LABORATORY EXPERIMENT ON LOGIC FOR SLOWER LEARNING ADOLESCENTS

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I lovingly and gratefully dedicate this report

to my mother, Mrs. Emma Haig,

and

to the memory of my father, Mr. Vahram Haig.

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CHAPTER I

INTRODUCTION

The slower learning adolescent usually finds mathematics difficult and uninteresting. He generally takes a course in mathematics either to gain a credit for graduation or to wait for the time when he can legally drop out of school.

Many times the slower learning adolescent's dislike of mathematics is a result of inadequate teaching methods and of an uninteresting and meaningless curriculum. Professional journals, mathematics method books, and reference books on the slower learning adolescent indicate that the traditional lecture-demonstration-drill approach together with the present grade placement of topics and the emphasis on perfecting computational skills does not meet the needs of this type of student. H. Van Engen, Herbert Hannon, Edward Krug, the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, and G. M. Wilson are stating that a special mathematics program designed for the learning capacity of the slower learning student should be developed. They state that it should not be a remedial arithmetic program, but instead a mathematics program which enables students to develop mathematical concepts and skills necessary for the solution of their present problems as well as those they will meet in the future. To help develop such a program, H. Van Engen and Douglas A. Pidgeon are suggesting that the personal-social topics such as insurance, investments, installment buying, and taxation be eliminated from the intermediate school curriculum. They explain that these topics are not of interest to these students because the students are not mature enough yet to apply them to their immediate situation.

To help improve the teaching methods used in the mathematics courses for the slower learning adolescent, Donovan A. Johnson and Charles Butler are suggesting the use of the mathematics laboratory. Title III of the National Defense Education Act of 1958 is promoting the laboratory method by enabling mathematics departments to remodel facilities and to purchase equipment and materials. In addition to facilities, equipment, and materials, the teacher needs to have guide sheets for the student to use in the laboratory. This study reviews the literature concerning the needs and characteristics of the slower learning adolescents, their ability to understand and use mathematical logic, and the laboratory method of teaching mathematics.

I. THE PROBLEM

Statement of the problem. This study was designed to review the literature related to the slower learning adolescent, logical reasoning patterns, and the laboratory method and to design a sample laboratory experiment to develop the concepts of logic. This experiment is presented in the form of a guide sheet to be used by seventh, eighth, and ninth grade slower learning adolescents. Similar experiments can be designed to enable teachers to make use of the laboratory method in the classroom.

It is hoped that the writer and other teachers will use this study, and design other studies to answer the following questions:

Are slower learning adolescents able to learn and to use logical reasoning patterns?

Does the slower learning adolescent learn the concepts discussed in a unit on logical reasoning patterns more quickly, more easily, and more completely when taught through the use of the laboratory method as compared to the lecture-demonstration-drill method?

Importance of the study. At the present time much written support is being given by mathematics consultants and specialists in the field of mathematics methodology to improve the quality of mathematics presented to the "average" and "above average" student in the intermediate school. However, very little has been done to improve the presentation of material to or the present curriculum of the slower learning adolescent. Many times teachers are exposed to some well-organized units and experiments, but lack the ideas, the time, or the material to develop them. Therefore, the review of literature and sample experiment in this report should prove useful to teachers of slower learning adolescents in the intermediate school.

II. DEFINITIONS OF TERMS USED

Adolescents. Adolescents are children in the last stage of the development from immaturity to maturity consisting of ages 12-15.

Guide sheet. The guide sheet contains the essential information which the student must have in order to carry out the experiment. Specifically, the guide sheet contains a list of objectives, equipment needed, directions, and questions which lead the student to the conclusions.

Intermediate school. Intermediate schools are middle schools and junior high schools which have an educational program designed to meet the needs, interests, and abilities of adolescents. The curriculum is a flexible aggregate of exploratory experiences in group and individual activities.

Laboratory. A room which is properly equipped for the performance of experiments. This may be a classroom, or a special room equipped for the purpose of carrying on laboratory experiments.

Laboratory demonstrations. Activities that either a student, group of students, or a teacher performs in order to explain or introduce topics through example.

Laboratory experiments. Activities that students can perform individually or in groups covering material related to the topics they are studying, will study, or have studied.

Laboratory method. The procedures and motivational devices used in a mathematics laboratory.

Logical reasoning or Logic. The science of the formal principles of correct reasoning including the ability to recognize invalidity and validity; to make logically correct inferences; to use inductive reasoning; to use deductive reasoning; to use and understand negations, conjunctions, disjunctions, implications, and definitions; to think consistently, clearly, precisely, and creatively; and to analyze and apply techniques and skills used in logical thinking.

Logical reasoning patterns. These patterns are understood to be the processes used in deducing new statements or propositions from one or more given statements or actions.

Slower learners or Slower learning adolescents. Students enrolled in the intermediate school who exhibit one or more of the following characteristics:

1. A number of college-capable students who have not as yet discovered their mathematical talent.
2. Those who have been insufficiently challenged.
3. Students from economically depressed areas or from disadvantaged homes.
4. Socially maladjusted children.
5. Those with emotional problems that prevent them from functioning at the level of their ability.

6. Lazy or reluctant students.

7. Unmotivated students.

8. Those with low natural ability.[1]

9. Those who rank below the 30th percentile in mathematical achievement.

10. Students who are inconsistent in growth development or are slow in maturing.

11. Students who dislike mathematics, have poor work habits, are repeated failures, or have an inadequate curriculum.

12. Students exposed to unwarranted educational pressure or unrealistic expectations.

13. Students with physical defects.[2]

14. Those who may be bored.

15. Those having family problems.

16. Those students who have a language problem--foreign or English.

17. Students having problems getting along with their peers.[3]

III. PROCEDURES EMPLOYED IN THE STUDY

The procedures employed in this study consisted of (1) analyzing textbooks, reference books, and magazine articles at Kansas State University on the slower learning adolescent, mathematical logic, and the laboratory method of teaching, (2) designing a format for the sample guide sheet to be

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[1]Sol Weiss, "Innovations and Research in the Teaching of Mathematics to the Terminal Student," The Mathematics Teacher, 60:611-12, October, 1967.

[2]Florence Elder, "Mathematics for the Below-Average Achiever in High School," The Mathematics Teacher, 60:235-39, March, 1967.

[3]Jane G. Stenzel, "Math for the Low, Slow, and Fidgety," The Arithmetic Teacher, 15:30-4, January, 1968.

used by the students in the laboratory, and by the teacher for future reference to develop more laboratory experiments, and (3) developing the appendices to be used by teachers wishing to expand this study to a unit on logical reasoning patterns.

CHAPTER II

REVIEW OF THE LITERATURE

A review of the literature pertaining to the slower learning adolescent, logical reasoning patterns as a unit of study in the intermediate school, and the laboratory method of teaching mathematics was conducted to answer the following questions:

1. How is a teacher able to identify a slower learning adolescent?
2. What are the needs and characteristics of a slower learning adolescent?
3. What is the teacher's role in teaching the slower learning adolescent?
4. What are the attitudes of mathematics consultants, specialists in the field of mathematics methodology, and teachers about the present mathematics curriculum for the slower learning adolescent in the intermediate school?
5. Do mathematics teachers, specialists in mathematics methodology, mathematics consultants, and psychologists advocate the teaching of logical reasoning patterns in the intermediate school?
6. Is there evidence that the slower learning adolescent in mathematics will be able to learn logical reasoning patterns?
7. What mathematical concepts should be covered in a unit on logical reasoning patterns?
8. Do specialists in the fields of mathematics, mathematics methodology, and psychology advocate the use of the laboratory method for the slower learning adolescent in mathematics or in teaching logical reasoning patterns?
9. How should the mathematics laboratory be organized for the slower learning adolescent?

Each of these questions is dealt with in the following sections of this chapter.

I. IDENTIFYING THE SLOWER LEARNING ADOLESCENT

Max A. Sobel[1] and Jane G. Stenzel[2] have stated that there have been many terms such as "low achiever," "slow learner," "nonacademic student," and "mathematically less gifted" used to describe adolescents who are below the average student in their ability to learn mathematics. These students are not always dull, but instead they may not like mathematics or may not understand it. As G. Orville Johnson[3] has stated, these "below average" students are usually one of two types of youngsters. They may be "late bloomers" who at a later time when they have achieved additional mental maturity will have the ability to learn the concepts and skills their "average" classmates are now learning, or they may never be able to learn these concepts and skills because of their lack of mental development.

Many teachers have made the mistake of trying to identify these slower learning adolescents by IQ scores. Max A. Sobel[4] explains that there is no fixed IQ score which will identify this type of student. Even though the average IQ score of a group of slower learners is usually about 85, there have been cases of students with low IQ scores doing well in class as well as cases of students with high IQ scores failing in class. To help the classroom teacher identify the slower learning adolescent, Max A. Sobel suggests giving attention to such items as "past performance in arithmetic,

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[1]Max A. Sobel, Teaching General Mathematics (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), p. 1.

[2]Jane G. Stenzel, "Math for the Low, Slow, and Fidgety," The Arithmetic Teacher, 15:30, January, 1968.

[3]Orville Johnson, Education for the Learners (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), cited by Sobel, op. cit., p. 2.

[4]Ibid.

teacher recommendations, reading ability, IQ scores, and results on standardized tests."[5] In 1964, a conference sponsored by the U. S. Office of Education in cooperation with the National Council of Teachers of Mathematics identified the slower learning student by citing the following definition:

> The low achiever in mathematics is that student who, by teacher estimates, achievement tests, or by whatever means the school uses for marking, grouping, or promotion, ranks below the 30th percentile of the student population in achievement in mathematics.[6]

II. NEEDS AND CHARACTERISTICS OF A SLOWER LEARNING ADOLESCENT

According to G. Orville Johnson, teachers and administrators expect slower learning adolescents to be discipline problems, to be disinterested in learning and in school, and to be harder to teach because of a shorter attention span. While many of these characteristics can be found among a group of slower learners, such characteristics are not part of a child. All too often the behavior of a student is a "reflection or result of what teachers and the school have done <u>to</u> him rather than <u>for</u> him."[7] The problems facing a slower learner are a combination of his background, ability, and environment. Mr. Strom[8] felt that the three fundamental things which a

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[5]Ibid.

[6]U.S., Office of Education in cooperation with the National Council of Teachers of Mathematics, <u>Preliminary Report of the Conference on the Low Achiever in Mathematics</u>, (Washington, D.C.: Government Printing Office, 1964), as cited by Sobel, op. cit., p. 1.

[7]G. Orville Johnson, "Motivation the Slow Learner," <u>The Inner-City Classroom: Teacher Behaviors</u>, (Columbus, Charles E. Merrill Books, Inc., 1966), p. 111.

[8]Ibid., p. 116.

school must do to develop a meaningful program for these slower learning adolescents are: employ qualified teachers and administrators; initiate a curriculum that takes into account the child's environment, learning activities, and academic, social, and occupational potentials; and use methods of teaching geared for success in learning. Young found in his study that a student loses interest in a subject because (a) he failed to see the need for the course, (b) the course was uninteresting, (c) he lacked a foundation for the course, or (d) the course was too difficult.[9]

For the slower learning adolescent in the intermediate school, the emphasis should be placed on the characteristics and needs of the student rather than on specific mathematical topics, according to Max A. Sobel.[10] Some of these needs and characteristics can be found in Appendix B. Abraham summarized these needs when he wrote these "three A's: Acceptance, Affection, Achievement."[11]

Briggs,[12] Hines,[13] and Sobel[14] suggested that a mathematics curriculum based on the characteristics and needs of a student should refer to prior mathematical experiences as well as have opportunities for the student to explore his interests, his aptitudes, and his capabilities, while leading

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[9]U.S., Department of Health, Education, and Welfare, Research Problems in Mathematics Education, Cooperative Research, Monograph No. 3 (Washington, D.C.: Government Printing Office, 1960), p. 15.

[10]Sobel, p. 3.

[11]Willard Abraham, "The Slow Learner--Surrounded and Alone," Today's Health, XLIII (September, 1965), p. 59.

[12]Harlan Cameron Hines, Junior High School Curricula (New York: Macmillan Co., 1924), p. 6.

[13]Ibid., p. 4.

[14]Sobel, op. cit., pp.3-4.

carefully and slowly from easier mathematical material to the more difficult. This curriculum should motivate the student, emphasize method rather than just finding the right answer, be a mathematical program rather than just socially applied mathematics, and present a variety of problems.

As Max Sobel[15] has stated, the slower learning adolescent in the intermediate school has the same characteristics, needs, and interests as other students his age. However, he has a greater need than the average child to experience success and approval as well as feel that he is a member of a group with a contribution to make. He needs his status identified, his confidence restored, his interest stimulated, his attitude toward mathematics made favorable, and his ego flattered.

III. TEACHER'S ROLE IN TEACHING THE SLOWER LEARNING ADOLESCENT

Sarah B. Greenholz[16] has stated that automation has eliminated many of the unskilled jobs that were once available to high school dropouts, many of whom are slow learners. Since these jobs have been eliminated, these students must be encouraged to remain in school. Therefore, an effective mathematics program for the slower learning adolescent should be created. According to Biggs,[17] Kindred,[18] Sobel,[19] and Stenzel[20] some of

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[15] Ibid., p. 4.

[16] Sarah B. Greenholz, "Reaching Low Achievers in High School Mathematics," Today's Education, 57:71, September, 1968.

[17] Edith E. Biggs, "Mathematics Laboratories and Teachers' Centres--the Mathematics Revolution in Britain," The Arithmetic Teacher, 15:408, May, 1968.

[18] Leslie W. Kindred, The Intermediate Schools (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968) p. 114.

[19] Sobel, op. cit., p. 7.

[20] Stenzel, op. cit., pp. 30-1.

the more recent ways of achieving greater understanding of mathematical concepts for the slower learner are to

1. Have a daily routine plan in which the student can think for himself and discover for himself the mathematical patterns which are to be found everywhere in his environment.

2. Develop concepts introduced earlier.

3. Introduce concepts which are appropriate to slower learning students.

4. Make certain the program blends smoothly.

5. Start teaching from where the student is and not where he is expected to be.

6. Present new ideas related to other subjects through the mathematics laboratory, field trips, and community resources.

7. Allow time each day for review of previous material.

Many times teachers expect these children to be discipline problems. Max A. Sobel suggested that the best way to prevent discipline problems from arising was "to keep the student occupied throughout the period with a meaningful, interesting, and challenging curriculum."[21]

Some slower learning students have learned to hate and fear mathematics. However, according to Krug, "mathematics in any form demands effort and application from students. Unnecessary tensions in the form of fear and hatred are destructive to mental and emotional health and to real learning."[22] A solution to this problem might result in the student reaching levels of

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[21] Sobel, loc. cit.

[22] Edward A. Krug, Curriculum Planning (New York: Harper and Brothers, 1957), pp. 152-53.

attainment believed to be unattainable before. Bigge,[23] Backman,[24] and Crowley[25] have stated that to help a student overcome this problem, a teacher can first try to restore the student's self-esteem. Too often by the time a slower learning adolescent reaches the intermediate school, his repeated failures have produced a self-image composed of defeat and frustration. The teacher can improve this image by giving the student simple tasks which are realistic in terms of his capabilities. The teacher should let the student know that he wants to help him if the student makes an effort. A teacher should never ridicule, but be aware and make an effort to understand the factors that contribute to the problems of the slower learning adolescent. According to Crowley[26] these factors include brain damage, cultural disadvantage, psychological disadvantage, low IQ, and physical defects.

> A teacher who accepts the fact that the low achievers are teachable; a teacher who has a missionary spirit and a respect for the worth of pupils with limited ability; a teacher who is concerned and interested in individuals; a teacher who can make a pupil feel he not only belongs but also is important; a teacher who can instill a sense of worth, responsibility, and desire to achieve; a teacher who cares enough to give his very best to the low achievers will make the program a success.[27]

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[23] Morris L. Bigge, Learning Theories for Teachers (New York: Harper and Row, Publishers, 1965), 292, 289.

[24] Alfred Morry Backman, "Factors Related to the Achievement of Junior High School Students in Mathematics," Dissertation Abstracts, 29:2139-A, January, 1969.

[25] Regis F. Crowley, "Teaching the Slow Learner," Today's Education, 58:48, January, 1969.

[26] Ibid., p. 49.

[27] U. S. Office of Education in cooperation with the National Council of Teachers of Mathematics, op. cit., p. 13.

IV. ATTITUDES CONCERNING THE PRESENT MATHEMATICS CURRICULUM

According to Charles H. Butler and F. Lynwood Wren,[28] the basic concern of general education is to help each student realize and accept his individual potential and to help him find success so that he will become a responsible citizen. However, as Harold H. Lerch and Francis J. Kelly have pointed out "approximately one-third of the students in grades seven through twelve are unsuccessful in meeting the requirements of our educational system."[29] Most of these students drop out of school before graduation and continue to fail in their vocational and civil obligations. Lerch and Kelly felt that the "two important factors contributing to this lack of adjustment are the inappropriateness of the student's educational experience and the lack of school climate for suitable personal development."[30] This quotation gives evidence to the fact that the school program as a whole and the mathematics program in particular are not fulfilling the purposes of general education and the principles of learning as listed in Appendix C.

It is interesting that children in the primary grades find numbers and numerical concepts fascinating, but by the time these same children reach the intermediate school, they have learned to hate and fear mathematics.

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[28] Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Co., 1965), p. 44.

[29] Harold H. Lerch and Francis J. Kelly, "A Mathematics Program for Slow Learners at the Junior High Level," The Arithmetic Teacher, 13:232, March, 1966.

[30] Ibid.

This was recognized by Herbert Hannon,[31] Edward A. Krug,[32] and stated in a research project by the U. S. Department of Health, Education, and Welfare.[33] Edward Krug[34] suggested that this situation might be due to the curriculum, because of the grade placement of topics or because of the emphasis placed on perfecting computational skills.

H. Van Engen[35] noted that the arithmetic in the intermediate school for those pupils in the lower-ability group must be carefully considered. However, H. Van Engen,[36] Herbert Hannon,[37] Edward Krug,[38] the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics,[39] and G. M. Wilson[40] stated that this consideration has not been met and that there is a need to develop a special mathematics program designed for the learning capacity of the slower learning student. Such a program

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[31]Herbert Hannon, "A Time to Appraise the New and Re-evaluate the Old in Upper Grades and Junior High School Mathematics," School Science and Mathematics, 63:171, March, 1963.

[32]Krug, op. cit., p. 152.

[33]U. S. Department of Health, Education, and Welfare, op. cit., p. 11.

[34]Krug, op. cit., p. 153.

[35]H. Van Engen, "Arithmetic in the Junior-Senior High School," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago, Illinois: The University of Chicago Press, 1951), p. 106.

[36]Ibid., p. 104.

[37]Hannon, loc. cit.

[38]Krug, op. cit., pp. 146-48.

[39]F. Lynwood Wren (ed.), "The Secondary Mathematics Curriculum," The Mathematics Teacher, 52:403, May, 1959.

[40]U. S. Department of Health, Education, and Welfare, op. cit., p. 5.

should be a mathematics program and not a remedial arithmetic program. This program should enable students to develop mathematical concepts and skills necessary for the solution of their present problems as well as those they will meet in the future. However, these mathematics-education consultants believe that the present curriculum which exists in the seventh and eighth grades is not designed for this purpose. Thus, personal-social topics such as insurance, investments, installment buying, and taxation which are being taught in the intermediate school are not of interest to the student because, as H. Van Engen[41] and Douglas A. Pidgeon[42] explain, these students are not mature enough yet to find these topics applicable to their immediate situations.

The research project[43] states, and Charles H. Butler and F. Lynwood Wren,[44] The Secondary School Curriculum Committee,[45] Max A. Sobel and Donovan A. Johnson,[46] and C. L. Thiele[47] agree, that the mathematics curriculum in the seventh and eighth grades is inadequate to meet the modern

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[41]Engen, op. cit., pp. 115-116.

[42]Douglas A. Pidgeon (ed.), Achievement in Mathematics (The Mere, England: National Foundation for Educational Research in England and Wales, 1967), p. 124.

[43]U. S. Department of Health, Education, and Welfare, op. cit., pp. 8-9.

[44]Butler, op. cit., p. 89.

[45]Wren, op. cit., 403, 409.

[46]Max A. Sobel and Donovan A. Johnson, "Analysis of Illustrative Test Items," Evaluation in Mathematics, National Council of Teachers of Mathematics Twenty-Sixth Yearbook (Washington, D.C.: National Council of Teachers of Mathematics, 1967), pp. 71-92.

[47]C. L. Thiele, "Arithmetic in the Middle Grades," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago, Illinois: The University of Chicago Press, 1951), p. 89.

needs of adolescents. Their major concern is that few new mathematical ideas are introduced in these grades and that the traditional courses at this level do not challenge the student because the emphasis is usually placed on drill rather than providing for deeper understanding of the topics. Therefore, they suggest that the mathematics program for the slower learning adolescent should contain the same basic concepts as those taught to the average and faster learners. The only major difference should be the teaching methods applied with social applications used only as an aid in the development of these fundamental concepts and principles.

V. SPECIALISTS WHO ADVOCATE THE TEACHING OF LOGICAL REASONING PATTERNS

Charles H. Butler and F. Lynwood Wren,[48] Walter R. Fuchs,[49] Edward A. Krug,[50] and the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics[51] are only a few of the many specialists who believe that the mathematics curriculum in the intermediate school is inadequate. They suggest that a modern mathematics program be introduced which would stress the structure, the concepts, and the understanding and meaning of basic mathematical processes. Max A Sobel and Donovan A. Johnson[52] have suggested that some of the objectives of a mathematics program should be:

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[48]Butler, loc. cit.

[49]Walter R. Fuchs, Mathematics for the Modern Mind (New York: Macmillan Company, 1967), p. 26.

[50]Krug, loc. cit.

[51]Wren, loc. cit.

[52]Sobel and Johnson, loc. cit.

1. The student should have an understanding of mathematical processes and concepts.

2. The student should associate mathematical concepts with applications in community situations.

3. The student should be stimulated to demonstrate creativeness and imagination.

4. The student should be able to think logically.

5. The student should use mathematical concepts to discover new generalizations or applications.

Coinciding with the beliefs of Max A. Sobel and Donovan A. Johnson that logical reasoning has its place in the intermediate school are Charles H. Butler and F. Lynwood Wren,[53] Lucian Blair Kinney and C. Richard Purdy,[54] G. Pólya,[55] Kenneth A. Retzer and Kenneth B. Henderson,[56] The Secondary School Curriculum Committee of the National Council of Teachers of Mathematics,[57] and Sol Weiss.[58] They agree that mathematics is an excellent subject for teaching understanding and usage of plausible reasoning patterns especially deduction. These specialists also realize that logical reasoning makes mathematics more meaningful. Frege asserted "the basic concepts of

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[53]Butler, op. cit., 70, 121, 273, 377.

[54]Lucian Blair Kinney and C. Richard Purdy, Teaching Mathematics in the Secondary School (New York: Rinehart and Company, 1952), 8-9, 227.

[55]G. Pólya, "Mathematics as a Subject for Learning Plausible Reasoning," The Mathematics Teacher, 52:7-9, January, 1959.

[56]Kenneth A. Retzer and Kenneth B. Henderson, "Effect of Teaching Concept of Logic on Verbalization of Discovered Mathematical Generalizations," The Mathematics Teacher, 60:707-09, November, 1967.

[57]Wren, op. cit., p. 400.

[58]Sol Weiss, "Innovations and Research in the Teaching of Mathematics to the Terminal Student," The Mathematics Teacher, 60:615, October, 1967.

arithmetic can be reduced to purely logical concepts, and the basic properties of arithmetic can be deduced from purely logical propositions."[59] This assertion has been examined by Kenneth A. Retzer and Kenneth B. Henderson,[60] and their conclusion was that logical concepts in relation with mathematical generalizations should be an explicit part of the curriculum.

Inhelder and Piaget have made the following statement:

> All authorities agree that adolescents show rapid progress in their capacity to generalize, to make deductions, and to draw conclusions from abstract situations. They learn fairly rapidly how to apply rules to specific situations and to look for causes and consequences in order to discover explanations for themselves.[61]

Studies of Piaget[62] support the belief that boys and girls twelve years of age are able to work with mathematical ideas at a considerably higher level of abstraction than is characteristic of traditional mathematical courses in grades seven and eight, and generally prefer intellectual activities that challenge their reasoning power.

A recent study at Wisconsin State University[63] and another at the University of Wisconsin[64] indicated that formal logic was successfully taught with some degree of abstraction to the eighth grade pupils. In another

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[59]Friedrich Waismann, Introduction to Mathematical Thinking (New York: Frederick Unger, 1951), pp. 69-70.

[60]Retzer, op. cit., p. 710.

[61]Barbel Inhelder and Jean Piaget, The Growth of Logical Thinking from Childhood to Adolescence (New York: Basic Books, Inc., 1958), p. 332.

[62]Wren, op. cit., p. 403.

[63]Ibid.

[64]William A. Miller, "A Unit in Sentenial Logic for Junior High School Students; Involving Both Valid and Invalid Inference Patterns," School Science and Mathematics, 69:548, June, 1969.

experiment by Patrick Suppes and Frederick Binford[65] a group of fifth-and sixth-grade students learned as much logic as a college class in the same course, but at a slower pace with considerably more direct teacher supervision. These studies indicate not only that logical reasoning patterns can be learned by adolescents in an intermediate school but that they are important and useful.

Applications of logical reasoning occur in everything in which man engages. Max Wertheimer[66] and the Thirtieth Yearbook of the National Council of Teachers of Mathematics recognized that "logic is not a substitute for precise reasoning or thinking, but the methods of logic clarify our patterns of thought, guide us to correct reasoning processes, and help us avoid errors"[67] through the understanding of the underlying assumptions in the events which take place in our everyday situations.

This review of literature definitely indicates that specialists in the fields of both mathematics and mathematics-education advocate the teaching of logical reasoning patterns.

VI. EVIDENCE SUPPORTING THE ABILITY OF THE SLOWER LEARNING ADOLESCENT TO LEARN AND USE LOGICAL REASONING PATTERNS

Finally mathematics educators are becoming aware of the fact that an

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[65] Patrick Suppes and Frederick Binford, Experimental Teaching of Mathematical Logic in the Elementary School (Stanford University Press, 1964), 16-9, 24.

[66] Max Wertheimer, Productive Thinking (New York: Harper and Brothers, Publishers, 1959), p. 259.

[67] More Topics in Mathematics, National Council of Teachers of Mathematics Thirtieth Yearbook, Booklet No. 12 (Washington, D.C.: National Council of Teachers of Mathematics, 1969), p. 187.

adequate mathematics program must be provided for all students, including the slower learning adolescent. It has been recognized by Florence Elder[68] and Jane G. Stenzel[69] that the slower learner has been mathematically deprived because our educational system has failed to provide him with courses which are exciting as well as applicable to his world. Instead, each year the same old stuff he has rejected for years is either spoonfed to him or shoved down his throat. By the time the slower learning adolescent has reached the seventh or eighth grade, he has already built up psychological blocks for at least seven or eight years against the addition, subtraction, multiplication, and division which has been drilled into him.

In 1962, the National Council of Teachers of Mathematics appointed a Committee on Mathematics for the Non-College Bound. As Max A. Sobel[70] explained, this committee decided that units on new independent topics which were designed to motivate the latent interests of the slower learning adolescent were to be developed. The content of these units should extend student's appreciation and understanding of the principles in areas such as symbolic representation, problem solving, and mathematical thinking.

In 1963, John Raymond Hodges[71] concluded from his study that certain eighth grade students can learn important basic mathematical concepts and terminology regardless of prior mathematical achievement. Inhelder and

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[68]Florence Elder, "Mathematics for the Below-Average Achiever in High School," The Mathematics Teacher, 60:235-39, March, 1967.

[69]Stenzel, loc. cit.

[70]Sobel, op. cit., p. 13.

[71]John Raymond Hodges, "A Study of the Ability of a Group of Eighth Grade Students to Learn and Use Certain Mathematical Concepts," Dissertation Abstracts, 24:5430, June, 1964.

and Piaget,[72] The Secondary School Curriculum Committee,[73] and Patrick Suppes and Frederick Binford[74] indicated that the adolescent in the intermediate school is capable of logical thinking and can be taught to use formal operations. When these students were enrolled in a course in logic, the course gave prestige to the students who were not socially accepted by others.

Although this is not much evidence supporting the ability of the slower learning adolescent to learn logical reasoning patterns, the investigator felt that this review of literature indicated that a slower learning adolescent with the help of a teacher who is interested in him and believes he is teachable will make the unit a success.

VII. MATHEMATICAL CONCEPTS TO BE COVERED IN A UNIT ON LOGICAL REASONING PATTERNS

The mathematics program for the slower learning adolescent, according to The Secondary School Curriculum Committee,[75] Kindred,[76] Sage,[77] and Stenzel,[78] should contain the same basic structure and concepts as the program for the average and faster learners, but should be geared to the needs of the slower learners in the class. The teacher should not try to

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[72]Inhelder, loc. cit.

[73]Wren, loc. cit.

[74]Suppes, op. cit., p. 23.

[75]Wren, op. cit., p. 409.

[76]Kindred, op. cit., p. 46.

[77]Edward Sage, "Elements of a Good Program in Arithmetic," School Science and Mathematics, 62:186-7, March, 1961.

[78]Stenzel, op. cit., pp. 31-2.

make mathematicians of these children, but provide them an opportunity to find the answers for themselves in intellectual activities that will challenge their reasoning powers. In this way, the child will want to continue studying mathematics.

According to H. Van Engen,[79] the ultimate goals of arithmetic are to establish patterns of thought, to develop generalizations, and to establish means of attacking real problem situations by Kinney's method of (1) understanding the problem, (2) analyzing and organizing the problem, (3) recognizing the process required, (4) solving and verifying the answer.[80] These goals can be met through the teaching of a unit on logical reasoning patterns taught as a set of fundamental principles and deductive applications.

Butler and Wren,[81] Bernard H. Gundlach and others,[82] William Miller,[83] Richard Mosier,[84] Douglas Pidgeon,[85] and Patrick Suppes and Frederick Binford[86] stated that these fundamental principles should include the ability to recognize invalidity and validity; to make logically correct inferences; to use inductive reasoning; to use deductive reasoning; to use and understand negations, conjunctions, disjunctions, implications, and definitions; to

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[79]Engen, op. cit., pp. 104-05.

[80]Kinney, op. cit., p. 234.

[81]Butler, op. cit., p. 70

[82]Bernard H. Gundlach and others, Junior High School Mathematics 8, River Forest, Illinois: Laidlaw Brothers Publishers, 1968, pp. 23-50.

[83]Miller, op. cit., pp. 549-52.

[84]Richard D. Mosier, "From Inquiry Logic to Symbolic Logic," Educational Theory, 18:33, Winter, 1968.

[85]Pidgeon, op. cit., p. 18.

[86]Suppes, op. cit., pp. 3, 5.

think consistently, clearly, precisely, and creatively; and to analyze and apply techniques and skills used in logical thinking.

This review of literature indicated to the investigator that the concepts taught in a unit on logical reasoning patterns should include Objectives in Teaching Mathematics to the Slower Learning Adolescent, Appendix C, and *Considerations in Developing a Lesson for the Slower Learner*, Appendix D, as well as including the fundamental principles of logical reasoning.

VIII. ADVOCATES FOR THE USE OF THE LABORATORY METHOD

A major problem that still remains is that of providing a meaningful curriculum for the slower learning adolescent. Although a student fails to learn the fundamentals in arithmetic, the curriculum in the intermediate school all too often repeats the same material in the same way. This constant drill fails to produce skill in these fundamentals as well as killing any interest these youngsters may have had for mathematics. Such a program for a slower learner is dull, destructive of all interests, and helps to create discipline problems as was observed by Max A. Sobel.[87] Agreeing with Sobel, H. Van Engen,[88] Butler and Wren,[89] and a research project by the U. S. Department of Health, Education, and Welfare[90] stated that these pupils should be freed from incessant drill and routine work. They suggested that these

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[87]Sobel, op. cit., p. 7.

[88]Engen, op. cit., p. 106.

[89]Butler, op. cit., pp. 136-38.

[90]U. S. Department of Health, Education and Welfare, op. cit., p. 9.

pupils be placed in a laboratory situation which requires some quantitative thinking to clarify mathematical concepts. In this laboratory situation the students should be exposed to experiences associating them with physical things and with more realistic and interesting situations.

Butler and Wren,[91] Elder,[92] Groenendyk,[93] Hartung,[94] Hines,[95] Larry Johnson,[96] Kindred,[97] Kinney and Purdy,[98] Phillips,[99] Pólya,[100] and Stenzel[101] stated that a group of slower learning adolescents are not necessarily dull, but appreciate topics which are taught with realistic expectations and deadlines, short range goals, motivational devices, opportunity and room for students to work together and move around, and content selected specifically

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[91]Butler, op. cit., p. 110.

[92]Elder, loc. cit.

[93]Eldert A. Groenendyk, "Lab for 9th Grade General Math.," Midland School, 78:13-4, May, 1964.

[94]Maurice L. Hartung, "Motivation for Education in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), pp. 53-4.

[95]Hines, op. cit., p. 4.

[96]Larry K. Johnson, "The Mathematics Laboratory in Today's School," School Science and Mathematics, 60:587, November, 1962.

[97]Kindred, op. cit., pp. 378, 470.

[98]Kinney, op. cit., p. 225.

[99]Harry L. Phillips, "The Mathematics Laboratory," American Education, 1:1-3, March, 1965.

[100]G. Pólya, "Method and Techniques of Explaining New Mathematical Concepts in the Lower Forms of Secondary Schools, Part I," The Mathematics Teacher, 58:345-52, April, 1965.

[101]Stenzel, op. cit., pp. 33-4.

for the slower learner. Sarah Greenholz[102] discussed the attempt at West Hempstead Junior-Senior High School in New York to provide a stimulating program for the "under-achiever" from grades K-12. The classes were small, the classroom was organized like a workshop, and the courses were problem oriented in the form of experiments. William Fitzgerald stated that "self-selected mathematics learning activities meet the individual needs of the slower learning students and that these students develop a better attitude toward mathematics than those in a conventional class."[103]

Maurice Hartung[104] found that these repeated successful experiences help to build the students' self-esteem and integrity, and Edith E. Biggs recommended the laboratory method by stating: "I hear and I forget. I see and I remember. I <u>do</u> and I <u>understand</u>."[105]

This review of literature suggested to the investigator that the laboratory method met the needs and characteristics as well as improved the learning attitude of slower learners and put into practice principles of learning which are commonly accepted by psychologists.[106] Although there was no information concerning the use of the laboratory method in teaching logical reasoning patterns, this review indicates that it has been successful

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[102]Sarah Greenholz, "Successful Practice in Teaching Mathematics to Low Achievers in Senior High School," <u>The Mathematics Teacher</u>, 60:330-31, April, 1967.

[103]William M. Fitzgerald, <u>Self-Selected Mathematics Learning Activities</u> (Ann Arbor: University of Michigan, 1965), 65, 68, 73.

[104]Hartung, op. cit., p. 56.

[105]Biggs, loc. cit.

[106]Glenda Gayle Travis, "Laboratory Experiments for Senior High General Mathematics" (unpublished Master's report, Kansas State University, 1969), pp. 11-6.

in teaching other topics to slower learning adolescents.

IX. ORGANIZING A MATHEMATICS LABORATORY

Adolescents like to investigate, experiment, and manipulate in order to gain a visual picture of the basic idea. Junior high schools today are providing these experiences through exploration and guidance which can be found in a mathematics laboratory. Teaching in a mathematics laboratory is informal and individualized with opportunities to discover or rediscover mathematical laws and truths by following these suggestions:

1. Provide the pupil with opportunities to acquire significant information about the field or activity being studied.
2. Explore the pupil's own interests and qualifications in that field.[107]
3. Encourage the pupil that it is better to form a wrong hypothesis, provided it can be tested and the errors discovered, than it is to be without a hypothesis.[108]
4. Help the pupil realize that there is no perfect device which solves every problem.[109]

According to Kinney and Purdy,[110] many mathematics teachers feel that the whole world is a laboratory in which the workings of mathematics is studied, and the classroom is the headquarters which furnishes the equipment and materials for carrying on this study and for providing stimulating and

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[107] Kinney, op. cit., p. 225.

[108] Kenneth B. Henderson and Robert E. Pingry, "Problem-Solving in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 118.

[109] Pólya, "Methods and Techniques of Explaining New Mathematical Concepts in the Lower Forms of Secondary Schools, Part I," loc. cit.

[110] Kinney, op. cit., p. 299.

worthwhile experiences.

The pamphlet, Mathematics Laboratories in Schools,[111] has stated that the mathematics laboratory is a place for learning by doing, a workshop, a mathematics library, a computation room, a mathematics room, and an environment in which a pupil learns efficiently and more meaningfully. However, before a teacher can use the laboratory method, he must know how to organize a mathematics laboratory. As Raymond Sweet[112] stated, the laboratory should not replace any classroom activity, but should be used to teach those concepts which are best learned through the laboratory method.

According to Larry Johnson[113] and Charles H. Butler and F. Lynwood Wren,[114] the laboratory experiments should be carefully planned, co-ordinated with classroom activities, closely supervised, and guided toward definite ends. Successful laboratory experiments can be devised by following these five procedures of Donovan Johnson:[115]

1. Prepare guide sheets for the students so that they will know what material they need and what they are to investigate.
2. The laboratory lesson needs adequate materials.
3. The laboratory lesson needs a classroom with some flexibility and certain special equipment.
4. Each student should participate in an activity in which he can have some success.

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[111]The Mathematical Association, Mathematics Laboratories in Schools (London: G. Bell and Sons, 1968), p. 113.

[112]Raymond Sweet, "Organizing a Mathematics Laboratory," The Mathematics Teacher, 60:117, February, 1967.

[113]Larry Johnson, op. cit., p. 588.

[114]Butler, op. cit., p. 137.

[115]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics (Belmont, California: Wadsworth Publishing Co., Inc., 1967), pp. 307-10.

5. The students must be properly prepared for the laboratory lesson.

The teacher may use several types of laboratory experiments. These types range from data collecting experiments to using mathematical games. A list of twenty-five types of experiments may be found in Appendix F and a criteria for judging procedures and motivational devices used in the experiments in Appendix G.

Several types of laboratories are found in junior high schools. The type usually depends on the facilities and materials the teacher has at his disposal. Four types of laboratories were discussed by J. S. Frame.[116] The simplest type is a section of a classroom where students can conduct laboratory experiments. Some junior high schools have a small room built between two classrooms which is used by students when conducting experiments. A third type that several schools have is a computation laboratory. In this type of laboratory, the students deal with the field of digital computers. The last type mentioned by Frame is equipped with several desk calculators and a high-speed electronic digital computer. This type of laboratory is used with advanced students.

Some teachers do not think that they have the proper facilities for experiments and demonstrations. However, laboratory materials vary from school to school and can be classified into real experiences, manipulative materials, pictorial materials, and symbolic materials.[117] A list of up-to-date items useful in a modern mathematics laboratory can be found in

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[116] J. S. Frame, "Facilities for Secondary School Mathematics," The Mathematics Teacher, 57:388-89, October, 1964.

[117] Foster Grossnickle, Charlotte Junge, and William Metzner, "Instructional Material for Teaching Arithmetic," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago: The University of Chicago Press, 1951), p. 161.

Appendix H.

CHAPTER III

SUGGESTED LABORATORY EXERCISE FOR A PROPOSED UNIT ON LOGIC

This section contains a representative experiment designed for seventh, eighth, or ninth grade slower learning adolescents. This experiment can be used as a guide to help teachers design similar experiments for each topic taught in a unit on logic.

The sample experiment presented is flexible. This experiment and all future experiments should be adjusted to the specific students who are being taught. In one class, it may be necessary for the teacher to supply more information than is given on the guide sheet or to give careful step-by-step instructions during the laboratory period, while in another class, there may be a need to design more stimulating experiments to replace the less challenging ones.

Mathematical recreations provide an excellent means of stimulating the interest of slower learners and of furnishing ideas for more experiments. These recreations may be in the form of tricks, games, or puzzles. Some of the numerous texts available that deal with mathematical recreations are listed in Appendix A.

The principles used in designing the following experiment are listed in Appendix I.

LABORATORY EXERCISE ON LOGICAL REASONING PATTERNS

OBJECTIVE: To discover the truth values of the conjunction and the disjunction of two propositions.

EQUIPMENT: Two simple circuits--one series and one parallel.

DIRECTIONS:

General:

Figure 1

Figure 2

The two figures above represent the same switch in two different positions. Figure 1 shows a switch when it is open. Figure 2 shows a switch when it is closed.

A closed switch allows the electric current to flow freely from point A to point B. An open switch will not allow the current to flow at all.

PART I.

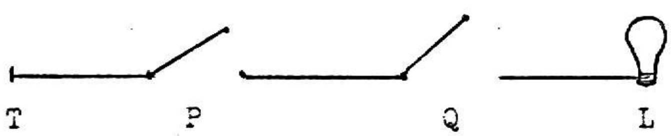

Figure 3

Consider the network of switches in Figure 3. Here we have two switches connected in a line. This type of a connection is called a <u>series</u> connection. The first switch is labeled P and the second switch is labeled Q. T labels the place where the switches connect with the source of electricity. L is the label for the light bulb.

Problem:

When will the light bulb light? Let us experiment to find out.

1. a. Close both switches in the series connection.
   b. Does the bulb light? ________
   c. Place your answer in the correct box in Figure 4.

2. a. Close only the switch labeled P. Leave the switch labeled Q open.
   b. Does the bulb light now? ________
   c. Place your answer in the correct box in Figure 4.

3. a. Do you think the bulb will light if you close only the Q switch and leave the P switch open? ________
   b. Try it. Does the bulb light? ________
   c. Place your answer to the question in part b in the correct box in Figure 4.

4. a. Now leave both switches open. What will happen?
   b. Does the bulb light? ________
   c. Place your answer in the table in Figure 4.

Table

Figure 4

Switch P	Switch Q	Does the bulb light?
closed	closed	
closed	open	
open	closed	
open	open	

Result:

From this table you can see that the bulb will light only when switch P is ________ and switch Q is ________.

PART II.

Now let us consider the conjunction of two propositions. Let $\overline{P}$ be a proposition and $\overline{Q}$ be a proposition. Define $\overline{P} \wedge \overline{Q}$ to be the conjunction of $\overline{P}$ and $\overline{Q}$.

Problem:

When will $\overline{P} \wedge \overline{Q}$ be a true statement?

Explanation:

To answer this question let us try to relate our findings in PART I to these propositions $\overline{P}$ and $\overline{Q}$. Define the lighted bulb to stand for a true conjunction.

1. Look at the switch in Figure 2.
2. The position of the switch is ________.
3. Can the current travel to point A to B with a switch in this position? ________ Will the bulb light? ________
4. Therefore, let a closed switch stand for a true proposition.
5. Now look at the switch in Figure 1. This switch is in the ________ position.
6. Can the current travel from point A to point B? ________ Will the bulb light? ________
7. Therefore, let an open switch stand for a false proposition.

Directions:

1. Look at Figure 4.
2. Review the definition of a true conjunction, statement 4, and the definition of a false conjunction, statement 7, under PART II.
3. Using 1 and 2 above complete the table in Figure 5.

Tables

Figure 4

Switch P	Switch Q	Does the bulb light?
closed	closed	yes
closed	open	no
open	closed	no
open	open	no

Figure 5

Proposition $\overline{P}$	Proposition $\overline{Q}$	Conjunction $\overline{P} \wedge \overline{Q}$
true	true	
true	false	
false	true	
false	false	

PART III.

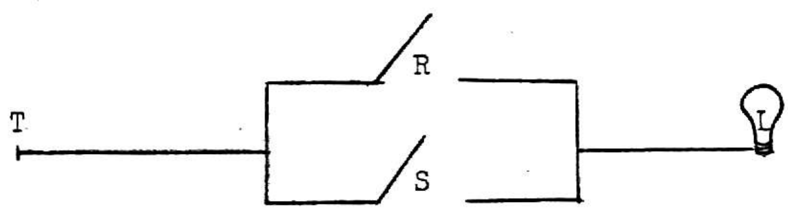

Figure 6

Consider the network of switches in Figure 6. Here we have two switches parallel to one another. This type of a connection is called a parallel connection. The first switch is labeled R and the second switch is labeled S. T labels the place where the switches connect with the source of electricity. L is the label for the light bulb.

Problem:

When will the light bulb light? Let us experiment to find out.

1. a. Close both switches in the parallel connection.
   b. Does the light bulb light? ________
   c. Place your answer in the correct box in Figure 7.

2. a. Close only the switch labeled R. Leave the switch labeled S open.
   b. Does the bulb light now? ________
   c. Place your answer in the correct box in Figure 7.

3. a. Do you think the bulb will light if you close only the S switch and leave the R switch open? ________

b. Try it. Does the bulb light? ______
c. Place your answer to the question in b in the correct box in Figure 7.

4. a. Now leave both switches open. What will happen?
b. Does the bulb light?
c. Place your answer in the table in Figure 7.

Table

Switch R	Switch S	Does the bulb light?
closed	closed	
closed	open	
open	closed	
open	open	

Figure 7

Result:

From this table you can see that the bulb will light when switch R is ______ and switch S is ______, when switch R is ______ and switch S is ______ or when switch R is ______ and switch S is ______.

PART IV.

Now let us consider the disjunction of two propositions. Let $\overline{R}$ be a proposition and $\overline{S}$ be a proposition. Define $\overline{R} \vee \overline{S}$ to be the disjunction of $\overline{R}$ and $\overline{S}$.

Problem:

When will $\overline{R} \vee \overline{Q}$ be a true statement?

Explanation:

To answer this question let us try to relate our findings in PART III. to these proposition $\overline{R}$ and $\overline{S}$. Define the lighted bulb to stand for a true disjunction.

1. Look at the switch in Figure 2.
2. The position of the switch is ______
3. Can the current travel from point A to B with a switch in this position? ______ Will the bulb light? ______
4. Therefore, let a closed switch stand for a true proposition.
5. Now look at the switch in Figure 1. This switch is in the ______ position.
6. Can the current travel from point A to point B? ______ Will the bulb light? ______
7. Therefore, let an open switch stand for a false proposition.

Directions:

1. Look at Figure 7.
2. Review statement 4 and statement 7 under PART IV.
3. Using 1 and 2 complete the table in Figure 8.

Tables

Switch R	Switch S	Does the bulb light?
closed	closed	yes
closed	open	yes
open	closed	yes
open	open	no

Figure 7

Proposition $\overline{R}$	Proposition $\overline{S}$	Disjunction $\overline{R} \vee \overline{S}$
true	true	
true	false	
false	true	
false	false	

Figure 8

CHAPTER IV

SUMMARY

The appendices to this report were included specifically for teachers interested in extending this report to a unit on logic for slower learning adolescents taught using the laboratory method.

Appendices B, D, E, and G were developed to help the teacher understand the needs of and organize lessons for the slower learning adolescent. Appendix B lists some of the needs and characteristics which a teacher must keep in mind when working with slower learning adolescents. Appendices D and E outline some of the considerations and objectives needed in developing lessons for the slower learning adolescent, while Appendix G examines criteria which can be used for judging procedures included in these lessons.

Appendices A, C, F, G, H, and I were developed to help teachers choose textbooks and prepared materials, as well as design their own materials and experiments for the laboratory. Appendix A is a bibliography of ideas for future laboratory experiments on logic. Appendices F and H expose the teacher to the types of materials and experiments which can be used in a laboratory situation, while Appendices C and G outline the considerations which must be taken into account when looking for or designing materials for the slower learning adolescent. Appendix I summarizes the criteria used in designing the sample experiment and was included to help teachers design future experiments.

ACKNOWLEDGMENTS

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APPENDIX A

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APPENDIX B

NEEDS AND CHARACTERISTICS OF SLOWER LEARNING ADOLESCENTS

1. Slower learners are usually slow in reading, arithmetic, and social studies.

2. They have a short attention span.

3. Their families usually have few if any books or magazines, and their parents are usually uninterested in school affairs.

4. These students need to experience success.

5. Their experiences and oral expression are limited.

6. The slower learner's future is usually a world of jobs, and during his school years may be alienated from his parents, teachers, and often fellow students.[1]

7. The adolescent years are the years of rapid, uneven growth and there is a need for both teachers and students alike to understand this growth and to realize that it is both variable and individual in nature.

8. Adolescents have a continual drive to seek personal independence from both parents and teachers.

9. They seek peer acceptance and have a need to belong to some social group.

10. They are insecure, primarily due to the tremendous physical change taking place at this age, and crave security and success.

11. They want recognition, approval, and status.

12. Their interests change rapidly; they crave new experiences while at the same time longing for the security of the old.[2]

13. They may have poor work habits and in need of an adequate curriculum because of poor preparation.

14. Each adolescent has individual problems, emotional, physical, family, language, cultural, brain damage, low I.Q., etc.[3]

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[1] William D. Boutwell, "Classroom Materials that Motivate the Slower Teen-age Learner," National Catholic Educational Association, 62:370-76, August, 1965.

[2] Max A. Sobel, Teaching General Mathematics (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), p. 3.

[3] Irving Allen Dodes, "Some Comments on General Mathematics," The Mathematics Teacher, 60:246-51, March, 1967.

15. Slower learners are less self-critical, poorer readers, more impulsive, and have a lower self-concept than "average" children.

16. They also need a sincere motivation for mathematics and to know that you care about them.[4]

Every child must also experience the developmental tasks of adolescents.[5]

17. Achieving new and more mature relations with agemates of both sexes.

18. Achieving a masculine or feminine social role.

19. Achieving one's physique and the use of the body effectively.

20. Achieving emotional independence of parents and other adults.

21. Achieving assurance of economic independence.

22. Selecting and preparing for an occupation.

23. Preparing for marriage and family life.

24. Developing intellectual skills and concepts necessary for civil competence.

25. Desiring and achieving socially responsible behavior.

26. Acquiring a set of values and an ethical system as a guide to behavior.

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[4]U. S. Department of Health, Education, and Welfare, Research Problems in Mathematics Education, Cooperative Research, Monograph No. 3 (Washington: Government Printing Office, 1960), p. 21

[5]Robert J. Havighurst, Developmental Tasks and Education (New York: David McKay Company, Inc., 1952),pp. 33-72.

APPENDIX C

PURPOSES OF GENERAL EDUCATION AND PRINCIPLES OF LEARNING

1. To contribute to the preparation for life needs, not only those which the student realizes but also those he must be taught to realize.

2. To establish basic relevance between knowledge and everyday experiences.

3. To provide a nonspecialized type of training characterized by wide application, universal value, and great intellectual appeal.

4. To lay the foundation of basic information essential to later intelligent pursuit of individual interests and special aptitudes.[1]

5. The individual learns most effectively in situations of a problem solving nature when he gains insight into the learning situation.

6. The individual learns most effectively when proceeding toward goals which he recognizes and accepts as his goals.

7. The individual learns most effectively through his own activity.

8. The individual uses his learnings most effectively when he sees relationships between previous learnings and new learnings and previous situations and new situations.

9. The individual learns most effectively when he has the freedom to determine his own pace.

10. The individual learns most effectively when he wants to do something to such an extent that he is willing to modify his behavior in order to do it.

11. The individual learns most effectively when he feels a reasonable degree of security, success, and worth.

12. The individual learns most effectively from a given learning activity when he is ready for the learnings involved.

13. The individual learns most effectively when he feels that his interests and present needs are being satisfied.

14. The individual retains and expands his learnings most effectively through repeated functional use rather than through isolated repetition.

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[1]Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Company, 1965), p. 44.

15. The individual learns most effectively when his whole self is involved.

16. The individual learns most effectively when he establishes new relationships among the learnings he has and between him and the factors in his environment.

17. The teacher must be sensitive to provide for the needs and abilities of each learner.

18. The teacher must define clearly his own purposes and plan carefully the procedures, activities, and methods of evaluation.

19. The teacher must help the learners establish worthwhile purposes.

20. The teacher must maintain an atmosphere in the classroom that is conducive to learning and marked by freedom and security.

21. The teacher must make the best use of his own and learners time, materials, and equipment.[2]

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[2]Statement from notes received in Dr. Marjorie Scherwitzky's education class at State University College at Oneonta, Oneonta, New York, Fall, 1965.

APPENDIX D

OBJECTIVES IN TEACHING MATHEMATICS TO THE SLOWER LEARNING ADOLESCENT

1. The student should have an understanding of mathematical processes and concepts.
2. The student should associate mathematical concepts with applications in community situations.
3. The student should be stimulated to demonstrate creativeness and imagination.
4. The student should understand the logical structure of mathematics and the nature of a proof.
5. The student should be stimulated to demonstrate such mental activities as visualization of problems.
6. The student should have the ability to use a general problem solving technique.
7. The student should use mathematics concepts to discover new generalizations or applications.[1]
8. The student should know that you care.
9. The student should be taught through the use of concrete experiences.
10. The student should have variety in his work with games, activities, use of audio visual aids, and supervised group practice at the chalkboard because of his short attention span.[2]
11. The students should have opportunities to succeed.
12. The students should have warm-up exercises to get him settled down.
13. The students should have an opportunity to learn through several senses at a time.[3]

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[1]Max A. Sobel and Donovan A. Johnson, "Analysis of Illustrative Test Items," Evaluation in Mathematics, Twenty-Sixth Yearbook of National Council of Teachers of Mathematics (Washington, D.C.: National Council of Teachers of Mathematics, 1961), pp. 71-92.

[2]Regis F. Crowley, "Teaching the Slow Learner," Today's Education, 58:49, January, 1969.

[3]Sarah B. Greenholz, "Reaching Low Achievers in High School Mathematics," Today's Education, 57:72, September, 1968.

14. Every student should also be taught to:

    a. think effectively
    b. communicate thought
    c. discriminate among values
    d. make relevant judgments
    f. do his part as an active and responsible citizen
    g. choose a vocation intelligently
    h. gain skill in adding to his previous knowledge
    i. find self-expression in and create an appreciation for things of beauty
    j. make sound emotional and social adjustments
    k. choose a vocation wisely
    l. realize and appreciate his cultural heritage
    m. understand his physical environment[4]

15. The teacher should try to avoid personality clashes with the students.

16. The teacher should overcome frustration and resentment with understanding, patience, and encouragement.

17. The teacher should try to project the slower learner into the future. There are many needed and respected occupations which require diligence and responsibility rather than great intelligence.[5]

18. The teacher should always be prepared for the day's lesson and try to make each lesson complete in itself, allowing time for supervised study and have the homework assignments representative of the class.[6]

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[4]Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book, Co., 1965), p. 44.

[5]Crowley, loc. cit.

[6]Greenholz, loc. cit.

APPENDIX E

CONSIDERATIONS IN DEVELOPING A LESSON FOR THE SLOWER LEARNER[1]

1. Assign problems which are real to the students.
2. Point out that the methods of attacking problems in life are similar to those in mathematics.
3. Show how mathematics helps us to understand our environment.
4. Give applications to domestic and vocational life.
5. Have students conduct surveys of their own on the relationships between mathematics and daily life.
6. Use field trips to explain concepts.
7. Use visual aids to help students understand the mathematical problems.
8. Consider the needs and characteristics of slower learners and the goals of education.

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[1] Maurice L. Hartung, "Motivation for Education in Mathematics," *The Learning of Mathematics: Its Theory and Practice*, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 48.

APPENDIX F

TYPES OF EXPERIMENTS AND ACTIVITIES

1. Make measurements of two variables in an everyday setting to determine the relationship between them.
2. Collect data by performing an operation.
3. Find a pattern by drawing or constructions.
4. Discover a scientific principle by performing an experiment with simple equipment.
5. Explore a relationship by manipulating simple objects.
6. Collect original data from a survey to determine what variables are related.
7. Extend the solution of a specific problem to a generalization.
8. Perform indirect measurements to determine unknown distances.
9. Represent a mathematical idea by building a simple model.
10. Complete an individual project or report.
11. Illustrate mathematical ideas by folding paper.
12. Build exhibits, charts, bulletin board displays.
13. Use commercial laboratory devices to perform experiments.
14. Use laboratory kits for laboratory work.
15. Plan, make, and use mathematical games, puzzles, and stunts.
16. Operate calculators and program computers.
17. Organize and operate a business enterprise.
18. Use an audio-visual aid as the basis for laboratory work.[1]
19. Ordering of objects in a series.
20. Examining the parts and the whole independently.[2]

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[1]Donovan A. Johnson and Gerald R. Rising, Guidelines for Teaching Mathematics, (Belmont, California: Wadsworth Publishing Company, Inc., 1967), pp. 302-306.

[2]Leslie W. Kindred, The Intermediate Schools (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968), p. 468.

21. Relating symbolic logic to basketball, checkers, and other games.

22. Teaching symbolic logic through the computer.[3]

23. Examine the logic of the *Life Career Games*.[4]

24. Examine mathematical tricks, games, and puzzles.

25. Work with diagrams and set theory.[5]

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[3]Julian Prince, "Jule Loves Susie," *American Education*, 4:24-5, April, 1968.

[4]Barbara Veranhorst, *Life Career Game* (Palo Alto, California: Palo Alto Univied School District, 1968).

[5]Maurice Hartung, "Motivation for Education in Mathematics," *The Learning of Mathematics: Its Theory and Practice*, Twenty-First Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 53.

APPENDIX G

CRITERIA FOR JUDGING PROCEDURES AND MATERIALS

Ideas of Maurice L. Hartung[1] to keep in mind while designing procedures and motivations:

1. Is the proposed procedure likely to be effective?

   a. Does it draw upon motives actually present in the learner?

   b. Is it designed to utilize a combination of several motives in the learner?

   c. Is it appropriate for the age level of the learner?

   d. Is it based upon recognition of a goal of the learner, and can the learner believe he can achieve the goal?

   e. Does it motivate many students or just a few?

   f. How long is the motivation likely to persist?

2. Is the motivation of desirable type?

   a. Does it lead the student to value the learning experience itself rather than external rewards?

   b. Will it widen and deepen the interests of the learner?

   c. Does it tend to develop desirable attitudes toward the content or skill and toward the teacher?

   d. Are the goals which are set actually attainable?

   e. Does the motivation tend to strengthen attitudes necessary for democratic citizenship?

   f. Is the motivation consistent with the promotion of good social relations between students?

3. Is the procedure practical?

   a. Is the required expenditure of time and money within the means of the school?

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[1]Maurice L. Hartung, "Motivation for Education in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), pp. 65-6.

b. How well can the procedure be controlled in practice?

c. Does the teacher know how to administer the procedure?

William D. Boutwell[2] suggests a list of things to watch for when teachers are searching for materials.

1. Look for sentences of length 10-15 words with simple structure. Avoid materials using hard words and contractions.

2. Look for materials with a conversation style that involves the reader as much as possible.

3. Look for materials that relate closely to the lives and personal interests of the students.

4. Look for child growth and development materials which are of interest to your particular grade level.

5. Look for materials which are humorous and look easy.

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[2]William D. Boutwell, "Classroom Materials that Motivate the Slower Teenage Learner," National Catholic Educational Association, 62:370-76, August, 1965.

APPENDIX H

LABORATORY MATERIALS[1]

1. An overhead projector complete with an acetate roll, transparency sheets, pencils, and other accessories.

2. Wall screens of appropriate size for use with the overhead projector and other audio-visual machines.

3. A complete library, including a large variety of books at various ability levels, magazines, experimental mathematics material, supplemental, colorful textbooks, recreational mathematics materials, pictures, posters, charts, programmed teaching materials, information on mathematics films and filmstrips, monographs, enrichment booklets, accompanying reading materials, occupational information relative to mathematics, and other reference materials.

4. Rigid models of various kinds to help in understanding the geometric designs and shapes, as well as becoming familiar with certain measuring instruments.

5. Storage cabinets, filing cabinets, exhibit cases, and bookcases to adequately store, protect, and exhibit the equipment and materials.

6. A tape recorder that may be used for individual or group instruction.

7. A combination slide and filmstrip projector accompanied with appropriate slides and filmstrips that may be used to clarify situations that arise in the course.

8. A controlled reading machine or tachistoscope with appropriate filmstrips that may be used to improve mental reading and computation abilities.

9. An electric calculator that may be used to perform lengthy manipulations.

10. A few simple hand adding machines that may be used, at first, to understand the concept of addition, and later, to free the pupil from boring computations.

11. Work tables that may be used during the experimental portions of the class period.

12. A variety of adjustable figures and models that are helpful in explaining geometric theorems.

13. A variety of instruments, such as pantographs, compasses, rulers, scissors, crayons, meter sticks, slide rules, drawing pencils, protractors, and T-squares that may be used in appropriate situations.

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[1]Eldert Groenendyk, "The Mathematics Laboratory," Midland Schools, 77:12, January, 1963.

14. A plainimeter to help create understanding of formulas for areas of regular and irregular figures.

15. A complete transit set that may be used in outdoor projects where angle measurement is necessary.

16. A stop watch that may be used in collecting data used in deriving generalizations and formulas.

17. A variety of manipulative devices and games to stimulate interest and help in clarifying basic concepts.

18. A magnetic chalkboard accompanied with magnetic devices, a pegboard, abacus, a flannel board may be used in explaining set theory, number lines, and numeration systems.

19. An adequate supply of chalkboard instruments that may be used in chalkboard work and demonstrations.

20. An opaque projector for showing materials that cannot be reproduced easily on transparencies.

21. The regular equipment and furniture required in classrooms, such as the teacher's desk, pupil desks, filing cabinets, typewriter, closet space, chalkboards, extension cords, convenient electrical outlets, proper light control for audio-visual equipment, and other minor items.

22. Special enrichment items, such as simple teaching machines, record player, simple tools, portable screen, and mathematics kits provide many enrichment possibilities.

23. Headphone sets may also be used for individual or group study.

24. The large electronic teaching machines with special booths may be used with prepared filmstrips for remedial and enrichment purposes.

25. A few drawing boards and instruments may be available for work in scale drawings.

26. Bulletin board and wall board displays for clarifying concepts and processes.

27. Paper and accessories such as construction paper, graph paper, filing cards, lined paper, and paste.

APPENDIX I

PRINCIPLES USED IN DESIGNING EXPERIMENTS

1. H. Van Engen[1] and Douglas Pidgeon[2] explain that the present curriculum of personal-social topics is not designed for the learning capacity of the slower learning student.

2. Max A. Sobel[3] and others agree that the present curriculum does not challenge the students since the emphasis is usually placed on drill rather than providing deeper understanding of the topics. One suggestion which was made was to teach the same basic concepts taught to average and faster learners in a more concrete situation.

3. Regis F. Crowley,[4] Max A. Sobel,[5] and others suggested that a mathematics curriculum based on the characteristics and needs of a slower learning student must:

    a. Motivate the student.
    b. Refer to prior mathematical experiences.
    c. Have opportunities to explore the interests, aptitudes, and capacity of pupils.
    d. Lead carefully and slowly from easier mathematical material to the difficult.
    e. Emphasize method rather than just finding the right answer.
    f. Be a mathematical program rather than just socially applied mathematics teaching mathematical processes and concepts.
    g. Present a variety of problems.
    h. Teach the student through the use of concrete experiences with specific and clear instructions.
    i. Take into account the rapidly changing interests and the craving for new experiences of these students by having a variety of games, activities, and audio-visual aids.
    j. Stimulate the student to demonstrate creativeness and imagination.
    k. Give the student opportunities to succeed.

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[1]H. Van Engen, "Arithmetic in the Junior-Senior High School," The Teaching of Arithmetic, National Society for the Study of Education Fiftieth Yearbook, Part II (Chicago, Illinois: The University of Chicago Press, 1951), pp. 115-116.

[2]Douglas A. Pidgeon (ed.), Achievement in Mathematics (The Mere, England: National Foundation for Educational Research in England and Wales, 1967), p. 124.

[3]Max A. Sobel, Teaching General Mathematics (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), p. 13.

[4]Regis F. Crowley, "Teaching the Slow Learner," Today's Education, 58:49, January, 1969.

[5]Sobel, op. cit., pp. 3-5.

4. According to Biggs[6] and Marjorie Scherwitzky[7] some of the more recent ways of achieving greater understanding of mathematical concepts for the slower learner are to:

   a. Develop concepts introduced earlier.
   b. Introduce concepts which are appropriate to the slower learning students.
   c. Start teaching from where the student is and not where he is expected to be.
   d. Present new ideas related to other subjects through the mathematics laboratory and community resources.
   e. Have the student learn through his own activity.
   f. Have the student see relationships between previous learnings and new learnings.
   g. Allow the student to learn at his own pace.
   h. Expose a student to a given learning activity when he is ready for the learning involved.
   i. Allow the student to involve his whole self in learning activities.
   j. Allow the student to establish new relationships between him and the factors in his environment.

5. Max A. Sobel and Donovan Johnson[8] and H. Van Engen[9] suggested that some of the objectives of a mathematics program should be:

   a. To help the student demonstrate creativeness and imagination.
   b. To help the student to think logically.
   c. To help the student to use mathematical concepts to discover new generalizations or applications.
   d. To help the student establish a means of attacking real problem situations by Kinney's method of
      1. understanding the problem
      2. analyzing and organizing the problem
      3. recognizing the process required
      4. solving and verifying the answer.

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[6] Edith E. Biggs, "Mathematics Laboratories and Teachers' Centres--the Mathematics Revolution in Britain," The Arithmetic Teacher, 15:408, May, 1968.

[7] Statement from notes taken in Dr. Marjorie Scherwitzky's education class at State University College at Oneonta, Oneonta, New York, Fall, 1965.

[8] Max A. Sobel and Donovan A. Johnson, "Analysis of Illustrative Test Items," Evaluation in Mathematics, Twenty-Sixth Yearbook of National Council of Teachers of Mathematics (Washington, D.C.: National Council of Teachers of Mathematics, 1961), pp. 71-92.

[9] Engen, op. cit., pp. 104-05.

6. The studies by Patrick Suppes and Frederick Binford[10] and John Hodges[11] indicate that logical reasoning patterns are important and useful and can be learned by adolescents in an intermediate school regardless of prior mathematical achievement.

7. Bernard H. Gundlach[12] and others state that the fundamental principles to be taught in a unit on logical reasoning patterns should include the ability:

   a. To recognize invalidity and validity.
   b. To make logically correct inferences.
   c. To use and understand negations, conjunctions, disjunctions, implications, and definitions.
   d. To use inductive reasoning and deductive reasoning.
   e. To analyze and apply techniques and skills used in logical thinking.
   f. To think consistently, clearly, precisely, and creatively.

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[10] Patrick Suppes and Frederick Binford, Experimental Teaching of Mathematical Logic in the Elementary School (Stanford: Stanford University Press, 1964), 16-9, 24.

[11] John Raymond Hodges, "A Study of the Abilities of a Group of Eighth Grade Students to Learn and Use Certain Mathematical Concepts," Dissertation Abstracts, 24:5430, June, 1964.

[12] Bernard H. Gundlach and others, Junior High School Mathematics 8, River Forest, Illinois: Laidlaw Brothers Publishers, 1968, pp. 23-50.

LABORATORY EXPERIMENT ON LOGIC FOR

SLOWER LEARNING ADOLESCENTS

by

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B.S., State University College at Oneonta, 1968

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mathematics

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

The two major purposes for designing this study were to review the available literature relating the needs and characteristics of the slower learning adolescent to his ability and need to understand the material presented in a unit on logical reasoning patterns taught using the mathematics laboratory method and to develop a sample laboratory experiment to enable teachers to make use of the laboratory method in the classroom. The experiment is presented in the form of a guide sheet designed for seventh, eighth, or ninth grade slower learning adolescents.

The procedures employed in this study consisted of (1) analyzing textbooks, reference books, and magazine articles at Kansas State University on the slower learning adolescent, mathematical logic, and the laboratory method of teaching, (2) designing a format for the sample guide sheet to be used by the students in the laboratory, and by the teacher for future reference to develop more laboratory experiments, and (3) developing the appendices to be used by teachers wishing to expand this study to a unit on logical reasoning patterns.

The review of literature covered the following topics:

1. Identifying the slower learning adolescent.
2. Needs and characteristics of a slower learning adolescent.
3. The teacher's role in teaching the slower learning adolescent.
4. Attitudes concerning the present mathematics curriculum.
5. Specialists who advocate the teaching of logical reasoning patterns.
6. Evidence supporting the ability of the slower learning adolescent to learn and to use logical reasoning patterns.
7. Mathematical concepts to be covered in a unit on logical reasoning patterns.

8. Advocates for the use of the laboratory method.

9. Organizing a mathematics laboratory.