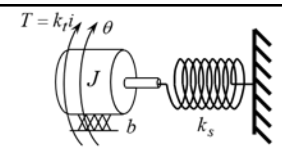
Laboratory #13

In this lab you are to experimentally determine five data points for the frequency response of the motor-and-spring configuration of the Motorlab. Then you are to estimate all parameters of the model except J and k_{dr}. We assume we know these two parameters accurately.

You are to use sine waves and utilize "Run Wave Autosave" for the input current on the Motorlab, since the input to the transfer function of interest is current. **You should use a magnitude of 0.25 Amp for sine wave frequencies near the natural frequency (~= resonance).** This will hopefully prevent fatigue failures of the spring. **Be careful near the resonance. It is easy to break the spring with the resonance.** For the other input frequencies, you are given an input amplitude to use (see table on next page).

To begin the lab you should experiment to find the actual natural frequency. You should find natural frequency by finding a frequency where the phase lag is very near 90 degrees. If you find a phase lag between 80 and 100 degrees that is sufficient to estimate the natural frequency given that the phase transition is very sharp for this lightly damped system. But try to do your best. Once you have found the natural frequency, then you should fill in the data table. Note that the frequencies you use for data collection are dependent on the natural frequency you find. You may round these other frequencies to the nearest Hz.



$i(t)$ = motor current

$\theta(t)$ = angular position

$k_{dr} = 180/\pi$ deg/rad

k_s = spring constant

k_t = motor torque constant (initial guess = 0.05 $(\mathrm{N \cdot m/A})$)

J = motor inertia + collar inertia = 1.29e-5 $(\mathrm{kg \cdot m^2})$

b = viscous friction coeficient (initial guess $\cong 2 \times 10^{-4}$ $\mathrm{N \cdot m \cdot s/rad}$)

$$G(s) = \frac{\theta(s)}{I(s)} = \frac{k_{dr} k_t}{Js^2 + bs + k_s} = \frac{K_{dc}\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

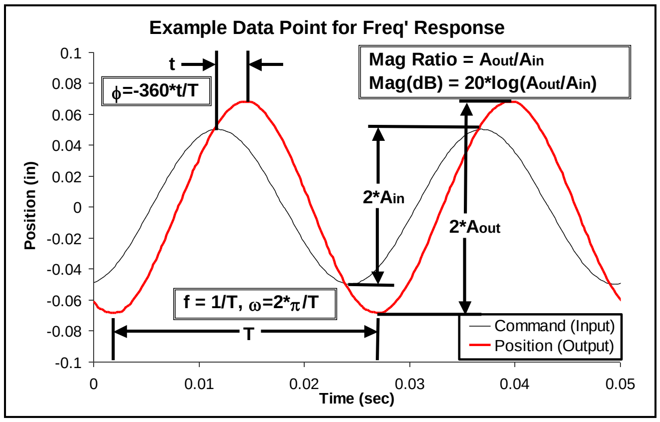
ς = damping ratio, ω_n = natural frequency

ω_d = damped frequency of oscillation $= \omega_n\sqrt{1-\varsigma^2}$

ω_r = resonant frequency $= \omega_n\sqrt{1-2\varsigma^2}$

Plotting The Responses to Input Sine Waves

You should use the mlolplots(data,Iscale) function. You may have to include the Iscale argument for the current to be visible on the same plot as the position.



Some Related MATLAB Functions

Helpful MATLAB functions:

log10() – log base 10

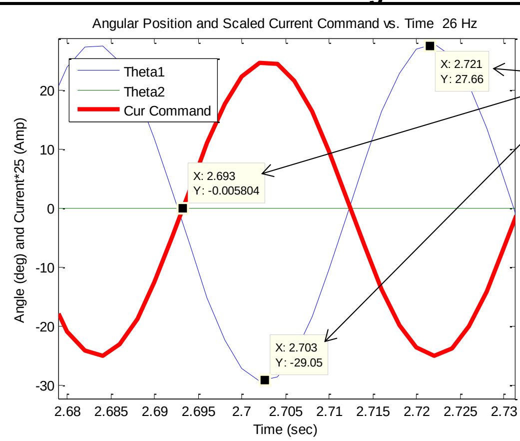
bode() – generates the bode (frequency response) plot of a tf – note you can change the freq' units to Hz by right clicking on the figure and choosing 'properties'

[m,p,w]=bode() – generates the data for a frequency response plot of a tf – note the mag (m) is a ratio not dB

loglog() – plotting routine for a log-log scale

semilogx() – plotting routine for a log scale on the x-axis

Taking data for the table and searching for the natural frequency.



Example Calculations from data cursors

amplitude of input = A_{in} = 1 Amp (known from motorlabGUI)
amplitude of output = A_{out} = (27.66 - (-29.05))/2 = 28.35 deg
amplitude ratio (magnitude) = A_{out}/A_{in} = 28.35 deg/A
magnitude (dB) = 20*$\log_{10}(A_{out}/A_{in})$ = 29.1 dB

input frequency = 26 Hz (known from motorlabGUI)
time at midpoint of output = t_{mid} = (2.703+2.721)/2 = 2.712 sec
time lag = t_{lag} = t_{mid} - t_{cross} = 2.712 - 2.693 = 0.019 sec
phase lag = 360* t_{lag} *$freq$ = 360*0.019*26 = 178 deg
phase shift = - phase lag

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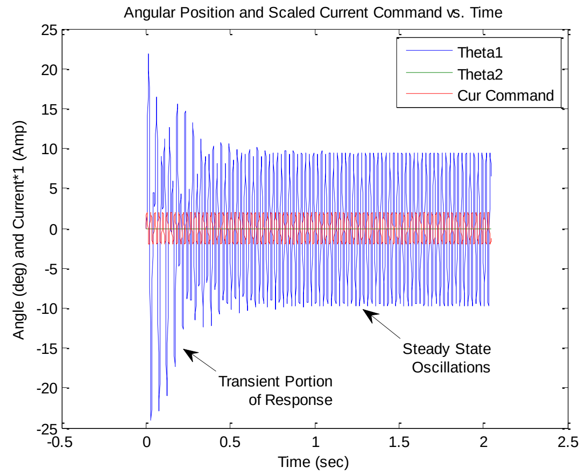
The sample frequency chosen should be small enough to allow sufficient time for the response to settle into steady state oscillations, but large enough to be at least 10 times larger than the input sine wave frequency. The data should be taken from the steady state oscillations



DATA TABLE

Freq'	ω_n	$\omega_n/10$	$0.75*\omega_n$	$1.25*\omega_n$	$2*\omega_n$
Input Amplitude (Amp)	0.25	1	1	1	2
Freq' Value (Hz)					
Mag' Ratio (deg/Amp)					
Mag' Ratio (dB)					
Phase Shift (deg)					

Improve your theoretical model

Use data from the table to find all the coefficients for the standard 2nd order form. **The magnitude ratio at one tenth of the natural frequency should give you the DC gain**. The damping ratio can be found by symbolically calculating the magnitude of the standard 2nd order form at the natural frequency and then using the actual magnitude at the natural frequency from the data.

Then you should equate the two forms of the model to determine the physical parameters (k_t, k_s, b) of the model.

Things to Turn In (check the quiz on Canvas)

- The completed data table.
- Five experimental plots (from mlolplots() like on the previous page) of the input and output showing the data cursors used for magnitude and phase calculations.
- A final Bode plot showing the initial model, the improved model, and the magnitude and phase data.
- Your completed lab 13 Matlab code.
- Your improved k_t, k_s, and b values (on the quiz).

Some useful equations for magnitude and phase of 2nd order underdamped TF.

$$G(s) = \frac{K_{dc}\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \qquad G(j\omega) = \frac{K_{dc}\omega_n^2}{\omega_n^2 - \omega^2 + j2\varsigma\omega_n\omega}$$

$$|G(j\omega)| = \frac{K_{dc}\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2}} \quad \text{and} \quad \angle G(j\omega) = -\tan^{-1}\left(\frac{2\varsigma\omega_n\omega}{\omega_n^2 - \omega^2}\right)$$

$$|G(j\omega_n)| = \frac{K_{dc}\omega_n^2}{\sqrt{(2\varsigma\omega_n^2)^2}} = \frac{K_{dc}\omega_n^2}{2\varsigma\omega_n^2} = \frac{K_{dc}}{2\varsigma} \quad \text{and} \quad \angle G(j\omega_n) = -\tan^{-1}\left(\frac{2\varsigma\omega_n^2}{0}\right) = -\pi/2 = -90\,\text{deg}$$