TRACTOR TRAILER CORNERING

by

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NOMENCLATURE

R	Radius of path of fifth wheel	Ft
a	x-Co-ordinate of center of curvature in x-y plane	Ft
b	y-Co-ordinate of center of curvature in x-y plane	Ft
x	x-Co-ordinate of path of Trailer in x-y plane	Ft
y	y-Co-ordinate of path of Trailer in x-y plane	Ft
ξ	x-Co-ordinate of path of Fifth wheel in x-y plane	Ft
η	y-Co-ordinate of path of Fifth wheel in x-y plane	Ft
γ	Angle indicated in Figure 2	Degrees
θ	Angle indicated in Figure 2	Degrees
L	Length of Trailer	Ft
V	Distance between the tractrix and the leading curve	Ft

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INTRODUCTION

Maneuvering present day tractor-trailer trucks about in the narrow streets of big cities is sometimes quite difficult. It has been suggested that the trucks be made more manageable by developing a servomechanism to "steer" the trailer wheels while cornering. The servo is to be automatically actuated by a signal from the main steering wheel in the tractor.

In this paper, a preliminary study is made of how trailers corner without servomechanisms. When a trailer moves around a corner, the hitch point of the trailer makes some curve called the "leading curve" and correspondingly the center of the rear axle of the trailer makes some other curve called the "tractrix." It is assumed for the purpose of analysis that the curve made by the fifth wheel, i.e., hitch point, of the trailer while cornering has a constant radius of curvature.

The tracking performance of trailers has previously been studied by Fazekas [1] in the U.S.A. and Schaar in Germany. Tracking performance was studied particularly for leading curves with constant radius of curvature.

In this paper the differential equation and its solution for the 90^0 circular tractrix has been derived. Numerical results have been determined for different radii of cornering, using a digital computer.

[ ] Numbers in brackets designate references at end of report.

DESCRIPTION OF CORNERING PROBLEM

When a four-wheeled trailer moves around a corner, the hitch point describes a curve called the "leading curve" as shown in Figure 1.

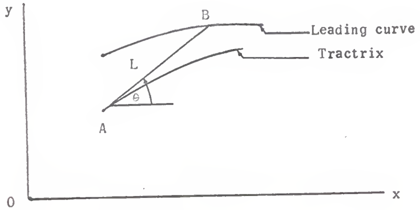

FIGURE 1

SCHEMATIC OF GENERAL TRACTRIX

The rear axle of the trailer lies normal to the curve described by the center of the rear axle. This curve is called the "general tractrix," of the "leading curve" as shown in Figure 1.

The general tractrix is thus characterized by the property that the distance of its tangent, taken from the point of tangency to the leading curve, is constant.

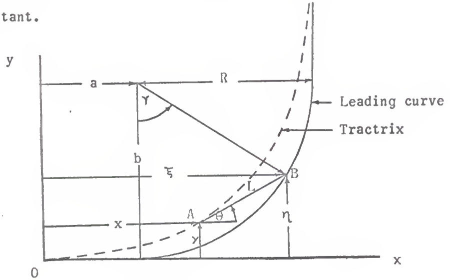

FIGURE 2

SCHEMATIC OF A 90^{0} CIRCULAR TURN

In the present study it is assumed that the path of the curve made by the hitch point has a constant radius of curvature while the trailer is performing a 90^{0} turn. This is shown in Figure 2.

The path made by the center of the rear axle of the trailer, i.e., "tractrix," is also shown in Figure 3.

The problem is to determine the equations which describe the tractrix.

DERIVATION OF EQUATIONS FOR FIRST PORTION OF TRACTRIX

The first portion of the tractrix is that part for which the maximum value of γ is 90^0. After γ reaches 90^0 the hitch point travels on the straight line. Equations for this are derived later on.

With the previous assumptions, a differential equation may be derived in terms of convenient variables using the principles of geometry.

From geometry,

$$\eta = y + L \sin \theta \tag{1}$$

$$\xi = x + L \cos \theta \tag{2}$$

$$\eta = b - R \cos \gamma \tag{3}$$

$$\xi = a + R \sin \gamma \tag{4}$$

Equating equations (1) and (3)

$$y \quad L \sin \theta = \quad b - R \cos \gamma \tag{5}$$

Similarly equating equations (2) and (4)

$$x + L \cos \theta = \quad a + R \sin \gamma \tag{6}$$

Now considering that x, y, θ are functions of γ and differentiating equations (5) and (6) with respect to γ leads to

$$\left(\frac{dy}{dx}\right) \left(\frac{dx}{d\gamma}\right) + L \cos \theta \frac{d\theta}{d\gamma} = R \sin \gamma \tag{7}$$

$$\left(\frac{dx}{d\gamma}\right) - L \sin \theta \frac{d\theta}{d\gamma} = R \cos \gamma \tag{8}$$

From equation (8),

$$\frac{dx}{d\gamma} = R \cos \gamma + L \sin\theta \frac{d\theta}{d\gamma} \tag{9}$$

Since the distance of the tractrix's tangent taken from the point of tangency to the leading curve is constant, the essential criteria of tracking is

$$\frac{dy}{dx} = \tan \theta$$

Substituting the values of $\frac{dy}{dx}$ from equation (10) and $\frac{dx}{d\gamma}$ from equation (9) into equation (7).

$$\left[\tan \theta\right] \left[R \cos \gamma\right] + \left[\tan \theta\right] \left[L \sin \theta \frac{d\theta}{d\gamma}\right] + L \cos \theta \frac{d\theta}{d\gamma} = R \sin \gamma \quad (11)$$

Multiplying equation (11) through by cos θ

$$R \sin \theta \cos \gamma + L (\sin^2 \theta + \cos^2 \theta) \frac{d\theta}{d\gamma} = R \cos \theta \sin \gamma$$

$$R \cos \theta \sin \gamma - R \sin \theta \cos \gamma = L \frac{d\theta}{d\gamma}$$

$$R (\sin \gamma \cos \theta - \cos \gamma \sin \theta) = L\frac{d\theta}{d\gamma}$$

Dividing by L,

$$\frac{d\theta}{d\gamma} = (\frac{R}{L}) (\sin \gamma \cos \theta - \cos \gamma \sin \theta)$$

$$\frac{d\theta}{d\gamma} = (\frac{R}{L}) (\sin (\gamma - \theta) \quad (12)$$

Let $\gamma - \theta = \lambda$ (13)

Differentiating equation (13) with respect to γ leads to

$$\frac{d\theta}{d\gamma} = 1 - \frac{d\lambda}{d\gamma} \quad (14)$$

Substituting $\frac{d\theta}{d\gamma}$ from equation (14) and $(\gamma - \theta)$ from equation (13) into equation (12),

$$(1 - \frac{d\lambda}{d\gamma}) = (\frac{R}{L}) \sin \lambda, \ \frac{d\lambda}{d\gamma} = 1 = \frac{R}{L} \sin \lambda.$$

Separating the equation.

$$\frac{d\lambda}{1 - (\frac{R}{L}) \sin} = d\gamma \quad (15)$$

This is the differential equation for the first portion of the tractrix. The appropriate boundry condition is that $\lambda = 0$ when $\gamma = 0$ and $\theta = 0$.

SOLUTION OF THE DIFFERENTIAL EQUATION

Integrating equation (15) the solution is obtained [4].

$$\left[\frac{1}{\sqrt{\left(\frac{R}{L}\right)^2 - 1}} \ln \left\{ \left| \frac{\tan\left(\frac{\lambda}{2}\right) - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{\tan\left(\frac{\lambda}{2}\right) - \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| \right\} \right]^{\lambda} = \left[\gamma\right]^{\gamma}$$

$$\lambda_0 = 0 \quad \gamma_0 = 0$$

For $\frac{R}{L} > 1$

$$\left[\ln \left\{ \left| \frac{\tan\left(\frac{\lambda}{2}\right) - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{\tan\left(\frac{\lambda}{2}\right) - \left(\frac{R}{L}\right) + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| \right\} \right]_0^{\lambda} = \gamma \sqrt{\left(\frac{R}{L}\right)^2 - 1}$$

$$\ln \left\{ \left| \frac{\tan\frac{\lambda}{2} - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| \right\} - \ln \left\{ \left| \frac{-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| \right\} = \gamma \sqrt{\left(\frac{R}{L}\right)^2 - 1}$$

Using the principle $\log A - \log B = \log\left(\frac{A}{B}\right)$

$$\ln \left\{ \left| \frac{\left(\tan\frac{\lambda}{2} - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)}{\left(\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)} \quad \frac{\left(-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)}{\left(-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)} \right| \right\} = \gamma \sqrt{\left(\frac{R}{L}\right)^2 - 1}$$

TAKING EXPONENTIALS OF BOTH SIDES

$$\left| \frac{\left(\tan\frac{\lambda}{2} - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)\left(-\left(\frac{R}{L}\right)^2 + \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)}{\left(\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)\left(-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}\right)} \right| = e^{\gamma \sqrt{\left(\frac{R}{L}\right)^2 - 1}}$$

Using the property $|ab| = |a|\,|b|$

$$\left| \frac{\tan\frac{\lambda}{2} - \frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| = \left| \frac{-\frac{R}{L} - \sqrt{\left(\frac{R}{L}\right)^2 - 1}}{-\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 - 1}} \right| e^{\gamma \sqrt{\left(\frac{R}{L}\right)^2 - 1}}$$

$$\left|\frac{(\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 -1}\,) - 2\sqrt{(\frac{R}{L})^2 -1}}{(\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 -1}\,)}\right| = \left|\frac{(-\frac{R}{L} - \sqrt{(\frac{R}{L})^2 -1})}{(-\frac{R}{L} + \sqrt{(\frac{R}{L})^2 -1})}\right| e^{\gamma\sqrt{(\frac{R}{L})^2 -1}}$$

$$\left|1 - \frac{2\sqrt{(\frac{R}{L})^2 -1}}{(\tan\frac{\lambda}{2} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 -1}\,)}\right| = \left|\frac{(-\frac{R}{L} - \sqrt{(\frac{R}{L})^2 -1})}{(-\frac{R}{L} + \sqrt{(\frac{R}{L})^2 -1})}\right| e^{\gamma\sqrt{(\frac{R}{L})^2 -1}} \qquad (16)$$

The absolute value signs may be omitted in equation (16) for the following reasons:

For $(\frac{R}{L}) > 1$

It is true that

$$(\frac{R}{L})^2 - 1 > 0$$

$$2\sqrt{(\frac{R}{L})^2 -1} > 0$$

The value of λ must not exceed that value of λ which causes equation (15) to become an improper integral. This value is given by the following equation.

$$\sin\lambda = \frac{L}{R}$$

Using trigometric relations,

$$\tan\frac{\lambda}{2} = \frac{\sin\lambda}{1+\cos\lambda}$$

$$\tan\frac{\lambda}{2} = \frac{(\frac{L}{R})}{1 + \sqrt{1 - (\frac{L}{R})^2}}$$

$$\tan\frac{\lambda}{2} = \frac{1}{(\frac{R}{L}) + \sqrt{(\frac{R}{L})^2 -1}}$$

Then the value of λ must always be less than $2 \tan^{-1} \dfrac{1}{(\frac{R}{L}) + \sqrt{(\frac{R}{L})^2 - 1}}$

Therefore, $\tan \frac{\lambda}{2} = \dfrac{1}{(\frac{R}{L})^2 + \sqrt{(\frac{R}{L})^2 - 1} + k}$ where $0 < k$

Considering,

$$\tan \frac{\lambda}{2} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1} = \frac{1}{(\frac{R}{L}) + \sqrt{(\frac{R}{L})^2 - 1} + k} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1}$$

$$\frac{-k\left[\frac{R}{L} - \sqrt{(\frac{R}{L})^2 - 1}\right]}{k + \frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1}}$$

but, k and $\frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1}$ are positive quantities, as the $\frac{R}{L}$ ratio is considered. greater than one.

Also,

$$(\frac{R}{L})^2 > (\frac{R}{L})^2 - 1$$

taking square root of both sides, $\frac{R}{L} > \sqrt{(\frac{R}{L})^2 - 1}$. So it is true that

$$\left[\frac{R}{L} - \sqrt{(\frac{R}{L})^2 - 1}\right] > 0.$$

It follows, therefore, that the quantity $\left[\tan \frac{\lambda}{2} - \frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1}\right]$ is always negative, and hence the left hand side of equation (16) is always positive.

Considering the right hand side of equation (16),

$$\frac{-(\frac{R}{L}) - \sqrt{(\frac{R}{L})^2 - 1}}{-(\frac{R}{L})^2 + \sqrt{(\frac{R}{L})^2 - 1}} = \frac{\frac{R}{L} + \sqrt{(\frac{R}{L})^2 - 1}}{\frac{R}{L} - \sqrt{(\frac{R}{L})^2 - 1}}$$

Since the numerator and denominator are positive, the right side of equation (16) is always positive. The absolute value signs in equation (16) can be omitted. The equation becomes,

$$\frac{2\sqrt{\left(\frac{R}{L}\right)^2-1}}{\left(\tan\frac{\lambda}{2}-\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right.} = \frac{\left(\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}{\left(\frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}\, e^{\gamma\sqrt{\left(\frac{R}{L}\right)^2-1}} \qquad (17)$$

Solving equation (17) for λ

$$\tan\frac{\lambda}{2}-\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1} = \frac{2\sqrt{\left(\frac{R}{L}\right)^2-1}}{1-\dfrac{\left(\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}{\left(\frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}\, e^{\gamma\sqrt{\left(\frac{R}{L}\right)^2-1}}}$$

So,

$$\tan\frac{\lambda}{2} = \frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}+\frac{2\sqrt{\left(\frac{R}{L}\right)^2-1}}{1-\dfrac{\left(\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}{\left(\frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}\, e^{\gamma\sqrt{\left(\frac{R}{L}\right)^2-1}}}$$

$$= \frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}+\frac{2\sqrt{\left(\frac{R}{L}\right)^2-1}}{1+\dfrac{\left(\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}{\left(-\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}\, e^{\gamma\sqrt{\left(\frac{R}{L}\right)^2-1}}}$$

Finally,

$$\lambda = 2\tan^{-1}\left[\frac{R}{L}-\sqrt{\left(\frac{R}{L}\right)^2-1}+\frac{2\sqrt{\left(\frac{R}{L}\right)^2-1}}{1+\dfrac{\left(\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}{\left(-\frac{R}{L}+\sqrt{\left(\frac{R}{L}\right)^2-1}\right)}\, e^{\gamma\sqrt{\left(\frac{R}{L}\right)^2-1}}}\right] \qquad (18)$$

For $\frac{R}{L} < 1$, a different solution is obtained as follows:

$$\frac{2}{\sqrt{1 - (\frac{R}{L})^2}} \tan^{-1} \left\{ \frac{\tan \frac{\lambda}{2} - \frac{R}{L}}{\sqrt{1 - (\frac{R}{L})^2}} \right\} = \gamma \tag{19}$$

Solving equation (19) for λ,

$$\tan^{-1} \left\{ \frac{\tan \frac{\lambda}{2} - \frac{R}{L}}{\sqrt{1 - (\frac{R}{L})^2}} \right\} = \frac{\gamma}{2} \sqrt{1 - (\frac{R}{L})^2}$$

$$\frac{\tan \frac{\lambda}{2} - \frac{R}{L}}{\sqrt{1 - (\frac{R}{L})^2}} = \tan \left[\frac{\gamma}{2} \sqrt{1 - (\frac{R}{L})^2} \right]$$

$$\tan \frac{\lambda}{2} = \frac{R}{L} + \sqrt{1 - (\frac{R}{L})^2} \tan \left[\frac{\gamma}{2} \sqrt{1 - (\frac{R}{L})^2} \right]$$

$$\lambda = 2 \tan^{-1} \left[\frac{R}{L} + \sqrt{1 - (\frac{R}{L})^2} \left\{ \tan \left(\frac{\gamma}{2} \sqrt{1 - (\frac{R}{L})^2} \right\} \right] \tag{20}$$

Equations (1), (2), (3), (4), (13), (18), and (20) describe the tractrix and the leading curve for the first portion of the tractrix.

EQUATIONS FOR SECOND PORTION OF TRACTRIX

The second portion of the tractrix is the curve described by the trailer when the hitch point moves on a straight line. Equations for this portion of the tractrix are well known [5]

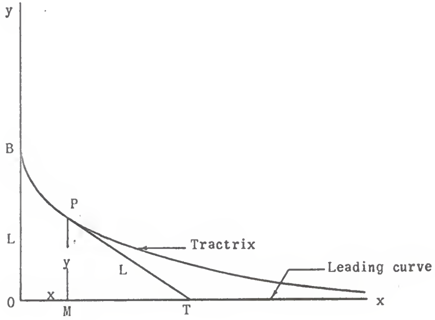

FIGURE 3

TRACTRIX OF A STRAIGHT LINE

The equations for this portion of the tractrix are expressed in parametric form as follows:

$$x = L\left[t - \tanh t\right] \tag{21}$$

$$y = \frac{L}{\cosh t} \tag{22}$$

These relations are to be joined to the first portion of the tractrix to get the total tractrix.

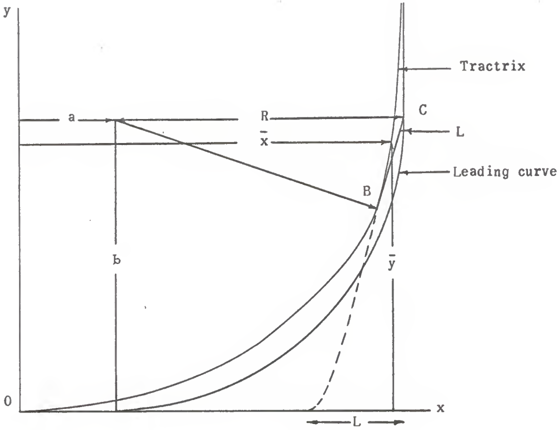

FIGURE 4

TOTAL TRACTRIX INCLUDING THE STRAIGHT LINE MOTION OF THE HITCH POINT

Equations for x and y coordinates of a point on the second portion of the tractrix will become,

$$\bar{x} = a + R - \frac{L}{\cosh t} \tag{23}$$

Similarly,

$$\bar{y} = L\,(t - \tanh t) \tag{24}$$

The end point of the first portion of the tractrix curve, is the beginning point of the second portion of the tractrix. Using this property, the x coordinate of the end point of the first portion of the tractrix is made equal to the x coordinate of the beginning point of the second portion of the tractrix. Then the value of the parameter "t" is calculated accordingly.

With this value of t, the value of the y-coordinate of the beginning point of the second portion of the tractrix is calculated using equation (24).

The difference between the y coordinate of the beginning point of the second portion of the tractrix and the y coordinate of the end point of the first portion of the tractrix is the constant to be added to equation (24).

With this constant C added to equation (24), it becomes,

$$\bar{y} = L\,(t - \tanh t) + C \qquad (25)$$

Explicit relations for $\bar{x}$, $\bar{y}$ coordinates of the tractrix and ξ, η coordinates of the leading curve have now been established.

With equations (23) and (25) the second portion of the tractrix can easily be determined.

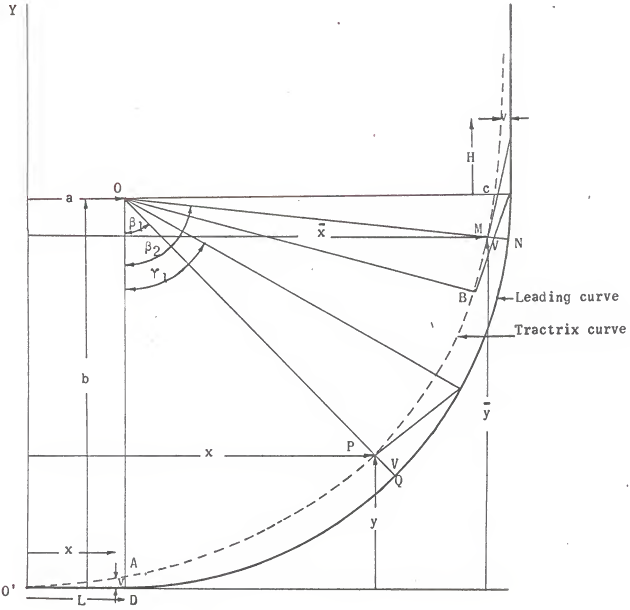

FIGURE 5

Diagram showing Parameters
Used in Presenting Results

PRESENTATION OF RESULTS

A convenient way to present the results is to plot the distance between the tractrix and the leading curve for one value of the ratio $\frac{R}{L}$. As the leading curve starts from the point D in Figure 5, we can plot the distance $\frac{V}{L}$ between the tractrix and the leading curve against $\frac{x}{L}$ up to the point A of the tractrix, where $\beta = 0$. From geometry $V = y$ for $\beta_1 \leq 0$.

To obtain the formula for the distance V when $\beta > 0$; $\Upsilon \leq \frac{\pi}{2}$ consider a point P on the tractrix and Q on the leading curve in the portion of arc AB.

$$\tan = \beta_1 \quad (\bar{x}-a)/(b-\bar{y})$$

$$\beta_1 = \quad \tan^{-1}\left[\frac{(x-a)}{(b-y)}\right]$$

where β_1 is indicated in Figure 4.

$$OP = \sqrt{(x-a)^2 + (b-y)^2}$$

V is the distance between two curves for a particular value of β.

$$V = (OQ - OP)$$
$$= R - \sqrt{(x-a)^2 + (b-y)^2}$$

$\frac{V}{L}$ is plotted against β_1 for this portion of the tractrix.

To obtain the formula for the distance V when $0 < \beta < \frac{\pi}{2}$; and the leading curve is a straight line, consider a point M after the point B.

From the point B onwards, the second portion of the tractrix starts.

The derivation for V is obtained in a similar way as follows:

$$\tan \beta_2 = (\bar{x} - a)/(b-\bar{y})$$

$$\beta_2 = \tan^{-1}\left[(\bar{x} - a)/(b-\bar{y})\right]$$

$$\text{Distance } OM = \sqrt{(\bar{x} - a)^2 + (b - \bar{y})^2}$$

$$V = (ON - OM)$$

$$V = R - \sqrt{(\bar{x} - a)^2 + (b - \bar{y})^2}$$

For this portion of the tractrix $\frac{V}{L}$ is plotted against β_2. For the remaining portion of the tractrix from the point C on the tractrix where $\beta \geq \frac{\pi}{2}$, $\frac{V}{L}$ is plotted against $\frac{H}{L}$. Equations for $\frac{V}{L}$ and $\frac{H}{L}$ are as follows:

$$\frac{V}{L} = (a + R - \bar{x})/L$$

and

$$\frac{H}{L} = (\bar{y} - b)/L$$

The graph of the distance between the tractric and the leading curve is roughly indicated in Figure 6.

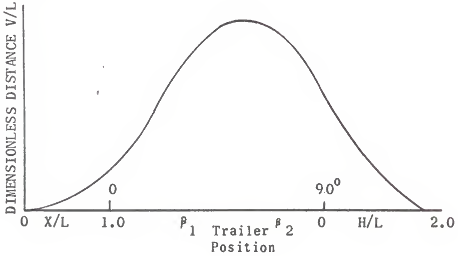

FIGURE 6

DISTANCE BETWEEN THE TRACTRIX
AND THE LEADING CURVE

For this curve to be continuous the scale for β is calculated as follows:

The maximum value of $\frac{X}{L}$ is 1.0 and is represented by 1 unit on graph paper.

The length of the quarter circle made by the hitch point to be represented on the graph is equal to $(\frac{\pi R}{2})$. Number of units on graph paper representing the length of the quarter circle are equal to $\frac{\pi R}{2L}$. Thus, the scale for β is obtained.

NUMERICAL RESULTS

An IBM 1410 electronic computer was used to obtain solutions to equations (18) through (25) for given values of the independent variables. These numerical results are given in Tables I and II. Figures 8, 9, 10, 11, 12, and 13 show the distance between the curves for different radii of cornering. Figure 7 shows the effect of increase of radius of cornering, on the distance between the curves.

The numerical work was done using 11 place arithmetic.

TABLE I

Distance Between the Tractrix and the Leading Curve

R = 45 Ft.   L = 30 Ft.

x/L	β_1(degrees) for $\beta > 0$, $\alpha \leq \frac{\pi}{2}$	β_2(degrees) for $\alpha \geq \frac{\pi}{2}$	H/L	V/L
0.00000				0.00000
0.02618				0.00000
0.13075				0.00024
0.23479				0.00136
0.41475				0.00724
0.59100				0.02027
0.71413				0.03506
0.81073				0.05058
0.90458				0.06961
0.99821				0.09229
	0.86051			0.09839
	9.36357			0.14756
	15.11604			0.17530
	18.98005			0.19186
	30.67758			0.23371
	41.49913			0.26365
	48.54246			0.27963
	48.42645			0.27938
	48.41873			0.27937

Table I (Con't)

X/L	β_1(degrees for $\beta > 0 \alpha \leq \frac{\pi}{2}$	β_2(degrees) for $\alpha \geq \frac{\pi}{2}$	H/L	V/L
		48.41834		0.27937
		60.50916		0.29872
		65.67774		0.30134
		70.96308		0.29933
		74.96940		0.29422
		77.64615		0.28891
		84.30138		0.26851
		86.92853		0.25735
		88.23091		0.25112
		89.52465		0.24445
			0.01859	0.23747
			0.51040	0.14391
			0.80572	0.10686
			1.01376	0.08671
			1.61113	0.04765
			2.51018	0.01938
			3.02006	0.01164
			3.50002	0.00720
			4.01000	0.00433
			4.27999	0.00330

TABLE II

Distance Between the Tractrix and the Leading Curve

$R = 300$ Ft. $L = 30$ Ft.

x/L	β_1(degrees) for $\beta > .0$, $\alpha \leq \frac{\pi}{2}$	β_2(degrees) for $\alpha \geq \frac{\pi}{2}$	H/L	V/L
0.00000				0.00000
0.17453				0.00008
0.34901				0.00065
0.52343				0.00221
0.69776				0.00480
0.87198				0.00901
	0.26429			0.01489
	5.25943			0.03535
	9.25743			0.04275
	26.25074			0.04974
	35.24728			0.05004
	40.24536			0.05009
	45.24343			0.05011
	51.24113			0.05012
	84.22846			0.05013
		84.22845		0.05015
		85.26020		0.05005
		86.81007		0.04878

Table II (Con't)

x/L	β_1(degrees) for $\beta>0, \alpha \geq \frac{\pi}{2}$	β_2(degrees) for $\alpha \geq \frac{\pi}{2}$	H/L	V/L
		87.67174		0.04701
		88.36116		0.04488
		88.87719		0.04279
		89.39513		0.04026
		89.56741		0.03931
		89.73967		0.03830
		89.91191		0.03724
			0.02065	0.03615
			0.20045	0.03019
			0.29038	0.02759
			0.50025	0.02237
			1.01009	0.01344
			1.49003	0.00831
			1.76001	0.00635
			2.00001	0.00499
			2.30000	0.00369
			2.42000	0.00328

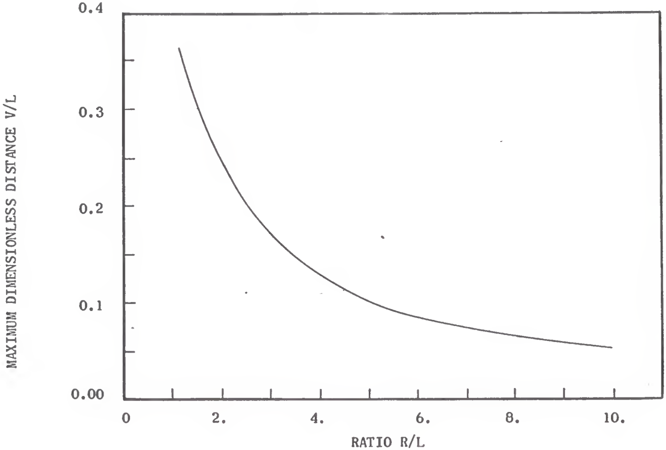

FIGURE 7

Maximum Dimensionless Distance V/L versus Ratio R/L

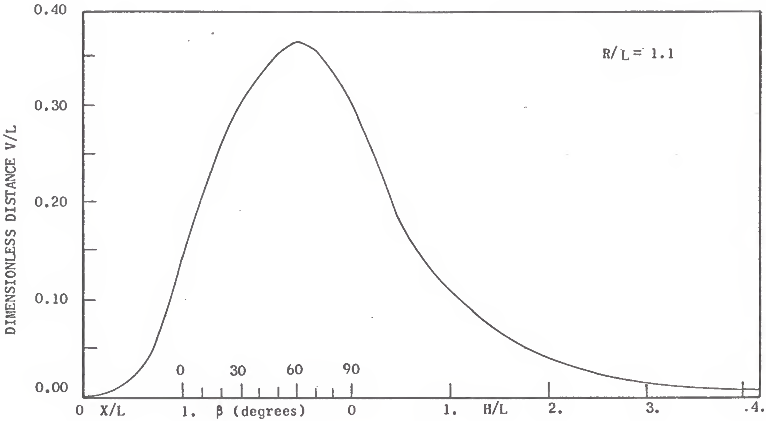

FIGURE 8

TRAILER'S POSITION

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

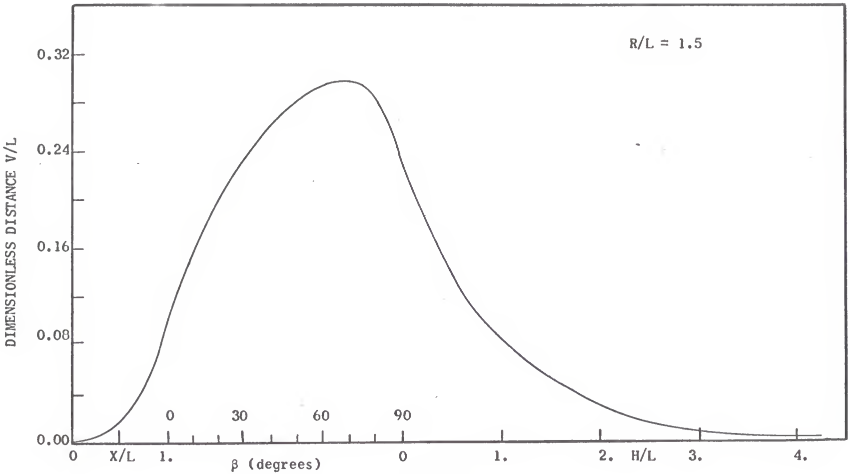

FIGURE 9

POSITION OF THE TRAILER

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

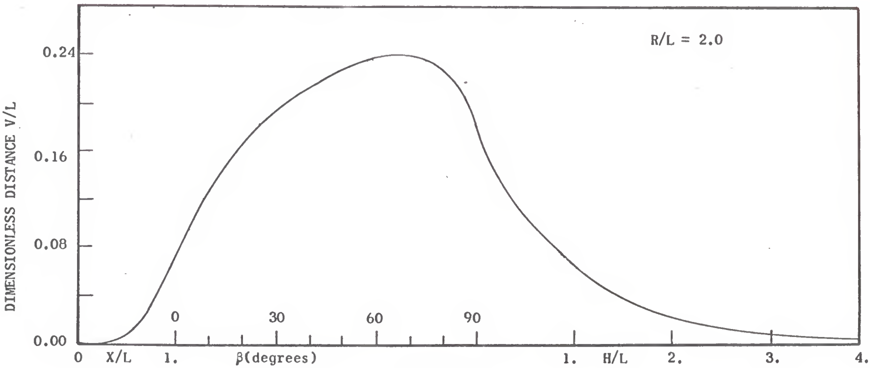

FIGURE 10

POSITION OF THE TRAILER

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

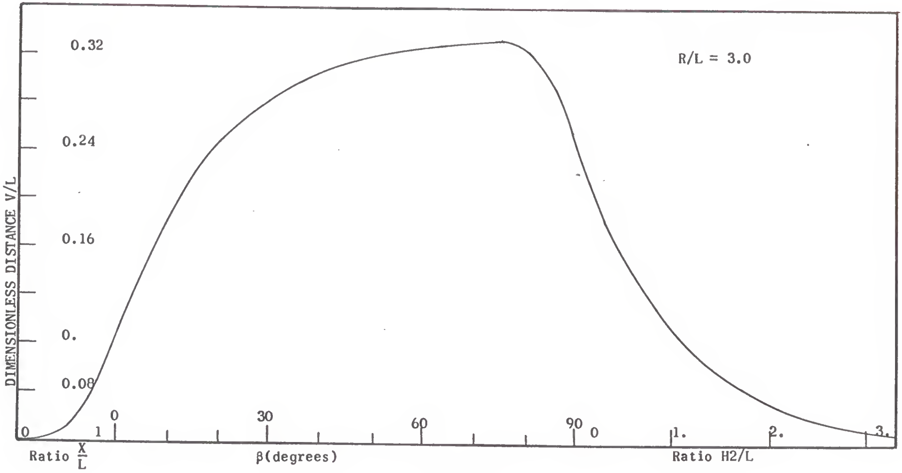

FIGURE 11

POSITION OF THE TRAILER

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

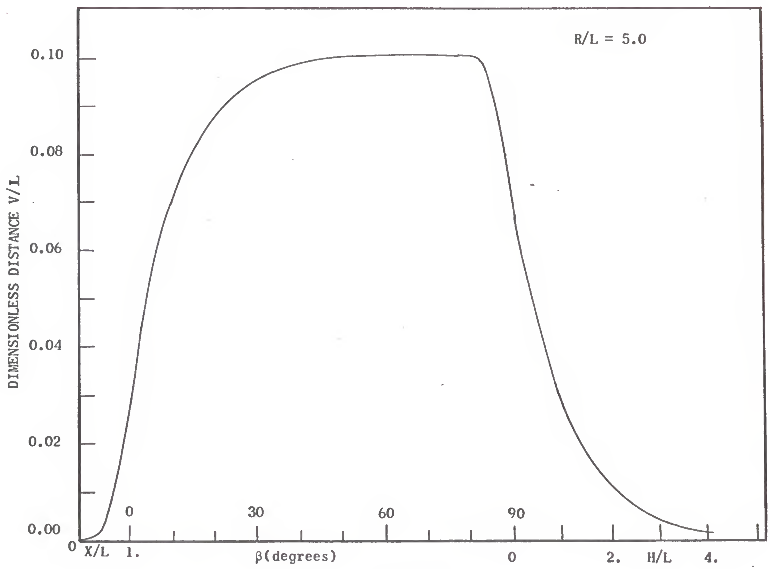

FIGURE 12

TRAILER'S POSITION

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

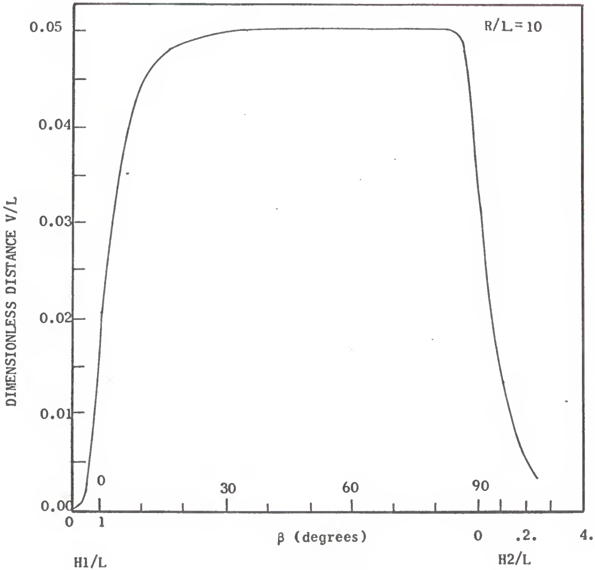

FIGURE 13

POSITION OF THE TRAILER

Dimensionless distance between the tractrix and the leading curve versus Trailer's position

DISCUSSION OF RESULTS

The present results indicate that the distance between the leading curve and tractrix increases up to some value of the angle β and then decreases to zero.

It can be seen as in Figure 7 that as the ratio $\frac{R}{L}$ increases, the maximum distance between the leading curve and the tractrix becomes less. Thus, it is concluded that for corners with larger radius of curvature, cornering can easily be done without any difficulty.

The radius of curvature of cornering, i.e., R, was assumed constant for simplicity of analysis. But, practically the radius of curvature of the leading curve may not be constant. Further, investigation is recommended to determine the distances between other leading curves and corresponding tractrix curves.

CONCLUSIONS

The results indicate that the distance between the leading curve and the tractrix will become smaller as the radius of curvature of the leading curve is increased. The procedure can be extended to other leading curves to determine the differential equation. However, the solution becomes more complex as the geometry of the problem is varied.

The degree of accuracy of the solution obtained by this procedure is dependent upon the efficiency and speed of present day high speed digital computing facilities. The method is practical and results are accurate.

ACKNOWLEDGMENT

I express my appreciation to Dr. F. C. Appl of the Department of Mechanical Engineering, Kansas State University, for his guidance and counsel throughout the course of this work as my adviser.

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APPENDIX A

Listing of Fortran Program to solve for the distance between the tractrix curve and the leading curve.

```
      MON$$      JOB  TRACTOR
      MON$$      COMT 15,40PAGES,,DR F C APPL  R BATTU  MECH
      MON$$      ASGN MJB,12
      MON$$      ASGN MGO,16
      MON$$      MODE GO,TEST
S     MON$$      EXEQ FORTRAN,,,12,,,,TRACTOR
    5 FORMAT(5E16.8)
    7 FORMAT (4E21.12)
      READ (1,5)C,DEL,D,ER
      RL=1.5
      E=30.
      ERR=0.001
   10 GAMA=0.
      B=(RL)*C
      DELTA=DEL
      ERB=0.1
   11 P=RL-(SQRT((RL)*(RL)-1.))
      Q=(RL+SQRT((RL)*(RL)-1.))/(-RL+SQRT((RL)*(RL)-1.))
      T=(2.*(SQRT((RL)*(RL)-1.)))
      S=(T)/(1.+(EXP(GAMA*(SQRT((RL)*(RL)-1.)))))*Q)
      A=2.*(ATAN(P+S))
      THETA=GAMA-A
      R=(RL)*C
      ETA=B-(R*(COS(GAMA)))
      CETA=D+(R*(SIN (GAMA)))
      X=CETA-(C*(COS (THETA)))
      Y=ETA-(C*(SIN(THETA)))
      WRITE(3,7)GAMA,THETA,ETA,CETA
      WRITE (3,7) X,Y,A,RL
      IF(X-30.)8,4,4
    4 BETA1=(ATAN((X-D)/(B-Y)))*(180.)/(3.1428)
      BETA2=(ATAN((X-D)/(B-Y)))/R
      R1=SQRT((X-D)*(X-D)+(B-Y)*(B-Y))
      DIS1=(R-R1)/C
      DIS2=(R-R1)/R
      GO TO 2
    8 VER1=Y/C
      HOR1=X/C
      VER2=Y/R
      HOR2=X/R
      WRITE(3,5)HOR1,VER1,HOR2,VER2
    2 WRITE(3,5)DIS1,BETA1,DIS2,BETA2
      ERROR=ABS(GAMA-1.57079633)
      IF(ERROR-ER)14,14,12
   12 IF(GAMA-1.57079633)13,14,15
   13 GAMA=GAMA+DELTA
      A1=ATAN(1./(SQRT((RL)*(RL)-1.)))
      IF(A-A1)16,14,14
   16 GO TO 11
   15 DELTA=DELTA/2.
```

```
      GAMA=GAMA-DELTA
      GO TO 11
   14 Z=0.
   17 COSHT=(EXP(Z)+EXP(-Z))/2.
      TANHT=(EXP(Z)-EXP(-Z))/(EXP(Z)+EXP(-Z))
      XB=D+R-(C/COSHT)
      WRITE(3,5)XB
      ER2=ABS(XB-X)
      IF(ER2-ERR)19,19,25
   25 IF(XB-X)18,19,26
   26 DELZ=DELZ/2.
      Z=Z-DELZ
      GO TO 17
   18 DELZ=0.05
      Z=Z+DELZ
      GO TO 17
   19 YA=C*(Z-TANHT)
      C1=Y-YA
   20 COSHT=(EXP(Z)+EXP(-Z))/2.
      TANHT=(EXP(Z)-EXP(-Z))/(EXP(Z)+EXP(-Z))
      XB=D+R-(C/COSHT)
      YB=C*(Z-TANHT)+C1
      IF(YB-B)27,28,28
   27 BETA3=(ATAN((XB-D)/(B-YB)))*(180.)/(3.1428)
      BETA4=(ATAN((XB-D)/(B-YB)))/R
      R2=SQRT((XB-D)*(XB-D)+(B-YB)*(B-YB))
      DIS3=(R-R2)/C
      DIS4=(R-R2)/R
      WRITE(3,5)DIS3,BETA3,DIS4,BETA4
      GO TO 21
   28 HOR3=(YB-B)/C
      HOR4=(YB-B)/R
      VER3=(D+R-XB)/C
      VER4=(D+R-XB)/R
   21 WRITE(3,5)HOR3,VER3,HOR4,VER4
      ER1=ABS(XB-(D+R))
      IF(ER1-ERB)23,23,22
   22 DELZ=0.03
      Z=Z+DELZ
      GO TO 20
   23 RL=RL+1.
      IF(RL-6.)10,10,24
   24 STOP
      END
S     MON$$      EXEQ LINKLOAD
                 CALL TRACTOR
     MON$$      EXEQ TRACTOR,MJB
   .30000000E 02   .17453290E-01   .30000000E 02   .10000000E-06
```

TRACTOR TRAILER CORNERING

by

RAMGOPAL BATTU

B. S., Osmania University, 1963

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas
1965

When a trailer moves around a corner, the hitch point of the trailer makes a curve called the "leading curve." In this paper it is assumed, for the purpose of analysis, that the leading curve has a constant radius of curvature. The differential equation for the tractrix is determined, and a solution found.

Numerical results are determined for six leading curves with different radii of curvature. Curves indicating the distance between the tractrix and the leading curve as a function of trailer position are plotted for all cases. The method of finding the distance between the curves is well adapted to analysis on high speed digital computers.

The differential equation derived in this paper for the tractrix is not valid when the hitch point makes other than circular leading curves.