_{i=1}^{4} z_i^{n,m} x_i^{n,m}$$

$$= z_1^{n,m}\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_2^{n,m}(1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_3^{n,m}[x_3^{n,m-1} + T_3^{n,m}(\theta^{n,m}, x_1^{n,m-1},$$

$$x_2^{n-1,m}, v^{n,m}, vh^{n,m})] +$$

$$z_4^{n,m}[x_4^{n-1,m} + T_4^{n,m}(\theta^{n,m}, x_1^{n,m-1},$$

$$x_2^{n-1,m}, v^{n,m}, vv^{n,m}]$$

$$= z_1^{n,m}\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_2^{n,m}(1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ z_3^{n,m}\{x_3^{n,m-1} + k_{10}^{n,m}[\theta^{n,m}(x_1^{n,m-1}$$

$$+ x_2^{n-1,m} + v^{n,m}) + vh^{n,m}] + k_{11}^{n,m}$$

$$[\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + vh^{n,m}]^2$$

$$+ k_{12}^{n,m}C_1^{n,m}[\{\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vh^{n,m}\}/C_1^{n,m}]^{11}\} + z_4^{n,m}\{x_4^{n-1,m} + k_{20}^{n,m}$$

$$[(1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + vv^{n,m}]$$

$$+ k_{21}^{n,m}[(1-\theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vv^{n,m}]^2 + k_{22}^{n,m}C_2^{n,m}[\{(1-\theta^{n,m})$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + vv^{n,m}\}/C_2^{n,m}]^{11}\}, \tag{8}$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m}\theta^{n,m} + z_2^{n,m}(1 - \theta^{n,m}) + z_3^{n,m}$$

$$\{k_{10}^{n,m}\theta^{n,m} + 2k_{11}^{n,m}\theta^{n,m}[\theta^{n,m}$$

$$11k_{12}^{n,m}(\theta^{n,m})^{11}[(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) +$$

$$vh^{n,m}/C_1^{n,m}]^{10}\} + z_4^{n,m}\{k_{20}^{n,m}$$

$$(1 - \theta^{n,m}) + 2k_{21}^{n,m}(1 - \theta^{n,m})$$

$$[(1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) +$$

$$vv^{n,m}] + 11k_{22}^{n,m}(1 - \theta^{n,m})^{11}[\{(x_1^{n,m-1}$$

$$+ x_2^{n-1,m} + v^{n,m}) + vv^{n,m}\}/C_2^{n,m}]^{10} \quad , \tag{9}$$

$$z_2^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_2^{n-1,m}}$$

$$= z_1^{n,m-1}, \tag{10}$$

$$z_3^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_3^{n,m-1}}$$

$$= z_3^{n,m}, \tag{11}$$

$$z_4^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_4^{n-1,m}}$$

$$= z_4^{n,m} \quad , \tag{12}$$

$$z_1^{n,M} = 0, \quad n = 1, 2, \ldots, N, \tag{13}$$

$$z_2^{N,m} = 0, \quad m = 1, 2, \ldots, M, \tag{14}$$

$$z_3^{n,M} = 1, \quad n = 1, 2, \ldots, N, \tag{15}$$

$$z_4^{N,m} = 1, \quad m = 1, 2, \ldots, M. \tag{16}$$

where

$k_{j,q}^{n,m}$ = the parameters in travel time - volume relationship in which $j = 1, 2$ denotes horizontal and vertical links respectively and $q = 0, 1, 2$ denotes coefficient numbers.

From equations (11), (12), (15) and (16), we obtain

$$z_3^{n,m} = z_4^{n,m} = 1\,, \qquad \begin{array}{l} n = 1, 2, \ldots, N, \\ m = 1, 2, \ldots, M. \end{array} \tag{17}$$

In order to determine an optimal sequence of decision variables $\theta^{n,m}$, to minimize the total accumulated travel time we use the following conditions;

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}} = 0\,, \tag{18a}$$

or

$$H^{n,m} = \text{minimum}, \tag{18b}$$

Equation (8) gives

$$\frac{\partial H^{n,m}}{\partial \theta^{n,m}} = 0$$

$$= (z_1^{n,m} - z_2^{n,m} + k_{10}^{n,m} - k_{20}^{n,m})$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + 2k_{11}^{n,m}[\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vh^{n,m}]\ (x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + 11k_{12}^{n,m}C_1^{n,m}[\{\theta^{n,m}$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + vh^{n,m}\}/C_1^{n,m}]^{10}$$

$$(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) - 2k_{21}^{n,m}[(1 - \theta^{n,m})$$

$$x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) + vv^{n,m}](x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$- 11k_{22}^{n,m}C_2^{n,m}[\{(1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vv^{n,m}\}/C_2^{n,m}]^{10}\ (x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})\ . \tag{19}$$

The second partial derivative of the Hamiltonian with respect to the decision variable, $\theta^{n,m}$ which is used at the computational procedure, is

$$\frac{\partial^2 H^{n,m}}{\partial(\theta^{n,m})^2} = 2(k_{11}^{n,m} + k_{21}^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})^2$$

$$+ 110k_{12}^{n,m}C_1^{n,m}[\{\theta^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vh^{n,m}\}/C_1^{n,m}]^9(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})^2$$

$$+ 110k_{22}^{n,m}C_2^{n,m}[\{(1 - \theta^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$+ vv^{n,m}\}/C_2^{n,m}]^9(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})^2. \tag{20}$$

This formulation of the problem also assumes essentially that all flows along the links, in the network under consideration are in the direction of inereasing superseript. This implies that the network destination node is the point (N,M): i.e., the node farthest to the right and down. The same multiquadrant proeedure is employed, as used in section 3 for single eopy.

COMPUTATIONAL PROCEDURE

The eomputational procedure used for eaeh copy is essentially the same as presented in seetion 3 for single copy problem. Also the proeedure to solve a multieopy problem, is the same as described before in this seetion for the problem with turn penalty.

NUMERICAL RESULTS

The total accumulated travel time for copy 1 is 31,710 minutes and convergence took place in 33 iterations. Fig. 8 shows the input volumes $v^{n,m}$ at different nodes and the optimal traffic assignment.

For copy 2, the link volumes $vv^{n,m}$ and $vh^{n,m}$ are obtained from the optimal traffic assignment of copy 1 which have the same traffic flow directions as for copy 2. These traffic flow volumes, $vv^{n,m}$ and $vh^{n,m}$ at each link and the input volumes $v^{n,m}$ for copy 2 are shown in Fig. 9. The total travel time for copy 2 is 49,346 minutes, which includes the total time of copy 1, and the convergence occurred after 39 iterations. Fig. 10 shows the input volumes and optimal traffic assignment on copy 2.

For copy 3 again, the link volumes $vv^{n,m}$ and $vh^{n,m}$ are obtained from the optimal traffic assignment of copy 1 and copy 2, which have the same flow directions as for copy 3. These traffic flow volumes, $vv^{n,m}$ and $vh^{n,m}$ at each link and the input volumes $v^{n,m}$ for copy 3 are shown in Fig. 11. The total travel time for copy 3 is 72,243 minutes and this includes the total time obtained on copies 1 and 2. The convergence occurred after 34 iterations. Fig. 12 shows the input volumes and the optimal traffic assignment on copy 3.

Fig. 13 shows the final traffic assignment for all the three copies.

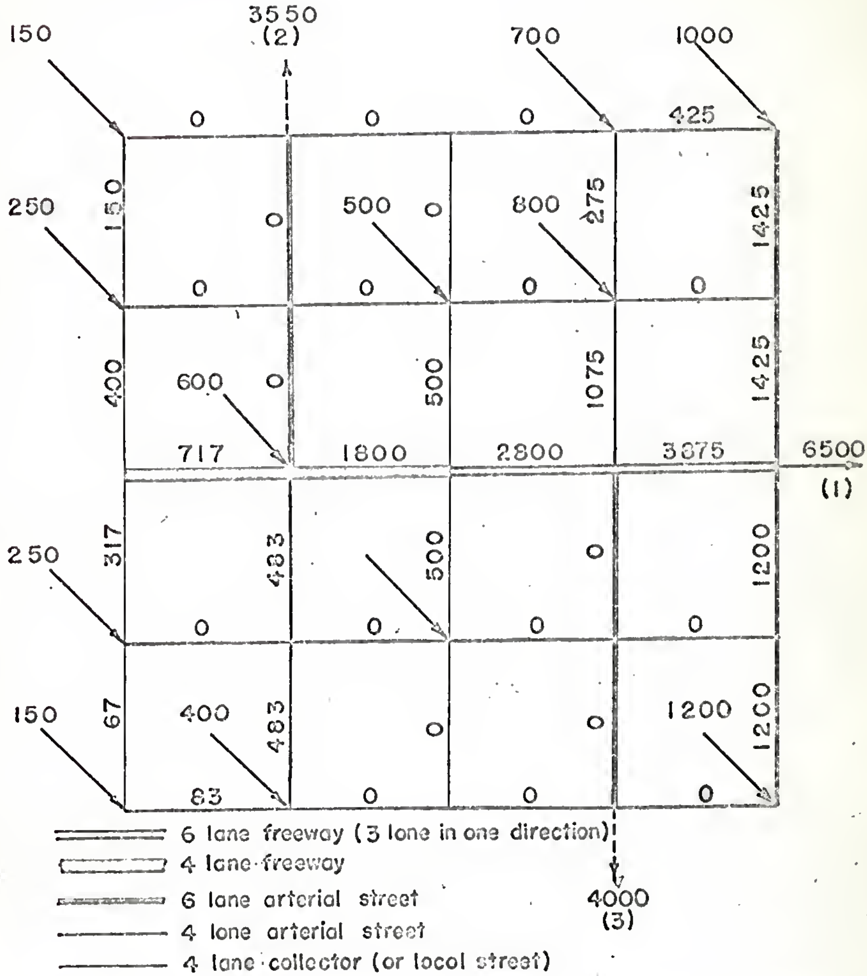


Figure 8 INPUT VOLUMES $v^{n,m}$ AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY I.

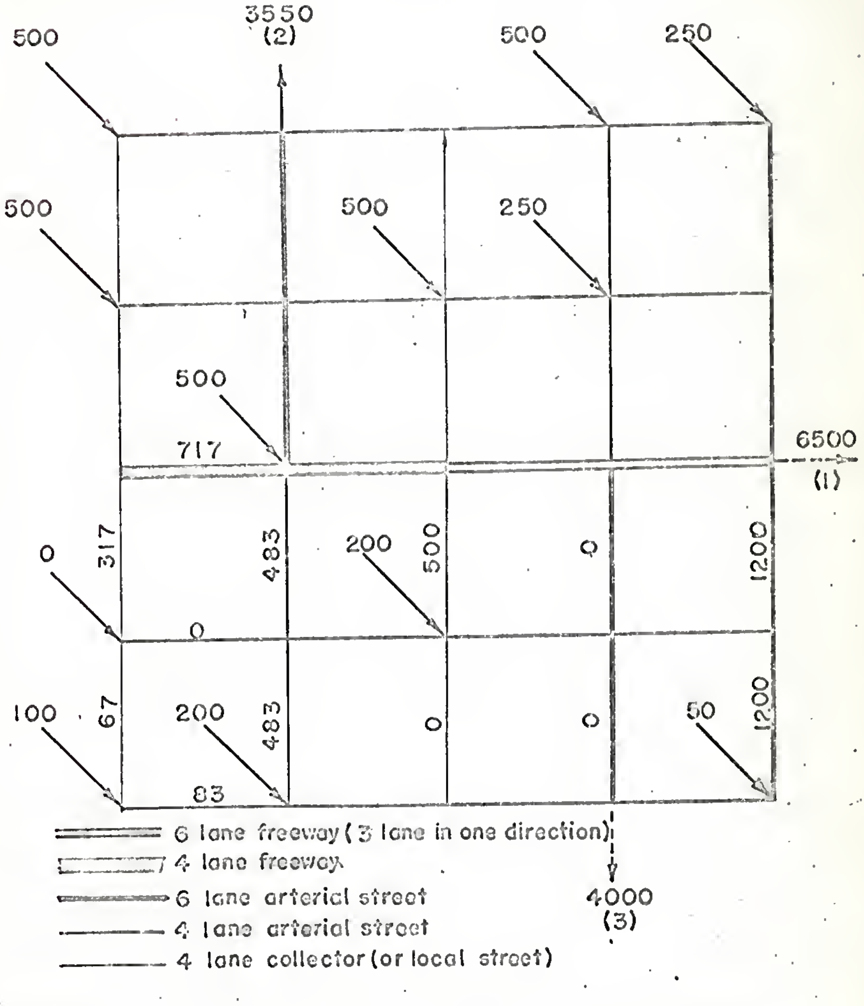


Figure 9 INPUT VOLUMES $v^{-n,m}, vv^{-n,m}$ AND vh^{nm} FOR COPY 2/1.

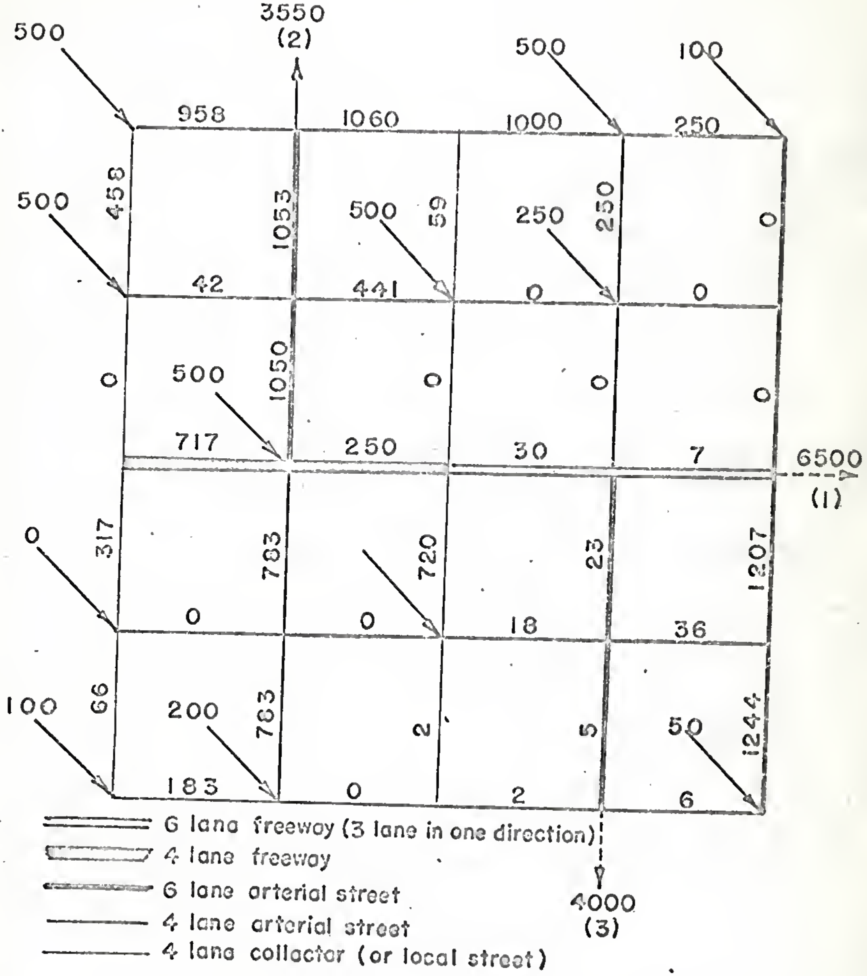


Figure 10 INPUT VOLUMES $v^{n,m}$ AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY 2/1.

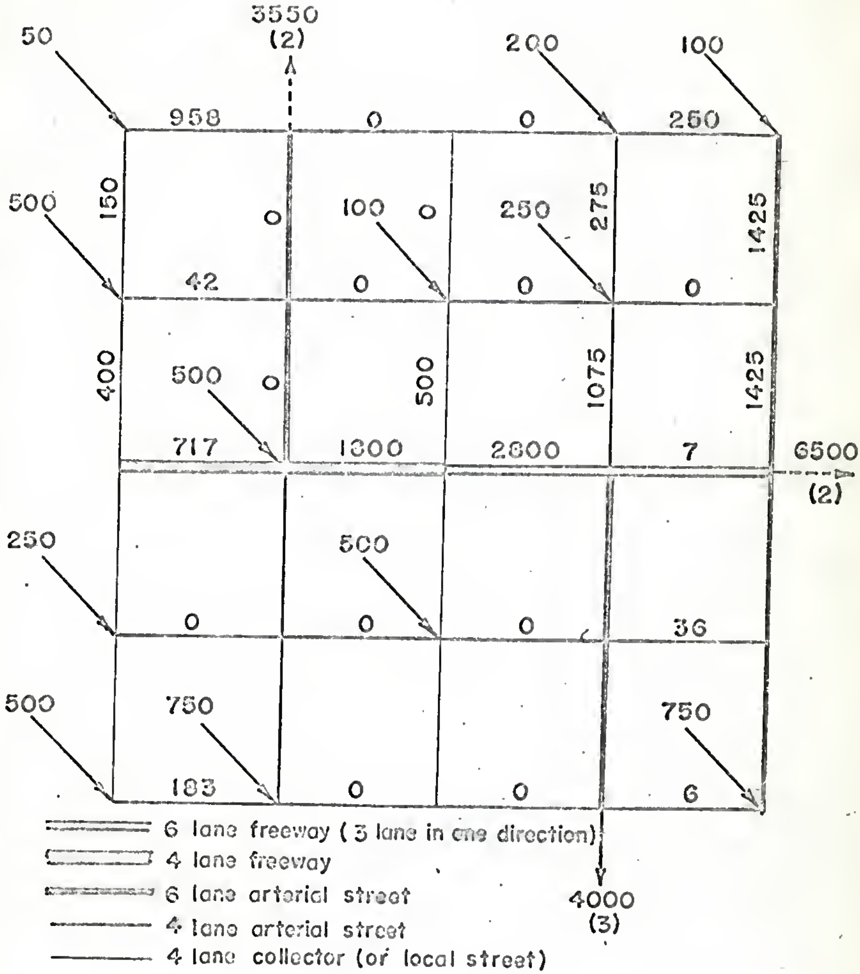


Figure 11 INPUT VOLUMES $v^{n,m}$, $vv^{n,m}$ AND $vh^{n,m}$ FOR COPY 3/2/1.

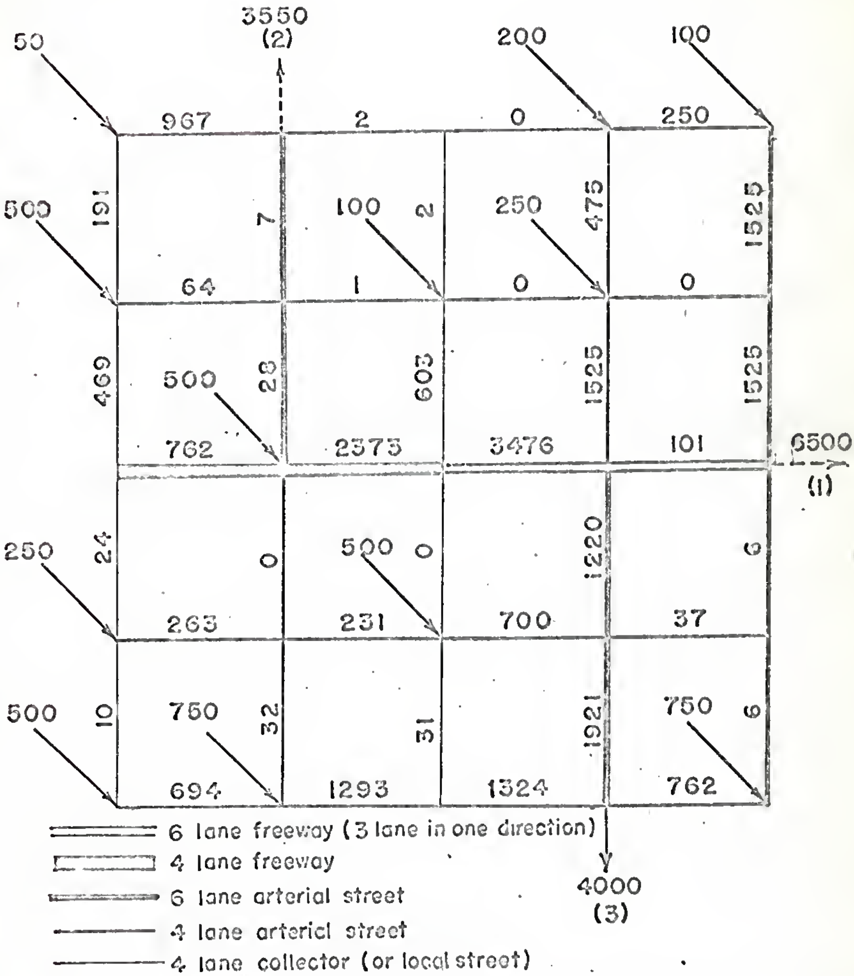
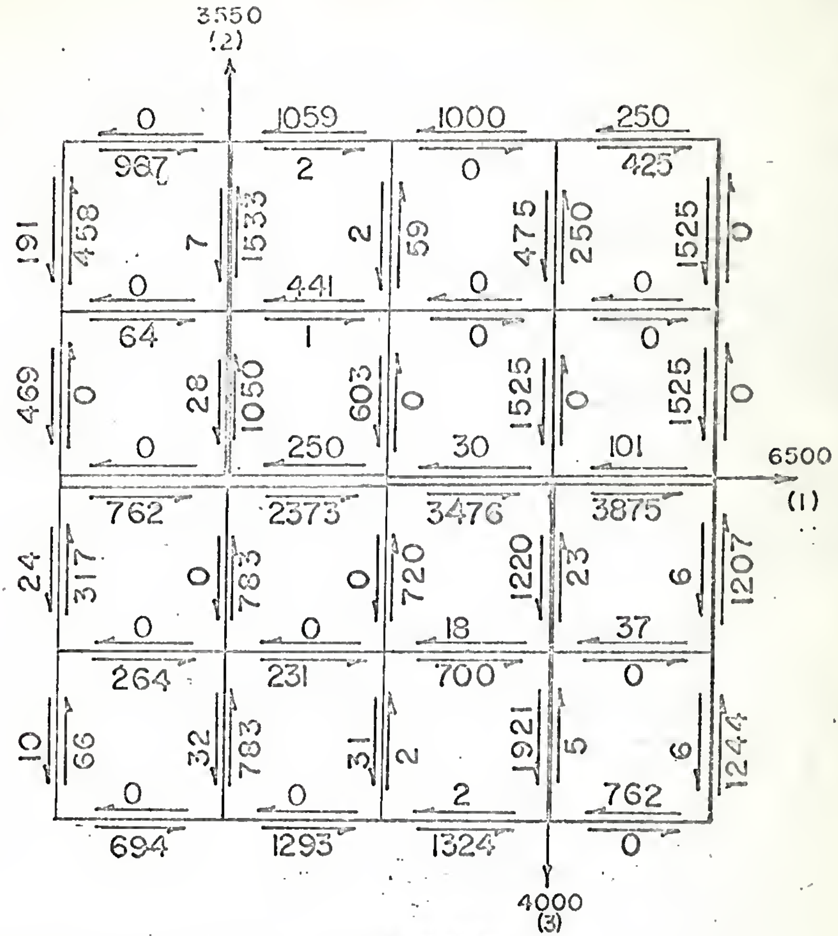


Figure 12 INPUT VOLUMES $v^{n,m}$ AND THE OPTIMAL TRAFFIC ASSIGNMENT FOR COPY 3/2/1.



6 lane freeway (3 lane in one direction)

4 lane freeway

6 lane arterial street

4 lane arterial street

4 lane collector (or local street)

Figure 13. FINAL TRAFFIC ASSIGNMENT FOR 3 COPIES.

COMPARISON BETWEEN THE RESULTS WITH PENALTY AND WITHOUT PENALTY

The total travel time for the traffic flow network as shown by Fig. 1 is 74,752 minutes when the left turn and right turn penalties are taken into account, and the total travel time for the same traffic flow network is 72,175 minutes when the turn penalties are assumed to be zero. This is quite obvious, because if we assume turn penalties to exist there is certain amount of time lost at every intersection (node) when a vehicle takes a right hand or left hand turn, but there is no loss of time at any node when the turn penalties are neglected. In fact it is this extra time at each node which causes an overall increase in the final travel time.

Fig. 7 shows the optimal traffic assignment when turn penalties are considered and Fig. 13 represents the optimal traffic assignment when turn penalties are not considered. Studying carefully the traffic assignment pattern at each node we find that there is a change in assignment characteristic, but this change is not appreciable. For example, consider node (3,3) of Figs. 7 and 13. In Fig. 7, 600 and 700 vehicles enter the node vertically, from above and below respectively. The corresponding numbers of vehicles in Fig. 13 are 603 and 720. Also no vehicles leave the node vertically above or below in both figures. 2400 and 50 vehicles enter the node horizontally, from left and right respectively, in Fig. 7, whereas corresponding numbers in Fig. 13 are 2373 and 30. Finally, in Fig. 7 the number of vehicles which leave the node horizontally to the left and right are 250 and 3500 respectively. The corresponding number of vehicles in Fig. 13 are 250 and 3500. Thus we find that there is no appreciable change in the trend of the traffic assignment.

The increase in total travel time is 3% when turn penalties are taken into account.

DISCUSSION

The multi-copy solution obtained through the computational procedure already described in this section is not an absolute or global optimum solution, but it is a suboptimum solution.

In order to obtain an absolute optimum solution the computational procedure has to be modified to one which is similar to solving a single copy multiquadrant problem. The proposed procedure may be as follows:

Suppose there is a 3-copy traffic flow network. The procedure described in section 3 for a single copy network is carried through one cycle for copy 1. Then one cycle is carried out for copy 2, using the volumes $vv^{n,m}$ and $vh^{n,m}$, previously obtained on copy 1 as fixed inputs to same links of copy 2. Finally, one cycle is again carried out on copy 3, using the volumes $vv^{n,m}$ and $vh^{n,m}$, obtained previously on copy 2 as fixed inputs to same links of copy 3. This makes one iteration for the network. Subsequent iterations are obtained and the iterative process is stopped when the new set of decision variables is sufficiently close to the previous set of all copies to indicate adequate convergence.

The present suboptimal solution is very close or may be the absolute optimum solution because of following reasons:

(i) The formulation of the problem restricts the operation not in a congested situation.

(ii) The calculation of the sum of the total travel time for copies 1 and 2 by the present method uses the link travel time-volume relationship in the lower portion of line A'B' of Fig. 2 in section 3, but in reality it should have used this relationship somewhere in the upper portion of A'B', because in the computation of travel time for copy 1 the volumes of copy 2 and copy 3 which are in the same direction and on the same links as for copy 1 are not considered, or in other words volumes $vv^{n,m}$ and $vh^{n,m}$ are assumed to be zero for copy 1. Even in this situation it never goes beyond point B' which is the point the links volume reaches its capacity. This is because of the restriction the operation in the portion A'B'. This implies that the sum of the total travel time for each copy may be proportional to the actual travel time.

5. MINIMIZATION OF THE SUM OF TRAVEL TIME COST AND INVESTMENT COST ON A TRANSPORTATION SYSTEM.

The basic objective of a transportation study as concluded by Zettel and Carll [25], is the economic analysis of such a transportation network which provides valuable guidance in developing a comprehensive long range transportation plan.

In this study a mathematical model has been developed for the economic evaluation of a transportation system. Like many other studies, a single objective, to minimize the transportation cost, is developed. The transportation cost of any transportation system consists of three basic costs. They are (1) travel time cost, (2) operating cost and (3) investment cost. It has been found through numerous surveys that travel time is dominant as a factor in selecting a particular route and operating cost does not contribute much in selecting a route. Therefore operating cost can be combined with the travel time cost. The value of the travel time cost, c_t, is assumed to be constant, namely \$1.55 per hour per vehicle, and the total travel time cost is obtained by multiplying the total travel time by this constant.

Studies have also shown that the travel time cost and the operating cost on a transportation system could be reduced if a proper amount of investment is made on the system. This means that there is also investment cost incurred on the transportation system. Hence the objective function is reduced to minimize the sum of the travel time cost and the investment cost.

THE TRAVEL TIME EQUATION

As discussed in sections 2 and 3 the unit travel time is in general dependent on traffic volume and roadway conditions. The objective of this study as explained before, is the minimization of the sum of the investment cost and the travel time cost of a given transportation system. The investment is an independent variable and it is assumed that it could be expressed in terms of dollars per mile. Since the roadway conditions depends entirely upon the investment made on the roadways, the unit travel time can be expressed as a function of both traffic volume and investment. The relationship among them is complex. In developing a mathematical model, Wang, Snell and Funk [20,21] made some assumptions to simplify the relationship in order to express the relationship by a relatively simple equation which is manageable and yet not too far from reality.

In order to express unit travel time as a function of traffic volume and investment, some basic characteristics were observed [20,21]. They were:

(1) Unit travel time was increased as the traffic volume increased.

(2) Unit travel time was decreased as the investment increased.

(3) Unit travel time had a lower limit (free flow travel time).

(4) If the travel time was held constant, the service volume increased as the investment increased.

Referring back to Fig. 2 of section 3 the dotted part of the curve shows the relationship under congested conditions. Therefore under normal operating conditions, it is logical to assume that unit travel time (in hours per vehicle per mile) is linearly related to traffic volume. This can be represented by following equation:

$$t = k + k'v \tag{1}$$

where

t = unit travel time (hr/mi/veh)

k = free flow travel time (hr/mi/veh)

k' = slope of the curve in Fig. 2 of section 3 (hr^2/mi/veh)

v = traffic volume per unit time (veh/hr)

Keeping basic characteristics in mind and further assuming that the free flow travel time is constant for each link and traffic volume served is proportional to investment for a constant travel time, an equation of the following form may be hypothesized [20,21]

$$t = k_1 + \frac{k_2}{\theta} v \tag{2}$$

where

t = unit travel time (hr/mi/veh)

k_1 = free flow travel time (hr/mi/veh). The magnitude depends on the maximum speed obtainable or regulated.

k_2 = coefficient of improvement (dollar-hr/mi^2/veh^2). Its magnitude depends on link location and reflects the difficulty of improvement.

θ = equivalent hourly investment per unit length (dollar/mi/hr).

v = traffic volume per unit time (veh/hr).

In the case where old facilities exist, the investment should be expressed as:

$$\theta = k_3 + \theta' \tag{3}$$

where, k_3, in dollars per mile per hour, represents the existing investment and θ', in dollars per mile per hour, is the additional investment.

The general form of the unit travel time equation then becomes

$$t = k_1 + \frac{k_2}{k_3 + \theta'} v \tag{4}$$

The charaeteristics of this equation are demonstrated in Figs. 1, 2 and 3.

Let L be the length of the link and c_t the cost of time. The objective function then becomes

$$S = \theta' L + (k_1 v + \frac{k_2}{k_3 + \theta'} v^2) L c_t \tag{5}$$

In this section two cases will be studied in detail.

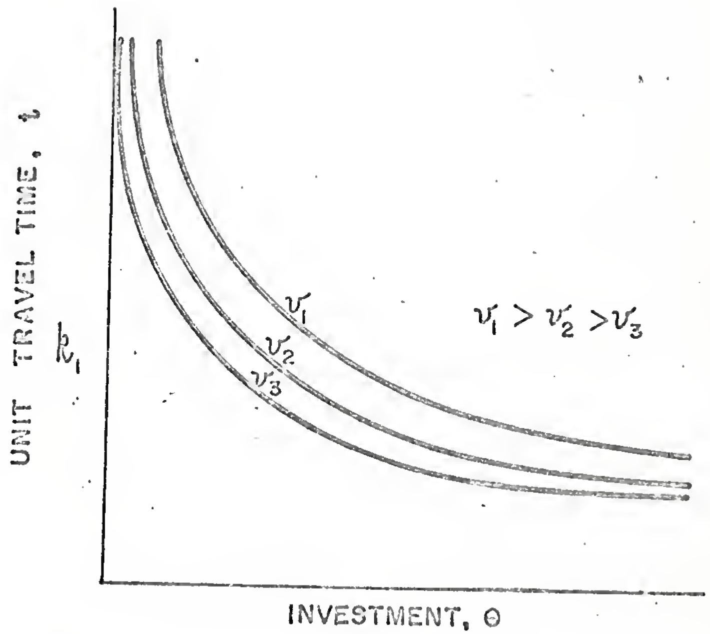


FIGURE I. TRAVEL TIME - INVESTMENT CURVE WITH FIXED VOLUME.

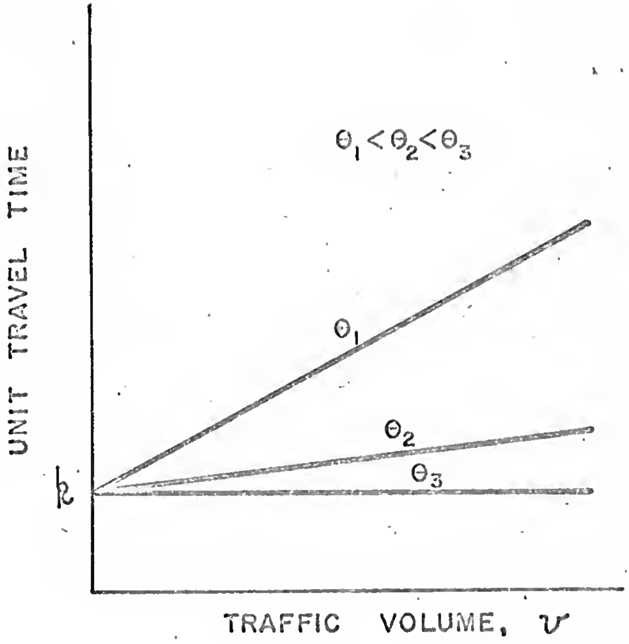


FIGURE 2. TRAVEL TIME-VOLUME CURVE WITH FIXED INVESTMENT.

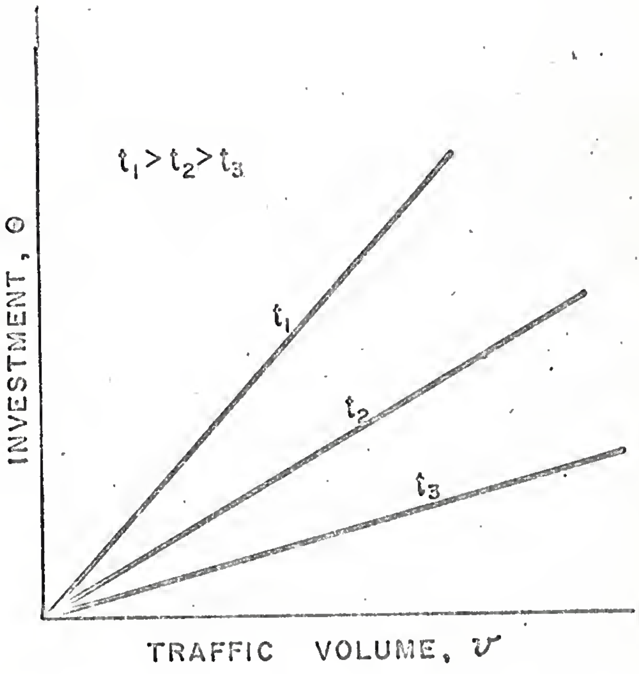


FIGURE 3. VOLUME-INVESTMENT CURVE WITH FIXED TRAVEL TIME

5-1. INVESTMENT WITH NO BUDGET CONSTRAINTS

STATEMENT OF THE PROBLEM

A general examole of optimal investment policy is studied. Fig. 2 of section 2 shows a basic N x M rectangular network with node (N,M) as the destination node, and all other nodes as origins. The input volumes at each node can be obtained from a traffic distribution study. In this particular case the overall system budget is assumed to be unlimited, but it has been considered that there are upper limit and lower limit for investment on each link. The problem is to find an investment policy under each investment condition such that the total cost is minimum under the following assumptions.

1. No turn penalties.
2. Zone centroids coincide with the nodes.
3. Traffic directions are preassigned.
4. Traffic distribution is fixed.
5. Transportation network can be represented by a rectangularly arranged combination of links.
6. Travel time is the only factor that influences the traffic assignment.

Unit travel time on each link can be exoressed as:

$$t_j^{n,m} = k_{j1}^{n,m} + \frac{k_{j2}^{n,m}}{\theta_j^{n,m} + k_{j3}^{n,m}} x_j^{n,m} \tag{6}$$

where

$j = 1$, for horizontal links,

$j = 2$, for vertical links.

FORMULATION OF THE PROBLEM

The performance equations for a typical interior node (n,m) as shown in Fig. 4 of section 3 can be written as follows:

$$x_1^{n,m} = \theta_3^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) \tag{7}$$

$$= \theta_3^{n,m}(A^{n,m}) \quad , \quad x_1^{n,0} = 0 \quad ,$$

$$x_2^{n,m} = (1-\theta_3^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m}) \tag{8}$$

$$= (1-\theta_3^{n,m})\ A^{n,m} \quad , \quad x_2^{0,m} = 0 \quad ,$$

$$x_3^{n,m} = x_3^{n,m-1} + \theta_1^{n,m}\ L_1^{n,m} \quad , \quad \theta_1^{n,m} \geq 0 \ ; \ x_3^{n,0} = 0 \ , \tag{9}$$

$$x_4^{n,m} = x_4^{n-1,m} + \theta_2^{n,m}\ L_2^{n,m} \quad , \quad \theta_2^{n,m} \geq 0 \ ; \ x_4^{0,m} = 0 \ , \tag{10}$$

$$x_5^{n,m} = x_5^{n,m-1} + k_{11}^{n,m}x_1^{n,m}L_1^{n,m}c_t + \frac{k_{12}^{n,m}L_1^{n,m}c_t}{\theta_1^{n,m} + k_{13}^{n,m}}(x_1^{n,m})^2 \quad ,$$

$$x_5^{n,0} = 0 \ . \tag{11}$$

Substituting the value of $x_1^{n,m}$ from equation (7) into equation (11), we have

$$x_5^{n,m} = x_5^{n,m-1} + k_{11}^{n,m}A^{n,m}\theta_3^{n,m}L_1^{n,m}c_t + \frac{k_{12}^{n,m}L_1^{n,m}c_t}{\theta_1^{n,m}+k_{13}^{n,m}}(A^{n,m}\theta_3^{n,m})^2$$

$$x_5^{n,0} = 0 \ , \qquad (12)$$

$$x_6^{n,m} = x_6^{n-1,m}+k_{21}^{n,m}x_2^{n,m}L_2^{n,m}c_t + \frac{k_{22}^{n,m}L_2^{n,m}c_t}{\theta_2^{n,m}+k_{23}^{n,m}}(x_2^{n,m})^2 \ ,$$

$$x_6^{0,m} = 0 \ . \qquad (13)$$

Substituting the value of $x_2^{n,m}$ from equation (8) into equation (13), we have

$$x_6^{n,m} = x_6^{n-1,m} + k_{21}^{n,m}A^{n,m}(1-\theta_3^{n,m})L_2^{n,m}c_t + \frac{k_{22}^{n,m}L_2^{n,m}c_t}{\theta_2^{n,m}+k_{23}^{n,m}}[A^{n,m}(1-\theta_3^{n,m})]^2 ,$$

$$x_6^{0,m} = 0 \ , \qquad (14)$$

$n = 1, 2, \ldots, N; \qquad m = 1,2,\ldots,M$

where

$x_j^{n,m}$ = a state variable representing the number of vehicles on link j immediately beyond node (n,m), j = 1,2 in which j = 1 denotes the horizontal link and j = 2 denotes the vertical link,

$x_3^{n,m}$ = a state variable representing the total investment on horizontal link from node (n,1) including the horizontal link immediately beyond node (n,m), dollars/hour,

$x_4^{n,m}$ = a state variable representing the total investment on vertical links from node (1,m) including the vertical link immediately beyond node (n,m), dollars/hour,

$x_5^{n,m}$ = a state variable representing the total travel time cost on horizontal links from node (n,1) including the horizontal link immediately beyond node (n,m),

$x_6^{n,m}$ = a state variable representing the total travel time cost on vertical links from node (1,m) including the vertical link immediately beyond node (n,m),

$v^{n,m}$ = the number of vehicles entering or leaving the network at node (n,m),

$\theta_j^{n,m}$ = the decision variable that represents investments on link j immediately beyond node (n,m), j = 1,2 in which j = 1 denotes the horizontal link and j = 2 denotes the vertical link, in dollars/mile/hour,

$\theta_3^{n,m}$ = the decision variable that represents the fraction of vehicles which enter the node and leave on the horizontal link, at node (n,m),

$k_{j1}^{n,m}$ = free flow time constant on link j immediately beyond node (n,m), j = 1,2, in which j = 1 denotes the horizontal link and j=2 denotes the vertical link,

$k_{j2}^{n,m}$ = coefficient of investment on link j immediately beyond node (n,m), j=1,2, in which j=1 denotes the horizontal link and j=2 denotes the vertical link,

$k_{j3}^{n,m}$ = existing investment on link j immediately beyond node (n,m), j=1,2, in which j=1 denotes the horizontal link and j=2 denotes the vertical link,

$L_j^{n,m}$ = length of the link j immediately beyond node (n,m), j=1,2, in which j=1 denotes the horizontal link and j=2 denotes the vertical link,

c_t = time cost, dollar/hour/veh.

The objective function S which is to be minimized represents the total accumulated travel time cost and the total accumulated investment cost all over the transportation system.

$$S = \sum_{n=1}^{N} x_3^{n,M} + \sum_{m=1}^{M} x_4^{n,m} + \sum_{n=1}^{N} x_5^{n,M} + \sum_{m=1}^{M} x_6^{N,m} \qquad (15)$$

where the first two terms represent the accumulated travel time cost incurred on horizontal and vertical links of the transportation system respectively, and the last two terms represent the accumulated investment cost incurred on the horizontal and vertical links respectively on the transportation system.

The Hamiltonian function and the adjoint variables at the node (n,m) can be written as follows:

$$H^{n,m} = \sum_{i=1}^{6} z_i^{n,m} x_i^{n,m} . \qquad (16)$$

Substituting equations (7) through (14) into equation (16), we have

$$H^{n,m} = z_1^{n,m}\theta_3^{n,m}A^{n,m}+z_2^{n,m}(1-\theta_3^{n,m})A^{n,m}+z_3^{n,m}(x_3^{n,m-1}+\theta_1^{n,m}L_1^{n,m})$$

$$+z_4^{n,m}(x_4^{n-1,m}+\theta_2^{n,m}L_2^{n,m})+z_5^{n,m}[x_5^{n,m-1}+k_{11}^{n,m}\theta_3^{n,m}A^{n,m}L_1^{n,m}c_t$$

$$+\frac{k_{12}^{n,m}L_1^{n,m}c_t}{\theta_1^{n,m}+k_{13}^{n,m}}(\theta_3^{n,m}A^{n,m})^2]+z_6^{n,m}[x_6^{n-1,m}+k_{21}^{n,m}(1-\theta_3^{n,m})A^{n,m}L_2^{n,m}c_t$$

$$+\frac{k_{22}^{n,m}L_2^{n,m}c_t}{\theta_2^{n,m}+k_{23}^{n,m}}\{(1-\theta_3^{n,m})A^{n,m}\}^2] \quad , \tag{17}$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m}\theta_3^{n,m} + z_2^{n,m}(1-\theta_3^{n,m}) + z_5^{n,m}k_{11}^{n,m}\theta_3^{n,m}L_1^{n,m}c_t$$

$$+ z_6^{n,m}k_{21}^{n,m}(1-\theta_3^{n,m})L_2^{n,m}c_t$$

$$+ 2z_5^{n,m}\frac{k_{12}^{n,m}L_1^{n,m}c_t}{\theta_1^{n,m}+k_{13}^{n,m}}A^{n,m}(\theta_3^{n,m})^2$$

$$+ 2z_6^{n,m}\frac{k_{22}^{n,m}L_2^{n,m}c_t}{\theta_2^{n,m}+k_{23}^{n,m}}A^{n,m}(1-\theta_3^{n,m})^2 \quad , \tag{18}$$

$$z_2^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_2^{n-1,m}}$$

$$= z_1^{n,m}\theta_3^{n,m} + z_2^{n,m}(1-\theta_3^{n,m}) + z_5^{n,m}k_{11}^{n,m}\theta_3^{n,m}L_1^{n,m}c_t$$

$$+ z_6^{n,m}k_{21}^{n,m}(1-\theta_3^{n,m})L_2^{n,m}c_t$$

$$+ 2z_5^{n,m}\frac{k_{12}^{n,m}L_1^{n,m}c_t}{\theta_1^{n,m}+k_{13}^{n,m}}A^{n,m}(\theta_3^{n,m})^2$$

$$+ 2z_6^{n,m}\frac{k_{22}^{n,m}L_2^{n,m}c_t}{\theta_2^{n,m}+k_{23}^{n,m}}A^{n,m}(1-\theta_3^{n,m})^2 \quad , \tag{19}$$

$$z_3^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_3^{n,m-1}}$$

$$= z_3^{n,m} \quad , \tag{20}$$

$$z_4^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_4^{n-1,m}}$$

$$= z_4^{n,m} \quad , \tag{21}$$

$$z_5^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_5^{n,m-1}}$$

$$= z_5^{n,m} \quad , \tag{22}$$

$$z_6^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_6^{n-1,m}}$$

$$= z_6^{n,m} \quad , \tag{23}$$

$$z_1^{n,M} = 0 \quad , \quad n = 1,2,\ldots,N, \tag{24}$$

$$z_2^{N,m} = 0 \quad , \quad m = 1,2,\ldots,M, \tag{25}$$

$$z_3^{n,M} = 1 \quad , \quad n = 1,2,\ldots,N, \tag{26}$$

$$z_4^{N,m} = 1 \quad , \quad m = 1,2,\ldots,M, \tag{27}$$

$$z_5^{n,M} = 1 \quad , \quad n = 1,2,\ldots,N, \tag{28}$$

and

$$z_6^{N,m} = 1 \quad , \quad m = 1,2,\ldots,M. \tag{29}$$

From equations (20) through (23) and (26) through (29), we obtain

$$z_3^{n,m} = z_4^{n,m} = z_5^{n,m} = z_6^{n,m} = 1, \qquad \begin{array}{l} n = 1,2,\ldots,N, \\ m = 1,2,\ldots,M. \end{array} \tag{30}$$

The Hamiltonian function then becomes:

$$H^{n,m} = z_1^{n,m}x_1^{n,m} + z_2^{n,m}x_2^{n,m} + x_3^{n,m} + x_4^{n,m} + x_5^{n,m} + x_6^{n,m} \ . \tag{31}$$

The necessary conditions to minimize the objective function, S, are:

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = 0 \quad , \qquad \theta_1^{n,m} > 0 \quad , \tag{32a}$$

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = 0 \quad , \qquad \theta_2^{n,m} > 0 \quad , \tag{32b}$$

$$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} = 0 \quad , \qquad 0 < \theta_3^{n,m} < 1 \quad , \tag{32c}$$

when $(\theta_1^{n,m}, \theta_2^{n,m}, \theta_3^{n,m})$ is an interior point of an admissible control, or $H^{n,m}$ = minimum with respect to those $\theta_j^{n,m}$ which are at a boundary point of the constraints.

Substituting equations (7) to (14) into equation (31) and taking derivatives with respect to the various decision variables, the following equations are obtained:

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = L_1^{n,m} - \frac{k_{12}^{n,m} L_1^{n,m} c_t}{(\theta_1^{n,m} + k_{13}^{n,m})^2} (A^{n,m} \theta_3^{n,m})^2 \quad , \tag{33}$$

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = L_2^{n,m} - \frac{k_{22}^{n,m} L_2^{n,m} c_t}{(\theta_2^{n,m} + k_{23}^{n,m})^2} [A^{n,m}(1 - \theta_3^{n,m})]^2 \quad , \tag{34}$$

$$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} = (z_1^{n,m} - z_2^{n,m}) A^{n,m} + (k_{11}^{n,m} L_1^{n,m} - k_{21}^{n,m} L_2^{n,m}) A^{n,m} c_t$$

$$+ 2 \frac{k_{12}^{n,m} L_1^{n,m} c_t}{\theta_1^{n,m} + k_{13}^{n,m}} (A^{n,m})^2 \theta_3^{n,m}$$

$$- 2 \frac{k_{22}^{n,m} L_2^{n,m} c_t}{\theta_2^{n,m} + k_{23}^{n,m}} (A^{n,m})^2 (1-\theta_3^{n,m}) \quad . \tag{35}$$

The second partial derivative of the Hamiltonian with respect to the decision variable, $\theta_3^{n,m}$, which is used at the computational procedure is,

$$\frac{\partial^2 H^{n,m}}{\partial(\theta_3^{n,m})^2} = 2 \frac{k_{12}^{n,m} L_1^{n,m} c_t}{\theta_1^{n,m} + k_{13}^{n,m}} (A^{n,m})^2 + 2 \frac{k_{22}^{n,m} L_2^{n,m} c_t}{\theta_2^{n,m} + k_{23}^{n,m}} (A^{n,m})^2 \quad . \tag{36}$$

Setting equations (33) and (34) equal to zero and applying the boundary conditions of the decision variables, the values of $\theta_1^{n,m}$ and $\theta_2^{n,m}$ can be obtained from the following equations:

$$\theta_1^{n,m} = \sqrt{k_{12}^{n,m} c_t} \quad A^{n,m} \theta_3^{n,m} - k_{13}^{n,m} \quad \text{when} \quad \theta_1^{n,m} > 0 \quad , \tag{37}$$

or

$$\theta_1^{n,m} = 0 \quad \text{when} \quad \sqrt{k_{12}^{n,m} c_t} \quad A^{n,m} \theta_3^{n,m} - k_{13}^{n,m} \leq 0 \quad , \tag{38}$$

$$\theta_2^{n,m} = \sqrt{k_{22}^{n,m} c_t} \quad A^{n,m} (1-\theta_3^{n,m}) - k_{23}^{n,m} \quad \text{when} \quad \theta_2^{n,m} > 0 \quad , \tag{39}$$

or

$$\theta_2^{n,m} = 0 \quad \text{when} \quad \sqrt{k_{22}^{n,m} c_t} \quad A^{n,m} (1-\theta_3^{n,m}) - k_{23}^{n,m} \leq 0 \quad . \tag{40}$$

When both $\theta_1^{n,m}$ and $\theta_2^{n,m}$ are greater than zero, equations (37) and (39) can be substituted into equation (35) to obtain the following equation

$$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} = (z_1^{n,m}-z_2^{n,m})\ A^{n,m} + (k_{11}^{n,m}L_1^{n,m}-k_{21}^{n,m}L_2^{n,m})\ A^{n,m}c_t$$

$$+ 2\ k_{12}^{n,m}c_t L_1^{n,m}A^{n,m} - 2\ k_{22}^{n,m}c_t L_2^{n,m}A^{n,m}$$

$$= A^{n,m}[(z_1^{n,m}-z_2^{n,m}) + (k_{11}^{n,m}L_1^{n,m}-k_{21}^{n,m}L_2^{n,m})c_t$$

$$+ 2(\ k_{12}^{n,m}c_t\ L_1^{n,m} - \ k_{22}^{n,m}c_t\ L_2^{n,m})]\ . \tag{41}$$

$\theta_3^{n,m}$ is eliminated by the substitution and the value of $\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}}$ becomes independent of $\theta_3^{n,m}$ as shown in equation (41). This implies that the value of $H^{n,m}$ is linearly related to $\theta_3^{n,m}$ and the extreme of $H^{n,m}$ with respect to $\theta_3^{n,m}$ occurs at a boundary. In this case, to obtain the minimum value of $H^{n,m}$,

$$\theta_3^{n,m} = 0 \quad \text{if} \quad \frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} > 0 \quad ,$$

$$= 1 \quad \text{if} \quad \frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} < 0 \quad ,$$

$$= \text{any value between 0 and 1} \quad \text{if} \quad \frac{\partial H^{n,m}}{\partial \theta^{n,m}} = 0 \quad .$$

When either $\theta_1^{n,m}$ or $\theta_2^{n,m}$ is equal to zero, or when both are equal to zero, equation (41) is no longer valid. Equation (35) is then set equal to zero and solved for the optimal value of $\theta_3^{n,m}$.

In an urban area, the available space for road construction is often limited. For example, a freeway with more than eight lanes would be very difficult to build near a CBD (central business district) area. It is, therefore, necessary to set an upper limit on the size of the links. This limit can be expressed as a limit on the investment on each link.

Also in developing an urban transportation network, it is sometimes required to provide a minimum level of service for the entire area. For example, arterial streets would be distributed uniformly throughout the whole area. This criterion can be fulfilled by requiring a minimum amount of investment on each link. These upper and lower limits can be expressed mathematically as follows:

$$\left(\theta_1^{n,m}\right)_{min} \leq k_{13}^{n,m} + \theta_1^{n,m} \leq \left(\theta_1^{n,m}\right)_{max} \quad , \tag{42}$$

$$\left(\theta_2^{n,m}\right)_{min} \leq k_{23}^{n,m} + \theta_2^{n,m} \leq \left(\theta_2^{n,m}\right)_{max} \quad , \tag{43}$$

where

$\left(\theta_j^{n,m}\right)_{max}$ = the upper limit on the investment on link j immediately beyond node (n,m) j=1,2, in which j=1 denotes the horizontal link and j=2 denotes the vertical link,

$\left(\theta_j^{n,m}\right)_{min}$ = the lower limit on the investment on link immediately beyond node (n,m) j=1,2, in which j=1 denotes the horizontal link and j=2 denotes the vertical link.

COMPUTATIONAL PROCEDURE

The above formulation provides the equations (1) through (36) to find the optimal sequence of decision variables $\theta_1^{n,m}$, $\theta_2^{n,m}$, and $\theta_3^{n,m}$. The particular procedure used to accomplish this is as follows:

Step 1. Assume a set of decision variables, $\theta_3^{n,m}$.

Step 2. Calculate $x_1^{n,m}$, $x_2^{n,m}$ and $A^{n,m}$ by equations (7) and (8), starting at n=m=1 and proceeding to n=N and m=M.

Step 3. Calculate decision variables, $\theta_1^{n,m}$ and $\theta_2^{n,m}$, by equations (37) and (39) and check the boundary conditions for each special case.

Step 4. Calculate the values of $x_i^{n,m}$, i = 3, 4, 5, 6, by equations (9) through (14) starting at n=m=1 and proceeding to n=N and m=M.

Step 5. Calculate the adjoint vectors, $z_i^{n,m}$, i = 1, 2, with the above $x_i^{n,m}$ values, by equations (18) and (19), starting at n=N, m=M and proceeding backward to n=m=1.

Step 6. Calculate $\dfrac{\partial H^{n,m}}{\partial \theta_3^{n,m}}$ and $\dfrac{\partial^2 H^{n,m}}{\partial (\theta_3^{n,m})^2}$ by equations (35) and (36) using the values of $x_i^{n,m}$ and $z_i^{n,m}$ obtained above.

Step 7. Compute a new sequence of decision variables $\theta_3^{n,m}$ from the following equation.

$$(\theta_3^{n,m})_{revised} = (\theta_3^{n,m})_{old} + \Delta\theta_3^{n,m} \tag{44}$$

where

$$\Delta\theta_3^{n,m} = - \frac{\dfrac{\partial H^{n,m}}{\partial \theta_3^{n,m}}}{\dfrac{\partial^2 H^{n,m}}{\partial (\theta_3^{n,m})^2}} \tag{49a}$$

and check the boundary condition.

Step 8. Return to step 2 and repeat the procedure until the value of the objective function, equation (9), is sufficiently close to the previous value to indicate adequate convergence.

5-2. INVESTMENT WITH FIXED SYSTEM BUDGET

Sometimes the total budget for a transportation system improvement is predetermined and fixed. Obviously, the total investment in this case, must be equal to the fixed system budget. This, then becomes a fixed end point problem.

FORMULATION OF THE PROBLEM

The performance equations for a typical interior node as shown in Fig. 4 of section 3 can be written as follows:

$$x_1^{n,m} = \theta_3^{n,m}(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$= \theta_3^{n,m}(A^{n,m}) \quad , \quad x_1^{n,0} = 0 \quad , \tag{45}$$

$$x_2^{n,m} = (1-\theta_3^{n,m})(x_1^{n,m-1} + x_2^{n-1,m} + v^{n,m})$$

$$= (1-\theta_3^{n,m})\ A^{n,m} \quad , \quad x_2^{0,m} = 0 \quad , \tag{46}$$

$$x_5^{n,m} = x_5^{n,m-1}+k_{11}^{n,m}L_1^{n,m}c_tA^{n,m}\theta_3^{n,m}+ \frac{k_{12}^{n,m}L_1^{n,m}c_t}{\frac{\theta_1^{n,m}}{L_1^{n,m}}+k_{13}^{n,m}} (A^{n,m}\theta_3^{n,m})^2 \;,$$

$$x_5^{n,0} = 0 \;, \qquad (47)$$

$$x_6^{n,m} = x_6^{n-1,m} + k_{21}^{n,m}L_2^{n,m}c_tA^{n,m}(1 - \theta_3^{n,m})$$

$$+ \frac{k_{22}^{n,m}L_2^{n,m}c_t}{\frac{\theta_2^{n,m}}{L_2^{n,m}}+k_{23}^{n,m}} [A^{n,m}(1 - \theta_3^{n,m})]^2 \;, \qquad x_6^{0,m} = 0 \;, \qquad (48)$$

$$x_7^{n,m} = x_7^{n,m-1} + \theta_1^{n,m} + \theta_2^{n,m} \;, \qquad \sum_{n=1}^{N} x_7^{n,M} = G \;, \qquad (49)$$

where

$x_7^{n,m}$ = a state variable representing the total investment on both links from node (n,1) including both links immediately beyond node (n,m),

G = total system budget.

Here, $\theta_1^{n,m}$ and $\theta_2^{n,m}$ are total investments on the horizontal and vertical links respectively at node (n,m), in dollars/hr.

Since total investment is a fixed amount, the objective function becomes:

$$S = \sum_{n=1}^{N} x_5^{n,M} + \sum_{m=1}^{M} x_6^{N,m} \;. \qquad (50)$$

The Hamiltonian function and the adjoint variables can be written as follows:

$$H^{n,m} = z_1^{n,m}x_1^{n,m} + z_2^{n,m}x_2^{n,m} + z_5^{n,m}x_5^{n,m} + z_6^{n,m}x_6^{n,m} + z_7^{n,m}x_7^{n,m}$$

$$= z_1^{n,m}\theta_3^{n,m}A^{n,m} + z_2^{n,m}(1-\theta_3^{n,m})A^{n,m}$$

$$+ z_5^{n,m}[x_5^{n,m-1} + k_{11}^{n,m}L_1^{n,m}c_t$$

$$+ \frac{k_{12}^{n,m}L_1^{n,m}c_t}{\dfrac{\theta_1^{n,m}}{L_1^{n,m}} + k_{13}^{n,m}}(\theta_3^{n,m}A^{n,m})^2]$$

$$+ z_6^{n,m}[x_6^{n-1,m} + k_{21}^{n,m}L_2^{n,m}c_t(1-\theta_3^{n,m})A^{n,m}$$

$$+ \frac{k_{22}^{n,m}L_2^{n,m}c_t}{\dfrac{\theta_2^{n,m}}{L_2^{n,m}} + k_{23}^{n,m}}\{(1-\theta_3^{n,m})\ A^{n,m}\}^2]$$

$$+ z_7^{n,m}[x_7^{n,m-1} + \theta_1^{n,m} + \theta_2^{n,m}] \quad , \tag{51}$$

$$z_1^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_1^{n,m-1}}$$

$$= z_1^{n,m}\theta_3^{n,m} + z_2^{n,m}(1-\theta_3^{n,m})$$

$$+ z_5^{n,m}(k_{11}^{n,m}\theta_3^{n,m}L_1^{n,m}c_t)$$

$$+ z_6^{n,m}[k_{21}^{n,m}(1-\theta_3^{n,m})L_2^{n,m}c_t]$$

$$+ 2z_5^{n,m} \frac{k_{12}^{n,m}L_1^{n,m}c_t}{\frac{\theta_1^{n,m}}{L_1^{n,m}} + k_{13}^{n,m}} (\theta_3^{n,m})^2 A^{n,m}$$

$$+ 2z_6^{n,m} \frac{k_{22}^{n,m}L_2^{n,m}c_t}{\frac{\theta_2^{n,m}}{L_2^{n,m}} + k_{23}^{n,m}} (1-\theta_3^{n,m})^2 A^{n,m} \quad , \tag{52}$$

$$z_2^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_2^{n-1,m}}$$

$$= z_1^{n,m-1} \quad , \tag{53}$$

$$z_5^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_5^{n,m-1}}$$

$$= z_5^{n,m} \quad , \tag{54}$$

$$z_6^{n-1,m} = \frac{\partial H^{n,m}}{\partial x_6^{n-1,m}}$$

$$= z_6^{n,m} \quad , \tag{55}$$

$$z_7^{n,m-1} = \frac{\partial H^{n,m}}{\partial x_7^{n,m-1}}$$

$$= z_7^{n,m} \quad , \tag{56}$$

$$z_1^{n,M} = 0 \quad , \quad n = 1, 2, \ldots, N, \tag{57}$$

$$z_2^{N,m} = 0 \quad , \quad m = 1, 2, \ldots, M, \tag{58}$$

$$z_5^{n,M} = 1 \quad , \quad n = 1, 2, \ldots, N, \tag{59}$$

$$z_6^{N,m} = 1 \quad , \quad m = 1, 2, \ldots, M. \tag{60}$$

From equations (54), (55), (57) and (60), we obtain

$$z_5^{n,m} = z_6^{n,m} = 1 \quad , \quad \begin{array}{l} n = 1, 2, \ldots, N, \\ m = 1, 2, \ldots, M. \end{array} \tag{61}$$

It has already been stated that $\sum_{n=1}^{N} x_7^{n,M}$ is fixed, which is the total system budget, so $z_7^{n,m}$, $n = 1, 2, \ldots, n$, remains unknown. However the following approach will enable us to find $z_7^{n,m}$, at any node (n,m).

At node (N,M),

$$\theta_1^{N,M} = \theta_2^{N,M} = 0$$

This gives

$$\sum_{n=1}^{N} x_7^{n,M} = \sum_{n=1}^{N} x_7^{n,M-1} = G \qquad (62)$$

Substituting the values of the adjoint variables obtained from equations (52) to (60) into equation (51) and then taking the derivative of the Hamiltonian function at node (N,M-1), partially with respect to $\theta_1^{N,M-1}$, we have

$$\frac{\partial H^{N,M-1}}{\partial \theta_1^{N,M-1}} = -\frac{k_{12}^{N,M-1} c_t}{\left(\frac{\theta_1^{N,M-1}}{L_1^{N,M-1}} + k_{13}^{N,M-1}\right)^2} (A^{N,M-1})^2$$

$$+ z_7^{N,M-1} = 0 \qquad (63)$$

We can write down the value of $x_7^{N,M-1}$ from equation (49)

$$G = \sum_{n=1}^{N} x_7^{n,M-2} + \theta_1^{N,M-1} + \theta_2^{N,M-1} \quad .$$

But $\theta_2^{N,M-1} = 0$, as no vertical link exists at node (N,M-1). This gives

$$G = \sum_{n=1}^{N} x_7^{n,M-2} + \theta_1^{N,M-1}$$

or

$$\theta_1^{N,M-1} = G - \sum_{n=1}^{N} x_7^{n,M-2} \qquad (64)$$

Substituting equation (64) into equation (63) gives

$$z_7^{N,M-1} = \frac{k_{12}^{N,M-1} c_t \, (A^{N,M-1})^2}{\left(\dfrac{G - \sum_{n=1}^{N} x_7^{n,M-2}}{L_1^{N,M-1}} + k_{13}^{N,M-1}\right)^2} \tag{64}$$

or from equation (56)

$$z_7^{n,m} = \frac{k_{12}^{N,M-1} c_t \, (A^{N,M-1})^2}{\left(\dfrac{G - \sum_{n=1}^{N} x_7^{n,M-2}}{L_1^{N,M-1}} + k_{13}^{N,M-1}\right)^2} \tag{65}$$

$$n = 1, 2, \ldots, N; \qquad m = 1, 2, \ldots, M.$$

The necessary conditions for S to be a local minimum is that:

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = 0 \quad , \qquad 0 < \theta_1^{n,m} < G - x_7^{n,m} \quad , \tag{66a}$$

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = 0 \quad , \qquad 0 < \theta_2^{n,m} < G - x_7^{n,m} - \theta_1^{n,m} \quad , \tag{66b}$$

$$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} = 0 \quad , \qquad 0 < \theta_3^{n,m} < 1 \quad , \tag{66c}$$

when $(\theta_1^{n,m}, \theta_2^{n,m}, \theta_3^{n,m})$ is an interior point, or

$$H^{n,m} = \text{minimum} \tag{66d}$$

when $(\theta_1^{n,m}, \theta_2^{n,m}, \theta_3^{n,m})$ is at a boundary point of the constraints.

Substituting equations (45) to (49) and equation (61) into equation (51) and taking derivatives with respect to various decision variables, the following equations are obtained:

$$\frac{\partial H^{n,m}}{\partial \theta_1^{n,m}} = - \frac{k_{12}^{n,m}(A^{n,m}\theta_3^{n,m})^2 c_t}{\left(\frac{\theta_1^{n,m}}{L_1^{n,m}} + k_{13}^{n,m}\right)^2} + z_7^{N,M-1} \quad , \tag{67}$$

$$\frac{\partial H^{n,m}}{\partial \theta_2^{n,m}} = - \frac{k_{22}^{n,m}A^{n,m}(1-\theta_3^{n,m})^2 c_t}{\left(\frac{\theta_2^{n,m}}{L_2^{n,m}} + k_{23}^{n,m}\right)^2} + z_7^{N,M-1} \quad , \tag{68}$$

$$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}} = (z_1^{n,m} - z_2^{n,m})A^{n,m} + (k_{11}^{n,m}L_1^{n,m} - k_{21}^{n,m}L_2^{n,m})c_t A^{n,m}$$

$$+ 2\,\frac{k_{12}^{n,m}(A^{n,m})^2\theta_3^{n,m}L_1^{n,m}c_t}{\frac{\theta_1^{n,m}}{L_1^{n,m}} + k_{13}^{n,m}}$$

$$- 2\,\frac{k_{22}^{n,m}(A^{n,m})^2(1-\theta_3^{n,m})L_2^{n,m}c_t}{\frac{\theta_2^{n,m}}{L_2^{n,m}} + k_{23}^{n,m}} \tag{69}$$

The second partial derivative of the Hamiltonian with respect to the decision variable, $\theta_3^{n,m}$, which is used at the computational procedure, is

$$\frac{\partial^2 H^{n,m}}{\partial(\theta_3^{n,m})^2} = 2\,\frac{k_{12}^{n,m}(A^{n,m})^2 L_1^{n,m} c_t}{\dfrac{\theta_1^{n,m}}{L_1^{n,m}} + k_{13}^{n,m}} + 2\,\frac{k_{22}^{n,m}(A^{n,m})^2 L_2^{n,m} c_t}{\dfrac{\theta_2^{n,m}}{L_2^{n,m}} + k_{23}^{n,m}} \quad . \tag{70}$$

Setting equations (67) and (68) equal to zero, we obtain

$$\theta_1^{n,m} = \frac{k_{12}^{n,m} c_t}{z_7^{N,M-1}} A^{n,m}\, \theta_3^{n,m}\, L_1^{n,m} - k_{13}^{n,m}\, L_1^{n,m} \quad . \tag{71}$$

COMPUTATIONAL PROCEDURE

By using equations (45) through (72) the optimal sequence of the decision variables $\theta_1^{n,m}$, $\theta_2^{n,m}$ and $\theta_3^{n,m}$ can be found. The following procedure is used to accomplish this.

Step 1. Assume a set of decision variables ($\theta_1^{n,m}$, $\theta_2^{n,m}$, $\theta_3^{n,m}$).

Step 2. Calculate values of $x_i^{n,m}$, i = 1, 2, 5, 6, 7 and $A^{n,m}$ starting at n=m=1 and proceeding to n=N, m=M.

Step 3. a.) For the first iteration, calculate $z_7^{N,M-1}$ by equation (64) with the above $x_i^{n,m}$ and $A^{n,m}$ values and go to step 4.

b.) For the second and the following iterations, calculate $z_7^{N,M-1}$ by equation (64) with the above $x_i^{n,m}$ and $A^{n,m}$ values. This $z_7^{N,M-1}$ value is then compared with the value obtained in the previous iteration. If the two values are sufficiently close, proceed to step 6. If they are not sufficiently close, proceed to step 4.

Step 4. Calculate new values of $\theta_1^{n,m}$ and $\theta_2^{n,m}$ using equations (71) and (72) and check the boundary conditions.

Step 5. Return to step 2.

Step 6. Calculate $z_1^{n,m}$ and $z_2^{n,m}$ starting at n=N, m=M and proceeding backward to n=m=1 by the use of equations (52) through (61).

Step 7. Calculate $\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}}$ and $\frac{\partial^2 H^{n,m}}{\partial (\theta_3^{n,m})^2}$ by equations (69) and (70), using the values of $x_i^{n,m}$ and $z_i^{n,m}$ obtained above.

Step 8. Compute a new sequence of decision variables $\theta_3^{n,m}$ from the following equation:

$$(\theta_3^{n,m})_{revised} = (\theta_3^{n,m})_{old} + \Delta\theta_3^{n,m} \tag{73}$$

where

$$\Delta\theta_3^{n,m} = - \frac{\dfrac{\partial H^{n,m}}{\partial \theta_3^{n,m}}}{\dfrac{\partial^2 H^{n,m}}{\partial (\theta_3^{n,m})^2}} \tag{73a}$$

and check the boundary conditions.

Step 9. Return to step 2 and repeat the procedure until the value of the objective function is sufficiently close to the previous value to indicate adequate convergence.

In the case where a minimum level of service is to be provided, the minimum investment can be treated as the existing facilities. The problem can then be solved by the general method without changing the algorithm. In other words, when the values of $k_{13}^{n,m}$ are less than the minimum required investment, set them equal to the minimum investment and deduct the difference from the total budget.

The above formulation provides solutions to a single-quadrant network, single-copy problem. To solve a multi-quadrant network, multi-copy problem, the procedures developed by Snell, et. al. [8,17] can be employed.

NUMERICAL EXAMPLES

Three numerical examples are presented in this section to demonstrate the use of the model. Examples 1 and 2 illustrate the first case of this section under different investment conditions and example 3 illustrates the second case.

A hypothetical network is developed as shown in Fig. 4. Node (4,4) is assumed to be the centroid of the CBD. The input volumes, $v^{n,m}$, are also shown in the figure. All links have an equal length of one mile. The area is divided into two parts by a diagonal line which passes through nodes (1,4) and (4,1). The lower part which is adjacent to the CBD was assumed to be densely developed. The upper part was assumed to be less densely developed. Assuming the maximum speed in the densely developed area to be 60 mph and in the less densely developed area 70 mph, minimum travel times in these two areas become 0.0167 hour per mile and 0.0143 hour per mile respectively. Single line links represent existing local streets and double line links represent existing arterial streets.

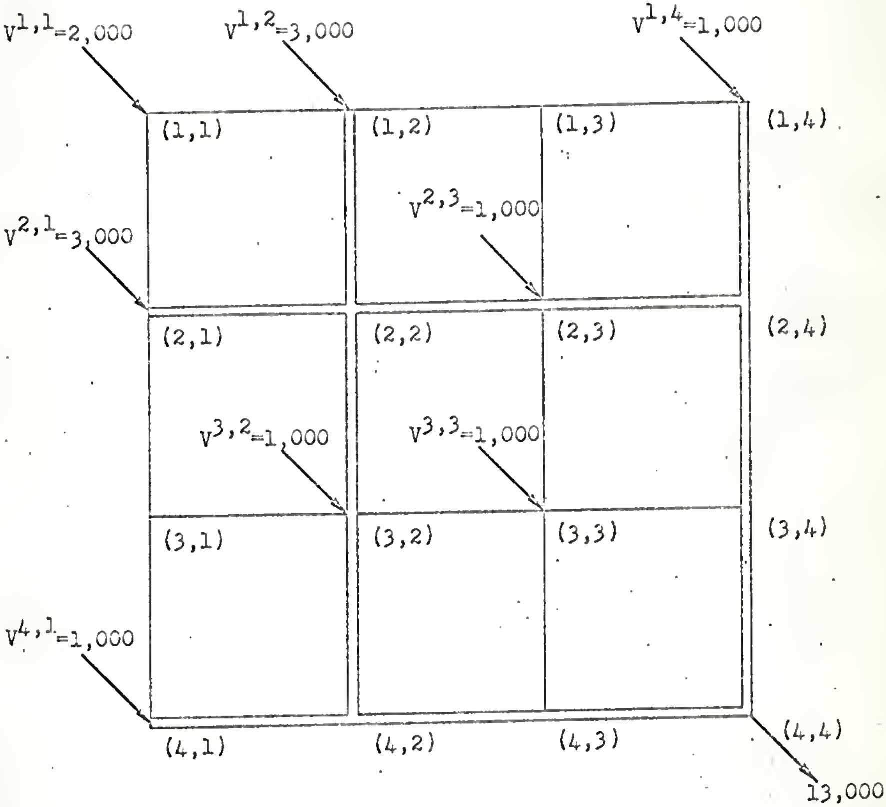


Fig. 4. Hypothetical Network and Input Volumes $v^{n,m}$ for Numerical Examples 1, 2 and 3.

Input data for the models are summarized in Table 1. Values of k_{i2} and k_{i3} are also indicated in Fig. 5 and Fig. 6 respectively. The time cost, c_t, is assumed to be \$1.55 per hour per vehicle as suggested by AASHO [15].

Example 1

Suppose we are planning for a completely undeveloped area where no facilities exist and there is no budget limitation on link investment. A theoretical optimal system can then be developed to accommodate the predicted trip demand. Using the formulation of "investment with no budget constraint" and letting $k_{i3}^{n,m} = 0$, for all (n,m), the resulting system is shown in Fig. 7. Notice that the system forms a shortest path tree in which only one route is built for each origin-destination pair and all trips are assigned to this route. This result coincides with the analysis discussed in page 120 which shows the linear characteristic of the problem under no limit condition.

Example 2

The hypothetical network shown in Fig. 4 is to be improved with the following conditions:

1. No system budget limit.
2. A minimum level of service (arterial street) is to be provided for the entire area.
3. Roadway space obtainable is restricted.

Nodes (n,m)	links (i)	$K^{n,m}_{i1}$	$K^{n,m}_{i2}$	$K^{n,m}_{i3}$	$\Theta^{n,m}_{i\max}$	$\Theta^{n,m}_{i\min}$	$V^{n,m}$
1,1	1	0.0143	0.00003	8.0	80	10	2,000
	2	0.0143	0.00004	8.0	80	10	
1,2	1	0.0143	0.00006	8.0	80	10	3,000
	2	0.0143	0.00005	10.0	80	10	
1,3	1	0.0143	0.00008	8.0	80	15	0
	2	0.0143	0.00006	8.0	80	10	
1,4	1	0.0167	1.000	0.00001	100	15	1,000
	2	0.0167	0.00010	15.0	100	15	
2,1	1	0.0143	0.00005	10.0	80	10	3,000
	2	0.0143	0.00005	8.0	80	10	
2,2	1	0.0143	0.00006	10.0	80	10	0
	2	0.0143	0.00005	10.0	80	10	
2,3	1	0.0167	0.00010	15.0	100	15	1,000
	2	0.0167	0.00008	12.0	100	15	
2,4	1	0.0167	1.000	0.00001	100	15	0
	2	0.0167	0.00015	15.0	100	15	
3,1	1	0.0143	0.00006	8.0	80	10	0
	2	0.0143	0.00006	8.0	80	10	
3,2	1	0.0167	0.00008	12.0	100	15	1,000
	2	0.0167	0.00010	15.0	100	15	
3,3	1	0.0167	0.00015	12.0	100	15	1,000
	2	0.0167	0.00015	12.0	100	15	
3,4	1	0.0167	1.000	0.00001	100	15	0
	2	0.0167	0.00025	15.0	100	15	
4,1	1	0.0167	0.00008	15.0	100	15	1,000
	2	0.0167	1.000	0.00001	100	15	
4,2	1	0.0167	0.00015	15.0	100	15	0
	2	0.0167	1.000	0.00001	100	15	
4,3	1	0.0167	0.00020	15.0	100	15	0
	2	0.0167	1.00	0.00001	100	15	
4,4	1	0.0167	0.00001	0.00001	100	15	0
	2	0.0167	0.00001	0.00001	100	15	

$i = 1$ for horizontal links $\qquad G = \$300.00$

$i = 2$ for vertical links $\qquad c_t = \$1.55/\text{hour}$

Table 1 Input Data of Numerical Examples 1, 2 and 3.

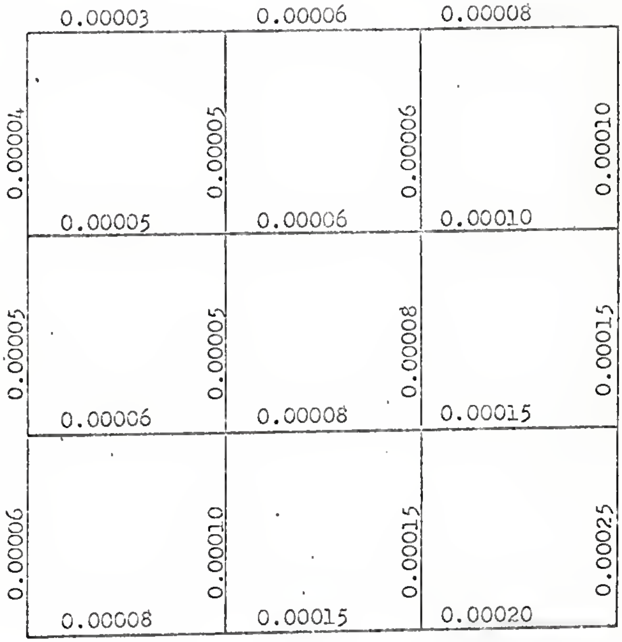


Fig. 5. k_{i2} Values for Numerical Examples 1, 2 and 3.

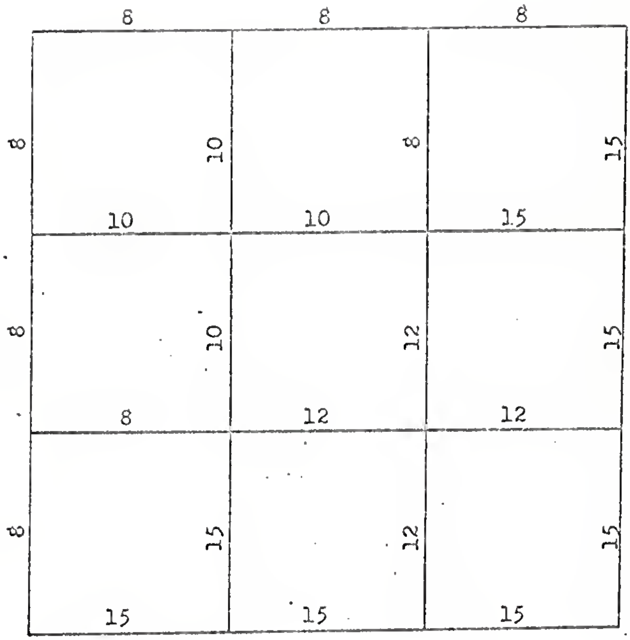
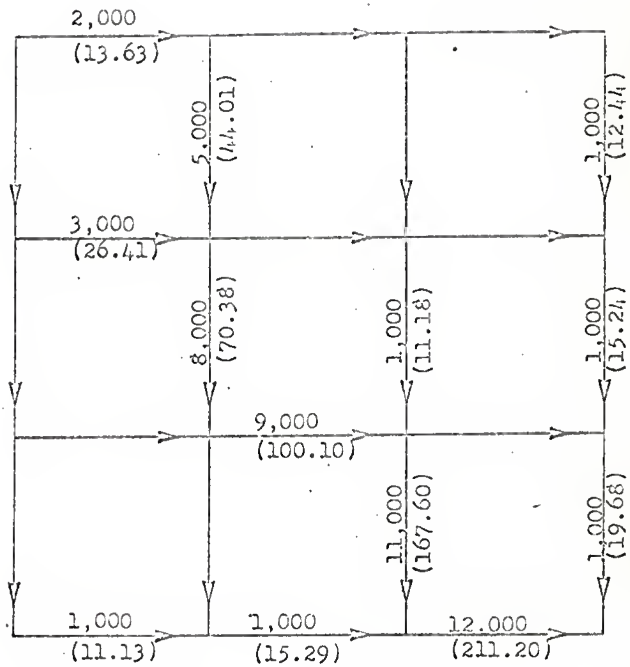


Fig. 6. k_{i3} Values for Numerical Examples 1, 2 and 3.



Total Investment = \$ 718.63
Travel Time Cost = \$2,101.23
Total Cost = \$2,819.86

2,000 : Traffic volume
(13.63) : Investment

Fig. 7. Optimal Investment and Traffic Assignment Results of Example 1

The investment limits, (θ_i)min, i=1,2, and (θ_i)max, i=1,2, associated with conditions 2 and 3 are listed in Table 1. The formulation of this problem has been developed in the previous section under the category, "investment with no budget constraint".

The results are obtained on an IBM 1620 computer and they are presented in Fig. 8. Note that with the minimum level of service provided for the entire area, trips are assigned rather uniformly to take advantage of all facilities. Considering existing facilities as part of the cost, total cost becomes \$2,875.99 (2,603.99 + 272.00). Comparing this cost with the total cost in example 1 (\$2,819.86), the difference is only about two percent. This indicates that providing a minimum level of service might be desirable in an urban area.

Example 3

The hypothetical network as shown in Fig. 6 is to be improved with a total system budget of \$300 (G = 300, equivalent peak hour budget). The resulting traffic assignment and link investments are shown in Fig. 9. Comparing the costs with those obtained in example 2, it is evident that although investment cost decreases more than 30 percent, total cost increases only 1.4 percent. This again points out the advantage of area-wise transportation system development.

The number of iterations and approximate computing time used for each example are summarized in Table 2.

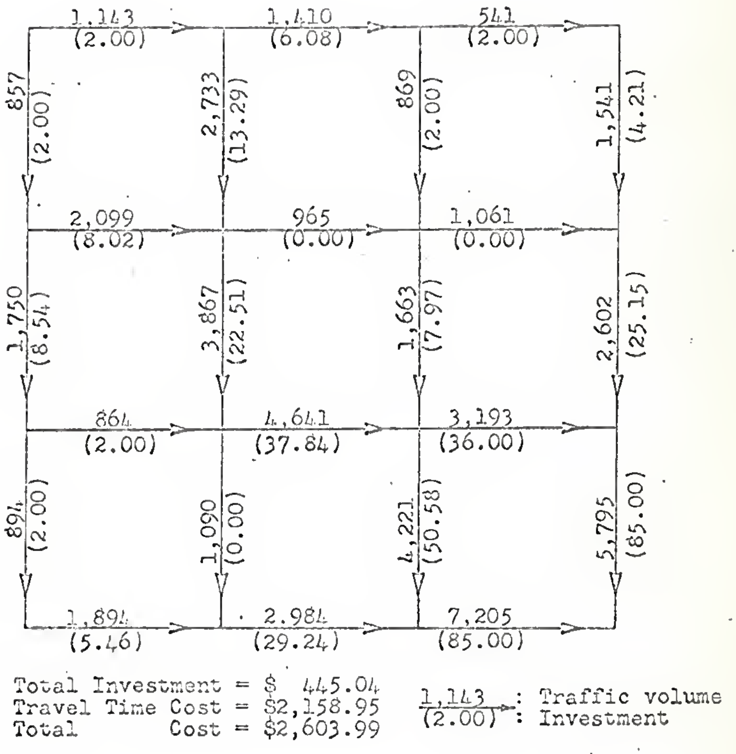


Fig. 8. Optimal Investment and Traffic Assignment Results of Example 2

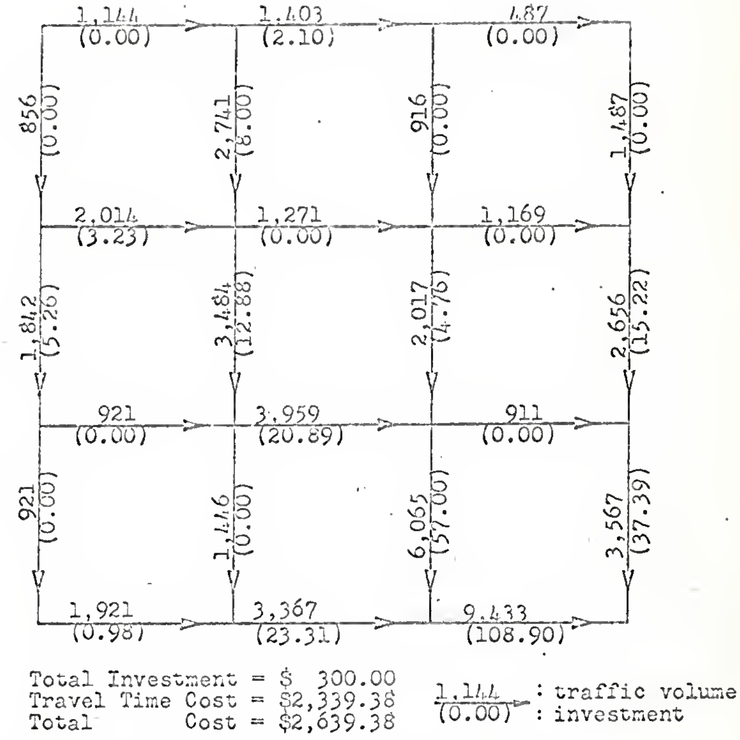


Fig. 9. Optimal Investment and Traffic Assignment Results of Example 3.

Table 2. Number of Iterations, Approximate Computing Time Used and Total Costs for Numerical Examples.

Example No.	Starting Point	Number of Iterations	Time Used (min.)	Total Cost ($)
1	$\theta_3^{n,m} = 0.7$	15	20	2,819.86
2	$\theta_3^{n,m} = 0.7$	18	25	2,603.99
3	$\theta_1^{n,m} = 15$ $\theta_2^{n,m} = 5$ $\theta_3^{n,m} = 0.3$	18	120	2,639.38

DISCUSSION

This section does not consider the taxing policy by including toll to divert the traffic.

If the travellers are told that a particular route will take minimum amount of travel time, they will all rush to that route and thus will cause a congestion on that route. In order to avoid this an optimum amount of toll can be fixed on that route, so that both the congestion situation and the free flow situation can be avoided.

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APPENDIX I COMPUTER PROGRAM FOR TRAFFIC ASSIGNMENT PROBLEMS

The computer flow chart which illustrates the computational procedure is presented in Fig. 1; the FORTRAN program symbols, their explanations and corresponding mathematical notations are summarized in Table 1. The computer program for IBM 360/50 follows the symbol table.

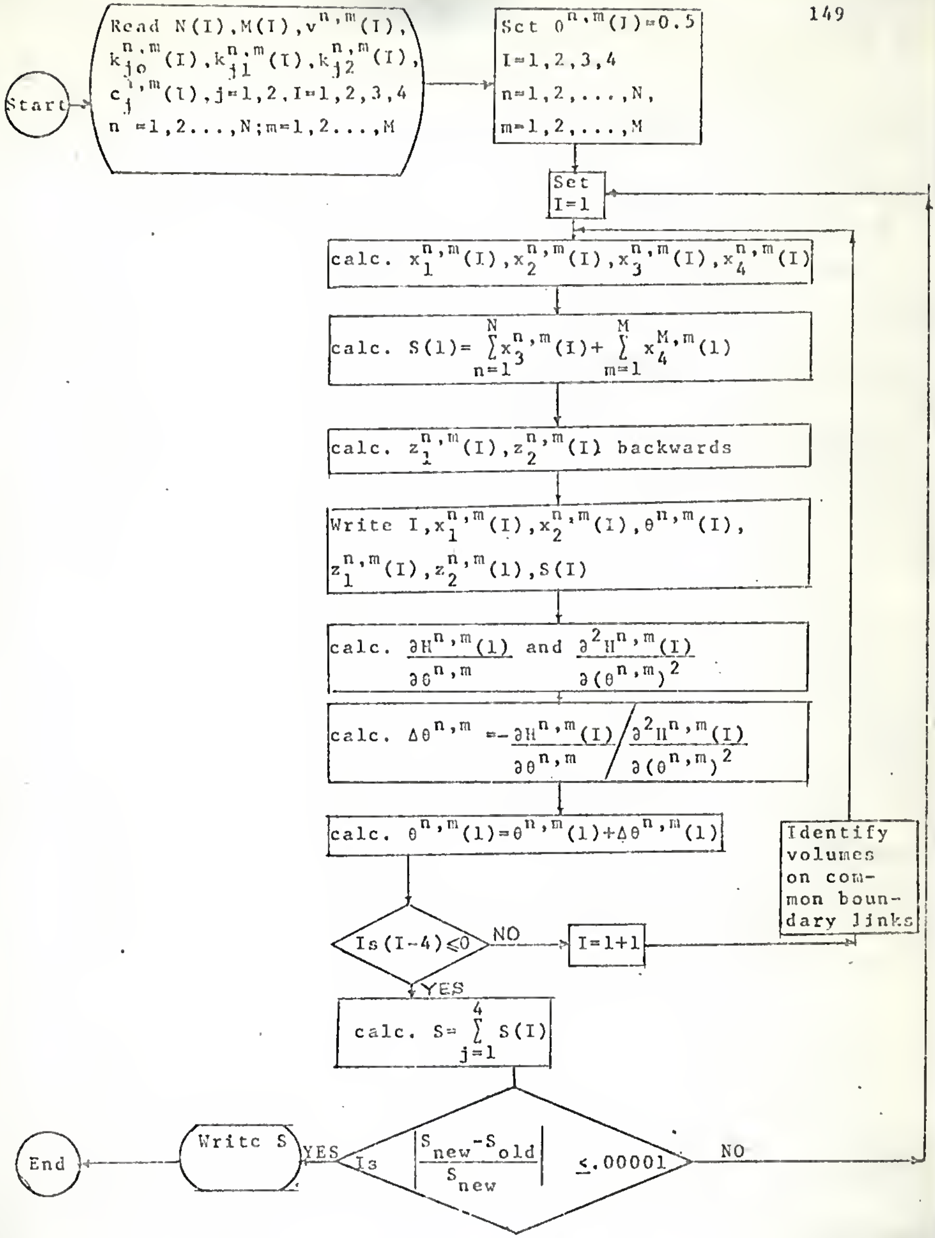

Fig. 1. Computer Flow Diagram.

Table 1. Program Symbols and Explanation

Program Symbols	Explanation	Mathematical Symbols
ACAP(I)	volume Capacity of link I	$c_j^{n,m}$
AHI1	first part of Hamiltonian required for numerical derivative	
AHI2	second part of Hamiltonian required for numerical derivative	
AHV		$vh^{n,m}$
AI(I,J)	total inflow volume at node (I,J)	$x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m}$
AK(I)		$k_{j1}^{n,m}$
AKK	decimal fraction of the calculated change in the decision variable	
AKO(I)		$k_{j0}^{n,m}$
AKI(I)		$k_{j2}^{n,m}$
ANON	conversion factor for $k_{j2}^{n,m}$	
ATEMPT	new value of the decision variable	$\theta^{n,m}+\Delta\theta^{n,m}$
AVV		$vv^{n,m}$
COP1,COP2	the order of copy loading	
D(I,J)		$\theta^{n,m}$
DDH	second derivative of Hamiltonian with respect to the decision variable	$\dfrac{\partial^2 H^{n,m}}{\partial(\theta^{n,m})^2}$
DELTA	the maximum percentage difference in total time between successive iterations which will stop the iterative process	

Table 1. Program Symbols and Explanation

Symbol	Explanation	Notation
DELTB	absolute value of percentage change in total time between successive iterations	
DELTAD		$-\dfrac{\partial H^{n,m}}{\partial \theta^{n,m}} \Big/ \dfrac{\partial^2 H^{n,m}}{\partial (\theta^{n,m})^2}$
DH	first derivative of Hamiltonian with respect to the decision variable	$\dfrac{\partial H^{n,m}}{\partial \theta^{n,m}}$
ITER	iteration number	
KEY(I)	denotes that if KEY(I) = 1, quadrant I is present and if KEY(I) = 0, quadrant I is absent	
KEYSO	denotes that if KEYSO = 0, print copy volumes and if KEYSO = 1, print total volumes	
LIN	denotes that if LIN = 1, time function is linear and if LIN = 0, time function is non-linear	
LMM(I)	M dimension of quadrant I	M
LNN(I)	N dimension of quadrant I	N
NN	a multiple of 10 that causes print out of copy volumes	
TEMPT	the total time for the previous iteration used to determine whether or not to cease the iterative process	
TIM	the total time for the quadrant for all vehicles upto this copy on the links in the appropriate direction	
TIME(I)	time on quadrant I excluding appropriate boundary links	
TIMM(I)	time on quadrant I (excluding appropriate boundary links) from previous copies in the same direction on the same links.	

Table 1. Program Symbols and Explanation (continued)

TIMP	the time for the quadrant for the vehicles from previous copies in the same direction and on the same links	
TLH	TPREVT - TTTP	
TMH		$x_3^{n,m}$
TMV		$x_4^{n,m}$
TOTT	total time including present copy	
TPREVT	total time for previous copies	
TTT	total time on the present iteration	
TTTP	TIMM(1) + TIMM(2) + TIMM(3) + TIMM(4)	
TTTT	the total time for the previous iteration used to decide whether or not the total time is oscillating	
V(I,J)		$v^{n,m}$
VH(I,J)		$x_1^{n,m}$
VHS(I,J)	vehicles horizontal at the boundaries between quadrants	
VV(I,J)		$x_2^{n,m}$
VVS(I,J)	vehicles vertical at the boundaries between quadrants	
ZH(I,J)		$z_1^{n,m}$
ZV(I,J)		$z_2^{n,m}$

```
C   TRAF ASSIGN. --I=K0*V+K*V**2+K1*V**11   -------NUM. DERIV.---MULTI-COPY
C   FRONTAGE ROADS ON BOUNDARY
C   ROAD CLASSIFIED, NO TURN PENALTY
C   PROGRAM FOR 360 COMPUTER
      DIMENSION VHS(9,9),VVS(9,9),KEY(9),ZH(9,9,9),ZV(9,9,9),D(9,9,9),
     1 AK0(9),AK(9),AK1(9),ACAP(9),VH(9,9,9),VV(9,9,9),V(9,9,9),LNN(9),
     2 LMM(9),LCH(9,9,9),LCV(9,9,9),AVV(9,9,9),AHV(9,9,9),AI(9,9,9),
     3 TIME(9),TIMM(9),AHI1(9),AHI2(9)
    1 FORMAT(4I3)
   11 FORMAT(1X,I2,3I3)
   12 FORMAT(5E20.8,2I5)
  937 FORMAT(1H1)
  963 FORMAT(2I8,6E16.6)
  310 FORMAT(A4,A4,A4)
  311 FORMAT(30H                              DATE   ,A4,A4,A4)
  312 FORMAT(I6,5X,F12.4,2E12.4,F12.2)
  313 FORMAT(59H RD.CLASS           K0          K               K1      CAPACIT
     1Y    )
    5 FORMAT(6F12.2)
 1111 FORMAT(46H     V      LCH  LCV      AHV             AVV        )
    2 FORMAT(1X,F7.0,2I5,F12.4,2X,F12.4,2X,2F12.4)
  658 FORMAT(94H      ROW      COL     VERT. VEH.         HOR. VEH.
     1DEC. VAR.             ZV                  ZH ,6X,A4,A4,A4,A4,A4,A4)
      READ(1,310)DATE1,DATE2
  100 READ(1,310)TITL1,TITL2,TITL3
      READ(1,310) COP1,COP2
      READ(1,1)(KEY(I),I=1,4)
      READ(1,5)DELTA,AKK,ANON
      READ(1,11)(LNN(I),I=1,4)
      READ(1,11)(LMM(I),I=1,4)
      READ(1,1)LIN
      READ(1,12)TPREVT
      WRITE(3,937)
      WRITE(3,311)DATE1,DATE2
      WRITE(3,310) COP1,COP2
      WRITE(3,11)(KEY(I),I=1,4)
      WRITE(3,11)(LNN(I),I=1,4)
      WRITE(3,11)(LMM(I),I=1,4)
      WRITE(3,12)DELTA,AKK
      WRITE(3,313)
      DO 6 I=1,5
      READ(1,5)AK0(I),AK(I),ACAP(I)
      AK1(I)=ANON/(ACAP(I)**10)
    6 WRITE(3,312)I,AK0(I),AK(I),AK1(I),ACAP(I)
      WRITE(3,937)
C     ZERO CORE STORAGE
      DDH=0.0
      ITER=0
      KEYS0=0
      NN=5
      TTTT=0.0
      TEMPT =0.0
      DO 675 I=1,4
      TIME(I)=0.0
        TIMM(I)=0.0
      DO 675 J=1,6
      VHS(I,J)=0.0
  675 VVS(I,J)=0.0
```

```
      DO 50 I=1,3
      AHI1(I)=0.0
   50 AHI2(I)=0.0
      WRITE(3,1111)
      DO 778 K=1,4
      IF(KEY(K))19,778,779
  779 LR=LNN(K)
      LC=LMM(K)
      WRITE(3,11)K,LR,LC
      DO 777 J=1,LC
      DO 777 I=1,LR
      READ(1,2)V(K,I,J),LCH(K,I,J),LCV(K,I,J),AHV(K,I,J),AVV(K,I,J)
      D(K,I,J)=.5
      WRITE(3,2)V(K,I,J),LCH(K,I,J),LCV(K,I,J),AHV(K,I,J),AVV(K,I,J)
  777 CONTINUE
  778 CONTINUE
      WRITE(3,937)
   17 ITER=ITER+1
      TTT=0.
      IF(ITER-90)384,383,19
  383 KEYSO=2
  384 DO 14 K=1,4
      LN=LNN(K)
      LM=LMM(K)
      IF(KEY(K))19,14,618
  618 WRITE(3,1)ITER,K
  625 DO 4 I=1,LN
      DO 4 J=1,LM
      JP=J+1
      JM=J-1
      IP=I+1
      IM=I-1
      IF(I-1)19,57,58
   57 IF(J-1)19,59,63
   58 IF(J-1)19,61,626
   61 IF(I-LN)602,603,19
   63 IF(J-LM)604,656,19
  626 IF(J-LM)650,615,19
  604 AI(K,I,J)=VH(K,I,JM)-V(K,I,J)
      GO TO 60
  650 IF(I-LN)600,651,19
  651 XX=VH(K,I,JM)+VV(K,I,JM)
      GO TO  669
  615 IF(I-LN)653,377,19
  377 VV(K,I,J)=0.
      VH(K,I,J)=0.
      AI(K,I,J)=0.
      GO TO 4
   59 AI(K,I,J)=-V(K,I,J)
      GO TO 60
  602 AI(K,I,J)=VV(K,IM,J)-V(K,I,J)
      GO TO 60
  600 AI(K,I,J)=VV(K,IM,J)-V(K,I,J)+VH(K,I,JM)
      GO TO 60
  603 XX=0.0
  669 AI(K,I,J)=VV(K,IM,J)-V(K,I,J)+VVS(K,J)+XX
      GO TO 60
  653 XX=VV(K,IM,J)+VH(K,IM,J)
```

```
      GO TO 662
  656 XX=0.6
  662 AI(K,I,J)=VH(K,I,JM)+VHS(K,I)+XX-V(K,I,J)
      GO TO 60
   60 VH(K,I,J)=D(K,I,J)*AI(K,I,J)
      VV(K,I,J)=(1.-D(K,I,J))*AI(K,I,J)
    4 CONTINUE
  687 GO TO (677,678,679,680),K
  677 L=4
      GO TO 681
  678 L=3
      GO TO 681
  679 L=2
      GO TO 681
  680 L=1
      GO TO 681
  681 DO 689 J=1,LM
      VVS(L,J)=VV(K,LN-1,J)-V(K,LN,J)
  689 CONTINUE
      GO TO (682,683,684,685),K
  682 L=2
      GO TO 686
  683 L=1
      GO TO 686
  684 L=4
      GO TO 686
  685 L=3
  686 DO 787 I=1,LN
      VHS(L,I)=VH(K,I,LM-1)-V(K,I,LM)
  787 CONTINUE
C     PUNCH VOLUMES , DECISION VECTORS,TOTAL TIME AND ITER
      TIMP=0.0
      TIM=0.0
      GO TO (186,187,188,189),K
  186 K1=LN-1
      K2=LM-1
      GO TO 190
  187 K1=LN-1
      K2=LM
      GO TO 190
  188 K1=LN
      K2=LM-1
      GO TO 190
  189 K1=LN
      K2=LM
  190 DO 182 I=1,K1
      DO 182 J=1,K2
      KH=LCH(K,I,J)
      KV=LCV(K,I,J)
      IF(ITER-1)19,49,48
   49 TIMP=TIMP+AK(KV)*(AVV(K,I,J)**2)+AK(KH)*(AHV(K,I,J)**2)+AKO(KH)*
     1 AHV(K,I,J)+AKO(KV)*AVV(K,I,J)+AK1(KH)*(AHV(K,I,J)**I1)+AK1(KV)*
     2 (AVV(K,I,J)**I1)
   48 IF(VH(K,I,J)+AHV(K,I,J)-.001)141,142,142
  141 TMH=0.
      GO TO 143
  142 TMH=AK(KH)*((VH(K,I,J)+AHV(K,I,J))**2)+AKO(KH)*(VH(K,I,J)+AHV
     1 (K,I,J))+AK1(KH)*(VH(K,I,J)+AHV(K,I,J))**I1
```

```
143 IF(VV(K,1,J)+AVV(K,1,J)-.001)144,145,145
144 TMV=0.
    GO TO 146
145 TMV=AK(KV)*((VV(K,1,J)+AVV(K,1,J))**2)+AKO(KV)*(VV(K,1,J)+
   1 AVV(K,1,J))+AK1(KV)*(VV(K,1,J)+AVV(K,1,J))**11
146 TIM=TIM+TMH+TMV
162 CONTINUE
181 TIME(K)=TIM
    WRITE(3,12)TIME(K)
    IF(ITER-1)19,47,553
 47 TIMM(K)=TIMP
553 GO TO (923,923,923,184),K
184 DO 185 J=1,4
185 TTT=TTT+TIME(J)
    WRITE(3,12)TTT,AKK
    IF(TTT-TTTT)379,923,380
379 TTTT=TTT
    GO TO 923
380 AKK=0.75*AKK
    IF(AKK-.05)1601,1602,1602
1601 AKK=0.05
    GO TO 1602
1602 TTTT=TTT
923 GO TO (720,720,720,999),K
999 IF(ITER-1)19,720,721
721 DELTE=ABS(ABS(TTT-TEMPT)/TTT)
    TEMPT=TTT
720 DO 370 I=1,LN
    DO 370 J=1,LM
    N=LN+1-I
    M=LM+1-J
    NM=N-1
    MM=M-1
    NP=N+1
    MP=M+1
    IF(N-LN)25,20,19
 20 IF(M-LM)25,21,19
 21 D(K,N,M)=1.
    ZH(K,N,M)=0.
    ZV(K,N,M)=0.
    GO TO 370
 25 IF(J-1)19,700,709
709 KH=LCH(K,N,MP)
    KV=LCV(K,N,MP)
    IF(VH(K,N,MP)+AHV(K,N,MP)-.001)707,702,702
707 TVH=0.
    GO TO 703
702 TVH=(VH(K,N,MP)+AHV(K,N,MP))**10
703 IF(VV(K,N,MP)+AVV(K,N,MP)-.001)704,705,705
704 TVV=0.
    GO TO 706
705 TVV=(VV(K,N,MP)+AVV(K,N,MP))**10
706 ZH(K,N,M)=ZH(K,N,MP)*D(K,N,MP)+ZV(K,N,MP)*(1.-D(K,N,MP))+2.*AK(KH)
   1 *((D(K,N,MP)**2)*AI(K,N,MP)+AHV(K,N,MP)*D(K,N,MP))+2.*AK(KV)*(((
   2 1.-D(K,N,MP))**2)*AI(K,N,MP)+AVV(K,N,MP)*(1.-D(K,N,MP)))+AKO(KH)*
   3 D(K,N,MP)+AKO(KV)*(1.-D(K,N,MP))+11.*AK1(KH)*D(K,N,MP)*TVH
   4 +11.*AK1(KV)*(1.-D(K,N,MP))*TVV
    IF(I-1)19,804,700
```

```
  804 ZV(K,N,M)=ZH(K,N,M)
      GO TO 370
  700 KV=LCV(K,NP,M)
      KH=LCH(K,NP,M)
      IF(VH(K,NP,M)+AHV(K,NP,M)-.001)711,712,712
  711 TVH=0.
      GO TO 713
  712 TVH=(VH(K,NP,M)+AHV(K,NP,M))**10
  713 IF(VV(K,NP,M)+AVV(K,NP,M)-.001)714,715,715
  714 TVV=0.
      GO TO 716
  715 TVV=(VV(K,NP,M)+AVV(K,NP,M))**10
  716 ZV(K,N,M)=ZH(K,NP,M)*D(K,NP,M)+ZV(K,NP,M)*(1.-D(K,NP,M))+2.*AK(KH)
     1 *((D(K,NP,M)**2)*A1(K,NP,M)+AHV(K,NP,M)*D(K,NP,M))+2.*AK(KV)*(((
     2 1.-D(K,NP,M))**2)*A1(K,NP,M)+AVV(K,NP,M)*(1.-D(K,NP,M)))+AKO(KH)*
     3 D(K,NP,M)+AKO(KV)*(1.-D(K,NP,M))+11.*AK1(KH)*D(K,NP,M)*TVH
     4 +11.*AK1(KV)*(1.-D(K,NP,M))*TVV
      IF(J-1)19,805,370
  805 ZH(K,N,M)=ZV(K,N,M)
  370 CONTINUE
      IF(ITER-NN*10)1001,1003,19
 1003 IF(K-3)1002,1002,1004
 1004 NN=NN+1
      GO TO 1002
 1001 IF(KEYSO-1)87,1002,1002
 1002 WRITE(3,658)TITL1,TITL2,TITL3,COP1,COP2
      DO 393 I=1,LN
      DO 393 J=1,LM
      IF(KEYSO-1)445,445,444
  445 X11=VV(K,I,J)
      X12=VH(K,I,J)
      GO TO 446
  444 X11=VV(K,I,J)+AVV(K,I,J)
      X12=VH(K,I,J)+AHV(K,I,J)
  446 WRITE(3,963)I,J,X11,X12,D(K,I,J),ZV(K,I,J),ZH(K,I,J)
  393 CONTINUE
   87 IF(ITER-1)19,18,119
  119 GO TO (18,18,18,120),K
  120 IF(DELTB-DELTA)90,15,15
   90 IF(KEYSO-1)89,89,84
   89 KEYSO=1+KEYSO
      GO TO 18
   84 TTTP=TIMM(1)+TIMM(2)+TIMM(3)+TIMM(4)
      TLH=TPREVT-TTTP
      TOTT=TLH+TTT
      WRITE(3,12)TLH
      WRITE(3,12)TOTT
      GO TO 19
   15 KEYSO=0
   18 DO 13 I=1,LN
      DO 13 J=1,LM
      N=LN+1-I
      M=LM+1-J
      KH=LCH(K,N,M)
      KV=LCV(K,N,M)
      IF(I-1)19,371,372
  371 IF(J-1)19,13,372
  372 DO 376 L=1,3
```

```
      BK=L
      IF(D(K,N,M)-.01)835,499,836
  835 D(K,N,M)=.01
      GO TO 499
  836 IF(D(K,N,M)-.99)499,499,838
  838 D(K,N,M)=.99
  499 DI=D(K,N,M)-.02+BK/100.
  931 IF(AI(K,N,M)-1.)932,932,930
  932 AI(K,N,M)=1.
  930 AHI1(L)=ZH(K,N,M)*DI*AI(K,N,M)+ZV(K,N,M)*(1.-DI)*AI(K,N,M)+AK(KH)*
     1 ((DI*AI(K,N,M))**2+2.*DI*AI(K,N,M)*AHV(K,N,M))+AK(KV)*(((1.-DI)*
     2 AI(K,N,M))**2+2.*(1.-DI)*AI(K,N,M)*AVV(K,N,M))+AKO(KH)*DI*
     3 AI(K,N,M)+AKO(KV)*(1.-DI)*AI(K,N,M)
      IF(DI*AI(K,N,M)+AHV(K,N,M)-.001)933,934,934
  933 TAHH=0.
      GO TO 935
  934 TAHH=AK1(KH)*(DI*AI(K,N,M)+AHV(K,N,M))**II
  935 IF((1.-DI)*AI(K,N,M)+AVV(K,N,M)-.001)936,938,938
  936 TAHV=0.
      GO TO 939
  938 TAHV=AK1(KV)*((1.-DI)*AI(K,N,M)+AVV(K,N,M))**II
  939 AHI2(L)=TAHH+TAHV
  376 CONTINUE
  701 DH=((AHI1(3)-AHI1(1))+(AHI2(3)-AHI2(1)))/.02
      IF(LIN-1)304,305,19
  305 IF(DH)302,303,303
  302 ATEMP=1.0
      GO TO 204
  303 ATEMP=0.0
      GO TO 204
  304 DDH=((AHI1(1)-2.*AHI1(2)+AHI1(3))+(AHI2(1)-2.*AHI2(2)+AHI2(3)))
     1 /.0001
  357 DELTAD=-DH/(DDH+0.000000001)
  334 IF(J-1)19,820,821
  821 IF(I-1)19,820,822
  820 ATEMP=D(K,N,M)+DELTAD
      GO TO 309
  822 IF(ABS(DELTAD)-.1)823,823,825
  825 DELTAD=(DELTAD/ABS(DELTAD))*.1
  823 ATEMP=D(K,N,M)+AKK*DELTAD
  309 IF(ATEMP-.9999999)201,202,202
  201 IF(ATEMP-.0000001)203,203,204
  202 D(K,N,M)=.9999999
      GO TO 710
  203 D(K,N,M)=.0000001
      GO TO 710
  204 IF(ATEMP-.01)308,203,307
  307 IF(ATEMP-.99)308,202,308
  308 D(K,N,M)=ATEMP
  710 GO TO 13
   13 CONTINUE
   14 CONTINUE
      GO TO 17
   19 GO TO 100
      END
```

APPENDIX II. COMPUTER PROGRAMS FOR COST MINIMIZATION PROBLEMS

The computer flow chart which illustrates the computational procedure to obtain optimal sequence of the decision variables $\theta_1^{n,m}$, $\theta_2^{n,m}$ and $\theta_3^{n,m}$ for section 5-1, is presented in Fig. 2. Fig. 3 presents the computer flow chart which illustrates the computational procedure to obtain optimal sequence of the decision variables $\theta_1^{n,m}$, $\theta_2^{n,m}$ and $\theta_3^{n,m}$ for section 5-2. The FORTRAN symbols, their explanations and corresponding mathematical notations are summarized in Table 2. The computer programs for IBM 1620 follow the symbol table.

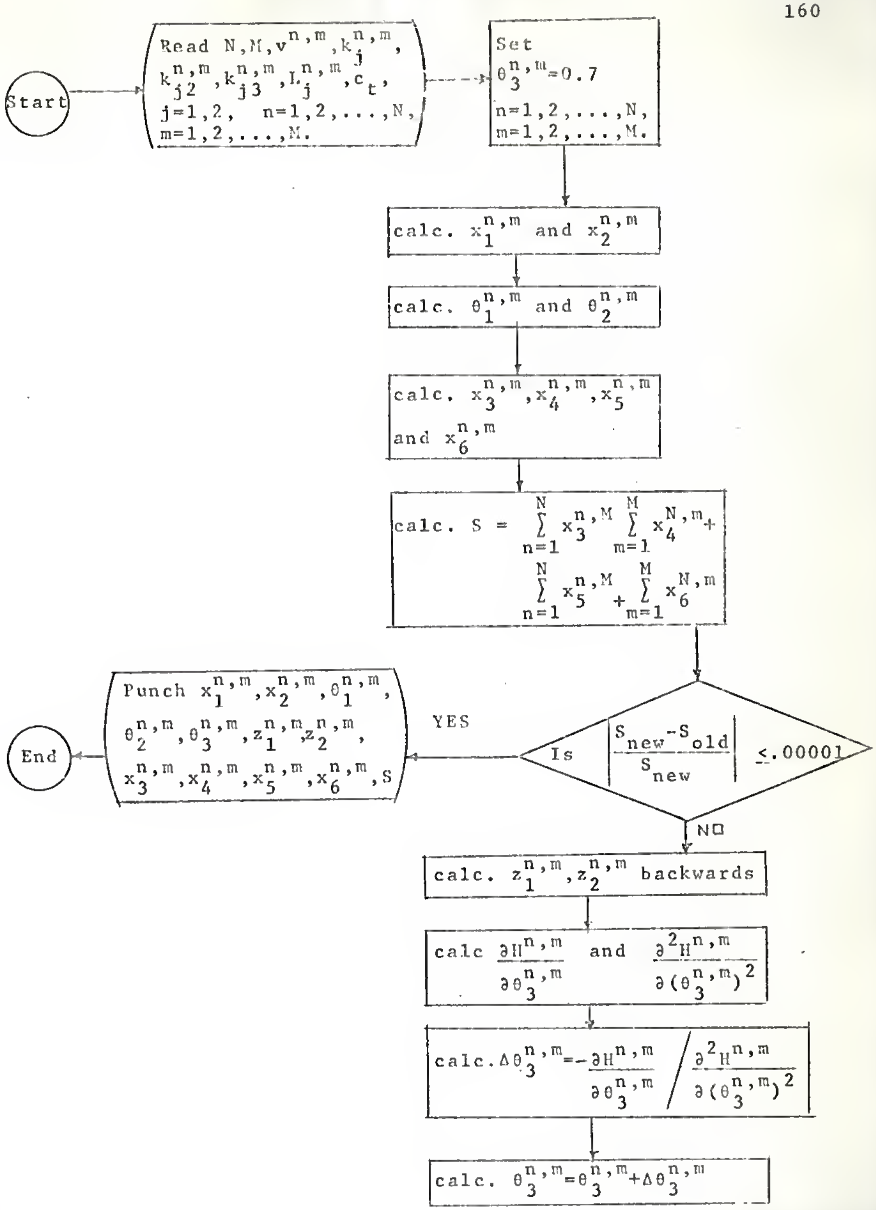

Fig. 2. Computer Flow Chart for Numerical Examples of Section 5-1.

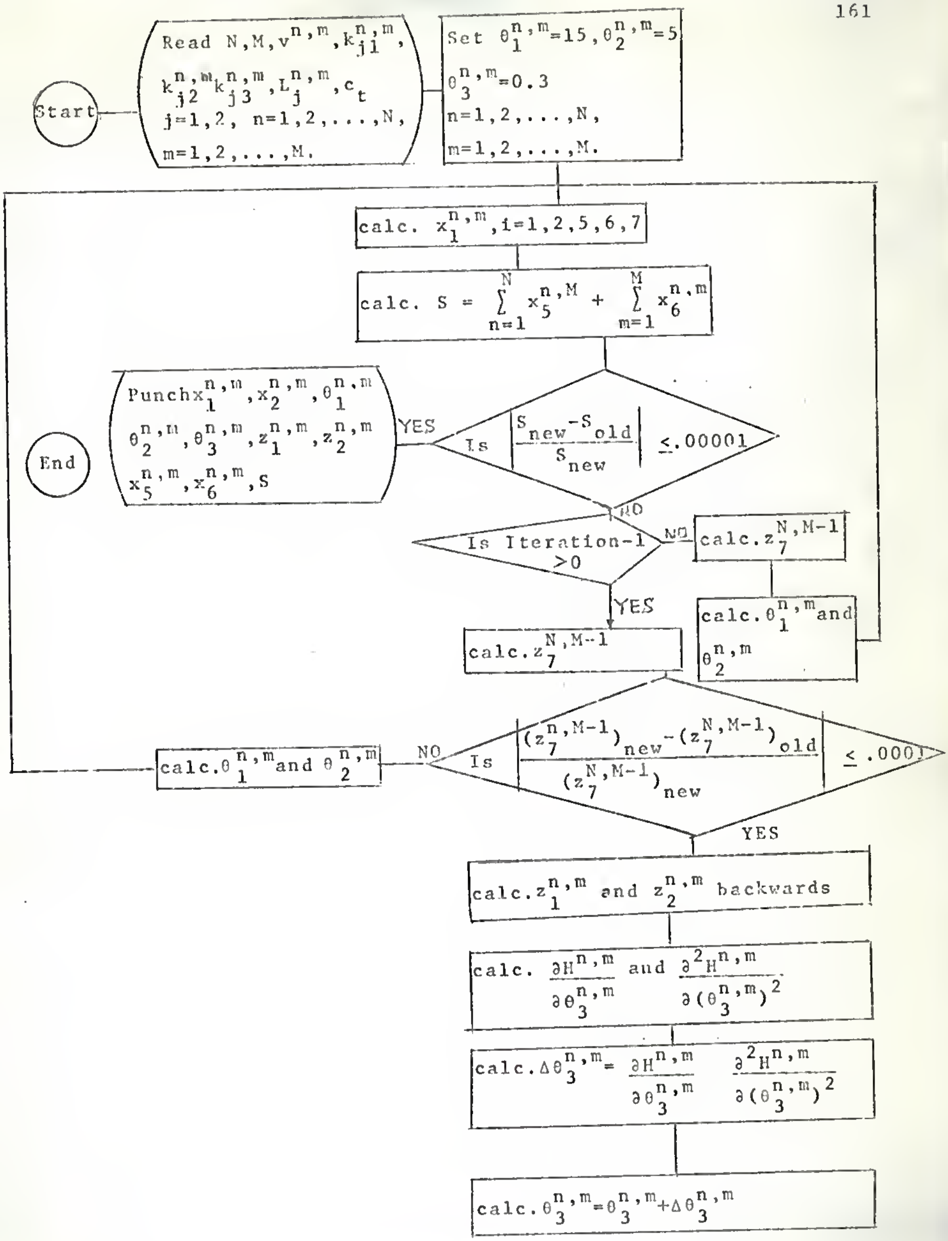

Fig. 3. Computer Flow Chart for Numerical Examples of Section 5-2.

Table 2. Program Symbols and Explanation

Program Symbols	Explanation	Mathematical Symbols
AD		$\theta_1^{n,m}+\theta_2^{n,m}$
AI(I,J)	total inflow volume at node (I,J)	$x_1^{n,m-1}+x_2^{n-1,m}+v^{n,m}$
AL1		L_1
AL2		L_2
CH1		$k_{11}^{n,m}$
CH2		$k_{12}^{n,m}$
CH3		$k_{13}^{n,m}$
COST	objective function	S
COSTH		$x_3^{n,m}$
COSTI	sum of travel time cost	$x_3^{n,m}+x_4^{n,m}$
COSTP	maximum value of total cost	
COSTT	sum of investment cost	$x_5^{n,m}+x_6^{n,m}$
COSTV		$x_4^{n,m}$
CV1		$k_{21}^{n,m}$
CV2		$k_{22}^{n,m}$
CV3		$k_{23}^{n,m}$
D1		$\theta_1^{n,m}$
D2		$\theta_2^{n,m}$
D3		$\theta_3^{n,m}$
DASM1	initial value of $\theta_1^{n,m}$	
DASM2	initial value of $\theta_2^{n,m}$	
DASM3	initial value of $\theta_3^{n,m}$	
DDH		$\dfrac{\partial^2 H^{n,m}}{\partial(\theta_3^{n,m})^2}$

Table 2. Program Symbols and Explanation (continued)

DELTB		$\left\lvert\frac{S-S_{max}}{S}\right\rvert$
DH		$\frac{\partial H^{n,m}}{\partial \theta_3^{n,m}}$
DZ		$\left\lvert\frac{(z_7^{N,M-1})_{revised}-(z_7^{N,M-1})_{old}}{(z_7^{N,M-1})_{revised}}\right\rvert$
GI		G
HV		$x_1^{n,m}$
IC1	maximum number of iterations	
ITER	iteration number	
KEYSO	denotes that if KEYSO = 0, print copy volumes and if KEYSO = 1, print total volumes	
M	N dimension of the network	
N	N dimension of the network	
SH1		$\theta_1^{n,m}$ min
SH2		$\theta_1^{n,m}$ max
SV1		$\theta_2^{n,m}$ min
SV2		$\theta_2^{n,m}$ max
T	time cost	c_t
V		$v^{n,m}$
VV		$x_2^{n,m}$
X5		$x_7^{n,m}$
Z5P		$z_7^{N,M-1}$
ZH		$z_1^{n,m}$
ZV		$z_2^{n,m}$

```
C     OPTIMAL TRAFFIC SYSTEM INVESTMENT WITHOUT BUDGET CONSTRAINT
      DIMENSION HV(4,4),VV(4,4),D1(4,4),D2(4,4),D3(4,4),
     1 AHH(3),  AI(4,4),V(4,4),ZH(4,4),ZV(4,4),AL1(4,4),AL2(4,4),
     2CH1(4,4),CH2(4,4),CH3(4,4),CV1(4,4),CV2(4,4),CV3(4,4)
     3 ,SH1(4,4),SH2(4,4),SV1(4,4),SV2(4,4)
    1 FORMAT(4I4)
    2 FORMAT(7F10.4)
    3 FORMAT(15H ENTER NEW DATA)
    4 FORMAT(2I4,7E10.4)
    5 FORMAT(14H ERROR RUN OUT)
    6 FORMAT(15H END OF PROGRAM)
    7 FORMAT(I3,4F15.4)
    8 FORMAT(3F20.6)
    9 FORMAT(74H  ROW COL     HV            VV           HINV         VINV        D.VAR
     1        ZH          ZV)
   10 FORMAT(3X,2I3,3X,4E15.6)
   11 FORMAT(7E11.4)
   12 FORMAT(6F12.5)
  100 READ 1, N, M, IC1
      READ 2, DELTA, AKK, T, AK,AFK
      READ 2, DASM1, DASM2, DASM3
      PUNCH 4, N,M,DELTA,AK
      COSTP=9999999999.99
      KEYSC=0
      DO 102 I=1,N
      DO 102 J=1,M
      READ 2, V(I,J), AL1(I,J),AL2(I,J)
      PUNCH 12, V(I,J),AL1(I,J),AL2(I,J)
      IF(I-N) 201,202,1112
  201 IF(J-M) 301,302,1112
  301 D3(I,J)=DASM3
      GO TO 102
  302 D3(I,J)=0.0
      GO TO 102
  202 D3(I,J)=1.0
  102 CONTINUE
      DO 103 I=1,N
      DO 103 J=1,M
      READ 2, CH1(I,J),CH2(I,J),CH3(I,J),SH1(I,J),SH2(I,J)
      READ 2, CV1(I,J),CV2(I,J),CV3(I,J),SV1(I,J),SV2(I,J)
      PUNCH 11, CH1(I,J),CH2(I,J),CH3(I,J),SH1(I,J),SH2(I,J)
  103 PUNCH 11, CV1(I,J),CV2(I,J),CV3(I,J),SV1(I,J),SV2(I,J)
      ITER=0
 1000 IF(ITER-IC1) 1001,1111,1112
 1001 ITER=ITER+1
      IF(ITER-IC1)321,322,1112
  322 KEYSC=1
  321 DO 211 I=1,N
      DO 211 J=1,M
      IF(I-1)1112,311,312
  311 IF(J-1) 1112,411,412
```

```
 411 AI(I,J)=V(I,J)
 430 HV(I,J)=AI(I,J)*D3(I,J)
     VV(I,J)=AI(I,J)*(1.-D3(I,J))
     GO TO 211
 412 AI(I,J)=HV(I,J-1)+V(I,J)
     GO TO 430
 312 IF(J-1) 1112,511,512
 511 AI(I,J)=VV(I-1,J)+V(I,J)
     GO TO 430
 512 AI(I,J)=HV(I,J-1)+VV(I-1,J)+V(I,J)
     GO TO 430
 211 CONTINUE
     DO 40 I=1,N
     DO 40 J=1,M
     IF(HV(I,J))1112,135,200
 135 D1(I,J)=0.
     GO TO 115
 200 D1(I,J)=HV(I,J)*(CH2(I,J)*T)**0.5-CH3(I,J)
     IF(D1(I,J)-SH1(I,J))106,106,107
 106 D1(I,J)=SH1(I,J)
     GO TO 115
 107 IF(D1(I,J)-SH2(I,J))115,108,108
 108 D1(I,J)=SH2(I,J)
 115 IF(VV(I,J))1112,505,175
 175 D2(I,J)=VV(I,J)*(CV2(I,J)*T)**0.5-CV3(I,J)
     IF(D2(I,J)-SV1(I,J))206,206,207
 206 D2(I,J)=SV1(I,J)
     GO TO 40
 207 IF(D2(I,J)-SV2(I,J))40,208,208
 208 D2(I,J)=SV2(I,J)
     GO TO 40
 505 D2(I,J)=0.
  40 CONTINUE
     COSTH=0.0
     COSTV=0.0
     COSTT=0.
     DO 221 I=1,N
     DO 221 J=1,M
     COSTH=COSTH+D1(I,J)*AL1(I,J)
     COSTV=COSTV+D2(I,J)*AL2(I,J)
 221 COSTT=COSTT+(CH1(I,J)*HV(I,J)+CH2(I,J)*HV(I,J)**2/(D1(I,J)+
    1 CH3(I,J)))*T*AL1(I,J)+(CV1(I,J)*VV(I,J)+CV2(I,J)*VV(I,J)**2/
    2 (D2(I,J)+CV3(I,J)))*T*AL2(I,J)
     COSTI=COSTH+COSTV
     COST=COSTI+COSTT
     PUNCH 7, ITER,T,AKK
     PUNCH 8, COST,COSTI,COSTT
     IF(SENSE SWITCH 2)125,243
 125 TYPE 8,COST
 243 DELTB=ABSF(ABSF(COST-COSTP)/COST)
     IF(DELTB-DELTA) 1101,1101,1102
1101 IF(KEYSO-1)1103,1111,1112
```

```
1103 KEYSO=1
     GO TO 1104
1102 KEYSO=0
1104 DO 231 I=1,N
     DO 231 J=1,M
     LN=N+1-I
     LM=M+1-J
     NP=LN+1
     MP=LM+1
     IF(LN-N)611,612,1112
 612 IF(LM-M)611,613,1112
 613 D3(LN,LM)=1.0
     ZH(LN,LM)=0.0
     ZV(LN,LM)=0.0
     GO TO 231
 611 IF(J-1)1111,711,712
 712 ZH(LN,LM)=ZH(LN,MP)*D3(LN,MP)+ZV(LN,MP)*(1.-D3(LN,MP))+
    1 (CH1(LN,MP)*D3(LN,MP)+2.*CH2(LN,MP)*HV(LN,MP)*D3(LN,MP)/
    2 (D1(LN,MP)+CH3(LN,MP)))*T*AL1(LN,MP)+(CV1(LN,MP)*(1.-D3(LN,MP))+
    3 2.*CV2(LN,MP)*VV(LN,MP)*(1.-D3(LN,MP))/(D2(LN,MP)+CV3(LN,MP)))*
    4 T*AL2(LN,MP)
     IF(I-1)1111,811,711
 811 ZV(LN,LM)=ZH(LN,LM)
     GO TO 231
 711 ZV(LN,LM)=ZH(NP,LM)*D3(NP,LM)+ZV(NP,LM)*(1.-D3(NP,LM))+
    1 (CH1(NP,LM)*D3(NP,LM)+2.*CH2(NP,LM)*HV(NP,LM)*D3(NP,LM)/
    2 (D1(NP,LM)+CH3(NP,LM)))*T*AL1(NP,LM)+(CV1(NP,LM)*(1.-D3(NP,LM))+
    3 2.*CV2(NP,LM)*VV(NP,LM)*(1.-D3(NP,LM))/(D2(NP,LM)+CV3(NP,LM)))*
    4 T*AL2(NP,LM)
     IF(J-1)1111,815,231
 815 ZH(LN,LM)=ZV(LN,LM)
 231 CONTINUE
     IF(SENSE SWITCH 3)1502,1503
1503 IF(KEYSO-1)1504,1502,1112
1502 PUNCH 9
     DO 300 I=1,N
     DO 300 J=1,M
     PUNCH 4, I,J,HV(I,J),VV(I,J),D1(I,J),D2(I,J),D3(I,J),
    1 ZH(I,J),ZV(I,J)
 300 CONTINUE
1504 IF(SENSE SWITCH 1)1501,1507
1501 READ 2, AKK
1507 DO 241 I=1,N
     DO 241 J=1,M
     IF(I-N)119,116,1112
 119 IF(J-M)117,118,1112
 116 D3(I,J)=1.0
     GO TO 241
 118 D3(I,J)=0.
     GO TO 241
 117 IF(AI(I,J))1112,121,122
 121 AI(I,J)=1.
```

```
122 DH=(ZH(I,J)-ZV(I,J))*AI(I,J)+(CH1(I,J)*AI(I,J)+2.*CH2(I,J)*D3(I,J)
   1 *AI(I,J)**2/(D1(I,J)+CH3(I,J)))*T*AL1(I,J)-(CV1(I,J)*AI(I,J)+2.
   2 *CV2(I,J)*(1.-D3(I,J))*AI(I,J)**2/(D2(I,J)+CV3(I,J)))*T*AL2(I,J)
    DDH=(2.*CH2(I,J)*AI(I,J)**2/(D1(I,J)+CH3(I,J)))*T*AL1(I,J)+
   1 (2.*CV2(I,J)*AI(I,J)**2/(D2(I,J)+CV3(I,J)))*T*AL2(I,J)
    IF(ABSF(DH)-AEK)217,217,251
251 D3(I,J)=D3(I,J)-AKK*DH/DDH
    IF(D3(I,J)-1.)257,262,262
262 D3(I,J)=1.
    GO TO 217
257 IF(D3(I,J))263,263,217
263 D3(I,J)=0.
217 GO TO 241
241 CONTINUE
    COSTP=COST
    GO TO 1000
1111 PUNCH 6
    TYPE 3
    PAUSE
    GO TO 100
1112 TYPE 5
    STOP
    END
```

```
C     OPTIMAL TRAFFIC SYSTEM INVESTMENT WITH FIXED SYSTEM BUDGET
      DIMENSION HV(4,4),VV(4,4),D1(4,4),D2(4,4),D3(4,4),X5(4,4),
     1          AI(4,4),V(4,4),ZH(4,4),ZV(4,4),AL1(4,4),AL2(4,4)
     2 ,CH1(4,4),CH2(4,4),CH3(4,4),CV1(4,4),CV2(4,4),CV3(4,4)
    1 FORMAT(4I4)
    2 FORMAT(7F10.4)
    3 FORMAT(15H ENTER NEW DATA)
    4 FORMAT(2I4,7E10.4)
    5 FORMAT(14H ERROR RUN OUT)
    6 FORMAT(15H END OF PROGRAM)
    7 FORMAT(I3,4F15.4)
    8 FORMAT(3E20.6)
    9 FORMAT(74H  ROW COL     HV          VV          HINV        VINV       D.VAR
     1       ZH          ZV)
   11 FORMAT(7E11.4)
  100 READ 1, N, M, IC,NC1
      READ 2, DELTA, T, GI, AKK1, AK2 ,DDZ
      READ 2, DASM1, DASM2, DASM3
      READ 2, S1,SS2,S3,S4
      PUNCH 4, N,M,DELTA,GI,S1,SS2,S4,DASM1,DASM3
      COSTP=0.
      KEYSO=0
      DO 30 I=1,N
      DO 30 J=1,M
      READ 2,  V(I,J),AL1(I,J),AL2(I,J)
      PUNCH 2, V(I,J),AL1(I,J),AL2(I,J)
      D1(I,J)=DASM1
      D2(I,J)=DASM2
   33 D3(I,J)=DASM3
   30 CONTINUE
      DO 40 I=1,N
      DO 40 J=1,M
      READ 2,  CH1(I,J),CH2(I,J),CH3(I,J),CV1(I,J),CV2(I,J),CV3(I,J)
   40 PUNCH 11,CH1(I,J),CH2(I,J),CH3(I,J),CV1(I,J),CV2(I,J),CV3(I,J)
      ITER=0
      NC=NC1
 1000 IF(ITER-IC)41,1111,1112
   41 ITER=ITER+1
      Z5P=0.
      IF(ITER-IC)42,43,1112
   43 KEYSO=1
   42 DO 50 I=1,N
      DO 50 J=1,M
      IF(I-N)21,22,1112
   22 IF(J-M)23,24,1112
   24 D3(I,J)=1.
      D1(I,J)=0.
      D2(I,J)=0.
      GO TO 59
   23 D3(I,J)=1.
      D2(I,J)=0.
```

```
      GO TO 59
   21 IF(J-M)59,26,1112
   26 D3(I,J)=0.
      D1(I,J)=0.
   59 IF(I-1)1112,44,48
   44 IF(J-1)1112,45,47
   45 AI(I,J)=V(I,J)
   46 HV(I,J)=AI(I,J)*D3(I,J)
      VV(I,J)=AI(I,J)*(1.-D3(I,J))
      GO TO 50
   47 AI(I,J)=HV(I,J-1)+V(I,J)
      GO TO 46
   48 IF(J-1)1112,51,52
   51 AI(I,J)=VV(I-1,J)+V(I,J)
      GO TO 46
   52 AI(I,J)=HV(I,J-1)+VV(I-1,J)+V(I,J)
      GO TO 46
   50 CONTINUE
      AKK2=AK2
      S2=SS2
  103 DO 60 I=1,N
      DO 60 J=1,M
      IF(I-N)201,202,1112
  202 IF(J-M+1)201,203,204
  203 D1(I,J)=GI-X5(I,J-1)
      GO TO 62
  204 X5(I,J)=X5(I,J-1)
      GO TO 60
  201 IF(I-1)1112,61,62
   61 IF(J-1)1112,63,64
   63 X5(I,J)=D1(I,J)+D2(I,J)
      GO TO 66
   62 IF(J-1)1112,65,64
   65 X5(I,J)=X5(I-1,M)+D1(I,J)+D2(I,J)
      GO TO 66
   64 X5(I,J)=X5(I,J-1)+D1(I,J)+D2(I,J)
   66 IF(X5(I,J)-GI)60,60,79
   79 AD=D1(I,J)+D2(I,J)
      D1(I,J)=D1(I,J)-(X5(I,J)-GI)*D1(I,J)/AD
      D2(I,J)=D2(I,J)-(X5(I,J)-GI)*D2(I,J)/AD
      X5(I,J)=GI
   60 CONTINUE
      COSTH=0.
      COSTV=0.
      COSTT=0.
      DO 70 I=1,N
      DO 70 J=1,M
      COSTH=COSTH+D1(I,J)
      COSTV=COSTV+D2(I,J)
   70 COSTT=COSTT+(CH2(I,J)*HV(I,J)**2/(D1(I,J)/AL1(I,J)+CH3(I,J)))*
     1 AL1(I,J)*T+(CV2(I,J)*VV(I,J)**2/(D2(I,J)/AL2(I,J)+CV3(I,J)))*
```

```
   2AL2(I,J)*T+(CH1(I,J)*HV(I,J)*AL1(I,J)+CV1(I,J)*VV(I,J)*AL2(I,J))*T
    COSTI=COSTH+COSTV
    COST=COSTT+COSTI
    PUNCH 7, ITER,I,AKK1, AKK2
    PUNCH 8, COST,COSTI,COSTT
    IF(SENSE SWITCH 2) 67,68
 67 TYPE 8, COST
 68 DELT=ABSF(ABSF(COST-COSTP)/COST)
    Z5=CH2(N,M-1)*HV(N,M-1)**2/((GI-X5(N,M-2))/AL1(N,M-1)+CH3(N,M-1))
   1 **2*T
    DZ=ABSF((Z5-Z5P)/Z5)
    IF(DZ-DDZ)211,211,212
212 Z5P=Z5
    DO 102 I=1,N
    DO 102 J=1,M
    DH2=Z5      -CH2(I,J)*HV(I,J)**2*T/(D1(I,J)/AL1(I,J)+CH3(I,J))**2
    DDH2=2.*CH2(I,J)*HV(I,J)**2*T/AL1(I,J)/(D1(I,J)/AL1(I,J)+
   1 CH3(I,J))**3
    DH3=Z5      -CV2(I,J)*VV(I,J)**2*T/(D2(I,J)/AL2(I,J)+CV3(I,J))**2
    DDH3=2.*CV2(I,J)*VV(I,J)**2*T/AL2(I,J)/(D2(I,J)/AL2(I,J)+
   1 CV3(I,J))**3
    AD2=AKK2*DH2/(DDH2+0.00000001)
    AD3=AKK2*DH3/(DDH3+0.00000001)
127 IF(AD2-S2)131,133,133
133 AD2=S2
    GO TO 135
131 IF(AD2+S2)134,135,135
134 AD2=-S2
135 D1(I,J)=D1(I,J)-AD2
    IF(D1(I,J))123,123,124
123 D1(I,J)=0.
124 IF(AD3-S2)136,137,137
137 AD3=S2
    GO TO 139
136 IF(AD3+S2)138,139,139
138 AD3=-S2
139 D2(I,J)=D2(I,J)-AD3
    IF(D2(I,J))125,125,126
125 D2(I,J)=0.
126 IF(D1(I,J)-S3)141,142,142
141 DT1=((CH2(I,J)*T/Z5      )**0.5*HV(I,J)-CH3(I,J))*AL1(I,J)
    IF(D1(I,J)-DT1)143,142,142
143 D1(I,J)=DT1
142  IF(D1(I,J)-S4)147,148,148
148 D1(I,J)=S4
147 IF(D2(I,J)-S3)144,145,145
144 DT2=((CV2(I,J)*T/Z5      )**0.5*VV(I,J)-CV3(I,J))*AL2(I,J)
    IF(D2(I,J)-DT2)146,145,145
146 D2(I,J)=DT2
145 IF(D2(I,J)-S4)149,150,150
150 D2(I,J)=S4
```

```
149 GO TO 102
102 CONTINUE
    IF(SENSE SWITCH 1)104,105
104 READ 2, AKS2,S2
105 GO TO 103
211 DO 80 I=1,N
    DO 80 J=1,M
    LN=N+1-I
    LM=M+1-J
    NP=LN+1
    MP=LM+1
    IF(LN-N)81,82,1112
 82 IF(LM-M)81,83,1112
 83 D3(LN,LM)=1.0
    ZH(LN,LM)=0.
    ZV(LN,LM)=0.
    GO TO 80
 81 IF(J-1)1112,84,85
 85 ZH(LN,LM)=ZH(LN,MP)*D3(LN,MP)+(CH1(LN,MP)*D3(LN,MP)+2.*CH2(LN,MP)*
   1 HV(LN,MP)*D3(LN,MP)/(D1(LN,MP)*AL1(LN,MP)+CH3(LN,MP)))*AL1(LN,MP)
   2 *T+ZV(LN,MP)*(1.-D3(LN,MP))+(CV1(LN,MP)*(1.-D3(LN,MP))+2.*
   3 CV2(LN,MP)*VV(LN,MP)*(1.-D3(LN,MP))/(D2(LN,MP)/AL2(LN,MP)+
   4 CV3(LN,MP)))*AL2(LN,MP)*T
    IF(I-1)1112,86,84
 86 ZV(LN,LM)=ZH(LN,LM)
    GO TO 80
 84 ZV(LN,LM)=ZH(NP,LM)*D3(NP,LM)+(CH1(NP,LM)*D3(NP,LM)+2.*CH2(NP,LM)*
   1 HV(NP,LM)*D3(NP,LM)/(D1(NP,LM)*AL1(NP,LM)+CH3(NP,LM)))*AL1(NP,LM)
   2 *T+ZV(NP,LM)*(1.-D3(NP,LM))+(CV1(NP,LM)*(1.-D3(NP,LM))+2.*
   3 CV2(NP,LM)*VV(NP,LM)*(1.-D3(NP,LM))/(D2(NP,LM)/AL2(NP,LM)+
   4 CV3(NP,LM)))*AL2(NP,LM)*T
    IF(J-1)1112,87,80
 87 ZH(LN,LM)=ZV(LN,LM)
 80 CONTINUE
    IF(ITER-NC)69,182,1112
 69 IF(DELTB-DELTA)183,183,99
 99 KEYSO=0
    IF(SENSE SWITCH 4)183,98
182 NC=NC+NC1
183 PUNCH 9
    DO 90 I=1,N
    DO 90 J=1,M
    PUNCH 4, I,J,HV(I,J),VV(I,J),D1(I,J),D2(I,J),D3(I,J),ZH(I,J),
   1 ZV(I,J)
 90 CONTINUE
    IF(KEYSO-1)75,97,1112
 75 IF(SENSE SWITCH 3)97,96
 97 DO 120 I=1,N
    DO 120 J=1,M
120 PUNCH 11, X5(I,J),Z5
 96 IF(DELTB-DELTA)71,71,72
```

```
 71 IF(KEYSC-1)73,1111,1112
 73 KEYSC=1
    GO TO 98
 72 KEYSC=0
 98 IF(SENSE SWITCH 1)94,95
 94 READ 2, AKK1,S1
 95 DO 101 I=1,N
    DO 101 J=1,M
107 IF(AI(I,J))1112,111,113
111 AII=0.
    GO TO 112
113 AII=AI(I,J)
112 DH1=ZH(I,J)-ZV(I,J)+(CH1(I,J)*AL1(I,J)-CV1(I,J)*AL2(I,J))*T+2.*
   1 CH2(I,J)*AII*D3(I,J)/(D1(I,J)/AL1(I,J)+CH3(I,J))*AL1(I,J)*T-2.*
   2 CV2(I,J)*AII*(1.-D3(I,J))/(D2(I,J)/AL2(I,J)+CV3(I,J))*AL2(I,J)*T
    DDH1=2.*CH2(I,J)*AII     *AL1(I,J)/(D1(I,J)/AL1(I,J)+CH3(I,J))*T+
   1 2.*CV2(I,J)*AII     *AL2(I,J)/(D2(I,J)/AL2(I,J)+CV3(I,J))*T
    AD1=AKK1*DH1/(DDH1+0.0000001)
    IF(ABSF(AD1)-S1)121,121,122
122 IF(AD1)162,162,172
162 AD1=0.-S1
    GO TO 121
172 AD1=S1
121 D3(I,J)=D3(I,J)-AD1
    IF(D3(I,J)-1.)151,152,152
152 D3(I,J)=1.
    GO TO 101
151 IF(D3(I,J))153,153,101
153 D3(I,J)=0.
101 CONTINUE
    COSTP=COST
    GO TO 1000
1111 PUNCH 6
    TYPE 3
    PAUSE
    GO TO 100
1112 TYPE 5
    STOP
    END
```

OPTIMAL TRAFFIC ASSIGNMENTS AND ECONOMIC ANALYSES OF TRANSPORTATION SYSTEMS BY THE DISCRETE MAXIMUM PRINCIPLE

by

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This report attempts a systematic, elementary and exhaustive presentation of the use of the discrete maximum principle to solve traffic assignment and economic analysis problems of transportation systems. Traffic assignment is the process of allocating personal or vehicular trips in an existing or proposed system of travel facilities, and economic analysis deals with the minimization of the sum of travel time cost, operating cost and the investment cost of the transportation system.

The optimal traffic assignment pattern is obtained in section 2, by considering the constant travel time-volume relationships. An optimal traffic assignment pattern, based on the nonlinear travel time-volume relationship, is presented and a single copy network is considered in section 3. Section 4 considers an optimal traffic assignment of a multicopy traffic flow network, that is, a multidestination network with a nonlinear travel time-volume relationship. In section 5, the economic analysis of the transportation system is studied.

Based on the results obtained from sections 3 and 4 of the report it is concluded that the maximum principle technique makes possible the use of nonlinear travel time-volume relationships. The technique is therefore considered to have the potential to represent a 'real world' situation, that is, it is possible to simulate congestions and delay resulting from increasing traffic volumes.

In section 5 a nonlinear travel time equation is developed, giving the relationships among travel time, traffic volume and investment cost. Using this equation, optimum-seeking procedures are developed. Two investment conditions, namely investment with no constraints on budget and investment with fixed budget on a transportation system, are considered.