INVESTIGATION OF THE QUALITY OF FREQUENCY MODULATION PRODUCED BY A SINUSOIDAL VARIABLE CONDENSER

by

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ABSTRACT

An investigation is made of the quality of frequency modulation that is produced by sinusoidaly varying the capacity of an ordinary radio-frequency oscillating circuit. A study is also made, by use of a receiving circuit coupled with the oscillating circuit, of the frequency modulation effect with respect to the resonance curve of the receiving circuit.

By the use of a sinusoidal variable condenser placed in the oscillator circuit of a radio transmitter, the natural frequency is found to be governed approximately by the relation,

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

The capacity is made to vary sinusoidaly by varying the area of coincidence of two superimposed brass plates sinusoidaly. $\underline{C} \propto \underline{A}$, where $\underline{C}$ is capacity, and $\underline{A}$ is area of coincidence. We thus have modulated radio frequency.

This method of varying the frequency sinusoidaly affords a means of studying the quality of frequency modulation produced, since the wave form of the resulting output of current, may be analyzed in relation to the known input variation of capacity.

It is found that the amount of distortion of the reception current, in a receiver which is coupled with the transmitter of modulated frequency, depends on the point of operation on the resonance curve of the receiver. The best point of reception on the resonance curve is on the straight portion on the side. Distortion is greatest near the peak and bottom of the curve. When reception is on the peak of the resonance curve, the second (double fundamental frequency) harmonic is quite prominent.

INTRODUCTION

In a previous experiment conducted by Professor Leo E. Hudiburg, Assistant Professor of Physics, Kansas State College, it was determined that frequency modulation was produced by varying the capacity of the oscillating circuit at audio frequency, which in turn varied the natural frequency of the circuit. It is the purpose of this thesis to determine the quality of frequency modulation produced by this method, and the factors affecting it.

There are three factors which might cause distortion of the current in the receiving circuit that is produced by frequency modulation in the transmitter. First, from the equation $n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ it is evident that the frequency n is not directly proportional to capacity C. Therefore, if C varies sinusoidaly it does not necessarily follow that n will. Second, the edge capacity effect of the plates of the sinusoidal variable condenser might cause distortion of the sinusoidal capacity variation. Third, some distortion will result if reception is not on the straight portion of the resonance curve.

We will determine then the amount of distortion present in the reception current, and how much of it is due to the above named factors.

THEORY

We shall consider first a simple case for obtaining sinusoidal capacity variation which depends on the area of coincidence between two geometrical figures in parallel planes when one is projected upon the other. This is taken from a paper on "An Electrostatic Alternator" by Professor E.R. Lyon.

A rectangular tooth is moving with uniform velocity across a stationary sinusoidal tooth shown above D, Plate I, Fig. I. In general let there be p such pairs of rotor and stator teeth, each pair with its adjacent space occupying the distance 2D in the direction of motion, and p being an integer. p = 1 in Fig. I. S_1 is the area of coincidence of the rotor and stator teeth.

$$S_1 = p\int z\,dx \qquad z = h \sin \pi x/D \qquad \text{--1}$$

$$S_1 = ph\int_0^x \sin \pi x/D = (phD/\pi)(1 - \cos \pi x/D) \qquad \text{--2}$$

$$S_1 = (phD/\pi)(1 - \cos \pi x'/D) =$$

$$(phD/\pi)(1 - \sin \pi x''/D) \qquad \text{--3}$$

It is readily seen that the foregoing equations continue to be true throughout the cycle. Proof: - When the trailing edge arrives at the present position of the leading edge, the area of coincidence becomes S_2, and $x'_2 = x$ in Fig. I if $x'_1 = x'$ in Fig. I, and

PLATE I

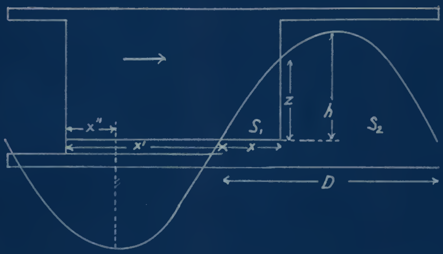


Fig. 1

Fig. 2

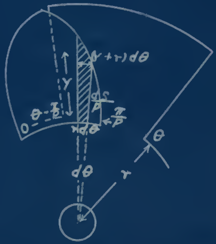


Fig. 3

$$S_2 = 2phD/\pi - S_1 = 2phD/\pi - (phD/\pi)(1 - \cos\pi x'_2/D)$$

$$S_2 = (phD/\pi)(1 + \cos\pi x'_2/D) =$$

$$(phD/\pi)(1 + \cos\pi x/D) \qquad --4$$

But $S_1 = (phD/\pi)(1 - \cos\pi x'_1/D)$. Therefore in general, if S is the area of coincidence of the rotor and stator teeth,

$$S = (phD/\pi)(1 + \cos\pi x'/D) = (phD/\pi)(1 - \cos\pi x/D)$$

$$S = (phD/\pi)(1 - \sin\pi x''/D) \qquad --5$$

A modification of the preceding case is used in the sinusoidal variable condenser as employed in this work. The rotor consists of a metal plate with ($p = 8$) teeth which have radial edges projecting outwards from a circle of radius r, to a circle of radius R, Plate II. The angular width of each tooth is π/p radians. The stator consists of ($p = 8$) petal shaped plates placed on an insulating plate in a circle of radius r and projecting outward to a circle of radius R. (See Plate III.) The design of the petal shaped plates is governed by the following equations. (Fig. 2, Plate I.)

$$dS/p = (1/2)(r + y)^2\, d\theta - (1/2)(r^2\, d\theta) =$$

$$ryd\theta + (1/2)\,(y^2 d\theta) \qquad d\theta = dx/r$$

$$dS = p(y + y^2/2r)\, dx \qquad --6$$

Where $\underline{S}$ is intended to represent the area of coincidence of the eight stator teeth with the eight rotor teeth.

By equation (1)

$$dS = pzdx \qquad \text{--7}$$

and by equations (7), (6) and (1)

$$z = y + y^2/2r = h \sin \pi x/D \qquad \text{--8}$$

In equation (8) $\underline{z}$ is permitted to assume positive values only for the determination of $\underline{y}$. When $\underline{z}$ is negative, $y = 0$ by construction. This also applies to equation (1). It can be seen from Fig. 1 that only the positive part of the sinusoid was used in the construction of the stator, it being impossible for a negative area of coincidence to exist. It was necessary for us to resort to the proof in equation (4) to sustain our general equation (5). In Fig. 1, and with a corresponding condition governing Fig. 2 and Fig. 3, the identity of $\underline{S}$, is $\underline{S_1}$ from the instant of contact of the leading edge of the rotor tooth with the extreme left edge of the sinusoid of the stator tooth, to the instant of contact of the trailing edge of the rotor tooth with this same point on the stator, after which the identity of $\underline{S}$ is the area designated by $\underline{S_2}$ in Fig. 1. Thus, by a 180° reversal of phase of the identity of $\underline{S}$ during the course of the cycle, we are able to complete the cycle and to preserve the generality of equation (5). Rewriting equation (8),

$$y^2 + 2ry + r^2 = 2hr \sin \pi x/D + r^2$$

$$y + r = \pm \sqrt{r^2 - 2hr \sin \pi x/D}$$

$$y = \sqrt{r^2 + 2hr \sin \pi x/D} - r, \text{ if positive. Otherwise}$$

$$y = 0 \qquad \text{--9}$$

Since

$$D = \pi r/p, \quad x = r\theta, \text{ and } \pi x/D = p\theta \qquad \text{--10}$$

$$y = \sqrt{r^2 + 2hr \sin \pi p\theta} - r \qquad \text{--11}$$

It must be noted that h in equation (11) is not the amplitude of y, but is instead the amplitude (the maximum value of the ordinate z) of the equivalent sinusoid. If the design of the petal shaped teeth of the stator is governed by equation (11) then equation (5) is correct for the area of coincidence. This can be shown as follows. From equation (10), (8) and (6),

$$dS = phr \sin p\theta d\theta \qquad \text{--12}$$

Referring to Fig. 2, when $0 \langle \theta \rangle \pi/p$

$$S = phr \int_0^{\theta} \sin p\theta' d\theta'$$

$$S = hr(1 - \cos p\theta) \qquad \text{--13}$$

Referring to Fig. 3, when $\pi/p \langle \theta \rangle 2\pi/p$

$$S = phr \int_{\theta - \frac{\pi}{p}}^{\frac{\pi}{p}} \sin p\theta' d\theta'$$

$$S = hr(1 - \cos p\theta) \qquad \text{--14}$$

PLATE II

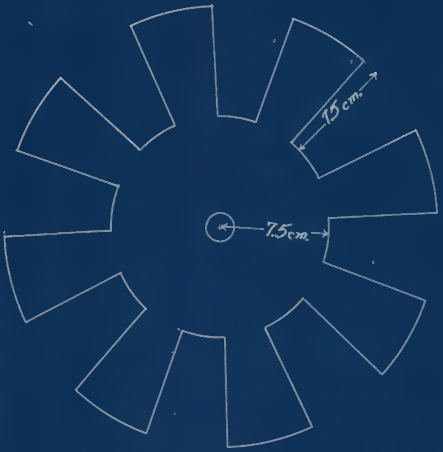
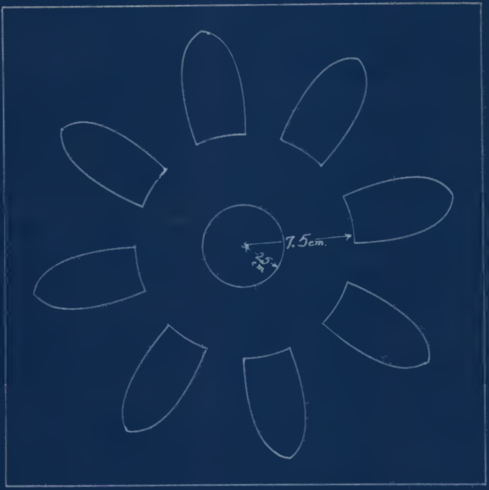


PLATE III



Let $p\theta - \pi/2 = p\phi = wt$, $\qquad w = 2\pi f$ --15

By equations (15), (14) and (13)

$$S = hr\,(1 + \sin p\phi = hr\,(1 + \sin wt) \qquad \text{--16}$$

By combining equations (15), (10) and (5) we have equation (16).

Because of the variable surface S of coincidence between the teeth, the device is a variable condenser. Also it has a certain amount of fixed capacity that is in parallel with its variable capacity. Therefore, its capacity C may be expressed as a function of time by the following equation

$$C = A + B\cos wt, \quad A \text{ and } B \text{ being constants} \qquad \text{--17}$$

A practical application of this sinusoidal variable condenser will now be considered, namely when it is placed in the oscillator circuit of a radio transmitter. The circuit diagram for this condition is represented by Fig. 4, Plate IV.

The natural or free oscillation frequency of a circuit containing constant resistance, constant inductance and constant capacitance in series is

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{1r^2}{4L^2}} \qquad \frac{\sqrt{1 - \frac{r^2C}{4L}}}{2\pi\sqrt{LC}} \qquad \text{--18}$$

When r is small compared with 2L/C, equation (18) reduces to

$$n = \frac{1}{2\pi\sqrt{LC}} \qquad \text{--19}$$

The above conditions are taken from Lawrence's "Principles of Alternating Current" Page 208.

When a circuit containing constant resistance, constant inductance, and constant capacitance in series, is in resonance, the inductive reactance is equal to the capacitive reactance. Consider a circuit containing a resistance r, an inductance L and a capacitance C in series.

$$I = \frac{E}{\sqrt{r^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}} \qquad \text{--20}$$

For resonance $2\pi nL = \frac{1}{2\pi nC}$. Resonance frequency is

$$n = \frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad n = K(1/\sqrt{LC}) \qquad \text{--21}$$

By equations (21) and (19), resonant frequency of a series circuit which has low resistance, compared with the ratio of its inductance and capacitance, is practically the same as its free oscillation frequency. Since its free oscillation frequency depends upon the product of L and C, governed by the condition of equation (19), it follows that the resonant frequency will vary as C is varied. This will result in a shifting of the resonance curve corresponding to every change of value of capacitance. In Fig. 6,

PLATE IV

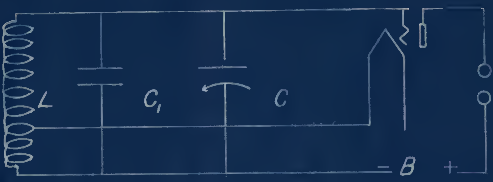


Fig. 4

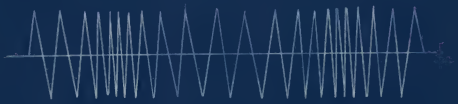


Fig. 5

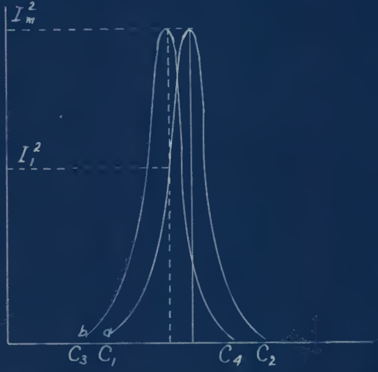


Fig. 6

Plate IV, the resonance curve a represents the behavior of the circuit under constant conditions. It is seen that any change in natural frequency, produced by a capacity change, will cause a shift in the resonance curve, as shown by curve b. By equations (17) and (19)

$$n = \frac{1}{2\pi\sqrt{LA + LB\cos 2\pi ft}} \qquad \text{--22}$$

if f/n_o is very small, where f = audio frequency, n_o = resonance frequency

It can be shown that the current in a receiving set tuned to a transmitter oscillating at constant frequency, will vary as the frequency of the transmitter changes. Let curve a Fig. 6, represent the resonance curve of the transmitter at constant frequency. Then I^2_m is the current of the receiver tuned to the transmitter. Now if the natural frequency of the transmitter changes the resonance curve will shift, the new resonance curve being represented by curve b. The receiving set however is still tuned to curve a, therefore its current will change to I^2_1. More will be said regarding the current in the receiver presently.

Let us now consider the sinusoidal capacity variation, and the resulting frequency variation. Differentiating equation (17) with respect to time,

$$\frac{dC}{dt} = -2\pi f \sin 2\pi ft \qquad \text{--23}$$

The resulting frequency variation is found by differentiating equation (22) with respect to time.

$$\frac{dn}{dt} = \frac{fLB \sin 2\pi ft}{(LA + LB \cos 2\pi ft)^{3/2}}$$

Since B/A is very small,

$(LA + LB \cos 2\pi ft)^{3/2} = (LA)^{3/2}$, approximately

but $n_o = \dfrac{1}{2\pi\sqrt{LA}}$

Therefore, approximately,

$$\frac{dn}{dt} = n_o \quad (2\pi fB/A) \sin 2\pi ft \qquad \text{--24}$$

The final wave train is represented by Fig. 5, as a cyclical and periodically progressing system of alterations of the radio frequency.

The resulting variation of the current in the receiving set that is coupled with the modulated frequency transmitter may be determined as follows:

$$I = \frac{E}{Z}, \text{ where } Z = \sqrt{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2}$$

$$I^2 = \frac{E^2}{Z^2}$$

$$I^2 = \frac{E^2}{R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2} \qquad \text{--25}$$

PLATE V

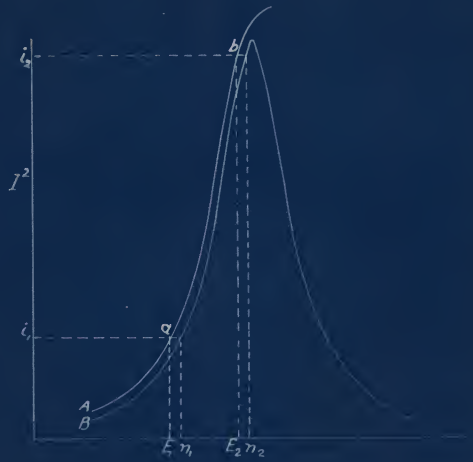


Fig. 7

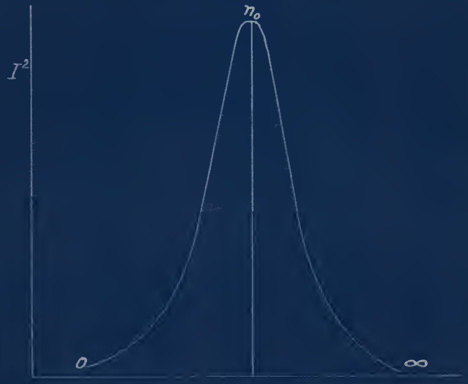


Fig. 8

Differentiating equation (25) with respect to frequency (n),

$$\frac{dI^2}{dn} = \frac{\left[(E)^2(-2)\left(2\pi nL - \frac{1}{2\pi nC}\right)\left(2\pi L - \frac{1}{2\pi n^2 C}\right)\right]}{\left[R^2 + \left(2\pi nL - \frac{1}{2\pi nC}\right)^2\right]^2} \qquad \text{--26}$$

In Fig. 11 it can be seen from the slope of the resonance curve that

$$dI^2 = 0 \quad \text{when } n = 0,\ n = n_0,\ n = \infty \qquad \text{--27}$$

When one-half of the resonance curve is compared with the characteristic curve of a radio vacuum tube we see that they are very similar, as shown by Fig. 7, where A is characteristic curve and B is the resonance curve of the receiving circuit. Now it is known that only the straight portion of the characteristic curve of a vacuum tube is useful for amplification; that is, if the grid voltage varies beyond E_1 and E_2, Fig. 7, causing the plate current to vary beyond i_1 or i_2, distortion of plate current takes place. We can apply the same conditions to the resonance curve. If the frequency varies beyond n_1 and n_2 (Fig. 6) on the resonance curve B, causing the current to go beyond i_1 and i_2, distortion of the current will result.

It is evident that the ideal resonance curve of a receiving set for the reception of a frequency modulated wave, would be one with sides of steep constant slope. The steepness with which curves rise as resonance is approached depends on the magnitude of the resistance of the circuit compared with the magnitudes of the inductive and capacitive reactances. For our purpose a circuit of low resistance is desired. This makes possible very fine tuning to the receiving set, because the shift of the operating point on the resonance curve, due to the small change of radio frequency that is caused by the change in capacity of the transmitter, can be made to yield a large current change for extremely small radio frequency changes.

From what has been said, and from Fig. 8, it is self-evident that the conditions governed by equation (27) would be very undesirable points of operation. The straight portion of the curve occurs when

$$\frac{dI^2}{dn} = C \quad \text{where } \underline{C} \text{ is a constant.}$$

$$\frac{dI^2}{dn} = C \quad \text{when } \frac{d^2I^2}{dn^2} = 0$$

This occurs when $n = 0$, $n = \infty$, and at some point between $n = 0$ and $n_o = n$, and again between $n = n_o$ and $n = \infty$ --28

The following is a mathematical derivation for the value of frequency (n) when the above condition is satisfied.

$$\frac{d^2I^2}{dn^2} = \frac{-2E^2\left\{\left[R^2 + (2\pi nL - 1/2\pi nC)^2\right]\left[(2\pi L + 1/2\pi Cn^2)^2 - (1/\pi Cn^3)(2\pi nL - 1/2\pi nC)\right]\right.}{\left[R^2 + (2\pi nL - 1/2\pi nC)^2\right]^3}$$

$$\frac{\left.-4(2\pi nL - 1/2\pi nC)^2(2\pi L - 1/2\pi Cn^2)^2\right\}}{\left[R^2 + (2\pi nL - 1/2\pi nC)^2\right]^3} \qquad \text{--29}$$

Let $\frac{d^2I^2}{dn^2} = 0$

$$\left[R^2 + (2\pi Ln - 1/2\pi nC)^2\right]\left[(2\pi L - 1/2\pi n^2C)^2 - (1/\pi n^3C)(2n\pi L - 1/2\pi nC)\right]$$

$$-4(2\pi nL - 1/2\pi nC)^2 \quad (2\pi L - 1/2\pi n^2C)^2 = 0 \qquad \text{--30}$$

Since $\underline{n}$ is radio frequency, $(1/n)$ approaches zero. We may rewrite the above equation to be approximately correct as follows,

$$\left[R^2 + (2\pi nL - 1/2\pi nC)^2\right]\left[2\pi L\right] \; -4(2\pi nL - 1/2\pi nC)^2 \quad (2\pi L)^2 = 0 \qquad \text{--31}$$

but $Z^2 = R^2 + (2\pi nL - 1/2\pi nC)^2$

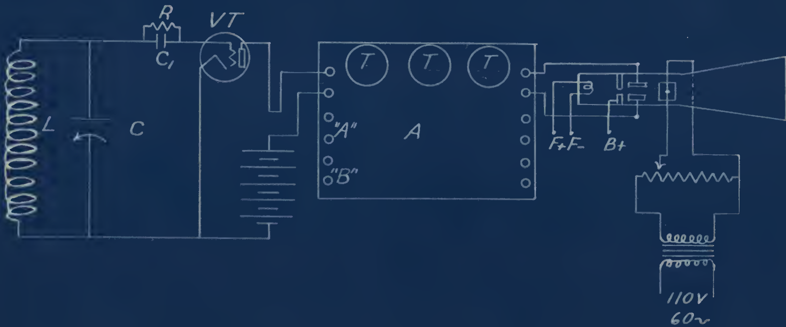
$$2\pi LZ^2 - 4(2\pi nL - 1/2\pi nC)^2 \quad (2\pi L)^2 = 0, \quad \frac{Z^2}{2\pi L} = 4(2\pi nL - 1/2\pi nC)^2 \qquad \text{--32}$$

Let $a = \sqrt{\frac{Z^2}{8\pi L}}$,

$a = (2\pi nL - 1/2\pi nC)$ --33

Multiply equation (33) by $n/2\pi L$. $\frac{an}{2\pi L} = n^2 - 1/4\pi^2 LC$ --34

PLATE VI



$$\frac{an}{2\pi L} - n^2 = 1/4\pi^2 LC = n_o^2 \qquad \text{--35}$$

Let $2b = a/2\pi L$

$$2bn - n^2 = n_o^2, \quad n^2 - 2bn + b^2 = n_o^2 + b^2 \qquad \text{--36}$$

$$n = \sqrt{n_o^2 + b^2} - b, \quad b = (1/4\pi L)\sqrt{z^2/8\pi L}$$

$$n = \sqrt{n_o^2 + z^2/128\pi^3 L^3} - (1/4\pi L)\sqrt{z^2/8\pi L} \qquad \text{--37}$$

It is to be understood that this value of n may be on either side of n_o .

The circuit diagram of the receiving set is shown in Plate VI., illustrating also the method of connecting the deflecting plates of the cathode ray oscillograph which was used in this investigation. The horizontal deflecting plates are connected across the plate circuit of the amplifier. Because of the alternating plate current, which we wish to study, the pencil of cathode rays passing between the plates will vibrate in a vertical line corresponding to the alternations of the current. If it is desired to compare the frequency with a known standard, the vertical plates may be connected to a 60 cycle 110 volt line. This will result in Lissajou's figures on the screen of the tube.

We will now consider the theory of operation of the cathode ray oscillograph. The following is taken from a paper, "The Cathode Ray Oscillograph" by J.B. Johnson, published in the Bell System Technical Journal, January 1932, Vol. XI, Number 1.

The principle of operation is quite simple. There are two electrodes in an elongated, evacuated glass tube, as in Fig. 9, Plate VII; one of them may be a heated filament, the other, a plate, with a small hole in it. When a potential is applied between the electrodes, making the filament cathode and the plate anode, the electrons emitted by the hot filament are drawn to the anode. Some of them pass through the fine hole in the anode, and continue as a thin pencil of electrons, a cathode ray down the length of the tube. At the end of the tube is a screen of fluorescent materials, which shines brightly at the point where the ray strikes it. We can therefore see where the ray ends on the screen. Another pair of electrodes in the form of two plates $\underline{P}$ and $\underline{P_1}$ is introduced so that the cathode ray passes between them. Now we apply a voltage between the plates, so that one is positive with respect to the other. The electrons of the ray, being negative charges, are, during their passage between the plates, drawn toward the positive plate and emerge in a different direction because of the applied voltage. The amount of deflection is a measure of the strength of the applied electric field. We have then in this cathode ray, a pointer which tells the magnitude of the field that deflects it. It is, furthermore, a pointer

PLATE VII

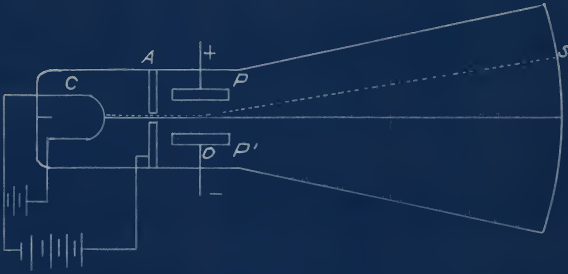


Fig. 9

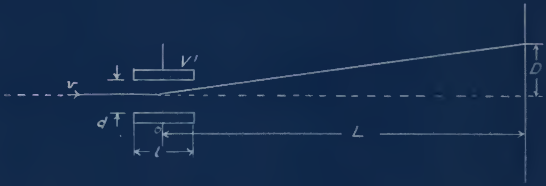


Fig. 10

that is almost without mass and sluggishness. Therefore, it responds to very high frequencies quite accurately.

The speed of the electrons as they emerge from the aperture in the anode, can be determined from the energy equation,

$$1/2\ mv^2 = eV \qquad \text{--38}$$

the energy of motion equaling the total work done on the electron by the field between cathode and anode. V is the potential between cathode and anode, e the electric charge constituting the electron, m its mass and v its speed. Solving for the speed we have the relation between the speed and the driving voltage.

$$v = \sqrt{2(e/m)\quad V} \qquad \text{--39}$$

It is found that if the applied voltage is 30,000 volts the speed of the electrons is 1/3 the volocity of light. A change of direction of the ray induced at the deflector plates is therefore transmitted to the end of the ray in a very short time.

Let us see how the ray responds to voltage applied to the plates. The ray normally travels with a speed v along the tube. Referring to figure (10), the ray now passes between two plates of length ℓ and separation d, between which a potential difference V' is maintained. While the ray is passing between the plates the electrons are subject to an acceleration

$$a = (e/m)E = (e/m)(V'/d) \qquad --40$$

This continues during the time $t = \ell/v$. The transverse velocity acquired is therefore;

$$v' = at = (e/m)(V'/d)(\ell/v) \qquad --41$$

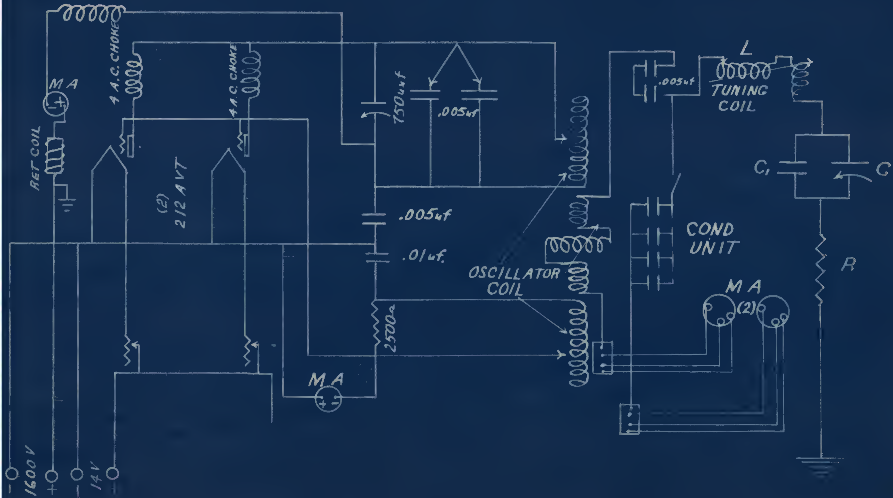
The ray then travels on in a straight line to the screen which it meets at a distance D from the normal position. The deflection D bears the same relation to the length of the beam, from the center of the deflecting plates, as the transverse velocity bears to the longitudinal.

$$D/L = v'/v = (e/m)(V'/d)\left(\frac{\ell}{2(e/m)V}\right) = (1/2)(\ell/d)(V'/V)$$

$$D = (1/2)(\ell L/d)(V'/V) \qquad --42$$

We see then that this is a very accurate method for obtaining the wave form of an alternating current. By tuning the receiving set at different points on its resonance curve the best point for reception may be determined. The frequency corresponding to this point should theoretically be the value n, as found by equation (37).

PLATE VIII



4 A.C. CHOKE
4 A.C. CHOKE
M A
RET COIL
(2)
212 AVT
750uuf
.005uf
.005uf
.01uf.
2500Ω
M A
OSCILLATOR
COIL
.005uf
L
TUNING
COIL
C1
C
COND
UNIT
R
M A
(2)
1600 V
14 V

PLATE IX

CONSTRUCTION OF APPARATUS

The apparatus used consisted of standard parts belonging to the department and that which was constructed in the shop.

TRANSMITTER

In the first part of the work the oscillator circuit of a Standard Western Electric 500 Watt radio transmitter Type I B, was used. The part of the transmitter used is shown by the circuit diagram, Plate VIII. All of the parts were standard parts of the transmitter except $\underline{C}$, $\underline{C_1}$ and $\underline{R}$ in the antenna circuit.

C_1 = Condenser for dummy antenna, .0005 μf.

C = Sinusoidal variable condenser.

R = Resistance for dummy antenna, 10 ohms.

The inductance of $\underline{L}$ was first calculated by Nagaoka's formula as found in any radio hand book.

$$L = \frac{4\pi^2 N^2 R^2 K}{\ell} \text{ cm}$$

in which

L = inductance in cm.

R = radius to center of wire of coil = 7.75 cm.

N = number of turns of winding = 32

ℓ = length of winding in cm. = 15.5 cm.

K = constant depending on the ratio of coil diameter to the coil length $2R/\ell$.

The values of K have been worked out by H. Nagaoka. The value of L inductance, worked out by the above formula for this particular coil, was found to be 100 h.

The desired value of capacity C may be calculated as follows. By equation (21)

$$n = \frac{1}{2\pi\sqrt{LC}}$$

Since frequency n is equal to the reciprocal of T, period of time in seconds of one oscillation, ($n = 1/T$),

$$T = 2\pi\sqrt{LC}$$

Where L and C are expressed in henries and farads respectively; or, expressing L and C in microhenries and microfarads,

$$T = \frac{2\pi\sqrt{LC}}{10^6}$$

Wave length λ is equal to speed, in meters per second, multiplied by the period T.

$$\lambda = \frac{2\pi(3 \times 10^8)}{10^6} \sqrt{LC}$$

$$\lambda = 1885 \sqrt{LC}$$

$$C = (\lambda/1885)^2 \ (1/L).$$

Since we know the wave length λ is to be between 400 and 600 meters, the value of C may be approximately determined.

$C = .0005 \ \mu f$

The resistance R was constructed of nichrome wire strung on a bakelite board with binding posts so that any value of resistance might be obtained from 3 ohms to 10 ohms. The construction of the sinusoidal variable condenser C will be given presently.

In the last part of this work a 50 watt Hartley oscillator was used. The various parts are listed corresponding to the letters used in the circuit diagram, Plate X.

L = Inductance coil.

C = Sinusoidal variable condenser.

C = Special Calibrated condenser.

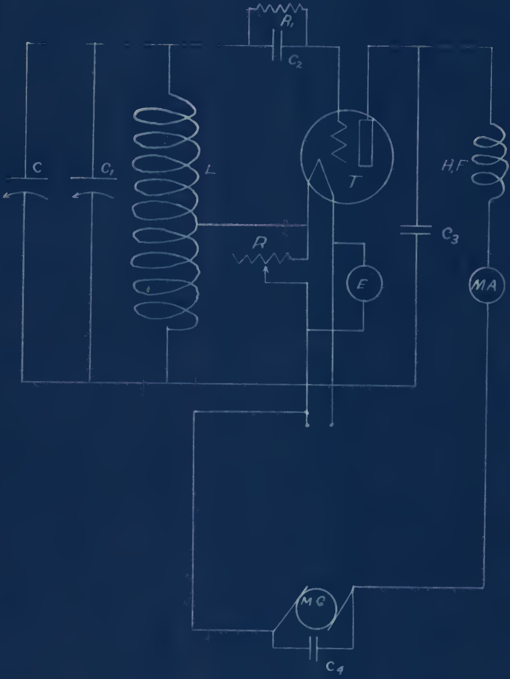
T = 50 Watt Model UV 203 Radiatron.

R = Filament rheostat, Model R.T. 537 (R.C.A.)

$C_{\lambda}C_3$ = Faradon Condensers, Model BC 1014 (.002 μf)

H.F. = About 200 μh choke.

PLATE X



E = 0-15 voltmeter.

M.A. = Microammeter.

The inductance $\underline{L}$ was found by the same method as given above. The capacity of $\underline{C_1}$ was calculated by applying the formula

$$C = \frac{KA}{4\pi d}$$

Where C = Capacity in centimeters,

K = The specific inductive capacity of the dielectric,

A = Area of one side of one plate in square centimeters,

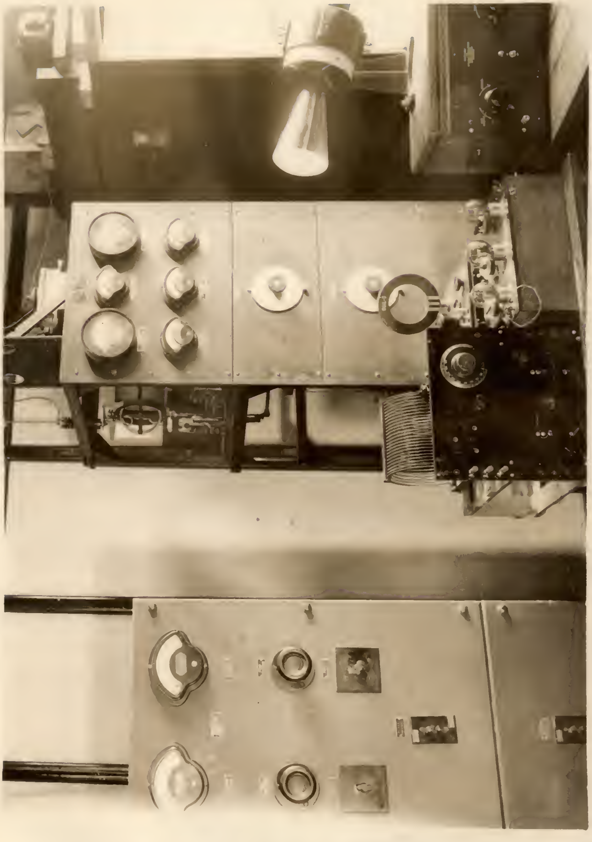
d = Separation of plates in centimeters.

RECEIVER

The receiving set consists of a tuned detector circuit followed by two stages of audio-frequency amplification, as shown by the circuit diagram, Plate VI. The tuned circuit was constructed from parts found in the department, with practically the same constants as the transmitter. The amplifier consisted of a push-pull, two-stage audio frequency amplifier.

The parts are, corresponding to the letters used on the circuit diagram Plate VI, as follows;

PLATE XI.



L = Inductance of practically the same value as transmitter's.

C = Variable condenser.

C_2 = By-pass condenser.

R = Grid leak resistance.

VT = Model 201 UV Radiotron.

A = Push-pull, two-stage, amplifier.

P_1P_2 = Horizontal plates of Cathode ray tube.

$P'_1P'_2$ = Vertical plates of Cathode ray tube.

P o t = Potentiometer to vary voltage.

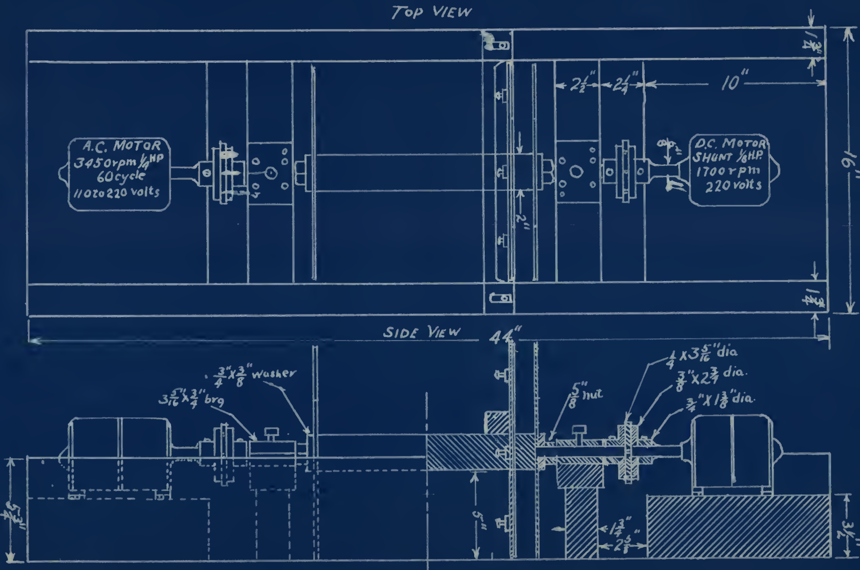
C T = Cathode Ray Oscillograph Tube, General Radio Co. Type 497.

The power unit used with the tube is not shown in the diagram.

SINUSOIDAL VARIABLE CONDENSER

The rotor of the sinusoidal variable condenser was constructed from a brass plate, cut in a circle of radius ($R = 15$ cm). Then from a circle of radius, $r = 1/2R = 7.5$cm, inscribed on the brass plate, ($p = 8$) apertures were cut projecting outwards radialy to circle of radius R, thus leaving ($p = 8$) teeth forming the rotor as explained in the

PLATE XII



SINUSOIDAL VARIABLE CONDENSER

theory. The angular width of each tooth or aperture may be calculated as follows. Since there are ($p = 8$) teeth and ($p = 8$) apertures, the angular width in degrees of each tooth or aperture is $22^{\circ}30'$. The rotor is shown on Plate II.

The petals for the stator were cut out of a brass plate, 1/8" thick, their shape being governed by equation (11). It is first necessary however to calculate value of the constant h in equation (11), which may be done as follows. The maximum value of y will be when $\sin p\theta = 1$. It is evident the maximum value is $y - r$. Making these substitutions, we may rewrite equation (11);

$$r = \sqrt{r^2 + 2hr} \quad -r \qquad 2r = \sqrt{r^2 - 2hr} \quad \text{or } 3r^2 = 2hr$$

$$h = (3/2)\, r$$

Substituting this value of h in equation (11).

$$y = r\sqrt{1 + 3 \sin p\theta} \quad -r$$

Let $r = 7.55$ cm $\quad p = 8$

Thus we see that the value of y for each degree change from 0° to $22^{\circ}.5$ may be calculated and plotted, giving the shape of the petals, as is shown by Plate III. It must be remembered that the angular width of each tooth corresponds to 180 electrical degrees.

We may calculate the maximum area of coincidence as follows. From equation (14) where

$$A = hr(1 - \cos p\theta)$$

The maximum area of coincidence is,

$$S_o = 2hr$$

Then substituting the value of h obtained above

$$S_o = 3r^2 \text{ , } r = 7.5 \text{ cm, } S_o = 171.07 \text{ cm}^2$$

Since the condenser formed by the plates of the rotor and stator is of the parallel plate type, the maximum capacitance may be calculated by the equation

$$C = \frac{KA}{4\pi d}$$

Where A is the effective area and d the distance separating the plates. Then

$$A = S_o = 171.07 \text{ cm}^2 \quad d = .2 \text{ cm} \qquad K = 1$$

$$C = \frac{171.07}{4\pi(.2)} = 68.046 \text{ cm}$$

To reduce centimeters of capacity to $\mu\mu$f. divide by .9

$$C = \frac{68.046 \text{ cm}}{.9} = 75.6 \ \mu\mu\text{f.}$$

The brass petals were superimposed on a plate glass ina circle of radius r, extending radialy outwards towards a circle of radius R. A top and side view of the sinusoidal variable condenser completely assembled is shown on Plate XII.

RESULTS AND CONCLUSIONS

The sinusoidal variable condenser was placed in the oscillator circuit of the transmitter, with the rotor on the grounded side of the circuit. Electrical connection was made by use of a copper brush on the shaft on which the rotor turned. The petals of the stator were electricaly connected by means of binding posts which extended through holes, bored in the glass, and screwed into the petals, thus serving the double purpose of making electrical connection and clamping the petals to the plate glass.

The receiving circuit was inductively coupled with the transmitter while the rotor of the condenser was turning. When the receiving set was tuned to the transmitter, a hum could be heard in the ear phones. When the rotor was stopped the hum could not be heard, indicating that frequency modulation was produced by the variable condenser.

Further indication, of frequency modulation, was determined by the use of a wave-meter, which was coupled with the transmitter. With the variable condenser at minimum capacity, a resonance curve (a, Plate XIII) was plotted. Another curve b was plotted with the variable condenser set at maximum capacity with equation (14), when

$$S - S_0 = 2hr$$

PLATE XIV

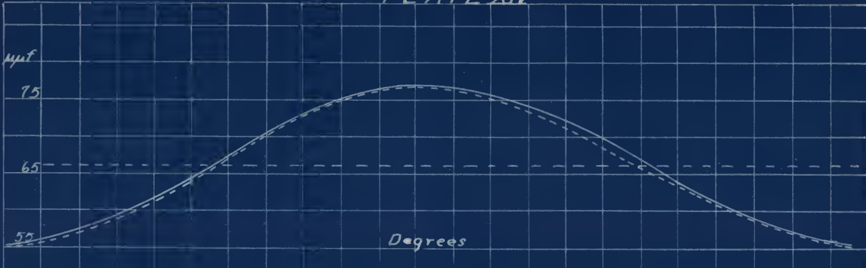


Fig. 11

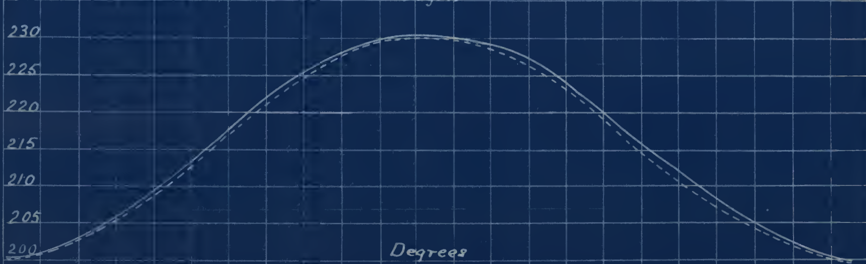


Fig. 12

It is obvious that the maximum change of frequency due to the maximum change of capacity in the transmitter is the difference between the resonance frequencies of curves a and b, which is approximately 10 cycles per second. With the rotor running, the resonance curve C was obtained. It would seem that its resonance frequency would be 1/2 of the difference of the frequencies of the curves a and b. This however does not prove to be true, experimentally, as can be seen. With the capacity of the variable condenser set at 1/2 maximum capacity, curve d was obtained.

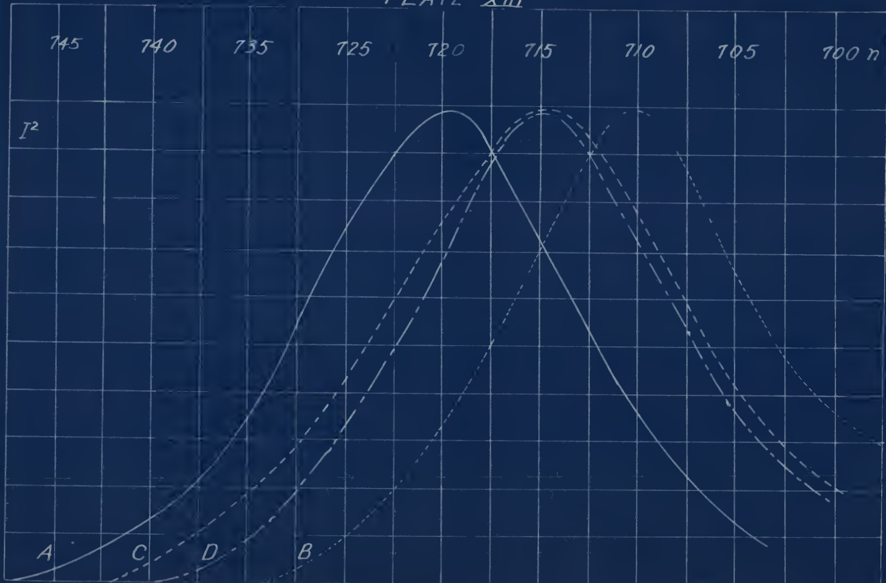
It is quite obvious that when the receiving set is tuned on curve C the current varies as the resonance frequency varies from curve a to curve b, which corresponds to the theory illustrated by Fig. 6.

Up to this point, we have found that frequency modulation is produced by the sinusoidal variable condenser in the oscillator circuit of the transmitter. Let us now investigate the quality of modulation produced.

From consideration of the facts involved it seems quite probable that distortion of the wave-form of the current produced by frequency modulation, which in turn is produced by the sinusoidal variable condenser, might result from three causes. First, the edge capacity effect of the plates of the variable condenser as they approach and leave each other, might be enough to distort the theoretical variation of capacity. Second, from the relation of capacity and frequency in the equation $n = \frac{1}{2\pi\sqrt{LC}}$ it is evident that frequency n does not vary directly with capacity C Therefore, if C does vary sinusoidaly it does not necessarily follow that n will. Third, unless reception is on the straight portion of the resonance curve distortion will result.

The amount of edge capacity effect was found as follows. The sinusoidal variable condenser was calibrated in degrees for one complete cycle, 360 electrical degrees, or $2\pi/p = 45$ mechanical degrees. The capacity was then measured for each degree change over the complete cycle by the resonance method, employing a standard condenser in parallel with the unknown, and a wave meter, as described in "Radio Frequency Electrical Measurements" by Brown, pages 6-8. The results thus obtained were plotted with degrees as abscissae and capacity as ordinates, resulting in the (solid) curve Fig. 11, which should theoreticaly be a true sine wave. Taking 1/2 the maximum difference of capacity as the amplitude B, a true sine wave was plotted, by the relation $A = B \sin \theta$

PLATE XIII



where A is instantaneous value. This curve (broken) was superimposed on the (solid) curve, Fig. 11, and the degree of accuracy determined. It can be seen that the curves nearly coincide, which indicates the edge capacity effect is negligible since the capacity variation is very nearly sinusoidal. There is a slight distortion between 134° and 182° which is undoubtedly due to some mechanical defect in the construction of the apparatus. It is found by experiment that a very small irregularity in the shape or thickness of the plates caused quite a perceptible distortion in the sinusoidal capacity variation.

To determine the amount of distortion due to the second cause mentioned above, much of the same method was employed. By the use of a wave-meter coupled with the transmitter, the frequency change for each degree change of the variable condenser was measured and the results plotted with degrees as abscissae and frequency as ordinates, (see solid curve, Fig. 12). A true sine wave (broken) was superimposed on it in order to determine its degree of accuracy. Here too, the frequency variation curve is practically sinusoidal. Any of the small irregularities of this curve, are not enough to produce strong harmonics in the reception current.

Up to this point in our investigation, frequency modulation, as produced by the sinusoidal variable condenser, is of a good quality, being free from distortion. Any distortion in the wave form of the received current is either due to the point of reception on the resonance curve, the third cause mentioned above, or to some outside electrical disturbance. Let us, then, investigate the resonance curve in relation to distortion.

With the ear phones connected in the receiver, it was found the pitch of the hum changed with change of speed of the rotor of the variable condenser. For experimental purposes the best results were obtained with the rotor turning at about 1200 R.P.M. The frequency of the current then heard through the ear phones should theoreticaly be

$$f = \frac{P \times R.P.M.}{60} = \frac{8 \times 1200}{60} = 160 \text{ cycles per second.}$$

When the receiver was tuned near the bottom of the resonance curve the tone was quite distorted, as it also was near the top. Best reception was on the side of the resonance curve. For more accurate work a cathode ray oscillograph was used in place of the ear phones, as explained above in theory.

PLATE XV

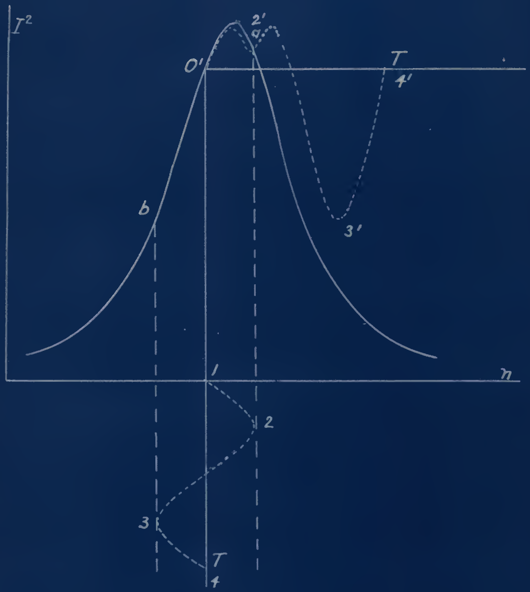
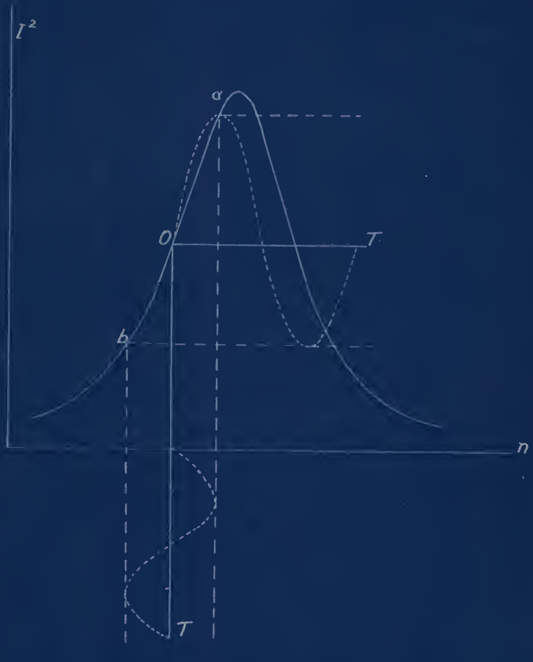


PLATE XVII



Let us consider for a moment, what effect the point of reception will have on the wave form of the current. In Plate XV is shown a resonance curve plotted with frequency f as abscissae and current squared I^2 as ordinates. The same method is employed here, as that used with the characteristic curve of a triode vacuum tube. That is, the plate current is a function of the grid potential. In this case however, the plate current is a function of frequency variation. Corresponding to each frequency, a definite value of plate current must flow. The frequency variation follows the sine curve 1,2,3,4. The plate current is then given by the curve 1',2',3',4'. In the particular case illustrated by Plate XV, the set is tuned at the point O, just off the peak of the resonance curve. The frequency varies between the points a and b, resulting in the plate current which, as can be seen, is non-sinusoidal, containing harmonics. This is an undesirable point of reception.

In Plate XVI is shown a case where the point of reception O, is exactly on the peak of the resonance curve. Here, as may be seen, the frequency of the plate current is double that of the fundamental. This, too, is an undesirable point of reception.

The ideal case is shown in Plate XVII. Here the point of reception O is on the straight portion of the resonance curve and the wave form of the plate current is sinusoidal, corresponding to the fundamental.

The results observed with the cathode ray oscillograph agree very closely with the cases illustrated above. In Fig. 13, is shown the wave form of the current when the set was tuned near the peak of the resonance curve. This corresponds to the case illustrated by Plate XV. When reception was at the peak of the resonance curve, the curve of the plate current is that shown by Fig. 14, corresponding to the case illustrated by Plate XVI. Although the wave form is not quite symmetrical, it can be seen that its frequency is double that shown in Fig. 13. The best wave form observed is that shown in Fig. 15, when the set was tuned on the side of the resonance curve. This corresponds to the case illustrated on Plate XVII. It can be seen that the curve is not entirely free from distortion, but the irregularities are probably due to undetemined electrical disturbances. It was found by experiment that such disturbances as sparking at the brushes of the motor affected the wave form quite considerably.

PLATE XVIII

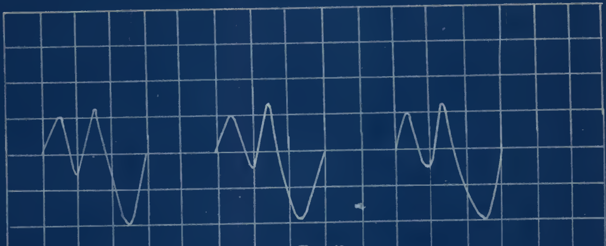


Fig. 13

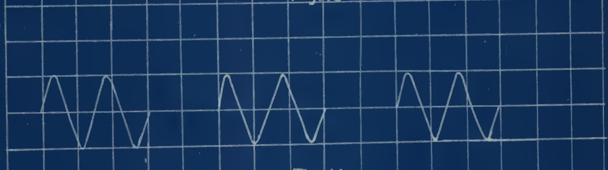


Fig. 14

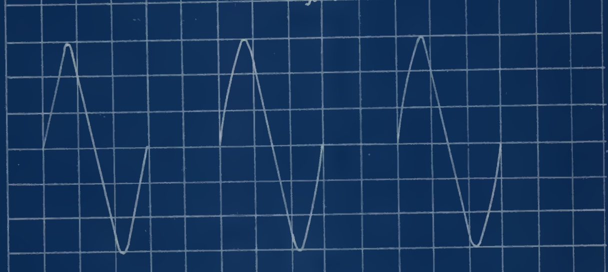


Fig. 15

We can say in conclusion, the quality of frequency modulation, as produced by a variable condenser placed in the oscillator circuit of a radio transmitter, is good. Distortion in reception is due to the point of operation on the resonance curve of the receiver, and not because of poor quality of modulation. We have shown this to be true since the capacity of the variable condenser was found to vary sinusoidaly, which in turn produced sinusoidal frequency variation. The best point of reception is on the straight part, on the side of the resonance curve, of the receiving set.

ACKNOWLEDGMENT

I wish to express my appreciation to my major instructor Professor E.R. Lyon, for directing, outlining and doing much of the theoretical calculations of this work; also, for the use of his diagrams and paper on "An Electrostatic Alternator", in the construction of the sinusoidal variable condenser; to Professor J.O Hamilton, for photographing the apparatus; to Professor Leo E. Hudiburg, for aiding and directing in the construction of the apparatus.

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