ECONOMICS OF INNOVATION: COMPETITION, CLUBS AND THE ENVIRONMENT

by

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B.S., North Dakota State University, 2007
M.S., North Dakota State University, 2010
M.A., Kansas State University, 2013

AN ABSTRACT OF A DISSERTATION

submitted in partial fulfillment of the requirements for the degree

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Abstract

Innovation is development of new ideas that leads to better solutions to current problems. From an economic standpoint, innovation is the engine of economic growth. The appearance of innovation is not uniform in the market, and neither are its affects. The development of new products and technology is significant in any industry. As a result, understanding the path of progress within an industry is necessary to maximize the benefit from innovation. The focus of this research is to further understand the relationship between producers, consumers, and the environment, in the context of innovation. Three scenarios are evaluated.

First, innovation evaluated in the context technology intensive industries with product differentiation. Using an optimal control approach with product differentiation and firm outlook we examine conditions that maximize social welfare. When firm(s) have the same discount rate regardless of market structure, a monopoly will develop more innovative products. However, it is shown that competition may increase innovation if firms alter their outlook in a duopoly market structure.

Next, influence of consumers on producer adoption of clean technology is evaluated. A spatial model is developed to analyze welfare implications of environmental policies in a competitive market with production and consumption heterogeneity. Consumers with heterogeneous preferences choose between non-green and certified green products, while firms with heterogeneous production costs decide whether to engage in green production. In order for green products to be recognized by consumers, firms must join a green club. The number of green firms, environmental standard, and overall welfare under the market solution are all found to be socially sub-optimal.

Finally, producer innovation in markets characterized by public policy due to emission concerns is evaluated. Using a dynamic approach, we derive a firm's optimal R&D investment strategy to develop clean technology. Explicitly allowing for the cumulative nature of R&D shows that emissions per unit of output are lowest when the firms cooperate in R&D, and show that a profit-maximizing merged entity will never choose the most efficient investment strategy in clean technology, which has implications for emission tax policy and environmental innovation to improve overall welfare.

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Dedication

To all the women in my family. To my three wonderful daughters, Faith, KayLynn and Konstance, you made me take breaks from my work (frequently it was involuntarily) and were the best motivation to complete my education. To my mother, DeEtte, you always believed I could do anything, even when I didn't think I could (your editing on hundreds of pages of my writing helped too). And most importantly to my wonderful wife, Ellyssa, you put your life on hold, so that I could successfully pursue my dreams. This was only possible because of your support.

Chapter 1 - Strategic Investment and Innovation of Products

1. Introduction

Understanding the role of innovation in the face of competition is fundamental to identifying an industry's progression. The ways that competition affects the incentives to innovate is fundamental to understanding not only specific markets, but also overall economic growth. Innovation leads to the creation new products that can yield greater demand and utility, providing a method for firms to increase profit and market share. In order to survive in a competitive industry, innovation becomes "a life or death matter for the firm" (Baumol 2002, pg.1).

These questions have been long debated in the economic literature. Dating as far back as Schumpeter (1943), the role of competition and firm size relative to innovation has been discussed. Understanding the importance of strategic and monopolistic positions to enhance innovation, Schumpeter noted that a perfectly competitive market "is in a less favorable position to evolve" (Schumpeter 1934, pg. 106). A counterargument by Arrow (1962) is that monopolists gain little advantage from innovation, so that incentives for innovation are highest in a competitive market.

Since the seminal works by Arrow and Schumpeter, a myriad of studies have compared both product and process innovation with varying degrees of competition. Berry and Waldfogel (2010) analyze the relationship between product quality and market size, and show how the structure of costs is an important aspect in determining the quality that persists in a market. Aghion et al. (2014) focus on the relationship between competition and innovation. Using patents and the Lerner index, they show that an inverted-U relationship is present, with the highest levels of innovation at an intermediate level of industry concentration.

Others have added new properties and structure to understand the incentive to innovate further. Gans and Stern (2004) examine the actions of incumbent/entrants when technology can be licensed. Bandyopadhyay and Acharyya (2004) examine the effects of consumer heterogeneity (high-end vs. low-end) on the complementary effects of product and process innovation. Their analysis shows that the prevalence of high-end users increases a firm's gains from product innovation. Chen and Schwartz (2013) incorporate consumers' heterogeneity by using the traditional Hotelling line to represent location in preference space, which increases a monopolist's gain from innovation.

While the majority of innovation literature has used a static approach, much of the contemporary analysis has employed a dynamic setting. Nocke (2007) evaluates R&D and product innovation under collusion. Cellini and Lamberini (2009) compare competition and cooperation of duopolies. Walter and Chang (2014) use a similar approach and apply it to clean technology, but include the monopoly outcome. Ouardighi et al. (2013) study R&D stock with spillovers. These approaches allow for temporal analysis with a cumulative building of knowledge, which more closely represents the R&D process. The work by Arrow and Schumpeter has also been re-evaluated by Cellini and Lamberini (2011). In a dynamic setting, they evaluated the relationship between R&D intensity and market structure in the presence of spillovers. Their results show that R&D investment increases with the number of firms, thus contradicting previous findings obtained from static analysis.

The distinction between product innovation (creating new goods) and process innovation (reducing production costs) is also important. In the majority of studies, process innovation is the focus (such as Arrow 1962, Cellini and Lambertini 2011) and is represented by decreasing marginal cost. In the case of product innovation, oftentimes the development of a new good is represented by either product differentiation (Lambertini and Mantovani 2010; Belleflamme and Vergart 2011) or a quality measure (Bandyopadhyay and Acharyya 2004; Chen and Schwartz

2013; Saha 2014). While these approaches provide useful insights about product innovation, they may not represent some industries accurately. With product differentiation, the structure inherently assumes that innovation diminishes the level of competition between products, whereas a quality measure assumes a maximum level of development (which usually is exogenous provided).

This paper develops a dynamic model of R&D, building on the approach of Cellini and Lambertini (2009), in which innovation is the result of an accumulation of knowledge. In addition, we expand on types of innovation that occurs in a market and provide additional insights into the competition/innovation debate. In particular, we divide innovation into three distinct types: appeal, differentiation, and process. While all three types of innovation are a result of R&D, each has a different effect in a market. Innovation can make a product: better (appeal)[1] thereby changing the utility users receive from the product, or have different uses (differentiation)[2] thereby changing its substitutability and target market, or finally, cheaper to produce (process)[3]. This distinction is necessary because both appeal and product differentiation affects the demand for a good, while process innovation affects the firms' costs and therefore supply. If there are two products and one has higher appeal, every consumer will strictly prefer it, *ceteris paribus*. With differentiated products having similar appeal, on the other hand, a subset of consumers will strictly prefer one of the products available.

Our results provide new perspectives on the Arrow-Schumpeter debate. We show that a monopoly will obtain a higher level of innovation as measured by steady state product appeal, but

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[1] Appeal defines the usefulness of a product. Examples of this are new features or being more ergonomic.
[2] Differentiation defines the substitutability of products. Examples are product's functionality or style of a specific subset of consumers. This could include quality, since a consumer preference depends on income (this possibility is omitted, as will be discussed). In our opinion, quality should be treated differently than innovation. One measure of quality is a products' longevity which is a property of the products inputs.
[3] Process innovation is treated the same as in the literature, decreasing the production costs of a product.

that the innovation advantage does not necessarily imply higher consumer or total surplus. Hence, there may be a tradeoff between innovation and welfare. Moreover, we show that the innovation advantage of monopolies may not hold if the level of industry competition affects the discount rate driving the firm's outlook decisions. In particular, if a monopolist tends to behave more myopically (make decisions consistent with a higher discount rate) then the resulting level of innovation will be lower than a duopolist's. We provide a justification for myopic behavior for monopolies, and show that duopoly firms have an incentive to make decisions based on a relatively lower discount rate. In addition, we provide condition under which it is possible for an entire industry to behave myopically.

This paper is organized as follows: Section 2 describes the basic structure of our model, while sections 3 and 4 provide the monopoly and duopoly derivations. We compare the results of each market structure in section 5. Section 6 concludes.

2. Model

In this paper, we incorporate all three types of innovation, but focus directly on appeal.[4] Consumers' utility function incorporates both the level of product differentiation and appeal. Horizontal product differentiation is represented by heterogeneous consumer preferences (similar to Chen and Schwartz 2013), which allows us to simultaneously incorporate a product's appeal into a firm's innovation decision. This approach also allows us to compare and measure product appeal. Finally, we indirectly integrate process innovation into the firm's production function.

To begin, we start with the supply of a good with initial appeal A_o, which represents the basic or fundamental product introduced into the marketplace. Firms are then able to improve the

[4] The firm can only invest in appeal innovation, while process and differentiation are accounted for indirectly. All three types of innovation could be included independently, but it is cumbersome, and is left for future study.

product by investing in R&D. Firms R&D can increase their product's appeal through innovation to make the product better and more valuable. Each new investment in R&D builds upon the previous work. At the same time, consumers become accustomed to new features (or the features become obsolete), so that the value of a product will eventually decay back to the initial appeal if the firm does not introduce new features into their product. These properties imply that product appeal evolves over time depending on the trajectory of investment and the rate at which features become obsolete.

2.1 Innovation and Appeal

We define the equation of motion for product appeal as:

$$\dot{A}_{i,t} = \gamma V_{i,t} - \delta A_{i,t} \tag{1}$$

where $A_{i,t}$ represents additional appeal of firm i's products due to innovation (in time period t), $V_{i,t}$ represents investment in product appeal (or similarly R&D), γ represents the effectiveness of investment on product development, and δ represents the rate of obsolescence.

As time passes, two effects are concurrently operating in equation (1). First, as firms invest in their product, the overall appeal improves. The second effect allows new features to decrease in novelty to the point of eventually becoming obsolete. Our representation does not imply removal of obsolete features from future products, but instead allows features to become a fundamental part of the good or characteristic of which is improved upon.[5]

Similar to Arrow (1962); Ouardighi (2013); and Chen and Schwartz (2013), we assume that innovation is "perfectly patentable" (or R&D is fully appropriated). Therefore, the equation

[5] Some examples are: low resolution cameras for cell phones, Wi-Fi for laptops, touch screens, or ESC (electronic stability control, although in some countries this was compulsory.

of motion is dependent on the firm's investment only, and void of any spillover effects.[6] This may seem like a limiting assumption, but here we define products as having a firm-specific style and design. Consumers' heterogeneity implies that some individuals have a preference for one firm's distinct design features, so that the firm would erode their brand distinction by copying a rival's product.

2.2 Firm Profits

Taking from the R&D literature, we model investment in appeal (V_i) as quadratic in cost. Therefore, each time period the firm's profits are:

$$\pi_{i,t} = x_{i,t} P_{i,t} - c A_{i,t}^{2} - \beta V_{i,t}^{2} \tag{2}$$

where c is a scaling parameter on the fixed cost of production and β is a scaling parameter on investment costs. Note that the structure of fixed cost along with the obsolescence term ($-\delta A_{i,t}$) in equation (1) represent process innovation in our model. Obsolescence can be treated as an indicator of process innovation because firms inevitably learn how to produce new features more efficiently as they become more commonplace. Equation (1) implies that if investments in new product features are small relative to the rate of obsolescence, A_i will fall and equation (2) implies that the fixed cost of production will fall as well.

Similar to Berry and Waldfogel (2010) we assume fixed costs are increasing in appeal.[7] This is appropriate because, unlike quality, more appealing products require new production methods. For convenience, we set the marginal cost of producing the good equal to zero, as we assume improvements in appeal are not caused by better or more expensive inputs. While

[6] The absence of spillovers means the closed-loop solutions will collapse into the open-loop solution.
[7] Note that Berry and Waldfogel (2010) discuss product quality not appeal. Our assumption is still appropriate since we are not concerned with the quality of inputs.

assuming "better" products do not have higher marginal cost may not be appropriate for some industries, inclusion of this property would yield little benefit for our purposes.

Our model structure captures several previously identified properties in the process and product innovation literature. Specifically, it precisely matches both the "early" and "growth" product life cycle, which represent two of the three stages outlined by Abernathy and Utterback (1978).[8] During the first two stages, the production cost of new product innovation outpaces cost reduction from process innovation. A case could also be made that our approach represents the "mature" stage as well, which Bayus (1995) characterizes as the period when "only small improvements in product and process are undertaken."[9]

2.3 Demand and User Utility

As is common in the optimal control literature, we assume that a new cohort of consumers enters the market each time period. Each consumer purchases no more than one good. Similar to Chen and Schwartz (2004), demand for the product(s) is derived from a cohort of uniformly distributed heterogeneous consumers. However, our approach differs in that we focus on each firm's product innovation in the context of horizontal product differentiation.

In order to identify the demand for each firm's product, we need to derive the consumer demand from their utility. If two firms provide the product (not necessarily with the same appeal),

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[8] First, in the early stage of development, the degree of process innovation occurring is smaller, due to the size of $A_{i,t}$. The incentive to invest in product innovation is relatively large because the amount of obsolescence is small when compared to new investments ($\gamma V_{i,t} >> \delta A_{i,t}$), representing the early stage. However, as product innovation occurs, $A_{i,t}$ increases from investment, the incentive to develop the product erodes ($\delta A_{i,t}$ increases and approaches $\gamma V_{i,t}$), representing the growth stage. At the same time, the degree of process innovation is increasing as $A_{i,t}$ increases. This means that product innovation is initially large, but continually decreases. Process innovation, on the other hand, is initially small, but continues to grow.

[9] We will show that, in the steady state, appeal and therefore costs remain constant, which shows that improvements cease. In addition, if a firm stopped investing in their product, once their appeal has reached its apex, process innovation would begin diminishing. This could also represent the "mature" stage since process innovation would outpace product innovation. Our focus is on product innovation, but if we continued our analysis beyond the point of product innovation, our representation of process innovation would match the bell shaped described by Utterback and Abernathy (1975).

then each firm's innovation or product improvements increases the demand and the associated utility derived from that firm's product. We denote the increased utility by ΘA_i, where the scaling parameter, θ, measures the increase in appeal or value to consumers from an increase in product development.

To represent the consumers heterogeneity via product differentiation, consumers are located along a Hotelling line of distance one, with consumers preferring firm *i* (*j*) located closer to zero (one). This yields a utility function for an arbitrary consumer *x* (where $x \in [0, 1]$) as:

$$U_x = \begin{cases} A_o + \theta A_i - P_i - \tau x & \text{if good "i" is purchased,} \\ A_o + \theta A_j - P_j - \tau(1-x) & \text{if good "j" is purchased,} \\ 0 & \text{if no good is purchased,} \end{cases} \quad (3)$$

Where τ represents the standard measure of product differentiation, and P_i and P_j represent the prices for firm *i* and *j*'s product, respectively.

Several details about the utility function require additional comments. Similar to Berry and Waldfogel (2010), we intentionally omit consumer income effects. Including income effects would unnecessarily complicate the model and yield little, if any, benefit.[10] It is possible to incorporate generic products, by representing other firms operating in the market providing a product with appeal A_o. While this may be a useful extension of the model depending on the market in question, for brevity we omit the discussion of this case.[11]

In addition, the structure of the utility function facilitates several market outcomes. First, by removing either firm, we can represent innovation solely by a monopoly. Second, if users exist that refrain from purchasing either good, then the market is segmented and each firm will operate

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[10] Bonanno and Haworth (1998) provide a static analysis which includes both income and quality in the context of process and product innovation. They also include heterogeneity in the firm's products (high and low).
[11] For a similar static version of this scenario, see Chen and Schwartz (2013). In our approach it amounts to setting $A_o = P_o = C_o$, where $P_1 = P_o + markup$. Our focus is innovation, so we simply assume existence of generic products.

as a monopolist.[12] Lastly, if every consumer purchases a product, the firms will be in direct competition with one another. As is the custom with the Hotelling spatial approach, we identify the marginal consumer (denoted by x^M for monopoly and x^* for duopoly) in each market structure in order to identify the market size for each product.

3. Monopoly Development

When firm i has no direct competitors, either because the firm is a monopolist or because the market is partially served, the marginal consumer (x_i^M) is indifferent between purchasing firm i's product and purchasing nothing. Setting $A_O + \theta A_i - P_i - \tau x = 0,$ we can identify the marginal consumer as:

$$x_i^M = \frac{A_O + \theta A_i - P_i}{\tau} \tag{4}$$

This implies that the market size for the monopolist is the interval [0, x_i^M]. We obtain the revenue of the firm's product, conditional on appeal in each period t, as $x_{i,t}^M P_{i,t} = \left(\frac{A_O + \theta A_{i,t} - P_{i,t}}{\tau} \right) P_{i,t}$.

The firm faces a dynamic optimization problem in which $P_{i,t}$ and $V_{i,t}$ are control variables and $A_{i,t}$ is a state variable, with (1) as the equation of motion. However, the optimal price in each period can be determined as a contemporaneous function of the other variables because it only affects the firm's current revenue and neither impacts current costs nor the future values of the state variable (equations (1) and (2)). The firm's problem has a nested structure, in which price is the solution to a static revenue maximization problem each period, and the optimized revenue

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[12] Example of industries where product demand is segmented: In the early cell phone industry, the Motorola Razr was aimed at younger market relative to blackberry which was designed for professional use.

function becomes part of the dynamic optimization problem in which investment $V_{i,t}$ is the sole control variable.

The monopoly revenue (as a function of price) for product with appeal $A_{i,t}$ is $R_{i,t}^{M}(P_{i,t}) = \underset{P_{i,t}}{Max}\ [(A_O + \theta A_{i,t} - P_{i,t})/\tau])P_{i,t}$. The first-order condition for a maximum is, $(A_O + \theta A_{i,t} - 2P_{i,t})/\tau = 0,$ which implies a revenue-maximizing price of $P_{i,t} = (A_O + \theta A_{i,t})/2$. Substituting this price into the objective function allows us to represent the firm's revenue function as: $R_{i,t}^{M}(A_{i,t}) = (1/4\tau)(A_O + \theta A_{i,t})^2$. Therefore, firm's revenue depends only on the dynamic investment choices, which are determined by the control problem:

$$\underset{V_{i,t}}{Max}\ \Pi = \int_0^{\infty}\left[\frac{1}{4\tau}\left(A_O + \theta A_{i,t}\right)^2 - cA_{i,t}^{\ 2} - \beta V_{i,t}^{\ 2}\right]e^{-\rho t}dt \qquad \text{s.t.} \quad \dot{A}_{i,t} = \gamma V_{i,t} - \delta A_{i,t} \tag{5}$$

where ρ represents the firm's discount rate on future profits. Equation (5) represents the monopolist's discounted stream of profits.

3.1 Hamiltonian & Monopolists' Dynamic System

From equation (5), we can derive the Hamiltonian[13] for firm *i* as:

$$H = \frac{1}{4\tau}\left(A_O + \theta A_i\right)^2 - cA_i^2 - \beta V_i^2 + \lambda_i\left(\gamma V_i - \delta A_i\right) \tag{6}$$

where λ_i is the co-state variable.

Next, we verify the sufficient condition for a constrained maximum.[14] This requires that $4c\tau > \theta^2$, which seems very plausible, as it states that the shift in demand will be smaller than the

[13] Note that the time subscript is suppressed for convenience.

[14] Taking the appropriate second order conditions, yields $H_{VV} = -2\beta < 0$ and $H_{AA} = (\theta^2/2\tau - 2c)$, thus $H_{VV}H_{AA} - H_{AV}H_{VA} = (-2\beta)(\theta^2/2\tau - 2c) - (0)(0) > 0$, as long as $4c\tau > \theta^2$. Thus, the sufficient conditions are satisfied for a constrained maximum. The stability of the dynamic system is also provided in appendix A.

change in production costs for very appealing products. If we think about the contrapositive of this condition, it becomes obvious that the condition will be satisfied in all cases of interest. If the shift in appeal from product development is so substantial that it exceeds the additional cost of product, this implies that the firm's product is so appealing that demand, and therefore profits, will become unbounded. For the remainder of our analysis, we assume this condition is satisfied.

In order to identify the optimal investment strategy and the resulting steady state, the maximum principle conditions are calculated from equation (6). This yields the following conditions:

$$\frac{\partial H}{\partial V_i} = \gamma\lambda_i - 2\beta V_i = 0 \tag{7A}$$

$$\dot{A}_i = \frac{\partial H}{\partial \lambda_i} = \gamma V_i - \delta A_i \tag{7B}$$

$$\dot{\lambda}_i - \rho\lambda_i = -\frac{\partial H}{\partial A_i} = \delta\lambda_i - \frac{1}{2\tau}\left(A_i\theta^2 + \theta A_o - 4c\tau A_i\right) \tag{7C}$$

Rearranging (7A) yields the firm's optimal investment function:

$$V_i = \frac{\gamma}{2\beta}\lambda_i \tag{8}$$

Observe that the firm's optimal investment function depends on the co-state variable. Taking the derivative of best optimal investment function with respect to time yields:

$$\dot{V}_i = \frac{\gamma}{2\beta}\dot{\lambda}_i \tag{9}$$

Substituting equation (7C) and λ_i, derived from (8), into equation (9), yields the monopoly (denoted by "M") firm's investment path over time, which characterizes the dynamics of investment efforts for firm *i*. This can be written in terms of the state (appeal) and control (investment) as:

$$\dot{V}_i^M = \frac{\gamma\left(\left(4c\tau - \theta^2\right)A_i - \theta A_o\right)}{4\beta\tau} + \left(\delta + \rho\right)V_i \ , \tag{10}$$

which when combined with equation (7B), summarizes the dynamic movements of investment and appeal.

We can observe in equation (10) that the initial appeal of the product negatively affects the level of investment, which implies that a monopolist will slow the development of better products. The slower development results in a larger stream of discounted profits. While these properties bring insight into the path of innovation, identifying the resulting level of investment is required to understand the consequences of the firm's decisions.

3.2 Monopoly Steady State Analysis

Setting equation (7B) and (10) equal to zero, we identify the dynamic system's stationary conditions, which are characterized by the following Proposition:

Proposition 1. *As long as* $4c\tau > \theta^2$ *holds, the monopolist steady state point is:*

$$A_i^M = \frac{\theta\gamma^2 A_o}{\left(4c\tau - \theta^2\right)\gamma^2 + 4\beta\tau\delta\left(\delta + \rho\right)}; \qquad V^M = \frac{\theta\gamma\delta A_o}{\left(4c\tau - \theta^2\right)\gamma^2 + 4\beta\tau\delta\left(\delta + \rho\right)}$$

which is a unique saddle point.

Proof. See Appendix A.

In addition, we can use the steady state values to identify the firm's steady state price and market size using equation (4), as:

$$P_i^M = \frac{2\tau A_o\left(c\gamma^2 + \beta\delta\left(\rho + \delta\right)\right)}{\left(4c\tau - \theta^2\right)\gamma^2 + 4\beta\tau\delta\left(\delta + \rho\right)} \tag{11A}$$

$$x_i^M = \frac{2A_o\left(c\gamma^2 + \beta\delta\left(\rho + \delta\right)\right)}{\left(4c\tau - \theta^2\right)\gamma^2 + 4\beta\tau\delta\left(\delta + \rho\right)} \tag{11B}$$

Substituting (11A) and (11B) into the firm's profit function provided in equation (2), yields the firm's steady state profit each period:

$$\pi^{M} = \frac{A_0^2\left(4c\tau\gamma^2\left(c\gamma^2+2\beta\delta(\rho+\delta)\right)-\theta^2\gamma^2\left(c\gamma^2+\beta\delta^2\right)+4\beta^2\tau\delta^2(\delta+\rho)^2\right)}{\left(\left(4c\tau-\theta^2\right)\gamma^2+4\beta\tau\delta(\delta+\rho)\right)^2} \tag{11C}$$

Next, we measure the steady state market benefits. We define the consumer surplus (CS) and total surplus (TS) in the market as:

$$CS = \int_0^x U_x dx \tag{12A}$$

$$TS = CS + \pi \tag{12B}$$

Evaluating equations (12A) and (12B) in the steady state using equations (11A), (11B), and (11C) we can calculate consumer surplus and total surplus in the monopoly steady state as:

$$CS^{M} = \frac{2\tau A_0^2\left(c\gamma^2+\beta\delta^2+\beta\rho\delta\right)^2}{\left(\left(4c\tau-\theta^2\right)\gamma^2+4\beta\tau\delta(\rho+\delta)\right)^2} \tag{13A}$$

$$TS^{M} = \frac{A_0^2\left(6\tau c^2\gamma^4-c\theta^2\gamma^4+12\tau c\beta\gamma^2\delta(\delta+\rho)-\theta^2\beta\gamma^2\delta^2+6\tau\beta^2\delta^2\left(\delta^2+\rho^2\right)+12\tau\beta^2\delta^3\rho\right)}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta(\rho+\delta)\right)^2} \tag{13B}$$

The monopolist's profit, consumer surplus, and total surplus (equations 11C, 13A, and 13B) will be used to evaluate how welfare changes with the addition of another firm into the marketplace.

3.3 Comparative Dynamics

Next, we evaluate the effects of product differentiation on the steady values for the firm's investment, appeal, and price as well as the effects on consumer surplus, profit and total surplus.

This is accomplished by taking the derivatives of equations (11) and (13), with respect to τ.[15] This allows us to state:

Corollary 1. *Higher levels of product differentiation cause the Monopolist's steady state investment, appeal, price, profit, consumer surplus, and total surplus all to decrease.*

As it becomes harder to attract consumers with disfavor for the monopolist's product, the monopolist's incentive to innovate erodes. This means that as a monopolist's product becomes less appealing due to location or consumer preferences, it decreases the firm's investment in the product.

Next, we evaluate the effects of product's initial appeal on the steady values for investment, appeal, price, and profit.[16] This allows us to state:

Corollary 2. *For products with greater initial appeal, the Monopolist increases its steady state investment, appeal, and price, which result in greater profit, consumer surplus, and total surplus.*

This result is a consequence of demand shifts caused by innovation. Not only do products with high initial demand allow the monopolist to charge a higher price, they also expand the market for the product. As a consequence, the monopolist has conditional resources to invest in R&D. The result is a higher level of both investment and product appeal in the monopoly market.

4. Duopoly Development

In this section, we evaluate how product innovation occurs in a fully served market serviced by duopoly firms that both invest in product innovation.[17] In a fully served market, the marginal

[15] These are provided in Appendix B.
[16] These are provided in Appendix B.
[17] Note that in the case that the market is not fully-served, each firm will operate as a monopolist according to our findings in section 2.

consumer (x^*) is indifferent between purchasing a product from either firm, therefore from equation (3), we have: $A_o + \theta A_i - P_i - \tau x = A_o + \theta A_j - P_j - \tau(1-x)$. In order to identify firm *i's* market size, we solve for *x*, which yields:

$$x^* = \frac{\tau + \theta\left(A_i - A_j\right) + \left(P_j - P_i\right)}{2\tau} \tag{14A}$$

Similarly, for firm *j*, we solve for (1- *x*), which yields:

$$\left(1 - x^*\right) = \frac{\tau + \theta\left(A_j - A_i\right) + \left(P_i - P_j\right)}{2\tau} \tag{14B}$$

We can represent the size of firm *i*'s market by the interval [0, x^*], and firm *j*'s is [x^*, 1]. Notice that equations (14A) and (14B) are symmetrical, thus for brevity, in the following section we focus solely on firm *i*.

As in the monopoly case, each duopolist faces a nested dynamic problem where revenue is determined each period conditional on the state of product appeal and the resulting revenue function becomes a component of the optimal control problem. In the duopoly case, however, the appeal of both firms are relevant state variables and the price obtained by each firm is the result of a Cournot price game played each period. Conditional on firm *j*'s price, P_j, and the observed states A_i and A_j, firm *i*'s best response function at each instant *t*, is the solution to the revenue maximization problem

$$\underset{P_{i,t}}{Max}\left(\frac{\tau+\theta\left(A_{i,t}-A_{j,t}\right)-\left(P_{i,t}-P_{j,t}\right)}{2\tau}\right)P_{i,t}$$

The first order condition to this problem is $(1/2\tau)(\tau - 2P_{i,t} + P_{j,t} + \theta A_{i,t} - \theta A_{j,t}) = 0$, which implies a best response function of: $P_{i,t} = (1/2)(\tau + P_{j,t} + \theta A_{i,t} - \theta A_{j,t})$. Given a symmetric best response function for firm *j*, the Nash equilibrium of the Cournot price game for firm *i* is

$P_{i,t} = \tau + (1/3)(\theta A_{i,t} - \theta A_{j,t})$. Substituting this expression for P_i and the symmetric expression for firm j into the revenue function gives a reduced-form revenue function for firm i of $R_{i,t}^* = (1/18\tau)(3\tau + \theta A_{i,t} - \theta A_{j,t})^2$. We can then write firm i's optimal control problem in a fully served market as:

$$\underset{V_{i,t}}{Max} \quad \Pi = \int_0^\infty \left[\frac{1}{18\tau}\left(3\tau + \theta A_{i,t} - \theta A_{j,t}\right)^2 - cA_{i,t}{}^2 - \beta V_{i,t}{}^2 \right] e^{-\rho t} dt$$

$$\text{Subject to: } \dot{A}_{i,t} = \gamma V_{i,t} - \delta A_{i,t} \tag{15}$$

As before, this represents the discounted stream of profits for the firm in the competitive setting.

4.1 Hamiltonian & Duopolists' Dynamic System

From equation (15), we derive the current-value Hamiltonian for firm i as: [18]

$$H = \frac{1}{18\tau}\left(3\tau + \theta A_i - \theta A_j\right)^2 - cA_i^2 - \beta V_i^2 + \lambda_i\left(\gamma V_i - \delta A_i\right) \tag{16}$$

The Hamiltonian reveals that the sufficient condition for a constrained maximum are satisfied.[19] The maximum principle conditions derived from equation (16) are as follows:

$$\frac{\partial H}{\partial V_i} = \gamma\lambda_i - 2\beta V_i = 0 \tag{17A}$$

$$\dot{A}_i = \frac{\partial H}{\partial \lambda_i} = \gamma V_i - \delta A_i \tag{17B}$$

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[18] The time subscript is again suppressed for convenience.

[19] As before, we take the appropriate second order conditions: $H_{VV} = -2\beta < 0$ and $H_{AA} = \theta^2/9\tau - \left(2c\right)$, using these, we obtain: $H_{VV}H_{AA} - H_{AV}H_{VA} = (-2\beta)[(\theta^2)/(9\tau) - (2c)] - (0)(0) > 0$, Therefore, as long as $18c\tau > \theta^2$, (which is satisfied based off the monopoly condition) the sufficient conditions are met for a constrained maximum. Note that firm j's investment (V_j) and appeal (A_j) are exogenous to firm i. However, if symmetry is assumed, then $H_{VV}H_{AA} - H_{AV}H_{VA} = \left(-2\beta\right)\left(-2c\right) > 0$. The stability of the dynamic system is provided in appendix D.

$$\dot{\lambda}_i - \rho\lambda_i = -\frac{\partial H}{\partial A_i} = \delta\lambda_i + \frac{1}{9\tau}\left(\theta^2 A_j - \theta^2 A_i - 3\theta\tau + 18c\tau A_i\right) \tag{17C}$$

Using equation (17A), we can identify the firm's optimal investment function as:[20]

$$V_i = \frac{\gamma}{2\beta}\lambda_i \tag{18}$$

Taking the derivative of optimal investment function from equation (18) with respect to time yields:

$$\dot{V}_i = \frac{\gamma}{2\beta}\dot{\lambda}_i \tag{19}$$

Note that the duopolist's investment function appears to be identical to the monopolist. However, the co-state variable, representing the shadow price of an additional unit of appeal, differs in the two cases. To facilitate our comparison between different market structures, we impose a symmetry condition on firms in the duopoly setting (and we denote the results with "*").[21] Imposing symmetry on equation (17C), we substitute it and the co-state variable in (18) into (19). As before, we can identify the firm's investment strategy, which yields:

$$\dot{V}_i^* = \frac{\gamma}{6\beta}\left(6cA_i - \theta\right) + \left(\delta + \rho\right)V_i \tag{20}$$

Using equation (20) and (17B), we can summarize the dynamic movements of investment and appeal when symmetrical duopolists serve the market. Specifically, we observe that the duopolist's investment function shows a slightly different response when compared to the monopolist. The duopolists are unconcerned with the level of differentiation or initial product appeal, which is appropriate due to competition.

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[20] Note that $\frac{\partial H_i}{\partial V_j}\frac{\partial V_j^*}{\partial A_i} = 0$, thus implying the absence of any feedback effect and that our results are time consistent.

[21] One extension of our analysis is to evaluate asymmetric firms, thus allowing an examination of incumbent and entrant actions similar to Gans and Stern (2004), however, we leave this for future study.

The optimal investment path now must account for rival products, with the result that both firms focus on maintaining pace with one another as opposed to solely focusing on profits. This makes intuitive sense, as inefficient or sub-optimal investment strategies result in a loss of market share, and therefore profit in the duopoly setting.

4.2 Duopoly Steady State Analysis

As before, we can solve for the stationary conditions by setting equation (17B) and (20) equal to zero, which allows us to state the following:

Proposition 2. *The duopolist steady state point in a fully served market under symmetry is:*

$$A_i^* = \frac{\theta\gamma^2}{6c\gamma^2+6\beta\delta(\delta+\rho)} \qquad V_i^* = \frac{\theta\gamma\delta}{6c\gamma^2+6\beta\delta(\delta+\rho)}$$

which is a unique saddle point.

Proof. See Appendix C.

One important observation from the duopoly steady state result, which is similar to the dynamical system, is the absence of the initial appeal and product differentiation. While the omission of product differentiation is expected due to the firms being symmetrical, the absence of initial appeal implies that development reaches the same steady state independently of the original product. This is a symptom of competition. In order to maintain market share, a firm's product appeal must "keep up" with their competitor's. Both firms are no longer focused solely on maximizing the profit stream; but must incorporate product improvements that occur in the market.

Next, we identify the firms' steady state price and market size by imposing symmetry on equations (14A) and (14B). This yields:

$$P^* = \tau \tag{21A}$$

$$x^* = \frac{1}{2} \tag{21B}$$

Substituting these results into the firm's profit function yields the firm's steady state profit:

$$\pi^* = \frac{\tau}{2} - \frac{\theta^2\gamma^2\left(c\gamma^2 - \beta\delta^2\right)}{36\left(c\gamma^2 + \beta\delta^2 \beta\rho\delta\right)^2} \tag{21C}$$

Next, we measure the steady state market benefits. We define the consumer surplus (CS) and total surplus (TS) in the duopoly market as:

$$CS = \int_0^x U_x dx + \int_{1-x}^1 U_x dx \tag{22A}$$

$$TS = CS + 2\pi \tag{22B}$$

Evaluating equation (22A) and (22B) in the steady state using equations (21A), (21B), and (21C) we can calculate consumer surplus and total surplus in the duopoly steady state as:

$$CS^* = \frac{4A_o - 5\tau}{8} + \frac{\theta^2\gamma^2}{22\left(c\gamma^2 + \beta\delta^2 + \beta\rho\delta\right)} \tag{23A}$$

$$TS^* = \frac{3\tau + 4A_o}{8} + \frac{\theta^2\gamma^2\left(2\beta\delta^2 - 2c\gamma^2 + 3\right)}{36\left(c\gamma^2 + \beta\delta^2 + \beta\rho\delta\right)^2} \tag{23B}$$

The next section compares duopoly's profit, consumer surplus, and total surplus (equations 21C, 23A, and 23B) to the monopoly result in order to ascertain the effects of competition.

4.3 Comparative Dynamics

As in the previous section, we evaluate the effects of product differentiation on the steady-state values for the firm's on consumer surplus, profit and total surplus. Note that product differentiation (τ) is absent from the steady-state appeal and investment expressions in both cases.

Evaluating the effects of product differentiation on equation (21) and (23), allows us to state the following[22]:

Corollary 3. *Higher levels of product differentiation increases the Duopolists' steady state price and profit, while decreasing consumer surplus. However, the net effect is an increase in total surplus.*

Although product differentiation is omitted from investment and appeal results, the firm recognizes greater product differentiation allows them to increase prices. The firm ignores product differentiation in their innovation decision, but exploits consumers' heterogeneous preferences to increase revenue. This means that greater product differentiation does not directly affect the firm's investment decision. Instead, greater product differentiation facilitates higher prices and by extension profit, thus providing resources for investment.

The product's initial appeal is not only absent from investment and appeal, but also pricing and profit. This means the firm ignores the products initial value for both the pricing and investment decision. Ultimately, the products' initial quality enters only into the consumer preferences; therefore, it (positively) affects only consumer surplus and total surplus.

5. Comparative Analysis

Using the results from section 3 and 4, we compare the duopoly and monopoly outcomes to determine the effects of competition on firms' decisions and welfare. We compare both market constructs in two distinct ways. In the first section, we assume that a firm's outlook is unaffected by the presence of a rival firm. The second section evaluates how outlook (as measured by the discount rate) affects a firm, and shows whether the presence of another firm can change a firm's

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[22] The appendix D contains the necessary calculations.

outlook. [23] Finally, in the spirit of Teece (1992) we examine potential cooperative actions within an industry.

In order to complete a relevant analysis, we assume that a monopoly in operation will service at least half the market. This condition is necessary to evaluate the effects of competition. In the scenario where a monopolist's market size is less than half, the introduction of a rival does not impact the monopolists operation.[24] Obviously in that scenario, the introduction of another firm increases consumer surplus and total surplus, while innovation, investment, and pricing of the incumbent firm is unaffected. The introduction of a "rival" unambiguously increases total welfare because new consumers are served, but not because it affects the incumbent monopolist or its consumers.

5.1 Uniform Outlook

In this section, we determine if the monopoly or duopoly market yields the highest: product appeal, consumer surplus, and total surplus. Our goal is not to determine the amount of investment, but evaluate which market has the highest level of innovation. This means that we treat innovation purely as a magnitude and ignore the level of product differentiation. We use this approach in order to identify the market that results in the best product.

We begin by examining the level of innovation that occurs in each market, and compare the steady state results from each scenario. For a duopoly to have a higher level of innovation, it means that $A^* > A^M$, or that:

[23] We differentiated a "myopic" firm from "forward-looking" firm by comparing their respective discount factor (ρ). Myopic (forward-looking) firms are those firms who put a large (small) discount on future profit, which is represented with a relatively large (small) ρ.

[24] The monopolist will serve at least half of the market if $4c\gamma^2(A_o-\tau)+4\beta(A_o-\tau\delta)(\delta+\rho)+\gamma^2\theta^2>0$.

$$\frac{\theta\gamma^2}{6c\gamma^2+6\beta\delta(\rho+\delta)} > \frac{\theta\gamma^2 A_0}{\left(4c\tau-\theta^2\right)\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta} \tag{24}$$

From this comparison, we can determine the market structure that maximizes innovation. Since this condition never holds, we can formally state:

Proposition 3. *The level of innovation will always be greater in a monopoly relative to a duopoly, ceteris paribus.*

Proof: See Appendix E.

Our result is similar to the static findings by Chen and Schwartz (2013), who concluded that competition does not increase the level of innovation in a market. These results clearly contrast the findings of Arrow. Interestingly, our results also contradict the previous finding of Cellini and Lambertini (2011) who used a dynamic R&D approach. However, their focus on process innovation and the inclusion of spillover effects could be responsible.[25] Another potential cause could be the market structure itself. As noted by Bonanno and Haworth (1998), Bertrand and Cournot competition lead to different innovation incentives.

Our model reveals that a monopolist will always be able to extract a higher profit, generating additional resources to fund research. However, the use of an optimal control approach provides additional details about why monopolists have a higher level of innovation. Specifically, our approach shows that the monopolist's path of investment differs depending on the product, as shown by the inclusion of the initial product's quality in the firm's decision. This does not occur

[25] Walter and Chang 2014 may provide some insights as to the cause. They evaluate duopoly with both research competition and cooperation, and then compare both settings to the monopoly case (although they are discussing clean technology). They find that cooperation yields the most "innovation," while competition yields the least, and monopoly in the middle. If we consider the cooperative and competitive duopoly case as the bounds on spillover, it shows that depending on the level of spillover, the duopoly case could exceed the monopoly case.

in the duopoly setting, where firms must focus on maintaining market share by developing fast enough to keep up with rivals.

5.2 Progressive and Myopic Outlooks

In this section, we evaluate conditions that may change a firm's outlook and, therefore, the market outcome. As shown by Fiat et al. (2014), a myopic vs. non-myopic (i.e. progressive) outlook yields different equilibrium. We first examine whether conditions exist for a duopoly to exceed a monopolist's innovations. If such a scenario is present, it would reinforce the results of Arrow (1962) and the dynamic results of Celini and Lambertini (2011), but provide alternative reason for the result. This would also contradict the recent results of Chen and Schwartz (2013) and Teece (1992).

To begin, we postulate that profitability disincentivizes a firm from being progressive. In a duopoly setting, myopic behavior could be disastrous when rivaling a progressive firm. However, if barriers to entry are high, a monopoly firm can maintain both profit and market share, thus incentivizing myopic behavior. Therefore, we assume the rate of discount may change between market structures. We denote the monopolist's discount rate as ρ_m and the duopolist's discount rate as ρ_f.

Setting the duopolist's appeal to be greater than the monopolist's, or $A^* > A^M$, we see that in order for this to occur the following condition must hold:

$$\rho_m > \frac{3A_O\rho_f}{2\tau} + \frac{6A_Oc\gamma^2 + \beta\delta^2(6A_O - 4\tau) - \gamma^2\left(4c\tau - \theta^2\right)}{4\tau\beta\delta} \tag{25}$$

Note that the discount rate (ρ) must lie in the interval [0, ∞]. Therefore, the condition outlined in equation (25) is plausible. This allows us to state:

Proposition 4. *If a monopolist is myopic relative to firms in a duopoly market as to satisfy the condition in (28), then duopoly innovation will exceed that of the monopolist.*

Our result shows that even though a monopoly has the greatest potential for innovation, myopic behavior may result in innovation below the potential level. The idea that firms may behave myopically is not new in the literature. Holmstrom (1989) notes, “Larger firms are at a comparative disadvantage in conducting highly innovative research…” and attributed this to the firm having to manage a “heterogeneous set of tasks.” This may loosely apply to our scenario, as the monopolist’s level of production will always exceed that of a duopolist. Another reason, as conjectured by Stein (1989), is that short-term stock pricing might incentivize myopic behavior.

While both explanations for myopia could apply to a monopoly, Stein’s explanation may not be as relevant to our scenario (duopoly firms should have the same incentive). Regardless, we provide an alternative explanation. Myopic strategies are reactive and favor the status quo, making them easier for firms to execute. Therefore, a profitable monopoly has an incentive to behave myopically. Obviously, there are limitations as to how myopic a firm can act even as a monopoly, but the absence of a rival clearly disincentives progressive behavior if it creates any hardship for a firm. Furthermore, it may be hard to detect a myopic behavior, since the firm still captures monopoly profits.

Do these results imply that duopolies or even oligopolies will not behave myopically, or does Stein (1989) apply to duopolists as well? If only one firm behaves myopically in a competitive market, the consequences are more obvious. A progressive firm will invest more in its product, relative to a myopic one. Eventually, the progressive firm will create a better product and will capture a greater market share, while the myopic firm is pushed out of the market. But what if incentive exists for both firms to behave myopically? We seek to identify the conditions under which oligopoly firms may also have an incentive to behave myopically.

To identify the effects of a firm's outlook, we take the derivative with respect to the discount rate of firm's profit (from equation 11C) and appeal, which yields:

$$\frac{\partial \pi^*}{\partial \rho} = \frac{\left(c\gamma^2 - \beta\delta^2\right)\theta^2\beta\gamma^2\delta}{18\left(c\gamma^2 + \beta\delta^2 + \beta\rho\delta\right)^3} \tag{26A}$$

$$\frac{\partial A^*}{\partial \rho} = \frac{-\theta\beta\gamma^2\delta}{6\left(c\gamma^2 + \beta\delta^2 + \beta\rho\delta\right)^3} < 0 \tag{26B}$$

If we assume that the industry is one with high fixed costs and low technological decay (such that $c\gamma^2 - \beta\delta^2 > 0$), then both firms' profits unambiguously increase, while their appeal decreases. If we assume this condition holds, this allows us to state the following:

Proposition 5. *In industries served by oligopolies and characterized by innovation, all firms earn the highest profits from industry-wide myopic behavior. However, the Nash equilibrium is for all firms to be progressive.*

Proof: Using the standard payoff matrix, we construct the potential payoffs for the possible outlook combinations (myopic or forward-looking) for our two firms. The result follows from a traditional prisoner's dilemma type outcome.

As already shown in equation (26A), if both firms use the same discount rate (we denote the symmetry with *) then both firms profit increases with myopic outlooks. This means if we compare the myopic outcome (denoted with *m*, so $\pi_{i,m}{}^* = \pi_{j,m}{}^* = \pi_m{}^*$) to the progressive outcome (denoted with *f*, so $\pi_{i,f}{}^* = \pi_{j,f}{}^* = \pi_f{}^*$), then $\pi_f{}^* < \pi_m{}^*$. In the event that firms have different outlooks, so that one firm behaves myopically and the other is progressive, we know from (26B) that the myopic firm's product will have lower appeal.[26] Using the firm optimal price strategy from

[26] We focus on the two firm case, but the proof could be expanded to N firms.

equation, we can solve for the myopic firm's market share as a function of both firms appeal using equation (14A):

$$x_{i,m} = \frac{\tau + \theta\left(A_i - A_j\right) + \left(P_j - P_i\right)}{2\tau} \tag{27}$$

Examining equations (27) and firm's optimal price shows us that when a firm behaves myopically relative to its competition, the result is that the firm develops a less appealing product with a lower price which results in a loss of market share.[27] Therefore, the progressive firm will charge a higher price and capture a greater share of the market, resulting in higher profit ($\pi_{i,m} < \pi_{j,f}$). This gives us the following payoff matrix:[28]

Table 1.1 Payoff Matrix

		Firm i	
		Myopic	Far-Sighted
Firm j	Myopic	$\pi_{i,m}^{*} = \pi_{j,m}^{*}$	$\pi_{i,f} > \pi_{j,m}$
	Far-Sighted	$\pi_{i,m} < \pi_{j,f}$	$\pi_{i,f}^{*} = \pi_{j,f}^{*}$

Note that if only one firm is behaving myopically, that firm will have an incentive to change its outlook, as the firm will recapture its lost market share and increase its profits with a progressive outlook. Therefore, we can conclude that the Nash equilibrium is for both firms to be progressive. However, the greatest payoff comes from both behaving myopically.

While our approach assumes consumer preferences for the two firm is perfectly split, if we expand our analysis to general case with N firms (each with an equal amount of product

[27] If we let $A_j = A_i + a$, then firms charge $P_i = \tau - (\theta a / 3)$ and $P_j = \tau + (a\theta / 3)$. The resulting market share is $x = (\tau / 2) - (a\theta / 6\tau)$ and $1 - x = (\tau / 2) + (a\theta / 6\tau)$.

[28] Myopic behavior corresponds to a big ρ, while progressive behavior corresponds to a small ρ.

differentiation), then we can examine the outlook of an individual firm relative to the industry's outlook and determine the effect via profit. If we assume that with the exception of firm *i,* the remaining N-1 firms act identically, then π_j (where where $j \in [1, N\text{-}1]$) represents an arbitrary firm's profits. Even though the magnitudes may change, the inequalities from the two-firm game hold in the case of N firms, thus our results are expandable to N firms. **Q.E.D.**

The question thus becomes: is it possible for firms to collude and maintain a myopic arrangement? In a duopoly market, the firms' dominant strategy is to be progressive. Therefore, if one firm maintains a myopic outlook, while the others become more progressive, the progressive firms will gain market share and increase their profits, at the myopic firm's expense. In a traditional single turn game, we could safely conclude that all the firms will maintain a progressive perspective. However, given that this game repeats continuously in the optimal control setting, there is evidence by Duffy and Ochs (2009) that firms will be able to cooperate and maintain an agreement.[29] This shows that cooperation in innovation may impede development, thus providing conditions, which contradicts the results by Teece (1992), which shows that cooperative behavior enhances innovation.

5.3 Industry Suspects

Depending on a firm's strategies, it is conceivable that a myopic arrangement can be maintained and that this strategy may occur in some industries. The difficultly comes in identifying myopic behavior in the marketplace due to the challenges of measuring innovation. However, we provide two possible suspects in order to help illustrate the potential for such arrangements. The

[29] For additional strategies that will yield this equilibrium see Bergin and Bernhardt (2009) which discusses how cooperation can occur through imitation.

U.S. automotive and telecommunications are industries with high fixed costs of production. Some business commentators have suggested both industries lacked foresight in recent years.

In 2008, the big three automakers (Ford, GM, Chrysler) experienced decreased demand for their vehicles. Several factors may have contributed to decreased demand, such as higher gas prices or demand shifts to different products. Both of these factors influenced automakers beyond the big three. However, the magnitude of these effects was not uniform. The disproportionately large effects on the big three could be a result of their failure to develop and innovate at the pace of competitors, specifically in fuel-efficient models. If the managers of these firms shared a belief that US-made vehicles guarantee a certain share of the market due to brand loyalty, then little investment in innovation may have been the optimal strategy.

The telecommunications industry is another example. Initially, U.S. internet speeds rivaled all other countries, but now lag other high and middle-income countries (Akamai Technologies, 2014). While tools necessary to increase speeds are available, internet service providers (ISPs) need to invest and incorporate new products into their network in order to increase speeds. With a shared belief in a secure domestic market insulated from foreign competitors, ISPs in the US may have benefitted collectively from a slow pace of development (Fung 2014). However, as noted above, this equilibrium is unstable and disrupted by a single innovative firm. The introduction of Google Fiber highlighted the limited amount of investment by major ISPs (Gustin 2012). After the introduction of Google's more innovative product, other ISPs have begun making upgrades of their own (Finley 2013).

Both automotive and telecommunications are industries requiring innovation to maintain competitiveness, thus requiring continuous and large investments in R&D. While the connection between our analysis and these markets is quite loose, we believe they provide examples of systematic myopic behavior in an industry, which is optimal for the firms involved but harmful to consumers and (potentially) welfare.[30] This also reinforces Stein (1989), since the incentive for the firm to behave myopically is present in both monopoly and duopoly settings. However, we are not implying that industry myopia is widespread. On the contrary, it seems more likely that rival firms would be unlikely to maintain a myopic cartel based on the pace of innovation. One thing we can conclude about the prevalence of myopic industries is their unlikelihood in industries with low/no barriers to entry.

6. Conclusion

Using an optimal control approach, we are able to obtain similar results to those of: Chen and Schwartz (2013), using a static model; and those by Cellini and Lambertini (2011), using a dynamic game. However, unlike previous papers, our structure accounts for traditional product innovation and incorporates product appeal in the presence of consumer heterogeneity. We evaluate additional temporal effects of R&D. We show conditions for a duopoly to be more innovative and strictly welfare enhancing, relative to a monopoly. This yields additional insights unattainable by previous models.

Our analysis does not end the Schumpeter-Arrow debate but adds perspective through a common framework in which both lines of argument have validity. The resources obtained by a monopoly certainly provide the means to invest more heavily in R&D relative to a competitive firm. However, the competitive firm must innovate in order maintain its market share. While our

[30] If the industry outlook becomes more myopic, we know that consumer surplus will decrease in both market types since $\partial CS^*/\partial\rho < 0$ and $\partial CS^M/\partial\rho < 0$, but total surplus may or may not decrease, since $\partial TS^M/\partial\rho < 0$ and $\partial TS^*/\partial\rho < 0$ (if $2c\gamma^2 - 2\beta\delta^2 - 3 > 0$).

approach provides no closure to this argument, it does explain why empirical analysis may yield results matching either Schumpeter or Arrow.

What we find is that the monopolist maximizes innovation, *ceteris paribus*. However, in our analysis we include welfare measure and find that competition is likely to increase total surplus. This adds a new dimension to the debate about innovation. As stated by Baumol (2002):

> *"Probably the most powerful force that may well lead to the continuation of the remarkable growth in innovation activity is the adoption of innovation as the prime weapon of competition in many of the leading oligopolistic sectors of the economy."*

There is no doubt that competition is a necessary component when discussing innovation. However, the industry structure and outlook have additional implications as well. Furthermore, we believe the reason this statement still holds is the indirect effect competition has on a firm's outlook.

While we cannot undermine the justification of industry performance provided by Holmstrom (1989) and Stein (1989), we are able to provide an alternative reason for firm myopia. The value of our approach is in the new depth provided in the innovation/competition debate by including the firm's outlook and its relationship to industry outlook. By using a dynamic game in continuous time, we find that not only is the structure of the market important for innovation, but also the outlook of the firms. Based off the potential on firms' strategies, we find reasons why myopic behavior may or may not be systemic to an industry. This highlights another angle for anti-trust regulation, with potential implication for both oligopoly and monopoly markets. The implications of systemic myopic groupthink are detrimental to both the consumer and overall innovation. However, the challenge comes in identifying the behavior.

Our analysis, of course, is a simplified representation of the complex and detailed innovation process. While it may not fully capture all the benefits of innovation in a market, our structure can be modified to integrate a myriad of other features (spillover, higher marginal cost, cooperation, regulation, etc.), which may yield additional interesting results. For now, we are able to show that if we ignore concerns about myopic behavior, that competition erodes innovation. However, we have also shown that maximizing welfare may come at the expense of innovation, or vice versa.

Chapter 2 - Green Product Certification, Heterogeneous Firms, and Green Consumers

1. Introduction

Environmental awareness has grown drastically over the last several decades. As concerns have developed, consumer taste and preference in the products they purchase have shifted. Preferences for greener products have become ubiquitous; as such the demand for green products continues to expand. This demand is what has driven the market for green products (Michels, 2008). Hamilton and Zilbermann (2006), in reference to a marketing intelligence service,[31] indicate that "green products account for approximately 9% of new-products introductions in the United States." Furthermore, consumer spending on LOHAS (lifestyles of Health and Sustainability) related products has already eclipsed $250 billion according to LOHAS journal (Dosey, 2010).

However, consumer preference for green products is far from uniform. The typical approach incorporates consumer heterogeneity either by location (e.g., Kurtyka and Mahenc 2011; Conrad 2005) or by the level of green preference (e.g., Amacher et al. 2004). Much of the contemporary literature analyzing green preferences assumes that consumers can directly observe a firm's emissions and the benefits from clean production, thus making government intervention straightforward.

The introduction of new "green" products adds additional utility for consumers with green preferences, but claims made by the producers of green products often comes into question. Similar to credence products discussed by Baron (2011), consumers do not have access to the necessary information about green products to verify the claims of producers. These asymmetries in the

[31] In their paper, Hamilton and Zilbermann (2006) refer to ProductScan Online, Marketing Intelligence Service Ltd. 1999.

market for green products have led to the development of third party verification or certification, by so called "green clubs." In the market where product quality information is asymmetric, green clubs represent an important tool for both green consumers and producers. Firms that voluntarily join green clubs are subjected to verification and additional standards. This differs from voluntary agreements and standards discussed by Segerson and Miceli (1998), which are proposed as an alternative to regulation or legislation.

But what benefits do firms receive from certification? As noted by Potoski and Prakash (2005), club participation is effective because "its broad positive standing with external audiences provides a reputational benefit…"[32] Basically, "…firms can differentiate their product from those of firms whose products do not meet the standard" (Baron 2011).As a result, socially responsible firms and their verified green products are capable of gaining a positive reputation and a premium in an expanding market. As green products have become more prevalent, so too have third party monitors. The EPA lists dozens of programs or "clubs" to verify and promote use of clean methods of production (EPA 2014).

New studies have begun evaluating green products when consumers are unable to directly identify a firm's environmental attributes. To inform consumers, firms require certification either by using eco-labeling or joining green clubs. These certifications have been evaluated under various market structures, such as product types (Hamilton and Zilberman 2006), available technologies (Mason 2006), and in the context of environmental innovation (Dosi and Moretto 2001). While it has been shown that emission standards may not necessarily increase social welfare (Moraga-Gonzalez and Pardon-Fumero 2002), others such as Grolleau et al. (2007) have

[32] This is in reference to ISO 14001, a green club with over 1,500 members in the United States. See Potoski and Prakash (2005).

analyzed the strategic aspect of imperfect certification. Mason (2011) assumes certification is a noisy test with potentially incorrect outcomes, while van't Veld and Kotchen (2011) evaluate several certification types and discusses how imperfect monitoring can affect market outcomes and product standards using implicit functions.

Examining firms with different costs for abatement or environmental friendliness is also commonly studied in the environmental literature. However, the analysis is frequently limited in variety. Doni and Ricchiuti (2013); Moraga-Gonzalez and Pardon-Fumero (2002); and Amacher et al. (2004) evaluate how heterogeneous costs affect market outcomes in the presence of heterogeneously concerned consumers. However, the number of firms is limited to two (high and low cost), while allowing consumers a range of preferences for the firms products. Because of restrictions places on market structures, policy work by Lombardini-Riipinen (2005) which shows that socially optimal outcome can be achieved using an emission tax, may not be applicable to all industries due to their structure.

Our goal is to further develop an analytical framework for analyzing the heterogeneity of firms, so we can expand the policy implications beyond the duopoly or high/low cost firm approach. Similar to Ben Youssef and Lahmandi-Ayed (2008); Baksi and Bose (2006), we focus our analysis on issues related to the use of eco-certification in the presence of heterogeneous consumers. However, we wish to take into account the decisions of heterogeneous firms in a competitive market, which is important for several reasons. First, environmental friendliness is not limited to duopoly or even the oligopoly case. Second, a firm's abatement costs and profits are certainly not uniform, especially when a market is served by heterogeneous firms with differential costs of production. Third, we can evaluate how eco-certification and environmental regulation affect the endogeneity of market structure in terms of the number of green and non-green firms. Specifically, our analysis allows for the exit of firms from a market.

Our approach shows that the number of green firms, the level of environmental standard, and the level of overall welfare under the competitive market solution are all socially sub-optimal. This leads us to examine what are possible measures by government to correct the Pareto sub-optimality. We find that the introduction of a subsidy policy for greener production or a tax charge for green certification by a club (which we refer to as an "eco-certification tax") generates a positive effect social welfare. Nevertheless, this welfare-improving policy is not Pareto optimal (i.e., it is a second-best outcome). This prompts us to analyze the efficacy of dual policy instruments that combine subsidizes for a greener production standard and an eco-certification tax. We show that this dual-tool policy helps achieve the first-best or Pareto-optimal outcome in environmental standards and overall welfare.

The remainder of this paper is structured as follows. In Section 2, we lay out the analytical framework for heterogeneous consumers, heterogeneous firms, and green clubs. We then derive the equilibrium outcome under perfect competition. In Section 3, we examine the socially optimal outcome, which serves as the benchmark to show the Pareto sub-optimality of the market equilibrium. In Sections 4 and 5 we focus our analyses on welfare implications of two environmental policies: one involves a single-tool policy on greener production or certification, and the other involves a double-tool policy of both greener production subsidies and a green certification tax. Section 6 concludes.

2. The Analytical Framework

We beginning our analysis by considering green production as a two-stage game in the absence of government intervention.[33] This allows us to examine the equilibrium outcome of the game under the market solution. In the first stage, a green club determines the certification standard that a firm's product should meet in order for the firm to be qualified as a member. Once the green product standard is set, the second stage occurs, and firms determine if they should join the green club and produce certified green products.

To characterize market interaction between firms and consumers, we first discuss the preferences of consumers.

2.1 Heterogeneous Consumers

Following Hotelling's spatial framework, we assume that consumers with heterogeneous preferences are uniformly distributed between zero and one of a market line. However, we determine the consumer's location on the Hotelling line by using the strength of their green preference. For analytical simplicity, we assume that each consumer purchases one unit of a product, whether it is green or non-green. The preference function of a consumer located at x, where $x \in [0,1]$, is specified as follows:

$$U(x) = \begin{cases} v + (1-x)\tau\theta + \beta\psi - (P + P_e) & \text{if purchasing green product} \\ v + \beta\psi - P & \text{if purchasing non-green product} \end{cases} \tag{1}$$

where v represents the utility from the non-green product, P represents market price of the non-green product, P_e represents the mark-up for the green product, θ is the abatement or "environmental friendliness" of the green product, and τ scales the "warm glow" or utility from

[33] Another stage of policy implementation is added when we evaluate regulatory implications.

consuming the green product.[34] Therefore, the degree of a consumers' environmental conscientiousness in purchasing the green product is represented by $(1-x)\tau\theta$. This means that consumers close to zero (one) place a high (low) value on green product. As in van't Veld and Kotchen (2011), we use β to capture the benefit to the public of having one unit of the green product. The overall benefit (i.e., positive externality) to the public of the green product market is then measured by $\beta\psi$, where $\psi = n\theta$ and n is the total quantity of the green product sold in the market. Note that the value of n remains to be determined in equilibrium.

Setting the utility from green consumption equal to the utility from non-green consumption, we have from (1) that $v+(1-x)\tau\theta+\beta\psi-(P+P_e)=v+\beta\psi-P$, which implies that the marginal green consumer or the quantity of the green product demanded (n_D) is:

$$n_D = x^D = 1 - \frac{P_e}{\theta t} \tag{2}$$

The number of green consumers is represented by the interval $[0, x^D]$, while the number of non-green consumers is $[x^D, 1]$.[35] Next, we discuss the production decisions of firms.

2.2 Heterogeneous Firms

Similar to Mason (2011), we examine the scenario where consumers cannot identify a firm's environmental friendliness, thus firms must join a club in order for environmental friendliness to be recognized. Borrowing from van't Veld and Kotchen (2011), we assume that the cost of managing a green club is increasing in the number of member firms. But, we explicitly

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[34]Here we assume that θ and τ are not correlated. However, Teisl et al.(2002) suggest that one aim of green labels is to "educate consumers about the environmental impacts of the product's manufacture, use, and disposal, thereby leading to a change in purchasing behavior…" Thus, it's possible that higher standards could actually shift user utility green products by increasing awareness of their negative impacts. However, we leave this for a future study.

[35] As the number of consumers is normalized to one, we have $0 \le 1-(P_e/\theta\tau) \le 1$, which implies that $\theta\tau \ge P_e \ge 0$.

represent club costs, and assume they are quadratic in the number of members: $\mathrm{Cl}(\mathrm{y}^S) = \gamma(y^S)^2$, where y^S is the number of firms that join the club. The club's costs thus represent the expenses of having products inspected and certified. Since costs are shared equally among all member firms, each individual's membership fee is γy^S. As a result, the membership fees received by the club are equal to the club's operating costs.

To reflect the heterogeneity of firms, we consider a case similar to Fischer and Lyon (2014), where firms are uniformly distributed according to their differing marginal costs of production. As in Baron (2011), we assume that each firm produces one unit of product (green or non-green). Following Lombardini-Riipinen (2005) and Garcia-Gallego and Georgantzis (2009), we further consider that abatement associated with the green product is quadratic in cost. The profit function for an arbitrary firm, y, is then represented by:

$$\Pi(y) = \begin{cases} P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y & \text{if producing a green product in a club} \\ P - c - yk & \text{if producing non-green product} \end{cases} \tag{3}$$

where k represents the highest level of Ricardian rent,[36] c represents the marginal cost of non-green production, and ε represents marginal cost of increasing a products' cleanliness. We assume that firms that are more efficient (or have higher Ricardian rents), also have lower abatement costs. The market is competitive, but all firms have non-negative profits with the non-green product, so $P = c + k$. Note that y is not bounded, thus allowing for free entry and exit, however, for any firm, y, where $y > 1$, then $\Pi(y) < 0$.

By equating the green product profit with the non-green profit, we can discern the marginal firm who is indifferent to either production type. We have from (3) that

[36] If firm located at 0, produced a non-green product, then it's profit would be $\Pi(0) = P - c - k(0) = (k + c) - c = k$. Thus, there Ricardian rent is k. Similarly, at firm located at "1", receives no profit or Ricardian rent.

$P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y = P - c - ky$. Solving the equation gives the marginal green firm[37] or the quantity of the green product supplied (n_S):

$$n_S = y^S = \frac{P_e}{\varepsilon\theta^2 + \gamma} \tag{4}$$

Thus the number of green firms is represented by the interval [0, y^S], while the number of non-green firms is $[y^S, 1]$.[38]

2.3 Market Solution with a Green Club

In equilibrium, the quantity of a green product demanded is equal to that of the green product produced.[39] Denoting n in the subsequent analysis as the *equilibrium* quantity of the green product sold, we have from (2) and (4) that

$$n_D = n_S = n \;\; \Rightarrow \;\; \frac{\theta\tau - P_e}{\theta\tau} = \frac{P_e}{\varepsilon\theta^2 + \gamma} \tag{5}$$

This implies that the equilibrium premium for the green product satisfies the following condition:

$$P_e = \frac{\tau\theta(\varepsilon\theta^2 + \gamma)}{\theta(\varepsilon\theta + \tau) + \gamma} \tag{6}$$

Substituting P_e from (6) back into (2) yields

$$n = \frac{\tau\theta}{\theta(\varepsilon\theta + \tau) + \gamma} \tag{7}$$

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[37] Note that the marginal green firm, y, has the property: $y = y^S$.

[38] We have the additional restriction that $0 \le P_e \big/ (\varepsilon\theta^2 + \gamma) \le 1$, which implies that $0 \le P_e \le \varepsilon\theta^2 + \gamma$.

[39] The number of green products consumed is $n = \min\{x^D, y^S\}$, where y^S is the number of green products produced, therefore in equilibrium $n = x^D = y^S$.

Instead of specifying one coordinator of the club such as industrial group, government, or environmentalist clubs, we consider a club that emphasizes product differentiation. Our approach is congruent to those of Baron (2011), who recognizes that an organization (or in our case, a club) "provides product differentiation that segments the market." This means that the club maximizes the green product's cleanliness standard. It is important to note that a club is likely to face multiple objectives. To gain validity and differentiate their member's products, a club must set and enforce higher standards. In addition, a club requires firm participation in order for their labeling to be recognizable in the marketplace, thus club has a desire to increase membership. Conversely, firms will not join a club unless there is sufficient demand for a green product.[40] Therefore, our modeling coincides with the incentives clubs and firms will realistically face.[41] Specifically, for the firms it requires both a green price premium ($P_e \geq y\varepsilon\theta^2 + \gamma y$) and sufficient demand for the green product ($n \geq y$).[42]

This means that a green club still maximizes its membership, but maintains an equilibrium in the green product market. Obviously, if firms cannot sell their green product they have no incentive to pay for club membership; similarly if a shortage of the green product exists, clubs will seek more members or higher standards. The cleanliness standard that maintains participation can be identified by taking the first-order condition for the club with respect to the level of environmental friendliness, which yields

$$\frac{\partial n}{\partial \theta} = \frac{\tau(\gamma - \varepsilon\theta^2)}{(\varepsilon\theta^2 + \tau\theta + \gamma)^2}$$

[40] This condition ensures that the green club's presence is welfare-increasing. This is not a necessary condition, but removing it could make the green clubs presence decrease social welfare (see Mattoo and Singh 1994).
[41] For analysis on different club types see van't Veld and Kotchen (2011). Additional club types can be examined in our structure, however, our focus is policy and welfare implication, thus we leave that topic for future study.
[42] The value of n should satisfy the following condition $0 \leq \theta\tau \big/ (\varepsilon\theta^2 + \tau\theta + \gamma) \leq 1 \Rightarrow$ $-(\gamma/\gamma) \leq \theta^2 \Rightarrow \theta > 0.$

Solving for the optimal standard specified by the green club (denoted by the superscript "*GC*") yields:

$$\theta^{GC} = \sqrt{\frac{\gamma}{\varepsilon}} \tag{8}$$

Substituting θ^{GC} in (8) back into n in (7), we have the equilibrium number of green firms in the market as:

$$n^{GC} = \frac{\tau}{(\tau + 2\sqrt{\gamma\varepsilon})} \tag{9}$$

It follows immediately from (9) that $0 < n^{GC} < 1$. Similarly, we substitute θ^{GC} from (8) into (6) to identify the green product premium:

$$P_e^{GC} = \frac{2\tau\gamma}{\tau + 2\sqrt{\gamma\varepsilon}} \tag{10}$$

Using θ^{GC} in (8) we obtain the following comparative-static derivatives:

$$\frac{\partial\theta^{GC}}{\partial\tau} = 0,\ \frac{\partial\theta^{GC}}{\partial\varepsilon} = -\frac{\sqrt{\gamma\varepsilon}}{2\varepsilon^2} < 0,\ \text{and}\ \frac{\partial\theta^{GC}}{\partial\gamma} = \frac{1}{2\sqrt{\gamma\varepsilon}} > 0 \tag{11}$$

From (11), there are several interesting observations. First, consumer's cleanliness preference does not influence the club standard. This could be interpreted as consumer's ability to encourage the existence of a standard, but not to influence the level. This may seem odd at first, but if we assume that green consumer would prefer all their products be clean, green clubs should be observed in every industries. However, green products will only be brought to market if firms can remain profitable while providing the products. All this depends firms' costs, so higher environmental cleanliness or club costs determine whether a firm is subject itself to the standard.

Next, from n^{GC} in (9), we obtain:[43]

$$\frac{\partial n^{GC}}{\partial \tau} = \frac{2[\tau^2\sqrt{\gamma\varepsilon} - 4\gamma\varepsilon(\tau + \sqrt{\gamma\varepsilon})]}{(\tau^2 - 4\gamma\varepsilon)^2} > 0 \tag{12A}$$

$$\frac{\partial n^{GC}}{\partial \varepsilon} = \frac{-\tau\gamma}{\sqrt{\gamma\varepsilon}(\tau + 2\sqrt{\gamma\varepsilon})^2} < 0 \tag{12B}$$

$$\frac{\partial n^{GC}}{\partial \gamma} = \frac{-\tau\varepsilon}{\sqrt{\gamma\varepsilon}(\tau + 2\sqrt{\gamma\varepsilon})^2} < 0 \tag{12C}$$

The signs in equations (12) come as no surprise. As preferences for green products increase so does club participation. For firms, as the club membership or abatement costs increase, it disincentives club membership for marginal firms.

Lastly, from P_e^{GC} in (10), we obtain:

$$\frac{\partial P_e^{GC}}{\partial \tau} = \frac{4\gamma(\tau^2\sqrt{\gamma\varepsilon} - 4\tau\gamma\varepsilon + 4\gamma\varepsilon\sqrt{\gamma\varepsilon})}{(\tau^2 - 4\gamma\varepsilon)^2} > 0 \tag{13A}$$

$$\frac{\partial P_e^{GC}}{\partial \varepsilon} = \frac{-2\tau\gamma^2}{\sqrt{\gamma\varepsilon}(\tau + 2\sqrt{\gamma\varepsilon})^2} < 0 \tag{13B}$$

$$\frac{\partial P_e^{GC}}{\partial \gamma} = \frac{2\tau(\tau + \sqrt{\gamma\varepsilon})}{(\tau + 2\sqrt{\gamma\varepsilon})^2} > 0 \tag{13C}$$

Equations (13) show some interesting results with regards to the green price premium. First, as club membership fees rise the green price premium increases. However, when abatement costs increase, the green price premium decreases. The reason is that increasing abatement costs leads to lower standards being set by the club which reduces the level of product differentiation between green and non-green products. As a consequence, the green price premium decreases.

[43] It will be shown that in order for the club to exist and have members $\tau > 2\sqrt{\varepsilon\gamma}$ must hold.

This, combined with our previous result, means that greater consumer preferences for a green product do not result in higher cleanliness standards set by the club, but instead affect the price of the green products.

With all the results from the above comparative statics, we establish the first Proposition:

Proposition 1. *In a Hotelling-type spatial economy in which consumers choose between green and non-green products and heterogeneous firms may join a green club in order for environmental friendliness to be recognized (through green product certification), we have the following results:*

(i) *An increase in* τ, *the degree of consumers' environmental conscientiousness, increases both the quantity and price of the green product sold in the market. But the optimal level of environmental standard set by the green club is unaffected by* τ.

(ii) *An increase in* ε, *the cost of abatement, reduces the quantity and price of the green product, while decreasing the green product's standard.*

(iii) *An increase in* γ, *the club membership cost, raises the green product's standard and price, but lowers the quantity of the green product sold.*

3. Evaluating the Market Solution from the Social Welfare Perspective

In this section, we derive the social welfare measures for the Hotelling's spatial market presented in the above sections. This allows us to calculate the benefits derived from the market solution with a green club and compare it to the social planner's solution. A welfare comparison between the alternative scenarios will allows us to identify whether the market solution can maximize social welfare, and help determine the regulatory role of the government (if any).

3.1 Social Welfare in the Market Solution

We begin with the calculation of consumer benefits. Note that $\beta\psi$ is the external benefit to the society from the sale of the green product, where $\psi = n\theta$. Integrating over all consumers buying either green or non-green products, the consumer surplus measure is given by

$$CS = \int_0^x [v + (1-x)\tau\theta + \beta\psi - (P + P_e)]dx + \int_x^1 (v + \beta\psi - P)dx \tag{14A}$$

In equilibrium, the quantity of green product sold (n) is equal to the number of green consumers (x). Using the competitive market property that the non-green product price is equal to its value or $P = v$, we simplify the expression in (14A) to be[44]

$$CS = \beta\theta n + (1/2)\theta\tau x(2-x) - P_e x \tag{14B}$$

The consumer surplus measure in (14B) has three terms: public green benefit, private green benefit, and green premium, respectively.

Similarly, integrating over all firms producing either green or non-green products, the producer surplus measure is given by

$$PS = \int_0^y (P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y)dy + \int_y^1 (P - c - yk)dy \tag{15A}$$

As before, we incorporate the competitive market property associated with the non-green product that its price is equal to the cost of production for the marginal firm, that is, $P = c + k$, we simplify the expression in (15A) to be[45]

$$PS = (1/2)\left(k + 2P_e y - (\varepsilon\theta^2 + \gamma)y^2\right) \tag{15B}$$

[44] For a detailed derivation of the consumer surplus measure, see Appendix F.
[45] For a detailed derivation of the producer surplus measure, see Appendix F.

The producer surplus measure in (15B) has three terms: Ricardian rents, green price premium, and green cost.

As in the literature, social welfare is taken as the sum of consumer and producer surplus, which yields:

$$SW = \left[\beta\theta n + (1/2)\theta\tau(2-x)x - P_e x\right] + (1/2)\left[k + 2P_e y - (\varepsilon\theta^2 + \gamma)y^2\right] \tag{16A}$$

Evaluating SW in (16A) at the market equilibrium where $x = y = n,$ we have:

$$SW = \theta\beta n + \frac{k}{2} + \frac{n\theta\tau(2-n)}{2} - \frac{(\gamma + \varepsilon\theta^2)n^2}{2} \tag{16B}$$

The four terms that constitute social welfare can be identified as: public green benefit, Ricardian rents, private green benefit, as well as green cost. Substituting θ^{GC} and n^{GC} from (8) and (9) into the welfare function in (16B), after arranging terms, we have:

$$SW^{GC} = \frac{\tau\sqrt{\gamma\varepsilon}(2\beta + \tau)}{2\varepsilon(\tau + 2\sqrt{\gamma\varepsilon})} + \frac{k}{2} \tag{17}$$

Based on SW^{GC} in (17), we have several interesting observations. First, as expected social welfare is strictly increasing with β and $k,$ thus greater public benefits and Ricardian rents result in greater social welfare. Moreover, the comparative static derivatives of SW^{GC} in (17) with respect to $\tau,$ $\varepsilon,$ and γ are:

$$\frac{\partial SW^{GC}}{\partial\tau} = \frac{(\tau - 2\sqrt{\gamma\varepsilon})^2(\tau^2\sqrt{\gamma\varepsilon} + 4\beta\gamma\varepsilon + 4\tau\gamma\varepsilon)}{2\varepsilon(\tau^2 - 4\gamma\varepsilon)^2} > 0 \tag{18A}$$

$$\frac{\partial SW^{GC}}{\partial\varepsilon} = -\frac{\tau\sqrt{\gamma\varepsilon}(2\beta + \tau)[\tau^3 + 4\gamma\varepsilon(4\sqrt{\gamma\varepsilon} - 3\tau)]}{4\varepsilon^2(\tau^2 - 4\gamma\varepsilon)^2} < 0 \tag{18B}$$

$$\frac{\partial SW^{GC}}{\partial\gamma} = \frac{\tau^2(2\beta + \tau)(\tau^2 + 4\gamma\varepsilon - 4\tau\sqrt{\gamma\varepsilon})\sqrt{\gamma\varepsilon}}{4\gamma\varepsilon(\tau^2 - 4\gamma\varepsilon)^2} > 0 \tag{18C}$$

The first two results in (18A) and (18B) are as expected. First, higher preferences for green products yield greater social welfare. Secondly, as the cost of abatement increases, social welfare decreases. The last derivative in (18C) is less intuitive and more significant. For that reason, we state:

Corollary 1. *In the presence of heterogeneous firms, social welfare increases with higher club membership costs.*

Normally, a higher club cost should decrease social welfare since it discourages firms from joining a club and producing green products. However, in the presence of heterogeneous firms the appeal of joining a club puts a pressure on the club to lower the standard and accept firms with higher abatement costs. As shown in (7) and (13C), this lowers the green price premium and increases demand for the green product. If the club membership fee were higher, only firms with low abatement costs would find it beneficial to join. Furthermore, the lower abatement cost firms are more likely to accept a higher standard, which yields higher price premiums. This result is analogous to Buchanan's (1965) Theory of a Club which he describes as a "theory of optimal exclusion, as well as one of inclusion." Basically, the argument is that the club needs members to operate, but the exclusivity of club is directly related to its effectiveness.

To evaluate the efficacy of the market solution with a green club, we need to identify the conditions (in terms of the number of firms producing the green product and the level of environmental standard) under which social welfare is maximized. This leads us to examine the environmental issues from the perspective of a social planner who seeks to maximize overall welfare.

3.2 Optimal Welfare in the Social Planner's Solution

In the framework we consider, the socially optimal outcome is found using the approach by van't Veld and Kotchen (2011). The social planner determines optimal club size and standard, or values of θ and n, that maximize overall welfare as given in (16B). The first-order Kuhn-Tucker conditions are:

$$\frac{\partial SW}{\partial \theta} = \frac{n\left(2\beta + 2\tau - n\tau - 2n\varepsilon\theta\right)}{2} \leq 0 \text{ and } \frac{\partial SW}{\partial n} = \theta\beta - n\gamma + \theta\tau - n\theta\tau - n\varepsilon\theta^2 \leq 0$$

Assuming temporarily that these conditions are binding, the optimal values of θ and n are:

$$\theta = \frac{2\beta + 2\tau - n\tau}{2n\varepsilon} \tag{20A}$$

$$n = \frac{\theta(\beta+\tau)}{\theta(\tau+\varepsilon\theta)+\gamma} \tag{20B}$$

Substituting θ from (20A) into n in (20B) and considering the boundary conditions on the number of consumer's purchasing the green product $(0 \leq n_D \leq 1)$, we obtain candidates for the social planner's (denoted with "*SP*") equilibrium number of green firms:

$$n^{SP} = \left\{0, \frac{2\tau(\beta+\tau)}{\tau^2 - 4\gamma\varepsilon}\right\} \tag{21}$$

Since $0 \leq n^{SP} \leq 1$, this implies that $0 \leq 2\tau^2 + 2\beta\tau \leq \tau^2 - 4\gamma\varepsilon$. In order for the club to exist, the condition that $\tau^2 - 4\gamma\varepsilon > 0$ requires that $\tau > 2\sqrt{\varepsilon\gamma}$. Evaluating the above condition $2\tau^2 + 2\beta\tau \leq \tau^2 - 4\gamma\varepsilon$, implies that $\tau^2 + 2\tau\beta \leq -4\gamma\varepsilon$, which obviously cannot happen. We thus have the inequality condition that $2\tau^2 + 2\beta\tau \geq \tau^2 - 4\gamma\varepsilon > 0$. This indicates that if the market contains a green club then the socially optimal number of green firms is:

$$n^{SP} = 1 \tag{22A}$$

Substituting $n^{SP} = 1$ from (22a) back into θ in (20) yields

$$\theta^{SP} = \frac{2\beta + \tau}{2\varepsilon} \tag{22B}$$

Based on n^{SP}, θ^{SP}, and the social welfare function in (16), we have

$$SW^{SP} = \frac{(2\beta + \tau)^2}{8\varepsilon} + \frac{k - \gamma}{2} \tag{22C}$$

From (22c), the comparative-static derivatives of the social planner's social welfare are:

$$\frac{\partial SW^{SP}}{\partial \beta} > 0, \quad \frac{\partial SW^{SP}}{\partial k} > 0, \quad \frac{\partial SW^{SP}}{\partial \tau} > 0, \quad \frac{\partial SW^{SP}}{\partial \varepsilon} < 0, \text{ and } \frac{\partial SW^{SP}}{\partial \gamma} < 0.$$

As before, public benefits from having a green product, Ricardian rents, and consumer preference for the green product all positively affect social welfare. Additionally, higher abatement costs negatively affect social welfare. One key implication departing from the market solution with a green club is the negative effect of higher club membership costs on social welfare. This seems appropriate, since the social planner decides the club participation and standard, the club costs no longer needs to disincentivize high cost firms from joining a club. To summarize this result:

Corollary 2. *Higher club membership costs decrease the maximum attainable level of social welfare in the social planner's solution. However, higher club costs increase social welfare in the market solution.*

In the previous section, we see that as the club become more exclusive by increasing the standard, members receive a higher green price premium. However, the social planner need not worry about the exclusiveness of the club-determined standard, since it can decide both the level of participation and the standard. Therefore, membership costs become a hurdle for a social planner that negatively affects social welfare.

3.3 Comparison

We begin by examining the differences between the market solution and the social planner's solution. We begin by evaluating the difference in the number of firms producing the green product. In view of (9) and (22A), we see immediately that the market solution yields a lower level of firm participation than the social planner, or to state another way: $n^{GC} < n^{SP}$.

Next, we compare the environmental standard in each scenario by using θ^{GC} in (8) and θ^{SP}, in (22B). Assuming that the market solution yields a higher standard in order to identify conditions where $\theta^{GC} \geq \theta^{SP}$, we have

$$\sqrt{\frac{\gamma}{\varepsilon}} \geq \frac{2\beta+\tau}{2\varepsilon} \tag{23}$$

It can easily be verified that this condition violates the constrained condition that $\tau > 2\sqrt{\gamma\varepsilon}$. Therefore, we conclude that the standard set by the green club in the market solution is strictly below that in the social planner's solution. We thus have

$$\theta^{GC} < \theta^{SP} \tag{24}$$

Finally, we verify differences in overall welfare using SW^{GC} in (17) and SW^{SP} in (22C). Again, we analyze whether the market solution is superior to the social planner's solution, or specifically, $SW^{GC} \geq SW^{SP}$. This yields the following condition:

$$\frac{\tau(\tau\sqrt{\gamma\varepsilon}-2\gamma\varepsilon)\left(2\beta+\tau\right)}{2\varepsilon(\tau^2-4\gamma\varepsilon)}+\frac{k}{2} \geq \frac{(2\beta+\tau)^2}{8\varepsilon}+\frac{k-\gamma}{2} \tag{25}$$

Since $\tau > 2\sqrt{\gamma\varepsilon}$ in the presence of green clubs, the social planner's welfare is relatively higher. That is, $SW^{GC} < SW^{SP}$.[46] We thus have the following Proposition:

Proposition 2. *The market solution has a lower environmental standard, a lower number of green firms, and a lower level of overall welfare relative to the social planner's solution. That is,* $\theta^{GC} < \theta^{SP}$, $n^{GC} < n^{SP}$, $SW^{GC} < SW^{SP}$.

4. Welfare Implications of a Single-Tool Environmental Policy

We have shown that the market solution with a green club is Pareto sub-optimal, so naturally, some regulatory questions arise: What regulatory measures that can be taken by government to correct the Pareto sub-optimality? Will subsidies for greener production standard be a socially optimal policy? In this section, we examine the efficacy of various policies in the presence of a green club. Our approach is similar to that of Heyes and Maxwell (2004), who analyze the effects of regulatory policy and non-government labeling when both occur concurrently in a market. However, our approach allows the club to act as a monitor of the firms actions, thus allowing the government to set regulation according to member firms actions.

We begin by constructing green production as a three-stage game. In the first stage, the government determines subsidies and taxes (either for abatement or club membership) to maximize social welfare. In the second stage, the club sets the maximum level of cleanliness standard that maintains equilibrium in the green product market. In the third and last stage, each

[46] Showing that $SW^{GC} < SW^{SP}$ requires proving that $[(2\beta+\tau)^2 - 4\gamma](\tau^2 - 4\gamma\varepsilon) > 4\tau(\tau\sqrt{\gamma\varepsilon} - 2\gamma)(2\beta+\tau)$. If we let $\tau = 2\sqrt{\gamma\varepsilon} + a$, this condition simplifies to: $a^3 + 4a(\beta^2 + a\beta + \gamma\varepsilon - \gamma) + 4\sqrt{\gamma\varepsilon}[(a+2\beta)^2 + 4\beta\sqrt{\gamma\varepsilon} - 4\gamma] > 0$, which is sufficiently positive whenever $\beta^2 + a\beta + \gamma\varepsilon > \gamma$ and $(a+2\beta)^2 + 4\beta\sqrt{\gamma\varepsilon} > 4\gamma$. We assume these conditions hold.

profit-maximizing firm decides on joining join the club according to the standard and price for the green product.

4.1 Green Production with Abatement Subsidies

In this case, we incorporate a tactic used by Segerson and Miceli (1998), where regulators use a "carrot" approach. The government provides a subsidy (denoted as s_A) to the firm for each unit of abatement. We can identify the number of green firms by solving by the following equality $P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y + s_A\theta = P - c - yk$. This yields the marginal green firm or the quantity of the green product supplied as:

$$n_{S,A} = y = \frac{P_e + s_A\theta}{\gamma + \varepsilon\theta^2} \tag{26}$$

One observation from (26) is that abatement subsidies increase the number of green firms.[47]

As before, we solve for the quantity of the green product sold in the market by setting n_D from (2) equal to $n_{S,A}$ from (26), which yields

$$\frac{\theta\tau - P_e}{\theta\tau} = \frac{P_e + s_A\theta}{\gamma + \varepsilon\theta^2}$$

In equilibrium, the green price premium must satisfy:

$$P_e = \frac{(\varepsilon\theta^2 - s_A\theta + \gamma)\theta\tau}{\gamma + \theta\tau + \varepsilon\theta^2} \tag{27}$$

Substituting P_e back into n_D yields[48]

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[47] The number of green firms must satisfy this condition: $0 \le y \le 1$. This implies that $0 \le (P_e + s\theta)/(\gamma + \theta^2\varepsilon) \le 1$ or to state another way: $0 \le P_e \le \theta^2\varepsilon + \gamma - s\theta$.

[48] Note the condition that $0 \le [(s+\tau)\theta]/(\varepsilon\theta^2 + \tau\theta + \gamma) \le 1$ which implies that: $\theta^2 \ge (s\theta - \gamma)/\varepsilon$.

$$n = \frac{(s_A + \tau)\theta}{(\varepsilon\theta + \tau)\theta + \gamma} \tag{28}$$

Taking the derivative of n with respect to θ and setting the resulting expressions to zero yields the optimal standard for the green club in the presence of an abatement subsidy (denoted by "C*A*") scenario. That is,

$$\theta^{CA} = \sqrt{\frac{\gamma}{\varepsilon}} \tag{29A}$$

Substituting θ^{CA} into n from (28), we have

$$n^{CA} = \frac{(s_A + \tau)}{\tau + 2\sqrt{\gamma\varepsilon}} \tag{29B}$$

With (29a), we calculate the equilibrium value of green premium,[49]

$$P_e^{CA} = \frac{\tau\left[2\gamma\varepsilon(s_A + \tau) - (s_A\tau + 4\gamma\varepsilon)\sqrt{\gamma\varepsilon}\right]}{\varepsilon(\tau^2 - 4\gamma\varepsilon)} \tag{29C}$$

Note the absence of the abatement subsidy in the clubs emission standard, while it is present in the green price premium and the equilibrium number of green firms. From (29B), we can see that a higher abatement subsidy leads to more firms joining a club which, in turn, results in a lower green price premium.

Next, we can determine if the abatement subsidy can yield the optimal number of green firms and the optimal emission standard. Setting $n^{SP} = n^{CA}$ leads to a subsidy of:

$$s_A^* = 2\sqrt{\gamma\varepsilon} \tag{30}$$

Therefore, we conclude that obtaining the socially optimal number of green firms is possible with an emissions subsidy.

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[49]The green product premium is positive, $P_e > 0$, if $(s_A\theta - \gamma)/\varepsilon < \theta^2$.

Using the same approach, we next determine the optimal abatement subsidy that generates the social planner's emission standard by solving $\theta^{SP} = \theta^{CA}$, which yields:

$$\frac{2\beta+\tau}{2\varepsilon} = \sqrt{\frac{\gamma}{\varepsilon}}$$

which doesn't hold since $\tau > 2\sqrt{\gamma\varepsilon}$. Therefore, the abatement subsidy can never lead to the same standard. The resulting standard is always below that of the social planner's. We thus have

Proposition 3. *While an emission subsidy can yield the optimal green club participation, there is no emission subsidy that will generate the Pareto-optimal green standard in the market solution. That is,* $\theta^{CA} < \theta^{SP}$, $n^{CA} = n^{SP}$, *and* $SW^{CA} < SW^{SP}$.

4.2 Green Production with Membership Subsidies/Taxes

We next examine the effects of a government subsidy (denoted as s_M) for firms when they join a club. Similar to the previous case, we can identify the number of green firms by solving the following equality: $P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y + s_M = P - c - yk$. Solving for y yields the supply of green products:

$$n_{S,M} = y = \frac{P_e + s_M}{\gamma + \varepsilon\theta^2} \tag{31}$$

Setting demand for green product, n_D, from (2) equal to the new supply of the green product, $n_{S,M}$, from (31), we have

$$\frac{\theta\tau - P_e}{\theta\tau} = \frac{P_e + s_M}{\gamma + \varepsilon\theta^2}$$

In equilibrium, the green product premium must satisfy:

$$P_e = \frac{\tau\theta(\varepsilon\theta^2 - s_M + \gamma)}{\theta(\tau + \varepsilon\theta) + \gamma} \tag{32}$$

Substituting P_e from (32) back into n_D yields the equilibrium number of green firms:

$$n = \frac{s_M + \tau\theta}{\theta(\tau + \varepsilon\theta) + \gamma} \tag{33}$$

Using (33), we solve for the optimal environmental standard for the green club with club membership tax or subsidy (denoted by "C*M*") scenario, which yields

$$\theta^{CM} = \sqrt{\frac{(\gamma - s_M)}{\varepsilon}} \tag{34A}$$

Substituting θ^{CM} from (36a) into n in (34), we have

$$n^{CM} = \frac{\tau[\tau - 2\sqrt{(\gamma - s_M)\varepsilon}]}{\tau^2 - 4\gamma\varepsilon} \tag{34B}$$

With equation (33), we calculate the green price premium as: [50]

$$P_e^{CM} = \frac{2\tau(\gamma - s_M)\sqrt{(\gamma - s)\varepsilon}}{(2\gamma - s_M)\varepsilon + \tau\sqrt{(\gamma - s)\varepsilon}} \tag{34C}$$

Note the presence of the membership subsidy in the club emission standard, green price premium, and the quantity of green firms, unlike the emission subsidy case.

Using the same approach as before, we determine the optimal membership subsidy/tax that results in the social planner's emission standard by solving $\theta^{SW} = \theta^{CM}$, which yields:[51]

$$s_M^* = \gamma - \frac{(2\beta + \tau)^2}{4\varepsilon} \tag{35}$$

[50]Note that $P_e > 0$ if $(s - \gamma)/\varepsilon < \theta^2$.

[51] The associated welfare calculations for the club membership case are provided in the appendix.

Careful examination of (35) shows that the optimal membership subsidy to ensure the social planner's standard is actually a tax.[52] While this may seem counter-intuitive, recall that a club must ensure its members can sell their products. Therefore, if the government taxes club membership, it reduces the number of firms that produce the green product, thus allowing the club to raise its standard. Unlike the emission subsidy scenario, a proper membership tax leads to the socially optimal emission standard. The tax can be considered a certification expense in the same spirit as Hamilton and Zilberman (2006), for that reason we refer to it as an eco-certification tax.

Next, we determine if the proper eco-certification tax can yield the optimal number of firms and emission standards. Assuming that $n^{SW} \le n^{CM}$, it must satisfy the condition $s_M \ge \gamma + \varepsilon\theta^2$. Given that $s_M < \gamma$, there are no values of s_M that satisfy the inequality condition. Therefore, we have

Proposition 4. *While a tax charge for green certification can yield the optimal green standard, there is no membership subsidy/tax that will generate the socially optimal level of green club participation. That is,* $\theta^{CM} = \theta^{SP}$, $n^{CM} < n^{SP}$, *and* $SW^{CM} < SW^{SP}$.

5. Welfare Implications of a Double-Tool Environmental Policy

We have shown that even with an emission subsidy or club membership tax, the equilibrium outcome is Pareto sub-optimal. This suggests that there is no single policy tool capable of achieving the socially optimal outcome in the presence of green clubs. In this section, we evaluate the use of dual tool by regulator, and examine if it can lead to the socially optimal outcome.

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[52] In order for $s_M^* > 0$, $4\gamma\varepsilon < (2\beta + \tau)^2$ must hold. If we let $\tau = 2\sqrt{\gamma\varepsilon} + \alpha$, where $\alpha > 0$, then we can rewrite the previous inequality as: $4\gamma\varepsilon < 4\beta\left(2\sqrt{\gamma\varepsilon} + \alpha + \beta\right) + 4\gamma\varepsilon + \alpha\left(4\sqrt{\gamma\varepsilon} + \alpha\right)$, which cannot hold, thus implies that $s_M^* < 0$.

As in the previous analyses, we construct green production as a three-stage game. However, the first stage differs from our previous set-up. In the first stage, the government determines both the abatement and club membership subsidies/taxes to maximize social welfare. The second and third stages of the game remain unchanged: the club sets the cleanliness standard while maintaining equilibrium in the green product market. In last stage, each firm decides on joining the club according to the demand for the green product.

5.1 A Double-Tool Approach

Similar to the previous section, we begin by introducing the subsidy for green production (S) and subsidy for club membership (ω) in the green firm profit function. As before, we identify the number of firms in the green product market by solving: $P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y + S\theta + \omega = P - c - yk$, for y. This gives the supply of green products:

$$n_{S,D} = y = \frac{P_e + S\theta + \omega}{\gamma + \varepsilon\theta^2} \tag{36}$$

To determine the equilibrium number of green firms, we set the supply and demand for the green product equal to one another using (36) and (2), respectively. This yields

$$n_D = n_{S,D} \;\Rightarrow\; \frac{\theta\tau - P_e}{\theta\tau} = \frac{P_e + S\theta + \omega}{\gamma + \varepsilon\theta^2}$$

Solving for the green product premium gives:

$$P_e = \frac{(\varepsilon\theta^2 - S\theta + \gamma - \omega)\,\theta\tau}{\theta(\tau + \varepsilon\theta) + \gamma} \tag{37}$$

Substituting P_e from (37) back into n_D in (2), we have the equilibrium number of green firms:[53]

$$n = \frac{(S+\tau)\theta + \omega}{\theta(\tau + \varepsilon\theta) + \gamma} \tag{38}$$

Taking the derivative of n with respect to θ and setting the resulting expression to zero, gives the optimal environmental standard for the green club with the dual policy (denoted by "CD") as:

$$\theta^{CD} = \frac{\sqrt{\Phi} - \varepsilon\omega}{\varepsilon(S+\tau)} \tag{39A}$$

where $\Phi \equiv \varepsilon[\gamma(S+\tau)^2 + \omega(\varepsilon\omega - S\tau - \tau^2)]$. Substituting θ^{CD} back into n from (38), we have

$$n^{CD} = \frac{\sqrt{\Phi}(S+\tau)^2}{2\Phi + \sqrt{\Phi}\,(S\tau + \tau^2 - 2\varepsilon\omega)} \tag{39B}$$

Making use of (37) and (39b), we calculate the green price premium as[54]

$$P_e^{CD} = \frac{\tau(\sqrt{\Phi} - \varepsilon\omega)[2\Phi - (\sqrt{\Phi} - \varepsilon\omega)(S\tau + S^2 + 2\varepsilon\omega)]}{\varepsilon(S+\tau)[2\Phi + (\sqrt{\Phi} - \varepsilon\omega)(S\tau + \tau^2 - 2\varepsilon\omega)]} \tag{39C}$$

5.2 Social Planner's Solution with Dual Policy

The eco-certification tax shows up in the club emission standard, green price premium, and the number of green firms, unlike the emission subsidy case. In addition, the abatement subsidy shows up in the green price premium and the number of green firms. This means that potentially, the eco-certification tax could be used to optimize the club standard, while the emission subsidy could be used to optimize club participation.

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[53]Note that $0 \le [(s+\tau)\theta+\omega] / [\varepsilon\theta^2+\tau\theta+\gamma] \le 1$, which implies that: $\theta^2 \ge (\omega+s\theta-\gamma)/\varepsilon$.

[54]Note that $P_e > 0$ if $(s\theta+\omega-\gamma)/\varepsilon < \theta^2$.

We begin by setting the dual policy club standard equal to social planner's club standard, or $\theta^{CD} = \theta^{SP}$, which yields:

$$\frac{\sqrt{\Phi} - \varepsilon\omega}{\varepsilon(S+\tau)} = \frac{(2\beta+\tau)}{2\varepsilon} \tag{40A}$$

Solving for the club eco-certification tax that yields the socially optimal club standard, we have:

$$\omega = \frac{(S+\tau)[4\gamma\varepsilon - (2\beta+\tau)^2]}{8(\beta+\tau)\varepsilon} \tag{40B}$$

Therefore, using (40A) as our club membership policy rule, we ensure the optimal club standard is obtainable. Substituting ω in (40B) back into (39B) yields

$$n^{CD} = \frac{S+\tau}{2(\beta+\tau)} \tag{41}$$

Setting n^{CD} in (41) equal to the socially optimal number of firms, i.e., $n^{CD} = n^{SP} = 1$, and solving for the optimal emission subsidy, we have

$$S^* = 2\beta + \tau \tag{42A}$$

Using S^* in (42A), we simplify the optimal eco-certification tax provided in (40B) to:

$$\omega^* = \gamma - \frac{(2\beta+\tau)^2}{4\varepsilon} \tag{42B}$$

Together, the results in (42A) and (42B) provide a dual policy rule that ensures the first-best solution. Or to state another way:

Proposition 5: *While a single tool policy cannot yield the socially optimal outcome, a dual tool policy is the first best or Pareto optimum, if the government sets dual policy of*

$\omega^{*}=\gamma-(2\beta+\tau)^{2}/4\varepsilon$ *and* $S^{*}=2\beta+\tau$. *That is, given* S^{*} *and* ω^{*}, *we have* $\theta^{CD}=\theta^{SP}$, $n^{CD}=n^{SP}$, *and* $SW^{CD}=SW^{SP}$.

If government adopts a dual policy which combines subsidizes for a greener production standard and taxes for the club membership of green firms, the policy is able to achieve Pareto optimality in environmental standards, the number of green firms, and overall welfare.

6. Concluding Remarks

In this paper, we have endeavored to analyze welfare implications of environmental regulations for an economy in which heterogeneous consumers choose between green and non-green products, and firms may join a green club in order for environmental friendliness to be recognized. In the analysis, we take into account the heterogeneity of firms in production and abatement costs. This allows us examine competitive markets, and analyze how eco-certifications and environmental regulation affect the endogeneity of green production. While previous work has primarily focused on evaluating oligopoly market structures, our results are distinctly different from previous studies and have implications for club and regulatory decisions within a competitive market.

We have shown that club operation in a competitive market is welfare-improving and, similar to Ibanez and Grolleau (2008), decreases the level of pollution. However, it results in a lower number of green firms with a lower environmental standard than is socially optimal. The implementation of environmental policies can help improve Pareto optimality or efficiency. In addition, the use of an abatement subsidy increases club participation, which is welfare-improving, but is not Pareto optimal. Applying an eco-certification tax is also welfare-improving, but is still sub-optimal. Unlike previous research analyzing duopoly markets in the context of eco-labels, our results show that there is no single policy which will yield the socially optimal outcome.

Finally, we suggest the implementation of a mixed policy, which combines subsidizes for a greener production standard and a certification tax for firms that want to join a green club. This policy mix is shown to be Pareto optimal (that is, the first-best optimum) in environmental standards and overall welfare, and therefore shows the potential gains from regulatory involvement in competitive markets with green clubs.

Chapter 3 - R&D Investment in Clean Technology

1. Introduction

The relationship between investment and clean technology has become an important topic in the regulatory and environmental literature. Using a Pigovian approach in high emission industries is not a new topic, but much of the new environmental research has shifted to identifying the incentives of firms to undertake development of clean technology in the context of environmental regulation. Initiating an emissions tax is a common approach to internalize the negative effects of production emissions. However, with the development of new emission-reducing technology identifying the best policy becomes even more challenging. Large emissions taxes may erode a firms' profit, but increases the benefits of developing clean-technology. Conversely, smaller tax rates allow firms to enjoy higher profits, providing firms additional resources to develop clean technology.

If we view clean technology development as a special case of innovation,[55] the effects of environmental regulation can extend beyond a firm's emissions, but influence the development of technology. Several papers have evaluated the relationship between innovation and competition. Arrow (1962) showed that the incentive to invest may be greater under monopoly than in a competitive setting. However, Schumpeter (1934) showed a positive relationship between innovation and market power. This demonstrates a glimpse of how challenging it is to identifying firm's incentive to innovate. Contemporary work such as Cellini and Lambertini (2009) has started to evaluate how R&D structure affects innovation. One interesting question that appears not to

[55] For a richer discussion about innovation and clean technology, see Jaffe et al. (2002).

have been systematically analyze concerns how the introduction of environmental policies change a firm's incentive to undertake R&D in clean technology.

Analyzing the impacts of environmental policy on clean technology development under imperfect competition has a renewed interest in the environmental literature. Contemporary research has expanded on previous findings by including important characteristic into the market framework, such as: evaluating the transfer of environmental technology between countries (Iida and Takeuchi; 2011), licensing technology (Kim et al., 2012) and how regulation affects a firm's decision to develop clean technology "in house" or license it (Heyes and Kapur, 2011).

Previous literature has overwhelming shown how environmental policy incentivizes clean technology development. This research points out an important relationship between taxes and investment, which facilitates a strategic approach, such as those by Ulph and Ulph (2007) and Greaker and Rosendahl (2008). Thus, our understanding of a firm's decision in the face of environmental policy helps identify optimal tax policies (Kurtyka and Mahenc, 2011). Work by Canton et al. (2008) indicates the challenges of identifying optimal tax rate in the presence of imperfect competition, while showing the relationship between abatement and production. Although the strategic approach can also include a myriad of details, the development of technology is frequently limited to the static case. As revealed by Beladi, Liu, and Oladi (2013), a dynamic approach has the benefit of showing the long-run environmental implications of a policy. Furthermore, studies by Malueg and Tsutsui (1997); and Cellini and Lambertini (2009) show how important a dynamic approach is to R&D.

We hope to show that by incorporating a dynamic structure into the development of clean technology, we obtain better understanding of a firm's response to regulation. By using an optimal control and dynamic game approach, we can determine a firm's investment "path" in the presence of a given policy, as opposed to the firm's one time investment decision. While the static approach can provide an insightful "snapshot," the additional details provided in a dynamic setting are paramount in the context of cumulative effects, such as those exhibited by R&D. Feichtinger et al. (2014) highlights the richness obtained by using a dynamic approach. They construct a non-cooperative differential game, where firm decide to undertake R&D projects for emission abatement, in the presence of a (constant endogenous) Pigovian tax policy. The authors' results show the existence of an open-loop equilibrium and show a concave relationship between investment and the number of firms thus creating an inverted-U investment curve. With the exception of Feichtinger et al. (2014), much of current environmental research treats clean technology development and pollution abatement in the static setting.

The purpose of the present paper is to identify a firm's clean-tech investment strategy in response to an emissions tax when R&D is represented dynamically. This approach has the benefit of capturing explicitly the cumulative effect of R&D investment within an environmental framework. Using this structure, we examine three different investment strategies (competition, cooperation, and merging) under duopoly, and identify each firm's optimal R&D response in the context of firm emissions, environmental damage, and social welfare. Furthermore, our research shows the relationship between emissions taxes and firm's clean-tech investment, thus providing details about incentives created by emissions taxes.

In contrast to the traditional static analysis, which does not allow for temporal or cumulative effects of R&D, our dynamic analysis has implications for emission tax policy and cooperation in environmental innovation to improve overall welfare. The key findings of the present study are as follows: (i) As an emission tax or the level of emissions decreases, the incentive to invest in clean technology decreases. (ii) A welfare-improving emissions tax policy requires that the tax rate adjust with the development of clean technology. (iii) Emissions per unit of output and total environmental damage are *lowest* when firms cooperate in R&D, relative to the scenarios when they compete in R&D or merge into a single entity. (iv) A forward-looking firm increases total surplus and has lower environmental damage than a myopic firm. (v) Social welfare is at the highest level under the cooperative R&D regime.

The remainder of the paper has the following structure. Section 2 first presents the set-up of each firm's dynamic optimization problem when duopolists compete in clean-technology R&D in the presence of an emissions tax. We then show the dynamic movements of emissions and investments, and discuss emission tax implications. In section 3, we further derive the steady state solutions for the three alternative regimes: R&D competition, R&D cooperation, and the merging into a single entity. Section 4 evaluates and compares the three different regimes in terms of their effects on environmental damage, firm profits, consumer surplus, and social welfare. Finally, Section 5 concludes.

2. Modeling Emissions & Clean Technology Innovation

As in Cellini and Lambertini (2009), we consider a market served by a Cournot duopoly, with inverse demand: $P = a - (q_1 + q_2)$, where q_i is the quantity of output produced by firm (i = 1,2) and a represents the choke price. We further assume that each unit of output by firm i yields E_i units of pollution, which is taxed at rate τ.[56] For convenience, we assume marginal cost is zero.

In addition, the government exogenously sets the emission tax, which remains constant. This is an important divergence from Feichtinger (2014), which uses an (constant) endogenous tax

[56] To guarantee that each firm produces a positive quantity of output, it will be shown that this emission tax rate satisfies the following condition: $0 < \tau < a/E_i$.

rate. By endogenizing the tax rate, they are forced to optimize social welfare in the steady-state only, whereas we make no assumption about the time period being optimized by the implemented policy. We do this for several reasons. First, environmental regulation is highly politicized, and therefore updates or changes occur infrequently and may not be optimal[57]. Second, even in the face of popular support for an environmental regulation, governments may still implement a constant (potentially sub-optimal) tax rate[58]. Finally, we will show the problems that arise from instituting a dynamic endogenous tax rate. However, our approach will provide several useful insights about the construction of a welfare enhancing emission policy.

2.1 Dynamic R&D Optimization: Problems of Duopolistic Firms

Each period firm *i*'s operating profit after paying emission taxes is $\pi_i = \left(P - \tau E_i\right) q_i$. Substituting in the inverse demand gives firm *i*'s profit as $\pi_i = \left[a - (q_1 + q_2) - \tau E_i\right] q_i$. This indicates that each firm will not produce if at any time $a \le \tau E_i$. From the profit function, we can obtain firm *i*'s best response function as $q_i = (a - q_j - \tau E_i)/2$ for $i, j = 1, 2$ and $i \ne j$. Substituting firm *j*'s best response function into firm *i*'s best response function, we have firm *i*'s optimal output equation as $q_i = (a - 2\tau E_i + \tau E_j)/3$. Since our focus is investment in clean technology substituting firm *i*'s optimal output into its profit function allows us to express each firm's Cournot profit as a function of both firms' emissions:: $\pi_i = (a - 2\tau E_i + \tau E_j)^2 / 9$.

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[57] Several examples exist where policies are enacted but haven't been updated, such as carbon taxes (of various levels) in European countries. The U.S. gasoline tax could also be viewed as an emission tax, which has not been updated since 1997 (Federal Highway Administration website, *Highway History* Retrieved May 2014) . In 1980, Sweden instituted a nitrogen oxide emissions tax, which wasn't updated until 2008 (OECD Environmental Policy Paper No.2 Dec. 2013). Regardless, these policies do not change without the creation and implementation of new policy.

[58] As mentioned in "We have a winner" article from *The Economist*, British Columbia instituted a carbon tax of C$10 in 2008, with incremental increases until it reached C$25 in 2012. A poll mentioned in the article states that " the tax is popular: it is backed by 54%." As of the writing of this article, there have been no changes to the carbon tax rate.

Firms invest in clean technology in order to reduce their tax burden and maximize their individual profits. Firms reduce their emissions through R&D in process innovation, which results in pollution abatement during production. For this reason, we would expect marginal cost of production to be unaffected by a firm's emission reductions, thus its omission is appropriate.[59] As frequently assumed in the R&D investment literature, each firm's expenditure on the development of clean technology is taken to be a quadratic function. That is, this expenditure function is V_i^2, where V_i is the level of emissions-reduction investment by firm *i*. Thus, firm i selects the level of investment in each period in order to maximize the current-value of its' profit function, according to the following dynamic optimization problem:

$$\max_{V_{i,t}} \Pi_i = \int_0^{\infty} \left[\frac{1}{9}(a - 2\tau E_{i,t} + \tau E_{j,t}) - V_{i,t}^2 \right] e^{-\rho t} dt \quad \text{s.t.} \quad \dot{E}_{i,t} = (\delta - \beta V_{i,t}) E_{i,t} \qquad E_i(0) = E_o \tag{1}$$

where $i = 1,2$ for $i \neq j$, ρ is the discount rate, δ is the cost of maintaining clean technology, and β is a parameter measuring the effectiveness of an investment to curbing emissions, E_o is the initial emission rate per-unit of production. We assume that each firm's technology development affects emissions during production and does not have any spillover effect, thus our approach omits any feedback effects.

The dynamic optimization framework as specified in (1) has several advantages over the traditional static case. First, by using this approach, each firm's decision to invest incorporates both the long-run profits and maintenance costs of (clean) technology. In addition, the inclusion of discount rate allows us to distinguish between a forward-looking or myopic firm, while also

[59] Emissions reduction through product innovation is certainly a topic of interest; however, it would require inclusion of marginal cost and would most likely further complicate any analysis. Furthermore, we would expect there to be spillover effects from any development, since rival firms can obtain, and therefore analyze/copy any product. For that reason, we leave this as a topic for future study.

embracing the cumulative nature of R&D and applying it to emission reducing technology. Lastly and most importantly, we can construct a firm's investment strategy (over time) when faced with an emission tax.

2.2 Dynamic Game of Firms under Duopoly

From the dynamic optimization problem in (1) for each firm, we can determine the optimal levels of investment by the firms. In order for our approach in a dynamic game to be useful, the equilibrium results need to be sub-game perfect. This means that each firm's R&D strategy is optimal in every period, regardless of the values of the state variable.[60] To begin, we first derive the firm's current-value Hamiltonian (for brevity the time subscript is omitted) as:

$$H_i = \frac{1}{9}(a - 2\tau E_i + \tau E_j) - V_i^2 + \lambda_i E_i(\delta - \beta V_i) \tag{2}$$

where λ_i is the costate variable associated with firm i's per-unit emissions, thus it represents the marginal value of an additional unit reduction of emissions for each good produced. Since we assume that firm i's R&D does not affect firm j's emissions, we can omit the equation of motion for firm j's emission.[61] From equation (2), we calculate the maximum principle conditions as:

$$\frac{\partial H_i}{\partial V_i} = -2V_i - \beta \lambda_i E_i \tag{3A}$$

$$\dot{E}_i = \frac{\partial H_i}{\partial \lambda_i} = (\delta - \beta V_i) E_i \tag{3B}$$

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[60] For a discussion on open loop and feedback strategies, see Kamien and Schwartz (2012 p. 275)

[61] To show that spillover is absent, we can evaluate: $H_i = (a - 2\tau E_i + \tau E_j)/9 - V_i^2 + \lambda_i E_i(\delta - \beta V_i) + \lambda_j E_j(\delta - \beta V_j)$. This yields one addition expression then those in equation (3): $\dot{\lambda}_j - \rho\lambda_j = -\frac{\partial H}{\partial E_j} = \lambda_j \beta V_j - \frac{\tau}{9} - \delta\lambda_j$. While both firms' emissions are present in equations (3a), (3b), and (3c), the co-state variable associated with firm j's emission (λ_j) and firm j's investment (V_j) are absent. This implies that firm i's investment decision is independent of firm j's investment, or equivalently, there is no research spillover present, and the transition equations for firm j's emission can be omitted from equation (2). Of course, the effects of spillover are a topic of interest, however, we leave this for future study.

and the associated co-state equations:

$$\dot{\lambda}_i - \rho\lambda_i = -\frac{\partial H_i}{\partial E_i} = \frac{4\tau}{9}(a - 2\tau E_i + \tau E_j) - \delta\lambda_i + \lambda_i \beta V_i \tag{3C}$$

Setting the derivative in equation (3A) to be zero, we find that the firm's optimal investment (denoted by superscript "*") is:

$$V_i^* = \frac{-\beta\lambda_i E_i}{2} \tag{4}$$

As noted by Feichtinger (1983), the absence of feedback effects ensures that the open loop solution is sub-game perfect. Evaluating the feedback effects using equations (2) and (4), yields: $-\frac{\partial H_i}{\partial V_j}\frac{\partial V_j^*}{\partial E_i} = (-\beta)(0) = 0,$ thus affirming the absence of feedback effects. With this condition satisfied, the open-loop Nash equilibrium is sub-game perfect.[62]

2.3 Dynamic R&D Optimization: Problem of a Monopoly

Next, we evaluate how tax policy affects a firm's investment strategy. In order to isolate the policy effects from those of competition, we evaluate the investment decisions of a monopoly. The firm's profit function can easily be derived using the same demand equation and setting $q_j = 0$. Therefore the monopolist's dynamic optimization problem is:

$$\max_{V_{i,t}} \Pi_i = \int_0^\infty \left[\frac{1}{4}(a - \tau E_{i,t})^2 - V_{i,t}^2 \right] e^{-\rho t} dt \qquad \text{s.t.} \quad \dot{E}_{i,t} = (\delta - \beta V_{i,t}) E_{i,t} \qquad E_i(0) = E_o \tag{5}$$

and the associated present-value Hamiltonian is:

$$H_i = \frac{1}{4}(a - 2\tau E_i)^2 - V_i^2 + \lambda_i E_i(\delta - \beta V_i) \tag{6}$$

---

[62] For additional information on open-loop games and the approach to prove an open-loop game is sub-game perfect see Cellini and Lambertini (2009).

From equation (6), we calculate the maximum principle conditions:

$$\frac{\partial H_i}{\partial V_i} = -2V_i - \beta \lambda_i E_i \tag{7A}$$

$$\dot{E}_i = \frac{\partial H_i}{\partial \lambda_i} = (\delta - \beta V_i) E_i \tag{7B}$$

and the associated co-state equation:

$$\dot{\lambda}_i - \rho \lambda_i = -\frac{\partial H_i}{\partial E_i} = \frac{\tau}{2}(a - \tau E_i) - \delta \lambda_i + \lambda_i \beta V_i \tag{7C}$$

As before, the firm's optimal investment (denoted by superscript "*M*") is found by setting the derivative in equation (7A) to zero:

$$V_i^M = \frac{-\beta \lambda_i E_i}{2} \tag{8}$$

Obviously, the absence of a rival firm removes concern about feedback effects, thus we can disregard concerns about the results being sub-game perfect. Taking the derivative of (8) with respect to time, yields

$$\dot{V}_i^M = \frac{\partial V_i^M}{\partial t} = -\frac{\beta}{2}(\lambda_i \dot{E}_i + \dot{\lambda}_i E_i) \tag{9}$$

Substituting (7B), (7C), and (8) into the firm's investment equation of motion from (9) yields the dynamic investment equation as a function of the state and control variables:

$$\dot{V}_i^M = \rho V_i^M - \frac{\beta \tau E_i}{4}(a - \tau E_i) \tag{10}$$

Equations (7B) and (10) constitute a complete system of a firm's investment and emissions in a dynamic analysis. Plotting the movement on a phase-plane diagram yields potential investment strategies for emission reducing technology.

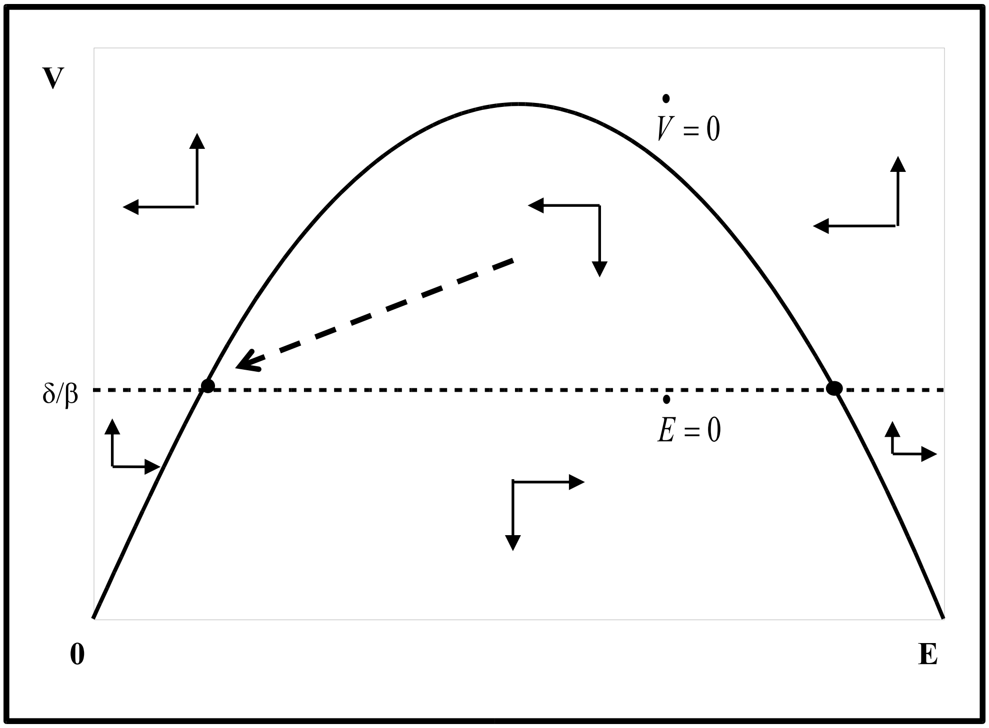

Figure 1 Phase-plane diagram showing optimal R&D investment path

Figure 1 presents a graphical illustration of how each firm's optimal investment path depends on the initial emissions level, with two (non-trivial) nodes[63]. The horizontal axis measures the level of per-unit emissions and the vertical axis measures the level of R&D investment. Two isoclines (emissions and investment) are drawn, which shows two points where a variable does not change over time. As illustrated in Figure 1, the unstable node is of little importance, since no optimal investment path lead to it. The stable node is a saddle-point equilibrium, which is optimally approached from the northeast saddle-path. This indicates that each firm invests heavily in clean technology initially and gradually investment tapers off. Eventually, additional investment no longer will yield a large enough reduction in emissions relative to its maintenance cost, so each firm invests only enough to sustain the current level of technology.

[63] A third and fourth node exist. However, they correspond to when the firm either: 1) does not invest in clean technology, or 2) shuts down. These are omitted for obvious reasons.

Setting the dynamic investment equation in (10) to zero, we determine the steady state relationship between investment and emissions as follows:

$$V_i = \frac{1}{4\rho}\beta\tau E_i(a-\tau E_i) \tag{11A}$$

It follows directly from (11A) that

$$\frac{\partial V_i}{\partial \tau} = \frac{\beta E_i(a-2\tau E_i)}{4\rho} \tag{11B}$$

This permits us to establish the following Proposition:

Proposition 1. *Higher emission taxes decrease the steady state level of investment in clean technology if the following condition holds:* $\tau > \frac{a}{2E_i}$; *otherwise increasing the tax rate increases investment.*

Proof: Examining the derivative in (11B), we see that

$$\frac{\partial V_i}{\partial \tau} < 0 \text{ if } \tau > \frac{a}{2E_i}, \text{ and that } \frac{\partial V_i}{\partial \tau} > 0 \text{ if } \tau < \frac{a}{2E_i}. \qquad \text{Q.E.D.}$$

As expected, an increase in the benefits from developing clean technology encourages higher investment, and vice versa. This result from Proposition 1 supports the conclusions of Porter and Linde (1995) which state that "regulation creates pressure that motivates innovation and progress." This holds true for either production with higher per-unit emissions or from higher emission tax rates. While the effects of emissions tax appear to mirror previous findings from the static case, our objective is to show how the results will diverge over time. However, at higher tax rates or per unit emissions, investment can be crowded out due to the tax burden, or to state another way:

Corollary 1. *As the level of emissions decline, the firm eventually reduces its level of investment.*

This shows that our approach already incorporates supplementary details omitted in static analysis. In addition, evaluating the effects of the discount rate in equation (11A), we can identify how a firm's outlook/behavior affects it's investment decision. Specifically,

$$\frac{\partial V_i}{\partial \rho} = -\frac{\beta\tau(a-\tau E_i)}{4\rho^2} < 0 \tag{11C}$$

From this, we have the following result:

Corollary 2. *Firms who are more myopic in their outlook, relative to forward looking firms, invest less in clean technology.*

While this result may be expected, it further illustrates the limitations of static analysis in the presence of cumulative effects. Furthermore, it shows that both firm behavior and profit can influences technological development. Identifying how environmental policy can alter the firm's incentive is of obvious importance; therefore, we proceed by evaluating the properties associated with the emission taxes.

2.4 Emission Taxes

Traditionally, the optimal tax rate is obtained by determining the social welfare maximizing point, where private benefits (firm revenue) equals marginal costs (firms cost and emission damage). The tax rate is then assigned so the firm's profit maximizing level of production coincides with social welfare. However, in the dynamic setting, the level of emissions changes intertemporally until it reaches the steady state.

In Figure 2, we show different steady-state levels of emissions, which correspond to different tax rates**.** As can easily be seen from Figure 2, the resulting steady-state level of emissions depends not only on the tax rate, but also on the productivity of emissions-reduction investment and the maintenance costs.

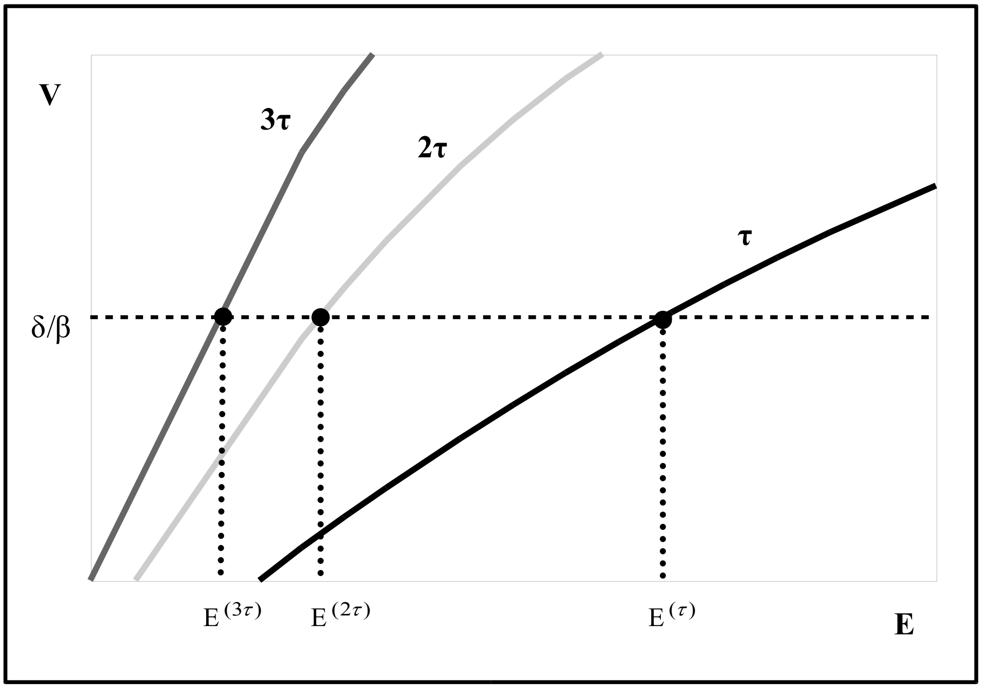

Figure 2 Effects of taxes on the steady-state equilibrium levels of emissions

At first glance, one can notice that any increase in the tax rate is met with a lower steady state per-unit emission level. Nevertheless, firms offset higher emissions taxes by further increasing investment for the development of clean technology. As illustrated in Figure 2, the tax rate influences a firm's investment strategy, and therefore its emissions. The dynamic investment equation from (8) provides additional policy implications, specifically:

Corollary 3. *Other things being equal, any positive tax rate on emissions ensures that a polluting firm has an incentive to develop clean technology.*

The tax rate certainly affects R&D investment, thereby influencing the progress of clean technology development. Any corrective (Pigovian) tax instituted in the initial period (or any period before the steady state) will only be equivalent to the negative emission externalities in that period. However, from an environmental standpoint, any (positive) emission tax will incentivize firms to reduce emissions. While the tax may not perfectly match the damage done by emissions, the additional cost the firm incurs encourages it to reduce the costs associated with any emissions. For that reason, creating the incentive to innovate and invest in clean technology could still lead to significant emission reductions, thus indirectly accomplishing potential emission-cutting objectives without being equivalent to the negative externality. But why not identify the optimal tax? The next section will help show the infeasibility of an optimal tax in a dynamic setting.

3. Steady-State Equilibrium Analyses of Three R&D Regimes

3.1 Optimization Problems

In this section, we present steady state analysis for three R&D investment strategies in the development of clean technology: competition, merging, and cooperation. In the R&D competition regime, firm i independently determines its investment (V_i), and both firms compete via the product market. This regime was constructed in Section 2.2, and the firm's current value Hamiltonian is represented in equation (2).

In the second regime, the firms merge to form a monopoly, which then chooses the total output and total investment. As with any monopoly, the merged entity has an incentive to decrease the level of production relative to the output under non-cooperative or cooperative R&D regime in order to obtain monopoly profit. Furthermore, with a lower level of production the merger's overall tax burden is relatively lower. It is then reasonable to expect that the monopoly has the least incentive to invest in clean technology. The dynamic model of monopoly was also previously constructed in Section 2.3, and the firm's current value Hamiltonian is provided in equation (6).

In the R&D cooperation regime, duopolistic firms cooperatively research and jointly determine their investment by agreeing and matching contributions (thus $V_i = V_j = V$).[64] Since firms work together, investment in clean technology development is made by both firms, and therefore both firms reap the rewards of the emissions-reducing technology that is developed. However, firm's still compete via the product market. Therefore firm *i*'s dynamic optimization problem is:

$$\max_{V_t} \Pi_i = \int_0^{\infty} \left[\frac{1}{9}(a - 2\tau E_{i,t} + \tau E_{j,t})^2 - V_t^2 \right] e^{-\rho t} dt \quad \text{s.t.} \quad \dot{E}_{i,t} = (\delta - 2\beta V) E_{i,t} \quad E_i(0) = E_o \tag{12}$$

and the associated present-value Hamiltonian is:

$$H_i = \frac{1}{9}(a - 2\tau E_i + \tau E_j)^2 - V^2 + \lambda_i E_i (\delta - 2\beta V) + \lambda_j E_j (\delta - 2\beta V) \tag{13}$$

Similar to the previous case, the open loop solution is sub-game perfect; however, the reason is distinctly different. Since the firms enter into an investment agreement, there is no concern about deviation from the equilibrium path, thus making the result time consistent.

From equation (13), we calculate the maximum principle conditions:

$$\frac{\partial H_i}{\partial V} = -2V - 2\beta \lambda_i E_i - 2\beta \lambda_j E_j \tag{14A}$$

$$\dot{E}_i = \frac{\partial H_i}{\partial \lambda_i} = (\delta - \beta V) E_i \tag{14B}$$

and the associated co-state equations:[65]

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[64] For a slightly different example of cooperative research, see Cellini and Lambertini (2009) and Poyago-Theotoky (2007). Both authors evaluate spillover in the context of innovation, and discuss when firms coordinate the level of research investment or form "research cartel." Our focus is cooperative efforts, thus the firms jointly invest in research to improve technology. This is synonymous to the case where spillover is perfect.

[65] Choosing the investment level (V) that jointly maximizes both firms' profit, yields the following Hamiltonian: $H_i = (1/9)(a - 2\tau E_i + \tau E_j)^2 + (1/9)(a - 2\tau E_j + \tau E_i)^2 - V^2 + \lambda_i E_i (\delta - 2\beta V) + \lambda_j E_j (\delta - 2\beta V)$. Taking into account firm symmetry $(E_i = E_j = E)$, gives $H_i = (2/9)(a^2 - 4\tau^2 E^2) - V^2 + 2\lambda E(\delta - \beta V)$, which yields the same level of emissions and output.

$$\dot{E}_i - \rho\lambda_i = -\frac{\partial H_i}{\partial E_i} = \frac{4\tau}{9}(a - 2\tau E_i + \tau E_j) - \delta\lambda_i + 2\lambda_i \beta V \tag{14C}$$

$$\dot{\lambda}_j - \rho\lambda_j = -\frac{\partial H_i}{\partial E_j} = \frac{-2\tau}{92}(a - 2\tau E_i + \tau E_j) - \delta\lambda_j + \lambda_j \beta V \tag{14D}$$

As before, the firm's optimal investment (denoted by superscript "C") is found by setting the derivative in equation (14A) to zero:

$$V^C = -\beta\lambda_i E_i - \beta\lambda_j E_j \tag{15}$$

In order to simplify the calculations, we assume that the firms are symmetrical (i.e., $E_i = E_j = E$). Thus far, we have constructed three different research scenarios, and validated the consistency of their results. The next step is a comparative analysis of each scenario.

3.2 Steady-State Values

With the derived Hamiltonians for the three R&D strategies (from eqn. 2, 6, and 13), we first solve for the steady-state values of control variables (V_i), the state variable (E_i), and output[66] for the R&D competition scenario (denoted with "$*$") using equations (3). These results are reported as follows:

$$V^* = \frac{\delta}{\beta} \tag{16A}$$

$$q^* = \frac{1}{6\beta}\left(a\beta + \sqrt{a^2\beta^2 - 18\delta\rho}\right) \tag{16B}$$

$$E^* = \frac{1}{2\beta\tau}\left(a\beta - \sqrt{a^2\beta^2 - 18\delta\rho}\right) \tag{16C}$$

Using equations (7) and (14), we can derive the maximum principle conditions for the remaining scenarios. From this, we calculate the steady-state equilibrium values for the R&D

[66] Note that $q_i = (a - 2\tau E_i + \tau E_j)/3$ t for duopolists and $q_i = (a - \tau E_i)/2$ for monopolist.

cooperation (denoted with "*C*") and merger (denoted with "*M*") scenarios. These two sets of results are reported, respectively, as follows:

$$V^C = \frac{\delta}{2\beta}; \qquad V^M = \frac{\delta}{\beta} \tag{17A}$$

$$q^C = \frac{1}{6\beta}(a\beta + \sqrt{a^2\beta^2 - 9\delta\rho}); \qquad q^M = \frac{1}{4\beta}(a\beta + \sqrt{a^2\beta - 16\delta\rho}) \tag{17B}$$

$$E^C = \frac{1}{2\beta_\tau}(a\beta - \sqrt{a^2\beta^2 - 9\delta\rho}); \quad E^M = \frac{1}{2\beta_\tau}(a\beta - \sqrt{a^2\beta^2 - 16\delta\rho}) \tag{17C}$$

From (16B) and (17B), we see that the tax rate is absent from the firms production level, which is different from the static case (see previous section for static output). The shows that the optimal steady-state production is independent of the emission tax rate. Note from equation (16C) that the discount rate and the cost of maintaining clean technology must satisfy the following constrained condition: $0 \le \rho\delta \le (a^2\beta^2)/18$, in order for each firm to undertake R&D investment in clean technology in each scenario. We assume that this condition holds, and conveniently define $X = (18\rho\delta)/(a^2\beta^2)$. Therefore, as the temporal effects potentially vary between zero and $a^2\beta^2/18$ in strength, the value of *X* falls within the following (unit) range: $0 \le X \le 1$. This normalization approach allows us to simplify the temporal effects, without ignoring them.[67]

4. Comparing Alternative R&D Regimes

Having derived the steady-state equilibrium solutions for the scenarios of R&D competition, cooperation, and merging, we evaluate and compare their differences in terms of effects on environmental damage, consumer surplus, firm profits, and social welfare.

[67] Values of *X* close to zero will be associated with either a forward-looking firm and/or technology with greater longevity. Furthermore, values close to one will be associated with myopic firms and/or technology that wears out quickly. Since we are not imposing any expectation or restrictions on the type of firm or technology, this will be useful to determine the results for a variety of scenarios.

Using the steady-state values for the control and state variables, we can calculate consumer surplus and social welfare according to the following equations:

$$CS = \frac{1}{2}(q_i + q_j)^2 \tag{18A}$$

$$SW = CS + \pi_i + \pi_j + \tau(E_i q_i + E_j q_j) - \psi(E_i q_i + E_j q_j)^2 \tag{18B}$$

where ψ is a positive parameter which is used to represent the marginal damage of pollution to the environment. Note that $T = \tau(E_i q_i + E_j q_j)$ is the total amount of emission taxes collected and $D = \psi(E_i q_i + E_j q_j)^2$ is total damage to the environment.

It is important to recognize that the consumer surplus and social welfare equations used here evaluate individual time periods. Given that our focus is on identifying the R&D strategy with the best steady state result this is appropriate; calculating the cumulative effects would yield the same result. Since values found in the steady state will hold for any future time period, any difference would be minimal and occur on the path to a steady-state.

4.1 Emissions & Environmental Damage

Using the equilibrium levels of emissions as shown in equations (16C) and (17C), we compare each clean technology investment strategy, which allows us to state the following:

Proposition 2. *Under an exogenous emissions tax policy, per-unit emissions are lowest when firms develop clean technology cooperatively.*

Proof: Substituting $\rho\delta = (Xa^2\beta^2)/18$ into equations (16C) and (17C), we compare emissions for all values of X $(0 \le X \le 1)$ and obtain the following inequality: $E^* > E^M > E^C$. Q.E.D.

From the standpoint of environmental innovation, Proposition 2 indicates that cooperative development unambiguously results in the "cleanest" products. Normally, a monopoly is beneficial in an environmental setting, specifically with conservation or resource extraction due to the limited production. However, our dynamic analysis shows that although a monopoly finds it profitable to reduce production, it has less incentive to undertake R&D investment in clean technology. Therefore, determining whether lower production or greater investment will yield lower overall emissions is important from an environmental stand-point. In addition, traditional environmental policies focus on minimizing the overall level of environmental damage needs to differentiate these effects.

Note that the environmental damage portion of social welfare in equation (18B) is $D = \psi(E_i q_i + E_j q_j)^2$. Making use of the market output and the per-unit emission in equations (16B), (17B), (16C), and (17C), we calculate the overall environmental damage for the three alternative regimes as follows:

$$D^* = \frac{\psi X^2 a^4}{36\tau^2}; \quad D^C = \frac{\psi X^2 a^4}{144\tau^2}; \quad D^M = \frac{\psi X^2 a^4}{81\tau^2} \tag{19}$$

The values from (19) show environmental damage is inversely related to the emission tax. This comes as no surprise, as the per-unit emissions tax increases, firms have a larger incentive to cut emissions. Comparing the values in (19) allow us to establish:

Proposition 3. *Total damage to the environment is lowest in the cooperative clean-technology, and highest in the competitive clean-technology investment cases.*

Proof: Comparing the values of environmental damage as shown in (19), we have $D^* > D^M > D^C$. Q.E.D.

If the sole purpose of the environment regulation is to diminish total emissions, competition in R&D investment should be discouraged. From the perspective of environmental innovation for reducing emissions, competition in the development of technology leads to higher investment expenditures (see equations (16A) and (17A)), but less effective clean technology. While our analysis assumes that the firms are symmetric, competition in the development of clean technology will not guarantee that the cleanest technology is utilized in a goods production. The benefit of cooperative or monopoly development is that it assures that any clean technology or innovation that is developed will be used in production of all goods.

4.2 Consumer Surplus

A policy's overall effect on a market needs to include both the costs and benefits to consumers and producers in order to determine its effectiveness. Substituting the steady-state values of outputs from (16B) and (17B) into equation (18A), we calculate consumer surplus for the three alternative regimes as follows:

$$CS^{*} = \frac{a^2}{18}(1+\sqrt{1-X})^2; \quad CS^{C} = \frac{a^2}{18}(1+\sqrt{1-\frac{X}{2}})^2; \quad CS^{M} = \frac{a^2}{288}(3+\sqrt{9-8X})^2 \qquad (20)$$

It follows from (20) that $CS^C > CS^* > CS^M$. While the merging or cooperation regime yields the lowest total emissions, consumers benefit the most from a cooperative R&D investment approach due to competition in the output market. We can separate the benefits from a cooperative R&D investment strategy into two different effects: first, the efficiency of joint-development of clean technology or the "public good" of cleaner air, which benefits consumers and non-market participants as an externality (environmental damage). The second is the price effect from market competition for output. Thus, cooperative strategy unambiguously bestows the greatest benefits for consumers.

The environmental benefits are also an important result in the context of consumers who may also be concerned with environmental issues such as emissions (so called "green consumers" Sengupta 2012) when purchasing a good (for other examples see Kurtyka and Mahenc 2011; Gori and Lambertini 2013). Since cooperation yields the highest consumer surplus and produces the cleanest product, this has implications for environmentally conscience consumers. Despite our exclusion of consumer's preferences for "clean" products, it's safe to assume that environmentally conscience consumer would unambiguous prefer the cooperative strategy.

4.3 Firm Profits

Next we identify the R&D strategy would yield the highest profit for a firm. Using equations (16A), (16C), (17A), and (17C), we calculate firm profits for the three alternative regimes as follows:

$$\pi^* = \frac{a^2(2-X+2\sqrt{1-X})}{36} - \frac{\delta^2}{\beta^2} \tag{21A}$$

$$\pi^C = \frac{a^2(4-X+2\sqrt{4-2X})}{72} - \frac{\delta^2}{4\beta^2} \tag{21B}$$

$$\pi^M = \frac{a^2(9-4X+3\sqrt{9-8X})}{72} - \frac{\delta^2}{\beta^2} \tag{21C}$$

A comparison of these values reveals that $\pi^* < \pi^M$ and $\pi^* < \pi^C$. We thus have:

Proposition 4. *Each firm's optimal investment strategy is dependent on the discount rate and the maintenance costs. If these rates are such that the following condition is satisfied:*

$$\left(5+3\sqrt{9-8X}+2\sqrt{4-2X}-3X\right)a^2\beta^2 < 54\delta^2 \quad or \quad 9\sqrt{1-\frac{16\rho\delta}{a^2\beta^2}}-4\sqrt{2-\frac{18\rho\delta}{a^2\beta^2}} < \frac{54\delta^2-5}{a^2\beta^2},$$

then $\pi^C > \pi^M > \pi^*$*, otherwise,* $\pi^M > \pi^C > \pi^*$.

Proof: This comes directly from equations (21), by comparing the values of X and δ. Note that X is positively related to δ, and only the negation of δ enters into the firm's profit equation. Thus, higher maintenance costs decreases revenue and therefore profit. Q.E.D.

With higher investment costs and lower output prices, it comes as no surprise that the R&D competition strategy has the lowest profit for each firm. However, the optimal strategy under duopolistic competition depends crucially on the nature of clean technology in an industry. As shown by Proposition 3, industries with higher costs of maintaining clean technology lend itself to joint-development in clean technology. This is directly related to the saving that occurs from the lower cost of R&D investment and maintenance.

As the cost of maintaining clean technology decreases, eventually, the benefits of monopoly pricing will exceed the additional costs of investment by a single firm relative to a cooperative approach. This suggests that polluting firms, which operate in an industry with a low maintenance costs in clean technology, *ceteris paribus*, have an incentive to merge.

4.4 Social Welfare

Using the social welfare equation in (18B), we calculate the steady-state values for the three alternative R&D regimes as follows:

$$SW^{*}=\frac{(4+X+4\sqrt{1-X})a^2}{18}-\frac{9\delta^2\psi\rho^2}{\beta^4\tau^2}-\frac{2\delta^2}{\beta^2} \tag{22A}$$

$$SW^{C}=\frac{(8+X+4\sqrt{4-2X})a^2}{36}-\frac{9\delta^2\psi\rho^2}{4\beta^4\tau^2}-\frac{\delta^2}{2\beta^2} \tag{22B}$$

$$SW^{M}=\frac{(27+4X+9\sqrt{9-8X})a^2}{144}-\frac{4\delta^2\psi\rho^2}{\beta^4\tau^2}-\frac{\delta^2}{\beta^2} \tag{22C}$$

Each of the social welfare equations has three distinct terms. The first term measures the sum of consumer surplus, firm profits, and tax revenue and hence can be referred to as market benefits. The second term measures pollution damage, and the third term is investment cost. Because the firm's cost of an emissions is perfectly offset by the tax revenue generated, the emission tax rate is absent from the market benefit expression. However, the tax rate is still present in the pollution damage expression. This is appropriate since at any finite tax rate, a firm will still pollute if the benefits exceed the cost of production and emission tax burden.

Looking at equations (22), we can easily rank the investment costs associated with each strategy, and determine that they are highest under R&D competition, but lowest under R&D cooperation. As previously determined, pollution damage is highest in the R&D competition regime, with the cooperative and monopoly setting yielding the same level of environmental damage. Combining these two results, we see that the combined R&D costs and pollution damage are unambiguously lowest in the cooperative setting. Furthermore, the monopoly strategy has lower cost and pollution damage relative to the R&D competition approach. However, this still omits the market benefits, and in order to identify the optimal strategy from the social welfare perspective, we must determine the strategy with the highest net benefit.

As shown in figure 3 the market benefits are not consistent for all values of X. While we can easily see that the cooperative R&D strategy yields the greatest market benefits for all values of X, we cannot identify the worst strategy. Looking at the temporal effects, we can state the following:

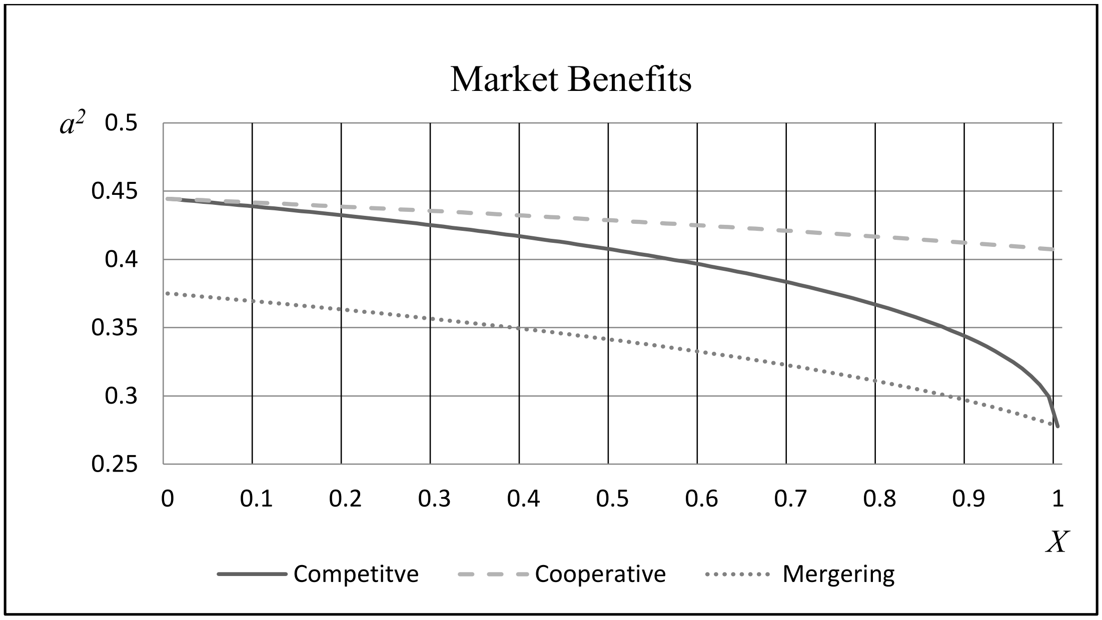

Figure 3 Market benefits under alternative R&D regimes

Proposition 5. *As the temporal effects increase due to the firm being more myopic or higher maintenance costs, the market benefits and social welfare unequivocally decrease.*

Proof: This follows directly from equation (22), taking derivative of the first term with respect to X yields negative values for any X, such that $0 \leq X \leq 1$. Q.E.D.

Proposition 5 shows a definite benefit for firms that are forward looking, as opposed to myopic. In addition, the maintenance of technology has an important role in environmental policy. Higher costs have a detrimental effect by reducing the market benefits through eroding the benefits of R&D investments. Regardless, we know that environmental damage and investment cost are lowest under R&D cooperation, using the results shown in figure 1, we have

Proposition 6. *Social Welfare is unambiguously the highest under the cooperative R&D strategy. However, the strategy that results in the lowest social welfare depends on the values of* ρ δ *and* ψ.

Proof: Evaluating equation (22) for every potential value of *X*, we know that market benefits are highest in the R&D cooperative setting, which was also previously shown to have the lowest cost. We can therefore conclude that, among the three scenarios we consider, social welfare is highest with R&D cooperation. Q.E.D.

While we have shown that R&D cooperation is unequivocally the best strategy for maximizing social welfare, the worst strategy cannot unequivocally be determined. For products with lower (higher) maintenance costs, merger (R&D competition) strategy yields the lowest social benefit. However, depending on investment cost and pollution damage, social welfare of the R&D competition strategy may potentially be smaller, regardless of the costs of maintaining clean technology. The R&D competition scenario, relative to the merger, yields the largest market benefits, but also the largest investment costs and environmental damage. This highlights an area of additional research since the results may contradict Feichtinger (2014) inverted-U investment function, which may be due to the clear distinction in policy construction.

One last result applies to introduction and governance of emission policies. Obviously, if new emission policies tax at a high enough rate, firms may be forced to exit the market (that is, $q_i = 0$ whenever $\tau > a/E_i$). However, low emission taxes create little incentive, thus an effective tax policy must encourage investment without pushing firms out of the market. Therefore, we can make one additional policy observation from equation (22) which shows that social welfare is monotonically increasing in τ.

Proposition 7. *In the presences of environmental innovation, a welfare-improving emission tax policy must take into account the rate of technological progress (and/or reductions in emissions).*

Proof: Making use of equation (22), we take the limit of social welfare with respect to the tax rate as $\tau \to \infty$ and find that the total environmental damage approaches zero, thus improving overall welfare.[68] Q.E.D.

This last result is important for environmental policy. This indicates that an emission tax that incorporates technological progress will enhance social welfare. Static analysis cannot identify the effects of a policy over time. By omitting the dynamic effects of clean technology innovation in the presence of an emission policy, firm investment strategies are not represented properly. In addition, without properly constructed environmental policies the firm incentive to invest corrodes over time, which is detrimental to social welfare.

The proof of Proposition 7 also shows the futility of deriving a dynamic endogenous tax policy. The optimal policy would maximize social welfare in each period (one could argue the appropriate weights to use in each time period), which would then maximize overall welfare. Regardless, any such policy would fail to have a terminal point besides abating all emissions, but as shown in the proof above emissions would be asymptotic to zero. This result, combined with additional insights provided by our approach, show the failings of using an endogenous policy (constant or dynamic) in continuous time.

5. Concluding Remarks

Our dynamic, albeit simple, design has several benefits over the traditional approach. First, we are able to incorporate the cumulative nature of technology development into the firm's investment decision under imperfect competition. Second, we obtain the firm's optimal investment

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[68] Note that this is evaluating the steady-state welfare, thus as τ increases each firm has made the necessary investment V_i in order to remain profitable. This further indicates that E_i has been sufficiently reduced by clean technology to satisfy $0 < \tau < a/E_i$.

"path." Finally, by identifying the dynamic investment equation, implications for environmental innovation and social welfare can be determined.

The analysis in this paper yields several important results. We can determine the optimal strategy to produce the cleanest product, reduce overall emissions, or maximize social welfare. We have shown in a dynamic setting that, among the three R&D regimes we consider, social welfare is highest under R&D cooperation. However, firm profits may be highest in the merger case, thus showing an incentive to merge. This creates obvious anti-trust application. Even though the merger policy yields the lowest possible level of total environmental damage, both consumer surplus and social welfare are higher when non-merging firms cooperative in clean technology R&D. One obvious extension is to identify policies to encourage the firms to select the optimal strategy to maximize welfare.

We have demonstrated how important dynamics is for analyzing clean technology and pollution abatement. With few exceptions, traditional environmental research has focused on static evaluations of clean technology. Frequently, the static representations show that emission taxes' significantly impact both the firms' output and investment decisions; however, our dynamic approach shows that the firm's steady state cost of clean technology and output is determined independently of the emissions tax. However, the emission tax still impacts the firm's decision to develop clean technology and its rate of investment.

Explicitly taking into account the temporal effects associated with R&D investment in clean technology, we find that the long-run impacts of environmental policies may starkly deviate from the immediate or short-run impacts. We have also identified several properties for welfare-improving emission policies in the presence of clean technology development under imperfect competition. Furthermore, the use of a dynamic approach highlights how a firm's response to policy may diverge from those identified in a static setting. The differences between dynamic and static analyses have important policy implications for environmental innovation, tax policy, and social welfare. The application of dynamic optimization in environmental policy is an interesting topic requiring additional research.

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Appendix A - Proof of Proposition 1

Using the assumption that $4c\tau > \theta^2$, we can evaluate the stability of the dynamic system created by equations (8B) and (9). This yields the following Jacobian matrix:

$$J^M = \begin{bmatrix} \frac{\partial \dot{V}}{\partial V} = (\delta + \rho) & \frac{\partial \dot{V}}{\partial A} = -\frac{1}{4\beta\tau}\gamma\left(\theta^2 - 4c\tau\right) \\ \frac{\partial \dot{A}}{\partial V} = \gamma & \frac{\partial \dot{A}}{\partial A} = -\delta \end{bmatrix}$$

Solving for the eigenvalues of the Jacobian matrix yields:

$$\lambda_1^M = \frac{1}{2}\rho - \frac{1}{2}\sqrt{\frac{1}{\beta\tau}\left(\gamma^2\left(4c\tau - \theta^2\right) + 4\beta\tau\delta^2 + 4\beta\tau\delta\rho + \beta\tau\rho^2\right)},$$

$$\lambda_2^M = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\frac{1}{\beta\tau}\left(\gamma^2\left(4c\tau - \theta^2\right) + 4\beta\tau\delta^2 + 4\beta\tau\delta\rho + \beta\tau\rho^2\right)}$$

With the restriction on Θ^2, we can verify that $\lambda_1^M > 0$ and $\lambda_2^M < 0$, thus indicating that the system reaches steady state corresponding to a saddle point.

Appendix B - Comparative Dynamics (Monopoly)

Taking the derivative of the monopolist's steady state values (note that $4c\tau > \theta^2$), yields:

$$\frac{\partial V_i^M}{\partial \tau} = -\theta\gamma\delta A_o \frac{4c\gamma^2+4\beta\delta^2+4\beta\rho\delta}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^2} < 0$$

$$\frac{\partial A_i^M}{\partial \tau} = -\theta\gamma^2 A_o \frac{4c\gamma^2+4\beta\delta^2+4\beta\rho\delta}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^2} < 0$$

$$\frac{\partial P_i^M}{\partial \tau} = -2\theta^2\gamma^2 A_o \frac{c\gamma^2+\beta\delta^2+\beta\rho\delta}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^2} < 0$$

$$\frac{\partial \pi_i^M}{\partial \tau} = -4A_o^2 \frac{\left(c\gamma^2+\beta\delta^2+\beta\rho\delta\right)\left(4\tau c^2\gamma^4-c\theta^2\gamma^4+8\tau c\beta\gamma^2\delta^2+8\tau c\beta\gamma^2\delta\rho-\theta^2\beta\gamma^2\delta^2+\theta^2\beta\gamma^2\delta\rho+4\tau\beta^2\delta^4+8\tau\beta^2\delta^3\rho+4\tau\beta^2\delta^2\rho^2\right)}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^3} < 0$$

$$\frac{\partial CS^M}{\partial \tau} = \frac{-4A_o^2\left(c\gamma^2+\beta\delta^2+\beta\rho\delta\right)^2\left(\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^3} < 0$$

Taking the derivative of the monopolist's steady state values w.r.t. initial product appeal (and again, noting that $4c\tau > \theta^2$), yields:

$$\frac{\partial V_i^M}{\partial A_o} = \frac{\theta\gamma\delta}{-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta} > 0$$

$$\frac{\partial A_i^M}{\partial A_o} = \frac{\theta\gamma^2}{-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta} > 0$$

$$\frac{\partial P_i^M}{\partial A_o} = \frac{2\tau\left(c\gamma^2+\beta\delta^2+\beta\rho\delta\right)}{-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta} > 0$$

$$\frac{\partial \pi_i^M}{\partial A_o} = \frac{2A_o\left(4\tau c^2\gamma^4-c\theta^2\gamma^4+8\tau c\beta\gamma^2\delta^2+8\tau c\beta\gamma^2\delta\rho-\theta^2\beta\gamma^2\delta^2+4\tau\beta^2\delta^4+8\tau\beta^2\delta^3\rho+4\tau\beta^2\delta^2\rho^2\right)}{\left(-\theta^2\gamma^2+4c\tau\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta\right)^2} > 0$$

Appendix C - Proof of Proposition 3

As before, we can evaluate the stability of the dynamic system created by equations (20B) and (23). This yields the following Jacobian matrix:

$$J^* = \begin{bmatrix} \frac{\partial \dot{V}}{\partial V} = (\delta + \rho) & \frac{\partial \dot{V}}{\partial A} = \frac{c\gamma}{\beta} \\ \frac{\partial \dot{A}}{\partial V} = \gamma & \frac{\partial \dot{A}}{\partial A} = -\delta \end{bmatrix}$$

Solving for the eigenvalues of the Jacobian matrix yields:

$$\lambda_1^* = \frac{1}{2}\rho - \frac{1}{2}\sqrt{\frac{1}{\beta\tau}\left(4c\gamma^2 + 4\beta\delta^2 + 4\beta\delta\rho + \beta\rho^2\right)},$$

$$\lambda_2^* = \frac{1}{2}\rho + \frac{1}{2}\sqrt{\frac{1}{\beta\tau}\left(4c\gamma^2 + 4\beta\delta^2 + 4\beta\delta\rho + \beta\rho^2\right)},$$

As before, it is easy to verify that $\lambda_1^* > 0$ and $\lambda_2^* < 0$, thus indicating that the system reaches steady state corresponding to a saddle point.

Appendix D - Comparative Dynamics (Duopoly)

Taking the derivative of the duopolists' steady state values, yields:

$$\frac{\partial P_i^*}{\partial \tau} = 1 > 0 \qquad \frac{\partial \pi_i^*}{\partial \tau} = \frac{1}{2} > 0 \qquad \frac{\partial CS^*}{\partial \tau} = \frac{-5}{8} < 0 \qquad \frac{\partial TS^*}{\partial \tau} = \frac{3}{8} > 0$$

Taking the derivative of the duopolists's steady state values w.r.t. initial product appeal

yields: $\frac{\partial CS^*}{\partial A_o} = \frac{1}{2} > 0$ $\frac{\partial TS^*}{\partial A_o} = \frac{1}{2} > 0$

Appendix E - Comparison of Product Appeal

In order to prove that monopolists innovation will always exceed a duopolists, we assume that $A^* > A^M$, this means: $\frac{\theta\gamma^2}{6c\gamma^2+6\beta\delta(\rho+\delta)} > \frac{\theta\gamma^2 A_o}{(4c\tau-\theta^2)\gamma^2+4\beta\tau\delta^2+4\beta\tau\rho\delta}$

Rewritten, this yields a condition on product differentiation: $\tau > \frac{\theta^2\gamma^2}{4(c\gamma^2+\beta(\rho+\delta))}+\frac{3A_o}{2}$

Next, we identify the necessary condition for the duopolist to fully-serve the market (otherwise both firms act like monopolists).To fully sever the market, the utility from both products must be greater than zero for indifferent user (this is where *x=1/2*), thus: $A_o+\theta A_i-P_i-\tau\left(\frac{1}{2}\right)>0$ or with steady state values: $\tau < \frac{\theta^2\gamma^2}{9(c\gamma^2+\beta(\rho+\delta))}+\frac{2A_o}{3}$

Substituting the maximum value for product differentiation into the condition for greater duopoly innovation, yields: $\frac{\theta^2\gamma^2}{9(c\gamma^2+\beta(\rho+\delta))}+\frac{2A_o}{3} > \frac{\theta^2\gamma^2}{4(c\gamma^2+\beta(\rho+\delta))}+\frac{3A_o}{2}$

Which clearly cannot happen, therefore we can conclude that monopolist will always exceed duopolist level of innovation.

Appendix F - Welfare Calculations

Consumer surplus

Given the preferences of heterogeneous consumers as specified in (1), we have

$$CS = \int_0^x [v + (1-x)\,\tau\theta + \beta\psi - (P + P_e)]dx + \int_x^1 (v + \beta\psi - P)dx$$

Note that $\beta\psi$ is the *external benefit* to the society from the green product's environmental friendliness or abatement, where $\psi = \theta n$ and n is the equilibrium quantity of the green product sold in the market. We then have

$$CS = \int_0^x [v + (1-x)\,\tau\theta + \beta\theta n - (P + P_e)]dx + \int_x^1 (v + \beta\theta n - P)dx$$

which is re-written as

$$\begin{aligned} CS &= \int_0^x [(1-x)\,\tau\theta - P_e]dx + \int_0^x (v + \beta\theta n - P)dx + \int_x^1 (v + \beta\theta n - P)dx \\ &= \left[\tau\theta x - \frac{\tau\theta x^2}{2} - P_e x\right]_0^x + \left[vx + \beta\theta n x - Px\right]_0^1 \\ &= \left[\frac{\theta\tau x(2-x)}{2} - P_e x\right] + (v + \beta\theta n - P) \end{aligned}$$

Competitive market for the non-green product implies that the equilibrium price for the good is equal to its price, that is, $P = v.$ In addition, the equilibrium quantity of the green product sold (n) is equal to the number of green consumers (x). It follows that

$$CS = \beta\theta n + \frac{\theta\tau x(2-x)}{2} - P_e x\ ,$$

where $\beta\theta n$ is public benefit from the green product, $[\theta\tau x(2-x)]/2$ is private benefit to green consumers, and $P_e x$ is the amount of premium to green firms.

Producer surplus

Given the profit functions of green and non-green firms as specified in (3), we have

$$PS = \int_0^y (P + P_e - y\varepsilon\theta^2 - ky - c - \gamma y)dy + \int_y^1 (P - c - yk)dy$$

which is re-written as

$$PS = \int_0^y (P_e - y\varepsilon\theta^2 - \gamma y)dy + \int_0^1 (-ky)dy + (P - c)$$

$$= \left[P_e - \frac{\varepsilon\theta^2 y^2}{2} - \frac{\gamma y^2}{2} \right]_0^y + \left[-\frac{ky^2}{2} \right]_0^1 + (P - c)$$

$$= P_e y - \frac{(\varepsilon\theta^2 + \gamma)y^2}{2} + \frac{k}{2} + (P - c - k)$$

Competitive market for the green product implies that the equilibrium price for the good is equal to the cost of production for the marginal firm, that is, $P = c + k$. We thus have

$$PS = \frac{k}{2} + P_e y - \frac{(\varepsilon\theta^2 + \gamma)y^2}{2}.$$

where $k/2$ is Ricardian rent, $P_e y$ is green price premium, and $(\varepsilon\theta^2 + \gamma)y^2/2$ is green cost.