DESIGN OF POST-TENSIONED FLAT PLATES

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INTRODUCTION

Prestressing can be defined in general terms as the preloading of a structure, before application of the required design loads, in such a way as to improve its overall performance. Although the principles and techniques of prestressing have been applied to structures of many types and materials, the most common application is in the design of structural concrete.

Post-tensioning is a method of prestressing in which the prestressed reinforcement, whether high-strength steel wires, strands or bars, generally known as "tendons", is tensioned after the concrete has hardened and gained sufficient strength. In many cases, particularly in slabs, post-tensioning tendons are asphalt-coated or greased and put in a plastic tube or duct. The duct prevents the concrete from bonding to the steel and that is why these type of tendons are called "unbonded-tendons". Anchorage and jacking hardware is provided. When the concrete has hardened, the tendons are stretched and anchored, and the jack removed.

Post-tensioning is most commonly applied to cast-in-place concrete members, and to those involving complex curvatures. Bridges, large girders, floor slabs, roofs, shells, pavements, and pressure vessels are among the constructions usually prestressed by post-tensioning.

The application of post-tensioning in the United States did not develop rapidly in the 1950's, but its usage increased 350 percent in the period from 1965 to 1980. A primary reason for this remarkable increase is the growing awareness of the advantages of post-tensioning among engineers, architects, and owners. The benefits include:

a) Reduced structural depth

b) Watertightness

c) Control of deflection

d) Esthetic potential of cast-in-place concrete

e) Longer spans at an economical cost

Prestressed concrete slabs are used in civil engineering structures of many types, to provide flat, useful surfaces such as for floors, roofs, decks, or walls. The floor system known as "Flat Plate" construction, in which there are no beams or other projections below the bottom surface of the slab and the slab is carried directly by columns, is well suited to prestressed concrete construction.

Post-tensioned flat plates have been found to be economical in parking structures, apartment buildings, office buildings, hospitals, and industrial buildings, both high-rise and low-rise type. The economy is achieved by less expensive form-work because of the lack of beams or drop panels below the bottom face of the slab,easy installation of electrical and mechanical equipment, smooth ceilings and floors, and reduction in story heights.

In the ten years between 1967 and 1976, surveys made by the Post-Tensioning Institute indicate that over 250 million square feet of post-tensioned slabs were completed, using about 115,000 tons of post-tensioning materials. To indicate the rapid increase in the use of post-tensioned floor systems, PTI statistics suggest that an additional 250 million square feet of post-tensioned floor slabs were completed in the five-year from 1977 through 1981. This later construction utilized only 90,000 tons of post-tensioning materials, which reflects the use of lower prestress levels in combination with larger amounts of bonded conventional reinforcement.

Post-tensioned flat plates have been used successfully with the lift-slab technique. The lift-slab type of construction resulted from an effort to reduce the cost of concrete construction by eliminating the expense of soffit forming and shoring.These slabs are cast and post-tensioned at ground level and then lifted into position by lifting rods connected to hydraulic jacks mounted on top of the columns. The prestressing of the slab reduces the slab's thickness, controls the deflection, eliminates cracks in the slab and cuts down the cost.

LITERATURE SURVEY

The stresses in prestressed flat plates are highly indeterminate. Until the advent of simplified design techniques developed in the late 1950's, design methods evolved chiefly from experience in the field and intuition rather than on established theory.

In the late 1950's several prestressed slab research projects were under taken in the United States. Scordelis and Lin[1], studied the elastic behavior and ultimate strength of continuous prestressed slabs and proposed several design recommendations. Later, Rice and Kulka[2] emphasized the need for deflection as an important criterion in the design of prestressed lift slabs.

Possibly, the largest stride in the design of prestressed slabs was the publication in 1963 of a paper by Lin[3] on the load balancing method. It was soon made apparent that the tendon profiles could be designed so that the upward cable force neutralized the vertical downward load. This approach by-passed a rigoruous analysis of the highly redundant stress system. Furthermore, the method provided for deflection control for the dead load which is generally the major portion of the load.

Koons and Schlegel[4] extended the load balancing approach and presented some practical aids for solving continuity and cable reversal. Rozvany and Hampson[5] and Brotchie and Russell[6] developed an elastic approach for the optimum design of prestressed flat plates. Both investigators arrived at the same results, the principal difference being that Rozvany and Hampson used load balancing whereas Brotchie and Russell used moment balancing.

In 1964, Candy[7] developed a procedure for designing flat plates using the load balancing method plus the ACI 318-63 ultimate strength provisions. Candy advocated using a column strip of width L/4 to L/3, rather than the customary L/2 width.

The trend towards high rise buildings and the commercial availability of high strength, light weight concrete re-focused attention on flat plates in the United States. In 1967, Grow and Vanderbilt[8] conducted an investigation into the shear strength of 10 post-tensioned light weight slabs using expanded shale aggregate. From this study a useful formula evolved for checking the shear strength of light weight prestressed slabs at columns.

In 1968, Wang[9] proposed a method for designing prestressed flat plates using working stresses. However, the method was unduly complicated and, furthermore, lacked any mention of ultimate strength checks. In late 1968, Parme[10] made a rigorous elastic analysis of the distribution of moments and direct forces induced by prestressing flat plates. He also included a set of useful design tables for finding prestressing moments.

Rozvany and Woods[11] emphasized the need for giving unbonded tendons a minimum level of average prestress in the event of high live loads or earthquake motions. However, in a subsequent discussion, Bondy[12] felt that introducing too high a level of average prestress would cause excessive shortening and camber problems. He said that the better solution would be to add bonded unprestressed reinforcement.

In 1971, Gerber and Burns[13] tested many post-tensioned flat plate specimens, simulating both lift-slab and cast-in-place construction. They reported that the shear design practice was conservative, providing factors of safety of the order of 3 to 4.5. They also reported that supplemental reinforcing steel was found to help control cracking, increase ductility of the structure , and greatly improve transmission of loads to columns.

ACI-ASCE Committee 423[14] has given a comprehensive report on design recommendations for concrete members prestressed with unbonded tendons. Much of the report is directly applicable to post-tensioned flat plates and its applicability will be shown later in this report.

Today, the majority of prestressed flat plate designs are based on load balancing plus service load and ultimate strength checks. In cases where ultimate strength provisions are not satisfied, it is general practice to furnish unprestressed bonded reinforcement.

GENERAL PRINCIPLES OF PRESTRESSED CONCRETE

Three different concepts may be applied to explain and analyze the basic behavior of prestressed concrete. These will be explained as follows:

FIRST CONCEPT - "STRESS CONCEPT"

This concept treats prestressed concrete as an elastic material so that it can be designed and analyzed with respect to its elastic stresses.

From this standpoint concrete is visualized as being subject to two systems of forces: internal prestress and external load, with the tensile stresses due to the external load counteracted by the compressive stresses due to the prestress. The prestress prevents cracking of the concrete and as long as there are no cracks, the stresses, strains, and deflections of the concrete due to the two systems of forces can be considered seperately and superimposed if necessary.

Consider a simple rectangular beam prestressed by a tendon placed eccentrically with respect to the centroid of the concrete section, Fig.1. The tensile prestress force F in the tendon produces an equal compressive force F in in the concrete, which also acts at the centroid of the tendon. Due to an eccentric prestress, the concrete is subject to a moment as well as a direct load. The moment produced by the prestress is"Fe", and the stress due to prestress across the section that has an area A, and moment of inertia I, is given by:

$$f = F/A \pm Fey/I$$

Now if M is the external moment at a section due to the load on and the weight of the beam, then the resulting stress distribution due to prestress and the external moment is given by:

$$f = F/A \pm Fey/I \pm My/I$$

as shown in Fig.2.

<u>SECOND CONCEPT - "STRENGTH CONCEPT"</u>

This concept is to consider prestressed concrete as a combination of steel and concrete, similar to reinforced concrete, with steel taking tension and concrete taking compression so that the two materials form a resisting couple against the external moment, Fig.3.

In prestressed concrete, high-tensile steel is used which will have to be elongated a great deal before its strength is fully utilized. By prestretching and anchoring the steel against the concrete, desirable stresses and strains in both materials are produced: compressive stresses and strains in concrete, and tensile stresses and strains in steel. This combined action permits the safe and economical utilization of the two materials which cannot be achieved by simply burying steel in the concrete as is done for ordinary reinforced concrete. The extreme fiber stresses at any section can be calculated by:

$$f = F/A \pm My/I$$

where

$$M = Ce\ , \quad C = F$$

e = eccentricity of the compressive force C from the center of gravity of concrete

THIRD CONCEPT - "LOAD BALANCING"

This concept sees prestressing as primarily an attempt to balance a portion of the load on the structure. While this load-balancing method often represents the simplest approach to prestressed design and analysis, its advantage over the previous two concepts is not significant for statically determinate structures. When dealing with statically indeterminate structures including flat slabs and certain thin shells, this load-balancing method offers tremendous advantages both in calculating and in visualizing.

Since this method is used in this report, it will be discussed in more detail.

ONE-DIMENSIONAL LOAD BALANCING

The fundamentals of the approach will be illustrated in the context of the simply supported uniformly loaded beam as shown in the Fig.4.1. The beam is to be designed for a balanced load consisting of its own weight W_o the superimposed dead load W_d

and some fractional part of the live load denoted by $K_b W_1$ Since the external load is uniformly distributed , it is reasonable to adopt a tendon having a parabolic shape. It is easily shown that a parabolic tendon will produce a uniformly distributed upward load equal to

$$Wb = 8Fz/L^2$$

where

F = magnitude of the prestress force

z = maximum sag of the tendon measured with respect to the chord between its end points

L = length of the span

If the downward load exactly equals the upward load from the tendon, these two loads cancel, and no bending stress is produced as shown in the Fig.4.2. The net resulting stress is uniform compression "f_a"equal to that produced by the axial force FCosθ. Under this stress distribution the beam would show no vertical deflection. This condition is called the "Balanced Condition".

However, if the live load is removed or increased, then bending stresses and deflections will result because of the "Unbalanced Portion" of the load. Stresses due to this differential loading must be calculated by the ordinary formulas in mechanics, $f = My/I$, and superimposed on the axial compression to obtain the net stresses for the "Unbalanced State".

Loads other than uniformly distributed would lead naturally to the selection of other tendon configurations. For example, if the external load is a single concentrated load at midspan, a harped tendon such as shown in Fig.5.1. would be chosen, with some eccentricity at midspan. A third-point loading would require a tendon to be harped at third points Fig.5.2 .

It should be remembered that, for simple spans designed by "Load Balancing Concept", it is necessary for the tendons to have zero eccentricity at the supports, because the external moment due to superimposed load is zero there.

LOAD BALANCING AS APPLIED TO CONTINUOUS MEMBERS

The load balancing concept is particularly advantageous for continuous members. The transverse forces that correspond to the particular tendon profile are found, making use of the geometry of the tendon profile. The structure can then be analyzed for the effects of these transverse forces by using any of the available methods for indeterminate analysis, such as method of superposition, moment distribution or matrix analysis.

For an indeterminate structure, the moments found from such an analysis are the "total moments" due to prestressing, and include the secondary moments due to support reactions as well as the primary moments due tendon eccentricity. Secondary moments may be found, if needed, by subtracting the primary moments, which are determined very easily, from the total moments obtained from the transverse forces. If the transverse forces exactly balances the applied loads, then we have a "balanced condition", that is only a uniform compressive force "F_a"given by $F_a = F/A$ is acting on any cross section of the member.

For any change from that balanced load condition, ordinary elastic analysis such as moment distribution can be applied to the load differential to obtain the moment at any section, and the resulting stresses computed from the formula $f = My/I$. The net stresses are obtained by superimposing these stresses on the uniform compressive stress F_a as before.

TWO DIMENSIONAL LOAD BALANCING FOR FLAT PLATES

The load balancing approach to design is specially useful in treating flat plates. There are two slightly different points of view regarding load balancing design for a flat plate system. These are discussed below.

The first approach is illustrated by the rectangular slab of Fig.6. showing a typical interior panel of a flat plate floor. The uniformly distributed load to be balanced is carried by a two-way network of tendons of parabolic shape, uniformly spaced along either side l_1 or l_2. The panel is considered to be supported at its edges along the column lines in each direction. The proportion of the load to be carried in one direction or the other is more or less arbitrary. Assume for this example that 60 percent of the load is assigned to the short direction l_2 and 40 percent to the long direction l_1. The required tendon network is shown in Fig.6.1 . Parabolic tendons are used , concave upward, near the top of the slab at the column lines and close to the bottom at midspan.

But the change in the slope of the tendons of the primary network, as they cross the column lines, produces a downward reaction on the slab strips along the column lines. A true line loading would be obtained for the sharply bent tendons shown, but this is not practical. In actual cases a transition curve, concave downward, would produce a strip loading of finite width along the column lines.

The downward load on the column strips must be resisted by a second set of tendons, placed along those column strips, as shown in Fig.6.2. If 40 percent of the load were carried in the long direction of the slab, then the column strips in the short direction would pick up that load and transmit it to the columns. Note that the 60 percent carried directly the short way by the slab, plus the 40 percent carried by the short direction strips, accounts for 100 percent of the load, as required by statics. A corresponding analysis holds in the perpendicular direction.

The final arrangement of tendons is found by superimposing the arrangements of Figs.6.1 and 6.2, resulting in a rather wide spacing of tendons in the central part of the panel and a concentrated band of tendons along the column lines in each direction, Fig.6.3 .

The second approach treats the slab as an orthogonal system of broad flat beams, each one of full panel width, and makes use directly of the fact that 100 percent of the load to be balanced must be carried in each direction. For purposes of analysis in the l_1 direction in Fig.7.1 the slab is considered to be supported continuously along the transverse column lines ab and cd. For the distributed loading, parabolic tendons are selected, with maximum sag controlled by the requirements of cover at the top and bottom of the slab.

However, on the basis of elastic analysis and tests, it is known, that the lateral distribution of bending moments due to the applied loading is not uniform across the width of the critical sections, but tends to concentrate near the column lines. According to current information, for simple spans, between 55 and 60 percent of the moment will be concentrated in the column strips, while for continuous spans between 65 and 75 percent will be within the column strips, the remainder being placed in the middle strips in each case. Due to this reason the tendons are placed in a like manner. The result is indicated in Fig.7.2, a large percentage of the tendons are concentrated in bands along the column lines.

A corresponding situation occurs in the direction l_2, the total number of tendons being sufficient to equilibrate 100 percent of the load to be balanced in that direction also.

The superposition of the tendons in each of the two perpendicular directions is shown in Fig.7.3. It is clear that the end results are the same as those of the analysis summarized in Fig.6.3, although the reasoning is different. The second approach is somewhat simpler to apply in practice, and so is more generally used.

ANALYSIS

BEHAVIOR OF FLAT PLATES

The behavior of a flat plate slab may be understood with reference to Fig.8, which shows a typical interior slab panel, together with portions of the adjacent panels. For purposes of design, a typical panel is divided into "COLUMN STRIPS" and "MIDDLE STRIPS".

A "COLUMN STRIP" is defined as a strip of slab having a width on each side of the column centerline equal to one-fourth the smaller of the panel dimensions l_1 and l_2.

A "MIDDLE STRIP" is a design strip bounded by two column strips. While, for flat plate slabs, there are no column line beams to provide edge support for the panels, column strips in each direction play the role of the missing beams.

A load applied to the central area, shown darkened in Fig.8 is shared between strips of slab spanning in the short and long directions of the panel. The division of the load between short and long direction strips depends on the aspect ratio of the panel, and on the boundary conditions.

Note that the portion of the load that is carried by the middle strip in the long direction is delivered to the column strips spanning in the short direction of the panel. This portion of the total load, plus that carried directly in the short direction by the middle strip, sums up to 100 percent of the load applied to the panel. Similarly, the short-direction middle strip delivers a part of the load to the long-direction column strips. This load, plus that carried directly in the long direction by the middle strip, includes 100 percent of the applied load. It is clearly the requirement of statics that, for column-supported slabs, 100 percent of the applied load must be accounted for in each direction, jointly by the column strips and middle strips.

Fig.9 shows a flat plate floor supported by columns at a,b,c, and d, and carrying load w per unit of surface area. Fig.10 indicates the moment diagram for the direction of span l_1. In this direction, the slab may be considered as broad, flat beam of width l_2. Accordingly, the load per unit length of span is wl_2.

In any span of a continuous beam, the sum of the midspan positive moment and the average of the negative moments at adjacent supports is equal to the midspan positive moment of a corresponding simply supported beam.

In terms of the slab, this requirement of statics may be written

$$1/2\ (M_{ab} + M_{cd}) + M_{ef} = 1/8\ wl_2 l_1^{\,2}$$

A similar requirement exists in the perpendicular direction, leading to

$$1/2\ (M_{ac} + M_{bd}) + M_{gh} = 1/8\ wl_1 l_2^{\,2}$$

The moments across the width of critical sections, such as across the lines ab or ef, are not constant, but vary as shown qualitatively in Fig.11. Along the column centerlines, where curvatures are greater,moments are greater, while along a line at the centerline of the panel, curvatures are more gradual and the corresponding moments are smaller. For design purposes, moments may be considered constant within the bounds of a middle strip or column strip as shown in the same Fig.11.

ACI-METHODS

Chapter 13 of the ACI Code deals in a unified way with two-way slab systems. While permitting design "by any procedure satisfying the conditions of equilibrium and geometrical compatibility", specific reference is made to two alternative approaches: a semiempirical "DIRECT DESIGN METHOD" and an approximate elastic analysis known as the "EQUIVALENT FRAME METHOD".

DIRECT DESIGN METHOD

In the "direct design method", the distribution between positive and negative moment zones of the total factored static moment along the span, as computed from the following equation,

$$M_o = W_u l_2 l_n^2/8$$

is made approximately using a set of coefficients prescribed by the ACI Code. This is done for panels centered on column lines, in each direction. After obtaining the maximum positive moment "Mpos" at midspan and "Mneg" at the faces of supports, it remains to distribute these moments across the width of the panel to the corresponding column and middle strips. This is accomplished by another set of coefficients prescribed by the ACI Code.

According to the ACI Code Section 18.12, elastic analysis is required for prestressed slab systems reinforced for flexure in more than one direction. The "DIRECT DESIGN METHOD" is based on moment coefficients obtained mainly by testing reinforced concrete slabs. The direct design method should not be used for prestressed concrete.

EQUIVALENT FRAME METHOD

The most widely used approximate method of analysis for post-tensioned flat plates is the "EQUIVALENT FRAME METHOD" which will be discussed in the following paragraphs.

The Equivalent Frame Method involves the representation of the three-dimensional slab system by a series of two-dimensional frames which are then analyzed for loads acting in the plane of the frames. The structure is divided for purposes of analysis into continuous frames, centered on the column lines and extending both longitudinally and transversely, as shown by the shaded strips in Fig.12.1.

Each frame is composed of a row of columns and a broad continuous beam consisting of the portion of slab bounded by panel centerlines on either side of the center line of columns.

For vertical loading, each floor with its columns may be analyzed separately, the columns assumed fixed at the floors above and below, Fig.12.2. In calculating bending moment at a support, it is convenient and sufficiently accurate to assume that the continuous frame is completely fixed at the support two panels distant from the one of interest. Regarding the method, the following may be stated:

MOMENTS OF INERTIA

When an elastic analysis is to be made, the moments of inertia are treated in a more accurate manner than when moment coefficients are used in the "direct design method".

The moment of inertia of the slab-beam between the center of the column is to be assumed equal to that of the slab-beam at the face of the column divided by the quantity $(1 - C_2/l_2)^2$ where C_2 and l_2 are measured transverse to the direction in which moments are being determined. (ACI-13.7.3.3), Fig.13.2. The moment of inertia of the column is to be assumed infinite from the top of the slab, to the bottom of slab-beam at the joint (ACI-13.7.4.5),Fig.13.1.

STIFFNESS, CARRY-OVER FACTORS, FIXED-END MOMENTS

Since no member of the equivalent frame has a uniform moment of inertia "I", the stiffness is not 4EI/L but something more than 4 for the constant and something more than 0.5 for the carry-over factor,C.O.F. Tables can tabulate "k" for use in kEI/L and C.O.F. Likewise the fixed-end moment is normally greater than for uniform I. Table 2. lists for flat plates the slab stiffness factors, moment factors and carry-over factors. The ACI-Code Commentary has similar tables for flat plates with $C_1/l_1 = C_2/l_2$ but accomodating unequal ratios at the two slab ends.

EQUIVALENT COLUMN STIFFNESS

The column flexural stiffness is modified to account for the torsional flexibility of the slab-to-column connection which reduces its efficiency for transmission of moments.

Fig.14 shows the condition at an interior column with the slab spanning in the direction l_1. According to the ACI Code, the flat plate is to be considered as supported by a transverse slab strip or beam of width "b" equal to the dimension of the column in the direction of the moment analysis, and height "h" equal to the depth of the slab. The rotational restraint provided for the slab is influenced, not only by the flexural stiffness of the column, but by the torsional stiffness of the transverse beam as well. With distributed torque "m_t" applied by the slab and resisting torque "M_t" provided by the column, the outer ends of the transverse slab strip will rotate to a greater degree than the central section, due to torsional deformation. To allow for this, the actual column and transverse slab strip are replaced by an "equivalent column", defined so that the total flexibility (inverse of stiffness) of the equivalent column is the sum of the flexibilities of the actual column and the slab strip. Thus,

$$1/K_{ec} = 1/K_c + 1/K_t$$

where

K_{ec} = flexural stiffness of equivalent column

$K_c = K_{c1} + K_{c2}$ = flexural stiffness of actual column

K_{c1} = stiffness at lower end of upper column

K_{c2} = stiffness at upper end of lower column

K_t = torsional stiffness of transverse slab strip

"K_c" values for the column are easily obtained from the coefficients in Table 1.

$$K_t = 9E_cC/l_2(1 - c_2/l_2)^3$$ for the transverse span on one side of the column.

where

c_2 = transverse column dimension

C = cross sectional constant for the transverse strip

$C = (1 - 0.63x/y)\ x^3y/3$ for the slab strip shown in Fig.14.

and x and y are, respectively, the smaller and larger dimensions of the rectangular cross section (h and b in Fig.14).

With the effective stiffness of the slab strip and columns found in this way, the analysis of the equivalent frame can proceed by any convenient means, the most common being the "MOMENT DISTRIBUTION METHOD".

LOADS ON THE EQUIVALENT FRAME

In keeping with the requirements of statics, equivalent beam strips in each direction must each carry 100 percent of the applied load.

If the service live load does not exceed three-quarters of the dead load, the maximum bending may be assumed to occur at all sections under full factored load (ACI-13.7.6.2) otherwise pattern loadings must be used to maximize positive and negative moments.

A provision of the ACI Code that the live load may be reduced to 3/4 its full value when considering the effects of alternate loadings is based on moment redistribution in reinforced concrete slabs, but should not be applied to prestressed designs.

If service live load exceed three quarters, then the maximum positive moment is calculated with full factored live load on alternate panels, while maximum negative moment at a support is calculated with full factored live load on the adjacent panels only.

REDUCTION OF NEGATIVE MOMENT TO FACE OF SUPPORT

Negative moments obtained from the analysis apply at the centerlines of supports. Since the support provided is not a knife-edge but a rather broad band of slab spanning in the transverse direction, some reduction in the negative design moment is proper. At interior supports, the critical section for negative bending, in both column and middle strips may be taken at the face of the supporting column, but in no case at a distance greater than 0.175 l_1 from the center of the column.

TRANSVERSE DISTRIBUTION OF LONGITUDINAL MOMENT

Having found the total moments at positive and negative critical sections from the equivalent frame analysis, it still remains to distribute those moments across the width of the critical section. For prestressed flat plates the following distribution of moments at the critical section is recommended.

Simple spans: 55 to 60 percent to the column strip with the remainder to the middle strip.

Continuous spans: 65 to 75 percent to the column strip with the remainder to the middle strip.

OTHER METHODS OF ANALYSIS

A more precise determination of the slab moments and deflections can be found by using the elastic plate theory. Analytical determination of deflections and moments in an elastic plate involves solution of the differential equation for plates, subject to the appropriate boundary conditions imposed by its loading and supports. For continuous slabs this requires considerable mathematical effort and for this reason this theory is not generally used directly in design. Various approximate methods are available for the solution of those differential equations, one of the simplest in concept is the "METHOD OF FINITE DIFFERENCE ", which reduces the solution of a differential equation to the solution of a set of linear simultaneous equations. The other approximate method of analysis is the "FINITE ELEMENT ANALYSIS".

DESIGN CONSIDERATIONS

SPAN LIMITATIONS

In-place post-tensioned plates have proven to be economical for 20 to 35 ft spans. For spans ranging between 35ft and 40ft, a prestressed flat plate system can still be used but drop panels around the columns must be provided to withstand the bending and shear stresses. When spans exceed 45ft, a prestressed beam-girder system or waffle slabs are found practical.

SPAN DEPTH RATIOS

ACI-ASCE Committee 423 recommends that for "prestressed slabs continuous over two or more spans in each direction, a span-depth ratio (for light loads, say about 50 psf) of 40 - 45 may be used for floors, and a ratio of 45 - 48 for roofs. These limits may be increased to 48 and 52, respectively, if calculations verify that deflection and camber are not objectionable". For practical purposes a ratio of 45 has been found to be appropriate.

To start with, most designers use a rule of thumb to determine the slab thickness. For the usual live loads and normal weight concrete the thickness h in inches can be determined from

$$h = 12 L/45$$

where

L = span length in feet.

AVERAGE PRESTRESS

Average prestress is defined as the amount of prestressing force divided by the area of slab concrete (F/A). This stress is a good indicator of how the design can proceed. It is obvious that this stress varies inversely with the slab thickness. It would seem, then, that to obtain the minimum thickness of slab, we should use the maximum average prestress. But there are some limitations for the "maximum average prestress" and the "minimum average prestress". ACI-ASCE Committee 423 recommends ;

Maximum average prestress

A high value of average prestress may induce excessive elastic shortening and creep. An arbitrary value of 500 psi is generally considered the maximum for solid slabs.

Minimum average prestress

A minimum average prestress is required, if it is desired to eliminate or minimize cracks in the slab. This minimum value ranges between 100 and 200 psi. Values of average prestress less than 125 psi are not recommended.

REQUIRED PRESTRESSING FORCE

In prestressed flat plates the tendon profile is not usually concordant. The least prestressing force will be obtained when the available tendon drape is a maximum in the controlling span. For an interior span the minimum prestressing force F is given by

$$F = W_b L^2/8z$$

and for a cantilever

$$F = W_b L^2/2z$$

where

L = length of the span

W_b = load balanced by prestressing

z = sag of parabolic tendon measured from the chord joining the ends of the parabola

Since the prestressing force will be the same throughout each slab span, there will be one governing span. The tendon profiles of the remaining spans will then be adjusted accordingly.

FIRE RESISTANCE AND COVER REQUIREMENTS

For fire protection purposes and for corrosion protection, building codes require a concrete cover for the top and bottom reinforcement.

Cover thicknesses for post-tensioned tendons in unrestrained slabs are determined by the elpsed time during a fire test until the tendons reach a critical temperature. For cold-drawn prestressing steel that temperature is 800 F. For restrained slabs there are no temperature limitations. Fire test of restrained slabs indicate that slabs with post-tensioned reinforcement behave about the same as reinforced concrete slabs of the same dimensions. Accordingly the cover for post-tensioned tendons in slabs should be the same as the cover for reinforcing steel in slabs. Cover thicknesses for post-tensioned tendons are suggested by Post Tensioning Institute and are given in Table 3.

LOSS OF PRESTRESS

According to the ACI Code, to determine effective prestress "f_{se}", allowance for the following sources of loss of prestress shall be considered :

a) Anchorage seating loss

b) Elastic shortening of concrete

c) Creep of concrete

d) Shrinkage of concrete

e) Friction loss due to intended or unintended curvature in post-tensioning tendons.

The commentary of the ACI 318-77 notes : "The lumpsum losses of 35,000 psi for pre-tensioning and 25,000 psi for post-tensioning generally give satisfactory results for many applications".

Recent investigations have indicated that some of the basic data on which these lumpsum prestress loss values were based was inaccurate. This is particularly applicable to the loss resulting from relaxation of prestressing steel.

Table 4. may be used for normal weight concrete and on an average values of concrete strength, prestress level, and exposure conditions. Actual values of losses may vary significantly above or below the table values in cases where the concrete is stressed at low strengths, where the concrete is highly prestressed, or in very dry or wet exposure conditions. The table values does not include losses due to friction.

ALLOWABLE WORKING STRESSES

When using equivalent frame method of analysis, following allowable stresses at service loads are recommended by the ACI-ASCE Committee 423 for the design of post-tensioned flat plates.

Compression in concrete

Negative moment areas around column	$0.3\ f_c'$
Other areas	$0.45\ f_c'$

Tension in concrete

In slabs with an average prestress of 125 psi or higher, the following concrete tensile stresses can be permitted at service loads, after allowance for all prestress losses :

Positive moment areas without the addition of non-prestressed reinforcement	$2f_c'^{0.5}$
Positive moment areas with the addition of non-prestressed reinforcement	$6f_c'^{0.5}$
Negative moment areas without the addition of non-prestressed reinforcement	0
Negative moment areas with the addition of non-prestressed reinforcement	$6f_c'^{0.5}$

ULTIMATE STRENGTH

Post-tensioned flat plates must meet the ultimate strength requirements of the ACI Code. This is achieved by comparing the nominal moment resistance multiplied by the strength reduction factor ϕ, at all critical sections, to the factored moments (or strength design moments) at these sections. The factored moment is calculated using the "equivalent frame method" of analysis with "moment distribution" with the usual load factors included. Secondary moments, if any, due to prestressing must be included, using a load factor of 1.0.

The nominal moment resistance at each critical section is calculated by the following equation

$$\phi M_n = \phi (A_{ps} f_{ps} + A_s f_y) (d - a/2)$$

where

M_n = nominal moment resistance

ϕ = strength reduction factor (0.9 for flexure)

A_{ps} = area of prestressed reinforcement (tendons)

A_s = area of non-prestressed reinforcement

d = distance from extreme compression fiber to centroid of prestressed reinforcement, or to combined centroid when non-prestressed reinforcement is included.

$a = (A_{ps} f_{ps} + A_s f_y) / 0.85 f_c' b$

b = width of compression face of member

f_c' = specified compressive strength of concrete

f_y = specified yield strength of non-prestressed reinforcement

f_{ps} = stress in prestressed reinforcement at nominal strength

The ACI Code recommends the use of the following approximate equation for "f_{ps}" if "f_{pe}", the effective prestress, is not less than $0.5f_{pu}$.

a) For members with bonded prestressing tendons :

$$f_{ps} = f_{pu}(1 - 0.5\ p\ f_{pu}/f_c')$$

b) For members with unbonded prestressing tendons :

$$f_{ps} = f_{se} + 10,000 + f_c'/100\ p$$

but "f_{ps}" shall not be taken greater than "f_{py}" or (f_{pe} + 60,000).

where

f_{pu} = specified tensile strength of prestressing tendons

f_{py} = specified yield strength of prestressing tendons

f_{pe} = effective prestress, after allowance for all prestress losses

p = ratio of prestressed reinforcement

= $A_{ps}/b\ d$

NON-PRESTRESSED REINFORCEMENT

According to the ACI Code non-prestressed reinforcement, when used in combination with prestressed steel, may be considered to contribute to the tensile force an amount equal to its area times its yield strength. According to the section 18.9 of the code, two-way flat plates prestressed with unbonded tendons should contain a minimum area of bonded reinforcement, as follows :

a) In positive moment areas where the magnitude of tensile stress in concrete at service loads does not exceed 2 $f_c'^{0.5}$ bonded reinforcement is not required, otherwise if it does exceed 2 $f_c'^{0.5}$ then the minimum area of bonded reinforcement shall be computed by

$$A_s = N_c/0.5\ f_y$$

where

N_c = tensile force in concrete due to unfactored dead load plus live load

f_y = yield strength of bonded reinforcement and shall not exceed 60,000 psi.

Bonded reinforcement shall be uniformly distributed over the precompressed tensile zone and as close as practicable to the extreme tension fiber. Its minimum length shall be one-third of the clear span. If the bonded reinforcement is considered in determining the nominal strength, its length should also be in accordance with the development length provisions of the ACI Code.

b) In negative moment areas at column supports, the minimum area of bonded reinforcement in each direction shall be computed :

$$A_s = 0.00075\ h.l$$

where

h = overall slab depth

l = length of span in direction parallel to that of the reinforcement being determined.

The above bonded reinforcement shall be distributed within slab width between lines that are 1.5 h outside oppposite faces of column support. At least 4 bars or wires shall be provided in each direction. The spacing of bonded reinforcement shall not exceed 12 inches and its length shall be at least one-sixth the clear span on each side of the support. If the bonded reinforcement is considered in determining the nominal strength, its length should also be in accordance with the development length provisions of the ACI Code.

MOMENT REDISTRIBUTION

The moment redistribution provisions of section 18.10.4 of the ACI Code are applicable to design of prestressed flat plates. In lieu of more exact calculations of rotation requirements and capicities, the code permits a limited amount of redistribution of elastic moments depending upon the reinforcement index ($w + w_p - w'$)

where

$$w = pf_y/f_c', \quad w' = p'f_y/f_c', \quad w_p = p_p f_{ps}/f_c'$$

$$p = A_s/bd \quad , \quad p' = A_s'/bd \quad , \quad p_p = A_{ps}/bd$$

A_s' = area of compression reinforcement

It is stated in the code that the negative moments due to factored dead and live loads calculated by elastic theory for any assumed loading arrangement, at the supports of continuous prestressed beams with sufficient bonded steel to insure control of cracking, may be increased or decreased by not more than

$$20 \{ 1 - (w + w_p - w'/0.3) \} \text{ percent}$$

Provided that these modified negative moments are also used for final calculations of the moments at other sections in the span corresponding to the same loading condition. Such an adjustment may only be made when the section at which the moment is reduced is so designed that $(w + w_p - w')$ is equal to or less than 0.2.

SHEAR STRENGTH

The shear strength of post-tensioned flat plates is governed by the more severe of the following two conditions :

a) Beam type shear

For beam type shear, the plate is considered to act as a wide beam with a potential diagonal crack extending a plane across the entire width as shown in the Fig.15. The critical section is taken a distance h/2 from the face of the column. As for beams,

$$V_u < \theta \, Vn$$

where

V_u = shear force at factored loads

V_n = nominal shear strength

θ = strength reduction factor(0.85 for shear)

Normally, shear reinforcement is not provided for beam-shear in slabs, and so $V_n = V_c$

where

$$V_c = (0.6 f_c'^{0.5} + 700\, V_u d/M_u) b\ d\ ,$$

equation 11-10 of the ACI Code.

but V_c need not be taken less than $2f_c'^{0.5}$ b.d nor shall V_c be taken greater than $5f_c'^{0.5}$b.d nor the value given in section 11.4.3 of the ACI Code.

The quantity "$V_u d/M_u$" shall not be taken greater than 1.0, where M_u is factored moment occuring simultaneously with V_u at section considered. When applying the above equation, d in the term "$V_u d/M_u$" shall be the distance from extreme compression fiber to centroid of prestressed reinforcement. Otherwise for the d outside the parantheses, it should not be less than 0.8 h. The above equation is valid for members with effective prestress force not less than 40 percent of the tensile strength of flexural reinforcement.

PUNCHING SHEAR

In punching shear, the potential diagonal crack follow the surface of a truncated cone or pyramid around the column, as shown in the Fig.16. The critical section is taken a distance d/2 from the face of the column, defining the shear perimeter b_o as shown in the same Fig. At that section, the basis for design is that

$$v_u < \phi v_c$$

where

v_u = shear stress due to factored loads

v_c = nominal shear stress carried by concrete

According to the ACI-ASCE Committee 423, the value of v_c can be taken equal to "v_{cw}" given by the equation 11-12 of the ACI Code 71, for slabs with average prestress in each direction not less than 125 psi.

$$v_{cw} = 3.5 f_c'^{0.5} + 0.3\ f_{pc} + V_p / \ b_w d$$

where

f_{pc} = compressive stress in concrete at the centroid of the section, not to be taken larger than 500 psi.

f_c' = specified compressive strength of concrete, not to exceed 5000 psi

b_o = perimeter of critical section

V_p = vertical component of effective prestress force at section

The term "$V_p/b_w d$" in the above equation usually contributes only a small amount, and for convenience this term may be conservatively neglected in the design of post-tensioned flat plates.

When the term "$V_p/b_w d$" is included in calculation of v_{cw}, it is necessary to use the actual reverse curvature tendon geometry and that only the value of V_p remaining in the tendon inside the critical section be used for determining the contribution of the tendon shear to the shear stress in the concrete. The shear stress due to factored loads v_u is calculated by the following equation :

$$v_u = V_u/\phi\, b_o d + aM_t c_3/\phi\, J_c$$

Section 11.12.2.4 ACI Code Commentary

where

V_u = factored shear force at section

b_o = perimeter of shear section at d/2 from face of column

d = distance from centroid of tendon to compression face in direction of moment, but need not be less than 0.8h, where h is the member thickness

ϕ = strength reduction factor, (0.85 for shear)

a = fraction of moment transferred by shear

$= 1 - 1/\{(1 + 2/3[(c_1 + d)/(c_2 + d)]^{0.5}\}$ for interior column

$= 1 - 1/\{(1 + 2/3(c_1 + 0.5\ d)/(c_2 + d)^{0.5}\}$ for edge column

c_1 = support dimension in the direction of moment transfer

c_2 = support dimension perpendicular to c_1

c_3 = distance from centroid of critical shear section to extreme fiber in direction of moment transfer

M_t = net moment to be transferred to column

J_c = polar moment of inertia of critical section

In order to understand the second term of the v_u equation, it is necessary to discuss "moment transfer" between column and slab.

MOMENT TRANSFER

When a rigid connection exists between column and slab, bending moments are transferred from slab to column and vice versa. Such moments are mainly generated by unbalanced gravity loads on either side of a column, lateral loadings due to winds and earthquakes, and temperature movements. Due to these effects the shear stress on the critical section is no longer uniformly distributed.

The situation can be modeled as shown in Fig.17. Here V_u represents the total vertical reaction to be transferred to the column, and M_u represents the unbalanced moment to be transferred, both at factored loads. The vertical force V_u causes shear stress distributed more or less uniformly around the perimeter of the critical section, represented by the inner pair of vertical arrows, acting downward. The unbalanced moment M_u causes horizontal flexural loading on the joint, represented by the forces T and C, and in addition causes shearing stresses represented by the outer pair of vertical arrows, which add to the shear stresses otherwise present on the right side, in the figure, and subtract on the left side.

According to the ACI Code, the moment considered to be transferred by flexure is over an effective slab width between lines that are one and one half slab thickness(1.5 h) outside opposite faces of the column, and is given by

$$M_{uf} = 1 /(1 + 2/3\{(c_1 + d)/(c_2 + d)\}^{0.5})M_u$$

for interior column

$$M_{uf} = 1 /(1 + 2/3\{(c_1 +0.5d)/(c_2+d)\}^{0.5})M_u$$

for edge column

While that assumed to be transferred by shear is as follows;

Muv = Mu - Muf, for both interior and edge columns

The moment Muv, together with the vertical reaction delivered to the column, causes shear stresses assumed to vary linearly with distance from the centroid of the critical section as shown in Fig.17. One thing is to be noted that for the case of an exterior column, the values of M_u and V_u are assumed to be taken at the centroid of the critical section. Hence, M_t is equal to the moment at the centroid of the column minus "$V_u g$", where g is the distance from the centroid of the column to the centroid of the critical section.

<u>TENDON REVERSAL</u>

The equations given under "required prestressing force" are theoretically true only if the prestressing tendons meet at a point over the supports. In practice, however, the tendons will gradually bend over the supports, so that at some point near the ends, the tendon curvature will be reversed. According to ACI-ASCE Committee 423 :

"The effects of reversed tendon curvature at supports are generally neglected in applying the load balancing method to design of flat plates since the reversed curvature has only a minor influence on the elastic moments (in the order of 5 to 10 percent), and does not affect the ultimate moment capacity".

TENDON DISTRIBUTION

The ultimate strength of a flat plate is controlled primarily by the total amount of tendons in each direction. However, tendons passing through columns or directly around column edges contribute more to load carrying capacity than tendons remote from the columns. ACI-ASCE Committee 423 suggests the following :

"As many tendons as practical should pass through the column with a minimum of 2 tendons. For panels with length/width ratios not exceeding 1.33, the following approximate distribution may be used :

Simple spans ; 55 to 60 percent of the tendons are placed in the column strip with the remainder in the middle strip

Continuous spans ; 65 to 75 percent of the tendons in the column strip with the remainder in the middle strip

TENDON SPACING

ACI-ASCE Committee 423 suggests that :
"The maximum spacing of tendons in column strips should not exceed four times the slab thickness. Maximum spacing of tendons in the middle strips should not exceed six times the slab thickness".

ALLOWABLE DEFLECTIONS

According to the section 9.5.4 of the ACI Code :
"Immediate deflection due to live load is limited to span/360 and the additional long time deflection due to sustained loads, creep and shrinkage of concrete, and relaxation of steel plus the immediate deflection due to live load is limited to span/240 ".

DESIGN PROCEDURE

1) Proportion slab thickness based on span-to-depth ratios.

2) Assume average prestress with maximum parabolic tendon profile for the initial estimate of balanced load

3) Find the number of tendons required, then again calculate the average prestress provided, (F/A).

4) Calculate the balanced load "W_{bal}" for each span and also the net load causing bending, which is equal to total service load minus the balanced load

5) Define for each principal direction the "Equivalent Frame".

6) Analyze the "Equivalent frame" for the unbalanced or net load and determine corresponding moments and stresses

7) Superimpose to the actual average prestress the stresses due to the net load and compare the resulting stresses with allowable stresses

8) Determine minimum non-prestressed reinforcement, if required.

9) Check ultimate flexural strength requirements.

10) Check shear strength requirements.

11) Compute deflection and compare with deflection limitations.

DESIGN EXAMPLE

For the purpose of this example[16] a typical transverse slab strip post-tensioned, of normal weight concrete and having unbonded tendons, will be designed. Fig.18.1. The interior columns are 20"x14" and the exterior columns are 12"x14", the story height being 8'-7", Fig.18.2. Normal weight concrete was also used for columns. The building is being designed for private apartments.

DEAD LOADS

Self weight of the slab + Load due to partitions (15 psf)

LIVE LOADS

40 psf   From Table 9.1, Page 409, Ref.18.

STRESSES   ACI 318-77 or ACI-ASCE Committee 423

Concrete

Specified compressive strength of concrete = f_c' = 4000 psi

Allowable compressive stresses

At negative moment areas around column   = $0.3f_c'$= 1200 psi

At positive moment areas   = $0.45f_c'$= 1800 psi

Allowable tensile stresses

At positive moment areas without the addition of non-prestressed reinforcement = $2f_c'^{0.5}$=126.5psi

At positive moment areas with the addition of non-prestressed reinforcement $= 6f_c'^{0.5} = 379.5$ psi

At negative moment areas with the addition of non-prestressed reinforcement $= 6f_c'^{0.5} = 379.5$ psi

<u>Steel</u>

<u>Prestressing steel</u>

Specified tensile strength of prestressing tendons $= f_{pu} = 270$ ksi

Allowable tensile stress in post-tensioning tendons immediately after tendon anchorage $= 0.7f_{pu} = 189$ ksi

<u>Ordinary reinforcing steel</u>

Specified yield strength of ordinary reinforcing steel $= f_y = 60{,}000$ psi

The design example is solved following the steps (1 to 11) mentioned under "Design Procedure".

1. Slab Thickness

For initial start, assume slab thickness as L/45

Longitudinal	=	20 x 12/45	=	5.33"
Transverse	=	17 x 12/45	=	4.55"
Transverse	=	25 x 12/45	=	6.67"

Use slab thickness as 6.5".

Loads

6.5" slab	=	81 psf				
Partitions	=	15 psf				
Dead Load	=	96 psf	x	1.4	=	134 psf
Live Load	=	40 psf	x	1.7	=	68 psf
Total service load	=	136 psf	Total factored load		=	202 psf

Service load design

2. Assume a prestressing force corresponding to an average compressive stress of 150 psi, with maximum parabolic tendon profile, for the initial estimate of balanced load. Then

$F_e = 0.150 \times 6.5 \times 12 = 11.7$ k/ft

Assume 1/2" diameter, 270 ksi strand tendons, and 30 ksi long term losses,(see Table.4), effective force per tendon is

$0.153 \times (0.7 \times 270 - 30) = 24.33$ k

3. For a 20-ft bay, number of tendons required are

$$20 \times 11.7/24.33 = 9.61, \text{ say 10 tendons}$$

Then, F_e provided = $10 \times 24.33/20 = 12.16$ k/ft

and average compressive stress (F_e/A) provided

$$= 12.16/(12 \times 6.5) = 155 \text{ psi} = 0.155 \text{ ksi}$$

4. Span 1 and Span 3 are the same, Fig.18.2

$$z_1 = (3.25 + 5.5)/2 - 1.75 = 2.625'' \text{ (see Fig.19.)}$$

$$W_{bal} = 8\ F\ z\ /\ L^2$$

$$= 8 \times 12.16 \times 2.625/(17^2 \times 12) = 0.074 \text{ ksf}$$

Net load causing bending = $0.136 - 0.074 = 0.062$ ksf = W_{net}

Span 2 :(see Fig.19.)

$$z_2 = 6.5 - 1 - 1 = 4.5''$$

$$W_{bal} = 8\ F\ z\ /\ L^2$$

$$= 8 \times 12.16 \times 4.5/(25^2 \times 12) = 0.058 \text{ ksf}$$

Net load causing bending = $0.136 - 0.058 = 0.078$ ksf

5.EQUIVALENT FRAME PROPERTIES

Geometry For the exterior column, half depth of the slab gives

$$a/l_c = b/l_c = 3.25/103 = 0.0316$$

Similarly for the interior column,

$$a/l_c = b/l_c = 0.0316$$

see the figure on Table.1 for a, b, and l_c

For the slab

Span 1 : $c_1/l_1 = 12/(17 \times 12) = 0.058$ @ Left End

$c_2/l_2 = 14/(20 \times 12) = 0.058$

$c_1/l_1 = 20/(17 \times 12) = 0.098$ @ Right End

Span 2 : $c_1/l_1 = 20/(25 \times 12) = 0.067$ @ Left and Right End

$c_2/l_2 = 14/(20 \times 12) = 0.058$

see the figure on Table.2. for c_1, c_2, l_1, and l_2

Stiffnesses

Column stiffness

Exterior column : (14"x12"), $I_c = 14 \times 12^3/12 = 2016$ in^4

$$E_{cc} = E_{cs} = E_c$$

Table.1 gives column stiffness coefficient "k_c"

For $a/l_c = b/l_c = 0.0316$, "k_c" = 4.71 (by interpolation)

$$K_c = k_c E_{cc} I_c / l_c$$

$$K_c = 4.71 \times 1 \times 2016/103 = 93.19$$

Where E_c is taken as 1, since only ratios of K will be used

Since column is continuous at the joint $K_c = 93.19 \times 2 = 184.38$

Torsional stiffness of slab in column line, K_t is calculated as follows :

$$C = (1 - 0.63x/y)x^3y/3$$

$$C = (1 - 0.63.6.5/12)6.5^3.12/3 = 724 \text{ in}^4$$

$K_t = 9EC/l_2(1 - c_2/l_2)^3$

$K_t = 2 \times 9 \times 1 \times 724/\{20x12(1 - 1.17/20)^3\} = 65in^3$

Equivalent column stiffness is then obtained using

$1 / K_{ec} = 1 / K_t + 1 / K_c$

$K_{ec} = (1/65 + 1/184.38)^{-1} = 48.1$

Interior column : (14"x20"), $I_c = 14 \times 20^3/12 = 9333.33$ in^4

$a/l_c = b/l_c = 0.0316$

$k_c = 4.71$ as before

$K_c = 4.71 \times 9333.33/103 = 426.8$

Since the column is continuous at the joint,

$K_c = 2 \times 426.8 = 853.6$

$C = (1 - 0.63x6.5/20)6.5^3x20/3 = 1456$

$K_t = 2 \times 9 \times 1456/\{20x12(1 - 1.17/20)^3\} = 131$ in^3

$K_{ec} = (1/131 + 1/853.6)^{-1} = 113.57$

<u>Slab Stiffness</u>

At exterior column : $c_1/l_1 = 0.058$, $c_2/l_2 = 0.058$

Using Table.2 and interpolating, we get

$M_{coeff} = 0.084$, $k = 4.05$, $C.O.F = 0.503$

At interior column span 1, and span 3 :

$c_1/l_1 = 0.098$, $c_2/l_2 = 0.058$

Using Table.2. again and interpolating we get

M_{coeff} = 0.084 , k = 4.091 , C.O.F = 0.506

For Span 2: c_1/l_1 = 0.067 , c_2/l_2 = 0.058, we get

M_{coeff} = 0.084 , k = 4.06 , C.O.F = 0.505

$$K_s = k\ E\ I_s/l_1$$

K_s @ Exterior Column = $4.05 x 1 x 20 x 12 x 6.5^3/17 x 12 x 12$ = 109.04

K_s @ Interior Column

(Span 1 and Span 3) = $4.091 x 1 x 20 x 12 x 6.5^3/17 x 12 x 12$ = 110.15

K_s for Span 2 = $4.06 x 1 x 20 x 12 x 6.5^3/25 x 12 x 12$ = 74.33

<u>Sum K</u>

Exterior column = 48.1(column) + 109.04(slab) = 157.14

Interior column: = 110.15 + 113.57 + 74.33 = 298.05

<u>Distribution Factors</u>

Exterior Joint : For column = 48.1/157.14 = 0.306 = 0.30

For slab = 109.04/157.14 = 0.694 = 0.70

Interior Joint : For slab(left) = 110.15/298.05 = 0.369 = 0.37

For column = 113.57/298.05 = 0.381 = 0.38

For slab(right) = 74.33/298.05 = 0.249 = 0.25

6.MOMENT DISTRIBUTION

Span 1 and Span 3: Net load = 0.062 k/ft = w

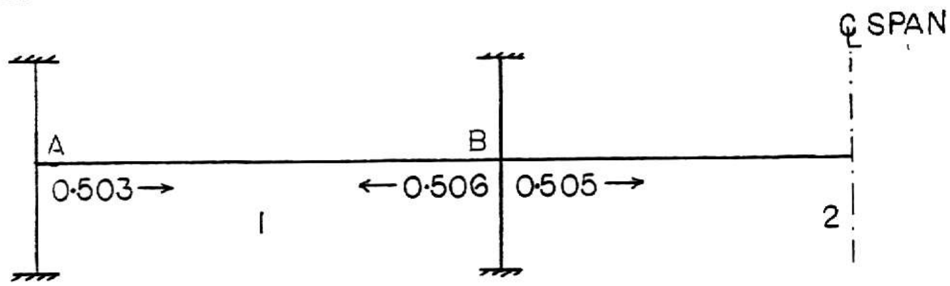
Fixed End Moment(FEM) = M_{coeff}x w x l^2

= $0.084 \times 0.062 \times 17^2$ = 1.51 k-ft

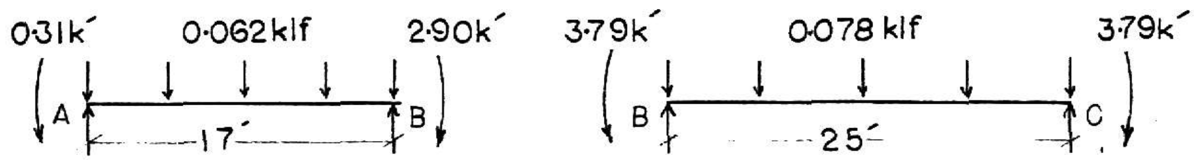
Span 2 : Net load = 0.078 k/ft = w

Fixed End Moment(FEM) = M_{coeff}x w x l^2=

= $0.084 \times 0.078 \times 25^2$ = 4.06 k-ft



	C	S	S	C	S
D.F	0.30	0.70	0.37	0.38	0.25
FEM		+1.51	-1.51		+4.06
BAL	-0.45	-1.06	-0.94	-0.97	-0.64
CO		-0.47	-0.53		+0.32
BAL	+0.14	+0.33	+0.08	+0.08	+0.05
TOTAL	-0.31	+0.31	-2.90	-0.89	+3.79



R_A = +0.527 V_{B1}= +0.527 V_{B2}= 0.975 V_{C1}= 0.975

-0.152   +0.152

+0.375   +0.679

Span AB

Moment @ the face of column = M_{cl} + Vc/3

M_{fcol} = -2.90 + 0.679x20/3x12 = -2.52 k-ft

Mid span positive moment = $-0.31 + 0.375x8.5 - 0.062x8.5^2/2$

= +0.638 k-ft

Span BC

Moment @ the face of column = -3.79 + 0.975x20/12x3 = -3.25 k-ft

Mid span positive moment = $-3.79 + 0.975x12.5 - 0.078x12.5^2/2$

= +2.30 k-ft

7.CHECK FOR NET TENSILE STRESSES

At face of column :

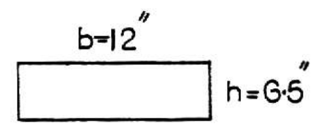
Since moment obtained from Span BC at the face of column B is greater than the moment obtained from Span AB therefore only that will be considered .

$f = M/S$

f = Bending stress at the section

M = moment at the section

S = section modulus = $bh^2/6$ (see figure)



$= 12 \times 6.5^2/6 = 84.5 \text{ in}^3$

$f = 3.25 \times 12 / 84.5 = 0.462$ ksi

The average compressive stress provided was = 0.155 ksi

Net tensile stress at the section = +0.462 - 0.155 = 0.307 ksi

Note that this stress is at the top most fiber of the section.

+0.307 ksi = 307 psi < 379.5 psi (Allowable) O.K

Check for compressive stress at the section:

-0.462 -0.155 = -0.617 ksi

-0.617 ksi = 617 psi < 1200 psi (Allowable) O.K

At Midspan BC :

$f = M/S$

$f = 2.30 \times 12 / 84.5 = 0.326$ ksi

Net tensile stress at the section = 0.326 - 0.155 = 0.171 ksi

0.171 ksi = 171 psi > 126.5 psi (Allowable) Not O.K

Therefore non-prestressed reinforcement is required.

At Midspan AB :

$f = 0.638 \times 12 / 84.5 = 0.090$ ksi ,

Net stress = 0.090 - 0.155 = -0.064 ksi (Compresion) O.K

8.NON-PRESTRESSED REINFORCEMENT

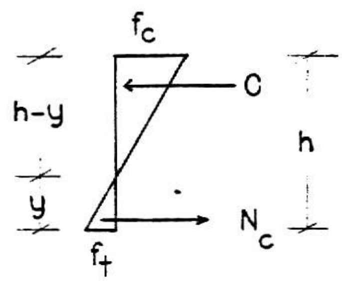
In positive moment areas where computed tensile stress in concrete at service load exceeds $2.f_c'^{0.5}$, minimum area of bonded reinforcement shall be computed by

$$A_s = N_c / 0.5 f_y$$

A_s = Minimum area of non-prestressed reinforcement required

N_c = Tensile force in concrete due to unfactored dead load plus live load.

Mid-span BC



f_c = -0.326 - 0.155 = -0.481 ksi, 481 psi < 1800 psi <u>O.K</u>

Now from similar triangles (see figure above),

$y / h - y = f_t / f_c$, $y / 6.5 - y = 0.171 / 0.481$

$y = 1.71"$

$N_c = 1.71 \times 0.171 \times 12 / 2 = 1.76$ k/ft

$A_s = 1.76 / 0.5 \times 60 = 0.058$ in^2/ft

$A_s = 0.058 \times 20 = 1.173$ in^2

Length of the bars = clear span/3 = (25 - 20/12)/3 = 7.77'say 8'

Provide 9 - # 4 x 8' @ 30" c/c , A_s provided = 1.77 in^2

These bars will be provided at the bottom of midspan BC.

The spacing is about 5 times the slab thickness.

9.ULTIMATE STRENGTH DESIGN

At ultimate, the balanced load moment is used to determine the secondary moment, by subtracting the primary moment, which is simply "F_e.e" at each support.

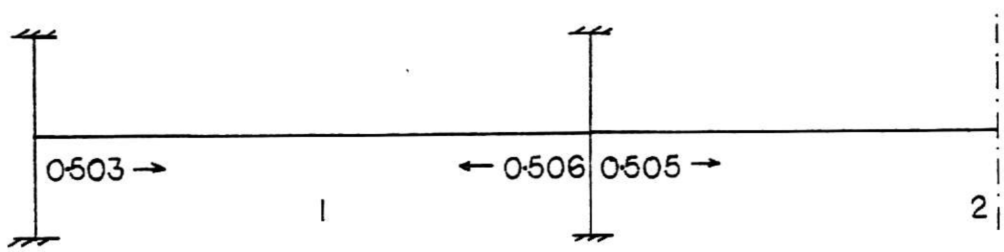
Balanced Load Moments

Span 1 and Span 3 : Balanced Load = 0.074 k/ft

FEM = $0.084 \times 0.074 \times 17^2$ = 1.8k'

Span 2 : Balanced Load = 0.058 k/ft

FEM = $0.084 \times 0.058 \times 25^2$ = 3.05k'



	C	S	S	C	S
D.F	0.30	0.70	0.37	0.38	0.25
FEM		-1.80	+1.80		-3.05
BAL	+0.54	+1.26	+0.46	+0.47	+0.31
CO		+0.23	+0.63		-0.16
BAL	-0.07	-0.16	-0.17	-0.17	-0.11
TOTAL	+0.47	-0.47	+2.72	+0.30	-3.01

Since load balancing accounts for both primary and secondary moments, secondary moments can be found from the following relationship:

$$M_{bal} = M_{prim} + M_{sec}$$

$$M_{prim} = F_e \times e \quad \text{at each support}$$

At exterior column : M_{prim} = 12.16 x 0" = 0 k'

At interior column : M_{prim} = 12.16 x (3.25-1)/12 = 2.28 k'

At exterior column : M_{sec} = 0.47 - 0 = +0.47 k'

At interior column

Span 1 and Span 3 : M_{sec} = 2.72 - 2.28 = +0.44 k'

Span 2 : M_{sec} = 3.01 - 2.28 = +0.73 k'

<u>Factored Load Moments</u>

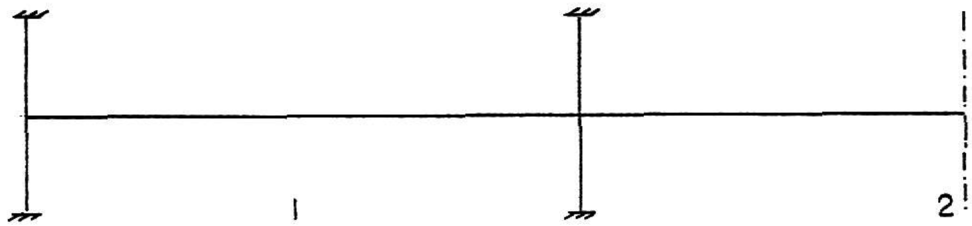
Factored Load = 1.4 Dead Load + 1.7 Live Load

= 134 + 68 = 202 psf = 0.202 ksf

Span 1 and Span 3 :

FEM = 0.084 x 0.202 x 17^2 = 4.90 k'

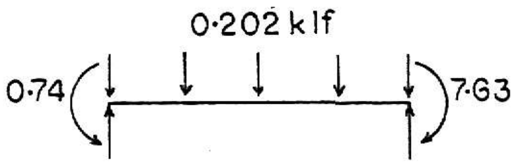
Span 2 : FEM = 0.084 x 0.202 x 25^2 = 10.61 k'

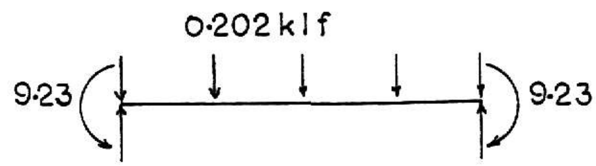


	C	S	S	C	S
D.F	0.30	0.70	0.37	0.38	0.25
FEM		+4.90	-4.90		+10.61
BAL	-1.47	-3.43	-2.11	-2.17	- 1.43
CO		-1.07	-1.72		+ 0.72
BAL	+0.32	+0.75	+0.37	+0.38	+ 0.25
CO		+0.19	+0.37		- 0.13
BA1	-0.06	-0.13	-0.08	-0.09	- 0.06
TOTAL	-1.21	+1.21	-8.07	-1.88	+ 9.96

<u>Design Moments At Column Centerline</u>

	SPAN 1	SPAN 2	
Factored Load Moments	-1.21	-8.07	-9.96
Secondary Moments	+0.47	+0.44	+0.73
Design Moments At Column Centerline	-0.74k'	-7.63k'	-9.23k'





$R_A =$ +1.717	V_{B1} = +1.717	V_{B2} = +2.525k	V_{C1} = 2.525k
−0.405	+0.405		
+1.311	+2.122		

Moment at face of column: $M_{fcol} = M_{ccol} + Vc/3$

At exterior column $= 0.74 + 1.311/3 = -0.30k'$

At interior column Span 1 $= -7.63 + 2.122 x 20/(12x3) = -6.45k'$

At interior column Span 2 $= -9.23 + 2.525 x 20/(12x3) = -7.83k'$

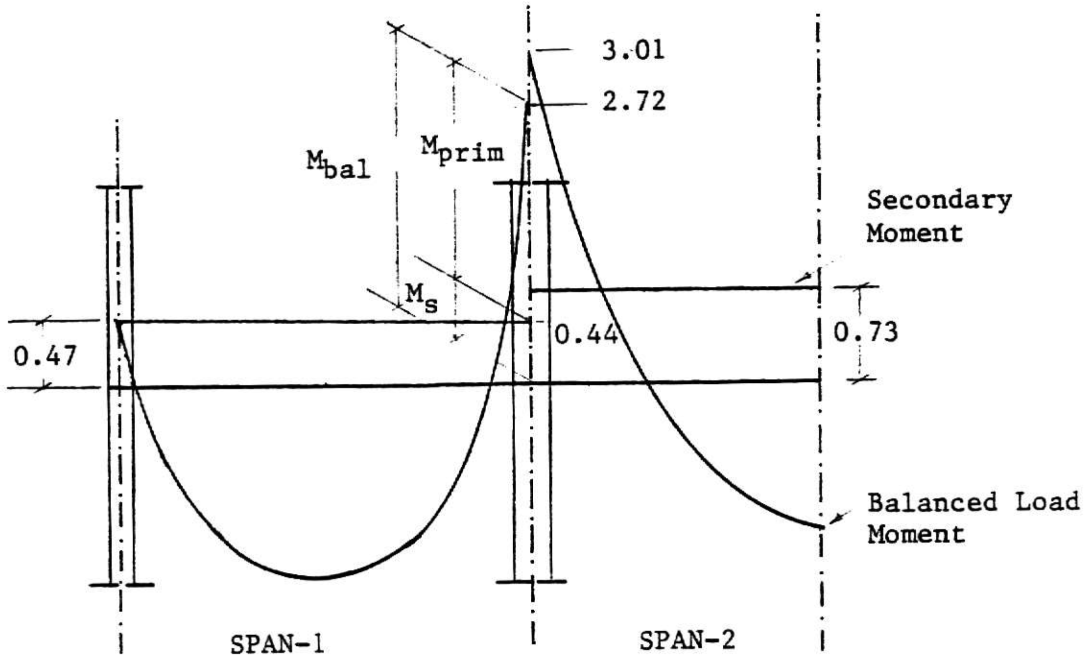
Midspan positive moments:

Span 1 : Point of zero shear $= 1.311/0.202 = 6.49'$ from A

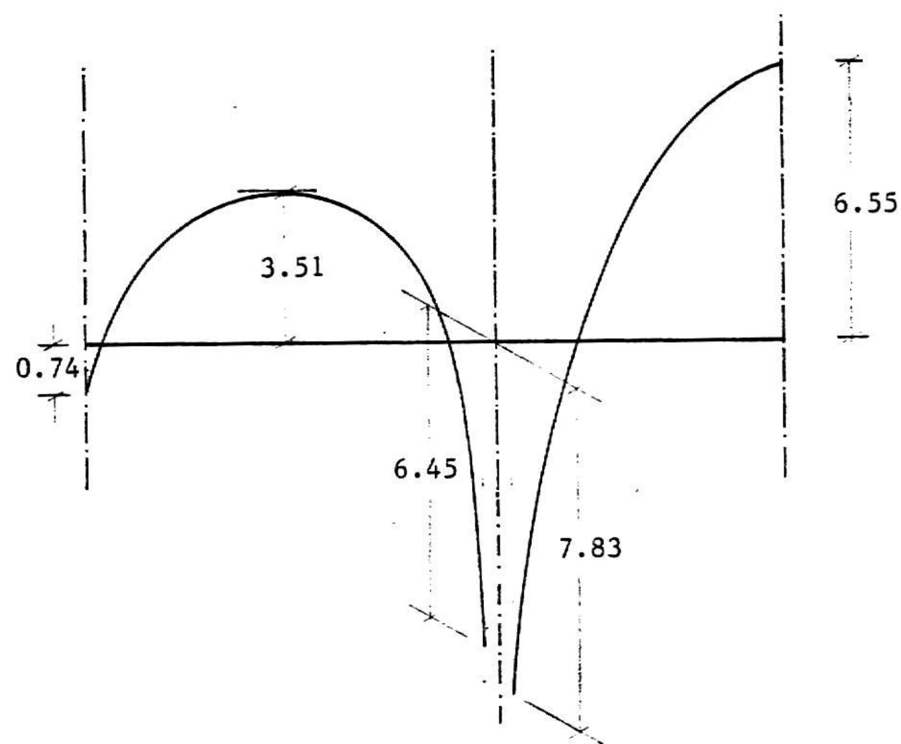
$M_{pos} = 1.311 x 6.49 - 0.74 - 0.202 x 6.49^2/2 = +3.51\ k'$

Span 2 : Point of zero shear $= 2.525/0.202 = 12.5'$ (@ midspan)

$M_{pos} = 2.525 x 12.5 - 9.23 - 0.202 x 12.5^2/2 = +6.55\ k'$



(1) Moments from Balanced Loads (in k-ft)



(2) Factored Load Design Moments (in k-ft)

Moment Diagram

<u>Calculation of flexural capacity at ultimate</u>

According to the ACI Code, bonded reinforcement (non-prestressed-reinforcement) should be provided in negative moment areas when unbonded tendons are used, and it may be considered to contribute to the tensile force and may be included in moment capacity calculations.

The minimum area of bonded reinforcement shall be computed by :

$$A_s = 0.00075\ h \times l$$

At interior column :

$$A_s = 0.00075 \times 6.5 \times (17+25/2) \times 12 = 1.23\ in^2$$

Length of bonded reinforcement = Clear Span/6 on each side of col

$$\text{Length} = (17+25)/(2 \times 6) + 20/12 = 8.67',\ \text{say } 9'$$

Provide 4 — # 5 x 9' Bars @ 10"c/c , A_s provided = 1.23 in^2

These bars should be placed within a width of column plus 1.5 x h on each side of the column.(33.5").

For average one foot strip, A_s = 1.23/20 = 0.061 in^2/ft

Calculating stress in tendon at ultimate, using ACI Code equation for unbonded tendons:

$$f_{ps} = f_{pe} + 10{,}000 + f_c'/\ 100\ p_p$$

We have provided 10-1/2"diameter tendons in a 20' bay, each with area = 0.153in^2

$p_p = A_{ps}/bd = (10x0.153)/(20x12x5.5) = 0.00116$

$f_{pe} = 0.7x270 - 30$(assumed losses)$= 159$ ksi

$f_{ps} = 159 + 10 + 4/(100x0.00116) = 203.5$ ksi

F_{su} = Force at ultimate in the tendons (k/ft)

$F_{su} = 203.5 \times 10 \times 0.153 / 20 = 15.57$ k/ft

F_y = Force in the bonded reinforcement at ultimate

$F_y = 0.061 \times 60 = 3.66$ k/ft

Total $F = F_{su} + F_y$

$F = 15.57 + 3.66 = 19.23$ k/ft

Depth of compression block :

$0.85\ f_c'b\ a = F$

$a = F/0.85\ f_c'b = 19.23/(0.85x4x12) = 0.471''$

Since bars and tendons are in the same layer,

Lever arm $= d - a/2 = 5.5 - 0.471/2 \cong 5.265'' = 0.439'$

Moment capacity at ultimate $= \phi M_n = 0.9x19.23x0.439 = 7.59$ k'

Since this value is less than the required capacity of 7.83 k', calculate available capacity at midspan and allowable inelastic moment redistribution at column.

Allowable redistribution $= 20\{1 - (w + w_p - w')/0.30\}$percent

$w = pf_y/f_c'$, $p = A_s/b\ d$

$w' = p'f_y/f_c'$, $p' = A_s'/b\ d$

$w_p = p_p f_{ps}/f_c'$, $p_p = A_{ps}/b\ d$

Here $w' = 0$, since no compression reinforcement is provided.

$p = 1.23/(20x12x5.5) = 0.00093$, $w = 0.00093x60/4 = 0.0139$

$p_p = 0.00116$, $w_p = 0.00116x203.5/4 = 0.059$

Sum $w = w + w_p = 0.0139 + 0.059 = 0.0729 < 0.20$ O.K

$20(1 - (0.0139 + 0.059)/0.30)) = 15.14$ or $15.2\ \%$

$M_{red} = 0.152 \times 7.83 = 1.19$ k'

Ultimate moment capacity at midspan 2 :

Since the midspan 2 requires 9 - # 4 bars from service load considerations, A_s provided = 1.77 in^2

$A_s = 1.77/20 = 0.088\ in^2/ft$

Exactly same calculations will follow as were done for interior column, therefore

$\phi M_n = 8.20$ k' at midspan 2 .

The required capacity at midspan 2 = 6.55 k'

8.20 - 6.55 = 1.65 k' is available to accomodate moment redistribution from the interior support section. Say 1 k' is reditributed :

$- M = -7.83 + 1.00 = -6.83$ k' < 7.59k' O.K

$+ M = +6.55 + 1.00 = +7.55$ k' < 8.20k' O.K

Thus minimum rebar and tendons are adequate for strength.

Midspan 1 and 3 :

$$F = F_{su} = 15.57 \text{ k/ft}$$

Depth of compression block:

$0.85f_c'b\ a = F$

$$a = F/(0.85xf_c'xb) = 15.57/(0.85x4x12) = 0.381''$$

Lever arm : $(d - a/2\ ) = 6.5 - 1.75 - 0.381/2 = 4.56'' = 0.379'$

$$M_u = 0.9 \times 15.57 \times 0.379 = 5.31 \text{ k}'$$

Since 5.31 k' > 3.51 k' O.K

Exterior column :

The flexural capacity at an exterior column is governed by moment transfer requirements. Since moment transfer also involves shear streses, the two aspects will be treated under the heading of shear.

10.SHEAR STRENGTH

See Figure.17 for various dimensions.

$$d = 0.8\ h = 0.8 \times 6.5 = 5.2''$$

$$c_1 = 12''$$

$$c_2 = 14''$$

$$c_t = 12 + 5.2/2 = 14.6''$$

$$c_m = 14 + 5.2 = 19.2''$$

Perimeter of the critical section = $b_o = c_m + 2c_t$

$$b_o = 19.2 + 2 \times 14.6 = 48.4''$$

Area of the critical section = A_c = b_o d = 48.4x5.2= 252in^2

Centroid of the critical section :

Taking moment of areas about edge "cd"

Areas of sides ad and cb = c_t x d = 14.6x5.2 = 75.92in^2

Lever arm = c_t / 2 = 14.6 / 2 = 7.3"

Area of side ab = c_m x d = 19.2 x 5.2 = 99.84 in^2

Lever arm = c_t = 14.6"

X =(2x75.92x7.3 + 99.84x14.6)/252 = 10.18"

$c_{ab} = c_t - X = 14.6 - 10.18 = 4.41"$

$c_{cd} = X = 10.18"$

g = eccentricity = 10.18 - 12/2 = 4.18"

$$J_c = dc_t^3/6 + c_t d^3/6 + c_m dc_{ab}^2 + 2c_t d(c_t/2 - c_{ab})^2$$

$$J_c = 5.2x14.6^3/6 + 14.6x5.2^3/6 + 19.2x5.2x4.41^2 + 2x14.6x5.2(14.6/2 - 4.41)^2 = 6249.21 \text{ in}^3$$

Total bay Moment at column centerline = -0.74x20 = -14.8 k'

Vertical shear at exterior column = 1.311 k/ft

Total vertical shear = 1.311x20 = 26.22 k = V

Assume exterior skin is masonry averaging 0.4 klf

Factored shear due to this masonry = 1.4x0.4x20 = 11.2 k

Total shear due to slab and masonry = 26.22 + 11.22

= 37.42 k = V_u

Moment transferred by eccentricity of shear reaction = V·g

$V = 26.22 \times 4.19/12 = 9.15$ k'

Net Moment to be transferred = $M_t = -14.8 + 9.15 = -5.65$ k'

Now finding the shear stress :

$v_u = V_u/A_c + a\, M_t c_{ab}/\, J_c$

$a = 1 - 1/\{1 + 2/3(c_m/c_t)^{1/2}\}$

$= 1 - 1/\{1 + 2/3(19.2/14.6)^{1/2}\} = 0.433$

$v_u = 37420/(0.85\text{x}252) + 0.433\text{x}5.65\text{x}4.41\text{x}1000/(6249.21\text{x}0.85)$

$= 174.70 + 20.3 = 195$ psi

Shear capacity of concrete is given by the following equation :

$v_c = 3.5\, f_c'^{1/2} + 0.3\, f_{pc} + V_p/A_c$(neglect it)

$v_c = 3.5 \times 4000^{1/2} + 0.3 \times 155 = 276.86$ psi

$v_u = 195$ psi $< v_c = 277$ psi <u>O.K</u>

Check beam shear :

$V = 202 \times 20 \times 7.73 = 31226$ lb

$v = V/(b_w\ d) = 31226/(20\text{x}12\text{x}3.25) = 40.03$ psi

$v = 40$ psi $< 2\ f_c'^{1/2} = 126.5$ psi <u>O.K</u>

Check flexural moment transfer :

The capacity of the section of width equal to the width of column plus 1.5 slab thickness each side will be investigated.

Assume that of the 10 tendons required for the 20' bay width, 2 are provided in the above mentioned region. As usual bonded reinforcement is required.

A_s = 0.00075 h 1 = 0.00075x6.5x17x12 = 1.0 in^2

Length of bars = clear span/6 + column width

= 15.67/6 + 12/12 = 3.61' say 4'

Provide 5 - # 4 x 4' bars , A_s provided = 1.0 in^2

f_{ps} = f_{pe} + 10 + f_c'/100 p_p

f_{pe} = 159 ksi (as before)

p_p = 0.153x2/(14 + 3x6.5)3.25 = 0.00281

f_{ps} = 159 + 10 + 4/(100x0.00281) = 183.2 ksi

F_p = 183.2 x 2 x 0.153 = 56.05 k

F_y = 1.0 x 60 = 60 k

Total F = 56.05 + 60 = 116.05 k

Depth of compression block :

a = F/ 0.85 f_c' b = 116.05/(0.85x4x33.5) = 1.02"

Lever arm :

For tendons, (d - a/2) = (3.25 - 1.02/2)/12 = 0.228'

For rebars , (d - a/2) = (5.5 - 1.02/2)/12 = 0.416'

ϕ M_n = 0.9(0.23x56 + 0.416x60) = 34.06 k'

We had NetMoment to be transfered = 5.65 k', from which ,

Moment transfered by shear = 0.433 x 5.65 = 2.43 k'

Moment transferred by flexure = 5.65 - 2.43 = 3.22 k'

Since capacity of the section = 34.06 k'>> 3.22 k'<u>O.K</u>

Shear at interior column :

Direct shear left and right of interior column

= 2.122+2.525 = 4.65k/'

Total direct shear = 20 x 4.65 = 93 k

Moment to be transferred = (9.23 - 7.63) x 20 = 32 k'

See Figure.17 for various dimensions.

$d = 6.5 - 1.0 = 5.5"$

$d = 0.8\ h = 0.8 \times 6.5 = 5.2"$

$c_1 = 20"$

$c_2 = 14"$

$d + c_1 = 5.5 + 20 = 25.5"$

$d + c_2 = 5.2 + 14 = 19.2"$

$A_c = b_o\ d = 2(25.5x5.2 + 19.2x5.5) = 476.4in^2$

$J_c = d(c_1 + d)^3/6 + (c_1 + d)d^3/6$

$+ d(c_2 + d)(c_1 + d)^2/2$

$J_c = 5.2(25.5)^3 + 25.5x5.2^3/6 +$

$+ 5.5(19.2)(25.5)^2/2 = 49301.32\ in^4$

Portion of Moment to be transfered by shear :

$$a = 1 - 1/\{1 + 2/3(c_1 + d / c_2 + d)^{1/2}\}$$

$$a = 1 - 1/\{1 + 2/3(25.5/19.2)^{1/2}\} = 0.434$$

Moment transferred by shear = M_{vf} = 0.434 x 32 = 13.8 k'

Moment transferred by flexure = M_{uf} = 32 - 13.8 = 18.2 k'

Shear stresses :

$$\phi\, v_u = V_u/A_c + a\, M_t\, c_{ab} / J_c$$

$$v_u = 93/(0.85x476.4) + 13.8x12x25.5/(0.85x49301.3x2)$$

$$= 0.279 \text{ ksi} = 279 \text{ psi}$$

$$v_c = 3.5 \times 4000^{1/2} + 0.3 \times 155 = 268 \text{ psi}$$

v_u = 279 psi > 268 psi , but V_p/A_c part of the v_c equation was neglected, if this term is considered then it will make up the deficiency.

Check beam shear:

V = 202 x 20 x 11.40 = 46056 lb

$v = V/(b_w\ d)$ = 46056/(20x12x5.5) = 34.89 psi

v = 34.89 psi < 126.5 psi (Allowable) <u>O.K</u>

Check flexural moment transfer :

Moment transferred by flexure is = M_{uf} = 18.2 k'

b = 14 + 3 x 6.5 = 33.5 , d = 5.5"

$$p_p = 2 \times 0.153 /(33.5 \times 5.5) = 0.00166$$

$$f_{ps} = 159 + 10 + 4/(100 \times 0.00166) = 59.0 \text{ k}$$

A_s = 1.23 in ^{2} (calculated before)

F_y = 1.23 x 60 = 73.8k

Total F = 59 + 73.8 = 132.8k

Depth of compression block :

a = F / (0.85 f_c'b) = 132.8/(0.85x4x33.5) = 1.16"

Lever arm = (d - a/2) = (5.5 - 1.16/2)/12 = 0.41'

ϕM_n = 0.9 x 0.41 x 131 = 48.38 k'

Since Moment transfer capacity in flexure = 48.38 k' is much greater than the required flexural moment transfer of 18.2 k'<u>OK</u>

<u>11. DEFLECTION</u>

Deflection due to live load of a 1 ft strip in 25 ft span by conjugate beam method will be investigated.

Live load = 40 psf = 0.04 ksf

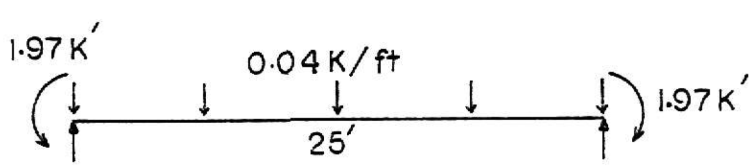
Moment of inertia = I = 12 x 6.5^3/12 = 275 in^4

Modulus of elasticity = E = $W^{1.5} 33\ f_c'^{0.5}$

= 3.8 x 10^3 ksi

EI = 1045 x 10^3 k-in^2

Support moments = 0.04 x (- 9.96)/ 0.202 = - 1.97 k-ft



R_B = 0.5 k R_C = 0.5 k

Deflection at midspan due to negative moment :

$y_1 = (1.97 x 12.5^2/1045 x 10^3 - 1.97 x 12.5^2/2 x 1045 x 10^3) = 0.254$

Deflection at midspan due to positive moment :

$y_2 = 5 \times w \times l^4/(384 \times EI) = 5 x 0.04 x 25^4 x 1728/384 x 1045 x 10^3$

$= 0.336"$

Net midspan deflection = 0.336" - 0.254" = 0.081"

Approximate live load deflection for 20' direction is similar to fixed end beam , $= w\ l^4/(384\ EI)$

$= 0.04 x 20^4 x 1728/(384 x 1045 x 10^3) = 0.027"$

Total deflection at the center of the panel = 0.081+0.027=0.108"

Allowable deflection = Span/360 = 25x12/360 = 0.833"

0.108" < 0.833" <u>O.K</u>

The net load in this span = 0.038 ksf,(0.078 - 0.04)

The long-term dead load deflection, assuming a creep factor of 2, is approximately = 2(0.038x0.108/0.04)= 0.205"

Long-term plus Short-term deflection = 0.205" + 0.108" = 0.313"

Allowable deflection = Span/480 = 25 x 12/480 = 0.625"

0.313" < 0.625"<u>O.K</u>

The reinforcement detail of the design strip and some construction details are shown in Fig.20 and Fig.21 respectively.

CONCLUSION

In recent years flat plate construction has become quite popular for medium and high rise buildings. In the five year period from 1977 through 1981, 250 million square feet of post-tensioned floor slabs were completed.

The simplification of design techniques is the most important reason for the accelerated growth of prestressed flat plate construction. Post-tensioned flat plates are indeterminate structures. The load-balancing method proposed by T.Y.Lin is well suited to these type of structures and offers tremendous advantages both in calculating and visualising. The Equivalent Frame Method of the ACI Code is an approximate and convenient method of analysis and should be used for post-tensioned flat plates.

Now a days, the majority of prestressed flat plate designs are based on the load balancing method plus service load and ultimate strength checks using ACI Code. When ultimate strength is exceeded, it is general practice to add unprestressed bonded reinforcement (when unbonded tendons are used) rather than increasing the level of average prestress. Ultimate strength is controlled by the total amount of tendons (plus any unprestressed reinforcement) rather than by the tendon distribution.

Beam shear is seldom critical in prestressed flat plates but punching shear is critical. To prevent any chance of an abrupt punching shear failure, a minimum of two tendons in each direction through the critical shear section over columns are provided.

Although the design procedure presented in this report is straight forward, it is sometimes lengthy and cumbersome for long hand calculations. Currently "Micro Computers" are solving many complicated design problems with the help of software packages, it is therefore desireable to have a computer programme based on a similar design procedure as disscussed in this report, for the design of post-tensioned flat plates.

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REFERENCES

1. Scordelis,A.C.,Lin,T.Y.and Itaya,R.,"Behavior of a continuous slab prestressed in two directions," Journal of the American Concrete Institute, proceedings Vol.56,No.6, December 1959, pp. 441-459.

2. Rice,Edward.K.and Kulka,F.,"Design of prestressed lift slabs for deflection control," Journal of the American Concrete Institute, proceedings Vol.59,No.8, February 1960, pp 681-693.

3. Lin,T.Y,"Load balancing method for design and analysis of prestressed concrete structures," Journal of the American Concrete Institute, proceedings Vol.60,No.6, June 1963, pp. 719-742.

4. Koons,Robert.L.and Schlegel,Gerald.J.,"A practical approach to the design of continuous structures in prestressed concrete," Journal of the Prestressed Concrete Institute, Vol.8,No.4 August 1963, pp. 35-56.

5. Rozvany,G.I.N.and Hampson,A.J.K.,"Optimum design of prestressed flat plates," Journal of the American Concrete Institute, proccedings Vol.60,No.8, August 1963 pp. 1065-1082.

6. Brotchie,J.F.and Russell,J.J.,"Flat plate structures," Journal of the American Concrete Institute, proceedings Vol.61,No.8, August 1964, pp. 959-996.

7. Candy,C.F.,"Prestressed flat slabs," N.Z.Engineering (Wellington),Vol.19,No.7,July 15,1964, pp.258-261.

8. Grow,J.B.and Vanderbilt,M.D.,"Shear strength of prestressed lightweight aggregate concrete flat plates," Journal of the Prestressed Concrete Institute,Vol.12,No.4,August 1967,pp.8-28

9. Wang,Chen-Hwa,"Direct design method for prestressed concrete slabs," Journal of the Prestressed Concrete Institute,Vol.13, No.3,June 1968, pp. 62-72.

10.Parme,A.L.,"Prestresed flat plates," Journal of the Prestressed Concrete Institute,Vol.13,No.6, December 1968, pp. 14-32.

11.Rozvany,G.I.N.and Woods,J.F.,"Sudden collapse of unbonded under prestressed structures," Journal of the American Concrete Institute, proceedings Vol.66,No.2, February 1969, pp. 129-135.

12.Bondy,Kenneth.B.,Discussion of "Sudden collapse of unbonded under prestressed structures," Journal of the American Concrete Institute, proceedings Vol.66,No.8, August 1969, pp. 680-681.

13.Gerber,Loris.L.and Burns,Ned.H.,"Ultimate strength tests of post-tensioned flat plates,"Journal of the Prestressed Concrete Institute,Vol.16,No.6,December 1971, pp. 40-58.

14. ACI-ASCE Committee 423,"Tentative recommendations for concrete members prestressed with unbonded tendons," Journal of the American Concrete Institute, proceedings Vol.71,No.2, February 1974, pp. 61-71.

15. Nasser,George.D.,"A look at prestressed flat plate design and construction," Journal of the Prestressed Concrete Institute, Vol.14,No.6, December 1969, pp. 62-77.

16. Lin,T.Y.and Burns, Ned.H.,"Design of prestressed concrete structures," John Wiley & Sons,Inc.,3rd Edition, 1981, p.643.

17. Nilson,Arthur.H.,"Design of prestressed concrete," John Wiley & Sons,Inc.,New York, 1978, p. 523.

18. Winter,George.and Nilson,Arthur.H.,"Design of concrete structures," McGraw-Hill Kogakusha,Ltd.,International Student Edition, 9th Edition, 1979, p. 637.

19. Ferguson,Phil.M.,"Reinforced concrete fundamentals," John Wiley & Sons, Inc. 3rd Edition, 1973, p. 743.

20. Rice,Paul.F.and Hoffman,Edward.S.,"Structural design guide to the ACI building Code," Van Nostrand Reinhold Company, 2nd Edition, 1979, p. 467.

21. Building code requirements for reinforced concrete,"ACI Standard 318-77".

22. Post-Tensioning Manual, "Post-Tensioning Institute," 1976. p. 288.

APPENDIX A FIGURES

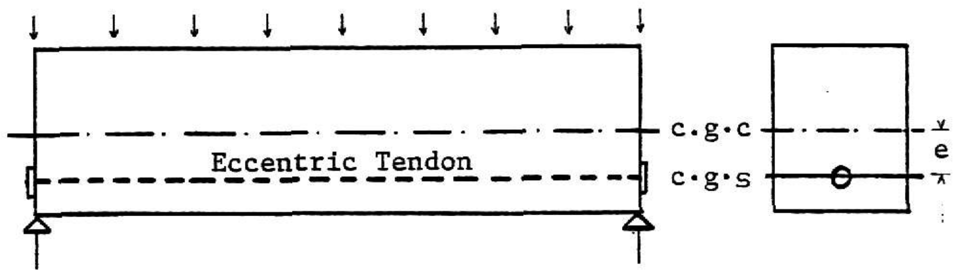


Fig. 1. Beam Eccentrically Prestressed and Loaded

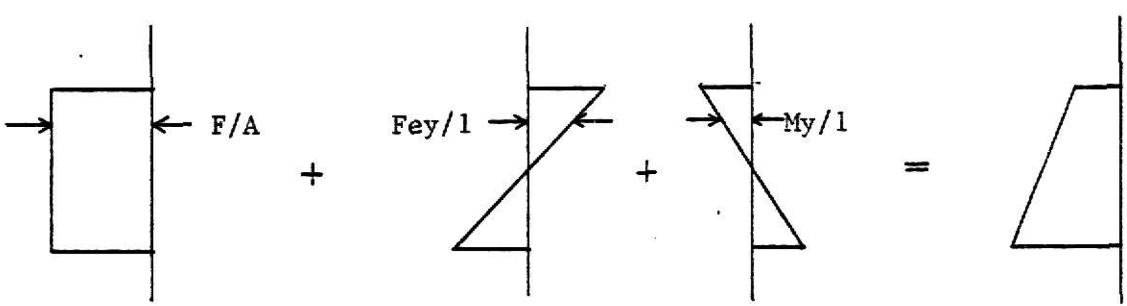


Fig. 2 Stress Distribution Across An Eccentrically Prestressed-Concrete Section

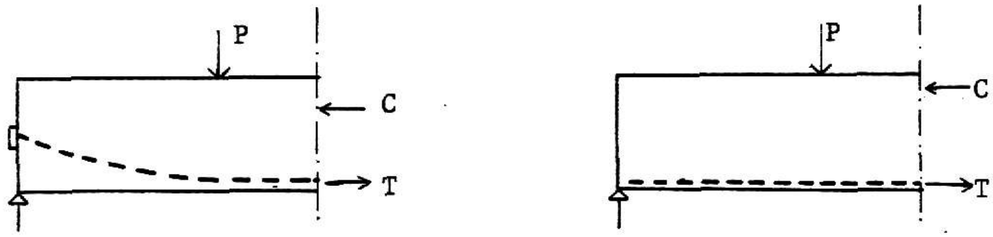
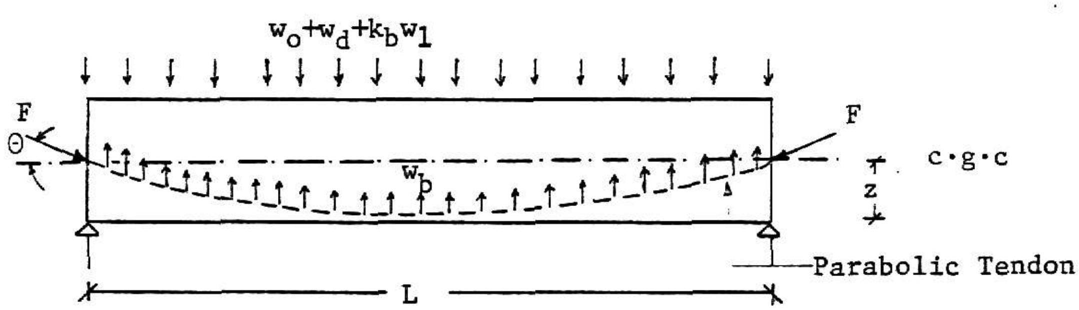
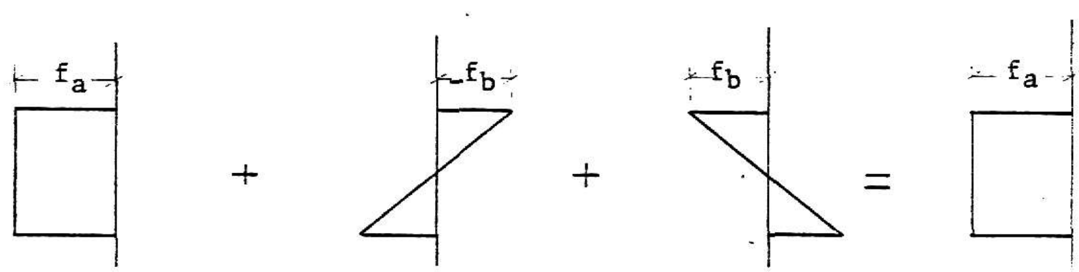


Fig. 3. Internal Resisting Moment in Prestressed and Reinforced Concrete Beams

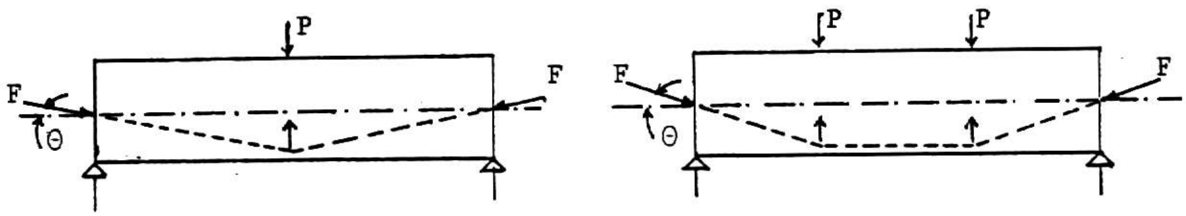


(1) External and Equivalent Loads



(2) Concrete Stresses Due to Axial and Bending Effects of Prestress Plus Bending Due to Balanced External Load

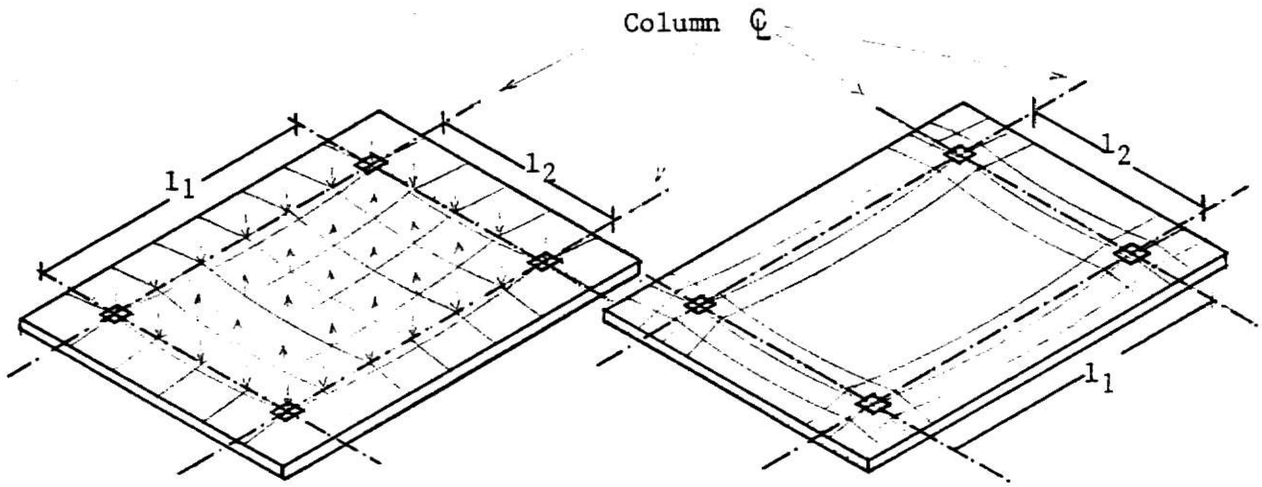
Fig. 4. Load Balancing for a Uniformly Loaded Beam



(1) Single Concentrated Load - One Point Harping

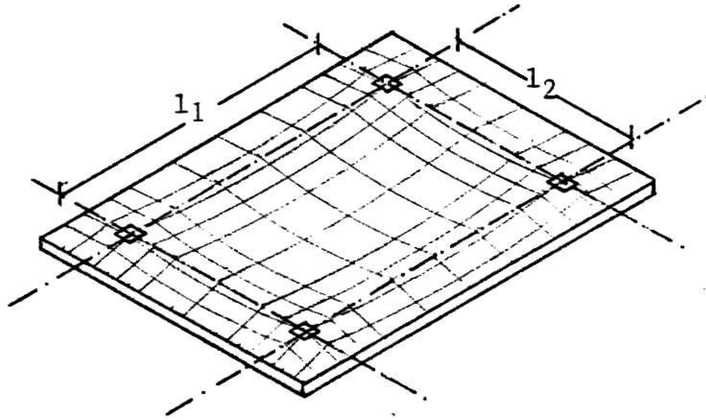
(2) Load at Third Points - Two Point Harping

Fig. 5. Prestress Beam with Harped Tendons



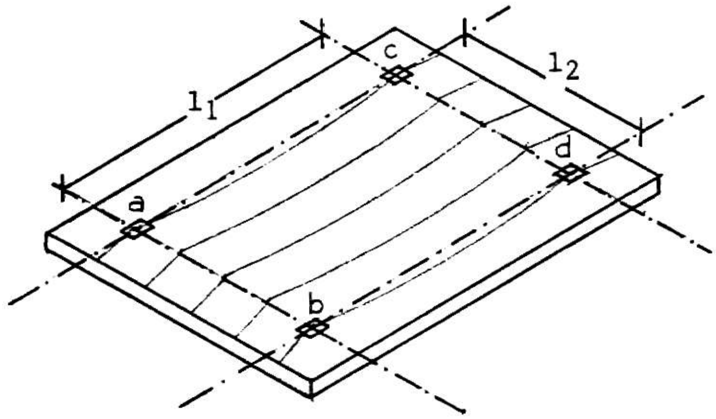
(1) Distributed Tendons

(2) Banded Tendons

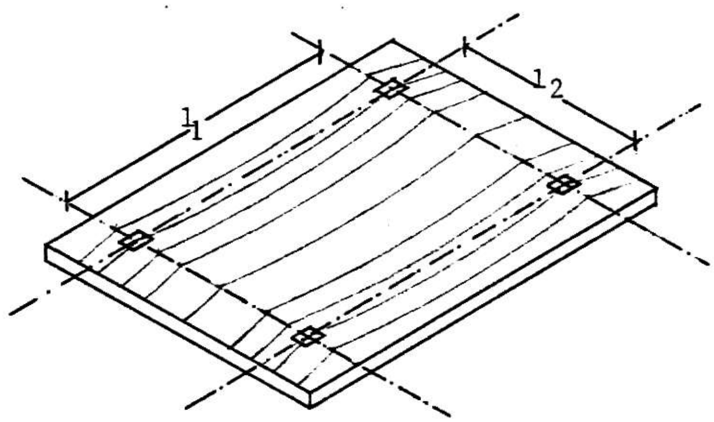


(3) Superposition of (1) and (2)

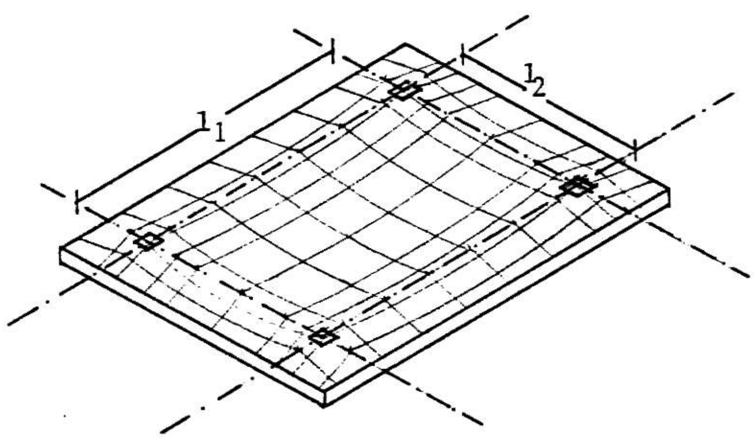
Fig. 6. Load Balancing for Flat Plates



(1) Tendons Uniformly Distributed



(2) Tendons Concentrated in Column Strips



(3) Two-Way Tendon Pattern

Fig. 7. Load Balancing for Flat Plate as Wide-Beam System

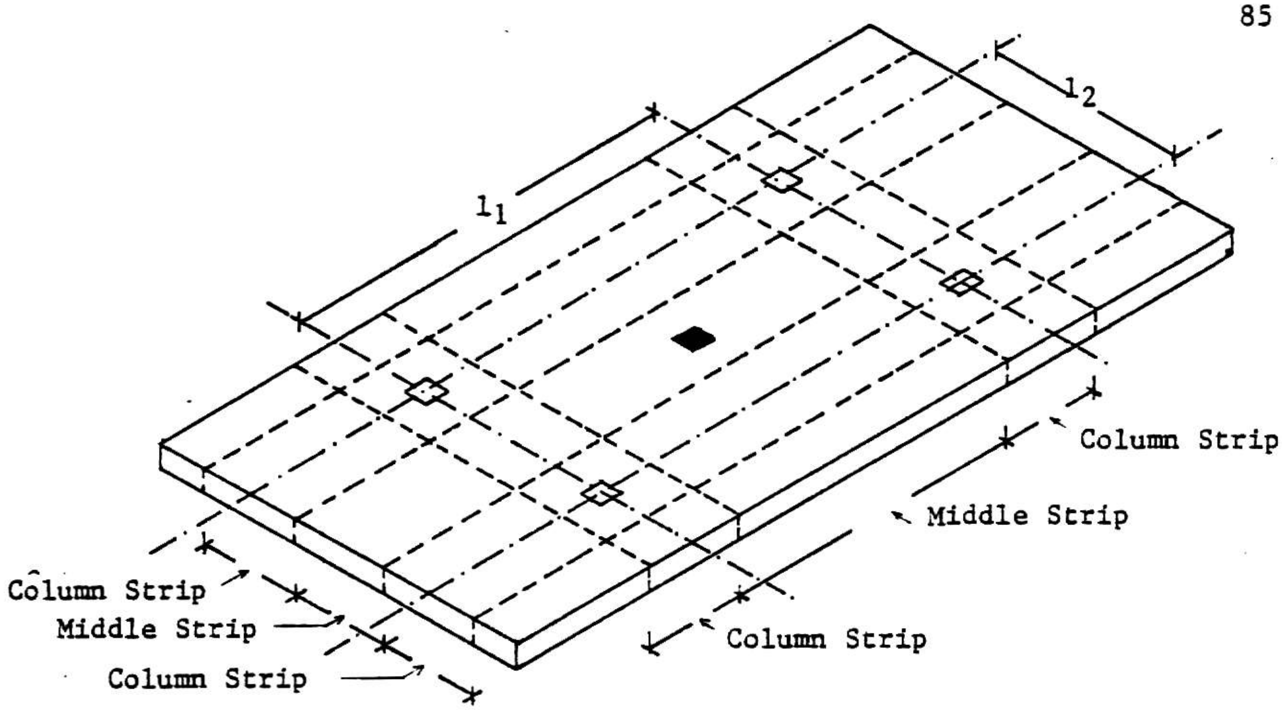


Fig. 8. Two-Way Flat Plate Floor System Showing Equivalent Beams

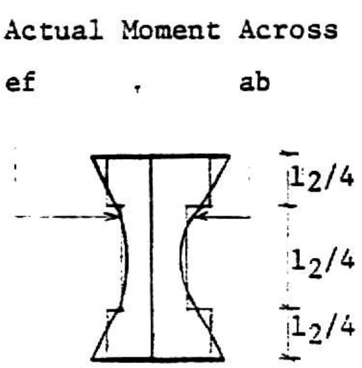


Fig. 11. Variation of Moment Across Width ab and ef

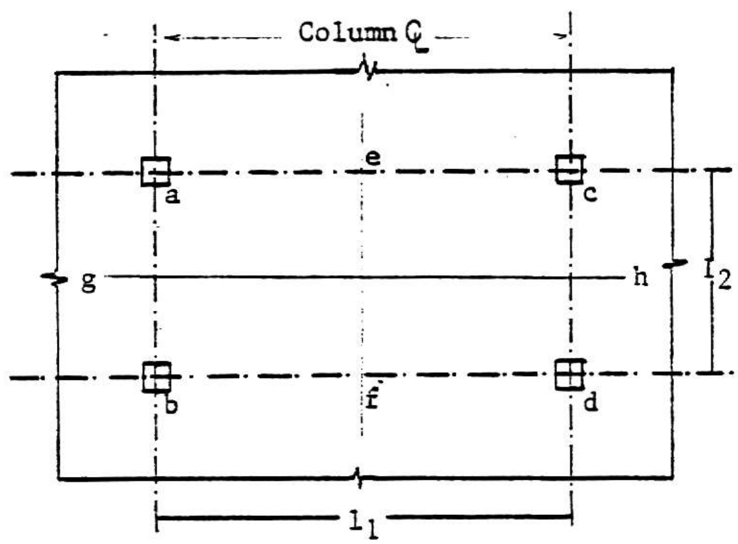


Fig. 9. Plan of an Interior Panel

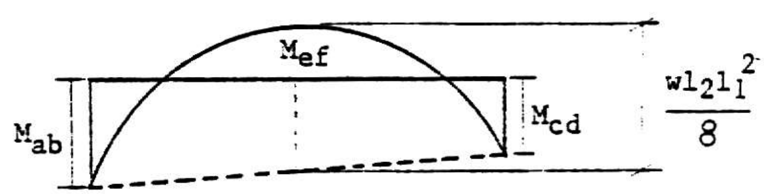
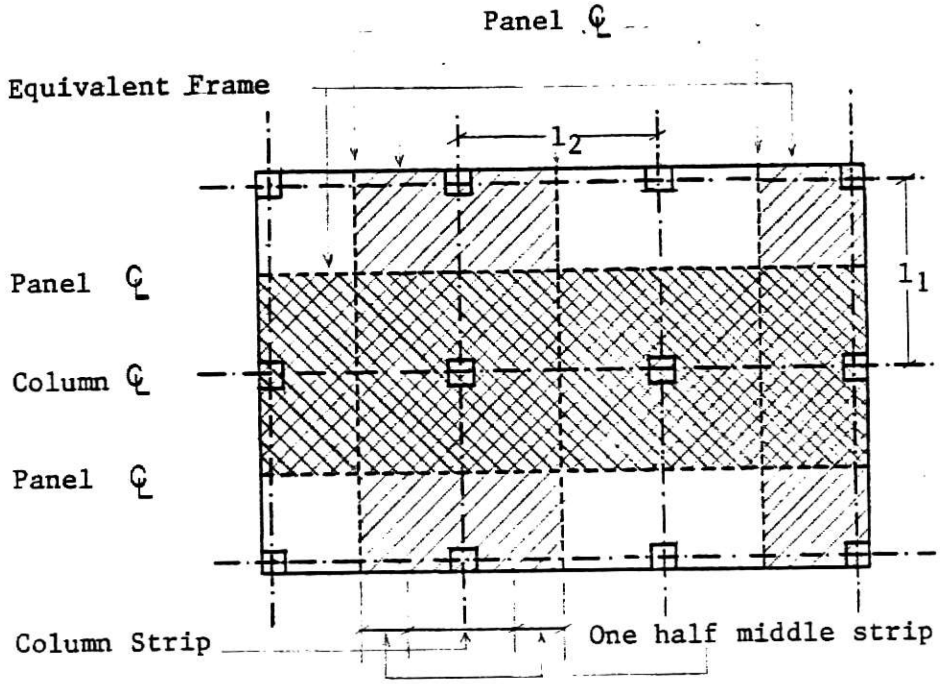
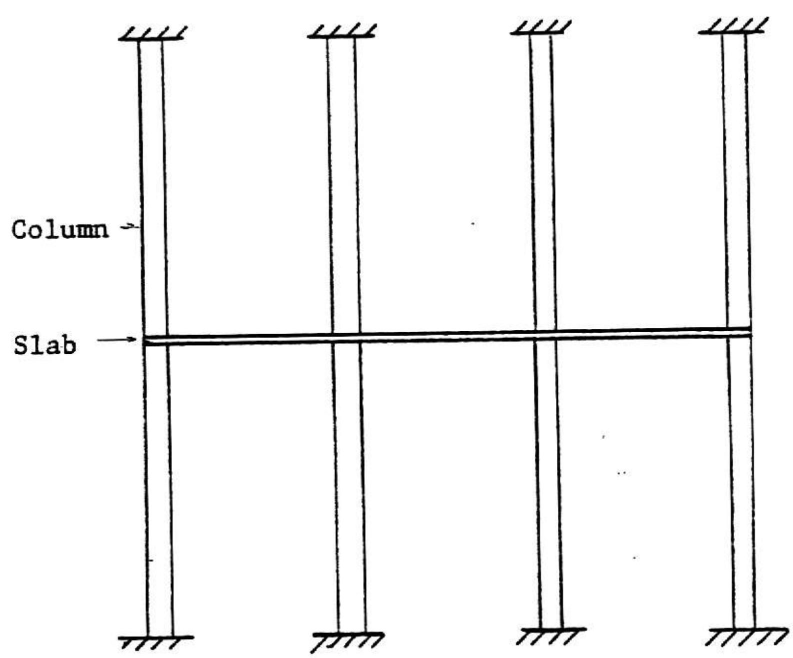


Fig. 10. Moments in l_1 Direction

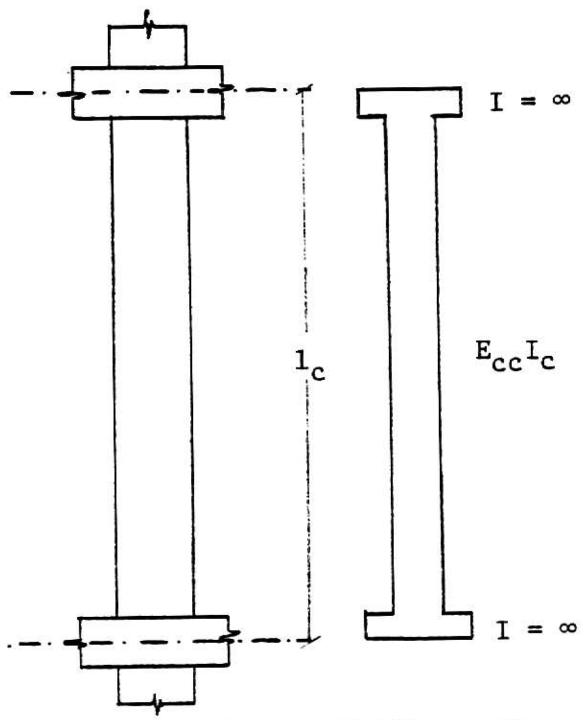


(1) Definition of Equivalent Frame

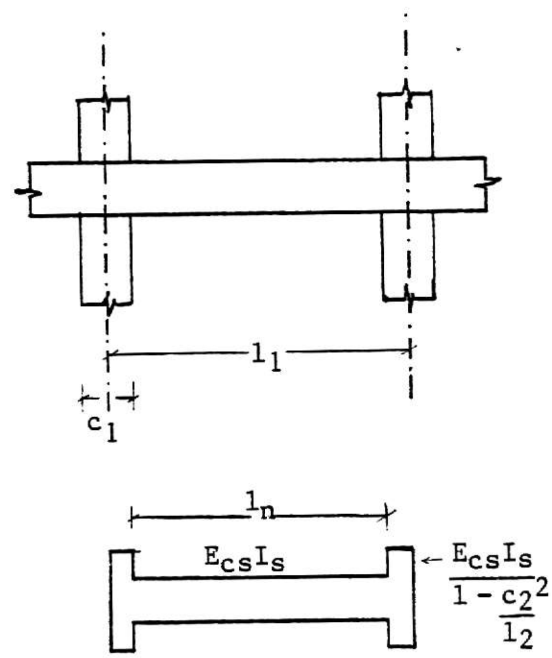


(2) Columns Assumed Fixed at the Floors Above and Below

Fig. 12. Equivalent Frame



(1) Of Column - Column Stiffness Diagram



(2) Of Slab Beam - Slab-Beam Stiffness Diagram

Fig. 13. Moments of Inertia

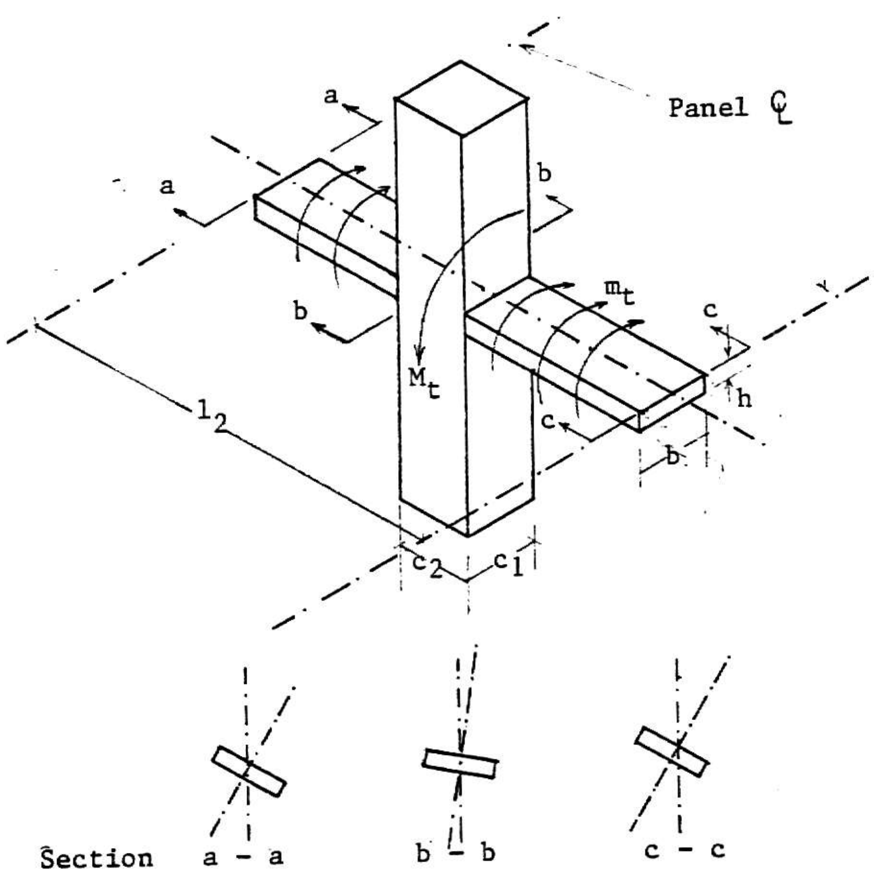


Fig. 14. Basis of Equivalent Column

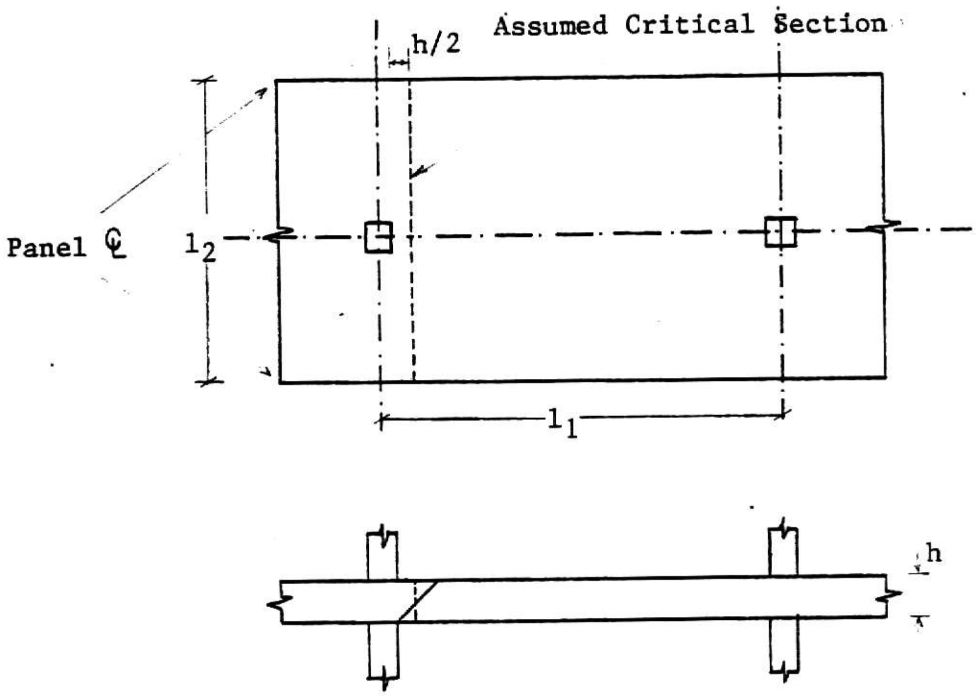


Fig. 15. Beam Shear in Rectangular Panel

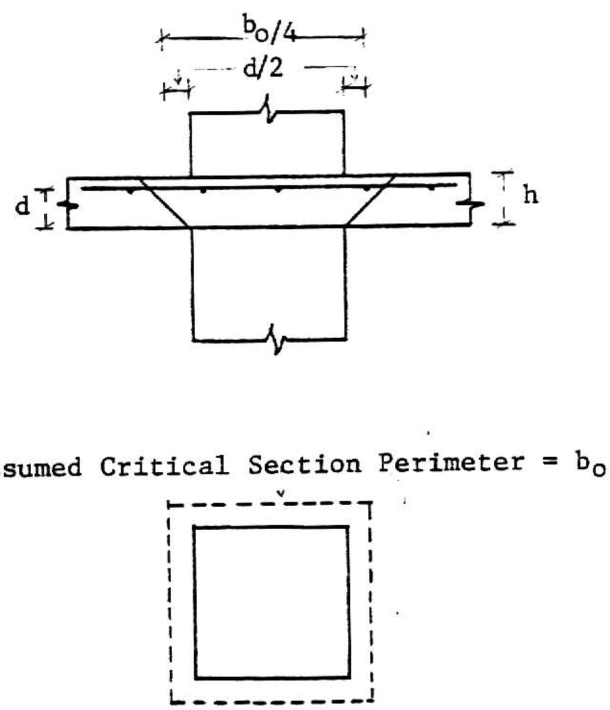
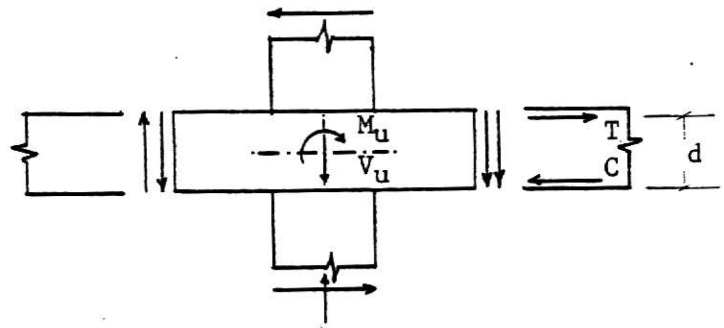
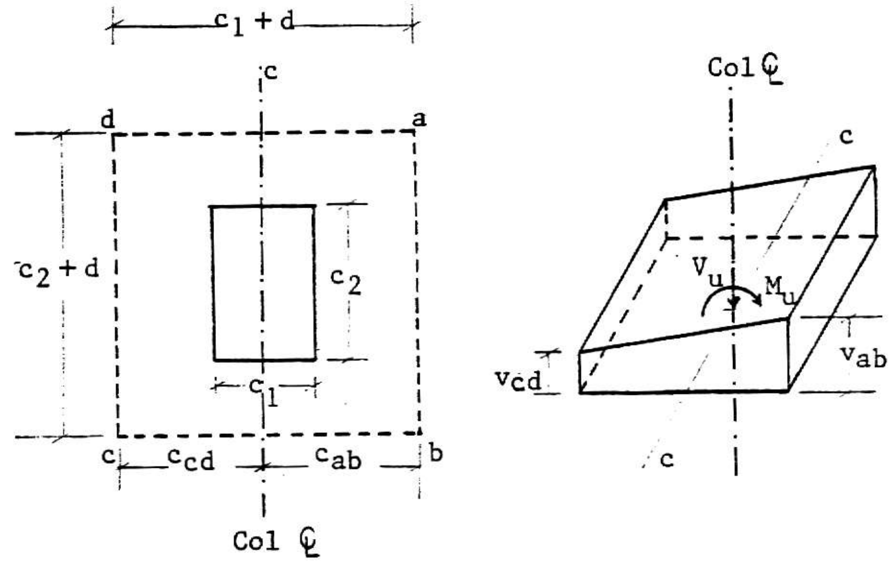


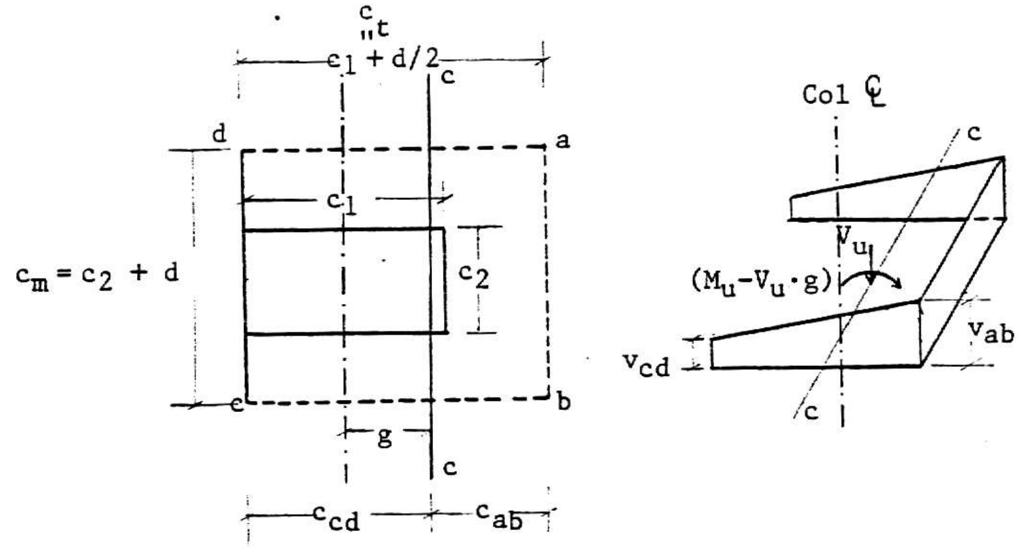
Fig. 16. Punching Shear





(1) Interior Column

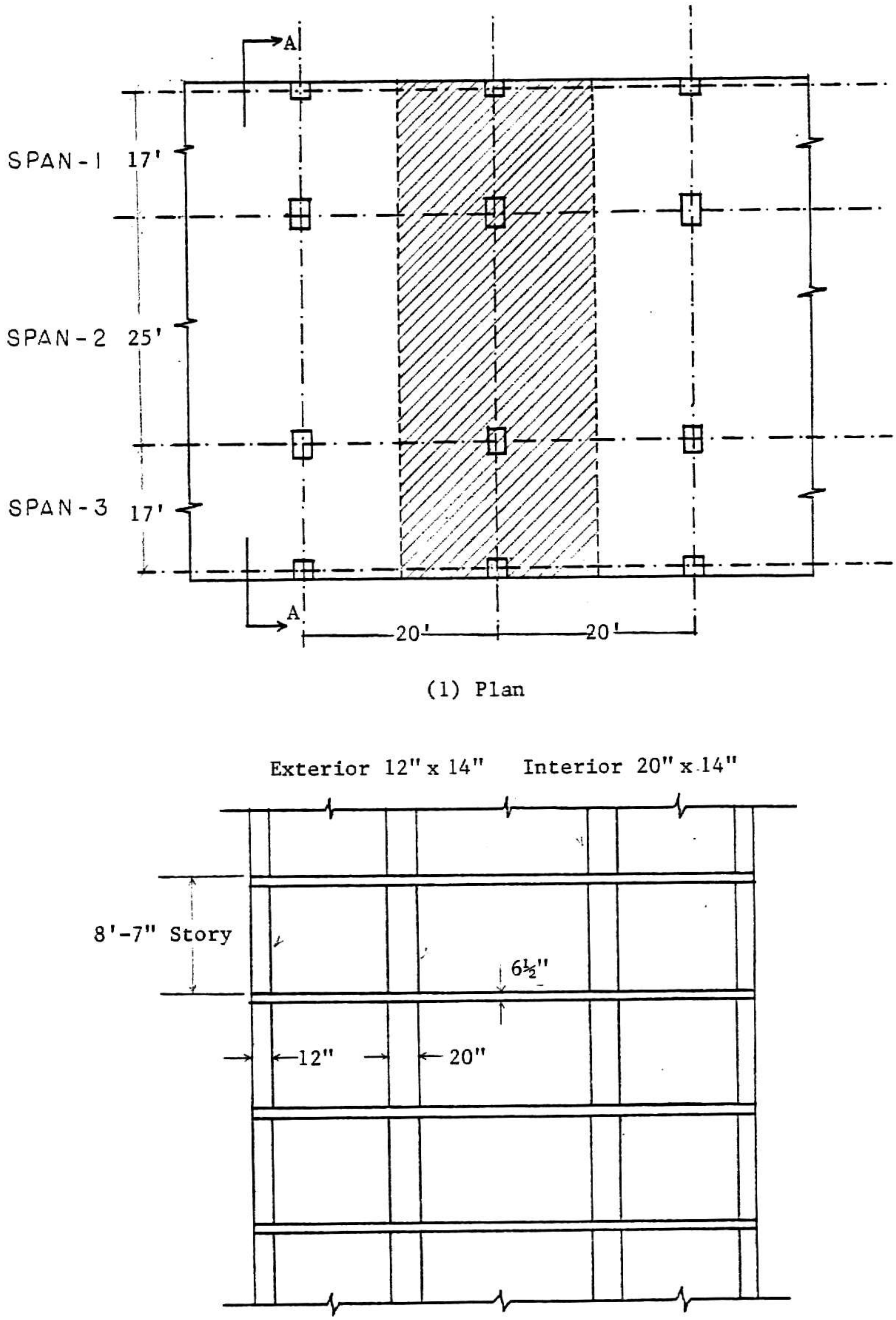
$$J_c = \frac{d}{6}(c_1+d)^3 + \frac{d^3}{6}(c_1+d) + \frac{1}{2}(c_2+d)(d)(c_1+d)^2$$



(2) Edge Column

$$J_c = \frac{dc_t^3}{6} + \frac{c_t d^3}{6} + c_m d c_{ab}^2 + 2c_t d\left(\frac{c_t}{2}-c_{ab}\right)^2$$

Fig. 17. Assumed Distribution of Shear Stress



(1) Plan

(2) Section AA

Fig. 18. Design Example

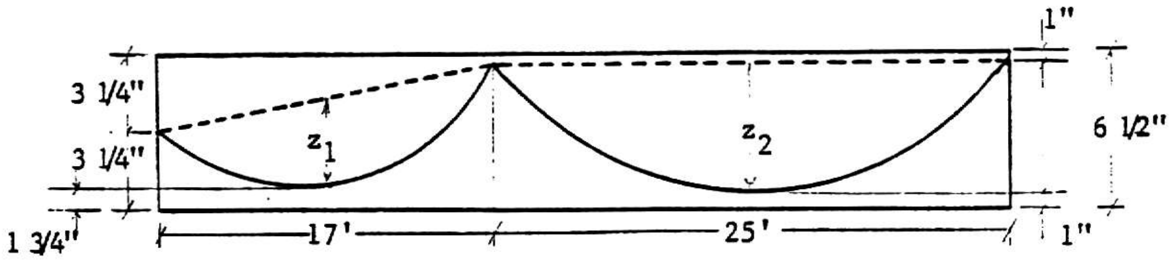


Fig. 19. Tendon Profile - Design Example

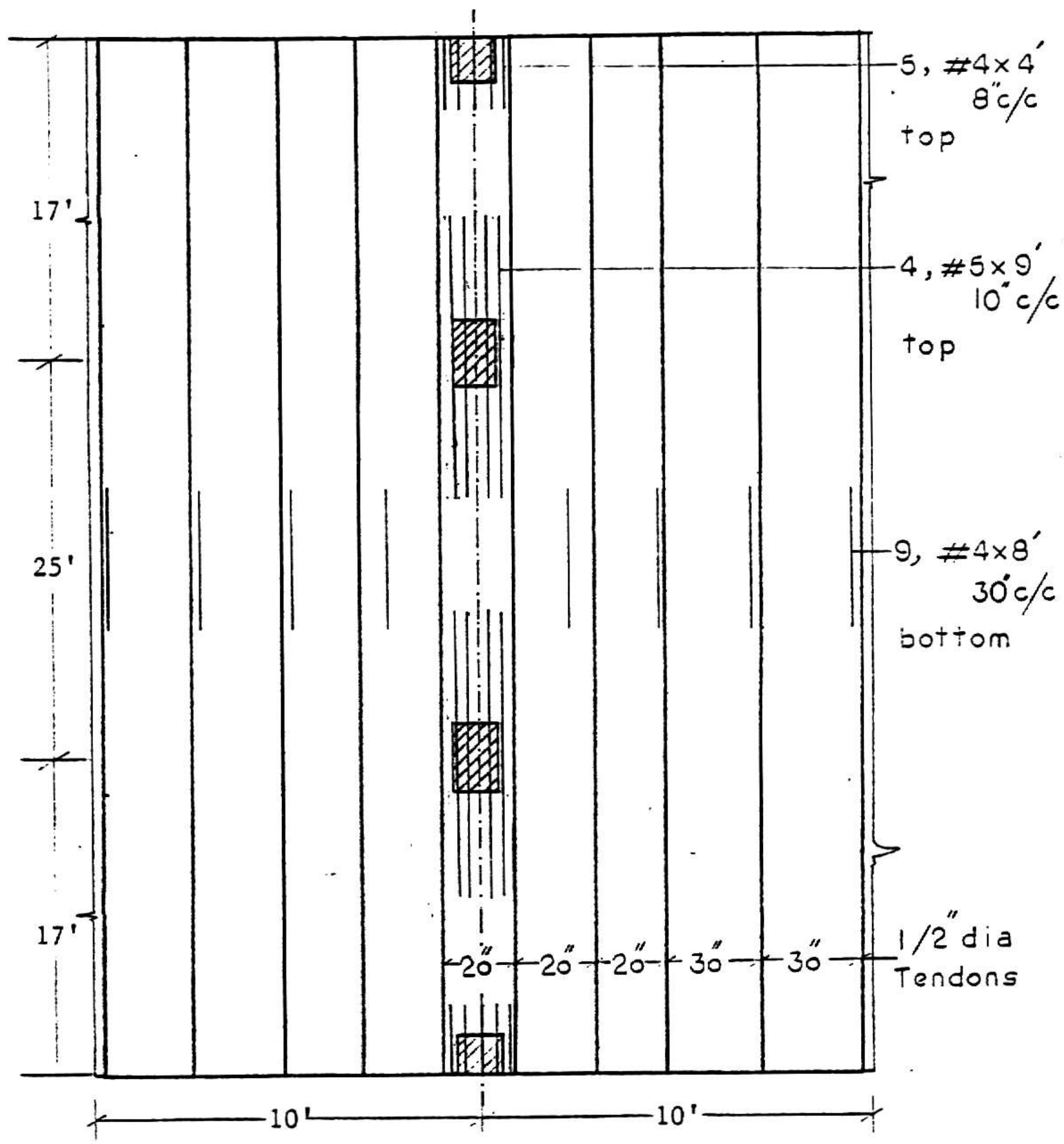
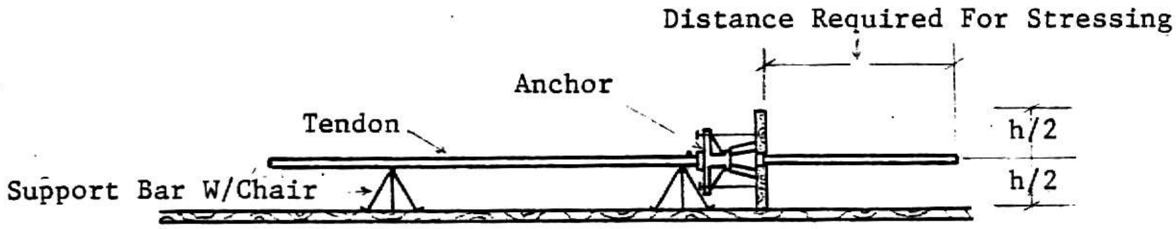
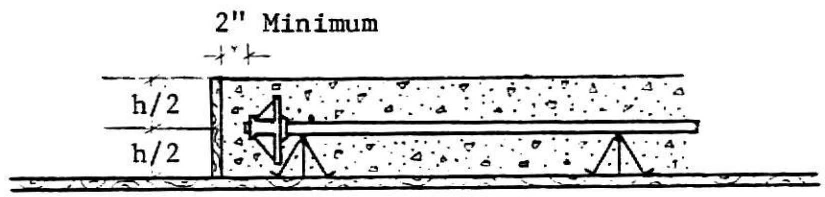


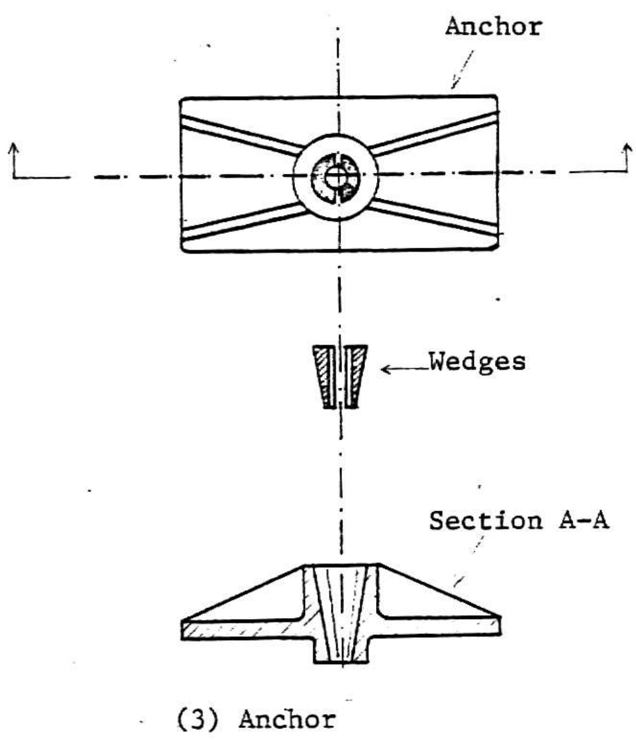
Fig. 20. Reinforcement Detail of the Design Strip



(1) Stressing End



(2) Dead End Anchorage



(3) Anchor

Fig. 21. Construction Detail

a/l_c \ b/l_c	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22
0.00	4.000	4.082	4.167	4.255	4.348	4.444	4.545	4.651	4.762	4.878	5.000	4.128
0.02	4.337	4.433	4.533	4.638	4.747	4.862	4.983	5.110	5.244	5.384	5.533	5.690
0.04	4.709	4.882	4.940	5.063	5.193	5.330	5.475	5.627	5.787	5.958	6.138	6.329
0.06	5.122	5.252	5.393	5.539	5.693	5.855	6.027	6.209	6.403	6.608	6.827	7.060
0.08	5.581	5.735	5.898	6.070	6.252	6.445	6.650	6.868	7.100	7.348	7.613	7.897
0.10	6.091	6.271	6.462	6.665	6.880	7.109	7.353	7.614	7.893	8.192	8.513	8.859
0.12	6.659	6.870	7.094	7.333	7.587	7.859	8.150	8.461	8.796	9.157	9.546	9.967
0.14	7.292	7.540	7.803	8.084	8.385	8.708	9.054	9.426	9.829	10.260	10.740	11.250
0.16	8.001	8.291	8.600	8.931	9.287	9.670	10.080	10.530	11.010	11.540	12.110	12.740
0.18	8.796	9.134	9.498	9.888	10.310	10.760	11.260	11.790	12.370	13.010	13.700	14.470
0.20	9.687	10.080	10.510	10.970	11.470	12.010	12.600	13.240	13.940	14.710	15.560	16.490
0.22	10.690	11.160	11.660	12.200	12.800	13.440	14.140	14.910	15.760	16.690	17.210	18.870
0.24	11.820	12.370	12.960	13.610	14.310	15.080	15.920	16.840	17.870	19.000	20.260	21.650

$\bullet K_c = \dfrac{kcEcIc}{l_c}$

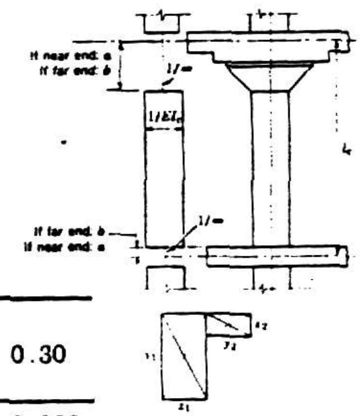


Table 1. Column Stiffness Coefficient

c_1/l_1 \ c_2/l_2		0.00	0.05	0.10	0.15	0.20	0.25	0.30
0.00	M	0.083	0.083	0.083	0.083	0.083	0.083	0.083
	k	4.000	4.000	4.000	4.000	4.000	4.000	4.000
	C_f	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.05	M	0.083	0.084	0.084	0.084	0.085	0.085	0.085
	k	4.000	4.047	4.093	4.138	4.181	4.222	4.261
	C_f	0.500	0.503	0.507	0.510	0.513	0.516	0.518
0.10	M	0.083	0.084	0.085	0.085	0.086	0.087	0.087
	k	4.000	4.091	4.182	4.272	4.362	4.449	4.535
	C_f	0.500	0.506	0.513	0.519	0.524	0.530	0.535
0.15	M	0.083	0.084	0.085	0.086	0.087	0.088	0.089
	k	4.000	4.132	4.276	4.403	4.541	4.680	4.818
	C_f	0.500	0.509	0.517	0.526	0.534	0.543	0.550
0.20	M	0.083	0.085	0.086	0.087	0.088	0.089	0.090
	k	4.000	4.170	4.346	4.529	4.717	4.910	5.108
	C_f	0.500	0.511	0.522	0.532	0.543	0.554	0.564
0.25	M	0.083	0.085	0.086	0.087	0.089	0.090	0.091
	k	4.000	4.204	4.420	4.648	4.887	5.138	5.401
	C_f	0.500	0.512	0.525	0.538	0.550	0.563	0.576
0.30	M	0.083	0.085	0.086	0.088	0.089	0.091	0.092
	k	4.000	4.235	4.488	4.760	5.050	5.361	5.692
	C_f	0.500	0.514	0.527	0.542	0.556	0.571	0.585
$X = (1 - c_2/l_2)^3$		1.000	0.856	0.729	0.613	0.512	0.421	0.343

$FEM = Mwl_2 l_1^2$
$K = kEI_s / l_1$ with
$I_s = l_2 h^3 / 12$
C.O.F. = C_F

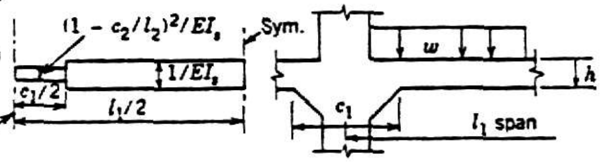


Table 2. Moment Distribution Factors for Slab-Beam Elements

Restrained or Unrestrained	Aggregate Type	Cover Thickness, in., for Fire Endurance of 1 hr	1-1/2 hr	2 hr	3 hr	4 hr
Unrestrained	Carbonate	3/4	1-1/16	1-3/8	1-7/8	–
Unrestrained	Siliceous	3/4	1-1/4	1-1/2	2-1/8	–
Unrestrained	Lightweight	3/4	1	1-1/4	1-5/8	–
Restrained	Carbonate	3/4	3/4	3/4	1	1-1/4
Restrained	Siliceous	3/4	3/4	3/4	1	1-1/4
Restrained	Lightweight	3/4	3/4	3/4	3/4	1

Table 3. Suggested Concrete Cover Thickness for Slabs Prestressed with Post-Tensioned Reinforcement

Post-Tensioning Tendon Material	Prestress Loss - psi Slabs	Beams and Joists
Stress Relieved 270k Strand and Stress Relieved 240k Wire	30,000	35,000
Bar	20,000	25,000

Table 4. Appropriate Prestress Loss Values

APPENDIX.C NOTATIONS

a = Depth of compression block

a = Fraction of moment transferred by shear

b = Width of the section = b_w

b_o = Perimeter of critical shear section

c_1 = Column dimension in the direction of l_1

c_2 = Column dimension transverse to c_1

c_3 = Distance from centroid of critical shear section to extreme fiber in direction of moment transfer

d = Distance from extreme compression fiber to centroid of prestressed reinforcement, or to combined centroid when non-prestressed reinforcement is included

f = Stress

f_c = Specified concrete strength

f_{pc} = Compressive stress in concrete at centroid of section

f_{pe} = Effective prestress after allowance for all losses

f_{ps} = Stress in prestress reinforcement at nominal strength

f_{pu} = Specified tensile strength of prestressed steel

f_y = Specified yield strength of non-prestressed steel

g = Distance from column centerline to centroid of critical secton

h = Thickness of the slab

k = Slab stiffness coefficient

k_c = Column stiffness coefficient

l_1 = Length of the span in the direction moments are computed

l_2 = Transverse span length

l_n = Clear span

p = Ratio of prestressed reinforcement

w = Load per unit of surface area

x = Smaller dimension of the rectangular cross section of the transverse slab strip

y = Larger dimension of the above mentioned cross section

z = Maximum sag of the parabolic tendon

A = Area of cross section

A_{ps} = Area of prestressed reinforcement

A_s = Area of bonded reinforcement

C = Cross sectional constant for the transverse slab strip

E = Modulus of elasticity

F = Effective prestress force

I = Moment of inertia

J_c = Polar moment of inertia of critical section

K = Stiffness

K_c = Flexural stiffness of actual column

K_{ec} = Flexural stiffness of equivalent column

K_t = Torsional stiffness of transverse slab strip

L = Length of the span, c/c distance between columns

M = Bending moment

M_o = Total factored static moment along the span

M_t = Net moment to be transferred to column

M_u =Factored Moment

M_{uf} =Moment transferred by flexure

M_{vf} =Moment transferred by shear

N_c =Tensile force in concrete due to unfactored dead plus live load

V_c = Shear carried by concrete

V_n = Nominal shear strength

V_p = Vertical component of effective prestress force at section

V_u = Shear force at factored loads

W_b = Balanced upward load

ϕ = Strength reduction factor

DESIGN OF POST-TENSIONED FLAT PLATES

by

Rais Mirza

B.E.(Civil), N.E.D University, Pakistan, 1981

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY

Manhattan, Kansas

1984

ABSTRACT

Post-tensioned concrete flat plates have become a major factor in floor systems for commercial and residential buildings of all types. They have found to be economical in parking structures, apartment buildings, office buildings, hospitals and industrial buildings, both high-rise and low-rise type.

Post-tensioned flat plates are highly indeterminate structures, therefore suitable procedures must be found for their analysis and design. Many engineers have done considerable research in this field and still many more are working on it.

In this report, "Load Balancing Method" proposed by T.Y.Lin , with "Equivalent Frame Method" of the ACI Code, for flat plates , is discussed thoroughly. This combination is well suited for the design office and is widely used. A design procedure is also given and a design example of a post-tensioned flat plate apartment is included. The flat plate floor was designed according to the provisions of the ACI 318-77 plus the recommendations given by ACI-ASCE Committee 423.

Unbonded tendons with bonded non-prestressed reinforcement have been used in the design example. The slab was analyzed for the gravity loads only, and was checked for service load stage, ultimate strength, shear strength and deflection.