

309

ON THE DISCRETE HILBERT TRANSFORM

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CHAPTER I

INTRODUCTION

In almost every field where Fourier techniques are used to represent and analyze physical processes, one finds that there are situations where there exist relationships between the real and imaginary parts or the magnitude and phase of the Fourier Transform. These relationships are known by different names, depending upon the field of interest, but often they are called Hilbert transform relations. In this respect, the field of digital signal processing is no exception.

This report concerns the discrete form of the Hilbert transform. It is known as the discrete Hilbert transform, and is conveniently represented in terms of a matrix equation. Our main objective is to implement the discrete Hilbert transform on a minicomputer system, and illustrate its frequency characteristics via examples. To this end, experimental results related to the following type of discrete Hilbert transform are included: lowpass, bandpass, highpass, and bandstop.

CHAPTER II

THE DISCRETE HILBERT TRANSFORM

2.1 Introduction

In this chapter, a development of the discrete Hilbert transform (DHT) is presented. It is shown that the DHT can be expressed in the form of a matrix equation.

2.2 Derivation of the DHT

The Hilbert transform (HT) of a continuous signal $f(t)$ is defined as [3]:

$$g(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(\tau)}{\tau - t} d\tau \quad (2-1)$$

where $g(t)$ denotes the HT.

An alternate form of writing (2-1) is:

$$g(t) = \frac{1}{\pi t} * f(t) \quad (2-2)$$

where the symbol "*" denotes convolution. Thus, the HT of $f(t)$ can be interpreted as a convolution between $f(t)$ and $1/\pi t$ to produce the desired signal $g(t)$.

Taking the Fourier transform (FT) of both sides of (2-2) yields:

$$G(jw) = H(jw) F(jw). \quad (2-3)$$

In (2-3), $F(jw)$ and $G(jw)$ denote the Fourier transforms of $f(t)$ and $g(t)$, respectively, and

$$H(jw) = FT(1/\pi t) = -j \operatorname{sgn}(w) \quad (2-4)$$

with

$$\text{sgn}(w) = \begin{cases} 1, & w > 0 \\ 0, & w = 0 \\ -1, & w < 0 \end{cases}$$

We now consider the case when the input is a discrete signal. Then corresponding to the input $f(t)$, we have a data sequence f_n , $n = 0, 1, 2, \dots, (N-1)$. Again, corresponding to $H(jw)$ in (2-4), one has a sequence of complex numbers, H_k , $k = 0, 1, 2, \dots, (N-1)$. We denote the resulting DHT sequence by g_n , $n = 0, 1, \dots, (N-1)$. In what follows, the cases when N is even or odd, one is treated separately.

Case 1: N is even

Equation (2-4) yields

$$H_k = \begin{cases} -j, & k = 1, 2, \dots, (N/2-1) \\ 0, & k = N/2, 0. \\ j, & k = (N/2 + 1), (N/2 + 2), \dots, (N-1). \end{cases} \quad (2-5)$$

Next, let F_k and G_k , $k = 0, 1, \dots, (N-1)$ denote the discrete Fourier transform (DFT) coefficients of the input data sequence $\{f_n\}$ i.e.,

$$F_k = \frac{1}{N} \sum_{n=0}^{N-1} f_n W^{-nk}, \quad k = 0, 1, \dots, (N-1) \quad (2-6)$$

and

$$G_k = \frac{1}{N} \sum_{n=0}^{N-1} g_n W^{-nk}, \quad k = 0, 1, \dots, (N-1) \quad (2-7)$$

where $W = e^{-j2\pi/N}$.

Thus, corresponding to (2-3) we have

$$G_k = H_k F_k, \quad k = 0, 1, \dots, (N-1). \quad (2-8)$$

Substituting (2-6) in (2-8) and then taking the inverse DFT, one obtains

$$\begin{aligned} g_n &= \frac{1}{N} \sum_{k=0}^{N-1} H_k \sum_{m=0}^{N-1} f_m W^{km} W^{-kn} \\ &= \frac{1}{N} \sum_{m=0}^{N-1} f_m \sum_{k=0}^{N-1} H_k W^{(m-n)k}, \quad n=0, 1, \dots, (N-1) \end{aligned} \quad (2-9)$$

Substitution for the H_k in (2-9) using (2-5) results in

$$g_n = \frac{-1}{N} \sum_{m=0}^{N-1} f_m [1 - (-1)^{n-m}] \left\{ \sum_{k=1}^{N-1} e^{-jk(n-m) 2\pi/N} \right\}, \quad (2-10)$$

for $n = 0, 1, 2, \dots, (N-1)$.

In (2-10) we use the result that

$$\sum_{k=1}^{N-1} e^{jk(n-m) 2\pi/N} = j \cot [(m-n)\pi/N] \quad (2-11)$$

to obtain

$$g_n = \frac{1}{N} \sum_{m=0}^{N-1} f_m [1 - (-1)^{n-m}] \cot [(m-n)\pi/N], \quad (2-12)$$

for $n = 0, 1, \dots, (N-1)$.

A more convenient way of writing (2-12) is as follows:

$$g_n = \frac{1}{N} \sum_{m=0}^{N-1} f_m c_{n,m}, \quad n=0, 1, \dots, (N-1) \quad (2-13)$$

where

$$c_{n,m} = \begin{cases} 0, & n=m, \text{ and } (n-m) \text{ even} \\ 2 \cot [(n-m)\pi/N], & (n-m) \text{ odd.} \end{cases}$$

Case 2: N is odd

Here (2-4) yields

$$H_k = \begin{cases} -j, & k = 1, 2, \dots, (N-1)/2 \\ 0, & k = 0 \\ j, & k = (N+1)/2, (N+3)/2, \dots, (N-1). \end{cases} \quad (2-14)$$

Proceeding in a manner similar to case 1, we obtain the following result

$$g_n = \frac{1}{N} \sum_{m=0}^{N-1} f_m c_{n,m}, \quad n = 0, 1, \dots, (N-1) \quad (2-15)$$

where

$$c_{n-m} = \begin{cases} \cot (n-m)\pi/N - \frac{(-1)^{n-m}}{\sin [(n-m)\pi/N]}, & n \neq m \\ 0, & n = m \end{cases}$$

Equations (2-13) and (2-15) are defined as the DHT of a given data sequence f_n , $n = 0, 1, \dots, (N-1)$, depending upon whether N is even or odd, respectively.

2.3 Matrix Representation

Either form of the DHT in (2-13) and (2-15) can be written in terms of a matrix notation as follows:

$$\underline{G}_N = \underline{H}_N \underline{F}_N$$

where:

$$\underline{F}_N' = [f_0, f_1, \dots, f_{N-1}] \text{ is the input data}$$

vector, prime denoting transpose;

$$\underline{G}_N' = [g_0, g_1, \dots, g_{N-1}] \text{ is the corresponding DHT vector, and}$$

$$\underline{H}_N \text{ is an } (NXN) \text{ DHT matrix.}$$

To illustrate, consider the case when N is even. Then (2-13) yields the following DHT matrix:

$$\underline{H}_N = \begin{bmatrix} 0 & -c_1 & 0 & -c_3 & 0 & -c_5 & \dots & -c_{N-1} \\ c_1 & 0 & -c_1 & 0 & -c_3 & 0 & \dots & 0 \\ 0 & c_1 & 0 & -c_1 & 0 & -c_3 & \dots & -c_{N-3} \\ & & . & & . & & & \\ & & . & & . & & & \\ c_{N-1} & 0 & c_{N-3} & 0 & c_{N-5} & 0 & & 0 \end{bmatrix} \quad (2-17)$$

where $c_k = 2 \cot (K\pi/N)$.

An examination of (2-17) reveals the following properties of the DHT matrix:

- 1) It is a square circulant matrix of order N , with only P distinct elements, (ignoring differences in sign), where

$$P = \text{integer value of } N/4. \quad (2-18)$$
- 2) It is a skew symmetric matrix.
- 3) Because of 1) and 2) above, only the first $N/2$ elements of the first row are sufficient to construct the entire matrix.

One may verify that properties very similar to the above are valid for the case when N is odd, by starting with (2-15).

CHAPTER III

DIGITAL FILTERING VIA THE DISCRETE
HILBERT TRANSFORM3.1 Introduction

The purpose of this chapter is to show that the DHT which was developed in the last chapter can be used to perform digital filtering operations--e.g., lowpass, bandpass, etc. We will present several DHT filter realizations and also present some experimental results.

3.2 Lowpass Filter Realization

The DHT lowpass filter realization is defined by the following matrix equation [7].

$$\underline{L}_N = [\underline{B}_N \underline{H}_N \underline{A}_N - \underline{A}_N \underline{H}_N \underline{B}_N] \underline{F}_N \quad (3-1)$$

where $\underline{H}_N$ is the DHT matrix defined in (2-13) or (2-15). The $N \times N$ matrices $\underline{A}_N$ and $\underline{B}_N$ are diagonal matrices as follows:

$$\underline{A}_N = \text{diag. } [1, \cos(a), \cos(2a), \dots, \cos\{(N-1)a\}]$$

and (3-2)

$$\underline{B}_N = \text{diag. } [0, \sin(a), \sin(2a), \dots, \sin\{(N-1)a\}]$$

where the parameter a is defined as

$$a = \frac{2\pi W}{N} \quad (3-3)$$

where W is the cut-off frequency expressed as a number of sample points in the frequency domain.

Equation (3-1) is written in a more convenient form as

$$\underline{L}_N = [LP] \underline{F}_N \quad (3-4)$$

$[LP] = [B_N \ H_N \ A_N - A_N \ H_N \ B_N]$ is the lowpass matrix.

It is clear that $[LP]$ represents the entire lowpass filtering process. It can be written in the form:

$$[LP] = \frac{1}{N} \begin{bmatrix} 0 & h_1 & 0 & h_3 & 0 & h_5 & \dots & h_{N-1} \\ h_1 & 0 & h_1 & 0 & h_3 & 0 & \dots & 0 \\ 0 & h_1 & 0 & h_1 & 0 & h_3 & \dots & h_{N-3} \\ & & & \cdot & & & & \\ & & & \cdot & & & & \\ h_{N-1} & 0 & h_{N-3} & 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} \quad (3-5)$$

where

$$h_k = 2 \cot(\pi k/N) \sin(ak), \text{ for } k \text{ odd, and } a = \frac{2\pi W}{N}; \text{ see } (3-3).$$

Substitution of (3-5) in (3-4) leads to the matrix equation:

$$\begin{bmatrix} l_1 \\ l_2 \\ \cdot \\ \cdot \\ l_N \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 0 & h_1 & 0 & h_3 & 0 & \dots & h_{N-1} \\ h_1 & 0 & h_1 & 0 & h_3 & \dots & 0 \\ 0 & h_1 & 0 & h_1 & 0 & \dots & h_{N-3} \\ & & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \cdot & \\ h_{N-1} & 0 & h_{N-3} & \cdot & \cdot & \cdot & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_N \end{bmatrix} \quad (3-6)$$

where l_i denotes the i -th DHT coefficient, and the input sequence is $f_1, f_2, \dots, f_N$.

Comparing (2-17) and (3-6), it follows that the DHT matrix $\underline{H}_N$ and the lowpass matrix in (3-6) have basically the same structure. However, the lowpass matrix is symmetric, while $\underline{H}_N$ is skew symmetric.

Next, let us consider the case that N is even. Then the middle row of (3-6) represents an FIR (finite impulse response) filter whose coefficients are given by:

$$\frac{1}{N} \{h_{m-1}, h_{m-3}, \dots, h_{m-3}, h_{m-1}\},$$

where $m = N/2$.

Thus, the transfer function of this filter is given by:

$$H(z) = \frac{1}{N} \left\{ h_{m-1} z^{-(m-1)} + h_{m-3} z^{-(m-3)} + \dots + h_{m-3} z^{m-3} + h_{m-1} z^{m-1} \right\} \quad (3-7)$$

Let v be a normalized frequency variable--i.e.,

$$v = f/f_N$$

where f_N is the Nyquist frequency. Then, evaluating

$$H(e^{j\pi v}) = h(z) \mid z = e^{j\pi v} \quad (3-8)$$

we obtain

$$H(e^{j\pi v}) = \frac{2}{N} \sum_{k, \text{odd}} h_k \cos(k\pi v), \quad 0 \leq v < 1 \quad (3-9)$$

where the h_k are defined in (3-5); i.e.,

$$h_k = 2 \cot(\pi k/N) \sin(ak)$$

where k is odd, and $\max(k) = N-1$.

Equation (3-9) is the desired DHT transfer function for the lowpass case.

As an illustrative example, we consider the case $N = 160$ and $W = 16$, where W is the cutoff frequency expressed as a number of sample parts in the frequency domain. Then the parameter "a" in (3-3) is

$$a = \frac{2\pi \times 16}{160} = 0.6283$$

Substituting this value of a in (3-9), we compute

$$10 \log |H(e^{j\pi v})|^2, \quad 0 \leq v < 1.$$

The resulting plot is shown in Figure 3.1 from which it is seen that a sharp cutoff is realized.

3.3 Bandpass Filter Realization

A DHT bandpass filter can be realized by connecting two DHT lowpass filters in parallel. The cutoff frequencies of these lowpass filters will be different. In matrix form, this operation is obtained as [7]:

$$\begin{aligned} \underline{B}_N &= \{[LP]_1 - [LP]_2\} \underline{F}_N \\ &= [BP] \underline{F}_N \end{aligned} \quad (3-10)$$

where $[LP]_1$ and $[LP]_2$ are two DHT lowpass filter matrices with cutoff frequencies W_1 and W_2 , respectively. Combining these matrices, we obtain the desired DHT bandpass filter matrix $[BP]$, and its elements h_k are as follows:

$$h_k = 2 \cot(\pi k/N) [\sin(a_1 k) - \sin(a_2 k)], \quad k \text{ odd} \quad (3-11)$$

where

$$a_i = 2\pi W_i/N, \quad i = 1, 2.$$

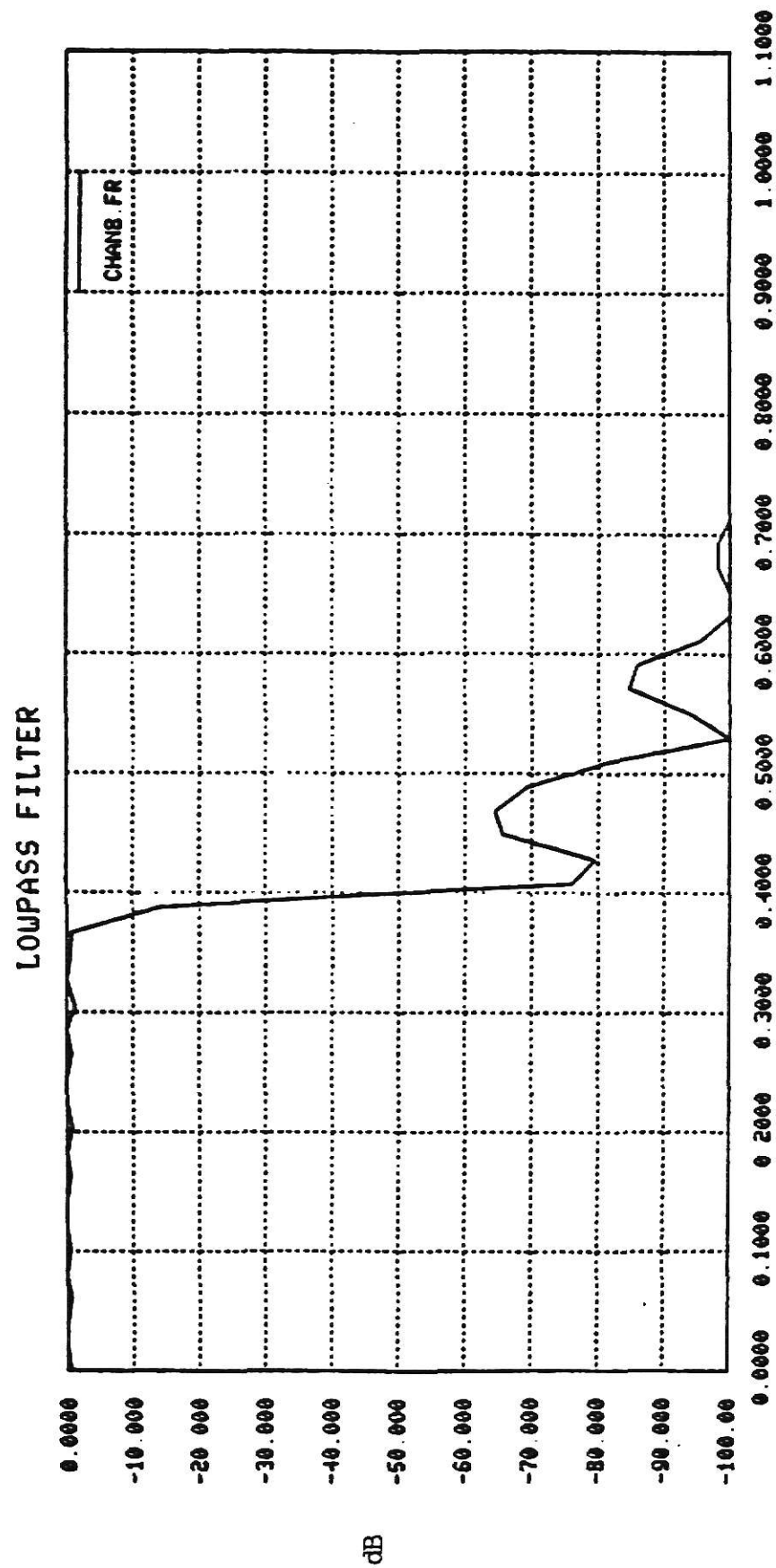


Figure 3.1. Gain function of a DHT lowpass filter; $N=160$, $W=16$.

For example, with $W_1 = 12$, $W_2 = 23$, and $N = 200$, the gain characteristic shown in Figure 3.2 is obtained. As was the case in the previous example, we see that sharp cutoffs are realized.

3.4 Highpass Filter Realization

This type of realization is obtained by connecting a direct path in parallel with a lowpass filter. In matrix form, this is described as

$$\underline{H}_N = \{[I] - [LP]\} \underline{F}_N \quad (3-12)$$

where I is an $N \times N$ identity matrix, and $[LP]$ is the DHT lowpass matrix defined in (3-4).

Let $[HP]$ denote the DHT lowpass matrix. Then, (3-12) can be written as

$$\underline{H}_N = [HP] \underline{F}_N \quad (3-13)$$

where

$$[HP] = [I] - [LP].$$

Substitution for $[LP]$ in (3-13) using (3-4) leads to

$$[HP] = \frac{1}{N} \begin{bmatrix} N & h_1 & 0 & h_3 & 0 & \dots & h_{N-1} \\ h_1 & N & h_1 & 0 & h_3 & & 0 \\ 0 & h_1 & N & h_1 & 0 & & h_{N-3} \\ & \cdot & \cdot & \cdot & & & \\ & \cdot & \cdot & \cdot & & & \\ h_{N-1} & 0 & h_{N-3} & 0 & \cdot & \dots & N \end{bmatrix} \quad (3-14)$$

where

$$h_k = -2 \cot(\pi k/N) \sin(ak), \text{ for } k \text{ odd, and}$$

$$a = \frac{2\pi W}{N}; \text{ see (3-3).}$$

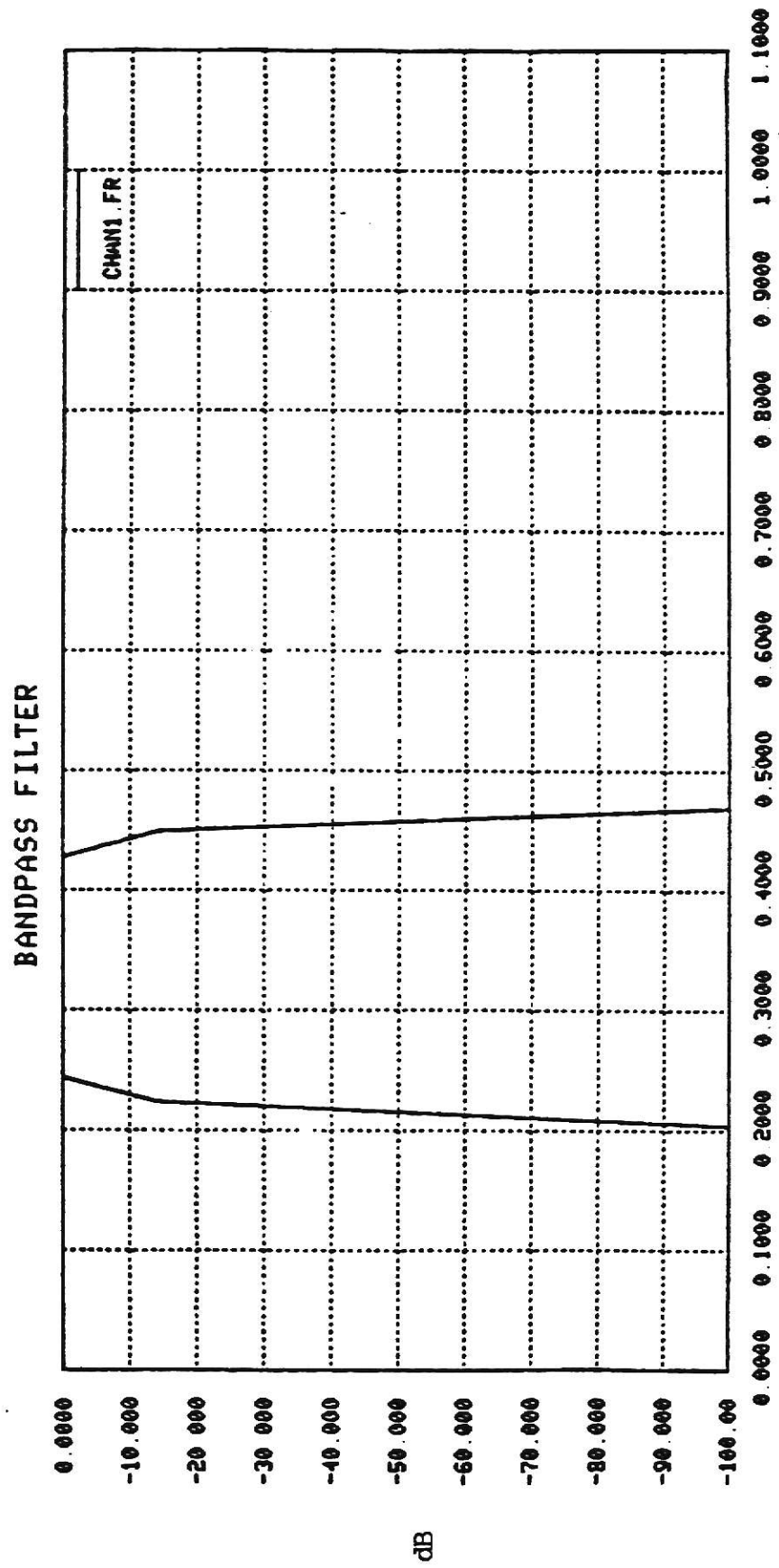


Figure 3.2. Gain function of a DHT bandpass filter; $N=200$, $W1=12$, $W2=23$

Substituting (3-14) in (3-13) we obtain

$$\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ \vdots \\ h_N \end{bmatrix} F \frac{1}{N} \begin{bmatrix} N & h_1 & 0 & h_3 & 0 & \dots & h_{N-1} \\ h_1 & N & h_1 & 0 & h_3 & \dots & 0 \\ 0 & h_1 & N & h_1 & 0 & \dots & h_{N-3} \\ & & & \cdot & \cdot & & \\ & & & \cdot & \cdot & & \\ h_{N-1} & 0 & h_{N-3} & 0 & & \dots & N \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_N \end{bmatrix} \quad (3-15)$$

For convenience, we consider the case when N is even. Then the middle row of (3-15) represents an FIR filter, whose coefficients are given by

$$\frac{1}{N} \{h_{m-1}, h_{m-3}, \dots, h_{m-3}, h_{m-1}\},$$

where $m = N/2$.

Then the transfer function of this function is obtained as

$$H(z) = \frac{1}{N} \left\{ h_{m-1} z^{-(m-1)} + h_{m-3} z^{-(m-3)} + \dots + h_{m-3} z^{m-3} + h_{m-1} z^{m-1} + 1 \right\}, \quad (3-16)$$

which yields

$$H(e^{j\pi v}) = \frac{2}{N} \sum_{k, \text{ odd}} h_k \cos(k\pi v) + 1 \quad (3-17)$$

where the h_k are defined in (3-14); i.e.,

$$h_k = 2 \cot(\pi k/N) \sin(ak)$$

where k is odd, and $\max(k) = N-1$.

Equation (3-17) is thus the transfer function of the DHT highpass filter.

To illustrate, let $N = 240$ and $W = 11$, where W is the cutoff frequency expressed as a number of sample points in the frequency domain. This corresponds to

$$a = \frac{2\pi \times 11}{240} = 0.2880; \text{ see (3-3).}$$

This value of a is substituted in (3-17) to compute

$$10 \log |H(e^{j\pi v})|^2, \quad 0 \leq v < 1$$

and obtain the plot shown in Figure 3.3. Once again, it is observed that a sharp cutoff is realized.

3.5 Bandstop Filter Realization

As in the highpass filter case, a bandstop filter can be realized by means of a direct connection in parallel with a bandpass filter.

This can be described in matrix form as follows [7]:

$$\underline{S}_N = [\text{BS}] \underline{F}_N \quad (3-18)$$

where

$[\text{BS}] = [\text{I}] - [\text{BP}]$ is the DHT bandstop matrix, and $[\text{BP}]$ is the bandpass matrix defined in (3-10).

The DHT bandstop matrix in (3-18) has the form and properties of the DHT highpass matrix in (3-13), except that its coefficients h_k are given by

$$h_k = -2 \cot(\pi k/N) [\sin(a_1 k) - \sin(a_2 k)], \quad k \text{ odd} \quad (3-19)$$

where

$$a_i = 2\pi W_i/N, \quad i = 1, 2$$

As examples, the gain characteristics with $W_1 = 16$, $W_2 = 24$, and $N = 200$ are shown in Figure 3.4, while Figure 3.5 shows the characteristic of a notch filter with $W_1 = 18$, $W_2 = 20$, and $N = 240$. It is clear that sharp cutoff frequencies are realized in both cases.

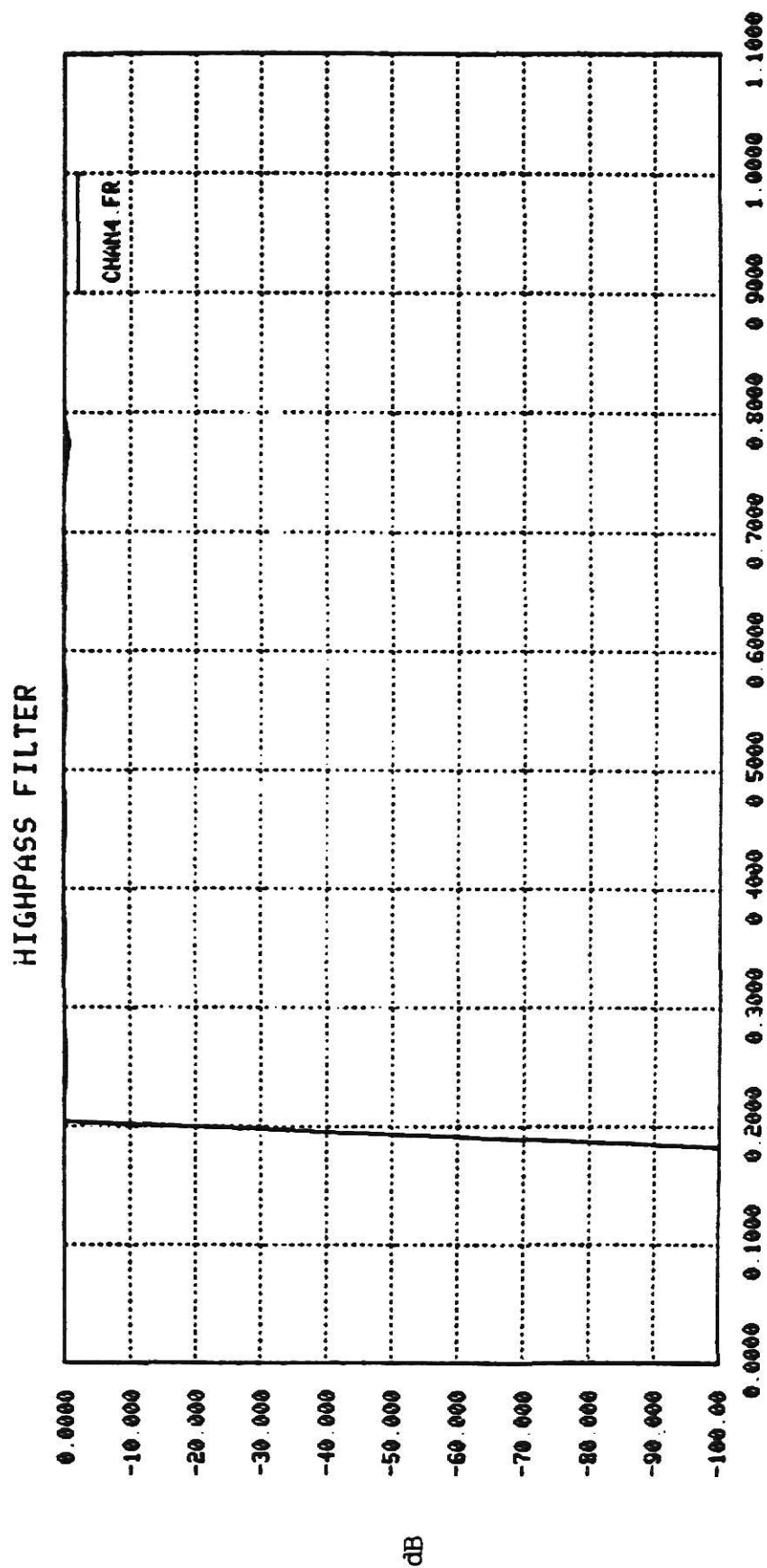


Figure 3.3. Gain function of a DHT highpass filter; $N=240$, $W=25$.

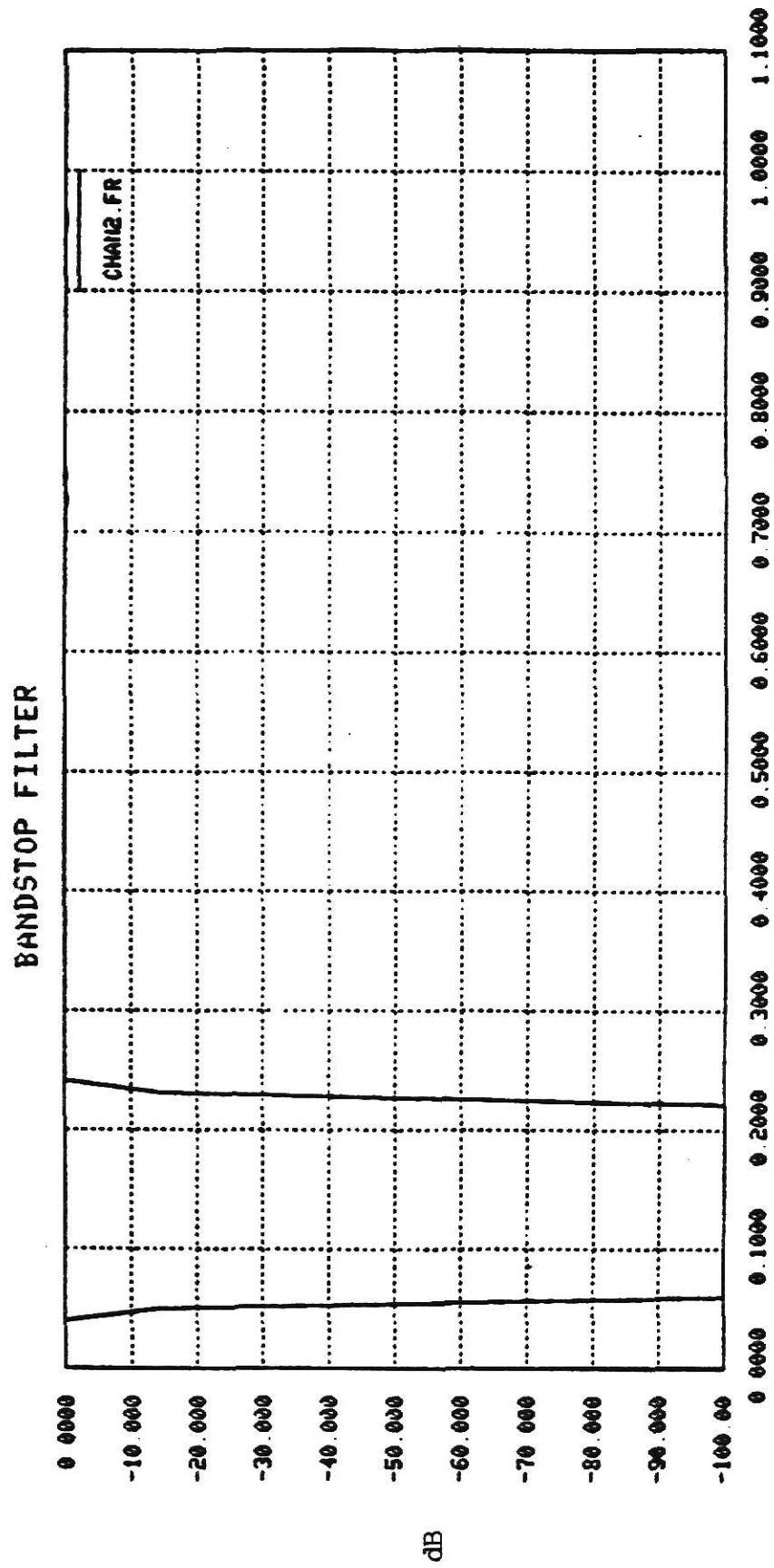


Figure 3.4. Gain function of a DHT bandstop filter; $N=200$, $W1=6$, $W2=24$

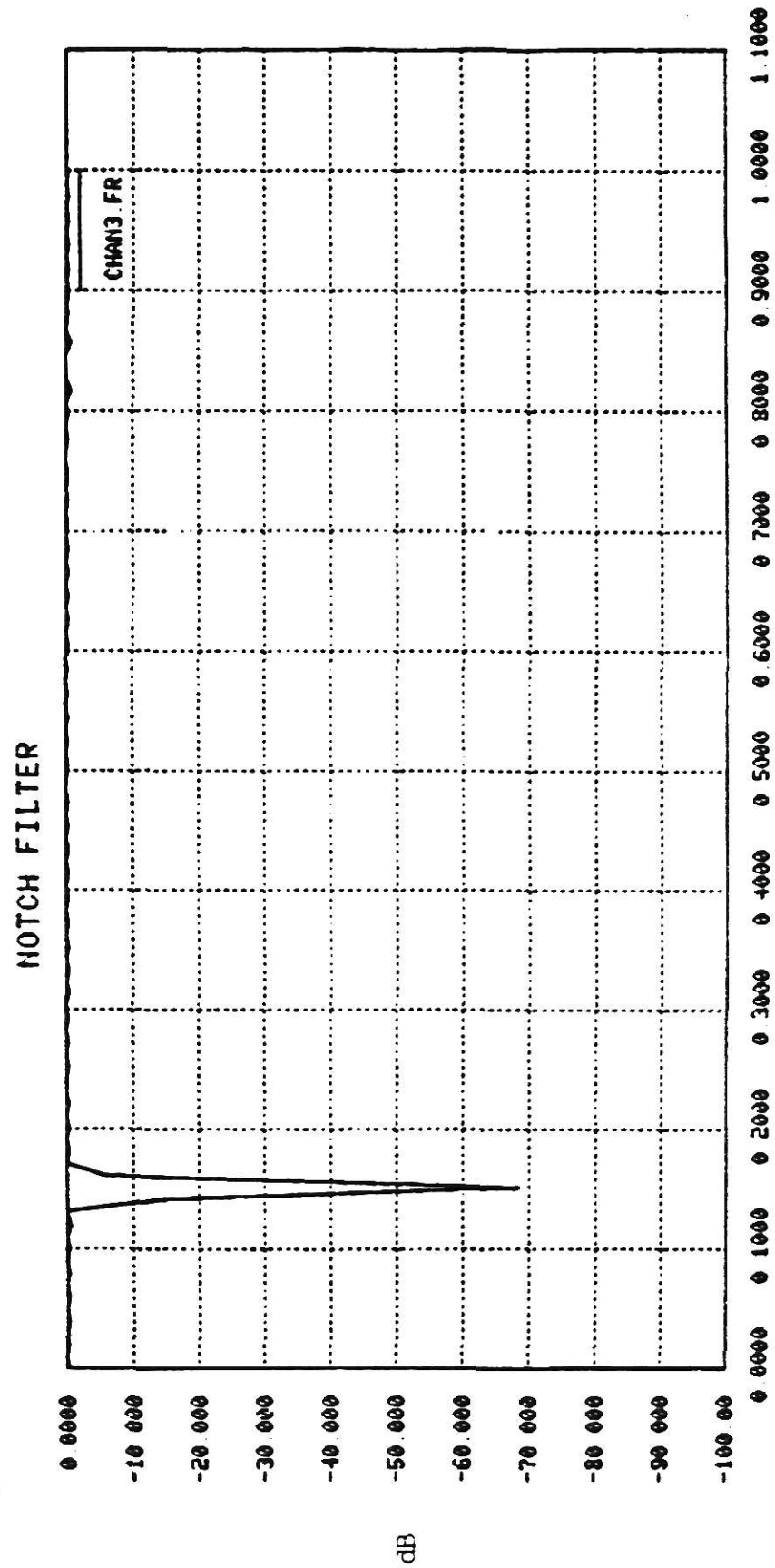


Figure 3.5. Gain function of a DHT notch filter; $N=240$, $W1=18$, $W2=20$.

CHAPTER IV

CONCLUSIONS

The experimental results that have been presented in this report demonstrate that near ideal filtering characteristics are obtainable using the DHT. Such filters can be implemented in terms of serial or parallel realizations using registers, read-only-memories, multipliers, and adders [8]. However, a major shortcoming with the DHT is that its transformation matrix is not orthogonal, although it is symmetric. Therefore, it may not be possible to develop an inverse DHT formulation.

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APPENDIX A

This appendix consists of listings of DHT computer programs that are available for use on the minicomputer system in the Department of Electrical Engineering's computer and signal processing laboratory.

HILBERT MATRIX

\$JOB

```

1      DIMENSION C(8,8),D(8,8)
2      N=8
3      B=1.
4      P=3,14159
5      DO 15 I=1,N
6      DO 15 J=1,N
7      K=(I+J)/2
8      E=I
9      G=J
10     W=(F+G)/2.
11     IE(K-W)13,12,13
12     12  C(I,J)=0.0
13     GO TO 15
14     13  M=I-J
15     14  C(I,J)= 2.*COS(P*M/N)*SIN(P*M/N)
16     15  CONTINUE
17     DO 16 I=1,N
18     DO 16 J=1,N
19     D(I,J)=0.0
20     DO 21 K = 1,N
21     21  D(I,J)=D(I,J)+C(I,K)*C(K,J)/64.
22     16  CONTINUE
23     WRITE(6,17) ((C(I,J),J=1,8),J=1,8)
24     17  FORMAT(LX,'IHI=1/8',8F14.4//,(9X,8F14.4/))
25     WRITE(6,18) ((C(I,J),I=1,8),J=1,8)
26     18  FORMAT(LX,'IHIT=1/8'.8F14.4//,(9X,8F14.4/))
27     WRITE (6,19)((D(I,J),J=1,8),I=1,8
28     19  FORMAT (LX,'IHIXIHIT=',8F14.4//,(10X,8E14.4/))
29     STOP
30     END

```

LOWPASS FILTER

\$JOB

```

1      DIMENSION C(8,8),D(8,8)
2      N=8
3      B=1.
4      P=3, 14159
5      DO 15 I=1,N
6      DO 15 J=1,N
7      K=(I+J)/2
8      E=I
9      G=J
10     W=(E+G)/2.
11     IE(K=W)13,12,13
12     12 C(I,J)=0.0
13     GO TO 15
14     13 M=I=J
15     C(I,J)=2.*COS(P*M/N)*SIN(2.*P*M/N)/SIN(P*M/N)
16     15 CONTINUE
17     DO 16 I=1,N
18     DO 16 J=1,N
19     D(I,J)=0.0
20     DO 21 K=1,N
21     21 D(I,J)=D(I,J)+C(I,K)*C(K,J)/64.
22     16 CONTINUE
23     WRITE (6,17)((C(I,J),J=1,8),J=1,8
24     17 FORMAT (IX,'IHI=1/8',8F14.4//,(8X,8F14.4/))
25     WRITE(6,18)((C(J,I),I=1,8),J=1,8)
26     18 FORMAT(1X,'IHIT=1/8',8F14.4//,(9X 8F14.4/))
27     WRITE(6,19)((D(I,J),J=1,8),J=1 8)
28     19 FORMAT(1X,'IHIXIHIT=',8F14.4//, (10X,8F14.4/))
29     STOP
30     END

```

BANDPASS FILTER

\$JOB

```

1      DIMENSION C(8,8),D(8,8)
2      N=8
3      BH=2.
4      BL=1.
5      P=3.14159
6      DO 15 I=1,N
7      DO 15 J=1,N
8      K=(I+J)/2
9      F=J
10     G=J
11     W=(F+G)/2.
12     IF(K=W)13,12,13
13     12  C(I,J)=0,0
14     GO TO 15
15     13  M=I-J
16     140 C(I J)=+2.*(COS(P*M/N)/SIN(P*M/N))*(SIN(2*P*BH
      *M/N)=SIN(2*P*BL*M/N
      1))
17     15  CONTINUE
18     DO 16 I=1,N
19     DO 16 J=1,N
20     D(I,J)=0.0
21     DO 21 K=1,N
22     21  D(I,J)=D(I,J)+C(I,K)*C(K,J)/64.
23     16  CONTINUE
24     WRITE (6,17)((C(I,J),J=1,8)..,I=1,8)
25     17  FORMAT(1X,'IHI=1/8',8F14.4//,(8X 8F14.4/))
26     WRITE(6,18)((C(J,I),J=1,8),J=1,8)
27     18  FORMAT(1X,'IHIT=1/8',8F14.4//,(9X 8F14.4/))
28     WRITE(6,19)((D(I,J),J=1,8),I=1,8)
29     19  FORMAT(1X,'IHIXIHIT=',8F14.4//,(10X 8F14.4/))
30     STOP
31     END

```

HIGHPASS FILTER

\$JOB

```

1      DIMENSION C(8,8),D(8,8)
2      N=8
3      B=1
4      P=3.14159
5      DO 15 I=1,N
6      DO 15 J=1,N
7      IF(I=J)11,10,11
8      10  C(I,J)=N
9      GO TO 15
10     11  K=(I+J)/2
11     E=I
12     G=J
13     W=(F+G)/2.
14     IF(K=W)13,12,13
15     12  C(I,J)=0.0
16     GO TO 15
17     13  M=I=J
18     14  C(I,J)=-2.*COS(P*M/N)*SIN(2.*P*B*M/N)/STN(P*M/N)
19     15  CONTINUE
20     DO 16 I=1,N
21     DO 16 J=1,N
22     D(I,J)=0.0
23     DO 21 K=1,N
24     21  D(I,J)=D(I,J)+C(I,K)*C(K,J)/64.
25     16  CONTINUE
26     WRITE(6,17)((C(I,J),J=1,8),*=1 8)
27     17  FORMAT(1X,'IHI=1/8',8F14.4//,(8X,8F14.4/))
28     WRITE(6,18)((C(J,I),I=1,8),J=1,8)
29     18  FORMAT(1X,'IHIT=1/8',8F14.4//,(9X 8F14.4/))
30     WRITE(6,19)((D(I,J),J=1,8),I=1,8)
31     19  FORMAT(1X,'THIXTHIT=',8F14.4//,(10X 8F14.4/))
32     STOP
33     END

```

BANDSTOP FILTER

\$JOB

```

1      DIMENSION C(8,8).D(8,8)
2      N=8
3      BH=2.
4      BL=1.
5      P=3.14159
6      DO 15 I=1,N
7      DO 15 J=1,N
8      IF(I=J)11,10,11
9      10  C(I,J)=N
10     GO TO 15
11     11  K=(I+J)/2
12         F=I
13         G=J
14         W=(F+G)/2.
15         IE(K=W)13,12,13
16     12  C(I,J)=0.0
17         GO TO 15
18     13  M=I=J
19     140 C(I,J)=-2.*(COS(P*M/N)/SIN(P*M/N))*(SIN(2*
        (BH*M/N)=SIN(2*P*BL*M/N 1))

20     15  CONTINUE
21         DO 16 I+1,N
22         DO 16 J=1,N
23         D(I=J)=0.0
24         DO 21 K=1,N
25     21  D(I,J)=D(I,J)+C(I,K)*C(K,J)/64.
26     16  CONTINUE
27         WRITE (6,17)((C(I,J),J=1,8),T=1,8)
28     17  FORMAT(1X',IHI=1/8',8F14.4//,(8X,8F14 4/))
29         WRITE(6,18)((C(J,I),I=1,8),J=1,8)
30     18  FORMAT(1X',IHIT=1/8',8F14.4//,(9X,8F14,4/))
31         WRITE(6,19)((D(I,J),J=1,8),T=1,8)
32     19  FORMAT(1X',IHIXIHIT=',8F14.4//,(10X,8F14.4/))
33         STOP
34         END

```

NOTCH FILTER

$$N=240 \quad B_1=18 \quad B_2=20$$

```

0001      DIMENSION H(119),A(100),Y(100),X(100)
0002      UC=20.
0003      DC=18.
0004      P=3.14159
0005      N=240
0006      S=N
0007      DO 15 L=1,100
0008      V=L
0009      A(L)=0.0
0010      DO 14 K=1,119,2
0011      T=K
0012      13  H(K)=2*COS(P*T/S)*(SIN(2*P*UC*T/S)=SIN(2*P*
          DC*T/S))/SIN(P*T/S)
0013      14  A(L)=A(L)=2.*H(K)*COS(T*P*V/100.)/S
0014      X(L)=(A(L)+1.)**2
0015      15  Y(L)=10.*ALOG(X(L))
0016      WRITE(6,16)(Y(L),L=1 100)
0017      16  FORMAT(LX,F12.6)
0018      STOP
0019      END

```

ON THE DISCRETE HILBERT TRANSFORM

by

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AN ABSTRACT OF A MASTER'S REPORT

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ABSTRACT

In many fields where Fourier techniques are used to represent and analyze physical processes, one finds that there exist relationships between the real and imaginary parts or the magnitude and phase of the Fourier transform. These relationships are known by different names, depending upon the field of interest, but often they are called Hilbert transform relations. In this respect, the field of digital signal processing is no exception.

This report concerns the discrete form of the Hilbert transform. It is known as the discrete Hilbert transform, and is conveniently represented in terms of a matrix equation. Our main objective is to implement the discrete Hilbert transform on a minicomputer system, and illustrate its frequency characteristics via examples. To this end, experimental results related to the following type of discrete Hilbert transform are included: lowpass, bandpass, highpass, and bandstop.