THE DESIGN PROCEDURES FOR PRESTRESSED REINFORCED CONCRETE

by

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By DEVENDRA M. DHARIA[1]

SYNOPSIS

Prestressed concrete design procedures are relatively new tools, with which designers will give more attention to the aspect of practical usage. The intent of this report is to show the proper procedures which an engineer must follow and the precautions which he must exercise in any reinforced concrete design so that the design can be done effectively.

"Prestressing" means the creation of stresses in a structure before it is loaded. These stresses are artificially imparted so as to counteract those occurring in the structure under loading. Thus, in a reinforced concrete beam, a counterbending is produced by the application of eccentric compression forces acting at the ends of the beam.

Prestressing systems are classified into two main groups, pre-tensioning and post-tensioning. The term pre-tensioning is used to describe any method of prestressing in which the tendons are tensioned before the concrete is placed. In contrast to pre-tensioning, post-tensioning is a method of prestressing

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in which the tendon is tensioned after the concrete has hardened.

The three main parts of the report are: (1) Analysis of sections for flexure, (2) Design of sections, and (3) Design of prestressed concrete bridge girder.

Numerous examples are solved to show the procedures.

INTRODUCTION

Prestressed concrete has become an important new material in structural and civil engineering and offers many advantages over ordinary reinforced concrete. With conventional reinforced concrete, the development of fine cracks cannot be avoided altogether, and they are not generally of serious consequence; but further development of ordinary reinforced concrete is impeded, since the permissible stresses for these materials are limited to such low values that their incorporation in the design would not be justified. With prestressed concrete, on the other hand, absence of fine hair cracks as a permanent condition can be assured up to maximum load.

When prestressed concrete was first conceived, essentially by Eugene Freyssinet of France, it was visualized as the transformation of a brittle material into an elastic material. Concrete which is weak in tension and strong in compression was precompressed by steel under high tension so that the brittle concrete would be able to withstand tensile stresses. Thus prestressed concrete was dealt with in terms of internal stresses, with "no-tension" being the general criterion for design and construction. This approach might properly be termed the "stress concept".

As prestressed concrete became widely produced and adopted, a second concept was formulated, commonly known as the ultimate strength theory. Under that concept, prestressed concrete is treated as a combination of high strength steel, with concrete to carry the compression and steel to carry the tension. Design formulas and code requirements were proposed as a result of that "strength-concept", and they were basically similar to those of conventionally reinforced concrete.

More recently, i.e., in Dec. 1961, a third concept has been developed. In the overall design of a prestressed concrete structure, prestressing is primarily inteded to balance a portion of the gravity loads so that flexural members, such as slabs, beams, and girders, will not be subjected to flexural stresses under a given loading condition. This transforms a member under bending into a member under axial stress and thus greatly simplifies the design and analysis of otherwise complicated structures. This is termed as the "balanced-concept".

LIFE-HISTORY OF PRESTRESSED MEMBER UNDER FLEXURE

The life-history of a prestressed member under flexure is briefly shown in Fig. 1. There are several critical points as follows:

(1) The point of no-deflection which usually indicates a rectangular stress block across all sections of a beam.

(2) The point of no-tention which indicates a triangular stress block with zero stress at either the top or the bottom fiber.

(3) The point of cracking which generally occurs when the extreme fiber is stressed to the modulus of rupture.

(4) The point of yielding at which the steel is stressed beyond its yield point so that complete recovery will not be obtained.

(5) The ultimate load which represents the maximum load carried by the member.

If the load-deflection or the moment-curvature relationship is of a definite shape, it is then possible to determine all five of the above points whenever one point is known. Actually, on account of the difference in the shape of the section, the amount and location of prestressed and nonprestressed steel, as well as different stress-strain relationships of both the concrete and the steel, these load-deflection or moment-curvature relationships may possess widely divergent forms. Thus, it is often necessary to determine more than one of the above five points in order to be sure that the beam will behave properly under various loading conditions.

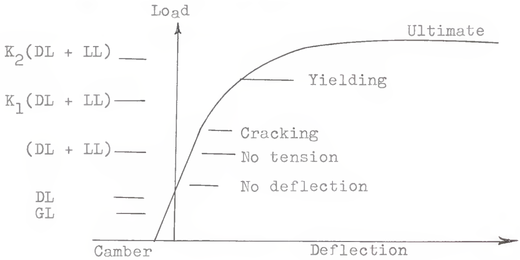


FIG. 1. LIFE HISTORY OF A PRESTRESSED MEMBER UNDER FLEXURE.

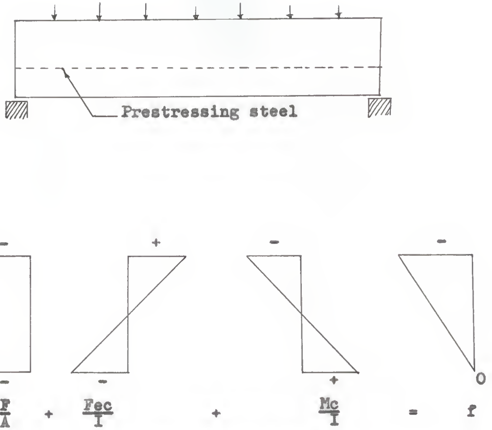
In Fig. 1, GL represents girder load, DL represents dead load, and LL represents live load.

Under the elastic stress concept, the point of no-tention or the point with a limited amount of tension is taken as the important criterion. Under this concept, Fig. 2, concrete is treated as an elastic material and a family of formulas is derived taking the shape of $F = Mc/I$. These formulas yield stresses in the beam under various loading conditions with special emphasis on the design live load.

Under the ultimate strength concept, a second family of formulas, Fig. 3, is developed taking the shape of $M = A_s f_s jd$ which is the familiar formula for reinforced concrete design. These formulas have also been extended to include elastic design by expressing the internal resisting moment as a T - C couple.

The concept of load balancing deals essentially with the first critical point in Fig. 1, i.e., the point of no-deflection. Based on this concept, the downward gravity load is balanced by the upward component of the prestressed steel. That upward component can be a concentrated force for a sharp bend, Fig. 4, or can be a distributed load for a curved cable, Fig. 5. A third family of formulas can be derived for the computation of these components. Thus, for a cable with a parabolic curve, we would have a uniform upward load $W = 8Fh/L^2$(1) where F is the prestressed force, h is the cable sag and L is the span length.

If the point of no-deflection, the point of no-tension,



Stress computations

FIG. 2. CONCEPT 1 - PRESTRESSED CONCRETE AS AN ELASTIC MATERIAL.

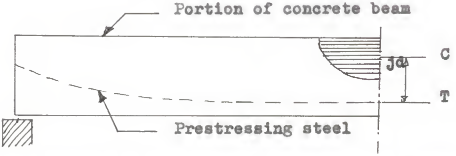


FIG. 3. CONCEPT 2 - PRESTRESSED CONCRETE AS A COMBINATION OF STEEL AND CONCRETE

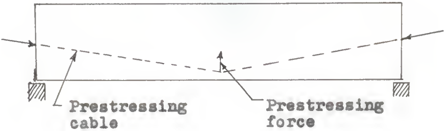


FIG. 4. BEAM WITH BENT CABLE.

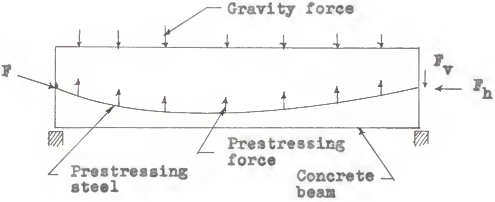


FIG. 5. BEAM WITH PARABOLIC CABLE.

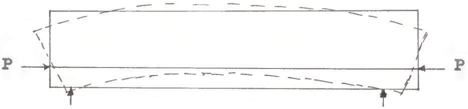


FIG. 6. COUNTER BENDING PRODUCED IN A BEAM IN WHICH ECCENTRIC COMPRESSION FORCES ARE APPLIED.

and the ultimate load are all determined for a beam, its life history is generally pretty well described. Since the behavior of many elements under their sustained load is often most significant, it becomes apparent that the criterion of no-deflection could often be the best controlling point.

The act of prestressing may be described as the introduction of stresses opposite in sense to those that the structural member will be expected to carry during its use. The most common way of introducing these stresses is with steel that has been stressed to some predetermined value and then restrained in the member from regaining its unstressed position. The restraint is accomplished either by bond, as in pretensioning, or by end-bearing devices as in post-tensioning.

The idea of prestressing has been employed in a great number of human activities. In making a wooden wheel, when the workman places the external steel hoop around it, properly heated, so as to put the wheel under compression after the temperature has returned to normal, that is prestressing.

The shortage of steel during the war resulted in Swiss engineers giving considerable attention to prestressed concrete. How long it will take for prestressed concrete to replace ordinary reinforced concrete completely is impossible to foretell, although this change would mean the creation of a product having improved properties and would lead to a saving of material, especially of steel.

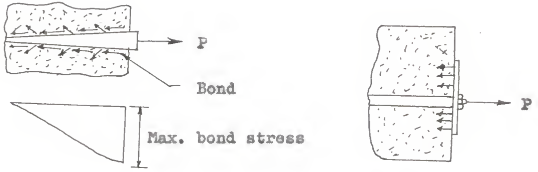
There are two basic methods of prestressing concrete, pretensioning and post-tensioning.

In pretensioning, long lengths of high-strength stress-relieved strands are tensioned before the concrete is poured in continuous forms. After the concrete has cured, the load is removed from the end abutments and transferred by bond resistance to the concrete member. The high-strength, stress-relieved materials are then cut off at the ends of each member, or in some cases the concrete section may be sawed into the required lengths by an abrasive cut-off wheel. Pretensioning is particularly adaptable to centrally located yards and widely used for mass production of smaller members.

One of the more recent developments in this country has been the use of pretensioned tendons which do not pass straight through the concrete member but which are deflected into a trajectory which approximates a parabolic curve.

In contrast to pretensioning, a member is said to be post-tensioned when it is fabricated in such a manner that the tendons are stressed and each end is anchored to the concrete section after the concrete has been cast and has attained sufficient strength to withstand safely the prestressing force. In this country, when using post-tensioning, the most common method used in preventing the tendon from bonding to the concrete during placing and curing of the concrete is to encase the tendon in a mortar-tight, flexible metal hose before placing it in the forms. The metal hose remains in the structure, and after the tendon is stressed, the void between the tendon and hose is filled with grout. This way the tendon becomes bonded to the concrete section and corrosion of the steel is prevented.

With pretensioning transmission of the force from wire to concrete is effected by bond resistance and friction together with a radial compression as indicated in Fig. 7a. Post-tensioning is carried out against the hardened concrete and compression is transmitted by such means as bearing or distribution plate, as indicated in Fig. 7b.



(a) Pretensioning - by bond and friction

(b) Post-tensioning - by bearing plate

FIG. 7. TRANSMISSION OF FORCE FROM STEEL TO CONCRETE.

PRESTRESSED VS. REINFORCED CONCRETE

Disadvantages of Reinforced Concrete

Concrete of good quality has relatively high compressive strength but is low in tensile strength. Therefore, steel reinforcing is so placed in reinforced concrete members as to take care of the tensile stresses which develop when a beam or slab deflects under load and the tensile strength of the concrete is exceeded. Generally speaking, about the lower two-thirds of the depth of a concrete beam (specifically, the portion below the neutral axis) is almost useless in resisting the tensile stresses resulting from the bending moments developed under load. When a reinforced concrete beam bends under load, cracks develop in the concrete, decreasing in size from the bottom to the neutral axis of the member. In areas where humidity is high, corrosion of the steel is certain to occur. Costs of labor and material are high.

Advantages of Prestressed Concrete

Prestressing combines and enhances the characteristics of the compression strength of concrete with the high tensile strength of stress-relieved cold drawn steel wire and strand.

Basically, economy is effected for the following reasons:

Steel for prestressing is six times stronger than ordinary steel, but only approximately three times more costly; concrete for prestressing is twice as strong, but only 10 to 20 percent more costly than ordinary concrete; and prestressing consumes less steel and concrete to attain equal or greater structural strength more economically. In the matter of design, economy is effected, as prestressed concrete makes possible thinner sections, lower depth-to-span ratios, longer cantilevering without ballast beams, and reduction in weight. Cracks, otherwise unavoidable in concrete, are eliminated by prestressing. Production of prestressed sections proceeds at top speed, affording the maximum utilization of labor and stockpiling against projected construction. Prestressed concrete eliminates construction delays by by-passing materials in short supply or on extended backlog delivery. Often it is possible to complete structures in half the time required by conventional methods. It is often possible to erect prestressed concrete in the time required to make, place, and shore up forms for poured-in-place concrete.

The durability of concrete is well known. Well known too, is its vulnerability to cracking under tensile forces. Prestressing makes concrete a flexible material, with the ability to withstand extraordinary deflection and recover without cracking.

The economy of maintaining a prestressed concrete structure is one of the principal advantages. Even under corrosive conditions, the cost of maintenance on prestressed concrete construction ranges from nil to the expense involved in painting, in cases where color effect is desired.

Example 1. (a) Plain concrete.

Consider a concrete column of cross-section 2 in. square, which is uniformly strained in tension.

Let tensile strength of concrete $= 600$ lb/in^2

Then the failing load of column $= 2 \times 2 \times 600 = \underline{2400}$ lb

(b) Reinforced concrete.

The above column is now reinforced by a ½ in. diameter bar of high tensile steel, bonded to the concrete.

Let modular ratio $n = 8$

Stress in steel $= n \times$ stress in concrete

When concrete is about to crack:

Stress in steel $= 8 \times 600 = 4800$ lb/in^2

Total load on column $= (A + (n-1)A_s)\, f_c$

$= (2 \times 2 + (8-1).196)\, 600$

$= \underline{3223}$ lb.

(c) Prestressed concrete.

If the bar in the example is initially tensioned and connected to the column either by bond or by special anchorages at the ends, the concrete will be put into compression. Let the stress in steel be 18000 lb/in^2, then initial compressive stress in concrete $= \dfrac{18000 \times .196}{2 \times 2 - .196} = 930$ lb/in^2

Tensile stress which may be imposed before cracking of concrete $= 930 + 600 = 1530$ lb/in^2

Final steel stress $= 18000 + 8 \times 1530 = 30240$ lb/in^2

Total load on column $= (2 \times 2 + (8-1).196) \times 1530 = \underline{8230}$ lb.

If the steel stress increased to 50,000 lb/in^2 then

Initial compressive stress in concrete $= \dfrac{50{,}000 \times .196}{2 \times 2 - .196}$

$= 2584$ lb/in^2

Tensile stress which may be imposed before cracking of concrete $= 2584 + 600 = 3184$ lb/in^2

Final steel stress $= 50{,}000 + 8 \times 3184 = 75{,}500$ lb/in^2

Total load on column $= (2 \times 2 + (8-1)\ .196)\ 3184 = \underline{17{,}100}$ lb.

This is more than five times the load on the equivalent reinforced-concrete column.

PRINCIPLES OF REINFORCED CONCRETE AND PRESTRESSED CONCRETE

Assumptions made in the calculations: (2)

(1) Sections which are plane before bending remains plane after bending. It follows from the assumption that the distribution of strain along a vertical section of a beam is linear and proportional to the distance from a fixed line known as the neutral axis.

(2) The modulii of elasticity of the concrete and steel are constant over the range of working stress.

(3) There is perfect adhesion between the steel and concrete (surrounding). When the steel and concrete are perfectly bonded, the strain in each material, at a given distance from the neutral axis, is the same. From this, the stress in the steel is equal to the corresponding concrete stress, multiplied by the modular ratio.

(4) The stress is uniform across the section of each reinforcing bar.

(5) In reinforced concrete calculations, the assistance afforded by the concrete in tension is neglected; in prestressed concrete calculations up to the working load it is always included.

It must be assumed in prestressed concrete that the stress-strain curve for concrete is identical in compression and tension, that is, that the modulii of elasticity have the same value.

ANALYSIS OF SECTIONS FOR FLEXURE

By analysis is meant the determination of stresses in the steel and the concrete when the form and the size of a section are already given or assumed.

Stress in a prestressed concrete beam results from the application of external loads and the prestressing force. In Fig. 9, the stress in any fiber due to prestressing equals the sum of the uniform stress F/A and a bending stress Fey/I.

Also acting on each fiber are the effects of the dead load moment of the girder and the live load. Summation of stresses due to all contributing loads is shown in Fig. 9. This relationship can be stated algebraically* as (3)

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

The main purpose in prestressing is to make the magnitude of $F/A \pm Fey/I$ such that it will counteract stresses due to My/I, leaving the beam principally in compression.

In order to counteract more closely the external load stresses with the effects of the prestressing force, the eccentricity of the prestressing force is sometimes varied by depress-

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*Negative sign indicates compression; positive indicates tension.

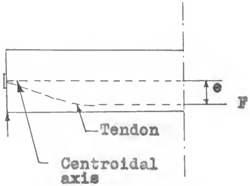


FIG. 8. ECCENTRIC PRESTRESS F.

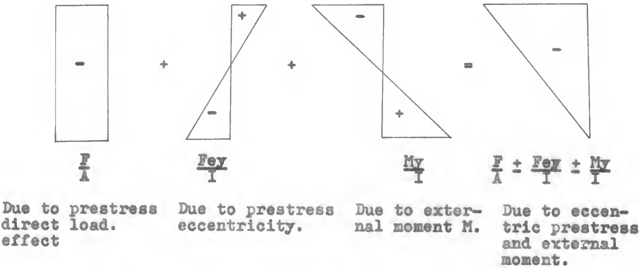


FIG. 9. STRESS DISTRIBUTION ACROSS AN ECCENTRICALLY PRESTRESSED CONCRETE SECTION.

ing the prestressing steel. By this means, extreme fiber stresses resulting from the moment Fe may be minimized in those parts of the beam where flexural stresses due to dead and live loads are small. In a pretensioned member with straight tendons, stresses at both midspan and at the ends of the beam are critical and must be investigated.

Example 2. A simply supported beam having cross section 10" x 12"; L = 10 ft., total load 15 K; F = 120 Kips; e = 2.5 in. Assume no loss of prestressing and neglecting the dead weight of the beam, draw stress diagrams for the beam section taken at the center of the span.

$$I = \frac{bd^3}{12} = \frac{10 \times 12 \times 12 \times 12}{12} = 1440 \text{ in}^4$$

$$M = \frac{15 \times 10}{4} = 37.5 \text{ K-ft.}$$

$$f = \frac{F}{A} \pm \frac{Fey}{I} \pm \frac{My}{I}$$

Top fibers $f = -\frac{F}{A} + \frac{Fey}{I} - \frac{My}{I}$

$$= -\frac{120{,}000}{120} + \frac{120{,}000 \times 2.5 \times 6}{1440} - \frac{37.5 \times 12 \times 6 \times 1000}{1440}$$

$$= -1000 + 1250 - 1880$$

$$= -1630 \text{ psi.}$$

Bottom fibers $f = -\frac{F}{A} - \frac{Fey}{I} + \frac{My}{I}$

$$= -1000 - 1250 + 1880$$

$$= -370 \text{ psi.}$$

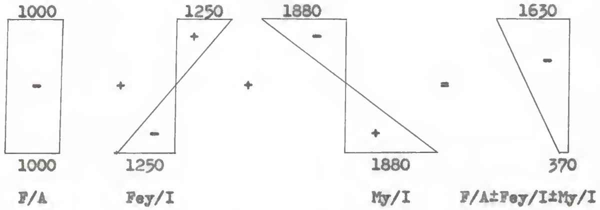


FIG. 10. STRESS DISTRIBUTION.

If the beam is unbonded, the stress in the steel will be different from the bonded beam.

The average stress is $f_s = E_s \dfrac{\Delta}{L} = \dfrac{n}{L} \displaystyle\int \dfrac{Mydx}{I}$

Example 3. A simply supported beam, span L ft., load P at the center, tendon is triangular shape and unbonded. Compare the stress in the steel if it was bonded. Neglect the weight of the beam.

$$M = M_0 \left(1 - \frac{x}{L/2}\right)$$

$$y = y_0 \left(1 - \frac{x}{L/2}\right)$$

$$f_s = \frac{n}{L} \int \frac{My}{I} \, dx$$

$$= \frac{nM_0y_0 \; 2}{LI} \int_{L/2}^{L/2} \left(1 - \frac{x}{L/2}\right)^2 dx$$

$$= \frac{2nM_0y_0}{LI} \int_0^{L/2} \left(1 - \frac{x^2}{L/2} - \frac{2x}{L/2}\right) dx$$

$$= \frac{2nM_0y_0}{LI} \left| x - \frac{x^3}{3\left(\frac{L}{2}\right)^2} - \frac{2x^2}{2\,(L/2)} \right|_0^{L/2}$$

$$= \frac{2nM_0y_0}{LI} \cdot \frac{1}{3} \cdot \frac{L}{2}$$

$$= \frac{(nM_0y_0)}{I} \cdot \frac{1}{3}$$ which is 1/3 of the stress for midspan of the bonded beam.

THE ULTIMATE STRENGTH OF PRESTRESSED BEAMS

It is not yet possible to calculate exactly the greatest bending moment which a prestressed beam can resist. In some theories, however, certain coefficients appear to be introduced so as to make the formula agree as nearly as possible with the results of experiments, which moreover, are sometimes made on beams which are too small to give results of real value.

An analysis of the rupture of beams shows that two possibilities exist: (1) Failure may occur by breaking of some of the wires before the top concrete is crushed. This kind of beam is named under-reinforced beam. (2) When the area of steel is greater than 0.2 percent, the concrete is generally crushed and the steel does not break. An over-reinforced beam is not desirable because its failure is sudden whereas the under-reinforced beam fails slowly.

Ultimate moment for under-reinforced bonded beam: (3)

$$C' = T' = A_s f'_s$$

$$M_u = T' a' = A_s f'_s a'$$

$$C' = K_1 f'_c K' bd$$

$$a' = d\ (1 - \frac{K'}{2})$$

$$K' = \frac{A_s f'_s}{K_1 f'_c bd}$$

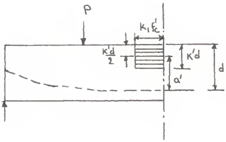


FIG. 11. ULTIMATE CAPACITY OF A SECTION.

According to Whitney's plastic theory of reinforced concrete beams, K_1 should be 0.85, based on cylinder strength. The ultimate resisting moment is:

$$M_u = A_s f'_s\ d\ (1 - \frac{K'}{2})$$

Example 4: A rectangular section 10 in. by 15 in. deep, A_s = 2.35 in^2, and the initial prestress = 150,000 psi. The c.g.s. of the wire is 4 in. above the bottom fiber of the beam,

$f'_s = 50{,}000$ psi; $f'_c = 4{,}000$ psi. Estimate the ultimate resisting moment.

$$K' = \frac{A_s\, f'_s}{.85\, f'_c bd} = \frac{2.35 \times 50{,}000}{.85 \times 4000 \times 10 \times 15} = 0.23 \text{ in.}$$

$$M_u = A_s f'_s\, d\, (1 - \frac{K'}{2}) = 2.35 \times 50{,}000 \times 15\, (1 - \frac{.23}{2})$$

$$= 1{,}560{,}000 \text{ in-lb.}$$

A beam would be called over-reinforced when the value of K' is greater than about 0.5.

DESIGN OF SECTIONS

A beam must be designed to fulfill three main conditions: (2)

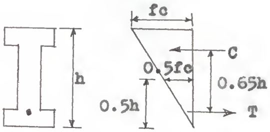
(1) The bending strength at all points must be in excess of the factored design moment, i.e., the working moment increased in the ratio of the chosen safety factor or load factor.

(2) The permissible stresses in the concrete must not be exceeded under the range of working load, having due regard to the variation of prestress with time.

(3) The shearing strength at all points must be in excess of the factored design shearing force.

Each of these conditions influences certain dimensions of the beam, and has to be considered in turn as the design proceeds.

Preliminary design:



Beam section. Resisting moment and stress distribution.

FIG. 12. PRELIMINARY DESIGN OF A BEAM SECTION.

In practice, the depth of the section is either given, known, or assumed, as is the total moment M_T on the section. Under the working load, the lever arm for the internal couple averages about 65 percent of the overall height h.

$$F = T = \frac{M_T}{0.65\ h}$$

$$A_s = \frac{F}{f_s} = \frac{M_T}{0.65\ hf_s}$$

The total prestress $A_s f_s$ is also the force C on the section. This force will produce an average unit stress on the concrete of

$$\frac{C}{A_c} = \frac{T}{A_c} = \frac{A_s f_s}{A_c}$$

For preliminary design, this average stress can be assumed to be about 50 percent of the maximum allowable stress f_c.

Hence,

$$\frac{A_s f_s}{A_c} = 0.5 f_c$$

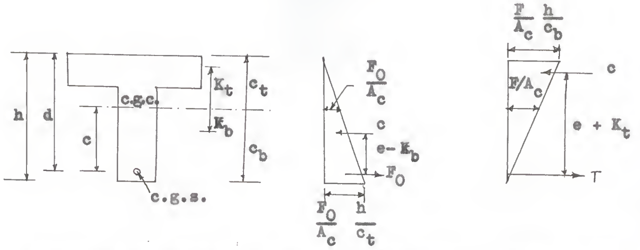
$$A_c = \frac{A_s f_s}{0.50 f_c}$$ area of concrete

In estimating the depth of the section, an approximate rule is to use 70 percent of the corresponding depth for conventional reinforced-concrete construction.

An accurate preliminary design can be made if the girder moment M_G is known in addition to the total moment M_T. When M_G is much greater than 20 to 30 percent of M_T, the initial condition under M_G generally will not control the design, and the preliminary design needs to be made only for M_T. When M_G is small relative to M_T, then the c.g.s. can not be located too far outside the kern point, and the design is controlled by $M_L = M_T - M_G$. In this case, the resisting lever arm for M_L is given approximately by $K_t + K_b$, which averages about 0.50 h. Hence, total effective prestress required is $F = M_L / 0.50$ h.

When M_G / M_T is small, this formula should be used instead of equation $F = M_T / 0.65$ h. When M_G / M_T is large, then use T - section and when M_G / M_T is small, then use I - section.

Elastic design, no tension in concrete: Case 1. Small ratios of M_G / M_T.



(a)Section properties. (b)Just after transfer c at bottom kern point. (c)Under working load, c at top kern point.

FIG. 13. STRESS DISTRIBUTION, NO TENSION IN CONCRETE, CASE 1.

The procedure of design is as follows: (3)

Step 1. From the preliminary design section, locate c.g.s. by $e - K_b = \frac{M_G}{F_0}$.

Step 2. Compute the effective prestress

$$F = \frac{M_T}{e + K_t}$$

then $F_0 = \frac{F\, f_0}{f_s}$

Step 3. Compute the required A_c by

$$A_c = F_0\, h / f_b c_t$$

and $A_c = F\, h / f_t c_b$

Step 4. Revise the preliminary section to meet the above requirements for F and A_c. Repeat steps 1 through 4 if necessary.

From the above discussion, the following observations regarding the properties of a section can be made:

1. $e + K_t$ is a measure of the total moment resisting capacity of the beam section. Hence, the greater this value, the more desirable is the section.

2. $e - K_b$ locates the c.g.s. for the section, and is determined by the value of M_G. Thus, within certain limits, the amount of M_G does not seriously affect the capacity of the section for carrying M_L.

3. h/c_b is the ratio of the maximum top fiber stress to the average stress on the section under working load. Thus, the smaller this ratio, the lower will be the maximum top fiber stress.

4. h/c_t is the ratio of the maximum bottom fiber stress to the average stress on the section at transfer. Hence, the smaller this ratio, the lower will be the maximum bottom fiber stress.

Example 5. Make a preliminary design and then final design for a section of prestressed concrete beam:

M_T = 240 k-ft., M_G = 30 k-ft., f_b = 1.8 ksi, f_0 = 150 ksi, f_s = 125 ksi, f_c = -1.60 ksi, f_t = 1.60 ksi, h = 30 in., F = 168 K.

Preliminary design:

$$\frac{M_G}{M_T} = \frac{30}{240} \times 100 = 12.5\%$$ is small, so using equation

$$F = M_L \;/\; 0.50\ h$$

$$M_L = M_T - M_G = 240 - 30 = 210 \text{ k-ft.}$$

$$F = M_L \;/\; 0.50\ h = 210 \times 12 \;/\; 0.50 \times 30 = 168 \text{ K}$$

$$A_s = F \;/\; f_s = 168 \;/\; 125 = 1.344 \text{ in.}^2$$

$$A_c = A_s f_s \;/\; 0.50\ f_c = 168 \;/\; 0.50 \times 1.6 = 210 \text{ in.}^2$$

Now a preliminary section can be sketched as shown in Fig. 14.

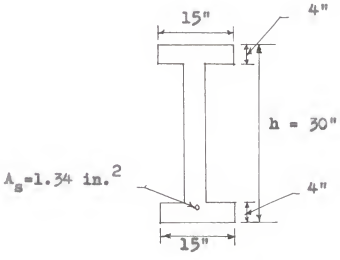


FIG. 14.

Final design:

$$A_c = 30 \times 15 - 22 \times 11 = 208 \text{ in.}^2$$

$$I = \frac{15 \times 30^3}{12} - \frac{11 \times 22^3}{12} = 23{,}983.33 \text{ in.}^4$$

$$K_t = K_b = \frac{I}{A_c \times c} = \frac{23{,}983.33}{208 \times 15} = 7.67 \text{ in.}$$

Step 1. For assumed $F = 168$ K

$$F_0 = F \times f_0 \;/\; f_s = 168 \times 150 \;/\; 125 = 201.6 \text{ K}$$

$$e - K_b = M_G \;/\; F_0 = 30 \times 12 \;/\; 201.6 = 1.79 \text{ in.}$$

$$e = 7.67 + 1.79 = 9.46 \text{ in.}$$

Step 2.

$$F = \frac{M_T}{e + K_t} = \frac{240 \times 12}{9.46 + 7.67} = 168 \text{ K}$$

$$F_0 = 168 \times \frac{150}{125} = 201.6 \text{ K}$$

Step 3. A_c required is

$$A_c = \frac{F_0\ h}{f_b c_t} = \frac{201.6 \times 30}{1.8 \times 15} = 224 \text{ in.}^2 \quad \text{(controlling)}$$

$$A_c = \frac{F\ h}{f_t c_b} = \frac{168 \times 30}{1.6 \times 15} = 210 \text{ in.}^2$$

Step 4. Try a new section as shown in Fig. 15.

$$A_c = 17 \times 30 - 13 \times 22 = 224 \text{ in.}^2$$

$$I = \frac{17 \times 30^3}{12} - \frac{13 \times 22^3}{12} = 26{,}700 \text{ in.}^4$$

$$K_t = K_b = \frac{26{,}700}{224 \times 15} = 7.94 \text{ in.}$$

$$e - K_b = M_G / F_0 = 30 \times 12 / 201.6 = 1.79 \text{ in.}$$

$$e = 1.79 + 7.94 = 9.73 \text{ in.}$$

$$F = \frac{M_T}{e + K_t} = \frac{240 \times 12}{9.73 + 7.94} = 163 \text{ K}$$

$$F_0 = 163 \times 150 / 125 = 195.6 \text{ K}$$

$$A_c = \frac{F_0\ h}{f_b c_t} = \frac{195.6 \times 30}{1.8 \times 15} = 217.33 \text{ in.}^2 \quad \text{(Bottom fiber)}$$

$$A_c = \frac{F\ h}{f_t c_b} = \frac{163 \times 30}{1.6 \times 15} = 203.75 \text{ in.}^2 \quad \text{(Top fiber)}$$

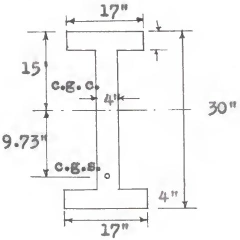
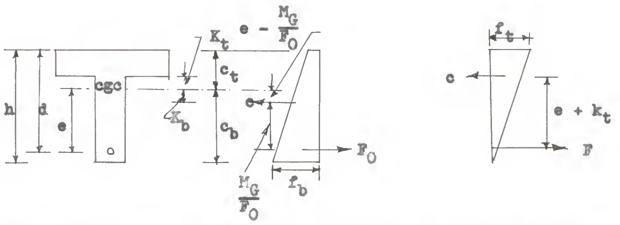


FIG. 15.

Hence, the section seems to be quite satisfactory, and no furthur revision is necessary.

Case 2. Large ratios of M_G / M_T



(a)Section properties. (b) Just after transfer, c above bottom kern point. (c)Under working load, c at top kern point.

FIG. 16. STRESS DISTRIBUTION, NO TENSION IN CONCRETE, CASE 2.

Design procedure is outlined as follows: (3)

Step 1. From the preliminary section, compute the theoretical location for c.g.s. by $e - K_b = M_G / F_O$; $K_b = I / A_c \times c_t$. If it is feasible to locate c.g.s. at the practical lower limit follow procedure for Case 1. If not, locate c.g.s. at the practical lower limit and proceed as follows:

Step 2. Compute F and F_O

$$F = \frac{M_T}{e + K_t} \qquad K_t = \frac{I}{A_c \times c_b}$$

$$F_O = \frac{F \times f_O}{f_s}$$

Step 3.

$$A_c = F\ h \ /\ f_t c_b$$

$$A_c = \frac{F_0}{f_b} \quad 1 + \frac{(e + M_G \ /\ F_0)}{K_t}$$

Step 4. Use the greater of the two A_c's and the new value of F, and revise the preliminary section. Repeat steps 1 through 4 if necessary.

Example 6:

Make a preliminary design and then final design for a section of prestressed-concrete beam. $M_T = 320$ k-ft., $M_G = 210$ k-ft., $f_b = -1.80$ ksi, $f_0 = 150$ ksi, $f_s = 125$ ksi, $f_t = -1.60$ ksi, $h = 36$ in.

Preliminary design:

$$F = T = M_T \ /\ 0.65\ h = 320 \times 12 \ /\ 0.65 \times 36 = 164\ K$$

$$A_s = F \ /\ f_s = 164 \ /\ 125 = 1.31 \text{ in.}^2$$

$$A_c = F \ /\ 0.50 \times f_c = 164 \ /\ 0.50 \times 1.6 = 205 \text{ in.}^2$$

Now a preliminary section can be sketched with a total concrete area of about 205 in.^{2}, a height of 36 in., and a steel area of 1.31 in.^{2}.

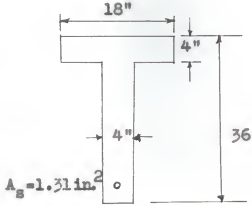


FIG. 17. PRELIMINARY T-SECTION.

Final design:

$$A_c = 18 \times 4 + 32 \times 4 = 200 \text{ in.}^2$$

$$c_t = \frac{18 \times 4 \times 2 + 32 \times 4 \times 20}{18 \times 4 + 32 \times 4} = 13.5;$$

$$c_b = 36 - 13.5 = 22.5 \text{ in.}$$

$$I = \frac{18 \times 4^3}{12} + 18 \times 4 \times 11.5^2 + 4 \times 32 \times 6.5^2 + \frac{4 \times 32^3}{12}$$

$$= 26{,}000 \text{ in.}^4$$

$$K_t = \frac{I}{A_c \times c_b} = \frac{26{,}000}{200 \times 22.5} = 5.8 \text{ in.}$$

$$K_b = \frac{I}{A_c \times c_t} = \frac{26{,}000}{200 \times 13.5} = 9.6 \text{ in.}$$

Step 1. Theoretical lowest location for c.g.s. is given by

$$e - K_b = M_G / F_O = 210 \times 12 / 197 = 12.8 \text{ in.}$$

$e = 12.8 + 9.6 = 22.4$ which is 0.1 in. above the bottom fiber.

Suppose that for practical reasons the c.g.s. has to be kept 3 in. above the bottom fiber to provide sufficient concrete protection. This problem then belongs to Case 2, and procedure is as follows:

$$a = 22.5 - 3 + 5.8 = 25.3 \text{ in.}$$

$$F = \frac{M_T}{e + K_t} = \frac{320 \times 12}{25.3} = 152 \text{ K}$$

$$F_O = 150 \times \frac{150}{125} = 182 \text{ K}$$

Step 3. Compute the area required by

$$A_c = \frac{F\ h}{f_t c_b} = \frac{152 \times 36}{1.60 \times 22.5} = 152 \text{ in.}^2$$

$$A_c = \frac{F_O}{f_b} \quad 1 + \frac{e - M_G / F_O}{K_t}$$

$$= \frac{182}{1.80} \quad 1 + \frac{(19.5 - 210 \times 12 / 182)}{5.8} = 199 \text{ in.}^2$$

(controlling)

A_c used = 200 in.^{2} o.k.

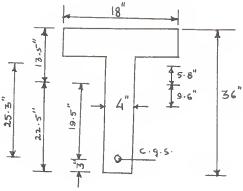


FIG. 18. TRIAL SECTION.

ULTIMATE DESIGN

Preliminary design:

For preliminary design, it can be assumed that the ultimate strength of bonded prestressed sections is given by the ultimate strength of steel acting with a lever arm of about 0.80 h

$$A_s = \frac{M_T \times m}{0.80\ h \times f_s{}'} \quad m = \text{factor of safety or the load factor}$$

The required ultimate concrete area under compression is

$$A_c{}' = \frac{M_T \times m}{0.80\ h \times 0.85\ f_c{}'}$$

Load factor of 2 can be assumed for steel and 2.5 for concrete.

Final design:

For final design the following factors must be considered: (3)

(1) Compressive stresses at transfer must be investigated for the tensile flange, generally by the elastic theory. In addition, the tensile flange should be capable of housing the steel properly.

(2) Proper and accurate load factors must be chosen for steel and concrete, related to the design load and possible overloads for the particular structure.

(3) The approximate location of the ultimate neutral axis may not be easily determined for certain sections.

(4) Design of web will depend on shear and other factors.

(5) The effective lever arm for the internal resisting

couple may have to be more accurately computed.

(6) Checks for excessive deflection and overstresses may have to be performed.

(7) The ultimate flexural strength for bonded sections is not so well known.

Example 8:

Make a preliminary design and then final design for a prestressed-concrete section 36 in. high to carry a total dead and live load moment of 320 k-ft., using steel with an ultimate strength of 220 ksi and concrete with $f_c' = 4$ ksi. Use ultimate design, and assume a bonded beam.

Preliminary design:

$$A_s = \frac{M_T \times m}{0.80\ h \times f_s'} = \frac{320 \times 12 \times 2}{0.80 \times 36 \times 220} = 1.21 \text{ in.}^2; \quad m = 2.0$$

$$A_c' = \frac{M_T \times m}{0.80\ h \times 0.85\ f_c'} = \frac{320 \times 12 \times 2.5}{0.80 \times 36 \times 0.85 \times 4} = 98 \text{ in.}^2;$$

$m = 2.5$

Thus, a preliminary section can be sketched as in Fig. 19 providing an ultimate area of 98 in.^{2} under compression, assuming the ultimate neutral axis to be 10 in. below the top fiber.

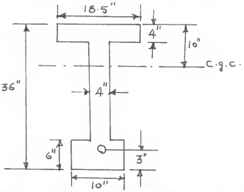


FIG. 19. PRELIMINARY SECTION.

The exact location of the ultimate neutral axis cannot and need not be obtained for a preliminary design but can be assumed to be about 30 percent of the effective depth of section.

Final design:

With the ultimate axis 10 in. below top fiber, the centroid of the ultimate compressive force is located by $\frac{74 \times 2 + 24 \times 7}{74 + 24}$ = 3.2 in. or 3.2 in. from top fiber. With the c.g.s. located 3 in. above bottom fiber, the ultimate lever arm for the resisting moment is 36 - 3.2 - 3 = 29.8 in. Now the area of steel required may be recomputed as

$$A_s = \frac{320 \times 12 \times 2}{29.8 \times 220} = 1.17 \text{ in.}^2$$

which is very near to the preliminary value of 1.21 in.^{2}, and no further trial is necessary.

SHEAR

The method of computing principal tensile stress in a prestressed-concrete beam is outlined as follows:

Step 1. From the total shear V across the section, deduct the shear V_s carried by the tendon to obtain the shear V_c carried by the concrete, thus,

$$V_c = V - V_s$$

Step 2. Compute the distribution of V_c across the concrete section by the usual formula, $v = V_c\, Q / I\, b$ where v = shearing unit stress at any given level, Q = statical moment of the cross-sectional area above (or below) that level about the centroidal axis, b = width of section at that level.

Step 3. Compute the fiber stress distribution for that section due to external moment M, the prestress F, and its eccentricity e by the formula

$$f_c = \frac{F}{A} \pm \frac{Fec}{I} \pm \frac{Mc}{I}$$

Step 4. The maximum principal tensile stress S_t corresponding to the above v and f_c is then given by the formula

$$S_t = \sqrt{v^2 + (f_c/2)^2} - f_c/2$$

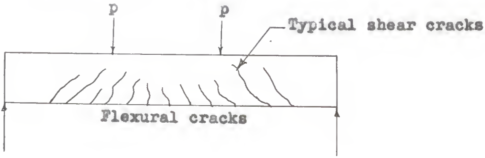


FIG. 20. SHEAR FAILURE IN PRESTRESSED BEAMS.

Example 9:

A prestressed concrete beam section under the action of a given moment has a fiber stress distribution as shown in Fig. 21. The total vertical shear in the concrete at the section is 140 kips. Compute and compare the principal tensile stresses at the centroidal axis N-N and at the junction of the web with the lower flange M-M.

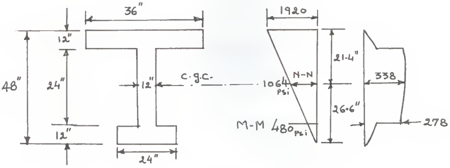


FIG. 21.

$I = 249{,}000 \text{ in.}^4$

	Section M-M	Section N-N
Q, in^3	$12 \times 24 \times 20.6 = 5940$	$14.6 \times 12 \times 7.3 + 5940 = 7220$
$v = \frac{V_c Q}{Ib}$, psi	$\frac{140{,}000 \times 5940}{249{,}000 \times 12} = 278$	$\frac{140{,}000 \times 7220}{249{,}000 \times 12} = 338$
f_c, psi	480	1064
$S_t = \sqrt{v^2 + (f_c/2)^2} - f_c/2$	$\sqrt{278^2 + (\frac{480}{2})^2} - \frac{480}{2} = 127$	$\sqrt{338^2 + \frac{1064}{2}^2} - \frac{1064}{2} = 88$

Shear, Web Reinforcement.

Theoretically speaking, if the principal tension in concrete is kept within the allowable limit, no web reinforcement is required. The method to compute spacing is outlined as follows:

Step 1. For each section of the beam, compute the maximum principal tension S_t, and the direction of the principal tensile plane, making an angle θ with the vertical plane.

Step 2. For vertical stirrups with spacing s, the force to be taken by each stirrup should be $S_t \; b \; s \; / \; \mathrm{Sin} \; \theta$, where b is the width of the beam.

Step 3. Equating the resisting and working forces

$$A_v f_v = S_t \; b \; s \; / \; \mathrm{Sin} \; \theta$$

and $$s = A_v f_v \; \mathrm{Sin} \; \theta \; / \; S_t \; b$$

Spacing of stirrups based on the ultimate design.

$$s = h A_v f_v' \; / \; V_c'$$

$$V_c' = V' - V_s'$$

FLEXURAL BOND AT INTERMEDIATE POINTS

For post-tensioned concrete, bond is supplied by grouting. For pretensioned concrete, bond is secured directly when placing the concrete. When a bonded beam is subject to shear, it is necessary to determine two things: first, the amount of bond stress existing between steel and concrete; second, the bond resistance between the two materials. (3)

$$U = \frac{n \; A_s \; y \; V}{\sum o \; I_t}$$

For round wires, $A_s / \sum_o = D/4$. We have $U = \frac{VynD}{4 \; I_t}$.

When wires are encased in metallic hoses, bond stress must be calculated for two contact areas; first, between wires and the grout; then between hoses and the concrete.

Bond stress after cracking,

$$U = \frac{V}{a \sum_o}$$

In the ultimate range, the value of 'a' can be approximated by 7/8 d as for reinforced concrete beam.

DESIGN OF A PRESTRESSED CONCRETE BRIDGE GIRDER

Two steps are involved in the design of a prestressed concrete beam for an actual structure:

(1) Choice of the shape and dimensions of the concrete member.

(2) Analysis of the member under the specified loading conditions to check unit stresses and determine the amount and details of the prestressing steel.

Steps in Design Procedure

Step 1. Compute the properties of the concrete cross section.

If the cross section is not uniform for the full length of the beam, it is usually best to analyze the section at the point of maximum moment and then check at any other points which might be critical because of the change in section. Properties to be computed are:

A_c = area of entire concrete section (steel area not deducted).

y_t = distance from top fiber to c.g.c. (center of gravity of entire concrete section).

y_b = distance from bottom fiber to c.g.c.

I_c = moment of inertia of entire concrete section about c.g.c.

Z_t = section modulus of top fiber referred to c.g.c.

Z_b = section modulus of bottom fiber referred to c.g.c.

w_G = dead load of member per unit length.

Step 2. Compute stresses in member due to its own dead weight.

M_G = bending moment due to w_G.

$f^t{}_G$ = stress in top fiber due to M_G.

$= M_G \div Z_t$

$f^b{}_G$ = stress in bottom fiber due to M_G.

$= M_G \div Z_b$

Step 3. Compute stresses in member due to applied loads.

In most cases these will be made up of additional dead load such as roof deck or highway wearing surface and live load.

w_s = additional dead load.

M_s = bending moment due to w_s.

$f^t{}_s$ = stress in top fiber due to M_s.

$= M_s \div Z_t$.

$f^b{}_s$ = stress in bottom fiber due to M_s.

$= M_s \div Z_b$.

w_L = distributed live load per unit length.

P_L = concentrated live load.

M_L = bending moment due to w_L and/or P_L.

$f^t{}_L$ = stress in top fiber due to M_L.

$= M_L \div Z_t$

$f^b{}_L$ = stress in bottom fiber due to M_L.

$= M_L \div Z_b$

Step 4. Determine the magnitude and location of the prestressing force at the point of maximum moment.

The prestressing force must meet two conditions:

(1) It must provide sufficient compressive stress to offset the tensile stresses which will be caused by bending moments.

(2) It must not create stresses either tensile or compressive which are in excess of those permitted by the specification.

Since we are considering a simple span beam, the moments due to the applied loads create compressive stresses in the top fiber and tensile stresses in the bottom fiber.

In order to meet the first condition, the prestressing force must create sufficient compressive stress in the bottom fiber to offset the tensile stresses from the bending moments. In other words $f^b{}_F$, stress in the bottom fiber due to the prestressing force F, must be equal in magnitude and of opposite sign to $f^b{}_G + f^b{}_S + f^b{}_L$.

Now $f^b{}_F = \frac{F}{A_c} + \frac{Fe}{Z_b}$

Setting this value of $f^b{}_F$ equal in magnitude to and of opposite sign to the sum of the bending moment stresses, we get

$$\frac{F}{A_c} + \frac{Fe}{Z_b} = - f^b{}_G - f^b{}_S - f^b{}_L$$

Use of this equation will give zero stress in the bottom fiber under full design load. For some structures, the specifications permit a small tensile stress f_{tp} under design load conditions. In this case, the magnitude of the compressive stress to be created by the prestressing force can be reduced by f_{tp} as shown in Equation (1).

$$\frac{F}{A_c} + \frac{Fe}{Z_b} = - f^b{}_G - f^b{}_S - f^b{}_L + f_{tp} \qquad (1)$$

When no tensile stress is permitted, f_{tp} equals zero.

In order to meet the second condition, the prestressing force must not create excessive stresses in the top fiber. For most designs the prestressing force, because of its eccentricity, causes a tensile stress in the top fiber. Since the dead load is always acting and the compressive stress it causes helps to offset the tensile stress, the net stress in the top fiber is the stress caused by the prestressing force plus the stress caused by the dead-load bending moment. To keep the tensile stress in the top fiber within allowable limits, we can write the following equation:

$$\frac{F}{A_c} - \frac{Fe}{Z_t} + f^t{}_G = f_{tp}$$

In this equation, $F/A_c - Fe/Z_t$ represents the stress due to the prestressing force, $f^t{}_G$ is the stress due to dead-load bending moment, and f_{tp} is the allowable tensile stress. This equation can be written as

$$\frac{F}{A_c} - \frac{Fe}{Z_t} = - f^t{}_G + f_{tp} \qquad (2)$$

Since in an actual design all the values except F and e are known, we can solve equation (1) and (2) and find F and e as follows:

Multiply Eq. (2) by Z_t/Z_b to get

$$\frac{Z_t F}{Z_b A_c} - \frac{Fe}{Z_b} = \frac{Z_t}{Z_b} (f^t{}_G + f_{tp}) \qquad (3)$$

Add Eq. (1) to Eq. (3) giving

$$(1 + \frac{Z_t}{Z_b}) \frac{F}{A_c} = \frac{Z_t}{Z_b} (f^t{}_G + f_{tp}) - f^b{}_G - f^b{}_S - f^b{}_L + f_{tp} \qquad (4)$$

Step 5. Select the tensioning elements to be used and work out the details of their location in the member.

Frequently e will be so large that it refers to a point below the bottom of the member or so close to the bottom that the tendons cannot be satisfactorily located. There are two remedies for this situation.

If the member being considered is symmetrical (one in which the section modulus of the top equals the section modulus of the bottom) and has stresses near the maximum allowed, a new section should be chosen. The new section should have more concrete in the top than in the bottom, thus raising the c.g.c. above the middle of its height, but the section modulus of the bottom should not be less than in the previous section.

For unsymmetrical sections in which the section modulus of the top is greater than that of the bottom or for symmetrical sections operating at low stresses, the value of e can be arbitrarily reduced until the tendons are far enough above the bottom of the member to permit satisfactory details. Changing the value of e will also change F. The new F should be found by substituting all known values including e in Eq. (1) and solve for F.

Decreasing e and increasing F will create a compressive stress in the top fiber. This can be computed by solving

$$f^t_F = \frac{F}{A_c} - \frac{Fe}{Z_t} \qquad (5)$$

Check the maximum stress in the top fiber by adding

$$f^t_F + f^t_G + f^t_S + f^t_L.$$

Step 6. Establish the concrete strength f'_{ci} at the time of prestressing and check stresses under the initial prestress condition.

The specifications permit higher unit stress in relation to f'_{ci} under this condition than with relation to f'_c under the final condition, so it is not always a governing factor but it must be checked.

Step 7. Establish the path of the tendons and check critical points along the member under initial and final conditions.

All the preceding calculations have been concerned with the point of maximum moment. In some members, other points may also be critical. These can usually be located by inspection of the moment diagrams, member properties, and location of prestressing steel.

There are two combinations of conditions which should be checked for maximum stress.

(1) Final prestress plus full design load.

(2) Initial prestress plus dead load only.

(it is seldom necessary to check prestress alone without the benefit of dead load. As the prestressing force is applied, it creates a negative moment and then develops a camber which raises the center portion off the form. Since the member is resting on each end, its dead-weight bending moment is effective.)

Step 8. Check ultimate strength to make sure it meets the requirements in the specification. Check percentage of prestressing steel.

There is no constant ratio between the design strength and the ultimate strength of a prestressed concrete member as there is in structural steel. Two prestressed concrete members of different cross section can have exactly the same load-carrying capacity based on allowable design stresses yet have entirely different ultimate strengths.

Use the method outlined in Sec. 209.2 of the ACI-ASCE Recommendations.

If too much prestressing steel is used, failure of the member will occur by crushing of the concrete. Check the amount of prestressing steel by the method shown in Sec. 209.2.3 of the ACI-ASCE Recommendations.

Step 9. Design of shear steel.

The most satisfactory results are obtained by using an empirical formula based on test data. The web reinforcement should be designed in accordance with Sec. 210 of the ACI-ASCE Recommendations. (4)

EXAMPLE:- PRESTRESSED CONCRETE BRIDGE GIRDER

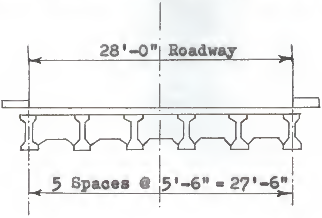


FIG. 22. CROSS SECTION OF BRIDGE AT A DIAPHRAGM.

BASIC DATA

Span: 75' - 0"

Width: 28' - 0" (Curb to curb)

ACI-ASCE 323 Report - AASHO Bridge Specs. (5)

$f_{ci} = 4000$ psi.

$f'_c = 5000$ psi.

$$\left.\begin{array}{l} {}^{*}E_c\text{(Girder)}:\ 1.8 \times 10^6 + 500 \times 5000 = 4.3 \times 10^6 \text{ psi.} \\ {}^{*}E_c\ \text{(Slab)}:\ 1.8 \times 10^6 + 500 \times 4000 = 3.8 \times 10^6 \text{ psi.} \end{array}\right\} \text{(Sec.203.2)}$$

Simple span

Slab thickness = 7 in.

Impact: 25%

Use Prestress precast AASHO type III girder. (6)

AASHO H 20 - S 16 - 44 Loading (7)

Wearing surface: 20 psf.

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*ACI-ASCE 323 Specifications.

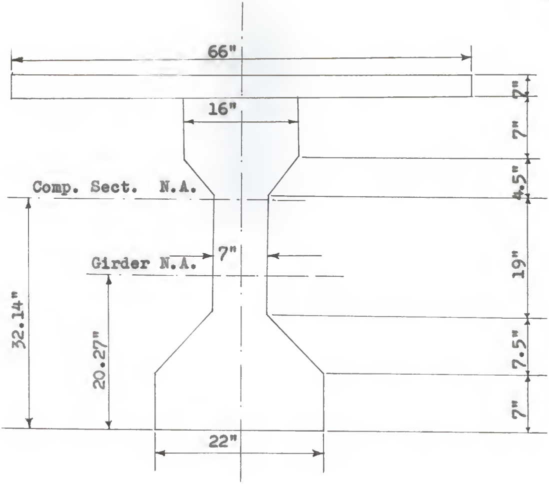


FIG. 23. DETAILS OF BEAM AND PORTION OF SLAB ACTING WITH BEAM.

STEP 1.

SECTION PROPERTIES

BEAM SECTION:

$A_c = 560$ in.^{2}
$Y_b = 20.27$ in. AASHO-PCI Publication
$I_c = 125{,}390$ in.^{4}

$Y_t = 45 - 20.27 = 24.73$ in.

$Z_t = 125{,}390 \div 24.73 = 5070$ in.^{3}

$Z_b = 125{,}390 \div 20.27 = 6186$ in.^{3}

Weight of beam $= \frac{560 \times 150}{144} = 583$ lb./ft.

COMPOSITE SECTION:

Correction for different E_c values beam and slab =

$$\frac{3.8 \times 10^6}{4.3 \times 10^6} = 0.88$$

Considering moment about top of slab.

Section	Area	y	Ay	Ay^2	I_0
66 x 7 x .88	406	3.5	1,421	4,974	1,660
Beam	560	31.73	17,769	563,810	125,390
			19,190	566,784	127,050
				127,050	
				695,834	

$y_t = 19{,}190 \;/\; 966 = 19.86$ in.

$y_b = 52 - 19.86 = 32.14$ in.

$I_c = 695{,}834 - 966\,(19.86)^2 = 314{,}825$ in.^{4}

$Z_t = 314{,}825 \;/\; 19.86 = 15{,}852$ in.^{3}

$Z_b = 314{,}825 \;/\; 32.14 = 9{,}795$ in.^{3}

Weight of slab = $\frac{66 \times 7 \times 150}{144} = 480$ lb./ft.

Dead weight of beam = 583 lb./ft.

STEP 2.

Compute stresses in the beam at the center of span due to its own dead weight.

$M_G = \frac{583\,(75)^2 \times 12}{8} = 4{,}915{,}000$ in.-lb.

$f^t{}_G = 4{,}915{,}000 \div 5070 = +\,969$ psi.

$f^b{}_G = 4{,}915{,}000 \div 6{,}186 = -\,794$ psi.

STEP 3.

Compute the stresses in the beam at the center of span due to applied loads.

Weight of slab = w_{ss} = 480 lb./ft.

$$M_{ss} = \frac{480\ (75)^2 \times 12}{8} = 4{,}055{,}000 \text{ in.-lb.}$$

The AASHO-PCI standards show diaphragms at one-third points of the span. Its size will be 8" wide by 30" x 66" i.e., weight carried by beam

$$= 2/3 \times 2.5 \times 5.5 \times 150 = 1375 \text{ lbs.}$$

The moment M_{sD} caused by this load at one-third point will be

$$M_{sD} = 1375 \times 25 \times 12 = 412{,}500 \text{ in.-lb.}$$

The total moment carried by the beam only due to superimposed load is

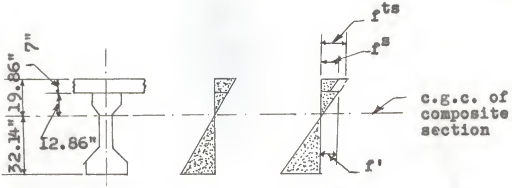
$$M_s = M_{ss} + M_{sD}$$

$$= 4{,}055{,}000 + 412{,}500 = 4{,}467{,}500 \text{ in.-lb.}$$

$$f^t{}_s = 4{,}467{,}500 \div 5070 = +\ 881 \text{ psi.}$$

$$f^b{}_s = 4{,}467{,}500 \div 6186 = -\ 722 \text{ psi.}$$

The load due to live load will be taken up by composite section. Fig. 24 shows elastic deformations developed when load is applied to composite section.



Composite section | Elastic Deformation | Stress distribution

FIG. 24. DISTRIBUTION OF STRAINS AND STRESSES IN COMPOSITE SECTION.

$$f^{ts} = \frac{M_s}{Z_t}$$

and because of difference in E_c of slab and beam

$$f^s = 0.88\ f^{ts}$$

f^s is less than f^{ts} because it is closer to the c.g.c. than the top of the slab.

$$f^t = \frac{12.86}{19.86}\ f^{ts} = 0.65\ f^{ts}.$$

Wearing surface weight $W_{ws} = 20 \times 5.5 = 110$ lb./lin. ft.

$$M_{ws} = \frac{110\ (75)^2 \times 12}{8} = 928{,}000 \text{ in.-lb.}$$

$$f^{ts}_{ws} = 928{,}000 \div 15{,}852 = +\ 59 \text{ psi.}$$

$$f^{b}_{ws} = 928{,}000 \div 9795 = -\ 95 \text{ psi.}$$

$$f^{t}_{ws} = +\ 59 \times 0.65 = +\ 38 \text{ psi.}$$

From AASHO tables p. 241, the live-load bending moment per lane is 1,075,100 ft.-lb.

Impact = 50 / (75 + 125) = 25%

Wheel load per stringer = 5.5 / 5 = 1.1

Wheel load = ½ lane load

Net live-load moment M_L per stringer is

M_L = 1,075,100 x 1.25 x 1.1 x ½ x 12 = 8,869,575 in.-lb.

f^{ts}_L = 8,869,575 ÷ 15,852 = + 560 psi.

f^{t}_L = + 560 x 0.65 = + 364 psi.

f^{b}_L = 8,869,575 ÷ 9795 = − 905 psi.

STEP 4.

Determine the magnitude and location of the prestressing force at the center of span.

Maximum allowable compression = 0.40 f'_c

(Sec. 207.3.2, a.l.a.)

= 0.40 x 5000 = 2,000 psi.

Maximum permissible tensile stress f_{tp} in the bottom fiber is zero. (Sec. 207.3.2, b.l.a.)

Equation for stress in bottom fiber in terms of F and e:

$$\frac{F}{A_c} + \frac{Fe}{Z_b} = - f^b{}_G - f^b{}_S - f^b{}_L + f_{tp} \qquad \text{(A-1)}$$

$$\frac{F}{560} + \frac{Fe}{6{,}186} = + 794 + 722 + 95 + 905 + 0$$

$$\frac{F}{560} + \frac{Fe}{6{,}186} = + 2{,}516 \qquad \text{(B-1)}$$

Under final conditions, the maximum permissible tensile stress in the top fiber is zero. This, however, is not the governing factor because under final conditions the top fiber of the girder has a compressive stress caused by the dead-weight bending moment of the poured-in-place slab.

The governing factor is the stress in the top fiber under prestress plus the dead weight of the girder only. The most critical combination of prestress and girder weight exists when the strands are first released from their anchorages, at which time the tension in the strand is maximum, F_O, and the strength of the concrete is minimum, f'_{ci}.

Allowable tension $f_{tp} = 3\sqrt{f'_{ci}} = 3\sqrt{4000} = -190$ psi. (Sec. 207.3.1,b.1.)

From Eq. (B-1) the prestressing force will create stress of + 2516 psi. in the bottom fiber at center of span. Since the maximum allowable stress is 2000 psi., some of the strands will slope up toward the end of beam. Critical points for tension in top fiber will be at hold-down points, where stresses due to prestress are the same as at the center of span. Let hold-down points be at 10' on either side of center. Call it "x". Stresses as in top fiber at hold-down points due to dead load will be

$$M_{Gx} = \frac{583 \times 27.5}{2}(75 - 27.5)\ 12 = 4{,}570{,}000 \text{ in.-lb.}$$

$$f^t_{Gx} = 4{,}570{,}000 \div 5070 = +901 \text{ psi.}$$

$$f^t_G - f^t_{Gx} = 969 - 901 = +68 \text{ psi.}$$

i.e., tensile stress at center of span must be 68 psi. less than allowable

$f_{tp} = -190 + 68 = -122$ psi.

Thus, allowable tensile stress in top fiber under initial prestress plus dead weight of the girder is 122 psi.

Stress condition at time of releasing strand is

$$\frac{F_0}{A_c} - \frac{F_0 e}{Z_t} = -f^t{}_G + f_{tp} \qquad \text{(B-2)}$$

By specification use 7/16" diameter strands at initial tension of 18,900 lb.

Unit stress = 18,900 / 0.1089 = 173,600 psi.

Considering losses of 35,000 psi. as per Sec. 208.3.2 Tentative recommendations.

Final stress = 173,600 - 35,000 = 138,600 psi. (f_F)

Compute stress losses.

Losses due to elastic shortening of concrete = 14,200 psi.

Shrinkage of concrete = 0.0001 E_s = 0.0001 x 27 x 10^6 = 2700 psi.

Relaxation of steel stress is 3% of initial tension = 0.03 x 173,600 = 5200 psi.

Total immediate losses will be

14,200 + 2700 + 5200 = 22,100 psi.

We shall use 20,000 psi. to remain on the conservative side.

Effective force = 173,600 - 20,000 = 153,600

and $\quad F_0 = (153,600 \div 138,600)\ F = 1.108\ F$

Substituting this value in Eq. (B-2)

$$\frac{1.108F}{A_c} - \frac{1.108Fe}{Z_t} = -f^t{}_G + f_{tp} \qquad \text{(B-3)}$$

Substituting values of A_c and Z_t in Eq. (B-3)

$$\frac{1.108F}{560} - \frac{1.108Fe}{5070} = -969 - 122$$

$$\frac{1.108F}{A_c} - \frac{1.108Fe}{5070} = -1091 \qquad (B\text{-}4)$$

Multiplying (B-4) by

$$\frac{Z_t}{1.108Z_b} = \frac{5070}{1.108 \times 6186} = 0.74$$

we get

$$\frac{0.82\ F}{560} - \frac{Fe}{6186} = -807 \qquad (B\text{-}5)$$

Adding (B-1) to (B-5)

$$\frac{1.82\ F}{560} = +1709$$

$F = 526{,}000$ lb.

Substituting this value of F in (B-1)

$$\frac{526{,}000}{560} + \frac{526{,}000\ e}{6186} = +2516$$

$e = 18.54$

$20.27 - 18.54 = 1.73$ in. from bottom of beam to c.g.s.

STEP 5.

Our final stress is 138,600 psi.

Tension available for 7/16" diameter strand will be

$138{,}600 \times 0.1089 = 15{,}120$ lb./strand

No. of strands = 526,000 / 15,120 = 34.8

35 strands are required for force of 526,000 lb.

With given eccentricity of 1.73" from bottom of beam, it is impossible to place 35 strands in a satisfactory manner.

Let us try for 38 strands arrangement.

$F = 15{,}120 \times 38 = 574{,}560$ lb.

e can be found from formula (B-1)

$$\frac{574{,}560}{560} + \frac{574{,}560\ e}{6186} = +\ 2516$$

$$1026 + 92.9\ e = 2516$$

$$e = 16.04 \text{ in.}$$

20.27 - 16.04 = 4.23 in. bottom of beam to c.g.s.

Minimum spacing = 4 x 7/16 = 1.75 in.

Clear space between strands = 1.75 - 7/16 = 1 5/16 in.

For this spacing we have to restrict aggregate size to 1 in. as per Sec. 216.2.1.

Maximum aggregate $= \frac{1\ 5/16}{1.33} = 0.99$ in. say 1 in. size.

For easier pouring of concrete spacing of 2" center to center in both directions can be used.

Try strand pattern shown in Fig. 25.

Center of gravity $= \frac{30 \times 4 + 4 \times 8 + 4 \times 11}{38}$

$= 5.17$ in. bottom to c.g.s.

This is 5.17 - 4.23 = 0.95 in. too high.

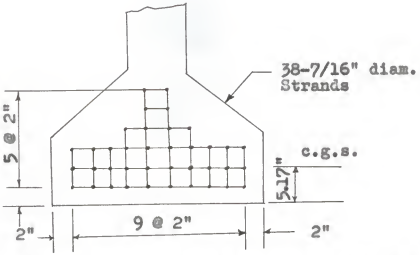


FIG. 25. TRIAL STRAND PATTERN AT CENTER OF SPAN.

Let us try strand pattern as shown in Fig. 26.

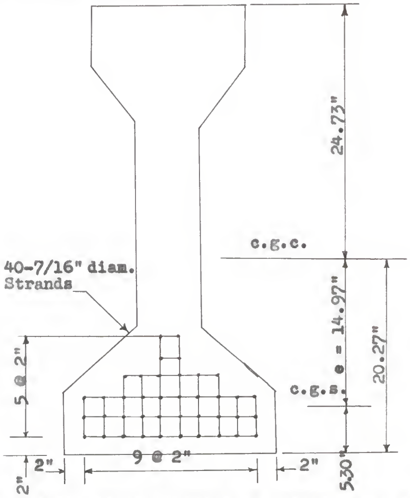


FIG. 26. FINAL STRAND PATTERN AT CENTER OF SPAN.

Compute c.g.s.

$$\frac{30 \times 4 + 6 \times 8 + 4 \times 11}{40} = 5.30 \text{ in. from bottom}$$

$e = 20.27 - 5.30 = 14.97$ in.

$F = 15{,}120 \times 40 = 604{,}800$ lb.

Find the stress in the bottom fiber under the new F:

$$f^b_F = \frac{604{,}800}{560} + \frac{604{,}800 \times 14.97}{6186}$$

$$= 1{,}080 + 1{,}464 = + 2{,}544 \text{ psi.}$$

Similarly stress in the top fiber will be

$$f^t_F = \frac{604{,}800}{560} - \frac{604{,}800 \times 14.97}{5070}$$

$$= 1{,}080 - 1{,}785 = - 705 \text{ psi.}$$

Now from step 4, $F_0 = 1.108$ F.

Therefore, stresses due to $F_0 = f^t_{F_0} = 1.108 \times (-705) = -781$ psi.

The stress in the top fiber under the critical condition of F_0 plus the dead weight of the beam is

$- 781 + 969 = + 188$ psi.

From step 4, permissible stress at top fiber is

$- 122$ psi. (f_{tp}).

Net stress in the top fibor of the beam under all applied loads is

$$f^t_F + f^t_G + f^t_S + f^t_{WS} + f^t_L = - 705 + 969 + 881 + 38 + 364$$

$= + 1547$ psi.

From above calculations, the stress in the top fiber at the center of span will vary between + 188 psi. to + 1547 psi. for F_0 plus the dead weight of girder under all applied loads.

These stresses are within the specified limits of - 122 to + 2000 psi.

Compute stress in the bottom fiber under prestress plus the weight of the beam only is

$f^{b}_{F} + f^{b}_{G} = + 2544 - 794 = + 1{,}750$ psi.

Net stress in the bottom fiber under all applied loads is

$f^{b}_{F} + f^{b}_{G} + f^{b}_{S} + f^{b}_{WS} + f^{b}_{L} = + 2544 - 794 - 722 - 95 - 905$

$= + 28$ psi.

From this, the stress in the bottom fiber will vary from + 1750 to + 28 psi., which is within the limits of + 2,000 to zero.

Net stress in top of the poured-in-place slab under all applied loads is

$f^{ts}_{WS} + f^{ts}_{L} = + 59 + 560 = + 619$ psi.

From step 1, E_c of the slab = 0.88 of the E_c of the beam. Therefore, the net stress in the top of slab is

$0.88 \times 619 = + 545$ psi.

From above it is clear that maximum compressive stresses in the top and bottom fibers of the beam are less than the allowable and there is no tensile stress in the top even under the conditions when it is allowable. Also, the maximum stress in poured-in-place slab is + 545 psi.

At this point, the economy of the section should be reviewed. So far, we have not taken advantage of permissible tensile stress in the top fiber. To do this, we have to use smaller prestressing forces with greater eccentricity, which we found impossible when working out the strand pattern. The eccentricity can only be increased by using one or more post-tensioned tendons, which will increase the cost.

Changing cross section of the beam is not feasible because the next AASKO-PCI standard section is too small. We cannot increase spacing of beams because unused stress in bottom fiber is not sufficient to take additional load.

Maximum stress in the slab is only + 545 psi. So, slab can be made thinner or of lower strength concrete or both. For use in a composite section, the final strength of the concrete slab should not be less than 3000 psi.

If lower strength concrete is used in the slab, the section modulus of the composite section will be slightly reduced. Reducing the slab thickness will reduce the section modulus, but it will also reduce the dead weight so that additional strands may not be necessary. In this case, however, the thinner slab will require more reinforcing steel to distribute the wheel loads to the beams.

From the above data, one can say that the section is quite economical.

STEP 6.

Check stresses under the initial prestress condition.

$f'_{ci} = +4000$ psi.

From Sec. 207.3.1

Allowable compression $= 0.60\ f'_{ci} = 0.60 \times 4000 = +2400$ psi.

Allowable tension $= 3\sqrt{f'_{ci}} = 3\sqrt{4000} = -190$ psi.

From step 4,

$$F_0 = 1.108\ F$$

$$= 1.108 \times 604{,}800 = 670{,}000 \text{ lb.}$$

From step 5,

$$f^t_{F_0} = -\ 781$$

$$f^t_{F_0} + f^t_G = -\ 781 + 969 = +\ 188 \text{ psi.}$$

The stress in the top fiber under initial prestress plus all applied loads is

$$f^t_G + f^t_{F_0} + f^t_S + f^t_{WS} + f^t_L = +\ 969 - 781 + 881 + 38 + 364 = +\ 1471 \text{ psi.}$$

$$f^b_{F_0} = \frac{F_0}{A_c} + \frac{F_0 e}{Z_b}$$

$$= \frac{670{,}000}{560} + \frac{670{,}000 \times 14.97}{6186}$$

$$= 1195 + 1621 = +\ 2816 \text{ psi.}$$

The stress in the bottom fiber under F_0 plus the girder only is

$$f^b_{F_0} + f^b_G = +\ 2816 - 794 = +\ 2022 \text{ psi.}$$

The stress in the bottom fiber under F_0 plus all applied loads is

$$f^b_{F_0} + f^b_G + f^b_{WS} + f^b_L = +\ 2816 - 794 - 722 - 95 - 905 = +\ 300 \text{ psi.}$$

From above calculations, all stresses at the center of span under initial prestress F_0 are within the allowable limits of $-\ 190$ to $+\ 2400$ psi.

STEP 7.

Establish the path of tendons and check any critical points along the member under initial and final conditions. From previous calculations

$$f^t_{F_0} = -781 \text{ psi.} \qquad f^b_{F_0} = +2816 \text{ psi.}$$

$$f^t_F = -705 \text{ psi.} \qquad f^b_F = +2544 \text{ psi.}$$

Since these stresses, which are all in excess of the allowable, would exist at the ends of the beam if strands were left in a straight line, it will be necessary to bend some of the strands up. One should compute e at the ends of the span to satisfy the requirement that $f^b_{F_0}$ shall not exceed + 2400 psi.

$$f^b_{F_0} = +2400 = \frac{670{,}000}{560} + \frac{670{,}000\ e}{6186}$$

$$= +2400 = +1196 + 108.3\ e$$

$$e = 11.12 \text{ in.}$$

$$f^b_F = 2400 \div 1.108 = +2166 \text{ psi.}$$

Since this exceeds the allowable of 2,000 psi for final conditions, the final condition will govern.

$$f^b_F = 2000 = \frac{604{,}800}{560} + \frac{604{,}800\ e}{6186}$$

$$2000 = 1{,}080 + 97.8\ e$$

$$e = 9.40 \text{ in.}$$

$$f^b_{F_0} = 2000 \times 1.108 = +2216 \text{ psi.}$$

$$f^t_F = \frac{604{,}800}{560} - \frac{604{,}800 \times 9.40}{5070}$$

$$= 1080 - 1120 + -40 \text{ psi.}$$

One has to select c.g.s. such that there should not be any tensile stress in the top fiber. Then

$$f^t_F = 0 = \frac{604{,}800}{560} - \frac{604{,}800 \times e}{5070}$$

$$1080 = 119.3\ e$$

$$e = 9.05 \text{ in.}$$

For this value of e

$$f^t_{F_0} = 0 \times 1.108 = 0$$

$$f^b_F = \frac{604{,}800}{560} + \frac{604{,}800 \times 9.05}{6186}$$

$$= 1080 + 885 = +\ 1965 \text{ psi.}$$

$$f^b_{F_0} = 1.108 \times 1965 = +\ 2177 \text{ psi.}$$

As these stresses are all within the allowable, e at the ends of the beam can be 9.05 in. or less.

20.27 - 9.05 = 11.22 in. from bottom to c.g.s.

The strand pattern in Fig. 27 is arranged so that 12 center strands can be sloped up in the web. The required moment of the strand group about the bottom at the ends is 40 x 11.22 = 449. The moment of the 28 strands which will be left in a straight line is

(24 x 4) + (4 x 8) = 96 + 32 = 128

Thus, the moment of the 12 raised strands must be 449 - 128 = 321.

The distance from the bottom to the center of gravity of the 12 strands is 321 ÷ 12 = 26.75 in.

Using higher c.g. as shown in Fig. 27, the c.g.s. of the arrangement of strands is

$$\text{Center of gravity} = \frac{24 \times 4 + 4 \times 8 + 12 \times 28.25}{24 + 4 + 12} = 11.68 \text{ in.}$$

$$e = 20.27 - 11.68 = 8.59 \text{ in.}$$

Check stresses with e = 8.95 in.

$$f^t_F = \frac{604,800}{560} \quad \frac{604,800 \times 8.59}{5070}$$

$$= 1080 - 1025 = + 55 \text{ psi.}$$

$$f^t_{F_0} = + 55 \times 1.108 = + 61 \text{ psi.}$$

$$f^b_F = \frac{604,800}{560} \quad \frac{604,800 \times 8.59}{6186}$$

$$= 1080 + 840 = + 1920 \text{ psi.}$$

$$f^b_{F_0} = + 1920 \times 1.108 = + 2127 \text{ psi.}$$

The strand pattern is now established at the center of span, Fig. 26, and at the ends of the beam, Fig. 27. The next step is to establish the path of the strands along the member. Let hold-down points be 10 ft. on either side of the center.

The stresses for the critical condition of bottom fiber stress under full load can be found out at various points along span, from the chart by Professor Gillan of Pennsylvania University, Engineering News Record, October 7, 1954, p. 320. These stresses are tabulated as in Table 1.

One must check to make sure that stresses are within the allowable at all points along the beam under all conditions of loading and either initial or final prestress. From Table 1, the maximum stresses will exist either under the dead load of the girder only or under all applied loads. The critical conditions are:

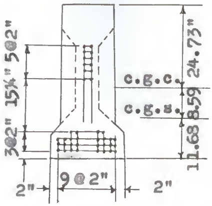


FIG. 27. STRAND PATTERN AT ENDS OF BEAM.

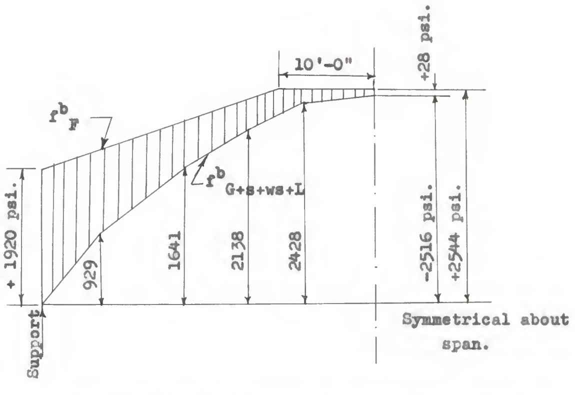


FIG. 28. DIAGRAM OF STRESSES IN BOTTOM FIBER UNDER FINAL PRESTRESS PLUS ALL APPLIED LOADS. (SHADED AREA REPRESENTS NET COMPRESSIVE STRESS.)

Table 1. Moments* and stresses from applied loads.

X**	Precast section						Composite section						$f^t_{(G+s+ws+L)}$	$f^b_{(G+s+ws+L)}$
	M_G	f^t_G	f^b_G	M_s	f^t_s	f^b_s	M_{ws}	f^t_{ws}	f^b_{ws}	M_L	f^t_L	f^b_L		
7.5'	147,500	+349	-286	132,000	+312	-256	27,800	+14	-34	288,000	+142	-353	+817	-929
15.0'	262,500	+620	-508	237,000	+561	-460	49,500	+24	-61	500,000	+246	-612	+1451	-1641
22.5'	344,500	+814	-667	314,000	+744	-610	65,000	+32	-80	638,000	+314	-781	+1904	-2138
30.0'	393,800	+930	-762	358,000	+848	-695	74,400	+36	-91	718,000	+354	-880	+2168	-2428
37.5'	410,000	+969	-794	372,000	+881	-722	77,400	+38	-95	739,000	+364	-905	+2252	-2516

*Moments are in foot-pounds.

**X is the distance from support to point being considered.

I. Final prestress plus

A. All applied loads

1. Top fiber -- Fig. 29.

2. Bottom fiber -- Fig. 28.

B. Dead load of beam only

1. Top fiber

2. Bottom fiber

II. Initial prestress (after immediate losses) plus

A. All applied loads

1. Top fiber

2. Bottom fiber

B. Dead load of beam only

1. Top fiber -- Fig. 31.

2. Bottom fiber -- Fig. 30.

Stresses are plotted as indicated in the diagrams. Study of all the above-mentioned stress diagrams tells us that all stresses are within the allowable limits.

STEP 8.

Check ultimate strength and also check percentage of prestressing steel.

From Sec. 209.2.1(b) ACI-ASCE Recommendations,

percentage of prestressing steel = $1.4\ d\ p\ f_{su} / f'_c$

where $d = 52 - 5.30 = 46.70$ in.

$A_s = 40 \times 0.1089 = 4.36$ in.^{2}

$p = A_s / bd = 4.36 / (66 \times 0.88) \times 46.70 = 0.00161$

$f'_c = 5000$ psi.

$f'_s = 27{,}000 / 0.1089 = 248{,}000$ psi.

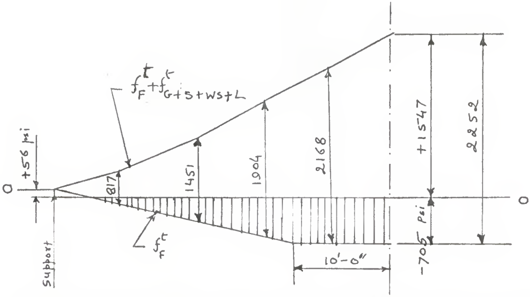


FIG.29 STRESSES IN TOP FIBER UNDER FINAL PRESTRESS PLUS ALL APPLIED LOADS.

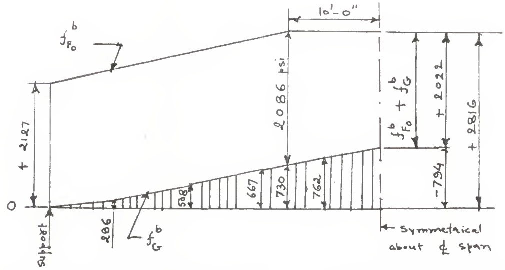


FIG.30 STRESSES IN BOTTOM FIBER UNDER INITIAL PRESTRESS PLUS DEAD LOAD.

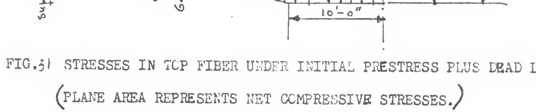


FIG.31 STRESSES IN TOP FIBER UNDER INITIAL PRESTRESS PLUS DEAD LOAD ONLY.

(PLANE AREA REPRESENTS NET COMPRESSIVE STRESSES.)

From Sec. 209.2.2.2.a

$$f_{su} = f'_s \left(1 - 0.5\, p \frac{f'_s}{f'_c} \right)$$

or $$f_{su} = 248{,}000 \left(1 - 0.5 \frac{0.00161 \times 248{,}000}{5000} \right)$$

$$= 248{,}000\ (1 - 0.040) = 238{,}000 \text{ psi.}$$

Substituting these values in the formula,

$$\frac{1.4 \times 46.70 \times 0.00161 \times 238{,}000}{5000} = 5.00$$

Since the flange thickness of composite section is 7 in. following formula for rectangular sections in Sec. 209.2.1.(a) applies:

$$M_u = A_s f_{su}\, d \left(1 - \frac{K_2}{K_1 K_3} \frac{p\, f_{su}}{f'_c} \right)$$

From Sec. 209.2.1(a)

$$\frac{K_2}{K_1 K_3} = 0.6$$

Substituting

$$M_u = 4.36 \times 238{,}000 \times 46.70 \left(1 - \frac{0.6 \times 0.00161 \times 238{,}000}{5000} \right)$$

$$= 46{,}200{,}000 \div 12 = 3{,}850{,}000 \text{ ft-lb.}$$

This is the ultimate moment the member can carry.

From Sec. 205.3.2 the minimum required ultimate is (1.5 D + 2.5 L.)

$$= 1.5\ (M_G + M_S + M_{WS}) + 2.5\ (M_L)$$

$$= 1.5\ (410{,}000 + 372{,}000 + 77{,}400) + 2.5\ (739{,}000)$$

$$= 3{,}136{,}600 \text{ ft.-lb.}$$

Since this is less than M_u, the member has sufficient ultimate strength.

From Sec. 209.2.3

Check percentage of prestressing steel.

$$\frac{p\ f_{su}}{f'_c} = \frac{0.00161 \times 238{,}000}{5000} = 0.0766$$

Since this is less than 0.30, the member is not over-reinforced.

STEP 9.

Design of shear steel.

From Sec. 210.2.5., one can find critical section for shear.

Dead weight is

Beam only:	583 lb. per ft.
Poured-in-place slab:	480 lb. per ft.
Wearing surface:	110 lb. per ft.
Total:	1173 lb. per ft.

Dead-load shear at the one-quarter point is

$$1173 \times \frac{75}{4} = 22{,}000$$

1,375 weight of diaphragm

23,375 lb.

Live-load shear at one-quarter point is V_L. (Fig. 32)

$$V_L = R_A = \frac{32 \times 56.25 + 32 \times 42.25 + 8 \times 28.25}{75} = 45.1 \text{ K.}$$

From step 3, one beam carries 0.55 lane load, so the live-load shear is 45,100 x 0.55 = 24,800 lb. The impact factor for loading in Fig. 32 is

$$I = \frac{50}{56.25 + 125} = 27.5\%$$

Total V_L = 1.275 x 24,800 = 31,600 lb.

Ultimate shear $V_u = 1.5\ V_D = 2.5\ V_L$

$$V_u = 1.5 \times 23{,}375 + 2.5 \times 31{,}600 = 114{,}000 \text{ lb.}$$

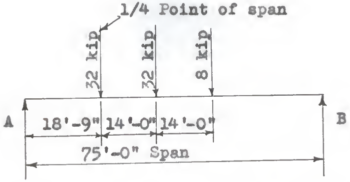


FIG. 32. TRUCK LOADING TO PRODUCE MAXIMUM LIVE LOAD SHEAR AT 1/4 POINT OF SPAN.

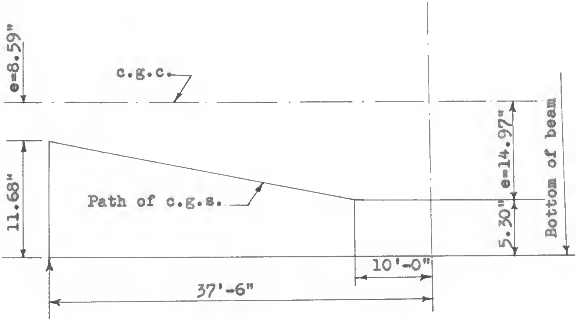


FIG. 33. PATH OF CENTER OF GRAVITY OF TENDONS.

From Fig. 33, the slope of the c.g.s. of the strands is

$$\frac{11.68 - 5.30}{[(37' - 6") - (10' - 0")] \times 12} = 0.0193$$

The shear carried by the strands is

$0.0193 \times 604{,}800 = 11{,}700$ lb.

Effective $V_u = 114{,}000 - 11{,}700 = 102{,}300$ lb.

From Sec. 210.2.2:

$$A_v = \frac{(V_u - V_c)\, s}{2f'_y\, jd} \qquad \text{where } V_c = 180\ b'\ jd.$$

$$\frac{18.75}{27.5}(11.68 - 5.30) = 4.35 \text{ in.}$$

8.59 = e at end from Fig. 33

24.73 = c.g.c. to top fiber from Fig. 26

$\underline{7.00}$ = poured-in-place slab

$d = 44.67$

$$p = \frac{A_s}{bd} = \frac{4.36}{(66 \times 0.88)\, 44.67} = 0.00168$$

$$f_{su} = 248{,}000\left(1 - \frac{0.5 \times 0.00168 \times 248{,}000}{5000}\right) = 238{,}000$$

From Sec. 209.2.1:

$$j = 1 - \frac{K_2\, p\, f_{su}}{K_1 K_3\, f'_c}$$

$$j = 1 - 0.6\left(\frac{0.00168 \times 238{,}000}{5000}\right) = 0.952$$

Then $V_c = 180 \times 7 \times 0.952 \times 44.67 = 53{,}600$ lb.

From Sec. 210.2.4 and Sec. 212.3.4, the maximum spacing of ties between the precast and poured-in-place sections is 24 inches. Let s = 24 in. Yield strength of reinforcing steel bar = f'_y = 40,000 psi.

Substituting known values,

$$A_v = \frac{(102,300 - 53,600)\ 24}{2 \times 40,000 \times 0.952 \times 44.67} = 0.344 \text{ in.}^2$$

From Sec. 210.2.3 the minimum amount of web reinforcement is

$$A_v = 0.0025\ b's$$

$$A_v = 0.0025 \times 7 \times 24 = 0.420 \text{ in.}^2$$

Provide 0.420 in.^{2} of structural steel bars every 24 in.

The area of two #4 bars is

$$2 \times 0.196 = 0.392 \text{ in.}^2$$

$$\frac{0.392}{0.420}\ (24) = 22.4 \text{ in.}$$

Use two #4 bars at a maximum spacing of 22 3/8" center to center for the full length of the beam.

Check the shear between precast section and poured-in-place section.

From Sec. 212.3.2

$$v = \frac{V_u Q}{I_c\ t'}$$

in which t' is the width of the contact surface between the two sections. From Sec. 104.2, Q = statical moment of cross section area, above or below the level being investigated for shear.

$$Q = 7 \times 66 \times 0.88\ (19.86 - 3.5) = 6,650$$

The shear carried by the composite section is only that due to the wearing surface and the live load, which is

Wearing surface = 110 lb. per ft. x 75/4 x 1.5 = 3,100 lb.

Live load = 31,600 lb. x 2.5 = 79,000 lb.

Applicable V_u = 82,100 lb.

(The 1.5 and 2.5 are ultimate load factors.) Substituting the values in the formula,

$$v = \frac{82,100 \times 6,650}{314,825 \times 16} = 109 \text{ psi.}$$

From Sec. 212.3.3 allowable is 150 psi. but Sec. 212.3.4 requires to provide minimum steel tie and according to this minimum reinforcement is two #3 bars at 12 in. center to center or $2 \times 0.11 = 0.22$ in.^{2} for 12 in. spacing.

If one uses two #4 bars having area of $2 \times 0.196 = 0.392$ in.^{2}, one can increase spacing to

$$\frac{0.392}{0.22} \times 12 = 21.4 \text{ in.}$$

So finally, use two #4 bars at 21 3/8" centers for stirrups and ties.

CONCLUSION

The full economy of any type of construction is never realized until a design specification or code of practice is available for the guidance of designers. As yet there is no recognized specification for prestressed concrete in this country. For this reason many engineers will be hesitant about undertaking a design. As long as this situation continues, the number of minds in the office and in the field at work on the development of improved prestressed concrete design and construction methods, keyed to the American economy, will be greatly limited.

For prestressed-concrete beams, no extensive tests are available at the moment. "Criteria for Prestressed Concrete Bridges", issued by the U. S. Bureau of Public Roads in 1959, and "Tentative Recommendations for Prestressed Concrete", by ACI-ASCE Joint Committee 323, has been accepted in the design of prestressed bridges and should be consulted also for other construction. It is important that such specifications be made available for prestressed concrete as soon as possible. Such a code of practice in its first draft should set up broad requirements which will insure safety without undue restrictions. Under no circumstances should such a specification prescribe methods of construction that will stifle development.

ACKNOWLEDGMENT

The writer wishes to express his sincere gratitude to Dr. John G. Mc Entyre and Professor Vernon H. Rosebraugh, for their kind guidance and assistance in the preparation of this report.

APPENDIX I - READING REFERENCES

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APPENDIX III - NOTATIONS

A = cross-sectional area in general.

A_c = net cross-sectional area of concrete.

A_g = gross cross-sectional area of concrete.

A_s = cross-sectional area of steel.

A_v = cross-sectional area of one set of steel stirrups.

a = lever arm between the centers of compression and tension in a beam section.

b = width of beam or its flange.

C = center of compressive force.

c = distance from c.g.c. to extreme fiber.

c_b, c_t = c for bottom (top) fibers.

c.g.s. = center of gravity for steel area.

c.g.c. = center of gravity of concrete section.

D = diameter of bars or wires.

d = depth of beam measured to c.g.s.

e = eccentricity in general.

E_c, E_s = modulus of elasticity for concrete (steel).

F = total effective prestress after deducting losses.

F_i = total initial prestress before transfer.

F_0 = total prestress, just after transfer.

f_c = unit stress in concrete.

f_s = unit stress in steel.

f_t, f_b = fiber stresses at top (bottom) fibers.

f_v = unit stress in steel stirrups

f_s = change in f_s.

h = overall depth of beam.

I = moment of inertia of section.

j = for resisting lever arm jd in a beam section.

k = coefficient for depth of compressive area kd in a beam section.

K_t, K_b = kern distances from c.g.s. for top (bottom).

L = length of member.

M,B.M. = bending moment in general.

n = modular ratio E_s/E_c.

P = force.

s = stirrup spacing.

S_t = principal tensile stress.

T = center of total tension.

U = unit bond stress.

V = total shear in beam.

V_c = total shear carried by concrete.

V_s = total shear carried by steel.

W = total weight.

w = weight per length.

y = perpendicular distance from c.g.c. line to said fiber.

Z = section modulus I/c.

( )' = refers to ultimate.

APPENDIX IV - LIST OF ILLUSTRATIONS

THE DESIGN PROCEDURES FOR
PRESTRESSED REINFORCED CONCRETE

by

DEVENDRA M. DHARIA

B. S., Stanford University, 1961

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1963

Approved by:

John G. McEntyre
Major Professor

Prestressed concrete design procedures are relatively new tools, with which designers will give more attention to the aspect of practical usage. The intent of this report is to show the proper procedures which an engineer must follow and the precautions which he must exercise in any reinforced concrete design so that the design can be done effectively.

"Prestressing" means the creation of stresses in a structure before it is loaded. These stresses are artificially imparted so as to counteract those occurring in the structure under loading. Thus in a reinforced concrete beam, a counter-bending is produced by the application of eccentric compression forces acting at the ends of the beam.

Prestressing systems are classified into two main groups, pre-tensioning and post-tensioning. The term pre-tensioning is used to describe any method of prestressing in which the tendons are tensioned before the concrete is placed. In contrast to pre-tensioning, post-tensioning is a method of prestressing in which the tendon is tensioned after the concrete has hardened.

The most outstanding difference between the prestressed and reinforced concrete is the employment of materials of higher strength for prestressed concrete. The entire section of the concrete becomes effective in prestressed concrete, whereas only the portion of the section above the neutral axis is supposed to act in flexure in the case of reinforced concrete. One-hundred pounds of mild steel required in ordinary reinforced concrete beam may be replaced by only twenty pounds of high strength wire in a construction of identical depth where prestressing is used.

The three main parts of the report are as follows:

i) Analysis of sections for flexure.

ii) Design of sections.

iii) Design of prestressed concrete bridge girder.

Numerous examples are solved to show the procedures.

Prestressed concrete provides a greater saving in material used in the construction, and perhaps in cost. It is believed that much research is still to be done on the procedures for the design of prestressed concrete.