

MODELING AND OPTIMIZATION OF A WATER
RESOURCE CONTROL SYSTEM

by 45

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INTRODUCTION

The purpose of this thesis was to devise an efficient approach to water resource control optimization. A test system has been devised and solved within an analytical framework.

This study concerns optimal policies regulating water resource structures already in existence. The majority of work in this field is directed toward simulation. Physical and regulatory system models are executed on a trial and error basis. The results of these runs are analyzed to determine optimal combinations. The techniques of simulation in this field are fully illustrated in texts written by Emery Castle (1), and Maynard Hufschmidt (2). With regard to resource control policies, an analytical approach has been proposed whereby objective functions may be constructed and solved directly. A definite need is recognized here since the installation of expensive structures may be eliminated by the efficient use of what is currently available.

When constructing the model for a test system, particular attention must be devoted to water supply, water demand, and the measures available for meeting these demands. The major elements of water supply are a part of surface and ground water flow as well as natural storage. Relevant characteristics are quantity, quality, and availability measured with respect to time and location. Water demands are points or

zones (present or potential) which draw upon the water resources of the system (2). Examples would include

- a. Municipal and industrial water supply.
- b. Hydroelectric power needs.
- c. Irrigation.
- d. Flood control.
- e. Water quality.
- f. Navigation.
- g. Water based recreation.

Measures for meeting these demands and alleviating water problems would include such examples as

- a. Multi-purpose reservoirs.
- b. Hydroelectric power plants.
- c. Irrigation canals.
- d. Water transmission and diversion works.
- e. Waste treatment facilities.

An initial model has been proposed and developed for study. A sketch of the physical system is illustrated in Figure 1. In constructing this model, data was drawn from the literature wherever possible. This practice was adopted to insure realistic response behavior. In addition, this data is in a form which subsequently may be determined for the analysis of real water resource systems.

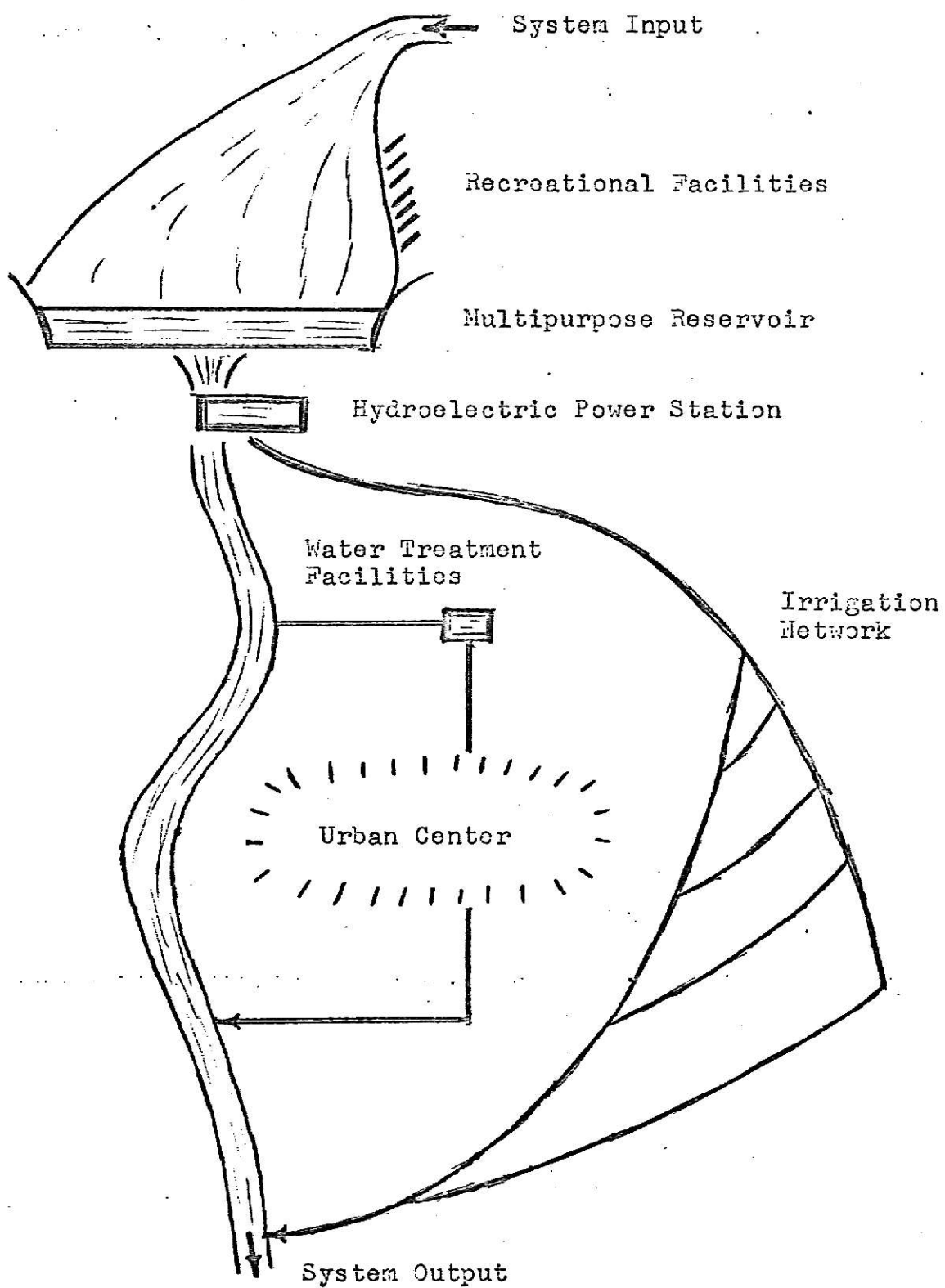


Fig. 1. Example of a water resource control system.

The basis of study for the model evolves about a form of static economic analysis. Assumptions made in this analysis are listed below.

- a. All capital facilities have been installed prior to the period of analysis.
- b. Target outputs remain fixed constants (or periodic functions) during the period of analysis. Target outputs represent minimal water supply commitments below which economic losses are suffered by a particular source of demand.
- c. Analysis shall be based on a stage-wise or finite difference approach. For the system in question, one month constitutes a single stage or increment of time.
- d. Total benefits accrued during one increment of time, less total economic losses or penalties, less operating and maintenance costs contracted during the same period of time and discounted to present value, constitutes the "cash-flow" for this period.
- e. The object of this study is to determine that set of operational policies which maximizes the cash flow stream over the period of economic analysis (3).

CONSTRUCTION OF A SYSTEM MODEL

WATER FLOW DISTRIBUTION

As illustrated in Figure 1, optimal policies for the system will affect water storage and releases for

- a. Municipal and industrial water supply.
- b. Hydroelectric power needs.
- c. Irrigation.
- d. Recreation.

Water input to the system occurs from

- a. Bulk flow from the river.
- b. Ground water.
- c. Precipitation.
- d. Drainage from the basin.

Water output from the system occurs from

- a. Bulk flow from the river.
- b. Ground water.
- c. Evaporation.
- d. Transpiration.

System accumulation occurs from

- a. Absorption.

b. Consumption (4).

With regard to the system in question, two assumptions are made to achieve some simplification. First, the loss from the system by evaporation and transpiration is equal to the gain by precipitation. Secondly, ground water effects are to be considered negligible.

The nature of water flow into a system during the period of analysis may be approximated with a hydrology generator. This is a mathematical model (either deterministic or stochastic in nature) which predicts future flow characteristics on the basis of past records and meteorological forecasts. A deterministic model could express water inflow as an explicit function of time and location. Under these circumstances, it may be proposed that a set of optimal policies be determined for the economic life of the system, and then re-determined at the end of each stage with the consequent updating of the hydrology generator. For this system then, the success of water allocation policies for a given month would depend upon the accuracy of the hydrology prediction made at the beginning of that month.

To account for the seasonal fluctuations of water flow, a Fourier series representation of the hydrology generator has been considered. A simplified version could be of the finite form

$$X_Q^{(n)} = q_0 + A \sin(\omega n) + B \sin(\omega' n) \quad (1)$$

where

$X_Q^{(n)}$ = Flow rate of water into the system during stage n,
acre ft./mo.

q_0 = Mean rate of flow, acre ft./mo.

ω = A frequency factor of short term fluctuations,
cyc./mo.

A = Expected deviation of short term fluctuations,
acre ft./mo.

ω' = A frequency factor of long term fluctuations,
cyc./mo.

B = Expected deviation of long term fluctuations,
acre ft./mo.

An advantage of a Fourier series expression is that it may be expanded to describe situations as complex as the programmer feels necessary. Flow data used for the model in this paper was obtained from a report by the U.S. Corps of Engineers (5). The form of this data is presented in Table 1. It was taken from the Jordan Creek area, a part of the Delaware River Basin. Observations were taken over a period of several years, and then tabulated statistically on a twelve month basis. The table may be approximated by the expression

$$X_Q^{(n)} = 7.0 \times 10^3 + 5.0 \times 10^3 \sin(\pi n/6) + 1.0 \times 10^3 \sin(\pi n/60) \quad (2)$$

TABLE 1. WATER FLOW DATA FOR THE JORDAN CREEK AREA

Month	Mean Flow Rate x 10^3 acre ft./mo.	Standard Deviation x 10^3 acre ft./mo.
January	9.04	6.07
February	10.05	4.72
March	12.91	4.35
April	10.34	5.16
May	8.16	4.77
June	4.34	2.77
July	4.75	4.06
August	4.56	7.08
September	3.49	3.50
October	3.46	4.69
November	6.56	4.33
December	8.93	4.80

The output of the hydrology generator in equation 2 is illustrated in Figure 1.1 for a time period of twelve months.

BENEFIT AND LOSS FUNCTIONS

As mentioned earlier, system analysis concerns the effect of operational policy on the monetary benefits and consequences accrued by each demand zone. Consequently, functions must be derived for each source of demand, relating economic gains or losses as a function of scheduled releases within the system.

For a given resource output, each point from a benefit function should measure the lesser of the following two commodities.

- a. The resource cost of providing equivalent goods and services by the least costly alternative.
- b. The "willingness" of consumers to pay for these goods and services.

The technique of constructing a benefit function for a given demand zone is illustrated in Figure 2. Each point from an economic loss function should measure the penalty of failure to meet a given target output. The target output for this demand source represents a minimal supply commitment. Failure to meet this commitment will invoke losses or penalties against the

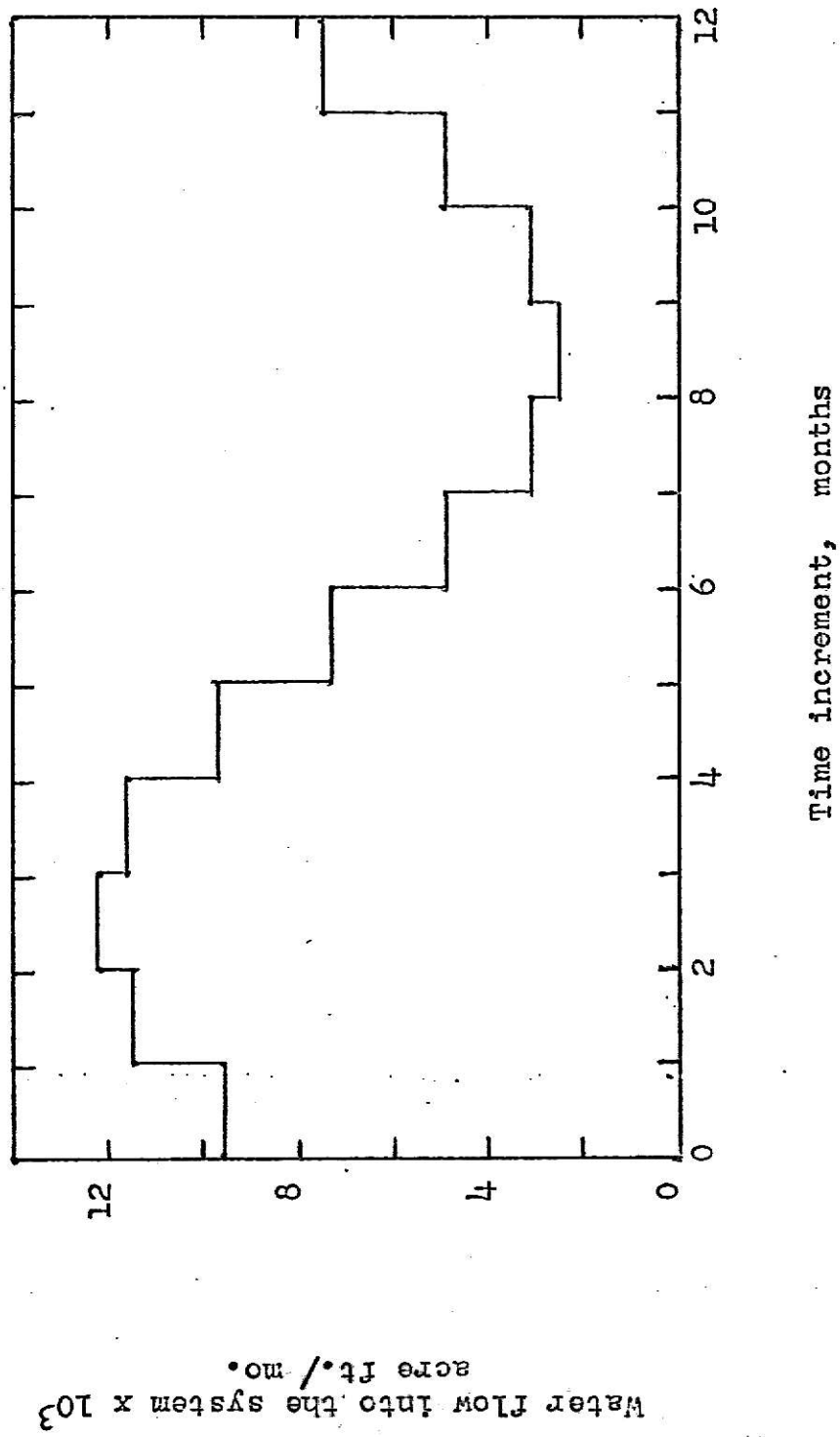
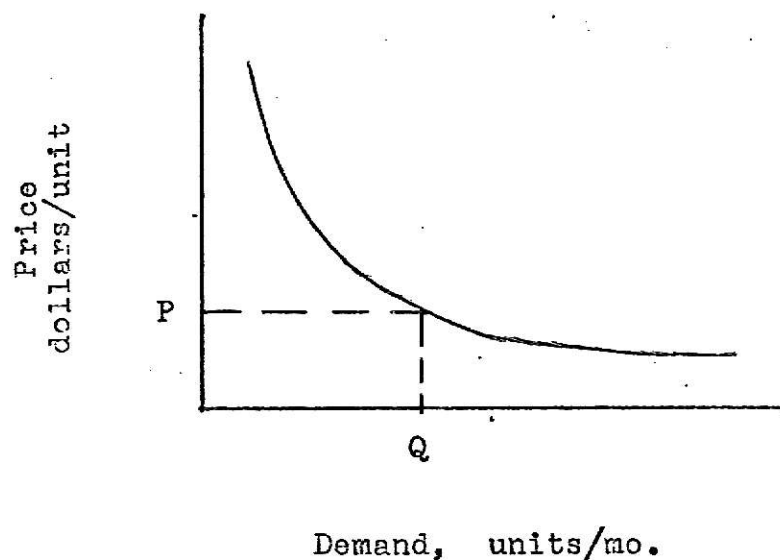


Fig. 1.1. Output of hydrology generator for a 12 month period.

- I. For a given commodity, determine the demand schedule for the zone or region in question.



- II. For each of a series of target outputs, Q , determine the total cost per month. Plot these values against demand. This relation constitutes a benefit function.

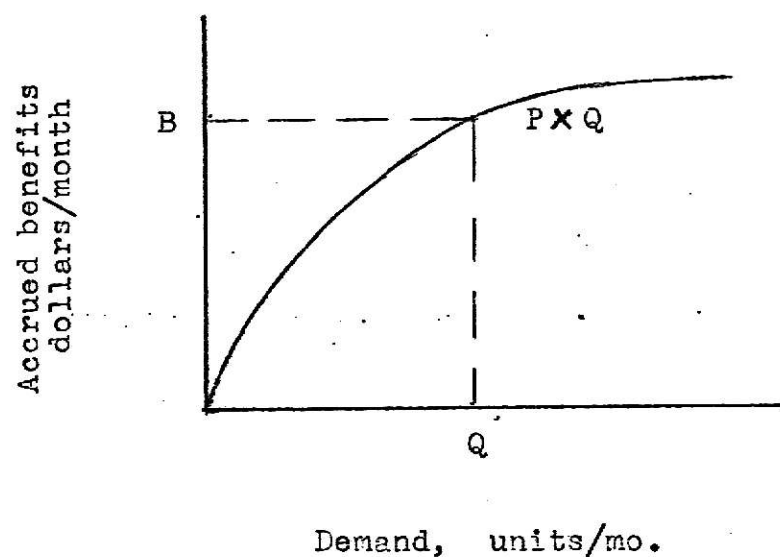


Fig. 2. Construction of a benefit function.

system. Examples of resource outputs with associated gain and loss functions are listed below (6).

- a. Municipal and industrial water supply at the time and location of need.
- b. Reservoir supply for hydroelectric power.
- c. Reservoir use for recreation.
- d. Water supply for irrigation.

On the pages that follow, benefit and economic loss functions are expressed for each demand source considered in the test system.

Water Supply For Urban Needs

Urban requirements include water supply for domestic, public, and commercial use. Some of the costs which must be faced involve storage, transport, purification, and administration (7). The marginal price of water in the Jordan Creek area has been found equal to \$12.10/acre foot under normal load conditions (5). This value would correspond to a single point on the demand schedule illustrated in Figure 2. In many cases, this may be the only information obtainable. If this is the case, the benefit function may be approximated by a straight line as shown in Figure 3. If, however, cost information is

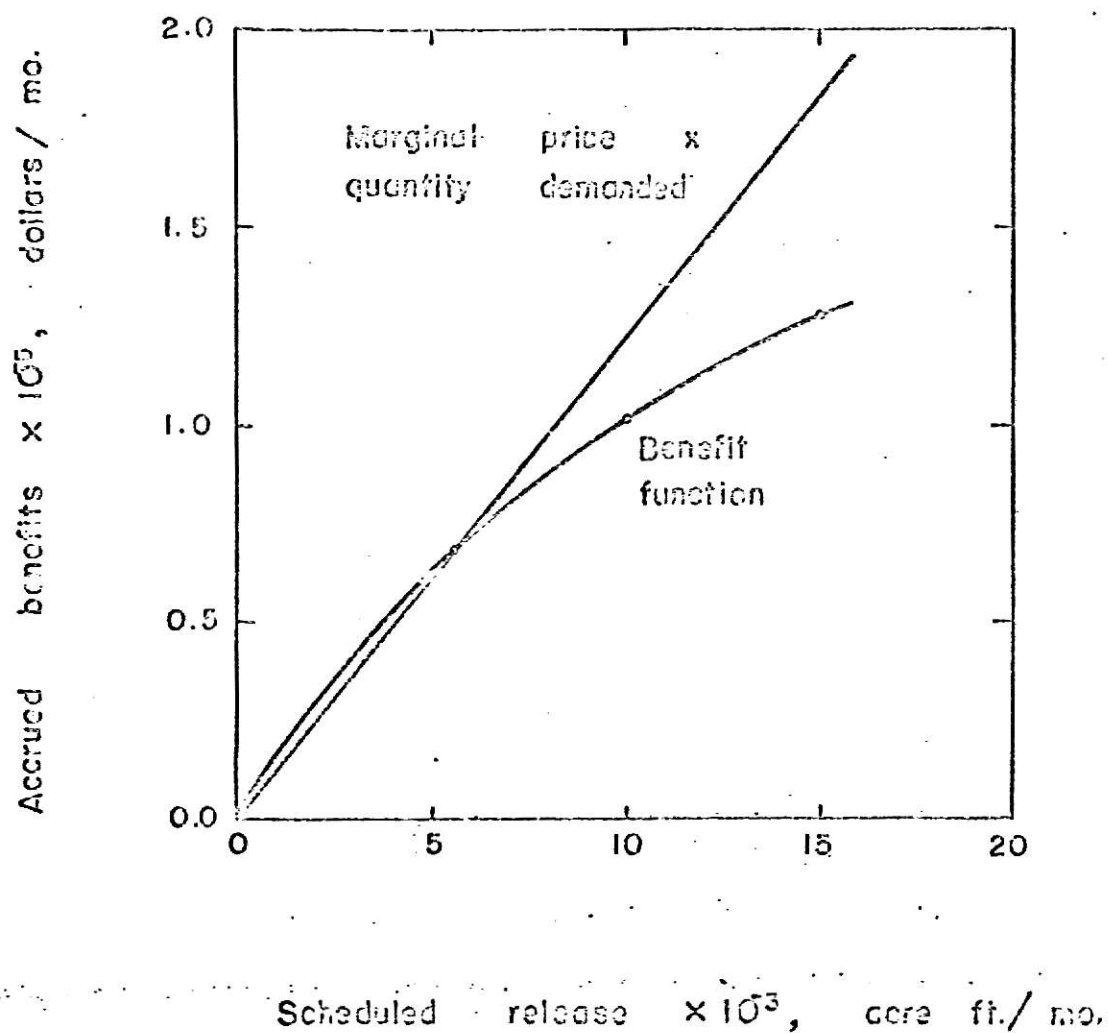


Fig. 3. A benefit function for urban water supply & quality.

available for varying load conditions, the benefit function would be plotted as a curve of the nature also shown in Figure 3. With this information given, a programmer can use a device such as Lagrange's Formula to fit data with an expression. The curve used in Figure 3, for example, is approximated by the function

$$\phi_{UB}^{(n)} = -4.0 \times 10^{-4} (\theta_{UR}^{(n)})^2 + 1.4 \times 10^1 (\theta_{UR}^{(n)}) \quad (3)$$

where

$\phi_{UB}^{(n)}$ = System benefit accrued from water supply to urban areas during stage n, dollars/mo.

$\theta_{UR}^{(n)}$ = Scheduled release of water for urban use during stage n, acre ft./mo.

The unit loss sustained from unexpected deficiencies below the target output demand has been found equal to \$16.40/acre foot (5). Data for loss functions will, at the very least, be more difficult to obtain than that for benefit functions.

Values here are connected with commercial and private losses contracted in a community when minimal expected commitments are not met. Average or point values such as the one above should, in most cases, be expected (2). Penalty or loss functions would, therefore, be linear. An example of such a function is

shown below.

$$\begin{aligned}\phi_{UL}^{(n)} &= 1.64 \times 10^1 (x_{UT}^{(n)} - \theta_{UR}^{(n)}) \\ &= 0, \quad x_{UT}^{(n)} \leq \theta_{UR}^{(n)}\end{aligned}\quad (4)$$

where

$\phi_{UL}^{(n)}$ = Loss sustained from urban supply shortage below the target output demand for stage n, dollars/mo.

$x_{UT}^{(n)}$ = Urban target output demand during stage n, acre ft./mo.

For the urban center in question, a constant target demand of 2.25×10^3 acre feet/month has been found a good approximation. It will be noted that the unit penalty cost is greater in magnitude than a corresponding decrease in the benefit function. The loss from an unanticipated deficit for an existing system will always be more costly than a reduction in planned output (8).

Water Supply For Irrigation

Under normal operating conditions in the Jordan Creek area, the marginal price of water for irrigation is \$2.25/acre foot (5). This is under urban supply costs since less is required

in purification and distribution facilities. An example of a benefit function is shown in Figure 4. This curve illustrates the connection between system gains and scheduled water releases for irrigation. Using Lagrange's Formula, the curve may be approximated by the following expression.

$$\phi_{IB}^{(n)} = -8.0 \times 10^{-5} (\theta_{IR}^{(n)})^2 + 2.7 (\theta_{IR}^{(n)}) \quad (5)$$

where

$\phi_{IB}^{(n)}$ = System benefit accrued from irrigation during stage n, dollars/mo.

$\theta_{IR}^{(n)}$ = Scheduled release of water for irrigation during stage n, acre ft./mo.

The unit loss sustained from unexpected deficiencies below the target output demand has been found equal to \$3.42/acre foot (5). A linear function expressing accrued loss as a function of this deficiency is shown below.

$$\begin{aligned} \phi_{IL}^{(n)} &= 3.42 (x_{IT}^{(n)} - \theta_{IR}^{(n)}) \\ &= 0, \quad x_{IT}^{(n)} \leq \theta_{IR}^{(n)} \end{aligned} \quad (6)$$

where

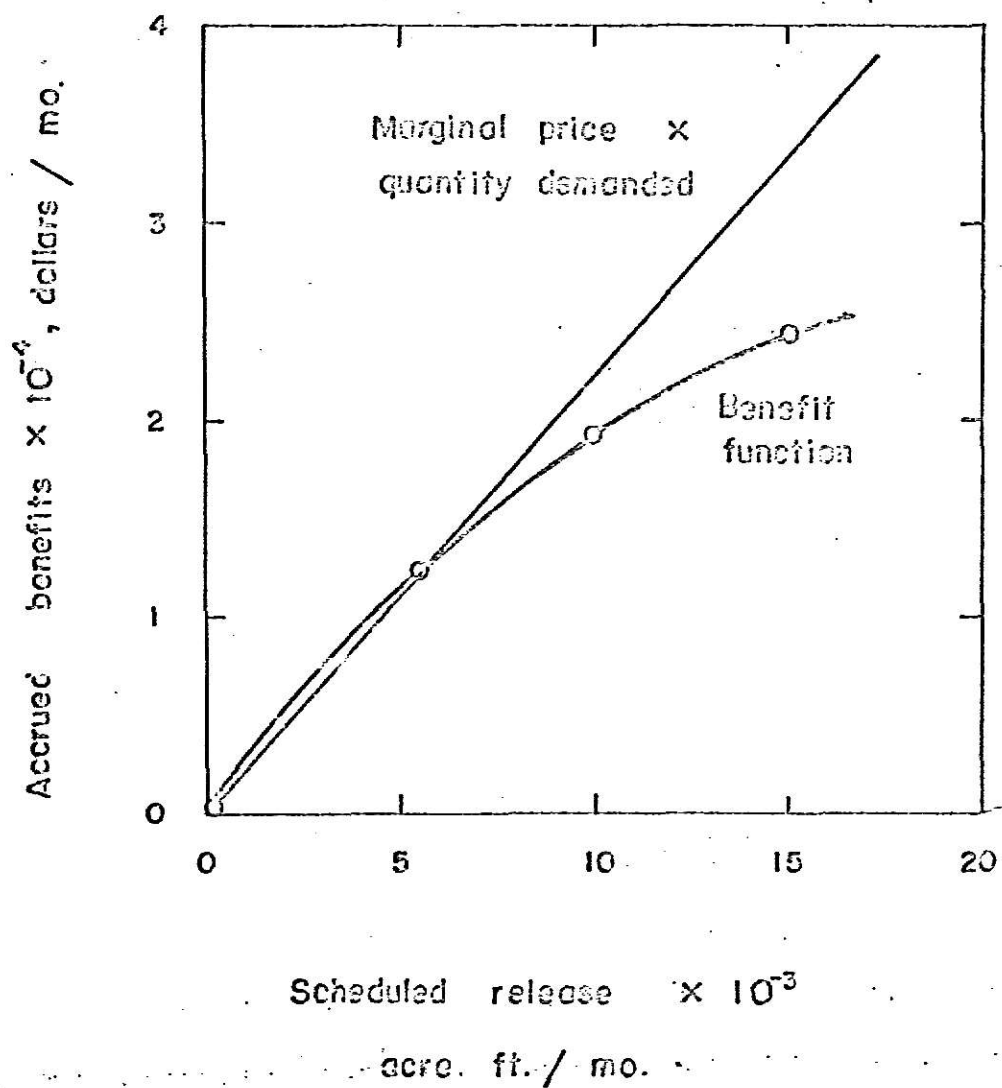


Fig. 4. A benefit function for irrigation water supply.

$\phi_{IL}^{(n)}$ = Loss sustained from irrigation supply shortage below the target output demand for stage n, dollars/mo.

$X_{IT}^{(n)}$ = Irrigation target output demand during stage n, acre ft./mo.

Unlike the urban center under study, the target output demand for irrigation can fluctuate widely from season to season. However, since the water supply needs of agriculture are cyclic in nature, target output demand may be expressed as a periodic function without encroaching into the field of dynamic analysis. A suggested form of this function is shown below.

$$\begin{aligned} X_{IT}^{(n)} &= d_o + D \sin(\omega''n) \\ &= 2.25 \times 10^3 + 1.0 \times 10^3 \sin(\pi n/6) \end{aligned} \quad (7)$$

where

d_o = Mean flow rate required by the irrigation target output demand, acre ft./mo.

ω'' = A frequency factor of seasonal fluctuations, cyc./mo.

D = Expected deviation of seasonal demand fluctuations, acre ft./mo.

Water Supply For Hydroelectric Power

Regarding power distribution within the test system, two

assumptions are made. First, hydroelectric power which is produced within the system is used by demand zones within the system. Power is not transported to other areas. Secondly, the widely fluctuating nature of water movement eliminates hydroelectric power as a continually reliable source. consequently, when power demands cannot be met internally, it is tapped from an external network. Common to the study of most areas, are relations which express unit benefits as a function of the hydroelectric plant output. Such a relation is plotted in Figure 5 for a power plant at one of the Jordan Creek reservoirs (5). Using Lagrange's Formula, this curve may be approximated by the function

$$\phi_{UnB}^{(n)} = 3.07 \times 10^{-16} (X_{po}^{(n)})^2 - 4.0 \times 10^{-9} (X_{po}^{(n)}) + 1.71 \times 10^{-2} \quad (8)$$

where

$\phi_{UnB}^{(n)}$ = Unit benefit from hydroelectric power output during stage n, dollars/k.w. hr.

$X_{po}^{(n)}$ = Average output of the power plant during stage n, k.w. hrs./mo.

$\phi_{HB}^{(n)}$, benefits accrued during the n^{th} month or stage from hydroelectric power, is then equal to

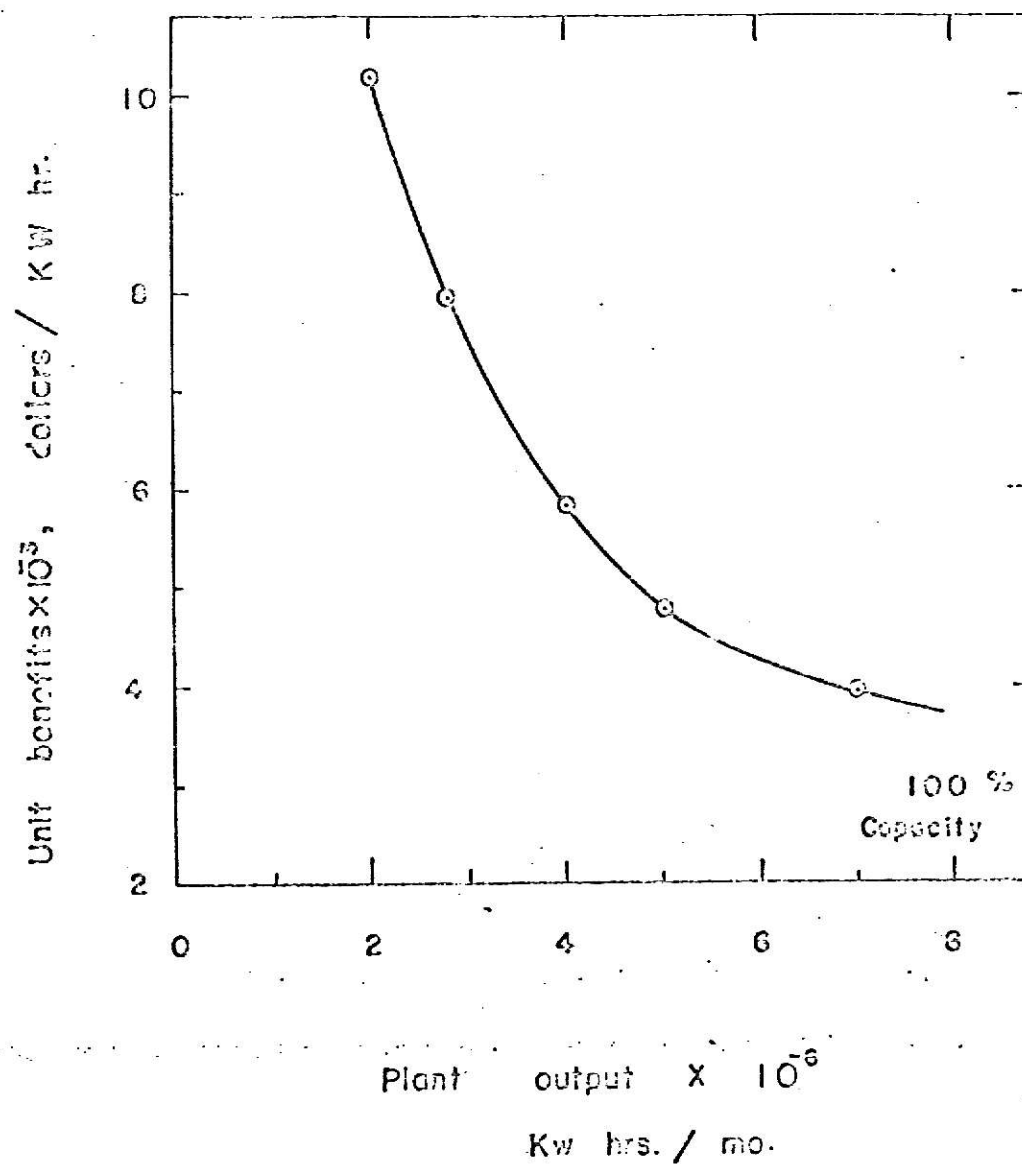


Fig. 5. Unit hydroelectric power benefits as a function of plant output.

$$\phi_{HB}^{(n)} = \phi_{UnB}^{(n)} X_{po}^{(n)} \quad (9)$$

To facilitate a solution later in the analysis, it is desirable that all functions be expressed in terms of water movement and time. With reference to the reservoir mentioned above (5), the power variable, $X_{po}^{(n)}$, is related to "head" (reservoir height at the dam), and water flow rate through the turbines by the expression

$$\begin{aligned} X_{po}^{(n)} &= K e X_{TR}^{(n)} X_h^{(n)} \\ &= 8.74 X_{TR}^{(n)} X_h^{(n)} \end{aligned} \quad (10)$$

where

K = A conversion factor relating hydroelectric power output to system parameters, k.w. hrs./acre ft.²

e = A plant efficiency factor.

$X_{TR}^{(n)}$ = Average flow rate utilized for the production of hydroelectric power during stage n , acre ft./mo.

$X_h^{(n)}$ = Average head during stage n , ft.

The stream diverted through power turbines, $X_{TR}^{(n)}$, is defined explicitly in the expression below.

$$X_{TR}^{(n)} = \theta_{UR}^{(n)} + \theta_{IR}^{(n)} + \theta_{ER}^{(n)} - \theta_{TB}^{(n)} \quad (11)$$

where

$\theta_{ER}^{(n)}$ = Scheduled release of water in excess of $\theta_{UR}^{(n)}$ and $\theta_{IR}^{(n)}$ during stage n, acre ft./mo.

$\theta_{TB}^{(n)}$ = Water flow from the reservoir which cannot be utilized for hydroelectric power, acre ft./mo.

$\theta_{TB}^{(n)}$ is, in effect, a slack variable. It permits the imposition of constraints on the power output level without affecting the other decision variables. Head, $X_h^{(n)}$, may in turn be described as a function of reservoir contents. Such an expression must be determined empirically. An example of such work is illustrated in Figure 6. Using Lagrange's Formula to fit this plot, approximation is made possible by the quadratic function

$$X_h^{(n)} = -6.70 \times 10^{-9} (X_v^{(n)})^2 + 1.97 \times 10^{-3} (X_v^{(n)}) \quad (12)$$

where

$X_v^{(n)}$ = Average reservoir volume during stage n, acre ft.

Combining equations 8, 9, 10, and 12, $\phi_{HB}^{(n)}$ may be expressed solely in terms of $X_v^{(n)}$ and $X_{TR}^{(n)}$.

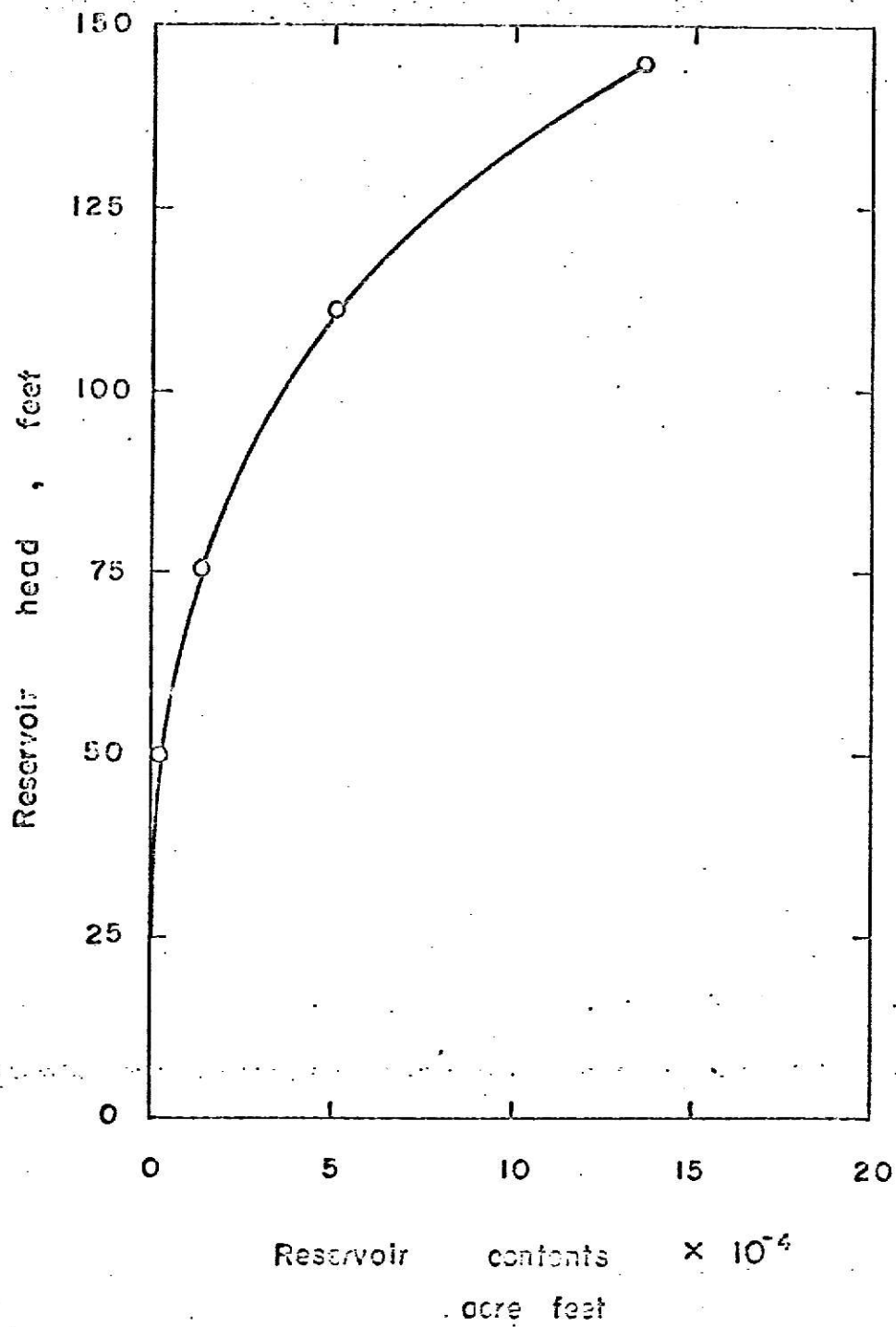


Fig. 6. Head vs. reservoir contents.

$$\begin{aligned}
\phi_{HB}^{(n)} = & -6.10 \times 10^{-42} (x_V^{(n)})^6 (x_{TR}^{(n)})^3 + 5.40 \times 10^{-36} (x_V^{(n)})^5 (x_{TR}^{(n)})^3 \\
& - 5.40 \times 10^{-29} (x_V^{(n)})^4 (x_{TR}^{(n)})^3 - 1.37 \times 10^{-27} (x_V^{(n)})^4 (x_{TR}^{(n)})^2 \\
& + 1.56 \times 10^{-23} (x_V^{(n)})^3 (x_{TR}^{(n)})^3 + 8.00 \times 10^{-20} (x_V^{(n)})^3 (x_{TR}^{(n)})^2 \\
& - 1.18 \times 10^{-14} (x_V^{(n)})^2 (x_{TR}^{(n)})^2 - 1.00 \times 10^{-11} (x_V^{(n)})^2 (x_{TR}^{(n)})^1 \\
& + 2.90 \times 10^{-4} (x_V^{(n)})^1 (x_{TR}^{(n)})^1
\end{aligned} \tag{13}$$

$x_V^{(n)}$, in addition, may be expressed in terms of flow variables by running a material balance about the reservoir. In continuous form, the rate of accumulation would be equal to the flow rate of water into the reservoir less the flow rate leaving. In finite difference form, this balance is represented by the transformation function that follows.

$$\begin{aligned}
x_V^{(n)} &= x_V^{(n-1)} + x_Q^{(n)} - x_{TOT}^{(n)} \\
&= x_V^{(n-1)} + x_Q^{(n)} - \theta_{UR}^{(n)} - \theta_{IR}^{(n)} - \theta_{ER}^{(n)}
\end{aligned} \tag{14}$$

where

$x_V^{(n-1)}$ = Average reservoir volume during stage n-1, acre ft.

$x_{TOT}^{(n)}$ = Total flow rate of water released from the reservoir during stage n, acre ft./mo.

The unit loss sustained from unexpected deficiencies below the target output demand has been found equal to \$0.02/k.w. hr.

(5). A linear function expressing accrued loss as a function of this deficiency is shown below.

$$\begin{aligned}\phi_{HL}^{(n)} &= 2.0 \times 10^{-2} (X_{HT}^{(n)} - X_{po}^{(n)}) \\ &= 0, \quad X_{HT}^{(n)} \leq X_{po}^{(n)}\end{aligned}\quad (14)$$

where

$\phi_{HL}^{(n)}$ = Loss sustained from a power shortage below the target output demand for stage n, dollars/mo.

$X_{HT}^{(n)}$ = Target output demand for hydroelectric power during stage n, k.w. hrs./mo.

For the test system under study, a constant target output demand of 5.0×10^6 k.w. hours/month has been found a good approximation. Again, since $\phi_{HL}^{(n)}$ must be expressed in terms of flow variables,

$$\begin{aligned}X_{po}^{(n)} &= f(X_{TR}^{(n)}, X_v^{(n)}) \\ &= K e^{X_{TR}^{(n)}} X_h^{(n)} (X_v^{(n)}) \\ &= -5.85 \times 10^{-8} (X_v^{(n)})^2 (X_{TR}^{(n)}) + 1.7 \times 10^{-2} (X_v^{(n)}) (X_{TR}^{(n)})\end{aligned}\quad (15)$$

Combining equations 14 and 15, $\phi_{HL}^{(n)}$ may be expressed in terms of $X_v^{(n)}$ and $X_{TR}^{(n)}$.

$$\begin{aligned} \phi_{HL}^{(n)} &= +1.17 \times 10^{-9} (X_V^{(n)})^2 (X_{TR}^{(n)}) - 3.4 \times 10^{-4} (X_V^{(n)}) (X_{TR}^{(n)}) + 2.0 \times 10^{-2} (X_{HT}^{(n)}) \\ &= 0, \quad X_{HT}^{(n)} \leq f(X_{TR}^{(n)}, X_V^{(n)}) \end{aligned} \quad (16)$$

Water Supply For Recreation

Water based recreation would include such activities as boating, swimming, and fishing as well as interest in adjacent shore facilities (9). The cost of alternative services has been calculated at \$1.05/visitor-day for the Jordan Creek area (5). The Corps of Engineers, in addition, has discovered that attendance may be plotted empirically as a function of reservoir capacity. An example of such a relationship is illustrated in Figure 7 for the Jordan Creek area. Using an appropriate curve fitting technique, Figure 7 may be approximated by the equation

$$X_{FA}^{(n)} = +1.53 \times 10^{-9} (X_V^{(n)})^2 - 3.68 \times 10^{-4} (X_V^{(n)}) + 3.89 \times 10^1 \quad (17)$$

where

$X_{FA}^{(n)}$ = Attendance for water based recreation during stage n,
visitor-days/acre of reservoir surface/month.

In a similar manner, the reservoir surface area, $X_{RS}^{(n)}$, may be plotted against its capacity. With this relationship known, system gains from recreational activities may be expressed as

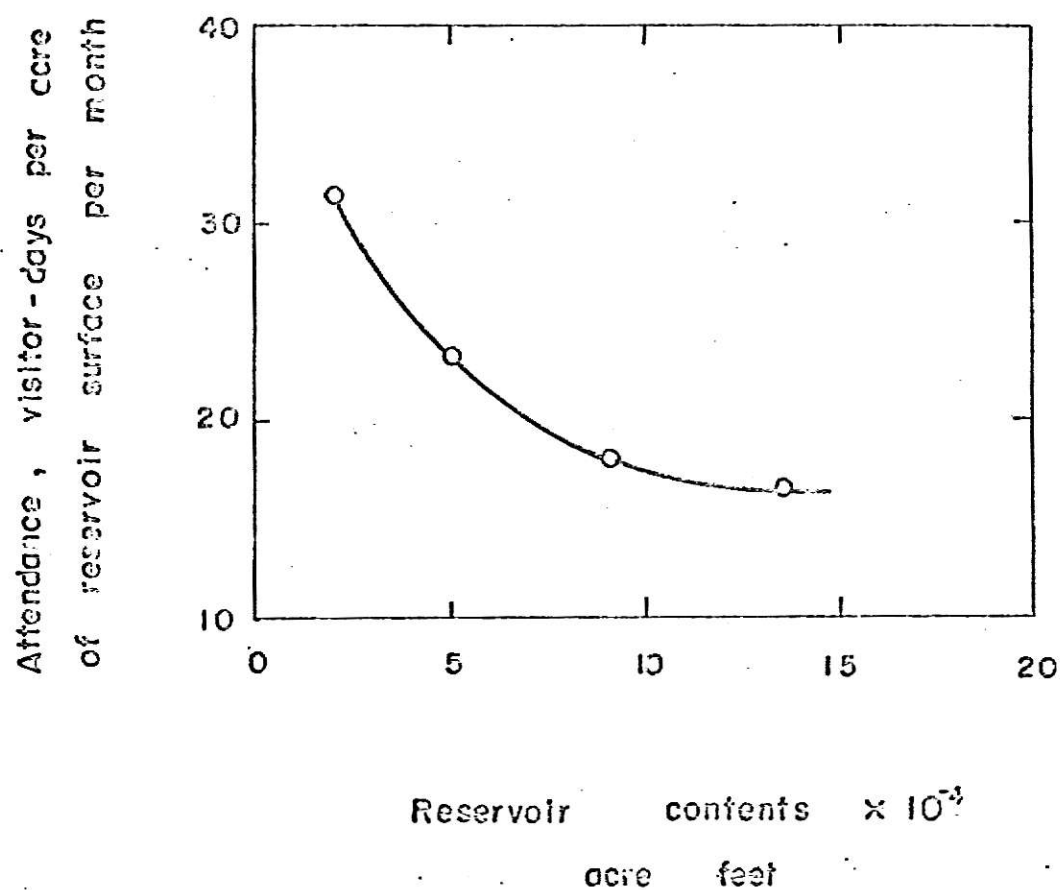


Fig. 7. Attendance as a function of reservoir capacity.

a function of $X_v^{(n)}$.

$$\begin{aligned}\phi_{RB}^{(n)} &= 1.05 X_{RA}^{(n)} (X_v^{(n)}) X_{RS}^{(n)} (X_v^{(n)}) \\ &= 3.2 \times 10^{-10} (X_v^{(n)})^3 - 7.70 \times 10^{-5} (X_v^{(n)})^2 + 8.1 \times 10^0 (X_v^{(n)})\end{aligned}\quad (18)$$

where

$\phi_{RB}^{(n)}$ = System benefit accrued from water based recreation during stage n, dollars/mo.

For this system, no target output or associated loss function is connected with recreation.

ECONOMIC ANALYSIS

As stated earlier, the object of a system analysis is to determine that set of operational policies which maximizes the cash flow stream over the period of analysis. Assuming an estimated economic life of Y years, this goal may be stated symbolically as follows.

$$\text{Maximize } V = \sum_{n=1}^{12Y} (S^{(n)} / (1 + r)^n) \quad (19)$$

where

V = The present value of net system gains accrued during the period of economic analysis, dollars.

$S^{(n)}$ = The net economic gain accrued by the system during stage n , dollars.

r = The interest rate of money prorated on a monthly basis.

$S^{(n)}$ in turn, may be represented by the expression

$$S^{(n)} = T_B^{(n)} - T_P^{(n)} - T_{om}^{(n)} \quad (20)$$

where

$$\begin{aligned} T_B^{(n)} &= \text{Total benefit accrued by the system during stage } n, \\ &\text{dollars.} \\ &= \phi_{UB}^{(n)} + \phi_{IB}^{(n)} + \phi_{HB}^{(n)} + \phi_{RB}^{(n)} \end{aligned} \quad (21)$$

$$\begin{aligned} T_P^{(n)} &= \text{Total penalty or loss contracted by the system during} \\ &\text{stage } n, \text{ dollars.} \\ &= \phi_{UL}^{(n)} + \phi_{IL}^{(n)} + \phi_{HL}^{(n)} \end{aligned} \quad (22)$$

$$T_{om}^{(n)} = \text{Total operating and maintenance costs contracted by the system during stage } n, \text{ dollars.}$$

As the objective function, equation (19) is subject to the following set of constraints.

$$a. \quad 0 \leq \theta_{UR}^{(n)} \leq 4.0 \times 10^3 \quad (23)$$

$$b. \quad 0 \leq \theta_{IR}^{(n)} \leq 4.0 \times 10^3 \quad (24)$$

$$c. \quad \theta_{ER}^{(n)} \geq 1.0 \times 10^3 \quad (25)$$

$$d. \quad \theta_{UR}^{(n)} + \theta_{IR}^{(n)} + \theta_{ER}^{(n)} \leq 1.3 \times 10^4 \quad (26)$$

$$e. \quad \theta_{TB}^{(n)} \geq 0 \quad (27)$$

$$f. \quad X_{po}^{(n)} (X_{TR}^{(n)}, X_v^{(n)}) \leq 7.2 \times 10^6 \quad (28)$$

$$g. \quad 2.0 \times 10^4 \leq X_v^{(n)} \leq 1.40 \times 10^5 \quad (29)$$

These constraints represent physical boundaries or limitations which must be observed within the system. Constraints a and b, for example, represent points beyond which benefits will not occur for an increase in available water supply. These limits define the maximum capacities of transport and purification systems as well as the demand zones they service. Constraints c and d are limitations imposed on systems to insure that water flow remains within the limits of existing river beds and channels. Constraint f represents an upper limit for the hydroelectric power plant. It is a reflection of physical size. Beyond this point, the plant can produce no more power, regardless of head or flow available. Constraint g defines the usable limits of the reservoir. The lower limit represents dead storage allocation for silting, and the accumulation of debris.

The upper limit represents the maximum capacity of the reservoir.

The control policies developed in this model are, in effect, rules for storing, releasing, and routing water through the system. In general, control policies affecting urban water supply, irrigation, and power generation conflict with flood control. Drawdowns for all of these purposes conflict with the maintenance of reservoir levels for recreational use. If it were not for these conflicts, there might possibly be little if any problem to consider. This study is orientated toward optimal water resource distribution with reference only to economic and physical considerations. If value judgements based on political or other frames of reference are inherent, additional priorities or constraints may be added to the model.

The objective function expressed in equation (19) relates net economic gains accrued by the system as a function of regulatory or operational policies. These policies affect flow releases from the reservoir during each increment of time. In this case, one month. For the system under study, four decision variables must be changed with time in such a way as to maximize the cash flow stream. For an analysis covering a period of Y years, the objective function would involve $4 \times 12 \times Y$ independent variables subject to the seven inequality constraints listed in equations (23) through (29). The size

of such a function would be formidable were it not for such techniques as dynamic programming. As a result, this function may be divided into a series of stage-wise problems, each involving four decision variables subject to the constraints listed above.

DISCUSSION OF RESULTS AND CONCLUSIONS

The objective function developed in the system model consists of four decision variables, $\theta_{UR}^{(n)}$, $\theta_{IR}^{(n)}$, $\theta_{ER}^{(n)}$, $\theta_{TB}^{(n)}$, and one state variable, $X_V^{(n)}$, applied at each of N stages. Solution, it was discovered, fell naturally in the realm of dynamic programming. A computer program was constructed which tied all subroutines into a dynamic program routine as center. A complete discussion of this program is included in Appendix A. In addition, a set of directions is included for anyone wishing to use this program, or an extended version of it. At each stage in the dynamic program routine, a search was conducted for an optimal set of variables which influenced the policies set at that stage. This was accomplished by the Hooke and Jeeves pattern search technique. The search subroutine was modified somewhat, so that it could handle constrained problems. A discussion of the Hooke and Jeeves technique is included in Appendix B.

In Appendix C, results are given for work directly related to the computer program. This includes flow diagrams, listings, and an explanation of all symbols used in the program. In addition, a sample solution is printed in table form for a twelve stage problem. With output data in this form, the user may select an initial (or final) state variable, $X_V^{(n-1)}$, from a list of grid values, and then trace an optimal path relating control

policies against time. The size of the list found in these tables is controlled by input parameter cards. The size, and hence the accuracy may be varied to fit the needs of the problem. An initial state variable ($X_V^{(0)} = 4.55 \times 10^4$ acre ft.) was chosen arbitrarily, and an optimal path plotted from these tables. Appendix D illustrates the nature of this path as it ranges through a time period of twelve months. The variation of the state variable as well as the decision variables are included in these plots. The system model was developed with the idea that it could be modified or extended to cover different situations. Consequently, the dynamic program routine and the search subroutine were written in as general a manner as possible. They can, if desired, be utilized by completely separate model and constraint subroutines.

To save time on the computer, a price must be paid in terms of precision. Computational accuracy may be adjusted in two ways for this program. First, instructions may be inserted, directing the use of double precision. By doing so, the computer will round off values to 16 rather than the usual 8 significant figures. This will help prevent round off error from affecting calculated results. A great number of calculations were made in this program. Double precision could not be used, however, since it increased execution time by a factor of 5. One half hour was required for the normal

execution of a 12 stage problem. Interpolation error may be reduced by increasing the list of grid values for the state variable. To minimize time, a list size of 10 elements was chosen for the sample output illustrated in this work. The magnitude of the range determined for the state variable should warrant a larger grid network. Time limitations, however, made this impractical.

Modifications were made in the Hooke and Jeeves search subroutine such that perturbations were checked against existing constraints as well as the profit function. In spite of this feature, the search pattern would hang up on certain constraints. Unable to back up sufficiently to find a new approach, the search would terminate at a point short of a global optimum. Changing the form of one of the constraints appeared to alleviate this condition. The nature of the problem is such, however, I must conclude that the best possible values have not been found. At this time, I do not recommend a pattern search for constrained problems. A random search technique, on the other hand, would be quite useful since it can not be thwarted by a set of constraints. It does not lay patterns which can be terminated short of the desired optimum.

DISCUSSION OF TOPICS FOR FUTURE RESEARCH

The system model constructed in this paper is a beginning only. It can be modified and extended to cover many different situations. If greater complexity is desired for future research, several changes may be considered. The number of resource demand zones, for example, can be expanded. Flood control and navigational requirements may be added to those already under study. Flood control should be thought of as a special case since time-movement studies are concerned with a period of hours and days as opposed to the general case of months and years. A special hydrology generator must be developed to handle this situation.

The system considered in this work was centered about a single multipurpose reservoir, and required one state variable, $X_v^{(n)}$, in the objective function. System models may include several reservoirs located in series or on separate tributaries. Several state variables will be involved. Special techniques such as quasi-linearization will be required to expedite a solution by dynamic programming.

In addition to varying the complexity of a system model, the mode of analysis may also be changed. In this paper, the system was stable and a form of static economic analysis employed. The assumptions made in this analysis have already been

listed. If the system is not stable, that is, continually growing, dynamic analysis should be considered. In addition to conditions already discussed, it would provide for capital facilities which may be installed anytime during the period of analysis. In addition, the corresponding target output demand levels can also be changed.

There is also no reason why a model must be deterministic in nature. It may also be stochastic. For those who are interested in this approach, serious consideration should be paid to the Markov model postulated by M. B. Fiering (2). This is a statistical expression of the hydrology generator as shown below.

$$X_{i+1} = \mu_j + \beta_j(X_i - \mu_{j-1}) + t_{i+1}\sigma_j(1 - \rho_j^2)^{1/2} \quad (30)$$

where

X_{i+1} = Estimated value of flow in the $(i+1)^{\text{st}}$ unit of time.

μ_j = Mean flow in the j^{th} season.

X_i = Flow in the i^{th} unit of time.

β_j = Regression coefficient of flow in season j on flow in season $(j-1)$.

ρ_j = Correlation coefficient between flows in seasons j and $(j-1)$.

σ_j = Standard deviation of flows in the j^{th} season.

t_{i+1} = A random deviate normally distributed with zero mean and unit variance.

This model has been used by Nathan Buras in his work with dynamic programming in the field of water resources development (10, 11).

APPENDIX A. DISCUSSION OF PROGRAM

The source program, SYMPROG, is written in Fortran IV language for the IBM 360 computer. It is designed to solve analytical models involving water resource control systems. This entails optimization of multistage systems with one state variable, and any given number of decision variables. SYMPROG actually consists of one program and three subroutine subprograms. The tie arrangement between these subroutines is illustrated in Figure A-1.

The control systems mentioned above require an optimal set of decisions occurring over a designated period of time. Time is therefore divided into increments, each of which is treated as a stage for solution by dynamic programming. Because of the irreversible nature of time, these stages are numbered in a forward manner as shown in Figure A-2. The optimal policy at each stage is determined in a relationship between total system gains and interval profit. The chronological nature of this system has necessitated some modification in the superscripts of the function as shown below.

$$f_k^{(N-k) (N-k+1)}(x_v, \underline{e}) = \max_{\substack{\underline{e} \\ (N-k+1)}}^{(N-k+1)} (P^{(N-k)}(x_v, \underline{e}^{(N-k+1)}) + f_{k-1}^{(N-k+1)}(x_v))$$

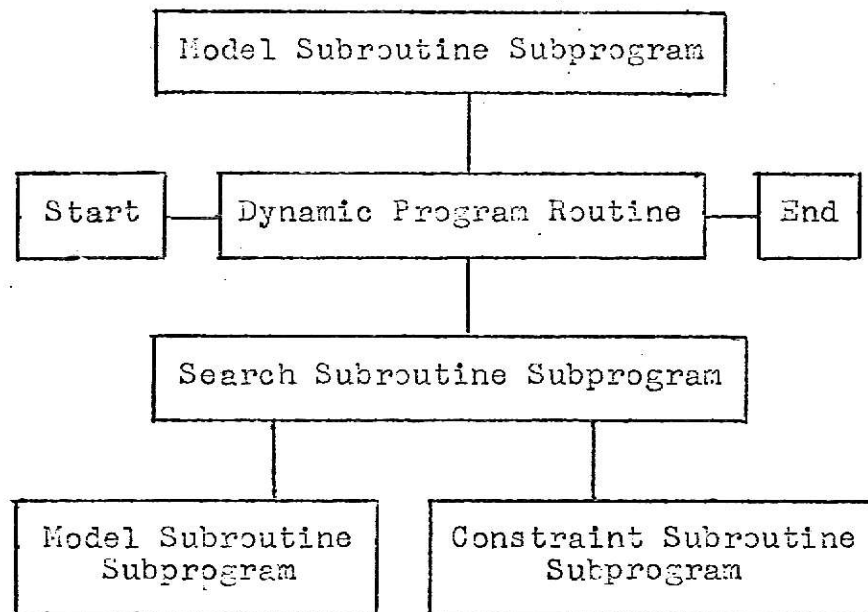


Fig. A-1. Tie arrangement between subroutines of the source program.

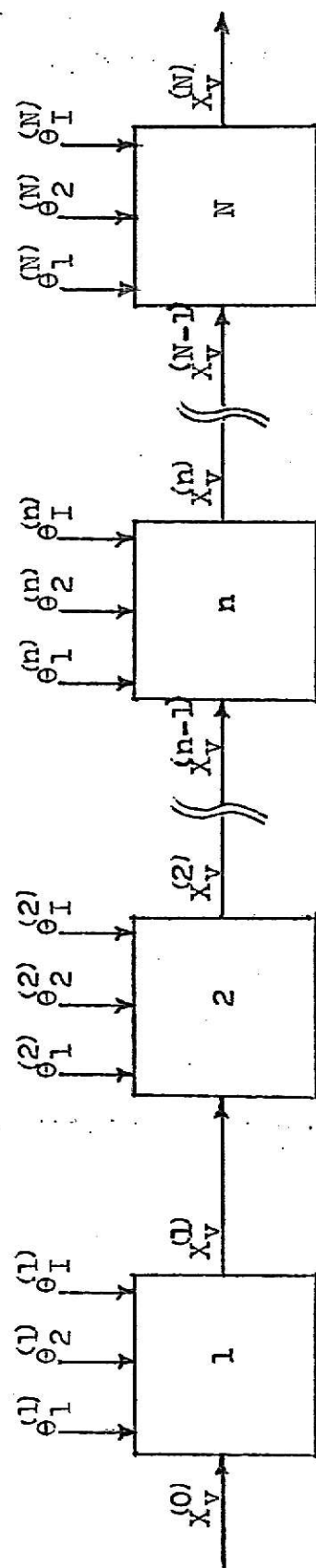


Fig. A-2. A multiple stage process with one state variable and I decision variables at each stage.

where

f_k = The total profit for a series of k remaining stages, dollars.

k = A counting index beginning with 1 at the last stage, N , and progressing by increments of one to N at the beginning stage, 1.

$P^{(N-k+1)}$ = The interval profit for a given stage, dollars.

$\underline{e}^{(N-k+1)}$ = The independent or decision variables involved at each stage, acre ft./mo.

In general, the dynamic program routine generates a set of grid points for the state variable, $X_V^{(n-1)}$, at the beginning of each stage. The size and nature of this set is dictated by input parameter cards. For each grid point, the search subroutine is called upon to find the optimal set of decision variables, $\theta_i^{(n)}$, $i = 1, 2, \dots, I$. With this information, the model subroutine is then called upon to calculate the corresponding values of $X_V^{(n)}$, and the interval profit, $P^{(n)}$. The dynamic program routine can then solve for the optimal policy, f_k . This routine has been generalized so that it may handle as many grid points of the state variable within as many stages as the programmer wishes. Any number of independent variables may be used.

The search subroutine includes the Hooke and Jeeves pattern search technique. This may be used to maximize an objective function with as many decision variables as desired. The sub-

routine is fed a base point consisting of an arbitrary set of values, one for each decision variable. The Hooke and Jeeves technique employs a perturbation pattern about this base point in search for an optimal set of values.

The value of the profit function is calculated in the model subroutine after each perturbation in the pattern search. This subroutine specifies the objective function connected with a specific model. If the function is subject to a set of constraints, a constraint subroutine may be added to the main program. The search subroutine has been modified to handle this possibility. In addition to calling the model subroutine after each perturbation, the search subroutine also draws the constraint subroutine to check the validity of each step. Success or failure of each evaluation will incur the same routing decisions specified for the model subroutine. In this manner, the search subroutine has the capability of moving along a boundary zone where an optimum point may exist.

The search subroutine has also been modified to accept any number of initial base points desired by the programmer. Before returning control to the dynamic program routine, the search subroutine will select the best result from several trials, each starting from a different base point. This feature is designed to achieve proper convergence in complex problems with local peaks or ridges in hyper-space. The result of each trial

is included in the printed output. This allows the programmer to monitor the success of the search subroutine.

In the following paragraphs, a set of directions has been supplied for prospective users of the SYMPROG program.

Modifications have also been suggested for those who wish to use specific subroutines from this program to solve different problems.

DIRECTIONS FOR USING THE SOURCE PROGRAM

The heart of SYMPROG is the dynamic program routine and the search subroutine subprogram. Other model subroutines with different parametric values and constraints may be tied into this center. The recommended order in which the source deck should be arranged is listed as follows.

Job cards

Dynamic program routine

Search subroutine subprogram

Model subroutine subprogram

Constraint subroutine subprogram

Job cards

Input data

Job card

Dynamic Program Routine

All data is read in at the beginning of this routine. The first three values are integer. They are placed in columns 1-5 on separate cards under the format (I5). This input corresponds to the following parameters.

NE = The number of times perturbation size is reduced before the search subroutine terminates a search pattern.

K = The number of independent or decision variables that are involved in the solution.

M = The number of initial base points which will be used in separate trials by the search subroutine.

The following data is floating point. Each value is placed in columns 1-12 on separate cards under the format (E 12.5). This input corresponds to the following parameters.

TOM = The operating and maintenance costs associated with a given system, dollars/mo.

S = The total number of stages considered in the dynamic program routine.

R = The interest rate of money prorated on a monthly basis.

D = A parameter controlling the number of grid points associated with the state variable x_v^{n-D} .

- ALOW = The lowest value considered in the grid network for the state variable, acre ft.
- AHIGH = The highest value considered in the grid network for the state variable, acre ft.
- BET = The factor by which perturbation size is cut when the search subroutine encounters a disruption in the search pattern.
- DEL(I) = The list of initial step sizes used for each decision variable in the perturbation pattern of the search subroutine.
I = 1,K
- BA(I,N) = The list of initial base points used by the search subroutine in separate trial search patterns.
I = 1,K
N = 1,M

The SYMPROG program will accept values of

- K up to 9
- M up to 9
- D up to 19

All other input parameters may be assigned any value necessary to accommodate the solution of a problem. If greater values are required for those parameters listed above, modify the dimension statements at locations 3, 4, and 83 such that

- DEL(9) is changed to DEL(K)
- BA(9,9) to BA(K,M)
- BAH(9,9) to BAH(K,M)
- XV(20) to XV(D+1)

XVO(20) to XVO(D+1)

X(9) to X(K)

FB(20) to FB(D+1)

FBO(20) to FBO(D+1)

DELP(9) to DELP(K)

TH(9) to TH(K)

BAP(9,9) to BAP(K,M)

BF(9) to BF(K)

where K, M, and D represent the higher values desired for the respective parameters.

The model used in this program requires four decision variables. These variables are labeled as X1, X2, X3, and X4 in the format at locations 6-8. If more than four independent variables are used in a given system, this format should be expanded.

At locations 31 and 32 are two call statements. One involves the search subroutine, and the other achieves connection with the model subroutine (labeled CRIT). In location 32, the four decision variables must again be referred to separately as X(1), X(2), X(3), and X(4). If more than four decision variables are used, this statement should be expanded. This change will also affect the number of dummy arguments in the subroutine statement at location 171. When values are calculated in a subroutine and brought back to the calling program,

the arguments used should be initialized. In this case the arguments are X (for the array $X(I)$, $I=1,K$), PI, $XV0(J)$, and XPO . They are initialized in locations 30.1 - 30.6. Different systems may involve different arguments. If so, they must be defined before engaging the call statement.

Search Subroutine Subprogram

In this subroutine, call statements are used to connect with the model and constraint subroutines at locations 98, 99, 99.1, 109, 110, 111, 121, 125, 126, 133.3, 133.4, 133.5, 135, 140.4, 140.5, 140.6, 154.1, and 154.2. In each case, the first four arguments are related to the four decision variables. These arguments must be expanded in problems calling for more than four independent variables. In addition, subroutine statements at locations 171 and 214 should have their arguments expanded in accordance with this change.

Model Subroutine Subprogram

Additional information required by this subroutine for calculations is supplied through arguments in the call statement. Two values which must be returned to the calling programs (regardless of the model used) are the calculated values of the state variable, $X_v^{(n)}$, and the interval profit, $P^{(n)}$, for each stage

n. They are required both by the dynamic program routine and the search subroutine in order to function.

Constraint Subroutine Subprogram

The method of testing a proposed perturbation against a series of constraints is illustrated by statements in locations 213 - 232. If the system under study is without constraints, the following modifications should be made.

1. Remove statements at locations 99.1, 111, 133.5, and 140.6.
2. Replace locations 133.5 and 140.6 with the statement: GO TO 191.

Output Data

The results are printed under a two line format for each grid point of the state variable $X_v^{(n-1)}$.

Line One

B = k. The counting index used to denote stage position in the dynamic program routine.

J = j. Grid point designation of the state variable $X_v^{(n-1)}$.

Line Two

XVJ = $(X_v^{(n-1)})_j$. The j^{th} grid value of a state variable. The

average reservoir volume during stage $n-1$, acre ft.

$X1 = \theta_{UR}^{(n)}$. Scheduled release of water for urban use during stage n , acre ft./mo.

$X2 = \theta_{IR}^{(n)}$. Scheduled release of water for irrigation during stage n , acre ft./mo.

$X3 = \theta_{ER}^{(n)}$. Scheduled release of water in excess of $\theta_{UR}^{(n)}$ and $\theta_{IR}^{(n)}$ during stage n , acre ft./mo.

$X4 = \theta_{TB}^{(n)}$. Water flow from the reservoir which cannot be utilized for hydroelectric power, acre ft./mo.

$XVOJ = (X_V^{(n)})_j$. The j^{th} grid value of a state variable. The average reservoir volume during stage n , acre ft.

$FBJ = f_k$. The total profit for a series of k remaining stages in the dynamic program routine, dollars.

Directly above this information are several lines of additional output. The number of lines corresponds to the number of initial base points inserted by the programmer. The results of each line represent the optimal values calculated by the search subroutine from a given base point. The best of several trials is selected for the final output listed below. Each line carries the following information from a given trial.

$PI = P^{(n)}$. The interval profit associated with a stage n , dollars.

$X(I) = \theta_i^{(n)}$, $i = 1, 4$. The four decision variables, $\theta_{UR}^{(n)}$, $\theta_{IR}^{(n)}$,

$\theta_{ER}^{(n)}$, and $\theta_{TB}^{(n)}$.

If more than four independent variables are involved, their values will be printed below this line.

APPENDIX B. DISCUSSION OF THE HOOKE AND JEEVES PATTERN SEARCH TECHNIQUE

Hooke and Jeeves (12) have devised an approach for staying on the crest of a ridge in hyper-space while searching for an optimum point. Their pattern search technique is based on the conjecture that any set of moves (adjustments of the independent variables) which have been successful during early experiments will be worth trying again. This strategy is successful on straight ridges because the only way an early pattern of moves can succeed is if it lies along the crest. Hence further moves in the same direction will be worthwhile if the ridge is straight.

Although the method starts cautiously with short excursions from the starting point, the steps grow with repeated success. Subsequent failure indicates that shorter steps are in order, and if a change in direction is required the technique will start over again with a new pattern. In the vicinity of the peak, the steps become very small and cannot overlook any new promising direction.

In visualizing what is meant by a pattern, it is useful to think of an arrow, its base at one end and its head at the other. The search begins at an initial base point, $\underline{b}_1$, which may be chosen arbitrarily. The experimenter then chooses a

step size δ_i , for each independent variable, θ_i , $i=1,2,\dots,I$. Let $\underline{\delta}_i$ be the vector whose i^{th} component is δ_i , all the rest being zero. After measuring the criterion at the initial base, one takes an observation at $\underline{b}_1 + \underline{\delta}_1$. If this new point is better than the base, it is registered as the temporary head $\underline{t}_{1,1}$. The double subscript shows that the first pattern is being developed, and that the first variable, θ_1 , has been perturbed. If $\underline{b}_1 + \underline{\delta}_1$ is not as good as $\underline{b}_1$ then $\underline{b}_1 - \underline{\delta}_1$ would be tried. If this new point is better than $\underline{b}_1$ it would be made the temporary head. If not, $\underline{b}_1$ is designated temporary head.

Perturbation of θ_2 , the next independent variable, is carried out in a similar manner, this time about the temporary head $\underline{t}_{1,1}$ instead of the original base, $\underline{b}_1$. In general, the i^{th} temporary head, $\underline{t}_{1,i}$, is obtained from the preceding one, $\underline{t}_{1,i-1}$, in the following manner (13).

$$\underline{t}_{1,i} = \begin{cases} \underline{t}_{1,i-1} + \underline{\delta}_i & \text{if } f(\underline{t}_{1,i-1} + \underline{\delta}_i) > f(\underline{t}_{1,i-1}) \\ \underline{t}_{1,i-1} - \underline{\delta}_i & \text{if } f(\underline{t}_{1,i-1} - \underline{\delta}_i) > f(\underline{t}_{1,i-1}) > f(\underline{t}_{1,i-1} + \underline{\delta}_i) \\ \underline{t}_{1,i-1} & \text{if } f(\underline{t}_{1,i-1}) \geq \max(f(\underline{t}_{1,i-1} + \underline{\delta}_i), f(\underline{t}_{1,i-1} - \underline{\delta}_i)) \end{cases}$$

This expression covers all values of i from 1 to I if the convention is adopted that

$$\underline{t}_{1,0} = \underline{b}_1$$

When all of the variables have been perturbed, the last temporary head point, $\underline{t}_{1,1}$, is designated the second base point, $\underline{b}_2$.

The original base point, $\underline{b}_1$, and the newly determined base point, $\underline{b}_2$, together establish the first pattern. It may be reasoned that if a similar exploration were conducted from $\underline{b}_2$ the results would likely be the same. If so, local excursions may be skipped, and the pattern extended immediately, doubling its length. This establishes a new temporary head, $\underline{t}_{2,0}$, for the second pattern based at $\underline{b}_2$. This initial temporary head is given by

$$\underline{t}_{2,0} = \underline{b}_1 + 2(\underline{b}_2 - \underline{b}_1) = 2\underline{b}_2 - \underline{b}_1$$

The double subscript, 2,0, indicates the beginning of the second pattern before any perturbations have occurred about the temporary head.

The search pattern is expanded until none of the temporary heads are any better than the preceding base point. In this case

$$\underline{b}_i = \underline{b}_{i+1}$$

The pattern is destroyed. Since this could mean it has reached

a peak or crossed a resolution ridge, new maneuvers are in order.

With the termination of one pattern, An attempt must be made to build a new one about the last successful base point, b_t . This point is made the initial temporary head, $t_{1,0}$ for a local exploration. If not, the perturbation steps must be shortened in an attempt to break the resolution ridge that may exist. The search pattern is ended when the step sizes fall below a preselected minimum. At this point it is concluded that a peak has been located, at least as far as can be determined with the finest resolution available.

This discussion has been in connection with the standard Hooke and Jeeves pattern search technique. Since no provision has been made for constrained problems, the computer search subroutine has been modified to allow for this. Discussion of this modification is included in Appendix A.

APPENDIX C. COMPUTER PROGRAM AND RESULTS

Within this section is a compilation of work related to the computer program, and the output from this program. Below, a list of the contents of the following pages has been submitted.

Figure (C-1). Computer Flow Diagram.

Table (C-1). Program Symbols And Explanation.

Table (C-2). Computer Program.

Table (C-3). Sample Input Data For Computer Program.

Table (C-4). Sample Output Of Computer Program For A 12 Stage System.

Start Dynamic Program Routine

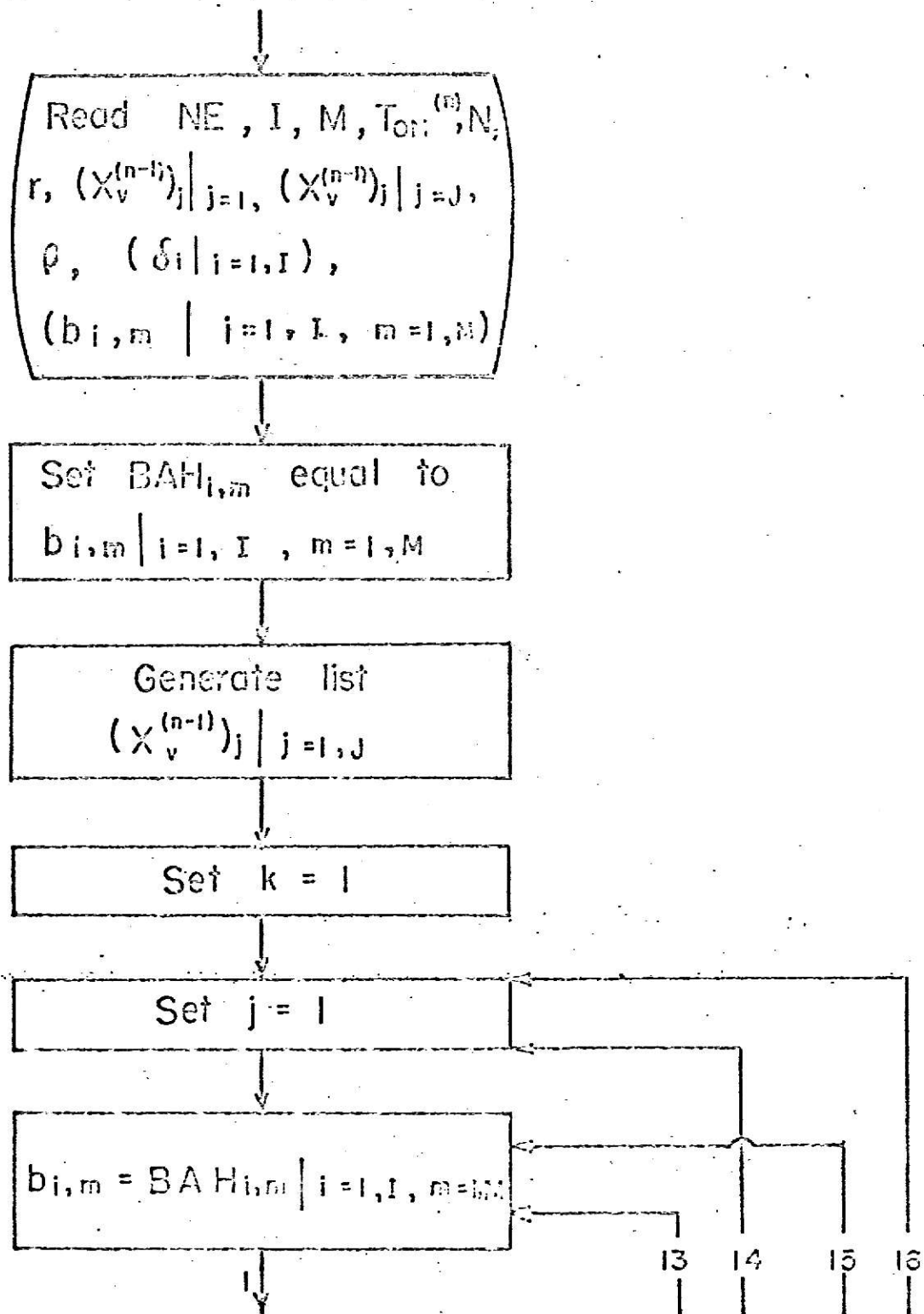


Fig. C-1. Computer flow diagram.

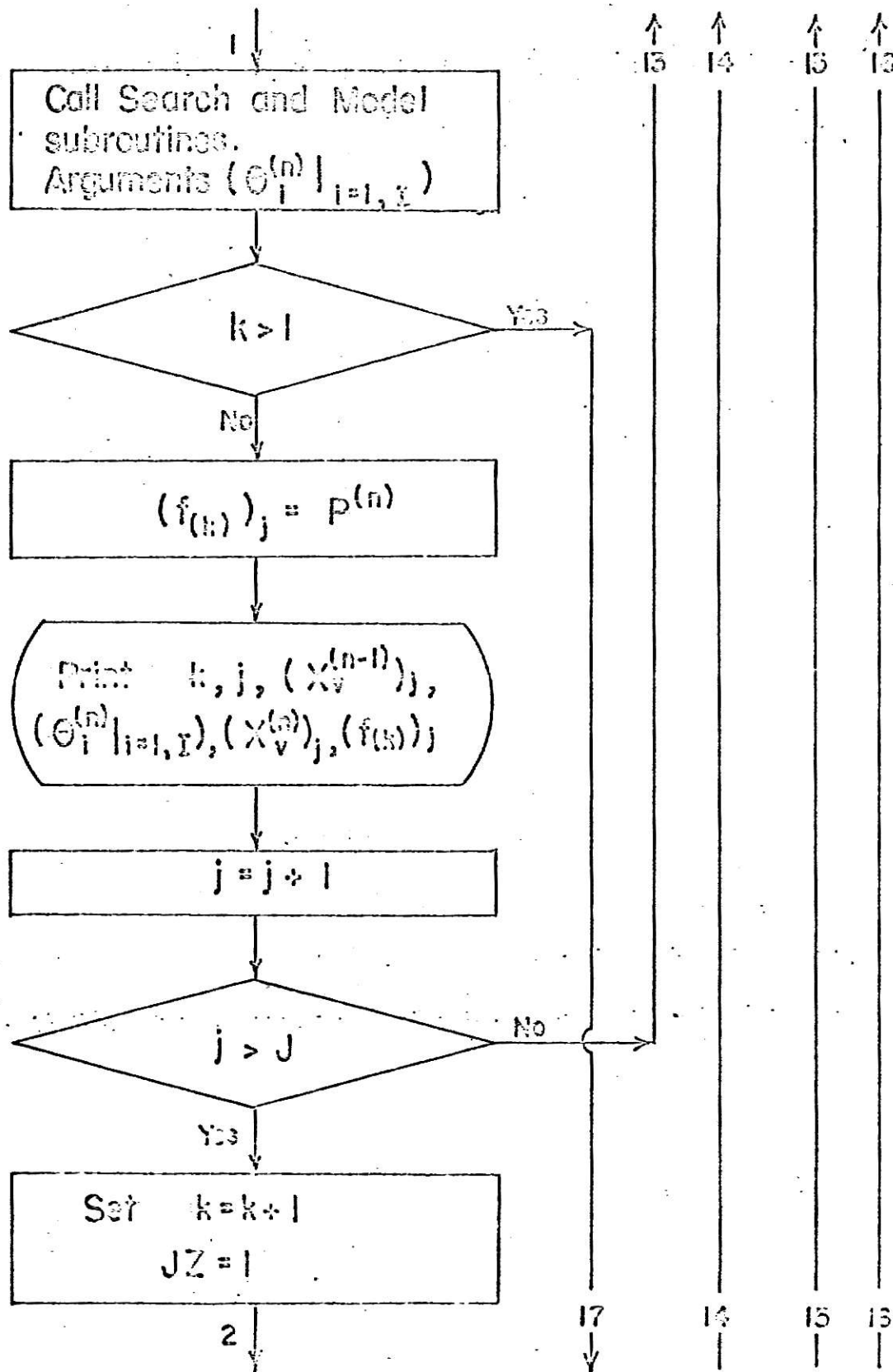


Fig. C-1. (Continued).

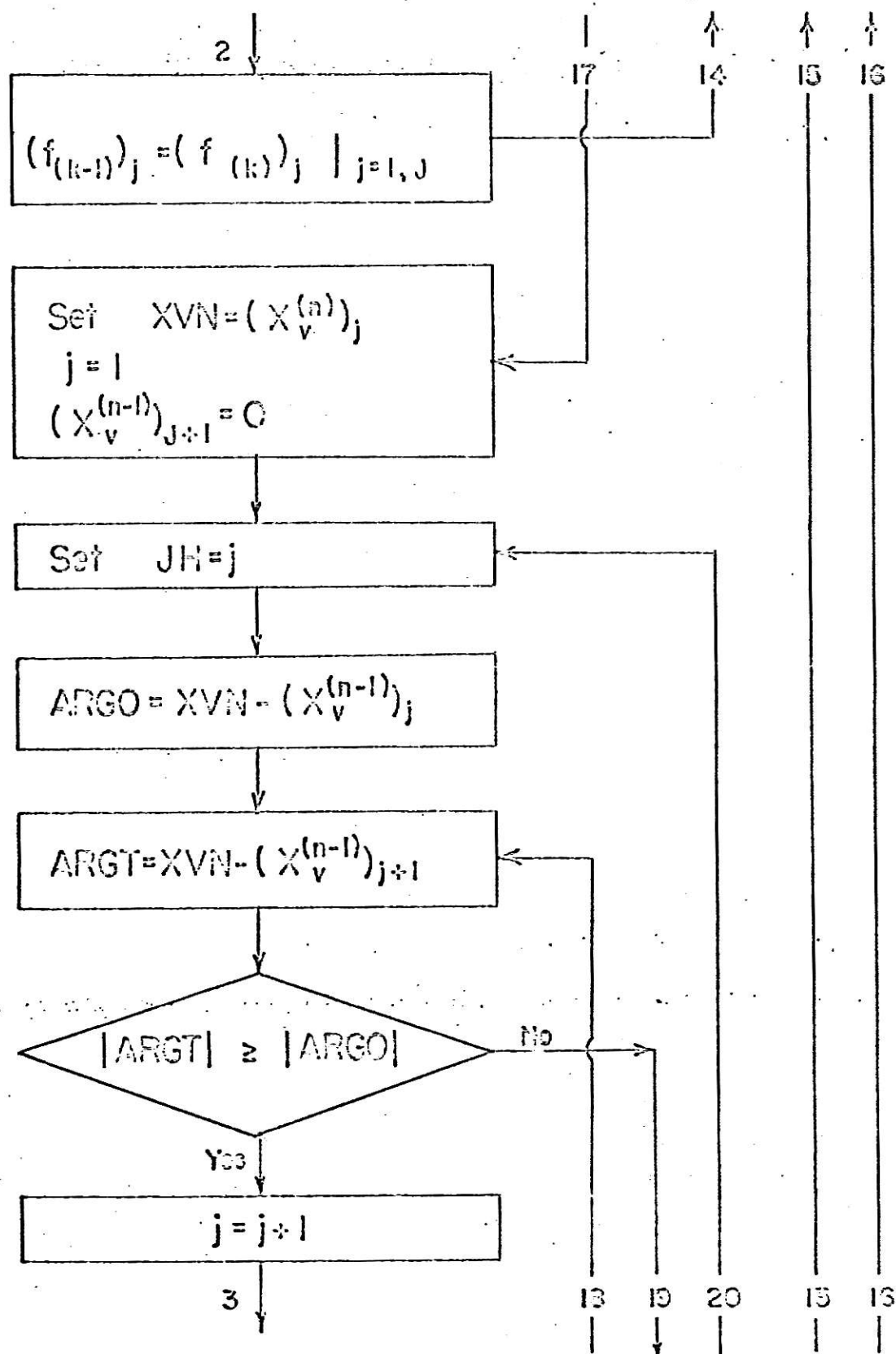


Fig.C-1. (Continued)

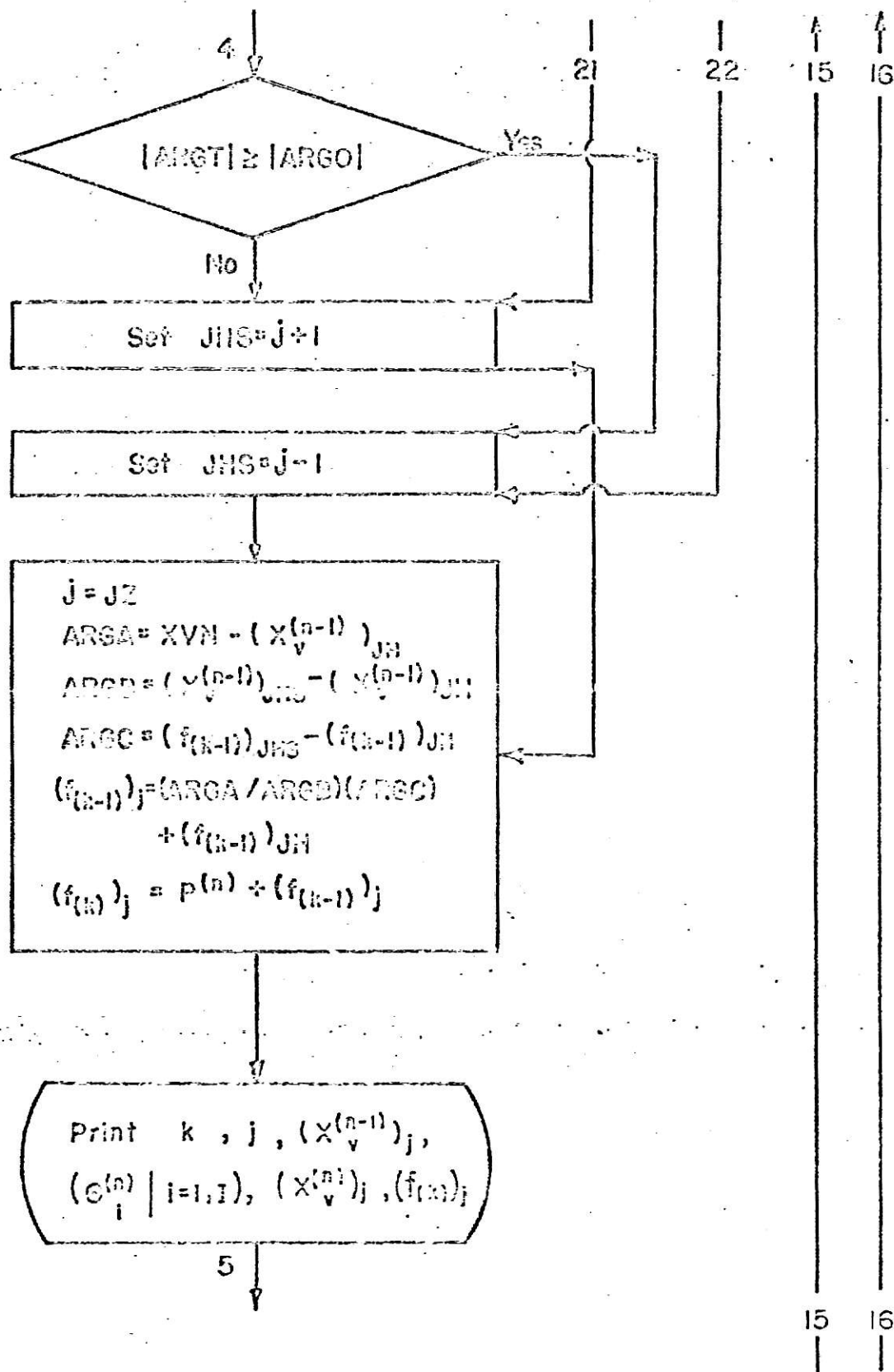


Fig. C-1. (Continued)

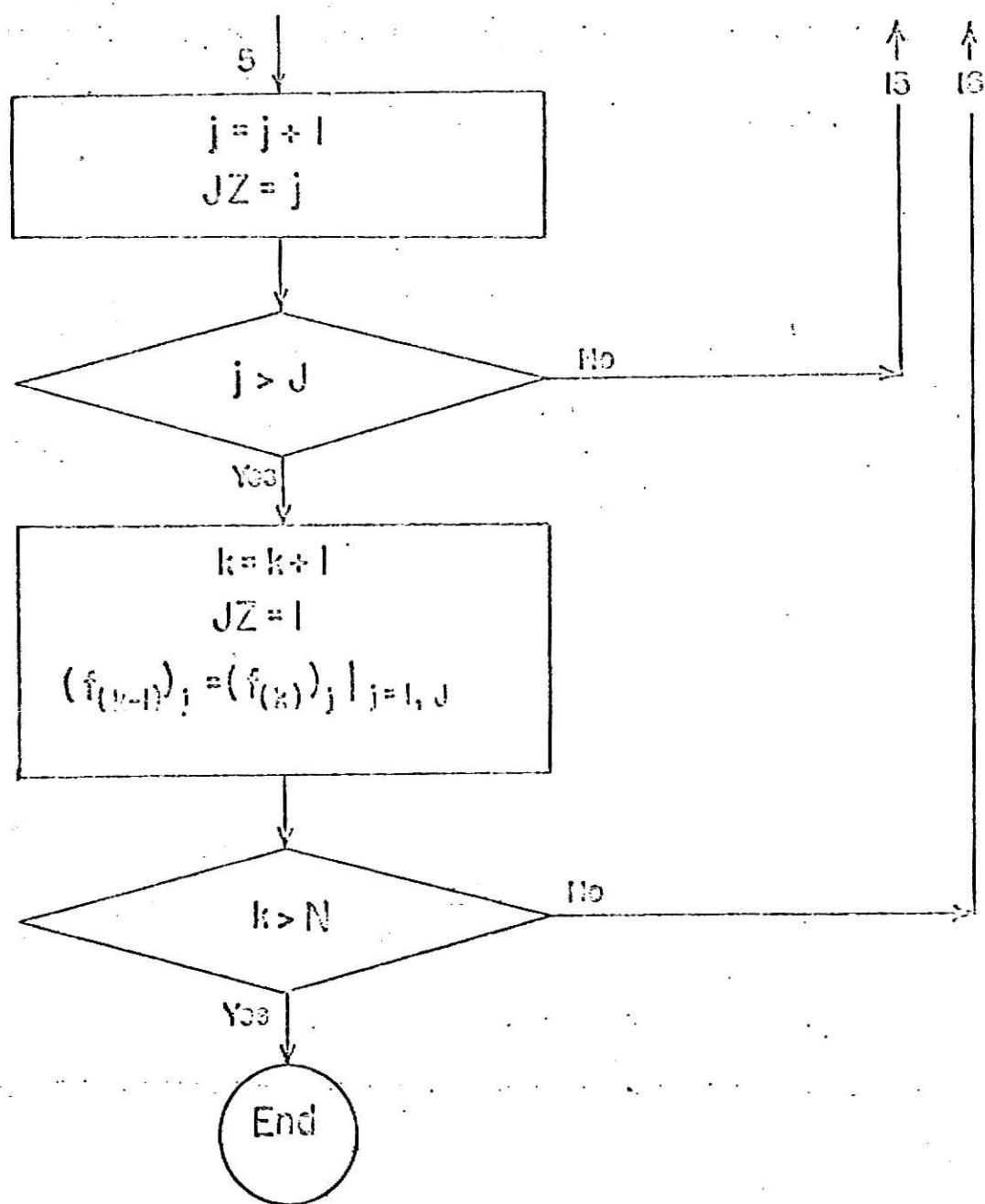


Fig.C-1. (Continued)

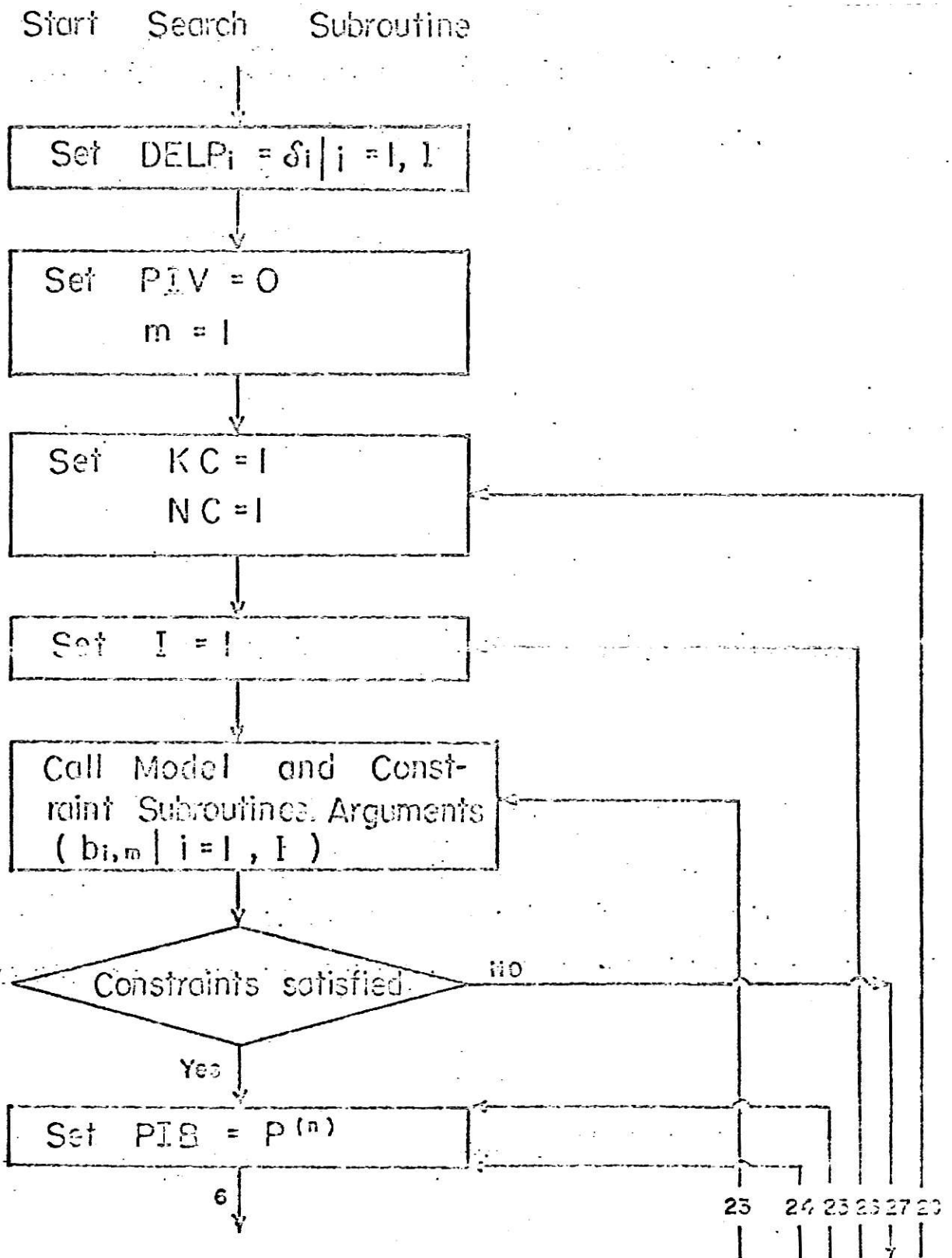


Fig. C-1. (Continued).

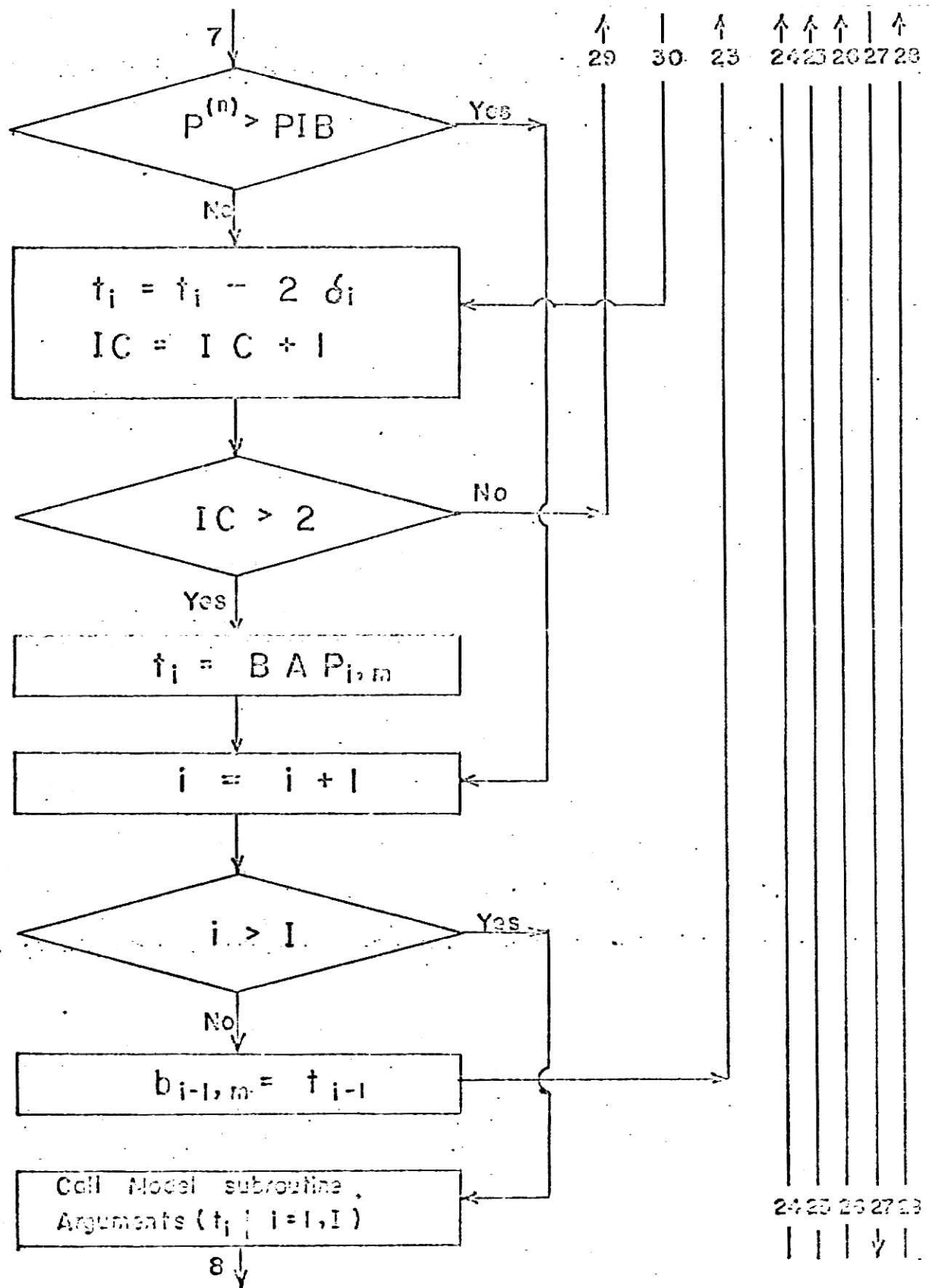


Fig. C-1 . (Continued).

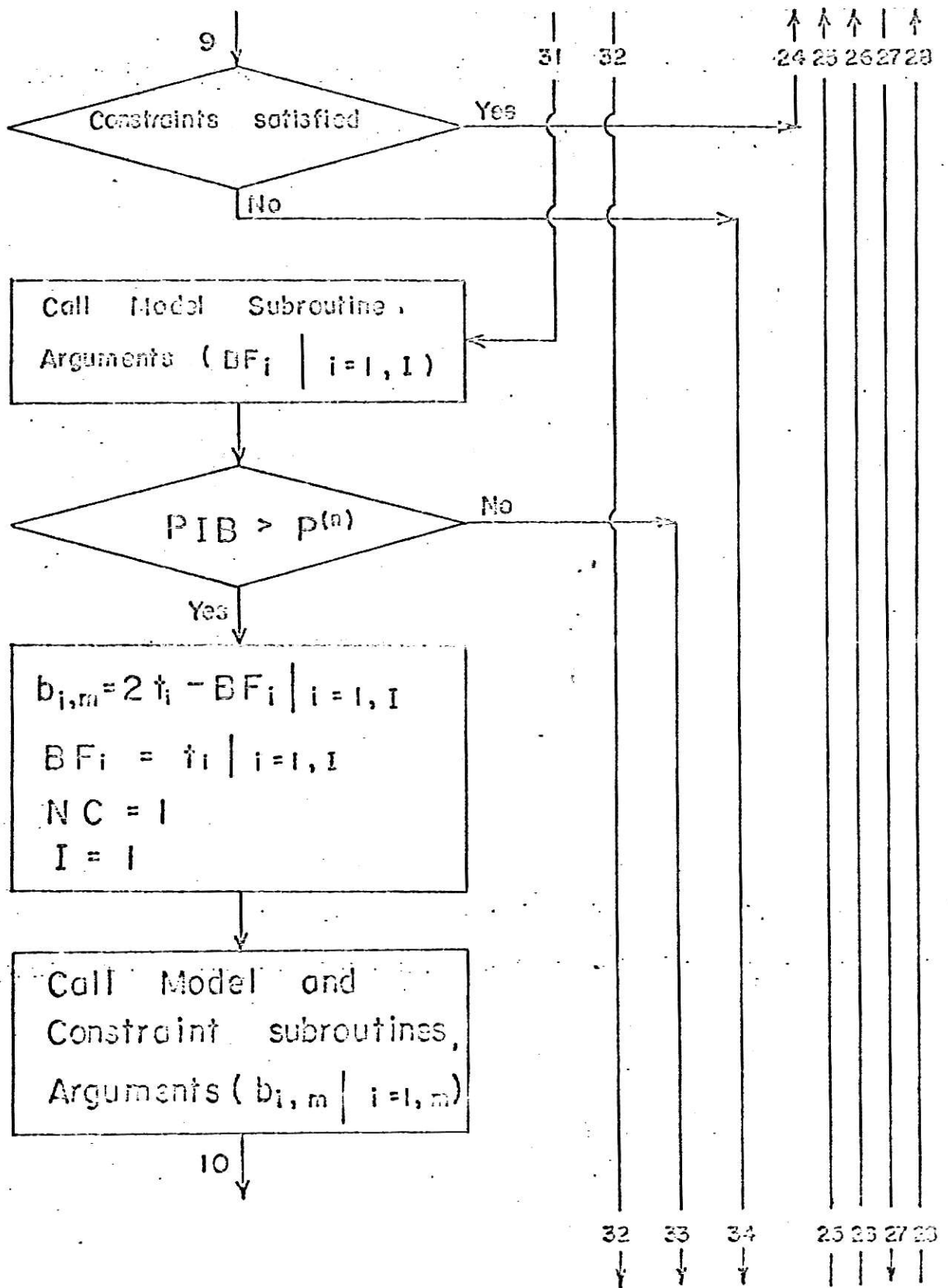


Fig. C-1. (Continued).

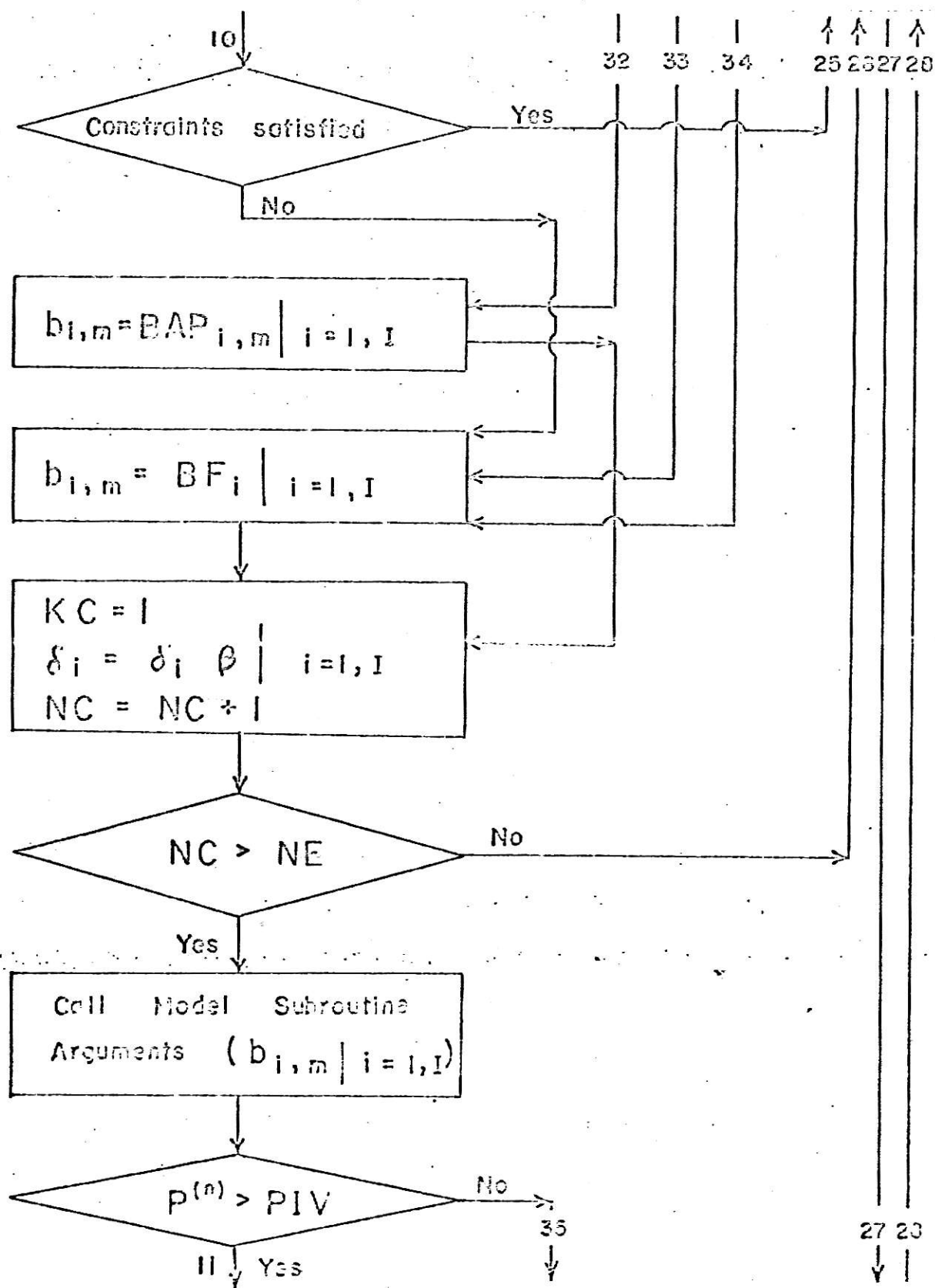


Fig. C - I. (Continued).

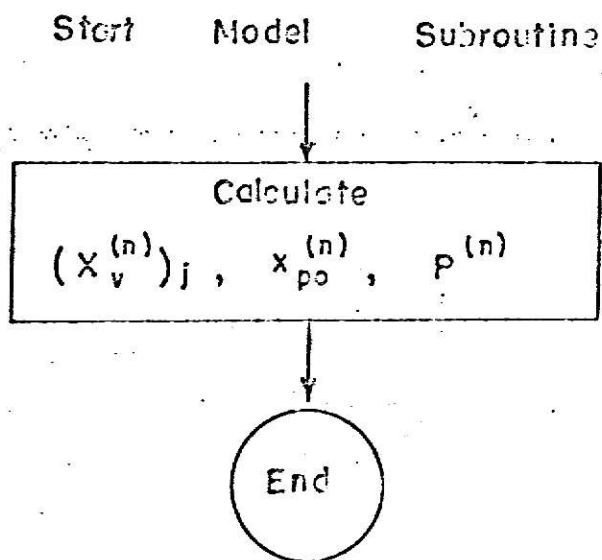
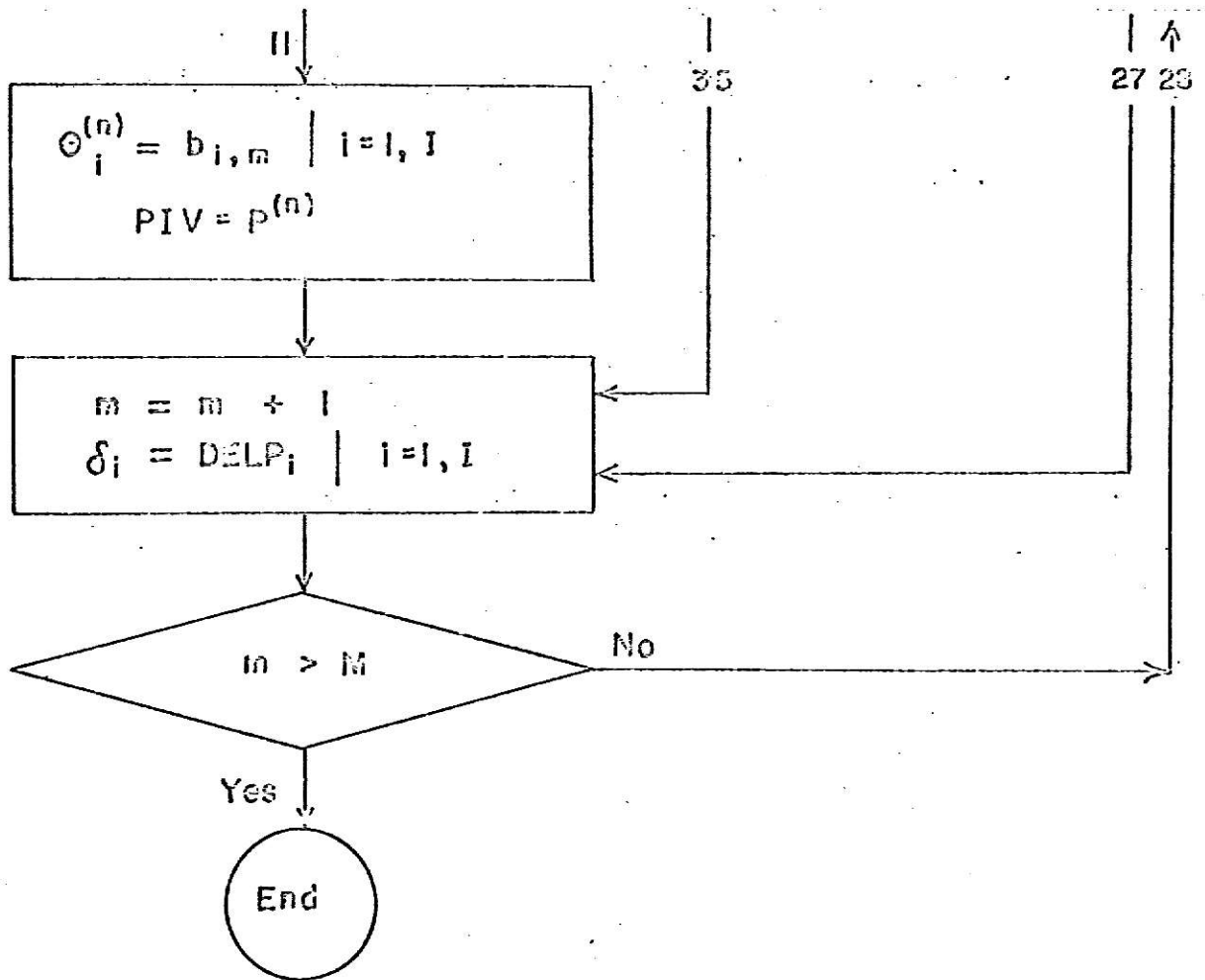


Fig. C - 1. (Continued)

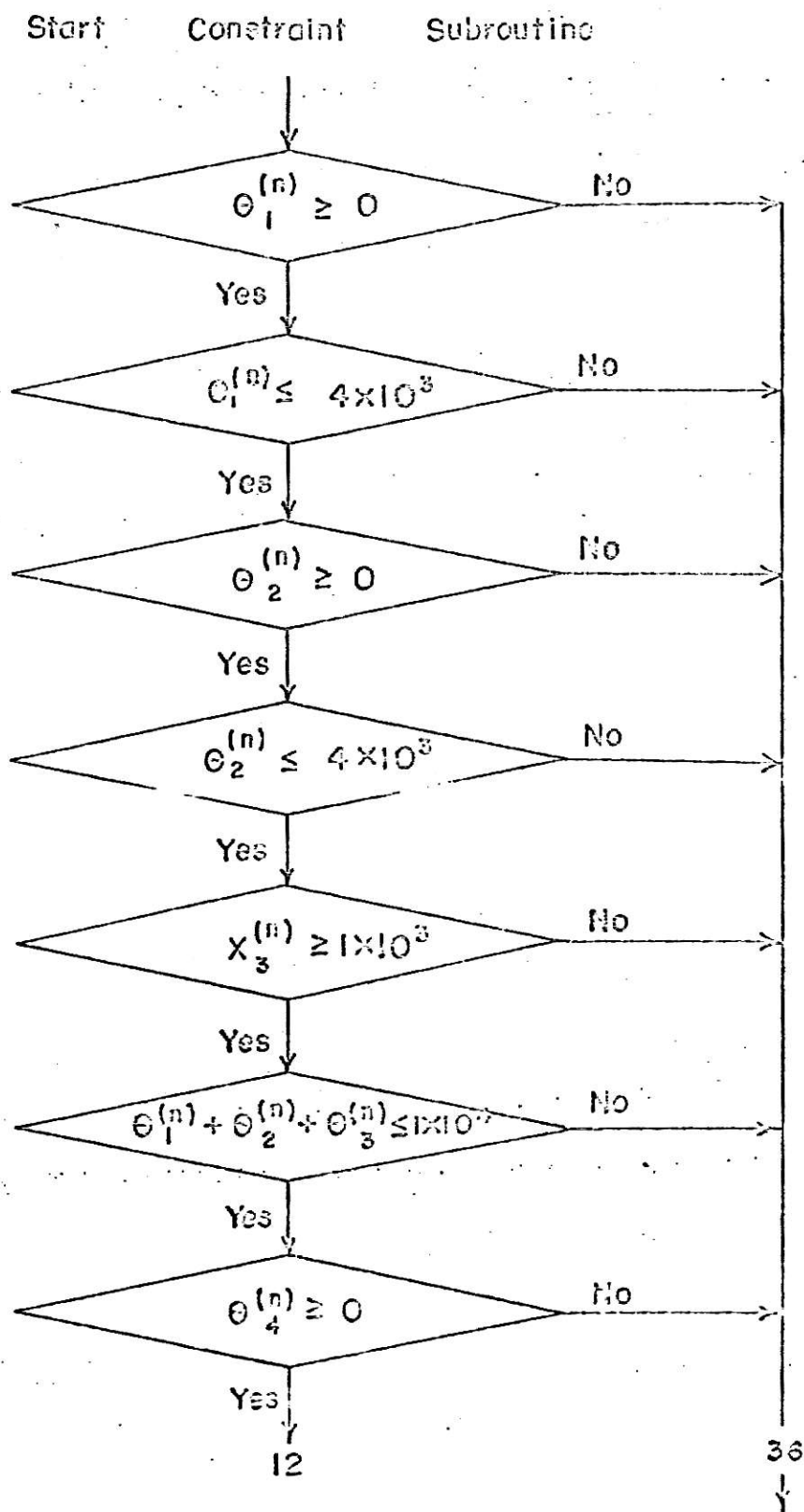


Fig. C-1. (Continued).

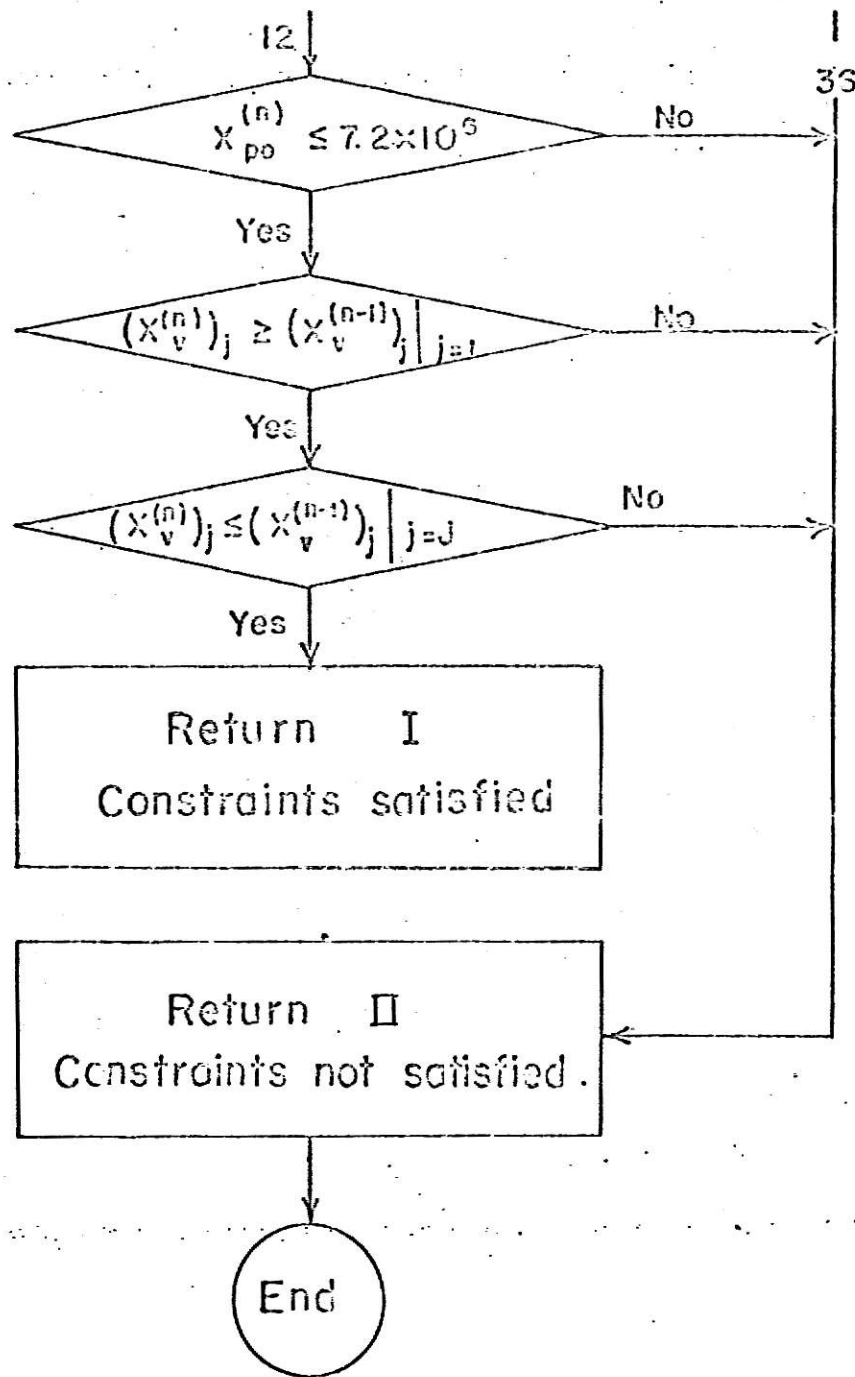


Fig. C-1 . (Continued).

TABLE C-1. PROGRAM SYMBOLS AND EXPLANATION

AHIGH	=	$(X_v^{(n-1)})_j \Big _{j=J}$	The highest value in the grid network for $X_v^{(n-1)}$, acre ft.
ALOW	=	$(X_v^{(n-1)})_j \Big _{j=1}$	The lowest value in grid network for $X_v^{(n-1)}$, acre ft.
ARGA	=		An intermediate function formed in the dynamic program routine; interpolation scheme.
ARGB	=		An intermediate function formed in the dynamic program routine; interpolation scheme.
ARGC	=		An intermediate function formed in the dynamic program routine; interpolation scheme.
ARGO	=		An intermediate function formed in the dynamic program routine; interpolation scheme.
ARGT	=		An intermediate function formed in the dynamic program routine; interpolation scheme.
B	=		k. The counting index used to denote stage position in the dynamic program routine.
BA(I,N)	=	$b_{l,i,m}$	A component of an initial base point in the search subroutine.
BAH(I,N)	=	BA(I,N)	This was devised to retain initial values in the dynamic program routine, acre ft./mo.
BAP(I,N)	=	BA(I,N)	This was devised to retain initial values in the search subroutine, acre ft./mo.

TABLE C-1 (CONTINUED)

BET	= θ_i . The factor by which perturbation size is cut when the search subroutine encounters a disruption in the search pattern.
BF(I)	= $b_{i+1,i,m}$. The i^{th} component of the base point following $b_{1,i,m}$ in the search subroutine.
D	= A parameter controlling the number of grid points associated with the state variable $x_v^{(n-1)}$.
DEL(I)	= δ_i . The i^{th} component of the perturbation size used in the Hooke and Jeeves pattern search.
DELP(I)	= DEL(I). This was devised to retain initial values in the search subroutine.
FB(J)	= f_k . The total profit for a series of k remaining stages in the dynamic program routine, dollars.
FBO(J)	= f_{k-1} . The total profit for a series of k-1 remaining stages in the dynamic program routine, dollars.
GRID	= Calculated grid size between values of $(x_v^{(n-1)})_j$, acre ft.
I	= i. Subscript designation of a specific decision variable.
IC	= A counter mechanism used in the search subroutine.
J	= j. Grid point designation of the state variable, $x_v^{(n-1)}$.

TABLE C-1 (CONTINUED)

JD	= J. The number of grid points associated with the state variable, x_{n-D}^v .
JH	= j. This was devised to retain initial values in the dynamic program routine.
JHS	= j+1. This was devised to retain initial values in the dynamic program routine.
JZ	= j. This was devised to retain initial values in the dynamic program routine.
K	= I. The number of independent or decision variables used in the system.
KC	= A counter mechanism used in the search subroutine.
M	= The total number of initial base points which will be used in the pattern search.
N	= m. Subscript designation of the initial base point, $b_{l,i,m}$, introduced into the Hooke and Jeeves pattern search.
NC	= A counter mechanism used in the search subroutine.
NE	= The number of times perturbation size is reduced before the search subroutine terminates a pattern.
OHB	= $\phi_{HB}^{(n)}$. System gain or benefit accrued from hydro-electric power during stage n, dollars/mo.
OHL	= $\phi_{HL}^{(n)}$. Loss sustained from power shortage below target output demand for stage n, dollars/mo.

TABLE C-1 (CONTINUED)

OIB	= $\phi_{IB}^{(n)}$. System benefit accrued from irrigation during stage n, dollars/mo.
OIL	= $\phi_{IL}^{(n)}$. Loss sustained from irrigation shortage below the target output demand for stage n, dollars/mo.
ORB	= $\phi_{RB}^{(n)}$. System benefit accrued from water based recreation during stage n, dollars/mo.
OUB	= $\phi_{UB}^{(n)}$. System benefit accrued from water supply to urban areas during stage n, dollars/mo.
OUL	= $\phi_{UL}^{(n)}$. Loss sustained from urban supply shortage below the target output demand for stage n, dollars/mo.
PI	= $P^{(n)}$. The interval profit associated with a stage n, dollars.
PIB	= PI. This was devised to retain initial values in the search subroutine, dollars.
PIV	= PI. This was devised to retain initial values in the search subroutine, dollars.
R	= r. The interest rate of money prorated on a monthly basis.
S	= N. The total number of stages considered in the dynamic program routine.
T	= $S^{(n)}$. Net economic gain accrued within the system during stage n, dollars.

TABLE C-1 (CONTINUED)

TH(I)	=	$t_{1,i}$	The i^{th} component of a temporary head used to extend the Hooke and Jeeves pattern search.
TOM	=	$\tau^{(n)}$	Total operating and maintenance costs incurred during stage n, dollars.
X(I)	=	$\theta_i^{(n)}$	Value of the i^{th} decision variable calculated in the search subroutine.
XV(J)	=	$(X_n^{(n-1)})_j$	The j^{th} grid value of a state variable. The average reservoir volume during stage n-1, acre ft.
XVO(J)	=	$(X_v^{(n)})_j$	The j^{th} grid value of a state variable. The average reservoir volume during stage n, acre ft.
XVJ	=	XV(J)	A dummy argument, acre ft.
XVOJ	=	XVO(J)	A dummy argument, acre ft.
XVN	=	XVO(J)	This was devised to retain initial values in the dynamic program routine, acre ft.
XTR	=	$X_{\text{TR}}^{(n)}$	Average flow rate utilized for production of hydroelectric power during stage n, acre ft./mo.
XUR	=	$\theta_{\text{UR}}^{(n)}$	Scheduled release of water for urban use during stage n, acre ft./mo.
XIR	=	$\theta_{\text{IR}}^{(n)}$	Scheduled release of water for irrigation during stage n, acre ft./mo.

TABLE C-1 (CONTINUED)

XER	=	$\theta_{ER}^{(n)}$	Scheduled release of water in excess of $\theta_{UR}^{(n)}$ and $\theta_{IR}^{(n)}$ during stage n, acre ft./mo.
XTB	=	$\theta_{TB}^{(n)}$	Water flow from the reservoir which cannot be utilized for hydroelectric power, acre ft./mo.
XIT	=	$X_{IT}^{(n)}$	Irrigation target output demand during stage n, acre ft./mo.
XHT	=	$X_{HT}^{(n)}$	Target demand for hydroelectric power during stage n, k.w. hrs./mo.
XUT	=	$X_{UT}^{(n)}$	Urban target output demand during stage n, acre ft./mo.
XPO	=	$X_{po}^{(n)}$	Output of power plant during stage n, k.w. hrs./mo.
XQ	=	$X_Q^{(n)}$	Flow rate of water into the system during stage n, acre ft./mo.
Y	=		An intermediate function formed in the model subroutine.
YP	=		An intermediate function formed in the model subroutine.

TABLE C-2. COMPUTER PROGRAM

```

DYNAMIC PROGRAM SECTION OF SYM PROG.
DIMENSION DEL(9),BA(9,9),BAH(9,9),XV(20),XVG(20),X(9),FB(20),
1   FBC(20),XP(9)
7C FORMAT (I5)
176 FORMAT (E12.5)
73 FORMAT(' ',7H      B=,E12.5,5H      J=,I5/' ',9H      XVJ=,E12.5,
1 5H X1=,E12.5,5H X2=,E12.5,5H X3=,E12.5,5H X4=,E12.5,
2 7H XVQJ=,E12.5,6H FBJ=,E12.5)
READ (1,70) NF,K,M
READ (1,176) TCM,S,R,D,ALOW,AHIGH,BET,(DEL(I),I=1,K),
1((BA(I,N),I=1,K),N=1,M)
6C DO 62 N=1,M
DO 61 I=1,K
BAH(I,N) = BA(I,N)
61 CONTINUE
62 CONTINUE
DO 350 J=1, D + 1.0
FBC(J) = 0.0
35C CONTINUE
JD = D + 1.0
GRID = (AHIGH - ALOW)/D
J = 1
XV(J) = ALOW
71 J = J+1
XV(J) = XV(J-1) + GRID
IF (J-JD) 71, 72, 72
72 B = 1.0
1 J = 1
2 DO 64 N=1,M
DO 63 I=1,K
BA(I,N) = BAH(I,N)
63 CONTINUE
64 CONTINUE
DO 300 I = 1,M
X(I) = 0.0
30C CONTINUE
PI = 0.0
XVG(J) = 0.0
XPO = 0.0
G = 0.0
CALL SEARCH (X,BA,DEL,BET,NF,M,XV(J),K,R,TOM,S,B,FBC,XV,JD)
CALL CRIT (X(1),X(2),X(3),X(4),S,B,XV(J),TCM,R,PI,XVG(J),XPO,
1 G, FBC, XV, JD)
GK=C
DO 355 I=1,4
XP(I) = X(I)

```

TABLE C-2 (CONTINUED)

```

355 CONTINUE
    IF (X(3)-1000.0) 352, 352, 351
351 SUM = (X(1)+X(2)+X(3)-1000.0)/(2.0)
    X(1) = SUM
    X(2) = SUM
    X(3) = 1000.0
    CALL CRIT (X(1),X(2),X(3),X(4),S,8,XV(J),TCM,R,PI,XVO(J),XPO,
1 G, FBO, XV, JD)
    IF (G-GK) 356, 352, 352
356 DO 357 I=1,4
    X(I) = XP(I)
357 CONTINUE
352 IF (B-1.0) 5, 3, 5
    3 FBO(J) = PI
    WRITE(3,73) B,J,XV(J),(X(I), I=1,4), XVO(J),FBO(J)
    J = J+1
    IF (J-JD) 2, 2, 4
    4 B = B + 1.0
    JZ = 1
    DO 74 J = 1,JE
    FBO(J) = FBO(J)
74 CONTINUE
    GO TO 1
    5 XVN = XVO(J)
    J = 1
    XV(JD + 1) = 0.0
    6 JH = J
    ARGO = XVN - XV(J)
    7 ARGJ = XVN - XV(J+1)
    IF (ABS(ARGJ)-ABS(ARGO)) 9, 8, 8
    8 J = J+1
    IF (J-JD) 7, 10, 10
    9 J = J+1
    IF (J-JD) 6, 10, 10
    10 J = JH
    IF (J-1) 11,11,102
    102 IF (J-JD) 101,12,12
    101 ARGO = XVN - XV(J-1)
    ARGJ = XVN - XV(J+1)
    IF (ABS(ARGJ)-ABS(ARGO)) 11, 12, 12
    11 JHS = J+1
    GO TO 13
    12 JHS = J-1
    13 J = JZ
    ARGA = XVN - XV(JH)
    ARGB = XV(JHS) - XV(JH)
    ARGC = FBO(JHS) - FBO(JH)
    FBO(J) = (ARGJ/ARGB)*(ARGC) + FBO(JH)

```


TABLE C-2 (CONTINUED)

```

      FB(J) = PI + FBC(J)
      WRITE(3,73) B,J,XV(J),(X(I), I=1,K),          XVC(J),FB(J)
      J = J+1
      JZ = J
      IF (J-JD) 2, 2, 14
14  B = B + 1.0
      JZ = 1
      DO 75 J = 1, JD
      FBC(J) = FB(J)
75  CONTINUE
      IF (B-S) 1, 1, 15
15  STOP
      END

```

```

      SEARCH SUBROUTINE SECTION OF SYM PROG.
      SUBROUTINE SEARCH ( X ,BA,DEL,BET,NE,M,XVJ,K,R,TCM,S,B,
1  FBO, XV, JD)
      DIMENSION BA(9,9), DEL(9), X(9), DELP(9), TH(9), BAP(9,9),
1  BF(9), FBO(20), XV(20)
80  FORMAT(' ',12H SEARCH N=, I5,5H PI=, E12.5, 7H X(1)=,
1  (4E12.5))
      DO 65 I = 1,K
      DELP(I) = DEL(I)
65  CONTINUE
      PIV = 0.0
      N = 1
16  KC = 1
17  NC = 1
18  I = 1
      PI = 0.0
      XVOJ = 0.0
      XPO = 0.0
      G = 0.0
19  CALL CRIT (BA(1,N),BA(2,N),BA(3,N),BA(4,N),S,B,XVJ,TCM,
1  R, PI, XVOJ, XPO, G, FBO, XV, JD)
      CALL CONST (BA(1,N),BA(2,N),BA(3,N),BA(4,N),XPO,XVOJ,
1  E191, E38)
191  PIB = G
      IF (KC - 1) 21, 21, 20
20  IT = I
      DO 66 I = 1,K
      DEL(I) = DELP(I)
66  CONTINUE
      I = IT
21  TH(I) = BA(I,N) + DEL(I)
      BAP(I,N) = BA(I,N)
      IC = 1

```

TABLE C-2 (CONTINUED)

```

22 BA(I,N) = TH(I)
   CALL CRIT (BA(1,N),BA(2,N),BA(3,N),BA(4,N),S,B,XVJ,TOM,R,
1   PI, XVQJ, XPC, G, FBO, XV, JD)
   CALL CONST (BA(1,N),BA(2,N),BA(3,N),BA(4,N),XPC,XVQJ,
1   &23, &24)
23 IF (G - PIB) 24, 24, 26
24 TH(I) = TH(I) - 2* DEL(I)
   IC = IC + 1
   IF (IC-2) 22, 22, 25
25 TH(I) = LAP(I,N)
26 I = I + 1
   IF (I-K) 27, 27, 28
27 BA(I-1,N) = TH(I-1)
   GO TO 19
28 CALL CRIT (TH(1),TH(2),TH(3),TH(4),S,B,XVJ,TOM,R,PI,XVQJ,XPC,
1   G, FBO, XV, JD)
   PIB = G
   KC = KC + 1
   IF (KC-2) 29, 29, 31
29 CALL CRIT (BAP(1,N),BAP(2,N),BAP(3,N),BAP(4,N),S,B,XVJ,
1   TOM, P, PI, XVQJ, XPC, G, FBO, XV, JD)
   IF (PIB-G) 33, 33, 30
30 DO 67 I = 1,K
   BA(I,N) = 2* TH(I) - BAP(I,N)
67 CONTINUE
   DO 68 I = 1,K
   BF(I) = TH(I)
68 CONTINUE
   NC= 1
   I = 1
   CALL CRIT (BA(1,N),BA(2,N),BA(3,N),BA(4,N),S,B,XVJ,TOM,R,
1   PI, XVQJ, XPC, G, FBO, XV, JD)
   CALL CONST (BA(1,N),BA(2,N),BA(3,N),BA(4,N),XPC,XVQJ,
1   &191, &34)
31 CALL CRIT (BF(1),BF(2),BF(3),BF(4),S,B,XVJ,TOM,R,PI,XVQJ,XPC,
1   G, FBO, XV, JD)
   IF (PIB-G) 34, 34, 32
32 DO 69 I = 1,K
   BA(I,N) = 2* TH(I) - BF(I)
69 CONTINUE
   DO 200 I = 1, K
   BF(I) = TH(I)
200 CONTINUE
   NC= 1
   I = 1
   CALL CRIT (BA(1,N),BA(2,N),BA(3,N),BA(4,N),S,B,XVJ,TOM,R,
1   PI, XVQJ, XPC, G, FBO, XV, JD)
   CALL CONST (BA(1,N),BA(2,N),BA(3,N),BA(4,N),XPC,XVQJ,

```

TABLE C-2 (CONTINUED)

```

1  &191, &34)
33 DO 76 I = 1,K
    BA(I,N) = BAP(I,N)
76 CONTINUE
    GO TO 35
34 DO 77 I = 1,K
    BA(I,N) = BF(I)
77 CONTINUE
35 KC = 1
    DO 78 I = 1,K
    DEL(I) = DEL(I)* BET
78 CONTINUE
    NC = NC + 1
    IF (NC-NE) 18, 18, 36
36 CALL CRIT (BA(1,N),BA(2,N),BA(3,N),BA(4,N),S,B,XVJ,TOM,R,
1  PI, XVOJ, XPC, G, FEO, XV, JD)
    IF (G -PIV) 38, 38, 37
37 DO 79 I =1,K
    X(I) = BA(I,N)
79 CONTINUE
    PIV = G
38 N = N + 1
    NT = N - 1
    WRITE (3,80) NT, PI, (X(I), I=1,K)
    DO 81 I = 1,K
    DEL(I) = DELP(I)
81 CONTINUE
    IF (N-M) 16, 16, 39
39 RETURN
    END

```

MODEL SUBROUTINE SECTION OF SYM PROG.

```

SUBROUTINE CRIT (XUR,XIR,XER,XTE,S,B,XVJ,TOM,R,PI,XVOJ,XPC,
1  G, FEO, XV, JD)
    DIMENSION FAC(20), XV(20)
    Y = (0.524)*(S - B + 1.0)
    YP = (0.1)*(Y)
    XQ = 7000.0 + 5000.0 * SIN(Y) + 1000.0 * SIN(YP)
    XIT = 2250.0 + 1000.0 * SIN(Y)
    XHT = 5000000.0
    XLT = 2250.0
    XVOJ = XVJ + XQ - XUR - XIR - XER
    XTR = XUR + XIR + XER - XTB
    XPC = (-0.000000585)*(XVOJ**2)*(XTR) + (0.017)*(XVOJ)*(XTR)
    CUB = (-0.0004)*(XUP**2) + (14.0)*(XUR)
    CUL = (16.4)*(XUT - XUR)
    CIB = (-0.00008)*(XIR**2) + (2.7)*(XIR)

```

TABLE C-2 (CONTINUED)

```

OIL = (3.42)*(XIT - XIR)
ORB = (0.0000000032)*(XVOJ**3) - (0.000077)*(XVOJ**2) +
1 (8.15)*(XVOJ)
OHL = (+0.0000000117)*(XVOJ**2)*(XTR) - (0.00034)*(XVOJ)*
1 (XTR) + (0.02) * (XHT)
OHB = (-6.1)*(10.0**(-42))*(XVOJ**6)*(XTR**3) + (5.40)*
1 (10.0**(-36))
2 *(XVOJ**5)*(XTR**3) - (5.4)*(10.0**(-29))*(XVOJ**4)*
2 (XTR**3) -
4 (1.37)*(10.0**(-27))*(XVOJ**4)*(XTR**2) + (1.56)*
5 (10.0**(-23))*
6 (XVOJ**3)*(XTR**3) + (8.00)*(10.0**(-20))*(XVOJ**3)*
7 (XTR**2) -
8 (0.000000000000012)*(XVOJ**2)*(XTR**2) - (0.00000000001)*
9 (XVOJ**2)*(XTR) + (0.000290)*(XVOJ)*(XTR)
IF (XUR - XUI) 41, 40, 40
40 OUL = 0.0
41 IF (XIR - XIT) 43, 42, 42
42 OIL = 0.0
43 IF (XPC - XHT) 45, 44, 44
44 OHL = 0.0
45 T = OUB + OIB + OHB + ORB - OUL - GIL - OHL - TOM
PI = (1)/(1.0 + R)**(S-B+1.0)
XVN = XVOJ
J = 1
XV(JD + 1) = 0.0
6 JH = J
ARGO = XVN - XV(J)
7 ARGJ = XVN - XV(J+1)
IF (ABS(ARGJ) - ABS(ARGO)) 9, 8, 8
8 J = J+1
IF (J-JD) 7, 10, 10
9 J = J+1
IF (J-JD) 6, 10, 10
10 J = JH
IF (J-1) 11, 11, 102
102 IF (J-JD) 101, 12, 12
101 ARGD = XVN - XV(J-1)
ARGT = XVN - XV(J+1)
IF (ABS(ARGJ) - ABS(ARGO)) 11, 12, 12
11 JHS = J+1
GO TO 13
12 JHS = J-1
13 J = JZ
ARGA = XVN - XV(JH)
ARGB = XV(JHS) - XV(JH)
ARGC = FEG(JHS) - FEG(JH)
FEG(J) = (ARGA/ARGB)*(ARGC) + FEG(JH)

```

TABLE C-2 (CONTINUED)

```
G = PI + FBO(J)
RETURN
END
```

CONSTRAINT SUBROUTINE SECTION OF SYM PROG.

SUBROUTINE CONST (XUR,XIR,XER,XTB,XPD,XVOJ, *,*)

```
IF (XUR - 0.0) 56, 46, 46
46 IF (XUR - 4000.0) 47, 47, 56
47 IF (XIR - 0.0) 56, 48, 48
48 IF (XIR - 4000.0) 49, 49, 56
49 IF (XER - 1000.0) 56, 50, 50
50 IF (XUR+XIR+XER-13000.0) 51,51,56
51 IF (XTB - 0.0) 56, 52, 52
52 IF (XPD - 720000.0) 53, 53, 56
53 IF (XVOJ - 20000.0) 56, 54, 54
54 IF (XVOJ - 140000.0) 55, 55, 56
55 RETURN 1
56 RETURN 2
END
```

TABLE C-3. SAMPLE INPUT DATA FOR COMPUTER PROGRAM

NE	=	5	BA(1,2)	=	0.400E04
K	=	4	BA(2,2)	=	0.0
M	=	5	BA(3,2)	=	0.600E04
TOM	=	0.173E05	BA(4,2)	=	0.0
S	=	0.300E01	BA(1,3)	=	0.0
R	=	0.500E-2	BA(2,3)	=	0.400E04
D	=	0.900E01	BA(3,3)	=	0.600E04
ALOW	=	0.200E05	BA(4,3)	=	0.0
AHIGH	=	0.135E06	BA(1,4)	=	0.300E04
BET	=	0.500	BA(2,4)	=	0.300E04
DEL(1)	=	0.100E02	BA(3,4)	=	0.300E04
DEL(2)	=	0.100E02	BA(4,4)	=	0.200E03
DEL(3)	=	0.100E02	BA(1,5)	=	0.500E03
DEL(4)	=	0.100E02	BA(2,5)	=	0.400E03
BA(1,1)	=	0.0	BA(3,5)	=	0.200E04
BA(2,1)	=	0.0	BA(4,5)	=	0.300E03
BA(3,1)	=	0.100E04			
BA(4,1)	=	0.0			
BA(1,2)	=	0.400E04			
BA(2,2)	=	0.0			
BA(3,2)	=	0.600E04			
BA(4,2)	=	0.0			

TABLE C-4. SAMPLE OUTPUT OF COMPUTER PROGRAM FOR A 12 STAGE SYSTEM

Remaining Stages, $k = 1$

Grid Value of Input State Variable	Value of Output State Variable	Optimal Values of Decision Variables				Interval Profit	Total Profit
		$x_v^{(n-1)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	
		$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	f_k
0.200 E5	0.200 E5	0.330 E4	0.330 E4	0.330 E4	0.100 E4	0.0	0.134 E6
0.327 E5	0.281 E5	0.400 E4	0.400 E4	0.400 E4	0.424 E4	0.0	0.298 E6
0.455 E5	0.411 E5	0.400 E4	0.400 E4	0.400 E4	0.398 E4	0.0	0.383 E6
0.583 E5	0.566 E5	0.400 E4	0.400 E4	0.400 E4	0.128 E4	0.0	0.423 E6
0.711 E5	0.708 E5	0.345 E4	0.345 E4	0.345 E4	0.100 E4	0.0	0.457 E6
0.838 E5	0.844 E5	0.304 E4	0.304 E4	0.302 E4	0.100 E4	0.0	0.485 E6
0.966 E5	0.977 E5	0.276 E4	0.276 E4	0.276 E4	0.100 E4	0.0	0.517 E6
0.109 E6	0.110 E6	0.259 E4	0.259 E4	0.257 E4	0.100 E4	0.0	0.558 E6
0.122 E6	0.123 E6	0.247 E4	0.247 E4	0.247 E4	0.100 E4	0.0	0.611 E6
0.135 E6	0.136 E6	0.242 E4	0.242 E4	0.239 E4	0.100 E4	0.0	0.681 E6

TABLE C-4 (CONTINUED)

Remaining Stages, $k = 2$

$x_{v}^{(n-1)}$	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	f_k
0.200 E5	0.200 E5	0.400 E4	0.0	0.100 E4	0.0	0.115 E6	0.251 E6
0.327 E5	0.310 E5	0.400 E4	0.175 E4	0.100 E4	0.0	0.236 E6	0.513 E6
0.455 E5	0.425 E5	0.400 E4	0.310 E4	0.100 E4	0.0	0.351 E6	0.709 E6
0.583 E5	0.537 E5	0.400 E4	0.400 E4	0.167 E4	0.0	0.418 E6	0.818 E6
0.711 E5	0.680 E5	0.355 E4	0.355 E4	0.100 E4	0.0	0.453 E6	0.896 E6
0.838 E5	0.817 E5	0.310 E4	0.310 E4	0.100 E4	0.0	0.482 E6	0.960 E6
0.966 E5	0.951 E5	0.280 E4	0.280 E4	0.100 E4	0.0	0.514 E6	0.102 E7
0.109 E6	0.108 E6	0.262 E4	0.260 E4	0.100 E4	0.0	0.553 E6	0.110 E7
0.122 E6	0.121 E6	0.249 E4	0.249 E4	0.100 E4	0.0	0.604 E6	0.121 E7
0.135 E6	0.134 E6	0.242 E4	0.242 E4	0.100 E4	0.0	0.670 E6	0.134 E7

TABLE C-4 (CONTINUED)

$x_v^{(n-1)}$	Remaining Stages, $k = 3$						f_k
	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	
0.200 E5	0.200 E5	0.218 E4	0.0	0.100 E4	0.0	0.749 E5	0.326 E6
0.327 E5	0.309 E5	0.400 E4	0.0	0.100 E4	0.0	0.197 E6	0.672 E6
0.455 E5	0.415 E5	0.400 E4	0.217 E4	0.100 E4	0.0	0.322 E6	0.959 E6
0.583 E5	0.549 E5	0.400 E4	0.155 E4	0.100 E4	0.0	0.389 E6	0.115 E7
0.711 E5	0.659 E5	0.366 E4	0.364 E4	0.100 E4	0.0	0.450 E6	0.129 E7
0.838 E5	0.797 E5	0.315 E4	0.315 E4	0.100 E4	0.0	0.480 E6	0.140 E7
0.966 E5	0.931 E5	0.284 E4	0.284 E4	0.100 E4	0.0	0.511 E6	0.150 E7
0.109 E6	0.106 E6	0.264 E4	0.264 E4	0.100 E4	0.0	0.549 E6	0.163 E7
0.122 E6	0.119 E6	0.246 E4	0.244 E4	0.100 E4	0.0	0.599 E6	0.178 E7
0.135 E6	0.132 E6	0.244 E4	0.242 E4	0.100 E4	0.0	0.663 E6	0.197 E7

TABLE C-4 (CONTINUED)

Remaining Stages, $k = 4$

$x_v^{(n-1)}$	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	f_k
0.200 E5	0.200 E5	0.145 E4	0.0	0.100 E4	0.0	0.472 E5	0.373 E6
0.327 E5	0.319 E5	0.225 E4	0.0	0.100 E4	0.0	0.154 E6	0.806 E6
0.455 E5	0.430 E5	0.400 E4	0.250 E4	0.100 E4	0.0	0.273 E6	0.117 E7
0.583 E5	0.541 E5	0.400 E4	0.167 E4	0.100 E4	0.0	0.389 E6	0.146 E7
0.711 E5	0.673 E5	0.400 E4	0.125 E4	0.100 E4	0.0	0.434 E6	0.166 E7
0.838 E5	0.789 E5	0.318 E4	0.318 E4	0.100 E4	0.0	0.481 E6	0.181 E7
0.966 E5	0.924 E5	0.285 E4	0.285 E4	0.100 E4	0.0	0.512 E6	0.196 E7
0.109 E6	0.105 E6	0.264 E4	0.264 E4	0.100 E4	0.0	0.550 E6	0.212 E7
0.122 E6	0.118 E6	0.247 E4	0.245 E4	0.100 E4	0.0	0.598 E6	0.232 E7
0.135 E6	0.131 E6	0.294 E4	0.294 E4	0.100 E4	0.0	0.662 E6	0.257 E7

TABLE C-4 (CONTINUED)

$x_v^{(n-1)}$	$x_v^{(n)}$	Remaining Stages, $k = 5$				$p^{(n)}$	f_k
		$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$		
0.200 E5	0.207 E5	0.129 E4	0.0	0.100 E4	0.0	0.456 E5	0.445 E6
0.327 E5	0.325 E5	0.225 E4	0.0	0.100 E4	0.0	0.158 E6	0.958 E6
0.455 E5	0.437 E5	0.392 E4	0.0	0.100 E4	0.0	0.275 E6	0.139 E7
0.583 E5	0.550 E5	0.400 E4	0.138 E4	0.100 E4	0.0	0.386 E6	0.175 E7
0.711 E5	0.685 E5	0.400 E4	0.614 E4	0.100 E4	0.0	0.425 E6	0.203 E7
0.838 E5	0.796 E5	0.315 E4	0.315 E4	0.100 E4	0.0	0.485 E6	0.223 E7
0.966 E5	0.930 E5	0.284 E4	0.284 E4	0.100 E4	0.0	0.516 E6	0.241 E7
0.109 E6	0.106 E6	0.264 E4	0.264 E4	0.100 E4	0.0	0.554 E6	0.262 E7
0.122 E6	0.119 E6	0.246 E4	0.244 E4	0.100 E4	0.0	0.604 E6	0.286 E7
0.135 E6	0.132 E6	0.244 E4	0.242 E4	0.100 E4	0.0	0.669 E6	0.317 E7

TABLE C-4 (CONTINUED)

Remaining Stages, $k = 6$

$x_v^{(n-1)}$	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	f_k
0.200 E5	0.238 E5	0.0	0.0	0.100 E4	0.0	0.876 E4	0.608 E6
0.327 E5	0.343 E5	0.225 E4	0.0	0.100 E4	0.0	0.168 E6	0.118 E7
0.455 E5	0.454 E5	0.399 E4	0.0	0.100 E4	0.0	0.287 E6	0.167 E7
0.583 E5	0.567 E5	0.400 E4	0.144 E4	0.100 E4	0.0	0.399 E6	0.211 E7
0.711 E5	0.704 E5	0.400 E4	0.511 E3	0.100 E4	0.0	0.430 E6	0.244 E7
0.838 E5	0.815 E5	0.313 E4	0.311 E4	0.100 E4	0.0	0.491 E6	0.268 E7
0.966 E5	0.948 E5	0.281 E4	0.279 E4	0.100 E4	0.0	0.523 E6	0.291 E7
0.109 E6	0.108 E6	0.261 E4	0.261 E4	0.100 E4	0.0	0.563 E6	0.316 E7
0.122 E6	0.121 E6	0.247 E4	0.247 E4	0.100 E4	0.0	0.615 E6	0.346 E7
0.135 E6	0.133 E6	0.243 E4	0.243 E4	0.100 E4	0.0	0.683 E6	0.383 E7

TABLE C-4 (CONTINUED)

$x_v^{(n-1)}$	Remaining Stages, $k = 7$					$p^{(n)}$	f_k
	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$		
0.200 E5	0.262 E5	0.0	0.0	0.100 E4	0.0	0.201 E5	0.910 E6
0.327 E5	0.368 E5	0.225 E4	0.0	0.100 E4	0.0	0.180 E6	0.151 E7
0.455 E5	0.491 E5	0.267 E4	0.0	0.100 E4	0.0	0.257 E6	0.205 E7
0.583 E5	0.594 E5	0.399 E4	0.122 E4	0.100 E4	0.0	0.405 E6	0.254 E7
0.711 E5	0.711 E5	0.400 E4	0.224 E4	0.100 E4	0.0	0.477 E6	0.292 E7
0.838 E5	0.840 E5	0.304 E4	0.304 E4	0.100 E4	0.0	0.499 E6	0.318 E7
0.966 E5	0.974 E5	0.276 E4	0.276 E4	0.100 E4	0.0	0.532 E6	0.345 E7
0.109 E6	0.110 E6	0.258 E4	0.258 E4	0.100 E4	0.0	0.573 E6	0.376 E7
0.122 E6	0.123 E6	0.248 E4	0.246 E4	0.100 E4	0.0	0.628 E6	0.412 E7
0.135 E6	0.136 E6	0.242 E4	0.242 E4	0.100 E4	0.0	0.700 E6	0.456 E7

TABLE C-4 (CONTINUED)

Remaining Stages, $k = 8$

$x_v^{(n-1)}$	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	f_k
0.200 E5	0.287 E5	0.0	0.0	0.100 E4	0.0	0.310 E5	0.135 E7
0.327 E5	0.392 E5	0.225 E4	0.0	0.100 E4	0.0	0.193 E6	0.198 E7
0.455 E5	0.520 E5	0.225 E4	0.0	0.100 E4	0.0	0.253 E6	0.255 E7
0.583 E5	0.620 E5	0.400 E4	0.102 E4	0.100 E4	0.0	0.412 E6	0.306 E7
0.711 E5	0.731 E5	0.400 E4	0.273 E4	0.100 E4	0.0	0.499 E6	0.346 E7
0.838 E5	0.866 E5	0.306 E4	0.289 E4	0.100 E4	0.0	0.527 E6	0.377 E7
0.966 E5	0.999 E5	0.272 E4	0.272 E4	0.100 E4	0.0	0.540 E6	0.407 E7
0.109 E6	0.113 E6	0.264 E4	0.248 E4	0.100 E4	0.0	0.608 E6	0.447 E7
0.122 E6	0.126 E6	0.254 E4	0.238 E4	0.100 E4	0.0	0.666 E6	0.492 E7
0.135 E6	0.138 E6	0.241 E4	0.241 E4	0.100 E4	0.0	0.718 E6	0.538 E7

TABLE C-4 (CONTINUED)

$x_v^{(n-1)}$	$x_v^{(n)}$	Remaining Stages, $k = 9$				$p^{(n)}$	f_k
		$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$		
0.200 E5	0.305 E5	0.0	0.0	0.100 E4	0.0	0.386 E5	0.191 E7
0.327 E5	0.410 E5	0.224 E4	0.0	0.100 E4	0.0	0.201 E6	0.255 E7
0.455 E5	0.538 E5	0.225 E4	0.0	0.100 E4	0.0	0.260 E6	0.315 E7
0.583 E5	0.639 E5	0.400 E4	0.895 E3	0.100 E4	0.0	0.417 E6	0.366 E7
0.711 E5	0.770 E5	0.400 E4	0.630 E3	0.100 E4	0.0	0.458 E6	0.406 E7
0.838 E5	0.885 E5	0.301 E4	0.286 E4	0.100 E4	0.0	0.533 E6	0.441 E7
0.966 E5	0.101 E6	0.363 E4	0.176 E4	0.100 E4	0.0	0.575 E6	0.481 E7
0.109 E6	0.115 E6	0.400 E4	0.0	0.100 E4	0.0	0.586 E6	0.529 E7
0.122 E6	0.127 E6	0.246 E4	0.245 E4	0.100 E4	0.0	0.678 E6	0.580 E7
0.135 E6	0.140 E6	0.241 E4	0.241 E4	0.100 E4	0.0	0.732 E6	0.622 E7

TABLE C-4 (CONTINUED)

$X_v^{(n-1)}$	$X_v^{(n)}$	Remaining Stages, $k = 10$				$p^{(n)}$	f_k
		$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$		
0.200 E5	0.311 E5	0.0	0.0	0.100 E4	0.0	0.413 E5	0.251 E7
0.327 E5	0.416 E5	0.225 E4	0.0	0.100 E4	0.0	0.205 E6	0.317 E7
0.455 E5	0.544 E5	0.225 E4	0.0	0.100 E4	0.0	0.264 E6	0.377 E7
0.583 E5	0.646 E5	0.400 E4	0.851 E3	0.100 E4	0.0	0.420 E6	0.428 E7
0.711 E5	0.781 E5	0.400 E4	0.149 E3	0.100 E4	0.0	0.450 E6	0.470 E7
0.838 E5	0.910 E5	0.400 E4	0.0	0.100 E4	0.0	0.490 E6	0.512 E7
0.966 E5	0.104 E6	0.339 E4	0.0	0.100 E4	0.0	0.515 E6	0.561 E7
0.109 E6	0.116 E6	0.381 E4	0.0	0.100 E4	0.0	0.584 E6	0.617 E7
0.122 E6	0.128 E6	0.245 E4	0.245 E4	0.100 E4	0.0	0.684 E6	0.669 E7
0.135 E6	0.141 E6	0.242 E4	0.242 E4	0.100 E4	0.0	0.740 E6	0.707 E7

TABLE C-4 (CONTINUED)

Remaining Stages, $k = 11$

$x_{V}^{(n-1)}$	$x_V^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$	$p^{(n)}$	f_k
0.200 E5	0.304 E5	0.0	0.0	0.100 E4	0.0	0.385 E5	0.309 E7
0.327 E5	0.409 E5	0.225 E4	0.0	0.100 E4	0.0	0.203 E6	0.376 E7
0.455 E5	0.537 E5	0.225 E4	0.0	0.100 E4	0.0	0.262 E6	0.436 E7
0.583 E5	0.638 E5	0.400 E4	0.902 E3	0.100 E4	0.0	0.421 E6	0.488 E7
0.711 E5	0.773 E5	0.400 E4	0.915 E2	0.100 E4	0.0	0.451 E6	0.536 E7
0.838 E5	0.906 E5	0.371 E4	0.0	0.100 E4	0.0	0.481 E6	0.586 E7
0.966 E5	0.103 E6	0.298 E4	0.425 E3	0.100 E4	0.0	0.513 E6	0.643 E7
0.109 E6	0.116 E6	0.321 E4	0.0	0.100 E4	0.0	0.560 E6	0.702 E7
0.122 E6	0.127 E6	0.294 E4	0.294 E4	0.100 E4	0.0	0.659 E6	0.751 E7
0.135 E6	0.140 E6	0.242 E4	0.242 E4	0.100 E4	0.0	0.739 E6	0.790 E7

TABLE C-4 (CONTINUED)

$x_v^{(n-1)}$	Remaining Stages, $k = 12$					$p^{(n)}$	f_k
	$x_v^{(n)}$	$\theta_1^{(n)}$	$\theta_2^{(n)}$	$\theta_3^{(n)}$	$\theta_4^{(n)}$		
0.200 E5	0.285 E5	0.0	0.0	0.100 E4	0.0	0.306 E5	0.357 E7
0.327 E5	0.396 E5	0.169 E4	0.0	0.100 E4	0.0	0.169 E6	0.425 E7
0.455 E5	0.518 E5	0.225 E4	0.0	0.100 E4	0.0	0.257 E6	0.487 E7
0.583 E5	0.628 E5	0.400 E4	0.0	0.100 E4	0.0	0.383 E6	0.544 E7
0.711 E5	0.753 E5	0.393 E4	0.335 E3	0.100 E4	0.0	0.450 E6	0.598 E7
0.838 E5	0.886 E5	0.377 E4	0.0	0.100 E4	0.0	0.480 E6	0.656 E7
0.966 E5	0.101 E6	0.344 E4	0.0	0.100 E4	0.0	0.514 E6	0.718 E7
0.109 E6	0.114 E6	0.384 E4	0.0	0.100 E4	0.0	0.587 E6	0.783 E7
0.122 E6	0.125 E6	0.249 E4	0.249 E4	0.100 E4	0.0	0.686 E6	0.818 E7
0.135 E6	0.138 E6	0.244 E4	0.244 E4	0.100 E4	0.0	0.742 E6	0.862 E7

APPENDIX D. PLOTS SHOWING RESULTS OF A SAMPLE OPTIMAL PATH

In Table C-4, a list of the output from the computer program was printed. This represents the solution of dynamic programming applied to a twelve stage system. As an example, the initial state variable ($\bar{x}_v^{(0)} = 4.55 \times 10^4$ acre ft.) was chosen arbitrarily, and an optimal path plotted from this table. Figures D-1 through D-6 show the nature of this path as it ranges through a time period of twelve months. The variation of the state variable as well as the decision variables are included in these plots. Because of the nature of the model, most of the variables were subject to constraints. The values of these constraints have been placed in the plots that follow. Doing so illustrates any tendency of the optimal path to move along boundary zones.

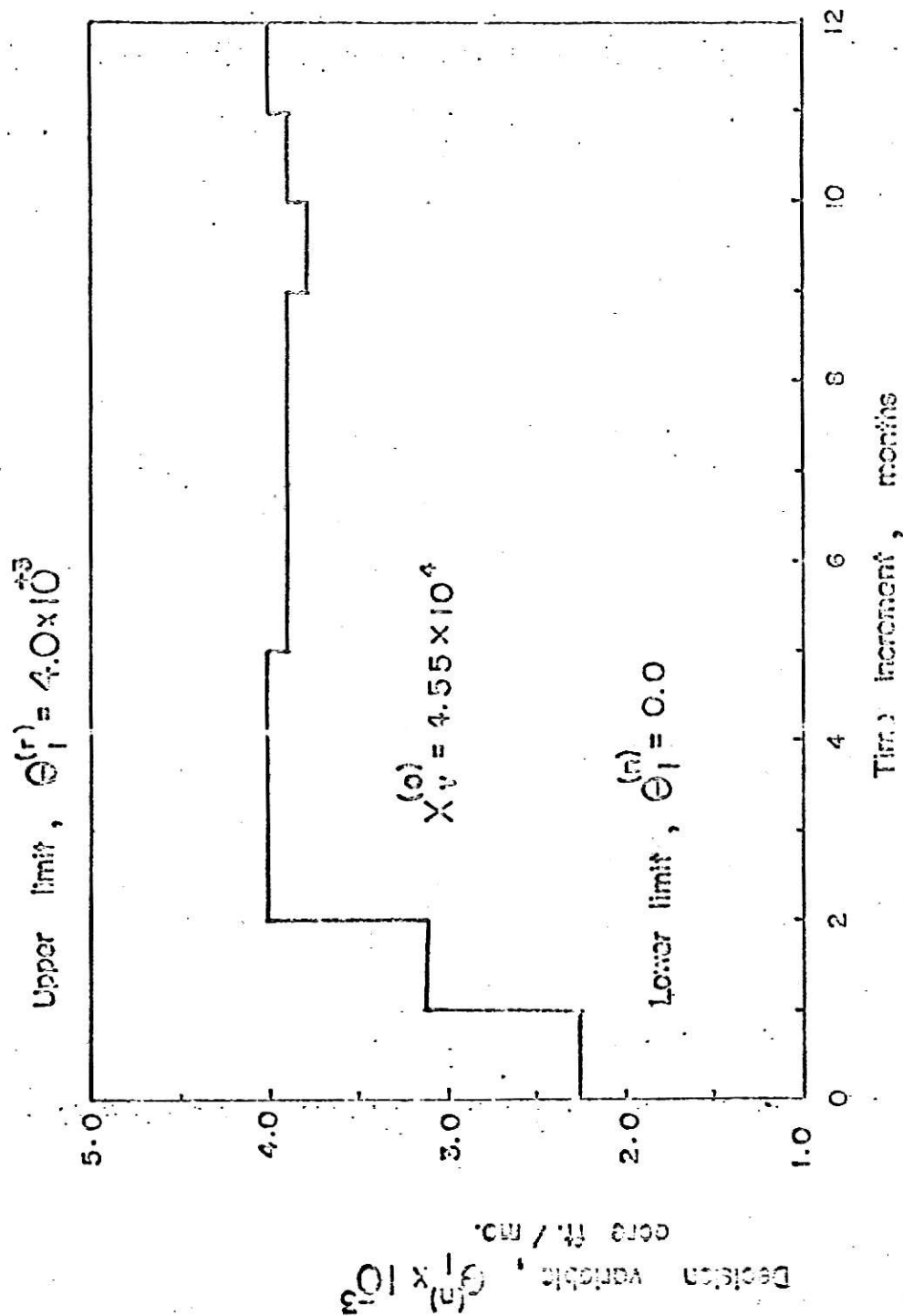


Fig. D-1. Example of optimal $\Theta_1^{(n)}$ for a 12 stage system.

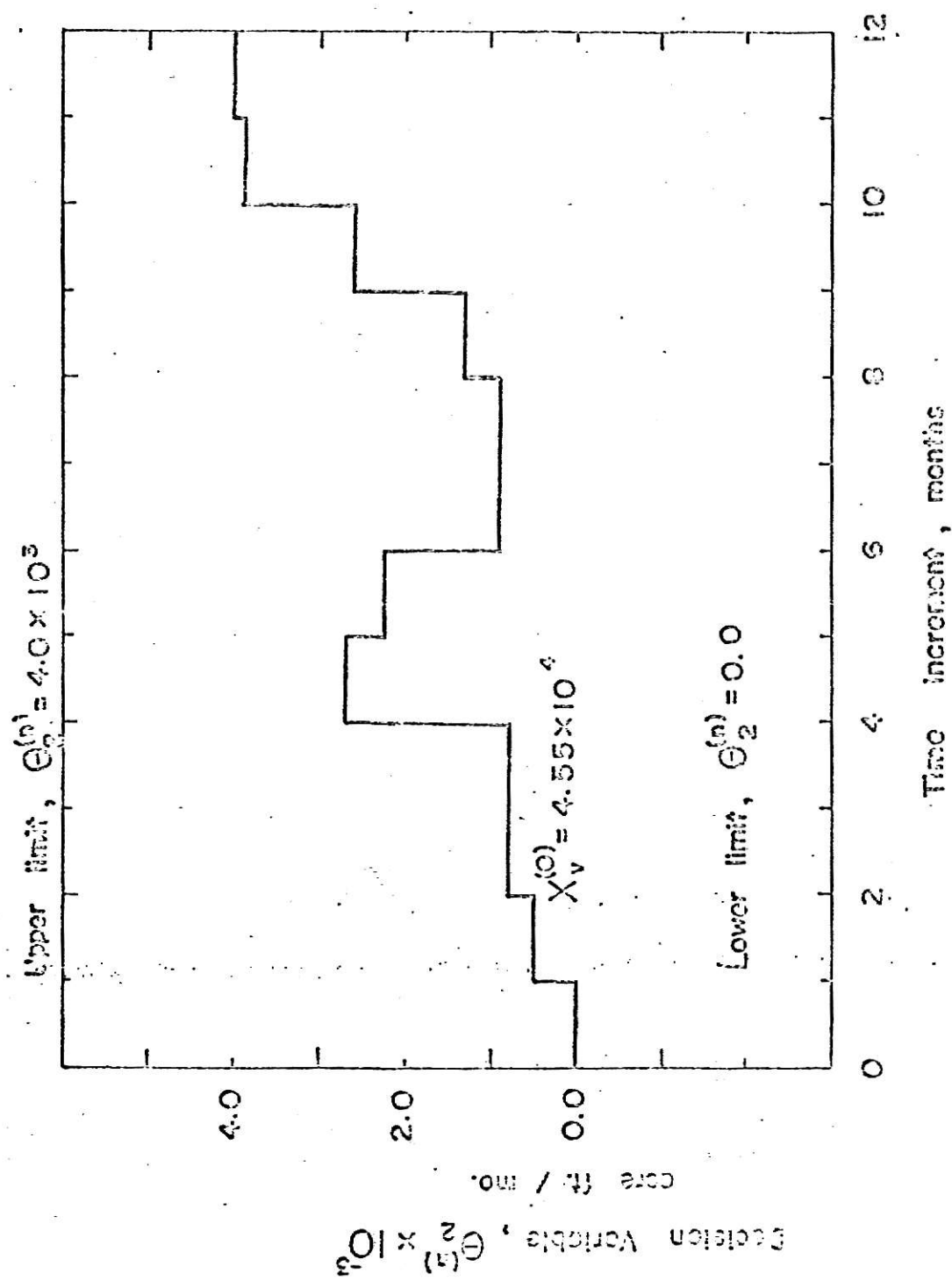


Fig.D-2. Example of optimal $\Theta_2^{(n)}$ for a 12 stage system.

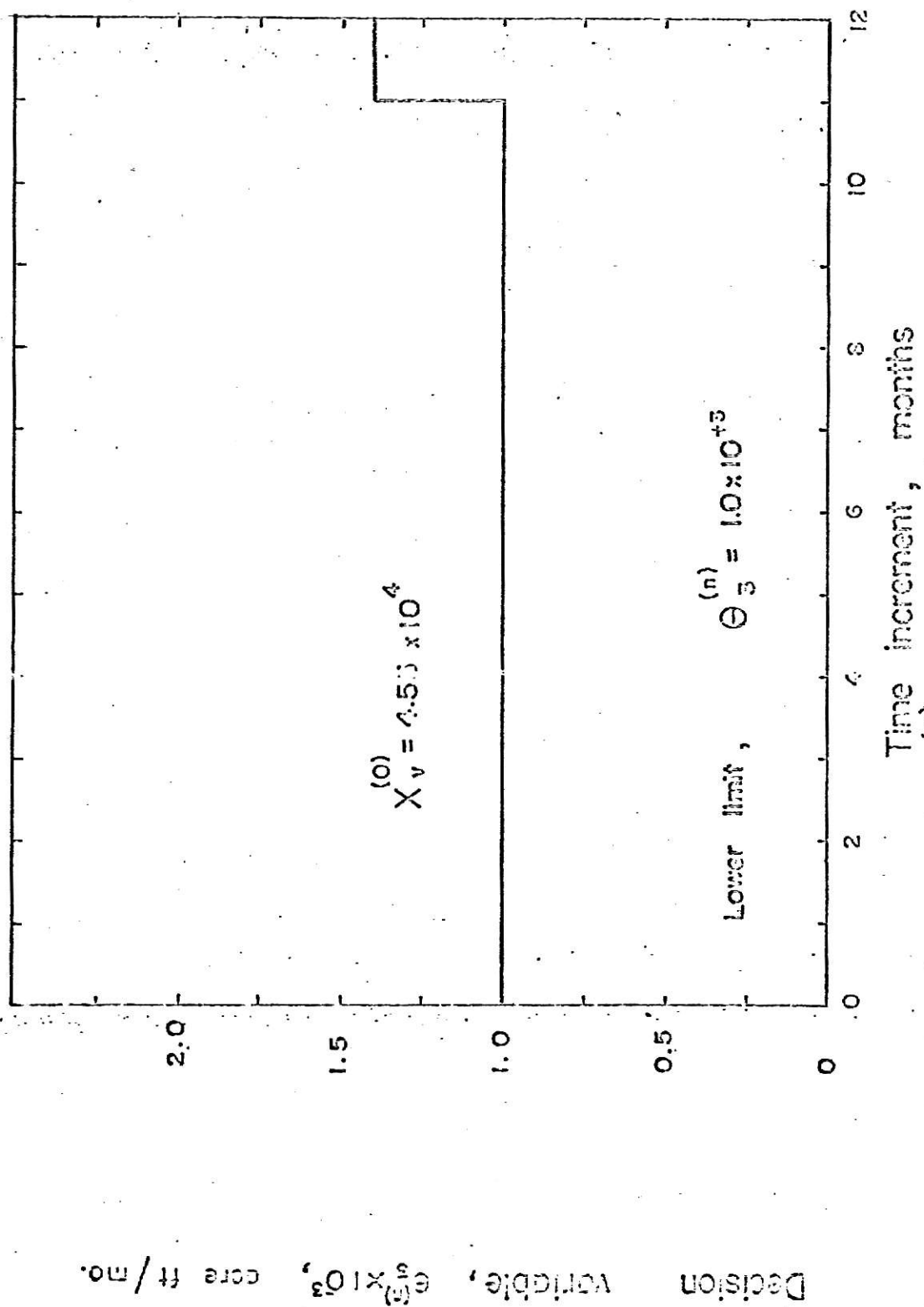


Fig. D-3. Example of optimal $\Theta_3^{(n)}$ for a 12 stage system.

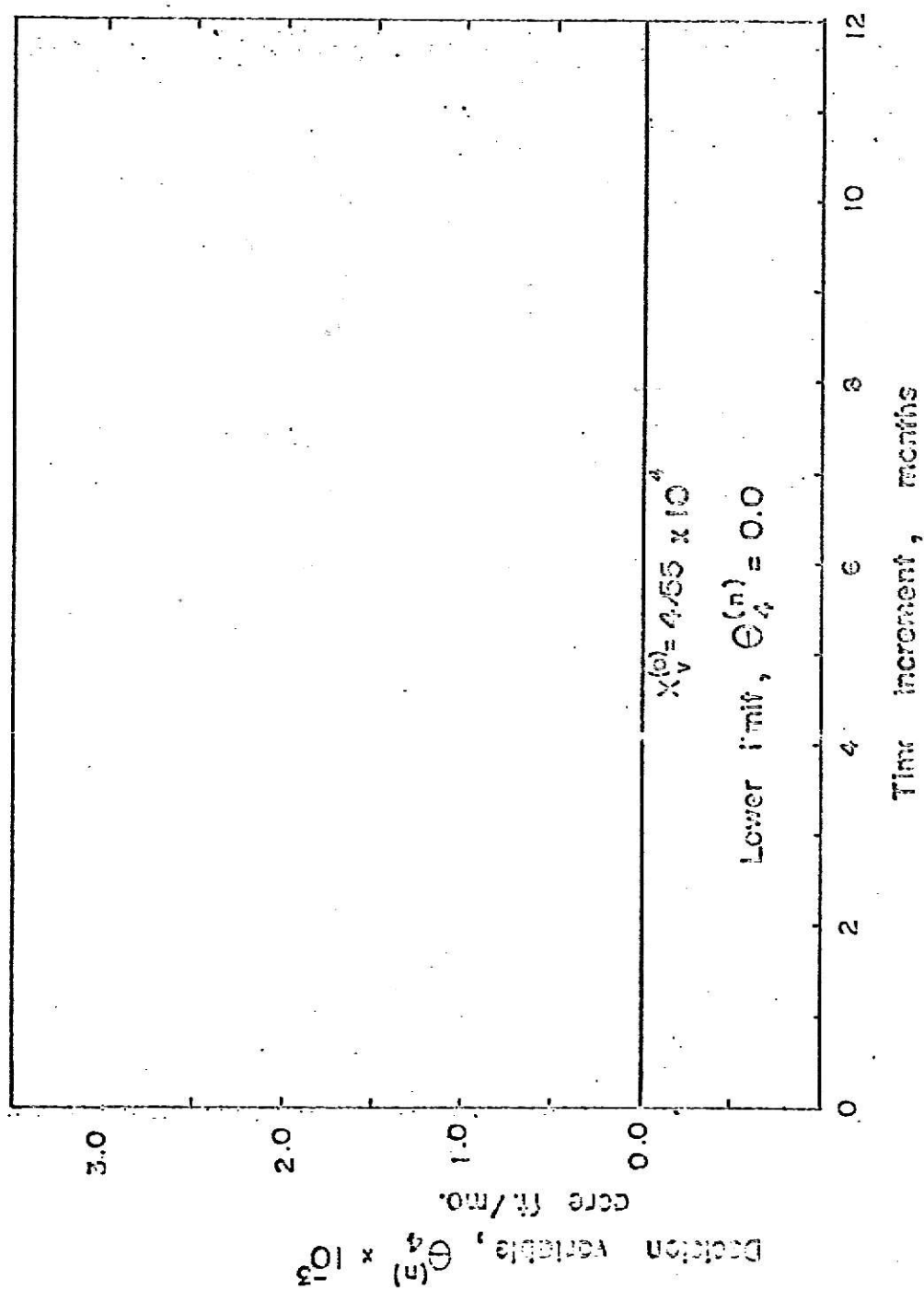


Fig. D-4. Example of optimal $\Theta_4^{(n)}$ for a 12 stage system.

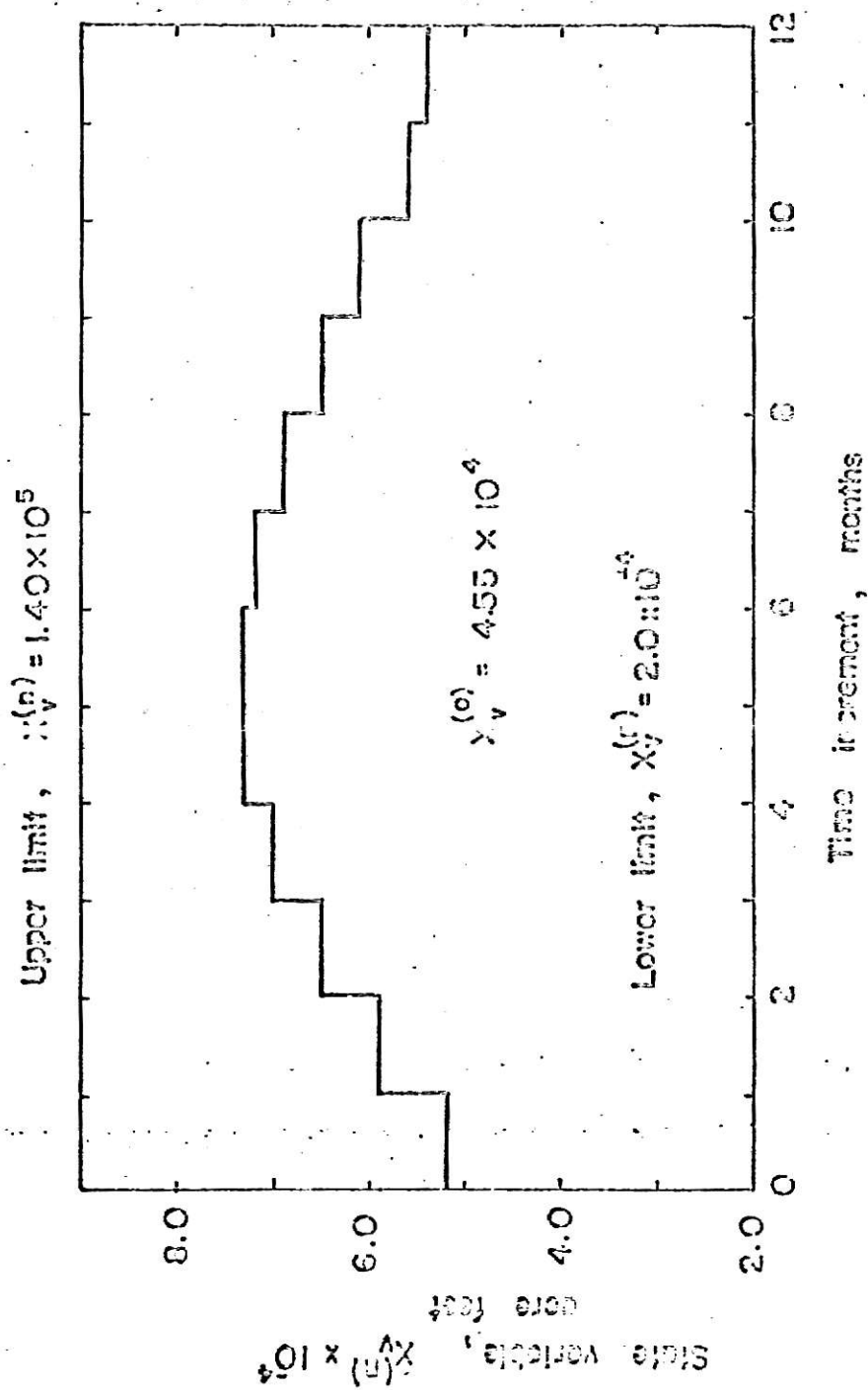


Fig. D-5. Example of optimal $X_V^{(n)}$ for a 12 stage system.

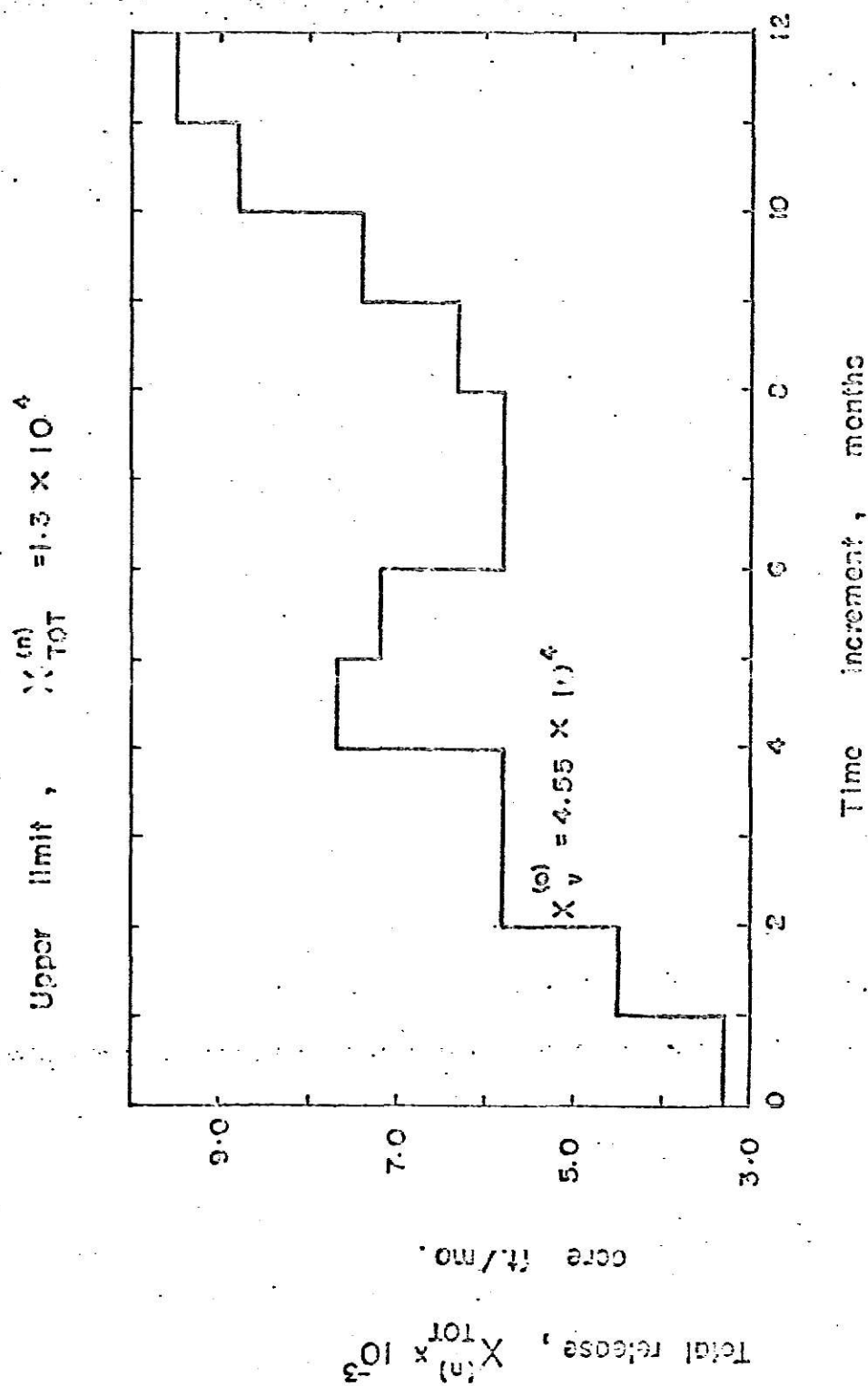


Fig. D-6. Example of optimal $X_{TOT}^{(n)}$ for a 12 stage system.

NOMENCLATURE

- A = Expected deviation of short term fluctuations. A coefficient of the hydrology generator, acre ft./mo.
- B = Expected deviation of long term fluctuations. A coefficient of the hydrology generator, acre ft./mo.
- $b_{l,i,m}$ = The i^{th} component of a base point used in the Hooke and Jeeves pattern search.
- D = Expected deviation of seasonal fluctuations. A coefficient of the irrigation target output demand, acre ft./mo.
- d_o = Mean flow rate required by the irrigation target output demand, acre ft./mo.
- e = The efficiency factor for a hydroelectric system.
- f_k = The total profit for a series of k remaining stages. Dynamic programming analysis, dollars.
- f_{k-1} = The total profit for a series of k-1 remaining stages, dollars.
- I = The number of independent or decision variables used in the system.
- i = Subscript designation of a specific decision variable.
- j = Grid point designation of the state variable $x_v^{(n-1)}$.
- J = The number of grid points associated with the state variable, $x_v^{(n-1)}$.

- K = A conversion factor relating hydroelectric power output to system parameters, k.w. hrs./acre ft.².
- k = A counting index indicating stage position in the dynamic programming analysis.
- l = Subscript designation of the number of iterations performed in the Hooke and Jeeves pattern search.
- m = Subscript designation of the initial base point $b_{l,i,m}$ introduced into the Hooke and Jeeves pattern search.
- M = The total number of initial base points which will be used in the pattern search.
- N = The total number of stages considered in the dynamic program routine.
- n = A superscript indicating a specific time increment or stage.
- $p^{(n)}$ = The interval profit associated with a stage n , dollars.
- q_o = Mean flow rate used by the hydrology generator, acre ft./mo.
- r = The interest rate of money prorated on a monthly basis.
- $s^{(n)}$ = Net economic gain accrued within the system during stage n , dollars.
- t = Time as a continuous variable, months.
- $t_{l,i}$ = The i^{th} component of a temporary head used to extend the Hooke and Jeeves pattern search.

- $T_B^{(n)}$ = Total benefit accrued within the system during stage n, dollars.
- $T_P^{(n)}$ = Total penalty or loss contracted within the system during stage n, dollars.
- $T_{om}^{(n)}$ = Total operating and maintenance costs incurred during stage n, dollars.
- V = Present value of net system gain accrued during the period of economic analysis, dollars.
- $X_h^{(n)}$ = Average head during stage n, ft.
- $X_{HT}^{(n)}$ = Target demand for hydroelectric power during stage n, k.w. hrs./mo.
- $X_{IT}^{(n)}$ = Irrigation target output demand during stage n, acre ft./mo.
- $X_{UT}^{(n)}$ = Urban target output demand during stage n, acre ft./mo.
- $X_{TR}^{(n)}$ = Average flow rate utilized for production of hydroelectric power during stage n, acre ft./mo.
- $X_{TOT}^{(n)}$ = Total flow rate of water released from the reservoir during stage n, acre ft./mo.
- $X_{po}^{(n)}$ = Output of power plant during stage n, k.w. hrs./mo.
- $X_{RA}^{(n)}$ = Attendance for water based recreation during stage n, visitor-days per acre of reservoir surface per month.
- $X_{RS}^{(n)}$ = Average reservoir surface area during stage n, acres.
- $X_v^{(n)}$ = Average reservoir volume during stage n, acre ft.

- $X_v^{(n-1)}$ = Average reservoir volume during stage n-1, acre ft.
- $X_Q^{(n)}$ = Flow rate of water into the system during stage n, acre ft./mo.

GREEK LETTERS

- β_i = The factor by which perturbation size is cut when the search subroutine encounters a disruption in the search pattern.
- δ_i = The perturbation size used in the Hooke and Jeeves pattern search.
- ω = A frequency factor of short term fluctuations in the hydrology generator, cyc./mo.
- ω' = A frequency factor of long term fluctuations in the hydrology generator, cyc./mo.
- ω'' = A frequency factor of seasonal fluctuations in the irrigation target output demand, cyc./mo.
- $\phi_{HB}^{(n)}$ = System gain or benefit accrued from hydroelectric power during stage n, dollars/mo.
- $\phi_{HL}^{(n)}$ = Loss sustained from power shortage below target output demand for stage n, dollars/mo.
- $\phi_{IB}^{(n)}$ = System benefit accrued from irrigation during stage n, dollars/mo.
- $\phi_{IL}^{(n)}$ = Loss sustained from irrigation shortage below the target output demand for stage n, dollars/mo.

$\phi_{RB}^{(n)}$ = System benefit accrued from water based recreation during stage n, dollars/mo.

$\phi_{VB}^{(n)}$ = System benefit accrued from water supply to urban areas during stage n, dollars/mo.

$\phi_{VL}^{(n)}$ = Loss sustained from urban supply shortage below the target output demand for stage n, dollars/mo.

$\phi_{VnB}^{(n)}$ = Unit benefit from hydroelectric power output, dollars/k.w. hr.

$\theta_{UR}^{(n)}$ = $\theta_1^{(n)}$, Scheduled release of water for urban use during stage n, acre ft./mo.

$\theta_{IR}^{(n)}$ = $\theta_2^{(n)}$, Scheduled release of water for irrigation during stage n, acre ft./mo.

$\theta_{ER}^{(n)}$ = $\theta_3^{(n)}$, Scheduled release of water in excess of $\theta_{UR}^{(n)}$ and θ_{IR} during stage n, acre ft./mo.

$\theta_{TB}^{(n)}$ = $\theta_4^{(n)}$, Water flow from the reservoir which cannot be utilized for hydroelectric power, acre ft./mo.

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MODELING AND OPTIMIZATION OF A WATER
RESOURCE CONTROL SYSTEM

by

DAVID LOUIS MEYER

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AN ABSTRACT OF A MASTER'S REPORT

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requirements for the degree

MASTER OF SCIENCE

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Manhattan, Kansas

1969

The purpose of this thesis was to devise an efficient approach to water resource control optimization. A test system has been constructed and solved within an analytical framework.

While constructing a model for the test system, particular attention was devoted to sources of water supply, water demand, and the measures available for meeting these demands. The major elements of water supply are components of surface and ground water flow as well as natural storage. Relevant characteristics are quantity, quality and availability measured with respect to time and location. Data used in the model was drawn from the literature wherever possible. This practice was adopted to insure realistic response behavior. In addition, this data is of a form which subsequently may be determined for the analysis of real systems.

The criteria against which operational policies are judged is their effect on the monetary benefits and consequences accrued by each demand source. Consequently, functions have been derived for each source, relating economic gains or losses as a function of scheduled releases within the system. Each point from a benefit function measures the resource cost of providing equivalent goods and services by the least costly alternative. Each point from a loss function measures the penalty of failure to meet a target output demand.

The objective function developed in the system model consists of four decision variables and one state variable applied at each of N stages. Solution, consequently, fell quite naturally in the realm of dynamic programming. At each stage in the dynamic program routine, a search was conducted by the computer for an optimal set of values which influenced the control policies for that stage. This was accomplished by the Hooke and Jeeves pattern search technique. The search subroutine was modified somewhat, so that it could handle constrained problems. The solution is printed in table form for a twelve stage problem. With output data in this form, one may select an initial (or final) state variable from a list of grid values, and then trace an optimal path relating control policies against time. The size of the list found in these tables is controlled by input parameter cards. The size, and hence accuracy may be varied to fit the needs of the problem. The system model was developed with the idea that it could be modified or extended to cover different situations. Consequently, the dynamic program routine and the search subroutine were written in as general a manner as possible. They can, if desired, be utilized by completely separate model and constraint subroutines.