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AN EXPERIMENTAL INVESTIGATION OF THE CUTTING
ACTION OF A SINGLE DIAMOND

by

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TABLE OF CONTENTS

Chapter	Page
LIST OF TABLES	iii
LIST OF FIGURES	iv
NOMENCLATURE AND SYMBOLS USED	vii
I. INTRODUCTION	1
II. DESCRIPTION OF EXPERIMENT	7
III. EXPERIMENTAL PROCEDURE	10
IV. EXPERIMENTAL RESULTS	12
V. TANGENTIAL FORCE	41
VI. DIAMOND WEAR	50
VII. CONCLUSION	55
VIII. RECOMMENDATIONS	56
References	57
Appendix	
1. Rock Properties	58
2. Curve Fitting Procedure and Program	60

LIST OF TABLES

Table	Page
1 Continuous Cuts.	22
2 Single Groove Cuts - Surface Preparation at .010" Feed, .020" Depth.	22
3 Single Groove Cuts - Surface Preparation at .005" Feed, .020 Depth	23
4 Effective Depth Constant	23
5 Single Groove Cuts - Surface Preparation at .0025" Feed, .002" Depth.	33
6 Tangential Force Constants - Single Groove Cuts, Surface Preparation at .005" Feed, .010" Depth	48
7 Tangential Force Constants - Single Groove Cuts, Surface Preparation at .010" Feed, .010" Depth	49
8 Tangential Force Constants - Single Groove Cuts, Surface Preparation at .0025" Feed, .002" Depth.	49
9 Various Wear Parameters.	50

LIST OF FIGURES

Figure		Page
1	Typical Diamond Mill Cutter.	3
2	Typical Petroleum Diamond Bit	3
3	A Single Diamond Tool	4
4	Test Dynamometer	6
5	Rock as Tested	6
6	Two Types of Diamond Cutting	9
7	Continuous Cut - Force vs. Speed	13
8	Single Groove Cuts - Force vs. Speed, Surface Preparation at .010" Feed, .020" Depth	16
9	Single Groove Cuts - Force vs. Depth, Surface Preparation at .010" Feed, .020" Depth	17
10	Single Groove Cuts - Force vs. Speed, Surface Preparation at .010" Feed, .005" Depth	18
11	Single Groove Cuts - Force vs. Depth, Surface Preparation at .010" Feed, .005" Depth	19
12	Effective Depth of Continuous Cuts	20
13	Procedure Used to get Effective Depth	21
14	The Exponent B vs. Depth	25
15	Relative Effects of Subsurface Damage	26
16	Single Groove Cuts - Force vs. Speed, Surface Preparation at .0025" Feed, .002" Depth	31

Figure		Page
17	Single Groove Cuts - Force vs. Depth, Surface Preparation at .0025" Feed, .002" Depth	32
18	Results for Single Groove Cuts - Force vs. Speed, Surface Preparation at .010" Feed, .020" Depth	34
19	Results for Single Groove Cuts - Force vs. Depth, Surface Preparation at .010" Feed, .020" Depth	35
20	Results for Single Groove Cuts - Force vs. Speed, Surface Preparation at .005" Feed, .010" Depth	36
21	Results for Single Groove Cuts - Force vs. Depth, Surface Preparation at .005" Feed, .010" Depth	37
22	Results for Single Groove Cuts - Force vs. Speed, Surface Preparation at .0025" Feed, .002" Depth	38
23	Results for Single Groove Cuts - Force vs. Depth, Surface Preparation at .0025" Feed, .002" Depth	39
24	Force vs. Speed Results for Continuous Cuts	40
25	Tangential Force vs. Speed - Surface Preparation at .010" Feed, .020" Depth	42
26	Tangential Force vs. Speed - Surface Preparation at .005" Feed, .010" Depth	43
27	Tangential Force vs. Speed - Surface Preparation at .0025" Feed, .002" Depth	44
28	Tangential Force vs. Speed Data for Continuous Cuts . . .	45
29	General Shape of Tangential Force Curve	46
30	Schematic Diagram of a Worn Diamond	51
31	Granite Test Fixture	52

Figure		Page
32	Worn Diamond Profile	52
33	Worn Diamond Flat Spot	52
34	Strength vs. Confining Pressure	58
35	Strength vs. Strain Rate	59

NOMENCLATURE AND SYMBOLS USED

A	Coefficient of force vs. speed relationship
A_o	Constant in tangential force equation
A_1	Constant in tangential force equation
A_w	Area of diamond flat spot
B	Exponent of speed relationship
B_o	Constant in tangential force equation
B_1	Constant in tangential force equation
C	Constant in effective depth equation
C_T	Constant in tangential force equation
C_1	Constant in force correction equation
C_2	Constant in force correction equation
FPM	Cutting speed in feet per minute
F_{ex}	Experimentally measured cutting force
F_{th}	Theoretical Cutting Force
H	Depth of single groove cuts
H_e	Effective depth of continuous cuts
H_{wd}	Depth of diamond wear
m	Slope of effective depth vs. speed line
r	Diamond radius
W	Constant in speed exponent equation
X	Abcissae
Y	Ordinate
X_1	Speed data points

Y_1	Force data points
Z	Constant in speed exponent equation
λ	Diamond wear angle

CHAPTER I

INTRODUCTION

Recently there has been increasing use of diamond tools to cut non-metallic materials. Diamond drill bits and core bits have long been used by the oil industry to drill hard formations. The increasing amount of off shore drilling and increasing depth of holes on land, where down time is very expensive, has put new interest in the use and performance of diamond bits. New refractories and ceramic materials require diamond tools for economical cutting and machining.

In many cases the costs involved are high and there is significant economic need to optimize the design and operation of diamond tools. A large diamond drill bit can cost \$20,000.00 and down time on an off shore drill rig can cost \$1,200.00 per hour. Even a small improvement in design and operating conditions can produce significant benefits.

So far, nearly all diamond tool design has been more of an art than a science. Most of this design has been based on trial and error and experience gained from previously built tools. Only recently has there been much progress in understanding the action of diamond tools. The amount of previous work is still limited, and much of the work done is primarily concerned with metal cutting i.e.; Loladze & Bokuchava [6] and Lindenbeck [5].

Most rock and many other non-metallic materials have a peculiar property which causes them to machine differently than other materials. Rock is ordinarily thought of as brittle, but the high pressure which occurs

under a diamond may cause rock failure to change from brittle to ductile with an accompanying increase in strength. This effect is shown in Appendix [1]. Custers [3] has produced continuous chips while machining glass. This is a dramatic example of ductile failure beneath a diamond tool.

Almost all diamond tools contain many diamonds set in some pattern, held in place by a matrix. Fig. [1] and [2] show typical diamond tools. The cutting action of diamond tools such as drill bits, milling cutters and saws depends on the cumulative cutting action of the individual diamonds. Therefore the study of cutting can be reduced to studying the action of a single diamond. A single diamond tool is shown in Fig. [3]. The basic problem is then to determine the cutting forces and the volume of material removed by a single diamond.

Using the concept of ductile failure of the rock Appl, etal. [1], [2], [8], and [9] have developed a theory of cutting for a single spherical diamond. A spherical diamond is an idealization of actual diamonds, which in an actual tool are of random shape, being roughly spherical. Whatever their actual shape they cut by plowing a groove, as a spherical tool does, rather than cutting with a sharp edge. The purpose of this work was to experimentally verify the theoretical work. The experiments consisted of cutting with a single spherical diamond on a cylindrical rock mounted in a lathe and measuring the cutting forces. This action of a sphere plowing a groove in rock is complex and not well understood. Therefore it is not surprising that some modifications to the theory were necessary to correlate theory and experiment. In the course of analysing the experimental data two important new factors were discovered. Although cutting speed was known to be a factor it was not included in previous theoretical analysis due to insufficient knowledge

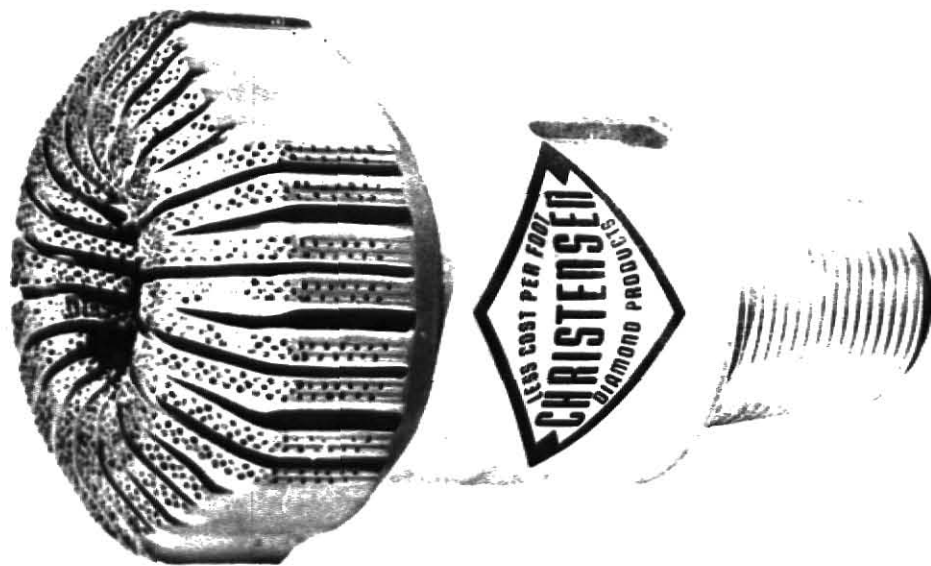
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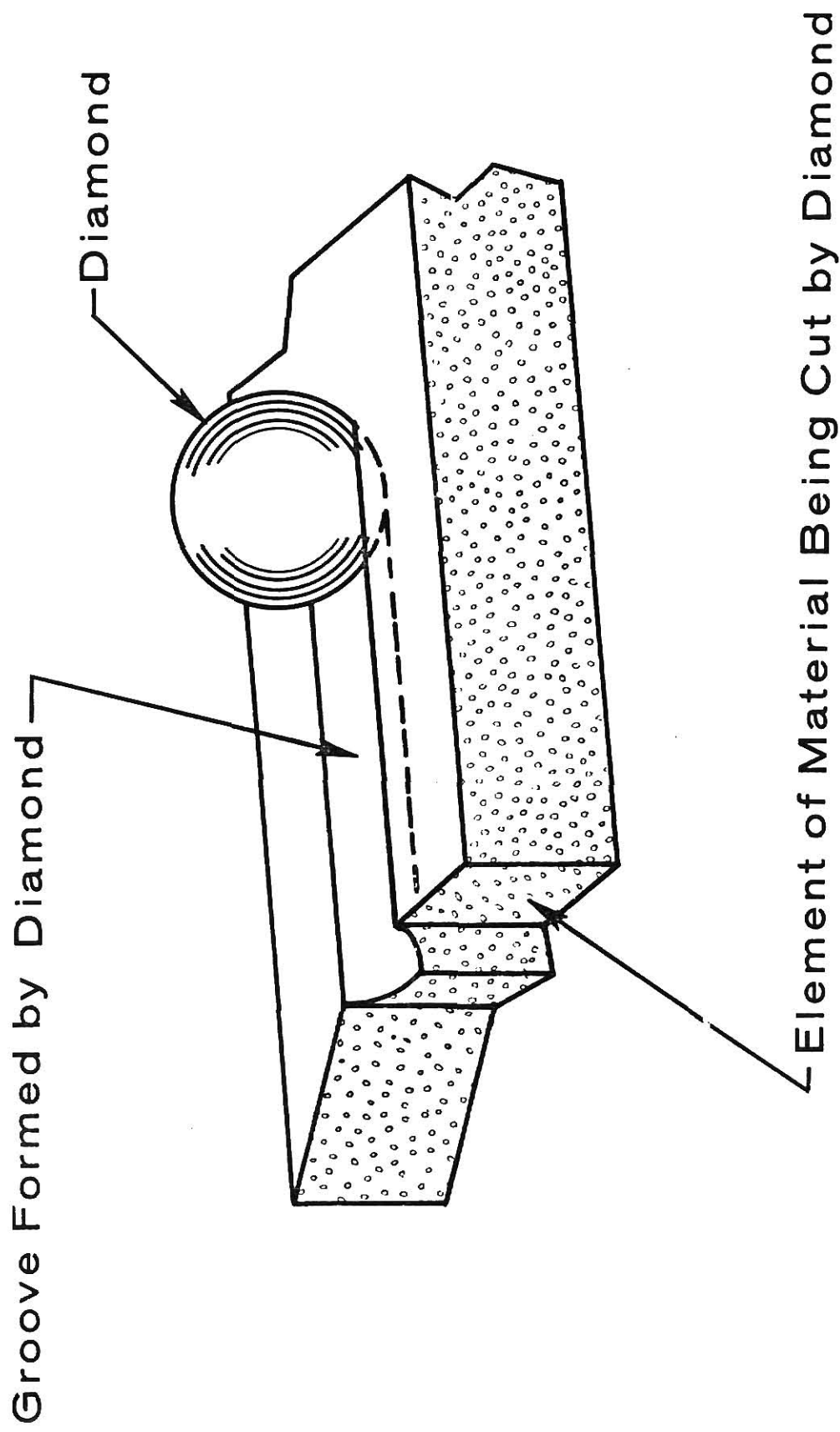
Typical Diamond Mill Cutter

Figure 1



Typical Petroleum Diamond Bit

Figure 2



Schematic Diagram of a Single Diamond Cutting Tool

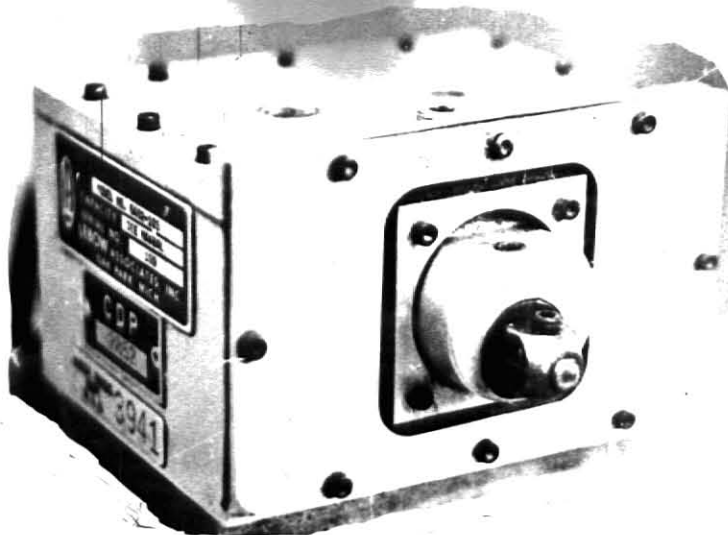
Figure 3

of rock properties. The experimental data obtained indicate a definite speed relationship. A factor depending on subsurface grain damage was also discovered. This was completely unexpected.

Ultimately it would be desirable to have sufficient information to mathematically design a diamond tool, possibly using a computer. Toward this end it was decided to attempt to describe the effects of these new factors by some mathematical equation.

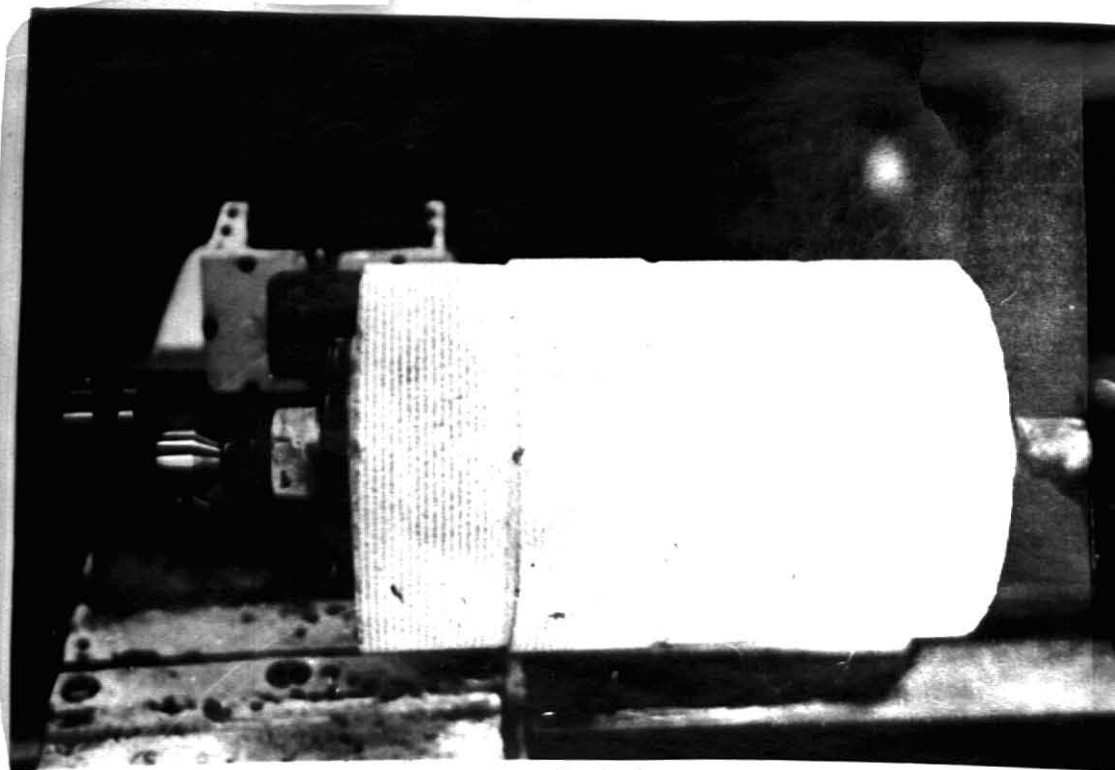
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Test Dynamometer

Figure 4



Rock as Tested

Figure 5

CHAPTER II

DESCRIPTION OF EXPERIMENT

The experimental procedure consisted of rotating a cylindrical piece of rock in a lathe and pressing a spherical diamond tool into the rock. Forces acting on the diamond were measured with a specially built dynamometer, Fig. [4]. The forces were recorded on two Sanborn recorders. A constant flow of coolant water was provided.

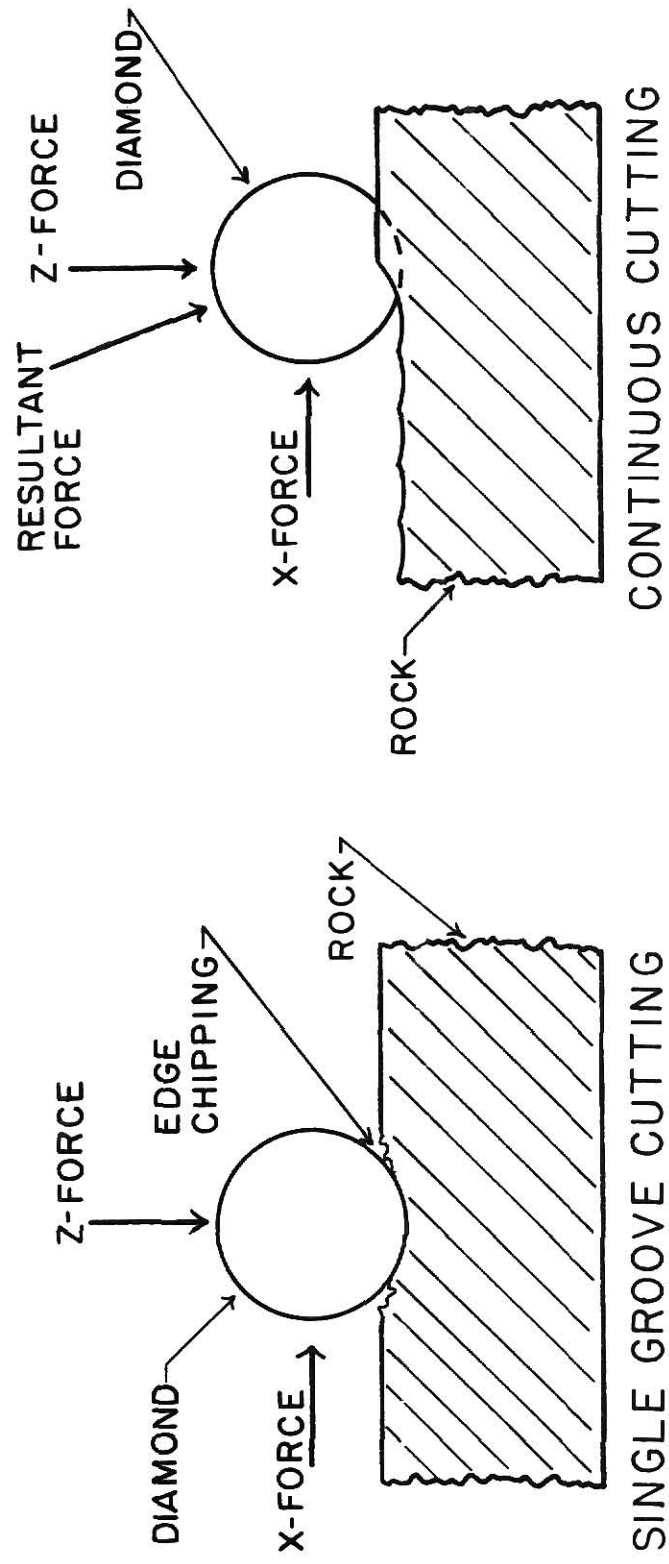
Some early work was done by Appl and Hansen with the rock chucked in a lathe and a carbide spherical tool cutting on the face of the rock. This was not entirely satisfactory since the surface speed was changing as the tool progressed toward the center of rotation and because the tool was not cutting a straight line. Some early work was also done using a carbide sphere and a cylinder of rock mounted as in Fig. [5]. This is the same set-up that was used in all subsequent work, with the exception that the carbide tool was replaced with a 1/8 inch spherical diamond tool. Unfortunately the lathe used had only fixed speeds of rotation and this proved to be somewhat of a handicap later on.

The rock used throughout the tests was Indiana Limestone. A large cylinder was cut out with a core bit and then a 1 inch hole was bored through the center to accept a mandrel. Three grooves were cut in the surface, as can be seen in Fig. [5]. This divided the rock into four segments, so that tests could be made on each segment separately. Time was saved since the surface of all four segments could be prepared with single passes.

The dynamometer used was especially designed and built for this project by Lebow. It had a 100 pound range in the normal direction and a 50 pound range in the axial and tangential directions. Calibration resistors could be switched into the strain gage bridge of each force component. These resistors produced an output corresponding to a known force. In this way the recorders could be easily calibrated. The normal and tangential forces were recorded on the two channels of one Sanborn recorder, the axial force was recorded on one channel of another Sanborn recorder.

The theoretical analysis is based on single groove cuts; that is the diamond is plowing a single symmetrical groove. In actual practice each diamond usually overlaps the path it made on previous passes. Tests can be easily made for each condition. Each of these conditions is shown in Fig. [6].

During single groove cutting the normal force is well defined and easily measured, but no accurate method has been found to measure the volume of rock removed. This is partly due to the fact that the edges of the groove are somewhat irregular due to chipping. During overlapping cutting the volume can be easily found from the depth, total distance traveled by the diamond, and the total volume removed, but the normal force cannot be directly compared to the single groove normal force since the cut is not symmetrical. Thus to correlate these two types of cuts a concept of effective depth was developed. The effective depth is defined to be that single groove depth which produces the same normal force as the resultant force for overlapping cuts. It is believed that when the normal cutting forces are the same the groove volumes will be nearly equal. Thus the volume removed during single cuts can be determined from the volume removed during continuous cuts at the same effective depth.



Two Types of Diamond Cutting

Figure 6

CHAPTER III

EXPERIMENTAL PROCEDURE

It was necessary to devise a procedure to obtain experimental results to compare with the single groove theory and in addition to verify the concept of effective depth. A number of different runs were made using both diamond and carbide tools. In all the tests the surface of the rock was prepared by making two or more cuts at a small depth, usually .005 inch or .010 inch, and a feed low enough to cause the cuts to overlap. Then a single groove cut was made at various depths and a feed large enough to insure that the cuts did not overlap or interfere with each other. The forces in three directions were recorded during the last surface preparation cut and during the single groove cut. In all cases a sufficient number of surface preparation cuts were made to completely remove all traces of the grooves left from the previous single cut runs.

As an example of this procedure take the following case. The lathe speed is set at 150 RPM, the diameter of the rock is measured to yield the surface speed, the depth of cut is set at .010 inch and the feed set at .005 inch/rev. Three cuts are made across the rock at these conditions with the forces recorded on the last pass. The depth is then set at .003 inch and the feed set at .120 inch and a cut is made across the first segment. Then the depth is changed to .005 inch and the second segment is cut and so on for .007 inch depth and .010 inch. A feed of .120 inch will produce single grooves at all depths.

After these tests were completed and the analysis begun an important effect of subsurface grain damage was discovered. This effect was taken into account in the expression that was developed. Direct comparison with the theory was difficult however since the theory was developed for zero subsurface damage. Therefore it was decided to make an additional test in which the subsurface damage was much lower.

The surface was prepared in three stages. First, two cuts were made at .005 inch feed and .010 inch depth, then three cuts of .0045 inch feed and .004 inch depth and then three cuts at .0025 inch feed and .002 inch depth. The single groove cut was then made. Forces were recorded during the finest surface preparation cuts and during the single groove cuts. By making finer and finer cuts it was hoped to get a surface near to virgin rock.

CHAPTER IV

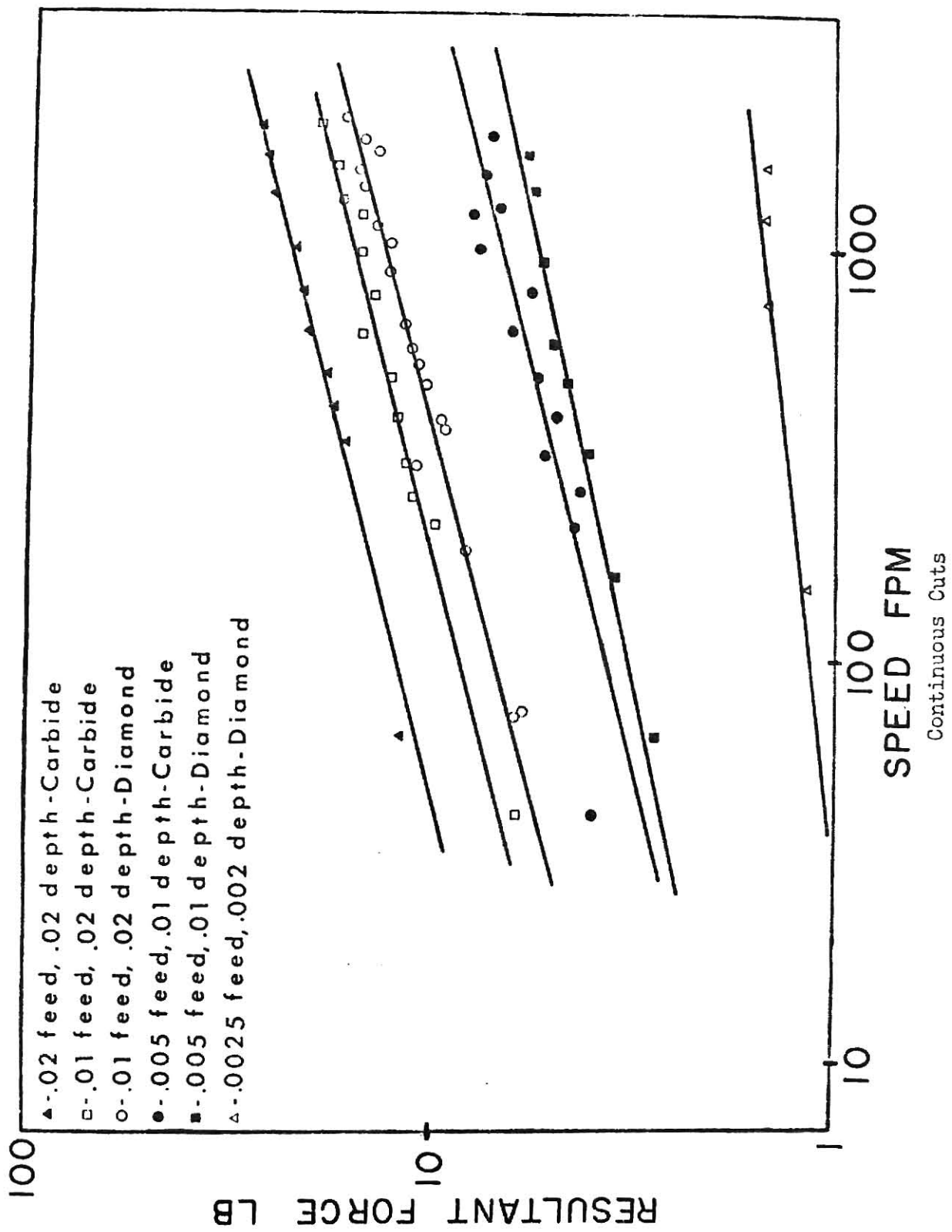
EXPERIMENTAL RESULTS

It was quickly discovered that the cutting forces in all cases were dependent on cutting speed. The theory had assumed that the force was independent of speed.

After some initial plotting and cross plotting it was decided that log (resultant force) vs. log (speed) for one combination of feed and depth came very close to being a straight line. Thus the equation describing the data was of the form $y = Ax^B$. The data had enough scatter so that it was questionable whether or not the straight lines corresponding to each surface preparation condition had the same slope. This question was felt to be important enough to mathematically fit the lines rather than trusting an eye fit. A type of least squares fitting procedure was employed. This procedure appears in Appendix [2].

A computer program was written to solve for the A and B of each line given the x_i 's and y_i 's. See Appendix [2]. This program was used to generate the lines shown in Fig. [7].

The data representing normal force of the single cuts was much more scattered, but still seemed to reduce to a straight line on log-log paper. There were two sets of data for single groove cuts under the same conditions of depth, feed and speed. At this time it was assumed that these two sets of data should be the same, within the limits of experimental error. Attempts were made to plot all the data together. Points were quite scattered and it



became clear that there were differences in the two data sets that had to be explained by means other than experimental error. It was later discovered that the difference was due to different amounts of subsurface damage. With the data separated scatter became less but was still enough to make line fitting somewhat questionable. Many attempts were made at using the same program as was used for the continuous cut data. In most cases the correlation coefficients were poor; due primarily to the low number of data points and high scatter. The computer based lines did not seem to have any definite relationship to each other at different depths. This was the final factor that led to abandoning the computer as a means of fitting.

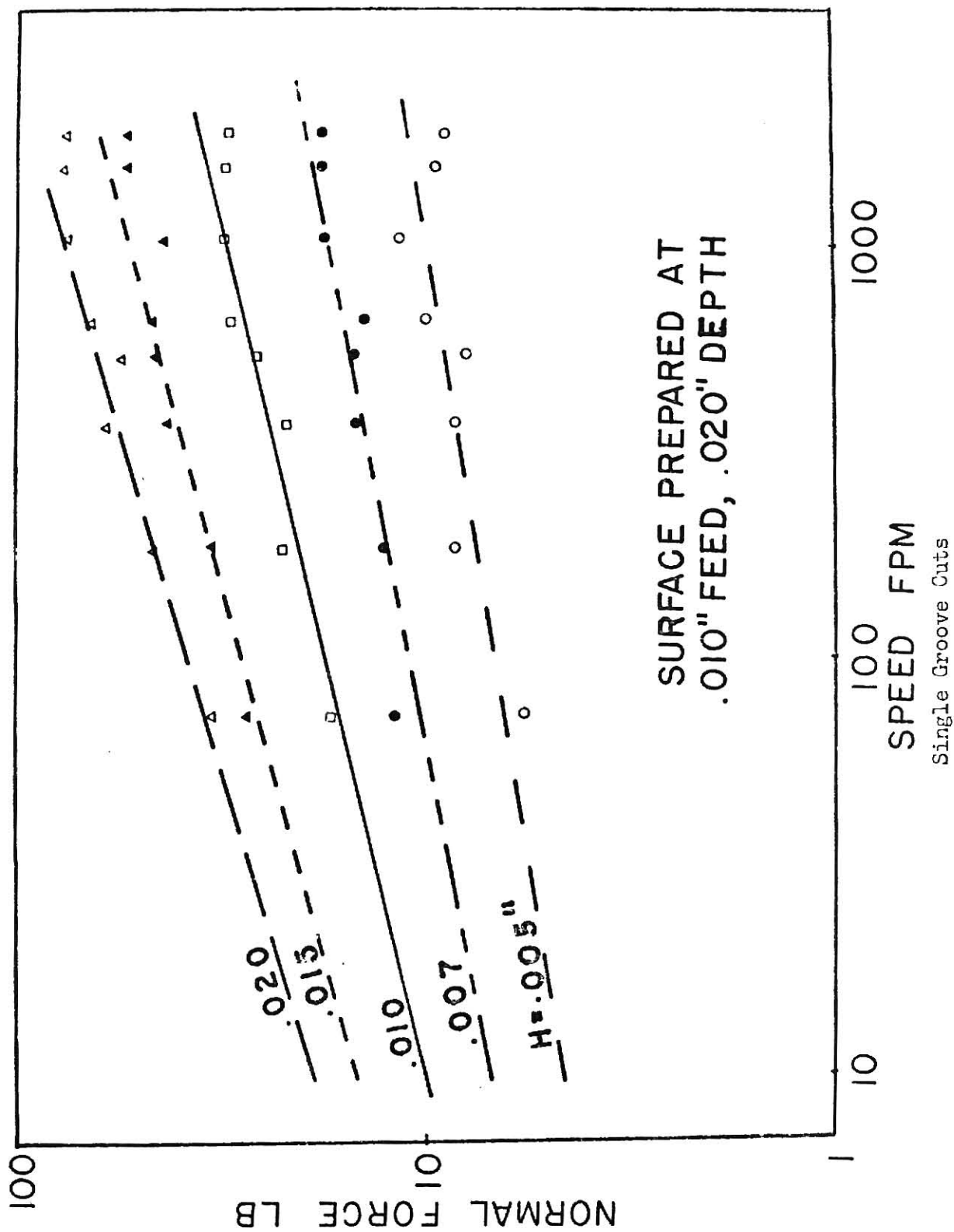
The next fitting procedure was graphical. The procedure began by plotting the data on log-log paper; force vs. speed. A trial straight line was then put through the points corresponding to each depth. A speed was then picked and a force was read from the trial line at each depth. Another plot was made with data from the trial lines, but this time the data was plotted with force vs. depth for a fixed speed. It was here assumed that some reasonably smooth relationship existed between all the data points of one data set.

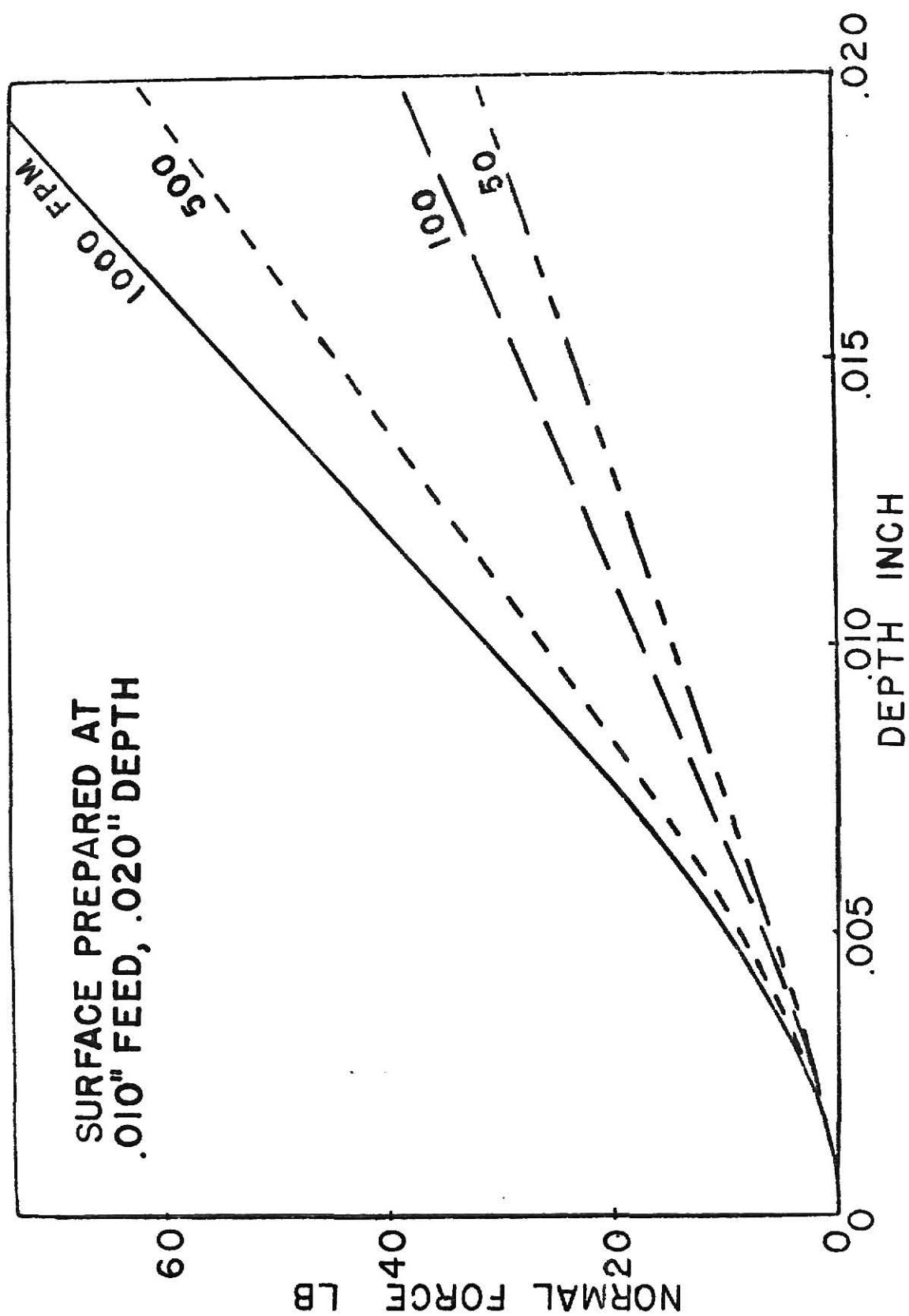
In this case the force is a function of two variables, speed and depth. If a smooth curve results from plotting force vs. speed holding depth constant a smooth curve should also result from plotting force vs. depth holding speed constant. Of course the curve will not necessarily be of the same form. This was done for one speed. The trial lines were then shifted till force vs. depth was a smooth curve and the trial lines still fit the data acceptably well. This was then repeated for several different speeds, covering the range of speeds in question. By trial and error shifting of the lines smooth curves

were created for all speeds on the force vs. depth graph while keeping a good fit on the data points. The results of this procedure are shown in Fig. [8] and Fig. [9]. This procedure was also applied to the second set of single groove data. These results are shown in Fig. [10] and Fig. [11].

The question then arose as to whether or not effective depth was constant with changing speed. The lines plotted for continuous cuts in Fig. [7] together with the plot of force vs. depth at different speeds in Fig. [9] and Fig. [11] were used to graphically determine the effect of changing speed on effective depth. At speeds corresponding to the speeds used in the single groove force vs. depth plots forces from the force vs. speed plot of continuous cuts were read off lines representing cutting conditions that were the same as the conditions of surface preparation for the single groove plots. These forces were then plotted on the force vs. depth plot to yield effective depth points. These depths were then plotted vs. speed. These points came close to lying on a straight line if plotted $\log(\text{force})$ vs. speed. Initially all the continuous cut lines were applied to each single groove plot and variations of effective depth for all the continuous cutting conditions were found. Only the line corresponding to the same subsurface damage was judged to be valid. The resulting plot of effective depth is shown in Fig. [12]. Figure [13] is a sketch of the procedure which may help to clarify the explanation. The results of applying these equations to the graphs are shown in Tables 1, 2, 3, and 4.

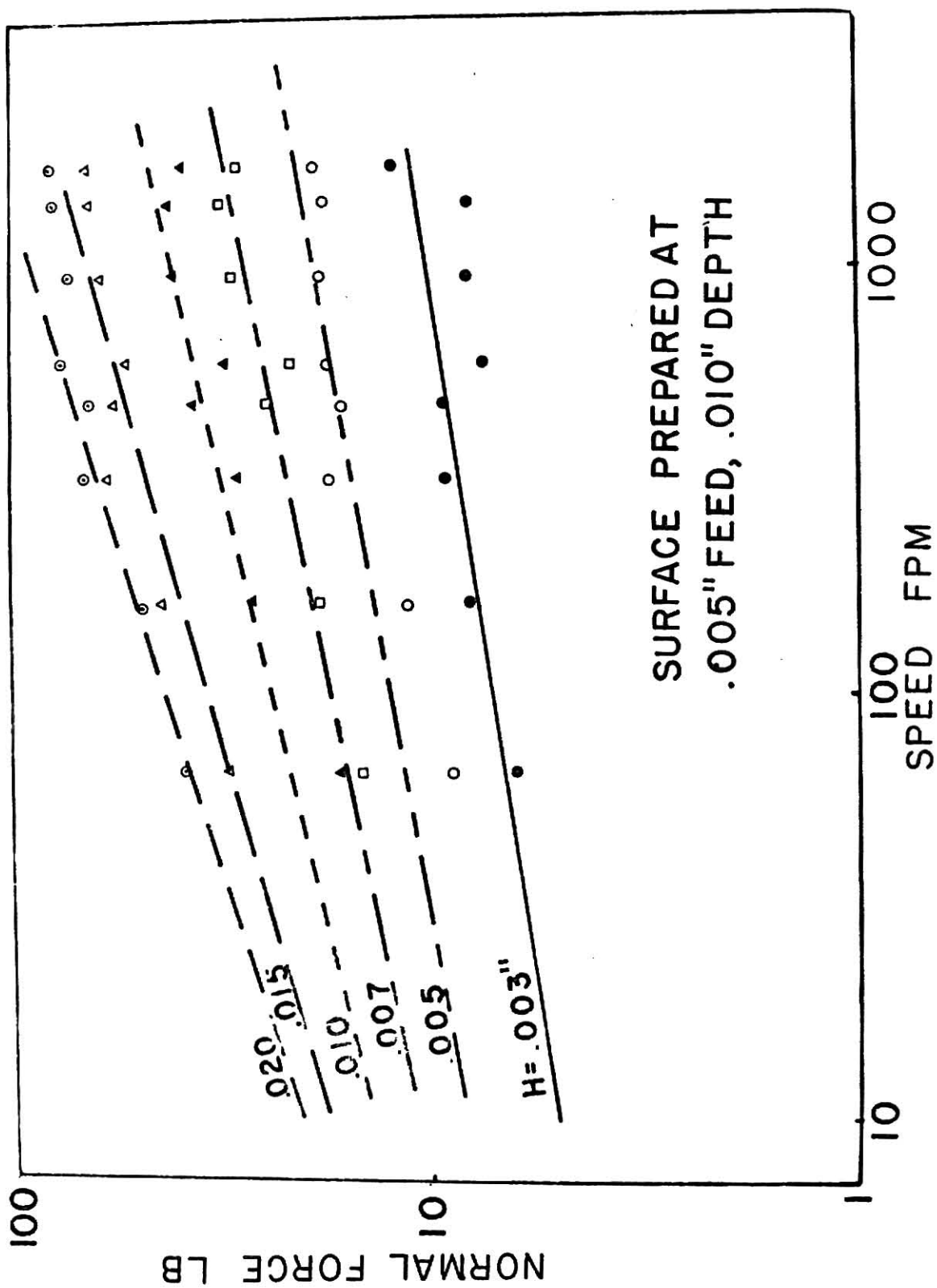
The sketch in Fig. [13] is exaggerated for clarity. Actually the speed curves lie close to each other in the region being used and the effective depth points all fall very close to each other, so the process is not very precise. However it was felt that the results were consistent enough to

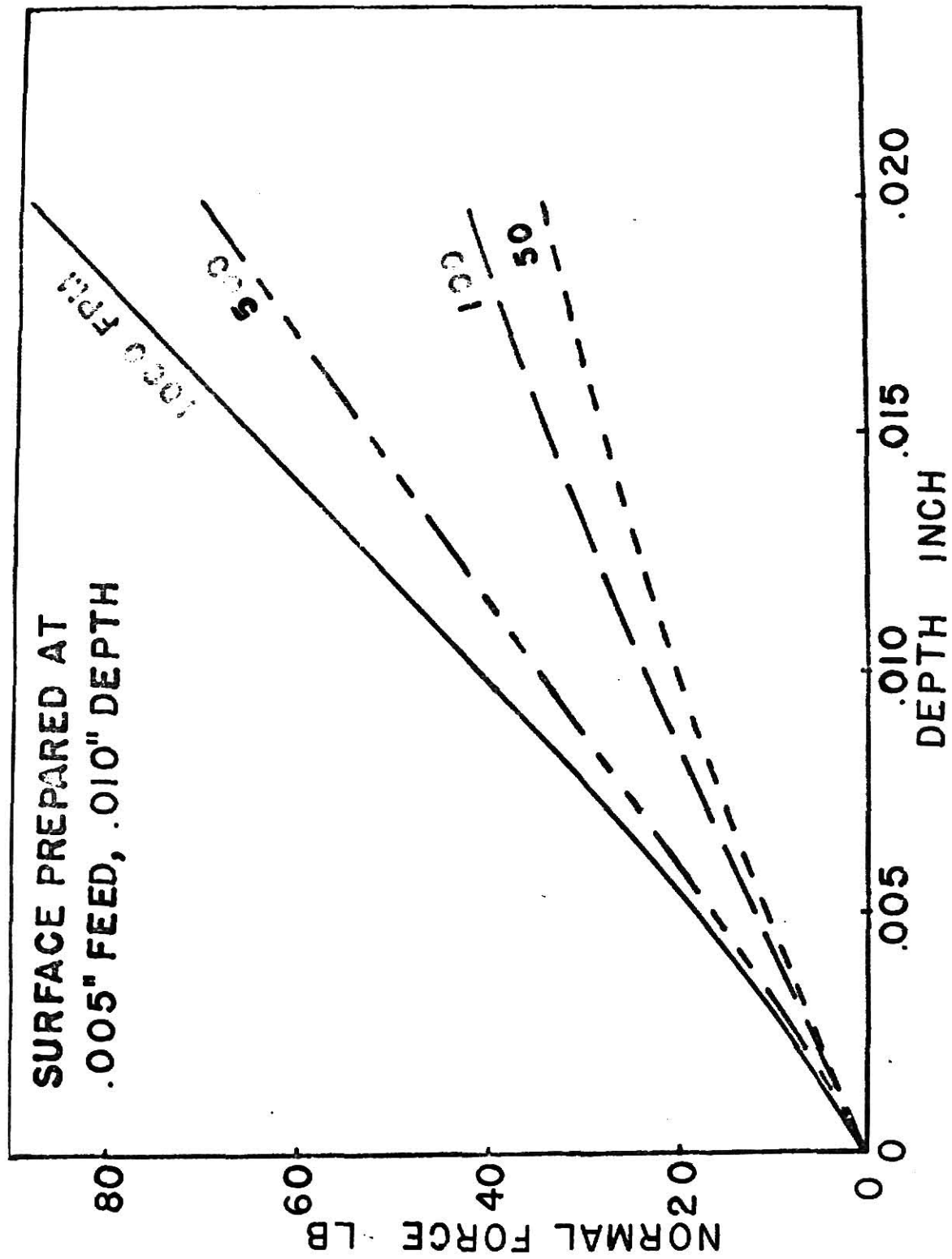




Single Groove Cuts

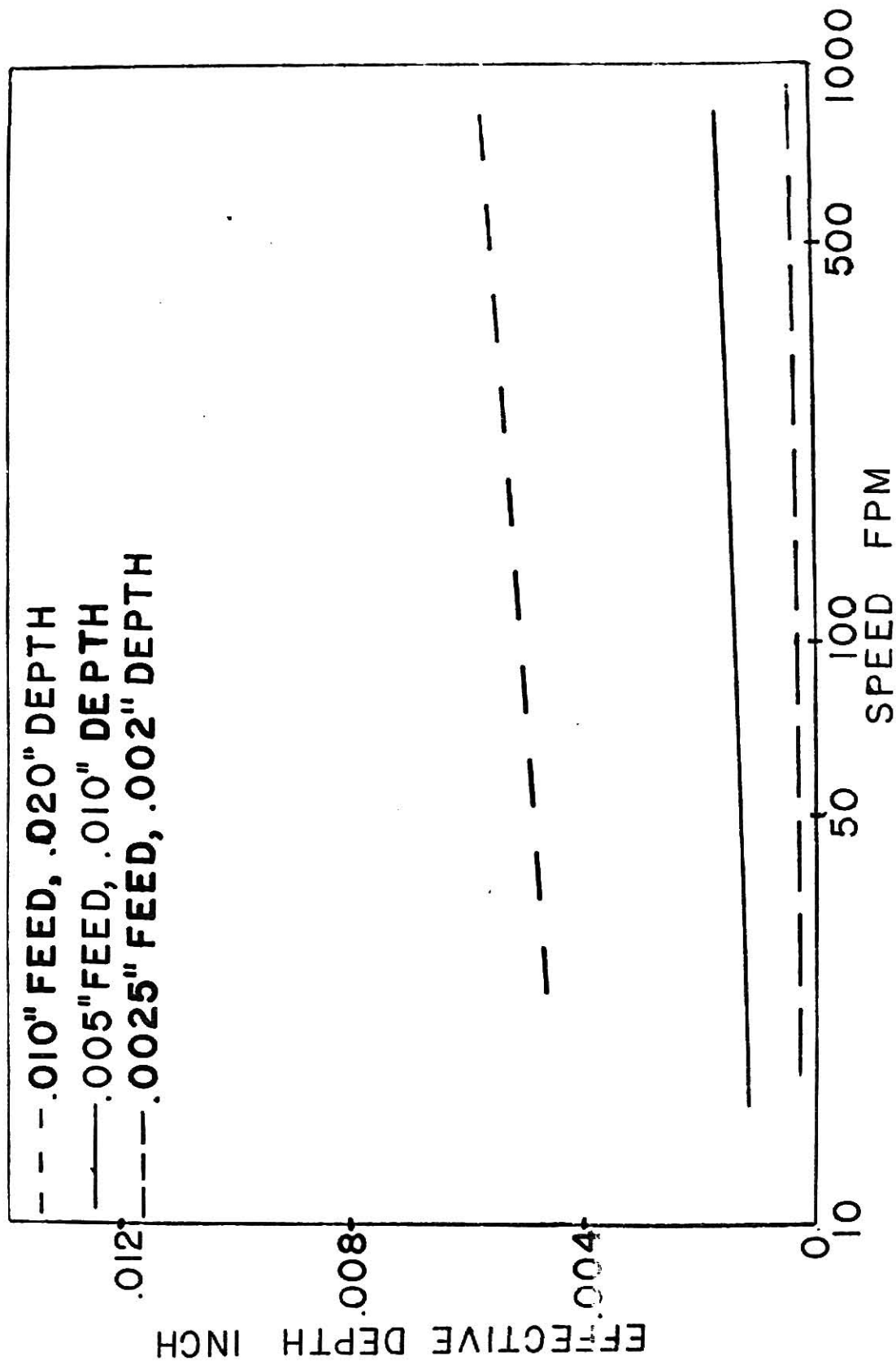
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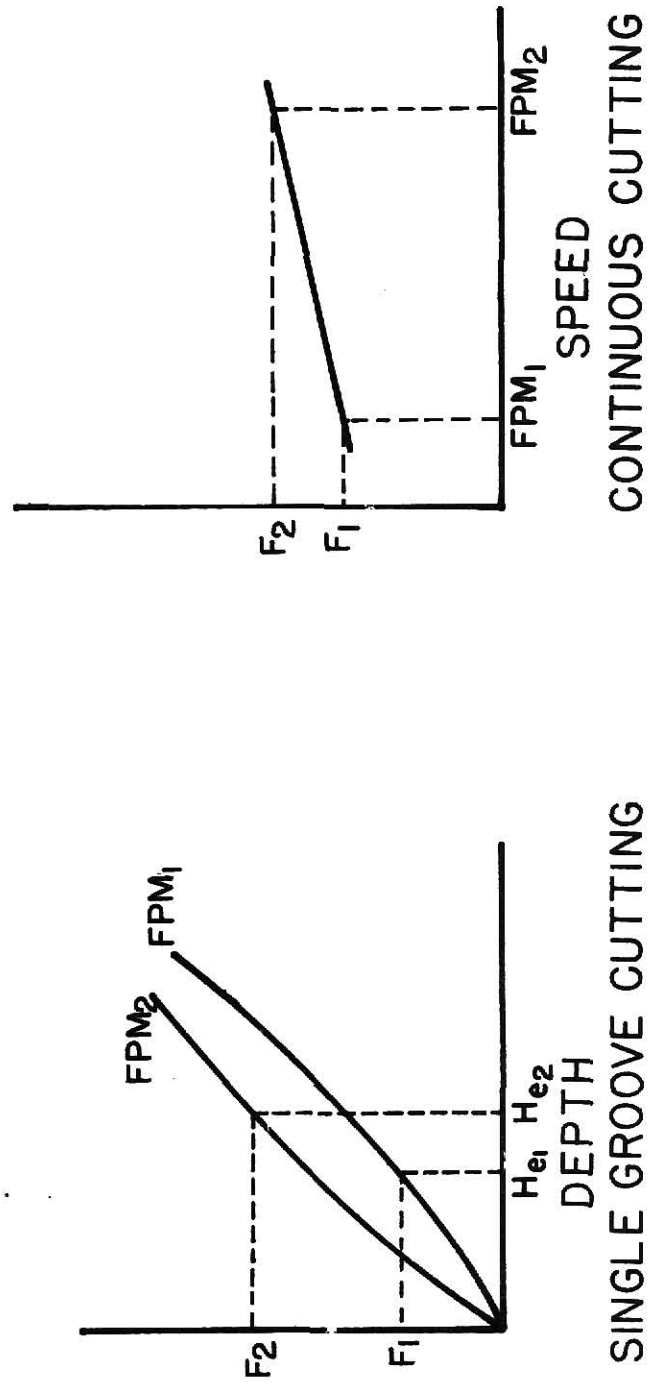
Single Groove Cuts

Figure 11



Effective Depth of Continuous Cuts

Figure 12



Procedure Used to Get Effective Depth

Figure 13

Table 1

$$F = A(FPM)^B \text{ - Continuous Cuts}$$

Tool	Conditions	A	B
carbide	.020" feed .020" depth	3.455	.273
carbide	.010" feed .020" depth	2.542	.265
diamond two data sets combined	.010" feed .020" depth	2.008	.273
carbide	.005" feed .010" depth	1.172	.255
diamond three bad points removed	.005" feed .010" depth	1.205	.2214
diamond	.0025" feed .002" depth	.685	.110

Table 2

$$F = A(FPM)^B \text{ - Single Groove Cuts}$$

Surface Preparation at .010" feed, .020" depth

$$\text{Feed} = .120"$$

depth	A	B
.020"	9.068	.3097
.015"	7.663	.2799
.010"	5.810	.2400
.007"	4.333	.2021
.005"	2.875	.1800

Table 3

$F = A(FPM)^B$ - Single Groove Cuts
 Surface Preparation at .005" Feed, .010" Depth

Feed = .120"

Depth in	A	B
.020	9.750	.318
.015	8.995	.284
.010	8.106	.231
.007	6.736	.201
.005	5.474	.1755
.003	3.589	.1483

Table 4

Effective Depth Constants
 $F = m \log(FPM) + C$

Conditions	m	C
.010" feed, .020" depth	.000755	.003645
.005" feed, .010" depth	.00045	.0007
.0025" feed, .002" depth	.000035	.000275

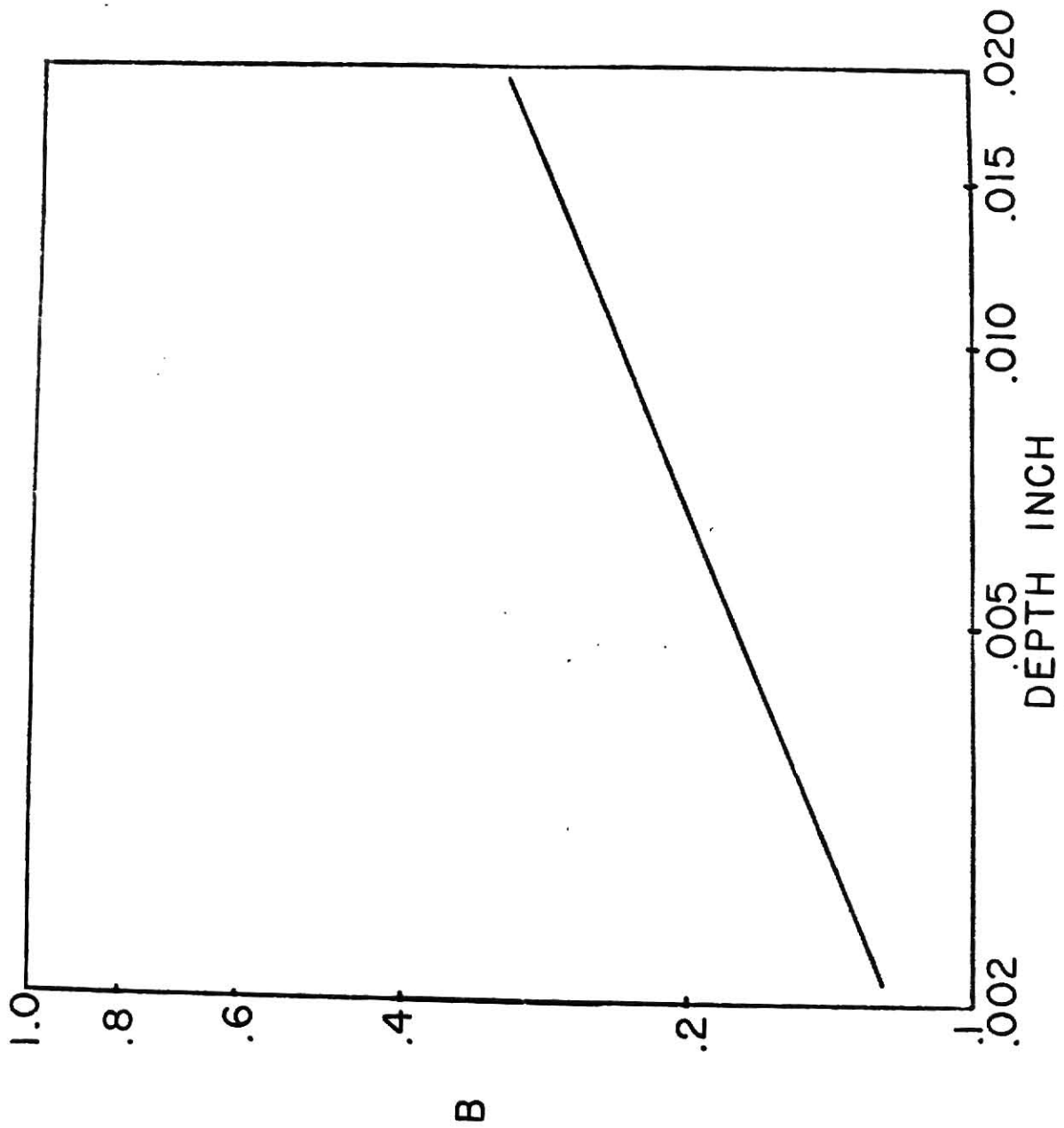
warrant reasonable confidence in the results.

The desired end result of the analysis was a mathematical equation giving force as a function of theoretical force and the various cutting parameters. In order to accomplish this the graphs involved had to be reduced to mathematical relationships. In regard to the continuous cuts; $\log(\text{force})$ vs. $\log(\text{speed})$ for different conditions of feed and depth plotted as a straight line. Thus an equation of the form $y = Ax^B$ will describe the data. An equation of the same form will describe the single groove data. In a similar fashion effective depth vs. $\log$ speed plotted as a straight line. This data can be described by an equation of the form $y = m\log x + C$.

The next step was to tie in the second variable, depth, in a mathematical way to the equation describing force and speed ie; $\text{force} = \text{function}(\text{speed}, \text{depth})$. In the cases of single groove cuts the plot of $\log(\text{force})$ vs. $\log(\text{speed})$ resulted in a straight line described by the equation $F = A(\text{FPM})^B$. The exponent B is the slope of these straight lines. It was found that B was the same for each similar depth regardless of the surface preparation. Furthermore it was found by plotting, that $\log(B)$ vs. $\log(\text{depth})$ resulted in a straight line. See Fig. [14]. Therefore the slope, B, can be expressed with an equation of the form $B = W(\text{depth})^Z$. W and Z were determined from the graph.

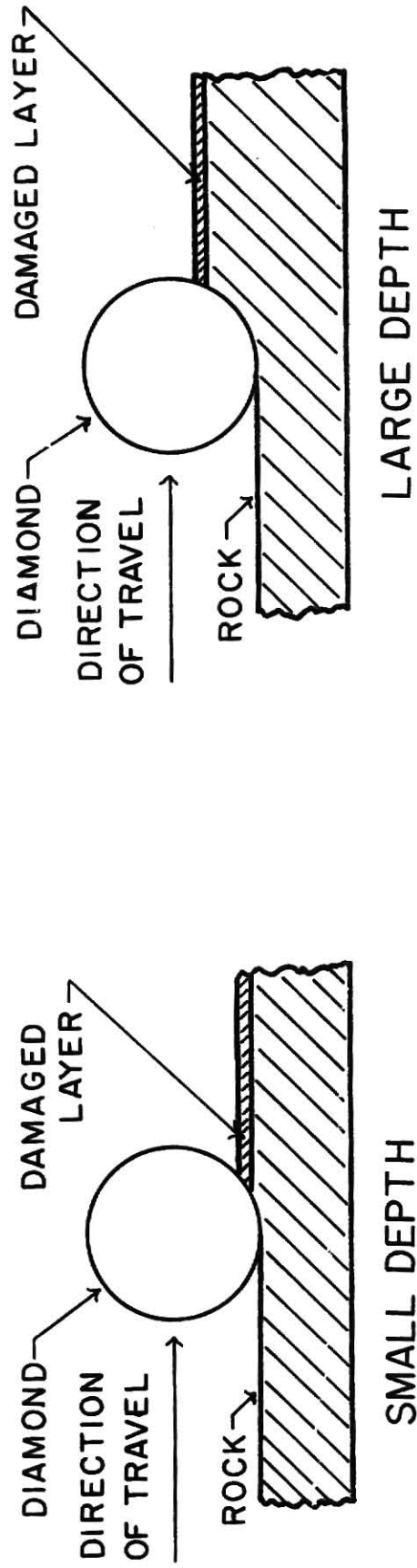
Various plots were made of the values of A and no general relationship could be found that described them very well. Fortunately it was not necessary to get a relationship between the A's.

As was pointed out previously the single groove data taken under the same conditions except for surface preparation did not agree well. This disagreement was particularly evident at small depths. Thus surface pre-



The Exponent B vs. Depth

Figure 14



Relative Effects of Subsurface Damage

Figure 15

paration had an important effect on the cutting force. The previous theory had not taken any such effects into account.

When the tool cuts a groove some of the grains of rock below the tool are damaged and crushed but still remain attached to the surface. See reference [7]. It was evident that the effects of subsurface damage caused differences in the single groove data. The fact that the effect was more pronounced at low depths of cut can be easily explained by Fig. [15]. It is evident that at a low depth of cut the relative amount of damaged material cut away compared to the undamaged material is much greater than in the case of a large depth. At very low depths the tool might not even reach undamaged rock.

It was believed that the depth to which subsurface damage extended was roughly proportional to the effective depth of the surface preparation cuts. Therefore effective depth was used as an indication of subsurface damage.

The desired end result of the analysis was to obtain the same type of correction factor which could be applied to the previous theory. Since speed and subsurface damage were not accounted for in the theory, they needed to be factors in the correction equation: single groove experimental force = a function of theoretical force, speed, effective depth, and depth. The feeling was that it would be best to have the theoretical force multiplied by an expression involving experimental variables: $F_{ex} = F_{th} \times \text{function}(FPM, h_e, h)$. The speed effect posed a problem. $F_{ex} = A(FPM)^B$ was used to describe the speed effect. This equation does not behave well in the neighborhood of zero speed. The ideal result would have been that at zero speed and zero effective depth the theoretical force equals the experimental force. The speed relationship chosen did not allow comparison at zero speed,

so another speed was needed as a starting point. It was assumed that the theoretical force should correspond to zero subsurface damage since the derivation of the theory was based on a homogeneous cutting medium. It was felt that the best hope of finding a suitable equation was to separate the various effects in order to make solving for the various constants easier:

$$F_{ex} = F_{th} \times \left(\text{terms to take care of the subsurface damage effect} \right) \times \left(\text{terms to take care of the speed effect} \right)$$

Since it was being assumed that at some speed and zero subsurface damage the experiment and theory should agree, the first bracketed term should reduce to 1 at zero effective depth and the last bracketed term should reduce to 1 at the same speed. Each of the speed effect relationships involved speed raised to the B power. Where $B = W H^Z$ for all cases. The speed effect that was decided upon is shown in the following equation:

$$\left[\frac{FPM}{42} \right]^B = \left[\frac{FPM}{42} \right]^{W H^Z}$$

This term reduces to 1 when speed equals 42 regardless of H(depth).

Many different expressions were tried to account for the subsurface damage. The requirements of this term were: it should equal 1 if $H_e = 0$ for all values of H, it should approach 1 as H increases, it should decrease if H_e is increased. After many tries on the computer the equation shown below was finally chosen as the best and simplest.

$$1 - A e^{\frac{-C_1}{H_e}} e^{-C_2 H}$$

The final general form of the equation is:

$$F_{ex} = F_{th} \left[1 - A e^{\frac{-C_1}{H_e}} e^{-C_2 H} \right] \left[\frac{FPM}{42} \right]^{W H^Z}$$

W and Z were found previously and the value for speed was found by trial and error to get the best fit. A computer program was then written to solve for the remaining constants, A, C_1 and C_2 . Points were picked off the graph and the constants solved for simultaneously.

It became questionable at this point as to whether there was sufficient information to justify the conclusions, so additional experiments were made. The best kind of test would have been to make single groove cuts on a surface with no subsurface damage, but this wasn't feasible. Hence to minimize subsurface damage several progressively lighter surface preparation cuts were made. The final surface preparation cuts were made at .0025 inch feed and .002 inch depth. Data was recorded the way as it was in the previous tests. The data was reduced in the same manner as before. Less data was taken on this test because of the large amount of time required to prepare the surface, especially at low speeds. Some of the data from this test is shown in Figs. [7], [12] and Table 4. Figure [16] and Fig. [17] show the single groove data for this test. The B values for these lines are given in Table 5. These values agree with the line in Fig. [14]. Applying the derived equation to the three cases of single groove cuts gives the results shown in Figs. [18], [19], [20], [21], [22] and [23]. If H in the equation is set equal to H_e , the equation should be applicable to continuous cuts with the surface prepared at the same conditions as the cutting conditions. This was done for the continuous cuts and the results are shown in Fig. [24].

The final form of the equation is:

$$F_{ex} = F_{th} \left[1 - Ae^{\frac{-C_1}{H_e}} e^{-C_2 H} \right] \left[\frac{FPM}{42} \right]^W H^Z$$

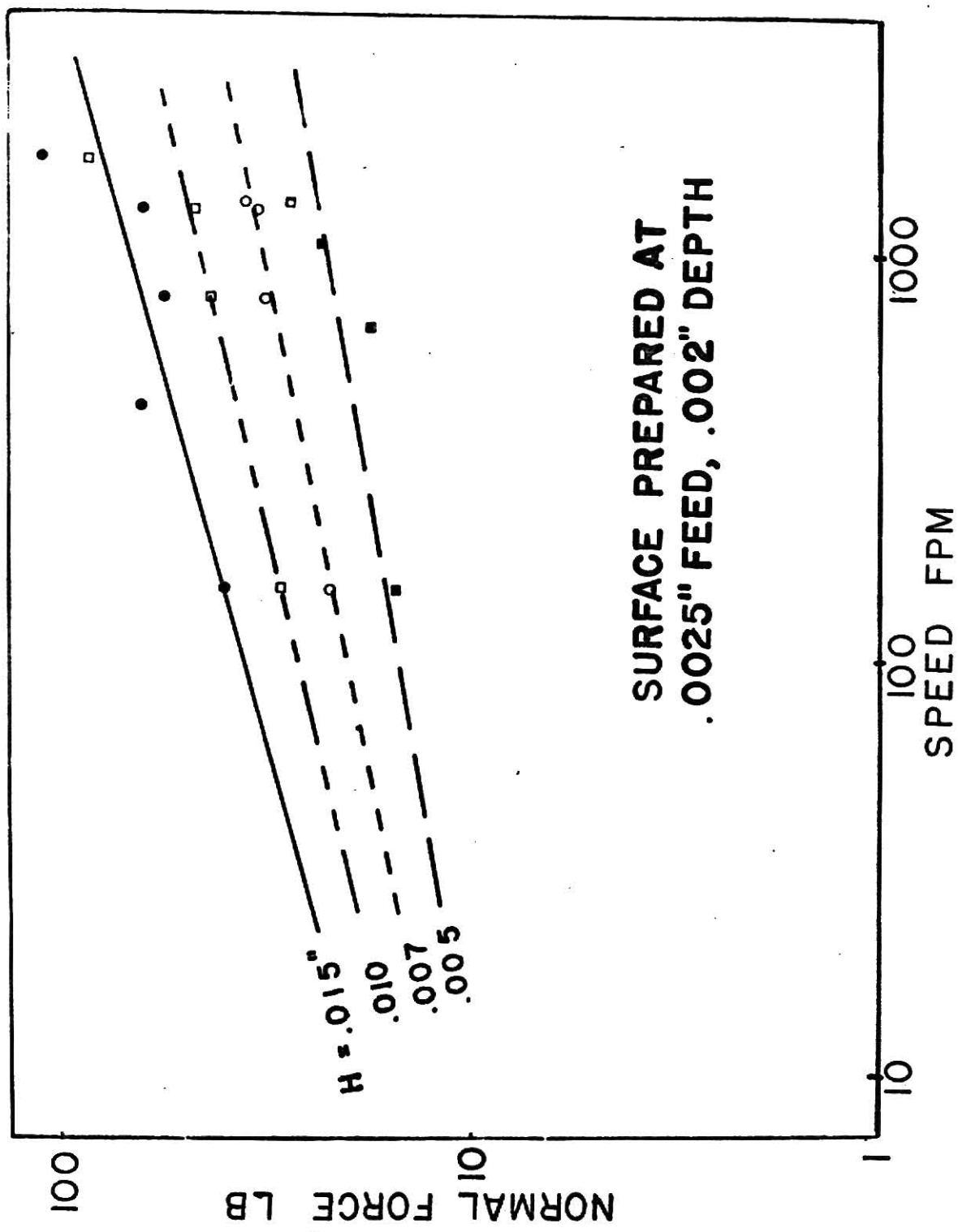
where: $C_1 = .001346328$

$$C_2 = 87.76093$$

$$A = 1.166724$$

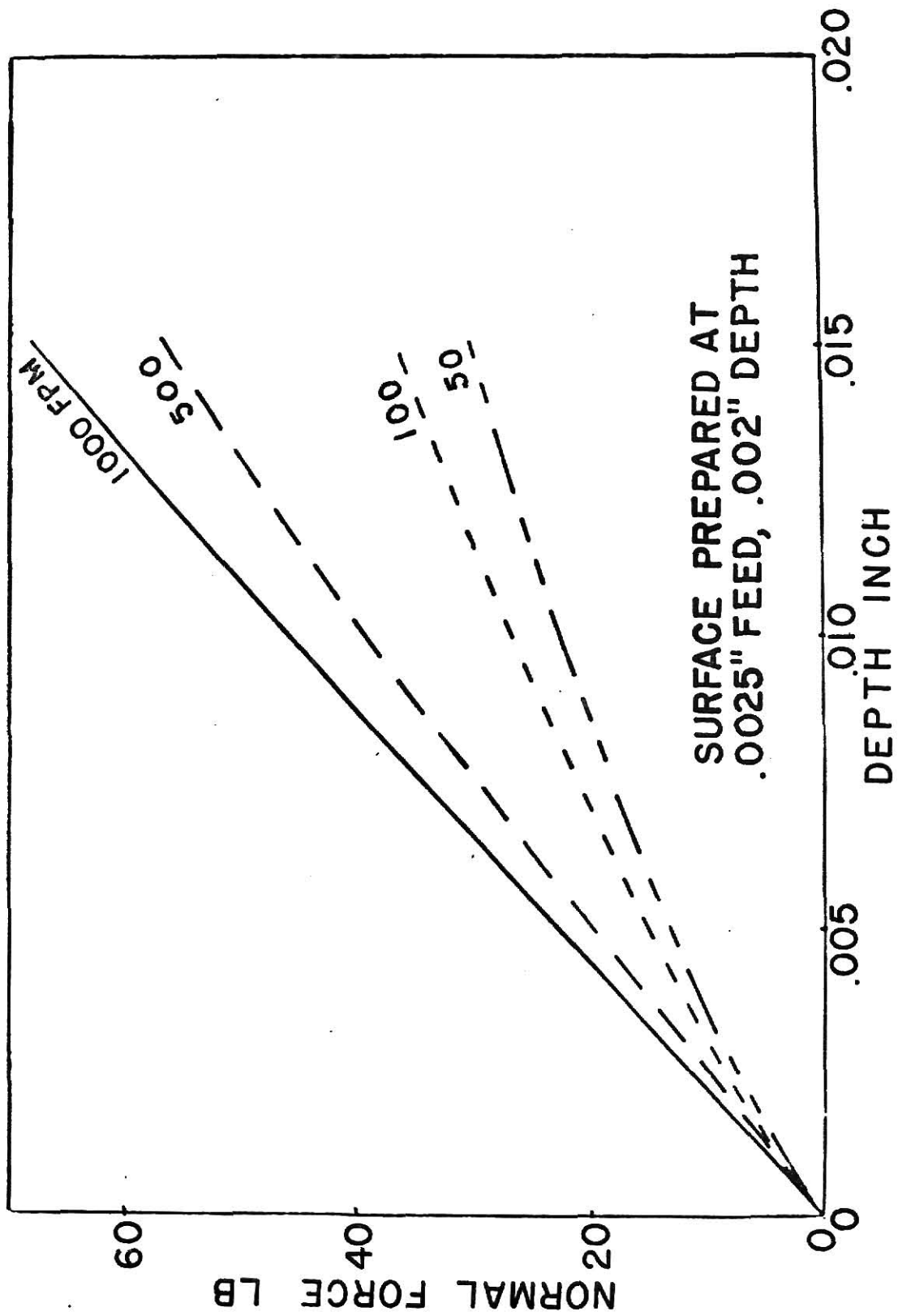
$$W = 1.575179$$

$$Z = .04106269$$



Single Groove Cuts

Figure 16



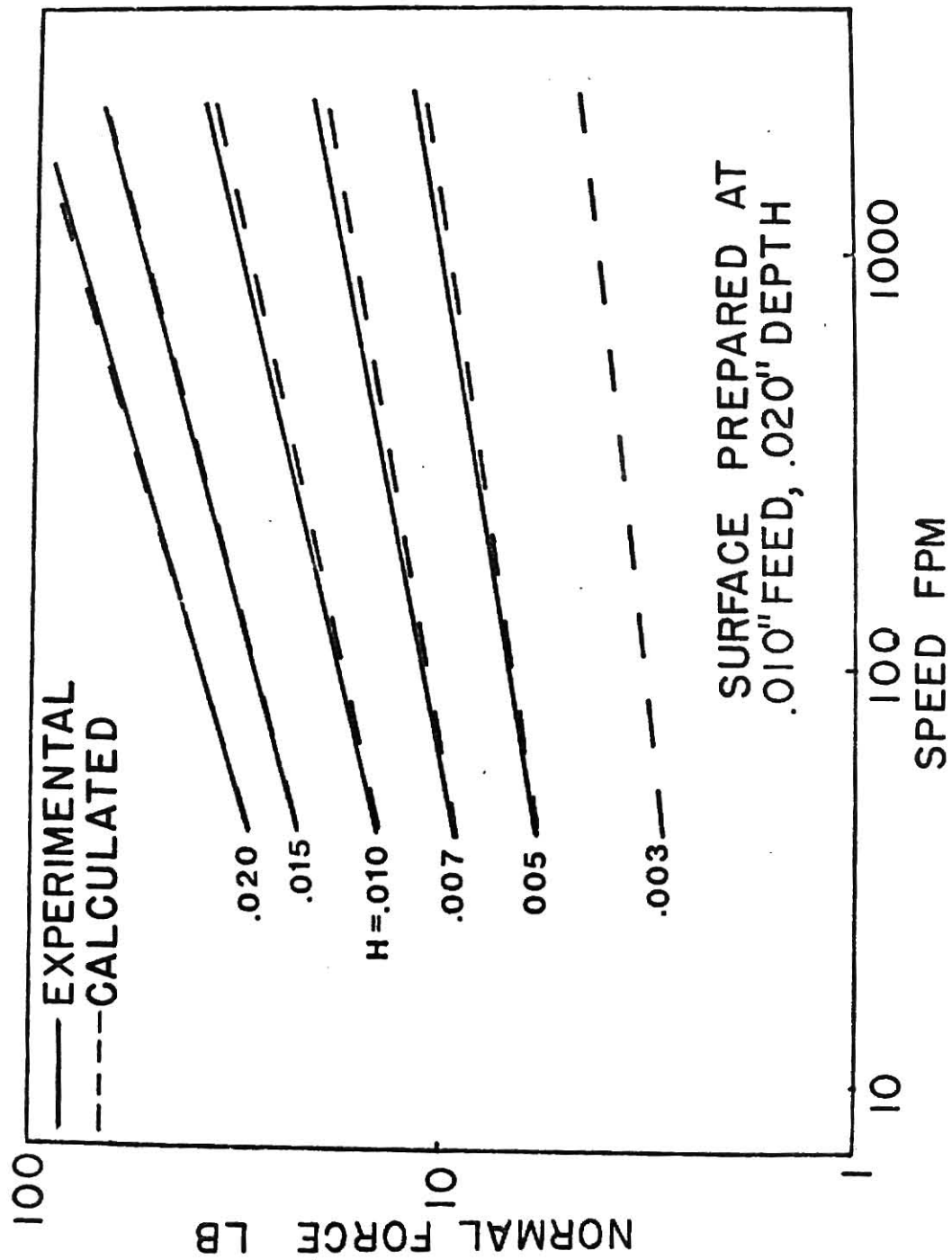
Single Groove Cuts

Figure 17

Table 5

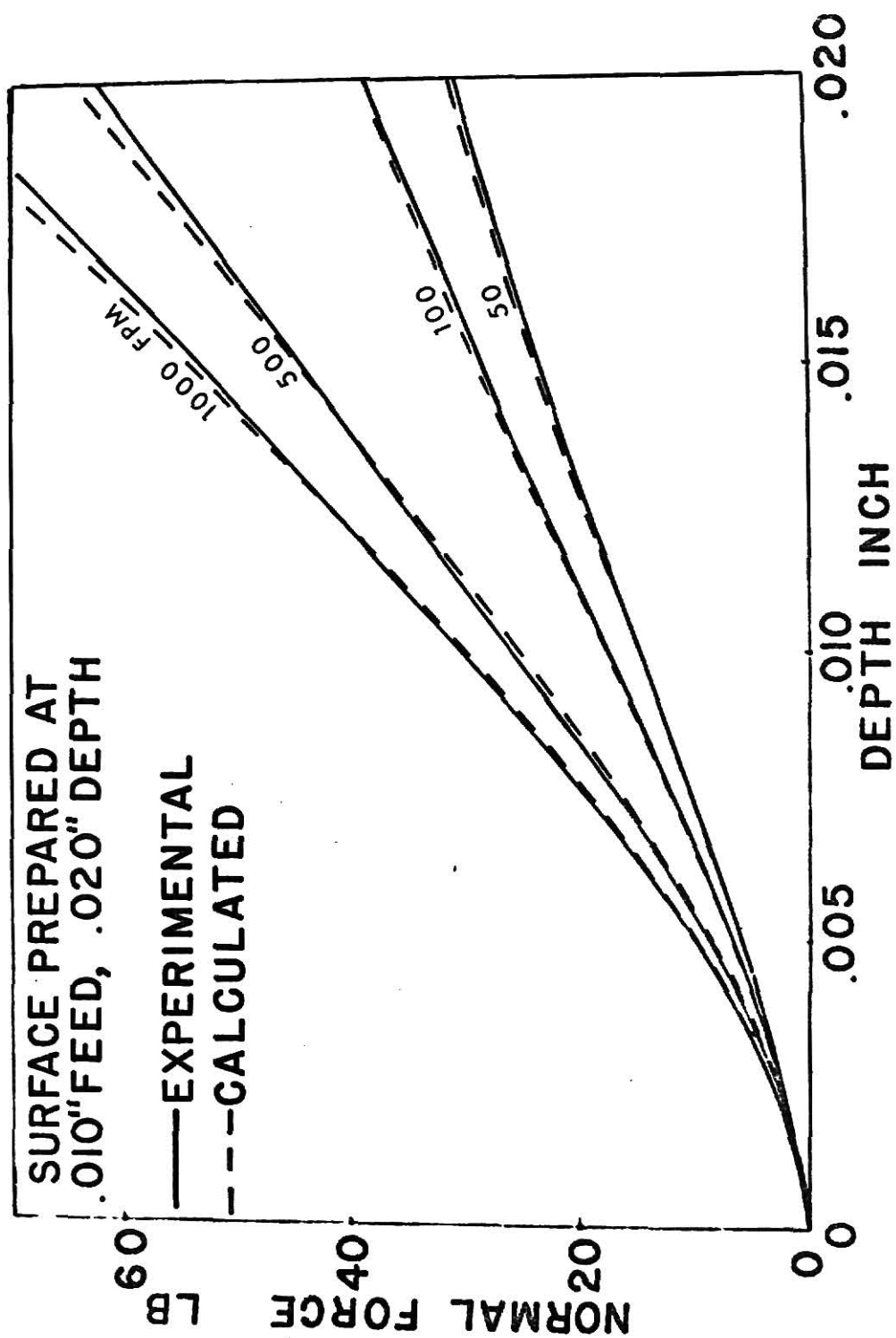
$F = A(\text{FPM})^B$ - Single Groove Cuts
Surface Preparation at .0025" Feed, .002" Depth

Depth in	B
.015	.282
.010	.239
.007	.204
.005	.176



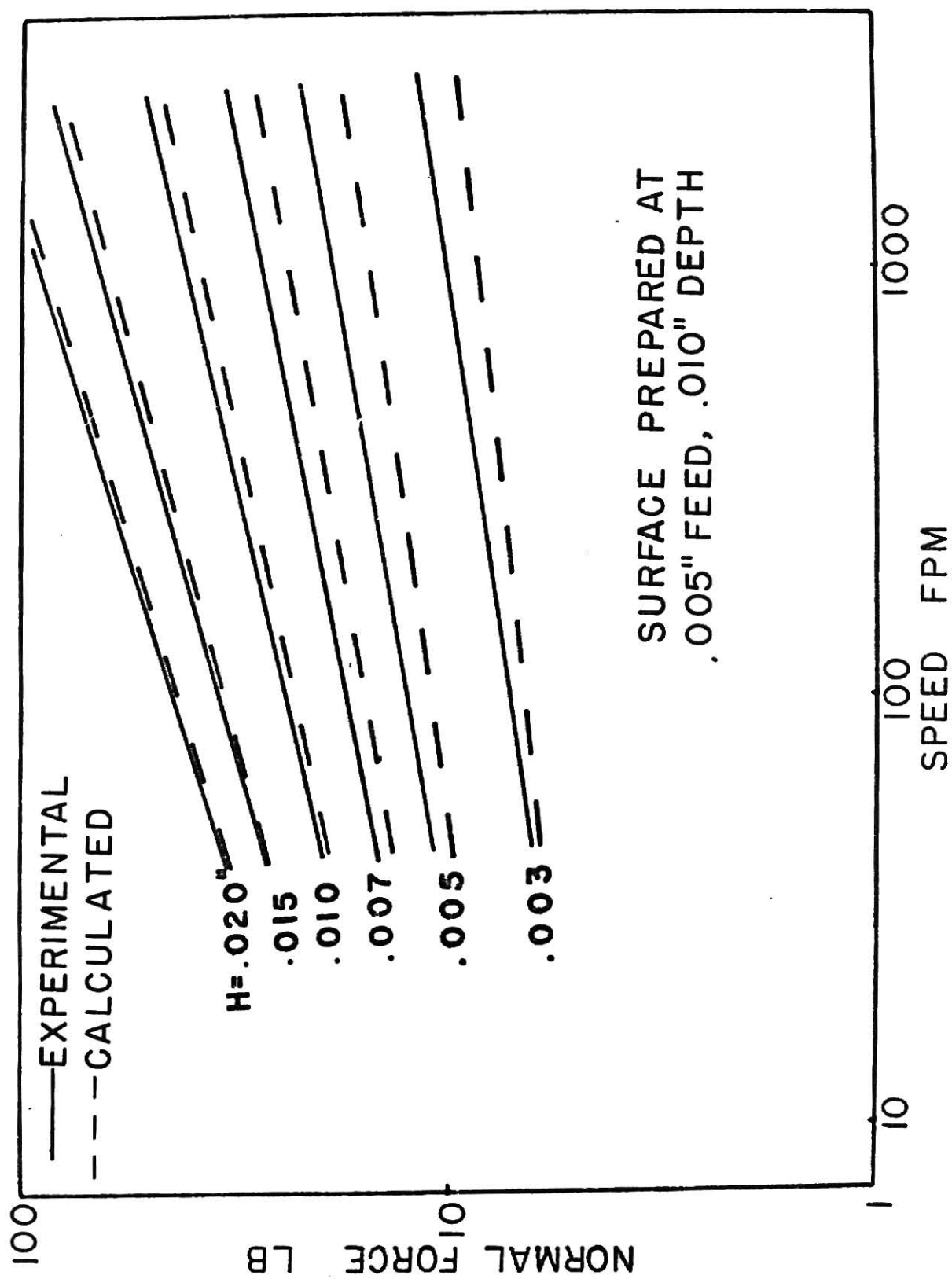
Results for Single Groove Cuts

Figure 18



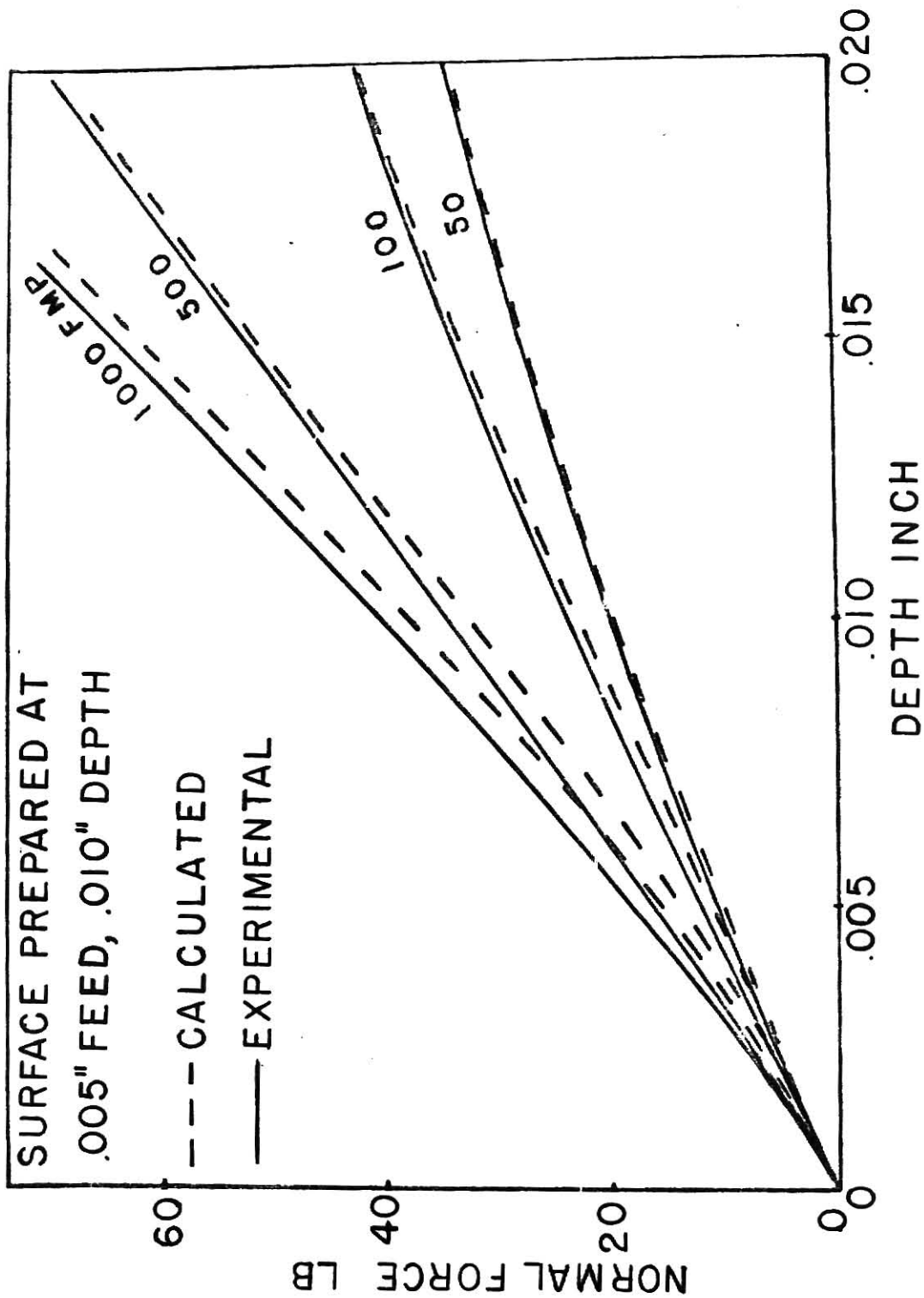
Results for Single Groove Cuts

Figure 19



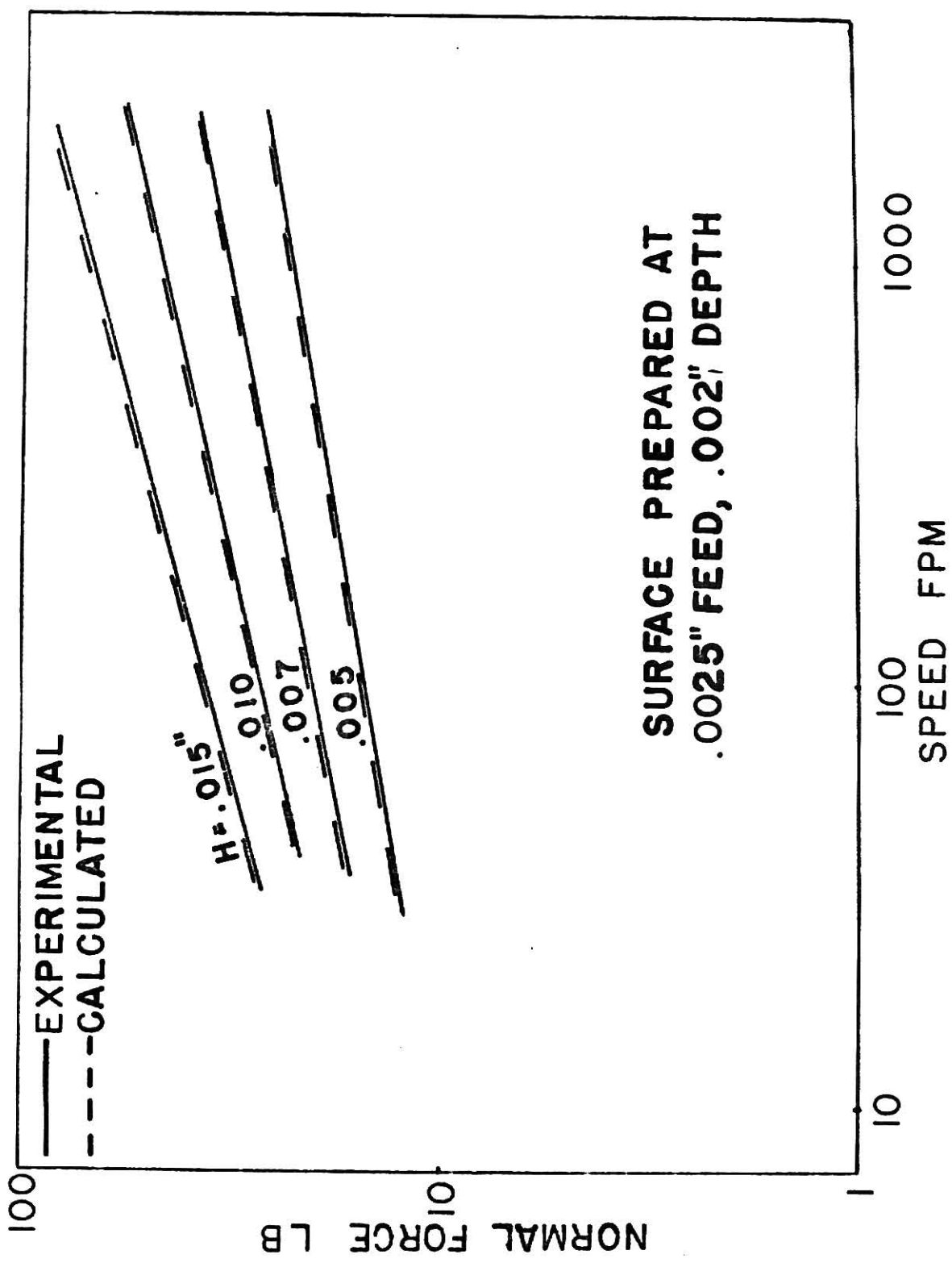
Results for Single Groove Cuts

Figure 20



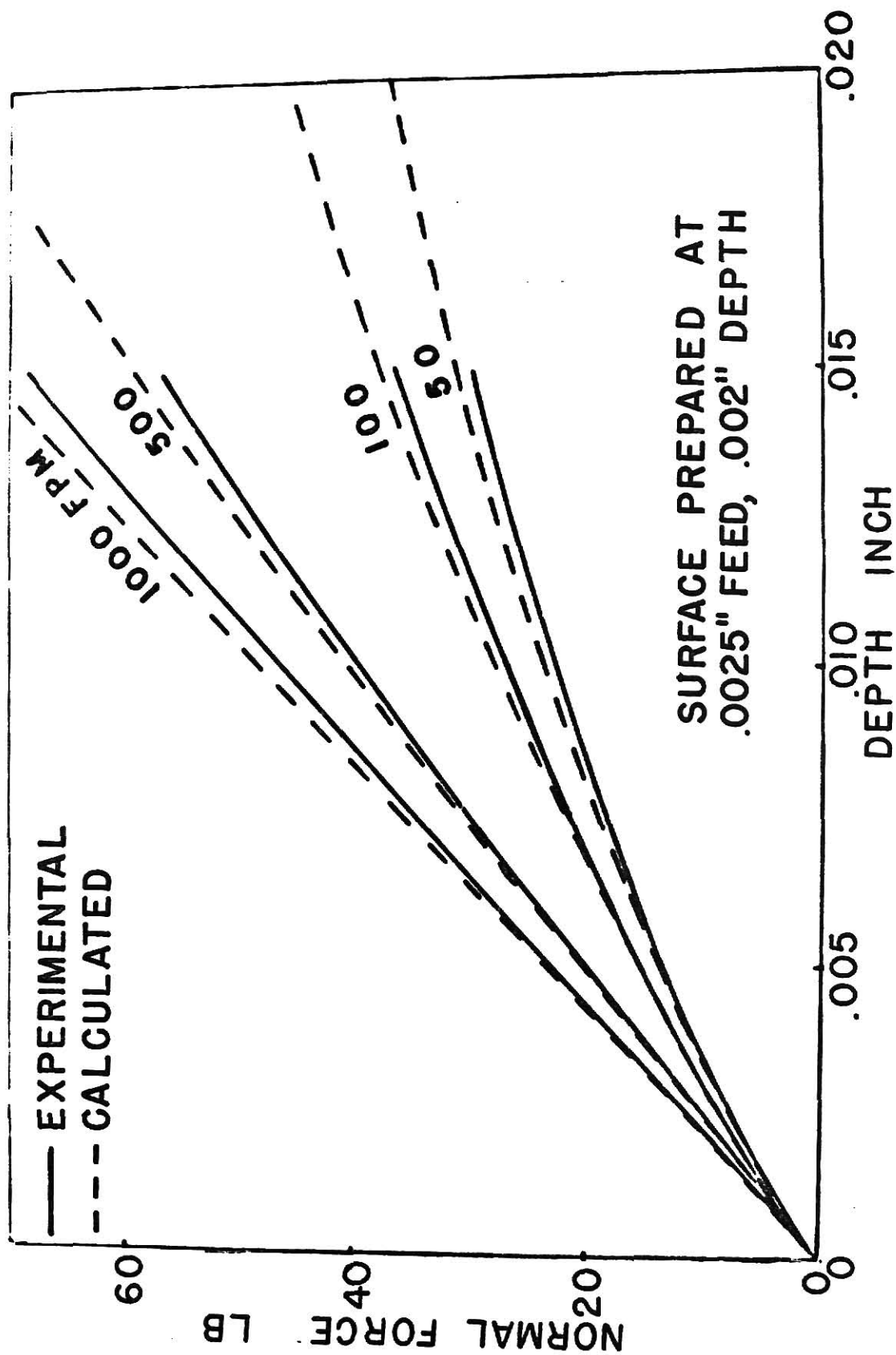
Results for Single Groove Cuts

Figure 21



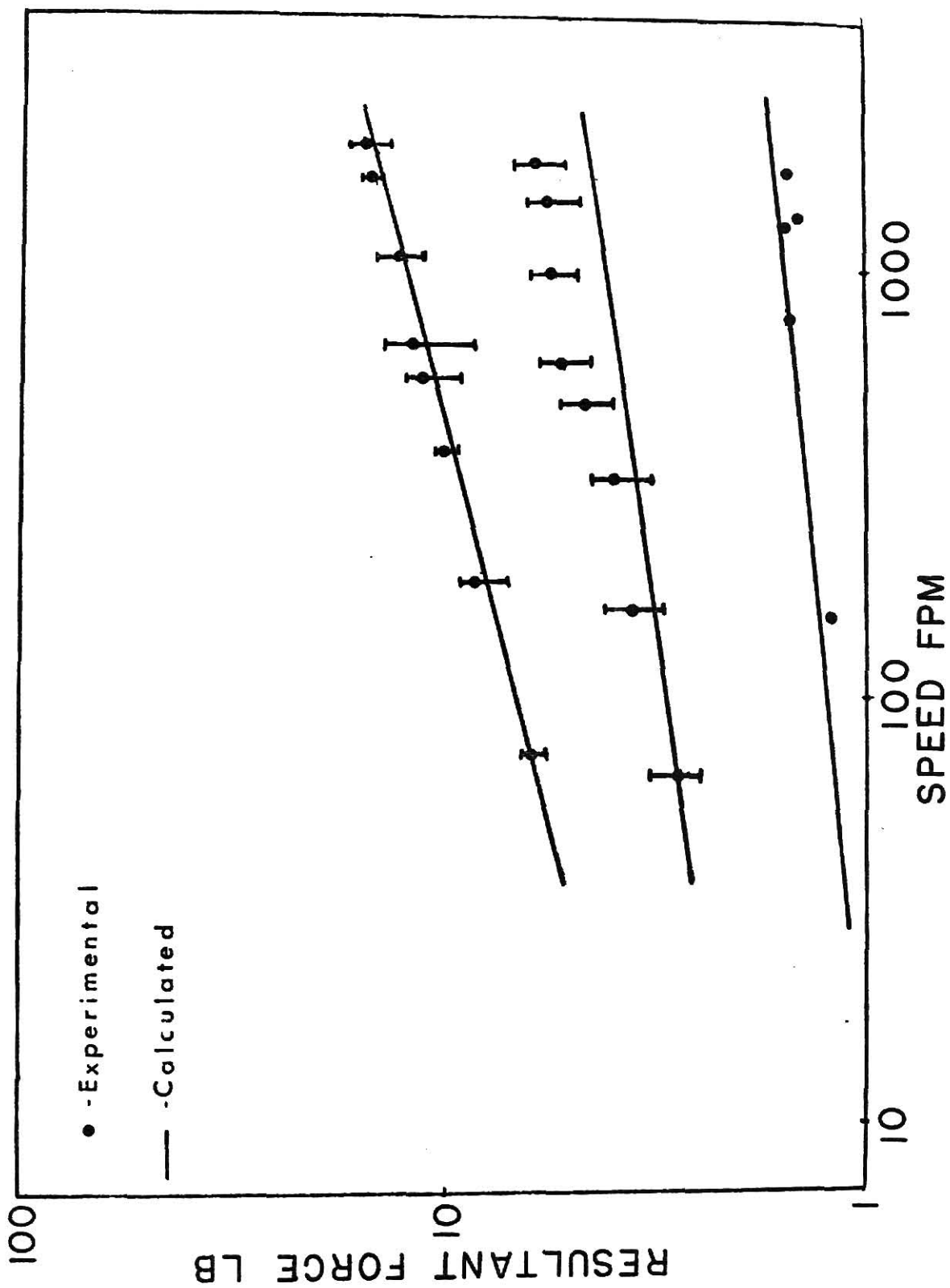
Results for Single Groove Cuts

Figure 22



Results for Single Groove Cuts

Figure 23



Results for Continuous Cuts

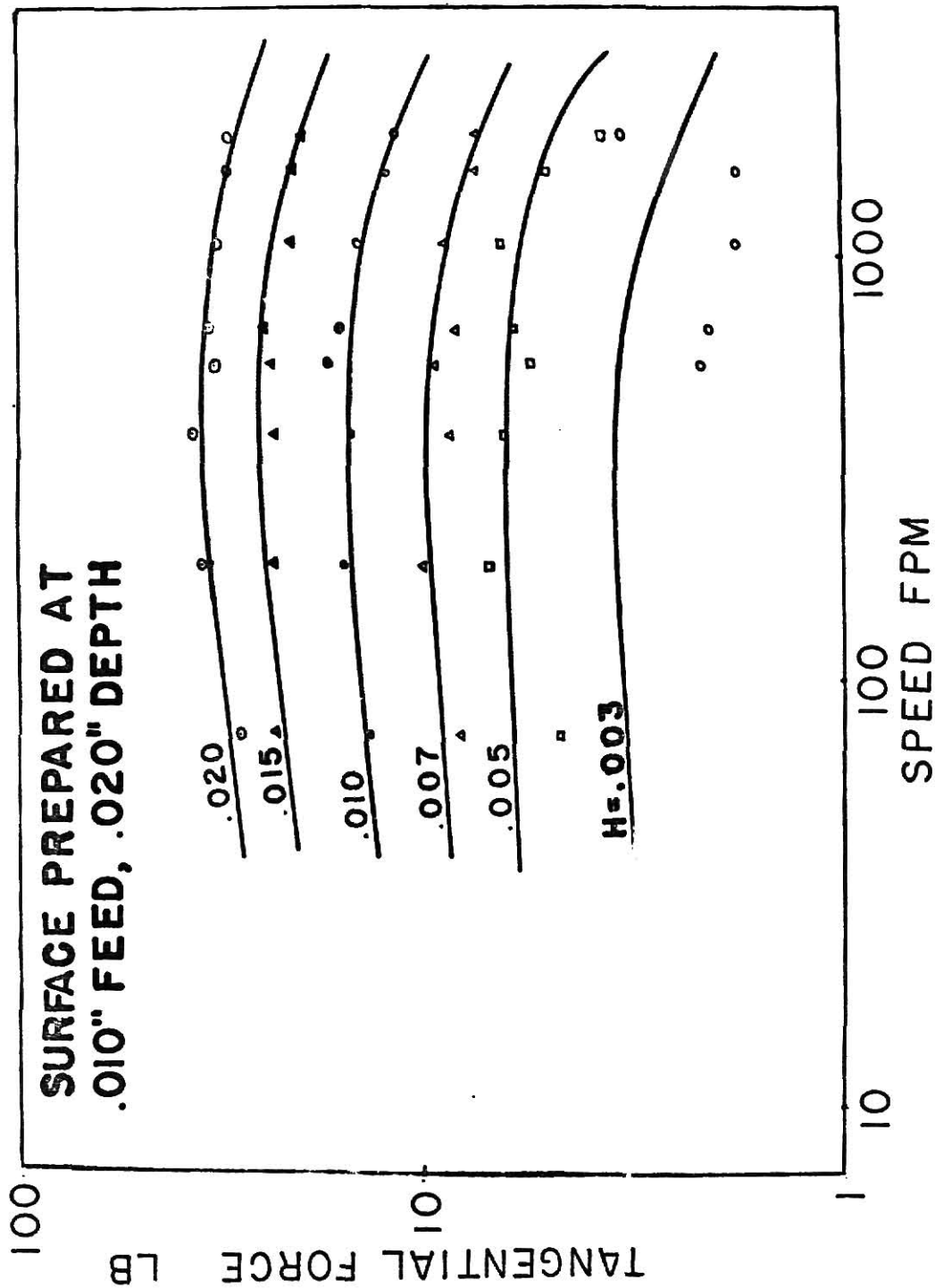
Figure 24

CHAPTER V

TANGENTIAL FORCE

The normal force component was of primary importance, however, to complete the study it was decided to attempt a similar analysis of the tangential force component. The data was approached in the same way as the normal force data. It became immediately clear that the $\log(\text{force})$ vs. $\log(\text{speed})$ plot would not result in a straight line as before. Since the form of equation describing these plots was not known the least squares fitting procedure could not be used. The same graphical fitting procedure was used that was used for the resultant force of the single grooves. The results of this procedure for the various cutting conditions are shown in Figs. [25], [26] and [27]. Data for the continuous cuts is shown in Fig. [28]. At first glance there appears to be little relationship between these curves. It was felt that if the speed range were large enough all the curves would have the same general shape (Fig. 29). As indicated in Fig. [29] there appears to be a transition from a straight line at one slope to a straight line at another slope. The transition seems to occur at different speed ranges depending on the amount of subsurface damage, and hence, all graphs would probably be similar if data were obtained over a wider range.

Difficulty was encountered in trying to find an equation that would describe these curves. A polynomial was tried but was given up because there was little hope of finding a physical explanation of the terms of the equa-



Single Groove Cuts

Figure 25

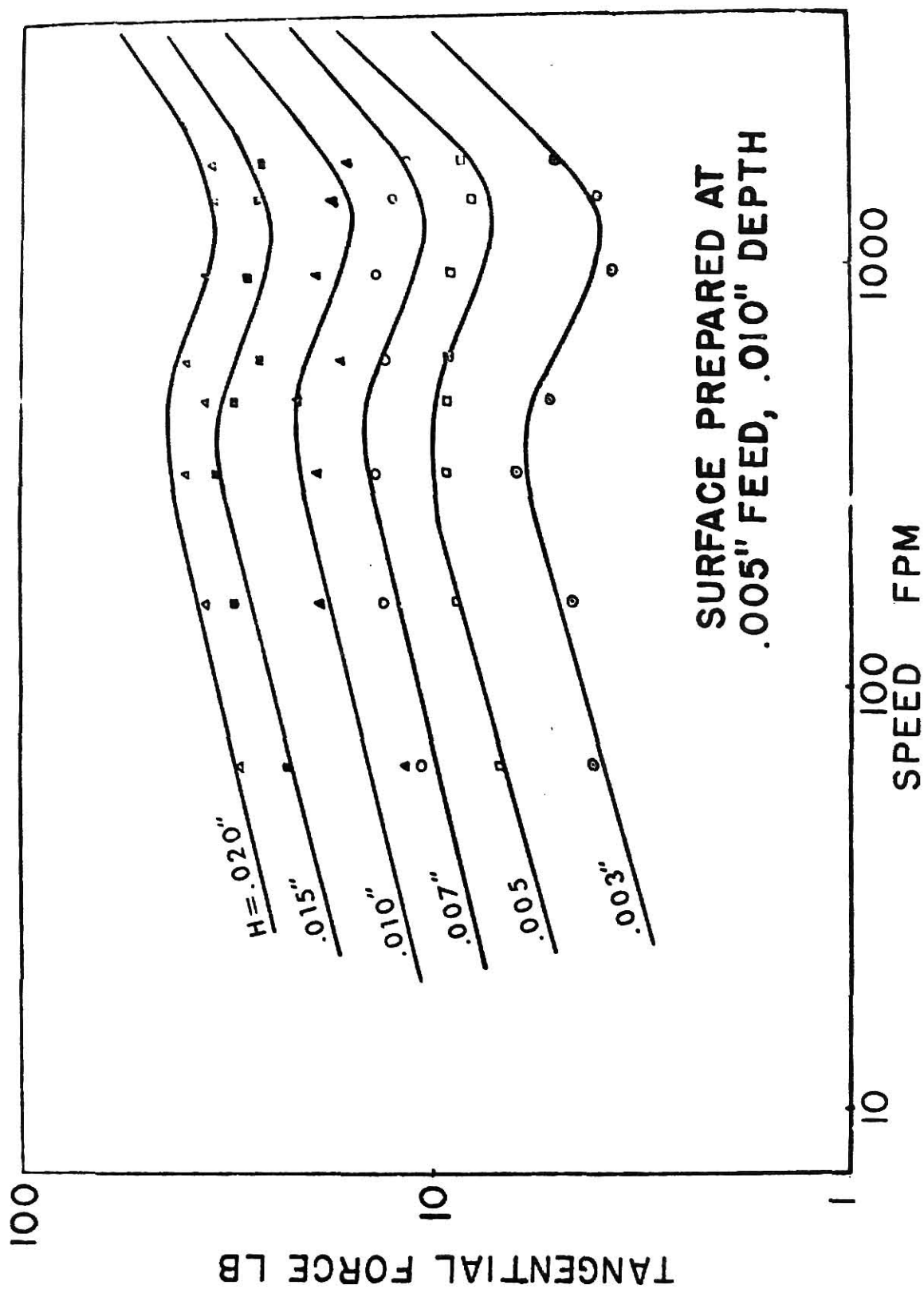
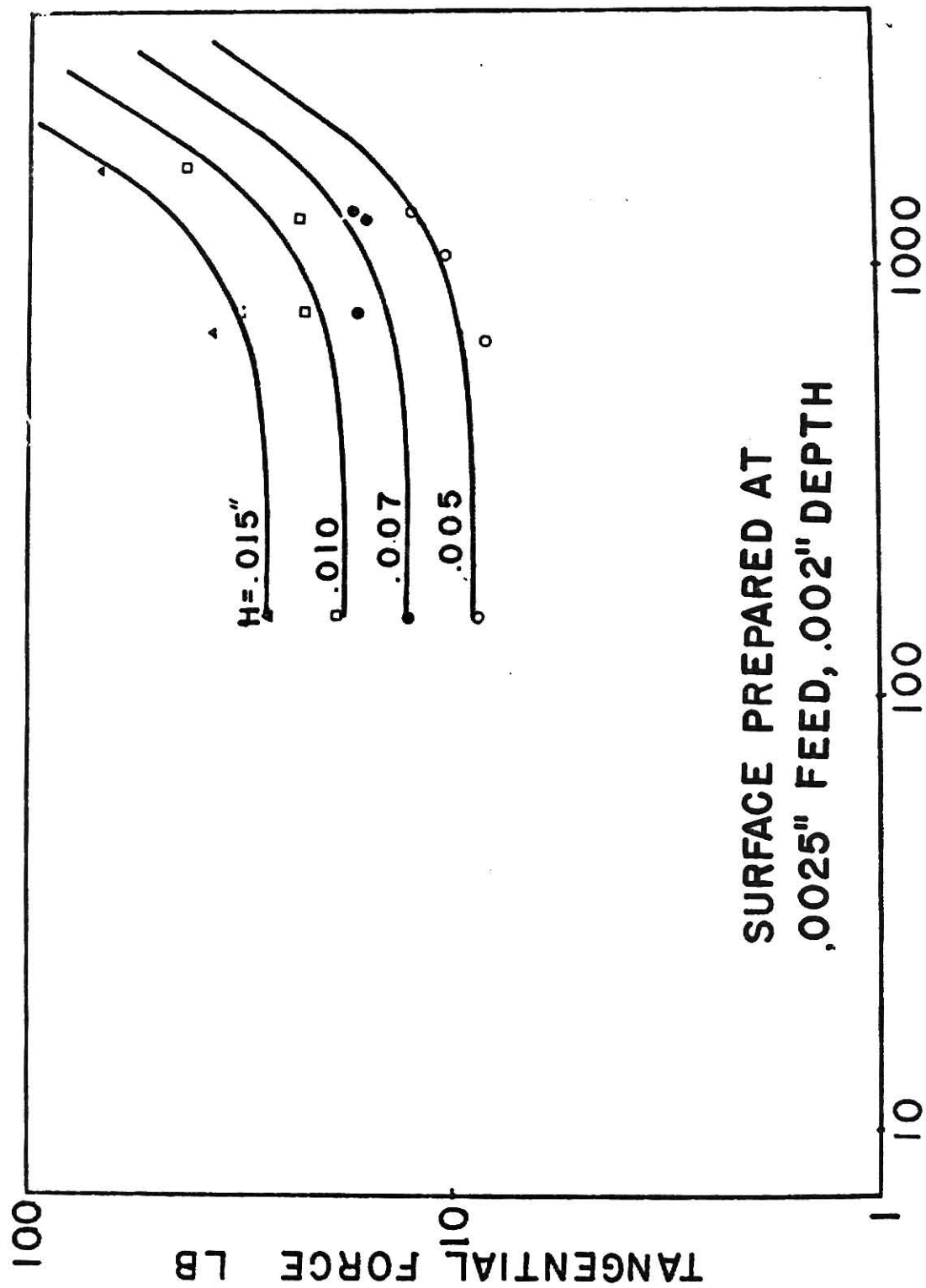
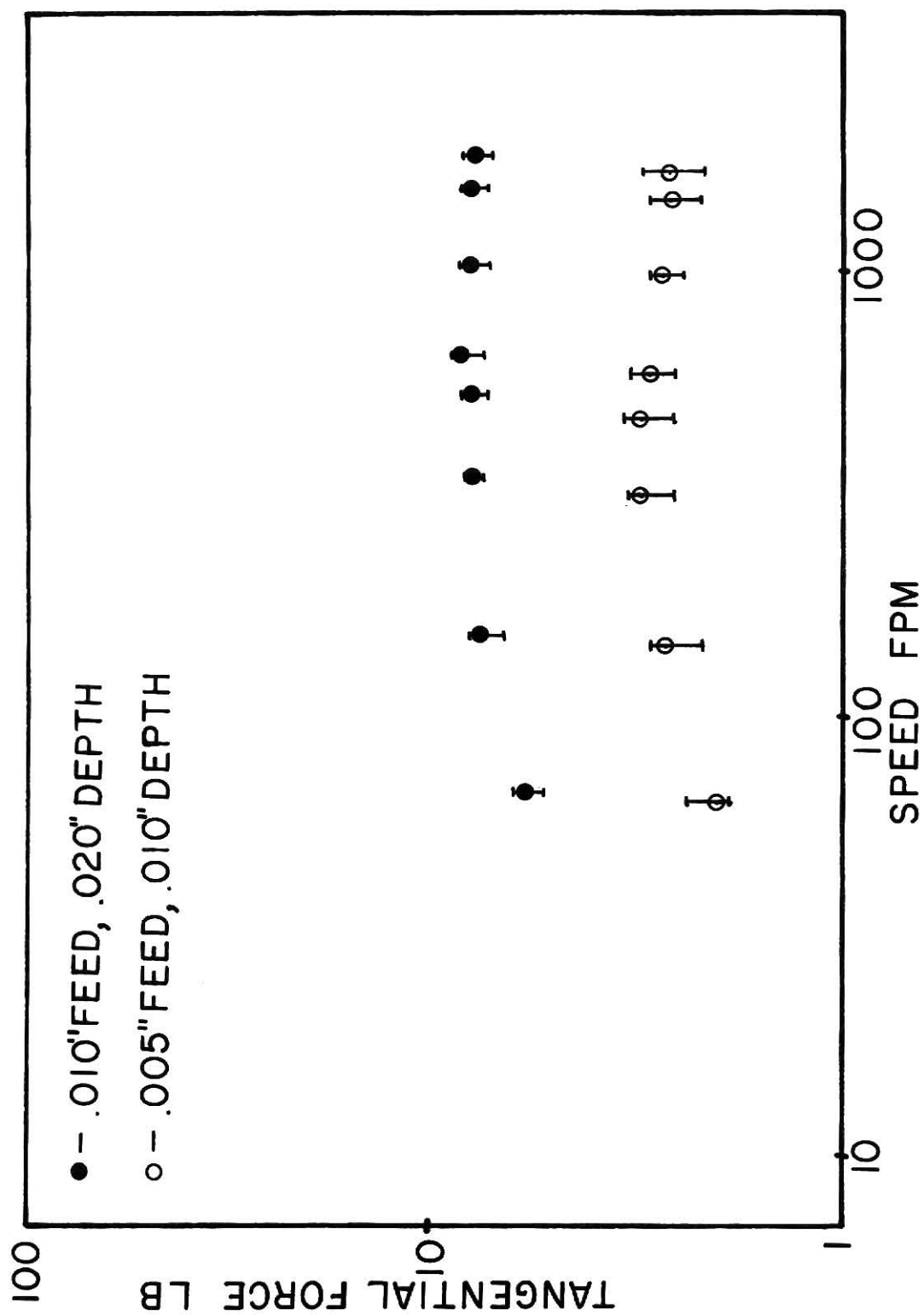


Figure 26



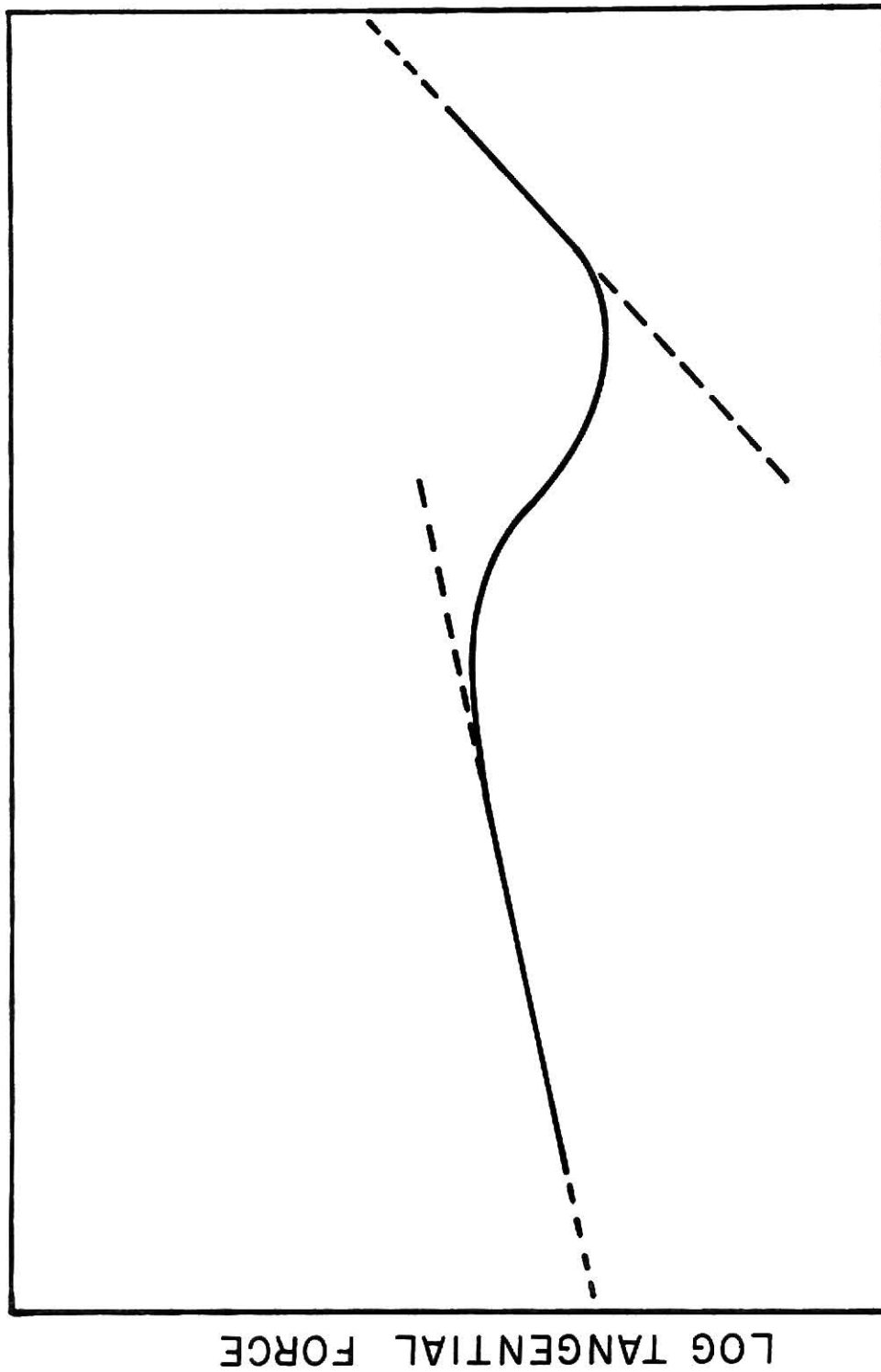
Single Groove Cuts

Figure 27



Force vs. Speed Results for Continuous Cuts

Figure 28



LOG SPEED
General Shape of Tangential Force Curve

Figure 29

tion. A reasonable form of equation was finally found by assuming that the curves have straight line portions as shown in Fig. 29. These portions can be described by equations of the form $y = Ax^B$. This is the same form as for the normal force analysis.

The final form of the equation decided upon is:

$$F = \left[A_o e^{-C_T (FPM)^D} (FPM)^{B_o} \right] + \left[A_1 (FPM)^{B_1} \right]$$

This form will give shapes similar to all the curves with appropriate constants. The curves in Fig. [26] contain enough information to solve for the constants. The values of the constants for this case are shown in Table 6. The curves of Fig. [25], [27], and [28] do not extend over a large enough range to permit solving for all the constants. The constants that can be found are given in Tables 7 and 8. In the case of Fig. [26] the following relations were found to apply.

$$\begin{aligned} A_o &= 1164.3 H^{1.1861} \\ A_1 &= 3395.9 H^{2.436} \\ B_o &= .1682 H^{-.08765} \\ B_1 &= .2780 H^{-.2733} \end{aligned}$$

Table 6

Tangential Force Constants - Single Groove Cuts
Surface Preparation at .005" Feed, .010" Depth

Depth in	A ₀	B ₀	D	A ₁	B ₁	C _t
.020	11.24	.237	1.59	.247	.666	2.95×10^{-5}
.015	7.99	.243	1.58	.122	.710	2.73×10^{-5}
.010	4.94	.252	1.66	.0456	.777	1.44×10^{-5}
.007	3.24	.260	1.70	.0191	.842	1.11×10^{-5}
.005	2.17	.268	1.69	.00843	.908	1.21×10^{-5}
.003	1.18	.280	2.08	.00243	1.02	1.00×10^{-6}

Table 7

Tangential Force Constants - Single Groove Cuts
Surface Preparation at .010" Feed, .020" Depth

Depth in	A ₀	B ₀	D	A ₁	B ₁	C _t
.020	17.4	.120	-	-	-	-
.015	12.8	.119	-	-	-	-
.010	9.08	.096	-	-	-	-
.007	6.70	.070	-	-	-	-
.005	5.10	.044	-	-	-	-
.003	2.54	.060	-	-	-	-

Table 8

Tangential Force Constants - Single Groove Cuts
Surface Preparation at .0025" Feed, .002" Depth

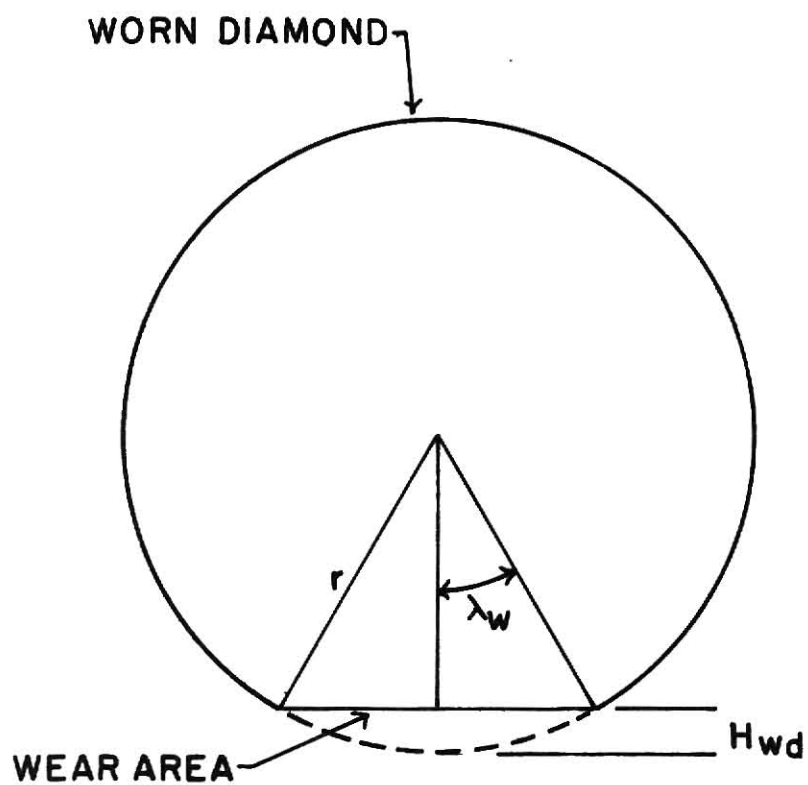
Depth in	A ₀	B ₀	D	A ₁	B ₁	C _t
.015	-	-	-	.000227	1.68	-
.010	-	-	-	.000221	1.61	-
.007	-	-	-	.000307	1.50	-
.005	-	-	-	.000663	1.32	-

CHAPTER VI

DIAMOND WEAR

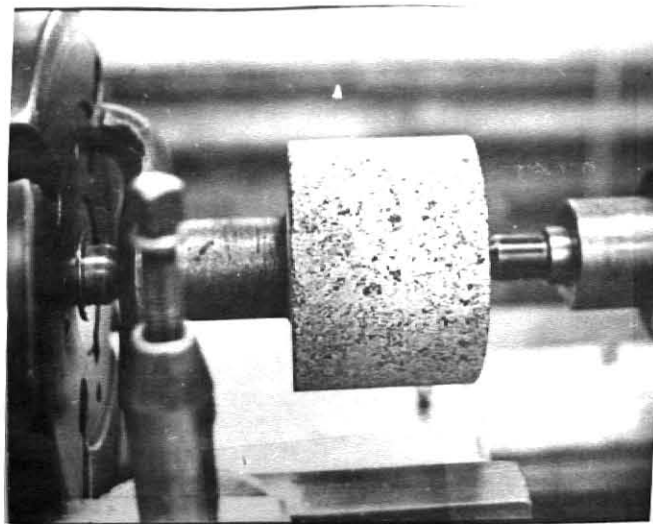
All the previous discussion has centered around cutting with a spherical diamond. When the diamond becomes worn, as it does in an actual tool, the cutting characteristics change. As the diamond wears the cutting forces increase till they become so large that the tool must be retired. For this reason it is important to study factors related to diamond wear. Limestone is not very abrasive and hence during the limestone test the diamond did not wear a perceivable amount. Granite is both strong and abrasive and causes severe wear problems so it was decided to conduct wear tests using granite.

A new spherical diamond 0.092 inch in diameter was mounted in the dynamometer. The granite was set up as in Fig. [31]. It was planned to keep the cutting parameters of speed, feed, and depth constant while recording forces as the diamond wore. The diamond was removed occasionally and photographed through a microscope. Figure [32] and [33] are examples of these pictures. These pictures were used to determine the amount of diamond wear. It was desired to find a relationship between the amount of wear and the amount of rock removed. As seen in Fig. [34] H_{wd} is the most obvious indicator of wear. However H_{wd} is small and hard to measure. The area of the flat was used as the indicator instead. Areas were determined in two ways. A planimeter was used to directly measure the area from the pictures. This was not entirely satisfactory since the diamond has small chipped out spots and the area is not easy to define. The other procedure was to find λw from



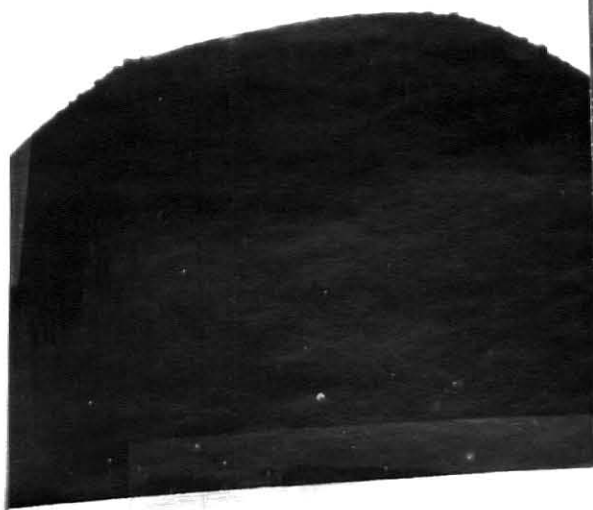
Schematic Diagram of a Worn Diamond

Figure 30



Granite Test Fixture

Figure 31



Worn Diamond Profile

Figure 32



Worn Diamond Flat Spot

Figure 33

profile pictures and use geometrical relations to get H_{wd} .

$$H_{wd} = r [1 - \cos \lambda w]$$

It is also possible to approximate the measured area as an equivalent circle to get H_{wd} .

$$H_{wd} = r [1 - \cos \lambda w]$$

$$\text{where } \lambda w = \sin^{-1} \frac{1}{r} \sqrt{\frac{A_w}{\pi}}$$

The volume of diamond worn away was computed from the following equation:

$$\text{Volume} = \pi \left[r H_{wd}^2 - \frac{H_{wd}^3}{3} \right]$$

These results are summarized in Table [9].

The wear tests were terminated after 52 runs because of time limitations. At this time the diamond had not worn enough to draw significant conclusions.

Table 9

Various Wear Parameters

distance traveled	wear area	vol. worn away	equivalent area method λ w	angle method λ w	H wd
5554 ft	.000317 in ³	1.76 X 10 ⁻⁷ in ³	12.6°	9.55°	.000658 in
19,767 ft	.000535 in ³	5.11 X 10 ⁻⁷ in ³	16.5°	12.42°	.00107 in
55,369 ft	.000612 in ³	6.61 X 10 ⁻⁷ in ³	17.6°	19.55°	.00265 in

CHAPTER VII

CONCLUSION

The normal component of force on a spherical diamond while cutting Indiana limestone has been analyzed. The final equation takes into account two important factors, speed and subsurface damage, heretofore not considered. The concept of effective depth was defined and is useful when working with continuous cuts or subsurface damage. The ranges of speed and depths of cut considered will cover most cases encountered in practice. Since, normal load is an important parameter in operation of diamond tools this analysis is worthwhile.

The speed effect that showed up in the normal load analysis also dominated in the tangential force analysis. However some factor caused the force to drop off then rise again. It is therefore important to learn more about this. If data were obtained over a wider speed range it may be possible to reach a definite conclusion regarding this phenomenon.

Little in the way of conclusions can be made from the wear data, except that the cutting conditions chosen were not severe enough to produce significant wear in a reasonable amount of time.

CHAPTER VIII

RECOMMENDATIONS

In order to apply this work several additional tests should be made. One variable not considered is diamond size. All the conclusions were based on a 1/8 inch diamond. It is expected that the analysis for other size diamonds will be similar except for the magnitudes involved. Also all the tests, except for wear, were done using Indiana limestone. Other types of rock should be tested to increase generality of the results. Again it is not expected that the analysis will change much since most rocks have somewhat similar properties.

This author would recommend more tests involving the tangential force to attempt to establish the reasons behind the strange speed effect. Tests conducted over a wider range of speed would be a good first step in this direction. A lathe with continuously variable speed would be an asset in this respect. This would make it possible to conduct tests at constant cutting speed.

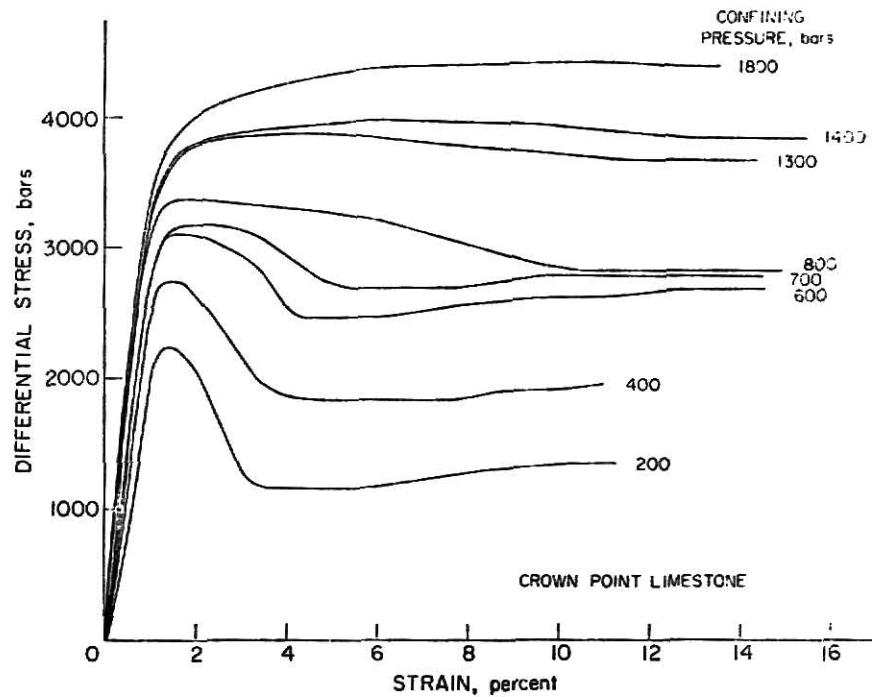
Diamond wear is of great economic importance to the diamond industry. Therefore a continuation of wear tests is recommended. It is suggested that a larger rock be used so that the diameter and the speed do not change so much during the test. It is also suggested that a larger speed, feed and depth be used so that excessive amounts of cutting time could be avoided. Also a better method of measuring diamond wear would be desirable.

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9. Rowley, D. S., and F. C. Appl. "Analysis of Surface Set Diamond Bit Performance." American Institute of Mining, Metallurgical and Petroleum Engineers, Inc., SPE 2242. 1968.

APPENDIX 1

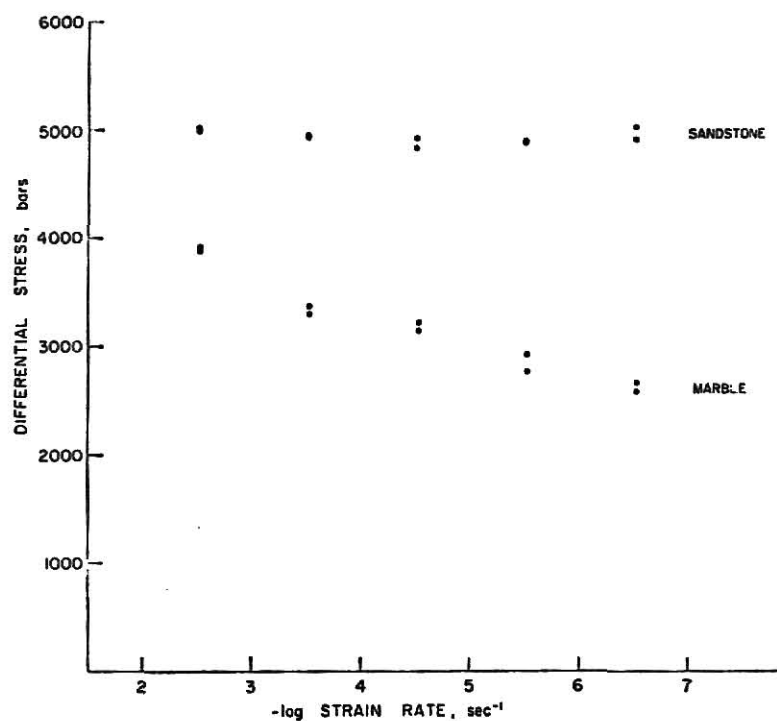
Rock Properties Taken from Reference 4



Effect of increased confining pressure on sustained differential stress (strength) of Crown Point limestone. Curves of differential stress versus longitudinal strain show the

increase in strength caused by increased pressure and a change in character as the deformation changes from brittle at low confining pressure to ductile at high pressure.

Figure 34



Effect of decreasing strain rate on the strength of sandstone and marble deformed at 2000 bars confining pressure. Although there is a marked decrease in strength of the marble, the strength of the

sandstone is unaffected over the same range of strain rate. The values indicated are for the differential stress sustained just beyond the knee of the stress-strain curve, at 2% for the marble and 3.5% for the sandstone.

Figure 35

APPENDIX 2

Analysis of Data - Effect of Cutting Speed

It appears that the data can be reasonably well represented in the form

$$Y = AX^B$$

where: Y = cutting force

X = cutting speed

For a specific set of experimental data

Y_i $i = 1, 2, 3, \dots, n$ force data

X_i $i = 1, 2, 3, \dots, n$ speed data

Let ϵ_i be the percent error for the i_{th} data point. The sum of the errors squared becomes

$$E = \sum_{i=1}^n \epsilon_i^2 = \frac{[AX_i^B - Y_i]^2}{Y_i^2}$$

For the best least squares fit E is to be minimized with respect to

A and B . Thus

$$\frac{\partial E}{\partial A} = 0$$

and

$$\frac{\partial E}{\partial B} = 0$$

Performing the differentiation and simplifying yields

$$A = \frac{\sum \frac{X_i^B}{Y_i}}{\sum \frac{X_i^{2B}}{Y_i^2}}$$

The second equation is nonlinear and hence B will be determined by iteration for the root of the equation:

$$G(B) = \sum \left[\frac{AX_1^B - Y_1}{Y_1^2} \right] X_1^B \ln X_1$$

Correlation Theory

Standard error of estimate of Y on X

$$S_{y,x} = \sqrt{\frac{\sum(Y - Y_1)^2}{n}}$$

Modified standard error of estimate (for small samples)

$$S_{y,x} = \sqrt{\frac{\sum(Y - Y_1)^2}{n-1}}$$

Non-linear correlation coefficient

$$\nu = \sqrt{\frac{\sum(Y - Y_1)^2}{\sum(Y_1 - Y_1)^2}}$$

ILLEGIBLE DOCUMENT

THE FOLLOWING
DOCUMENT(S) IS OF
POOR LEGIBILITY IN
THE ORIGINAL

THIS IS THE BEST
COPY AVAILABLE

The following program solves for A and B in the equation $Y = Ax^B$.

This program was originally written for an equation of the form $Y = Ax^B + C$.

In this case C is set equal to zero.

```

100 PROGRAM CUFIT (INPUT,OUTPUT)
110 DIMENSION X(50),Y(50)
120*** NCPT=0 TO OPTIMIZE W.R.T. C ; NCPT=1 FOR SPECIFIED C
130 DATA NCPT /1/
135 DATA C /0.0/
140 DATA N
141+ /18/
150 DATA X
151+/73.82,184.60,373.50,548.00,673.00,1062.00,1614.0,1955.0,
152+738.34,1471.86,473.79,913.90,1798.34,384.43,579.01,1174.60
153+ ,74.88,2199.6,32*0.0/
170 DATA Y
171+/6.28,8.40,9.81,11.14,12.04,13.28,16.00,15.47,11.50,15.96,
172+10.64,13.25,14.25,9.80,11.60,14.32,6.01,17.12,32*0.0/
190 DATA BIN,DELB,BMAX
191+ /0.01,0.1,10.0/
192 B=BIN
194 J=0
196 50 CONTINUE
200 SYI=0.0
210 SXIB=0.0
220 SYIXIB=0.0
230 SXI2B=0.0
240 SUMYI=0.0
250 DO 100 I=1,N
260 SYI=SYI+Y(I)
270 SXIB=SXIB+X(I)**B
280 SYIXIB=SYIXIB+Y(I)*X(I)**B
290 SXI2B=SXI2B+X(I)**(2.0*B)
295 SUMYI=SUMYI+Y(I)
300 100 CONTINUE
340 YIBAR=SUMYI/N
345 IF(NCPT.EQ.1) GO TO 120
350 A=(SYI*SXIB-N*SYIXIB)/(SXIB*SXIB-N*SXI2B)
360 C=(SYI-A*SXIB)/N
370 120 IF(NCPT.EQ.1) A=(SYIXIB-C*SXIB)/SXI2B
390 G=0.0
400 DO 150 I=1,N
402 G=G+(A*X(I)**B+C-Y(I))*X(I)**B*ALOG(X(I))
404 150 CONTINUE
406 G=G*A
411 IF(J.EQ.0.AND.(GPREV/G).LT.0.0) STOP
412 IF(J.EQ.0.OR.(GPREV/G).GT.0.0) GO TO 300
413 IF(ABS(DELB/B).LT.0.000001) GO TO 400

```

```

414 DELB=DELB/10.0
415 B=BPREV+DELB
416 GO TO 50
550 300 J=J+1
552 BPREV=B
554 GPREV=G
560 B=B+DELB
570 IF(B.GT.BMAX) STOP
580 GO TO 50
590 400 CONTINUE
600 PRINT 160
602 160 FORMAT(//////////,* DATA ANALYSIS*)
604 PRINT 180,A,B,C,G
606 180 FORMAT(/,* A =*,F10.5,4X,*B =*,F9.6,4X,*C =*,F10.5,
607+ 4X,*G =*,F14.6)
608 PRINT 190
609 190 FORMAT(//,3X,*X(I)*,6X,*Y(I)*,8X,*Y*,9X,*ERROR*,3X,*7 ERROR*)
615 SUMS=0.0
616 SUMN=0.0
618 SUMD=0.0
620 DO 205 I=1,N
625 YEST=A*X(I)**B+C
627 ERR=Y(I)-YEST
628 PERER=100.0*ERR/Y(I)
630 ERRN=YEST-YIBAR
640 ERRD=Y(I)-YIBAR
650 SUMS=SUMS+ERR*ERR
660 SUMN=SUMN+ERRN*ERRN
670 SUMD=SUMD+ERRD*ERRD
700 PRINT 210,X(I),Y(I),YEST,ERR,PERER
705 210 FORMAT(F8.2,4X,F6.2,4X,F7.3,4X,F7.4,4X,F6.2)
710 205 CONTINUE
750 STERES=SQRT(SUMS/(N-1))
760 CORCOF=SQRT(SUMN/SUMD)
770 PRINT 220,STERES
780 220 FORMAT(//,* MCD STD ERR OF EST =*,F10.5)
790 PRINT 230,CORCOF
800 230 FORMAT(* NON-LIN CORR COEF =*,F9.5)
810 PRINT 240,YIBAR
820 240 FORMAT(* AVE FORCE =*,F10.5)
830 PRINT 250
840 250 FORMAT(//////////)
900 END

```

ACKNOWLEDGEMENTS

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VITA

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Master of Science

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AN EXPERIMENTAL INVESTIGATION OF THE CUTTING
ACTION OF A SINGLE DIAMOND

by

James H. Reese

B.S., Kansas State University, 1970

AN ABSTRACT OF A THESIS

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

Name: James H. Reese

Date of Degree: June, 1972

Institution: Kansas State University

Location: Manhattan, Kansas

Title of Study: AN EXPERIMENTAL INVESTIGATION OF THE CUTTING ACTION OF
A SINGLE DIAMOND

Candidate for Degree of Master of Science

Major Field: Mechanical Engineering

Abstract:

The action of a single spherical diamond tool cutting limestone has been experimentally investigated. A modification to a previous theory of cutting has produced reasonably good agreement with regard to the experimentally measured normal force. A limited analysis of the tangential force component has been done. The effects of subsurface grain damage and cutting speed are discussed.

Preliminary experiments to determine diamond wear while cutting granite were conducted and a limited amount of data is presented.