

Holistic resilience analyses of power systems

by

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B. Tech., JNTUA College of Engineering Pulivendula, 2014

M. Tech., Indian Institute of Technology Gandhinagar, 2018

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the
requirements for the degree

DOCTOR OF PHILOSOPHY

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KANSAS STATE UNIVERSITY
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Abstract

Power systems are undergoing a rapid transformation driven by the proliferation of distributed energy resources (DERs), increasing penetration of renewable energy, electrification of transportation, and integration of cyber infrastructure for sensing and control. With increasing frequency of weather hazards, significant challenges are emerging in planning, operation, and resilience assessment, particularly under conditions of limited observability, heterogeneous data, and strong interdependencies with other critical infrastructures. This dissertation develops a unified community-centric framework for the planning, operation, and resilience analysis of power distribution systems by integrating optimal transport theory, advanced statistical learning, and hetero-functional graph modeling.

This dissertation proposes a novel optimal transport (OT)-based framework for planning distributed energy resources (DERs) to support resilient and community-aware power system operation. Strategic DER deployment enhances resilience by reducing reliance on centralized infrastructure and enabling localized service during outages. Conventional DER planning methods primarily optimize technical objectives while overlooking disparities in community-level energy outcomes. In this work, DER planning is mapped to distributional transformation of a community-centric metric using both unconstrained and constrained formulations to support socio-technical planning. The proposed approach enables balanced trade-offs between grid performance and community-level considerations, contributing to more resilient and inclusive energy system planning.

The second part of this dissertation addresses robust operation and control of distribution systems under limited observability. Accurate distribution system state estimation (DSSE) and phase identification (PI) are critical for reliable grid operation but are hindered by sparse, noisy, and heterogeneous measurements. To overcome these challenges, this dissertation introduces a novel Fréchet regression-based framework for DSSE and PI that operates

on distributions rather than point estimates, improving robustness to measurement noise, missing data, and model uncertainties. A joint framework is further developed to simultaneously perform PI and state estimation, improving accuracy in low-observability regimes. In addition, community-centric operational strategies such as coordinated microgrid operation and transmission–distribution (TSO/DSO) interaction frameworks, are developed to incorporate community-level objectives into real-time grid control. These contributions enhance situational awareness and enable adaptive, resilient operation of modern distribution systems.

The third part of the dissertation develops a hetero-functional graph (HFG) theory-based framework for resilience analysis of interdependent infrastructure systems. Conventional graph-based models primarily capture structural connectivity and fail to represent functional dependencies among system components, limiting their ability to model cascading failures. To address this gap, HFG theory is employed to represent multi-layered functional interactions among power systems and other critical infrastructures. The dissertation introduces analytical tools for modeling failure propagation across functionalities and develops sensitivity-based techniques to identify critical components influencing system resilience. The proposed framework enables a realistic and computationally efficient assessment of infrastructure robustness by capturing structural and functional interdependencies that govern disruption propagation and system degradation.

Building upon this foundation, the final part of the dissertation investigates higher-order functional dependencies using topological data analysis (TDA). While conventional graph models capture only pairwise interactions, real-world infrastructure systems exhibit higher-order relationships among multiple entities. To model these interactions, persistent homology and Hodge Laplacian-based spectral analysis are integrated into the hetero-functional graph framework. A percolation-based robustness analysis is developed to quantify connectivity and higher-order cycles under random and targeted disruptions. The results show that higher-order topological metrics provide earlier indicators of system degradation than traditional graph measures, enabling proactive resilience enhancement.

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Nomenclature

DER	Distributed energy resources
PV	Photovoltaic
OT	Optimal transport
COT	Constrained Optimal Transport
EDE	Equal distribution equivalent
OPF	Optimal power flow
PMD	Power Model Distribution
IV	Current Voltage
DSSE	Distribution system state estimation
PI	Phase identification
PMU	Phasor measurement unit
WLS	Weighted least square
FAD	Fraction of available data
MMG	Multi-microgrid
PDS	Power distribution system
VR	Voltage regulator
SCADA	Supervisory control and data acquisition
SVC	Static VAR compensator
TSO	Transmission System Operator
DSO	Distribution System Operator
VSA	Voltage sensitivity analysis
HFGT	Hetero-functional graph theory
MBSE	model-based systems engineering
QoL	Quality of life
VoLL	Value of lost load

TDA Topological data analysis
IUN Interdependent urban networks
NCC Number of connected components

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Preface

This dissertation, “Holistic Resilience Analyses of Power Systems,” is submitted for the degree of Doctor of Philosophy in the Mike Weigers Department of Electrical and Computer Engineering at Kansas State University. The research was conducted under the supervision of Professor Balasubramaniam Natarajan. This work is original to the best of my knowledge, except where acknowledgments and references are indicated. Most of the work has been presented in the following published peer-reviewed journals and conferences:

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2. P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Equity-driven Planning of Distributed Solar PV using Optimal Transport, 2025 IEEE Power & Energy Society General Meeting (PESGM), pp.1-5, IEEE,2025.
3. P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Hetero-functional Graph Theoretic Framework for Holistic Resiliency Analysis of Power Systems, IEEE Transactions on Systems, Man and Cybernetics: Systems, pp. 1-6, 2026.
4. P. Dhanapala, K.K. Challa, K. Jhala, and B. Natarajan, Distribution System Congestion Management with Various DSO and Distributed Resource Coordination Architectures, (Accepted in 2026 IEEE Power & Energy Society General Meeting).
5. A. Sreejith, P. Dhanapala, A. Melagoda, SA. Enderami, V. Aravinthan and A. Pahwa, ComPSim: A Community based Smart Grid Testbed for Holistic Resilience Analysis, 2023 IEEE International Conference on Communications, Control, and Computing

- Technologies for Smart Grids (SmartGridComm), Glasgow, United Kingdom, 2023, pp. 1-6, doi:10.1109/SmartGridComm57358.2023.10333936.
6. A. Sreejith, P. Dhanapala, B. Natarajan, A. Pahwa, and J.S. Bergtold, Socioeconomic Vulnerability-Based Resilience Assessment Metric for Power Distribution Systems, *IEEE Access*, vol. 14, pp. 2647-2660, 2025.
 7. P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Optimal Transport Theory for Solar PV Planning: Balancing Community and Grid Priorities, (submitted to *IEEE Transactions on Sustainable Energy*).
 8. P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Constrained Optimal Transport Theory for Distributed Generation Planning, (submitted to *IEEE Systems Journal*).
 9. R. Madbhavi, P. Dhanapala, B. Natarajan, A. Pahwa, et al., CISCs: A Socio-Technical Co-Simulator for Critical Infrastructure Interdependencies and Community Resilience, (submitted to *IEEE Transactions on Systems, Man and Cybernetics*)
 10. K. Jhala, K. K. Challa, P. Algikar, & P. Dhanapala, Distribution System Congestion Management-A Survey of Reliable Integration for Aggregated Resources and Microgrids. *IEEE Access* (2026).
 11. P. Dhanapala, A. Sreejith, B. Natarajan and W. Song, Robust Phase Identification and State Estimation in Unobservable Distribution Systems using Fréchet Regression, (manuscript submitted to *IEEE Transactions on Power Systems*).
 12. P. Dhanapala and B. Natarajan, Robustness Assessment of Interdependent Infrastructure Systems: A Topological Perspective, (manuscript submitted to *IEEE Transactions on Systems, Man and Cybernetics*).

Chapter 1

Introduction

Power systems transformation is driven by increasing electricity demand, widespread integration of renewable energy resources, aging infrastructure, and the growing interdependence between infrastructure systems. Unlike traditional power systems dominated by centralized resources, modern power systems are characterized by decentralized resources, bidirectional power flows, and dynamically evolving operating conditions. These changes introduce both opportunities for improved efficiency and new challenges related to reliability, stability, and resilience.

With increasing frequency and intensity of severe weather events, the concept of resilience has emerged as a critical performance attribute for modern power systems. The resilience behavior of an infrastructure system is often represented through a performance–time relationship, as illustrated in Fig. 1.1, where system functionality degrades following an extreme event and subsequently recovers through corrective actions and restoration processes. The depth of performance degradation, duration of service disruption, and rate of recovery collectively characterize the resilience of the system. These characteristics are particularly important in power systems serving diverse communities, where prolonged service interruptions can significantly impact critical services and socioeconomic well-being.

Ideal resilience performance depends on slower degradation and faster recovery rates, which are primarily enabled through adequate resource availability and accurate situational

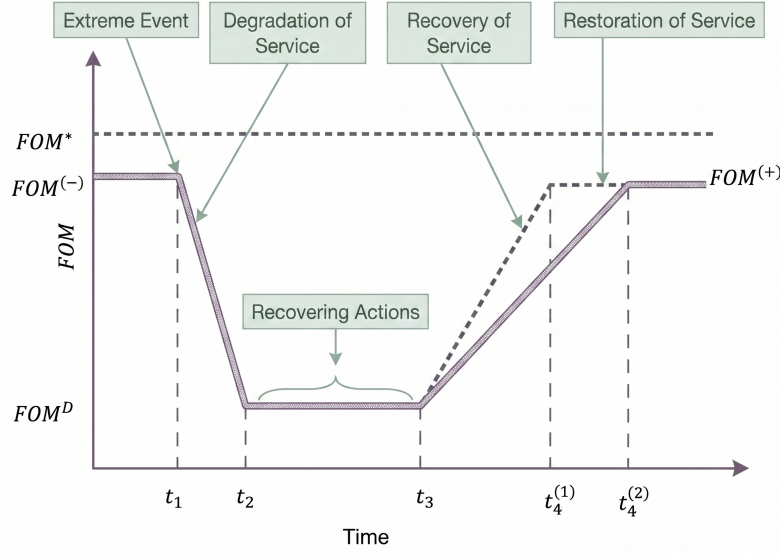


Figure 1.1: Evolution of the performance/service (figure of merit) of a system in the presence of an extreme event and recovering actions.

awareness. Historically, planning and operational practices in power systems have largely focused on improving efficiency and reliability, often without fully addressing resilience needs or disparities in impacts among consumers [1]. As power systems transition toward smart and sustainable infrastructures, utilities and policymakers must ensure that system benefits and burdens are distributed in a balanced manner through community-centric planning and operational approaches. This transition is particularly important as extreme events and cyber disruptions continue to increase in frequency and intensity. Infrastructure systems that lack redundancy, adaptability, and community awareness often experience catastrophic disruptions, disproportionately affecting socially vulnerable populations [2, 3]. These concerns highlight the need for holistic resilience planning and operation that is community-centric. This dissertation attempts to address this critical need for holistic resilience analyses.

1.1 Motivation

Resilience of the power system can be improved by both long term planning efforts (such as DER installations, system hardening efforts etc.) as well as operational efforts (such as better

managing the available DER resources, optimal control and management of microgrids etc.). These planning and operational efforts help reduce the degradation level experienced by a community when there is an extreme weather event by enabling the system to better absorb or adapt to the disruption. This ability is related to the idea of robustness of the system and corresponds to the first phase of the resilience trapezoid in Fig. 1.1. In the following subsections, we will discuss issues related to holistic resilience planning and operations from both a technical and societal perspectives - these issues serve as the motivation for the research questions addressed in this dissertation.

DER Planning Issues: Planning and executing system upgrades and redundancies to improve resilience and/or reliability is a long process that involves many stakeholders. In this dissertation, we are interested in planning for DER investments and its impact on the community from a socio-economic perspective.

The availability and strategic deployment of distributed energy resources (DERs) play a critical role in enhancing system robustness and resilience. Effective DER planning reduces reliance on centralized infrastructure and enables localized energy support during disturbances, thereby limiting the depth of system performance degradation. Power flow-based planning methods are commonly used to evaluate system reliability, efficiency, and operational feasibility. However, conventional planning frameworks often evaluate system performance primarily through aggregate metrics, overlooking differences in consumer-level outcomes. In practice, consumer experiences vary significantly across communities, particularly in terms of energy affordability and energy burden. Therefore, it is important to incorporate distribution of community impacts into DER planning to avoid reinforcing historical inequities.

DER Operational Issues: The integration of DERs across the distribution grid introduces additional operational complexity that necessitates enhanced situational awareness. Variability in load demand, renewable generation, and feeder-level conditions introduces measurement uncertainties and operational variability. Furthermore, measurements are typically available only at limited locations in the distribution grid, creating significant challenges for accurate monitoring and proactive decision-making. A fundamental challenge in distribu-

tion system operation is estimating network states and parameters from sparse and uncertain observations.

Many existing state estimation methods rely on predefined probabilistic assumptions and Euclidean representations, which do not fully capture the intrinsic non-Euclidean structure of power system data. Additionally, incomplete or inaccurate network models further complicate operational decision-making. Data-driven learning approaches that operate on structured and non-Euclidean representations provide an effective alternative for addressing these limitations.

Microgrid Management Issues: Microgrids have been shown to be an effective investment to improve resilience. For large-scale systems, coordinated microgrid control strategies play a critical role in maintaining system stability and operational flexibility. Multi-microgrid coordination and transmission-distribution system interaction frameworks enable effective congestion management and voltage regulation across interconnected networks but can be challenging. Modern coordination strategies must also support inclusive participation of distributed resources while maintaining operational reliability.

Interdependency Issues: Critical infrastructure systems are becoming increasingly interconnected. While this coupled operation has optimized efficiency, it has also stressed robustness and resiliency. As extreme events and cyber disruptions impact power systems and other critical infrastructures, resilience analysis must be conducted in a holistic and integrated manner. A deeper understanding of complex infrastructure behavior can be achieved through surrogate graphical models that capture both primary and secondary impacts during disruptions. Multilayer modeling frameworks enable the representation of interconnected infrastructure systems, including power, communication, transportation, and community services, within a unified structure. However, conventional multilayer graph representations often face ontological limitations in capturing functional dependencies across heterogeneous systems. Additionally, scalable modeling approaches are essential for capturing cascading failures in large-scale networks and enabling rapid assessment of disruption impacts.

In addition to pairwise interactions among system components, complex infrastructure networks exhibit higher-order dependencies involving multiple interacting entities. Tradi-

tional graph-based robustness metrics are typically based on pairwise connectivity structures and may fail to capture multi-component interactions such as feedback loops and redundancy cycles. These higher-order interactions often govern the stability and robustness of interconnected systems.

In summary, although several approaches have been developed for improving and assessing resilience, existing approaches suffer from several key limitations:

1. **Limited Flexibility:** Conventional DER planning methods typically rely on optimal power flow (OPF)-based formulations that primarily focus on network-centric technical metrics such as voltage regulation and power loss minimization, often neglecting community-level perspectives. In practice, the placement of a DER within a distribution network can disproportionately influence consumer-level energy costs depending on feeder topology and load distribution. Therefore, planning methodologies must incorporate flexible and target-oriented formulations capable of prioritizing both utility-driven performance metrics and community-driven outcomes (e.g., minimizing energy burden disparities).
2. **Scalability challenges:** Power flow-based planning and operational methods frequently involve mixed-integer nonlinear programming (MINLP) formulations that are computationally intractable for large-scale networks. As the number of network nodes and candidate DER locations increases, solving such optimization problems becomes increasingly difficult due to combinatorial complexity and non-convex power flow constraints. For example, during large-scale DER planning or outage assessment scenarios, it is necessary to evaluate system-wide impacts across thousands of nodes. Traditional solution approaches often fail to provide tractable solutions for practical large-scale distribution systems, thereby limiting their usability in real-world resilience planning studies.
3. **Inability to Handle Non-Euclidean Data Structures:** Accurate estimation of distribution system states, including voltage magnitudes, voltage angles, currents, and

active and reactive power flows, from limited measurements, remains a significant challenge. Many existing estimation techniques rely on Euclidean representations of inherently structured network data, such as adjacency matrices, covariance operators, and measurement vectors. However, power system measurements exhibit non-Euclidean characteristics due to network topology, phase coupling, and physical constraints. Representing such structured data using conventional Euclidean frameworks can distort the underlying system geometry, resulting in reduced estimation accuracy and poor robustness under measurement uncertainty and model mismatches.

4. **High Computational Complexity:** Most of the conventional cascading failure analysis methods rely on repeated power flow simulations and iterative nonlinear computations, which significantly increase computational burden as network size grows. In complex infrastructure systems, the impact of a change in the operational state of one functionality must be propagated through interconnected components to capture both primary and secondary failure effects. For large-scale distribution networks, solving nonlinear power flow equations under multiple contingency scenarios becomes computationally expensive, limiting the feasibility of real-time or large-scale resilience analysis.
5. **Inefficient Coordinated Control:** Existing coordination mechanisms among transmission system operators (TSOs), distribution system operators (DSOs), microgrid operators, and DER owners are often designed primarily for revenue maximization within electricity market environments. However, effective coordination strategies must balance economic objectives with operational reliability and equitable resource participation. For example, DER curtailment is commonly used as a congestion management strategy to maintain system reliability. While effective from a utility perspective, such practices may lead to opportunity losses for DER owners and reduced overall system efficiency. Therefore, coordinated control frameworks must incorporate mechanisms that enable equitable DER participation while maintaining voltage and congestion limits within acceptable operating ranges.

6. **Higher-Order Dependency:** Traditional graph-based resilience studies primarily rely on node-to-node pairwise interactions to evaluate network robustness under failure scenarios. Many widely used robustness metrics are constructed based on zero-dimensional (0-simplicial) connectivity representations, which capture only basic structural connectivity information. However, real-world interdependent infrastructure systems exhibit higher-order functional dependencies, including redundancy pathways, feedback loops, and multi-component interactions. Ignoring such higher-order structural features can lead to incomplete characterization of system vulnerability. For example, in interdependent infrastructure systems, functional cycles often govern the propagation and containment of cascading failures. Failure to capture these interactions during contingency analysis can result in inaccurate vulnerability assessment and suboptimal resilience planning decisions.

Based on these observations, there is a growing need for unified methodologies that integrate optimization, statistical learning, graph-based modeling, and topological analysis into scalable frameworks for resilience-driven power system design. Such integrated approaches enable proactive identification of vulnerabilities, improved situational awareness, and enhanced recovery performance under diverse disturbance scenarios. Collectively, these developments form the foundation for holistic resilience analysis of modern power and community infrastructure systems.

1.2 Research Questions

To address the challenges highlighted in the previous section, we focus on a subset of problems. Specifically, related to the challenge of flexibility and scalability in DER planning, we seek to address following question:

Question 1: *How can scalable and community-centric DER planning methods be developed to balance power system performance goals and community-centric objectives?*

Related to the challenges of situational awareness and control with non-Euclidean data

structures, we ask the following questions:

Question 2: *How can distribution system state and network topology information be jointly estimated under non-Euclidean and limited measurement conditions?*

Question 3: *What coordinated control strategies can enhance operational resilience while supporting equitable participation of distributed resources in a multi-microgrid setup?*

Related to the challenges of cascading failure analysis in interdependent system, we seek to address the following questions:

Question 4: *How can cascading failure impacts be efficiently and scalably evaluated across interdependent system functionalities?*

Question 5: *How can higher-order relationships among system functionalities be identified to improve assessment of robustness to failures?*

1.3 Contributions

To address Research Question 1, Chapter 3 of this dissertation develops an optimal transport (OT)-based distributed generation (DG) planning framework. Strategic DER deployment plays a critical role in enhancing system resilience by reducing dependence on centralized infrastructure and enabling localized energy support during disruptions. This chapter first introduces an OT formulation for transforming the household energy burden distribution toward a fairer target based on inequity theory and subsequently develops advanced multi-objective OT-based planning methods. The major contributions of this chapter are summarized below.

- A novel optimal transport-based framework is developed to incorporate the statistical characteristics of socioeconomic metrics into solar PV siting and sizing problems in large-scale distribution systems. The proposed formulation enables target-oriented DER allocation while simultaneously accounting for network-level performance metrics such as active power losses.
- The Equal Distribution Equivalent (EDE) from inequality theory is adopted as a rep-

representative target for energy burden redistribution. An OT-based formulation is developed to compute the minimum-cost transformation from the existing energy burden distribution to a desired target distribution, enabling target-oriented PV deployment beyond conventional average or worst-case optimization approaches.

- A Bregman barycenter-based formulation is introduced to integrate OT-EDE-based energy burden distribution with optimal power flow (OPF) constraints. This formulation provides a mathematically principled mechanism for balancing community-focused objectives with network operational efficiency. Simulation results demonstrate reductions in high-percentile energy burdens and inequality, as quantified using the Gini coefficient, while maintaining competitive active power loss performance under both revenue-agnostic and revenue-neutral conditions.
- A constrained optimal transport (COT)-based planning framework is developed to systematically explore trade-offs between loss minimization and energy burden redistribution, enabling flexible multi-objective planning under operational constraints.
- Simulations on IEEE 37-bus test system and a 559-node distribution system validate the scalability and effectiveness of the proposed methods. The results demonstrate significant reductions in high-percentile energy burden and Gini coefficient while preserving low active power losses.

More details on OT-based DG planning are provided in Chapter 3 and in the associated publications listed below.

- P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Equity-driven Planning of Distributed Solar PV using Optimal Transport, 2025 IEEE Power & Energy Society General Meeting (PESGM), pp.1-5, IEEE, 2025.
- P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Optimal Transport Theory for Solar PV Planning: Balancing Community and Grid Priorities, (submitted to IEEE Transactions on Sustainable Energy).

- P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Constrained Optimal Transport Theory for Distributed Generation Planning, (submitted to IEEE Systems Journal).

To address Research Questions 2 and 3, Chapter 4 develops robust situational awareness and coordination control frameworks for distribution systems operating under limited observability and high penetration of distributed energy resources (DERs). Accurate situational awareness and coordinated control are critical for maintaining resilient system operation, enabling faster detection of disturbances and improved recovery under uncertain operating conditions. The key contributions of this chapter are summarized below.

- A Fréchet regression-based formulation is developed to infer the distribution network adjacency matrix from limited active and reactive power measurements, enabling robust phase identification under low-observability conditions. This formulation accounts for the non-Euclidean structure of distribution system data, improving reliability under measurement scarcity.
- A Fréchet regression-based distribution system state estimation (DSSE) framework is proposed to estimate probability distributions of voltage states from active and reactive power measurements. The proposed method incorporates network line admittances and power flow relationships to enforce physical consistency while improving robustness to noisy and incomplete measurements.
- A joint phase identification (PI) and DSSE framework is developed to simultaneously estimate network topology and system states under limited observability. The proposed method demonstrates strong robustness to measurement uncertainties and model inaccuracies, including incomplete observability and incorrect topology information, achieving improved phase identification accuracy and reduced state estimation errors on IEEE 37-bus and IEEE 123-bus test systems.
- A holistic operational framework for interconnected microgrids is developed to optimize power scheduling under both coordinated and non-coordinated operating conditions.

The framework explicitly incorporates community-level energy burden metrics into the optimization process, enabling community-aware and resilience-oriented operational planning.

- A Nash bargaining-based game-theoretic mechanism is proposed for energy trading among interconnected microgrids. This mechanism leverages supply-demand characteristics to achieve mutually beneficial operating strategies, improving resource utilization and operational flexibility under dynamic system conditions.
- A comprehensive DSO–DER coordination and control architecture is developed for congestion management under both minimal-DSO and total-DSO operational paradigms. The framework systematically contrasts limited-control and fully coordinated environments, identifying structural and informational requirements necessary for scalable distribution-level decision-making.
- The proposed coordination architectures are implemented and validated on the IEEE 123-bus test system. Simulation results demonstrate improved DER dispatch efficiency, enhanced voltage regulation, reduced curtailment requirements, and improved overall operational performance under high-DER-penetration conditions.

More details on Fréchet regression-based situational awareness and coordinated control strategies are provided in Chapter 4 and the associated publications listed below.

- P. Dhanapala, A. Sreejith, B. Natarajan, and W. Song, Robust Phase Identification and State Estimation in Unobservable Distribution Systems using Fréchet Regression, (manuscript submitted to IEEE Transactions on Power Systems).
- P. Dhanapala, A. Sreejith, B. Natarajan, and A. Pahwa, Game Theoretic Coordination of Microgrids with Equity Considerations, 2024 IEEE Kansas Power and Energy Conference (KPEC), Manhattan, KS, USA, 2024, pp. 1-6, doi: 10.1109/KPEC61529.2024.10676236.

- P. Dhanapala, K.K. Challa, K. Jhala, and B. Natarajan, Distribution System Congestion Management with Various DSO and Distributed Resource Coordination Architectures, (Accepted in 2026 IEEE Power & Energy Society General Meeting).
- A. Sreejith, P. Dhanapala, A. Melagoda, SA. Enderami, V. Aravinthan and A. Pahwa, ComPSim: A Community based Smart Grid Testbed for Holistic Resilience Analysis, 2023 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm), Glasgow, United Kingdom, 2023, pp. 1-6, doi: 10.1109/SmartGridComm57358.2023.10333936.
- A. Sreejith, P. Dhanapala, B. Natarajan, A. Pahwa, and J.S. Bergtold, Socioeconomic Vulnerability-Based Resilience Assessment Metric for Power Distribution Systems, IEEE Access, vol. 14, pp. 2647-2660, 2025.
- K. Jhala, P. Algikar, P. Dhanapala, and K.K. Challa, "Distribution System Congestion Management - A Survey of Reliable Integration for Aggregated Resources and Microgrids," IEEE Access (2026).
- R. Madbhavi, P. Dhanapala, B. Natarajan, A. Pahwa, et al., CISCs: A Socio-Technical Co-Simulator for Critical Infrastructure Interdependencies and Community Resilience, (submitted to IEEE Transactions on Systems, Man and Cybernetics)

To address Research Question 4, 5 develops an analytical hetero-functional graph theory (HFGT)-based framework to model and propagate cascading failure impacts across interdependent infrastructure systems. Efficient and scalable propagation of disruption impacts is essential for resilience assessment, particularly in large-scale systems where traditional simulation-based approaches become computationally prohibitive. The proposed framework explicitly models system functionalities and their interactions, enabling the capture of both primary and secondary impacts during infrastructure outages. The key contributions of this chapter are summarized below.

- A HFGT-based framework is introduced for comprehensive integrated contingency analysis involving interdependent power distribution, communication, and community

networks. This approach leverages the structural and computational advantages of HFGT to significantly reduce the computational burden associated with large-scale resiliency analysis, enabling faster and more scalable impact assessment of critical infrastructure systems during failures or outages.

- A hetero-functional graph theory (HFGT)-based modeling framework is developed for integrated contingency analysis of interdependent power distribution, communication, and community systems. By representing functional relationships rather than purely structural connectivity, the proposed framework enables more realistic modeling of cascading disruptions while significantly reducing the computational burden associated with large-scale resilience assessment.
- A sensitivity-inspired analytical formulation is introduced to efficiently capture and propagate the effects of component failures across interconnected functionalities. The proposed formulation enables systematic quantification of failure impacts both within individual infrastructure layers and across interdependent systems, while maintaining modularity and adaptability for different infrastructure configurations.
- The proposed HFGT-based framework is validated using two representative case studies involving communication network failures and power line outages. The results demonstrate that the proposed approach accurately captures cascading effects and community-level impacts while achieving consistency with conventional simulation-based methods. Notably, the analytical formulation reduces simulation time by approximately 83%, highlighting its scalability and practical applicability for large-scale resilience analysis.

More details on HFGT-based contingency modeling and analytical sensitivity propagation are provided in Chapter 5 and the associated publications listed below.

- P. Dhanapala, A. Sreejith, B. Natarajan and A. Pahwa, Hetero-functional Graph Theoretic Framework for Holistic Resiliency Analysis of Power Systems, IEEE Transactions on Systems, Man and Cybernetics: Systems, pp. 1-6, 2026.

To address Research Question 5, Chapter 6 develops a higher-order topological analyses framework for identifying complex functional dependencies in interdependent infrastructure systems. The key contributions of this chapter are summarized below.

- The hetero-functional graph (HFG) framework is extended to encode multi-functional interactions using simplicial complexes, enabling systematic representation of higher-order dependencies beyond conventional pairwise connectivity models.
- Persistent homology and Hodge Laplacian-based spectral analysis techniques are integrated to identify and quantify cyclical structures within HFGs. These cycles represent higher-order functional dependencies that capture redundancy, feedback mechanisms, and latent structural relationships influencing system robustness.
- A percolation-based robustness analysis framework is developed to evaluate both connectivity (pairwise relationships) and higher-order topological features under random and targeted attacks. Results demonstrate that cycle-based topological metrics provide earlier and more informative indicators of system vulnerability compared to conventional graph-theoretic measures, enabling proactive resilience enhancement strategies.
- The extracted higher-order topological indicators are utilized to support resilience-oriented decision-making by identifying critical functional cycles that can be prioritized for targeted infrastructure hardening and restoration following disruptions.

More details on higher-order dependency modeling and topological resilience analysis are provided in Chapter 6 and the associated publications listed below.

- P. Dhanapala and B. Natarajan, Robustness Assessment of Interdependent Infrastructure Systems: A Topological Perspective, (submitted to IEEE Transactions on Systems, Man and Cybernetics).

1.4 Organization of the Dissertation

Figure 1.2 outlines the structure of this dissertation. Chapter 2 provides a literature review of related work. Chapter 3 develops OT-based planning methods that can effectively address the community and utility concerns in DER proliferation. Chapter 4 provides the necessary situation awareness and coordination control strategies that are robust and community-centric in the realm of high penetration of DERs. Chapter 5 advances the HGT modeling to capture the primary and secondary impacts for any disruptions in power systems to offer a more comprehensive and computationally efficient resiliency analysis. Chapter 6 uncovers the higher-order interactions in HFGs that are concerning to the proper function of interdependent infrastructure systems. Finally, Chapter 7 presents key conclusions of this dissertation and suggests avenues for future research.

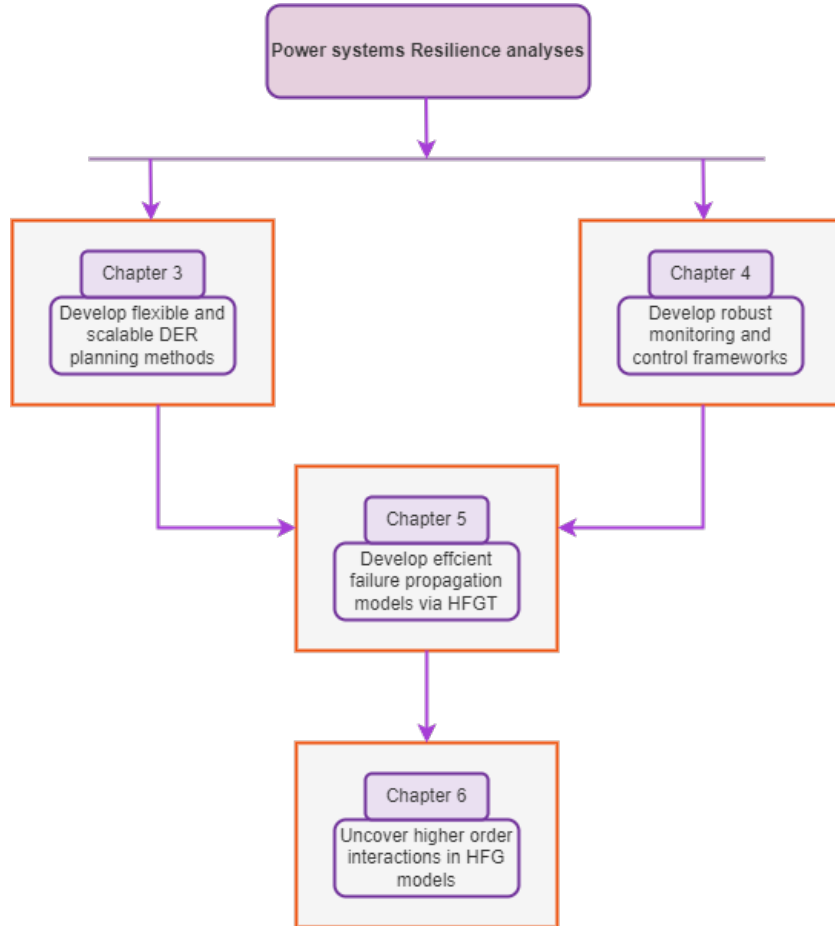


Figure 1.2: Outline of the dissertation.

Chapter 2

Literature review

This chapter highlights the gaps in the existing works and provides a literature review on the topics covered in this dissertation.

2.1 Distributed Energy Resources Planning

The power distribution system is undergoing a significant transformation with the increasing integration of distributed energy resources (DERs). These resources, such as solar photovoltaic (PV) systems, wind generation, energy storage, and electric vehicle charging infrastructure, are commonly planned based on optimal power flow (OPF)-based methodologies with the goal of minimizing losses, enhancing voltage profiles, or reducing energy costs [4–7]. As DER penetration grows, it becomes increasingly important to consider how these resources impact the communities served by the distribution utility. Federal, state, and local policy efforts have increasingly emphasized addressing unfair burdens within the built environment [8], prompting greater attention to community-focused planning and decision making in the energy-sector. In the context of distribution systems, this may include fair allocation of the benefits and burdens associated with energy production, delivery, and consumption infrastructure. Consequently, planners and operators must move beyond conventional efficiency-driven approaches and incorporate community-relevant metrics into future

investment decisions [9]. However, community-centric grid planning remains a relatively new area, and there is limited guidance on how to integrate fairness, or other socio-economic considerations into DER siting and sizing. Existing metrics, such as energy burden [10], value of lost load [11], and social infrastructure service burden [12], offer consumer-focused characterizations of accessibility and service quality, but they have not been systematically embedded within distribution system planning frameworks. While classical OPF-based planning methods effectively address operational efficiency, multiobjective variation of these methods primarily consider community impacts using aggregate performance metrics [13]. As a result, heterogeneous impacts of DER investments on consumers are not well captured. To address this limitation, this paper for the first time integrates a distributional perspective for capturing community impact indicators into the DER siting and sizing problem, enabling a more holistic planning framework. Specifically, we develop planning frameworks that incorporate community-level metrics into PV planning, with a focus on improving the distribution of household energy burdens. By explicitly considering the distribution of energy burden across the entire community, rather than relying solely on averages or extrema, the proposed method is first to support a holistic allocation of DER benefits while maintaining network efficiency.

Community-centric considerations in distribution system planning have received increasing attention among researchers and utilities. Recently, the authors in [14] highlight the need to account for community impacts to ensure that planning and operational decisions equitably benefit the populations served by distribution networks. Similarly, [15] introduces a simulation framework that integrates community characteristics into smart grid infrastructure planning, enabling utilities to assess how disruptions impact different communities and to prioritize load restoration accordingly. Motivated by emerging policies aimed at addressing disparities in the built environment, several utilities have begun setting targets related to fair service provision [16]. These efforts, supported by technological advancements and policy guidance, underscore the growing emphasis on ensuring clean, resilient, and fair power delivery while maintaining traditional goals of reliability and affordability. Despite this emerging focus, the methods for translating community objectives into actionable plan-

ning tools remain underdeveloped. Achieving fair outcomes through distribution system planning is challenging, particularly because high-level policy directives must be converted into quantitative, system-level criteria. For example, although residential rooftop PV potential represents approximately 65% of the U.S. national rooftop capacity [14], constraints in distribution networks often restrict access to DER adoption [17]. Energy affordability also varies significantly: approximately 16% of U.S. households experience energy burdens exceeding 6% of their income, disproportionately affecting minority communities [18].

Existing community-focused analyses, such as spatial studies, statistical regression, and simulation-based assessments, primarily reveal disparities in outage extent, duration, and resilience [19]. While informative, these approaches typically quantify impacts rather than guide DER planning. The purpose of the current community-focused studies is to quantitatively demonstrate the variability in power system resilience. Specifically, power outages often occur due to extreme events. Satellite images, outage complaint calls, and historical outage data reveal the disparities in the extent and duration of power outages experienced by different communities. Some recent works have begun incorporating fairness considerations into planning studies. For instance, [20] integrates energy access and outage risk into hosting capacity analysis by distinguishing between disadvantaged and non-disadvantaged communities, revealing poorer service quality in disadvantaged areas due to aging infrastructure and historically unfair planning decisions. The methodology in [21] assigns PV capacity and installation probabilities to customers before impact assessment, offering a probabilistic fairness-aware view but without optimizing DER siting and sizing. Optimization-based planning methods remain widely used for DER integration, yet most ignore community-level objectives [22]. The primary challenge lies in operationalizing community metrics, such as energy burden, into quantitative criteria suitable for planning and operations [23]. In [24], the authors apply a game-theoretic framework to reduce community energy burdens through coordinated microgrids, illustrating the potential of community solar. However, optimal PV siting and sizing is not addressed.

In our previous work [25], OT theory was applied to the targeted siting and sizing of solar PV, demonstrating how distributional perspectives can guide DER planning. However, that

work did not incorporate network power losses and therefore did not fully capture the multi-objective nature of distributed generation planning. Given that DG planning inherently requires balancing both network efficiency and consumer energy costs, there remains a need for rigorous, distribution-aware frameworks that translate community considerations into actionable planning decisions. To address this gap, the present work introduces a comprehensive OT-based planning approach that explicitly incorporates distribution of community-centric metrics (e.g., household energy burden) and offers a flexible mechanism for shaping the distribution of the community indicator toward any desired target, including, but not limited to, inequality/fairness focused targets such as the equal distribution equivalent (EDE).

2.2 Distribution System State Estimation

Advanced developments in modern electric power distribution systems have significantly increased the complexity and variability of distribution network operations. While transmission networks are routinely monitored, distribution systems typically operate with limited sensing infrastructure and incomplete network information. Consequently, maintaining accurate situational awareness of system operating conditions remains a fundamental challenge for utilities [26, 27].

Distribution system state estimation (DSSE) plays a vital role in revealing system operating conditions by estimating key electrical variables, such as bus voltages, line currents, and active and reactive power flows, from limited measurements [28]. However, many existing DSSE approaches assume that the underlying network topology is known a priori. In practice, this assumption is often violated due to inaccurate or unavailable topology information, due to frequent and unreported changes arising from network reconfiguration, maintenance activities, repairs, and load balancing operations [29]. Topology inaccuracies can lead to significant state estimation errors, potentially resulting in unreliable monitoring and control actions. Furthermore, the time-varying nature of distribution network configurations necessitates the development of integrated approaches capable of jointly estimating network topology (including phase connectivity) and system states under measurement un-

certainty. In addition, several conventional methods implicitly treat power system variables within a Euclidean framework. While quantities such as active and reactive power injections and voltage magnitudes can be naturally represented as Euclidean vectors, other important system descriptors, including voltage correlation matrices, covariance representations of system states, and graph-based network adjacency structures, inherently reside in structured non-Euclidean spaces. Ignoring this intrinsic geometry may distort physical relationships and degrade estimation performance, particularly under low-observability conditions. Motivated by these challenges, this paper proposes a scalable joint phase identification and state estimation framework that explicitly accounts for measurement uncertainties while incorporating non-Euclidean representations of network structure, making it suitable for real-time monitoring and control of large-scale distribution systems.

2.3 Phase Identification

Phase connectivity information is necessary to accurately determine the network topology and utilize it for the DSSE. In a power distribution system, node voltages and branch currents are generally considered to be the system states [30]. A complete survey of DSSE with its associated challenges is presented in [31]. Due to limited availability of measurements in distribution system, weighted least square (WLS) methods use pseudo-measurements and suffer from low accuracy and high parameter sensitivity. Recently, sparsity-based approaches, namely matrix completion (MC) and compressive sensing (CS), have shown better performance in DSSE applications. These methods take advantage of the underlying smoothness of the measurement data [32, 33]. However, these methods are computationally expensive and therefore not scalable for real-time state estimation of large distribution networks. Moreover, these methods assume a prior distribution for the measurement errors to quantify estimation uncertainty.

Complementary to state estimation, accurate phase identification (PI) is essential for reliable distribution system operation. Existing phase-identification methods can be classified into several categories. Mainly, these approaches employ additional hardware and available

measurements for the phase identification. The hardware-based techniques have been most widely used for phase identification. In the signal injection approach, a signal is injected into a particular phase at the substation, and a device installed at each customer meter is used to detect the injected signal [34, 35]. This process is performed separately for each phase and each customer. The primary advantage of hardware-based methods is their high accuracy. In particular, the signal injection and PMU-based approaches are well-established and provide reliable phase identification results. However, these methods require specialized hardware devices, leading to significantly high costs. In addition, considerable time and manpower are needed for utility field personnel to collect measurements at individual metering devices. Despite the substantial investment required, these approaches may remain the only viable option when AMI data are not available. Measurement-based methods can be broadly classified into two categories: power-based approaches and voltage-based approaches. Many data-driven approaches use a full set of measurements, including active power p , reactive power q , voltage magnitude v , and voltage angle θ information to identify network topology [36]. Smart meters measure and record p, q, v under the coordinated control of a central system, whereas μ PMUs measure the voltage phase angle θ at buses using GPS-synchronized signals [37]. However, μ PMUs require high-frequency synchronized measurements and communication signals. These measurements are then transmitted to data centers through either wireless or optical fiber communication networks. Several methods utilize the real power AMI time-series data for phase identification. These approaches exploit the fact that the sum of all customer loads connected to a given phase should equal the corresponding power measurement at the substation, accounting for line losses. In [38], the authors adopt this load-summing principle and formulate an optimization problem to determine the customer phase assignments. In [39], principal component analysis (PCA) and graph-theoretic techniques are incorporated to leverage the load-summing concept and construct a tree structure connecting individual customers to the substation. In [40], the power flow equations are solved, and a Tabu search is employed over all possible phase assignments to identify the configuration that best matches the observed power flows. However, this approach requires complete information necessary to solve the power flow equations. A

key limitation of load-summing-based approaches is their performance degradation when the system is poorly observable. In contrast, voltage-based approaches rely on voltage measurements, where PMU devices measure the voltage phase angle at individual customer locations and reference these measurements to the voltage phase angles at the substation. [41, 42]. Apart from this work, many voltage-based measurement methods leverage the fact that customers connected to the same phase exhibit voltage time series that are more strongly correlated than those of customers connected to different phases. The methods in [43, 44] uses correlations between customer voltage time series, applies the Fisher Z transform to correlation coefficients, and then performs statistical tests to decide whether two customer nodes are connected on the same phase. A constrained multi-tree algorithm is proposed for the phase identification problem [45]. This approach constructs a tree structure for each phase, with the substation serving as the root node, thereby exploiting the radial topology of the distribution system. Correlation coefficients are computed between the remaining unassigned customers and the existing nodes in each tree. At each step, the customer exhibiting the highest correlation is iteratively added to the corresponding tree until all customers are assigned. Spectral clustering-based formulation using voltage time-series data are proposed in [46, 47]. The authors in [46] employ an ensemble strategy in which spectral clustering is performed over multiple data windows to form a co-association matrix, which is then used as input to a subsequent spectral clustering step for phase assignment. [47] presents a graph clustering method for phase identification in an unobservable distribution system. The authors used an alternating minimization strategy for joint phase identification and state estimation. The aforementioned approaches are sensitive to measurement errors and can be computationally expensive for real-time implementation in large-scale distribution systems. In addition, many existing methods assume that power system data can be represented in a Euclidean vector space, where standard linear operations and distance metrics are directly applicable. However, several important power system quantities, such as correlation matrices, network adjacency structures, and covariance representations of system states, inherently lie on nonlinear manifolds or structured spaces [48, 49]. Treating such data as Euclidean may distort the underlying geometric relationships, leading to biased estimates,

loss of physical interpretability, and degraded algorithmic performance, particularly under low-observability and noisy measurement conditions. Consequently, there is a growing need for methodologies that explicitly account for the non-Euclidean nature of power system data, enabling more accurate modeling of network structures and improved robustness in topology identification and state estimation tasks.

Despite significant progress, existing phase identification and distribution system state estimation methods face several critical limitations. Hardware-based approaches, while accurate, incur high deployment and operational costs. Measurement-driven methods, including load-summing and voltage-correlation techniques, often suffer from degraded performance under low observability and are sensitive to measurement noise. Sparsity-based DSSE approaches, such as matrix completion and compressive sensing, are computationally intensive and may not scale well for real-time applications. Moreover, many existing methods rely on Euclidean representations of inherently structured data, such as adjacency matrices and covariance operators, which can distort the underlying system geometry and lead to suboptimal performance. These limitations motivate the need for a unified framework that is robust to low observability, computationally efficient, and capable of capturing the non-Euclidean structure of power system data.

2.4 Distribution System Operation Frameworks

2.4.1 Microgrid Energy Management

Microgrids (MGs) are becoming a key building block of smart power distribution systems. Their adoption is accelerated due to distributed energy resources (DER), demand response programs, and flexibility to be grid-connected or islanded [50]. MGs can enhance reliability and resiliency during disruptions, faults, and voltage collapse [51]. Microgrid-based resilience enhancement methods and various existing MGs to deal with tsunamis, wildfires, winter storms, hurricanes, and earthquakes show the importance of microgrid (MG) as a way forward for a reliable and efficient power delivery system [52], [53] and [54].

Most of the existing works that discuss the adoption of MG as a resilient and beneficial approach for future power grids, do not consider the impact of its operation on the communities that MG serves. MGs may serve diverse communities with varying socio-economic status. MG operation should enhance the operational resiliency of these communities while providing them affordable electricity. Optimizing MG operations to reduce energy burden is especially critical for vulnerable or disadvantaged communities. Hence, holistic management of MGs considering the energy burden of the communities served can help promote energy equity.

Increasing solar PV generation assets at the distribution system is discussed and the impact on disadvantaged communities is highlighted in [20]. A community can be considered disadvantageous based on the income level, and their location (e.g., being situated at the end of the feeder in a distribution system experiencing significantly more voltage issues relative to other customers upstream). In [55], [56] the authors consider energy equity in transmission and distribution system resiliency planning. It is equally important to consider energy equity in operations and management processes. Single MG energy management has been addressed by many authors in [57], [58] and [59], these works consider various demand-side management approaches like demand response, peak shaving, power smoothing, siting and sizing etc. In interconnected MGs, operators try to maximize revenues through their scheduling strategies and joint energy trading. To support the frequency at the grid level, the MGs are coordinated and participate in the demand response program [60]. By participating in the demand response programs, the MGs achieve overall peak reduction and cost benefits for the customers. A non-cooperative game theoretic framework is proposed in [61] to analyze the participation of MGs with renewable energy generation in the energy markets. In [62], coordination among MGs is formulated by bargaining principles, where the neighboring MGs share their information locally and decide the electricity prices while trading. However, the energy burden of the communities served by the MGs is not considered. Occasionally, when participating in energy markets, MGs may deviate from committed power schedules, resulting in pricing penalties. If these penalties are passed on to the customers, they can increase the energy burden of the communities. To mitigate this added energy burden and maintain

energy equity among the communities served by the MGs, coordination among the MGs could be a potential solution and of great value. However, DSO intervention is required to facilitate the necessary services for the MGs to operate in coordination [63].

2.4.2 Transmission and Distribution System Coordination

Power distribution systems are witnessing a rapid proliferation of behind-the-meter (BTM) flexible loads, distributed generations (DGs), battery energy storage systems, and demand response resources. These resources, if managed properly, can provide market and grid services to transmission and distribution systems. This transformation is driven by the increasing need for grid flexibility, technological advancements, and evolving regulatory frameworks. In particular, FERC Order No. 2222 [64] allows aggregated resources located in distribution system (DS) with a minimum capacity of 100 kW to participate in wholesale markets[65]. This regulatory development has opened new revenue streams for consumers and aggregators, potentially making energy more affordable. However, while such bulk market participation frameworks enhance system flexibility and efficiency at the transmission level, they may simultaneously introduce operational challenges at the distribution level. One of the most significant challenges is distribution system congestion, which arises from the fact that these resources are dispatched over the DS and causes uncertain bidirectional power flows due to the stochastic nature of DERs. Congestion management for the distribution system involves strategies to balance supply and demand across the network, ensuring that the power flow remains within the physical constraints of the grid.

These operational complexities necessitate new coordination mechanisms between transmission system operators (TSOs), distribution system operators (DSOs), aggregators, and consumers to ensure reliable and efficient operation across grid layers. Consequently, the role of the utilities have become increasingly critical as a network operator responsible for congestion management, DER coordination, and seamless TSO–DSO interaction[66]. Effective congestion management in distribution networks requires proactive rather than reactive operational strategies. Day-ahead operational planning plays a pivotal role in anticipat-

ing potential network congestion and allocating DERs in a manner that ensures both grid reliability and economic efficiency [67]. Managing network congestion while maximizing aggregator revenues is typically achieved by solving the Optimal Power Flow (OPF) problem, which is inherently challenging due to its nonlinear and nonconvex characteristics [68]. Several approaches, ranging from conventional parametric formulations to modern data-driven methods, have been proposed to address OPF in distribution systems [68]. While OPF itself is a computationally intensive task, incorporating TSO–DSO interactions further increases its complexity. This hierarchical coordination problem is commonly formulated as a bi-level optimization, where the TSO defines a feasible operating region that implicitly constrains the DSO-level OPF. In [69], this complex bi-level problem was addressed using a decomposition-based technique with a convex AC-OPF formulation to guarantee convergence. Similarly, [70], employs the Alternating Direction Method of Multipliers to solve the TSO–DSO coordination problem based on iterative information exchange between the two entities. In [71], the bi-level optimization is reformulated as a Mathematical Program with Complementarity Constraints and solved using an interior–exterior penalty approach.

As the penetration of DERs continues to rise, solving this hierarchical coordination problem efficiently has motivated the development of both data-driven [72] and analytical [73] methods for OPF. Within the TSO–DSO coordination context, capturing real and reactive power flexibility while achieving economic objectives is critical. For example, [74] quantifies PQV-based distribution system flexibility using a two-stage sampling strategy combining bounding-box projection and Fibonacci direction techniques to characterize the feasible operating region. Other studies have focused on specific coordination goals: [75] models reactive power capabilities as probabilistic capability charts using probabilistic curve representations, while [76] proposes analytical methods to quantify probabilistic line loading, a key metric in volatile market participation.

However, most existing OPF-based coordination frameworks implicitly assume traditional roles and responsibilities for the TSO and DSO. With the rapid growth of DERs and aggregators, new operational challenges have emerged, necessitating redefinition of the DSO construct. The DSO, conceived not merely as an organizational entity but as a functional

and market-oriented framework, represents a paradigm shift toward actively managed distribution networks [77]. Modern DSOs are expected to manage bidirectional power flows, integrate distributed flexibility resources, and operate the transmission–distribution (T–D) interface in near real time. By integrating planning, market, and operational functions, DSOs can unlock the value of flexibility services while deferring costly infrastructure upgrades [78]. Therefore, this work analyzes congestion management under various DSO architectures and evaluates their performance in terms of operational cost and DER utilization. Specifically, we model and assess congestion management in two representative DSO architectures—the Minimal DSO and the Total DSO.

2.5 Modeling Interdependent Systems

The energy infrastructure is undergoing significant transformation with the integration of distributed energy resources (DERs), advanced control systems, and analytics supported by communication networks [79],[80], [81]. While these advances have improved operational efficiency, they have also exposed the power system to cyber threats and multiple failure modes. Extreme weather events and cyberattacks, considered high-impact, low-frequency events, can significantly disrupt power system operations. In August 2017, Hurricane Harvey struck the Texas coast in the USA, resulting in power outages for more than 300,000 customers, as reported by the U.S. Department of Energy [82]. The power distribution network also faces new challenges due to vulnerabilities in communication networks. Many authors have examined how these vulnerabilities can lead to cascading failures across both transmission and distribution systems [83], [84]. This phenomenon not only affects power systems but also impacts other interdependent systems. Despite the increasing interplay of power, communication, and other infrastructure networks [85], [86], only a few papers have focused on holistic resiliency studies on interdependent network. There is a need to design and develop modular, efficient, and computationally inexpensive models that can effectively assess the impact of failures/cyber-attacks on complex interdependent physical, social, and economic infrastructures.

Several approaches have been developed to analyze the impact of component failures or outages in interdependent networks. In [87], a simulation-based approach is used to generate what-if scenarios, focusing on the reliability assessment of multiple hierarchical subsystems within interdependent networks. Similarly, [88] employs a Bayesian networks approach to evaluate the resilience of system-of-systems design alternatives. This approach accounts for operational failures and introduces a conditional resilience metric that quantifies the contribution of each constituent system to overall resilience of the network. In [89], network properties are used to quantify the robustness of interdependent networks against failures. Specifically, it looks at the degree distribution in power and communication interdependent networks to assess resiliency; a broader degree distribution increases the vulnerability of interdependent networks to random failures. The paper also highlights how failures in power stations can trigger cascading effects, disrupting internet communication networks, which in turn lead to further breakdowns in the power system. In all these studies, interdependent networks are modeled as a multilayer graph. However, multilayer graph models have been shown to exhibit ontological limitations [90], such as ambiguities in defining layer boundaries and entity identities across layers. The modeling constraints and challenges associated with multi-layer graphs for system-of-systems analysis are examined in [91]. The study highlights key limitations such as the complexity of accurately capturing inter-dependencies across layers, difficulties in defining consistent node and edge semantics across diverse subsystems. Furthermore, as the number of layers and nodes increases, computational scalability becomes a significant challenge, limiting the applicability of such models to large-scale interdependent networks. Additionally, in many of these studies, the impact of component failures in one layer on another is modeled sequentially, making it more complex and less efficient than modeling the interdependent networks as a single-layer graph.

Hetero-functional graph theory (HFGT) offers a robust platform for modeling complex interdependent systems. In [92], the authors addressed and proved the ontological constraints that impede the applicability of multi-layer networks, thereby positioning HFGT as a viable approach to mathematically model systems-of-systems with arbitrary structure and function. As demonstrated by [93], multilayer networks face inherent ontological limitations

rooted in their metadata definitions. Their tensor-based formulation shows that, without a fundamental redesign of this metadata structure, these models are theoretically incapable of representing systems-of-systems with full functional and structural generality. A key aspect of the HFGT framework is the concept of structural degrees of freedom, which represent the combinations of system processes and resources. These degrees of freedom can be analyzed individually to assess system capabilities or sequentially to describe the behavior of the system under various conditions. The underlying modeling philosophy in conventional graph modeling, as mentioned earlier, assumes that nodes represent resources and edges denote their physical connections. While conventional graph representations of interdependent infrastructure networks reveal topological information and fundamental relationships among system components, they fail to provide detailed insights into system processes and resource interdependencies in complex systems. Moreover, multilayer graphs modeled using conventional approaches may not adequately represent the dynamics of the system of systems interactions. To address these limitations, the authors in [94] introduce an HFGT-based framework to rigorously model interdependent infrastructure networks, overcoming key challenges associated with traditional multilayer graph theory. This modeling approach is extended to model smart cities in [95], where the authors model integrated electricity, water, and transportation systems as a hetero-functional graph within a single-layer structure, laying a foundation for the engineering of interconnected urban infrastructure systems. Building on this, the authors in [96] assess the robustness of interdependent networks using HFGT-based models for complete/partial and random/targeted attacks. The supporting infrastructure considered in [96] includes electricity, water, heating, natural gas, and road transportation. In [97], HFGT modeling is adapted for coordination and control applications in future power systems, highlighting its relevance to grid modernization efforts. Moreover, the versatility of HFGT has led to its adoption across multiple domains, including transportation [98], healthcare [99], and production systems [100]. Focusing on the electric power system, the authors in [101], [102], and [103] comprehensively discuss resilience analysis using HFGT-based models. This paper further explores the potential of HFGT framework by incorporating analytical methods inspired by sensitivity-based techniques to capture the

propagation of impacts across nodes within the network. This enhancement enables not only the modeling of interdependencies among critical infrastructure systems but also the interactions among community services, thereby supporting a more holistic and integrated impact analysis.

2.6 Robustness Analysis of Interdependent Systems

Modern infrastructure systems, including electric power, water, transportation, and community services, are increasingly interconnected and interdependent. While such interdependencies enhance operational efficiency and resource utilization, they also introduce significant vulnerabilities. In particular, disruptions in one infrastructure can propagate across others, resulting in cascading failures and widespread service disruptions [104, 105]. Recent extreme events and cyber-physical attacks have further highlighted the need for advanced modeling and analysis tools capable of capturing the complex interactions underlying such systems [106, 107].

A variety of approaches have been developed to model and evaluate the robustness of interdependent infrastructure systems to natural hazard and cyber attack-induced failures, including methods based on hierarchical graph representations [108], dynamic network flow model [109], agent-based modeling [110], among others. Traditional graph-theoretic approaches model infrastructure systems as networks of nodes and edges, where nodes represent physical components and edges denote structural connectivity. While these models are effective for analyzing pairwise interactions, they inherently assume that system behavior is governed solely by direct connections. However, real-world infrastructure systems exhibit complex functional dependencies that extend beyond pairwise relationships. For example, the operation of a water treatment facility depends not only on electrical supply but also on transportation access, communication systems, and control processes. Such multi-domain interactions introduce higher-order dependencies that are not adequately captured by conventional graph representations. Heterofunctional graphs (HFGs) have been recently proposed as an approach to capture complex functional dependencies across multiple infras-

structures. In this paper, we posit that topological metrics are more effective in characterizing the robustness of complex interconnected infrastructure networks represented as HFGs. The topological network analyses presented in the paper enable the systematic characterization of higher-order functional dependencies, thereby providing a more comprehensive understanding of system behavior under disruptions.

Graph theory has served as a foundational framework for analyzing networked systems, including power grids [111], transportation systems [112], water networks [113], supply chains [114], and healthcare systems [115]. In conventional graph-based representations, nodes correspond to infrastructure or service components, while edges represent the physical interconnections among them. To extend these approaches for modeling interdependent infrastructures, the network science community introduced the concept of multilayer networks. This framework enables the representation and analysis of “systems of systems” that span multiple interconnected domains. Several prior studies have made important contributions in this area [116–118]. In [116] and [117], novel graph-based models are proposed to capture complex interactions among infrastructures and to extract interdependencies among building systems under external shocks and stresses. In [118], a graph-based modeling approach is developed to analyze topological characteristics of coupled power-law networks for robustness assessment, while accounting for different types of interdependencies. However, these approaches largely rely on conventional modeling paradigms, wherein nodes represent resources and edges denote physical connections. Although such representations effectively capture topological properties and fundamental relationships among system components, multi-layer networks have ontological constraints and are limited in their ability to characterize system-level processes and the intricate functional dependencies that govern the operation of complex interdependent infrastructures [93]. In other words, questions such as “What are the dependencies of a particular functionality on others?”, “What processes are carried out within nodes and edges?”, and “How critical are nodes and edges to overall system performance?” cannot be adequately addressed using conventional graph representations [91]. To overcome these limitations, hetero-functional graph (HFG) theory has emerged as a powerful framework for modeling interdependent infrastructure systems [119–121]. In HFGs,

nodes represent functionalities rather than merely physical components, while edges encode functional dependencies among them. This abstraction enables a unified representation of both intra- and inter-domain interactions. Most existing HFG-based robustness analyses rely on conventional graph-theoretic metrics that primarily capture node-level connectivity. In particular, spectral analysis defined over node-level interactions provides valuable insights into global connectivity, diffusion dynamics, and structural cohesion of the network [122]. The associated eigenvalues and eigenvectors encode critical information regarding robustness, including connectivity strength, community structure, and vulnerability to node or edge removals. For instance, in [123], robustness analysis of an interdependent urban infrastructure network is performed using a weighted HFG framework under various contingency scenarios. The study evaluates conventional connectivity-based metrics, including the number of connected components, largest connected component, flow robustness, and service robustness. However, the analyses in [122] and [123] remain fundamentally limited to pairwise interactions and do not explicitly capture higher-order dependencies arising from multi-functional interactions across interconnected infrastructures.

To move beyond pairwise connectivity in HFGs, additional analytical tools are required to capture higher-order interactions among functionalities [124]. In this context, topological data analysis (TDA) has emerged as a powerful framework for systematically extracting structural information across multiple levels of abstraction. Rooted in algebraic topology, TDA provides a rigorous mathematical foundation for analyzing complex systems with higher-order relationships [125]. A key construct in TDA is the simplicial complex, which generalizes graph representations by incorporating higher-dimensional building blocks, known as simplices, to model multi-way interactions [126]. In particular, simplicial complex representation easily encodes higher-order interactions by the inclusion of 2-simplices (faces consisting of 3 nodes) and 3-simplices (volumes consisting of 4 nodes) to graphs. Although such structures arise naturally in interdependent infrastructure systems, their identification and characterization are nontrivial using conventional graph-based methods. Traditional approaches often rely on combinatorial or search-based techniques, such as depth-first search, which are computationally intensive and do not readily generalize to higher dimensions [127].

Persistent homology, a fundamental tool in TDA, addresses this challenge by analyzing the evolution of topological features across multiple scales through algebraic manipulation of boundary operators [128]. Building upon this foundation, the Hodge Laplacian provides a spectral framework for identifying and characterizing higher-order structures in simplicial complexes. In particular, the eigenvectors associated with the zero eigenvalues of the Hodge Laplacian correspond to harmonic components that capture independent cycles in the network. By leveraging this property, the analyses presented in this paper enables systematic extraction and quantification of higher-order functional dependencies in HFGs, facilitating a deeper understanding of interdependent infrastructure behavior beyond pairwise interactions.

Chapter 3

Optimal Transport-Based Planning of Distributed Energy Resources

The increasing penetration of distributed energy resources (DERs), including rooftop solar photovoltaic systems, battery energy storage systems, and flexible demand technologies, has fundamentally transformed the planning and operation of modern power distribution systems. Beyond improving operational efficiency, DERs provide new opportunities to enhance system resilience by enabling localized generation support, reducing dependency on centralized infrastructure, and supporting rapid recovery during disturbance events. Strategic planning of DER placement and capacity therefore plays a critical role in shaping the resilience characteristics of distribution networks, particularly in reducing service disruptions and improving recovery performance under adverse operating conditions.

3.1 Technical Background

Conventional DER planning strategies are typically formulated using optimal power flow (OPF)-based approaches that optimize aggregate technical objectives such as loss minimization, voltage regulation, and operational cost reduction. While these methods have proven effective in improving steady-state performance, they often rely on deterministic metrics

that do not fully capture the variability and heterogeneity present across different regions and communities. As a result, such approaches may produce solutions that perform well on average but exhibit uneven performance across locations, potentially increasing vulnerability in certain areas and amplifying the depth of service degradation during disruptive events.

3.1.1 OPF-Based DG planning

Consider a radial distribution system comprising n load nodes, where each load may represent an individual household, an aggregation of households, or a commercial or industrial consumer, depending on the desired modeling granularity. We use energy burden as a representative community-oriented metric to quantify disparities in access to affordable electricity services. Let I_i denote the monthly income of the i^{th} household, C denote the cost of energy (¢/kWh), and P_i^g denote the total monthly electricity consumption drawn from the grid. Energy burden is a commonly used metric to quantify electricity-related inequities in access to energy services within a community [129]. The Energy burden (as it relates to electricity) E_i for the i^{th} consumer is defined as the ratio of electricity cost/bill to the households' income, i.e., $E_i = \frac{CP_i^g}{I_i}$. A higher energy burden signifies that a larger proportion of income is spent on energy, posing significant challenges for low-income households. The deployment of distributed solar PV can alleviate this burden by offsetting a portion of grid-supplied electricity with locally generated energy, thereby reducing net energy consumption from the grid and associated costs.

Here, we discuss an OPF-based method for optimal solar PV siting and sizing. This approach aims to determine the best locations and capacities of solar PV units in a distribution system to minimize power losses while ensuring system stability and operational constraints. The OPF-based approach can be mathematically formulated as an optimization problem corresponding to,

$$\min_{P_i^{pv}} C \left(\sum_{i=1}^n (P_i^g - P_i^{pv}) + \sum_{i=1, i \neq j}^n P_{ij}^{loss} \right) \quad (3.1)$$

subject to

$$P_i^g + \beta_i P_i^{pv} - P_i^l = V_i \sum_{j=1}^n V_j Y_{i,j} \cos(-\theta_{i,j} + \delta_i - \delta_j) \quad (3.2)$$

$$Q_i^g + \beta_i Q_i^{pv} - Q_i^l = V_i \sum_{j=1}^n V_j Y_{i,j} \sin(-\theta_{i,j} + \delta_i - \delta_j) \quad (3.3)$$

$$\sum_{i=1}^m P_i^{pv} = \eta \%_0 \sum_{i=1}^l P_i^l \quad (3.4)$$

$$I_{ij} = Y_{i,j}(V_i - V_j) \quad (3.5)$$

$$P_{ij}^{loss} = I_{ij}^2 R_{i,j} \quad (3.6)$$

$$V^{\min} \leq V_i \leq V^{\max} \quad (3.7)$$

$$0 \leq P_i^{pv} \leq P_{\max}^l \quad (3.8)$$

$$0 \leq \sum_i^n \beta_i \leq N^{pv} \quad (3.9)$$

Here, P_i^g , Q_i^g , and P_{ij}^{loss} are the active, reactive power from the grid and active power loss across the branches. (3.2) and (3.3) are the power flow equations with β_i being a binary decision variable to choose location for solar PV. V_i is i^{th} node voltage; $I_{i,j}$ is the branch current between i and j ; $V_{\min}$ and $V_{\max}$ are the allowable limits for node voltages; $Y_{i,j}$ and $R_{i,j}$ are the branch admittance and resistance between i^{th} node and j^{th} node. The above formulation is a non-linear mixed integer optimization problem, and [130] elaborates on various methods to solve such problems. Broadly, these methods include MILP and Monte Carlo methods, heuristic algorithms such as genetic algorithms, and other hybrid approaches.

The OPF-based solar PV planning problem is formulated using the current-voltage (IV) representation, which provides a robust modeling of voltage and current variables suitable for unbalanced distribution networks. The optimization is carried out using the PowerModels-Distribution (PMD) package, an open-source Julia-based framework for scalable distribution-system optimization [131]. From this formulation, we obtain the optimal size and locations

of solar PV that minimize system losses. These results are then used to calculate the new energy burden distribution of population after placing the solar PV in the distribution system. This allows us to directly assess how a conventional, efficiency-driven PV planning strategy impacts community.

3.1.2 Optimal Transport Theory Fundamentals

Optimal transport theory provides a mathematically rigorous mechanism for transforming one probability distribution into another while minimizing an associated cost function. By representing community-level energy outcomes as probability distributions, the proposed framework enables systematic reshaping of system-level performance characteristics toward desirable operational and resilience objectives. This formulation provides a flexible foundation for balancing electrical performance requirements with distributionally consistent outcomes across diverse communities.

While conventional PV planning approaches minimize aggregate statistics of energy burden (e.g., average or maximum values), this work adopts a distributional viewpoint that explicitly models how PV deployment reshapes the entire population-level distribution of energy burden. To this end, we formulate the optimal solar PV siting and sizing problem as an optimal transport-based distribution alignment problem. The objective is to determine the most efficient transformation that maps an initial (source) energy-burden distribution μ , corresponding to the system without PV, to a desired (target) distribution ν , representing fairness-oriented outcomes under PV deployment.

Formally, let T denote a transport map that transforms the source distribution μ into the target distribution ν . The PV planning problem seeks a transformation T that minimizes a prescribed measure of dissimilarity between $T(\mu)$ and ν , subject to power system operational constraints. This formulation enables target-driven PV allocation based on general distributional fairness criteria, rather than being restricted to conventional average- or worst-case-based planning objectives. The resulting optimization problem is expressed as follows:

$$\min_T W(T(\mu), \nu) \quad (3.10)$$

where W is a measure of dissimilarity between distributions, often chosen to be the Wasserstein distance in optimal transport formulations [132], i.e.,

$$W(\mu, \nu) = \min_{\gamma \in \pi(\mu, \nu)} \int_{X \times Y} c(x, y) d\gamma(x, y). \quad (3.11)$$

Here, $\gamma(x, y)$ is the transportation plan or amount of mass to move from point x in μ to point y in ν ; $c(x, y)$ is the cost associated with the respective transportation plan. With this basic understanding of OT, we will explain the proposed PV planning method in the next section.

3.2 Community-Centric DG Planning

Recent advances in resilience-oriented power system planning highlight the importance of incorporating community-level characteristics into infrastructure design. In distribution networks serving diverse populations, variations in energy demand, economic capability, and access to distributed resources can significantly influence the resilience outcomes experienced by individual communities. Therefore, planning methodologies that explicitly account for distributional characteristics of energy costs and resource availability can improve both system-level robustness and community-level recovery potential following disturbances.

3.3 Multi-Objective DG Planning Methods Using Optimal Transport

This section describes the key steps of the proposed solar PV planning framework that integrates optimal transport (OT) with optimal power flow (OPF) to enable community-aware PV allocation. An overview of the integrated OT-OPF methodology is shown in Fig. 3.1. The framework proceeds in three stages. First, solar PV siting and sizing are

determined using an OT-based equal distribution equivalent (OT-EDE) approach, This stage explicitly targets distributional fairness without considering network losses. Second, an OPF-based planning model is solved to identify PV allocations that minimize active power losses while satisfying network constraints. Finally, the two planning outcomes are integrated by computing the barycenter of the energy-burden distributions obtained from the OT-EDE and OPF stages. This barycentric integration balances community-centric objectives with operational efficiency, resulting in the proposed integrated OT-OPF PV planning method.

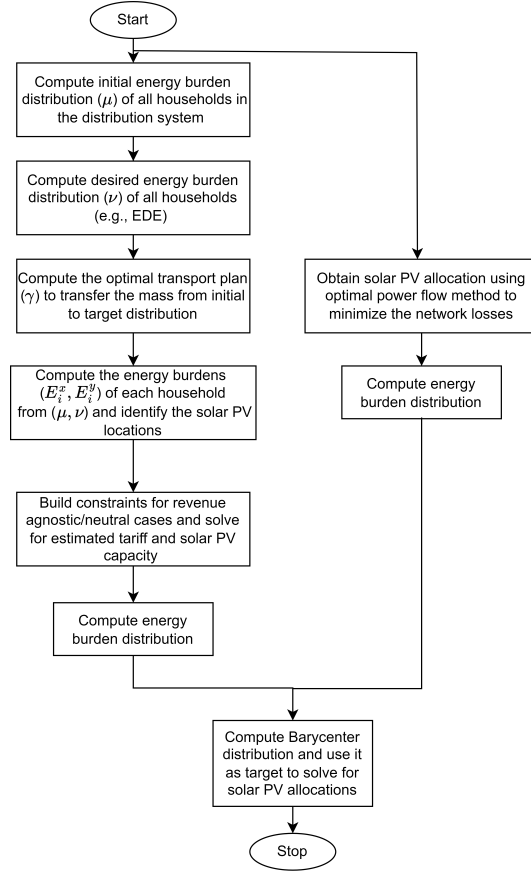


Figure 3.1: Flow chart of the proposed method.

3.3.1 OT-EDE based PV Planning

The steps involved in the optimal transport and equal distribution equivalent-based PV planning are described below:

Step 1 (Defining EDE Distribution): The target energy burden distribution ν is defined

using the concept of EDE as a representative and illustrative choice of a fairness-oriented target distribution. The EDE is a welfare-based measure of the central tendency of a distribution that accounts for inequality through a social welfare function. It represents the uniform value which, if assigned equally to all individuals, would yield the same level of social welfare as the original, unequal distribution. As demonstrated in [133], EDE provides a principled way to incorporate distributional justice into resource allocation problems. Specifically, the average energy burden across the population is computed and interpreted as the EDE-based target level for each consumer. For households whose actual energy burden exceeds this target, their burden is adjusted toward the target, while those already at or below the target remain unchanged. This process yields the target distribution ν , which serves as a community-oriented objective for planning.

It is important to emphasize that EDE is used here primarily as a representative example of a target distribution that satisfies desirable fairness properties. The proposed framework is not restricted to EDE; any alternative target distribution that satisfies the axiomatic properties of EDE as discussed in [8] can be seamlessly incorporated. Thus, the framework is general and flexible, allowing utilities and policymakers to specify different notions of fairness by selecting appropriate target distributions, while the optimal transport formulation provides a systematic mechanism to guide PV siting and sizing toward those objectives.

Step 2 (Optimal Transport Plan): Once the target distribution ν is obtained, we employ OT to obtain the most cost-effective transformation from the initial distribution, μ to the target ν . The resulting optimal transport plan is then utilized to determine the locations and capacities of solar PV installations on household rooftops within the distribution system. To solve the OT problem, we use the computationally efficient Sinkhorn algorithm [134], which introduces entropy regularization to ensure the necessary spread in the OT plan. The detailed procedure is presented in Algorithm 1. The entropy-regularized OT formulation corresponding to (2) is given by:

$$\gamma^* = \min_{\gamma \in \pi(\mu, \nu)} \sum_{x, y} \gamma(x, y) c(x, y)$$

$$+\lambda \sum_{x,y} \gamma(x,y)(\log \gamma(x,y) - 1) \quad (3.12)$$

subject to

$$\sum_y \gamma(x,y) = \mu_x \quad \forall \quad x \quad (3.13)$$

$$\sum_x \gamma(x,y) = \nu_y \quad \forall \quad y \quad (3.14)$$

$$\gamma \geq 0 \quad \forall \quad x,y \quad (3.15)$$

Here, λ is the regularization coefficient, and μ_x and ν_y are the column-wise and row-wise mass balance constraints, where the total mass moved from the sources must equal the mass received by the sinks, and the total mass received by the sinks must equal the mass sent from the sources.

Step 3 (Solar PV capacity and tariff estimation): The OT plan from Step 2 provides the number of consumers from a point x in the energy burden distribution μ that should be moved to a point y in the energy burden distribution ν . At this stage, we solve a feasibility problem to determine the tariff (i.e., the new cost per unit) and the size of solar PV installations at identified locations for both revenue-agnostic and revenue-neutral cases. In the revenue-agnostic scenario, constraints are applied only to households whose energy burden is reduced by the OT plan, leading to a revenue deficit for utilities. In the revenue-neutral scenario, constraints are also applied to households whose energy burden increases, ensuring revenue neutrality for utilities. It is important to note that the proposed method is flexible to incorporate other business models and cost-related constraints. To find the updated cost per unit and solar PV capacity required at the desired locations, we mathematically formulate the problem as a feasibility problem,

$$\text{Find } C, P_1^{pv}, P_2^{pv}, \dots, P_n^{pv} \quad (3.16)$$

Algorithm 1 Sinkhorn Algorithm for Entropy-Regularized Optimal Transport Plan

Require:

Cost matrix $C \in \mathbb{R}^{n \times m}$ where $C_{ij} = c(x_i, y_j)$
Source distribution $\mu \in \Sigma_n$ (where Σ_n is the n -simplex)
Target distribution $\nu \in \Sigma_m$
Entropy regularization parameter $\lambda > 0$
Convergence threshold $\epsilon > 0$
Maximum iterations N

Ensure:

Optimal transport plan $\gamma \in \mathbb{R}^{n \times m}$

Initialize:

$K \leftarrow \exp(-C/\lambda)$

▷ Element-wise exponential

$\mathbf{u} \leftarrow \mathbf{1}_n/n$

▷ Initialize scaling factors

$\mathbf{v} \leftarrow \mathbf{1}_m/m$

$k \leftarrow 0$

1: **while** $k < N$ **do**

 Update $\mathbf{u}$:

$\mathbf{u} \leftarrow \mathbf{a}/(K\mathbf{v})$

▷ Element-wise division

 Update $\mathbf{v}$:

$\mathbf{v} \leftarrow \mathbf{b}/(K^\top \mathbf{u})$

 Check convergence:

$\gamma \leftarrow \text{diag}(\mathbf{u})K\text{diag}(\mathbf{v})$

2: **if** $\|\gamma\mathbf{1}_m - \mathbf{a}\|_1 + \|\gamma^\top \mathbf{1}_n - \mathbf{b}\|_1 < \epsilon$ **then**

break

3: **end if**

$k \leftarrow k + 1$

4: **end while**

return γ

subject to

$$C(P_i^l - P_i^{pv}) \leq E_i^y I_i \quad \forall \quad (E_i^x - E_i^y) > 0 \quad (3.17)$$

$$(CP_i^l) \geq E_i^y I_i \quad \forall \quad (E_i^x - E_i^y) < 0 \quad (3.18)$$

$$0 \leq P_i^{pv} \leq P_i^l \quad (3.19)$$

$$\sum_{i=1}^m P_i^{pv} \leq \eta \sum_{i=1}^n P_i^l \quad (3.20)$$

$$C_{min} \leq C \leq C_{max} \quad (3.21)$$

Here, C, E_i^y are estimated cost of energy and updated energy burden at i^{th} node; P_i^l is load active power and P_i^{pv} is solar PV active power at the i^{th} node; η is the fraction corresponding to the allowed penetration percentage in the distribution system; C_{min} and C_{max} are the lower and upper limits of cost of energy. Constraint (3.17) is associated with i^{th} consumer whose energy burden is reduced through optimal transport plan, constraints (3.19), (3.20) bound the individual and cumulative solar PV generation on rooftops to ensure system reliability by preventing reverse power flows. The constraint (3.17) is nonlinear due to the bilinear term CP_i^{pv} , this can be solved by introducing an auxiliary variable z_i and bounding it with McCormick envelopes [135], where $z_i = CP_i^{pv}$. Therefore, the feasibility problem can be converted into a minimization problem as:

$$\min_{P_i^{pv}} C \quad (3.22)$$

subject to

$$(CP_i^l - z_i) \leq E_i^y I_i \quad \forall \quad (E_i^x - E_i^y) > 0 \quad (3.23)$$

$$z_i \geq C_{min} P_i^{pv} \quad (3.24)$$

$$z_i \geq C_{max} P_i^{pv} + CP_i^l - C_{max} P_i^l \quad (3.25)$$

$$z_i \leq C_{max} P_i^{pv} \quad (3.26)$$

$$z_i \geq C P_i^l + C_{min} P_i^{pv} - C_{min} P_i^l \quad (3.27)$$

(3.19) - (3.21)

Since the objective C and constraints (3.19) - (3.21) and (3.23) - (3.27) are linear, the considered optimization problem is convex and a solution can be obtained by existing convex optimization solvers. It is important to note that we have not considered the capital cost for PV installation within the analysis. It is possible to include those considerations as well, and it will be part of our future work. By following the procedure outlined in Subsection 3.3.1, we obtain the energy-burden distribution based on the OT-EDE formulation.

3.3.2 OPF-based PV Planning

In this subsection, we discuss an OPF-based method for loss-minimizing solar PV siting and sizing. This approach aims to determine the best locations and capacities of solar PV units in a distribution system to minimize power losses while ensuring system stability and operational constraints. The OPF-based approach can be mathematically formulated as an optimization problem corresponding to,

$$\min_{P_i^{pv}} \sum_{i=1, i \neq j}^n P_{ij}^{loss} \quad (3.28)$$

subject to

$$P_i^g + \beta_i P_i^{pv} - P_i^l = V_i \sum_{j=1}^n V_j Y_{i,j} \cos(-\theta_{i,j} + \delta_i - \delta_j) \quad (3.29)$$

$$Q_i^g + \beta_i Q_i^{pv} - Q_i^l = V_i \sum_{j=1}^n V_j Y_{i,j} \sin(-\theta_{i,j} + \delta_i - \delta_j) \quad (3.30)$$

$$\sum_{i=1}^m P_i^{pv} = \eta \gamma_0 \sum_{i=1}^l P_i^l \quad (3.31)$$

$$I_{ij} = Y_{i,j} (V_i - V_j) \quad (3.32)$$

$$P_{ij}^{loss} = I_{ij}^2 R_{i,j} \quad (3.33)$$

$$V^{\min} \leq V_i \leq V^{\max} \quad (3.34)$$

$$0 \leq P_i^{pv} \leq P_{\max}^l \quad (3.35)$$

$$0 \leq \sum_i^n \beta_i \leq N^{pv} \quad (3.36)$$

Here, P_i^g , Q_i^g , and P_{ij}^{loss} are the active, reactive power from the grid and active power loss across the branches. (3.29), (3.30) are the power flow equations with β_i being a binary decision variable to choose location for solar PV. V_i is i^{th} node voltage; $I_{i,j}$ is the branch current between i and j ; $V_{\min}$ and $V_{\max}$ are the allowable limits for node voltages; $Y_{i,j}$ and $R_{i,j}$ are the branch admittance and resistance between i^{th} node and j^{th} node. The above formulation is a non-linear mixed integer optimization problem, and [130] elaborates on various methods to solve such problems. Broadly, these methods include MILP and Monte Carlo methods, heuristic algorithms such as genetic algorithms, and other hybrid approaches. In this work, the OPF problem is formulated using the current–voltage (IV) representation, which provides a robust modeling of voltage and current variables suitable for unbalanced distribution networks. The optimization is carried out using the PowerModelsDistribution (PMD) package, an open-source Julia-based framework for scalable distribution-system optimization [131].

From this formulation, we obtain the optimal size and locations of solar PV that minimize system losses. These results are then used to calculate the new energy burden distribution of population after placing the solar PV in the distribution system. This allows us to directly assess how a conventional, efficiency-driven PV planning strategy impacts community and to compare it with the proposed OT-EDE-based fairness-oriented planning framework.

3.3.3 Integrated OT-OPF based PV Planning

In this subsection, we discuss the integrated OT-OPF method, which utilizes the energy burden distributions obtained from Subsections A and B. By incorporating results from both

OT-EDE and OPF, this approach provides a more practical and statistically rich approach to PV planning. Typically, DG allocation approaches consider cost and loss minimization as a multi-objective formulation, i.e.,

$$\min_{P_i^{pv}} C\left(\sum_{i=1}^n (P_i^g - P_i^{pv}) + \sum_{i=1, i \neq j}^n P_{ij}^{loss}\right) \quad (3.37)$$

Subject to constraints (3.29)–(3.36), the optimization ensures network reliability. However, (3.37) optimizes only average cost and losses, which limits its ability to capture the full variability or distributional characteristics of community related metrics. For a balanced approach that accounts for both the distribution of community metrics and optimal losses, a multi-objective formulation is required. Such a formulation must jointly consider distributional measures for community indicators (e.g., energy burden) and scalar objectives (e.g., network losses). Therefore, we propose an integrated OT-OPF-based PV planning framework that explicitly incorporates the distribution of relevant community metrics and network losses, while preserving overall system feasibility.

In this work, we exploit the key property of optimal multiple transport that is validated in [136].

Theorem 1. *Let $\{\mu_i\}_{i=1}^n \subset \mathcal{P}_2(\mathcal{X})$ denote a collection of demand distributions defined on a metric space $(\mathcal{X}, d)$, and let $\lambda_i \geq 0$ satisfy*

$$\sum_{i=1}^n \lambda_i = 1.$$

Define the total transportation cost functional

$$J(\nu) = \sum_{i=1}^n \lambda_i W_2^2(\nu, \mu_i),$$

where $W_2(\nu, \mu_i)$ is the 2-Wasserstein distance between the allocation distribution ν and demand distribution μ_i .

Then the Wasserstein barycenter

$$\mu^* = \arg \min_{\nu \in \mathcal{P}_2(\mathcal{X})} \sum_{i=1}^n \lambda_i W_2^2(\nu, \mu_i)$$

is the optimal resource allocation distribution that minimizes the total weighted transport cost from a common source distribution to all demand distributions.

Proof. By definition, the squared Wasserstein distance between two probability measures ν and μ_i is

$$W_2^2(\nu, \mu_i) = \inf_{\gamma_i \in \Pi(\nu, \mu_i)} \int_{\mathcal{X} \times \mathcal{X}} \|x - y\|^2 d\gamma_i(x, y),$$

where $\Pi(\nu, \mu_i)$ denotes the set of admissible transport plans between ν and μ_i .

The total transportation cost associated with an allocation distribution ν is therefore

$$J(\nu) = \sum_{i=1}^n \lambda_i \inf_{\gamma_i \in \Pi(\nu, \mu_i)} \int_{\mathcal{X} \times \mathcal{X}} \|x - y\|^2 d\gamma_i(x, y).$$

The Wasserstein barycenter is defined as the probability measure μ^* satisfying

$$\mu^* = \arg \min_{\nu \in \mathcal{P}_2(\mathcal{X})} J(\nu).$$

Hence, for any feasible allocation distribution $\nu \in \mathcal{P}_2(\mathcal{X})$,

$$J(\mu^*) \leq J(\nu).$$

That is,

$$\sum_{i=1}^n \lambda_i W_2^2(\mu^*, \mu_i) \leq \sum_{i=1}^n \lambda_i W_2^2(\nu, \mu_i).$$

Therefore, the barycenter μ^* minimizes the aggregate weighted transportation cost required to distribute resources from a common allocation distribution to multiple demand distributions. Consequently, the Wasserstein barycenter represents the optimal “average” distribution for multi-demand resource allocation under the Wasserstein transport metric

[136].

□

Theorem 1 establishes that the Wasserstein barycenter minimizes the aggregate weighted transportation cost across multiple demand distributions. This property enables the barycenter to serve as a unified reference distribution that simultaneously accounts for multiple planning objectives. In the proposed framework, these objectives correspond to the energy-burden (cost) distributions from OT-EDE and OPF, allowing the resulting barycenter to balance both cost- and loss-related aspects of the allocation process. First, we determine the barycenter distribution of the energy burden distributions obtained from the OT-EDE (μ_1) and OPF (μ_2) methods. The Bregman barycenter can be obtained as follows:

$$\nu_B = \min_{\nu} wW(\nu, \mu_1) + (1 - w)W(\nu, \mu_2) \quad (3.38)$$

where ν_B is the Bregman barycenter obtained through a cost-effective transformation of μ_1, μ_2 distributions; w is a tuning parameter that allows utilities to prioritize between energy access and network efficiency. Since the barycenter incorporates both non-technical and technical aspects of the OT formulation, we assume the barycenter ν_B as the target distribution to obtain the optimal transport plan using equations (3.12)-(3.15) and Algorithm 1. Finally, the feasibility problem elaborated in Subsection 3.3.1 is used to obtain the sizes and locations of the solar PV. We formulate both a revenue-agnostic and a revenue-neutral case to solve the feasibility problem and achieve the target distribution.

3.3.4 Constrained Optimal Transport-Based Planning

Extending the basic OT formulation, we now introduce a Constrained Optimal Transport (COT) framework that explicitly embeds power system performance metrics directly into the transportation cost. The entropy-regularized COT formulation for DG planning is given by,

- **COT formulation:**

$$\begin{aligned}
& \min_{\gamma \in \pi(\mu, \nu)} \sum_{x, y} \gamma(x, y) c(x, y) \\
& + \lambda \sum_{x, y} \gamma(x, y) (\log \gamma(x, y) - 1)
\end{aligned} \tag{3.39}$$

subject to

$$c(x, y) = f(\phi_{x, y}, \delta_{x, y}) = (1 - \alpha)\phi(x, y) + \alpha\delta(x, y) \tag{3.40}$$

$$\phi(x, y) = \|x - y\|^2 \tag{3.41}$$

$$\delta(x, y) = \|\delta_x - \delta_y\|^2 \tag{3.42}$$

$$\sum_y \gamma(x, y) = \mu_x \quad \forall \quad x \tag{3.43}$$

$$\sum_x \gamma(x, y) = \nu_y \quad \forall \quad y \tag{3.44}$$

$$\gamma(x, y) \geq 0 \quad \forall \quad x, y \tag{3.45}$$

$$\delta_x = \sum_{i=1}^k \frac{\partial P^{loss}}{\partial P_i^{pv}} = \sum_{i=1}^k \sum_{j=1}^n \beta_{ij} P_j^{pv} \quad \forall \quad B_i^x \tag{3.46}$$

$$\delta_y = \sum_{i=1}^k \frac{\partial P^{loss}}{\partial P_i^{pv}} = \sum_{i=1}^k \sum_{j=1}^n \beta_{ij} P_j^{pv} \quad \forall \quad y \tag{3.47}$$

where, λ is regularization parameter; $\phi(x, y)$ is Euclidean cost; $\delta(x, y)$ is cost proportional to network loss sensitivity factor; α is weight parameter; (3.43), (3.44) are mass balance constraints associated to source distribution (μ) and target distribution (ν); (3.45) ensures that transport plan is non-negative; δ_x is sum of loss sensitivity factors of consumers corresponds to B^x ; β_{ij} is loss coefficient matrix; P_i^{pv} is solar PV generation at i^{th} location in the

network. The network parameter-dependent term β_{ij} is used to compute the loss sensitivity of all buses [130]. Thus, substituting equation (3.46) into the transportation cost equation in (3.40) incorporates the active power loss component of the network. Consequently, while transforming the distribution, the transportation plan minimizes both the energy burden and the active power loss in DG planning. In standard OT formulations, the transportation cost $c(x, y)$ is typically defined purely in terms of the geometric or statistical distance between x and y , and therefore reflects only distributional objectives such as fairness or similarity between probability measures. In contrast, the proposed COT formulation augments this cost with a network-aware component $\delta(x, y)$ that depends on active power loss sensitivities. The term δ_x in (3.46) represents the marginal impact of PV injection at a given location on the total network losses. *By incorporating $\delta(x, y)$ into the transportation cost function $c(x, y)$, each unit of “mass” moved in the energy burden distribution is weighted not only by its impact on household energy costs but also by its effect on system active power losses. As a result, the optimal transport plan naturally favors DG siting and sizing decisions that both reduce disparities in energy burden and improve network efficiency.*

Convexity Analysis: In the objective function in (3.39); the cost term $\sum_{x,y} \gamma(x, y)c(x, y)$ is linear in $\gamma(x, y)$, since $c(x, y)$ is a fixed cost for given x, y . The second entropy regularization term $\lambda \sum_{x,y} \gamma(x, y)(\log \gamma(x, y) - 1)$ is convex for $\gamma(x, y) > 0$ [137]. Therefore, the objective function is convex. The constraints in (3.43), (3.44), and (3.46), (3.47) are affine; (3.45) is a convex inequality constraint. Therefore, the resulting entropy-regularized COT problem is convex and can be solved using the Sinkhorn algorithm [134].

Complexity Analysis: From a computational perspective, the proposed COT-based method offers significant advantages over traditional OPF-based approaches. Standard OPF formulations, even with linearization, typically exhibit complexity on the order of $\mathcal{O}(n^2)$ [138], where n is the number of network nodes, due to the need to evaluate nodal interactions in power flow equations. In contrast, the COT formulation operates on the support of the energy-burden distribution, with complexity $\mathcal{O}(m^2)$, where m is the number of points in the discretized energy burden distribution, often much smaller than n . As a result, the proposed

Table 3.1: Income and energy consumption of households

Region	Income (\$/year)	No.of houses	units(kWh)/ house	Energy burden(%)
Cluster-a	24000	7	500	3
Cluster-b	50000	8	600	1.728
Cluster-c	70000	10	800	1.645
Cluster-d	85000	7	900	1.524
Cluster-e	125000	5	1200	1.382

approach can efficiently handle large networks and high-resolution distributional modeling, providing a substantial computational advantage for planning studies.

3.4 Experimental Results and Discussion

3.4.1 IEEE 37 Bus Test System

This section presents the results of the proposed method evaluated on a modified IEEE 37-bus distribution system. The considered system has various load regions, as shown in Fig. 3.2, with each load region representing a group of consumers. The load regions are depicted as clusters in the figure, with each color code representing a cluster of loads or households with the same income and energy consumption. For simplicity of representation, this work assumes that each node is connected to a household. The details of the number of consumers in each cluster, along with their energy consumption and income levels, are shown in Table 3.1. The energy burden (%) per month is calculated and presented based on assumed income level and energy consumption.

The details above are considered as the base case for the distribution system. The population energy burden distribution μ is obtained for the base case, where no solar PV is installed in the system. Using the equal distribution equivalent and optimal transport, we define the desired distribution and find the best way to achieve it. Figures 3.3 and 3.4 show the source and target distributions. After defining the source and target distributions, the optimal transport plan provides the number of houses to be transported from the i^{th} point

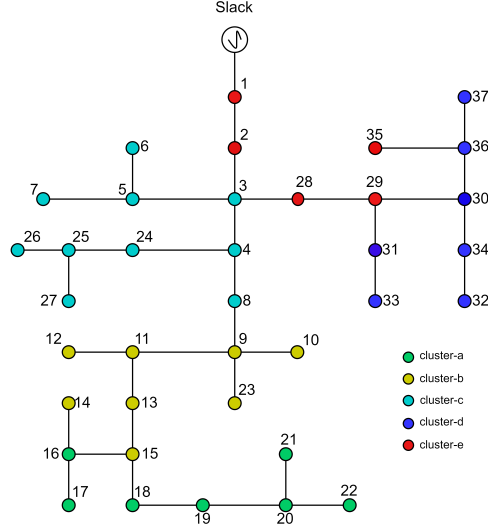


Figure 3.2: Diagram of IEEE 37 bus system with load clusters.

in source distribution to the j^{th} point in target distribution. Thus, the optimal plan using the Sinkhorn algorithm provides the population whose energy burden is altered. Later, the feasibility problem for the revenue-agnostic and revenue-neutral cases determines the solar PV ratings at the desired locations and estimates the cost of energy. Table 3.2 provides the kWp to be installed at each location in the distribution system. The nodes with a high initial energy burden are assigned solar PV with the respective ratings mentioned in Table 3.2. Figures 3.5 and 3.6 show the energy burden distribution for the revenue-agnostic and revenue-neutral cases, with costs of energy at 12 ¢/kWh and 15 ¢/kWh, respectively. It is noted that the mean value of the distribution is less than that of the target distribution, which satisfies the principle of the transfer property as per the EDE approach. Additionally, the Wasserstein distances between the target distribution and the revenue-agnostic and revenue-neutral cases are 0.170 and 0.4265, respectively, indicating that the two distributions are quite similar. Thus, the obtained distributions satisfy the properties of the EDE for energy burden distribution.

The total distributed solar PV installed is nearly 35% of the total connected load in the distribution system, which effectively achieves the target energy burden distribution, which is an equal distribution equivalent. Therefore, the proposed method for planning

Table 3.2: Solar PV allocation at various nodes

Node	Revenue Ag- nostic $C_{opt} = 12 \text{ ¢/kWh}$ kWp	Revenue Neu- tral $C_{opt} = 15 \text{ ¢/kWh}$ kWp
32	2.35	2.48
7	2.36	2.45
26	2.44	2.75
27	2.44	2.75
10	2.45	2.31
12	2.47	2.73
23	2.47	2.73
14	2.48	2.72
15	2.48	2.72
16	2.26	2.51
17	2.35	2.45
18	2.43	2.33
19	2.39	2.73
20	2.39	2.73
21	2.41	2.74
22	2.41	2.74

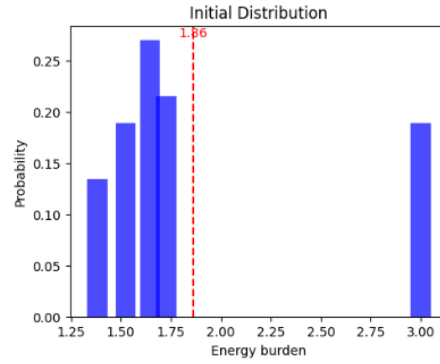


Figure 3.3: Distribution of energy burden over consumers.

distributed solar PV generation has the potential to address the inequities present in the current distribution system.

3.4.2 559 Bus Test System

This section presents the results of the proposed method, evaluated on a 559-node test system built upon the IEEE 37-node test system. This expanded test system is designed to study

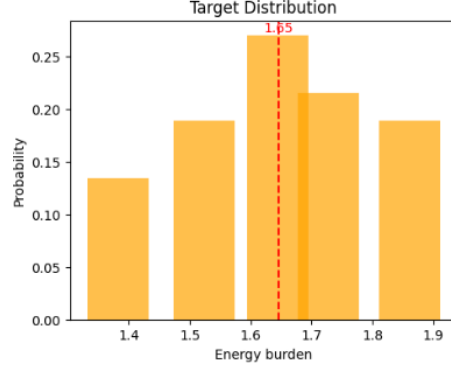


Figure 3.4: Distribution of energy burden over consumers.

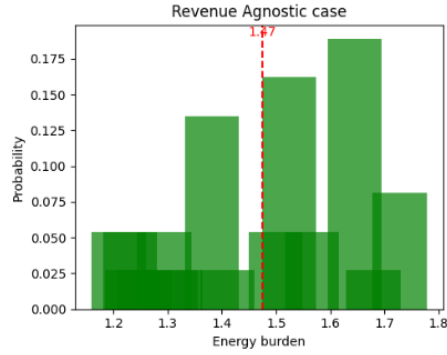


Figure 3.5: Distribution of energy burden over consumers.

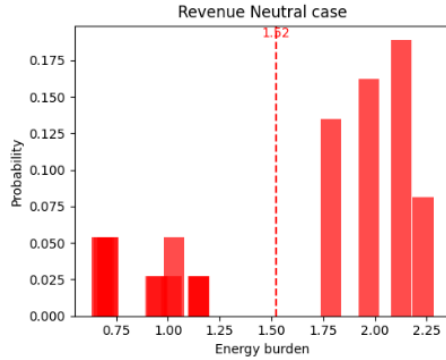


Figure 3.6: Distribution of energy burden over consumers.

the challenges associated with distributed energy resources [139], particularly in relation to household consumption in the distribution system. For better visualization, the 559 node test system is shown as a graph in Fig. 3.7, the details of loads, primary feeders, lateral feeders, and secondary feeders are available in [139]. The total active power supply to loads is 2.457 MW. The entire load is assumed to be distributed across five clusters with each cluster representing customers of various income groups. Table. 3.3 presents the various

Table 3.3: Income and energy consumption of households

Region	Income (\$/year)	No.of houses	units(kWh)/ house	Energy bur- den(%)
Orange	24000	113	500	6
Yellow	50000	120	600	1.728
Cyan	70000	80	800	1.645
Magenta	85000	34	900	1.524
Red	125000	101	1200	1.382

clusters in Fig. 3.7 and details of number of houses in each cluster and their respective units consumed, income, and energy burden. The energy burden distribution among the consumers is obtained from Table. 3.3 and this is the initial energy burden distribution without installing distributed solar PV generation. Using the proposed PV planning method, we aim to help households experiencing a high energy burden. It is important to note that the proposed framework is general and can be applied to arbitrary distributions of customers and their energy burdens.

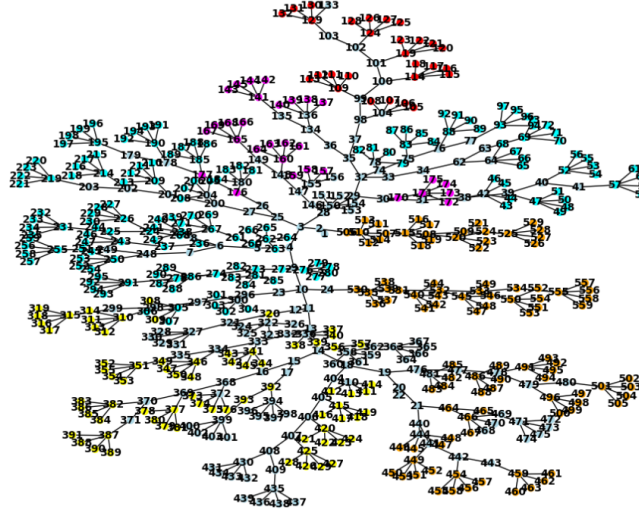


Figure 3.7: 559 bus system graph with various consumer clusters.

As discussed earlier, the population energy burden distribution μ is obtained for the base case, where no solar PV is installed in the system. Using the equal distribution equivalent

and optimal transport, we define the desired distribution and find the best way to achieve it. Figures 3.8 and 3.9 show the source and target distributions. After defining the source and target distributions, the optimal transport plan using the Sinkhorn algorithm provides the population whose energy burden needs to be altered. Next, a feasibility analysis is conducted for both revenue-agnostic and revenue-neutral cases to determine the feasible ratings of solar PV at the desired locations and to estimate the corresponding energy costs. Using these estimated PV ratings and costs, the energy-burden distribution is updated for each case to reflect the effect of PV deployment. Finally, the computed PV capacities and their locations serve as inputs for the optimal placement stage, where PV allocation is adjusted to minimize active power losses across the network. Lastly, we integrate the energy burden distributions obtained from OT-EDE revenue-agnostic and revenue-neutral cases with the respective Loss minimization method to find a Bregman barycenter distribution. This distribution captures both the OT-EDE and active power loss into the formulations, thus we can strategically give priority to efficiency and energy access while siting and sizing solar PV. Further, to benchmark the integrated method, a multi-objective optimization is implemented on the 559-bus test system to jointly minimize energy cost and active power losses. The problem is formulated using the current-voltage (IV) representation, which provides a robust modeling of voltage and current variables suitable for unbalanced distribution networks. The optimization is carried out using the PowerModelsDistribution (PMD) package, an open-source Julia-based framework for scalable distribution-system optimization [131].

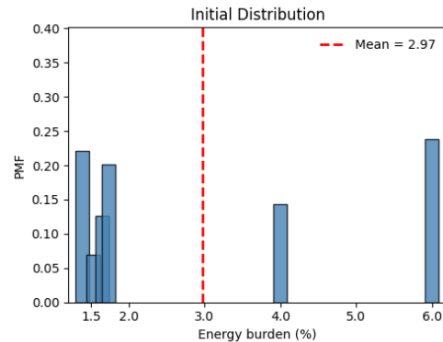


Figure 3.8: Initial energy burden distribution of consumers.

Tables 3.4 and 3.5 compare the performance of various DG planning strategies under

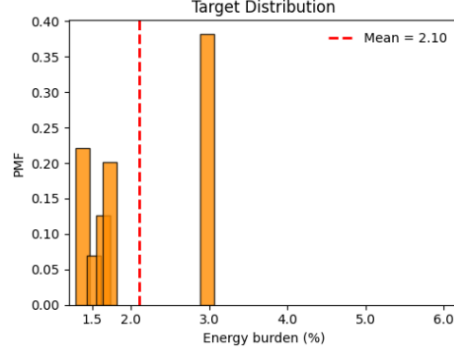


Figure 3.9: Target energy burden distribution of consumers.

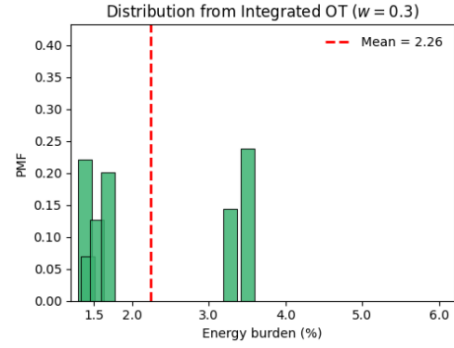


Figure 3.10: Energy burden distribution: OT-EDE method.

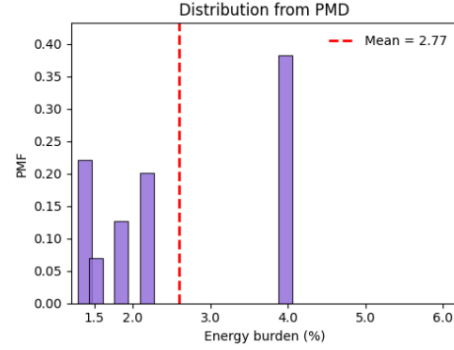


Figure 3.11: Energy burden distribution: OT-EDE method.

revenue-agnostic and revenue-neutral cases, respectively. At equivalent levels of solar PV penetration with respect to the total system load, the proposed integrated method consistently demonstrates a favorable balance between operational efficiency and distributionally measured performance, as reflected in the mean energy burden, 95th percentile energy burden, and Gini coefficient. The Gini coefficient, widely used in economics to quantify inequality within a population [140], provides a scalar measure of the degree of inequality in

Table 3.4: Comparison of various DG planning methods with Revenue Agnostic Model

Method	% Solar PV	% Active power loss	% Mean	95th per- centile	Gini coeff	Theil index	Palma ratio	P- gap
Base case	0	5.88	2.97	6	0.175	0.194	1.032	2.35
OT-EDE	5.7	5.35	2.21	3.15	0.067	0.058	0.051	1.73
Integrated method ($w = 0.9$)	5.7	4.88	2.33	3.32	0.082	0.061	0.054	1.79
Integrated method ($w = 0.3$)	5.7	4.62	2.54	3.57	0.095	0.077	0.066	1.83
Integrated method ($w = 0.1$)	5.7	4.35	2.79	3.93	0.124	0.104	0.099	1.89
PMD framework	5.7	3.97	2.99	4.32	0.137	0.128	0.114	1.98

Table 3.5: Comparison of various DG planning methods with Revenue Neutral Model

Method	% Solar PV	% Active power loss	% Mean	95th per- centile	Gini coeff	Theil index	Palma ratio	P- gap
Base case	0	5.88	2.97	6	0.175	0.194	1.032	2.35
OT-EDE	6.5	5.12	2.08	2.9	0.054	0.046	0.042	1.53
Integrated method ($w = 0.9$)	6.5	4.79	2.11	3.25	0.063	0.049	0.047	1.56
Integrated method ($w = 0.3$)	6.5	4.55	2.26	3.34	0.087	0.072	0.061	1.78
Integrated method ($w = 0.1$)	6.5	4.04	2.67	3.85	0.101	0.098	0.092	1.85
PMD framework	6.5	3.78	2.77	4.1	0.124	0.112	0.103	1.92

household energy burdens, with lower values indicating a more uniform distribution. In the context of community-oriented power system planning, it enables a systematic assessment of how DG deployment reshapes energy burden heterogeneity across households, thereby complementing traditional technical metrics with a population-level measure of distributional impact.

Compared to the loss-minimization approach, the integrated method substantially reduces inequality in energy burden outcomes. While loss minimization achieves the lowest active power losses, it results in relatively high 95th percentile values and Gini coefficients, indicating that benefits are unevenly distributed and that highly burdened households experience limited relief. In contrast, the integrated method, particularly for higher weighting factors (e.g., $w = 0.9$ and $w = 0.3$), achieves markedly lower 95th percentile energy burdens

and Gini coefficients, demonstrating its effectiveness in alleviating extreme energy burdens and improving overall energy access.

Relative to the OT-EDE-only approach, the integrated method incurs modestly higher power losses but achieves comparable reductions in the mean energy burden and inequality metrics. This highlights the value of incorporating network constraints into the planning process: the integrated framework preserves most of the community-oriented distributional gains of OT-EDE while significantly improving operational feasibility and reducing losses relative to purely distribution-driven planning.

The PMD framework improves upon loss minimization in terms of mean energy burden but exhibits Gini coefficients and upper-tail (95th percentile) values close to those of the loss-minimization approach. This suggests that PMD-based trade-offs, while effective at improving average outcomes, are less successful at targeting households experiencing the highest energy burdens. By contrast, the integrated method explicitly shapes the energy-burden distribution, resulting in lower upper-tail exposure and a more equitable allocation of benefits. Lower Gini coefficients indicate reduced inequality in energy burden across households, while lower 95th percentile values reflect direct mitigation of extreme energy burdens among the most vulnerable consumers. The integrated method consistently achieves improvements in both metrics, demonstrating its ability to deliver community-level affordability gains not only on average but also at the distributional extremes.

Overall, the results from both tables show that the integrated method provides a tunable and systematic mechanism to navigate the trade-off between efficiency and energy access. By adjusting the weighting parameter w , utilities can tailor PV planning strategies to prioritize loss reduction, distributional fairness, or a balanced combination of both, an adaptability that is not available in conventional DG planning approaches.

As discussed earlier, the population energy burden distribution is to be obtained for the base case, where no solar PV is installed in the system. After defining the source and target distributions, the transport plan provides the number of houses to be transported from the i^{th} point in source distribution to the j^{th} point in target distribution. Thus, the optimal plan using the Sinkhorn algorithm provides a population whose energy burden is altered by

Table 3.6: Comparison of various DG planning methods for COT framework

Method	% Solar PV	% Active power loss	% Mean	95th per- centile	Gini coeff	Theil index	Palma ratio	P- gap
Base case	0	5.37	2.27	3	0.125	0.188	1.021	2.252
COT ($\alpha = 0$)	5.73	4.51	1.65	1.83	0.045	0.038	0.036	1.41
COT ($\alpha = 0.2$)	5.73	4.32	1.68	1.87	0.048	0.049	0.043	1.52
COT ($\alpha = 0.6$)	5.73	3.87	1.73	1.96	0.057	0.057	0.054	1.63
COT ($\alpha = 0.8$)	5.73	3.55	1.75	2.08	0.061	0.063	0.059	1.67
COT ($\alpha = 1$)	5.73	3.04	1.79	2.81	0.121	0.072	0.069	1.85
PMD frame- work	5.73	3.65	1.77	2.70	0.118	0.068	0.066	1.73

allocating the solar PV. Further, to benchmark the COT-based framework, a multi-objective optimization is implemented on the 559-bus test system to jointly minimize energy cost and active power losses. The problem is formulated using the current-voltage (IV) representation, which provides a robust modeling of voltage and current variables suitable for unbalanced distribution networks. The optimization is carried out using the PowerModelsDistribution (PMD) package, an open-source Julia-based framework for scalable distribution-system optimization [131].

The comparative results are summarized in Table 3.6. The base case, without solar PV deployment, exhibits the highest active power losses (5.37%) along with a mean energy burden of 1.87, a 95th-percentile value of 3, and a Gini coefficient of 0.125. Incorporating the proposed COT framework results in noticeable improvements across both network and distributional metrics. As the weighting parameter α increases, the solution gradually shifts from affordability-oriented outcomes toward loss minimization. For lower values of α (e.g., $\alpha = 0 - 0.4$), the method achieves the lowest energy burden, with Gini coefficients ranging from 0.045 to 0.051 and substantially reduced high-percentile values (1.83-1.92). Although these settings result in relatively higher losses compared to loss-focused configurations, they remain below the base-case level. Conversely, higher values of α improve network efficiency,

reducing active power losses to as low as 3.04% at $\alpha = 1$. However, this comes with increased energy burden, reflected by a higher Gini coefficient (0.121) and a rise in the 95th percentile. The PMD-based framework prioritizes loss reduction (3.65%) but produces comparatively higher inequality metrics than most COT configurations.

Notably, $\alpha = 0.6$ and 0.8 provide well-balanced outcomes. At $\alpha = 0.6$, active power losses drop to 3.87% while maintaining a low Gini coefficient of 0.057 and a 95th-percentile burden of 1.96. Similarly, $\alpha = 0.8$ further reduces losses to 3.55% with only modest increases in dispersion metrics (95th percentile of 2.08 and Gini of 0.061). Overall, the results show that the proposed COT framework enables planners to systematically regulate the trade-off between network efficiency and affordability metrics, with intermediate α values emerging as practical operating regions. The proposed COT-based framework is implemented in Python and executed on a standard Intel i9 processor-based PC, and results show a 33% reduction in computation time compared to Julia-based PMD framework. These findings demonstrate that the proposed COT-based planning framework effectively coordinates loss reduction with improved energy-burden distribution while remaining computationally efficient for large-scale studies. The ability to regulate planning outcomes through the parameter α further supports its suitability for practical distribution system planning and scenario evaluation.

3.5 Summary

This chapter establishes the foundational planning layer of the proposed holistic resilience framework. By enabling distribution-aware DER placement strategies, the methodologies developed herein contribute to reducing the severity of performance degradation during disturbance events and improving the preparedness of distribution systems for resilient operation. The subsequent chapters build upon this planning foundation to address operational robustness, functional interdependencies, and higher-order resilience characteristics.

Chapter 4

Robust Operation and Control of Distribution Systems

Modern power distribution systems are increasingly required to operate under uncertain and dynamic conditions driven by the proliferation of distributed energy resources, variable demand patterns, and evolving grid architectures. In such environments, the ability of the system to maintain reliable operation and recover rapidly from disturbances has become a defining characteristic of resilient infrastructure. Achieving rapid restoration of system functionality following abnormal events requires accurate situational awareness, timely decision-making, and coordinated control strategies. Consequently, operational resilience of distribution systems depends heavily on the availability of reliable system state information and robust control mechanisms.

Distribution system state estimation (DSSE) plays a central role in enabling real-time situational awareness by providing estimates of critical system variables such as voltage magnitudes, phase angles, and power flows. Accurate state information allows operators to detect abnormal conditions, initiate corrective actions, and guide system recovery processes following disturbances. However, distribution systems are typically characterized by limited measurement infrastructure, heterogeneous data sources, and incomplete network information. These limitations significantly degrade observability and increase uncertainty in state

estimation, potentially delaying corrective actions and prolonging recovery times during fault or contingency events.

Another critical component of operational awareness is accurate phase identification, which ensures correct association of loads and measurements with their respective phases. Incorrect phase labeling can lead to inaccurate state estimation, improper control actions, and cascading operational errors. In the context of resilient system operation, reliable phase identification enhances the integrity of measurement data and improves the accuracy of downstream decision-making processes. Therefore, integrated approaches that jointly address state estimation and phase identification are essential for maintaining robust operation in low-observability environments.

Beyond monitoring and estimation, coordinated control strategies are necessary to actively stabilize the system and restore service following disruptions. Multi-layered control mechanisms, including primary, secondary, and tertiary control levels, enable distributed resources and network devices to respond dynamically to disturbances. Furthermore, community-centric operational frameworks, such as coordinated microgrid management and transmission–distribution interactions, provide additional flexibility by leveraging localized resources to support recovery efforts. These strategies contribute to minimizing the duration of service interruptions and enhancing the system’s ability to recover from disturbances in a timely manner.

This chapter develops a unified framework for robust distribution system operation that integrates advanced statistical learning techniques with coordinated control strategies. Specifically, Fréchet regression-based methodologies are introduced to perform distribution-aware state estimation and phase identification under limited observability conditions. A joint framework is further developed to simultaneously address phase identification and DSSE, improving reliability and computational efficiency. In addition, hierarchical control strategies and community-centric operational frameworks are examined to support adaptive and resilient system recovery. Collectively, the methodologies presented in this chapter contribute to reducing operational uncertainty, improving situational awareness, and enhancing the recovery capability of modern distribution systems.

4.1 Background

The distribution system can be represented as a tree-structured graph consisting of substations, feeders, laterals, transformers, and end-use customers Fig. 4.1. The electrical paths from the substation to individual customers are unique due to the radial topology. Typically, primary feeders operate as three-phase circuits, while laterals branching from the feeders may be single-phase, two-phase, or three-phase. Distribution transformers connected at the ends of laterals step down voltages to supply customers. In practice, the phase labels of these laterals are often incorrectly documented, outdated, or entirely unknown to the utility. Consequently, the true phase connectivity of customers downstream of a lateral may be uncertain. This lack of accurate phase information affects system balancing, state estimation, and operational planning.

4.1.1 State Estimation

A power distribution system is defined by the electrical interconnections among buses and phases, which determine the flow of power throughout the network. Let N denote the total number of phases across all buses in the system. The system behavior is fully characterized by its nodal voltage and branch current phasors, which constitute the state variables. In distribution system state estimation (DSSE), the primary objective is to estimate the nodal voltage phasors V_i using the available measurements. The relationship between the state variables $\mathbf{y}$ and the measurements $\mathbf{x}$ is nonlinear and can be expressed as follows:

$$\mathbf{x} = \mathcal{H}(\mathbf{y}) + \epsilon \quad (4.1)$$

where $\mathbf{y} = [|\mathbf{v}|, \theta, \mathbf{p}, \mathbf{q}]' \in \mathbb{R}^{4N}$, is the measurement vector containing voltage magnitude $|\mathbf{v}|' \in \mathbb{R}^N$, phase angle $\theta' \in \mathbb{R}^N$, real power $\mathbf{p}' \in \mathbb{R}^N$ and reactive power $\mathbf{q}' \in \mathbb{R}^N$, respectively, $\mathbf{x} \in \mathbb{R}^M$ is the measurement vector, $\mathcal{H} : \mathbb{R}^{4N} \rightarrow \mathbb{R}^M$ is the nonlinear relationship between the state and measurement vector; ϵ is the measurement noise. Ideally, the Newton–Raphson

(NR) method can be used to obtain an exact solution to the system of equations in (4.1) when it is fully determined (i.e., the number of equations M equals the number of unknowns $4N$) [28]. However, in practical distribution systems, real and reactive power measurements are not available at all customer nodes, resulting in an underdetermined problem. Therefore, the main goal of DSSE is to estimate $\mathbf{y}$ with the knowledge of $\mathbf{x}$ and the non-linear function $\mathcal{H}(\mathbf{y})$. This can be achieved by linearizing $\mathcal{H}(\mathbf{y})$ around an operating point [141], as

$$\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{w} \quad (4.2)$$

where $\mathbf{y} \in \mathbb{R}^{2N}$ is the vector of voltage magnitudes and phase angles of all the N nodes. The matrix $\mathbf{M}$ is given by,

$$\mathbf{M} = (Y_{LL}^{-1}diag(\bar{\mathbf{w}})^{-1}, -jY_{LL}^{-1}diag(\bar{\mathbf{w}})^{-1})$$

where $\mathbf{w} = -Y_{LL}^{-1}Y_{L0}\mathbf{v}_0$ is the zero-load voltage, $\mathbf{v}_0$ denotes the slack bus three-phase nodal voltage. $Y_{LL} \in \mathbb{C}^{N \times N}$, $Y_{L0} \in \mathbb{C}^{N \times 3}$, $Y_{00} \in \mathbb{C}^{3 \times 3}$ and Y_{0L} are the block matrices of the network admittance matrix Y defined as,

$$Y = \begin{bmatrix} Y_{00} & Y_{0L} \\ Y_{L0} & Y_{LL} \end{bmatrix} \in \mathbb{C}^{N \times N}$$

The solution of (4.2) can be obtained using the weighted least square (WLS) method [142]. However, for an underdetermined system (2), the WLS method is unsuitable due to convergence issues and the need to compute the inverse of the Jacobian matrix. This inversion is computationally expensive and, in some cases, infeasible because of the limited availability of real-time measurements. Therefore, compressive sensing-based techniques find their application in DSSE [143]. In these techniques, low observability in DSSE is addressed by exploiting the correlation among the measurements and system states. In particular, the

distribution system states reveal spatial and temporal correlation due to geographical proximity and correlations in power injections and consumption patterns at each bus [144]. However, compressive sensing-based methods often do not scale well to large-scale distribution systems and typically rely on assumed prior distributions to quantify uncertainty, which may be unsuitable for real-time operation. In addition, such approaches generally model system variables within a Euclidean framework, implicitly assuming that standard linear operations and distance metrics are valid. This assumption is restrictive, as several key quantities, such as correlation matrices and graph-based representations of network structure, naturally reside in non-Euclidean spaces with inherent geometric constraints. Ignoring this structure can distort the underlying relationships and degrade estimation performance, particularly under conditions of low observability and measurement noise.

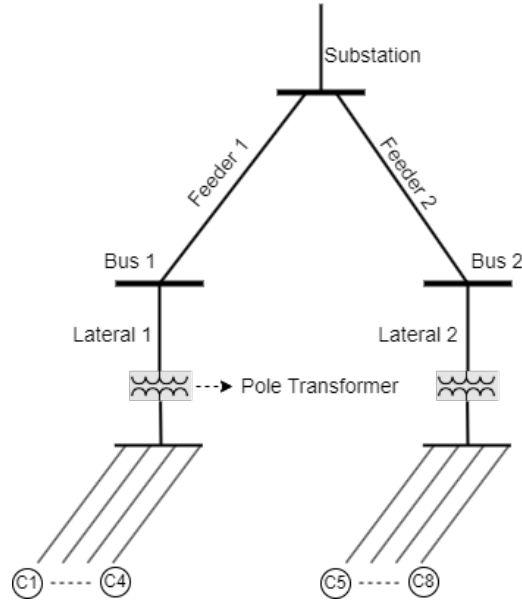


Figure 4.1: Representation of Network Topology with Laterals.

4.1.2 Phase Identification

The power distribution system as shown in Fig. 4.1 can be assumed as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{A})$, where $\mathcal{V} \in \mathbb{R}^N$ denotes the vertices, $\mathcal{E}$ are the edges, and $\mathbf{A} \in \mathbb{R}^{N \times N}$ is the adjacency matrix. The adjacency matrix for an undirected graph is a symmetric matrix. The

problem of phase identification can be mapped to the problem of discovering the adjacency matrix $\mathbf{A}$.

Consider a matrix of voltage magnitude measurements $V \in \mathbb{R}^{N \times K}$, where N is the total number of phases and K represents the number of time samples. The matrix $V = [V_1^{p_1}, \dots, V_n^{p_n}]$, where $V_i^{p_i}$ denote the voltage measurements at bus i corresponding to its set of phase p_i . For example, for a three-phase bus, $V_i^{p_i} = [V_i^a, V_i^b, V_i^c]$, where V_i^ϕ represents the phase-to-neutral voltage magnitude time-series for phase $\phi \in \{a, b, c\}$. To characterize the phase connectivity between buses, let q_i and t_j specify the two terminals between bus i and bus j , $q_i \in \mathbb{R}^{p_i}$ is the source terminal and $t_j \in \mathbb{R}^{p_j}$ is the target terminal. For instance, in Fig. 4.2, $q_i = [q_i(1), q_i(2), q_i(3)]$ and $t_j = [t_j(1), t_j(2), t_j(3)]$. The vectors q and t are the concatenated vectors of q_i and t_j will correspond to the phases p_i and p_j on the buses i and j , respectively. The phase connectivity between buses i and j varies based on the type of line configuration, which may be single-phase, two-phase, or three-phase. Several existing methods exploit voltage correlations and spectral properties to infer phase labels in distribution systems [145]. However, these approaches typically treat voltage measurements as Euclidean data and rely on linear distance metrics. In reality, many power system quantities, such as correlation structures and network connectivity representations, reside on non-Euclidean manifolds with inherent geometric constraints. Ignoring this structure can distort the underlying relationships and lead to inaccurate phase identification, ultimately resulting in erroneous reconstruction of the network connectivity.

4.1.3 Fréchet Regression Fundamentals

Many power system datasets are more naturally represented as probability distributions or structured matrices rather than Euclidean vectors. Consequently, classical regression models, whether parametric or nonparametric, are often inadequate for capturing the underlying relationships between such structured responses and the associated covariates in power system applications. In contrast, Fréchet regression for random objects with Euclidean predictors provides a principled framework to model these relationships by operating directly in non-

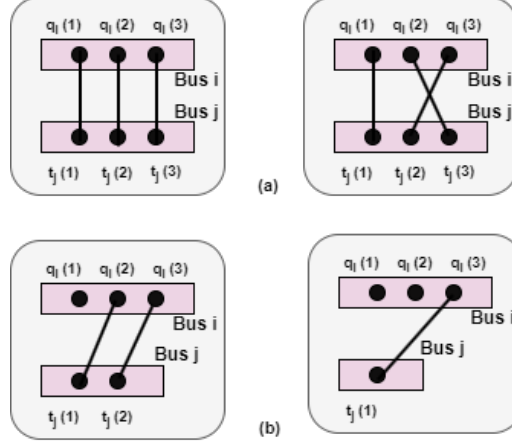


Figure 4.2: Different possible configurations for a two-bus connection. (a) Three-phase end (b) Two-phase end and One-phase end .

Euclidean spaces. In particular, distributional regression based on optimal transport theory establishes a connection between the conditional Fréchet mean of the response distribution and the covariates through an optimal transport mapping, enabling a geometry-aware characterization of the dependence structure.

To formalize the problem, let $\mathbf{x}$ denote a vector of covariates that influence a random power system object $\mathbf{F}$. For instance, $\mathbf{F}$ may represent a complete state measurement matrix $\mathbf{y}$, a time-varying voltage vector $\mathbf{v}_t$ or a correlation matrix $\mathbf{C}$. The covariate vector $\mathbf{x}$ may include available system measurements, load forecasts, and/or renewable generation forecasts.

Given historical observations $\{(\mathbf{x}_i, \mathbf{F}_i)\}_{i=1}^K$, Fréchet regression models the relationship between $\mathbf{x}$ and $\mathbf{F}$ by solving the following optimization problem [146]:

$$\mathbf{F}(\mathbf{x}) = \arg \min_{\mathbf{F} \in \mathcal{F}} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d^2(\mathbf{F}, \mathbf{F}_i) \quad (4.3)$$

where $\mathcal{F}$ is the probability distribution or matrix family in which the power grid data resides, and $d(.,.)$ is the Wasserstein metric defined on $\mathcal{F}$, $s(\mathbf{x}, \mathbf{x}_i)$ is a weight function to evaluate the influence of $(\mathbf{x}_i, \mathbf{F}_i)$ on the fitted $\mathbf{F}(\mathbf{x})$.

By preserving the intrinsic geometric structure of non-Euclidean data and accurately capturing the relationships between covariates and structured responses, Fréchet regression

provides a principled framework for both state estimation and phase identification. In the following section, this framework is developed and applied to the proposed problem setting.

4.2 Joint Phase Identification and DSSE Using Fréchet Regression

Power distribution systems are typically partially observable due to the limited availability of measurements. In this section, we propose a unified framework to estimate both the system states and phase labels. Estimation of spatiotemporal states in an unobservable distribution system can be achieved using the Fréchet regression as discussed in subsection 4.1. The regression for DSSE needs accurate phase labels to identify the system states accurately. First, phase identification is performed using Fréchet regression to estimate the network adjacency matrix, which reveals the underlying feeder topology and phase connectivity. This inferred topology is then used to construct the covariance matrix of the voltage states $\mathbf{y}$. Next, DSSE is carried out using the Fréchet Regression with the available topology and estimated covariance information, to obtain the voltages at all the nodes in the distribution system.

In this work, we aim to estimate the system states using Fréchet regression [146]. Consider $\mathbf{x}_i = [\mathbf{p}, \mathbf{q}]' \in \mathbb{R}^M$ as the vector of active power and reactive power measurements, and let $\mathbf{y}_i = \mathcal{N}_N(\mu, \Sigma)$ be the voltage magnitude distributions that are estimated from the historical data. The DSSE using Fréchet Regression is formulated as;

$$\arg \min_{\mathbf{y} \in \omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d^2(\mathbf{y}, \mathbf{y}_i) \quad (4.4)$$

where $\mathbf{x}$ is a vector of available active and reactive power measurements and $\mathbf{y}$ is the vector of voltage magnitude distributions at each node for the given $\mathbf{x}$; ω is the space of node voltage magnitude probability distributions. For probability distributions, d^2 is often the Wasserstein-2 distance, $s(\mathbf{x}, \mathbf{x}_i)$ is influence of $\mathbf{x}$ on $\mathbf{y}$ formulated as a Gaussian kernel, given by,

$$s(\mathbf{x}, \mathbf{x}_i) = \frac{\exp\left(-\frac{2}{h^2} \|\mathbf{x} - \mathbf{x}_i\|_2^2\right)}{\sum_{j=1}^K \exp\left(-\frac{2}{h^2} \|\mathbf{x} - \mathbf{x}_j\|_2^2\right)} \quad (4.5)$$

Accurate state estimation in distribution systems critically depends on the availability of correct network connectivity and phase labeling information. The relationship among the system states is inherently governed by the underlying network structure, including phase connections and line parameters. Any inaccuracies in the structural information can propagate through the estimation process, leading to biased or physically inconsistent voltage estimates. This issue is particularly pronounced in low-observability settings, where limited measurements make the estimation problem more sensitive to modeling errors. Therefore, accurate identification of network connectivity and phase labels is critical, as errors in these directly propagate to state estimation and lead to unreliable system states.

Motivated by this interdependence, the phase identification problem can be formulated as the problem of discovering the underlying network connectivity structure. The phase identification problem can be reduced to the problem of recovering the adjacency matrix $\mathbf{A}$. Consider a vector of active power and reactive power measurements $\mathbf{x}_i = [\mathbf{p}, \mathbf{q}]' \in \mathbb{R}^M$, and $\mathbf{Z}_i \in \mathbb{R}^{N \times N}$ be the adjacency matrix, this matrix $\mathbf{Z}_i$ reveals the network phase connectivity. The valid adjacency matrix $\mathbf{A}$ for a given $\mathbf{x}$ can be obtained using the Fréchet Regression as follows [146];

$$\arg \min_{\mathbf{A} \in \Omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d_F^2(\mathbf{A} - \mathbf{Z}_i) \quad (4.6)$$

where K represents the total number of samples, Ω is the space of physically valid adjacency matrices (PSD, symmetric, sparse), and d_F^2 is Frobenius metric ($\|\mathbf{A} - \mathbf{Z}_i\|_F^2$); $s(\mathbf{x}, \mathbf{x}_i)$ is influence of $\mathbf{x}$ on $\mathbf{A}$ formulated as Gaussian kernel, given by,

$$s(\mathbf{x}, \mathbf{x}_i) = \frac{\exp\left(-\frac{2}{h^2} \|\mathbf{x} - \mathbf{x}_i\|_2^2\right)}{\sum_{j=1}^K \exp\left(-\frac{2}{h^2} \|\mathbf{x} - \mathbf{x}_j\|_2^2\right)} \quad (4.7)$$

This approach is equivalent to projecting the sample adjacency matrices onto the space of the true adjacency matrix. In general, the distribution system graph is not fully connected

and is usually sparse. Thus, the original graph is obtained by solving the optimization problem as follows:

$$\arg \min_{\mathbf{A} \in \Omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d_F^2(\mathbf{A} - \mathbf{Z}_i) + \lambda \rho(\mathbf{A}) \quad (4.8)$$

where λ is a regularization parameter, $\rho(\mathbf{A})$ includes the sparsity term $\|\text{vec}(\mathbf{A})\|_1$. Additionally, some phase labels ($\mathbf{A}_{known} \in \mathbb{R}^{m \times n}$) are known to the utility along with the link constraints. The Link constraints include must-link constraints and cannot-link constraints. Based on the network topological information known to the utilities, must-link constraints require that there be exactly one connection between any two connected buses [147]. Similarly, cannot-link constraints specify that different phases on the same bus cannot be connected i.e., the corresponding elements in the adjacency matrix should be set to zero. Including these necessary constraints, the optimization problem is given as;

$$\arg \min_{\mathbf{A} \in \Omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d_F^2(\mathbf{A} - \mathbf{Z}_i) + \lambda \rho(\mathbf{A}) + \beta \|P_{\Omega}(\mathbf{A} - \mathbf{A}_{known})\|_F^2 \quad (4.9)$$

Subject to

$$0.5 \leq \sum_{m=1}^{p_j} \mathbf{A}(q_i(l), t_j(m)) \leq 1, \quad \forall i, \forall j, i \sim j \quad (4.10)$$

$$0.5 \leq \sum_{l=1}^{p_i} \mathbf{A}(q_i(l), t_j(m)) \leq 1, \quad \forall i, \forall j, i \sim j \quad (4.11)$$

$$\mathbf{A}(q_i(l), q_i(m)) \leq 0, \quad \mathbf{A}(t_i(l), t_i(m)) \leq 0 \quad (4.12)$$

Here, (4.10), (4.11) are the must-link constraints between two connected buses i and j as shown in Fig. 4.2, (4.12) imposes the cannot-link constraints. The operator P_{Ω} is defined as,

$$[P_{\Omega}(\mathbf{A}_{\text{known}})]_{mn} = \begin{cases} [\mathbf{A}_{\text{known}}]_{mn}, & \text{if } (m, n) \in \Omega, \\ 0, & \text{otherwise.} \end{cases} \quad (4.13)$$

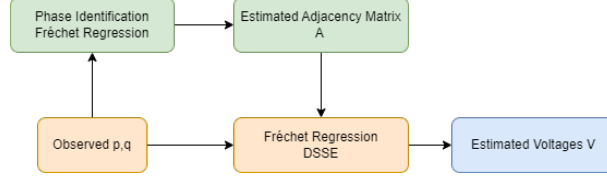


Figure 4.3: DSSE and Topology coupled learning.

The proposed formulation highlights the strong coupling between phase identification and state estimation in low-observable distribution systems. By leveraging Fréchet regression, both the network connectivity structure and the system states are modeled as random objects, enabling a unified treatment of topology and state estimation within a non-Euclidean framework as shown in Fig. 4.3. The joint or coupled learning using the Fréchet regression is particularly advantageous, as the adjacency structure and voltage distributions inherently reside in structured spaces where conventional Euclidean methods may fail to capture the underlying geometry. The joint estimation procedure, summarized in Algorithm 2, iteratively refines the phase labels and voltage states, where the inferred topology improves state estimation accuracy, and the updated state distributions, in turn, enhance topology recovery. This mutual reinforcement leads to a consistent and physically meaningful solution even under limited measurements and model uncertainties. Consequently, the proposed Fréchet regression-based joint framework provides a robust, scalable, and geometry-aware approach for reliable monitoring of distribution systems under low-observability conditions.

4.3 Control Strategies in Distribution Systems

4.3.1 Primary, Secondary, and Tertiary Controls

Modern microgrids (MGs) comprise diverse distributed generation (DG) units, including renewable resources and conventional generators, each characterized by distinct generation

Algorithm 2 Joint Topology and State Estimation in Unobservable Distribution Systems

- 1: **Input:** Limited measurements $\mathbf{x} = [\mathbf{p}, \mathbf{q}]^T$, known phase labels $\mathbf{A}_{known}$, regularization parameters λ, β .
- 2: Initialize adjacency matrix $\mathbf{A}^{(0)}$.
- 3: **for** $k = 0, 1, 2, \dots$ until convergence **do**
- 4: **Topology Update:** Solve Fréchet regression:

$$\mathbf{A}^{(k+1)} = \arg \min_{\mathbf{A} \in \Omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d_F^2(\mathbf{A}, \mathbf{Z}_i) + \lambda \rho(\mathbf{A})$$

$$+ \beta \|P_{\Omega}(\mathbf{A} - \mathbf{A}_{known})\|_F^2$$

subject to

$$0.5 \leq \sum_{m=1}^{p_j} \mathbf{A}(q_i(l), t_j(m)) \leq 1, \quad \forall i, j, i \sim j$$

$$0.5 \leq \sum_{l=1}^{p_i} \mathbf{A}(q_i(l), t_j(m)) \leq 1, \quad \forall i, \forall j, i \sim j$$

$$\mathbf{A}(q_i(l), s_i(m)) \leq 0, \quad \mathbf{A}(q_i(l), t_i(m)) \leq 0$$

- 5: **Covariance Update:**
 Construct the topology-aware covariance matrix:
 $\Sigma^{(k+1)} = f(\mathbf{A}^{(k+1)})$
 and update voltage distributions:

$$\mathbf{y}_i^{(k+1)} \sim \mathcal{N}(\boldsymbol{\mu}_i, \Sigma^{(k+1)})$$

- 6: **State Update:**
 Solve Fréchet regression for DSSE:

$$\mathbf{y}^{(k+1)} = \arg \min_{\mathbf{y} \in \omega} \sum_{i=1}^K s(\mathbf{x}, \mathbf{x}_i) d^2(\mathbf{y}, \mathbf{y}_i^{(k+1)})$$

- 7: Check convergence:

$$\|\mathbf{A}^{(k+1)} - \mathbf{A}^{(k)}\|_F < \epsilon$$

- 8: **end for**
 - 9: **Return:** $\mathbf{A}^*, \mathbf{y}^*$
-

capacities, operational costs, and reliability characteristics. Ensuring stable and efficient operation of such heterogeneous systems requires hierarchical control architectures that coordinate power sharing, voltage regulation, and system-wide optimization. Typically, MG operation is structured into three control layers: primary, secondary, and tertiary, each operating over different time scales and levels of system awareness. Primary control provides fast local regulation of voltage and frequency using decentralized mechanisms such as droop control, enabling immediate response to disturbances without requiring communication. Secondary control restores system variables to nominal operating conditions through coordinated adjustments across distributed resources, while tertiary control performs higher-level optimization by incorporating system demand, resource availability, and economic objectives to determine optimal operating setpoints.

While these hierarchical control strategies ensure stable MG operation under normal conditions, emerging distribution systems with high penetration of DERs and community-driven objectives require enhanced coordination mechanisms that extend beyond traditional operational goals. In particular, resilience-oriented operation demands coordinated decision-making across interconnected microgrids, where resource scheduling, energy sharing, and operational priorities must reflect both system-level constraints and community-level needs. Motivated by these requirements, the following sections develop community-centric operational frameworks that build upon conventional hierarchical control structures to enable coordinated microgrid operation with explicit consideration of community-level performance and resilience objectives.

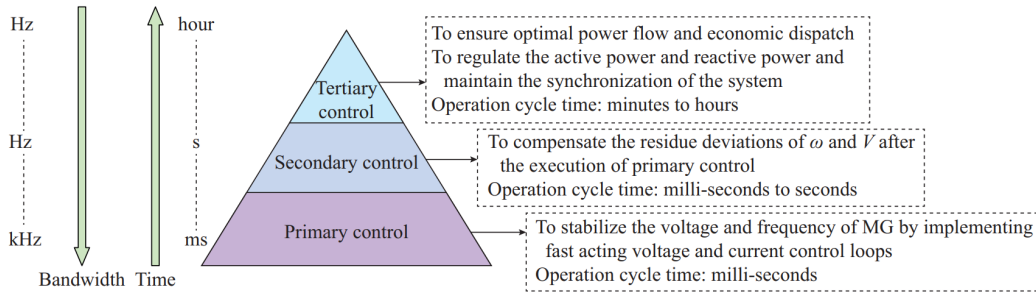


Figure 4.4: Hierarchical control of MG.

4.4 Community-Centric Distribution Systems Operational Frameworks

4.5 Microgrids Coordination with Community Considerations

Consider a network of $\mathcal{M}$ interconnected MGs $\mathcal{M} = \{1, 2, \dots, M\}$ that are individually managed and operated. Each microgrid $m \in \mathcal{M}$ has local solar PV generation and battery energy storage system (BESS). Also, assume each MG serves communities with diverse income levels. The MG operator can procure energy from various resources to meet their respective community loads. We assume that trading among the MGs as well as coordination with the distribution utility is enabled via a communication network infrastructure. The multi-microgrid (MMG) energy management framework is conceptually illustrated in Fig. 4.5.

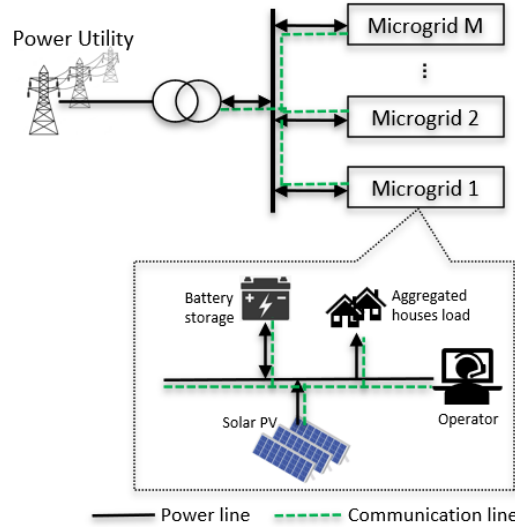


Figure 4.5: Energy management architecture in a MMG setup.

The goal of MMG energy management is to optimize hourly power dispatch decisions to balance supply and demand and minimize the total cost of energy in each microgrid. We assume that the scheduling of microgrid resources is based on the predictions of daily profiles

of utility electricity price, load, and solar PV profile. We consider an operation period of one day, on an hourly basis $T = 24$ equal time slots denoted as $\mathcal{T} = \{1, 2, \dots, T\}$ and estimate the schedules day ahead. In the process of optimizing dispatch decisions, the energy burden of the communities that are served by MGs is also taken into consideration. This is a unique aspect of this work. Energy burden is defined as the ratio of the total cost of energy to the total income level of that considered community, i.e.,

$$E = \frac{\text{Total cost of energy for a microgrid community}}{\text{Total income level of the microgrid community}} \quad (4.14)$$

Before presenting the non-coordinated and coordinated modes of operations in the subsequent sections, we discuss power scheduling aspects within each microgrid.

The power supply in each MG m can be categorized into local renewable power and power from the main grid. Various renewable energy technologies, such as solar PV, wind, and biomass, exist. However, we will focus on solar PV generation due to its low cost and availability of data. It is important to note that the mathematical framework can be easily extended to include other renewable energy resources. We assume that each MG m has installed solar PV generation of $P_{max,m}^{PV}(kW)$ and the constraint for the solar power supply is $0 \leq P_{t,m}^{PV} \leq P_{max,m}^{PV} \forall t \in \mathcal{T} \quad \forall m \in \mathcal{M}$. If the local solar PV generation is not able to meet the served community demand, MG m can buy power from the main grid. Let $P_{t,m}^u$ denote the power bought from the main grid by MG m , $\forall t \in \mathcal{T}$.

We assume that each MG m has installed BESS with a maximum state of charge as $SoC_{max,m}$ and at any given time, the battery will either charge or discharge. The respective charging and discharging constraints in each MG m are

$$P_{t,m}^{\text{batt}} = P_{t,m}^{\text{cha}} - P_{t,m}^{\text{dis}} \quad (4.15)$$

$$P_{t,m}^{\text{cha}} \leq b_{t,m} P_{\max,m}^{\text{cha}} \quad (4.16)$$

$$P_{t,m}^{\text{dis}} \leq (1 - b_{t,m}) P_{\max,m}^{\text{dis}} \quad (4.17)$$

$$SoC_{t,m} = SoC_{t-1,m} + \left(\eta^{\text{cha}} P_{t,m}^{\text{cha}} - \frac{P_{t,m}^{\text{dis}}}{\eta^{\text{dis}}} \right) \Delta t \quad (4.18)$$

$$SoC_{\min,m} \leq SoC_{t,m} \leq SoC_{\max,m} \quad (4.19)$$

$$b_{t,m} \in [0, 1] \quad (4.20)$$

where, $P_{t,m}^{\text{batt}}$ and $SoC_{t,m}$ are the net battery power and state of charge (SoC) of the battery for microgrid m at time t ; $P_{t,m}^{\text{cha}}$ and $P_{t,m}^{\text{dis}}$ are the charging and discharging battery powers; the charging and discharging efficiencies are captured by η^{cha} and η^{dis} ; $b_{t,m}$ is the binary variable that allows the battery to either charge or discharge; $P_{\max,m}^{\text{cha}}$ and $P_{\max,m}^{\text{dis}}$ are the maximum allowable charging and discharging powers.

Typically, MG energy management formulations consider the demand response while modeling the MG load. Here we consider the simple day ahead load profile $P_{t,m}^L, \forall t \in \mathcal{T}, \forall m \in \mathcal{M}$.

In the next section, two energy trading scenarios are presented. In both scenarios, the utility provides power to all the MGs in the network, and above a certain power level $P_{\max}^u$ the penalty will be levied. However, it is not equitable to penalize customers who have less demand and lower income levels. This aspect is explicitly considered in the proposed formulation.

We consider two modes of operation for MMG energy management. In the first mode, energy trading is allowed between utility and MGs. In the second mode, energy trading is allowed among the MGs along with the utility. For each mode of operation, the equitable scheduling of utility power (P_m^u), selling power to utility from MG (P_u^m), selling/buying power among the microgrids (P_n^m) and with the explicit consideration of energy burden (E) for each community is discussed.

4.5.1 Energy Trading Between Utility and Microgrids

In this mode of operation, each microgrid operator has the opportunity to buy or sell power on an hourly basis from/to the utility only. This is considered as the non-coordinated case. Usually, the utilities sell power at a higher price to the MG operator and buy power at lower prices from the MG operator. The respective costs are represented as $C_{t,m}^u$ and $C_{t,u}^m$. If the MGs aggregate demand exceeds a specified limit, the cost incurred on the excess power $P_{t,m}^p$ is $C_{t,m}^p$. This formulation effectively minimizes the overall cost of energy in a day. The optimization formulation is sum of operational cost of a microgrid over a period of T (typically 24 hours).

$$C(P) = \sum_{t=1}^T \left(C_{t,m}^u P_{t,m}^u + C_{t,m}^p P_{t,m}^p - C_{t,u}^m P_{t,u}^m \right) \quad \forall m \in \mathcal{M} \quad (4.21)$$

Where $P_{t,m}^u$ is the power bought by m^{th} MG from the utility to meet the local demand, and $P_{t,u}^m$ is the excess power sold to the utility in time slot t . Note that the MG cannot buy and sell power at the same time. Therefore, either $P_{t,m}^u$ or $P_{t,u}^m$ will be zero at all times.

The goal of energy management for noncoordinated MGs is $\mathbf{P}^* = \text{argmin} \quad C(P)$. Here, P^* is the set of optimal power schedules of the MGs i.e., optimal values of $P_{t,m}^u, P_{t,u}^m, P_{t,m}^{cha}, P_{t,m}^{dis}$ that minimizes $C(P)$.

Each microgrid should satisfy the power balance constraint at all time intervals, with the available resources. That is,

$$P_{t,m}^{PV} + P_{t,m}^u + P_{t,u}^m - P_{t,m}^{batt} = P_{t,m}^L \quad \forall t \in \mathcal{T}, \forall m \in \mathcal{M} \quad (4.22)$$

Where $P_{t,m}^{PV}$, $P_{t,m}^{batt}$ and $P_{t,m}^L$ are the solar PV power, battery power and load of m^{th} microgrid. The battery and solar PV should satisfy the constraints. From the power balance equation, if the sum of solar PV generation and the battery schedule is greater than the power demand in MG m , this excess amount of power is sold to the utility grid at a price of $C_{t,u}^m$.

Let C_m^a be the total cost of energy incurred by each MG m within a considered time

duration after buying and selling power to the utility. When this cost is passed on to the customers, the effect of this transaction varies with the type of community served by the MGs. For example, we can categorize the communities based on the income level of people served. To quantify the effect of C_m^a on the community based on income level I_m , we use the energy burden index E_m^a that is defined as the ratio of cost of energy to the income level of microgrid community. That is, $E_m^a = \frac{C_m^a}{I_m}, \forall m \in \mathcal{M}$. From the literature on energy equity [20], energy burden is one of the important indices that can be used to quantify the operational resiliency of the MG. Hence, MG operator can directly address and enhance the operational resiliency of the particular community on a daily basis.

To reduce the energy burden of the community, we explicitly introduce the constraint on energy burden in the energy management formulation. Specifically, we enforce a limit for the energy burden represented as E^{max} , which should not exceed for all the considered MG communities. The final formulation for this noncoordinated case is summarized below:

$$\mathbf{P}^* = \underset{P}{\operatorname{argmin}} \quad C(P) \quad (4.23)$$

$$\begin{aligned} \text{subject to} \quad & E_m^a \leq E^{max} \quad \forall m \in \mathcal{M} \\ & (4.15) - (4.22) \end{aligned}$$

Note that E^{max} has to be chosen carefully for the convergence of the solution.

4.5.2 Energy Trading Among Microgrids

In this mode of operation, the MG operators can trade energy among themselves along with the utility. This means that the excess power from each MG m can be sold to the utility or to other MG's $\mathcal{M} \setminus m$. Thus the overall operational cost of energy in a day can be further minimized by coordination among MGs. We formulate the energy trading interactions among interconnected microgrids based on the principles of Nash bargaining theory [62].

Consider that each MG $m \in \mathcal{M}$ bargains with all the other interconnected MGs $n \in \mathcal{M} \setminus m$ to determine the amount of traded power $P_n^m = \{P_{t,n}^m \forall t \in \mathcal{T} \quad \forall n \in \mathcal{M} \setminus m\}, \forall m \in \mathcal{M}$. The associated cost of unit energy is $C_n^m = \{C_{t,n}^m \forall t \in \mathcal{T} \quad \forall n \in \mathcal{M} \setminus m\}, \forall m \in \mathcal{M}$. Here $P_{t,n}^m$

is the amount of power that microgrid m exchanges with MG n in time slot t , and $C_{t,n}^m$ is the associated price of power traded between the MGs m and n . If MG m sells power to MG n , in time slot t , then $P_{t,n}^m > 0$; otherwise, if MG m purchases power from MG n then $P_{t,n}^m < 0$. The energy trading among the microgrids should satisfy the market clearing constraints i.e.,

$$P_{t,n}^m + P_{t,m}^n = 0 \quad \forall t \in \mathcal{T}, \forall n \in \mathcal{M} \setminus m, \forall m \in \mathcal{M} \quad (4.24)$$

$$C_{t,n}^m P_{t,n}^m + C_{t,m}^n P_{t,m}^n = 0 \quad \forall t \in \mathcal{T}, \forall n \in \mathcal{M} \setminus m, \forall m \in \mathcal{M} \quad (4.25)$$

The new power balance constraint for each microgrid is expressed as

$$P_{t,m}^{PV} + P_{t,m}^u - P_{t,m}^{batt} - \sum_{n=1, n \neq m}^M P_{t,n}^m = P_{t,m}^L \quad \forall t \in \mathcal{T}, \forall m \in \mathcal{M} \quad (4.26)$$

Here, $\sum_{n=1, n \neq m}^M P_{t,n}^m$ is the net power traded between MG m and all other MGs $\mathcal{M} \setminus m$ in time slot t . If $\sum_{n=1, n \neq m}^M P_{t,n}^m < 0$ then MG m purchases energy from other MGs to serve its local demand; otherwise $\sum_{n=1, n \neq m}^M P_{t,n}^m > 0$ then MG m sells power to make a profit.

Though the interconnected microgrids cooperate and trade energy with each other, they are still independent. We assume that each MG is a self-interested rational decision maker who aims to minimize its total operating cost. The new optimization formulation for each MG is:

$$C'(P) = \sum_{t=1}^T \left(C_{t,m}^u P_{t,m}^u + C_{t,m}^p P_{t,m}^p - C_{t,u}^m P_{t,u}^m - \sum_{n=1, n \neq m}^M C_{t,n}^m P_{t,n}^m \right) \quad \forall m \in \mathcal{M} \quad (4.27)$$

The goal of energy management for coordinated MGs is $\mathbf{P}^* = \operatorname{argmin} C'(P)$. Here, P^* is

the set of optimal power schedules of the MGs i.e., optimal values of $P_{t,m}^u, P_{t,u}^m, P_{t,n}^m, P_{t,m}^{cha}, P_{t,m}^{dis}$ that minimizes $C'(P)$.

Let C_m^b be the total cost of energy of a day for each MG m , the corresponding energy burden in this case for each MG will be $E_m^b = \frac{C_m^b}{I_m}$. From the formulations (4.27), it is clear that an MG m will only participate in the trading if it can reduce the energy burden of the community compared to a non-coordinated mode of operation. This means that some MGs may choose not to participate in the trading. Thus, we have the constraint $E_m^b \leq E_m^a$. Where E_m^a and E_m^b are the energy burdens of the community MG m in modes of operation presented in subsections 4.51 and 4.52. In the bargaining literature, we can also call E_m^a the disagreement point. In the next section, we analyze the Nash Bargaining solution.

4.5.3 Nash Bargaining Formulation Analysis

Nash bargaining game is a cooperative game in which the players seek to distribute a surplus value among themselves through negotiation. Let players be $\mathcal{M}' \subseteq \mathcal{M}$ denote the set of MGs who are willing to trade energy with each other. For the MGs $j \in \mathcal{M} \setminus \mathcal{M}'$, they cannot benefit from energy trading and thus have no incentive to participate in the energy trading. Therefore, we focus on the MGs in set $\mathcal{M}'$, and formulate and study their energy trading as a Nash bargaining problem, which fulfills the axioms mentioned in [148]: A strategic cooperative game Γ is expressed as $\Gamma = \{\Omega, A, U\}$ and consists of following components:

1. Player set $\Omega : \Omega = 1, 2, \dots, M$ where M is the number of MG operators
2. Action set $A : a \in A = \prod_{m=1}^M A_m = A_1 \times A_2 \times \dots \times A_n$, where each component, a_m , of the action vector a belongs to set A_m , the action set of player m . In this case, the action set for each MG is schedules of powers $P_m = (P_m^u, P_m^{cha}, P_m^{dis}, P_n^m)$.
3. Utility $U : U_m : A \rightarrow \mathcal{R}$ is the utility (payoff) function of player m , which depends on the strategies of all players and $U = (U_1, U_2, \dots, U_M) : A \rightarrow \mathcal{R}^M$ denotes the utility vector of utility functions. In this case, the utility is to reduce the energy burden of the MG communities. That is, $U_m = f(P_m^u, P_u^m, P_m^{cha}, P_m^{dis}, P_n^m)$.

Nash Bargaining Solution (NBS) is the most common property of interest in bargaining game theory, NBS represents a unique action/allocation vector corresponding to the mutual best response for all players. In other words, at NBS, no individual player can benefit from unilateral deviation.

An action vector $\hat{a}$ is a NBS if, for every player m and every action vector a

$$U_m(\hat{a}_m, \hat{a}_{M \setminus m}) \geq U_m(a_m, \hat{a}_{M \setminus m}) \quad (4.28)$$

Theorem 2. *The Nash Bargaining Solution $(U_1^*, U_2^*, \dots, U_{M'}^*)$ is the point in the bargaining set B that maximizes the product of the players utilities:*

$$\mathbf{P}^* = \underset{P}{\operatorname{argmax}} \quad \left(U_1^* U_2^* \dots U_{M'}^* \right) \quad U_m \in B \quad (4.29)$$

subject to all the constraints from subsections II.A, III.A and III.B. Here, $\mathbf{P}^*$ is set of optimal values of $P_{t,m}^u, P_{t,u}^m, P_{t,n}^m, P_{t,m}^{cha}, P_{t,m}^{dis}, U_m = E_m^a - E_m^b$ corresponds to the energy burden reduction in MG m , i.e., the difference between disagreement point E_m^a from non-coordinated case and E_m^b through bargaining in coordinated case. Compared with the summation of performance improvement of MGs, the Nash product can guarantee that the benefits of cooperation are shared by each MG in a fair manner [62]. Each player from participating MGs $m \in \mathcal{M}'$ bargains with MGs $n \in \mathcal{M}' \setminus m$ to determine the amount of power trading $P_n^m = \{P_{t,n}^m \forall t \in \mathcal{T} \quad \forall n \in \mathcal{M}' \setminus m\}, \forall m \in \mathcal{M}'$ and the associated cost of unit energy is $C_n^m = \{C_{t,n}^m \forall t \in \mathcal{T} \quad \forall n \in \mathcal{M}' \setminus m\}, \forall m \in \mathcal{M}'$. The energy trading among the microgrids should satisfy the market clearing constraints (4.24),(4.25). The utility function defined is an affine function and is concave. As a concave game admits at least one NBS [149], players involved in the cooperative bargaining game settle at an equilibrium point. The NBS can be characterized using a set of axioms.

Theorem 3. *Nash's bargaining solution is the unique bargaining solution that is Pareto*

efficient, symmetric, invariant under positive affine transformations and independent of irrelevant alternatives.

Proof: Formally, a bargaining problem is a pair (F, d) where $F \subseteq \mathcal{R}^N$ is a convex set of feasible outcomes and $d \in F$ (E_m^b) is a disagreement point that results if players fail to agree on an outcome. The objective function presented in (4.28) is concave and therefore has a unique maximum.

We denote the NBS by f^N and begin by showing that it satisfies the axioms. Also, we assume two MG operators in proving the NBS.

For Pareto efficiency, this follows directly from the fact that the objective function is increasing in E_1^b & E_2^b , let them be v_1 & v_2 , thus $F = \{(v_1, v_2) \in \mathcal{R}^2\}$.

For symmetry, assume that $d_1 = d_2$ and let $v^* = (v_1^*, v_2^*) = f^N(F, d)$. Clearly, v_2^*, v_1^* maximizes the objective function, and by the uniqueness of the optimal solution $(v_2^*, v_1^*) = (v_1^*, v_2^*)$ and thus $f_1^N(F, d) = f_2^N(F, d)$.

For invariance under positive affine transformation, define F' and d' as above, and observe that $f^N(F', d')$ is an optimal solution of the problem to maximize $(v_1 - \alpha_1 d_1 - \beta_1)(v_2 - \alpha_2 d_2 - \beta_2)$ subject to $v \in F, v_1 \geq d_1$, and $v_2 \geq d_2$. By setting $v' = \alpha \circ v + \beta$, it follows that $f^N(F', d') = \alpha \circ f^N(F, d) + \beta$.

For independence of irrelevant alternatives, let $v^* = f^N(F, d)$ and $F' \subseteq F$. If $v^* \in F'$, it remains optimal and thus $v^* = f^N(F', d)$.

Thus the proposed bargaining formulation satisfies all the necessary axioms and hence there exists an NBS for the coordinated MG energy management problem.

4.6 TSO/DSO Frameworks for Fair DERs Participation

Here, we discuss the Minimal DSO and Total DSO architectures, which primarily differ in their levels of coordination, communication, and control among the TSO, the distribution utility, and DERs. These differences determine how economic optimization and network

feasibility are integrated within the overall operational framework.

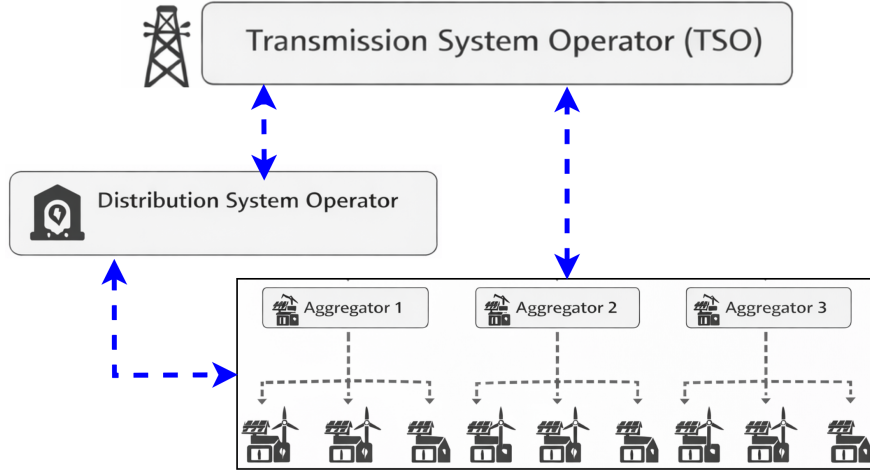


Figure 4.6: Minimal DSO Architecture

As shown in Fig. 4.6, the Minimal DSO architecture, communication between the TSO and the DSO/utility is limited and largely unidirectional. The TSO clears the wholesale market based on aggregated DER offers, and the DSO subsequently validates these dispatch results against the physical constraints of the distribution network. Any infeasibilities, such as voltage or thermal limit violations, are handled locally through curtailment or adjustment of DER outputs. Hence, this approach emphasizes simplicity and institutional separation but at the expense of full DER utilization and overall economic efficiency.

Conversely, the Total DSO architecture in Fig. 4.7 features a more advanced coordination mechanism, characterized by bidirectional communication and integrated decision-making between market and network layers. Here, the DSO directly incorporates both economic objectives (such as revenue maximization) and network constraints (such as voltage stability and line loading) into a single optimization framework. This unified control structure enables simultaneous congestion management and optimal DER dispatch, ensuring efficient resource utilization and minimizing post-market interventions.

Despite their structural differences, both architectures share a common goal, to maximize DER participation and economic returns while maintaining secure and reliable network operation. The choice between them depends on the desired level of institutional integration, data exchange, and operational complexity that a utility or regulatory framework can

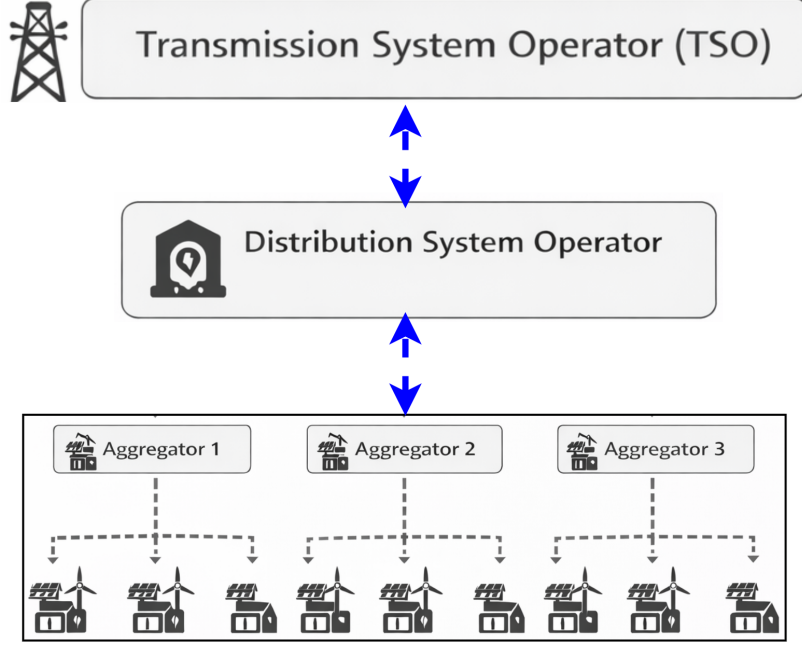


Figure 4.7: Total DSO Architecture

support. In practice, the Minimal DSO represents an incremental step toward active distribution management, while the Total DSO reflects a more comprehensive and future-oriented model aligned with high DER penetration scenarios.

Next, we details the optimization formulation for the Minimal and Total DSO architectures. Firstly, various DERs in the system are modeled, later these DER models will be considered as constraints in the OPF formulations. This work considers price response demand response, solar PV, and battery energy storage systems as DERs. The price-responsive demand response loads are modeled as;

$$p_{i,\phi}^{dr} = P_{i,\phi}^{dr} \quad \text{if } \rho_t \leq \rho_{b_1} \quad (4.30)$$

$$p_{i,\phi}^{dr} = \epsilon_{i,\phi}^1 P_{i,\phi}^{dr} \quad \text{if } \rho_{b_1} \leq \rho_t \leq \rho_{b_2} \quad (4.31)$$

$$p_{i,\phi}^{dr} = \epsilon_{i,\phi}^2 P_{i,\phi}^{dr} \quad \text{if } \rho_t \geq \rho_{b_2} \quad (4.32)$$

where $P_{i,\phi}^{dr}$ and $p_{i,\phi}^{dr}$ are the demand response load dispatches and flexible load availability at i^{th} node of ϕ^{th} phase. $\epsilon_{i,\phi}^1$ and $\epsilon_{i,\phi}^2$ are the fraction of flexible demand response load possible through the day-ahead optimization. ρ_{b_1} and ρ_{b_2} price bands. Model for the solar PVs with

the respective constraints is;

$$0 \leq p_{i,\phi}^s \leq P_{i,\phi}^s \quad (4.33)$$

where $p_{i,\phi}^s$ and $P_{i,\phi}^s$ are the possible solar PV dispatch and available capacity of generation at i^{th} node of ϕ^{th} phase. The BESS capacity and charge/discharge constraints are;

$$p_{i,\phi}^b = p_{i,\phi}^{dis} - p_{i,\phi}^{cha} \quad (4.34)$$

$$p_{i,\phi}^{cha} \leq \beta_{i,\phi} P_{i,\phi}^{cha} \quad (4.35)$$

$$p_{i,\phi}^{dis} \leq (1 - \beta_{i,\phi}) P_{i,\phi}^{dis} \quad (4.36)$$

$$SoC_{i,\phi} = SoC_{i,\phi}^{init} + \left(\eta^{cha} p_{i,\phi}^{cha} - \frac{p_{i,\phi}^{dis}}{\eta^{dis}} \right) \quad (4.37)$$

$$SoC_{i,\phi}^{min} \leq SoC_{i,\phi} \leq SoC_{i,\phi}^{max} \quad (4.38)$$

$$\beta_{i,\phi} \in [0, 1] \quad (4.39)$$

Here, $p_{i,\phi}^b$ is the BESS dispatch. $p_{i,\phi}^{dis}$ and $p_{i,\phi}^{cha}$ are the charging and discharging schedules; $\beta_{i,\phi}$ is a binary variable decides either to charge or discharge of BESS at i^{th} node of ϕ^{th} phase; $P_{i,\phi}^{dis}$ and $P_{i,\phi}^{cha}$ are the maximum discharging and charging capabilities of respective BESS; following their respective charge discharge efficiencies η^{cha} and η^{dis} ; State of charge $SoC_{i,\phi}$ follows the respective $SoC_{i,\phi}^{min}$ and $SoC_{i,\phi}^{max}$ constraints.

OPF Formulation:

$$\underset{P^{DER}}{\operatorname{argmin}} \sum_{i=1}^N \sum_{\phi=1}^{\Psi} \left(\rho(p_{\phi}^g) \right) \quad (4.40)$$

subject to

$$p_{i,\phi}^g + \sum_{i=1}^N p_{i,\phi}^b + \sum_{i=1}^N p_{i,\phi}^s = \sum_{i=1}^N p_{i,\phi}^l + \sum_{i=1}^N \sum_{j=1}^N p_{i,j,\phi} \quad \forall \quad t \quad (4.41)$$

$$p_{i,\phi}^l = p_{i,\phi}^{fix} + p_{i,\phi}^{dr} \quad (4.42)$$

$$p_{i,\phi}^g + p_{i,\phi}^b + p_{i,\phi}^s = p_{i,\phi}^l + \sum_{j=1}^N p_{i,j,\phi} \quad (4.43)$$

$$P_{i,\phi} = V_{i,\phi} \sum_{j=1}^N V_{j,\phi} Y_{i,j,\phi} \cos(-\theta_{i,j,\phi} + \delta_{i,\phi} - \delta_{j,\phi}) \quad (4.44)$$

$$Q_{i,\phi} = V_{i,\phi} \sum_{j=1}^N V_{j,\phi} Y_{i,j,\phi} \sin(-\theta_{i,j,\phi} + \delta_{i,\phi} - \delta_{j,\phi}) \quad (4.45)$$

$$P_{i,\phi} = p_{i,\phi}^g + p_{i,\phi}^b + p_{i,\phi}^s - p_{i,\phi}^l \quad (4.46)$$

$$p_{i,j,\phi} = V_{i,\phi} V_{j,\phi} Y_{i,j,\phi} \cos(-\theta_{i,j,\phi} + \delta_{i,\phi} - \delta_{j,\phi}) \quad (4.47)$$

$$V_{min} \leq V_{i,\phi} \leq V_{max} \quad (4.48)$$

Here, the objective (4.40) is to maximize the utility revenue by maximizing the utilization of DERs. ρ is the market price that decides the power derived from the grid $p_{i,\phi}^g$ for all the nodes $i \in N$ and phases $\phi \in \Psi$. $P^{DER} \in p^{dr}, p^b, p^s$ are the DER outcomes. The Constraints (4.41)-(4.48) ensure the network constraints, such as power balance, power flows, and voltage limits; $P_{i,\phi}, Q_{i,\phi}$ are the active and reactive power injections at i^{th} node of ϕ^{th} phase. $V_{i,\phi}$ is the voltage magnitude of i^{th} node of ϕ^{th} phase. The above optimization formulation is NP hard, and it is computationally intensive. In this work, the DSO models follow different approaches to solve the OPF based on the architecture limitations

4.6.1 Minimal DSO

In the Minimal DSO architecture, the distribution utility primarily functions as a network validator, ensuring that the operation of DERs does not compromise local network reliability. In this configuration, DERs or their aggregators directly participate in the wholesale market operated by the TSO/ISO, submitting bids and dispatch quantities based on market clearing results. The DSO subsequently receives the aggregated DER schedules or outputs and validates them against the distribution network constraints, such as voltage limits, thermal

ratings, and power flow feasibility.

If the validated dispatch results in network congestion, typically observed as voltage violations or line overloads, the DSO communicates the curtailed DER outputs back to the TSO/ISO. This feedback ensures that system reliability is maintained, albeit at the cost of reduced DER utilization. The congestion mitigation process in this architecture is often governed by rule-based or heuristic curtailment strategies, applied locally by the utility without co-optimization of market and network objectives.

Because the revenue optimization (by the market operator) and network validation (performed by the DSO) are decoupled, the overall decision-making process follows a sequential or decoupled OPF framework. While this approach simplifies coordination and preserves the autonomy of both entities, it leads to suboptimal economic outcomes and under-utilization of DER flexibility. As a result, the Minimal DSO architecture represents a pragmatic but limited coordination model, suitable for systems with moderate DER penetration where real-time congestion is infrequent. However, as DER proliferation increases, its limitations in achieving system-wide efficiency and flexibility utilization become more pronounced.

4.6.2 Total DSO

In the Total DSO architecture, the distribution utility assumes an integrated operational and market coordination role, performing both revenue maximization and network validation within a unified optimization framework. Unlike the Minimal DSO, where economic and physical feasibility analyses are performed sequentially and independently, the Total DSO model formulates a co-optimized problem that simultaneously considers market clearing objectives and distribution network constraints. This integrated approach ensures that DER dispatch decisions are both economically optimal and electrically feasible, thereby eliminating the need for post-market curtailment or rule-based adjustments.

By jointly optimizing DER participation and network operation, the Total DSO architecture enables efficient utilization of distributed flexibility resources, including generation, storage, and demand response. This results in improved system-wide efficiency, reduced con-

gestion, and enhanced reliability across the distribution network. Moreover, since the DSO manages both the technical and economic dimensions of DER coordination, it can implement fair and transparent allocation mechanisms, ensuring equal opportunity for all participating DERs. Such mechanisms promote equitable access to market revenues while maintaining adherence to local operational limits.

Additionally, the Total DSO model enhances the coordination between transmission and distribution systems by providing aggregated, network-aware flexibility offers to the TSO/ISO. This facilitates a more accurate representation of the distribution network’s operational capabilities in the wholesale market. Through this holistic framework, the Total DSO architecture not only improves DER owners’ satisfaction and market participation equity, but also contributes to the broader objectives of system reliability, economic efficiency, and sustainable grid operation.

4.7 Simulation Results and Discussion

In this section, the proposed approach is validated using the IEEE 37-bus and IEEE 123-bus test systems. For data generation, simulations are conducted over a 24-hour period with a 1-minute resolution. Multiple network configurations are considered to generate corresponding adjacency matrices, each representing a distinct historical network configuration. For instance, variations are introduced by modifying phase connections, operating alternate tie switches, and removing selected lines to represent maintenance conditions. The loads connected to each phase in both test systems are unequal, resulting in inherently unbalanced network conditions. These test feeders are selected due to their unbalanced nature, comprising a mix of single-phase, two-phase, and three-phase buses, thereby closely reflecting practical distribution systems. The proposed joint framework was implemented in Python and executed on a standard workstation equipped with an Intel i9 processor. Efficient numerical libraries were utilized to ensure scalability and computational efficiency. The obtained results demonstrate that the proposed approach is suitable for real-time or near real-time applications in large-scale distribution systems.

The performance of the proposed method is evaluated against the joint phase identification and state estimation framework based on the alternating minimization (AM) algorithm in [47], using the IEEE 37-bus and IEEE 123-bus systems under low-observability conditions. For 37-bus system, limited active and reactive power measurements are assumed to be available only at a subset of nodes, representing a partially observable network. As the measurement availability increases from 10% to 50%, while maintaining 60% known phase labels, the phase identification accuracy improves, as shown in Fig. 4.8. It can be seen that the proposed method performs better than the AM method across all measurement availability levels. This improvement is primarily due to the ability of the proposed Fréchet regression framework to exploit the underlying geometric structure of correlation relationships among measurements, enabling more robust topology inference even when measurements are sparse.

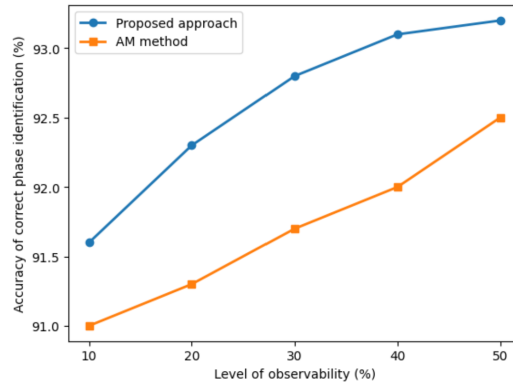


Figure 4.8: Performance of phase identification approaches in unobservable conditions- IEEE 37-bus test system.

For 123-bus system, we assume that limited active and reactive power measurements are available at only a subset of nodes, representing a partially observable network. The measurement availability is varied from 10% to 80%, with 40% of the phase labels assumed to be known. The corresponding phase identification accuracy is shown in Fig. 4.9, demonstrating improved performance as system observability increases.

Now, the performance of Algorithm 1 is compared with the AM method operating under incorrect model information. The incorrect model is generated by randomly perturbing the phase assignments associated with the entries of the network admittance matrix. The resulting estimation errors in voltage magnitude for different fractions of available data

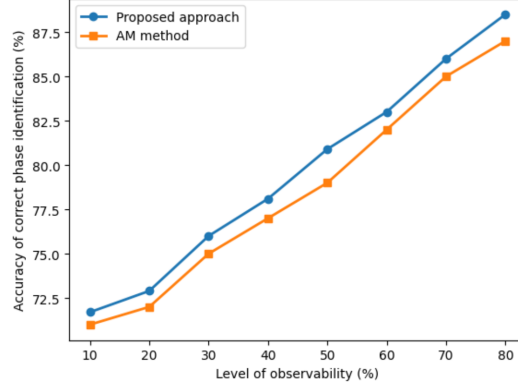


Figure 4.9: Performance of phase identification approaches in unobservable conditions- IEEE 123-bus test system.

(FAD) are presented in Figs. 4.10 and 4.11, respectively, for IEEE 37-bus and IEEE 123-bus test systems. As observed from these figures, the proposed approach achieves consistently lower state estimation errors across all levels of data availability compared with the recovery obtained using the AM method. For instance, reductions of up to 10% in voltage magnitude estimation error are achieved at 10% FAD, highlighting the robustness of the proposed approach to inaccuracies in network information.

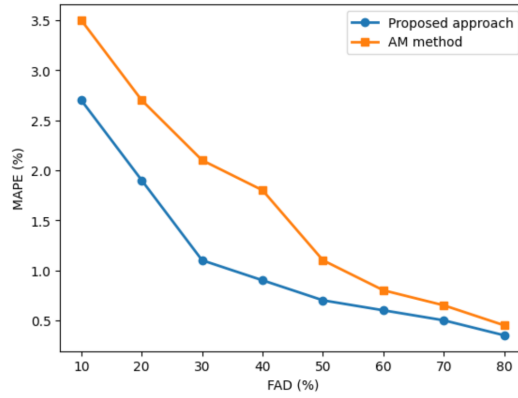


Figure 4.10: Voltage magnitude recovery for IEEE 37-bus system.

The results demonstrate that the proposed framework can provide reliable topology identification and state estimation even when only a small fraction of measurements is available, which reflects realistic operating conditions in many distribution systems. By leveraging historical operating data and the geometric structure of system variables, the proposed approach reduces dependence on extensive sensing infrastructure, such as widespread μ PMU

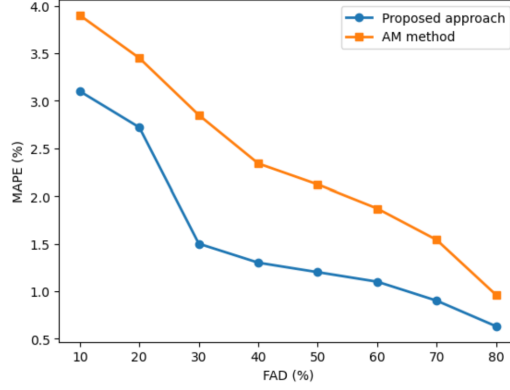


Figure 4.11: Voltage magnitude recovery for IEEE 123-bus system.

deployment. This makes the method particularly attractive for utilities seeking to enhance distribution system visibility and monitoring capabilities using existing measurement data while maintaining robustness to topology inaccuracies and measurement uncertainty.

Two operational scenarios of MG energy management are simulated, namely the non-coordinated and coordinated cases. In coordinated cases, energy trading is allowed between MGs and the DSO. In non-coordinated cases, the MGs can trade only with DSO. Then we solve for the hourly schedules for the power drawn by the MG's from the utility grid, the power traded among the MG's, and the BESS charging and discharging power available in their respective portfolios. This is a mixed integer non-linear problem (MINLP) and in general NP-hard. We have implemented the described formulation in Python by using cvxpy package GLPKMI solver. For problem simplicity, we have formulated the energy transactions in terms of active power on an hourly basis.

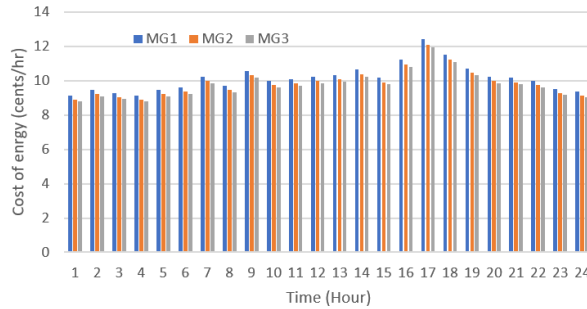


Figure 4.12: Utility electricity prices for microgrids.

We consider three MGs, each with solar PV, battery and an aggregated load of 100 houses with various mean income levels (\$ 25K, \$70K and \$115K per annum). The BESS capacity is 1000 kWh in each MG with charging and discharging efficiency of 95% and 90%. The solar PV peaks are 600 kW, 660 kW, and 720 kW respectively in each MG shown in Fig. 4.14. Based on the hourly electricity price and availability of solar PV, the schedule of BESS charging and discharging power, utility power and trading power among MGs to minimize the overall cumulative cost of energy in each MG. The electricity prices are borrowed from real-time hourly prices published by ComEd [150] and scaled to match the distribution system energy prices. The hourly utility price profile is shown in Fig. 4.12, with an additional 4 cents per kWh for the penalized prices compared to normal prices.

The aggregated power demand of 400 kW is considered as the allowable limit for all the MGs. The utility will penalize the MGs based on their power demand for excess power usage. The aggregated load profiles for the MG with 100 houses are shown in Fig. 4.13. Here, the load profiles are considered based on income levels. For example, the average income level of MG3 community is higher than that of the MG1 average income level and the respective load profiles capture their energy consumption.

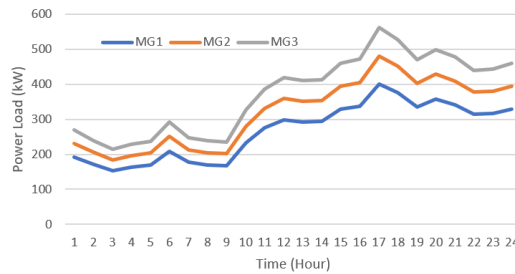


Figure 4.13: MG aggregated load.

Four cases have been simulated to observe the impact of the including of energy burden constraint on the power scheduling of MGs:

- Case-1: In this case, microgrid coordination is not established and energy burden constraint is not included. MGs can buy and sell power from/to the utility only.

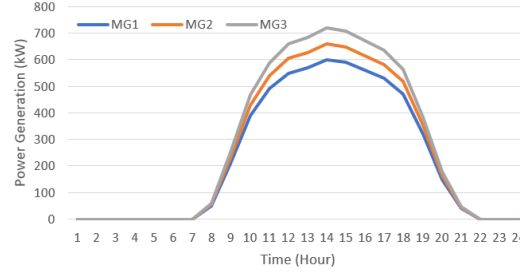


Figure 4.14: MG solar PV generation.

- Case-2: In this case, microgrid coordination is not established and energy burden constraint is included. MGs can buy and sell power from/to the utility only.
- Case-3: In this case, microgrid coordination is established and energy burden constraint is not included. MGs can buy and sell power from/to the utility as well as neighboring MGs.
- Case-4: In this case, microgrid coordination is established and energy burden constraint is included. MGs can buy and sell power from/to the utility as well as neighboring MGs.

These four cases were simulated and the results are presented. In Table. 4.1, it is observed that the average energy burden of communities decreases from 1.902% in case-1 to 1.394% in case-4. By comparing case-1 and case-2, there is a reduction of 14.3% in the average energy burden. This reduction in energy burden in the non-coordinated cases is attributed to enforcing the maximum energy burden constraint (i.e., 2%) in the formulation. As a result of this constraint, the penalized power is distributed fairly among the served communities. For instance, in MG1 where the income level of the community is low, to balance the load, MG1 has to draw the power from the utility by paying the penalty shown in Fig. 4.15. However, it is not fair to share the penalties among all the communities equally since their respective power requirement and consumption vary based on income levels. Therefore, there is a need to equitably distribute the penalized energy to operate the energy infrastructure fairly among the served communities. In Case-2, penalized utility power is not allocated to the highly burdened MG1 community. Consequently, the MG1 energy burden decreases

Table 4.1: Energy Burden Comparison of MGs For Four Cases

S.no.	Case-1	Case-2	Case-3	Case-4
E1(%)	3.091	1.999	1.387	1.387
E2(%)	1.543	1.634	1.390	1.382
E3(%)	1.074	1.259	1.412	1.414
Avg.(%)	1.902	1.630	1.396	1.394

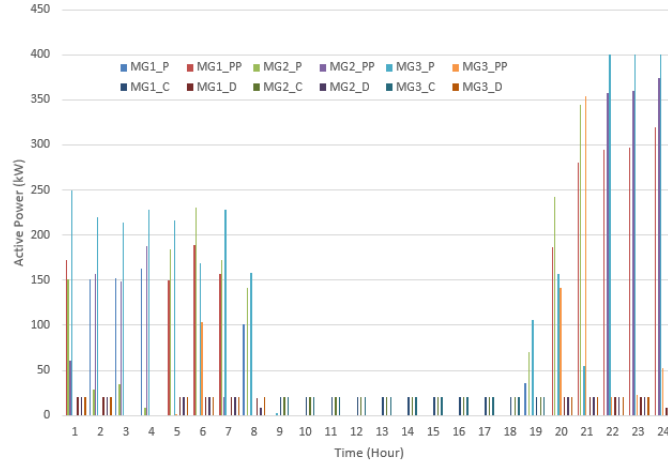


Figure 4.15: Schedules of utility power and BESS in Case-1.

from 3.091% in case-1 to 1.999% in case-2. Additionally, for the other MG communities, the energy burden remains below the maximum value. The BESS profile demonstrates typical charging and discharging patterns according to utility price and solar PV generation. The BESS charges during peak solar PV generation and discharges during other time intervals.

The energy burden of communities is further reduced through a bargaining-based mechanism that facilitates energy trading among MGs. A comparison between case-1 and case-3 reveals a 26.6% reduction in average energy burden. This reduction is attributed to the coordination among MGs. Instead of selling excess solar PV generation from MGs to the utility, MGs engage in trading among themselves. This operation benefits both the utility and MGs by enhancing reliability and affordability in the distribution system. Fig. 4.16 illustrates that no penalized power is allotted to MG1, thanks to the energy trading between MG3 and MG1.

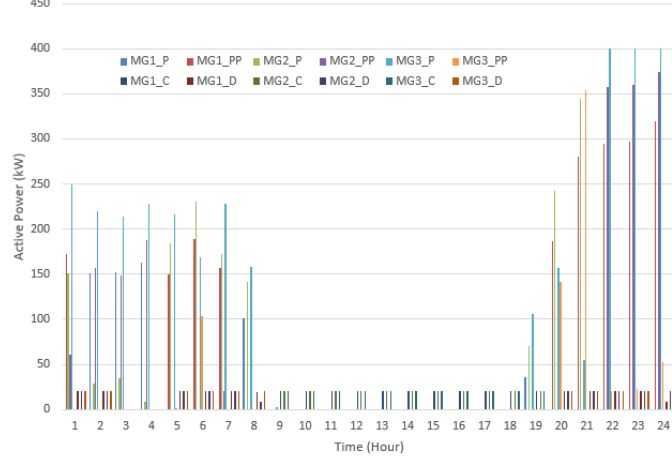


Figure 4.16: Schedules of utility power and BESS in Case-3.

The proposed DSO–DER coordination architectures are evaluated on the IEEE 123-bus test system for both the minimal DSO and Total DSO configurations. In the minimal DSO architecture, operational limitations restrict the controller’s ability to perform location-specific actions; consequently, the utility relies on a uniform DER-curtailment scheme to preserve network reliability under varying DER operating conditions. Under normal, uncongested conditions, no DER curtailment is necessary. A congested operating scenario is identified by simulating the system over a range of day-ahead loading conditions and DER dispatch levels. During peak DER production, particularly around 2 PM in the day-ahead schedule, nodal voltages exceed the acceptable operating band of 0.95–1.05 p.u. Although some utilities permit a wider voltage margin (0.9–1.1 p.u.), the simulation results demonstrate that under high DER penetration, node voltages can surpass even the 1.1 p.u. upper bound, as illustrated in Fig. 4.17. This highlights the increasing need for coordinated DER management in distribution networks with high renewable adoption.

In the minimal DSO architecture, acceptable voltage levels are traditionally restored by uniformly curtailing DER outputs across the network. However, when the spatial distribution of DERs is known, voltage-sensitivity information can be leveraged to identify the most influential DER locations for targeted curtailment. The latter strategy is adopted in this work to avoid overly conservative system-wide DER reductions. Using the voltage-sensitivity-based approach, nodal voltage magnitudes are brought back to within the ac-

ceptable range (≤ 1.05 p.u.) by proportionally reducing DER outputs at nodes exhibiting higher voltage impacts. The resulting nodal voltages and DER curtailment profiles are illustrated in Fig. 4.17. As shown, DERs located near nodes with voltages exceeding 1.05 p.u. experience greater curtailment, while DERs at non-critical nodes remain largely unaffected. Following the targeted curtailment, all nodal voltages are successfully regulated to ≤ 1.05 p.u., confirming the effectiveness of the proposed approach.

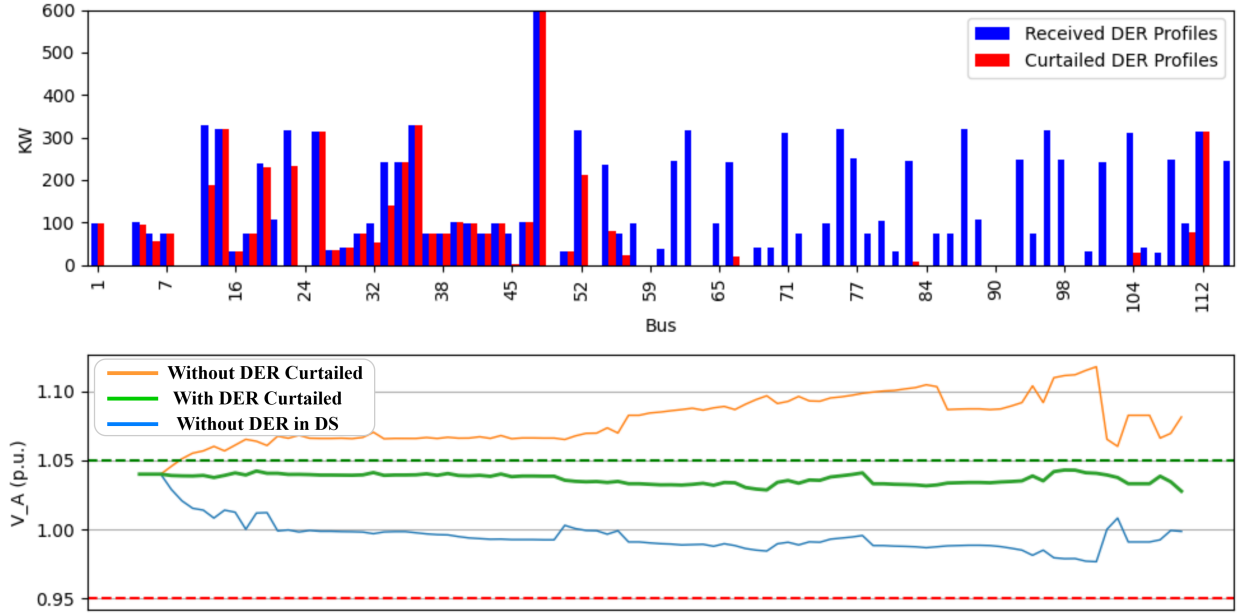


Figure 4.17: Generator power and nodal voltages for Minimal DSO

In the total DSO architecture, the utility solves the optimal power flows (OPF) to maximize DER utilization and revenue. OPF for total DSO the proposed studies is formulated using current-voltage (IV) formulation, which enables a robust representation of voltage and current variables suitable for unbalanced distribution networks. The OPF is implemented using the PowerModelsDistribution package, an open-source Julia-based framework for scalable distribution-system optimization [151]. The proposed modeling framework enables flexible experimentation with various DSO architectures and DER coordination strategies within unbalanced distribution networks. In the Total DSO architecture, the operator leverages detailed network information, DER capacities, and voltage-sensitivity relationships to maintain nodal voltages within the acceptable operating band of 0.95–1.05 p.u as shown in Fig.

4.18, while minimizing total DER curtailment. This enhanced flexibility allows the DSO to distribute the required curtailment optimally across multiple DERs, thereby reducing the overall curtailment burden compared to rigid or location-specific methods. Unlike the minimal DSO architecture—where curtailment is applied only to DERs located at nodes exhibiting overvoltage conditions—the Total DSO strategy coordinates DER outputs system-wide to achieve voltage regulation with significantly lower aggregate curtailment. As a result, the system attains improved voltage compliance and higher renewable utilization.

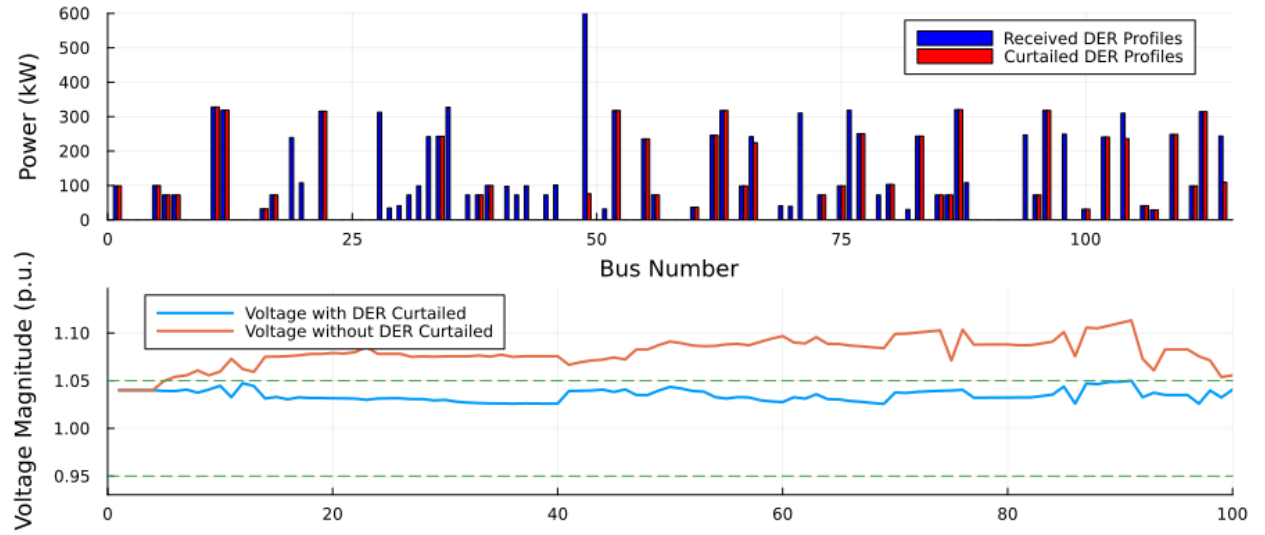


Figure 4.18: Generator power and nodal voltages for Total DSO

A comparison of the total DG real-power output and the corresponding curtailment percentages for both architectures is provided in Table 4.2. Prior to any congestion management, the aggregate available DG capacity in the system is 10,749 kW. After applying the respective congestion-management strategies, the minimal DSO architecture yields a total DG output of 4,625 kW, whereas the Total DSO architecture delivers a substantially higher output of 6,425 kW. These results clearly demonstrate the advantages of the increased operational flexibility offered by the Total DSO approach. Specifically, by coordinating curtailment decisions across the entire network, rather than targeting only DGs located at overvoltage nodes, the Total DSO achieves nearly double the usable DG real-power contribution compared to the minimal DSO. This improvement reflects a more efficient utilization of DGs and highlights

the importance of system-wide optimization in high-DG-penetrated distribution networks.

Table 4.2: Comparison of Min DSO and Total DSO

	Total P	% Curtailment
Min DSO	4,625.08	56.97
Total DSO	6,425	39.68

These results demonstrate that the enhanced flexibility of the Total DSO architecture enables significantly more efficient DER coordination compared to the minimal DSO approach. By leveraging detailed network models, voltage sensitivities, and system-wide optimization, the Total DSO maintains all nodal voltages within acceptable limits while minimizing overall curtailment. Consequently, the usable DER real-power output nearly doubles relative to the minimal DSO, highlighting the critical value of coordinated, distribution-level optimization in high-DER environments.

4.8 Summary

The methodologies presented in this chapter enhance the operational resilience of distribution systems by improving situational awareness, enabling reliable decision-making, and supporting coordinated recovery following disturbances.

Chapter 5

Resilience Analysis Using Hetero-Functional Graph Theory

Modern infrastructure systems, including electric power networks, communication systems, and community services, are increasingly interconnected and interdependent. While such interconnections improve operational efficiency and flexibility, they also introduce new vulnerabilities that can amplify the impact of localized disturbances. In particular, failures occurring within one component of the system can propagate through functional dependencies, resulting in cascading disruptions that degrade overall system performance. Understanding how these dependencies influence system behavior is essential for accurately characterizing infrastructure resilience.

Resilience of power systems is commonly evaluated using performance-based metrics that describe the system's ability to withstand disturbances and recover functionality over time. The depth and duration of performance degradation following a disturbance depend not only on physical connectivity but also on the functional relationships among system components. These relationships determine how failures propagate and how quickly the system can restore critical services. Therefore, modeling functional interactions among infrastructure components is fundamental to analyzing resilience characteristics such as disruption severity and recovery behavior.

Traditional network-based modeling approaches primarily rely on node-edge representations that capture structural connectivity but often neglect the underlying functional processes occurring within system components. As a result, conventional graph models may underestimate the extent of failure propagation and fail to capture complex dependency patterns among interconnected systems. These limitations restrict the ability of existing models to accurately assess resilience, particularly in systems where operational processes and service dependencies play a dominant role.

To address these challenges, this chapter adopts HFGT as a modeling framework for representing functional relationships among interconnected infrastructure systems. HFG theory enables explicit modeling of system functionalities and their interactions, allowing failure propagation to be analyzed at the functional level rather than solely at the structural level. By incorporating both power system components and community-level services into the modeling framework, the proposed approach provides a comprehensive representation of system dependencies and enables detailed resilience analysis under various disturbance scenarios.

This chapter develops analytical tools for modeling failure propagation and identifying critical functionalities that significantly influence system resilience. Sensitivity analysis techniques are introduced to quantify the relative importance of individual components in shaping system recovery behavior. The methodologies presented herein provide the foundation for functional resilience assessment and serve as a precursor to the higher-order dependency analysis introduced in the subsequent chapter.

5.1 HFGT Preliminaries

HFGT can be viewed as an integration of model-based systems engineering (MBSE) and network science, enabling structured representation of complex and interdependent engineering systems. The HFGT framework has been successfully applied across multiple domains, including transportation networks, production systems, and power grids. By merging system engineering principles with network-based representations, HFGT provides a unified mod-

eling paradigm capable of capturing structural, functional, and operational relationships within interconnected infrastructure systems.

The HFGT framework consists of a set of interconnected graph-based models that extend traditional network representations by explicitly distinguishing system functionalities from physical components. These models collectively enable holistic representation of system structure, control logic, and service behavior. The seven primary models within the HFGT framework include: (1) *System concept model*; (2) *Hetero-functional adjacency graph*; (3) *Controller agency graph*; (4) *Controller adjacency graph*; (5) *Service-as-operand behavior model*; (6) *Service feasibility graph*, and (7) *System adjacency matrix*.

Among these, the system concept model and hetero-functional adjacency graph form the structural foundation of the framework. These models enable integration of heterogeneous subsystems with distinct functionalities into a unified mathematical representation. Unlike conventional graph representations, where nodes represent physical components and edges denote physical connections, nodes in hetero-functional graphs represent system functionalities, while edges represent logical sequences or functional dependencies among those operations. This functional representation enables modeling of interactions that extend beyond purely structural connectivity.

The controller agency graph and controller adjacency graph represent the control architecture of the system. These models describe how decision-making entities interact and coordinate system operations, enabling differentiation of systems based on their control structures and communication patterns. Such representations are particularly important in modern infrastructure networks, where distributed control and coordinated decision-making play a critical role in system performance and resilience.

The service-as-operand behavior model and service feasibility graph characterize the operational behavior of system services and their functional constraints. These models enable representation of how system resources are utilized to deliver services under varying operating conditions. Finally, all individual models are integrated into a unified system adjacency matrix, which provides a compact mathematical representation for analyzing functional dependencies and propagating system-level interactions.

The structured representation provided by HFGT enables scalable and computationally efficient modeling of interdependent infrastructure systems. By capturing both structural and functional relationships, the framework supports detailed analysis of cascading disruptions and system-level dependencies, making it particularly suitable for resilience-oriented modeling of modern power and community infrastructure systems.

5.2 HFG Modeling of Power and Community-Centric Interdependent Systems

This section presents the system under consideration, which comprises three interdependent networks: the power distribution, the communication network, and the community network. We outline the interplay between these networks by describing their components and subsequently detail how these dependencies are modeled using the HFGT-based approach. Later, we discuss existing network simulation tools and highlight their limitations when applied to resilience analysis of interdependent infrastructure systems.

Power distribution system:

A simplified conceptual radial distribution system is illustrated in Fig. 5.1. In this study, we consider a low-voltage, n -bus power distribution system (PDS) comprising l loads and m voltage regulators (VR). The m VRs are connected to selective buses to help maintain voltage levels across all n nodes. Generally, to perform critical operations like voltage regulation, monitoring power flows, and state estimation, the PDS is typically equipped with smart sensors and intelligent electronic devices that can communicate with a central control center via a supervisory control and data acquisition (SCADA) system [152]. In this work, we consider the use of static VAR compensators (SVCs), which regulate voltage by receiving set points from the control center. These set points are computed based on system states [153]. The steady-state operating conditions of the distribution system are obtained by solving the power flow equations. For simplicity, this study adopts a single-phase equivalent model and performs a $1\text{-}\phi$ load flow analysis. The corresponding power flow equations are presented

below [154]:

$$P_i = V_i \sum_{j=1}^n V_j Y_{i,j} \cos(-\theta_{i,j} + \delta_i - \delta_j) \quad (5.1)$$

$$Q_i = V_i \sum_{j=1}^n V_j Y_{i,j} \sin(-\theta_{i,j} + \delta_i - \delta_j) \quad (5.2)$$

$$V^{\min} \leq V_i \leq V^{\max} \quad (5.3)$$

where, n is the number of nodes excluding the slack bus; P_i and Q_i are the net active and reactive power injected at bus i ; V_i , δ_i , V_j , and δ_j are the voltage magnitudes and phase angles at nodes i and j respectively. $Y_{i,j}$ and $\theta_{i,j}$ are the admittance and impedance angle between buses i and j . Except for the slack bus, all the other buses are assumed to be load buses in this analysis. As indicated in (5.3), the voltage level has to be maintained within the limits $V^{\min}$ and $V^{\max}$. The desired voltage level is achieved by regulating the reactive power by placing SVCs at optimal locations, and command signals are issued via the communication network. The typical droop equation for voltage control is

$$V_i = V^{ref} - k(Q_i - Q_i^{ref}) \quad (5.4)$$

where, V^{ref} is the target voltage, and Q_i^{ref} is the reactive power reference. If Q_i^{ref} command signal is not issued to the i^{th} SVC due to communication failure, the power distribution system can experience voltage fluctuations at some nodes in the network.

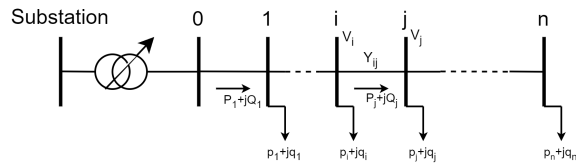


Figure 5.1: Diagram of radial distribution system.

Communication network:

Typically, SCADA systems have been implemented to monitor and control electrical power grids at high voltage levels for decades. To support the growing needs of distribution

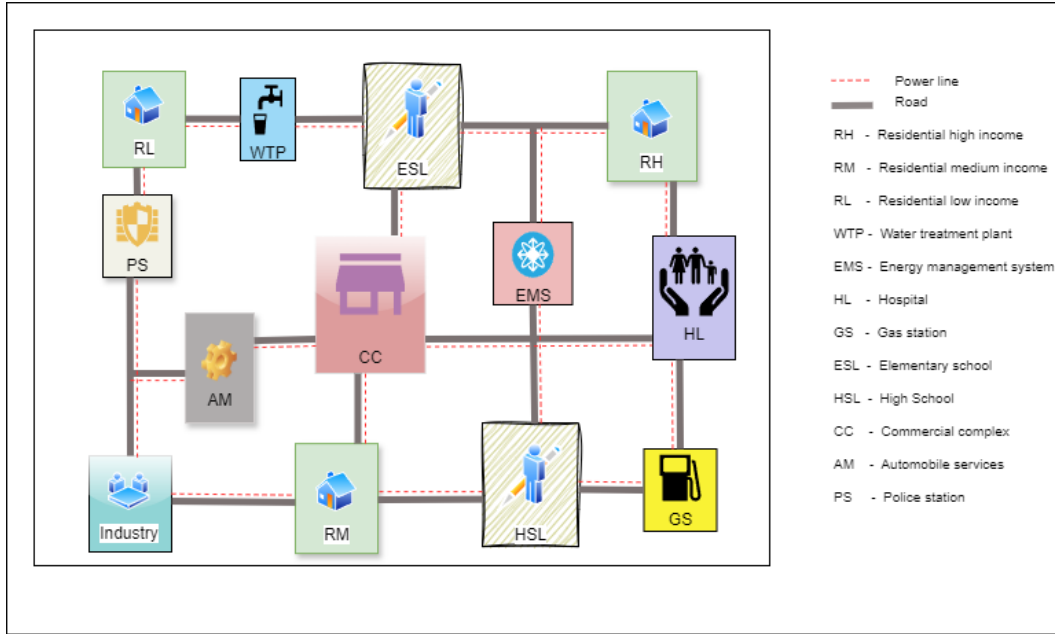


Figure 5.2: Diagram of a community with services that depend on a power network

systems, researchers have been proposing SCADA for lower voltage levels [152]. Thus, monitoring and controlling of PDS via the communication infrastructure has proven necessary. In this work, we assume that a communication network is employed to receive field data from sensors and send setpoints to m SVCs in the n -bus PDS. In voltage control operations, the PDS must collect measurements from remote locations, solve the power flow equations, and send command signals (reactive power setpoints) to voltage controllers. Hence, the communication infrastructure must be reliable to support the needs of the PDS. To model this interaction, we assume a star topology for the communication network. In this configuration, a central control center receives measurement data from smart meters at n buses via respective gateways and dispatches setpoints to the m SVCs through their corresponding gateways. Note that the decision-making entities in a communication network that supports the services of another network can be incorporated in HFGT-based modeling [92].

Community model:

A community is assumed to be a group of customers being served by the PDS. The PDS, along with interdependent critical infrastructures, delivers essential services that support the quality of life (QoL) of the community. While there are many approaches to quantify QoL of

a community, in this work, we model it as a weighted sum of community services [155]. The community model includes elements such as residences, hospitals, educational institutions, water treatment plants, commercial complexes, gas stations, energy management systems, as illustrated in Fig. 5.2. These community components are the loads in the PDS as discussed. Also, within the community, these components are interdependent as shown in Fig. 5.2. Thus the reliability of the PDS directly affects the functionality of these components, and a less reliable PDS compromises the QoL of the community. To capture the impact of disruptions, we focus on the value of lost load (VoLL). The VoLL is a measure of the economic impact of electricity outages on consumers and it is measured in \$/kWh [156]. A higher VoLL indicates a greater economic impact on consumers during outages, underscoring the critical importance of energy for some consumers. Traditionally, the economic impacts of infrastructure damage on the community are determined using computable general equilibrium models [157]. In this work, we quantify the economic impact using the VoLL parameter, which is particularly relevant for interdependent systems driven by PDS.

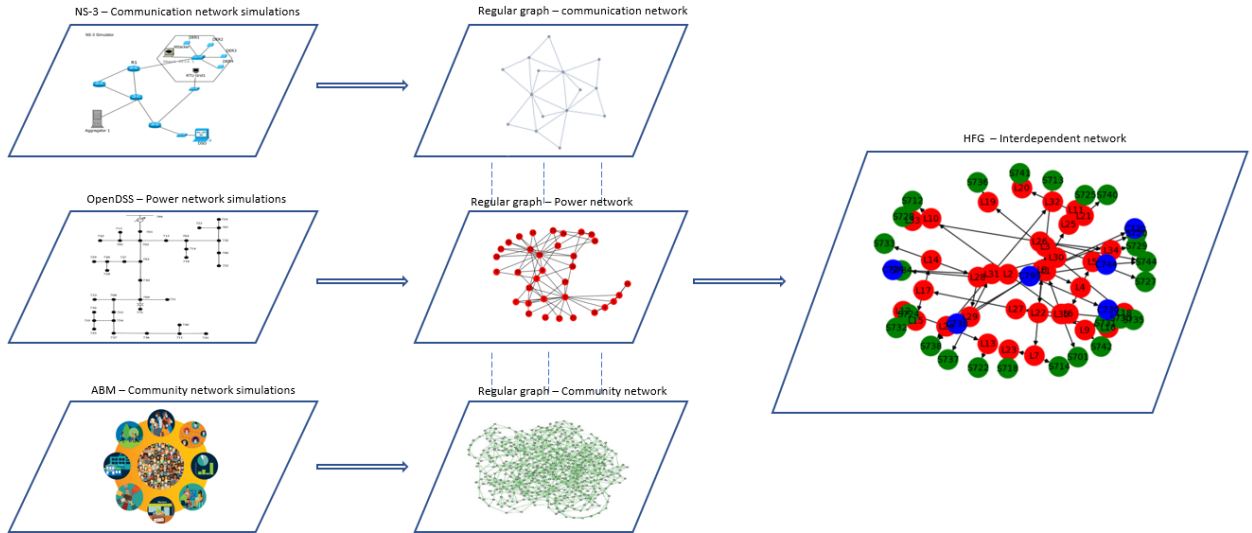


Figure 5.3: Hetero-functional graphical modeling

Simulation platforms:

Generally, PDS are simulated using tools such as OpenDSS, MATLAB, or Octave [158]. Communication networks, including studies of traffic flow, network delays, and congestion,

are commonly modeled using software packages like NS-3 and OMNeT++ [159]. Meanwhile, Community networks are often represented using multi-agent simulation platforms such as Agent-Based Modeling (ABM) tools and AnyLogic [160], [161]. These tools enable the evaluation of various impacts such as income loss, QoL, or quality of service (QoS) through simulation. In traditional approaches, each infrastructure is modeled and simulated independently using specialized tools. The interdependencies are then examined in a sequential manner, evaluating the impact of one network on another through layered simulations. This fragmented approach leads to increased complexity and longer simulation times, particularly as the number of interdependent infrastructures increases. Consequently, conducting comprehensive resiliency studies using multiple disconnected tools becomes increasingly time-consuming and inefficient. In this work, we assume static operating conditions, which are sufficient to illustrate the merits of the proposed method. It is important to note that no historical data is used in the simulations.

5.3 Sensitivity Analysis in HFGs

So far, we have introduced the networks under consideration. This section presents a hetero-functional graph theory (HFGT)-based modeling approach to propagate disruptions across networks and evaluate the Value of Lost Load (VoLL) associated with community services. We provide a detailed discussion on the construction of HFGT models tailored for interdependent infrastructure systems and integrate sensitivity analysis techniques to enhance impact quantification. The resulting framework is referred to as the Hetero-Functional Graph for Power Distribution Systems (HFG-PDS), which enables holistic analysis of outage impacts across power, communication, and community networks.

The HFG-based models are powerful yet simple representations of interdependent infrastructure networks. These models are developed by domain experts and are useful for critical operations in the systems. As illustrated in Fig. 5.3, the traditional approaches consider each network as a graph, and interdependencies are captured by representing it as a multilayer network. In contrast, in HFG models, all the networks are represented in a single

layer that results from the individual networks, as shown in Fig. 6.1. In the HFG, nodes are represented as functions, and the edges are the dependencies among functionalities. The operands of each node capture the function states, type, etc., to further enrich the graph representation. Through the knowledge of the type of dependency, the impact of change in operands of one functionality on others is propagated across the HFG. Hence, this unique modeling approach is effective for the PDS operators to observe the system states and to execute critical operations efficiently.

Traditionally, the power distribution system is represented as a regular graph (G) with a set of buses and branches as nodes ($\mathcal{V}$) and edges ($\mathcal{E}$), $G = (\mathcal{V}, \mathcal{E})$; where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. In this representation, nodes correspond to physical components (e.g., buses), and edges capture their physical interconnections. In contrast, hetero-functional graphs (HFGs) are different from the regular graphs, where the nodes are the functionalities and edges are their dependencies. Let $G' = (\mathcal{V}', \mathcal{E}')$ be the hetero-functional graph, where $\mathcal{V}' = \{1, 2, \dots, n\}$ is the set of functionalities, $\mathcal{E}' = \mathcal{V}' \times \mathcal{V}'$ is the set of edges. These functionalities $\mathcal{F}$ corresponding to $\mathcal{V}'$ are defined for all the components of the power distribution system, communication, and community networks. The architecture in Fig. 5.3 clearly demonstrates how the dependencies are modeled among heterogeneous infrastructures within the HFGT-based modeling framework.

We define features and operands for various functionalities in the HFG. Let $\mathbf{f}_i$ be the feature vector of i th functionality, and $\mathcal{X}_k$ represent the k th feature in it. Based on the specific functionality in HFG, the operands include type of functionality, function availability, command signal, apparent power, voltage, impedance etc., are defined. For example, the type of functionality feature explicitly describes the infrastructure from which the functionality is obtained. For a node from PDS, the features such as apparent power, voltage, and impedance are defined. For a node from the communication infrastructure, the command signal is defined as a feature. To explain further, let us observe Fig. 5.4. It depicts the functionalities and their dependencies in the interdependent network of power, communication, and community. The functionalities are the nodes with feature vectors $\mathbf{f}_i$ and the dependencies are edge connections between them.

For example, in a power network, there are functionalities associated with power lines, voltage controllers and loads. The power lines transport power between the buses, so the power line has 'transport power' functionality. The load in PDS consumes power, and this consumption of power at load nodes delivers services to the community. Therefore, the load has 'consumption of power' as functionality. Similarly, voltage controllers regulate the voltage in the network, so they have 'voltage regulation' as functionality. A power line is characterized by its type ($\mathcal{X}_t$), availability ($\mathcal{X}_a$), impedance ($\mathcal{X}_z$), and apparent power ($\mathcal{X}_s$): $f_i\{\mathcal{X}_t, \mathcal{X}_a, \mathcal{X}_z, \mathcal{X}_s\}$. A load or voltage regulator is characterized by type, availability, apparent power, and voltage ($\mathcal{X}_v$): $f_i\{\mathcal{X}_t, \mathcal{X}_a, \mathcal{X}_s, \mathcal{X}_v\}$. Similarly, the node from the communication network is characterized by type, availability, reactive power set point; $f_i\{\mathcal{X}_t, \mathcal{X}_a, \mathcal{X}_q\}$. Also, to characterize the community, the load features are extended by adding value of lost load $\mathcal{X}_{vll}$. Thus the complete community load functionality is defined as $f_i\{\mathcal{X}_t, \mathcal{X}_a, \mathcal{X}_s, \mathcal{X}_v, \mathcal{X}_{vll}\}$. Table 5.1 provides a simple example to demonstrate the connection between the regular interconnected graph Fig. 5.6 and HFG Fig. 5.4. The Fig. 5.6 represents a multi-layer graph of power, communication, and community networks. As discussed in the HFG modeling, the resources in interdependent networks are mapped as functionalities in HFG, with the edges representing the dependencies between these functionalities. Dependencies can exist among functionalities within a single infrastructure or across multiple infrastructures. For example, a dependency within a power distribution system may occur when the operation of a power line in a radial network depends on the proper functioning of an upstream line. In this case, the power transfer capability of a line is contingent upon the capacity of the preceding line. Similarly, cross-infrastructure dependencies can be observed, for instance, the operation of a water treatment plant may rely on the availability of electric power, illustrating the interdependence between the water and power systems. Additionally, the functioning of the water treatment plant affects water supply to residential consumers, representing a secondary impact. Therefore, the Hierarchical Functional Graph (HFG) of an interdependent network is constructed using system functionalities and their dependencies to capture both primary and secondary impacts.

Table 5.1: Various functionalities in interdependent networks

Name	Description	functionality
f1	Power line	Transport power from node 1 to 2
f2	Power line	Transport power from node 2 to 3
f3	SVC	Regulate reactive power at node 30
f4	Gateway	Send command signals to SVC at node 30
f6	GS	Provide gas to consumers at node 13
f11	Hospital	Treating patients at node 15

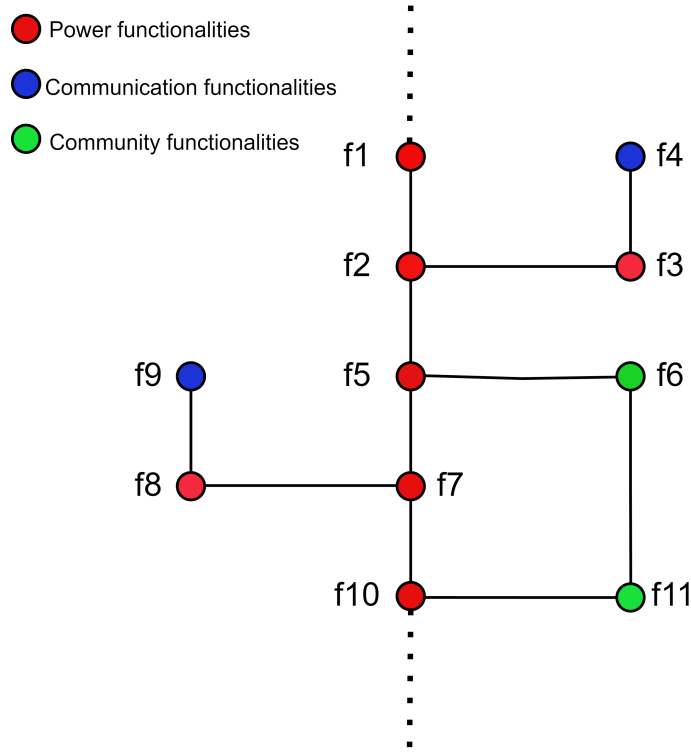


Figure 5.4: Propagation of impacts among functionalities.

With the functionalities defined as above, we proceed to analytically model the propagation of the effect of a change in one functionality on the other within the interdependent network. Assume a critical operation, such as voltage control in PDS. This operation depends on the availability of the communication functionality, which is responsible for delivering command signals to the voltage regulation functionality within the power infrastructure.

The seamless operation of voltage controllers hinges on the reliable transmission of these control signals to maintain voltage levels at all nodes and ensure safe and continuous operation of the system. Let the i th communication node be responsible for sending a command signal to the j th power node. This interaction can be formally expressed as;

$$\mathcal{X}_{js} = \mathcal{X}_{ia}\mathcal{X}_{is} \quad (5.5)$$

where, $\mathcal{X}_{ia}, \mathcal{X}_{is}$ are the availability and command signal features associated with the communication node i , $\mathcal{X}_{is}$ is the command signal received by the power node j .

To analytically quantify the effect of a change in one functionality on another within the HFG, we adopt the voltage sensitivity analysis method as presented in our prior work [162]. This method evaluates the voltage variation at an observation node o due to a change in complex power at an actor node a . Specifically, a change of complex power at a from S_a to $S_a + \Delta S_a$ results in a voltage change at observation node o from V_o to $V_o + \Delta V_o$. In the context of HFG, a change in complex power at an actor node, defined as functionality, propagates and influences an observing node, which represents another functionality. More clearly, for a transport power functionality, the operand $\mathcal{X}_s$ is affected, and for a consume power functionality, $\mathcal{X}_v$ is affected. The impact of change in feature $\mathcal{X}_{is}$ in consume power functionality on feature $\mathcal{X}_{js}$ in transport power functionality is inspired by sensitivity analysis and calculated using the following equation;

$$\Delta \mathcal{X}_{js} = \frac{\Delta \mathcal{X}_{is}}{\mathcal{X}_{iv}} \mathcal{X}_{jz} \quad (5.6)$$

Similarly, the relationship between $\mathcal{X}_{is}$ of transport power functionality and $\mathcal{X}_{jv}$ of consume power functionality is written as;

$$\Delta \mathcal{X}_{jv} = \frac{\Delta \mathcal{X}_{is}}{\mathcal{X}_{iv}} \sum_i^n \mathcal{X}_{ijz} \quad (5.7)$$

Here, the impact of a change in features from the i th functionality is computed on j th functionality using the sensitivity analysis. The features $\Delta \mathcal{X}_{is}, \mathcal{X}_{iv}$ are associated with changes

in apparent power and voltage of the actor functionality. $\Delta\mathcal{X}_{js}$ change in apparent power of the observed transport power functionality, $\Delta\mathcal{X}_{jv}$ change in voltage of the observed consume power functionality. $\mathcal{X}_{jz}$ and $\mathcal{X}_{ijz}$ impedance of observed power line functionality and the sum of impedances between actor and observed functionalities. Thus (5.7) captures the result of the change in voltage at an observation node due to a change in apparent power at an actor node. Therefore, the proposed analytical method can effectively incorporate the propagation of impacts over the functionalities in the HFG.

The final voltage state of an observation functionality, resulting from a change in complex power at an actor functionality, can be written as;

$$\mathcal{X}_{jv}^f = \mathcal{X}_{jv}^p + \Delta\mathcal{X}_{jv} \quad (5.8)$$

where $\mathcal{X}_{jv}^p$ and $\mathcal{X}_{jv}^f$ are present and final voltage states of the loads in the network.

These formulations illustrate one example of how the HFG modeling framework can be used for efficient impact analyses within the functionality space. In the next section, we analyze the computational complexity associated with resilience assessment in interdependent networks, comparing traditional methods with the proposed HFGE-based approach.

5.4 Computational Complexity

The power flows in the distribution system can be solved using the Newton-Raphson method [163], which has a computational complexity $\mathcal{O}(n^3)$. Communication network simulations, typically conducted using tools such as NS3, exhibit a complexity of $\mathcal{O}(m^2)$. Community networks modeled through agent-based approaches have a computational complexity of $\mathcal{O}(p^2)$, where n, m, p represent the number of nodes in the power, communication, and community networks, respectively. In contrast, the proposed HFGE-based modeling framework computes and propagates the impacts of changes across interdependent infrastructures using sensitivity-based techniques. This approach has a significantly reduced computational complexity of $\mathcal{O}(k)$, where k is the total number of functionalities (nodes) across all net-

works. Hence, the HFG model offers a modular and computationally efficient alternative for resilience analysis in complex interdependent systems.

In the next section, we will discuss the case studies to validate the merits of the proposed modeling.

5.5 Simulation Results and Discussions

This section evaluates the performance of the proposed HFGT-based modeling approach through two distinct case studies. Each case leverages the HFGT modeling framework for impact analysis in interdependent networks. The block diagram shown in Fig. 5.5 outlines the range of studies enabled by the HFGT model, which integrates the distribution system, communication infrastructure, and community networks. These models are utilized for community-centric outage analysis and voltage control. The power network is based on a modified IEEE 37-bus distribution system, with an overlaid communication network to support voltage regulation via SCADA. The community model includes a range of services, such as residential areas, hospitals, commercial centers, fire stations, gas stations, and water treatment plants, which are represented as loads within the distribution system. These services also interact with one another, forming an interconnected community layer. The combined interdependent system is illustrated in Fig. 5.6, where five voltage control and communication nodes, along with various community nodes, are denoted using blue and red colors, respectively. In the HFG, services provided by each network resource are represented as functionalities, and the edges denote their dependencies.

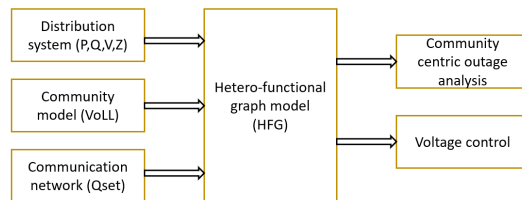


Figure 5.5: Applications of proposed HFG modeling.

Two case studies are conducted to validate and highlight the advantages of the proposed

HFG-PDS modeling framework. The first case study focuses on validating the HFGT model by comparing the results of outage analysis obtained through a simulation-based approach (using OpenDSS) with those derived from the HFGT model. The second case study demonstrates the benefits of the proposed HFGT approach by assessing the secondary impacts on community services resulting from communication and power line outages.

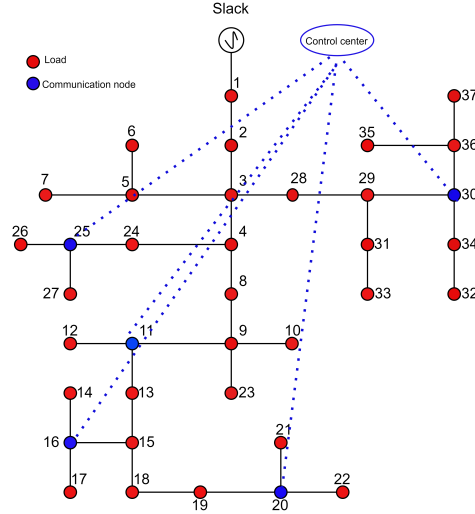


Figure 5.6: Diagram of modified IEEE 37 bus distribution system with communication network.

5.5.1 Outage Analysis - HFGT Validation

In this case study, we analyze the impact of a line outage using both OpenDSS simulations and the proposed HFGT approach to validate the modeling framework. As shown in Fig. 5.6, we consider a failure of the line between nodes 11 and 13 caused by an extreme weather event. This outage results in the disconnection of bulk loads on node 13 and downstream. Fig. 5.7 presents the voltage profiles at various nodes during the outage, computed using both OpenDSS and HFGT. From the results, it is observed that nodes 1, 2, 3, 5, and 7 experience voltage violations exceeding 1.05 p.u., while node 6 is close to a critical threshold. The voltage profiles obtained from the HFGT model closely match those from the OpenDSS simulation, validating the accuracy of the proposed approach. In the traditional simulation-based method (OpenDSS), the voltage violations are determined sequentially for each line

outage. In contrast, the HFGT approach models the outage as a loss of bulk load at the originating node of the failed line and analytically propagates the resulting changes in apparent power throughout the network. This enables efficient modeling of cascading impacts in the system.

From a community perspective, nodes experiencing voltage violations are unable to deliver their respective services. Assuming a one-hour outage duration, the lost services and the associated value of lost load (VoLL) are summarized in Table 5.2. The VoLL values, adopted as median estimates from [164], highlight significant economic losses for certain consumer categories. In addition to economic impacts, the table also presents the effects of service outages on community Quality of Life (QoL). Arbitrary coefficients are assigned to each service to reflect their relative contributions to QoL. These impacts underscore the importance of proactive voltage regulation strategies. Sophisticated models such as HFGT can facilitate rapid impact analysis and enable timely voltage control actions, such as reactive power support and transformer tap adjustments, to mitigate voltage-related disruptions.

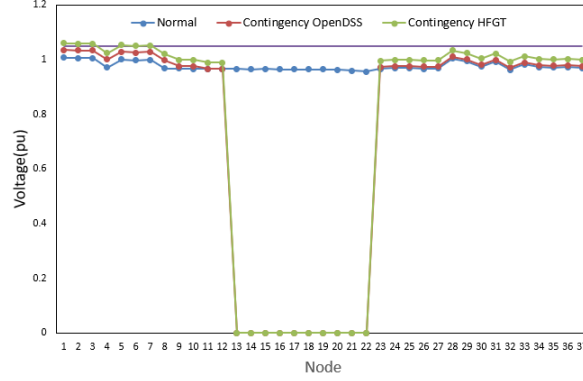


Figure 5.7: Impact of line outage.

5.5.2 Communication Failure Analysis

A key advantage of HFG modeling is its ability to capture secondary impacts across interdependent systems, impacts that are often overlooked by traditional simulation-based models. For example, communication network failures, which are relatively frequent due to their inherent vulnerabilities [165], can severely affect the PDS, and consequently, the community it serves. To illustrate this, we analyze the impact of a communication node failure on both the

Table 5.2: Value of lost load of community services

Node	Service type	VoLL (\$/kWh)	QoL Coeff.
1	Fire station	15.25	-0.03
2	Water treatment plant	29.3	-0.05
3	Residential (30 houses)	180	0
5	Residential (30 houses)	180	0
7	High school	33.35	-0.001
13	Gas station	49.51	-0.05
14	Police station	15.25	-0.05
15	Hospital	33.35	-0.05
17	Automobile services	49.51	-0.03
18	Commercial complex	49.51	-0.03
19	Residential (30 houses)	180	0
21	Residential (30 houses)	180	0
22	Residential (30 houses)	180	0

PDS and the community. Fig. 5.8 presents the results of a communication node (node 11) failure. The red curve represents the outcome obtained using a traditional simulation-based sequential method, where the corresponding PDS node receives an incorrect reactive power setpoint due to the communication fault. The green curve, on the other hand, is derived using the HFG approach, which models interdependencies between communication and power networks within a unified graph structure. The similarity in results between both methods validates the accuracy of the HFG model. However, the HFG approach offers a significant advantage in terms of computational efficiency, as it avoids the need for iterative simulations across different infrastructures. This makes HFG particularly effective for rapid assessment and response in complex interdependent systems.

We next analyze the impact on community services resulting from voltage violations observed in Fig. 5.8. Assuming an interruption duration of one hour, the simulation-based model shows that only loads at nodes 13-15, 17-19, 21 & 22 are impacted by the communication failure, with an average per-unit voltage of 0.946 across these nodes. This results in a total VOLL of 737 \$/kWh. However, the HFG model encapsulates the secondary impacts that are typically missed in traditional simulations. Due to service unavailability at nodes 13, 15, 17, & 18, there is an additional economic impact on residential houses at nodes 3, 5, 19, 21,

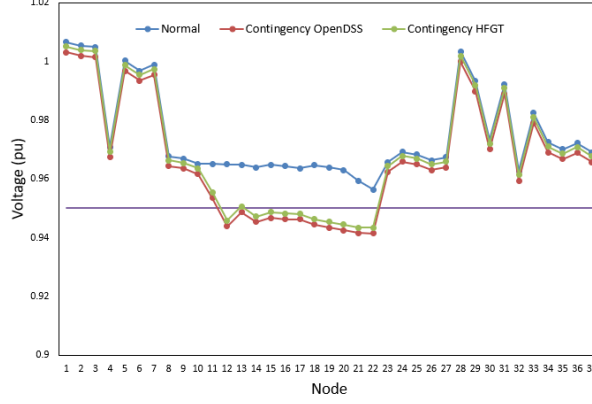


Figure 5.8: Impact of failure of communication.

& 22, resulting in a total VoLL of 1097 \$/kWh. Typically, the total VoLL is calculated based on direct and indirect outage costs, as mentioned in [164]. In this work, we assume an additional 10% of the original VoLL for each residential node, accounting for other service interruptions as an indirect outage cost. Similarly, for the line outage scenario discussed in Subsection VI.A, in addition to VoLL details from the Table. 5.2, the additional indirect cost on nodes 3,5,19,21, & 22 result in a total VoLL of 1367 \$/kWh. These results demonstrate that the proposed HFG modeling approach provides a more comprehensive assessment of outage impacts by encapsulating both direct and indirect interdependencies across infrastructures, an essential feature for realistic resiliency planning and impact mitigation.

The proposed HFG-PDS modeling approach enables utility operators to rapidly assess system states as failures propagate across interdependent networks. This situational awareness supports proactive decision-making, allowing operators to minimize service disruptions and reduce VoLL during outage events. Beyond economic loss, these service disruptions directly impact QoL within the community. In this framework, the QoL for a fully operational community is assumed to be 1. As shown in Table 5.3, during the line outage scenario, the QoL is reduced to 0.709, while for the communication failure scenario, it drops to 0.79. These values are computed by subtracting the sum of lost service QoL coefficients from the baseline value. Typically, QoL coefficients are derived from models that consider service availability and population proximity to services, as described in [166]. In this study, arbitrary QoL coefficients are assumed for illustrative purposes and are listed in Table II.

Thus, the HFGT-based interdependent network modeling not only supports efficient failure impact propagation analysis but also enables quantitative assessment of QoL degradation in communities, offering a comprehensive framework for both technical and societal resilience planning. It is important to note that the VoLL and QoL values used in this study are intended to illustrate the capability of the proposed HFGT-based framework to integrate economic and societal impacts into interdependent infrastructure analysis. While the framework explicitly supports heterogeneous, location-specific, and user-dependent parameterization, the accurate quantification of societal impacts requires careful calibration of VoLL and QoL coefficients using region-specific socioeconomic data, service criticality, and stakeholder priorities. Such calibration is essential for ensuring credibility and policy relevance of the results, particularly for adoption by utilities, local governments, and energy regulators.

Table 5.3: Effect of component failures on community services

Scenario	VoLL (\$/kWh) OpenDSS	VoLL (\$/kWh) HFGT	QoL
Line outage	1174	1804	0.709
Communication failure	737	1097	0.79

The analysis and case studies presented above clearly demonstrate the advantages of using the HFG modeling framework for rapidly observing system-wide changes during contingencies. This approach is not only effective in capturing interdependencies across infrastructures, but also numerically efficient in quantifying the cascading impacts of changes in one system on another. The proposed HFG modeling was executed on a standard Intel i9 processor-based PC, and results show an 83% reduction in computation time compared to traditional OpenDSS-based simulations. These findings underscore that the HFG-based models are modular, computationally lightweight, and scalable, thereby offering a practical and efficient alternative for abstracting and analyzing complex interdependent infrastructure networks.

5.6 Summary

This chapter introduced a hetero-functional graph-based framework for analyzing resilience in interdependent power and community systems. The proposed approach models functional dependencies and enables the study of failure propagation across interconnected components. Sensitivity analysis was used to identify critical functionalities influencing system disruption behavior. The results highlight the importance of functional interactions in shaping resilience outcomes and motivate the higher-order dependency analysis presented in the next chapter.

Chapter 6

Uncovering Higher-Order Functional Dependencies for Resilience

Infrastructure resilience is increasingly influenced by complex interactions that extend beyond simple pairwise relationships among system components. While conventional network models capture direct dependencies between individual elements, real-world systems often exhibit higher-order interactions involving multiple interconnected functionalities. These interactions can create feedback loops and multi-functional dependencies that significantly affect how failures propagate and how recovery processes evolve. Consequently, identifying and analyzing higher-order dependencies is essential for understanding systemic resilience behavior.

In traditional graph-based analysis, resilience assessment primarily focuses on node and edge failures, assuming that system behavior is governed by pairwise relationships. However, such assumptions may overlook coordinated disruptions arising from multi-component interactions, leading to an incomplete characterization of system vulnerabilities. In particular, higher-order structures such as cycles and loops can serve as critical pathways for cascading failures, influencing both the severity of disruption and the time required for system recovery. Therefore, extending resilience analysis to capture higher-order topological structures provides deeper insight into hidden vulnerabilities within interconnected infrastructure

systems.

To address these limitations, this chapter integrates topological data analysis (TDA) techniques into the hetero-functional graph framework developed in the previous chapter. Specifically, persistent homology is employed to identify multi-scale topological features, while Hodge Laplacian-based spectral analysis is used to characterize functional dependencies across different dimensions. These tools enable systematic detection of higher-order structures that influence failure propagation and recovery dynamics.

This chapter further develops contingency analysis frameworks to evaluate system performance under both random and targeted disruption scenarios. Comparative analyses between conventional pairwise models and higher-order representations demonstrate the enhanced capability of the proposed approach to detect critical vulnerabilities and quantify resilience degradation. The insights derived from this analysis provide a deeper understanding of systemic resilience and support the development of robust infrastructure planning and recovery strategies.

6.1 Cascading Failures in HFGs

The increasing interdependence of modern infrastructure systems necessitates modeling frameworks beyond conventional graph representations. Power systems, in particular, are tightly coupled with water, transportation, and community infrastructures through functional dependencies. To capture these interactions, this section presents a HFGT framework, followed by spectral analysis to quantify network connectivity and robustness. The HFG enables explicit representation of cross-domain interactions, while the graph Laplacian provides insights into node-level connectivity and the emergence of disconnected components under disturbances.

6.1.1 Hetero-Functional Graph Representation

Traditional graph-theoretic models represent power systems, water networks, transportation, and other community services as simple graphs. $\mathcal{G} = (V, E)$, where the node set V represents buses, resources, or other infrastructure components, and the edge set E denotes physical interconnections such as transmission lines, distribution lines, roads, and other links. While this abstraction is effective for analyzing structural properties (e.g., connectivity, shortest paths, and power flow feasibility), it inherently assumes that system behavior is governed solely by topological adjacency. Such an assumption is increasingly inadequate for modern infrastructure systems, where reliable operation depends on tightly coupled interactions across multiple domains.

In practice, power systems are deeply interdependent with external infrastructures and services, including water distribution (for cooling and hydropower), transportation networks (for fuel logistics and maintenance access), communication systems (for monitoring, protection, and control), and human/community-centric services (such as hospitals, emergency response, and critical loads). Furthermore, operational processes, such as protection schemes, control actions, market operations, and maintenance scheduling, introduce functional dependencies that are not explicitly captured by purely structural edges. Although multilayer graph models have been introduced to represent interactions across multiple infrastructure domains, these approaches primarily focus on inter-layer connectivity and still lack the capability to explicitly represent complex functional relationships among processes. As a result, traditional graph models fail to represent how disruptions propagate across domains, how functionality degrades under stress, and how recovery actions unfold in real-world scenarios.

To address these limitations, we adopt a hetero-functional graph (HFG) representation, which generalizes conventional graphs by incorporating multiple types of nodes, edges, and functional relationships. In an HFG, nodes represent not only physical components (e.g., generators, buses, substations) but also abstract entities such as control functions, communication layers, and community services. Edges extend beyond physical connectivity to encode functional dependencies, information flows, resource exchanges, and operational con-

straints. This allows the model to explicitly represent both intra-domain and inter-domain interactions in a unified framework.

More formally, an HFG can be defined as, $\mathcal{G}_H = (\mathcal{F}, \mathcal{D})$, where $\mathcal{F} = f_1, f_2, \dots, f_N$ is a set of functionalities, $\mathcal{D} \subseteq \mathcal{F} \times \mathcal{F}$. Each node and edge is associated with attributes that capture its physical, functional, or socio-technical role. This structure enables the modeling of higher-order relationships, including one-to-many and many-to-many dependencies, conditional interactions, and temporal evolution of system states. As illustrated in Fig. 6.1, an HFG framework can represent power, transportation, water, and community service systems as distinct yet interconnected functional layers, each identified using specific color codings for clarity. Within this representation, individual nodes correspond to functionalities rather than merely physical components, thereby enabling a more expressive description of system behavior. Importantly, dependencies exist not only within the power system but also across different infrastructure domains, highlighting the tightly coupled nature of modern critical infrastructures.

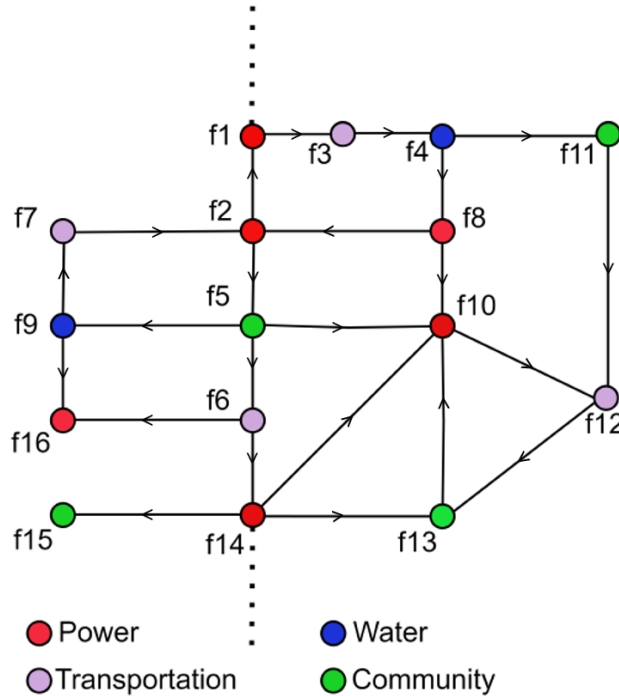


Figure 6.1: Functionalities of various networks as nodes in HFG assigned with respective colors

For instance, functionalities f_1, f_2 , and f_8 correspond to core power system operations, including substation transformation (voltage step-down), power transmission between buses (e.g., from bus 1 to bus 2), and localized power generation at a bus. These functionalities capture distinct operational roles within the grid rather than simply representing physical assets. Similarly, functionalities f_4 , and f_9 are associated with the water infrastructure, representing processes such as water treatment and pumping operations required for cooling in thermal generation systems. The transportation network is represented through functionalities such as f_3, f_7 , and f_{12} , which denote road links that facilitate maintenance crew mobility, enable the deployment of mobile generation units, and support services such as electric vehicle charging. Crucially, the HFG framework can also incorporate community-centric functionalities, such as f_5, f_{11}, f_{15} , which represent essential services including education, healthcare, and residential energy consumption. These services are inherently dependent on multiple infrastructure systems simultaneously, making them particularly vulnerable to cascading disruptions. By explicitly modeling such cross-domain and community-level dependencies, the HFG captures the multi-layered interactions that govern system performance, failure propagation, and recovery dynamics in a comprehensive manner.

By embedding these heterogeneous and functional aspects into the graph structure, the HFG framework provides several key advantages for complex system analysis. First, it enables a more realistic characterization of cascading failures, where disruptions in one domain (e.g., an overflow issue in a water treatment facility) can trigger failures in another (e.g., protection maloperation in power systems). Second, it facilitates resilience assessment by capturing not only structural robustness but also functional adaptability and recovery pathways. Finally, it provides a natural foundation for integrating advanced analytical tools, including topological data analysis, spectral methods, and machine learning, to study complex system behavior under disruptions.

6.2 Topological Data Analysis in HFGs

The fundamental mechanism underlying the efficient operation of HFGs can be attributed to higher-order interactions that occur across multiple topological scales. Unlike traditional pairwise interactions captured by edges, higher-order interactions emerge when three or more functionalities are interdependent, forming closed loops or cycles that encode feedback mechanisms and mutual dependencies among system components.

To systematically analyze these higher-order functional interactions, this section presents a framework for topological data analysis (TDA) of HFGs. In particular, we discuss simplicial complexes and the Hodge Laplacian to characterize network robustness and uncover higher-order functional interactions inherent in complex interdependent systems.

6.2.1 Conventional Pairwise Graph Analysis

Spectral analysis of the graph Laplacian enables the characterization of global network structure through its eigenvalues and eigenvectors. Formally, for any graph $\mathcal{G} = (V, E)$, let L denote the graph Laplacian constructed from the adjacency matrix A encoding functional interactions. The eigenvalue problem associated with L reveals that:

$$Lu_k = \lambda_k u_k \tag{6.1}$$

where λ_k denotes the k -th eigen value and u_k is the corresponding eigenvector. The multiplicity of the zero eigenvalue (i.e., $\lambda_k = 0$) equals the number of connected components in the graph. The Laplacian is defined as, $L = D - A$, where D is the diagonal degree matrix. This property provides a direct and computationally efficient way to quantify network connectivity and detect structural disintegration under failures.

In the HFG framework, this general graph formulation directly applies by interpreting the node set V as functional entities and the edge set E as functional dependencies among heterogeneous infrastructure components. Accordingly, the adjacency matrix A represents functional interactions across interconnected systems such as power, water, transportation,

and community services, while the Laplacian matrix L captures the structural organization of these dependencies. From an HFG perspective, this interpretation is particularly meaningful because node-node interactions correspond to functional dependencies across heterogeneous systems. A reduction in connectivity, reflected by an increase in the number of connected components, indicates a breakdown of these interdependencies, which may arise due to outages, disruptions, or targeted attacks. Conversely, a fully connected HFG implies strong integration and coordination across multiple infrastructure layers.

Thus, spectral analysis based on the graph Laplacian offers a foundational tool to assess the integrity of node-level interactions in HFGs. It provides critical insights into how disturbances affect system-wide connectivity, serving as a baseline for more advanced analyses involving higher-order interactions captured through topological data analysis.

6.2.2 Persistent Homology and Hodge Laplacian-Based Functional Dependency Analysis

6.2.3 Simplicial Complex

Consider an HFG $\mathcal{G}_H = (\mathcal{F}, \mathcal{D})$, where $\mathcal{F}$ denotes the set of functionalities (nodes) and $\mathcal{D}$ represents the set of pairwise functional interactions (edges), with $|\mathcal{F}| = n$. To capture higher-order interactions beyond pairwise dependencies, we extend this representation using the notion of a simplicial complex.

An n -simplex, denoted by $\sigma_n = [f_0, f_1, \dots, f_n]$, is defined as the convex hull of $n + 1$ affinely independent points $f_0, f_1, \dots, f_n$. Lower-dimensional simplices include nodes (0-simplices), edges (1-simplices), triangles (2-simplices), and tetrahedra (3-simplices), with higher-dimensional counterparts representing multi-way interactions among functionalities. A simplicial complex is then defined as a collection of such simplices that is closed under the inclusion of faces, i.e., if a simplex is included, all its lower-dimensional faces are also included. This framework provides a natural generalization of graphs by enabling the representation of higher-order functional dependencies that cannot be captured through pairwise

edges alone [167]. In the context of HFGs, simplicial complexes allow modeling of multi-functional interactions, such as feedback loops and collective dependencies among power, water, transportation, and community services. Figs. 6.1 and 6.2 illustrate the distinction between graph-based and simplicial complex representations for capturing such interactions in HFGs.

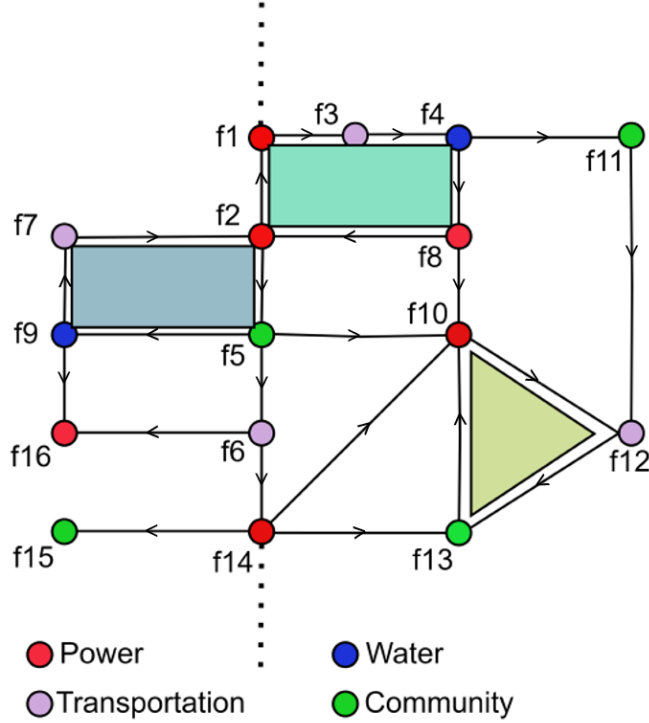


Figure 6.2: Representation of simplicial complex of HFG.

6.2.4 Chain Complex and Boundary Operators

Given the simplicial complex constructed from the HFG $\mathcal{G}_H = (\mathcal{F}, \mathcal{D})$, we define algebraic structures that enable the analysis of higher-order interactions. An n -chain is defined as a formal linear combination of n -simplicials, expressed as [168],

$$C = \sum_{i=0}^n \alpha_i \sigma_i \quad (6.2)$$

where σ_i denotes a n - simplex and $\alpha_i \in \{0, 1\}$ indicates its inclusion. The collection of

all n -chains forms a group C_n , and the sequence of these groups connected via boundary operators constitutes a chain complex. To relate chain groups across different dimensions, we define the boundary operator $\partial_n : C_n \rightarrow C_{n-1}$, which maps an n -simplex to its set of $(n-1)$ -dimensional faces. For an oriented n -simplex $\sigma_n = [f_0, f_1, \dots, f_n]$, the boundary operator is given by

$$\partial_n \sigma_n = \sum_{i=0}^n (-1)^i [f_0, f_1, \dots, \hat{f}_i, \dots, f_n] \quad (6.3)$$

where $\hat{f}_i$ denotes the omission of vertex f_i . This operation systematically extracts all lower-dimensional faces of a simplex. For example, applying ∂_2 to a 2-simplex (triangle) yields its three constituent edges. A fundamental property of boundary operators is that the boundary of a boundary is zero [167], i.e.,

$$\partial_{n-1} \partial_n \sigma_n = 0 \quad (6.4)$$

which implies that higher-order structures such as cycles are closed and do not have boundaries. In the context of HFGs, this formulation enables the identification of higher-order functional interactions. For instance, consider a 2-simplex representing a closed interaction among three functionalities $[f_{10}, f_{12}, f_{13}]$. Applying the boundary operator yields;

$$\partial_2 [f_{10}, f_{12}, f_{13}] = [f_{10}, f_{12}] + [f_{12}, f_{13}] - [f_{10}, f_{13}] \quad (6.5)$$

which corresponds to a set of directed pairwise functional dependencies forming a closed loop (1-cycle). Such cycles capture feedback mechanisms and interdependencies among functionalities in power, water, transportation, and community systems. Thus, the chain complex and boundary operators provide a rigorous algebraic framework to analyze higher-order connectivity patterns in HFGs, forming the foundation for subsequent topological and spectral analysis using the Hodge Laplacian.

6.2.5 Cycles

A n -cycle is defined as an n -chain whose boundary is zero, i.e., elements in the kernel of the boundary operator. Let Z_n denote the space of n -cycles, defines as [169],

$$Z_n = Ker(\partial_n) = \{\sigma_n \in C_n | \partial_n \sigma_n = 0\} \quad (6.6)$$

Similarly, let B_n denote the space of n -boundaries, given by,

$$B_n = img(\partial_{n+1}) = \{\sigma_n \in C_n | \partial_{n+1} \sigma_{n+1} \in C_n\} \quad (6.7)$$

Since the boundary of a boundary is zero ($\partial_n \partial_{n+1} = 0$), it follows that $B_n \subseteq Z_n$. The n^{th} homology group is then defined as the quotient space as,

$$H_n = Z_n / B_n \quad (6.8)$$

which characterizes equivalence classes of cycles that are not boundaries. The rank of H_n , known as the n^{th} Betti number β_n , quantifies the number of independent n -dimensional topological features:

$$\beta_n = rank(H_n) = rank(Z_n) - rank(B_n) \quad (6.9)$$

In the context of graph structures (1-skeletons), β_0 corresponds to the number of connected components, while β_1 represents the number of independent cycles (loops). Higher-order Betti numbers are zero for graphs but become significant in simplicial complexes, enabling the characterization of higher-order interactions in HFGs.

6.2.6 Hodge Laplacian over Simplicial Complexes of HFG

The Hodge Laplacian generalizes the classical graph Laplacian from node-level (0-simplices) interactions to higher-dimensional simplices, thereby enabling the analysis of multi-way functional dependencies in HFGs. While the standard graph Laplacian captures pairwise (node-

node) interactions, the Hodge Laplacian extends this notion to n -simplices to account for higher-order structures such as cycles and cavities.

- **Hodge Laplacian:** For a simplicial complex constructed from the HFG, the n^{th} Hodge Laplacian is defined using boundary operators that relate simplices across adjacent dimensions. Let B_n denote the boundary matrix corresponding to the boundary operator ∂_n , mapping n -simplices to $(n+1)$ -(upper adjacency) and $(n-1)$ -simplices (lower adjacency). The entries of B_n encode the incidence relationship between simplices along with their relative orientations [170].

$$(B_n)_{i,j} = \begin{cases} +1 & \text{if } \sigma_{n-1}^i \subset \sigma_n^j \text{ and } \sigma_{p-1}^i \sim \sigma_k^j \\ -1 & \text{if } \sigma_{n-1}^i \subset \sigma_n^j \text{ and } \sigma_{p-1}^i \approx \sigma_k^j \\ 0 & \text{otherwise.} \end{cases} \quad (6.10)$$

The n^{th} Hodge Laplacian is then given by,

$$L_n = B_n^T B_n + B_{n+1} B_{n+1}^T \quad (6.11)$$

where the first term $B_n^T B_n$ represents the lower adjacency (interaction with $(n-1)$ -simplices), and the second term $B_{n+1} B_{n+1}^T$ represents the upper adjacency (interaction with $(n+1)$ -simplices [171]).

In the context of HFGs, this formulation enables the analysis of functional interactions at multiple levels. The 0-th Hodge Laplacian $L_0 = B_1 B_1^T$ reduces to the standard graph Laplacian, capturing node-level (pairwise) dependencies among functionalities. The boundary matrix B_1 , equivalent to the incidence matrix in graph theory, encodes how functionalities are connected through edges.

More importantly, the 1st Hodge Laplacian L_1 operates on edges (1-simplices) and captures interactions among them through shared nodes (lower adjacency) and higher-order structures such as triangles (upper adjacency). For a pure graph (1-skeleton),

where no higher-order simplices exist, $B_2 = 0$, and the Hodge Laplacian simplifies to:

$$L_1 = B_1^T B_1 \quad (6.12)$$

However, in the HFG-induced simplicial complex, the presence of higher-order simplices (e.g., triangles representing multi-functional dependencies) results in a non-zero B_2 , allowing L_1 cycle-level interactions and feedback structures.

- **Functional cycle interpretation in HFG:**

The spectral decomposition of the Hodge Laplacian provides a principled approach to identify higher-order topological features, such as cycles, in HFGs. In particular, the n^{th} homology group H_n is given by the kernel of the n^{th} Hodge Laplacian, i.e.,

$$H_n = Ker(L_p) \quad (6.13)$$

Thus, the eigenvectors corresponding to the zero eigenvalues of L_n span the kernel space and characterize the n -dimensional topological features of the underlying simplicial complex [172]. To compute these features, we consider the spectral decomposition of the Hodge Laplacian:

$$L_n U_n = \Lambda_n U_n \quad (6.14)$$

where Λ_n is a diagonal matrix of eigen values and U_n contains the corresponding eigenvectors. The multiplicity of the zero eigenvalue of L_n equals the n^{th} Betti number β_n , which quantifies the number of independent n -cycles in the HFG. This generalizes the well-known result for graph Laplacians, where the number of zero eigenvalues of L_0 corresponds to the number of connected components. In particular, for simplicial complexes derived from HFGs, the zero eigenvalues of L_0, L_1 , and L_2 correspond to connected components (0-cycles), closed loops (1-cycles), and higher-order cavities (2-cycles), respectively.

Focusing on 1-cycles, which capture closed-loop functional dependencies across infrastructures, let U_1 denote the matrix whose columns are the eigenvectors of L_1 corresponding to zero eigenvalues. If β_1 is the first Betti number, then $U_1 \in \mathbb{R}^{|\mathcal{D}| \times \beta_1}$, where $|\mathcal{D}|$ is the number of edges (1-simplices) in the HFG. Each column of U_1 represents an independent 1-cycle [173].

Accordingly, the m^{th} 1-cycle C_m can be expressed as a linear combination of edges:

$$C_m = \sum_{e_{i,j} \in \mathcal{D}} a_{i,j}^m e_{i,j} \quad (6.15)$$

$a_{i,j}^m$ is the coefficient associated with edge $e_{i,j}$ in the m^{th} eigenvector of the Hodge Laplacian corresponding to edge $e_{i,j}$. These coefficients encode both the presence and orientation of edges within the cycle. For computational purposes, each cycle can be represented as a vector in the edge space by arranging the coefficients according to a fixed ordering of edges.

In the context of HFGs, these 1-cycles correspond to closed-loop functional interactions involving multiple infrastructures (e.g., power–water–transportation–community feedback loops). Identifying and analyzing such cycles provides critical insights into redundancy, feedback mechanisms, and potential pathways for cascading failures, thereby enabling a deeper understanding of system robustness beyond pairwise interactions.

Thus, the Hodge Laplacian provides a unified framework to analyze both pairwise and higher-order functional relationships in HFGs. It enables the decomposition of flows and interactions across multiple topological scales, offering deeper insights into the robustness, redundancy, and vulnerability of interdependent infrastructure systems under disturbances. In particular, identifying and analyzing higher-order functional relationships through cycles reveals critical information about feedback mechanisms and alternative pathways that sustain system functionality. The degradation of these structures serves as an early indicator of system fragility beyond what can be captured through pairwise connectivity alone.

To systematically quantify these effects, we employ a centrality-driven, percolation-based

analysis, wherein functionalities are progressively removed, and the resulting impact on both connectivity and higher-order topological features is evaluated. The following section presents the procedure for robustness analysis of HFGs based on this framework.

6.3 Contingency Analysis in the HFG Framework

To evaluate the robustness of the HFG, we adopt a percolation-based framework in which system functionalities are subjected to sequential removal, simulating contingencies such as cyber-attacks, component failures, or extreme events. In this context, nodes in the HFG represent functionalities, and their removal corresponds to the disruption or failure of associated processes. Since edges represent functional dependencies, they are not directly targeted; instead, their effects are implicitly captured through node removals. Two categories of attack strategies are considered: random attacks and targeted attacks. In random attacks, nodes are removed uniformly at random, representing limited system knowledge for an attacker. In contrast, targeted attacks assume full knowledge of the network structure for the attacker, where nodes are removed based on their structural importance. To quantify node importance, standard graph-theoretic centrality measures are employed, including degree centrality, betweenness centrality, eigenvector centrality, and PageRank centrality [174]. In particular, nodes with high betweenness centrality are prioritized for removal, as they play a critical role in facilitating functional dependencies and information flow across the network.

The percolation process proceeds in stages, where at each stage a node is removed according to the selected attack strategy. Additionally, the removal of a node may trigger cascading effects on dependent functionalities. Specifically, nodes that rely exclusively on the removed node may become non-operational and are subsequently eliminated, capturing the propagation of failures across interdependent infrastructures. At each stage of node removal, the resulting HFG is analyzed using both graph-theoretic and topological metrics. Conventional measures such as the number of connected components (β_0) are used to quantify network fragmentation. However, these metrics primarily capture pairwise connectivity and may fail to detect early-stage degradation of system functionality. To overcome this limitation, we

incorporate higher-order topological measures based on the Hodge Laplacian. In particular, the first Betti number (β_1) is used to quantify the number of independent cycles, representing closed-loop functional dependencies and redundancy within the network. These cycles capture critical feedback mechanisms and multi-domain interactions across power, water, transportation, and community systems.

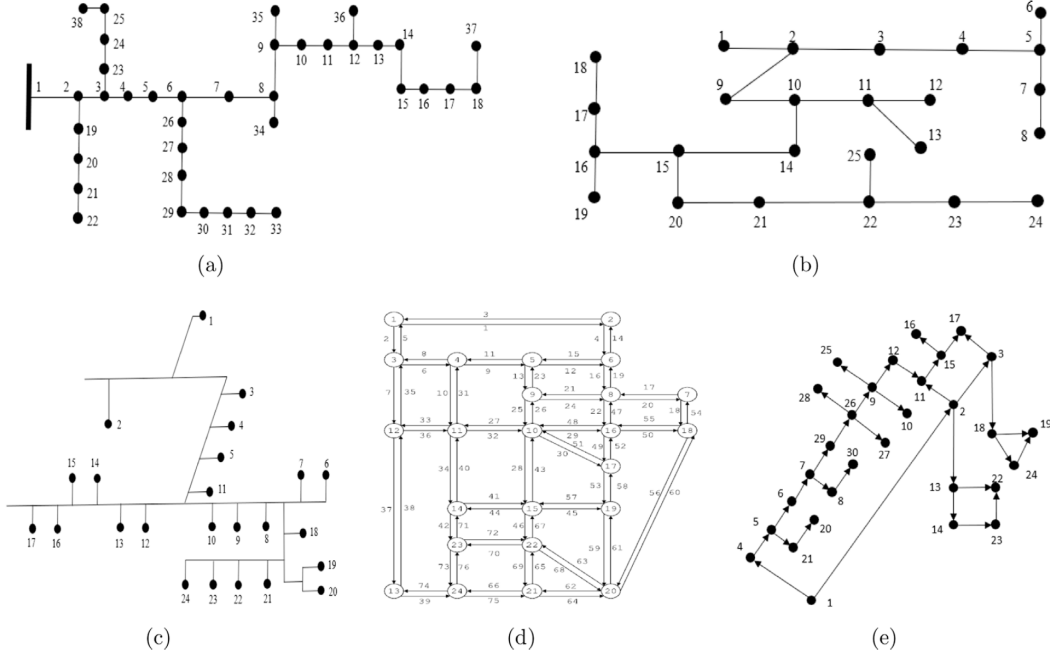


Figure 6.3: Integrated infrastructure with (a)electricity, (b)gas, (c)district heating, (d)transportation, (e)water networks.

6.4 Comparative Results and Discussions

We conduct experiments to test our hypothesis that topological metrics better characterize interdependent infrastructure robustness to failures. Specifically, we investigate the impact of random and targeted attacks on HFGs using both classic graph-theoretic and topological metrics. In particular, we evaluate system robustness by jointly analyzing connectivity-based measures and higher-order topological features across successive percolation stages. Furthermore, we examine how these metrics can inform the optimization strategies to degrade dependency relationships among functionalities that can be used to enhance overall system

resilience.

To demonstrate the proposed TDA framework, we consider two interdependent infrastructure systems: (i) an integrated urban infrastructure network (IUN), adapted from the framework in [123], and (ii) the Customizable Artificial Community (CLARC) dataset, a large-scale interdependent infrastructure system introduced in [175]. IUN models the interactions among multiple critical infrastructures using an HFGT representation. The network comprises five interdependent infrastructure systems, namely the electric power network, natural gas network, water distribution network, district heating network, and transportation network. The integrated network includes a gas-fired power plant (GPP), a solar power plant (SPP), a combined heat and power (CHP) unit, two water treatment plants (WTPs), three district heating plants (HPLs), two gas supply stations (GSs), two industrial parks (IPs), three commercial zones (CZs), six residential zones (RZs), and dedicated repair facilities for water, power, and natural gas infrastructures. These infrastructures collectively support 121 functionalities that describe resource production, transmission, distribution, consumption, and repair activities. Electricity demands of industrial, commercial, and residential consumers are supplied through multiple generation sources, including the GPP, CHP, and SPP, while the water, gas, and district heating networks provide essential supporting services required for the operation of other infrastructures. The model explicitly captures cross-sector dependencies among infrastructures. For example, the gas-fired power plant depends on natural gas supply, water availability, and repair services for continuous operation, while water treatment facilities require electrical energy to process and distribute water. Similarly, district heating systems depend on both electricity and gas resources, and repair facilities rely on transportation infrastructure to restore damaged assets. These interactions create a tightly coupled system in which disruptions in one infrastructure can propagate across multiple sectors. The corresponding hetero-functional graph is constructed from the process relation matrix, which encodes the dependency structure among system functionalities. To evaluate robustness, functionalities are sequentially removed through a percolation process, and the resulting network topology is analyzed after each removal stage. Two attack strategies are considered: random attacks, where functionalities are removed

uniformly at random, and targeted attacks, where functionalities are removed according to their structural importance. At each stage, the evolution of both the number of connected components (NCC) β_0 and the number of independent cycles β_1 is analyzed to capture the degradation of pairwise and higher-order functional interactions, respectively.

6.4.1 Random Attacks

In a random attack scenario, functionalities (nodes) are removed uniformly at random, representing contingencies with limited or no prior knowledge of the system structure. The evolution of robustness metrics across percolation stages is illustrated in Fig. 6.4, where the horizontal axis denotes the number of sequential node removals and the vertical axis represents the corresponding metric values evaluated on the residual HFG. This staged removal process is consistent with the classical definition of graph percolation and enables the simulation of progressive disruptions such as sequential cyber-attacks or cascading failures induced by natural hazards [176].

As expected, the zeroth Betti number β_0 corresponding to the NCC, initially increases as the network begins to fragment into smaller subgraphs. This reflects the progressive loss of global connectivity due to node removals. Beyond a certain stage (approximately stage 17 in this case), the fragmented subgraphs further decompose into isolated nodes, leading to a subsequent decline in β_0 . In contrast, the first Betti number β_1 , which quantifies the number of independent cycles, exhibits a monotonic decrease throughout the percolation process. Notably, β_1 vanishes at a much earlier stage (around stage 35), indicating the complete loss of higher-order functional interactions within the network. The disappearance of cycles signifies the breakdown of feedback mechanisms and redundancy pathways that are essential for sustaining system functionality under disturbances. For instance, functionalities associated with water treatment at node 11 from the water network, no longer receive power from the solar power plant at node 15 and the gas power plant at node 5, as per IUN in [123], thereby eliminating critical interdependencies required for reliable operation.

This observation underscores a fundamental limitation of connectivity-based metrics such

as β_0 , which primarily capture pairwise reachability and may still indicate residual structural connectivity even after the network has lost critical functional redundancy. In contrast, the rapid decay of the β_1 reflects the early disruption of higher-order topological structures, specifically independent cycles that encode feedback mechanisms and alternative functional pathways. Therefore, the evolution of β_1 provides a more sensitive indicator of systemic vulnerability, capturing the degradation of higher-order interdependencies that are not observable through node-level connectivity alone.

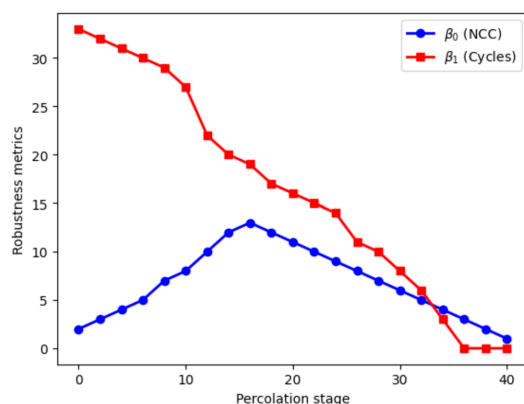


Figure 6.4: Trajectory of robustness metrics across various stages of percolation with random attack.

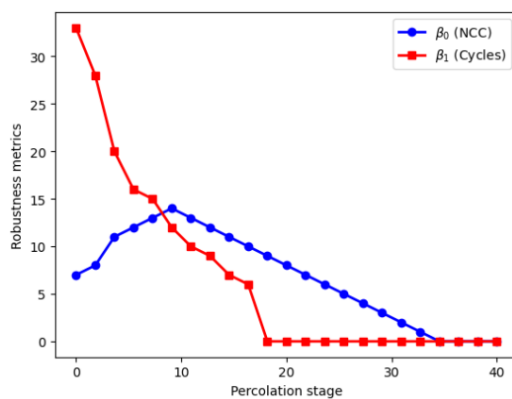


Figure 6.5: Trajectory of robustness metrics across various stages of percolation with targeted attack.

6.4.2 Targeted Attacks

To evaluate the impact of informed attacks, we consider a targeted attack strategy in which nodes are removed based on betweenness centrality, reflecting the attacker’s knowledge of the network structure. At each stage of the percolation process, the node with the highest betweenness centrality in the residual graph is removed, thereby prioritizing functionalities that play a critical role in mediating interactions across the network. The evolution of robustness metrics β_0 and β_1 , illustrated in Fig. 6.5 across successive percolation stages.

As expected, targeted attacks result in a significantly faster degradation of the system compared to random attacks. The connectivity metric β_0 exhibits an initial sharp increase due to rapid fragmentation, followed by fluctuations and eventual stabilization as the network decomposes into isolated nodes. In contrast, β_1 , representing the number of independent cycles, decays at a substantially faster rate, indicating an accelerated loss of higher-order functional interactions and redundancy. This behavior highlights the vulnerability of cycle-based structures to informed attacks. Since nodes with high betweenness centrality often lie on multiple shortest paths, their removal disrupts critical interaction pathways and collapses cycles more efficiently. Consequently, an attacker with access to structural information can strategically degrade system functionality by targeting nodes that contribute to higher-order dependencies, rather than merely fragmenting the network.

To further investigate this effect, we implement an additional attack strategy aimed explicitly at minimizing β_1 within a limited number of percolation stages. In this approach, nodes are selected based on their participation in cycle-forming edges, thereby directly targeting the simplicial structures responsible for higher-order interactions. By identifying and removing nodes associated with these critical edges, the number of independent cycles can be reduced more rapidly than with centrality-based strategies. Simulation results indicate an approximate 8% reduction in the number of stages required to eliminate all cycles in the HFG compared to betweenness-based attacks. These findings demonstrate that higher-order topological features provide a more effective basis for designing adversarial strategies and, conversely, for developing robustness enhancement mechanisms. In particular, protecting

nodes that sustain cycle structures can significantly improve the resilience of interdependent infrastructure systems against targeted disruptions.

6.4.3 Robustness Analysis on Large-scale Interdependent System

The CLARC dataset is a robust, multi-layered synthetic network representing a coastal footprint of 1,065 square miles and 77 census tracts [175]. Formulated as a system of systems containing 1,305 nodes and 4,764 directional arcs, it explicitly maps the physical and functional interdependencies between five core civil infrastructure layers, electric power (including thermal plants and substations), water supply, wastewater, telecommunications, and transportation (roads and bridges), and vital social infrastructures like public safety, healthcare, banking, and fuel distribution. Designed with precise directional topology (From ID- Master-Node to To ID-Master-Node) and embedded cross-layer constraints, the dataset is uniquely suited for topological robustness and network connectivity analyses. Specifically, it allows researchers to model how direct hazard damage to a single asset propagates through defined physical dependencies, such as 73 water nodes and 224 wastewater pump stations strictly requiring electric power to operate, thereby causing cascading failures that structurally fragment the coupled layers into isolated, non-functional connected components. This data serves as a large-scale functional dependency network for the proposed robustness analysis framework. Both the random and targeted attacks are performed on the CLARC dataset to analyze the trajectories of the NCC and Cycles.

A key observation from the Figures 6.6 and 6.7 is that higher-order topological structures, particularly cycles, degrade more rapidly than connectivity under both random and targeted attacks. While the network may remain structurally connected (i.e., β_0 remains unchanged or slowly varying), the rapid decline or disappearance of 1-cycles (β_1) signifies the early loss of independent cycles that encode redundancy and alternative functional pathways. This highlights that cycle-based metrics provide a more sensitive and informative characterization of system vulnerability compared to traditional connectivity measures, enabling the detection of functional degradation at earlier stages of disruption. These results confirm our hypothesis

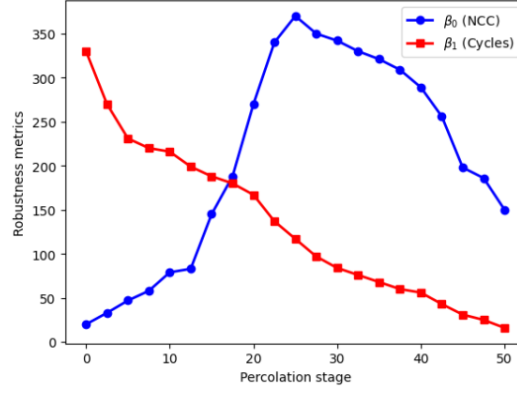


Figure 6.6: Trajectory of robustness metrics across various stages of percolation with random attack in CLARC system.

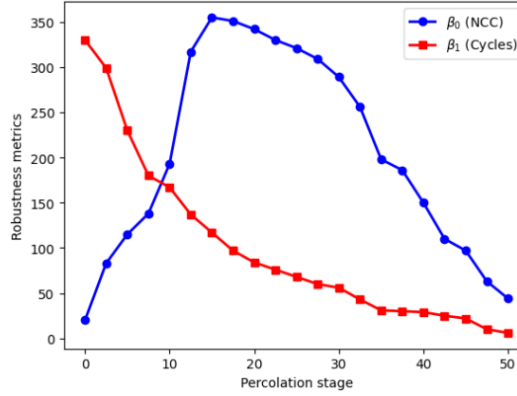


Figure 6.7: Trajectory of robustness metrics across various stages of percolation with targeted attack in CLARC system.

that relying purely on connectivity-based metric β_0 may result in an overly optimistic view of the robustness of system functionality.

6.5 Summary

The higher-order dependency analysis presented in this chapter enhances the understanding of systemic resilience by revealing hidden interaction patterns that significantly influence disruption propagation and recovery dynamics in interconnected infrastructure systems.

Chapter 7

Conclusions and Future work

7.1 Conclusions

This dissertation presented a comprehensive framework for holistic resilience analyses of modern power distribution systems by integrating planning, operational, and structural modeling perspectives. As power systems continue to evolve toward distributed, interconnected, and community-integrated infrastructures, ensuring resilient performance under disturbances has become a fundamental requirement. The methodologies developed in this work collectively address multiple dimensions of resilience, including disruption mitigation, recovery enhancement, and vulnerability identification across interconnected systems.

Chapter 3 of this dissertation focused on resilience-oriented planning of distributed energy resources using optimal transport-based methodologies. By modeling community-level energy characteristics as probability distributions, the proposed framework enabled distribution-aware resource allocation strategies that balance operational performance with community-level considerations. The resulting planning strategies demonstrated improved distributional consistency and reduced variability in system-level outcomes, thereby contributing to limiting the severity of performance degradation during disturbance events.

Chapter 4 addressed the operational dimension of resilience through robust distribution system state estimation and phase identification using Fréchet regression-based approaches.

The developed methodologies improved situational awareness under limited observability conditions and enabled more reliable estimation of system states in the presence of uncertainty and heterogeneous data sources. In addition, coordinated control strategies and community-centric operational frameworks were investigated to support adaptive system response and restoration following disturbances. These contributions enhance the system’s capability to detect abnormal conditions and initiate timely corrective actions, thereby supporting faster recovery and improved operational stability.

Chapter 5 of this dissertation introduced hetero-functional graph theory as a functional modeling framework for resilience analysis of interdependent infrastructure systems. By explicitly representing functional relationships among power and community components, the proposed framework enabled detailed characterization of failure propagation pathways and disruption dynamics. Sensitivity analysis techniques were developed to identify critical functionalities that disproportionately influence system performance degradation, providing valuable insights for targeted resilience enhancement strategies.

Building upon this foundation, Chapter 6, the final component extended the resilience analysis to capture higher-order functional dependencies using topological data analysis techniques. Persistent homology and Hodge Laplacian-based methods were employed to uncover latent structural patterns and hidden interaction pathways that are not observable through conventional pairwise analysis. Comparative studies demonstrated that higher-order representations provide deeper insights into systemic vulnerabilities and reveal additional risk pathways influencing disruption behavior and recovery processes.

Overall, this dissertation establishes a unified and multi-layered framework for analyzing resilience in modern power and community infrastructure systems. By integrating distribution-aware planning, robust operational strategies, functional modeling, and higher-order dependency analysis, the proposed methodologies contribute to a deeper understanding of the mechanisms governing system degradation and recovery. The outcomes of this work provide actionable insights for designing resilient infrastructure systems capable of maintaining reliable service under diverse operating conditions and disturbance scenarios.

7.2 Future work

While this dissertation establishes a holistic framework for resilient and community-centric power system planning and operation, several promising directions remain for future investigation. The following are the potential extensions to the work presented in this dissertation:

Chapter 3 develops OT-based DG planning methods. A primary research direction is to extend the current OT and COT formulations by incorporating additional community-centric metrics and enabling the planning of multiple distributed energy resources. This includes not only rooftop solar PV but also distributed energy storage systems, demand response resources, and electric vehicle charging flexibility. This will require developing coupled transport formulations that can jointly model spatial-temporal allocation of heterogeneous distributed energy resources while preserving a tractable optimization structure.

Moreover, practical adoption of community-centric DER planning requires alignment with regulatory policies and utility business models. Future work can explore the development of policy-aware optimization frameworks that incorporate tariff structures, incentive mechanisms, and market participation rules. In particular, the integration of market-compatible coordination mechanisms between utilities, DER aggregators, and communities will be investigated to ensure that resilience-enhancing strategies remain economically viable and operationally feasible.

Chapter 4 proposes the Fréchet regression-based DSSE and phase identification methods, which demonstrate robustness under limited observability. Future research can explore the integration of machine learning and physics-informed learning architectures, including graph neural networks and surrogate models, to enhance scalability and adaptability for large-scale distribution systems. For community-centric coordination control strategies, future work may explore deep reinforcement learning approaches to fully utilize the potential of DERs under dynamically varying and heterogeneous resource conditions.

In Chapter 5, the HFGT framework provides an efficient mechanism for modeling cascading failures across interdependent infrastructures. Future work can extend this framework to incorporate stochastic failure propagation models and time-dependent recovery dynamics,

enabling dynamic simulation of disruption and restoration processes.

Chapter 6 The integration of persistent homology and Hodge Laplacian-based analysis demonstrates the value of higher-order dependency identification. Future research can focus on developing learning-based topological frameworks that combine topological data analysis with machine learning models to automatically identify emerging vulnerability patterns. Such methods could enable early detection of structural degradation in complex networks and support predictive resilience management under evolving system conditions.

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