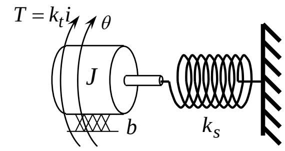
Laboratory #4

In this lab you are to experimentally determine the coefficients for a model of the motor-and-spring system in the Motorlab. You should be able to determine the inertia and motor torque constant from the Motorlab handout. Then you should be able to estimate the other parameters (b, k_s) using experimental data.

You are also to compare the step response of your model to experimental data. With the spring coupling in place and the load inertia locked down, the Motorlab apparatus will have the dynamic system described by the equations and schematic model to the right. This is the dynamic system we are studying in this laboratory.



$T(t) = J\ddot{\theta}(t) + b\dot{\theta}(t) + k_s\theta(t)$

$T(t) = k_t i(t)$

J = motor inertia + collar inertia

b = viscous friction coeficient

k_s = spring constant

$T(t)$ = torque (is proportion al to motor current)

$i(t)$ = motor current

$\theta(t)$ = angular position (best if measured in *rad* for dynamics)

$$G(s) = \frac{\theta(s)}{I(s)} = \frac{k_t}{Js^2 + bs + k_s} = \frac{k_{dc}\omega_n^{\ 2}}{s^2 + 2\varsigma\omega_n s + \omega_n^{\ 2}}$$

ς = damping ratio, ω_n = natural frequency, k_{dc} = dc gain

ω_d = damped frequency of oscillation = $\omega_n\sqrt{1-\varsigma^2}$

Estimating the Spring Rate and Natural Frequency

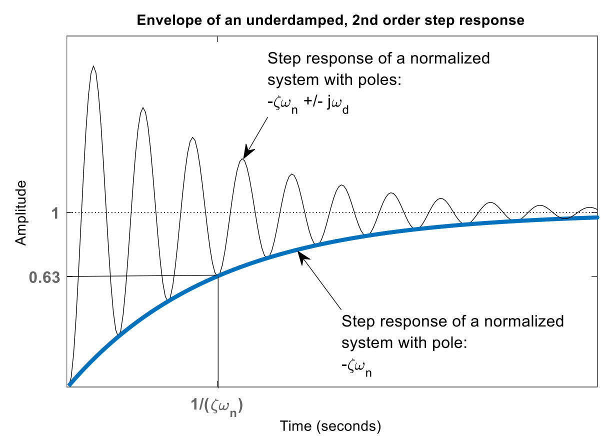
Looking at the differential equation in the model it can be seen that in steady state with a constant torque $T = k_s\theta$. Therefore, we should be able to estimate the spring's coefficient by obtaining steady state position and current data. It is important to note that θ is measured from the equilibrium position with zero torque. Fill in the table below and use the data to estimate the spring rate.

Current (A)	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0
Position (deg)									

Using the data from the table above obtain a plot of torque vs. angular position. On this same plot, draw the "best fit" line through the data by playing with the slope of the line described by $T = k_{estimate}\theta$. Generate this plot by completing the top part of the m-file and running it with successive guesses of $k_{estimate}$. Then, knowing the spring rate and the mass moment of inertia you should be able to estimate the natural frequency, ω_n.

Estimating the Damping Ratio and Friction Coefficient

Using a square wave input with an amplitude of 1 Amp, obtain a step response that can be used to estimate the damping ratio. Do this by fitting an envelope to the decay of the oscillations. The poles of a second-order, under-damped system are $-\varsigma\omega_n \pm j\omega_d$, and the envelope of the oscillations is formed by using the exponential $e^{-\varsigma\omega_n t}$. So, the envelope of the step response of a second order system may be plotted using the step response of a first order system with a pole of $-\varsigma\omega_n$, the real part of the second order poles. Knowing ω_n from above, you can vary ς to fit an envelope to the second order step response.



Comparing the Actual Step Response to the Theoretical

Using your coefficients, you should now be able to generate a TF in MATLAB that approximates the system. You could either use the (k_t, J, b, k_s) set of coefficients or the $(k_{dc}, \varsigma\omega_n, \omega_n^{\ 2})$ set of coefficients. Both should give you the same TF. Try it. Complete the bottom section of the m-file which plots data obtained from the Motorlab square wave along with the theoretical data.

Things To Turn in in Canvas:

- Plot 1: Straight line fit to Torque vs. Displacement data.
- Plot 2: Experimental step response and estimate of the envelope. This should have four data cursors: 1) steady state value, 2) value of the envelope at one time constant, and 3) & 4) two cursors at peaks to calculate the period of the oscillations.
- Plot 3: Experimental and theoretical step responses together.
- Completed m-file with comments.
- Table of coefficients.
- Completed fill-in-the-blanks.

Our coefficients (***with units***) for the two forms of the transfer function (TF) model found in this lab are given in the table below.

Coefficients of the TF with physical coefficients		**Coefficients of TF in standard second order form**	
k_t =		k_{dc} =	
J =		ς =	
b =		ω_n =	
k_s =			