ULTIMATE STRENGTH OF WIDE-FLANGE BEAM-COLUMNS

by 45

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I. INTRODUCTION

The problem that is considered in this report is the determination of the maximum amount of end moment that a member can sustain when it is subjected to a given axial thrust. The purpose of the report is to calculate the exact interaction curves for a steel wide-flange beam-column, to compare the results with the interaction formulas used in the AISC Specification, and to illustrate the use of the interaction curves. It is assumed that the plane of the applied moment is that of the web of the section and that failure is the result of excessive bending in this plane. Meanwhile, residual stresses and strain-hardening are not considered in the report.

There are a variety of methods available for the determination of the ultimate strength of wide-flange beam-columns, such as Newmark's integration method (Ref. 1), the stepwise integration method (Ref. 1) and the nomographic method (Ref. 2). The interaction curves in this report were calculated using Newmark's numerical integration method. The investigation is limited in scope to the case of axial thrust plus moment applied at only one end of the member, although the method used is applicable to other loading conditions.

The idealized stress-strain relationship of Fig. 1-1 is assumed to apply to steel wide-flange members. The corresponding stress and strain distributions due to the presence of axial force and moment are shown in Fig. 1-2.

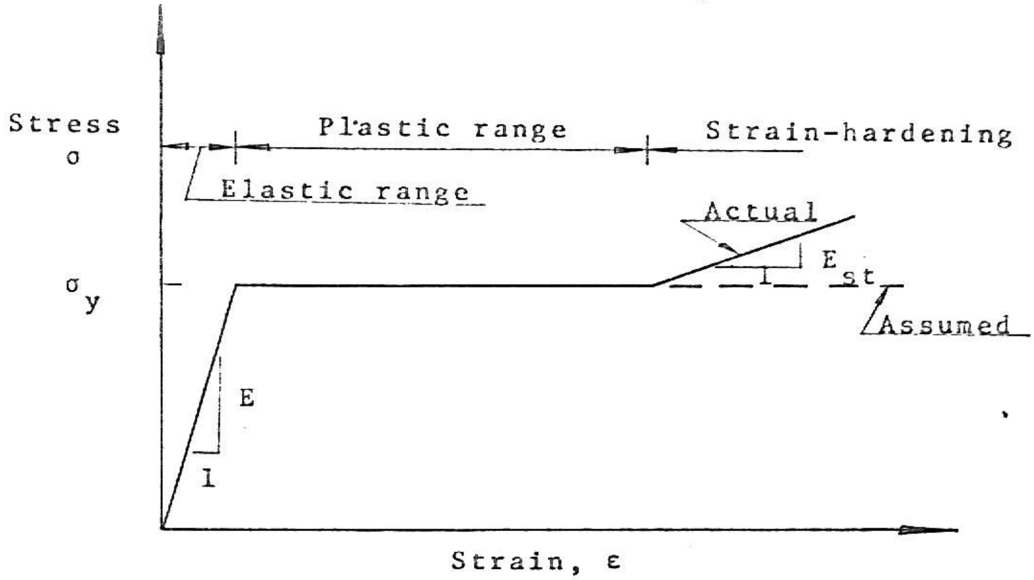


Fig. 1-1. Idealized stress-strain curve

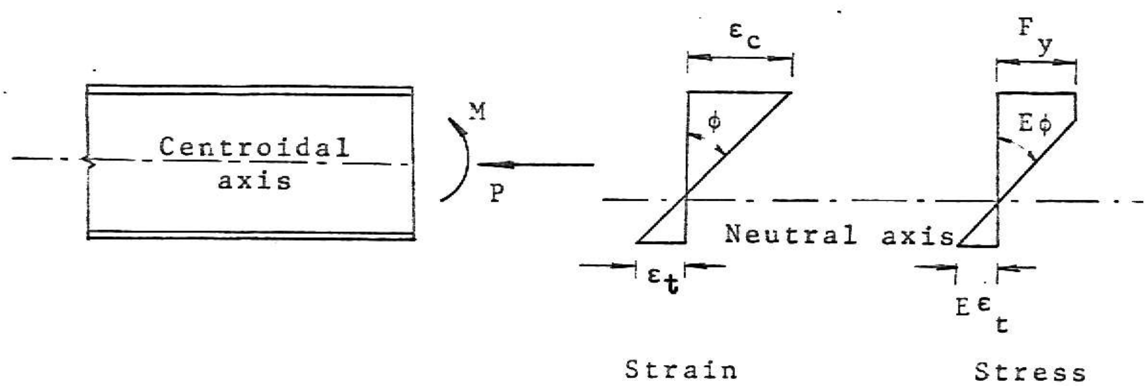


Fig. 1-2. Stress and strain distribution on the cross section at partial yielding

II. MOMENT-ROTATION BEHAVIOR

For the sake of illustration, the following description will be restricted to structural steel wide-flange beam-columns bent by end moment(s) about their major axis. Failure may be caused by one of the following instability phenomena:

1. Lateral-torsional buckling.
2. Local buckling.
3. Excessive bending in the plane of the applied moment(s).

Fig. 2-1 shows the theoretically determined relationship existing between the end-moment M_o and the end-slope θ for most wide-flange beam-columns. The solid line curve in Fig. 2-1

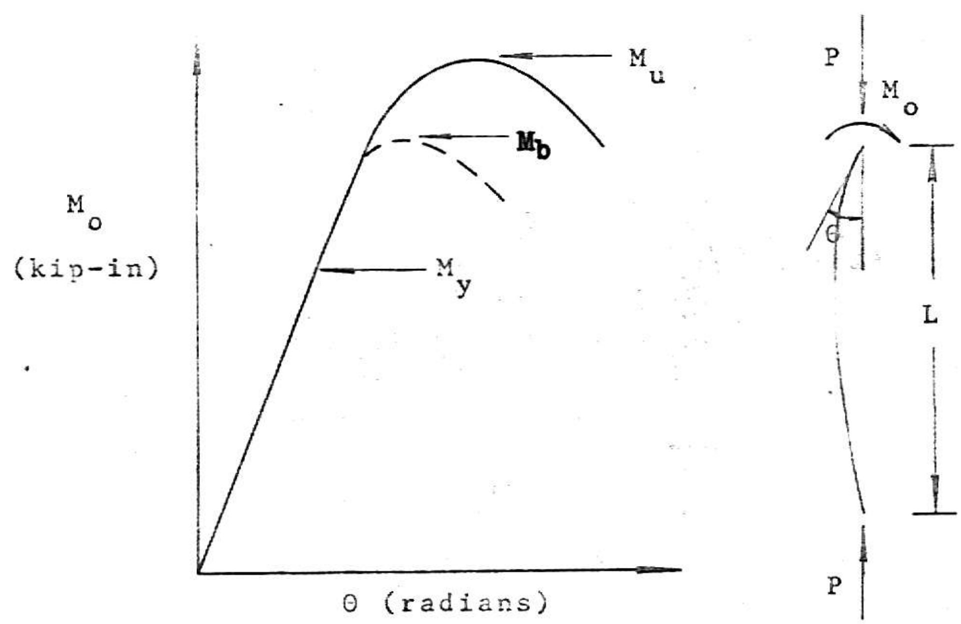


Fig. 2-1. End moment vs. end slope curve for wide-flange beam-columns

represents the optimal behavior of the wide-flange beam-column; no lateral-torsional or local buckling effects influence the situation in this case. This means that all deformations take place in the plane of bending and that the only weakening effect is due to yielding. The M_o-θ curve consists of the following two parts:

1. The ascending stable part, where an increase of deflection is accompanied by an increase in the moment.

2. The descending unstable part, where an increase of deformation results in a decrease in M_o.

The peak point corresponds to the ultimate moment, and the attainment of this moment constitutes failure. The M_o-θ curve is a straight line if the axial thrust is maintained constant and the material is elastic.

Failure in the plane of bending is possible only if the member is bent about its weak axis or if it is adequately braced against lateral-torsional buckling when bent about its strong axis. In the absence of adequate bracing an initially straight member will start to deflect laterally, as well as to twist, at a certain "critical" moment M_b which is lower than the ultimate moment M_u. (See dashed line in Fig. 2-1) In addition to lateral-torsional buckling, the M-θ relationship, and thus the ultimate strength, is also influenced by local buckling. In the design routine one must check that the flange width-thickness ratio does not exceed a certain maximum value specified in the appropriate specification (Ref. 3) to eliminate the possibility of local buckling.

III. MOMENT-THRUST-CURVATURE DIAGRAM

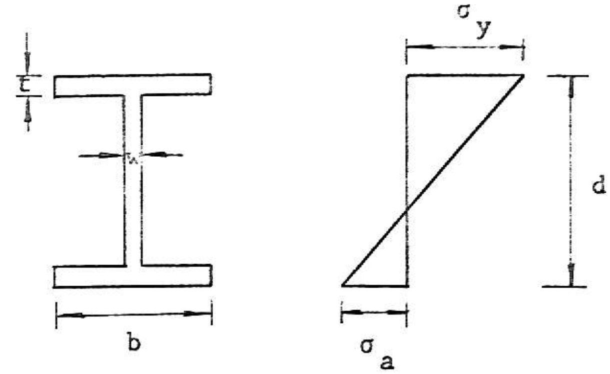
The results of an analytical study of the elastic-plastic deformation of wide-flange beam-columns are presented in this article. In the analysis, the influence of residual stresses is ignored; thus, the moment-curvature relationship is developed as a function of axial thrust only.

1. Elastic and Plastic Deformation

For flexure in the elastic range, $\phi = \frac{M}{EI} = \frac{d^2y}{dx^2}$ in which ϕ is the curvature, M denotes the bending moment, E is the Modulus of Elasticity, I represents the moment of inertia of the cross section about an axis perpendicular to the plane of the applied moment; x is the distance along the member and y denotes deflection in the plane of the applied moment. Above the elastic limit, however, an easy solution does not exist because stress is no longer a linear function of strain, and therefore ϕ does not vary linearly with M. An important assumption made in the following derivation is that the bending strain is proportional to the distance from the neutral axis. If strain-hardening is considered, it is once again necessary to make an assumption with regard to strain distribution in the inelastic range. Strain-hardening, however, is not considered herein.

2. Analysis of Elastic-Plastic Bending (Ref. 4)

A. Case I - elastic



Limits $-\sigma_y \leq \sigma_a \leq +\sigma_y$

$$R_a = \frac{\sigma_a}{\sigma_y}$$

$$-1 \leq R_a \leq +1$$

Axial thrust

$$\frac{P}{\sigma_y} = \frac{(1+R_a)}{2}\,[2bt+w(d-2t)]$$

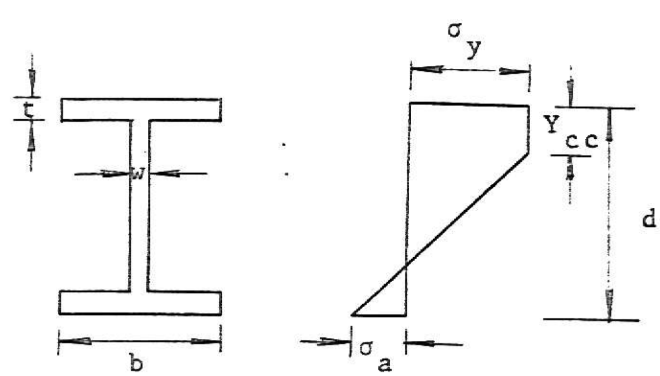
Moment

$$\frac{M}{\sigma_y} = \frac{(1+R_a)}{12d}\,[2bt(3d^2-6dt+4t^2)+w(d-2t)^3]$$

Curvature

$$\frac{\phi}{\phi_y} = \frac{(1+R_a)}{2}$$

B. Case II - top side gets yield penetration



Limits $R_a = \dfrac{\sigma_a}{\sigma_y}$

$$-1 \leq R_a \leq +1$$

$$\alpha = \frac{Y_{cc}}{d}$$

$$\frac{t}{d} \leq \alpha \leq \left(1 - \frac{t}{d}\right)$$

Axial thrust

$$\frac{P}{\sigma_y} = (1-R_a)(bt-wt)+\frac{(1+R_a)}{2d(1-\alpha)}[bt^2-wd^2(1-\alpha)^2-wt^2]+wd$$

Moment

$$\frac{M}{\sigma_y} = \frac{(1+R_a)}{12d(1-\alpha)}[w\{d(1-\alpha-\frac{t}{d})\}^2\{d(1+2\alpha)-4t\}$$

$$+ bt\{6d(1-\alpha)(d-t)-3dt+4t^2\}]$$

Curvature

$$\frac{\phi}{\phi_y} = \frac{(1+R_a)}{2(1-\alpha)}$$

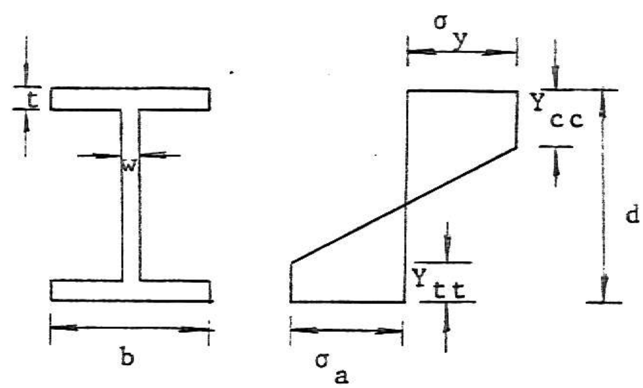
C. Case III - both sides have started to yield

Limits $\alpha = \frac{Y_{cc}}{d}$

$\frac{t}{d} \le \alpha \le (1-\frac{t}{d})$

$\beta = \frac{Y_{tt}}{d}$

$\frac{t}{d} \le \beta \le \alpha$



Axial thrust

$$\frac{P}{\sigma_y} = wd(\alpha-\beta)$$

Moment

$$\frac{M}{\sigma_y} = bt(d-t)+ \frac{wd^2}{6}[6(\beta-\frac{t}{d})(1-\beta-\frac{t}{d})+(1-\alpha-\beta)(1+2\alpha-4\beta)]$$

Curvature

$$\frac{\phi}{\phi_y} = \frac{1}{1-\alpha-\beta}$$

The derivations of the above expressions are given in Appendix A.

3. Procedure for Finding M-P-ϕ Curves for a Given Member

Material: ASTM A36 steel, F_y=36ksi, E=30000ksi

No strain-hardening, bending about its strong axis

No residual stresses

Section: 10 WF 39 (Compact section for A36 steel)

Procedure: From the above-mentioned three cases, we can observe that there are three equations (Axial thrust, moment, curvature) and only two unknowns in any one of them. Therefore, we are able to solve for these two unknowns using two assumed values of axial thrust and curvature. Substituting these two unknowns into the moment equation, we can obtain M. Axial thrust is considered to range from 0 to P_y in increments of $0.2P_y$.

a. Set $\frac{\phi}{\phi_y} = 0$ first, $\frac{P}{P_y} = 0$.

b. Compute moment by Case I for $\sigma \leq \sigma_y$.

c. If $\sigma > \sigma_y$, it belongs to Case II. Compute moment by Case II for $R_a \leq 1$.

d. If $R_a > 1$, it belongs to Case III. Compute moment by Case III.

e. Repeat the process for many other values of $\frac{\phi}{\phi_y}$ until the complete M-Θ curve is determined for zero axial thrust (P=0). Repeat the whole process for other axial thrust values until enough information is obtained for a complete set of M-P-ϕ curves.

f. The M-P-ϕ curves are plotted in Fig. 3-1.

A flow diagram and the Fortran program for finding the M-P-ϕ curves are given in Appendix B.

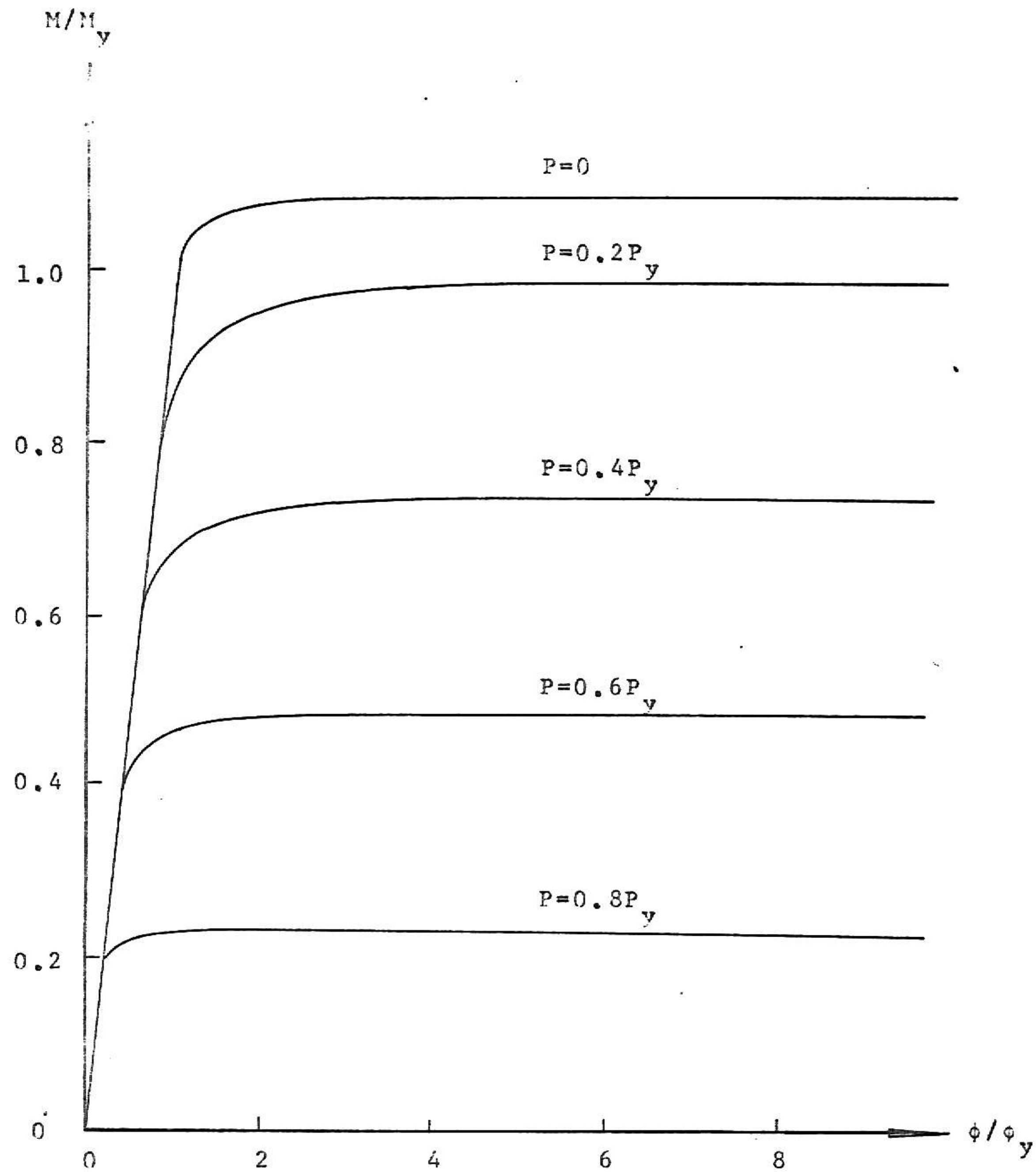


Fig. 3-1. Moment-thrust-curvature diagram for section 10 WF 39

IV. DETERMINATION OF THE ULTIMATE MOMENT CARRYING CAPACITY OF A GIVEN MEMBER

1. Procedure for Determination of the Ultimate Strength by Newmark's Numerical Integration Method

The steps used in determining each of the interaction curves are as follows:

Given loading condition, slenderness ratio and constant axial thrust value for the 10 W= 39 section.

a. Assume an end moment, M_o which is greater than the initial yield value.

b. Assume a possible deflection configuration as a first approximation.

c. Compute the moment values at ten equally spaced stations along the length of the member ($M_x = M_o + P \cdot y$) and obtain the curvature values from the M-P-ϕ diagram. (Fig. 3-1)

d. Correct the assumed deflections based on the values obtained from this numerical integration and repeat step c.

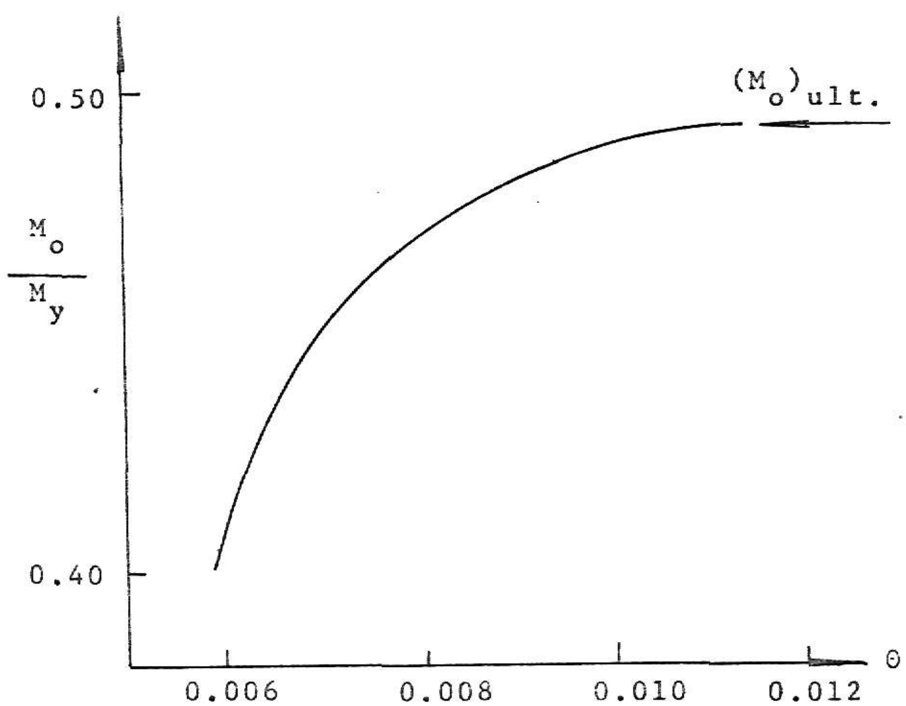
e. Repeat step d until desired accuracy is obtained. (0.001 in. is used in the following example.)

f. Determine the end rotation for the final deflection values of step e. If it is assumed that the deflection curve of the member within the three end segments can be represented by a parabola, then the end slope can be expressed in terms of the known deflection as

$\theta = \dfrac{4D_2 - D_3}{2\lambda}$ in which D_2 is the deflection at the first station away from the applied moment end of the member, D_3 is the deflection at the second station away from the applied moment end of the member, and λ is the grid spacing. (Assumed to be L/10 for the case considered.)

g. Assume greater values of the end moment M_o and repeat the same process as outlined previously. If a M_o greater than or equal to $M_{critical}$ is assumed, the numerical integration process diverges.

h. Plot M_o versus θ from step g and determine the maximum value of M_o from the resulting curve. (See Fig. 4-1)



M_o/M_y	θ
0.400	0.00590
0.410	0.00605
0.420	0.00620
0.430	0.00638
0.440	0.00671
0.450	0.00706
0.460	0.00750
0.470	0.00814
0.480	0.00909
0.490	0.01061
0.492	0.01116

Fig. 4-1. Typical moment-end slope curve

An example illustrating the determination of moments and end-slopes is given in Appendix C. A typical moment versus end rotation curve is given in Fig. 4-1. A flow diagram and the Fortran program for finding moment and end-slope curves are presented in Appendix D.

From the above diagram, $(\frac{M_o}{M_y})_{critical}=0.492$. Therefore, the ultimate moment carrying capacity M_u is 745.4 kip-in.

2. M-Θ curves for $\frac{P}{P_y} = 0.6$

The above mentioned Newmark's numerical integration procedure is well suited for determining the ultimate moment of a beam-column, but it furnishes only the stable branch of the M-Θ curves. In many applications, it is important to know also the descending (unstable) branch of this curves. However, this problem is beyond the scope of this report. By repeating Newmark's numerical integration procedure for different values of slenderness ratios, the M-Θ curves can be constructed as shown in Fig. 4-2.

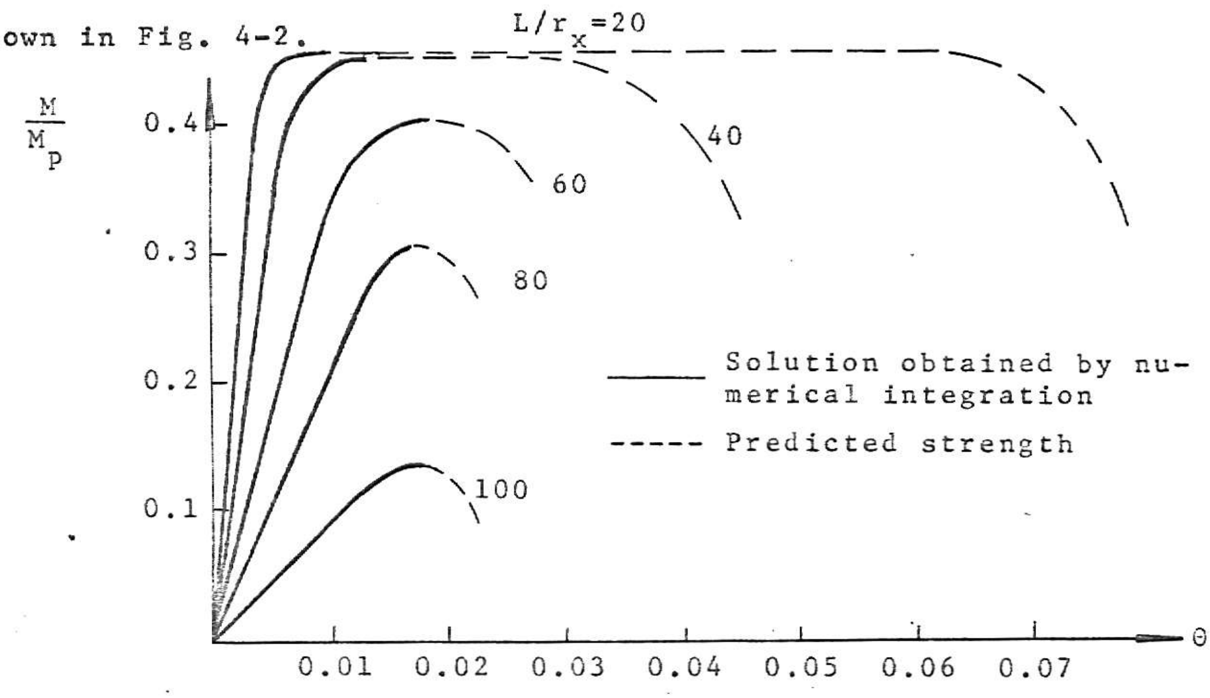


Fig. 4-2. Family of M-Θ curves for $P/P_y=0.6$

3. Ultimate Strength Interaction Curves (M-P-L curves)

The results of the ultimate strength calculations for beam-columns are represented best in the form of P-M-L interaction curves. A set of these is given in Fig. 4-3. These particular interaction curves are for the case of axial thrust plus an end-moment applied at one end of the member causing single-curvature deformation about the strong axis of an 10 WF 39 member. Each curve in Fig. 4-3 shows the relationship between P (nondimensionalized by P_y) and M (nondimensionalized by M_p) for a given slenderness ratio L/r. The particular curves in Fig. 4-3 do not include the effect of residual stress.

The following observations can be made about these interaction curves:

1. When P=0, the member is a beam and can support a moment equal to M_p.

2. When M=0, the member is a column which is able to carry a load equal to its own critical load.

3. The situations when P=0 and M=0 are two extreme cases. Between these extremes, beam-column action takes place.

4. For a given value of P, the member having L/r=0 can carry considerably more moment than the member having L/r=100. Thus, short members are stronger than long members.

5. Up to L/r=60 the interaction curves are nearly straight lines. For higher slenderness ratios the curves sag

downward, thus showing the larger influence of secondary moments due to deflection.

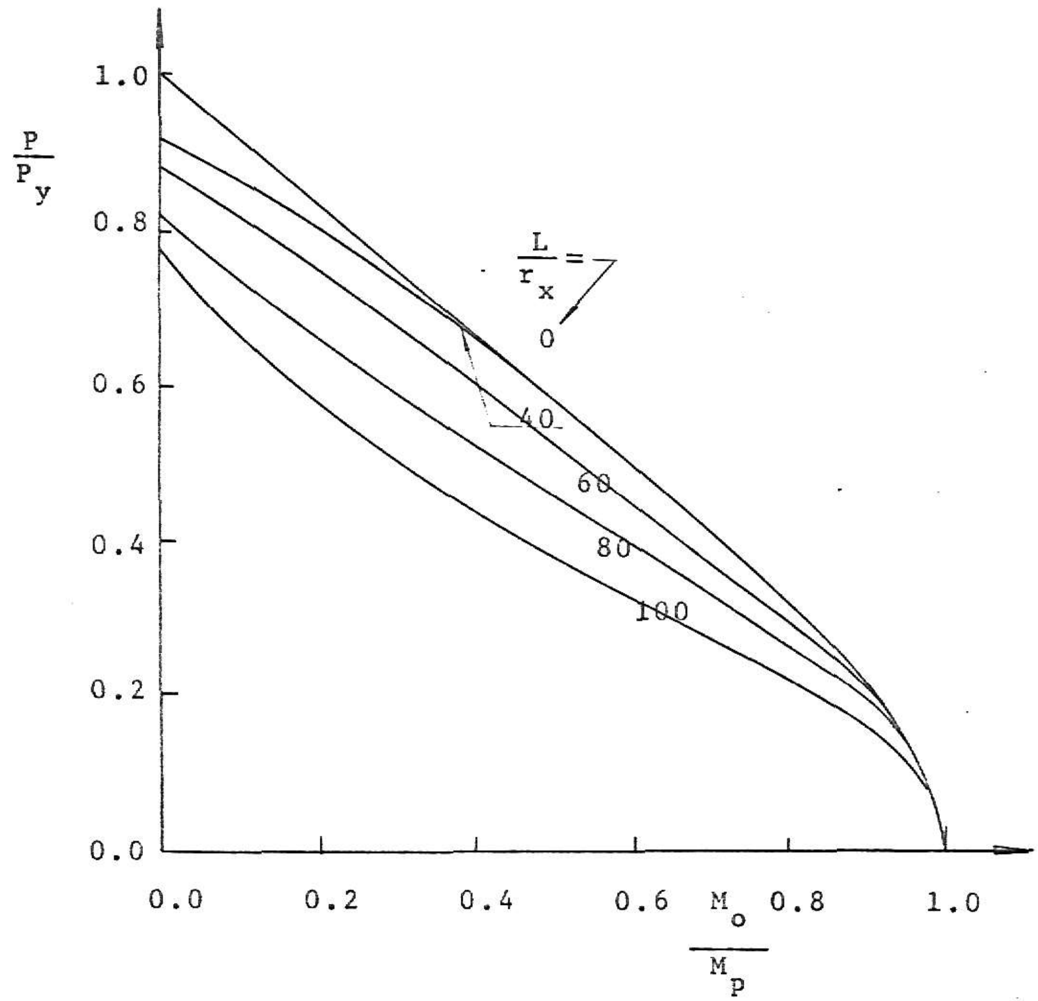


Fig. 4-3. Ultimate strength interaction curves for one end-moment only

V. APPROXIMATE ULTIMATE STRENGTH INTERACTION CURVES - AISC SPECIFICATION

In the AISC Specification (Ref. 3) three formulas referring to three distinct loading cases are given for the plastic design of beam-columns. These formulas were developed by fitting the curves into cubic and quadratic equations. All of the limitations of the original curves are, therefore, presented in these approximations. In general, the range of application was chosen as $0 \leq L/r \leq 120$ and $0 \leq P_o/P_y \leq 0.6$. It was considered that these covered the major range of pratical applications. The details of these formulas, together with their range of applicability, are given in Appendix E.

Pin-ended column subject to axial thrust plus an end-moment applied only at one end of the member will be discussed in this chapter. Assume an equation of the form

$$M_o/M_p=B-G(P_o/P_y)$$

in which B and G are assumed to be functions of the slenderness ratio only; the coefficients for an A7 steel wide-flange member are found from the specification (Ref. 3) to be

$$G=1.11+(L/r)/190-(L/r)^2/9000+(L/r)^3/720000$$

$$B=1.133+(L/r)/3080+(L/r)^2/185000$$

For A36 steel, the values of L/r which are used in the above mentioned equations are modified by the corrective term $\sqrt{36/33}$ (the square root of the ratio of the yield strengths of A36 and A7 steels). It should be noted that when the AISC beam-column formulas predict a value of M_o/M_p greater than 1.0

(that is, for small values of P_o/P_y), M_o/M_p=1.0 should be used. Meanwhile, these formulas are based on failure due to excessive bending in the plane of the applied moment. In plastic design failure due to local and lateral-torsional buckling is not permitted, because the member must deliver a considerable inelastic rotation in addition to being strong enough to support the loads. Both local and lateral-torsional buckling tend to reduce rotation capacity. For this reason, the geometry of the cross section of the member must fulfill certain minimum thickness provisions to prevent local buckling, and lateral bracing must also be provided.

A flow diagram and the Fortran program for finding ultimate strength interaction curves are provided in Appendix F. Approximate ultimate strength interaction curves are given in the next page. (See Fig. 5-1)

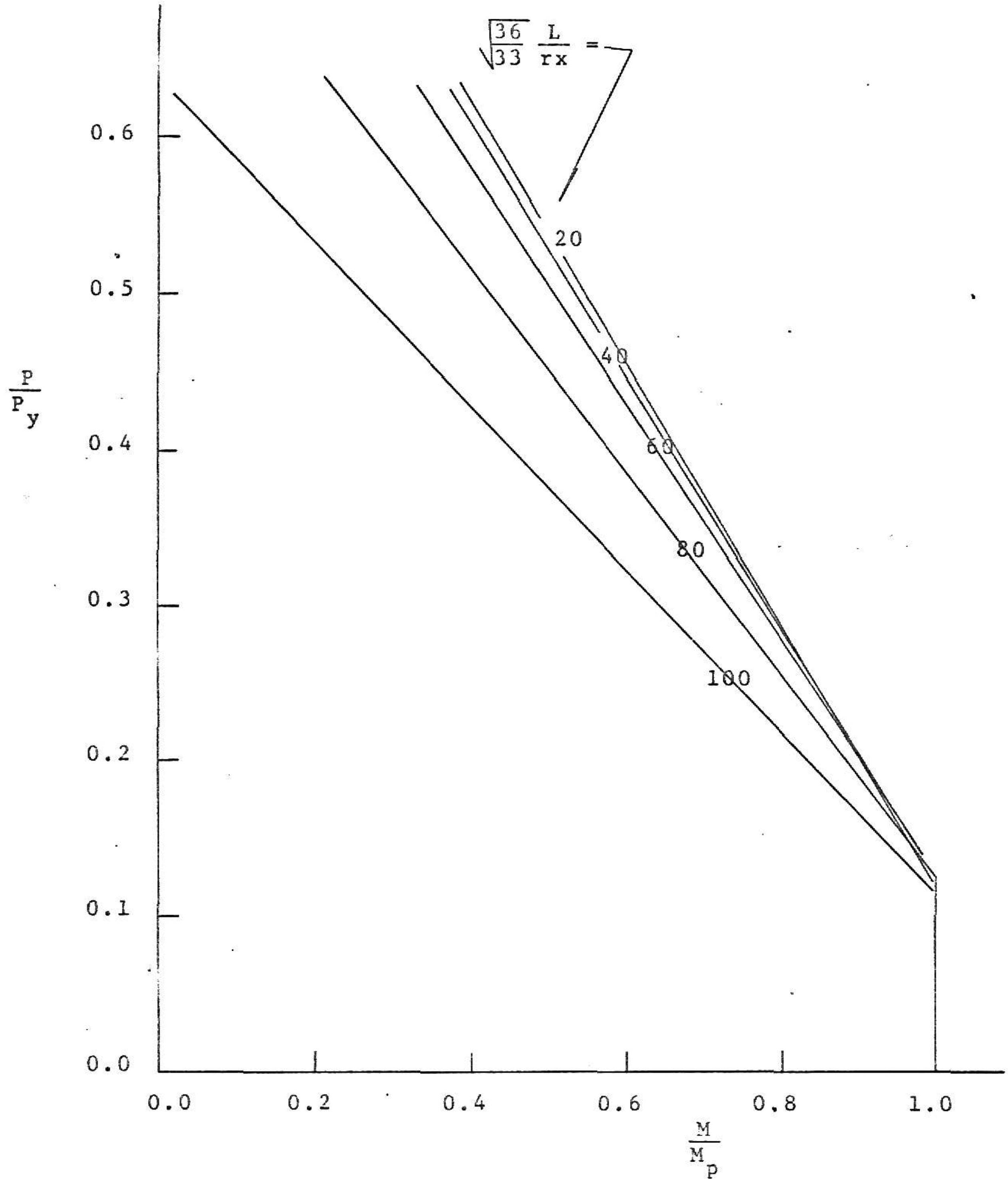


Fig. 5-1. Ultimate strength interaction curves from the AISC Specification

VI. DISCUSSION

1. Comparison between "Exact" and "Approximate" Interaction Curves

The agreement between the approximate interaction curves (Fig. 5-1) and the relationship determined numerically (Fig. 4-3) is shown in Fig. 6-1. The solid lines of Fig. 6-1 are obtained from Newmark's numerical integration procedure and the dashed lines are predicted by the AISC plastic design formulas.

2. Application of M-Θ Curves

A simple application of M-Θ curves is given in the following example where a restrained beam-column is analyzed. The structural system consists of a beam-column which is pinned at one end and restrained by a beam at the other end. An axial force of $0.6P_y$ (248 Kips) is applied to the beam-column and the upper joint is subjected to an external moment M_o. Equilibrium at this joint requires that the sum of the end-moments resisted by the beam and the beam-column be equal to M_o, and compatibility requires that the end slopes of the two members be equal to each other. The M-Θ curve of the beam-column is taken from Fig. 4-2, and the M-Θ curve for the beam is assumed to be ideally elastic-plastic.

Example 7-1 Determine the ultimate moment M_o for the restrained column. Bending is about the strong axis of the members.

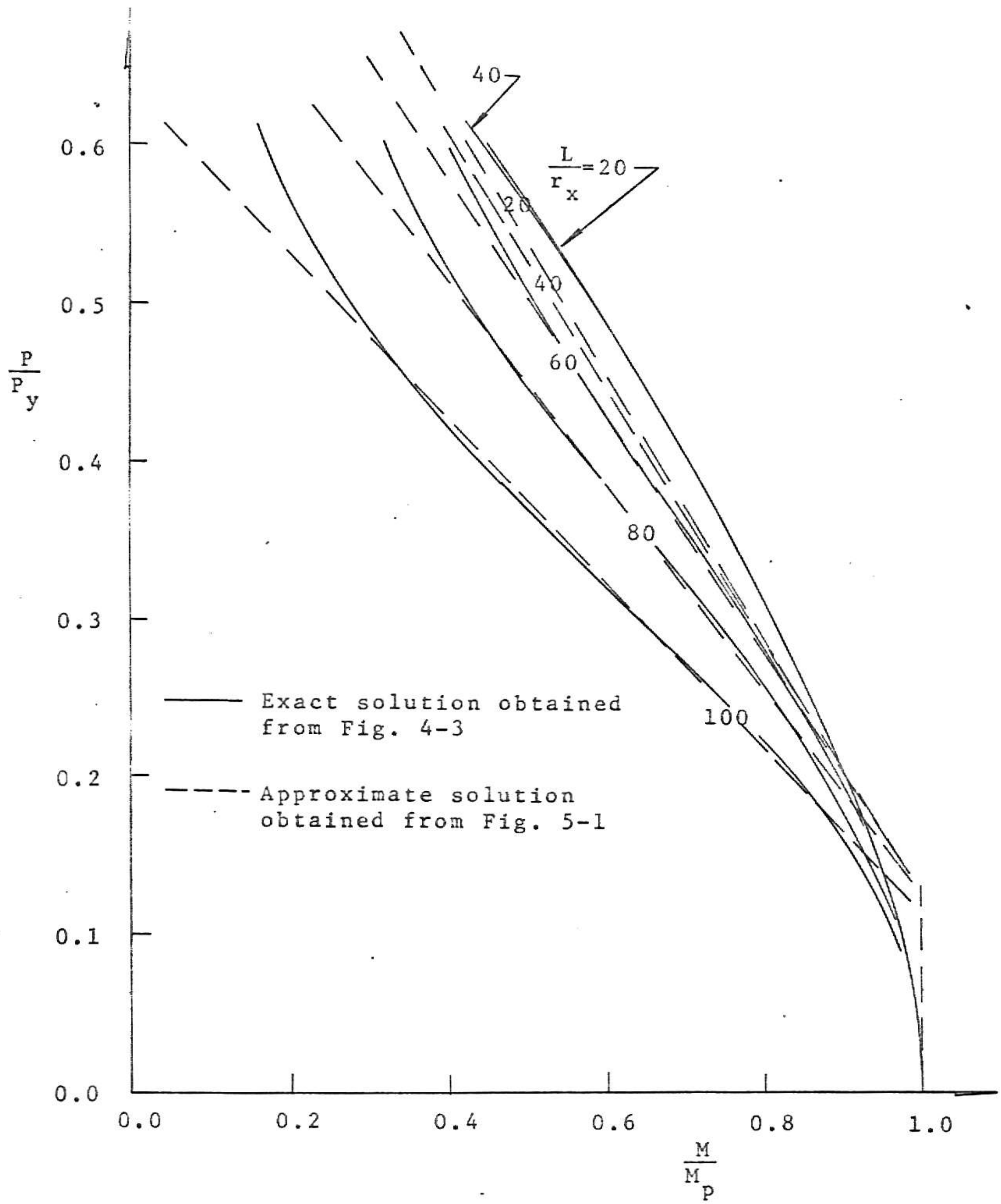
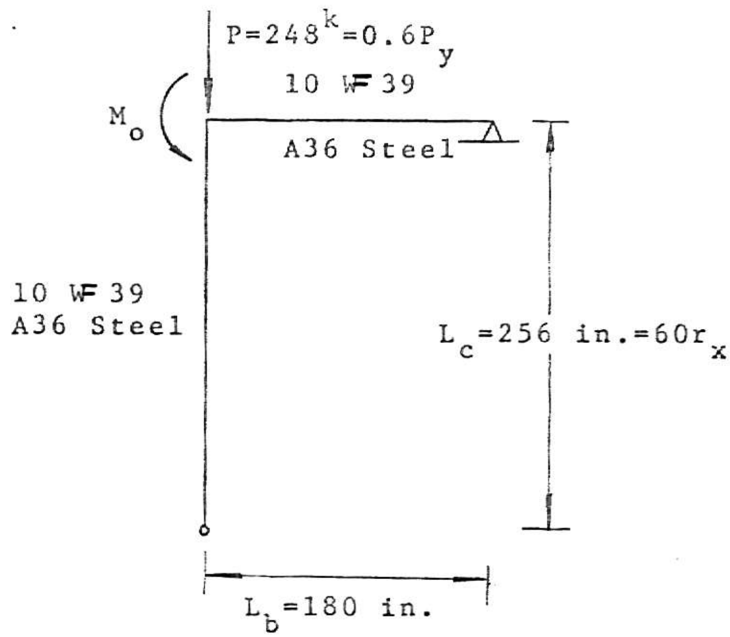
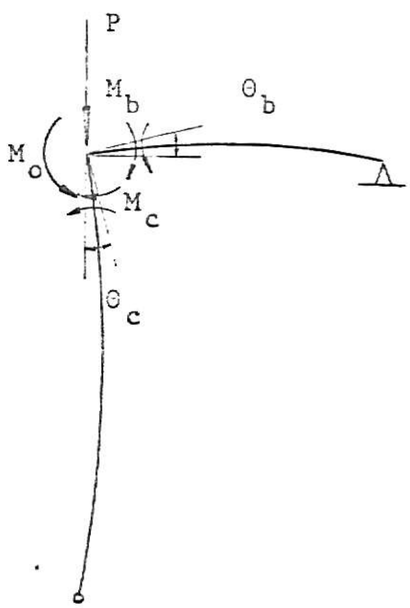


Fig. 6-1. Comparison between "Exact" and "Approximate" interaction curves



Solution:



<u>Equilibrium Condition</u>:

$M_b+M_c=M_o$

<u>Compatibility Condition</u>:

$\theta_b=\theta_c$

<u>M_b - θ_b relationship</u>:

Assumed ideal elastic-plastic relationship; that is, beam is elastic until $M_b=M_p$, and in the plastic range $M_b=M_p$, for any value of θ_b.

In the elastic range: $M_b = \dfrac{3EI_b\theta_b}{L_b}$

At the plastic limit: $M_b = M_p = \dfrac{3EI_b\theta_p}{I_c}$

For A36 steel, 10 WF 39 $Z=47.0\ in.^3$; $F_y=36$ ksi; $E=30000$ksi

$$M_p=ZF_y=47\times36=1692.0\text{kip-in.}$$

$$I_b=209.7\ in.^4$$

$$(M_b)_{elastic}=\frac{3EI_b\Theta_b}{L_b}=\frac{3X30000X209.8\Theta_b}{180}$$

$$=104900\Theta_b$$

$$\Theta_p=\frac{M_pL_b}{3EI_b}=\frac{1692X180}{3X30000X209.7}=0.0161\text{ rad.}$$

<u>$M_c - \Theta_c$ relationship:</u>

For $L/r_x=60$ and $P/P_y=0.6$, the M_c-Θ_c relationship is given in curve form in Fig. 4-2.

In Fig. 6-2, the curves for the beam and the beam-column are shown separately. The upper curve in this sketch is the M-Θ curve of the complete restrained structure; each point on this curve is constructed by adding the moments carried by the beam and the beam-column at the corresponding rotation Θ. It seems that in the beginning of the load application, the major share of the moment is resisted by the beam-column. However, as the external moment is increased beyond the capacity of the beam-column, more and more of the moment is resisted by the beam.

The above-mentioned example illustrates how members in a structure assist each other in carrying the load, and it also shows that not only the ultimate strength but also the deformation behavior of each element is of importance in determining the ultimate strength of the whole structure.

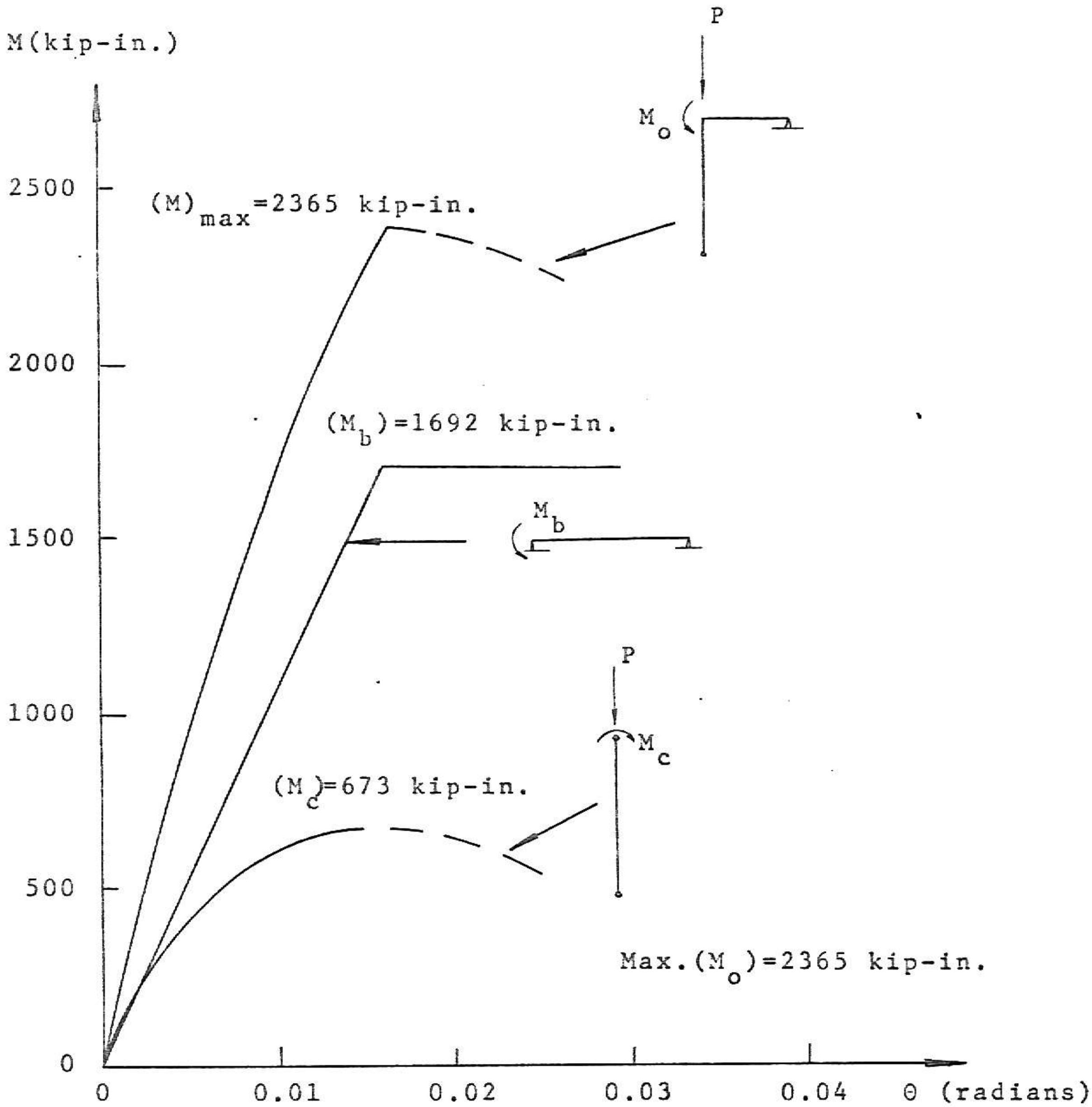


Fig. 6-2. The curves for the beam and the beam-column

3. Factors in Determining the Ultimate Strength of Wide-Flange Beam-Columns

The ultimate strength of wide-flange beam-columns will be affected by any one of the following factors:

a. Material of the section. (Yielding stress, stress and strain relationship.)

b. Properties of section.

c. Local buckling.

d. Lateral-torsional buckling. (Failure is in the plane of the applied moment(s) and that of the web of the beam-column.)

e. Slenderness ratio. (L/r)

f. Axial thrust.

g. End restraints of the beam-columns.

h. Residual stress (Not considered in this report.)

According to the previous chapters we know that each one of the factors listed above plays an important role in determining the ultimate moment carrying capacity of wide-flange beam-columns.

VII. CONCLUSIONS

Solutions to the problem of the determination of the maximum moment carrying capacity of a simply supported wide-flange beam-column (10 WF 39) loaded by axial force and a single end moment applied in the plane of the web have been presented. In obtaining these solutions, it has been assumed that the member in question will fail by excessive bending in the plane of the applied moment. Failure due to lateral-torsional or local buckling has not been considered. The resulting interaction curves (Fig. 4-3) do not include the influence of a typical cooling-type residual stress pattern. Another set of elastic-plastic bending equations including residual stresses are given in Ref. 4.

An important assumption made in the derivations of Appendix A is that the Bernoulli-Navier hypothesis (bending strain is proportional to the distance from the neutral axis) can be extended to include the case of plastic deformations. Actually this assumption is an idealization for steel members. As has been described previously, the only requirements are that the strain be proportional to the distance from the neutral axis in the elastic range and the stress be equal to the yield-point stress in the plastic range. Thus, the curvature at any section is a function of the part of the cross section which remains elastic. If the strain-hardening range is considered, it would be necessary to make another assumption with regard to strain distribution in the inelastic range.

The solution to the problem considered in this report was obtained using Newmark's numerical integration method, which can easily be applied to loading conditions other than the one considered. The determination of the descending (unstable) branch of the M-θ Curves (Fig. 4-2) is beyond the scope of this report. However, it can be obtained by either the stepwise integration method (Ref. 1) or the nomographical method (Ref. 2).

Fig. 6-1 shows the agreement between the "exact" interaction curves and "approximate" interaction curves. The approximate formulas (AISC Specification, Ref. 3) are quite conservative for L/r=20 and L/r=40. Agreement for other slenderness ratios is quite close when P/P_y is less than 0.5. Otherwise, the approximate solutions are conservative. When P/P_y is smaller than 0.15, the AISC Specification formulas for any slenderness ratio gives also provide a good approximate solution.

A set of charts (M-P-φ Curves, M-θ Curves and M-P-L Curves) is required for each type of material and cross-sectional shape. Therefore, the results obtained for a 10 WF 39 section of A36 steel are only approximately applicable to other sections and materials.

Further work is currently underway by other investigators to include the influence of lateral-torsional instability in the strength calculations, and preliminary results of these studies indicate that good correlation can be achieved when this type of failure is considered.

ACKNOWLEDGMENTS

The author would like to express his sincere gratitude to his major advisor, Dr. P. B. Cooper, for his constant encouragement, advice and valuable suggestions on this report. Meanwhile, the author is also grateful to Dr. J. B. Blackburn, Professor V. H. Rosebraugh and Dr. S. T. Parker for being members of the author's advisory committee.

NOMENCLATURE

A	Area of cross-section (in^2);
B, G, J, K	Non-dimensional constants;
D	Deflection (in.);
D_2	Deflection at the first station away from the applied moment end of the member;
D_3	Deflection at the second station away from the applied moment end of the member;
E	Young's Modulus of Elasticity (30000ksi);
E_{st}	Strain-hardening modulus;
F_1	Resultant of stress diagram 1 (kips);
F_2	Resultant of stress diagram 2;
I	Moment of inertia (in^4);
L	Length of member (in.);
L/r_x	Slenderness ratio;
M	Bending moment (kip-in.);
M_o	Applied moment at the end of member;
M_p	Fully plastic moment value under pure moment;
M_u	Ultimate moment carrying capacity;
M_y	Initial yield moment value under pure moment;
P	Axial thrust (kips);
P_o	Axial thrust at maximum load capacity for beam-columns;
P_y	Axial load corresponding to compressive yield stress over entire section;
R_a	Non-dimensional ratio ($R_a=\sigma_a/\sigma_y$);

S	Section modulus about the strong axis (in^3);
Y_{cc}	Yield penetration in top of member (in.);
Y_{tt}	Yield penetration in bottom of member;
Z	Plastic modulus about the strong axis (in^3);
b	Flange width (in.);
d	Depth of section (in.);
f	Shape factor ($f=Z/S$);
r_x	Radius of gyration about the strong axis;
t	Thickness of flange;
w	Thickness of web;
x	Distance along the member;
y	Lateral deflection
y_2	Distance between centroidal axis of section and the force F_2;
α	Non-dimensional ratio ($\alpha=Y_{cc}/d$);
β	Non-dimensional ratio ($\beta=Y_{tt}/d$);
β_1	Modified factor;
ε	Strain (in./in.);
ε_c	Strain in compression flange;
ε_t	Strain in tension flange;
Θ	End rotation (radians);
λ	Length of equally spaced segments of total member length;
σ	Stress ($kips/in^2$);
σ_a	Bottom fiber stress of section;
σ_y	Yield stress ($_y$=36ksi);

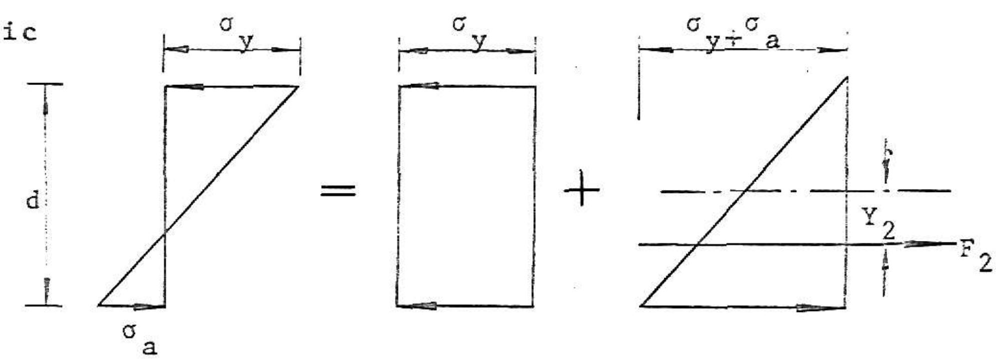
ϕ	Curvature (radians per inch);
ϕ_y	Curvature corresponding to first yield in flexure;
WF	Wide-flange;

APPENDIX A

Derivation of Elastic and Plastic Bending Equations

Wide-Flange Bending - Strong Axis (No residual stresses)

Case I - Elastic

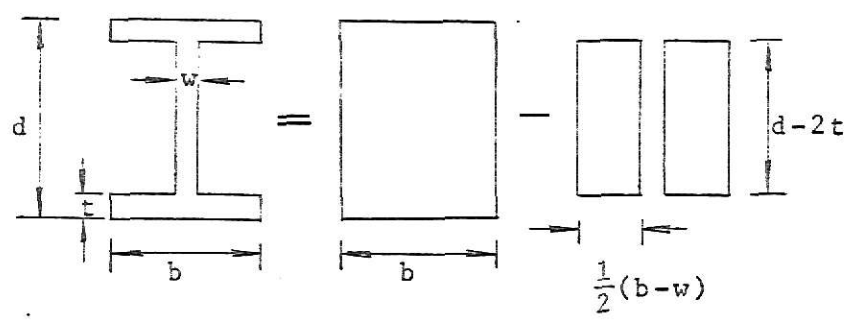


Limits

$-\sigma_y \le \sigma_a \le \sigma_y$

$R_a = \frac{\sigma_a}{\sigma_y}$

$-1 \le R_a \le 1$



AXIAL THRUST

$P = F_1 + F_2$

$= \sigma_y (2bt + w(d-2t)) - \frac{(\sigma_y + \sigma_a)}{2} (2bt + w(d-2t))$

$$\frac{P}{\sigma_y} = \frac{1+R_a}{2} (2bt + w(d-2t))$$

MOMENT

$$y_2 = \frac{\frac{1}{2}(\sigma_y + \sigma_a)(bd)\frac{d}{6} - \frac{1}{2}(\sigma_y + \sigma_a)\frac{1}{d}(d-2t)^3 (b-w)\frac{1}{6}}{F_2}$$

$M = F_2 \cdot y_2$

$$= \frac{1}{2}(\sigma_y + \sigma_a)(bd)\frac{d}{6} - \frac{1}{2}(\sigma_y + \sigma_a)\frac{(d-2t)^3}{d}(b-w)\frac{1}{6}$$

$$= \frac{(\sigma_y + \sigma_a)}{12d}\left(bd^3 - (d-2t)^3(b-w)\right)$$

$$\frac{M}{\sigma_y} = \frac{(1+R_a)}{12d}\left(2bt(3d^2 - 6dt + 4t^2) + w(d-2t)^3\right)$$

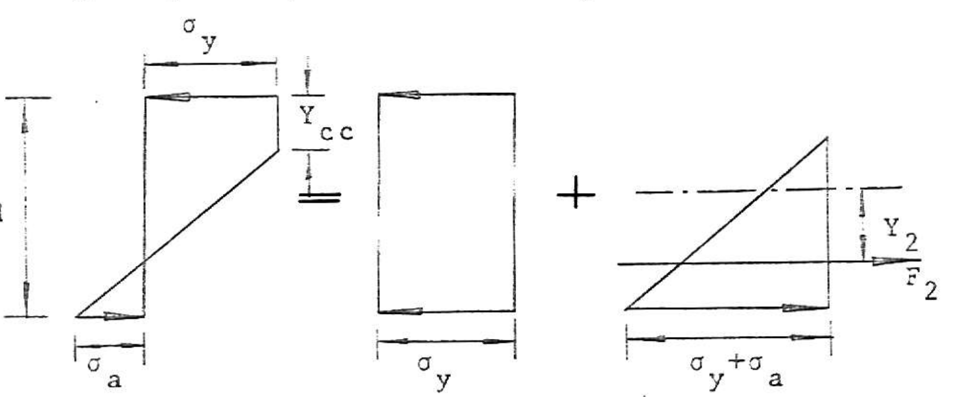
CURVATURE

$$\phi = \frac{(\sigma_y + \sigma_a)}{Ed}$$

$$\phi_y = \frac{2\sigma_y}{Ed}$$

$$\frac{\phi}{\phi_y} = \frac{1+R_a}{2}$$

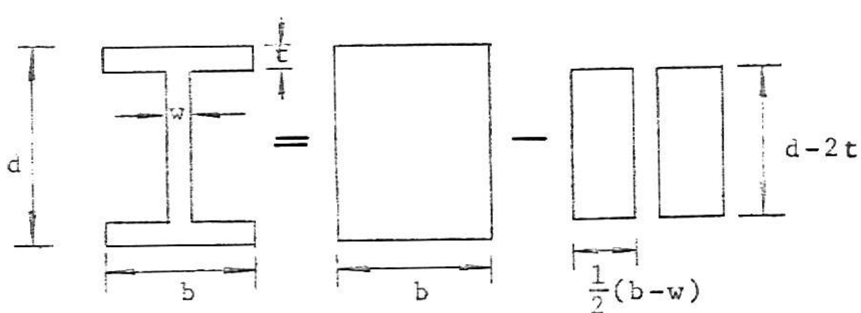
Case II - Top side gets yield penetration only



LIMITS

$$R_a = \frac{\sigma_a}{\sigma_y}$$

$$\alpha = \frac{Y_{cc}}{d}$$



$$\frac{t}{d} \leq \alpha \leq \left(1 - \frac{t}{d}\right)$$

$$-1 \leq R_a \leq 1$$

AXIAL THRUST

$$P=F_1+F_2$$

$$=\sigma_y(2bt+w(d-2t))-(\sigma_y+\sigma_a)(\tfrac{1}{2}(d-Y_{cc})b-\frac{(d-Y_{cc}-t)^2}{2(d-Ycc)}(b-w))$$

$$=\sigma_y(2bt+w(d-2t))-\frac{(\sigma_y+\sigma_a)}{2(d-Ycc)}((d-Ycc)b-(d-Ycc-t)^2(b-w))$$

$$=\sigma_y(2bt+w(d-2t))-\frac{(\sigma_y+\sigma_a)}{2(d-Ycc)}(2(d-Ycc)bt-t^2b+(d-Ycc-t)^2w)$$

$$=\sigma_y(2bt+w(d-2t))-\frac{(\sigma_y+\sigma_a)}{2(d-Ycc)}(bt^2(-1)+w(d-Ycc)^2+wt^2)$$

$$-\tfrac{1}{2}(\sigma_y+\sigma_a)(2b-2w)t$$

$$\frac{P}{\sigma_y} = (1-R_a)(bt-wt)+\frac{1+Ra}{2d(1-\alpha)}(bt^2-wd^2(1-\alpha)^2-wt^2)+wd$$

MOMENT

$$y_2=\frac{(\sigma_y+\sigma_a)\frac{1}{2}(d-Ycc)b(\frac{d+2Ycc}{6})-\frac{(\sigma_y+\sigma_a)(d-Ycc-t)^2}{2(d-Ycc)}(b-w)\frac{(d+2Ycc-4t)}{6}}{F_2}$$

$$M=F_2\cdot y_2$$

$$=\frac{(\sigma_y+\sigma_a)}{12(d-Ycc)}(b(d-Ycc)^2(d+2Ycc)-(d-Ycc-t)^2(b-w)(d+2Ycc-4t))$$

$$=\frac{(\sigma_y+\sigma_a)}{12(d-Ycc)}(((d-Ycc-t)^2 w(d+2Ycc-4t))+bt(4d^2+4Ycc^2+4t^2-8dYcc$$

$$+8Ycct-8dt+2d^2+2Y_{cc}d-4Y_{cc}^2-dt-2Y_{cc}t))$$

$$=\frac{(\sigma_y+\sigma_a)}{12(d-Y_{cc})}(w(d(1-\alpha-\frac{t}{d}))^2(d(1+2\alpha)-4t)+bt(6d(1-\alpha)(d-t)-3dt+4t^2))$$

$$\frac{M}{\sigma_y}=\frac{1+R_a}{12(1-\alpha)d}(w(d(1-\alpha-\frac{t}{d}))^2(d(1+2\alpha)-4t)+bt(6d(1-\alpha)(d-t)-3dt+4t^2))$$

CURVATURE

$$\phi=\frac{\sigma_y+\sigma_a}{E(d-Y_{cc})}$$

$$\phi_y=\frac{2\sigma_y}{Ed}$$

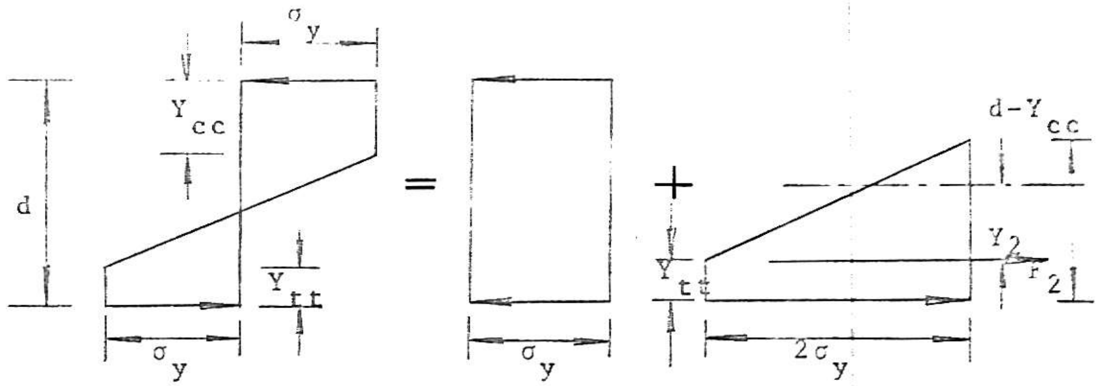
$$\frac{\phi}{\phi y}=\frac{1+Ra}{2(1-\alpha)}$$

Case III - Both sides have started to yield

LIMITS

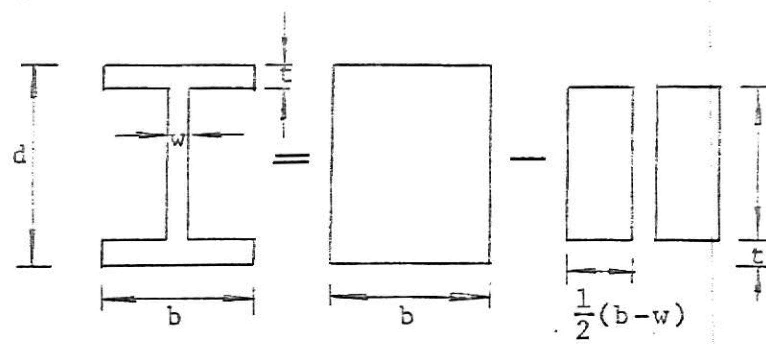
$\alpha = \frac{Y_{cc}}{d}$

$\beta = \frac{Y_{tt}}{d}$



$\frac{t}{d} \leq \alpha \leq (1 - \frac{t}{d})$

$\frac{t}{d} \leq \beta \leq \alpha$



AXIAL THRUST

$$P = F_1 + F_2$$

$$= \sigma_y(2bt + w(d-2t)) - 2\sigma_y bt - 2\sigma_y w(Y_{tt} - t) - 2\sigma_y w\frac{1}{2}(d - Y_{cc} - Y_{tt})$$

$$= \sigma_y(w(d-2t) - 2wY_{tt} + 2wt - w(d - Y_{cc} - Y_{tt}))$$

$$\frac{P}{\sigma_y} = wd(\alpha - \beta)$$

MOMENT

$$y_2 = \frac{2\sigma_y bt(\frac{d}{t} - \frac{t}{2}) + 2\ {}_y(Y_{tt} - t)w(\frac{d}{2} - t - \frac{Y_{tt}}{2} + \frac{t}{2}) + 2\ {}_y w\frac{(d - Y_{tt} - Y_{cc})(d - 4Y_{tt} - Y_{cc})}{12}}{F_2}$$

$$M = F_2 \cdot y_2$$

$$=\sigma_y\left(bt(d-t)+\frac{wd^2}{6}\left(6\left(\beta-\frac{t}{d}\right)\left(1-\alpha-\frac{t}{d}\right)+(1-\alpha-\beta)(1+2\alpha-4\beta)\right)\right)$$

$$=\sigma_y\left(bt(d-t)+\frac{wd^2}{6}\left(6\left(\beta-\frac{t}{d}\right)\left(1-\alpha-\frac{t}{d}\right)+(1-\alpha-\beta)(1+2\alpha-4\beta)\right)\right)$$

$$\frac{M}{\sigma_y}=bt(d-t)+\frac{wd^2}{6}\left(6\left(\beta-\frac{t}{d}\right)\left(1-\beta-\frac{t}{d}\right)+(1-\alpha-\beta)(1+2\alpha-4\beta)\right)$$

CURVATURE

$$\phi=\frac{2\sigma_y}{E(d-Y_{cc}-Y_{tt})}$$

$$\phi_y=\frac{2\sigma_y}{Ed}$$

$$\frac{\phi}{\phi_y}=\frac{1}{(1-\alpha-\beta)}$$

APPENDIX B

Flow Diagram and Fortran Program for Finding M-P-ϕ Curves

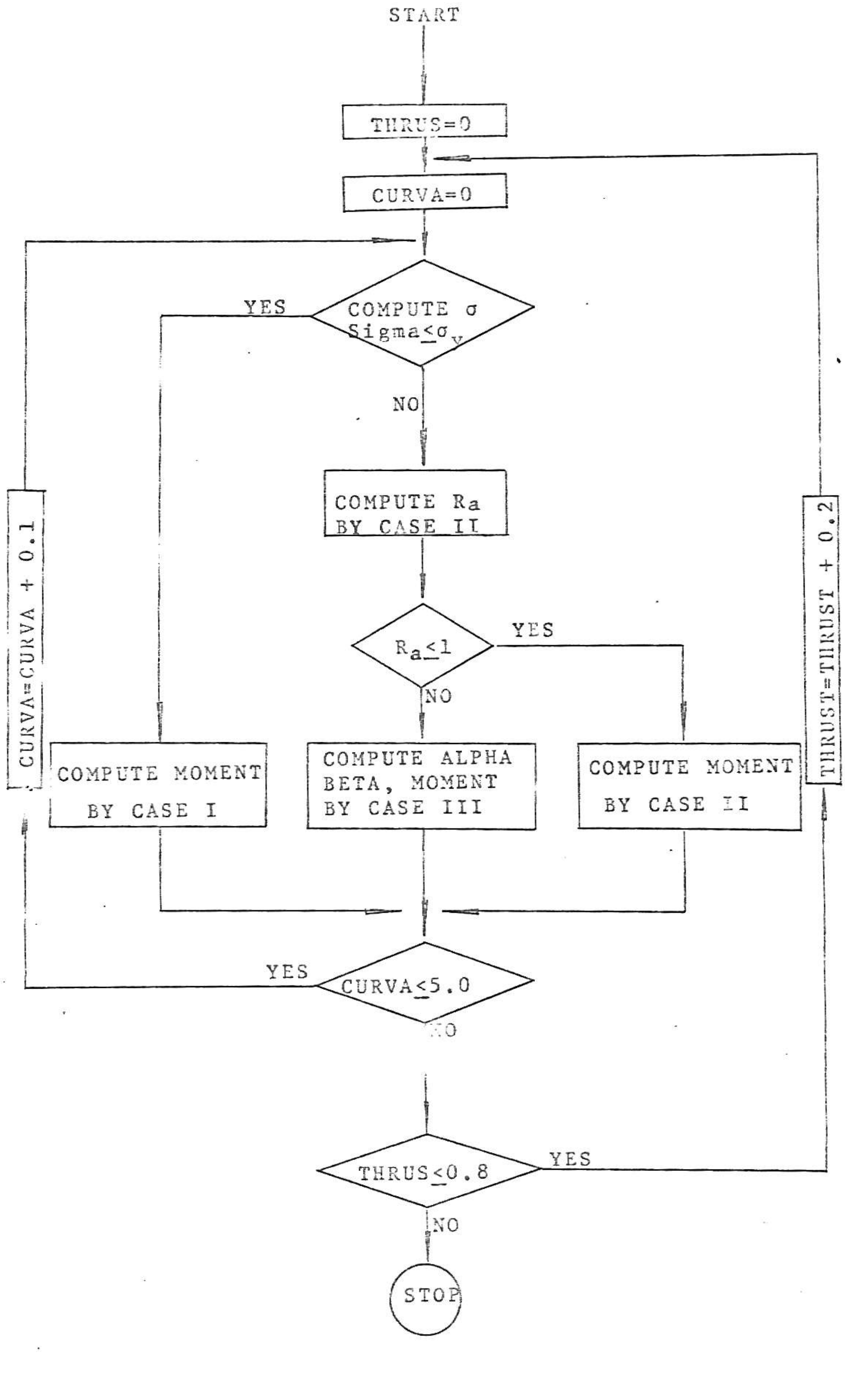
```
C   C   PROGRAM FOR MOMENT-THRUST-CURVATURE CURVES
      1 FORMAT ( 6F10.0 )
        READ 1,A,D,B,T,W,S
        THRUS=0.
        DO 10 I=1,5,1
        CURVA=0.
        DO 11 J=1,51,1
        SIGMA=THRUS+CURVA
        IF(1.-SIGMA) 7,2,2
      2 AMOME=CURVA
        PUNCH 3, THRUS,CURVA,AMOME,SIGMA
      3 FORMAT (9H CASE   I,5X,2F10.1,F10.4,F6.2)
        GO TO 12
      7 Q=14.5316*THRUS*CURVA-9.13*CURVA-0.272*CURVA**2+1.
        RA=(-(2.+5.13*CURVA)+((2.+5.13*CURVA)**2-4.*Q)**0.5)/2.
        ALPHA=(2.*CURVA-1.-RA)/(2.*CURVA)
        IF(1.-RA) 4,5,5
      5 U=W*(D*(1.-ALPHA-T/D))**2
        V=6.*D*(1.-ALPHA)*(D-T)
        AMOME=(CURVA/(6.*D)*U* D*(1.+2.*ALPHA-4.*T/D))/S
        AMOME=AMOME+(CURVA/(6.*D)*B*T*(V-3.*D*T+4.*T**2))/S
        PUNCH 6, THRUS,CURVA,AMOME,RA,ALPHA
      6 FORMAT (9H CASE II,5X,2F10.5,F10.4,6X,2F6.2)
        GO TO 12
      4 ALPHA=(THRUS*A/(W*D)+1.-1./CURVA)/2.
```

```
      BETA=(1.-1./CURVA-THRUS*A/(W*D))/2.
      G=(BETA-T/D)*(1.-BETA-T/D)
      AMOME=(B*T*(D-T)+W*D**2*G)/S
      AMOME=AMOME+(W*D**2/(6.*CURVA)*(1.+2.*ALPHA-4.*BETA))/S
12    PUNCH 8,THRUS,CURVA,AMOME,ALPHA,BETA
 8    FORMAT (9H CASE III,5X,2F10.1,F10.4,12X,2F6.2)
      CURVA=CURVA+0.1
11    CONTINUE
10    THRUS=THRUS+0.2
      STOP
      END
```

Notation of Program for Finding M-P-ϕ Curves

A	Area of section;
ALPHA	α (Non-dimensional ratio);
AMOME	Moment;
B	Flange width;
BETA	β (Non-dimensional ratio);
CURVA	Curvature;
D	Depth of section;
G,Q,U,V	Mathematical computation terms;
RA	R_a;
S	Section modulus about the strong axis;
SIGMA	σ;
T	Thickness of flange;
THRUS	Thrust;
W	Thickness of web;

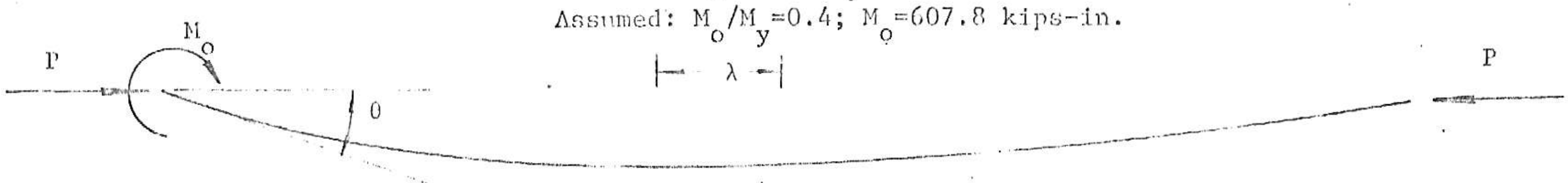
Flow Chart for Finding M-P-ϕ Curves



APPENDIX C Example of Determination of Moment and End-slope Curves

For example: Given: $L/r_x=40$; $P/P_y=0.6$; Section 10 WF 39; L=170.8 in.; λ=17.08in.

Assumed: $M_o/M_y=0.4$; M_o=607.8 kips-in.



	1	2	3	4	5	6	7	8	9	10	11	MF*	Notation
a	607.8	547.1	496.3	425.5	364.7	303.9	243.1	182.2	121.6	60.8	0		Moment due to M_o
b	0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0		Assumed deflection
c	0	24.80	24.80	24.80	24.80	24.80	24.80	24.80	24.80	24.80	0		Moment due to P
d	607.8	571.9	511.0	450.0	389.5	328.7	267.9	207.1	146.6	85.60	0		Total moment (a+c)
e	0.400	0.376	0.336	0.296	0.256	0.216	0.176	0.136	0.096	0.056	0		M_o/M_y
f	0.400	0.376	0.336	0.296	0.256	0.216	0.176	0.136	0.096	0.056	0	ϕ_y	Concen. angle changes**
g	0.400	0.776	1.112	1.408	1.664	1.880	2.056	2.192	2.288	2.344		$\lambda\phi_y$	Slope
h	0	0.400	1.176	2.288	3.696	5.360	7.240	9.296	11.488	13.776	16.12	$\lambda^2\phi_y$	Deflection
i	0	1.612	3.244	4.836	6.448	8.060	9.672	11.248	12.896	14.508	16.12	$\lambda^2\phi_y$	Correction to Deflection
j	0	1.212	2.048	2.548	2.752	2.700	2.432	1.988	1.408	0.732	0	$\lambda^2\phi_y$	Final deflection
k	0	0.0849	0.1435	0.1786	0.1929	0.1893	0.1705	0.1394	0.0987	0.0513	0		Final deflection in in.*
				End of first cycle Second and third cycle Fourth cycle									
a'	0	0.0876	0.1491	0.1864	0.2021	0.1986	0.1789	0.1459	0.1029	0.0532	0		Assumed Deflection****
k'	0	0.0876	0.1491	0.1865	0.2022	0.1987	0.1790	0.1460	0.1030	0.0532	0		Final deflection in inch

*Multiplication Factor

**From Fig 3-1, corresponding to M_o/M_y

***$\phi_y=2F_y/(dE)=0.00024$, $\lambda^2\phi_y=0.07$

****Line k from the third trial = line a' of the fourth trial

THE CORRESPONDING ENDSLOPE $\phi = \frac{4X0.0876 - 0.1491}{2X17.08} = 0.0059$ rad.

APPENDIX D

Flow Diagram and Fortran Program for Determining Moment and End-slope Curves

```
C C  PROGRAM FOR DETERMINATION OF ULTIMATE STRENGTH OF A BEAM-COLUMN
C    FY=36 KSI, E=30000000 PSI,P/PY=0.6, L/RX=40.
     DIMENSION DEFLE(20),CURVA(20),X(20),Y(20)
  21 FORMAT ( 7F10.5 )
     READ 21, (DEFLE(I),I=1,11 )
     P=11.48*36.*0.6
     YIELM=36.*42.2
     SPAN=4.27*40.
     FACT=0.4
     B=DEFLE(5)
     DO 26 K=1,12,1
     ENDMO=FACT*YIELM
     DO 25 M=1,20,1
     S=10.
     DO 22 I=1,11,1
     APPMO=(ENDMO*S/10.+P*DEFLE(I))/YIELM
     IF(0.4-APPMO) 1,2,2
   2 CURVA(I)=APPMO
     GO TO 11
   1 IF(0.43-APPMO) 3,4,4
   4 V=100.*(APPMO-0.4)
     CURVA(I)=0.4+0.0291*V+0.0183*V**2-0.00917*V**3+0.001667*V**4
     GO TO 11
```

```
  3 IF(0.45-APPMO) 5,6,6
  6 V=100.*(APPMO-0.43)
    CURVA(I)=0.542+0.057*V+0.016*V**2
    GO TO 11
  5 IF(APPMO-0.48) 12,8,8
  8 V=100.*(APPMO-0.44)
    CURVA(I)=0.65-0.0125*V+0.1212*V**2-0.0475*V**3+0.00875*V**4
    GO TO 11
 12 IF(APPMO-0,48) 13,13,14
 14 V=1000.*(APPMO-0.48)/3.
    CURVA(I)=1.74+0.171*V+0.3935*V**2-0.1487*V**3+0.04*V**4
    GO TO 11
 13 IF(APPMO-0.4956) 16,16.15
 16 V=10000.*(AM-0.49)/19.
    CURVA(I)=6.3+2.9168*V+0.6998*V**2-0.1166*V**3
 11 S=S-1.
 22 CONTINUE
    X(1)=0.
    Y(1)=0.
    DO 23 I=1,10,1
    X(I+1)=X(I)+CURVA(I)
    Y(I+1)=Y(I)+X(I+1)
 23 CONTINUE
    Z=0.
    DO 24 I=1,11,1
    DEFLE(I)=(Y(11)/10.*Z-Y(I))/14.288
```

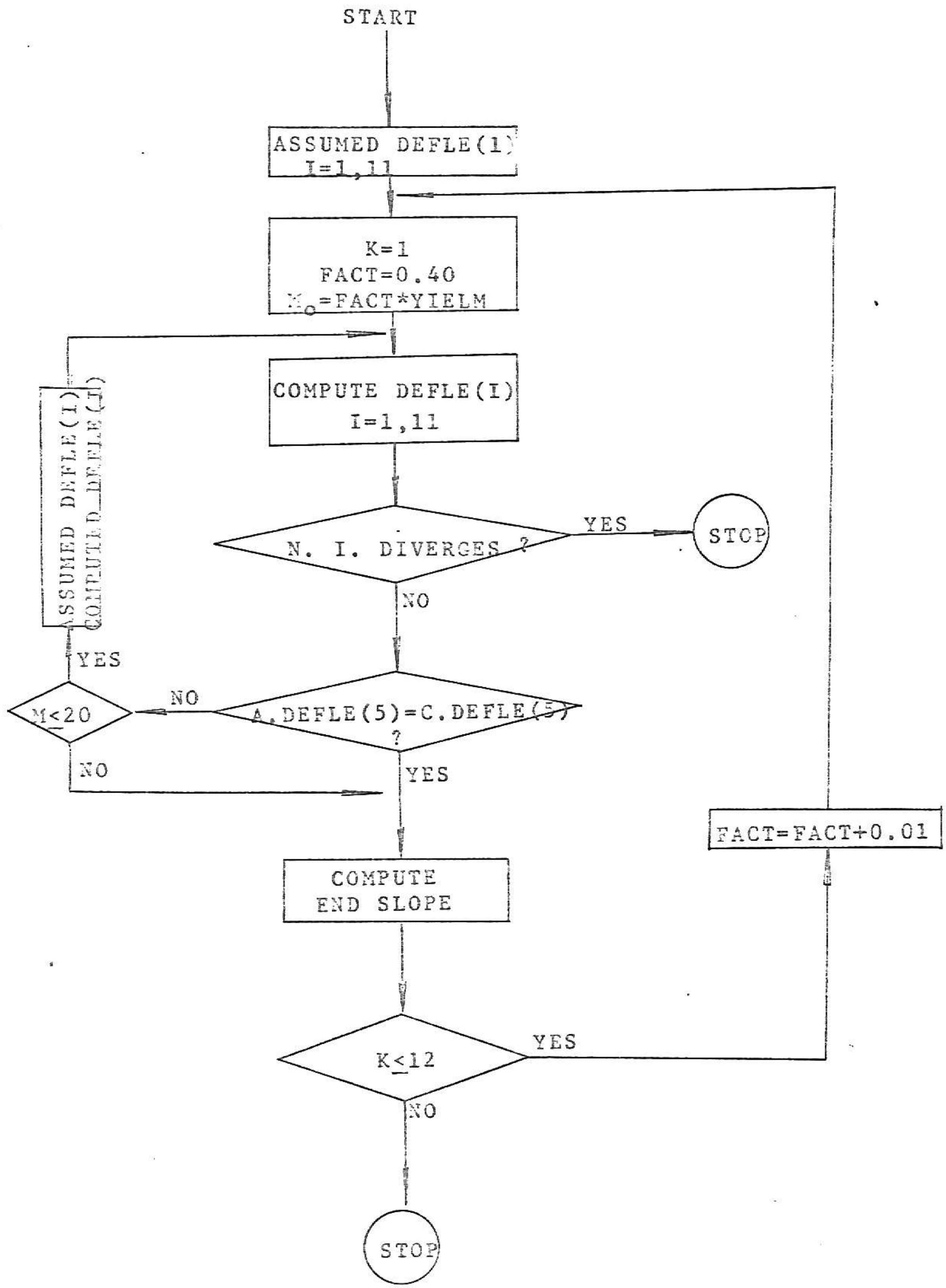
```
      Z=Z+1.
      IF(I-5) 24,28,24
   28 IF(B-DEFLE(5)) 30,29,30
   30 B=DEFLE(5)
   24 CONTINUE
   25 CONTINUE
   29 PUNCH 21,(DEFLE(I),I=1,11)
      ENDSL=(4.*DEFLE(2)-DEFLE(3))/(0.2*SPAN)
      PUNCH 33, FACT,ENDSL
   33 FORMAT (F8.4,5X,18HANGLE OF ROTATION=,F7.5)
      FACT=FACT+0.01
   26 CONTINUE
   15 PUNCH 9,FACT
    9 FORMAT (47H   NUMERICAL INTEGRATION DIVERGES WHEN M/MY = ,F6.4)
      STOP
      END
```

Notation of Program for Determining Moment and End-slope Curves

APPMO	Total moment divided by M_y;
CURVA	Curvature;
DEFLE	Deflection;
ENDMO	End moment;
ENDSL	End slope;
FACT	ENDMO/YIELM
P	Axial thrust;
S,V,Z	Modified factors;
X	Slope;
Y	Deflection in rad.;
YIELM	Initial yield moment value under pure moment;

*Statement 2 through statement 16 consist of an approximate interpolating polynomial for $M-\theta$ curves when $P/P_y=0.6$. The polynomial is obtained by Newton forward-difference formula. (Ref. 5)

Flow Chart for Determining Moment and End-Slope Curves



APPENDIX E

Formulas for Beam-Columns in Plastic Design-AISC Specification (Section 2.3, Ref. 3)

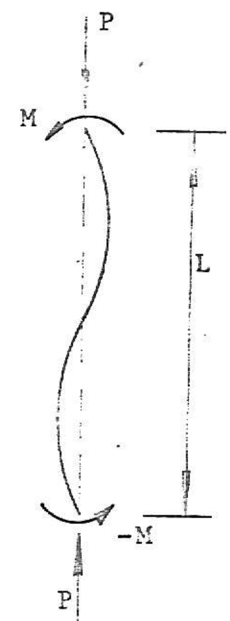
Case I beam-columns are those for which equal end-moments cause double-curvature deformation. The interaction equation is independent of length, and it is used when $0.15P_y < P \leq 0.6P_y$ and $L \leq 100r_x$.

Range of application: $0 \leq \frac{L}{r_x} \leq 100$

$$0 \leq \frac{P}{P_y} \leq 0.6$$

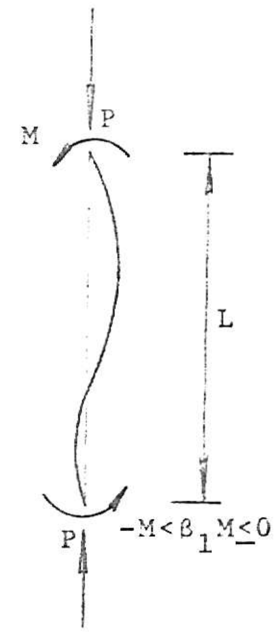
For $0 \leq \frac{P}{P_y} \leq 0.15$, $M = M_y$

For $0.15 < \frac{P}{P_y} \leq 0.6$, $M = 1.18\left(1 - \frac{P}{P_y}\right)M_p$



Case II beam-columns are members for which the smaller of the two end moments ranges in value from zero to just less than the larger end-moment (that is, $-M < \beta_1 M \leq 0$, where β_1 is the ratio of end moments). The moments would, except at the limit where one end moment is zero, cause double-curvature deformation. The interaction equation is an approximation of the case where one end-moment is zero. For moment ratios between -1.0 and 0 the formula is conservative.

Range of application: $0 \leq \frac{L}{r_x} \leq 120$



$0 \leq \frac{P}{P_y} \leq 0.6$

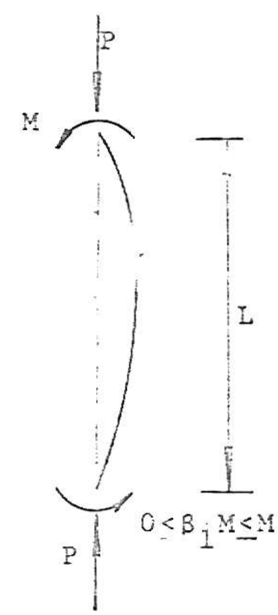
$-1.0 < \beta_1 \leq 0$

$M = 1.00$ or $M = M_p(B - G(\frac{P}{P_y}))$

whichever is smaller

Case III loading is an analytical approximation of the interaction curves for $\beta_1 = +1.0$ (equal end-moments causing single-curvature deformation), but it is to be used for all ratios between 0 (only one end-moment) and +1.0. For all but the case of $\beta_1 = +1.0$ where it is nearly exact, the application of the formula is conservative.

Range of application: $0 \leq \frac{L}{r_x} \leq 120$



$0 \leq \frac{P}{P_y} \leq 0.6$

$0 < \beta_1 \leq +1.0$

$M = M_p(1.0 - K(\frac{P}{P_y}) - J(\frac{P}{P_y})^2)$

Unlike the Case I formula which is independent of the slenderness ratio, the formulas for Case II and Case III are a function of the slenderness ratio. The coefficients B, G, K, and J are defined by the following formulas for A7 steel.

$$B=1.133+\frac{L/r_x}{3080}+\frac{(L/r_x)^2}{185000}$$

$$G=1.11+\frac{L/r_x}{190}-\frac{(L/r_x)^2}{9000}+\frac{(L/r_x)^3}{720000}$$

$$K=0.42+\frac{L/r_x}{70}-\frac{(L/r_x)^2}{29000}+\frac{(L/r_x)^3}{1160000}$$

$$J=0.77-\frac{L/r_x}{60}+\frac{(L/r_x)^2}{8700}+\frac{(L/r_x)^3}{606000}$$

APPENDIX F

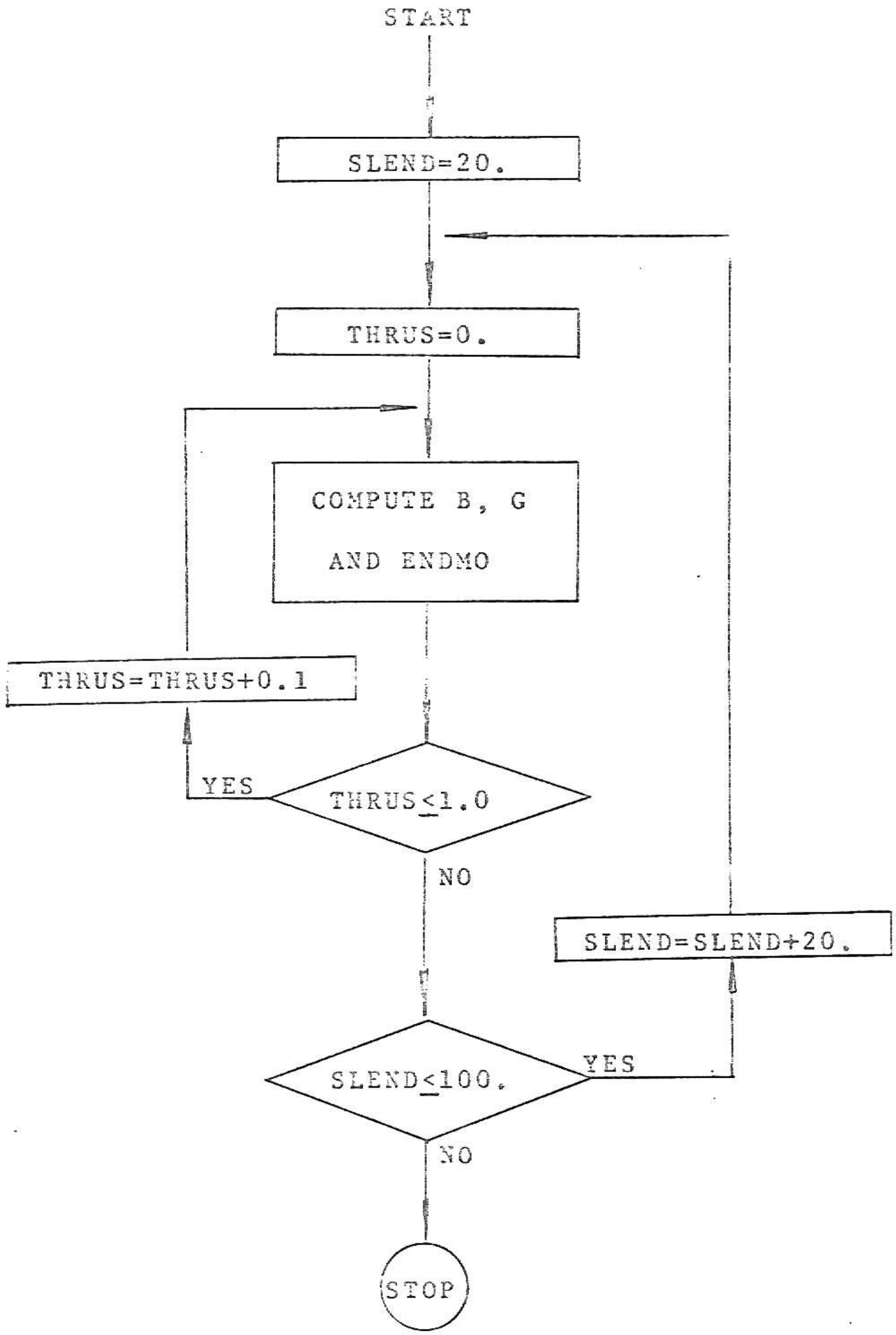
Flow Diagram and Fortran Program for Finding Ultimate Strength Interaction Curves

```
C  C   PROGRAM FOR ULTIMATE STRENGTH INTERACTION CURVES
C      DUE TO AISC PLASTIC DESIGN FORMULAS
    8  FORMAT (5X,F10.0,F10.1,F10.5)
       SLEND=20.
       DO 9 I=1,5,1
       THRUS=0.
       DO 10 J=1,11,1
       B=1.133+SLEND/3080.+SLEND**2/185000.
       G=1.11+SLEND/190.-SLEND**2/9000.+SLEND**3/720000.
       ENDMO=B-G*THRUS
       PUNCH 8,SLEND,THRUS,ENDMO
       THRUS=THRUS+0.1
   10  CONTINUE
       SLEND=SLEND+20.
    9  CONTINUE
       STOP
       END
```

Notation of Program of Finding Ultimate Strength Interaction Curves

B,G	Non-dimensional constants;
ENDMO	End moment;
SLEND	Slenderness ratio;
THRUS	Axial thrust;

Flow Chart for Finding Ultimate Strength
Interaction Curves



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ULTIMATE STRENGTH OF WIDE-FLANGE BEAM COLUMNS

by

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Taipei, China, 1964

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

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The problem that is considered in this report is the determination of the maximum amount of end moment that a member can sustain when it is subjected to a given axial thrust.

The purpose of the report is to represent each step in the determination of a Moment-Thrust-Curvature diagram and the determination of the ultimate strength of a given wide-flange beam-column (10 WF 39) in detail. A comparison between the exact interaction curves obtained in the report and the approximate interaction curves used in the AISC Specification is also presented. Finally, an example is provided to illustrate the use of moment-curvature diagrams, to illustrate how members in a structure assist each other in carrying the load, and to show that not only the ultimate strength but also the deformation behavior of each element is important in determining the ultimate strength of the whole structure.

The solution to the problem of this report was obtained using Newmark's numerical integration method. The problem was limited in scope to the loading case of axial thrust plus moment applied at only one end of the member. Also, it was assumed that the plane of the applied moment is that of the web of the section and that failure is the result of excessive bending in this plane. Residual stresses and strain-hardening were not considered in the paper. Because a set of charts are required for each type of material and cross-sectional shape to solve beam-column problems, the contents of this report are of necessity limited to mild structural steel and to as-rolled wide-flange sections with axial thrust plus moment applied at one end of the member. The results obtained for a 10 WF 39 section are only approximately applicable to other sections.