A COMPARISON OF THE POWER OF THE WILCOXON TEST TO THAT OF THE T-TEST UNDER LEHMANN'S ALTERNATIVES

by

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CONTENTS

CHAPTER I

INTRODUCTION

A statistical test is begun by stating two hypotheses about a parameter of the population. In the context of designed experiments, the null hypothesis and alternative hypothesis describe the behavior of experimental observations in the absence or presence, respectively, of treatment effects. However, if one does not know the precise way in which the treatments affect the observations, the choice of alternative is somewhat arbitrary.

In comparing two populations, one usually assumes that the two population distributions have the same shapes but have different locations, in which case the difference in means or medians between these two populations is the parameter for the statistical test. Such alternatives are called shift alternatives. Let F(x) be the cumulative distribution function (c.d.f.) of population one, G(x) the c.d.f. of population two, and let 'd' represent the difference between the locations of these two populations. The statistical hypotheses for shift alternatives are :

$$H_o: F(x) = G(x) \qquad\qquad H_a: F(x+d) = G(x)$$

The nature of the problem

In many contexts, the shift alternative presented above may be inappropriate. For example observations in educational or psychological studies sometimes reflect a 'ceiling effect' or 'floor effect'. In the other words, possible maximum (or minimum) values, as well as values that are near them, occur with far greater frequency than would be predicted by the shift model. For instance, concerning the scores of an examination in a class, the minimum of the score is zero and maximum 100. The scores of the exam will be bounded by zero and 100 regardless of the treatment effects such as teaching technique, or new facilities, etc. Therefore, the application of $F(x+d)$ may give values that exceed the boundary of the test at one end and present no frequency at the other end. Under this consideration, there is a need to consider other alternatives. Two feasible alternatives which were considered by Lehmann(1953) in dealing with rank tests are as follows:

K-alternative model,

$$G(X) = [F(X)]^k \qquad \text{, where k is a constant.}$$

P-alternative model,

$$G(X) = p\ [F(X)]^2 + (1-p)\ F(X) \qquad \text{,where } 0 <= p <= 1.$$

An example will be given to show that these models satisfy the ceiling effect or floor effect consideration. Select the scores earned on a certain test to be integers ranging from 1 to 5, set k=2.5 in K-model, and p=.45 in P-model. The cumulative probabilities are shown in the table 1-1.

Table 1-1 Example of the K-model and P-model

Score:x	F(x)	$G_k(x)=[F(x)]^k$	$G_p(x)=p[F(x)]^2+(1-p)F(x)$
1	.1	.0032	.0595
2	.3	.0493	.2055
3	.5	.1768	.3875
4	.9	.7648	.8595
5	1.0	1.0000	1.0000

The distribution of $G_k(X)$ and $G_p(X)$ have the desired characteristics, i.e. no impossible scores (less than 1 or greater than 5) is observed, and no score frequency is necessarily set to zero. By analogy with this example, X can also be a continuous random variable.

The significance of problem

The Student-t test is commonly used in testing shift alternatives even in circumstances in which its assumptions are violated. Boneau(1960:63) claimed that the t-test is a remarkably robust test, and concluded

' (as) the sample sizes reach 25 or 30, the approach (to asymptotic normality) should be close enough that one can, in effect, ignore the effects of violation of assumptions except extremes. Since this is so, the t-test is seen to be functionally non-parametric or distribution free.'

Any test or estimate that performs well under violations of underlying assumptions is usually referred to as robust. Some have taken Boneau's assertion of the robustness with respect to type I error to imply optimal power properties for the t-test in nonnormal circumstances, but as Scheffé(1959) has pointed out,

'... robustness for type I error is not sufficient recommendation for a test, the power must also be considered against some alternatives of interest.'.

Blair and Higgins (1980) argued that optimality of the t-test is not a consequence of the Central Limit Theorem, and no mathematical or statistical imperatives ensure the power superiority of the t-test if the shape of the sample population is unspecified. Blair and Higgins (1980: 325) used computer simulation to compare the power of the Wilcoxon test to that of the t-test under shift alternatives and for a variety of distributions and found

' ... (That while the) t-test is sometimes more powerful than the non-parametric procedure, the magnitude of that advantage is never very large and is usually quite modest. On the other hand, the Wilcoxon test often shows power advantages that are very large.'

Both the Boneau and the Blair and Higgins studies were limited to shift alternatives. There appears to be little knowledge about the relative power of the t-test and nonparametric tests under non-shift alternatives.

A comparison of rank tests against K and P alternatives was considered by Lehmann(1953:27). He showed that the power of rank tests in testing these alternatives is independent of the form of F(x) as long as F(x) is continuous. Table 1-2 contains selected power values for 3 tests under K alternatives, under significance level .10.

Table 1-2 Power Comparison Among The Non-parametric tests

(Source : Lehmann (1953))

	m = n = 4		m = n = 6	
	k = 2	k = 3	k = 2	k = 3
T-1	.23	.33	.29	.45
T-2	.31	.47	.38	.59
T-3	.32	.49	.41	.64

T-1: One sided median test.

T-2: One sided Wilcoxon rank sum test.

T-3: Most powerful rank test.

It is interesting to note that the Wilcoxon test has power values comparable to the most powerful rank test while it is considerably more powerful than the median test.

The purpose of this study is to determine whether the t-test or the Wilcoxon rank-sum test is the more powerful test when the alternatives are of the types considered by Lehmann (e.g. K-models, P-models). The distributions for F(x) chosen for this study were: uniform, normal, Laplace, Student-t with 3 degrees of freedom, and Cauchy. This choice provided a range of tail weights under Ho ranging from light tailed to heavy tailed.

CHAPTER II

METHODOLOGY

The methodology employed in this study was computer simulation. Through this technique, samples of various sizes were drawn from populations with known characteristics. The statistics of the Wilcoxon test and the t-test were calculated from the drawn samples, test of significance were carried out, and the rejection or non-rejection of Ho was recorded. The proportion of samples that resulted in the rejection of the null hypothesis is an estimate of the power of the test.(In this study all tests are one-tailed.)

Simulation of the alternatives

Let X be a random variable with continuous c.d.f. F(x). It is well known that the variable

$$U = F(X)$$

has a uniform [0,1] distribution. And the relationship

$$X = F^{-1}(U) \qquad (4)$$

can be applied to simulate the random numbers from F(x). Similarly, since G(x) has the form G(x)=H(F(x)) for the K-models and P-models, we have

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$$G(X) = H(F(X)) = U$$

or

$$X = F^{-1}H^{-1}(U) \tag{5}$$

The numbers from G(X) can be obtained by eqn.(5)

The function F and the inverse F^{-1} of various distributions will be described in the coming section. The inverse of H is derived as follows:

(i) in K-model :

$$G(X) = [F(X)]^{k} = U$$

$$F(X) = U^{1/k} = H^{-1}(U), \qquad (\text{ note: } H(x) = x^{k}.)$$

$$X = F^{-1}(U^{1/k}). \tag{6}$$

(ii) in P-model :

$$G(X) = p \ [F(X)]^{2} + (1-p)\ F(X) = U$$

$$F(X) = \frac{-(1-p)+\sqrt{(1-p)^{2}+ 4\ U\ p}}{2\ p} = H^{-1}(U),$$

$$(\text{ note: } H(x) = p\ x^{2}+ (1-p)\ x.)$$

$$X = F^{-1}(\ H^{-1}(U)\ ) \tag{7}$$

Varieties of distributions

In this study, the populations are selected according to the characteristics of the tails of the distributions. Five symmetric and continuous distributions from light tail to heavy tail (uniform, normal, Laplace, Student-t, and Cauchy) are investigated. The detail of the distribution function and its inverse function are given below.

(1) Uniform (rectangular) distribution

$$f(x) = 1 \qquad ,\text{where } 0 <= x <= 1$$

$$U = F(X) = X$$

(2) Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x/2}$$

$$U = F(X) = \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} e^{-x/2}\, dx$$

The approximation for $F^{-1}(U)$ used here is the one given by Hastings (1955). First obtain

$$Z = F^{-1}(U) = t - \frac{C0 + C1 \cdot t + C2 \cdot t^2}{1 + D1 \cdot t + D2 \cdot t^2 + D3 \cdot t^3}$$

where $t = (-2 \ln p)$,

$p = 1.0 - U$, for $U > .5$

$p = U$, for $0 <= U <= .5$

$C0 = 2.515517$

$C1 = 0.802853$

$C2 = 0.010328$

$D1 = 1.432788$

$D2 = 0.189269$

$D3 = 0.001308$

if $U > .5$, then $X = F^{-1}(U) = Z$, and if $U <= .5$, then $X = F^{-1}(U) = -Z$.

(3) Double exponential (Laplace) distribution

$$f(x) = \frac{1}{2} e^{-|x|}, \qquad -\infty < x < +\infty$$

$$U = F(X) = \begin{cases} \frac{1}{2} e^{x} & ,\text{for } X < 0 \\ 1 - \frac{1}{2} e^{-x} & ,\text{for } X >= 0. \end{cases}$$

$$X = F^{-1}(U) = \begin{cases} \ln(2U) & ,\text{for } 0 <= U < .5 \\ -\ln(2 - 2U) & ,\text{for } .5 <= U <= 1.0 \end{cases}$$

(4) Student-t distribution with 3 d.f.

$$f(x) = \frac{\Gamma(2)}{\Gamma(2/3)\sqrt{3\pi}\,(1+x^2/3)^2},$$

$$U = F(x) = \int_{-\infty}^{x} f(x)\, dx$$

The approximation used here is the one obtained by Wallace(1959). First, approximate F(x) by

$$F(x) = \Phi\left[\frac{(8\ v+1)}{(8\ v+3)} \cdot (v \cdot \ln(1+x^2 \cdot v^{-1}))^{1/2} \right]$$

where Φ is standard normal function and v is a degree of freedom of t-distribution. Since d.f. is set at 3, then the inverse function can be simplified as

$$X = F^{-1}(U) = \sqrt{(e^{z^2 \cdot .3888} - 1) \cdot 3}$$

where Z is the function of U described in the Normal approximation given by Hastings.

(5) Cauchy distribution

$$f(x) = (\ 2\ \pi \cdot (\ 1 + (x/2)^2\ )\ )^{-1},$$

$$U = F(X) = \frac{1}{2} + \frac{1}{\pi} \arctan(x/2),$$

$$X = F^{-1}(U) = 2 \cdot \tan[\ \pi \cdot (U-1/2)],$$

Sample sizes

The sample sizes chosen for this study are (10,10), (5,15), (10,30), (20,20) and (40,40) where the first value is the size of the sample drawn from F(x), the second value, G(x). These particular sample sizes were selected according to the following considerations:

(i) a broad range of sample sizes.

(ii) both balanced and unbalanced data.

Calculation of statistics

(i) t-test

$$t = \frac{\bar{x}_2 - \bar{x}_1}{S_p \sqrt{1/m + 1/n}}$$

where $S_p^{\,2}$ is the pooled variance of s_1^2 and s_2^2. $\bar{x}_1$, m are the mean and the size of the sample of the population in null hypothesis. $\bar{x}_2$, n are of the population in alternative.

(ii) Wilcoxon test

S_n = sum of the rank of the samples drawn from the population whose distribution is G(x)

It is assumed that tied observations do not occur.

Critical values and significance level

(i) t-test

The critical values of t-test are determined from Student t-distribution with (m + n - 2) d.f.

(ii) Wilcoxon test

The critical values of the Wilcoxon test are obtained from tabled values (Bradley 1968), for sample sizes (10,10), (5,15), (10,30), and (20,20). The critical value for sample size (40,40) is obtained from the normal approximation

$$P(\, W_s <= c \,) = \Phi\left[\frac{c - 1/2\ n\ (N+1) + 1/2}{\sqrt{m \cdot n \cdot (N+1)/12}}\right]$$

where Ws is the rank sum statistic and N = m + n . The significance level is .05, so

$$\frac{c - 1/2\ n\ (N+1) + 1/2}{\sqrt{m \cdot n \cdot (N+1)/12}} = 1.645$$

Thus, when m=40, n=40, we find $c = 1790.7534 \doteq 1791$.

Algorithm of simulation

(1) Specify F(x) and G(x).

(2) Select two independent samples of size m and n from the populations being studied.

(3) Compute the statistics of the t-test and the Wilcoxon test from the selected samples.

(4) Compare the statistics with appropriate critical values, and record reject/accept decisions.

(5) After 1029 iterations, the proportion of rejections registered by two statistics are noted as power.

(The data for the uniform distribution over [0,1] are generated by means of the function RANU on the Harris 700 computer.)

CHAPTER III

TABLE OF POWER FUNCTIONS

The results of simulation described in preceeding chapter are presented in tabular form. Table 3-1 indicates the power functions of the t-test and the Wilcoxon test under null hypothesis.(i.e. k=1 in K alternative.). The rest of the tables are identified with 3-K(m,n), or 3-P(m,n),where P and K represents the models of alternatives being studied, and m, n are the sizes of the samples. Every table has the same format. The values of the parameters (k or p) are placed in the first column. The power of the various population distribution are listed by column. In a table, 'T' means the t statistics and the following value is the power of the t-test, 'W' the Wilcoxon test. 'D' is the difference of power between the t-test and the Wilcoxon test. The tables are presented in the remainder of this chapter. Analysis and conclusions are presented in Chapter 4.

Table 3-1
One-tailed power probabilities of the t-test and the Wilcoxon test for the parameter k=1. the significance level at .05.

Sample size	Uniform	Normal	Laplace	Student-t	Cauchy
(10,10)	T .0437	T .0515	T .0457	T .0476	T .0340
	W .0486	W .0583	W .0525	W .0486	W .0622
(5, 15)	T .0525	T .0525	T .0496	T .0447	T .0622
	W .0466	W .0534	W .0525	W .0428	W .0515
(10,30)	T .0408	T .0457	T .0466	T .0457	T .0709
	W .0398	W .0496	W .0476	W .0583	W .0612
(20,20)	T .0457	T .0544	T .0476	T .0476	T .0262
	W .0476	W .0544	W .0515	W .0515	W .0525
(40,40)	T .0398	T .0437	T .0408	T .0505	T .0233
	W .0369	W .0496	W .0408	W .0534	W .0379

Table 3-K(10,10)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (10,10), the significance level at .05.

K	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
1.05	T .0564	T .0593	T .0709	T .0525	T .0301
	W .0603	W .0632	W .0777	W .0603	W .0525
	D(-.0039)	D(-.0039)	D(-.0068)	D(-.0078)	D(-.0224)
1.25	T .1205	T .1127	T .1040	T .1001	T .0641
	W .1186	W .1195	W .1088	W .1244	W .1098
	D(.0019)	D(-.0068)	D(-.0049)	D(-.0243)	D(-.0457)
1.50	T .2080	T .1798	T .1817	T .1905	T .1040
	W .2070	W .1817	W .1924	W .2012	W .1914
	D(.0010)	D(-.0019)	D(-.0107)	D(-.0107)	D(.-0875)
1.75	T .2818	T .2653	T .2692	T .2332	T .1854
	W .2847	W .2692	W .2731	W .2566	W .2536
	D(-.0029)	D(-.0039)	D(-.0039)	D(-.0233)	D(-.0952)
2.00	T .3703	T .3722	T .3421	T .3304	T .2080
	W .3644	W .3712	W .3547	W .3703	W .3703
	D(.0058)	D(.0010)	D(-.0126)	D(-.0398)	D(-.1623)
4.00	T .8620	T .8338	T .7736	T .7415	T .4043
	W .8086	W .8358	W .8047	W .8192	W .7843
	D(.0534)	D(-.0019)	D(-.0311)	D(-.0777)	D(-.3800)
6.00	T .9738	T .9397	T .9048	T .8688	T .5277
	W .9271	W .9213	W .9232	W .9271	W .9203
	D(.0466)	D(.0185)	D(-.0185)	D(-.0583)	D(-.3926)
8.00	T .9932	T .9806	T .9582	T .9213	T .5355
	W .9757	W .9776	W .9864	W .9670	W .9611
	D(.0175)	D(.0029)	D(-.0282)	D(-.0457)	D(-.4257)
10.00	T .9951	T .9893	T .9728	T .9407	T .5996
	W .9835	W .9874	W .9874	W .9786	W .9874
	D(.0117)	D(.0019)	D(-.0146)	D(-.0379)	D(-.3878)

Table 3-K(5,15)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (5,15), the significance level at .05.

K	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
1.05	T .0651	T .0554	T .0583	T .0583	T .0680
	W .0593	W .0515	W .0603	W .0525	W .0457
	D(.0058)	D(.0039)	D(-.0019)	D(.0058)	D(.0224)
1.25	T .1137	T .0962	T .1069	T .1050	T .0923
	W .1040	W .0943	W .0962	W .1020	W .1011
	D(.0097)	D(.0019)	D(.0107)	D(.0029)	D(-.0087)
1.50	T .1953	T .1613	T .1778	T .1817	T .1370
	W .1817	W .1467	W .1759	W .1759	W .1574
	D(.0136)	D(.0146)	D(.0019)	D(.0058)	D(-.0204)
1.75	T .2760	T .2799	T .2507	T .2362	T .1584
	W .2595	W .2420	W .2245	W .2255	W .1992
	D(.0165)	D(.0379)	D(.0262)	D(.0107)	D(-.0408)
2.00	T .3304	T .3450	T .3061	T .3207	T .2041
	W .2279	W .3090	W .3042	W .3265	W .3071
	D(.0525)	D(.0360)	D(.0019)	D(-.0058)	D(-.1030)
4.00	T .8086	T .7687	T .6997	T .6472	T .2682
	W .6599	W .6715	W .6676	W .6910	W .6628
	D(.1487)	D(.0972)	D(.0321)	D(-.0437)	D(-.3946)
6.00	T .9495	T .8999	T .8270	T .7687	T .3022
	W .8358	W .8319	W .8212	W .8270	W .8173
	D(.1137)	D(.0680)	D(.0058)	D(-.0583)	D(-.5151)
8.00	T .9699	T .9397	T .9048	T .8445	T .2750
	W .8989	W .8892	W .8921	W .8873	W .8892
	D(.0709)	D(.0505)	D(.0126)	D(-.0428)	D(-.6142)
10.00	T .9864	T .9631	T .9339	T .8601	T .3022
	W .9281	W .9135	W .9310	W .9164	W .9310
	D(.0583)	D(.0496)	D(.0029)	D(-.0564)	D(-.6288)

Table 3-K(10,30)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (10,30), the significance level at .05.

K	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
1.05	T .0505	T .0729	T .0661	T .0680	T .0632
	W .0496	W .0641	W .0564	W .0612	W .0564
	D(.0010)	D(.0087)	D(.0097)	D(.0068)	D(.0068)
1.15	T .0982	T .0923	T .1215	T .0952	T .0807
	W .0943	W .0972	W .1001	W .1050	W .1001
	D(.0039)	D(-.0049)	D(.0214)	D(-.0097)	D(-.0194)
1.25	T .1467	T .1458	T .1535	T .1331	T .1176
	W .1448	W .1380	W .1477	W .1361	W .1234
	D(.0019)	D(.0078)	D(.0058)	D(-.0029)	D(-.0058)
1.35	T .1798	T .1701	T .1788	T .1973	T .1322
	W .1720	W .1681	W .1740	W .2002	W .1594
	D(.0078)	D(.0019)	D(.0049)	D(-.0029)	D(-.0272)
1.50	T .2838	T .2886	T .2643	T .2410	T .1740
	W .2682	W .2634	W .2556	W .2527	W .2177
	D(.0155)	D(.0253)	D(.0087)	D(-.0117)	D(-.0437)
2.50	T .7541	T .7172	T .6842	T .6511	T .3353
	W .6589	W .6472	W .6676	W .6822	W .6707
	D(.0952)	D(.0700)	D(.0165)	D(-.0311)	D(-.3353)
3.50	T .9310	T .8999	T .8494	T .8260	T .3479
	W .8571	W .8542	W .8387	W .8669	W .8581
	D(.0739)	D(.0457)	D(.0107)	D(-.0408)	D(-.5102)
4.50	T .9806	T .9689	T .9417	T .8756	T .3741
	W .9300	W .9359	W .9436	W .9310	W .9232
	D(.0505)	D(.0330)	D(-.0019)	D(-.0554)	D(-.5491)
5.50	T .9951	T .9864	T .9689	T .9164	T .3800
	W .9650	W .9728	W .9689	W .9679	W .9747
	D(.0301)	D(.0136)	D(.0000)	D(-.0515)	D(-.5948)

Table 3-K(20,20)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (20,20), the significance level at .05.

K	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
1.05	T .0573	T .0641	T .0457	T .0641	T .0408
	W .0603	W .0622	W .0447	W .0564	W .0709
	D(-.0029)	D(.0019)	D(.0010)	D(.0078)	D(-.0301)
1.15	T .1011	T .0982	T .1127	T .1020	T .0564
	W .0952	W .0991	W .1176	W .1020	W .0904
	D(.0058)	D(-.0010)	D(-.0049)	D(.0000)	D(-.0340)
1.25	T .1616	T .1535	T .1642	T .1681	T .0748
	W .1409	W .1545	W .1506	W .1594	W .1016
	D(.0107)	D(-.0010)	D(.0136)	D(.0087)	D(-.0768)
1.35	T .2002	T .2021	T .1895	T .2041	T .0884
	W .1905	W .1992	W .2041	W .2109	W .1652
	D(.0097)	D(.0029)	D(-.0146)	D(-.0068)	D(-.0768)
1.50	T .3207	T .2974	T .3120	T .2731	T .1633
	W .3110	W .2974	W .3032	W .2954	W .2877
	D(.0097)	D(.0000)	D(.0087)	D(-.0224)	D(-.1244)
2.50	T .8192	T .8086	T .7386	T .7250	T .3955
	W .7765	W .7765	W .7813	W .7823	W .7716
	D(.0428)	D(.0321)	D(-.0428)	D(-.0573)	D(-.3761)
3.50	T .9621	T .9572	T .9135	T .9096	T .5053
	W .9368	W .9378	W .9407	W .9524	W .9534
	D(.0253)	D(.0194)	D(-.0272)	D(-.0428)	D(-.4480)
4.50	T .9990	T .9922	T .9767	T .9300	T .5627
	W .9932	W .9854	W .9922	W .9796	W .9776
	D(.0058)	D(.0068)	D(-.0155)	D(-.0496)	D(-.4151)
5.50	T .9990	T .9981	T .9903	T .9670	T .6064
	W .9951	W .9951	W .9961	W .9990	W .9971
	D(.0039)	D(.0029)	D(-.0058)	D(-.0321)	D(-.3907)

Table 3-K(40,40)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (40,40), the significance level at .05.

K	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
1.05	T .0641	T .0705	T .0709	T .0641	T .0437
	W .0690	W .0690	W .0826	W .0807	W .0641
	D(.0049)	D(.0019)	D(-.0117)	D(.0165)	D(-.0204)
1.10	T .0982	T .1098	T .1156	T .0904	T .0505
	W .0972	W .1079	W .1079	W .0884	W .1001
	D(.0010)	D(.0019)	D(.0078)	D(.0019)	D(-.0496)
1.15	T .1118	T .1409	T .1467	T .1409	T .0758
	W .1050	W .1419	W .1273	W .1254	W .1351
	D(.0068)	D(-.0010)	D(.0194)	D(.0155)	D(-.0593)
1.20	T .1769	T .1691	T .1691	T .2002	T .1040
	W .1730	W .1642	W .1652	W .1730	W .1730
	D(.0039)	D(.0049)	D(.0039)	D(.0272)	D(-.0690)
1.25	T .1983	T .2138	T .2138	T .1693	T .1108
	W .2012	W .2080	W .2196	W .1924	W .2206
	D(-.0029)	D(.0058)	D(-.0058)	D(.0039)	D(-.1098)
1.50	T .4869	T .4636	T .4500	T .4315	T .2245
	W .4597	W .4305	W .4538	W .4684	W .4752
	D(.0272)	D(.0330)	D(-.0039)	D(-.0369)	D(-.2507)
1.75	T .7153	T .7201	T .6929	T .6696	T .3392
	W .6890	W .6968	W .7162	W .7026	W .7007
	D(.0262)	D(.0233)	D(-.0233)	D(-.0330)	D(-.3615)
2.00	T .8892	T .8513	T .8105	T .7784	T .3780
	W .8562	W .8251	W .8416	W .8435	W .8542
	D(.0330)	D(.0262)	D(-.0311)	D(-.0651)	D(-.4762)
2.25	T .9524	T .9388	T .8912	T .8591	T .4606
	W .9291	W .9203	W .9135	W .9184	W .9213
	D(.0233)	D(.0185)	D(-.0224)	D(-.0953)	D(-.4606)
2.50	T .9835	T .9738	T .9592	T .9164	T .4927
	W .9650	W .9621	W .9660	W .9621	W .9611
	D(.0185)	D(.0117)	D(-.0068)	D(-.0457)	D(-.4684)

Table 3-P(10,10)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (10,10) the significance level at .05.

P-VALUE	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
0.10	T .0914	T .0544	T .0632	T .0671	T .0350
	W .0923	W .0651	W .0719	W .0719	W .0680
	D(-.0010)	D(-.0107)	D(.0087)	D(.0049)	D(-.0330)
0.25	T .0894	T .0982	T .0768	T .0884	T .0515
	W .0943	W .1059	W .0836	W .0923	W .0962
	D(-.0049)	D(.0078)	D(-.0068)	D(-.0039)	D(-.0447)
0.50	T .1603	T .1710	T .1594	T .1312	T .0923
	W .1672	W .1740	W .1623	W .1633	W .1497
	D(-.0068)	D(-.0029)	D(-.0029)	D(-.0321)	D(-.0573)
0.75	T .2721	T .2624	T .2498	T .2323	T .1438
	W .2585	W .2653	W .2536	W .2750	W .2420
	D(.0136)	D(-.0029)	D(-.0039)	D(-.0428)	D(-.0982)
0.85	T .2838	T .2828	T .2828	T .2983	T .1390
	W .2838	W .2818	W .2838	W .3246	W .2770
	D(.0000)	D(.0010)	D(-.0010)	D(-.0262)	D(-.1380)
0.95	T .3547	T .3333	T .3431	T .3120	T .1924
	W .3236	W .3362	W .3596	W .3431	W .3343
	D(.0311)	D(-.0029)	D(-.0165)	D(-.0311)	D(-.1419)

Table 3-P(5,15)

One-Tailed power probabilities of the t-test
and Wilcoxon test for sample sizes of (5,15),
the significance level at .05.

P-VALUE	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
0.10	T .0680	T .0593	T .0622	T .0690	T .0583
	W .0748	W .0573	W .0496	W .0632	W .0476
	D(-.0068)	D(.0019)	D(.0126)	D(.0058)	D(.0107)
0.25	T .0758	T .0768	T .0807	T .0943	T .0758
	W .0758	W .0758	W .0904	W .0933	W .0680
	D(.0000)	D(.0010)	D(-.0097)	D(.0010)	D(.0078)
0.50	T .1565	T .1409	T .1429	T .1156	T .1040
	W .1370	W .1409	W .1487	W .1322	W .1263
	D(.0194)	D(.0000)	D(-.0058)	D(-.0165)	D(-.0224)
0.75	T .2323	T .2264	T .2109	T .2177	T .1390
	W .2148	W .2089	W .2109	W .2167	W .2274
	D(.0175)	D(.0175)	D(.0000)	D(.0010)	D(-.0884)
0.85	T .2721	T .2741	T .2595	T .2420	T .1555
	W .2459	W .2362	W .2517	W .2430	W .2478
	D(.0262)	D(.0379)	D(.0078)	D(-.0010)	D(-.0923)
0.95	T .3246	T .2983	T .3149	T .2915	T .1613
	W .2809	W .2721	W .3149	W .2925	W .2653
	D(.0437)	D(.0262)	D(.0000)	D(-.0010)	D(-.1040)

Table 3-P(10,30)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (10,30) the significance level at .05.

P-VALUE	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
0.10	T .0680	T .0787	T .0777	T .0515	T .0739
	W .0651	W .0758	W .0729	W .0729	W .0777
	D(.0029)	D(.0029)	D(.0049)	D(-.0214)	D(-.0039)
0.25	T .0991	T .1059	T .1195	T .0923	T .0748
	W .0991	W .1079	W .1176	W .0982	W .1059
	D(.0000)	D(-.0019)	D(.0019)	D(-.0058)	D(-.0311)
0.50	T .2225	T .1837	T .1963	T .1905	T .1254
	W .2216	W .1905	W .1963	W .2099	W .1905
	D(.0010)	D(-.0068)	D(.0000)	D(-.0194)	D(-.0651)
0.75	T .3508	T .3324	T .3226	T .3003	T .1963
	W .3304	W .3236	W .3207	W .3246	W .3362
	D(.0204)	D(.0087)	D(.0019)	D(-.0243)	D(-.1399)
0.85	T .4393	T .4257	T .4004	T .3936	T .2177
	W .3848	W .3955	W .4004	W .4257	W .3848
	D(.0544)	D(.0301)	D(.0000)	D(-.0321)	D(-.1672)
0.95	T .5034	T .5121	T .4791	T .4674	T .2313
	W .4227	W .4529	W .4568	W .4548	W .4587
	D(.0807)	D(.0593)	D(.0224)	D(.0126)	D(-.2274)

Table 3-P(20,20)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (20,20) the significance level at .05.

P-VALUE	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
0.10	T .0729	T .0671	T .0544	T .0671	T .0398
	W .0729	W .0719	W .0544	W .0661	W .0690
	D(.0000)	D(-.0049)	D(.0000)	D(.0010)	D(-.0292)
0.25	T .1147	T .1147	T .1176	T .1156	T .0554
	W .1108	W .1186	W .1293	W .1283	W .1050
	D(.0039)	D(-.0039)	D(-.0117)	D(-.0126)	D(-.0496)
0.50	T .2381	T .2352	T .2284	T .2021	T .1166
	W .2381	W .2381	W .2284	W .2303	W .2313
	D(.0000)	D(-.0029)	D(.0000)	D(-.0282)	D(-.1147)
0.75	T .3887	T .3916	T .3596	T .3615	T .1944
	W .3810	W .4043	W .3868	W .3916	W .3790
	D(.0078)	D(-.0126)	D(-.0272)	D(-.0301)	D(-.1846)
0.85	T .5015	T .4752	T .4791	T .4402	T .2527
	W .4859	W .4665	W .5180	W .4762	W .4393
	D(.0155)	D(.0087)	D(-.0389)	D(-.0360)	D(-.1866)
0.95	T .5646	T .5549	T .5005	T .5121	T .2809
	W .5228	W .5238	W .5112	W .5607	W .5462
	D(.0418)	D(.0311)	D(-.0107)	D(-.0486)	D(-.2653)

Table 3-P(40,40)

One-Tailed power probabilities and the difference of the t-test and Wilcoxon test for sample sizes of (40,40) the significance level at .05.

P-VALUE	UNIFORM	NORMAL	LAPLACE	STUDENT-T	CAUCHY
0.10	T .0758	T .0845	T .0816	T .0777	T .0447
	W .0758	W .0836	W .0758	W .0845	W .0894
	D(.0000)	D(.0010)	D(.0058)	D(-.0068)	D(-.0447)
0.25	T .1613	T .1740	T .1419	T .1574	T .0855
	W .1652	W .1623	W .1652	W .1633	W .1642
	D(-.0039)	D(.0117)	D(-.0233)	D(-.0058)	D(-.0787)
0.50	T .3858	T .3469	T .3324	T .2925	T .1662
	W .3722	W .3547	W .3722	W .3537	W .3673
	D(.0136)	D(-.0078)	D(-.0398)	D(-.0612)	D(-.2012)
0.75	T .6385	T .6171	T .5879	T .5258	T .2663
	W .6249	W .6103	W .6249	W .6074	W .6346
	D(.0136)	D(.0068)	D(-.0369)	D(-.0816)	D(-.3683)
0.85	T .7454	T .6997	T .6880	T .6424	T .2964
	W .7221	W .6900	W .7211	W .6939	W .6978
	D(.0233)	D(.0097)	D(-.0330)	D(-.0515)	D(-.4014)
0.95	T .8406	T .8280	T .7901	T .7405	T .3605
	W .8066	W .8134	W .8076	W .8134	W .8037
	D(.0340)	D(.0146)	D(-.0175)	D(-.0729)	D(-.4431)

CHAPTER IV

ANALYSIS AND CONCLUSIONS

The K and P alternative models have a very close relationship. That is, k=2 in the K model is the extreme case of the P model with parameter p=1.0. The results for these models were very similar in the simulation study. Therefore, the results of K and P model will be discussed together.

There is no dramatic difference in power between the t-test and the Wilcoxon test. Except for the Cauchy distribution, most of the differences are less than .07, but it can still be found that some patterns in these differences exist. Obviously, we cannot determine which test is more powerful than the other without considering the types of distributions. Therefore, the effects of each of these factors will be discussed.

Analysis of the results

The sample size combinations were classified into two groups: balanced ((10,10), (20,20), (40,40)), and unbalanced ((5,15), (10,30)). The distributions were arranged according to the weight of the tail: very light tail (Uniform), light tail (Normal), mid-size tail (Laplace), heavy tail (Student-t with d.f. 3), and very heavy tail (Cauchy). Table 4-1, and 4-2, indicate the frequency of power advantage occurring in the magnitude A for both tests. The first column of the table is the range of the power advantage magnitude (A). The numbers following T (or W) are the frequencies of power superiority occurrences of T (or W) in various distributions. The frequencies for each distribution were obtained from the 46 estimated power values of the K model(Table 3K-(10,10) - 3K(40,40)), and 30 estimated power

Table 4-1
Frequency of power advantage A of the t-test and the Wilcoxon test at various distribution/magnitudes
(In K model)

A	Stat.	Uniform	Normal	Laplace	Student-t	Cauchy
	T	18	16	12	8	1
.00<=A<=.01	W	4	9	10	6	2
	T	7	6	6	3	0
.01<A<=.02	W	0	0	7	2	1
	T	4	3	2	1	1
.02<A<=.03	W	0	0	4	3	4
	T	2	5	1	0	0
.03<A<=.04	W	0	0	2	6	2
	T	2	2	0	0	0
.04<A<=.05	W	0	0	1	7	4
	T	4	1	0	0	0
.05<A<=.06	W	0	0	0	6	1
	T	0	2	0	0	0
.06<A<=.07	W	0	0	0	1	1
	T	2	0	0	0	0
.07<A<=.08	W	0	0	0	1	2
	T	0	0	0	0	0
.08<A<=.09	W	0	0	0	0	1
	T	1	1	0	0	0
.09<A<=.10	W	0	0	0	1	1
	T	2	0	0	0	0
.10<A	W	0	0	0	0	25
	T	42	36	21	12	2
TOTAL	W	4	9	23	33	44
	T	91.3	90.0	47.7	26.7	4.4
%	W	8.7	10.0	52.3	73.3	95.6

Table 4-2
Frequency of power advantage A of the t-test and the Wilcoxon test at various distribution/magnitudes
(In P model)

A	Stat.	Uniform	Normal	Laplace	Student-t	Cauchy
.00<A<=.01	T	4	10	6	5	1
	W	5	9	6	6	1
.01<A<=.02	T	6	3	1	1	1
	W	0	2	4	3	0
.02<A<=.03	T	3	1	1	0	0
	W	0	0	2	4	2
.03<A<=.04	T	2	3	0	0	0
	W	0	0	4	5	2
.04<A<=.05	T	2	0	0	0	0
	W	0	0	0	2	3
.05<A<=.06	T	1	1	0	0	0
	W	0	0	0	1	1
.06<A<=.07	T	0	0	0	0	0
	W	0	0	0	1	1
.07<A<=.08	T	0	0	0	0	0
	W	0	0	0	1	1
.08<A<=.09	T	1	0	0	0	0
	W	0	0	0	1	1
.09<A<=.10	T	0	0	0	0	0
	W	0	0	0	0	2
.10<A	T	0	0	0	0	0
	W	0	0	0	0	14
TOTAL	T	19	18	8	6	2
	W	5	11	16	24	28
%	T	79.2	62.1	33.3	20.0	6.7
	W	20.8	37.9	66.7	80.0	93.3

Table 4-3
Average and maximum power advantage
attained by the t-test and the Wilcoxon test
at various sample size (K alternative)

Sample size	Advantage of the test	Uniform	Normal	Laplace	Student-t	Cauchy
(10,10)	Avg. of T	.0197	.0061	.0000	.0000	.0000
	Max. of T	.0534	.0185	.0000	.0000	.0000
	Avg. of W	.0034	.0037	.0146	.0362	.2221
	Max. of W	.0039	.0068	.0311	.0777	.4257
(5,15)	Avg. of T	.0544	.0400	.0118	.0063	.0224
	Max. of T	.1487	.0972	.0321	.0107	.0224
	Avg. of W	.0000	.0000	.0019	.0414	.2907
	Max. of W	.0000	.0000	.0019	.0583	.6288
(10,30)	Avg. of T	.0311	.0258	.0097	.0068	.0068
	Max. of T	.0952	.0700	.0214	.0068	.0068
	Avg. of W	.0000	.0049	.0019	.0258	.2607
	Max. of W	.0000	.0049	.0019	.0554	.5948
(20,20)	Avg. of T	.0142	.0094	.0078	.0055	.0000
	Max. of T	.0428	.0321	.0136	.0087	.0000
	Avg. of W	.0029	.0010	.0185	.0352	.2191
	Max. of W	.0029	.0010	.0428	.0573	.4480
(40,40)	Avg. of T	.0161	.0141	.0104	.0130	.0000
	Max. of T	.0330	.0330	.0194	.0272	.0000
	Avg. of W	.0029	.0010	.0150	.0552	.2326
	Max. of W	.0029	.0010	.0311	.0953	.4762

Table 4-4

Average and maximum power advantage attained by the t-test and the Wilcoxon test at various sample size (P alternative)

Sample size	Advantage of the test	Uniform	Normal	Laplace	Student-t	Cauchy
(10,10)	Avg. of T	.0149	.0044	.0087	.0049	.0000
	Max. of T	.0311	.0078	.0087	.0049	.0000
	Avg. of W	.0042	.0049	.0062	.0272	.0855
	Max. of W	.0068	.0107	.0165	.0428	.1419
(5,15)	Avg. of T	.0214	.0141	.0051	.0026	.0093
	Max. of T	.0437	.0379	.0126	.0058	.0107
	Avg. of W	.0068	.0000	.0078	.0062	.0768
	Max. of W	.0068	.0000	.0097	.0165	.1040
(10,30)	Avg. of T	.0266	.0253	.0052	.0126	.0000
	Max. of T	.0807	.0593	.0224	.0126	.0000
	Avg. of W	.0000	.0044	.0000	.0206	.1058
	Max. of W	.0000	.0068	.0000	.0321	.2274
(20,20)	Avg. of T	.0115	.0199	.0000	.0010	.0000
	Max. of T	.0418	.0311	.0000	.0010	.0000
	Avg. of W	.0000	.0061	.0221	.0311	.1383
	Max. of W	.0000	.0126	.0389	.0486	.2653
(40,40)	Avg. of T	.0169	.0088	.0058	.0000	.0000
	Max. of T	.0340	.0146	.0058	.0000	.0000
	Avg. of W	.0039	.0078	.0301	.0466	.2562
	Max. of W	.0039	.0078	.0398	.0816	.4431

values of the P model(Table 3P-(10,10) - 3P(40,40)). Tied power values were omitted. For example in table 4-1, at the Laplace column and .02<A<=.03 row, '2' corresponds to T and '4' to W. This means, in Laplace distribution, there were 2 times that the t-test showed a power advantage of .02 to .03 over the Wilcoxon test while there were 4 times that the Wilcoxon test held this advantage over the t-test. The averages and maxima of the power advantage magnitudes showed in Table 4-3 and 4-4 were obtained from the specified sample size and distribution. For example in Table 4-3, at Laplace column and (20,20) row, the avg. value for T of .0078 and the max. for T of .0136 represent the average and maximum magnitude of power advantage of the t-test under (20,20) sample size and Laplace distribution (see the positive D values in Table 3-K(20,20) Laplace column.). Similarly, from the negative D values, the average .0185 and maximum .0428 of those of the Wilcoxon test also can be found. Details follow.

(1) Uniform distribution:

This is an extreme case of a symmetric light tailed distribution. The power of the t-test dominates that of the Wilcoxon test for both alternatives. In 91.3% of the cases in K alternative and 79.2% in P alternative the t-test has more power than the Wilcoxon test. The t-test has greater power advantage in testing the unbalanced data than it does in testing the balanced data, but the magnitude of power advantage of the t-test is modest (less than .1487 in K model, less than .0807 in P model).

(2) Normal distribution:

The t-test had a power advantage over the Wilcoxon test in 90% of the cases considered for the K model and 62.1% for the P model. Generally, the t-test has greater power advantage in testing unbalanced data while it has less power advantage for the balanced data case. The maximum power advantage magnitude of the t-test is .0972 in K model and .0593 in P model. These maximum power advantages are less than those of Uniform (very light tailed) distribution. The magnitude and frequency of power advantage of the Wilcoxon test are trivial.

(3) Laplace distribution:

In contrast to the lighter tailed distributions, the Wilcoxon test has somewhat greater power than the t-test for this distribution. The percentage of the cases for which the Wilcoxon test has a power advantage is 52.3% in K model and 66.7% in P model. The t-test still has somewhat more power in testing unbalanced data than balanced data for the same total sample size. The maximum magnitude of the power advantage of the Wilcoxon test is .0428 in K model .0398 in P model, while the maximum power advantage of the t-test is .0321 in K model and .0126 in P model.

(4) Student-t distribution with 3 d.f.:

In this distribution, the Wilcoxon test has greater power than the t-test. The percentage of the cases of power advantage for the Wilcoxon test is 73.3% in K model and 80.0% in P model. The power advantage does not appear to be affected by lack of balance in the data sets. The maximum magnitude of power advantage of the t-test is .0272 in K model and .0126 in P model while for the Wilcoxon test, it is .0953 in K model and .0816 in P model. In heavy tailed distributions, the power advantage of one test over the other appear to depend on the values of p or k. Generally, the larger values of p and k favor the Wilcoxon test while the small values favor the t-test.

(5) Cauchy distribution:

The power of the Wilcoxon test dominates that of the t-test in frequency and magnitude. The percentage of cases of power advantage for the Wilcoxon test is 95.6% in K model, 93.3% in P model. Also, the magnitude of power advantage of the Wilcoxon test is large. The maximum is .6288 in K model and .4431 in P model. There are some cases of power advantage for the t-test in testing unbalanced data, but the magnitude is modest.

<u>The power of the Wilcoxon test</u>

The estimate of the power functions of the Wilcoxon test, obtained by averaging the powers from distributions, are shown in Table 4-5, and 4-6. As derived by Lehmann(1953) the power of the Wilcoxon test is independent of the form of F(x) in K and P models. These two tables indicate that the Wilcoxon test has more power in testing balanced data than that of testing unbalanced data in both K and P models.

<u>Discussion</u>

(1) Generally speaking, the t-test has greater power advantage than the Wilcoxon test when the distribution tail weight is light. The power advantage of Wilcoxon test increases and that of the t-test decreases as the tail weight goes up.

Table 4-5
The estimate power function of the Wilcoxon test
(K model)

k	(10,10)	(5,15)	(10,30)	(20,20)	(40,40)
1.05	.0628	.0539	.0575	.0589	.0731
1.10	----	----	----	----	.1003
1.15	----	----	.0993	.1009	.1269
1.20	----	----	----	----	.1697
1.25	.1162	.0995	.1380	.1514	.2084
1.35	----	----	.1747	.1940	----
1.50	.1947	.1675	.2515	.2989	.4575
1.75	.2674	.2301	----	----	.7011
2.00	.3662	.2949	----	----	.8441
2.25	----	----	----	----	.9205
2.50	----	----	.6653	.7776	.9633
3.50	----	----	.8550	.9442	----
4.00	.8105	.6706	----	----	----
4.50	----	----	.9327	.9840	----
5.50	----	----	.9699	.9965	----
6.00	.9238	.8266	----	----	----
8.00	.9738	.8913	----	----	----
10.00	.9849	.9240	----	----	----

Table 4-6
The estimate power function of the Wilcoxon test
(P model)

p	(10,10)	(5,15)	(10,30)	(20,20)	(40,40)
.10	.0738	.0585	.0729	.0669	.0818
.25	.0945	.0807	.1057	.1184	.1640
.50	.1633	.1370	.2018	.2332	.3640
.75	.2589	.2157	.3271	.3885	.6204
.85	.2902	.2449	.3982	.4772	.7050
.95	.3394	.2851	.4492	.5329	.8089

Table 4-7
Maximum power advantages attained by the t and Wilcoxon statistics when small and moderate samples are drawn from non-normal distributions
(shift alternatives)

Distribution	Statistics	small samples	moderate samples
Uniform	T	.13	.09
	W	.00	.01
Laplace	T	.13	.00
	W	.04	.17
Truncated Normal	T	.10	.14
	W	.08	.00
Exponential	T	.11	.00
	W	.12	.44
Mixed Normal	T	.17	.00
	W	.30	.94
Mixed Uniform	T	.16	.06
	W	.08	.44

Note: (1) This is the case of the shift alternative model. Source from Blair and Higgins (1980:320).

(2) The moderate sample size is defined as n + m = 36 and n + m = 108. The small sample size is defined as n + m = 12.

(2) It is interesting to compare the magnitude of the power advantages under the Lehmann's alternatives to those under the shift alternatives. Asymptotic relative efficiencies indicate that the Wilcoxon test may have substantial power advantages over the t-test for heavy tailed distributions and shift alternatives. Actual power values were obtaned by simulation by Blair and Higgins (1980) for small and moderate sample size, and brief summary of their findings is given in Table 4-7. Generally, the Wilcoxon test had considerably larger power than the t-test for heavy tailed distributions. For instance, for the Laplace distribution, the maximum power advantage was .17 for moderate sample sizes. In contrast to this, the magnitudes of the power advantage of the Wilcoxon test over the t-test under Lehmann's alternatives appears to be relatively modest except for the Cauchy distribution. For example, for the K alternative under the Laplace distribution, the maximum power advantage of the Wilcoxon test over the t-test was only .0428, while the maximum advantage of the t-test over the Wilcoxon test was .0321. In general, the powers of these two tests appear to be much closer under Lehmann's alternatives than under shift alternatives.

(3) If the Lehmann's alternatives are appropriate for real data, then it would appear that there is little reason to choose one test over the other for most distributions. Some advantage appears to go to the t-test for the lighter tailed distributions while there is a slight advantage to the Wilcoxon test for the heavier tailed distributions.

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A COMPARISON OF THE POWER OF
THE WILCOXON TEST TO THAT OF THE T-TEST
UNDER LEHMANN'S ALTERNATIVES

by

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ABSTRACT

This report considers the problem of statistical test under non-shift alternatives. The purpose is to compare the power of the Student's t-test to that of the Wilcoxon's rank sum test under two alternatives of the forms

$$G(x) = [F(x)]^k ,$$

and

$$G(x) = p\ [F(x)]^2 + (1-p)\ [F(x)].$$

The methodology employed in the study was computer simulation. Combinations of 5 distributions (uniform, normal, Laplace, Student-t with 3 d.f., and Cauchy) and 5 sample sizes ((10,10), (5,15), (10,30), (20,20), and (40,40)) were investigated.

The simulation results indicate, generally, that the t-test is more powerful for light tailed distributions, while the Wilcoxon test has more power for heavy tailed distributions. However, the magnitude of the power advantage of the Wilcoxon test over the t-test are not, generally, as great as found with shift alternatives.