DIRECT METHOD OF MEASURING INTENSITY OF DIFFRACTION PATTERN BY PHOTOELECTRIC CELL

by

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TABLE OF CONTENTS

INTRODUCTION

Application of the wave theory to the explanation of diffraction phenomena was first made by Dr. Young[1]. He attributed the fringes to the interference of the direct light which passes very close to the edge with the light reflected by the edge at grazing incidence. However, it was soon found that Youngs explanation was incorrect, and it is to Fresnel that we owe the true solution of the problem.

According to Fresnel the phenomena of diffraction are to be attributed to the mutual interference of the secondary wavelets which diverge from the wave front. He expressed the resultant of all the secondary waves by means of two integrals taken within limits determined by the particular nature of the problem. The expression for the intensity

$$I = \frac{ab\lambda}{2(a+b)} \left(\int \cos\frac{\pi}{2} v^2\, dv \right)^2 + \left(\int \sin\frac{\pi}{2} v^2\, dv \right)^2$$

contains both of these integrals. Integrating them between certain values of v gives the resultant of the secondary disturbances from a corresponding portion of the wave front, v varying with s the distance of the wave front. The values of these integrals between 0 and upper limits of various values have been evaluated by Fresnel, Knochenhauer, Canchy, and Gilbert and the result given in tables known as Fresnel's integrals.

The graphical representation[2] of the resultant of a large number of vibrations of continuously varying phase and amplitude was employed by Cornu. This treatment is for the Fresnel class, and depends upon the application of results obtained from the mathematical treatment of the subject. The resultant effect of a number of disturbances of different amplitude and phase can be represented graphically as the closing side of a polygon the sides of which are proportional in length to the amplitudes produced by the disturbances acting separately and making angles with a fixed line equal to the phases of the disturbances. The Cornu Spiral which was constructed by Cornu from the tables of Fresnel's integrals is used in practice to plot the diffraction patterns above mentioned.

The photographical method has also been used in representing intensity of diffraction phenomena in two different ways. In the oldest method a photographic plate was used in place of the screen, and a photograph taken of the diffraction pattern in this way. Then by comparing the darkening of the photograph with a color wedge the relative intensity at various points in the pattern may be determined. In the other method the photographic plate or rather the negative is used as a screen between the light source and a photoelectric cell. A narrow slit is used in front of the photoelectric cell so as to admit only a narrow band of light to the cell. The negative with the diffraction pattern is then examined in front of this slit by observing the response of the photoelectric cell. From these responses and the displacement of the negative a curve can be plotted.

The most common way of determining the maximum and minimum of a diffraction pattern is the direct optical way. Here a diffraction slit system is set upon an optical bench and in place of the screen a traveling micrometer eye piece is used. Then by focusing the cross hair of the micrometer eye piece first on the maximum and then on the minimum the relative distance between these can be measured. This method however, is of no value in determining the relative intensity at any point.

PURPOSE OF THESIS

The purpose of this thesis was to investigate the possibility of using a photoelectric cell in back of a traveling slit to measure the distribution of intensity along a diffraction pattern.

METHOD

The greatest difficulty encountered in this problem was to secure a photoelectric cell sensitive enough and yet stable enough to permit accurate measurements of the variation in the current of the out-put circuit as produced by a change of intensity of illumination on the cell.

The first cell used was the Rayfoto, a liquid or photovoltaic cell. Measurements of change in electromotive force with change in illumination were made. The results showed that the conditions of the cell were very unstable and a variable electromotive force with time was obtained.

Quite a number of tests were made with the P4 Arcturus Photolytic cell, which was also a liquid cell. The first tests on this cell were made to determine the voltage change with a change in intensity of illumination. The e.m.f. terminal of a type K potentiometer were directly connected to the terminals of the cell. The cell was inclosed in a box, the front of which was fitted with an adjustable slit provided with a micrometer screw adjustment. Curves were made plotting potentiometer reading against slit openings. A representative curve is shown in Fig. I

Tests were also made to determine the constancy of response of the cell to intensity of illumination. Here

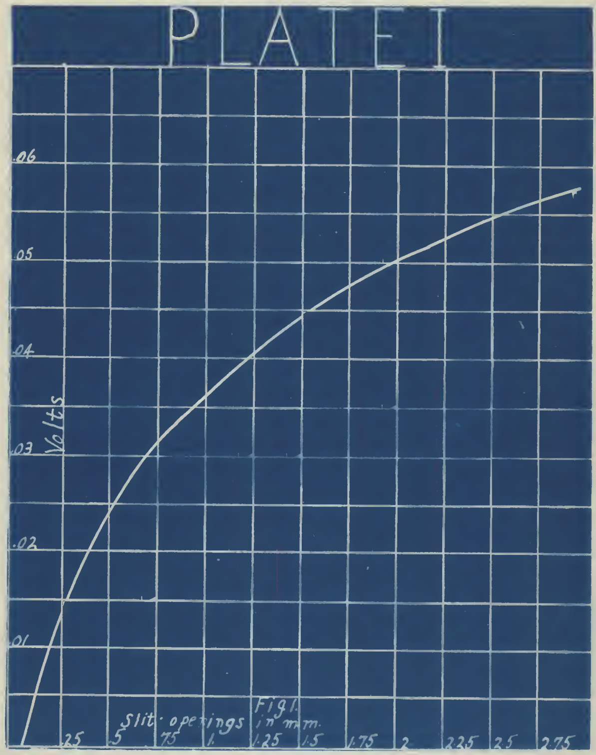


PLATE I
.06
.05
.04
.03
.02
.01
Volts
Fig 1.
Slit openings in m.m.
.25
.5
.75
1.
1.25
1.5
1.75
2
2.25
2.5
2.75

it was found, if the zero reading was checked each time, that a definite response was produced by each intensity of illumination. It was also found that the response of the cell was very small and that in order to measure very faint intensities or very small change in intensities of light some form of amplification was necessary.

A single 224 screen grid tube was first used for amplification. The circuit diagram is shown in Fig. II. This single stage amplification was not sufficient. Next the amplifing unit, of which a circuit diagram is shown in Fig. IV, was tried. In this circuit variations in plate current were so pronounced that it made it useless for accurate measurements, or even permit the use of a microammeter. Some of this variation was probably due to the amplifier but a large part of it was due to the Photolytic cell.

The final work was done with a Westinghouse Photoelectric Cell and Amplifier unit. The circuit and wiring diagram of this unit is shown in Fig. V. The unit uses a special type vacuum tube which has a high amplification constant especially designed for use in direct current amplification. With this unit the illumination intensity of about 7.5 foot candle should produce a plate current of

PLATE II

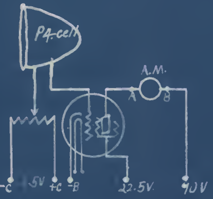


Fig. II

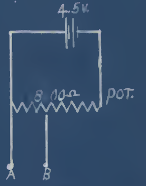


Fig. III

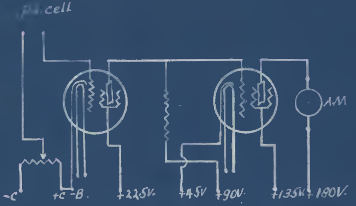


Fig. IV

PLATE III

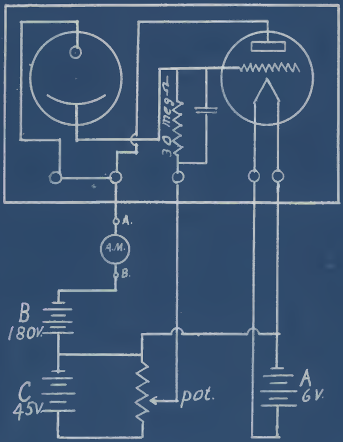


Fig V

Schematic diagram of Amplifier Unit.

approximately eight to ten milliamperes. The out-put is controlled by the grid voltage (C). This should be so adjusted that when the photoelectric cell is in the dark about one milliampere of plate current flows in the out-put circuit. To measure the change in plate current a sensitive D'Arsonval galvanometer was used. This made it necessary to employ a current cancelling device across the galvanometer. The diagram of this device is shown in Fig. III. By means of this it was possible to cancel the initial current of one milliampere or so in the plate circuit to zero. Thus it was possible to operate the amplifier tube on the most sensitive part of the tube characteristic curve and at the same time permit the use of a sensitive measuring meter.

The photoelectric cell used was a V.B. Vacuum type cell. Tests were made with different filters to determine its color sensitivity. These tests showed that the cell was most sensitive to yellow light and least sensitive to blue light.

The main part of the diffraction apparatus consisted of a box ten inches square and about the same height. A schematic diagram is shown in Fig. VI. This box was made to fit over the photoelectric cell and amplifier unit.

PLATE IV

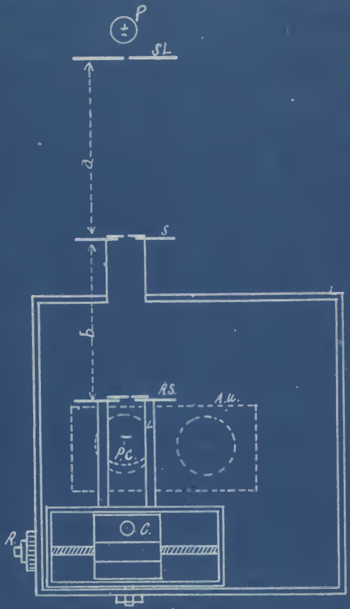


Fig VI
Schematic diagram of Apparatus

To one side of this box in a position even with and on the face side of the photoelectric cell a circular opening of two inches in diameter was made. In front of this opening a slit holder from an optical bench was mounted. This served as the holder for the diffraction aperture (S). The receiving slit (RS) was mounted on an adjustable L bracket in front of the photoelectric cell and parallel with the diffraction aperture. The bracket holding the receiving slit was mounted to a micrometer slide (C) of a comparator at the top of the box. This enabled the receiving slit to be moved horizontally in a plane of the light and diffraction slit. Accurate measurement of the displacements of this slit could be made by the micrometer screw (R).

The L bracket to which this receiving slit was mounted also permitted adjustments so that the distance between this slit and the diffraction aperture could be changed. In most of the experiments this distance was about twenty centimeters.

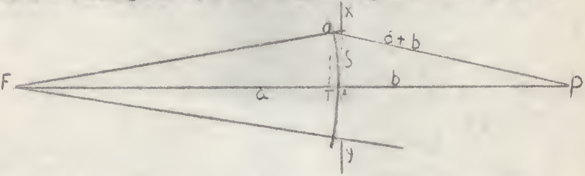
For the light source a 150 C.P. A.C. Royal Ediswan Pointolite was used. In some of the experiments a third slit of about .1m.m. in width was used in front of the light and parallel with the diffraction aperture. The function of this slit was to exclude all the light except a narrow band or line.

This line then served as the source or point of light. Some of the light emerging from this source would pass through the diffraction aperture and produce the diffraction pattern on the screen (i.e. the receiving slit.) The receiving slit then would allow very narrow bands of this diffracted light fall on the photoelectric cell. Thus by moving the receiving slit along the diffraction pattern the intensities at various points could be measured, provided that the response of the photoelectric cell was proportional to the intensity of illumination. The vacuum type photoelectric cell used for this work had, according to manufactures data, a linear response to illumination. This is also generally concluded for vacuum type cells by Zworykin and Wilson[4] in the "Photocell and their Application".

PLATE V.

THEORY

In the figure below let F be a long narrow slit from which a cylindrical wave diverges. Let xy be a thin plate with a rectangular aperture at A and equal in width to 2S.



The plate xy is parallel to the slit F and at a distance a from it. The distance AP = b and OP = $\delta + b$. The distance OT is approximately equal to S where the aperture is very narrow in comparison with b.

The distance AT can be found approximately by using the theorem from plane geometry; that if two chords intersect within a circle the product of the segments of the one is equal to the product of the segments of the other, thus AT = $S^2/2a$ neglecting the small quantity $(AT)^2$ because no appreciable error will result.

Then to find the retardation, we have

$$S^2 + (b + \frac{S^2}{2a})^2 = (\delta + b)^2$$

$$S^2 + b^2 + \frac{2bS^2}{2a} + \frac{S^4}{4a^2} = \delta^2 + 2\delta b + b^2$$

As δ^2 is very small in comparison with "b" also the term $S^4/4a^2$, these terms can be omitted without any appreciable error.

Thus from the above equation $\delta = \dfrac{S^2\,(a+b)}{2ab}$

Let $\sin 2\pi t/\theta$ represent the displacement of the vibration on the cylindrical surface at A. Every point on the surface may be regarded as a secondary source sending out waves to P. Thus the displacement at P due to a small element ds at A will be proportional to

$$\sin 2\pi\left(\frac{t}{\theta} - \frac{b}{\lambda}\right) ds \qquad (1)$$

while an element ds at O will contribute a displacement at P proportional to

$$\sin 2\pi\left(\frac{t}{\theta} - \frac{b+\delta}{\lambda}\right) ds \qquad (2)$$

The displacement at p due to the simultaneous action of all the elements ds of the circular arc corresponding to the aperture width 2s will be proportional to

$$\int \sin 2\pi\left(\frac{t}{\theta} - \frac{b+\delta}{\lambda}\right) ds \qquad (3)$$

According to Houstoun[3] the displacement at p should depend also on the distance of the secondary source and to be perfectly accurate the expression (2) should be divided by 1/OP Fresnel makes the assumption that it is only the strips in the neighborhood of T that matter, that for these OP can be considered constant, and that the other strips can be neglected. This assumption is justified by his results.

The only quantity in (3) depending on s is , therefore the integral may be written

$$\sin 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right) \int \cos \frac{2\pi\delta}{\lambda}\, ds - \cos 2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right) \int \sin \frac{2\pi\delta}{\lambda}\, ds$$

This reduces to K sin $\left\{2\pi\left(\frac{t}{T} - \frac{b}{\lambda}\right) - \phi\right\}$

By letting $K \cos \phi = \int \cos \frac{2\pi\delta}{\lambda}\, ds$, and $K \sin \phi = \int \sin \frac{2\pi\delta}{\lambda}\, ds$

The intensity of the resultant displacement of the vibration at p is thus proportional to

$$\left(\int \cos \frac{2\pi\delta}{\lambda}\, ds\right)^2 + \left(\int \sin \frac{2\pi\delta}{\lambda}\, ds\right)^2 \qquad (4)$$

From the preceeding work we have found $\delta = \frac{s^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$

Then if we introduce a new variable v such that

$$s^2 = v^2 \frac{\lambda}{2}\left(\frac{ab}{a+b}\right)$$

$$\text{or } \frac{2\pi\delta}{\lambda} = \frac{\pi v^2}{2} \quad \text{and} \quad v = 2\sqrt{\frac{\delta}{\lambda}} \qquad (5)$$

substituting this in (4) we have that the intensity is proportional to

$$\frac{ab}{2(a+b)} \quad \left(\int \cos \frac{\pi}{2} v^2 \, dv\right)^2 + \left(\int \sin \frac{\pi v^2}{2} \, dv\right)^2 \qquad (6)$$

The two integrals occurring in this expression are the basis of the Cornu spiral. Its application to Fig. VIII will be briefly outlined. In the case of a narrow rectangular aperture the amplitude of the vibration is measured by an arc of the spiral, the length of which is proportional to the width of the slit. The intensity will be represented by the square of a line joining the extremities of a constant length of the arc of the spiral. This length is calculated by using equations (1) and (5) and solving for v

$$\delta = \frac{S^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$a = 10 \text{ cm.} \quad b = 20 \text{ cm.} \quad \text{average } \lambda = .000059$$

$$S = .15 \text{ m.m.} = .015 \text{ cm.}$$

$$S = \frac{(.015)}{2}\left(\frac{1}{10} + \frac{1}{20}\right) = .0000165$$

$$v = 2\sqrt{\frac{\delta}{\lambda}} = 2\sqrt{\frac{.0000165}{.000059}} = 1. \text{ approximately}$$

Equal distances are marked off on the spiral. From the calculated value for v it is found that the length of the total constant arc is equal to 20 of these equal distances. Then to find the intensity at the center of the aperture ten of these equal distances are taken on each side of the center of the arc. The square of the distances between these two points will represent the intensity. This corresponds to zero abscissae on the curve refered to. As we proceed from the center, we push the arc of constant length along the spiral, squaring the line joining its extremities at regular intervals, plotting these values as ordinates, at abscissae corresponding to the distances advanced along the spiral.

From the proceeding discussion the length of the constant arc of the spiral for curve #1 was found to be 20 equal distances marked off on the spiral. The data for one side of the curve is given below. In plotting, the units for this curve were converted to the units of the experimental curve. The point of maximum intensity was chosen to coincide with the point of maximum galvanometer deflection, and all other ordinates were changed to the same ratio. For abscissae the points for the edge of the geometrical shadow were chosen to coincide and all other points were converted to the same ratio. This was done to see how the curves would compare when both plotted to the same scale.

l = length of chord of the constant arc, as measured on log scale of slide rule, D = distance advanced along spiral.

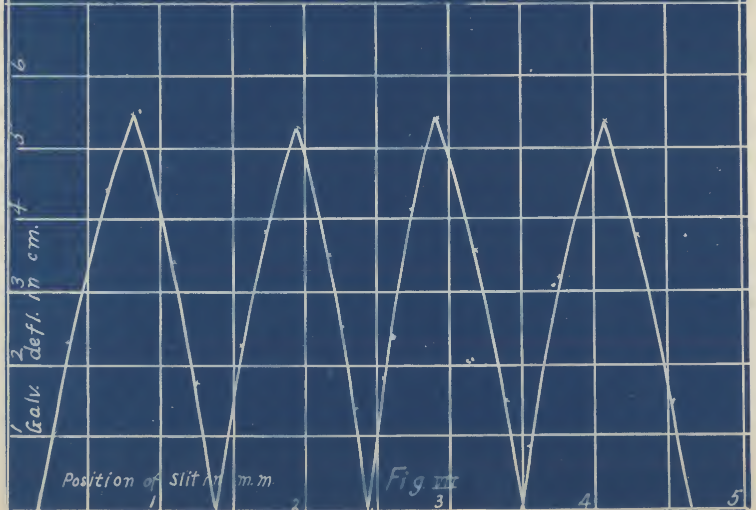
Theoretical Data for Fig. VIII

D	I	I^2	D	I	I^2
.0	9.1	82.[illegible]	.7	4.3	18.49
.1	9.0	81.	.8	3.58	12.82
.2	8.53	72.76	.9	3.16	09.99
.3	7.9	62.41	1.0	3.04	9.24
.4	7.1	50.41	1.1	3.1	09.61
.5	6.13	37.85	1.2	3.18	10.11
.6	5.10	26.83	1.3	3.2	10.24

RESULTS AND DATA

Fig. VII shows the typical curve of a diffraction pattern of a coarse wire grating. This part of the work was done to see if the response of the photoelectric cell was very definite. As the spacing of the wire on the grating is approximately equal to the thickness of the wire, the maxima and the minima should be distributed fairly uniformly. The curve shows this to be approximately so. No attempt was made to check this by any mathematical theory or equation, because only the general outline was desired. The grating used was made entirely of brass. The wire was wound on two 8-32 machine screws and the thickness of the wire was approximately half that of the pitch of the screw. A double convex lens was set at some distance from and parallel to the wire grating. At the focal point of this lens the

PLATE VI



pointolite source was located. Thus approximately parallel light fell upon the wire grating.

In the experimental curves the symbols used are as follows:

a = distance from light source to diffraction slit.

b = distance from diffraction slit to receiving slit.

s = one half the width of the diffraction slit.

So = width of receiving slit.

g = for galvonometer deflection in cm.

d = for displacement in m.m. of receiving slit.

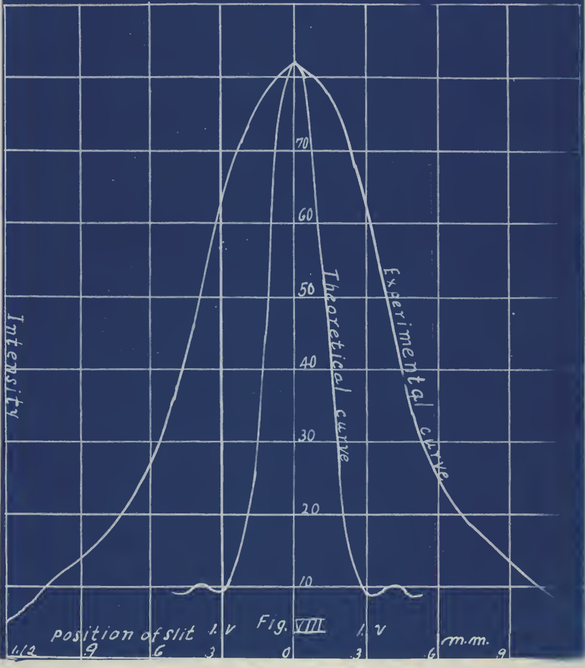
Experimental Data for Fig. VIII

d	g	d	g
.0	82.	.0	82.
.125	76.5	.12	78.
.250	67.	.225	68.
.375	54.	.3	62.
.500	38.	.4	51.5
.625	22.	.55	30.
.750	16.	.7	21.
.875	12.	.8	16.
1.	10.	1.	11.
1.2	5.	1.2	5.

CONCLUSIONS

From Fig. VIII which is a representative curve of the data taken it is evident that the experimental curve is some what broader than the theoretical curve. This was

PLATE VII


70
60
50
40
30
20
10
Intensity
Theoretical curve
Experimental curve
position of slit
1.v
Fig. VIII
1.v
m.m.
1.12
9
6
3
0
3
6
9

expected at the outset, because an extended source had been used. The slit at the source was .6 m.m. wide which has a broadening affect on the central part of the curve. All mathematical theory on which the Cornu spiral is based assures a point source or a very narrow line which has a negligible width. It can readily be seen that if the source has extended width that this can be divided up in small elements of the wave fronts. Each one of these elements would have its own theoretical maxima and minima. If these could be determined in some way mathematically and then adding intensities from these various elements, it seems possible that the resulting curve would approach the experimental curve very closely. However as far as the writer knows, there is no mathematical theory available for such a curve.

The reason for believing that the light source was mostly responsible for the broadening of the central part of the curve was that the very first experiments were performed with a so called pointolite source without the slit. This diffraction pattern of the central bright image was distributed over a much wider space. Then the source was narrowed by admitting the light through a slit. This reduced the width of the space over which the central bright image of the diffraction pattern was distributed. Here it was found that about .6 m.m. was the minimum slit useable, because any slit smaller than that would not admit enough illumination for the photoelectric cell to register any appreciable response.

Another cause of the broadening of the curve was the use of a rather wide receiving slit. In practically all the experiments it was about .1 m.m. wide. This was wide in comparison with the width of the diffraction aperture. A smaller slit would not admit sufficient illumination to permit the use of the photoelectric cell. The receiving slit would admit to the photoelectric cell the illumination of a narrow band in the diffraction pattern. The result was that as the slit was moved along from one edge of the central bright image of the pattern to the other edge, it would spread the resulting width in which the photoelectric cell registered responses, by about twice the slit width. This would, however, account for only a small part of the broadening of the curve.

The photoelectric cell and amplifier set was not especially adapted for work of this kind, as it was designed for the operation of relays. Thus it could not be expected to get the best results possible. It was also necessary to use white light and then use the wave length for which the cell was most sensitive in computing the theoretical pattern. It was necessary to do this on account of insufficient measn of amplification of the photoelectric cell out-put current.

Although this representative experimental curve is broader than the theoretical curve, as has been expected for reasons above mentioned, it is a representation of the intensity of the diffraction pattern because the photoelectric cell responses are directly proportional to the intensity of illumination that fall upon it. Also the symmetry of the curve shows that the responses were quite regular. From this it may be expected that it is a practical method for measuring the distribution of the intensity of a diffraction pattern, provided the pattern is not too faint.

ACKNOWLEDGMENT

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Date Due
