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PLANE STRESS FINITE ELEMENT ANALYSIS OF BEAMS WITH WEB OPENINGS

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9984

A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1972

Approved by :

Major Professor

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I. Introduction

In present construction practice, holes are frequently cut in the webs of W shape beams to permit the passage of utility components or to provide access to the inside of box beams. Sometimes the holes are cut in the web without any attempt to locally reinforce the web. While at other times the beam is locally reinforced with doubler plates, angles welded to the web, or bars or flats welded to the periphery of the hole.

When an opening is cut in the web of the beam, the beam may be weakened in the vicinity of the opening to the extent that reinforcing is required. Tests have shown that stresses predicted on the basis of modified elementary beam theory may lead to an unsafe design of beams with holes.

Therefore, to get an indication of the stress distribution that actually exists in beams with holes, a more accurate and more powerful method, the finite element method, was used in this report to analyze the elastic stresses around a rectangular opening in the web of a W shape beam.

The beam [Fig. 1] is simply supported with a concentrated load applied at the center of the span. With various M/V ratios and with and without reinforcing bars, the stresses in a portion 30 inches wide, centered about the hole [Fig. 3] were determined using the finite element method as incorporated in the ICES-STRUDLE computer program.

In this report, the finite element method will be briefly reviewed and the results calculated by the finite element method will be compared with those obtained both experimentally and by the Vierendeel method.

II. Literature Review

In the 1920's Muskelishvili (1)* developed a practical method of solving the so-called plane problem of the theory of elasticity and in particular, the problem of the stress distribution in a plane or thin plate which is weakened by any type of hole. Since then the problem of stress concentration due to such openings has been much studied and many papers concerning this problem have been published (2, 3, 4, 5, 6).

In the past few years a concentrated effort has been made by steel industry and university investigators (7) to develop analytical and experimental information on steel beams with web openings.

In 1966, Bower (11), using the theory of elasticity method incorporating complex variable techniques investigated the stresses around an hole in a W shape beam. From this investigation, it was concluded that : a). the applicability of the analysis depends on the size of the web hole and on the *magnitude of the moment-shear ratio* at the hole ; b.) the stress distributions near the hole in uniformly loaded beams are widely different in magnitude and in appearance from the distributions occuring in beams without holes ; c.) the solution is valid for predicting stresses near the opening providing the opening depth does not exceed half the web depth.

Later Bower (12) used the Vierendeel method to calculate the elastic stresses around rectangular holes in the webs of W shape beams. It was concluded that this method provided a reasonably accurate prediction of the stresses in the vicinity of a rectangular hole except for stress concentrations near the corners.

From Bower's (12) experimental study of the stresses in W shape beams

* Numbers in parentheses refer to corresponding item in the References.

with web-openings, it was concluded that : a.) the theoretical results based on the theory of elasticity and Vierendeel method ore reasonably accurate, and b.) the elasticity analysis is complex and requires a computer solution, while the Vierendeel analysis is relatively simple to perform.

III. Method of Analysis

A. Introduction

The finite element method initially proposed by Turner et al. (13) in 1956 has proved to be quite convenient, from an automation point of view, for the solution of problems in continuum mechanics. The first applications were in plane stress problems (14). The finite element method has since been extended to axi-symmetric stress analysis, flat plate bending, three-dimensional stress analysis, and shell analysis.

The basic concept of the finite element method is that every structure may be considered to be an assemblage of individual structural components or elements. The structure must consist of a finite number of joints or nodes, which approximates the essential characteristics of the actual structure. When the structure is idealized in this manner, it can be analyzed by standard methods of structural analysis.

B. Analysis Procedure (15, 16)

The finite element analysis of an elastic continuum may be divided into three basic phases:

a. Structural idealization : The first phase of the finite element technique is the structural idealization or the subdivision of the actual structure into a finite number of discrete elements that form the substitute structute. Judgement is required in making the subdivision because the analysis actually is performed on this substitute structure and the results can be valid only to the extent that the substitute structure simulates the behavior of the actual structure.

b. Evaluation of the element properties : Since the elements are assumed to be interconnected only at a limited number of nodal points, the essential elastic characteristics of an element are represented by the relationships between forces applied at the nodal points and the deflections resulting therefrom. The force-deflection relationships may be expressed most conveniently by the flexibility or stiffness matrix of the elements as follows :

$$\{Q\} = [K]\ \{u\}$$

$$\{u\} = [F]\ \{Q\}$$

where $[K]$ = stiffness matrix,

$[F]$ = flexibility matrix,

$\{Q\}$ = external forces, and

$\{u\}$ = external displacements.

c. Structural analysis of the element assemblage : When the element properties have been defined, the calculation of the unknown forces and deflections (and the corresponding stresses and strains) that are caused by the imposed loads and prescribed displacements along the boundaries of the structures can be carried out by a standard structural analysis. There are three conditions which these unknown and known forces and deflections must satisfy simultaneously. They are :

1. Equilibrium : This condition, involving only force quantities, simply requires that individual elements of the structure remain in equilibrium.

2. Compatibility : All deformations must be such that the structure remains continuous in its deformed configuration.

3. Constitutive relations: The internal forces and displacements of

the structure must be related as required by the material properties i. e. the stress-strain relation of the material must be satisfied.

Either of the two basic approaches to structural analysis, the force method or the displacement method, may be applied in satisfying these requirements. In general, it has been found that for highly complex structures of arbitary form the displacement method provides the simpler formulation and computer programming task, so only that method will be described in this report.

C. Displacement Method

The basic operations of the displacement method of analysis of any structure follow :

a. Evaluation of the stiffness properties of the individual structural elements, expressed in a convenient set of local coordinate axes.

$$\{\bar{q}\}_i = [\bar{k}]_i \{\bar{v}\}_i \tag{1}$$

where $\{\bar{q}\}_i$ = matrix of the forces acting on all the nodes of the i^{th} element,

$[\bar{k}]_i$ = stiffness matrix of the i^{th} element referred to the local coordinate system, and

$\{\bar{v}\}_i$ = nodal displacement matrix of the i^{th} element.

b. Transformation of the element stiffness matrix from the local coordinate system to a common datum related to the global coordinate system of the complete, assembled structure.

$$[k]_i = [T']_i [\bar{k}]_i [T]_i \tag{2}$$

where $(k)_i$ = stiffness matrix of the i^{th} element referred to the global coordinate system,

$(T)_i$ = transformation matrix which relates the local directions to the global directions, and

$(T')_i$ = the transpose of $(T)_i$.

c. Assembly of the individual element stiffnesses contributing to each nodal point to obtain the total nodal stiffness matrix (K) : This involves only simple additions when all element stiffnesses have been expressed in the same global coordinate system.

$$(K) = \sum_{i=1}^{n} (k)_i \tag{3}$$

$i = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, n$

n = number of element

d. Formulation of the equilibrium equations expressing the relationship between the applied nodal forces $\{Q\}$ and the resulting nodal displacements $\{u\}$:

$$\{Q\} = (K)\{u\} \ . \tag{4}$$

A more general form of the stiffness equation can be written as

$$\{Q\}' = (K)\{u\} + \{Q\}^{\circ}$$

where $\{Q\}'$ = column matrix of applied loads,

$\{Q\}^{\circ}$ = column matrix of forces introduced to maintain initial structural shape in the presence of thermal gradients or equivalent effects,

(K) = total stiffness matrix, and

$\{u\}$ = total set of nodal displacements for the unsupported structure.

This equation is the same as equation (4) in which

$$\{Q\} = \{Q\}' - \{Q\}^{\circ}.$$

e. Determination of the nodal displacements by solving the equilibrium equation as shown below.

$$\{u\} = [K]^{-1}\{Q\} \tag{5}$$

f. Determination of the element strains by operating on the nodal displacements, and determination of the stresses through the use of the stress-strain relations.

These operations are common to any structural analysis by the displacement method. The special and critical feature of the finite element method is the analysis of the stiffness characteristics of the arbitary two-and three-dimensional elements.

IV. Finite Element Stiffness Analysis

The standard element stiffness analysis procedure is carried out as follows (outlined here for a two-dimensional element):

A. Express internal displacement field in terms of displacement functions:

$$\{v\} = [N]\ \{f\} \tag{6}$$

where [v] is the vector of internal displacements in the element, [N] is the matrix of assumed displacement functions, and {f} is a vector of generalized coordinates.

The displacement functions should satisfy internal compatibility within the element and maintain displacement continuity along the common element interfaces.

B. Evaluate nodal displacement components in terms of the generalized coordinates:

$$\{v\}_i = [A]\ \{f\}\ . \tag{7}$$

The matrix [A] is obtained by substituting the coordinates of the nodal points into the displacement function matrix [N]. It will be square if the number of displacement functions has been taken equal to the number of nodal displacement components.

C. Express the generalized coordinates in terms of the nodal displacements:

$$\{f\} = [A]^{-1}\ \{v\}_i\ . \tag{8}$$

In rare instances, the matrix [A] may be singular, requiring a modification of the element coordinate system or the deformation patterns. In general, the inversion is a simple computer process.

D. Evaluate the element strains:

$$\{\epsilon\} = [B]\{f\} . \tag{9}$$

The matrix [B] is obtained from [N] by appropriate differentiation of the displacement functions.

E. Evaluate the element stresses:

$$\{\sigma\} = [D]\{\epsilon\} = [D][B]\{f\} . \tag{10}$$

Where [D] is the matrix of material constants which relate internal stress to strain. These may be isotropic, elasto-plastic, or any other specified characteristics.

F. Compute the generalized coordinate stiffness of the element:

By applying the principle of virtual displacements the following equality must be satisfied for all arbitary increments in the independent displacements,

$$\delta W_e = \delta W_i . \tag{11}$$

Where δW_i is the virtual work done by internal restraint stresses as a result of the displacement increments and δW_e is the virtual work done by the external forces (body and surface forces).

The internal work is equal to

$$\delta W = \int_V \{\sigma\}^T \{\delta\epsilon\} \, dV = \int_V \{\delta\epsilon\}^T \{\sigma\} \, dV .$$

Using equations (9) and (10)

$$\{\delta\epsilon\} = [B]\{\delta f\} \qquad \{\delta\epsilon\}^T = \{\delta f\}^T [B]^T$$

$$W_i = \{\delta f\}^T \int_V [B]^T [D]\ [B]\ \{f\}\ dV \tag{12}$$

$$W_e = \{\delta f\}^T \{Q\} \tag{13}$$

where $\{Q\}$ represents the generalized forces corresponding to the displacements $\{f\}$.

Substituting equations (12) and (13) into equation (11)

$$\{Q\} = \left(\int_V [B]^T [D]\ [B]\ dV \right) \{f\} \tag{14}$$

By definition of equation (4), the bracketed term represents the generalized coordinate stiffness of the element :

$$[\bar{k}] = \int_V [B]^T\ [D]\ [B]\ dV \tag{15}$$

G. Transform to the desired nodal point stiffness, $[K]$:

$$[K] = [A]^{-1}\ [\bar{k}]\ [A] \tag{16}$$

where the transformation matrix $[A]^{-1}$ relates the generalized coordinates to the nodal point displacements.

V. Plane Stress Element Stiffnesses

Various shapes of finite elements have been employed in plane stress and plane strain analyses. In general, rectangular elements (where applicable) appear to yield slightly better approximations of stresses and deflections for a given nodal pattern than triangular elements because they employ a more refined deformation approximation. However, because of their greater adaptability in fitting arbitary boundary geometries, triangular elements have been used more widely in the development of general purpose analysis programs. The stiffness analysis of a triangular plane stress element will be discussed in some detail here.

A. Displacement functions

Consider an arbitary triangular element as shown in Fig. V-1. Assume that the element is in a state of plane stress with the three nodal points at the vertices and with the six nodal displacements or degrees of freedom shown.

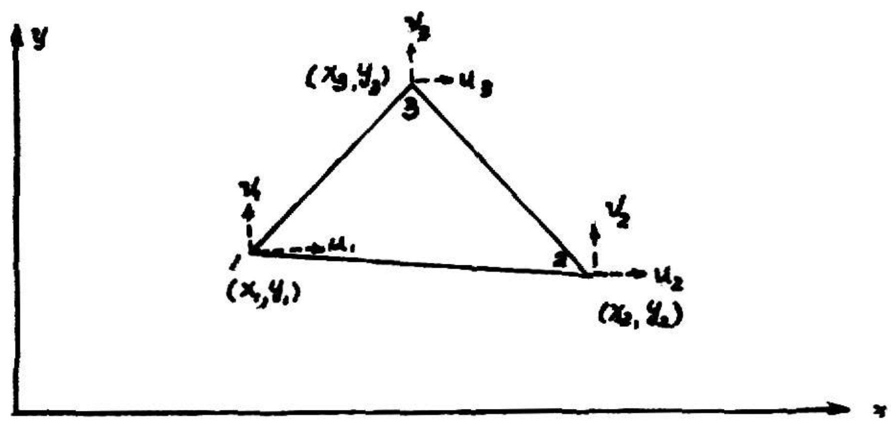

Fig. V-1 Triangular element in plane stress

As indicated in step (a) of the standard procedure, we can begin the formulation by assuming a linear displacement field for the element.

$$u = f_1 + f_2 x + f_3 y$$
$$v = f_4 + f_5 x + f_6 y \tag{17}$$

On substituting the boundary conditions,

$$u_1 = f_1 + f_2 x_1 + f_3 y_1 \ ,$$
$$u_2 = f_1 + f_2 x_2 + f_3 y_2 \ ,$$
$$u_3 = f_1 + f_2 x_3 + f_3 y_3 \ ,$$
$$v_1 = f_4 + f_5 x_1 + f_6 y_1 \ , \tag{18}$$
$$v_2 = f_4 + f_5 x_2 + f_6 y_2 \ , \text{ and}$$
$$v_3 = f_4 + f_5 x_3 + f_6 y_3 \ .$$

Solving equations (18) in terms of f_i , $i = 1, \ldots, 6$

$$u = \frac{1}{2\Delta}\left\{(a_i + b_i x + c_i y)u_i + (a_j + b_j x + c_j y)u_j + (a_m + b_m x + c_m y)u_m\right\} \text{ ,and}$$
$$v = \frac{1}{2\Delta}\left\{(a_i + b_i x + c_i y)v_i + (a_j + b_j x + c_j y)v_j + (a_m + b_m x + c_m y)v_m\right\}. \tag{19}$$

in which

$$\left.\begin{aligned} a_i &= x_j y_m - x_m y_j \\ b_i &= y_j - y_m = y_{jm} \\ c_i &= x_m - x_j = x_{mj} \end{aligned}\right\} \text{ for } \left\{\begin{aligned} &i=1,\ j=2,\ m=3; \\ &i=2,\ j=3,\ m=1; \\ &i=3,\ j=1,\ m=2; \text{ and} \end{aligned}\right.$$

$$2\Delta = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} = 2(\text{area of the triangular element}) .$$

If the coordinates are taken **from** the centroid of the element then

$x_i + x_j + x_m = y_i + y_j + y_m = 0$, and

$a_i = 2/3\,\Delta = a_j = a_m$.

The chosen displacement function automatically guarantees continuity of displacements with adjacent elements because the displacements vary linearly along any side of the triangular element and, with identical displacement imposed at the nodes, the same displacement will clearly exist all along an interface.

B. Strain : The [B] matrix of equation (9) representing the internal strain is obtained by differentiating the displacement functions appropriately, as follows :

$$\{\epsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{Bmatrix} = \frac{1}{2\Delta} \begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} . \quad (20)$$

It will be noted that in this case the [B] matrix is independent of the position within the element, and hence the strains are constant throughout it.

In general, the initial strains which may be caused by temperature changes, shrinkage and so on may be taken into account. The initial strains ϵ_0 are not associated with stress and are usually defined by average constant values. This is consistent with the constant strain conditions imposed by the prescribed displacement function.

C. Stresses and Elasticity Matrix

In general elastic behaviour, the relationship between stresses and strains will be linear and of the form

$$\{\sigma\} = [D]\left(\{\epsilon\} - \{\epsilon_0\}\right) \tag{20}$$

For plane stress in an isotropic material, the form of $[D]$ is

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{21}$$

where E = elastic modulus, and

ν = Poission's ratio .

D. The Stiffness Matrix $[K]$

$$[K] = \int_v [B]^T [D][B]\ t\ dxdy \tag{22}$$

This is the stiffness matrix of the element i, j, m with thickness t. If the thickness of the element is assumed to be constant, an assumption convergent to the truth as size of elements decreases, then, as neither of the matrices contains x or y we have simply

$$[K] = [B]^T[D][B]t\ . \tag{23}$$

This form is now sufficiently explicit for computation with the actual matrix operations being left to the computer.

E. The equivalent nodal forces can be represented as follows :

$$\{Q\} = \begin{Bmatrix} Q_i \\ Q_j \\ Q_m \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} .$$

The nodal forces are equivalent statically to the boundary stress and distributed loads of the element.

1. The external forces can be represented as follows :

$$\{Q'\} = \begin{Bmatrix} Q'_1 \\ \cdot \\ \cdot \\ \cdot \\ Q'_n \end{Bmatrix} .$$

These concentrated forces are applied at the nodes. Any one of them must have the same number of components as that of the element reactions considered.

In the plane stress case,

$$\{Q'_i\} = \begin{Bmatrix} X_i \\ Y_i \end{Bmatrix} .$$

2. The distributed loads can be represented as follows :

$$\{P\} = \begin{Bmatrix} X \\ Y \end{Bmatrix} ,$$

in which X and Y are the body force or surface force components.

Nodal forces due to distributed loads can be expressed as

$$\{Q^{\circ}\}_b = -\int_V [A]^T \{P\}\, dV$$

for the body forces and as

$$\{Q^{\circ}\}_s = -\int_V [A]^T \{P\}\, dA$$

for the surface forces.

If the origin of coordinates is taken at the centroid of the element, the equivalent forces due to body force are distributed equally to the nodes of the element. In other words

$$\{Q^{\circ}\}_b = \begin{Bmatrix} X \\ Y \\ X \\ Y \\ X \\ Y \end{Bmatrix} \Delta/3 \;.$$

The equivalent forces due to initial strain are given by

$$\{Q^{\circ}\}_{\epsilon_o} = -\int_V [B]^T [D]\{\epsilon_o\} dV$$

or

$$\{Q^{\circ}\}_{\epsilon_o} = -[B]^T [D]\{\epsilon_o\} t \;.$$

VI. Numerical Example

A. Problem Set Up

In 1970 a series of 12 W 45 beams which were simple supported on the ends and subjected to a concentrated load applied at the midspan were tested at Kansas State University (7). Each beam had a 9" x 6" rectangular web opening with its neutral axis centered at the middepth of the beam. The centerline of the opening, in the longitudinal direction, was 20" from the midspan. By varying the length of the shear span four different M/V ratios, shown in Fig. 1, were tested.

The beams were tested, using the four set ups of Fig. 1, with and without reinforcement. The reinforcement was of rectangular cross section oriented parallel to the beam flanges and welded to the web above and below the opening as shown in Fig. 2.

B. Simplification of The Problem

In the finite element method the structure is discretized into finite elements which simulate its behavior. Since there was more interest in the stresses around the opening, there was no need to model the whole beam. Therefore only a portion of the beam was studied. (Fig. 3)

Form the theory of elasticity (17), it is known that if a hole is made in the middle of a plate, the stress distribution in the neighborhood of the hole will be changed. But it can be concluded from Saint-Venant's principle that the change is negligible at distances which are large compared with the half depth of the hole. The length of the portion cut form the beam was 30" long. It can be seen from the results that the stress changes due to the opening beyond this portion of the beam have a negligible effect.

C. Plane Stress Analysis

In using the plane stress finite element method for the analysis, we have to modify the three-dimensional structure into a two-dimensional structure. The procedure followed was to substitute equivalent bar elements pin-connected to the appropriate nodes of the plate elements for the top and bottom flanges. These bar elements have one-dimensional material properties which can only transfer axial forces.

For the reinforcing bars, the equivalent bar elements of the reinforcement were put on the appropriate node points of the plate elements as shown in Fig. 4.

D. Superposition

The forces acting on a 30" wide plate section, as shown in Fig. 5, may be regarded as made up of two different force fields. They are : a). pure bending, b). end shear and bending. By superimposing, the results can be combined for various M/V ratios. The equations for superposition are also shown in Fig. 5.

A 12 kip concentrated load was applied at the midspan when the web was without reinforcement and a 24 kip load was applied when the web was reinforced.

E. Finite Element Discretization

1. Rectangular plate subjected to in-plane pure bending

a. Boundary conditions

The pattern of the displacement is symmetrical with respect to the Y axis and antisymmetrical with respect to the X axis. We therefore have to work with only one quarter of the plate. The boundary conditions were intrduced by restraining the X translation, u, for nodes on the Y and X axes as shown in Fig. 6a.

b. Equivalent nodal loads

Since the displacement varies linearly along the boundary for a constant strain triangle, the equivalent nodal forces are just the static resultants. These are shown in Fig. 7.

2. Plate subjected to in-plane end shear and bending

a. Boundary conditions

The patterns of the displacements are antisymmetrical with respect to both the X and Y axes. And again we have only to consider one quarter of the plate. The boundary conditions were introduced by restraining the X translation, u, for nodes on the X axis and the Y translation, v, for nodes on the Y axis as shown in Fig. 6b.

b. Equivalent nodal forces

The theoretical surface forces caused by shear and bending were replaced by two sets of concentrated forces as shown in Fig. 7. The equivalent nodal forces for both cases are the static resultants of the theoretical force distributions.

3. Dimensions and material properties

The finite elements shown in Fig. 8, with the following properties, were used in the analysis.

$t = 0.336''$ ------------------ thickness of the plate

$E = 29000$ ksi --------------- Young's modulus

$\nu = 0.3$ --------------------- Poisson's ratio

$G = 11150$ ksi --------------- modulus of rigidity

$A_f = 4.20\ in^2$ ---------------- effective area of equivalent bar elements of the top or the bottom flange.

$A_r = 1.446 \text{ in}^2$ ----------------- effective area of equivalent bar element of the reinforcement.

These values of A_f and A_r were calculated as shown below:

a. A_f (Fig. 4b)

$$I_{xx} = \frac{b\,h^3}{12} + 2(A_f\,y^2)$$

$$A_f = \frac{351-0.336(12)^3/12}{2(6)^2} = 4.20 \text{ in}^2$$

b. A_r (Fig. 4d)

$$A\,y^2 = A_r\,y_r^2$$

$$A_r = \frac{1.5(3.4375)^2}{(3.5)^2} = 1.446 \text{ in}^2.$$

4. Discretization

The plate was divided into 270 triangles and nodes were taken at the corner points. There are 16 bar elements for the flange and 13 bar elements for reinforcement(Fig. 8).

VII. Comparison and Discussion of The Results

A. Location of calculated stresses

The stresses determined by a finite element representation using constant strain triangles are constant throught an element. Provided that the material properties of the elements incident on a node are the same, the corresponding stresses in these elements may be averaged and the result attributed to the common node. Alternatively, the stresses determined for an element may be assigned to a particular point within the element and usually are assigned to the centroid. In these examples we determined the stress distributions across the various sections by using the second procedure.

B. Normal Stresses

In Fig. 9 through Fig. 24, the normal stresses calculated by the finite element method are compared with stresses calculated by the Vierendeel method and with the experimental stresses. These stresses were plotted at sections x=0, -4.5", -4.875", -7.5", -10.5" measured from the center of the opening.

Figure 9 through Fig. 16 show normal stresses for the unreinforced case, while Fig. 17 through Fig. 24 are for the reinforced case.

The finite element method predictions are in very good agreement with both the experimental and the theoretical values at the centerline (x=0) of the opening, especially for the low M/V ratios.

At the section x=-3", the finite element method predictions are in excellent agreement with the experimental values. The normal stresses at the edge of opening (x=-4.5") are not all in good agreement with those obtained by either the Vierendeel or the experimental method. However, the normal stresses predicted by the finite element method are more reasonable than those predicted by the Vierendeel method as they compare with the experimental values. Theoretically, the discrepancy between the finite element and experimental method is due to the edge effect of the web opening and in order to obtain better accurcy, we would have to use smaller elements in the regions of high stress gradient. However, this rapidly increases the computational costs and the improvement in accuracy must then be balanced against those costs.

Examining the normal stress distribution across sections x = -4.875", and -10.5", it is apparent that the stress concentrations attenuate rapidly as the transverse distance from the hole increases and also attenuate rapidly at the corner as the longitudinal distance from the corner increases. This local high stress was decreased for the reinforced opening where the normal stress, at a point x = - 3/8" from the edge of the opening and 3" above the middepth, was reduced 33 - 38 % from the value of the stress at the same location for the unreinforced case.

C. Shear Stresses

The shear stresses (Fig. 25 and Fig. 26) were also plotted at sections x = 0", -3.375", -4.5", -5.625", -7.5" and -10.5 measured from the center of the opening.

Since the same load magnitude was applied for each example in a series, the shear force at any section in the beam should have been constant, and the corresponding shear stress distribution should also have been constant. Therefore the average of the shear stress values from the individual examples have been used to represent the results for the series.

It can be seen from Fig. 25 that, for the unreinforced opening, the predicted shear stress at the center of the opening is in poor agreement with the experimental value. However, the predicted shear stress is in very reasonable agreement with the results calculated by the Vierendeel method. As can be noted, the measured normal stress at this section for this particular gage was also quite low. It might reasonably be assumed that the gage was defective. At the same section for the reinforced case the results of the three methods are in good agreement.

The agreement between the predicted and the experimental results are also very good at the edge of opening for the unreinforced case and at the section x=-7.5" for both unreinforced and reinforced cases. No experimental result was measured at the edge of the opening for the reinforced opening.

The predictions at the section x=-10.5" are in reasonable agreement with both the experimental values and results evaluated by the Vierendeel method.

Comparing the shear stresses, at sections x = -1.125", -3.375 and -5.625" where no experimental or theoretical results are available, for the unreinforced and reinforced cases, it can be seen the shear stress distributions changed rapidly due to the presence of the reinforcing bars. At the sections x = -7.5" and -10.5", the shear stress sign reversed at the reinforcing bars with poistive shear below the reinforcement and negative shear above the reinforcement.

VIII. Conclusions

From the examples analyzed by the finite element method the following conclusions can be reached:

1. At the center of the opening, the normal stresses predicted by the finite element method had very good agreement with those determined experimentally and by the Vierendeel method.

2. At the edges of the opening, the finite element method predicted the normal stresses in more reasonable agreement with the experimental stresses than did the Vierendeel method.

3. In general, the agreement of the predicted shear stresses with the experimental results was good.

4. The stress concentrations around the edge of the opening, especially at the corner were significantly decreased when the reinforcement was added.

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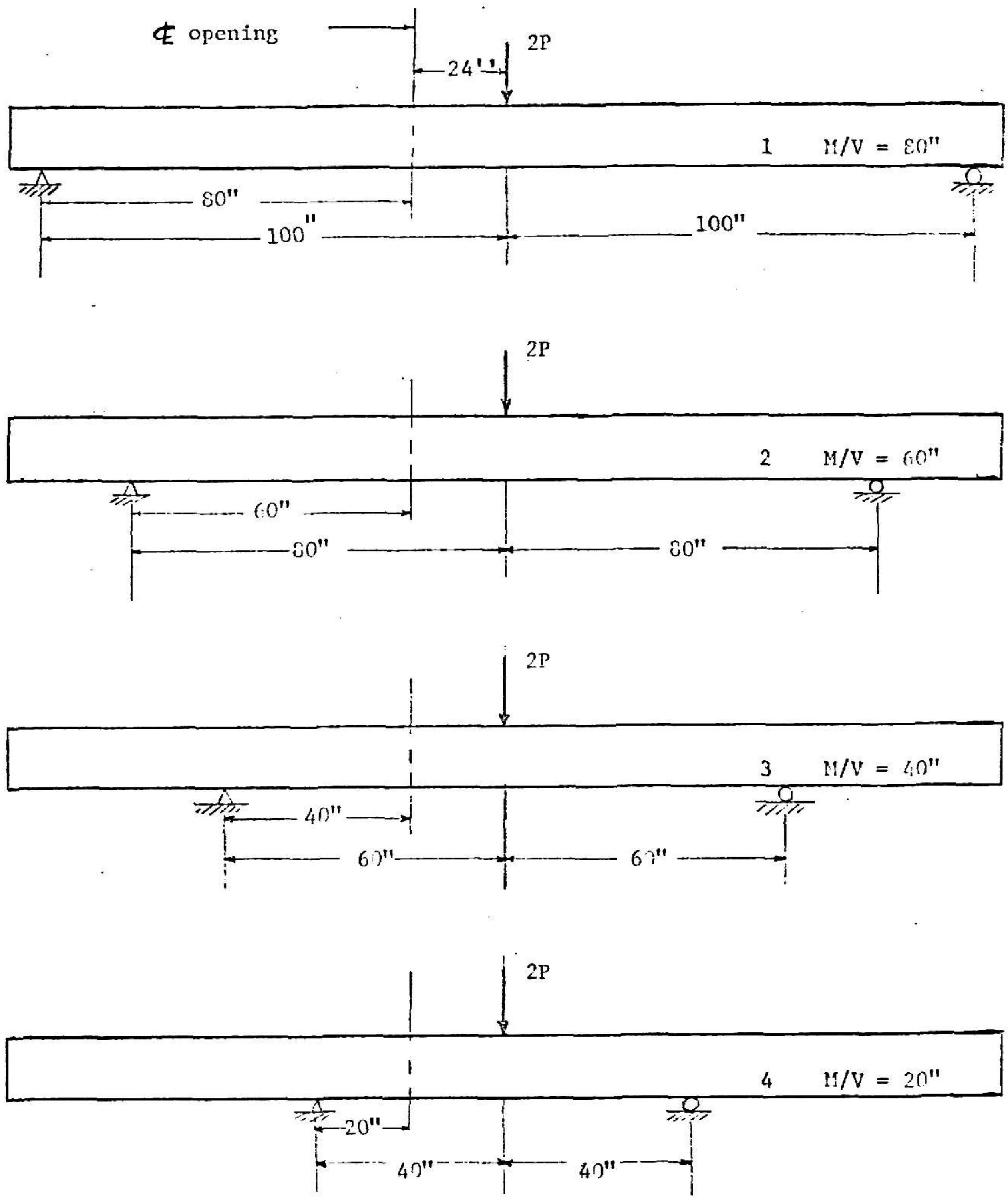


Fig.1 Simply supported beams with various M/V ratios

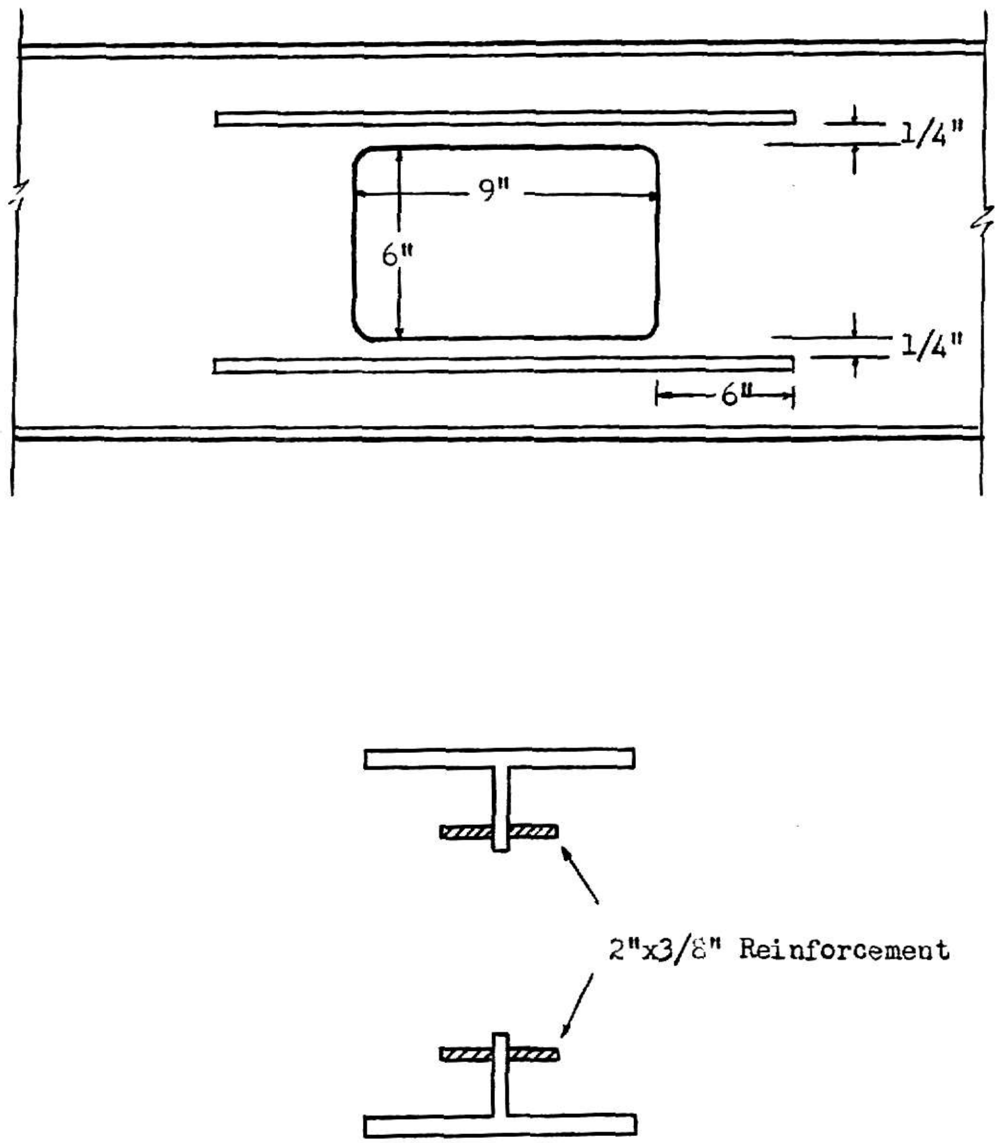


Fig. 2 Web Opening and Reinforcing Details

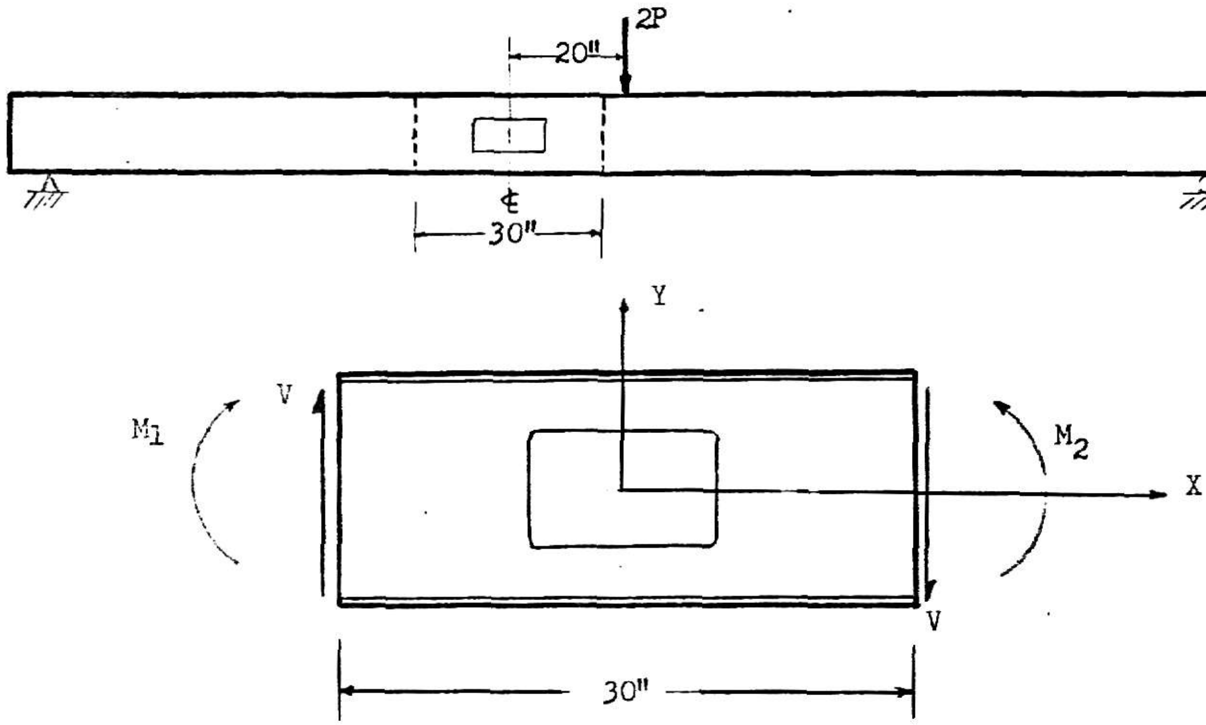


Fig.3 Portion of Beam Used in Analysis

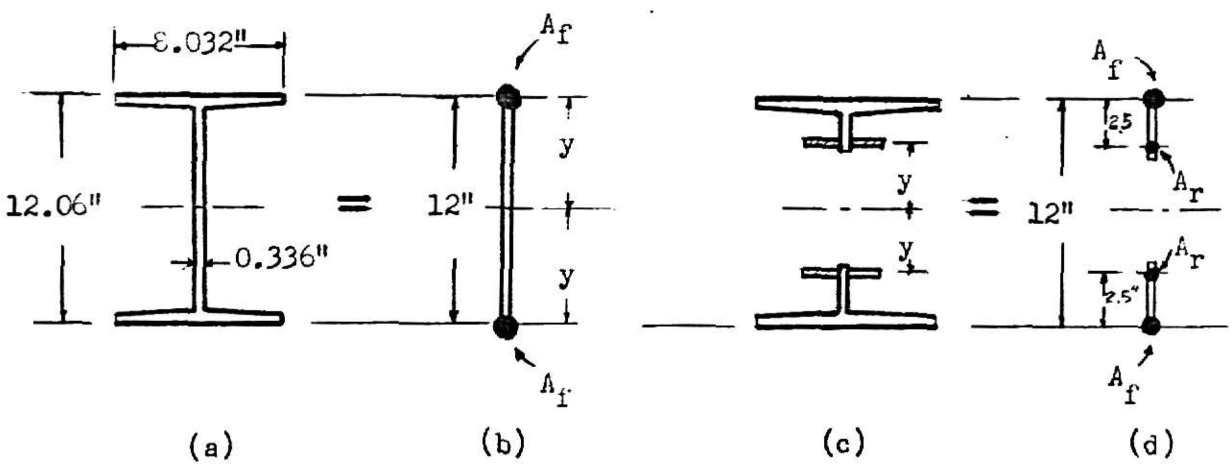
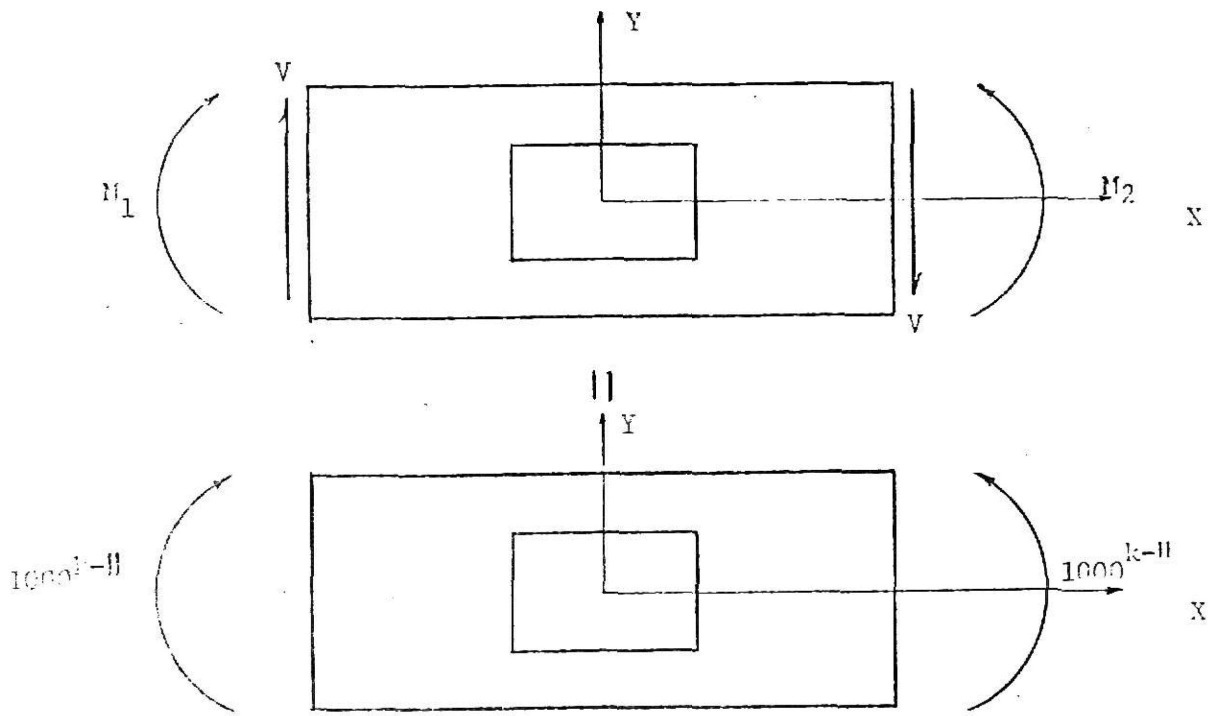
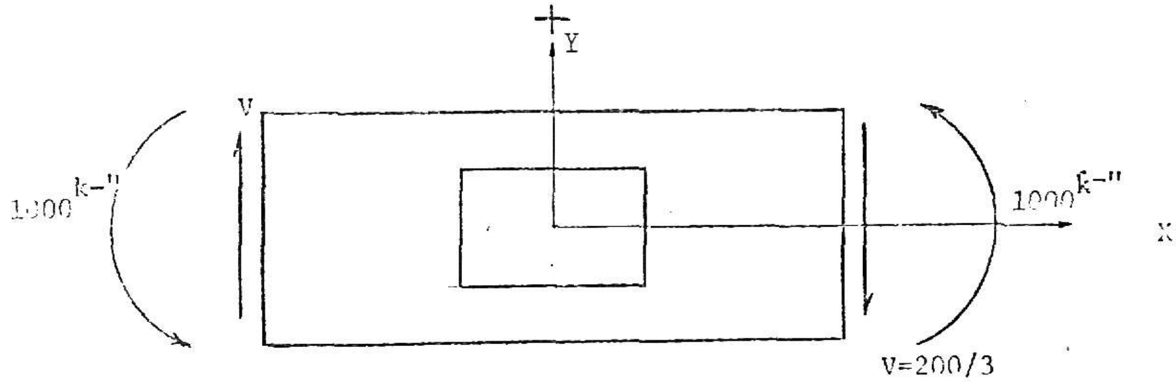


Fig. 4 Equivalent Bar Elements For Flanges And Reinforcing Bars



a. pure bending



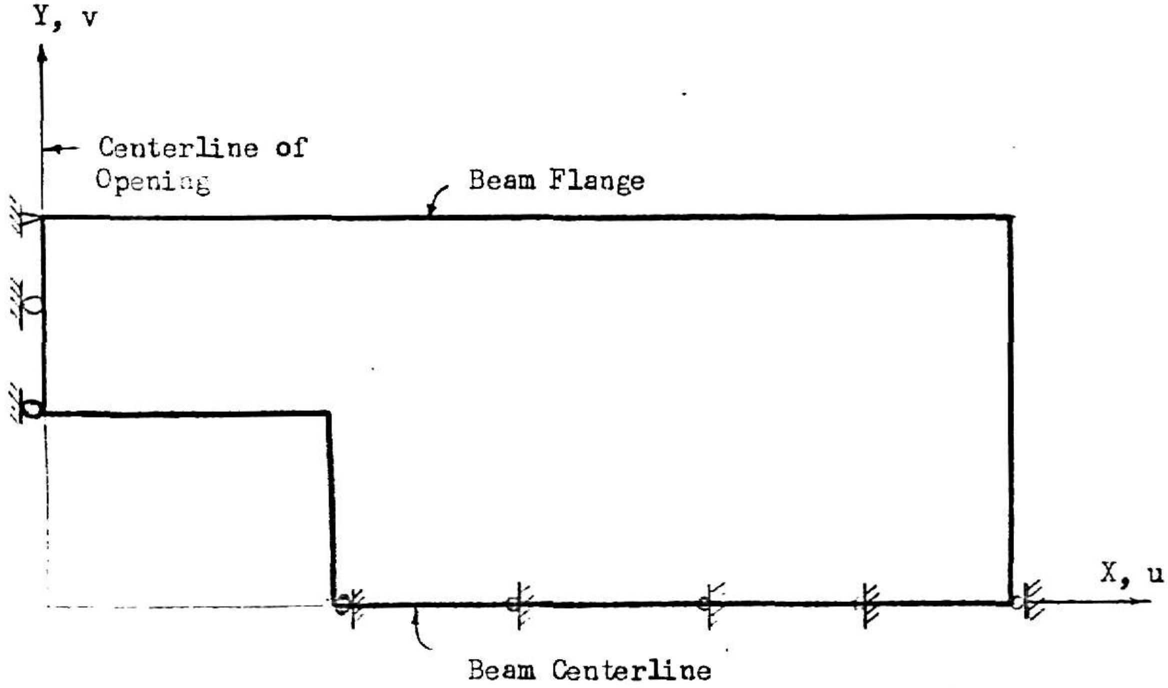
b. end shear with bending

$$M_1 = 1000x_1 - 1000x_2$$

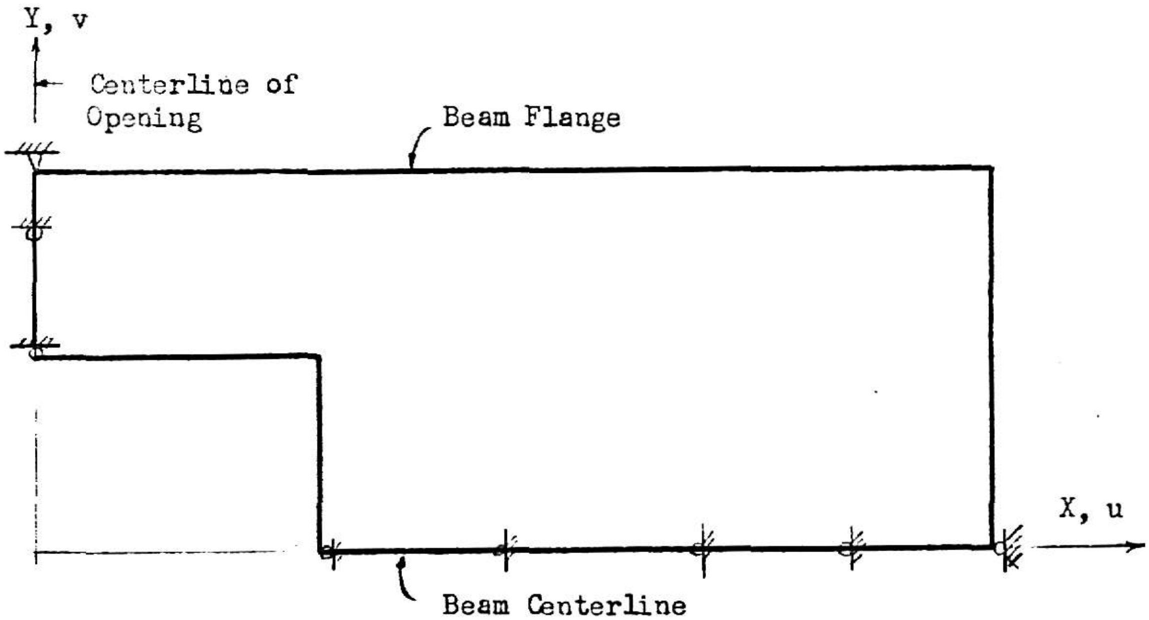
$$M_2 = 1000x_1 + 1000x_2$$

$$V = 200/3\ x_2$$

M/V (in)	x_1		x_2	
	2p=12k	2p=24k	2p=12k	2p=24k
80	0.48	0.96	0.09	0.18
60	0.36	0.72	0.09	0.18
40	0.24	0.48	0.09	0.18
20	0.12	0.24	0.09	0.18



a. Pure Bending



b. End Snear with Bending

Fig. 6 Boundary Conditions & Constraints

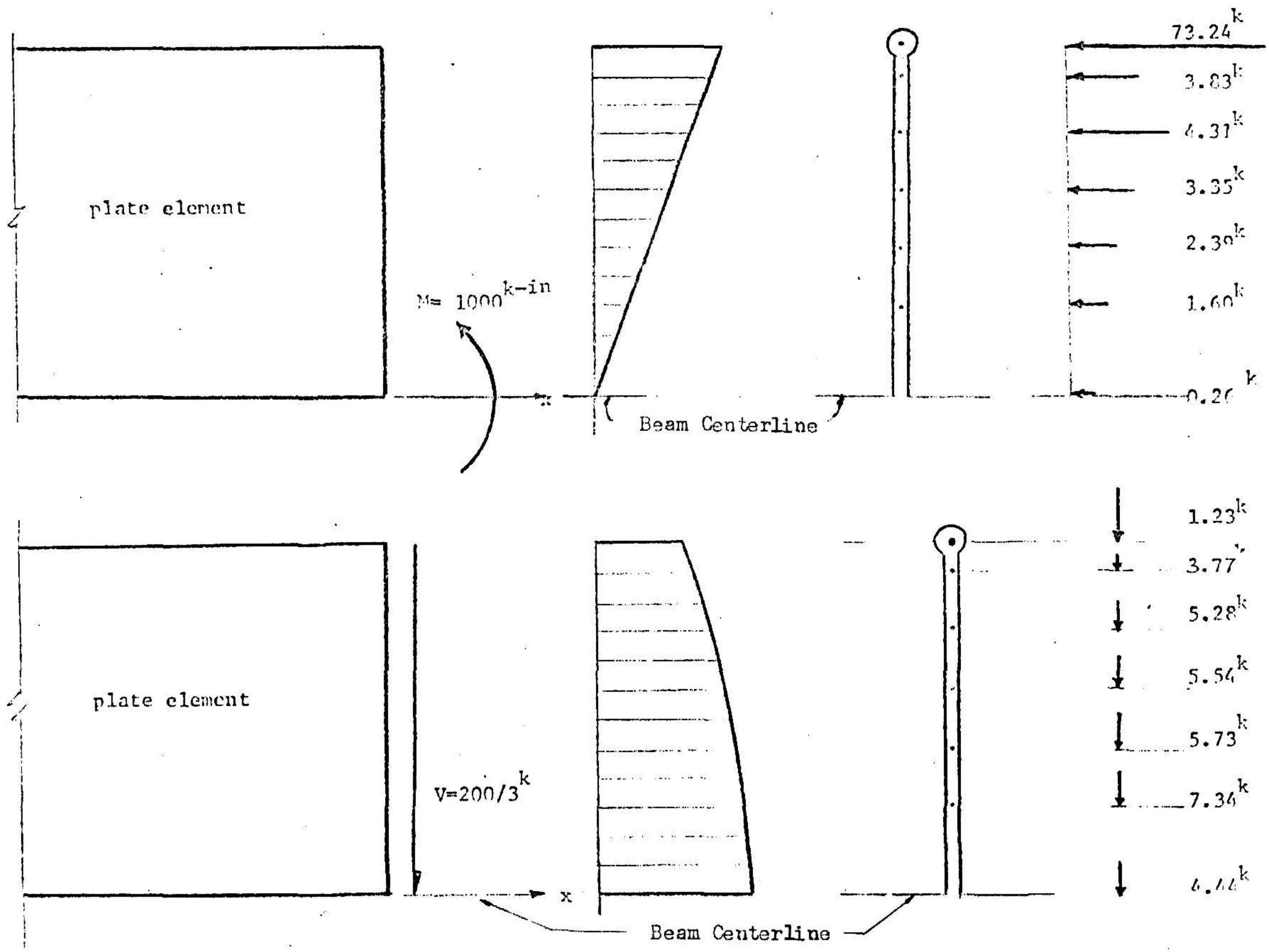


Fig.7 Surface Force Distribution & Equivalent Nodal Forces

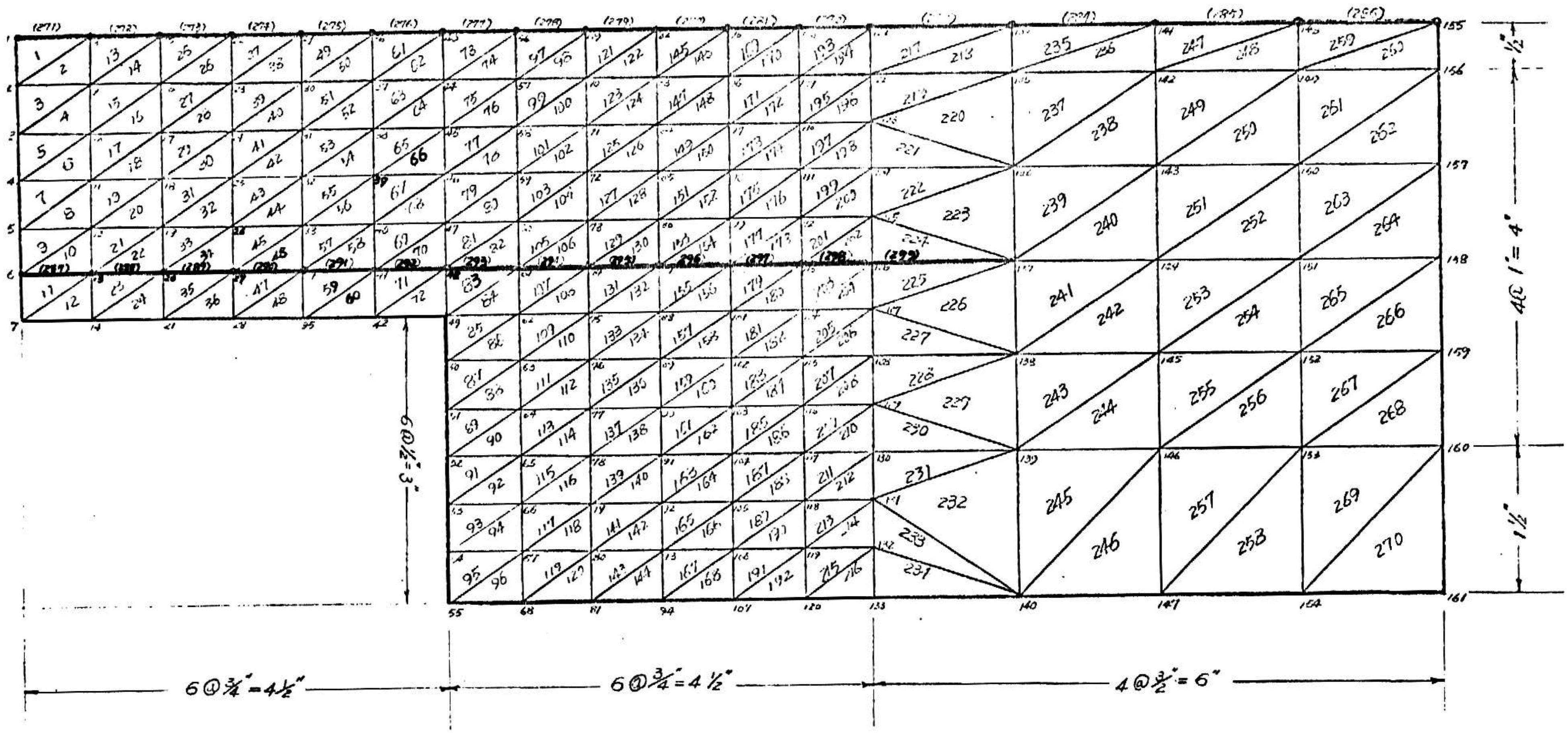


Fig. 8 Finite Element Discretization

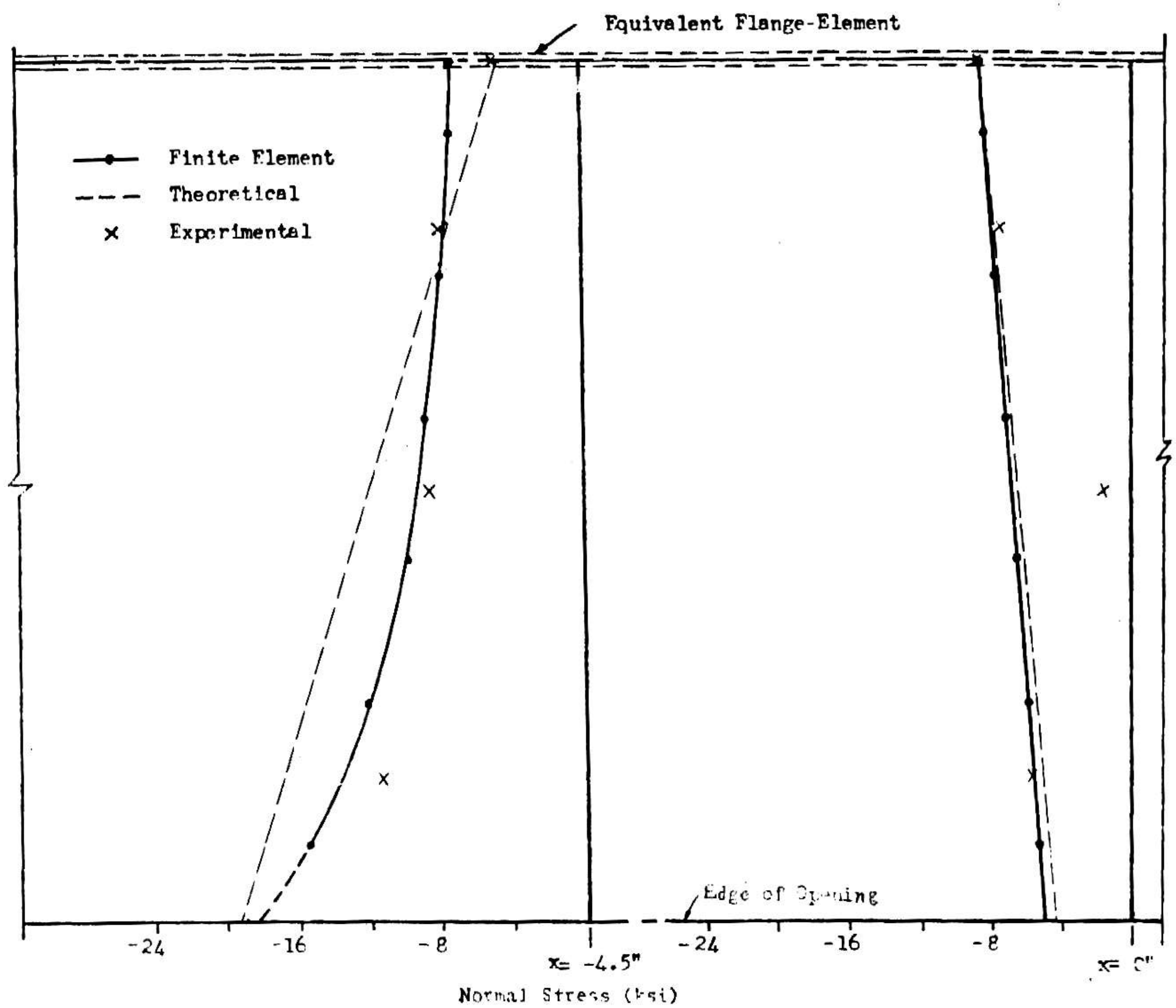


Fig. 9 Normal Stress (Unreinforced Opening) M/V = 80"

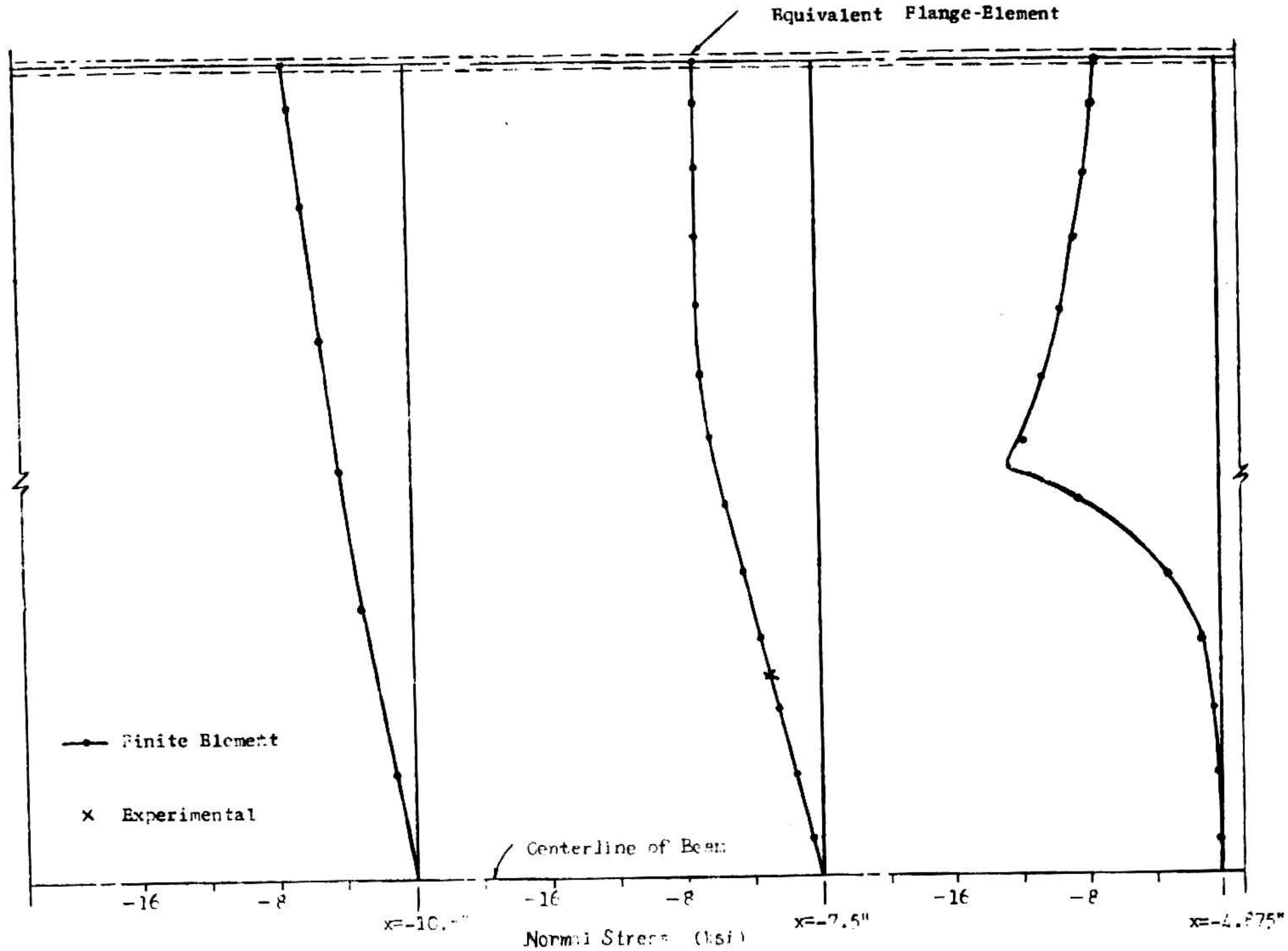


Fig. 10 Normal Stress (Unreinforced Opening) M/V = 80"

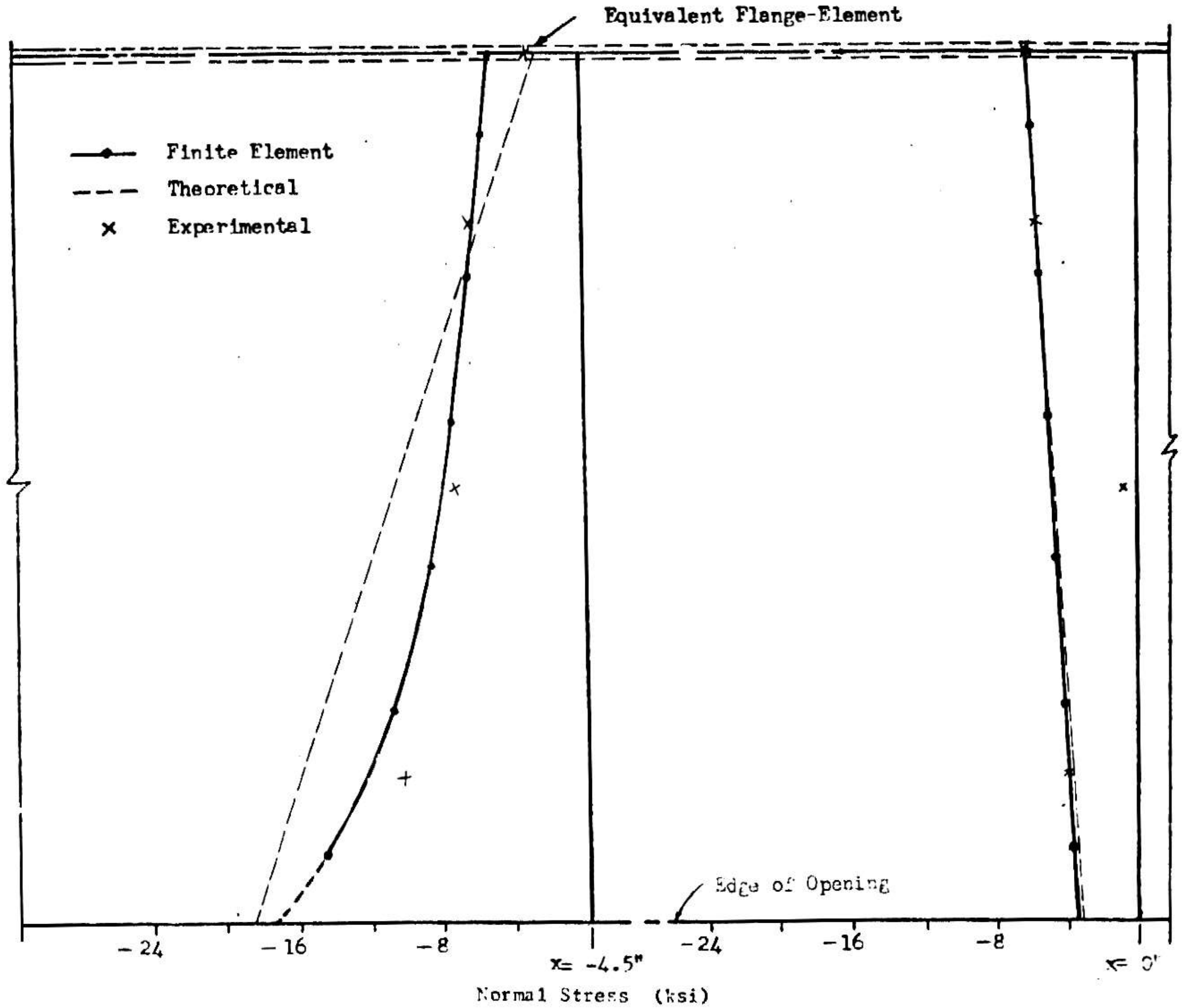


Fig. 11 Normal Stress (Unreinforced Opening) M/V = 60"

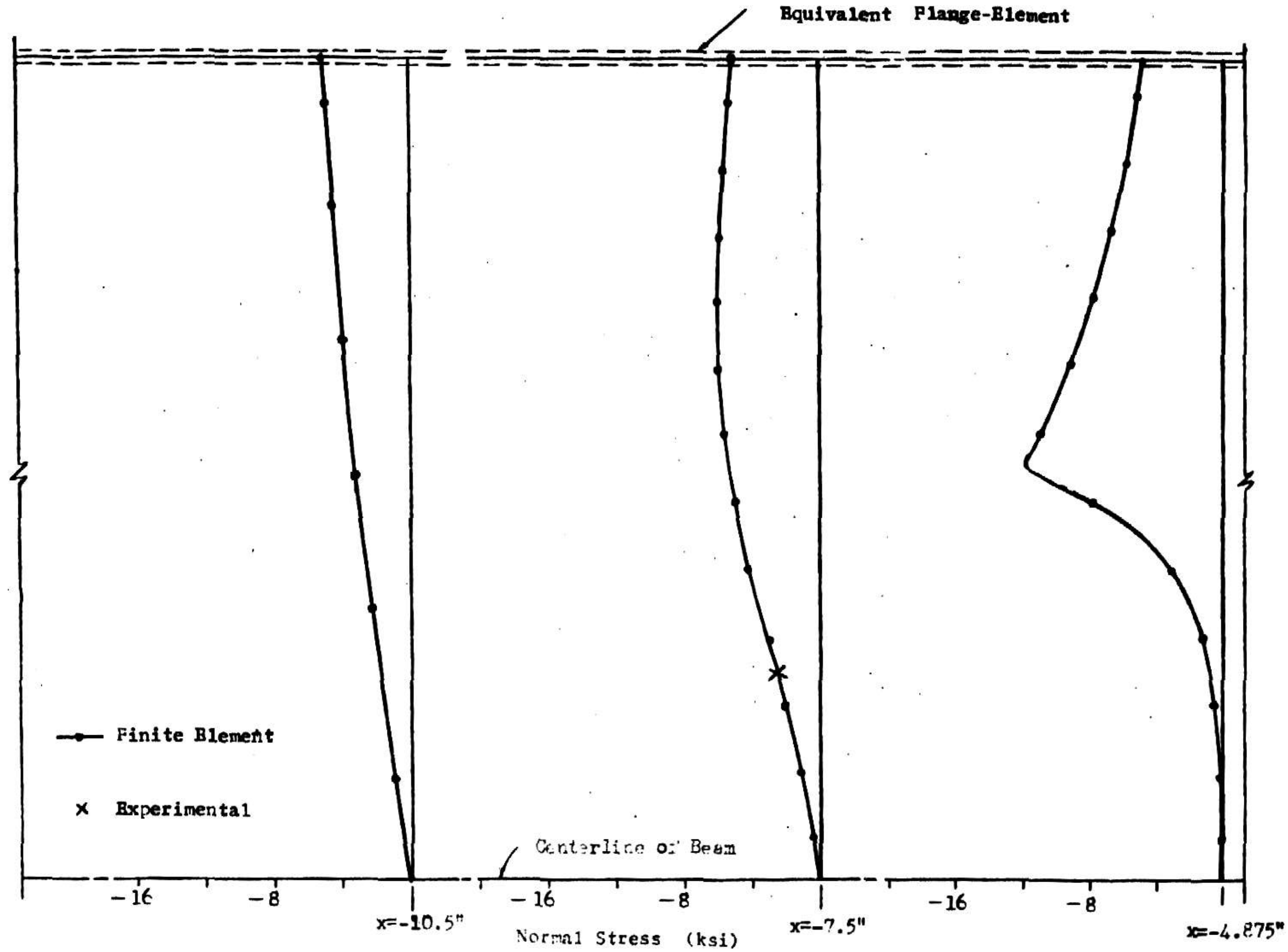


Fig. 12 Normal Stress (Unreinforced Opening) M/V = 60"

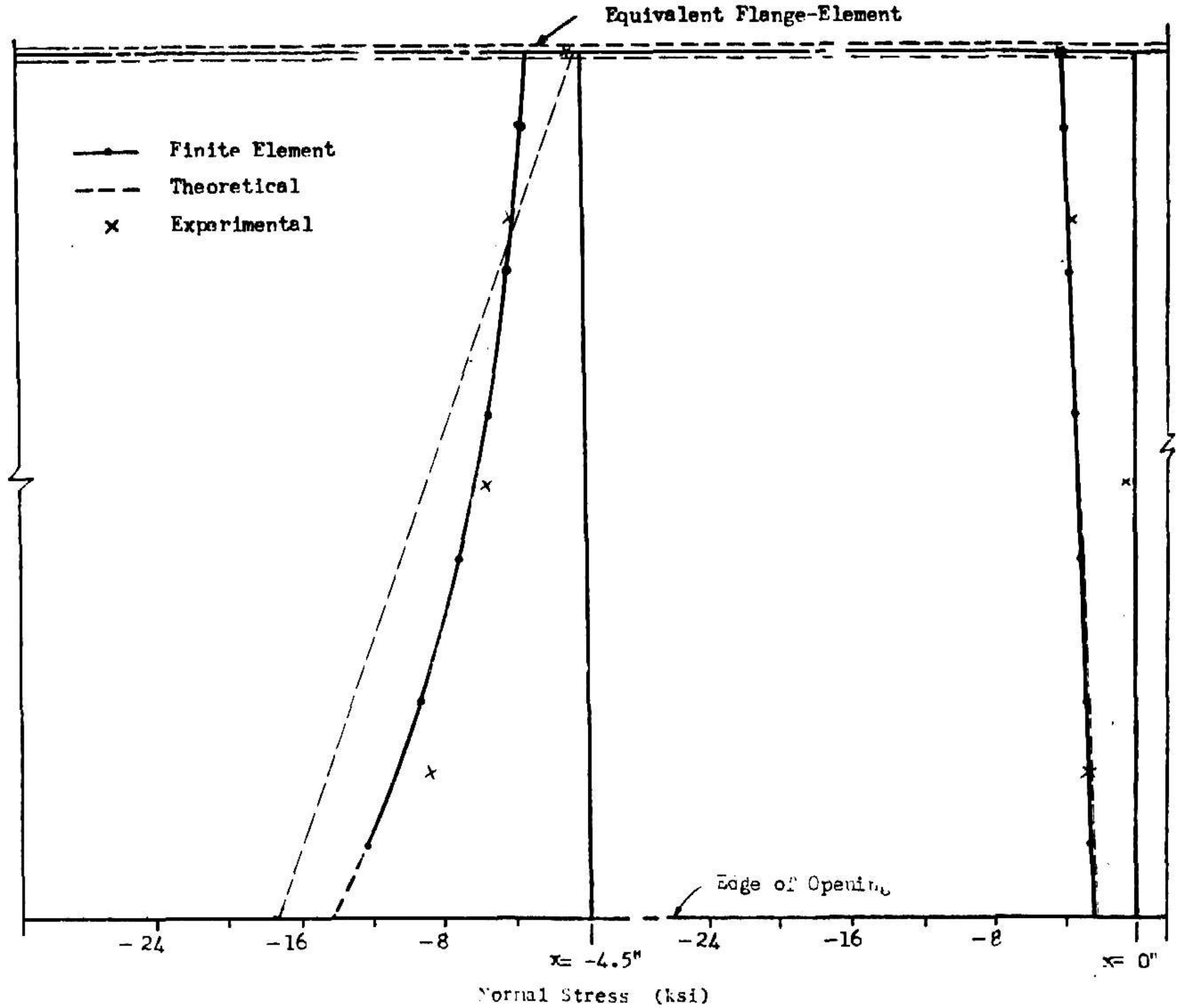


Fig. 13 Normal Stress (Unreinforced Opening) M/V = 40"

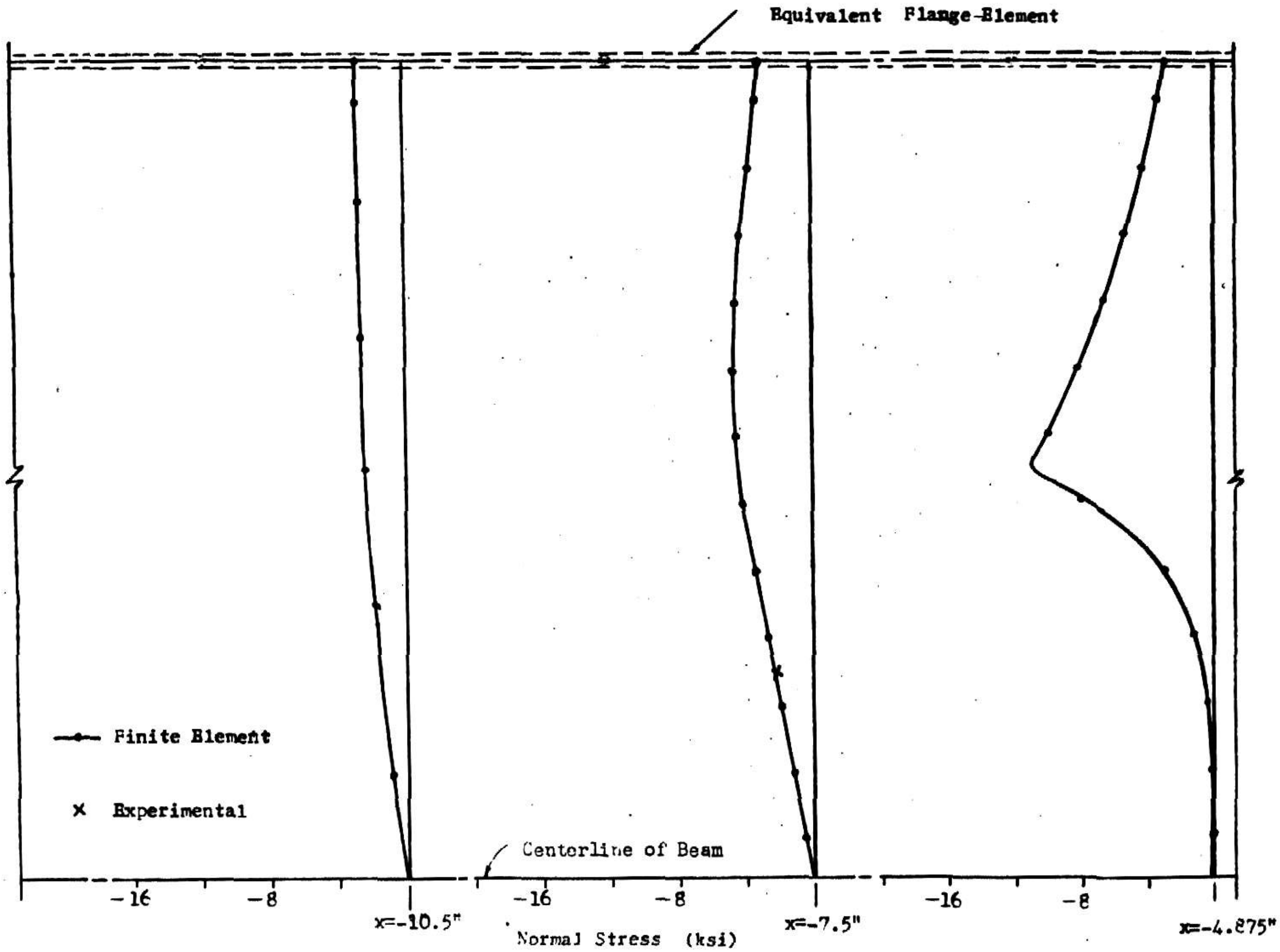


Fig. 14 Normal Stress (Unreinforced Opening) M/V = 40"

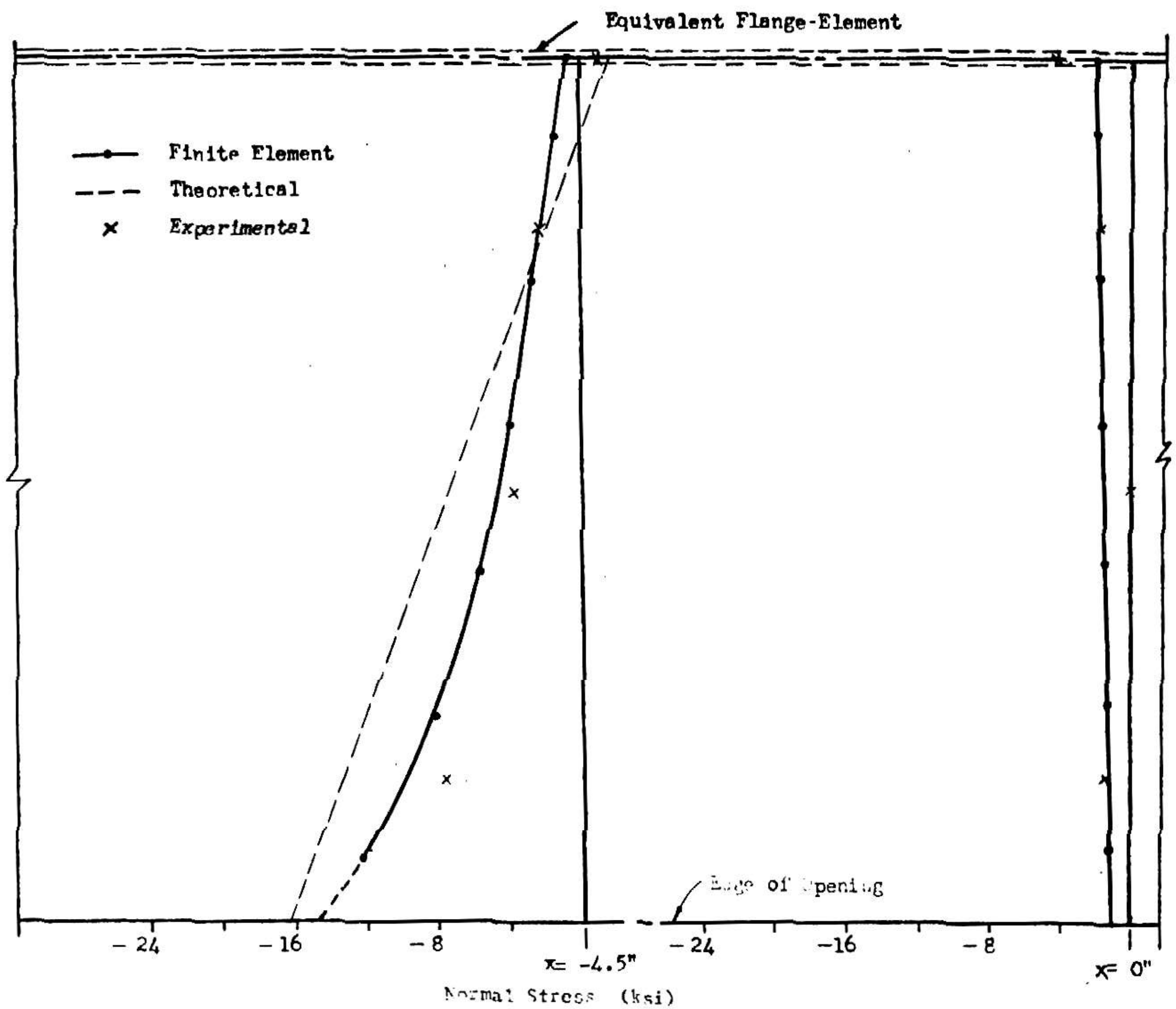


Fig. 15 Normal Stress (Unreinforced Opening) M/V =20"

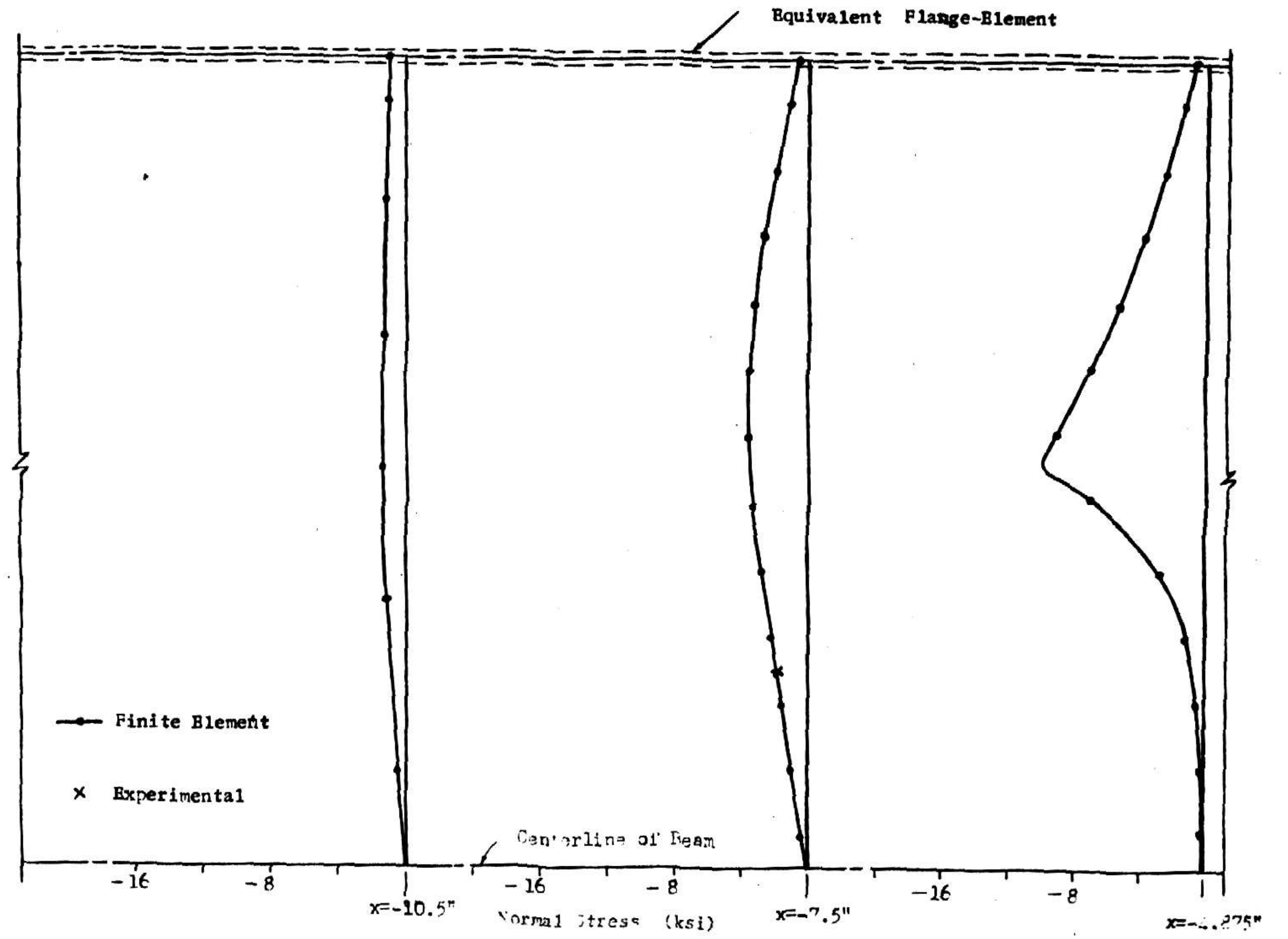


Fig. 16 Normal Stress (Unreinforced Opening) M/V = 20"

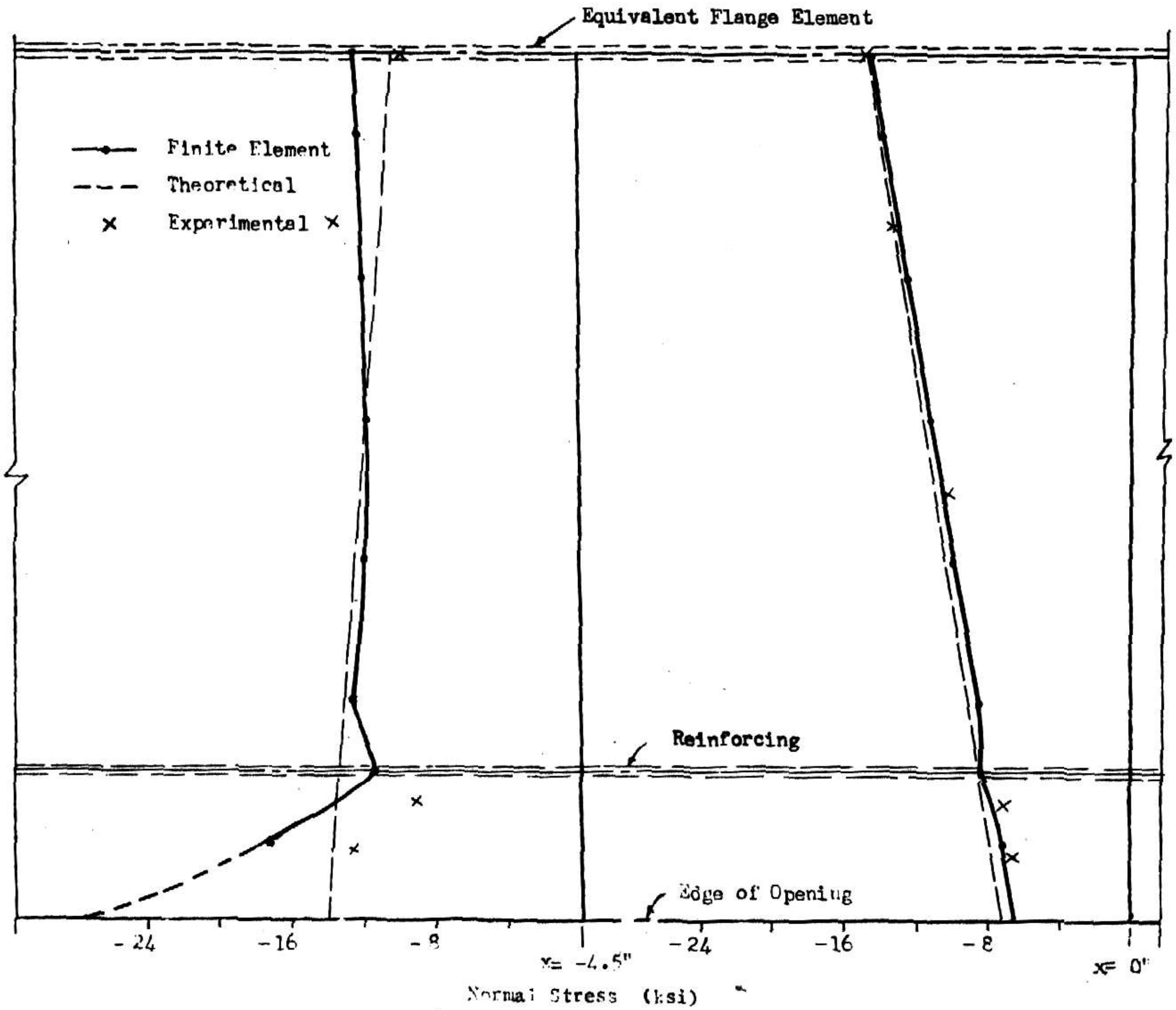


Fig. 17 Normal Stress (Reinforced Opening) M/V = 80"

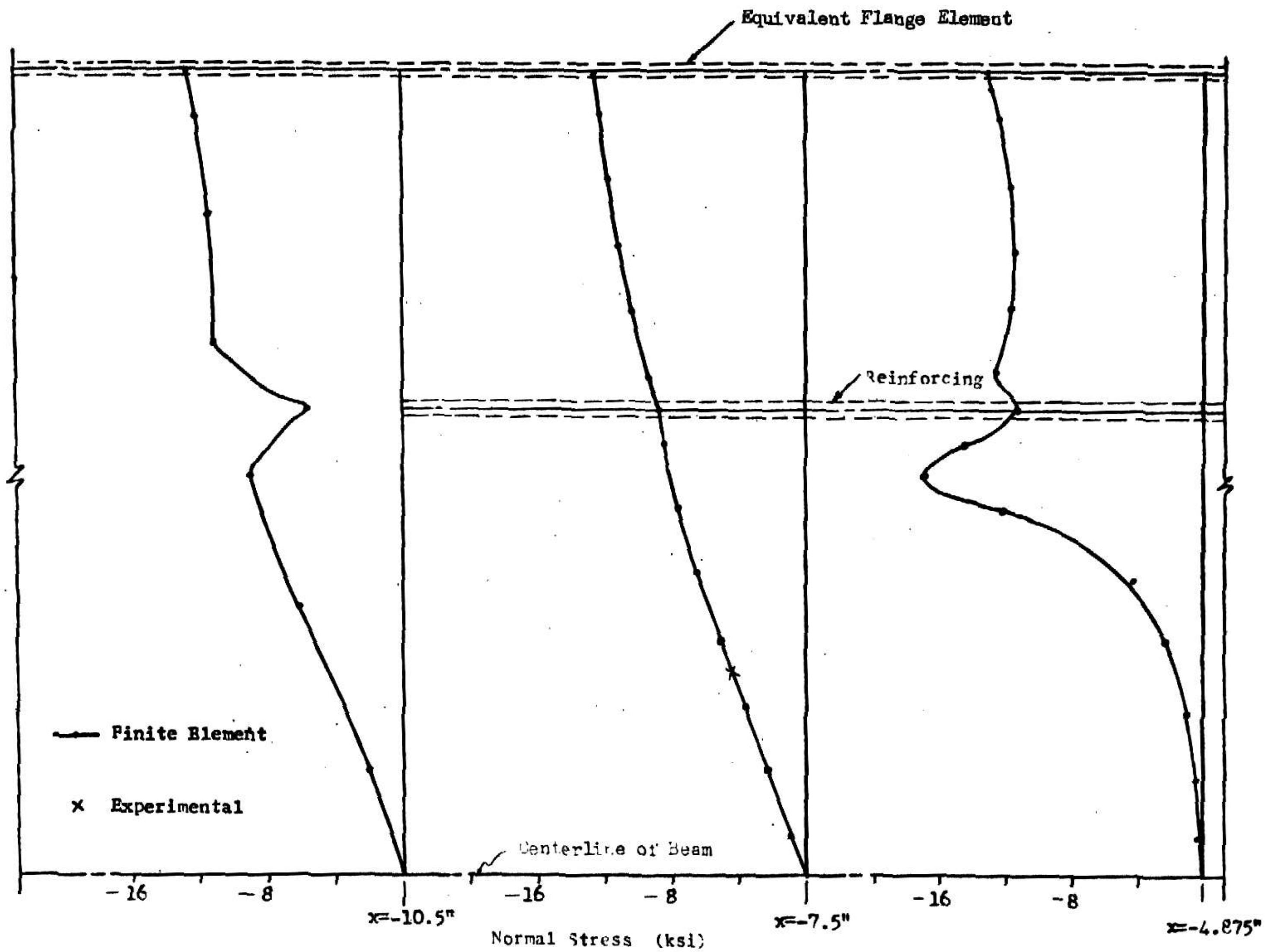


Fig. 18 Normal Stress (Reinforced Opening) M/V = 80"

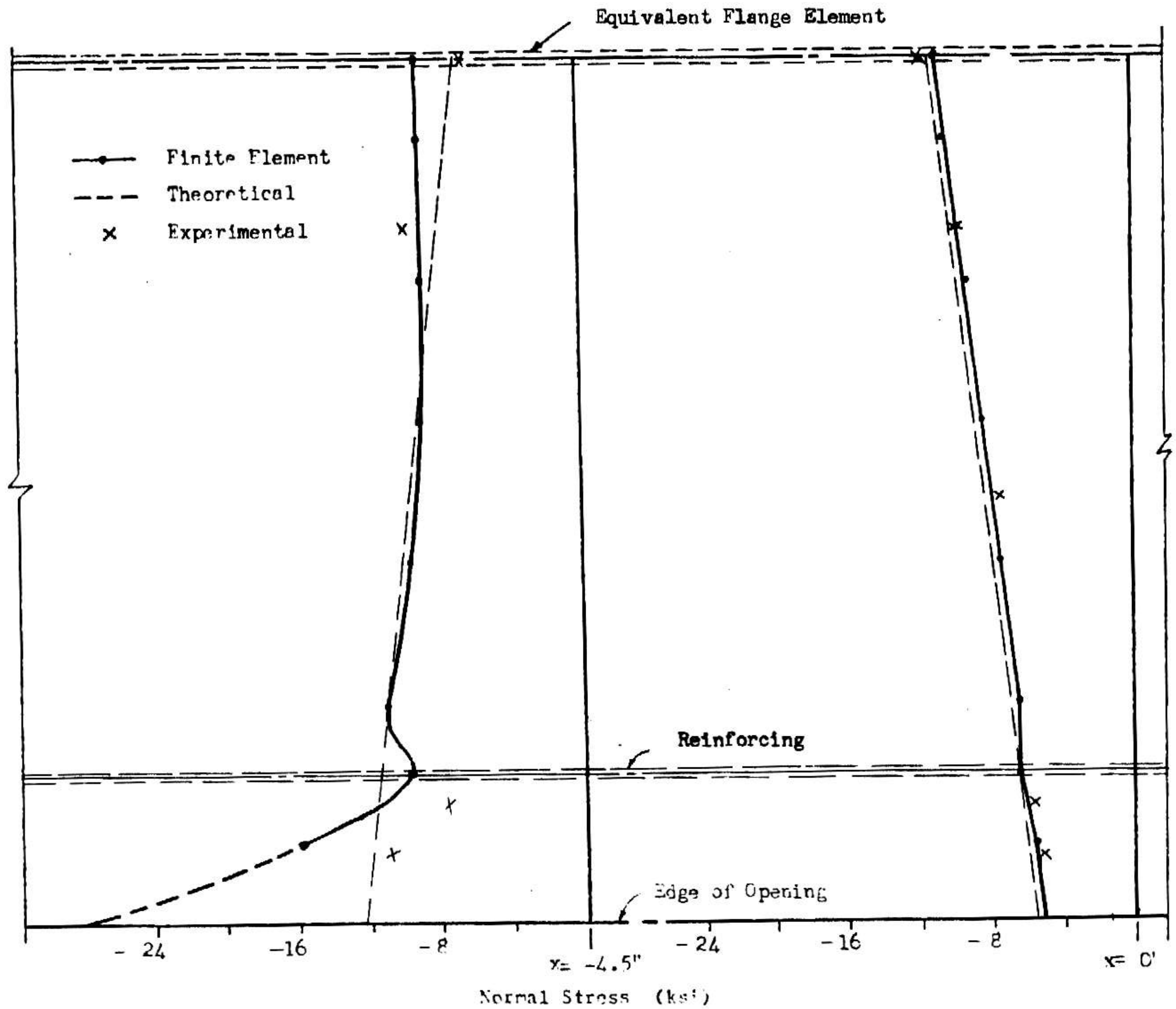


Fig. 19 Normal Stress (Reinforced Opening) M/V = 60"

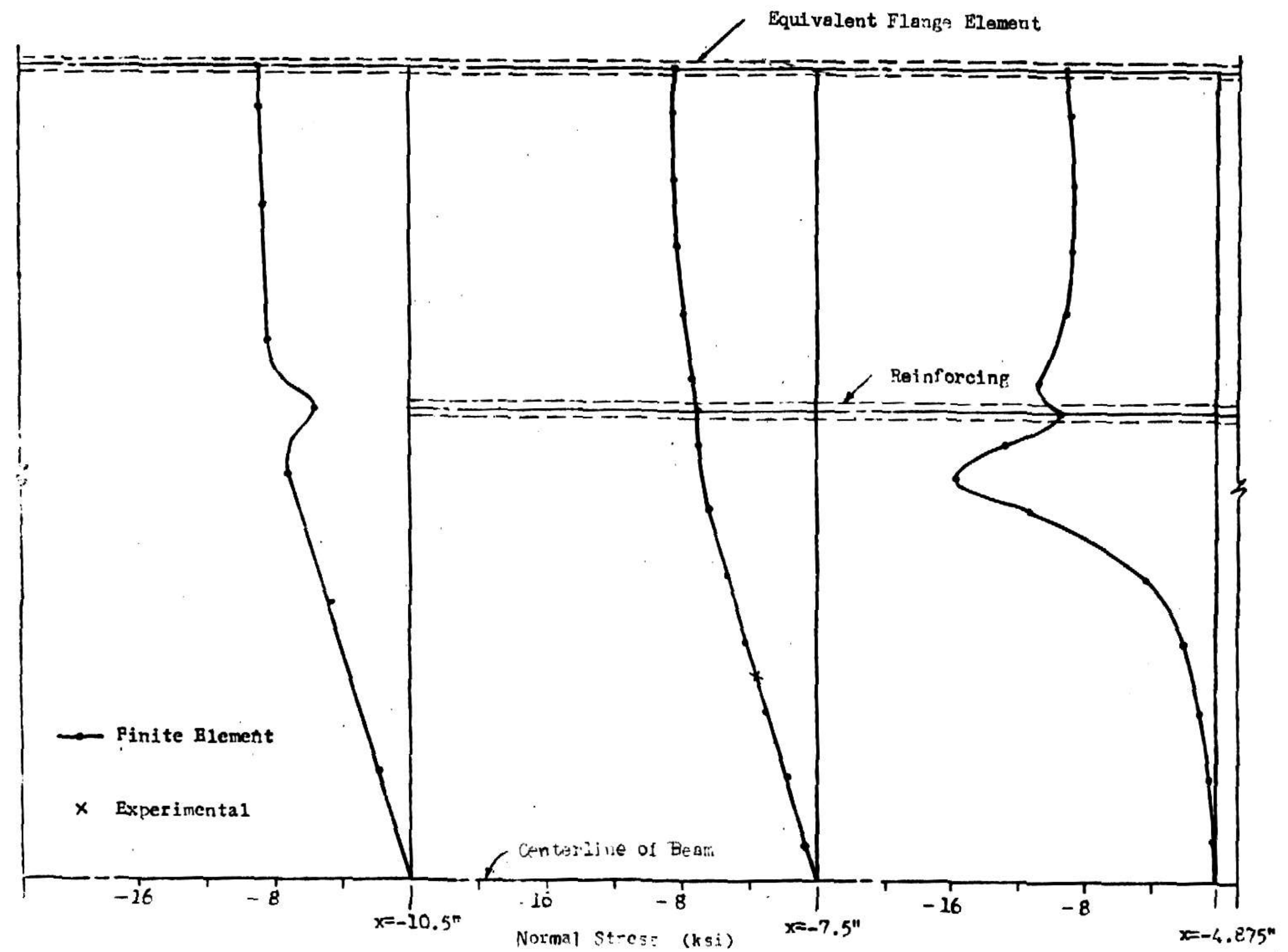


Fig. 20 Normal Stress (Reinforced Opening) M/V = 60"

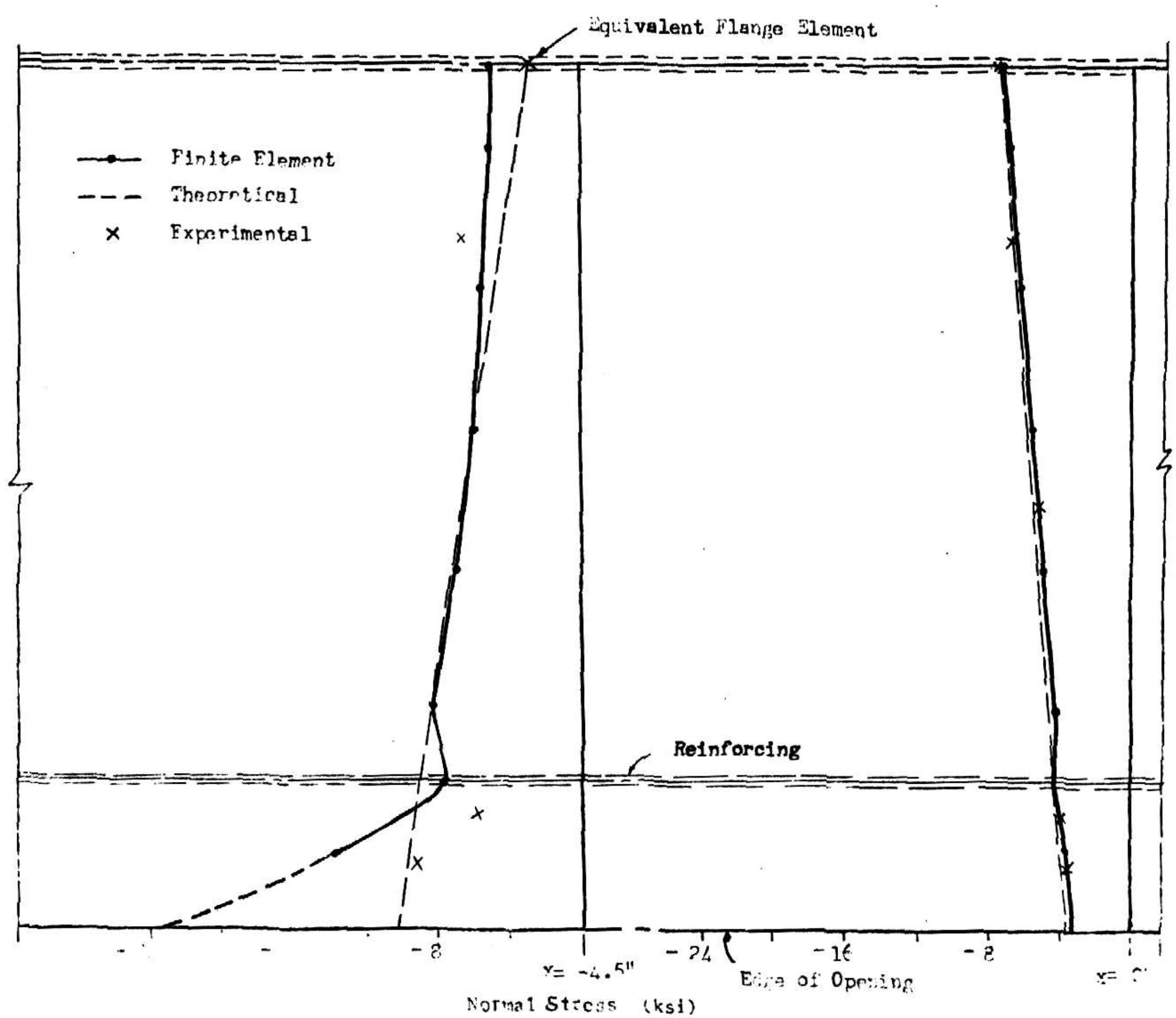


Fig. 21 Normal Stress (Reinforced Opening) M/V = 40"

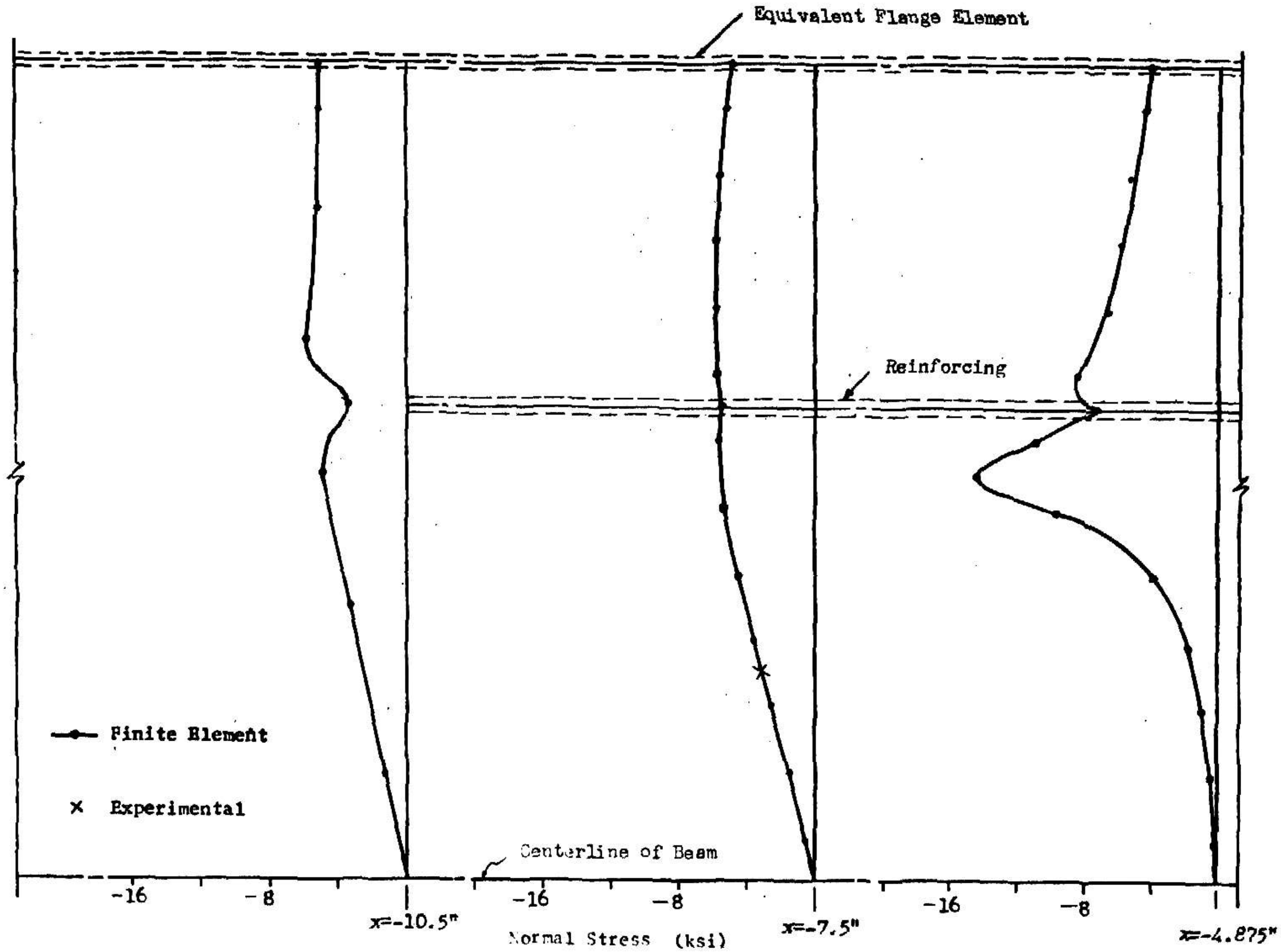


Fig. 22 Normal Stress (Reinforced Opening) M/V = 40"

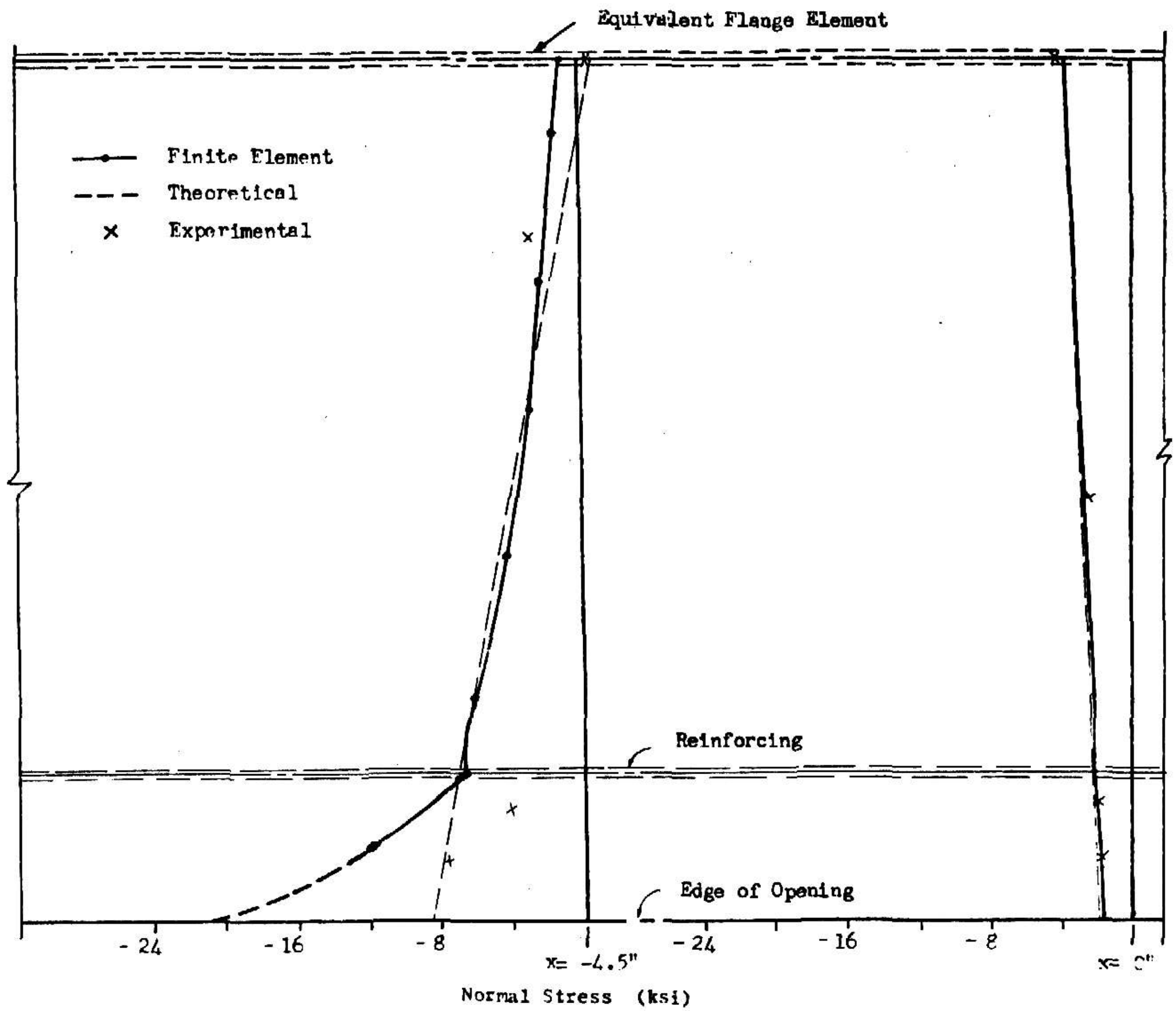


Fig. 23 Normal Stress (Reinforced Opening) M/V = 20"

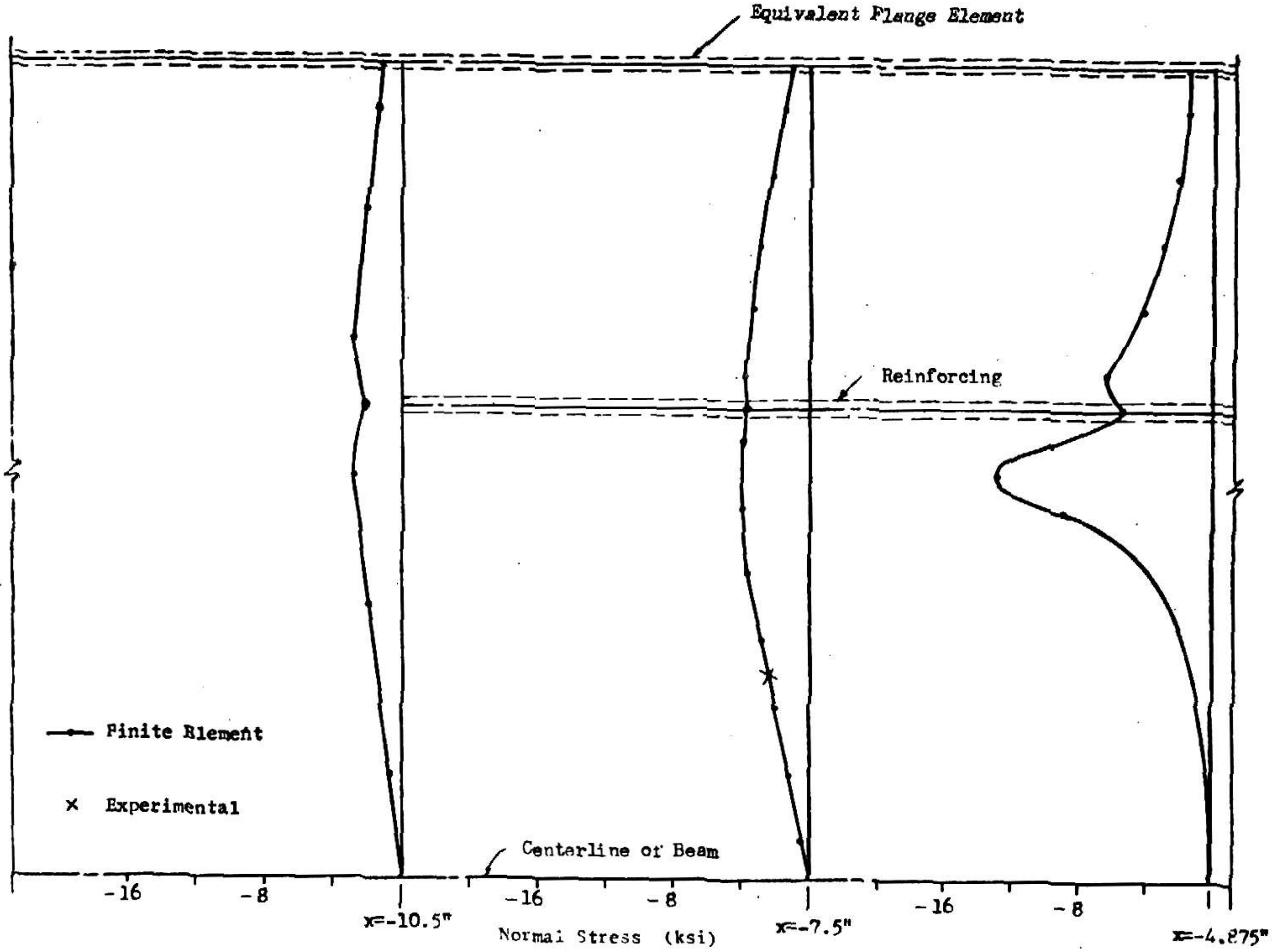


Fig. 24 Normal Stress (Reinforced Opening) M/V = 20"

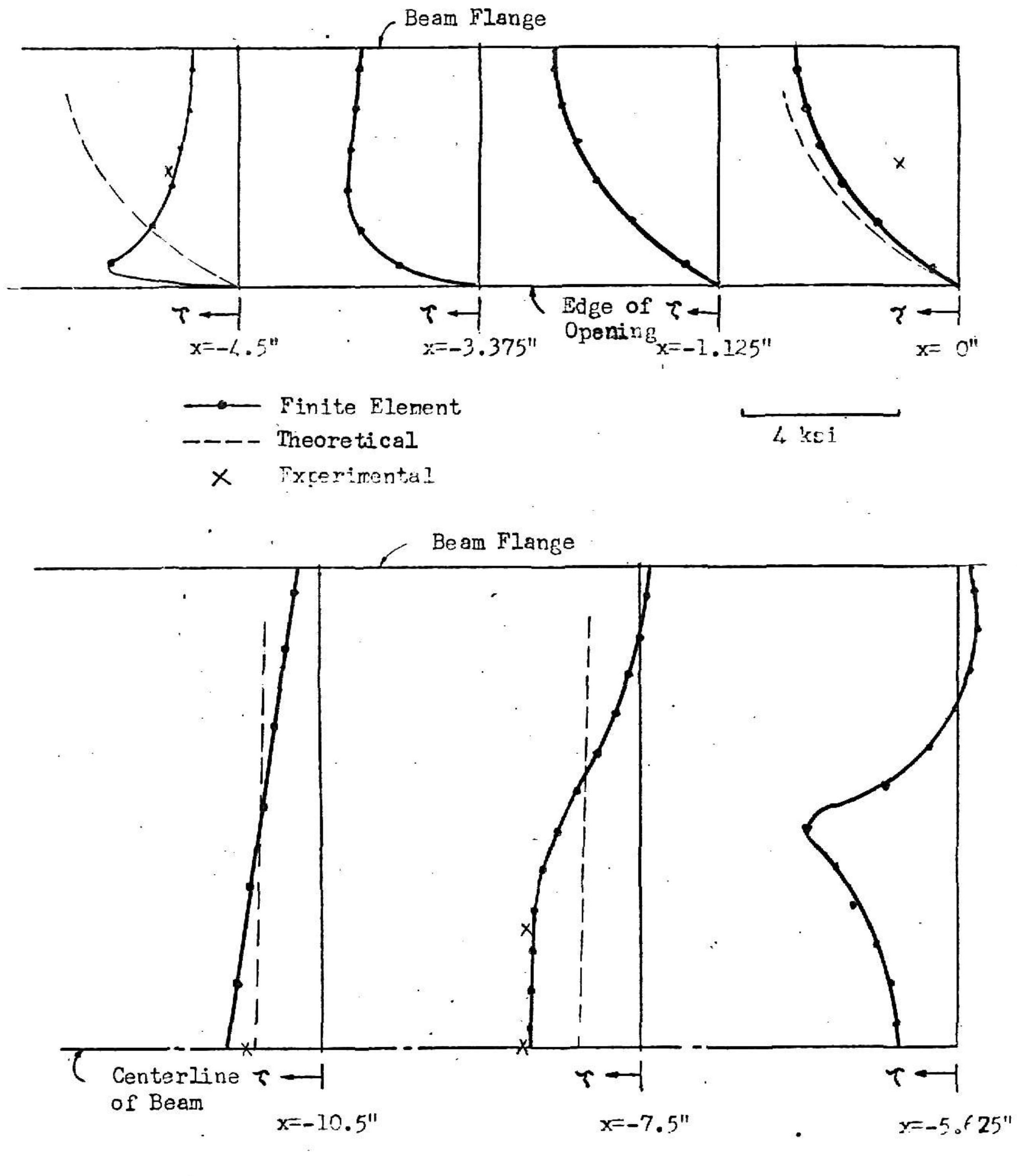


Fig. 25 Shear Stress (Unreinforced Opening)

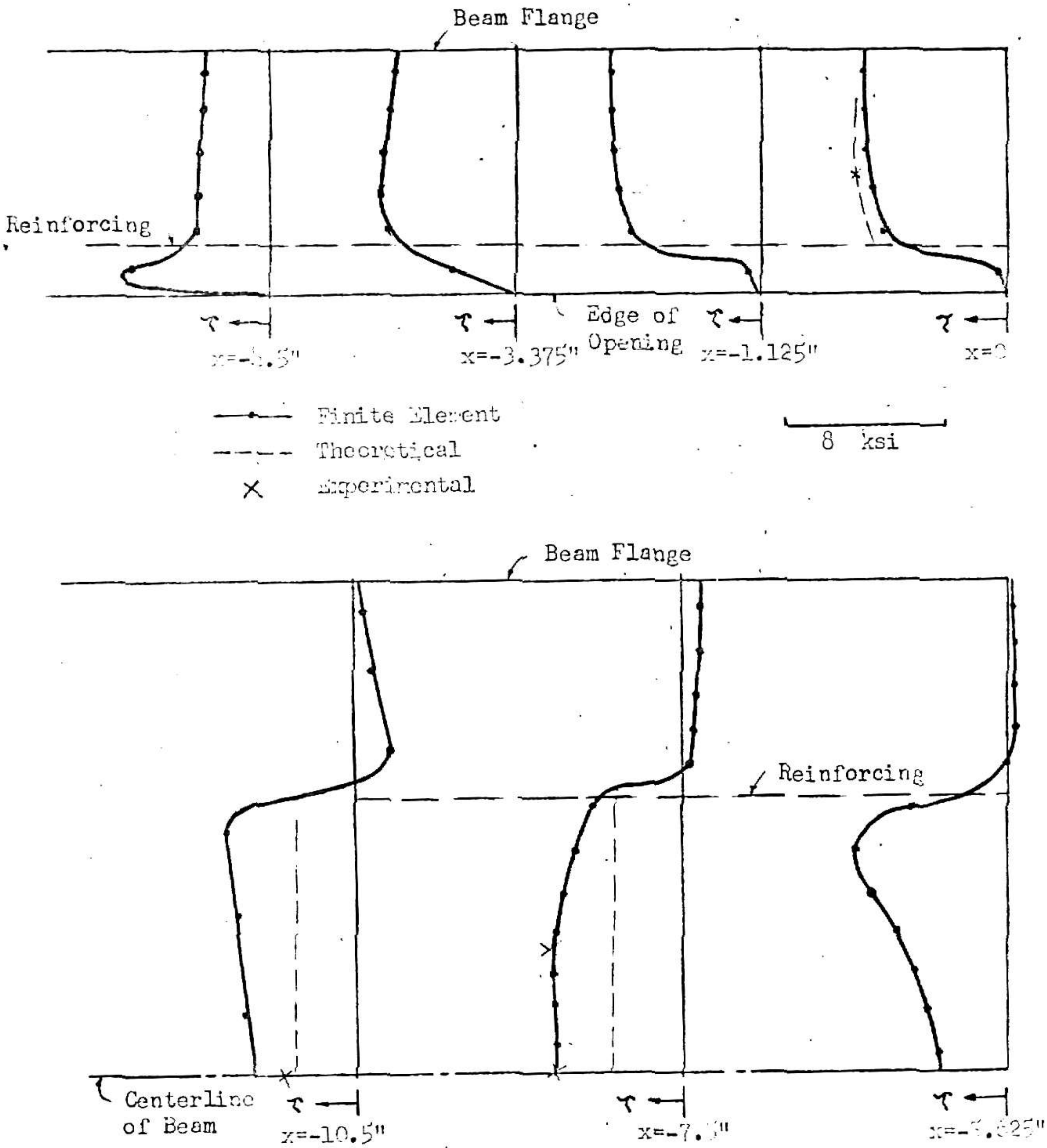


Fig. 26 Shear Stress (Reinforced Opening)

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ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation and gratitude to Dr. Robert R. Snell, professor of Civil Engineering at Kansas State University, for his invaluable advice and assistance during the writing of this report.

Appreciation is also due to Dr. Jack B. Blackburn, Head of the Civil Engineering Department, Dr. Harry D, Knostman, Dr. Stanley J. Clark, and Dr. John M. Marr for serving on the advisory committee.

PLANE STRESS FINITE ELEMENT ANALYSIS OF BEAM WITH WEB OPENING

by

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DIPLOMA, TAIPEI INSTITUTE OF TECHNOLOGY, 1965
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AN ABSTRACT OF A MASTER'S REPORT

Submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

KANSAS STATE UNIVERSITY
Manhattan, Kansas

ABSTRACT

This report presents a study of the stress distribution in a W shape beam with a rectangular web opening centered at the middepth of the beam.

The method of analysis used in this study was the finite element method. Results for the numerical examples were obtained using the ICES-STRUDLE computer program.

The results calculated by the finite element method were compared with those obtained experimentally and by the Vierendeel method.

It was concluded that the finite element method gave results reasonably close to both experimental and theoretical results. And the finite element method was very convenient to apply, from an automation point of view.