PLASTIC DESIGN OF BEAMS WITH REINFORCED
RECTANGULAR WEB OPENINGS

by 632

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INTRODUCTION

Problem:

In modern steel structures, openings are often provided in the webs of steel beams to provide for passage of utility components. In a multistory steel frame a considerable height differential is achieved by passing the utility components through web openings. This will considerably reduce the cost of the structure. Reinforcing may be required around the web openings depending on values of moment and shear at the opening. A designer needs a convenient method for determining when reinforcing is required and how much reinforcing is required.

Purpose:

The purpose of this report is to provide designers with a method for designing steel W-shape beams with reinforced rectangular openings in the webs.

Scope:

The method presented herein is based on an ultimate strength analysis and is applicable only to W-shape beams with:

1. Rectangular or square openings;
2. Openings centered on the longitudinal axis of the beams;
3. Reinforcing bars above and below the openings and parallel to the longitudinal axes of the beams as shown in Appendix A.

BRIEF SURVEY OF THE LITERATURE

A considerable amount of research has been directed toward determining the elastic stresses around openings in the webs of W-shape beams. Theoretical results were published by Muskhelishvili[8] in 1930, Heller[7] in 1962 and Bower[1] in 1966. An experimental analysis of circular openings was presented by So in 1963. Bower[2] also published the results of experimental investigations of rectangular and circular openings in 1966. An experimental elastic study was conducted at Kansas State University by Chang[4] in 1969.

An analytical study of the ultimate strength of beams with circular, rectangular and oval openings was reported by Redwood and McCutcheon[9] in 1968. Bower[3] summarized the results of another ultimate strength analysis in 1968. The plastic behavior and the ultimate strength design of beams with web openings were further discussed by Redwood in 1968.[10] All of these publications were restricted to web holes without reinforcing. Design charts for unreinforced web openings were prepared and published by U. S. Steel[12] in 1968.

A comparison of the economy of various reinforcing types was carried out by Segner.[11] Theoretical and experimental investigations on reinforced rectangular web openings were conducted by Cogdon[5] at McGill University and were published in 1969. Additional experimental work on the same problem has been reported by Cooper and Snell.[6]

SUMMARY OF DESIGN FORMULAS

Introduction:

(a) Interaction Diagrams: The plastic moment capacity of steel W-shape beams is effected by the presence of shear. The interaction diagram shown in Fig. 1a represents the effect of shear on the moment capacity of a beam and is based on Von Mises' yield criteria. The values of the applied shear V and applied moment M have been non-dimensionalised by dividing by M_p, the plastic moment, and V_p, the plastic shear force. The introduction of an opening in the web of a W-shape beam changes the interaction curve due to a change in the moment and shear capacity by the reduction of area available, and due to a stronger interaction between moment and shear. Figure 1b represents an interaction diagram for a beam with a web opening. This report presents interaction diagrams for the following three conditions:

(1) Unreinforced opening

(2) Opening reinforced to reach maximum shear capacity

(3) Opening reinforced for a further increase in moment capacity beyond that associated with the maximum shear capacity.

Beams with the following properties are considered for the above conditions:

(1) A_f/A_w = 0.5, 1.0 and 1.5; h/a = 0.5

(2) A_f/A_w = 0.5, 1.0 and 1.5; h/a = 1.0

where

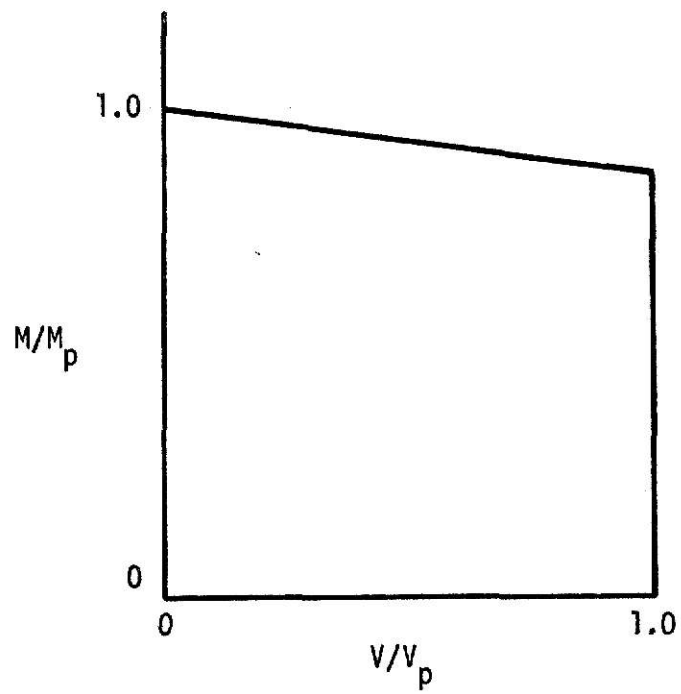A_f = area of one flange

A_w = area of web

4

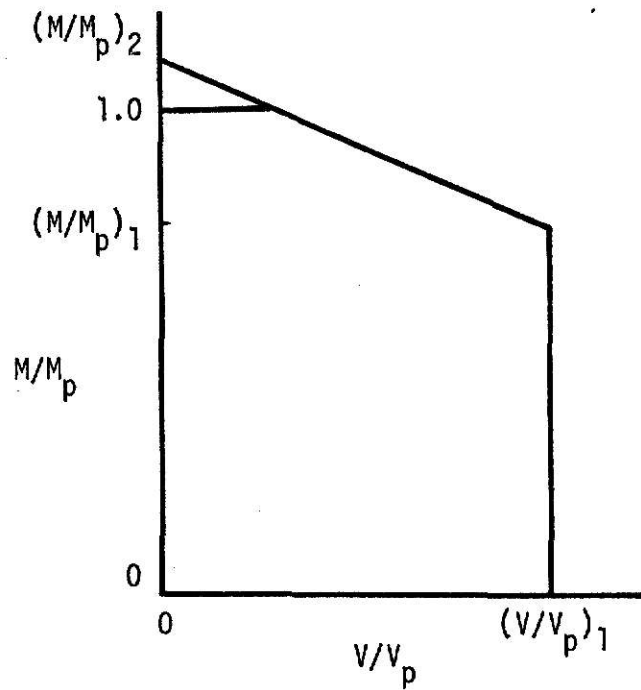Fig. 1a.   Interaction Diagram for a Beam
Without an Opening

Fig. 1b.   Interaction Diagram for a Beam
With an Opening

h = half depth of opening

a = half length of opening

(b) <u>Tables</u>: Tables are prepared for the above conditions and beam properties which provide values of $(M/M_p)_1$, $(V/V_p)_1$ and $(M/M_p)_2$ for values of $2h/d = 0.0, 0.2, 0.4, 0.6$, where

$(V/V_p)_1$ = ratio of the maximum shear capacity of a beam with an unreinforced or a reinforced opening to the plastic shear capacity of the beam without an opening (see Fig. 1b),

$(M/M_p)_1$ = ratio of the maximum moment which can be attained with the above shear acting on a beam with the unreinforced or the reinforced opening to the plastic moment of the beam without an opening,

$(M/M_p)_2$ = ratio of the maximum moment which can be attained for pure bending for the beam with the unreinforced or reinforced opening to the plastic moment of the beam without an opening, and

d = the depth of the beam.

(c) <u>Limits of Interaction Curves</u>: Beams with reinforced web openings can be stronger than unperforated beams for pure bending and for bending with low shear. However, this does not have practical significance, so 1.0 is taken as the maximum value of M/M_p and a horizontal line is drawn at this value from the (M/M_p) axis to intersect the interaction curve (see Fig. 1b).

The maximum shear capacity at the opening is equal to V_p, the plastic shear capacity of uncut section, times the ratio of the remaining web area to the initial web area, $(V/V_p)_1$.

Design Formulas:

The following equations are taken from a report on research conducted at McGill University by Congdon under the direction of Dr. Richard Redwood (Reference 5). Limitations on the use of the equations have been discussed previously.

(a) <u>Interaction diagram for unreinforced openings</u>: If openings are located in regions of low stress no reinforcing is required. The following equations are used to plot interaction diagrams (Figs. 1.1 to 1.6) for unreinforced openings. Interaction diagrams based on the equations enable the designer to decide if reinforcing is necessary.

$$\frac{M}{M_p}\bigg|_1 = \frac{1 - \frac{A_w}{2A_f}(1 - \frac{2h}{d}) \times \frac{1}{\sqrt{1+\alpha}}}{1 + \frac{A_w}{4A_f}} \tag{1}$$

$$\frac{V}{V_p}\bigg|_1 = (1 - \frac{2h}{d})\sqrt{\frac{\alpha}{1+\alpha}} \tag{2}$$

where $\quad \alpha = \frac{3}{4}(\frac{h}{a})^2 (\frac{d}{2h} - 1)^2$

$$\frac{M}{M_p}\bigg|_2 = 1 - \frac{\frac{A_w}{4A_f}(\frac{2h}{d})^2}{1 + \frac{A_w}{4A_f}} \tag{3}$$

(b) <u>Minimum reinforcing required to reach maximum shear capacity</u>: If openings are located in regions of high stress reinforcing is required. The minimum reinforcing area required to reach the maximum shear capacity is given by Eq. 4. If the beam is subjected to high shear and it is desired to develop the maximum shear capacity, reinforcing according to Fig. 2 (Appendix B) must be provided. Interaction diagrams for openings reinforced according to Fig. 2 are given by Figs. 2.1 to 2.6 (Appendix B), and are based on Eqs. 5, 6 and 7.

$$A_r = \frac{A_w}{4} (1 - \frac{2h}{d}) \sqrt{\frac{1}{\alpha}} \tag{4}$$

where A_r = area of reinforcing cone pair of bars

Interaction diagram for opening reinforced according to Eq. 4:

$$\frac{M}{M_p}_1 = \frac{1 - \frac{A_r}{A_f}}{1 + \frac{A_w}{A_f}} \tag{5}$$

$$\frac{V}{V_p}_1 = 1 - \frac{2h}{d} \tag{6}$$

$$\frac{M}{M_p}_2 = \frac{1 + \frac{A_w}{4A_f} [1 - (\frac{2h}{d})^2] + \frac{A_r}{A_f} (\frac{2h}{d})}{1 + \frac{A_w}{4A_f}} \tag{7}$$

(c) <u>Reinforcing required to further increase moment capacity</u>: Increasing the reinforcing area above the value given by Eq. 4 will not increase the shear capacity. (Shear capacity can be increased by

providing web doubler plates. This type of reinforcing is beyond the scope of this report.) If it is desired to increase the moment capacity Fig. 3.0 (Appendix B) based on Eq. 8 should be used to determine the required amount of reinforcing. Interaction diagrams for beams reinforced according to Fig. 3.0 are given by Figs. 3.1 to 3.6 (Appendix B), and are based on Eqs. 9, 6 and 7.

$$A_r = \sqrt{\frac{(aw)^3}{3} + (\frac{d - 2h}{4})^2 \, w^2} \tag{8}$$

Interaction diagram for opening reinforced according to Eq. 8:

$$\frac{M}{M_p}_1 = \frac{1 - \frac{A_r}{A_f} - \beta}{1 + \frac{A_w}{4A_f}} \tag{9}$$

$$\beta = - \frac{2\alpha}{1 + \alpha} \wedge \frac{A_r}{A_f}$$

$\frac{V}{V_p}_1$ given by Eq. 6

$\frac{M}{M_p}_2$ given by Eq. 7

(d) <u>Interaction diagram for opening reinforced between Eqs. 4 and 8:</u>

$$\frac{M}{M_p}_1 = \frac{1 - \frac{A_r}{A_f} - \beta}{1 + \frac{A_w}{4A_f}} \tag{10}$$

$$\beta = -\frac{2\alpha}{1+\alpha}\frac{A_r}{A_f} + \frac{1}{2}\left(\frac{A_w}{A_f}\right)\sqrt{\left(1-\frac{2h}{d}\right)^2\frac{1}{1+\alpha} - 16\left(\frac{A_r}{A_w}\right)^2\frac{\alpha}{(1+\alpha)^2}}$$

$\dfrac{V}{V_p}_1$ given by Eq. 6

$\dfrac{M}{M_p}_2$ given by Eq. 7

(e) <u>Interaction diagram for opening with reinforcing greater than that given by Eq. 8</u>:

$$\frac{M}{M_p}_1 = \frac{1 + \frac{A_r}{A_f}\left(\frac{2h}{d}\right) - \frac{1}{2\sqrt{3}}\frac{A_w}{A_f}\left(\frac{a}{h}\right)\left(\frac{2h}{d}\right)\left(1+\frac{2h}{d}\right)}{1 + \frac{A_w}{4A_f}} \qquad (11)$$

$\dfrac{V}{V_p}_1$ given by Eq. 6

$\dfrac{M}{M_p}_2$ given by Eq. 7

Equations 6, 7 and 11 are applicable only if

$$a \leq \frac{\sqrt{3}\,A_f}{A_w}$$

and $\qquad a \leq \dfrac{\sqrt{3}\,A_r}{w}$.

Reinforcing Details

The above formulas enable the designers to decide when reinforcing is required and how much reinforcing is required. Appendix C gives the

criteria for determining the nominal gap between the reinforcing bars and the horizontal edges of the opening, and for determining the extension of the reinforcing bars beyond the vertical edges of the opening.
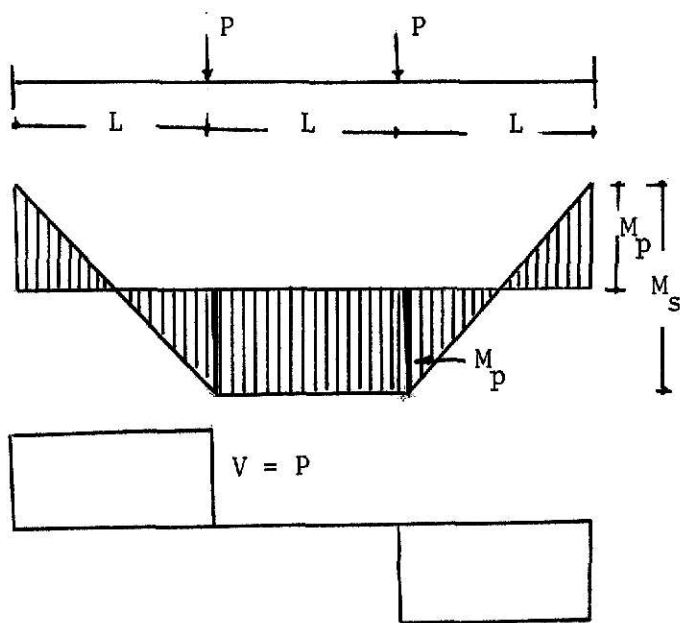
DESIGN EXAMPLES

1. Example 1

Fig. E1.1

A 36 steel

P_w = 48 kips
    (gravity only)

L.F = 1.7

L = 9'

M_s = PL

$2M_p$ = PL

M_p = PL/2

$M_p = \frac{1}{2}(48)(1.7)(9)$

    = 367 kip-ft.

Z req'd $= \frac{367 \times 12}{36}$

    = 122.3 in.^{3}

Try W21 x 53   (Z = 125.4 in.^{3})

Check shear:

V_u = 0.55(36)(0.375)(20.80)

    = 154.3 kips > 1.7 x 48   O.K.

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w = 0.375" A_w = 20.80(0.375) = 7.80 in.^{2}

d = 20.80"

b_f = 8.215" A_f = 8.215(0.522) = 4.29 in.^{2}

t = 0.522"

opening   2a = 18"   2h = 9"

$\frac{A_w}{A_f}$ = 1.82 $\frac{h}{a}$ = 0.5 $\frac{2h}{d}$ = 0.433

$$M_p = F_y Z = 36(125.4) = 4520 \text{ k-in.}$$

$$V_p = w(d-2t) \, F_y/\sqrt{3} = 0.375(20.80 - 2 \times 0.522) \, 36/\sqrt{3} = 153.8 \text{ k.}$$

For center span $\dfrac{V}{V_p} = 0$ $\dfrac{M}{M_p} = \dfrac{367 \times 12}{4520} = 0.975$

For end spans $\dfrac{V}{V_p} = \dfrac{1.7 \times 48}{153.8} = 0.531$

Note: $\dfrac{A_w}{A_f} = 1.82$ which is very close to 2, use Fig. 1.2 with

$\dfrac{A_f}{A_w} = 0.5$ $\dfrac{h}{a} = 0.5$

Interpolate for $\dfrac{2h}{d} = 0.433$

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1. For center span, where $\dfrac{V}{V_p} = 0$

$$\dfrac{M}{M_p} = 0.975$$

$\dfrac{M}{M_p}_2$ from Fig. 1.2 $= .936 < 0.975$

$\therefore$ Reinforcing is required.

For end span $\dfrac{V}{V_p} = 0.531$

$\dfrac{V}{V_p}_1$ from Fig. 1.2 $= 0.29 < 0.531$

Reinforcing is required.

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2. Reinforcing for opening in center span:

Since $\dfrac{V}{V_p} = 0$, set $\dfrac{M}{M_p}_2 = 0.975$ in Eq. 7 and solve for A_r

$$A_r = 0.506 \text{ in.}^2$$

Two $2'' \times \dfrac{1''}{4}$ bars providing $A_r = 1.0$ in.^{2} could be used.

3. Reinforcing for opening in end span:

Minimum reinforcing required to reach maximum shear capacity. (Use Fig. 2)

$$A_r \geq 0.25 \ A_w$$

Since $A_w = 7.8$ in.^{2}

$$A_r \geq 1.95 \ \text{in.}^2$$

Use two $2'' \ x \ \frac{1''}{2}$ bars providing $A_r = 2.0$ in.^{2}

Since A_r provided is $\simeq A_r$ minimum, use of graph for minimum A_r will not greatly underestimate the value of $\frac{M}{M_p}$.

Interpolate for $\frac{2h}{d} = 0.433$

$$\frac{M}{M_p}_1 = 0.38 \ \text{for} \ \frac{A_f}{A_w} = 0.5$$

$$\frac{M}{M_p}_1 = 0.62 \ \text{for} \ \frac{A_f}{A_w} = 1.00$$

$$\text{for} \ \frac{A_f}{A_w} = 0.55 \qquad \frac{M}{M_p}_1 = .38 + .024$$

$$= .404$$

Therefore maximum moment at centerline of opening with $2'' \ x \ \frac{1''}{2}$ reinforcing bar = 152 k-ft.

The corresponding distance from point of zero moment ($4 \frac{1'}{2}$ from the ends) is 1.86' or 22.3".

If opening must be located in end spans at a point of higher moment, larger reinforcing must be used.

4. Larger reinforcing for opening in end spans:

Use Fig. 3.

$$\frac{A_r}{A_w} = .286$$

$$A_r = .286 \times 7.8 \text{ in.}^2$$
$$= 2.23 \text{ in.}^2$$

Use bars $2\,\frac{1''}{2} \times \frac{1''}{2}$ ($A_r = 2.5$ in.^{2}) *Graphs will greatly underestimate; use Eq. 11 to draw interaction curve.*

From Eq. 11

$$\frac{M}{M_p}_{1} = 0.412$$

Check: $\dfrac{\sqrt{3} \times A_r}{W} = \dfrac{1.73 \times 2.5}{0.375} = 11.54'' > a$ O.K.

From

Eq. 6: $\dfrac{V}{V_p}_{1} = 0.567$

Eq. 7: $\dfrac{M}{M_p}_{2} = 1.114$

Since $\dfrac{V}{V_p} = 0.531$, max. $\dfrac{M}{M_p} = 0.446$ or max. moment at center line of opening = 168 k-ft. Therefore maximum distance from point of zero moment to center line of opening = 2.06'' or 24.7''. If the opening must be located in the end span at the point of higher moment, larger reinforcing bars must be used.
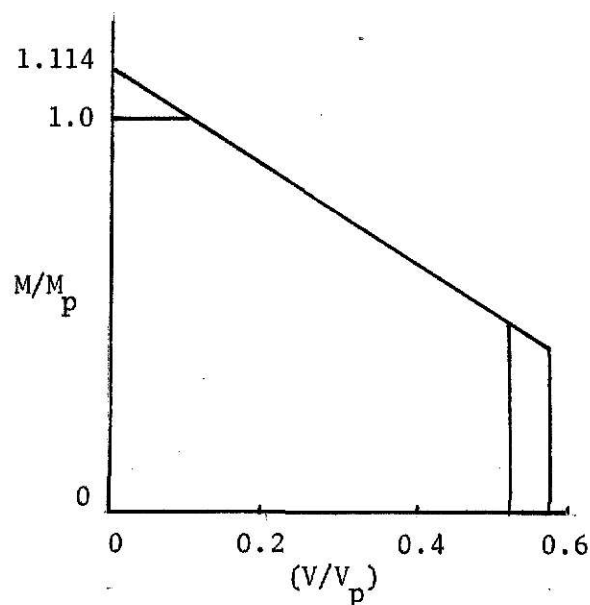

Fig. E 1.2
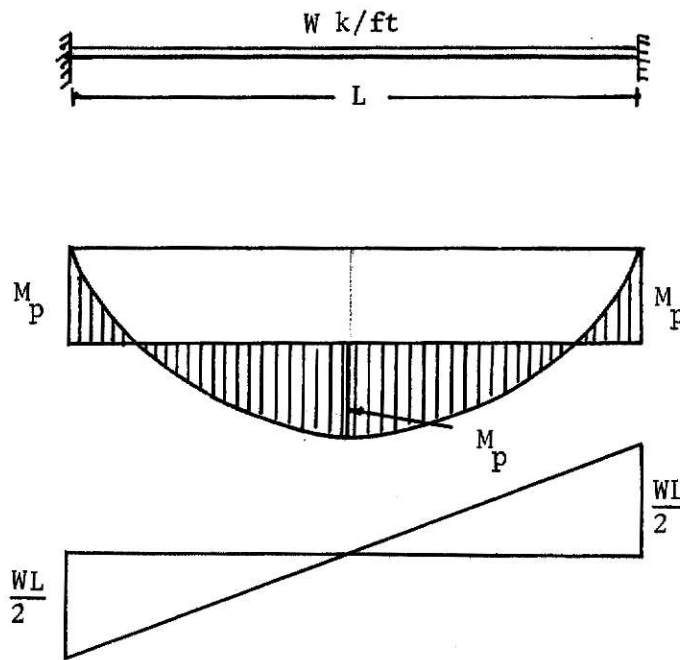
2. Example 2

W k/ft

L

M_p M_p

M_p

$\dfrac{WL}{2}$ $\dfrac{WL}{2}$

Fig. E2

A 36 steel

$W_w = 5$ k/ft

$L = 27'$

$L.F. = 1.7$

$$M_s = \frac{WL^2}{8}$$

$$2M_p = \frac{WL^2}{8}$$

$$M_p = \frac{WL^2}{16}$$

$$= (\frac{5 \times 27 \times 27}{16}) \times 1.7$$

$$= 387 \text{ k-ft}$$

Z required

$$\frac{387 \times 12}{36} = 129 \text{ in.}^3$$

Try W12 x 85

$$V_u = 0.55 \times (36)(12.5)(.495) = 125 \text{ k} > \frac{5 \times 27}{2} \times 1.7$$

$$> 104.5$$

O.K

w = .495″		
d = 12.5″	— $A_w = (.495)(12.5) = 6.18$ in.^{2}	$\dfrac{A_f}{A_w} = 1.44$
$t_f = .736″$		$\dfrac{h}{a} = 0.5$
	— $A_f = (.736)(12.08) = 8.9$ in.^{2}	
$b_f = 12.08″$		$\dfrac{2h}{d} = \dfrac{6}{12.08} = .497$
		$\simeq .5$

2a = 12″, 2h = 6″

$$M_p = F_y Z = 36(129.1) = 4647.6 \text{ k-in.}$$

$$V_p = w(d-2\ t)\ F_y/\sqrt{3} = 0.495(12.08 - 2 \times .736) \times 36/\sqrt{3}$$

$$= .495 \times 10.508 \times 36/1.73 = 108.2 \text{ k}$$

For end span $\dfrac{V}{V_p} = \dfrac{104.5}{108.2} = .965$

For center span $\dfrac{V}{V_p} = 0.$

<u>Note</u>: Maximum $\dfrac{V}{V_p}$ that can be reached with reinforcing $= 1 - \dfrac{2h}{d} = 0.5$

Opening can only be located in center 14' span.

Value of moment at 6.5' from end $= .46\ M_p$

$$\frac{M}{M_p} = .46$$

We will consider two cases.

(1) For center span $\dfrac{V}{V_p} = 0 \qquad \dfrac{M}{M_p} = 1$

(2) At 6.5' from end $\dfrac{V}{V_p} = 0.5 \qquad \dfrac{M}{M_p} = .46$

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1. For unreinforced opening, look in the Fig. 1.6

$$\frac{A_f}{A_w} = 1.5 \qquad \frac{h}{a} = 0.5$$

For (1) $\dfrac{V}{V_p} = 0 \qquad \dfrac{M}{M_p} = .962$

Reinforcing is required.

For (2) $\dfrac{V}{V_p} = .5$

Reinforcing is required.

---

2. Reinforcing for opening in center span.

Since $\frac{V}{V_p} = 0$, set $\frac{M}{M_p}_2 = 1$ in the Eq. 7 and solve for A_r

$$1 = \frac{1 + \frac{6.18}{4 \times 8.9}[1 - (0.5)^2] + \frac{A_r}{8.9} \times 0.5}{1 + \frac{6.18}{4 \times 8.9}}$$

$$= \frac{1 + .13 + \frac{A_r \times 0.5}{8.9}}{1.1735}$$

$$A_r = \frac{(1.1735 - 1.13)}{0.5} \times 8.9 = .775 \text{ in.}^2$$

Two $2'' \times \frac{1''}{4}$ bars providing $A_r = 1.0$ in.^{2} could be used.

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3. Reinforcing for opening 6.5' from ends to reach max. shear capacity.

From Fig. 2.0

$$\frac{A_r}{A_w} \geq .288$$

$$A_r = .288 \times 6.18 = 1.78 \text{ in.}^2$$

Use two $2'' \times \frac{1''}{2}$ bar providing

$$A_r = 2.0 \text{ in.}^2$$

Consider Fig. 2.6 for opening, reinforced to reach maximum shear capacity, $\frac{A_f}{A_w} = 1.5$, $\frac{h}{a} = 0.5$.

Note: Actual moment capacity will be more than given by this figure, but it will give a good hint for the next step.

$$\frac{M}{M_p} = 0.65 \text{ with } A_r = 1.78 \text{ in.}^2$$

No need to draw interaction diagram for $A_r = 0.2$ in.^{2}, above reinforcing is sufficient.

CONCLUSIONS

[1]   The method presented in this report is extremely practical and can be used for design purposes provided that a suitable load factor is used.

[2]   Reinforcing considered in this report resists high bending stresses but it can not increase the shear capacity above that given by the shear capacity of the cut section.

[3]   For a given beam, the strength of the beam decreases with a decrease in h/a ratio.

[4]   The strength of a beam decreases with an increase in 2h/d ratio, if all other variables are constant.

RECOMMENDATIONS FOR FURTHER STUDY

[1]   Similar charts and tables could be prepared for beams with other values of A_f/A_w and for openings with other values of $2h/d$ and h/a.

[2]   A similar design method for beams with opening not centered on the longitudinal axis could be developed.
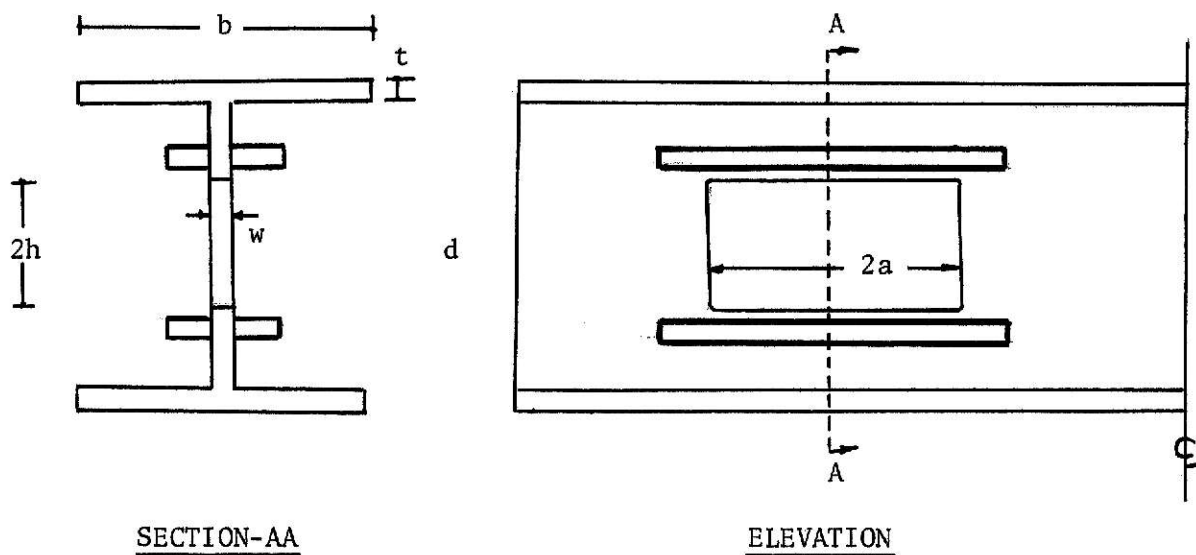
APPENDIX A

NOTATION

SECTION-AA   ELEVATION

Fig. A1

a = half the length of opening

A_f = area of one flange = bxt

A_r = area of reinforcing (one pair of bars)

A_w = area of web = dxw

b = flange width

d = depth of beam

F_y = yield point of steel

h = half the depth of opening

M = applied bending moment

M_p = plastic bending moment of the section
        without an opening = F_y x Z

t = flange thickness

V = applied shear

V_p = plastic shear force of section without
an opening = $w(d - 2t) F_y/\sqrt{3}$

w = web thickness

Z = plastic section modulus of gross section

APPENDIX B

DESIGN CHARTS AND TABLES

Page

A. Tables:

$A_f/A_w = 0.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.514	0.768	0.987
0.40	0.418	0.484	0.947
0.60	0.435	0.200	0.880

Table 1.1

$A_f/A_w = 0.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.000	1.00	1.00
0.20	0.400	0.692	0.987
0.40	0.328	0.327	0.947
0.60	0.410	0.111	0.880

Table 1.2

Tables for Unreinforced Openings
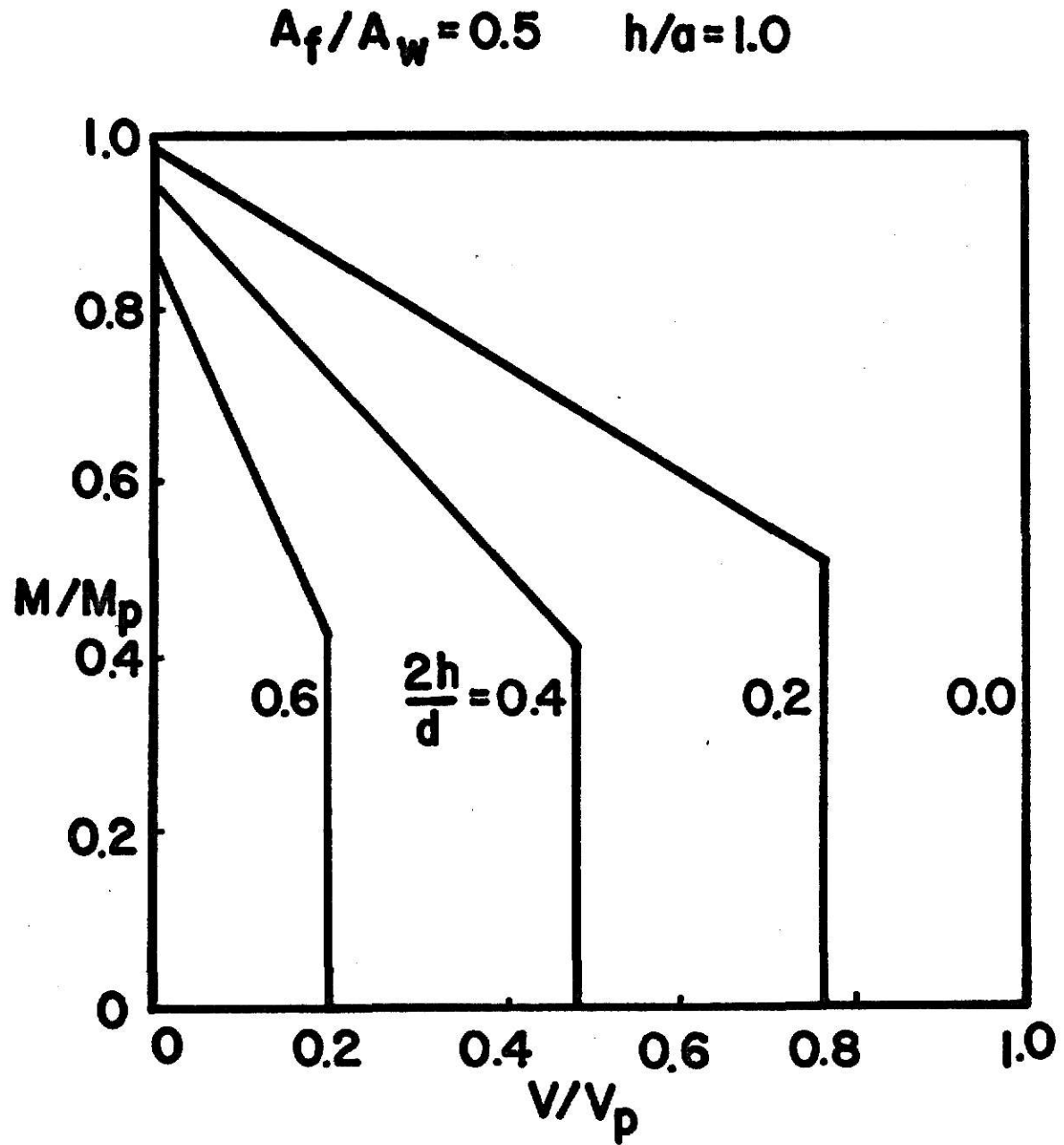
$$A_f/A_w = 0.5 \qquad h/a = 1.0$$

Fig. 1.1. Interaction Diagram for Unreinforced Opening
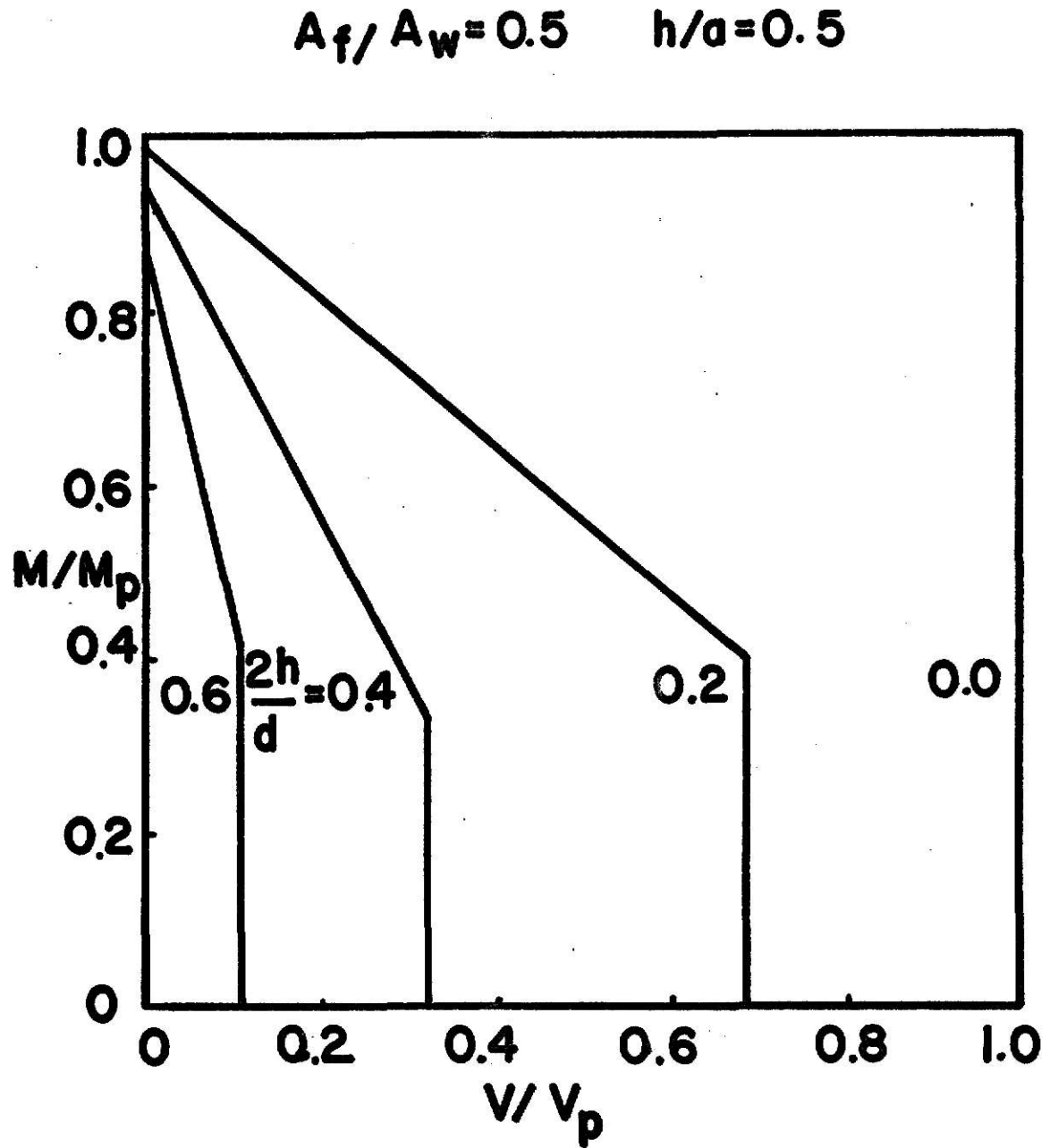
Fig. 1.2. Interaction Diagram for Unreinforced Opening

$A_f/A_w = 1.0$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.000	1.000	1.000
0.20	0.711	0.768	0.992
0.40	0.651	0.484	0.973
0.60	0.661	0.200	0.928

Table 1.3

$A_f/A_w = 1.0$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.000	1.000	1.000
0.20	0.640	0.692	0.992
0.40	0.666	0.327	0.973
0.60	0.569	0.111	0.928

Table 1.4

Tables for Unreinforced Openings

$A_f/A_w = 1.0$ $h/a = 1.0$

M/M_p vs V/V_p chart with curves labeled $\frac{2h}{d} = 0.4$, 0.6, 0.2, 0.0

Fig. 1.3. Interaction Diagram for Unreinforced Opening

$$A_f/A_w = 1.0 \qquad h/a = 0.5$$
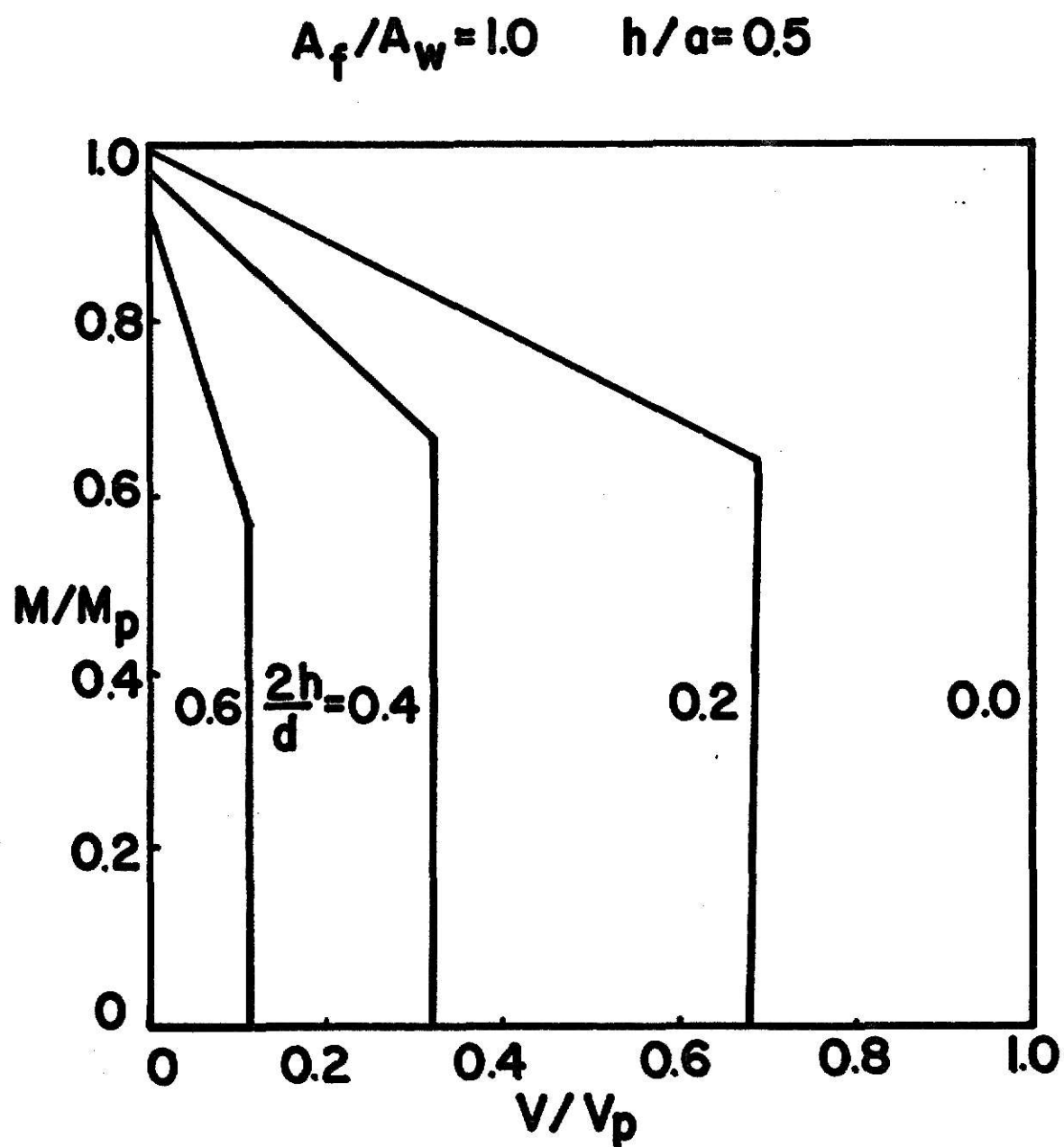
Fig. 1.4. Interaction Diagram for Unreinforced Opening

$A_f/A_w = 1.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.000	1.000	1.000
0.20	0.794	0.768	0.994
0.40	0.752	0.418	0.977
0.60	0.758	0.200	0.949

Table 1.5

$A_f/A_w = 1.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.000	1.000	1.000
0.20	0.743	0.692	0.994
0.40	0.714	0.327	0.977
0.60	0.693	0.111	0.949

Table 1.6

Tables for Unreinforced Openings

$A_f/A_w = 1.5$ $h/a = 1.0$
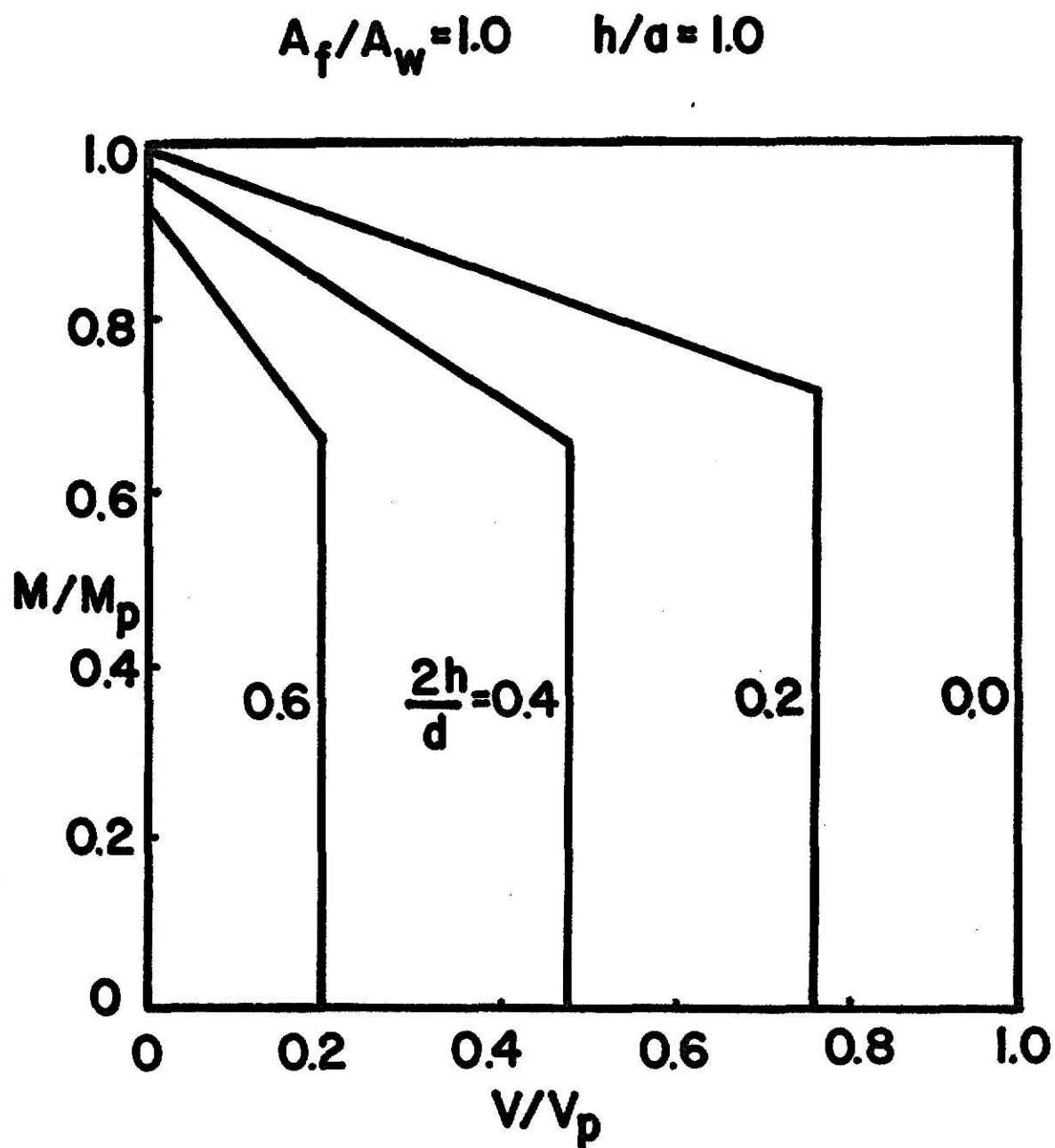

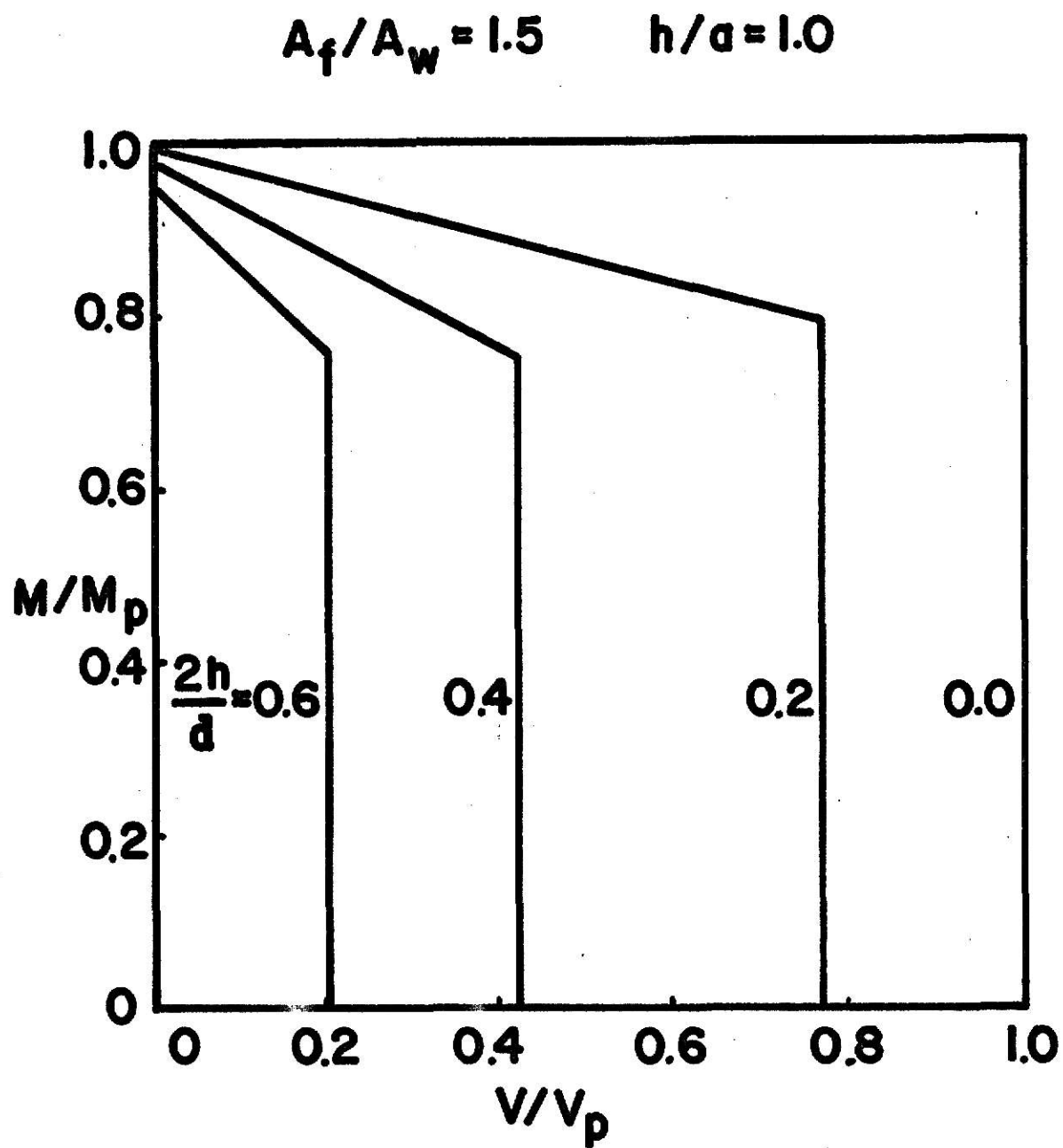Fig. 1.5. Interaction Diagram for Unreinforced Opening
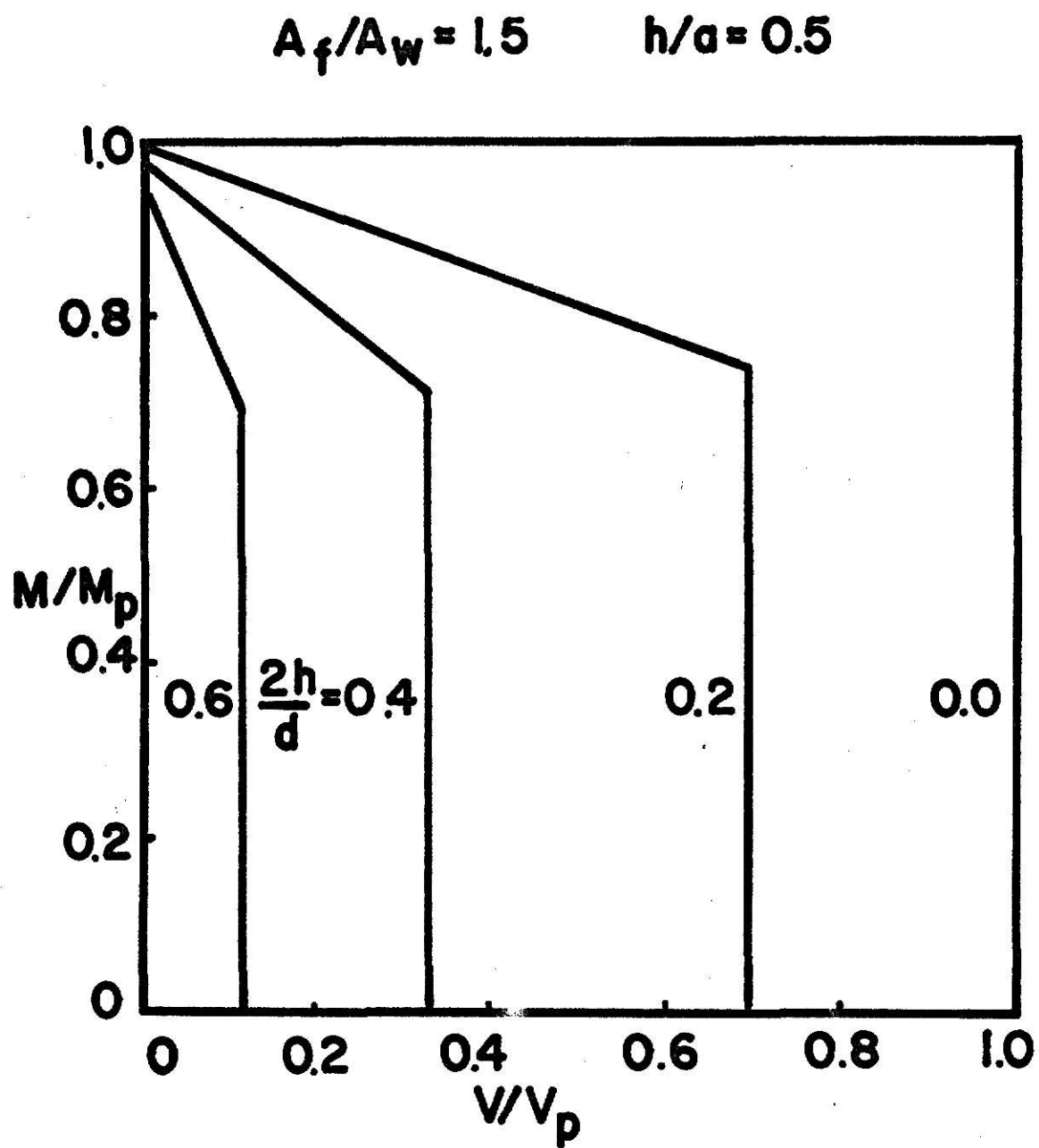
Fig. 1.6. Interaction Diagram for Unreinforced Opening

2h/d	A_r/A_w	
	h/a = 1.0	h/a = 1/2
0.0	0.00	0.00
0.2	0.058	0.116
0.4	0.115	0.231
0.6	0.173	0.346

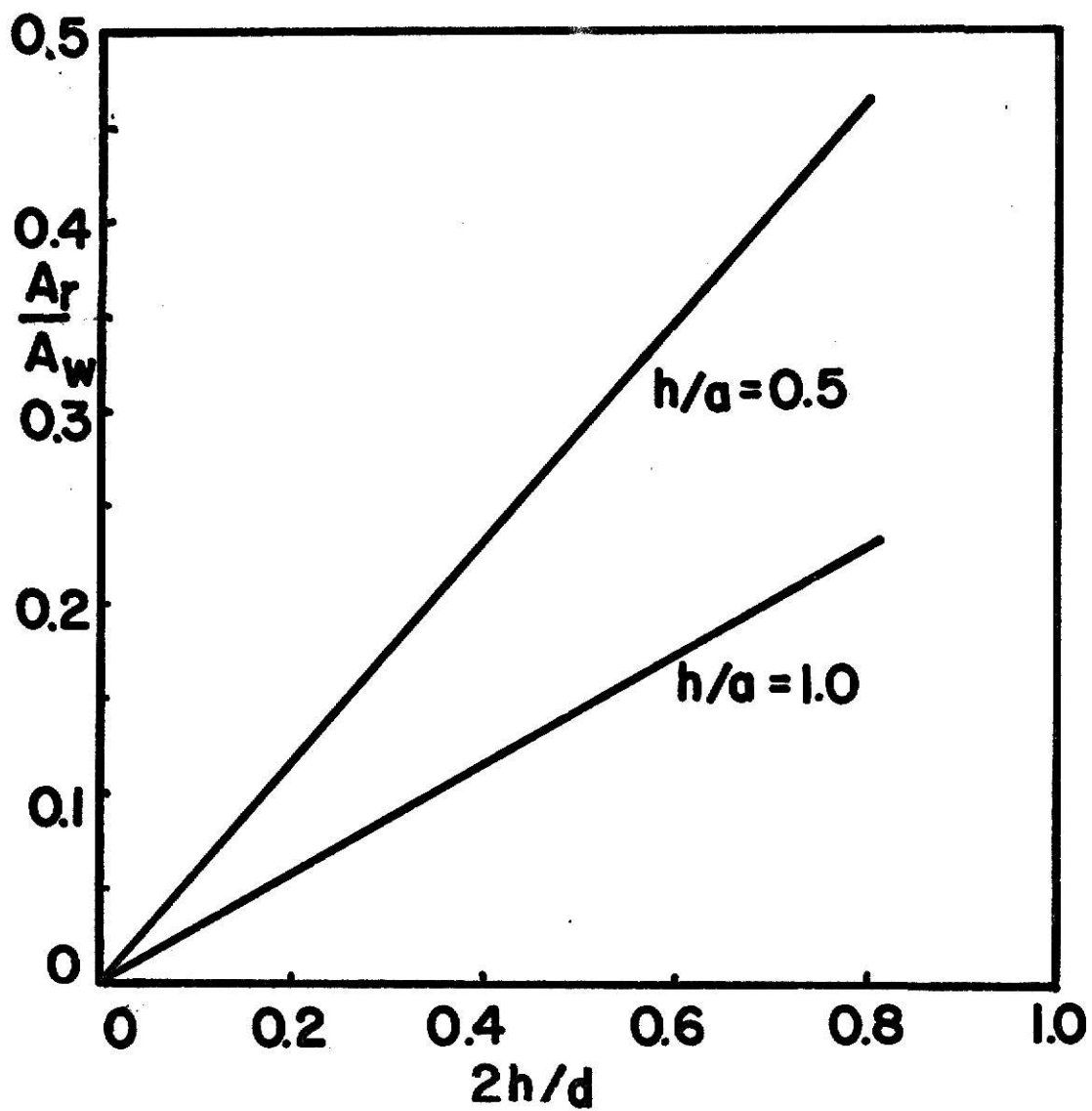Reinforcing Required to Reach

Maximum Shear Capacity

Table 2

Fig. 2.0. Reinforcing Required to Reach Maximum Shear Capacity

$A_f/A_w = 0.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.589	0.80	1.002
0.40	0.513	0.60	1.008
0.60	0.436	0.40	1.018

Table 2.1

$A_f/A_w = 0.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.512	0.80	1.017
0.40	0.358	0.60	1.069
0.60	0.205	0.40	1.156

Table 2.2

<u>Tables for Openings Reinforced to Reach</u>

<u>Maximum Shear Capacity</u>

Fig. 2.1. Interaction Diagram for Opening Reinforced
According to Fig. 2

Fig. 2.2. Interaction Diagram for Opening Reinforced
According to Fig. 2

$A_f/A_w = 1.0$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.753	0.80	1.001
0.40	0.708	0.60	1.004
0.60	0.661	0.40	1.011

Table 2.3

$A_f/A_w = 1.0$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.707	0.80	1.01
0.40	0.615	0.60	1.041
0.60	0.523	0.40	1.094

Table 2.4

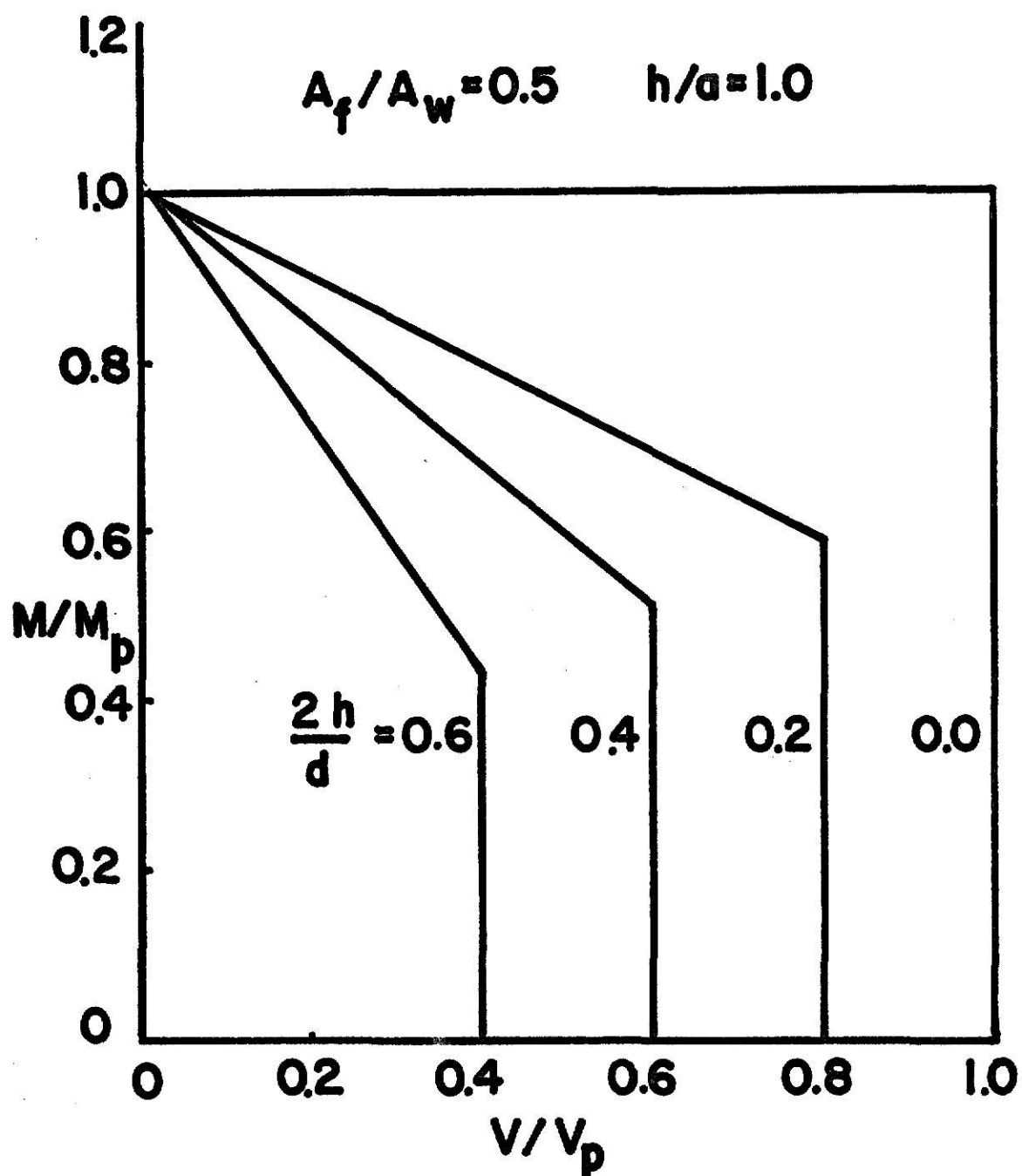Tables for Openings Reinforced to Reach

Maximum Shear Capacity

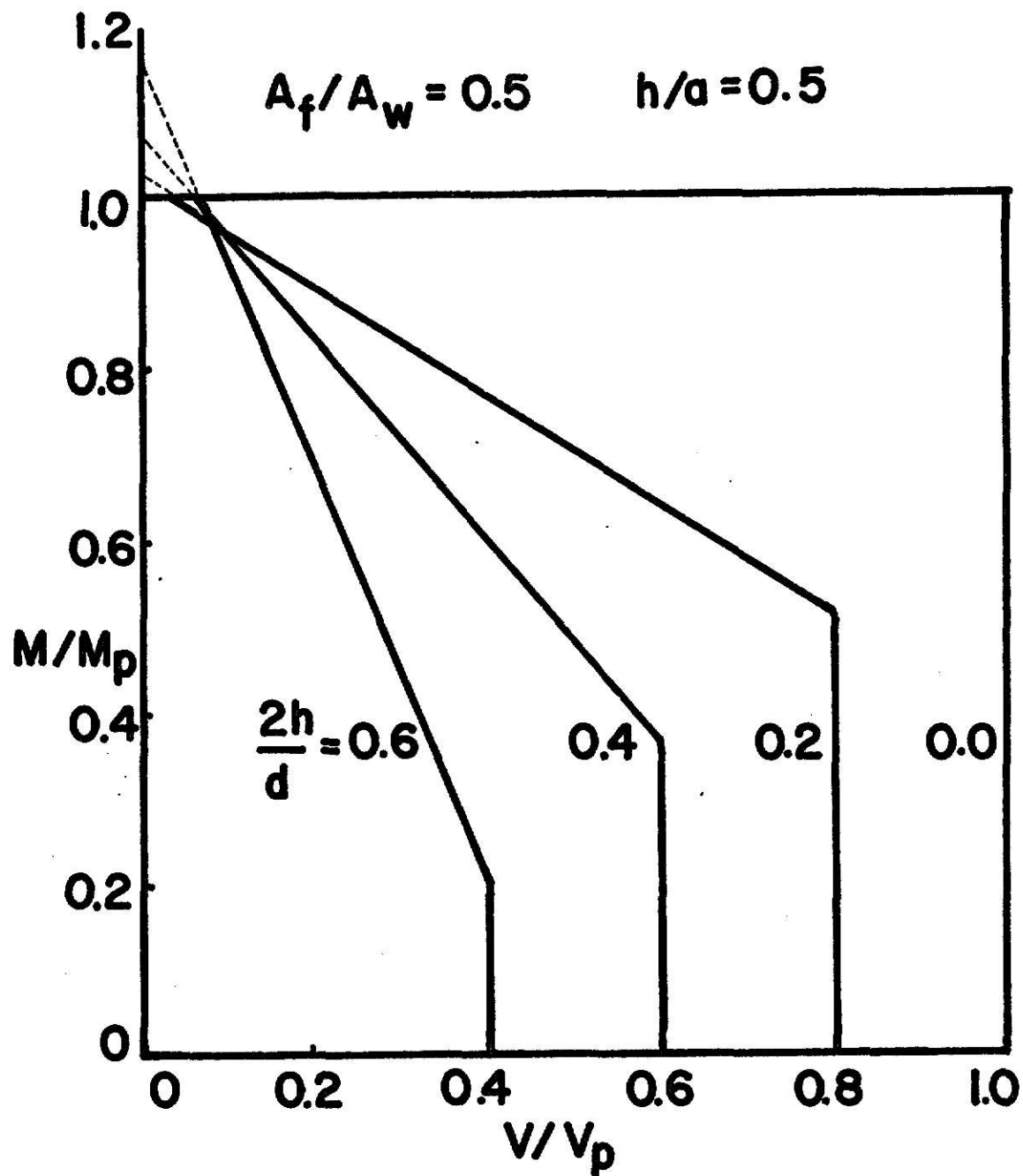Fig. 2.3. Interaction Diagram for Opening Reinforced
According to Fig. 2

Fig. 2.4. Interaction Diagram for Opening Reinforced
According to Fig. 2

$A_f/A_w = 1.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.824	0.80	0.997
0.40	0.792	0.60	1.003
0.60	0.758	0.40	1.008

Table 2.5

$A_f/A_w = 1.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.791	0.80	1.008
0.40	0.752	0.60	1.034
0.60	0.660	0.40	1.067

Table 2.6

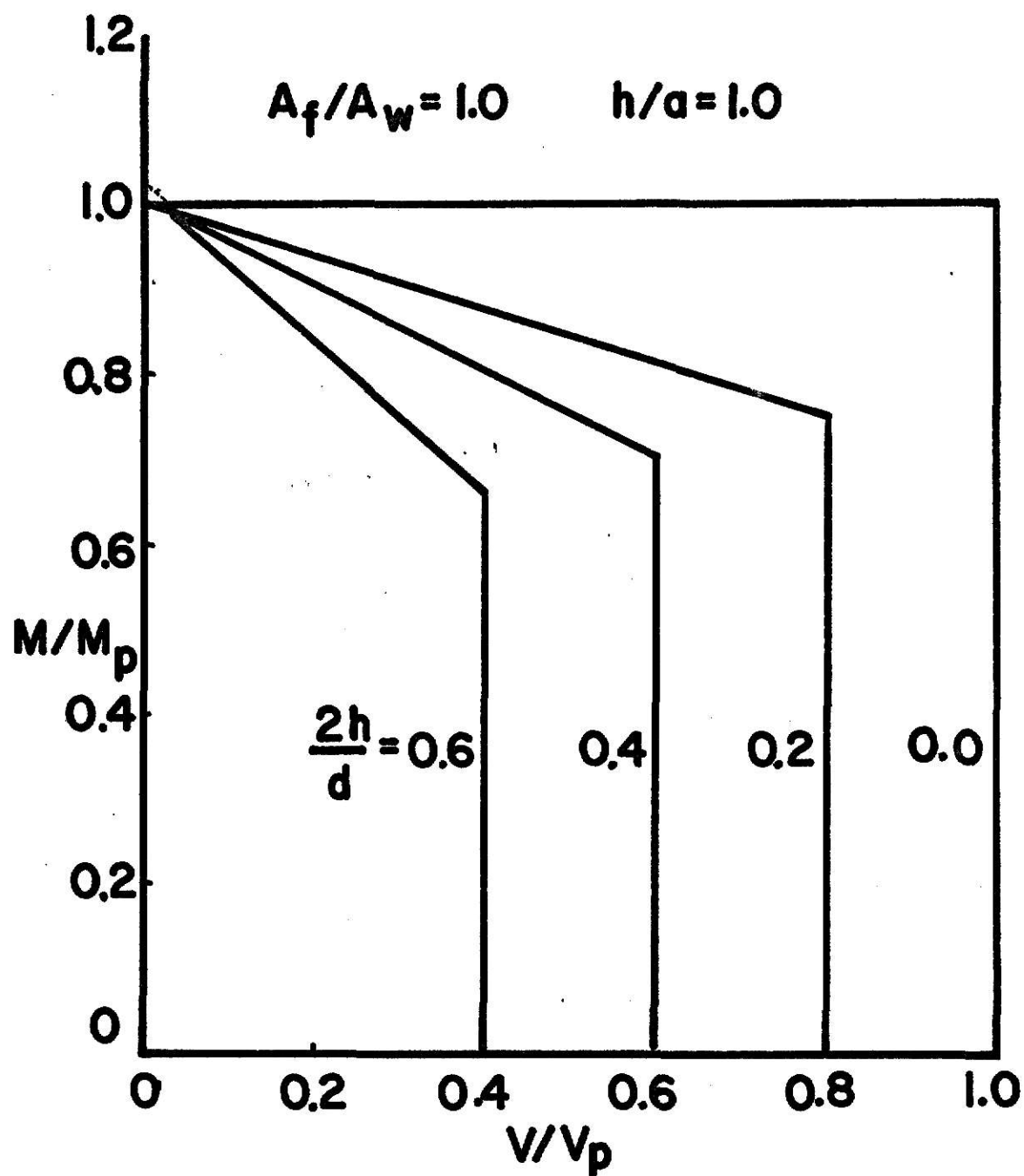Tables for Openings Reinforced to Reach

Maximum Shear Capacity

Fig. 2.5. Interaction Diagram for Opening Reinforced According to Fig. 2
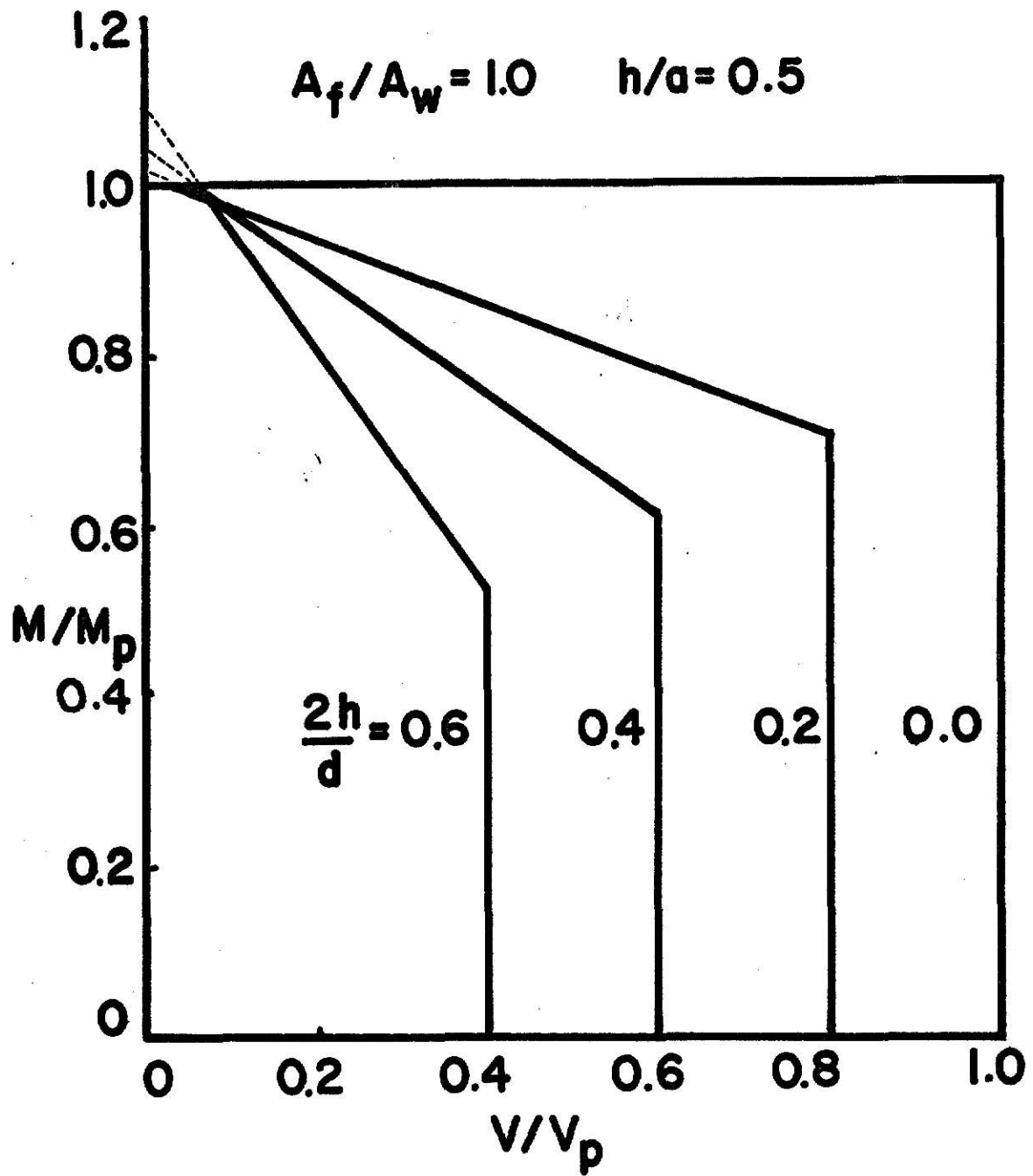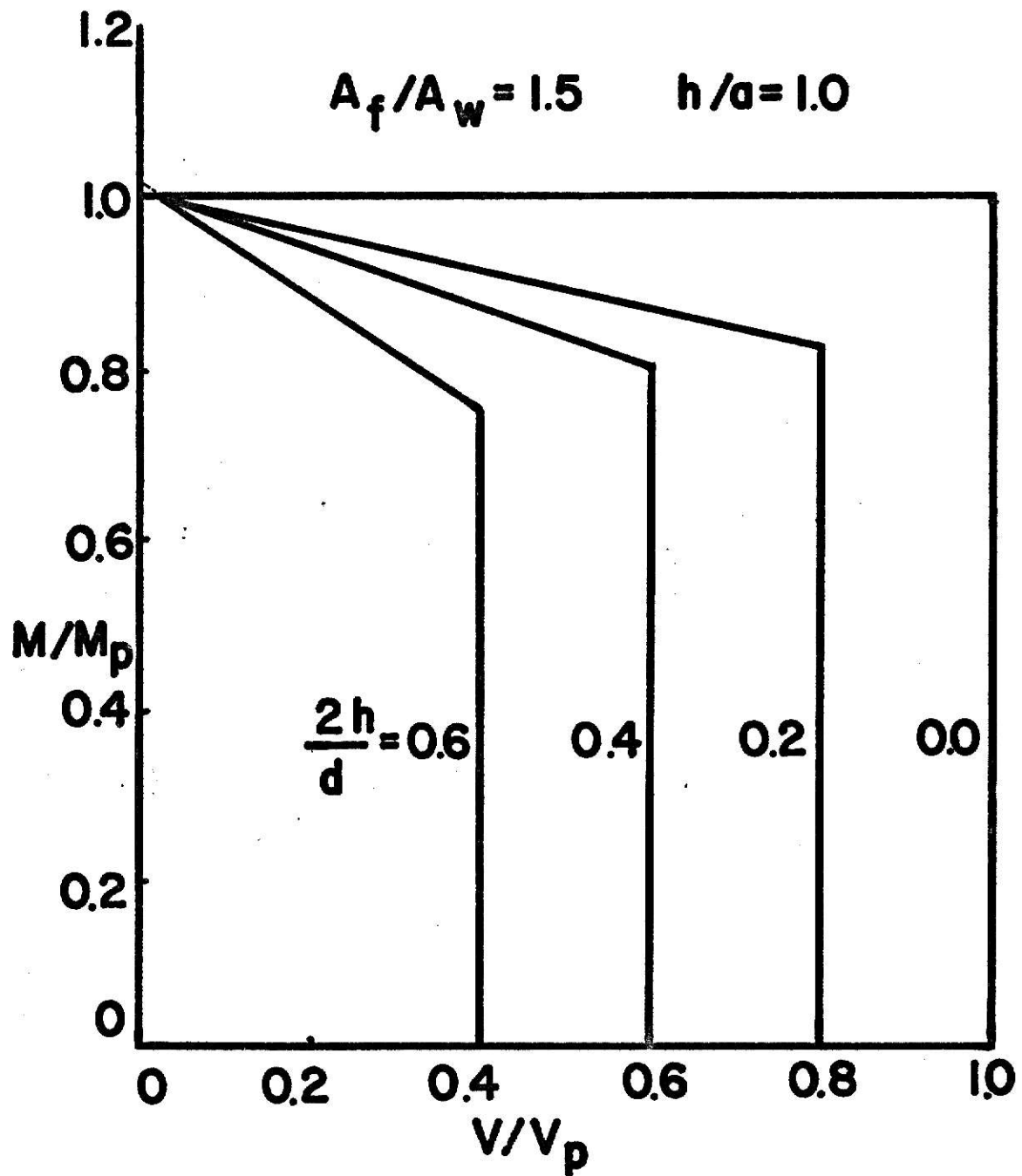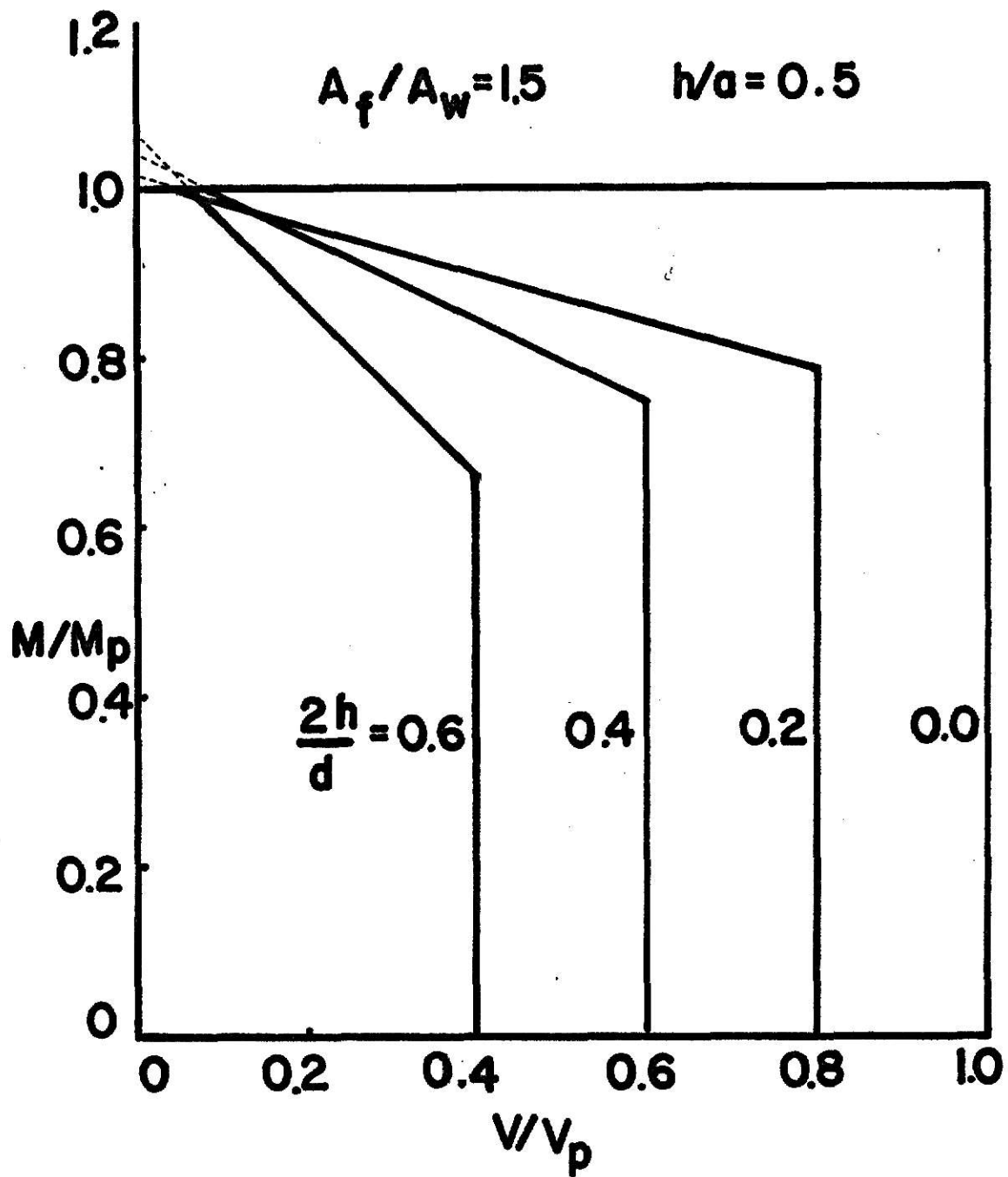
Fig. 2.6. Interaction Diagram for Opening Reinforced
According to Fig. 2

2h/d	A_r/A_w	
	h/a = 1.0	h/a = 1/2
0.0	0.00	0.00
0.1	0.227	0.232
0.2	0.208	0.231
0.4	0.189	0.276
0.6	0.200	0.361

<u>Reinforcing Required to Further</u>

<u>Increase Moment Capacity</u>

Table 3

Fig. 3.0. Reinforcing Required to Further Increase Moment Capacity

$A_f/A_w = 0.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.867	0.80	1.0405
0.40	0.655	0.60	1.047
0.60	0.533	0.40	1.053

Table 3.1

$A_f/A_w = 0.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.8206	0.80	1.048
0.40	0.409	0.60	1.093
0.60	0.259	0.40	1.168

Table 3.2

<u>Tables for Opening Reinforced for Further</u>

<u>Increase in Moment Capacity</u>

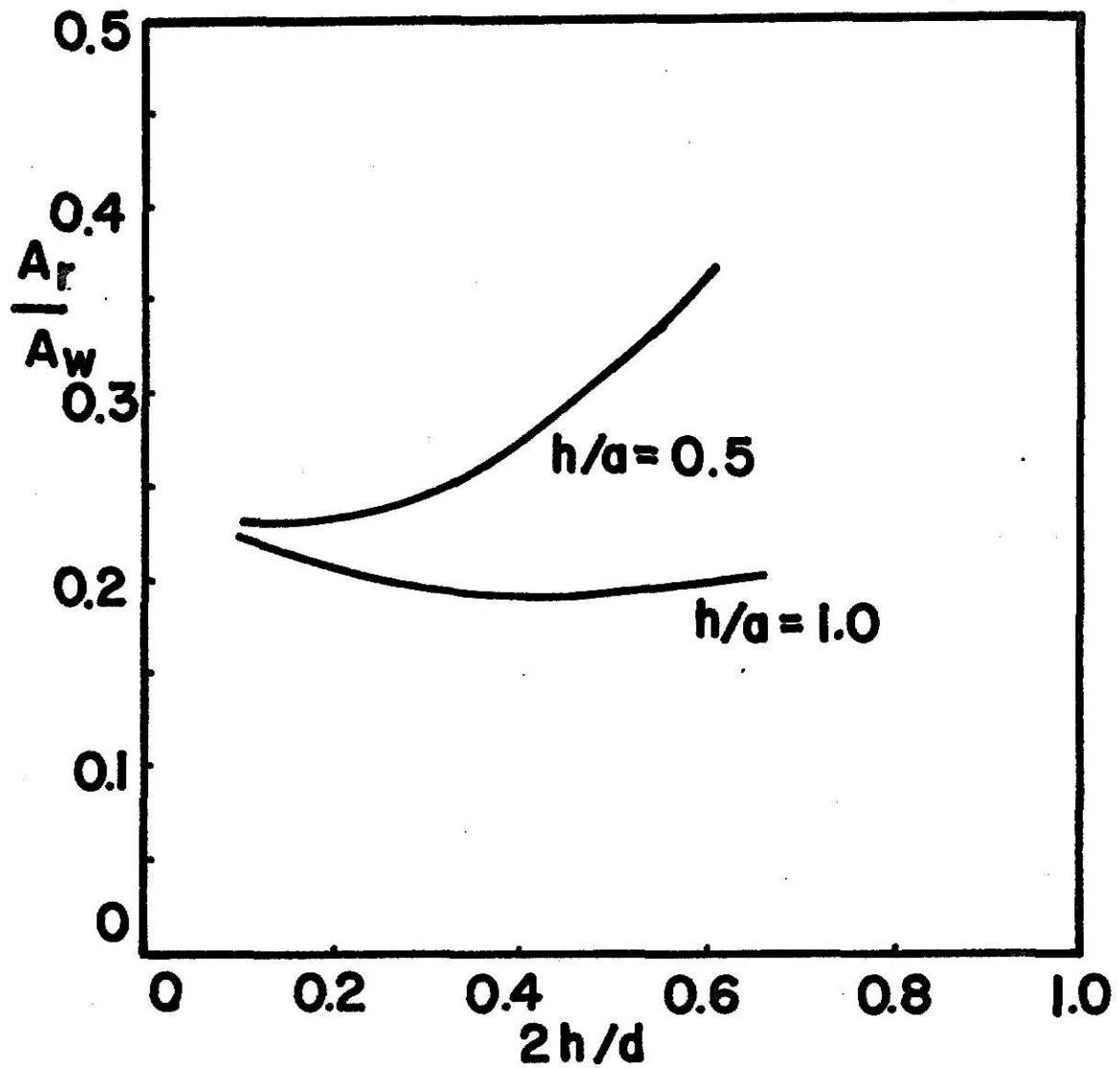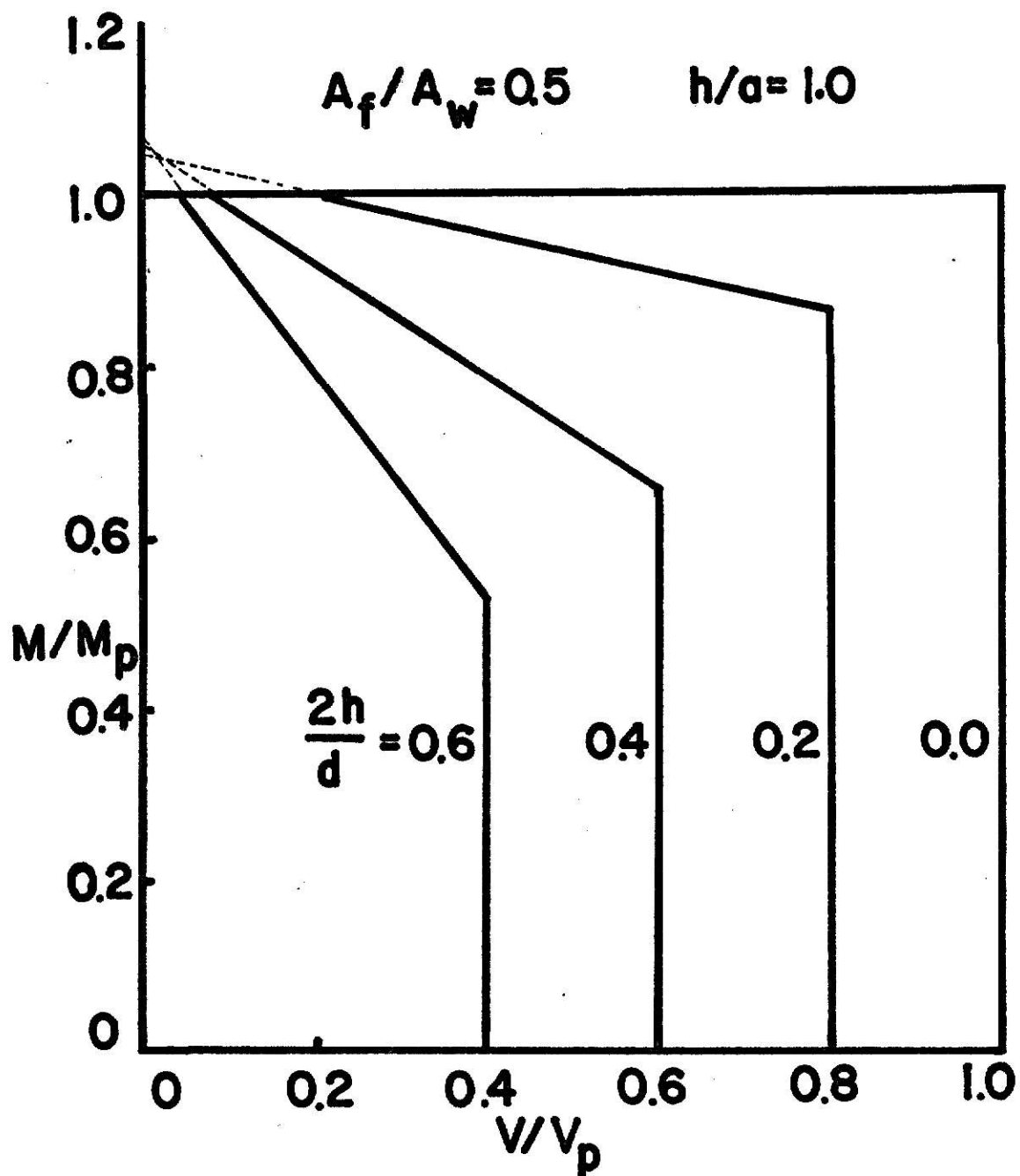Fig. 3.1.   Interaction Diagram for Opening Reinforced
According to Fig. 3

Fig. 3.2. Interaction Diagram for Opening Reinforced
According to Fig. 3

A_f/A_w = 1.0   h/a = 1

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.932	0.80	1.025
0.40	0.793	0.60	1.028
0.60	0.720	0.40	1.024

Table 3.3

A_f/A_w = 1.0   h/a = 0.5

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.892	0.80	1.028
0.40	0.645	0.60	1.056
0.60	0.555	0.40	1.109

Table 3.4

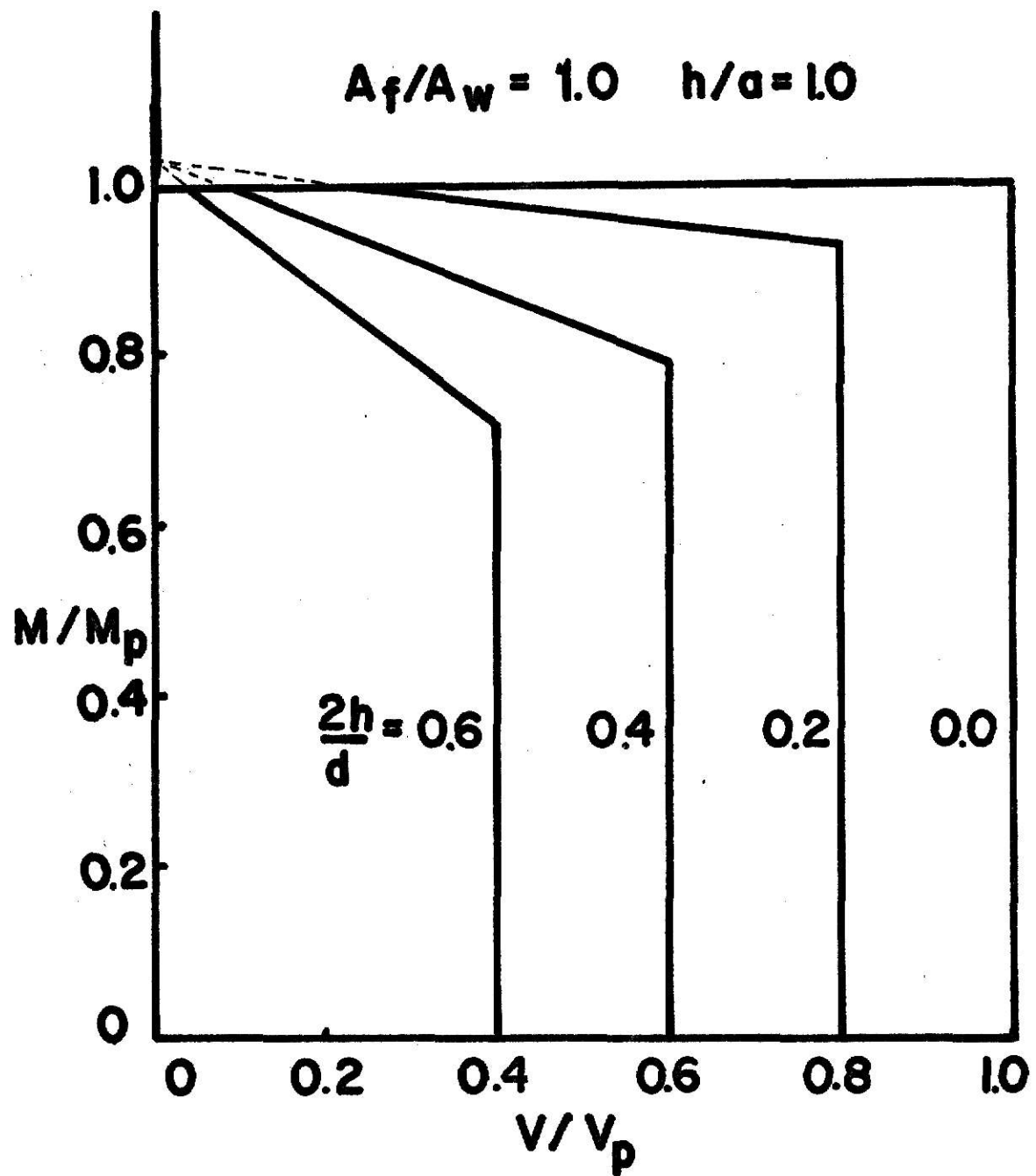Tables for Opening Reinforced for Further

Increase in Moment Capacity

Fig. 3.3. Interaction Diagram for Opening Reinforced
According to Fig. 3

Fig. 3.4. Interaction Diagram for Opening Reinforced
According to Fig. 3

$A_f/A_w = 1.5$ $h/a = 1$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.952	0.80	1.019
0.40	0.852	0.60	1.021
0.60	0.800	0.40	1.017

Table 3.5

$A_f/A_w = 1.5$ $h/a = 0.5$

2h/d	$(M/M_p)_1$	$(V/V_p)_1$	$(M/M_p)_2$
0.00	1.00	1.00	1.00
0.20	0.923	0.80	1.021
0.40	0.747	0.60	1.040
0.60	0.683	0.40	1.072

Table 3.6

Tables for Opening Reinforced for Further

Increase in Moment Capacity

Fig. 3.5. Interaction Diagram for Opening Reinforced
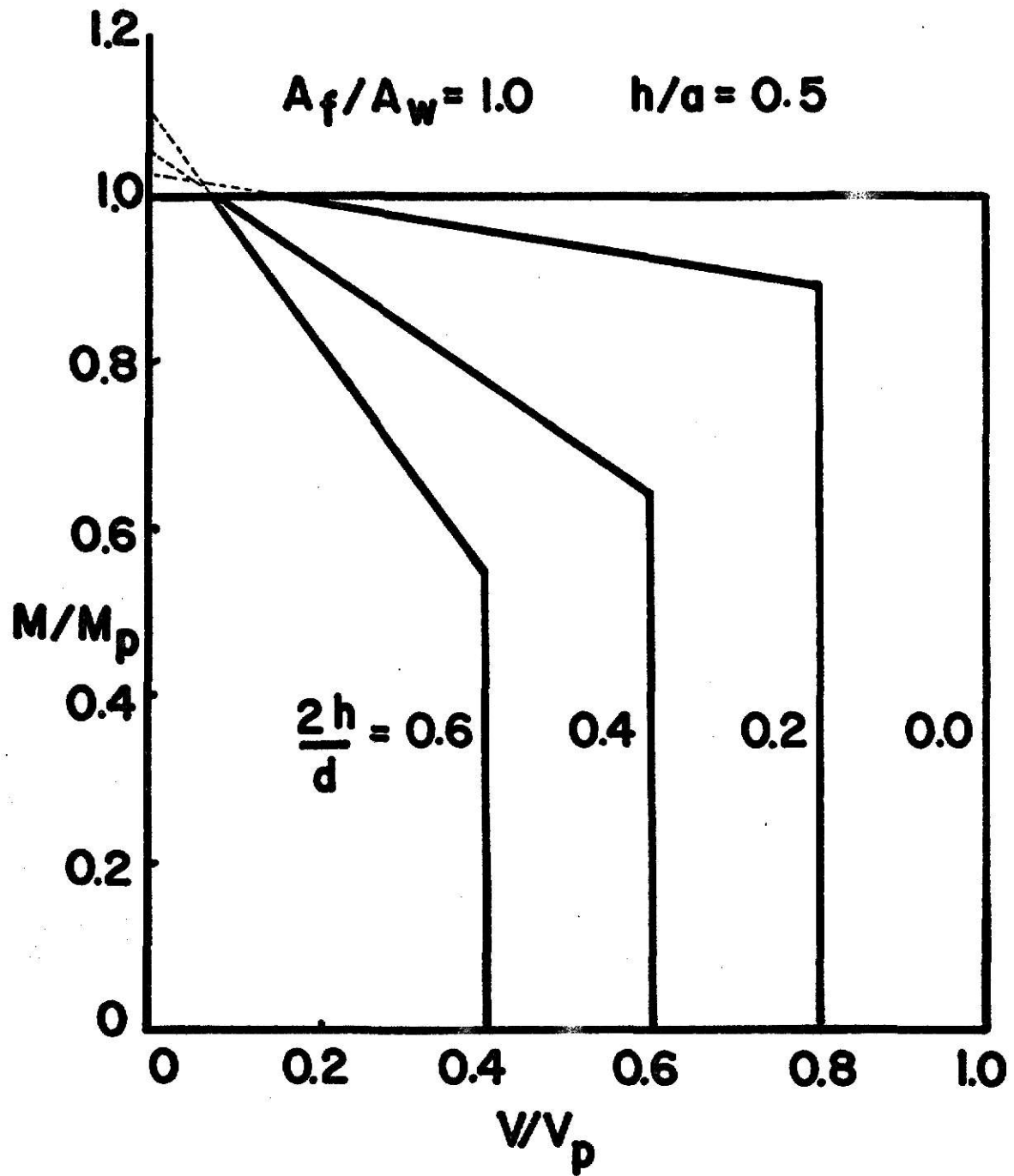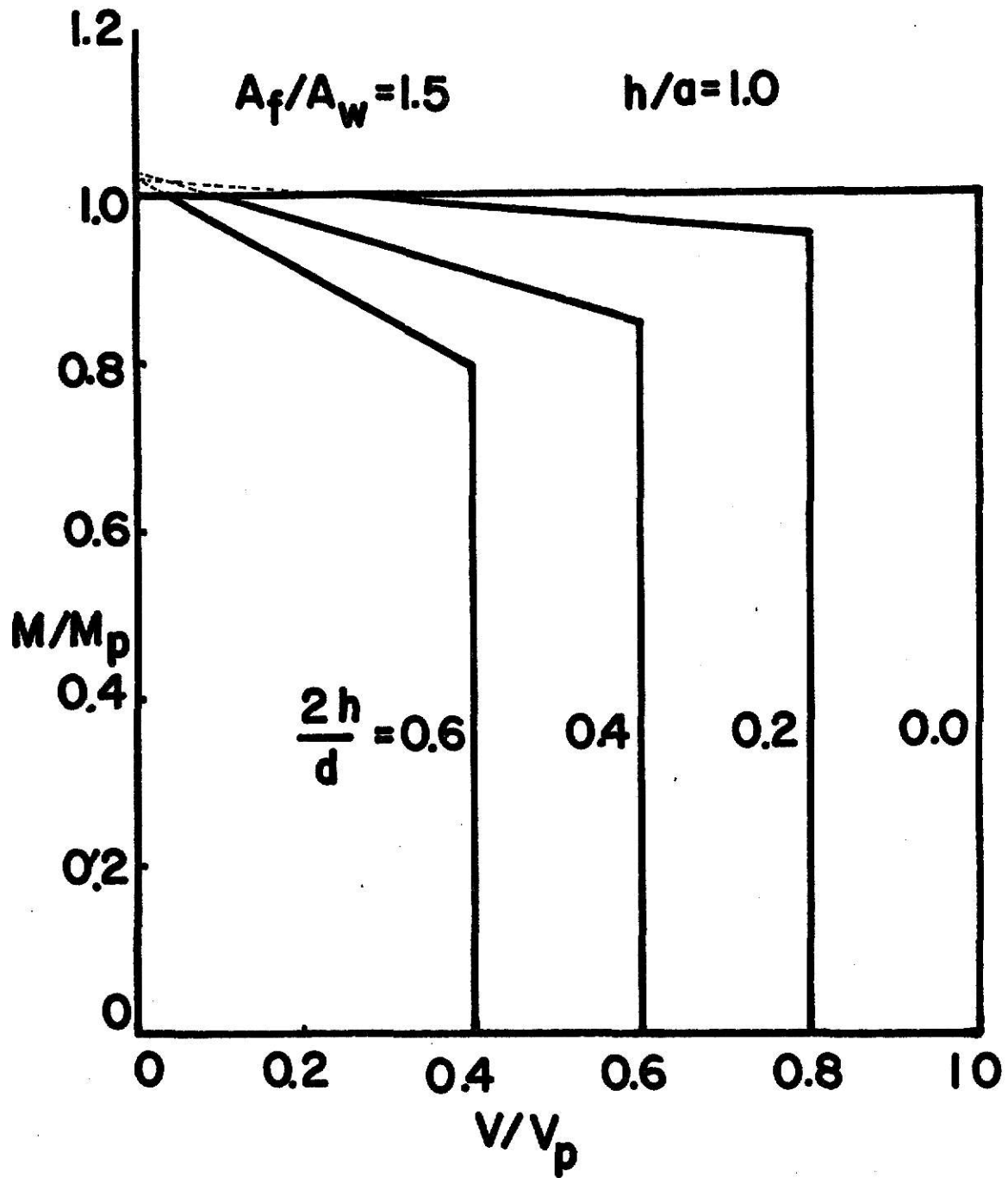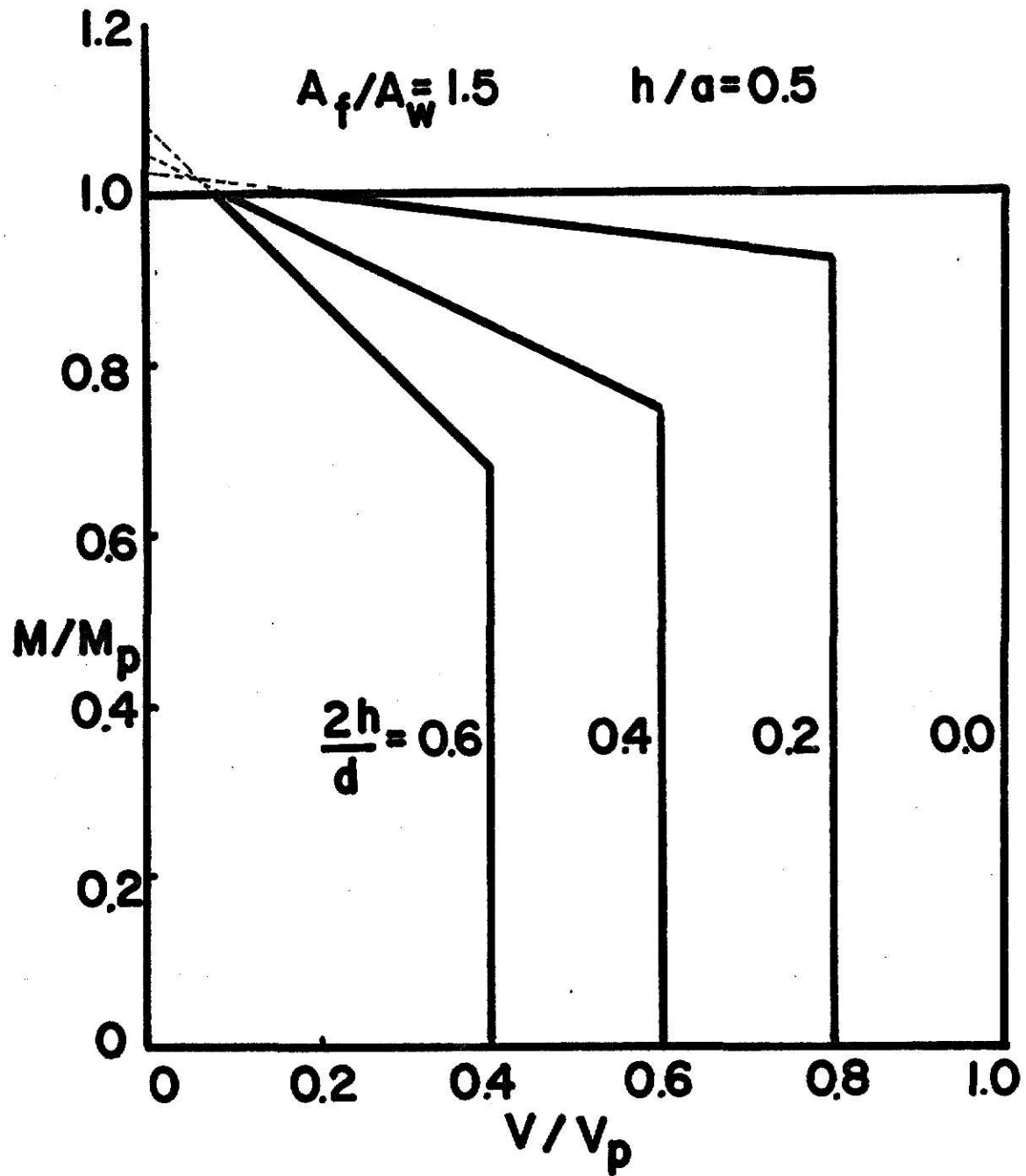According to Fig. 3

Fig. 3.6. Interaction Diagram for Opening Reinforced According to Fig. 3

APPENDIX C

CRITERIA FOR PROVIDING REINFORCING[*]

[1]   The nominal gap between the reinforcing bars and the horizontal edges of the opening should be at least 1/16" larger than the nominal weld size in order to avoid burning of the opening edges.

[2]   Extension of the reinforcing bars beyond the edges of the opening should be equal to the length required to develope the strength of the bars in web-to-reinforcing bar welds.

[3]   Fillet welds placed on only one side of the reinforcing bars will provide an adequate connection to the web, provided that the welds are proportioned to develop the shear strength of the web.

[4]   Reinforcing bars on only one side of the web may be used if the required A_r is provided and the above criteria are fulfilled.

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[*]From Ref. 6.

REFERENCES

(1) Bower, J. E., "Elastic Stresses Around Holes in Wide-Flange Beams," Journal of the Structural Division, A.S.C.E., Vol. 92, No. ST2, Proc. Paper 4773, April 1966.

(2) Bower, J. E., "Experimental Stresses in Wide-Flange Beams with Holes," Journal of the Structural Division, A.S.C.E., Vol. 92, No. ST5, Proc. Paper 4945, October 1966.

(3) Bower, J. E., "Ultimate Strength of Beams with Rectangular Holes," Journal of the Structural Division, A.S.C.E., Vol. 94, No. ST6, Proc. Paper 5982, June 1968.

(4) Chang, Kho Shu, "Experimental Study of Beam with Web Openings," M.S. Thesis, Kansas State University, 1969.

(5) Congdon, Judith G., "Ultimate Strength of Beams with Reinforced Rectangular Opening," M.S. Thesis, McGill University, July 1969.

(6) Cooper, Peter B. and Snell, Robert R., "Test on Beams with Reinforced Web Openings," January 1970.

(7) Heller, S. R., Jr., Brock, J. S. and Bart, R., "The Stresses Around a Rectangular Opening with Rounded Corners in a Beam Subjected to Bending with Shear," Proceeding, Vol. 1, Fourth U.S. National Congress on Applied Mechanics, Berkeley, Calif., 1962.

(8) Muskhelishvili, N. I., "Some Basic Problems of the Mathematical Theory of Elasticity," 2nd Edition P. Noordhoff Ltd, 1963.

(9) Redwood, R. G. and McCutcheon, J. O., "Beam Tests with Unreinforced Web Openings," Journal of the Structural Division, A.S.C.E., Vol. 94, No. ST1, Proc. Paper 5706, Jan. 1968.

(10) Redwood, R. G., "Plastic Behavior and Design of Beams with Rectangular Holes," Journal of the Structural Division, A.S.C.E., Vol. 94, No. ST6, Proc. Paper 5982, June 1968.

(11) Segner, E. P., Jr., "Reinforcement Requirements for Girder Web Opening," Journal of the Structural Division, A.S.C.E., Vol. 90, No. ST3, Proc. Paper 3919, June 1964.

(12) USS Building Design Data, "Design of Beams with Web Openings," United State Steel Publication.

ACKNOWLEDGEMENTS

PLASTIC DESIGN OF BEAMS WITH REINFORCED
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by

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AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1970

ABSTRACT

The objective of this report is to provide practising engineers with a method for determining the strength of a steel W-shape beam with a reinforced rectangular opening in the web for various values of moment and shear at the opening, and for determining the amount of reinforcing required around the opening. Design tables and charts have been prepared based on equations developed at McGill University. Design examples serve as a guide to the use of the charts.